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# Structural Behaviour and Design Methods of Tensegrity Domes

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## *ABSTRACT*

A comprehensive study on the structural behavior and structural types of Tensegrity domes is presented. The numerical analysis method of Tensegrity structure is also discussed. The first Tensegrity dome --Georgia Dome is analyzed as a prototype through a non-linear software using the numerical method presented in the paper. Based on the analysis, the structural behavior of Tensegrity dome is sumarized and therefore, some design methods for Tensegrty dome are proposed. Based on above studies, several new types of Tensegrity domes with different geometric grids are proposed by the author. A comparison of the structural behavior between Georgia Dome and the domes proposed by the author is also made.

*Keywords:* Tensegrity, Non-linear, Equilibrium, Static

## *1. INTRODUCTION*

In the process of designing long span space structures, the way to reduce the self-weight of the structure and consequently the cost of the building is the key issue. Among different types of structures, the ‘Tensegrity System’, that is a self-equilibrium system composed of continuous prestressed cables and individual compression bars, is one of the most promising solutions.

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The concept of ‘Tensegrity’ was first conceived by B. Fuller (1975), which reflected his idea of ‘nature relies on continuous tension to embrace islanded compression elements’. Unfortunately, his ‘tensegrity dome’ has never been executed in engineering project.

It was D.H. Geiger (1986), who made use of Fuller’s thought and designed an innovative structure ‘cable dome’. It has been successfully put into practice in the circular roof structures of Gymnastic and Fencing Arenas for the Seoul Olympic Games in 1986. However, it is could not be considered as an actual tensegrity system, because the compressed ring is not inside the set of cables.

In 1992, M. Levy (1989, 1991) further improved the layout of the cable dome and built the Georgia Dome in quasi-elliptical shape for Atlanta Olympic Games. R.Motro(1992,2003) and A.Hananor(1993) did much research on double layer grid Tensegrity system. Some researchers also proposed a kind of RP system for Tensegrity dome grid. The dome is made of self-stressed equilibrium reciprocal prisms. Base on this idea, B.B.Wang (1998) proposed his own RP grid dome. J. Rebielak (2003) proposed several new structural system of cable dome shaped by means of simple form of spatial hoops.

In addition to above research toward the geometry grid of the system, some researchers also did some research on the numerical analysis of the system. K. Kebiche (1999) performed the Geometrical non-linear analysis of Tensegrity systems. C. Sultan, et al. (2001) discussed the prestressability problem of Tensegrity structures. D Williamson, et al. (2003) discussed the Equilibrium conditions of a Tensegrity structure. Through the research of the researchers, Algorithm considering geometrical nonlinear is widely developed, and the dominant role of initial equilibrium state and prestressed force is also widely recognized.

Among the domes in Tensegrity system built to date and the domes proposed by the researchers, there exits different variations of the network geometries. For designers, it is interesting to know the correspondent structural features with different layout of the network, which will influence the weight and the cost of the structure. For application purpose, the way to design Tensegrity dome also becomes necessary.

In this paper, the design methods of such kind of dome are discussed. Several structural types of Tensegrity domes with different geometric grid are proposed by the author. The numerical analysis method of Tensegrity structure is also discussed. The first Tensegrity dome --Georgia Dome is analyzed as a prototype through a non-linear software using the numerical method presented in the paper. Based on the analysis, the structural behavior of Tensegrity dome is sumarized and some design methods for Tensegrty dome are proposed. A comparison of the structural behavior between Georgia Dome and the domes proposed by the author is also made.

Since most long span sports halls are non-circular in plan and usually have the configuration in oval shape, the layouts of domes proposed in this paper are all presented elliptical.

## *2. NUMERIACAL ANALYSIS METHOD*

The Tensegrity structure is a geometrical non-linear system, the structural analysis can be divided into two phases: the first phase is the initial equilibrium; the second phase is static analysis. The software used to analyze the structures in the paper is based on above numerical method. The two phases are presented below:

### 2.1 The initial geometrical equilibrium

When the boundary condition is determined, the distribution and magnitude of the prestressed force applied to the Tensegrity structure is correspondent to the initial geometrical equilibrium of the structure. Therefore, how to determine the initial geometrical equilibrium are the key issues. Iterative method is used in the paper to determine the initial geometrical equilibrium state of the structure.

In the procedure of the determination of the initial equilibrium, the coordinate can be firstly presumed with an ideal distribution of the prestressed force. But the node in the structural grid will not be balanced under this condition, imbalance force will be resulted, hence, the displacement of the node will also be resulted. Thus, the coordinate and the prestressed force need to be adjusted step by step until the whole structure is balanced.

The formula to determine the initial equilibrium is

$$[K]^0\{\Delta U\}^0 = -\{P\}^0 + \{R\}^0 \qquad (1)$$

Where:

| | |
|---|---|
| $[K]^0$ | initial stiffness matrix |
| $\{\Delta U\}^0$ | variation of the coordinate |
| $\{P\}^0$ | Prestressed force |
| $\{R\}^0$ | Residual force |

### 2.2 Static analysis

After the determination of the initial equilibrium of the structure, the structure can be analyzed under the load. The fundamental equation for Static analysis is:

$$[K]\{U\}=-\{P\} +\{R\} \qquad (2)$$

| | |
|---|---|
| $[K]$ | Total stiffness matrix |
| $\{U\}$ | Displacement vector of the node |
| $\{P\}$ | Load vector |
| $\{R\}$ | Residual force |

The Newton Raphson approach is used here for solving the solution of the equation. The total load is divided into small increments and the calculation procedure is divided into correspondent steps, and for each increment a new $[K]_i$ is used. The non-linearity is therefore treated as piece-wise linearity and a constant $[K]_i$ is used in all increments. After each iteration, the "unbalanced" portion of the external force is estimated and applied in the next increment.

For each step:

$$[K]_i\{\Delta U\}_i=\{\Delta P\}_i \tag{3}$$

$$\{U\}_i=\{U\}_{i-1}+\{\Delta U\}_i \tag{4}$$

Where：

| | |
|---|---|
| $[K]_i$ | Stiffness matrix when n=i, |
| $\{\Delta U\}_i$ | Increment of the displacement when n= i |
| $\{\Delta P\}_i$ | Imbalance load when n= i |
| $\{U\}_i$ | displacement when n= i |
| $\{U\}_{i-1}$ | displacement when n= i-1 |

As $\{\Delta U\}_i$ is obtained, $\{\Delta F\}_i$ , the increment of the internal force when n=i can be therefore obtained. For each step the internal force can be obtained as:

$$\{F\}_i=\{F\}_{i-1}+\{\Delta F\}_i \tag{5}$$

Where：

| | |
|---|---|
| $\{F\}_i$ | the internal force of member n= *i* |
| $\{F\}_{i-1}$ | the internal force of member n= *i*-1 |
| $\{\Delta F\}_i$ | the increment of member when n=*i* |

When all the steps are finished, the increment of the displacement and the increment of the internal force in different step will be added together, so the final result can be got.

$$\{U\}=\sum_{i=1}^{n}\{\Delta U\}_i=\{\Delta U\}_1+\{\Delta U\}_2+\cdots+\{\Delta U\}_n \tag{6}$$

$$\{F\}=\sum_{i=1}^{n}\{\Delta F\}_i=\{\Delta F\}_1+\{\Delta F\}_2+\cdots+\{\Delta F\}_n \tag{7}$$

## *3. STRUCTURAL BEHAVIOUR OF THE GEORGIA DOME*

### 3.1 The layout and material used for Georgia Dome

In order to design different types of Tensegrity domes in elliptical plans, the static behavior is worth investigating by analyzing the force and deformation in the structure. In this part, the structural behavior of Georgia Dome is analyzed.

Fig.1 shows the structural layout of the Georgia Dome. It is noticed that instead of using the radial cable-and-strut trusses as Geiger designed in his cable domes, Levy preferred the triangulated geometry for the network.

The layout and dimension of the dome is shown in Fig.1. In Fig.1-d, letter a, b, c, d denotes the ridge cable in each layer; e, f, g, h denotes the diagonal cable; l, m, n denotes the tension hoop cable, while P1, P2, P3, P4 denotes the compression strut, all ascending from the bottom to top. The following materials are used for calculation: Φ 5mm high strength tensile wires for cables with a tensile strength of 1670N/mm$^2$ and Q345 circular tubes for struts with a yielding strength of 345N/mm$^2$. Uniform superimposed load of 0.6 kN/m$^2$ is applied to the top surface of the dome.

In the analysis, the following cross-section of the cables is used for the structural analysis in the paper. For ridge cables on a,b,c,d layers, the areas are 85.4cm$^2$,46.2cm$^2$,21.8cm$^2$ and 21.8cm$^2$ respectively. For diagonal cables on e,f,g,h layers, the areas are 47.4cm$^2$,56cm$^2$,23.9cm$^2$ and 23.9cm$^2$ respectively. For tension hoop cables on l,m,n layers , the areas are 189.7cm$^2$,267.8cm$^2$ and 75.6cm$^2$ respectively. For struts P1, P2, P3, P4, the areas are 270.8cm$^2$,270.8cm$^2$, 115cm$^2$ and 400cm$^2$ respectively.

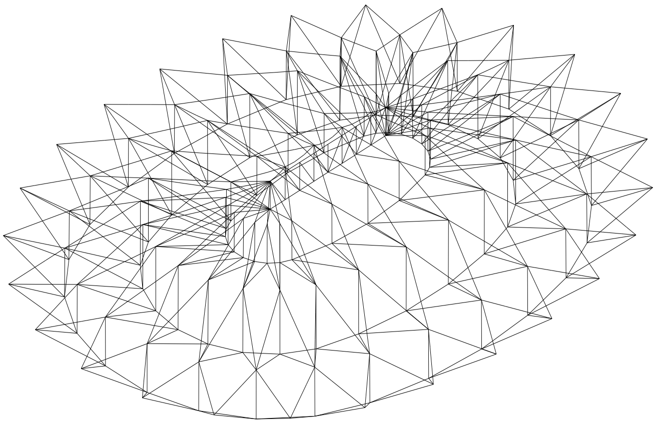
### 3.2 The structural behavior

Using a nonlinear finite element analysis program, it was found that the variation of member forces and deformation of a Tensegrity dome depicts certain special features. The variation of forces with loading is basically linear. When the load increases, each type of the cable responds in different manner. The forces in ridge cables decrease; with those in inner and top layer decrease more rapidly than the outer ones. The variation of forces in diagonal cables depends on its position. For outer and lower layer, it increases, while for inner and upper layer, it decreases. Forces in tension hoop cables increase with the load, the rate of increasing is much larger in the bottom layer.

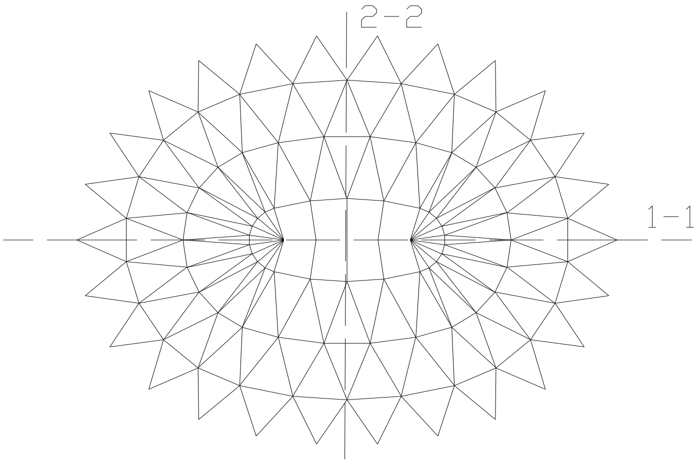
It is interesting to note the failure mode of the dome. When the load keeps increasing, the forces in the ridge cables on d layer (shown in Fig.1-d) as well as the diagonal cables on h layer (shown in Fig.1-d) all decrease. When the load attains certain value, the force in one of the ridge cables, usually in the central section of the dome, will decrease to zero and thus the cable becomes slack. However, the forces in hoop cables and part of the diagonal cables are still increasing. The structure can still maintain its bearing capacity, but the deformation is increasing significantly. If the load increases further until one of the diagonal cables on h layer also becomes slack, then failure occurs to the whole structure. The failure modes of Tensegrity dome are neither the breaking of the tension hoops and diagonal cables, nor the buckling of the struts. It is the slackening of the ridge cable and diagonal cable in the central section of the dome that determines the bearing strength.

### 3.3 The design method

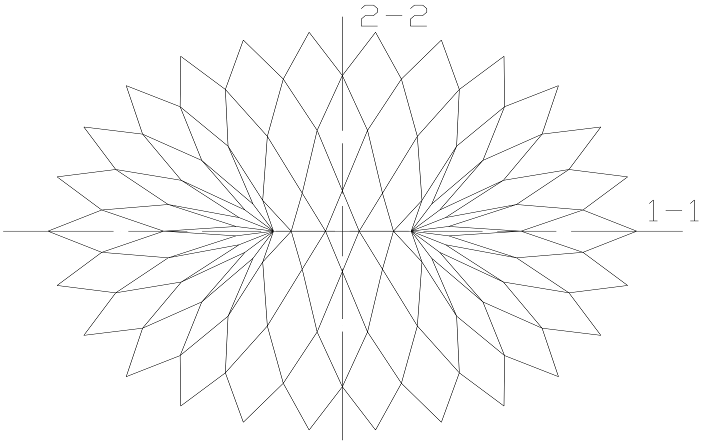
From the analysis of the Georgia Dome, an efficient way to increase the bearing capacity can be found. It is to increase the prestressed forces in the ridge and diagonal cables on the inner and upper layer of the central section of the dome. In another word, strengthen the central section and to simplify the semi-circular sectors. And all the domes proposed in this paper by the author are based on this design method.



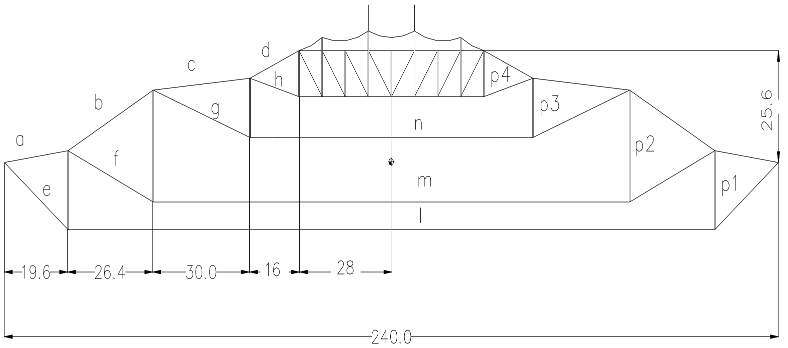
a. Isometric View



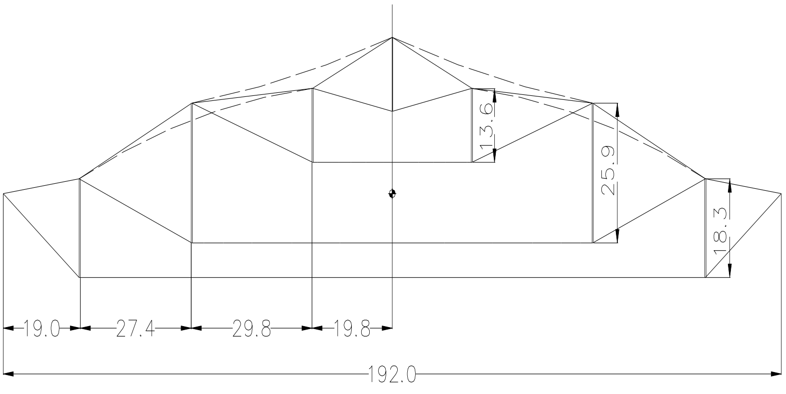
b. Plan of Diagonal and Hoop Cables



c. Plan of Ridge Cable



d. Section 1-1



e. Section 2-2

Fig.1 Georgia Dome---Prototype

## *4. STRUCTURAL TYPES PROPOSED BY THE AUTHOR*

Based on the structural behavior of the Georgia Dome, several structural schemes of Tensegrity domes are designed and analyzed by the author. For the purpose of comparison, all schemes are designed in an elliptical plan with a longitudinal span of 240 m, and same materials are chosen as the Georgia Dome. The schemes are analyzed under the same load as well.

### 4.1 Structural Type 1

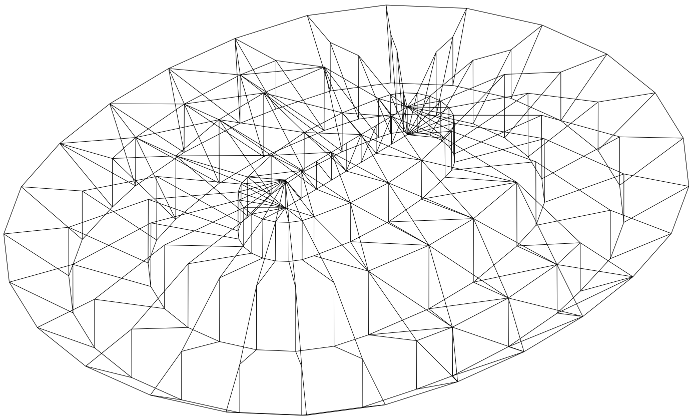
Fu F. (2000) proposed that the circular cable dome designed by Geiger demonstrates some significant advantages. The ways of forming networks in wedge shape of the cable dome is simpler than the triangulated networks. The number of cable elements is less, and hence less weight. As there are less cables connecting at a node, the construction of a joint is more or less easy. The advantages of such network are shown on circular domes, since the construction of the joints in each layer is the same, so the types of joints are less. However, the stiffness of cable dome is smaller when compared with the triangulated dome system, especially in the horizontal direction. There are no links between the top chords joints in the circumferential direction of the cable dome. For triangulated networks, all the top chord joints are connected by the ridge cables, thus a greater horizontal stiffness can be obtained.

Therefore, for the layout of structural Type 1 (shown in Fig. 2), networks of triangulated dome and cable dome are used simultaneously so as to utilize their respective advantages. The quasi-elliptical configuration is composed of two semi-circles at both ends using the same networks of cable dome, and a rectangular central section using triangulated networks.

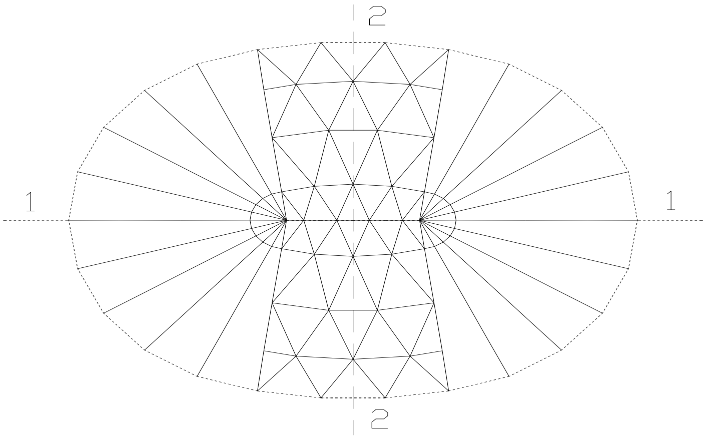
From the studies in the previous part, it was found that the weakest position in an elliptical Tensegrity dome is in the central section, where the slackening of ridge cables always occurs and the displacements are larger. Therefore, the design principal of structural type 1 is to strengthen the central section of the dome. Under the action of the vertical load, the semi-circles at both ends tend to pull away the central section. It is required a stiffer central section to resist the horizontal displacements, thus triangulated networks with strong horizontal stiffness are used. Furthermore, the top chord in the central section tends to resist compression, in order to increase the load bearing capacity; circumferential bars are established in the top chord.

For the section of two semi-circles at both ends, networks of cable dome are used, in order to simplify the construction of connecting joints and to lessen the types of the joint.

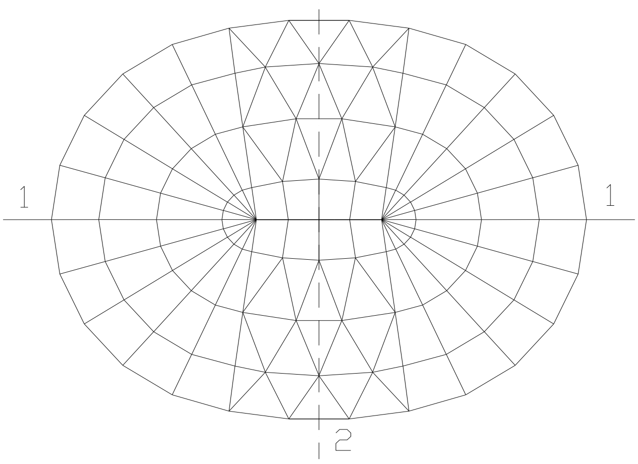
The layout and dimension of the dome is shown in Fig.2. In Fig.2, the same letters are chosen to denote the different layer of member. The cross sections of structural member are taken as follows. For ridge cables - 100cm$^2$, for diagonal cables - 60 cm$^2$, for tension hoop cables on n layer - 80cm$^2$, on other layers - 200cm$^2$, for top chord bars - 115cm$^2$, for strut P3 - 96.7cm$^2$, other struts - 213.8cm$^2$.



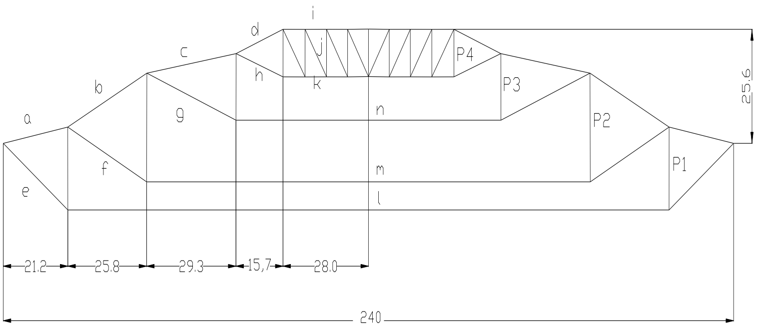
a. Isometric View



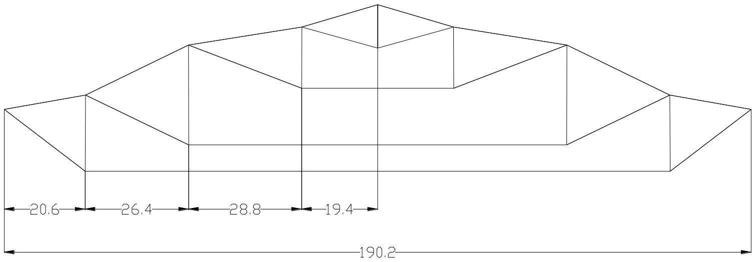
b. Plan of Ridge Cable and
Top Chord Member



c. Plan of Diagonal and Hoop Cables



**d.**



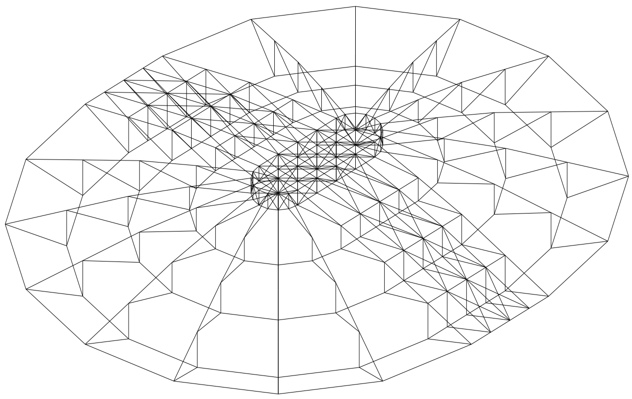
**e.Section2-**

**Fig.2** Structural Type 1

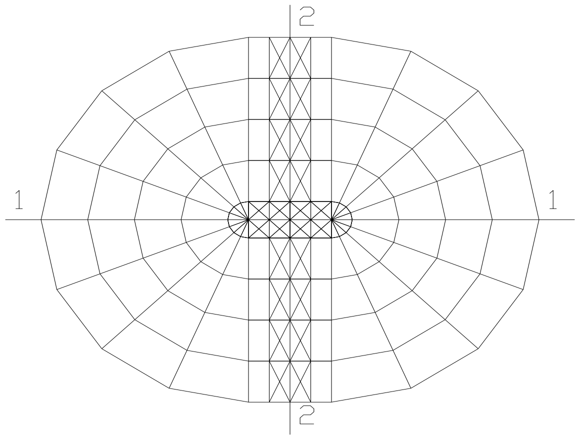
## 4.2 Structural Type 2

As can be seen from Fig. 3, this structural type is similar to Type 1, except the central section is all constructed by rigid bars instead of cables. This structural scheme can be imagined as a cable dome to be divided into two halves and connected by a transverse truss system. The central truss (shown is Fig.3-e) acts as a center tension ring in a cable dome.

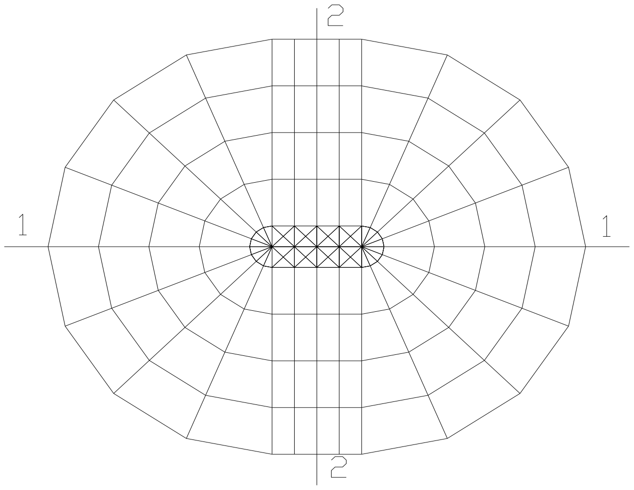
The layout and dimension of the dome is shown in Fig.3. In Fig.3, the same letters are chosen to denote the different layer of member. The calculated cross-section for ridge cables is 100cm$^2$, for diagonal cables - 80cm$^2$, for tension hoop cables - 200cm$^2$. The areas of all bar elements in the central truss are 203.4cm$^2$.



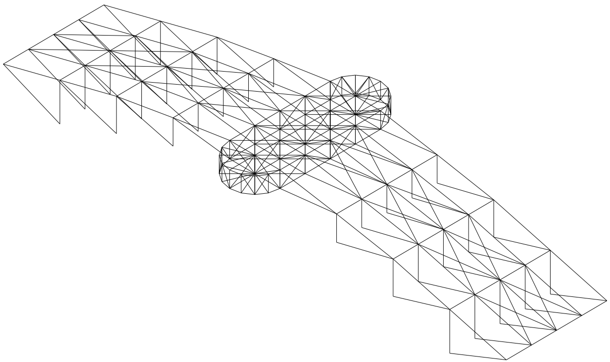
a. Isometric View



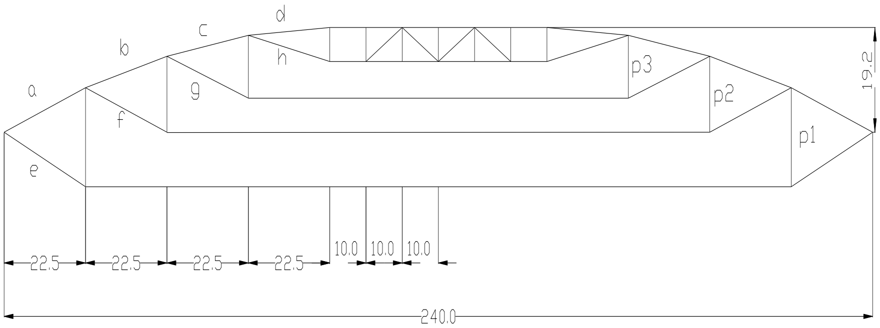
b. Plan of Top Chord Member And Hoop Cable



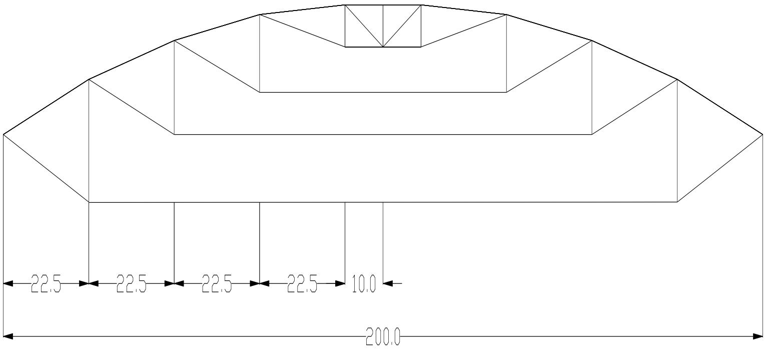
c. Plan of Bottom Chord Member



**e.Central Truss**



**g.Section 1-1**



**f.Section 2-2**

Fig.3 Structural Type 2

## *5. COMPARISON OF THE STRUCTURAL TYPES*

The two structural types that have been proposed by the author are compared with the prototype – Georgia Dome in terms of nominal steel weight and maximum vertical displacement in Table 1. As the cost of the cables is approximately twice the cost of the steel sections, the weight of the cable is multiplied by 2 and then added to the weight of the steel sections to give the nominal steel weight of the corresponding structural type. This will reflect the cost of the structure in a more objective sense.

Table 1 Comparison of Structural Types

| Structural type | Steel wt (kg/m$^{2)}$ | Nominal steel. wt (kg/m$^2$) | Max. vertical displacement (mm) |
|---|---|---|---|
| Georgia dome | 23.3 | 37.8 | 706 |
| Type 1 | 20.5 | 33.4 | 846 |
| Type 2 | 31.5 | 42.2 | 407 |

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From Table 1, it can be seen that structural types 1 and 2 all demonstrate a low steel consumption. For Type 1, it is even lower than that of Georgia Dome, but it is less stiff. However, the maximum vertical displacements of Type 1 and Georgia Dome, around 1/227 to 1/272 of the shorter span, are acceptable. This proves that the concept of designing quasi-elliptical domes, i.e. to strengthen the central section and to simplify the semi-circular sectors, is effective.

## *6. CONCLUSIONS*

From the comparison, the main conclusions are obtained as follows.

1. The structural behavior of a Tensegrity dome is different from conventional dome structures. The failure mode is characterized by the slackening of the ridge and diagonal cables in the central section of the dome.
2. The weakest position in an elliptical Tensegrity dome is in the central section. The concept of design is to strengthen this part of the dome, especially the horizontal stiffness.
3. For long span roofs around 200m, either the networks of cable dome in wedge shape or the triangulated network is appropriate. Each type of network geometry has its advantages and disadvantages. The combination of these two will provide a satisfactory solution.
4. The structural types proposed by the author are practical, as it is checked by the finite element software.

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