Optimal Dispatch of Pumped Storage Hydro Cascade under Uncertainty

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***Abstract*—In this paper, we propose an optimal dispatch scheme for a pumped storage hydro cascade that maximizes the energy per cubic meter of water in the system taking into account uncertainty in the net load variations. To this end, we introduce a model to describe the behaviour of a pumped storage hydro cascade and formulate its optimal dispatch. We then incorporate forecast scenarios in the optimal dispatch, and define a robust variant of the developed system. The resulting optimization problem is intractable due to the infinite number of constraints. Using tools from robust optimization, we reformulate the resulting problem in a tractable form that is amenable to existing numerical tools and show that the computed dispatch is immunised against uncertainty. The efficacy of the proposed approach is demonstrated by means of a realistic case study based on the Seven Forks system located in Kenya.**

***Index Terms*—Robust optimization, Hydroelectric system, Pumped storage hydro cascade, Optimal Dispatch Scheme.**

I. INTRODUCTION

Renewable-based resources have been integrated into power systems at a very high pace the last decades making the reliable operation of power systems more challenging. Due to the inherent variable and intermittent nature of renewable resources adequate generation from traditional resources needs to be scheduled to ensure the load generation balance and the secure operation of power systems. One way to make the integration of renewable resources to the grid more smooth is to better utilize existing system resources, e.g., through coupling of resources that have complimentary characteristics. Such candidates are hydro and solar resources in the sense that when the power output of one, e.g., solar generation is high, the potential power output of the other is lower, e.g., hydro generation (e.g. [1], [2]). This is true when the sun irradiance is high and there is no rainfall thus reducing the water inflows to the hydroelectric system and the potential output of the hydroelectric resource. Besides the complementarity effect pumped hydro and solar resources are nicely combined together because the first may serve as a "storage" device for the latter. When the solar generation exceeds the net load then the remaining power may be used to move water upstream in the pumped hydroelectric power system. Moreover, when the solar generation is not sufficient to meet the net load the pumped hydro has satisfactory ramping capabilities to quickly ramp up to meet the load.

In order to integrate solar generation with hydro resources and maximize their benefits mathematical models and algorithms that are able to effectively deal with the uncertainty need to be developed. In [3], dynamic programming is used to determine hydroelectric power scheduling. The authors in [4] use Monte Carlo techniques for the short-term operation of the Itaipu hydroelectric power system subject to inflow uncertainties. Another case study is presented in [5] where a model predictive control scheme for the Mid-Columbia hydropower system is proposed.

In this work: i) We develop a deterministic model for the operation of a pumped storage hydro cascade, ii) We incorporate forecast scenarios in an optimization and define a robust variant of the developed pumped storage hydro cascade system. Using tools from robust optimization we reformulate the resulting problem in a tractable form that is amenable to existing numerical tools, while offering immunization of the computed dispatch against uncertainty, iii) We demonstrate the proposed approach through a realistic system based on the Seven Forks system located in Kenya.

II. HYDROELECTRIC SYSTEM MODEL

In this section, we introduce the hydropower function, the pumped storage hydro modelling and the scheduling constraints that are utilised to develop our framework.

We consider a hydroelectric power system with N hydroelectric power plants indexed by $\mathscr{N} = \{1, \ldots, N\}$ that we wish to schedule for a time period $\mathscr{T} = \{1, \ldots, T\}$. Let us assume that K plants are pumped storage hydro and comprise the set $\mathscr{K} = \{1, 2, \ldots, K\}$, and the remaining plants are in $\tilde{\mathscr{N}} = \mathscr{N} \setminus \mathscr{K}$. We model the behavior of the system in discrete time which is a valid assumption since we consider the steady state operation of the hydroelectric system (e.g. [6], [7]).

A. Hydroelectric Power Output

A hydroelectric power plant $i \in \tilde{\mathscr{N}}$ output is a function of the water discharge and the head level of the plant. The head is the difference between the level of the reservoir and the tail water. In particular, the power of a hydroelectric power plant i at time t is defined as

$$P_i(t) = \eta_i \, \rho \, g \, h_i(t) \, q_i(t), \, \forall i \in \tilde{\mathscr{N}}, \, \forall t \in \mathscr{T}, \tag{1}$$

where ρ is the density of the water in kg/m^3; g is the gravitational acceleration in m/s^2; $h_i(t)$ is the net head of water of hydropower plant i at time t in m; $q_i(t)$ is the discharge of water of plant i during time t in m^3/s; η_i is the efficiency of the turbine generator.

The dispatch of a hydroelectric power system is usually formulated as an optimization problem. The use of (1) as a constraint in the dispatch algorithm makes the optimization problem non-convex since it is a non-linear constraint. Thus, several works are dedicated into determining approximations of (1) (e.g., [5]). A linear function fit to the three-dimensional hydropower production function denoted by

$$\tilde{P}_i(t) = \alpha_i + \beta_i h_i(t) + \gamma_i q_i(t), \forall i \in \tilde{\mathscr{N}}, \tag{2}$$

is a good approximation of (1) as shown in [8].

The power output of each hydroelectric power plant $i \in \tilde{\mathscr{N}}$ is constrained by a minimum and a maximum output, i.e., $P_i^{\mathrm{m}} \leq \tilde{P}_i(t) \leq P_i^{\mathrm{M}}$, for all $t \in \mathscr{T}$. Similar statements are true for the head levels and the water discharge rates. Thus, we have that

$$P_i^{\mathrm{m}} \leq \tilde{P}_i(t) \leq P_i^{\mathrm{M}},\ \forall i \in \tilde{\mathscr{N}},\ \forall t \in \mathscr{T}, \tag{3}$$
$$h_i^{\mathrm{m}} \leq h_i(t) \leq h_i^{\mathrm{M}},\ \forall i \in \tilde{\mathscr{N}},\ \forall t \in \mathscr{T}, \tag{4}$$
$$q_i^{\mathrm{m}} \leq q_i(t) \leq q_i^{\mathrm{M}},\ \forall i \in \tilde{\mathscr{N}},\ \forall t \in \mathscr{T}. \tag{5}$$

B. Cascading Effect of Hydroelectric System

The cascading effect of a hydroelectric system refers to the water balance between reservoirs. In other words the water at a downstream dam is affected by the discharge and spillage at upstream dams, and inflows. In the inflow parameters, evaporation and percolation losses are taken into account. Furthermore, in the water balance equation the time that the water needs to travel from one dam to the other should be considered.

A mathematical formulation of the water balance of the hydroelectric power system may be expressed as

$$\begin{aligned} V_1(t) &= V_1(t-1) + (r_1(t) - q_1(t) - s_1(t))\Delta t, && (6)\\ V_2(t) &= V_2(t-1) + (r_2(t) + q_1(t-\tau_1) + s_1(t-\tau_1) \\ &\quad -q_2(t) - s_2(t))\Delta t, && (7)\\ &\ \ \vdots \\ V_N(t) &= V_N(t-1) + (r_{N-1}(t) + q_{N-1}(t-\tau_{N-1}) \\ &\quad +s_{N-1}(t-\tau_{N-1}) - q_N(t) - s_N(t))\Delta t, && (8) \end{aligned}$$

where $V_i(t)$ is the live volume of hydroelectric power plant i at the end of time t in m^3; τ_i is the time delay between reservoir i and $i+1$, i.e., the time water needs to travel from one to the other ; $r_i(t)$ is the inflow into hydroelectric power plant i during time to t ; $s_i(t)$ is the spillage discharge of hydroelectric power plant i during time to t; and Δt the time interval between the decision making, e.g., one hour.

There are constraints associated with the reservoir storage volume limits of each hydroelectric power plant $i \in \mathscr{N}$, which are defined as

$$V_i^{\mathrm{m}} \leq V_i(t) \leq V_i^{\mathrm{M}},\ \forall i \in \mathscr{N},\ t \in \mathscr{T}. \tag{9}$$

C. Pumped storage hydro modelling

Pumped storage hydroelectric plants are designed to serve the peak load at certain times, e.g., peak hours, with hydroelectric energy and then pumping the water back up into the reservoir at other times, e.g., light load periods. Thus, two intervals need to be considered when modelling the operation of pumped storage hydroelectric systems: intervals of generation and intervals of pumping. In any one interval, the plant can be (i) pumping or (ii) generating. The idle case may be presented as either pump or generate .

Let us assume that hydro electric plant k in the cascade is a pumped storage hydro plant. Then, for the generation intervals t we have:

$$\begin{aligned} \tilde{P}_{q_k}(t) &= \alpha_k + \beta_k h_k(t) + \gamma_k q_k(t), && (10)\\ V_k(t) &= V_k(t-1) + (r_k(t) + q_{k-1}(t-\tau_{k-1}) \\ &\quad +s_{k-1}(t-\tau_{k-1}) - q_k(t) - s_k(t))\Delta t, && (11) \end{aligned}$$

where $\tilde{P}_{q_k}(t)$ is the power output of pumped storage hydro plant k at time t.

For the pump intervals t' we have:

$$\begin{aligned} \tilde{P}_{w_k}(t') &= -\alpha_k - \beta_k h_k(t') - \gamma_k w_k(t'), && (12)\\ V_k(t') &= V_k(t'-1) + (r_k(t') + q_{k-1}(t'-\tau_{k-1}) \\ &\quad +s_{k-1}(t'-\tau_{k-1}) + w_k(t') \\ &\quad -s_k(t'))\Delta t', && (13) \end{aligned}$$

where $\tilde{P}_{w_k}(t')$ is the power needed to pump the water of pumped storage hydro plant k at time t' and $w_k(t')$ is the pumping rate at time t'.

We represent the intertemporal water balance constraints that relate the charge/discharge decisions, combining (11) and (13) with

$$\begin{aligned} V_k(t) &= V_k(t-1) + (r_k(t) + q_{k-1}(t-\tau_{k-1}) \\ &\quad +s_{k-1}(t-\tau_{k-1}) + w_k(t) - q_k(t) \\ &\quad -s_k(t))\Delta t, && (14) \end{aligned}$$

These equalities serve to ensure that the reservoir accumulates water during the light load conditions so as to discharge water in subsequent peak load hours. The constraints associated with the ranges with discharge and pumping rates are

$$u_{q_k}(t)P_{q_k}^m \leq \tilde{P}_{q_k}(t) \leq u_{q_k}(t)P_{q_k}^M, \forall k \in \mathscr{K}, t \in \mathscr{T}, \tag{15}$$
$$u_{q_k}(t)q_k^m \leq q_k(t) \leq u_{q_k}(t)q_k^M, \forall k \in \mathscr{K}, t \in \mathscr{T}, \tag{16}$$
$$u_{w_k}(t)P_{w_k}^m \leq \tilde{P}_{w_k}(t) \leq u_{w_k}(t)P_{w_k}^M, \forall k \in \mathscr{K}, t \in \mathscr{T}, \tag{17}$$
$$u_{w_k}(t)w_k^m \leq w_k(t) \leq u_{w_k}(t)w_k^M, \forall k \in \mathscr{K}, t \in \mathscr{T}. \tag{18}$$

Note that the minimum and maximum discharge (pumping) rates are both multiplied by the operational state status variable $u_{q_k}(t)$ ($u_{w_k}(t)$). Whenever the pumped storage hydro generates (pumps) at time t, the associated status variable $u_{w_k}(t)$ ($u_{q_k}(t)$) is 0, resulting in the pumping output $w_k(t)$ (discharge output $q_k(t)$) being 0. This formulation preserves the linearity of the constraints, which helps with the computational tractability. Furthermore, in order to ensure that the pumped storage hydro does not both pump and generate at the same t we include (e.g., [9])

$$0 \leq u_{q_k}(t) + u_{w_k}(t) \leq 1, \forall t \in \mathscr{T}, \tag{19}$$
$$u_{q_k}(t), u_{w_k}(t) \in \{0, 1\}, \forall t \in \mathscr{T}. \tag{20}$$

D. Reservoir geometry

There is a relationship that connects the volume of a reservoir at time t, i.e., $V_i(t)$ with a certain head level, i.e., $h_i(t)$. This mapping may be approximated by a linear function when referring to short-term operation of hydroelectric systems since for small head level differences we have small volume differences. We denote this relationship by

$$h_i(t) = \zeta_1 V_i(t) + \zeta_2, \forall i \in \mathscr{N},\ t \in \mathscr{T}, \tag{21}$$

where ζ_1, ζ_2 some coefficients.

III. Optimal dispatch of pumped storage hydro cascade

In this section, we formulate the optimal dispatch of pumped storage hydro cascade. To this end, we introduce the power balance constraint; justify what is the objective of the optimal dispatch and define it; and determine the optimization problem.

A. Power Balance Constraint

The output of a hydroelectric power system is used to meet the net load at every time instant $t \in \mathscr{T}$. In this regard, we have

$$\sum_{i \in \tilde{\mathscr{N}}} \tilde{P}_i(t) + \sum_{k \in \mathscr{K}} (\tilde{P}_{q_k}(t) - \tilde{P}_{w_k}(t)) = \Delta P_L(t), \forall t \in \mathscr{T}, \quad (22)$$

where $\Delta P_L(t)$ is the net load at time t. We use the net load definition since we wish to include in our formulation the effects of renewable resources.

B. Objective Function Formulation

When formulating the optimal dispatch of a pumped storage hydro cascade we wish to maximise the energy per cubic meter of water in the system, i.e., system efficiency. To this end, we wish to operate each dam at the highest possible head and minimise the spillage effects [10]. The rationale behind this statement is that for the same discharge rate of water q for a higher head level, i.e., $h_1 > h_2$, the power output is higher, i.e., $P_1 > P_2$, as it may be easily seen through (1).

In this regard, we wish to maximise the head of each reservoir at every time instant, i.e., $h_i(t)$, for all $i \in \mathscr{N}, t \in \mathscr{T}$. We have

$$\sum_{t \in \mathscr{T}} \sum_{i \in \mathscr{N}} h_i(t). \quad (23)$$

The spilling of water may be seen as the discharge of a water amount without any power generation. In this regard, water spilled is water that is not used by the hydroelectric power system. So we wish to minimize spillage effects: $\overline{M} \sum_{t \in \mathscr{T}} \sum_{i \in \mathscr{N}} s_i(t)$. We have:

$$\overline{M} \sum_{t \in \mathscr{T}} \sum_{i \in \mathscr{N}} s_i(t), \quad (24)$$

with $\overline{M}$ a large positive constant.

C. Optimal Dispatch Formulation

In the previous sections, we have identified the objective function and the constraints that will be included in the optimal dispatch formulation of the pumped storage hydro cascade. In this regard, we use (23) and (24) to construct the objective function. The decision variables of the optimal dispatch are the live volumes $V_i(t)$; the head levels $h_i(t)$; the spillage $s_i(t)$; and the water discharge rates $q_i(t)$, for all $i \in \mathscr{N}$ and $t \in \mathscr{T}$, the pumping rates of the pumped storage hydro units $w_k(t)$, the operational state status variables $u_{q_k}(t)$ ($u_{w_k}(t)$) for $k \in \mathscr{K}$ and $t \in \mathscr{T}$. Once, the head level and water discharge rate of each dam are determined we may calculate the power output using (2) and (10). The power needed to pump may be calculated using (12). The power balance constraint now becomes:

$$\begin{aligned} &\textstyle\sum_{i \in \mathscr{N}} (\alpha_i + \beta_i h_i(t) + \gamma_i q_i(t)) \\ &- \textstyle\sum_{k \in \mathscr{K}} (\alpha_k + \beta_k h_k(t) + \gamma_k w_k(t)) = \Delta P_L(t), \forall t \in \mathscr{T}. \end{aligned} \quad (25)$$

We represent the cascading constraints by (6)-(8) and (14) for the hydroelectric plants and the pumped storage hydro plants, respectively; the relationship between the head level and live volume by (21); the power balance by (25). The lower and upper bounds of decision variables are included through (4)-(5), and (9). The power output limits constraints in (3) are now

$$P_i^m \leq \alpha_i + \beta_i h_i(t) + \gamma_i q_i(t) \leq P_i^M, \forall i \in \tilde{\mathscr{N}}, t \in \mathscr{T}. \quad (26)$$

Moreover, (15), (17) are rewritten as:

$$\begin{aligned} u_{q_k}(t) P_{q_k}^m \leq \alpha_k + \beta_k h_k(t) + \gamma_k q_k(t) \leq u_{q_k}(t) P_{q_k}^M, \\ \forall k \in \mathscr{K}, t \in \mathscr{T}. \end{aligned} \quad (27)$$

$$\begin{aligned} u_{w_k}(t) P_{w_k}^m \leq \alpha_k + \beta_k h_k(t) + \gamma_k w_k(t) \leq u_{w_k}(t) P_{w_k}^M, \\ \forall k \in \mathscr{K}, t \in \mathscr{T}. \end{aligned} \quad (28)$$

Note that the constraints for the discharge and pumping rates in a pumped storage hydro plant are taken into account through (16), (18). The constraints that ensure that the pumped storage hydro does not both pump and generate are (19), (20).

$$\begin{aligned} \max_{\substack{h_i(t), s_i(t), q_i(t), w_k(t), \\ u_{q_k}(t), u_{w_k}(t), V_i(t)}} & \sum_{t \in \mathscr{T}} \sum_{i \in \mathscr{N}} h_i(t) - \overline{M} \sum_{t \in \mathscr{T}} \sum_{i \in \mathscr{N}} s_i(t) \\ \text{subject to } & (4) - (9), (14), (16), (18) - (21), \\ & (25) - (28). \end{aligned} \quad (29)$$

The resulting optimization problem given in (29) is a mixed-integer linear program (MILP) due to the presence of the binary variables $u_{q_k}(t)$, $u_{w_k}(t)$, for all $k \in \mathscr{K}, t \in \mathscr{T}$. The output of (29) determines the head levels, power output, volume, spillage and water discharge for every hydroelectric power plant at every time instant in the period of interest. It also determines the pumping and generating intervals, pumping rates, and the power needed for pumping for the pumped storage hydro at every time instant in the period of interest.

IV. Robust Optimal Dispatch

The optimal dispatch of the pumped storage hydro cascade is used for the short term operation of a hydroelectric power system. However, the net load is usually a forecast of the actual net load and thus, contains some forecast error. This situation is exacerbated when the hydroelectric power system is coupled with some renewable resource, such as solar generation, which is intermittent and variable. In this regard, it is useful to make the optimal dispatch robust to uncertainty. In this section, we introduce uncertainty into the net load and perform a stochastic analysis, taking forecast errors into account. The resulting optimization problem is intractable; thus, we recast it into a tractable form that is immune to uncertainty. In a similar manner uncertainty in time delays or inflows to the system could be taken into account.

A. Uncertainty Modelling

We model the net load ΔP_L with two components: (i) the nominal prediction, i.e., $\overline{\Delta P}_L$; and (ii) a random forecast error vector $\delta = [\delta(1), \ldots, \delta(T)]^\top \in \Delta_t \subset \mathbb{R}^T$. Thus we have: $\Delta P_L = \overline{\Delta P}_L + \delta$. We use the vector δ to construct bounds of the forecast error, which are modelled as follows:

$$\delta(t) \in \Delta_t = [-\theta(t)\overline{\Delta P}_L(t), \theta(t)\overline{\Delta P}_L(t)], \forall t \in \mathscr{T}, \quad (30)$$

with $\theta(t) > 0$, $\forall t \in \mathscr{T}$. The power balance constraint given in (25) may now be written as

$$\begin{aligned}&\textstyle\sum_{i\in\mathscr{N}}(\alpha_i+\beta_i h_i(t)+\gamma_i q_i(t))\\&-\textstyle\sum_{k\in\mathscr{K}}(\alpha_k+\beta_k h_k(t)+\gamma_k w_k(t))\\&=\overline{\Delta P}_L(t)+\delta(t),\forall\delta(t)\in\Delta_t, t\in\mathscr{T}.\end{aligned}\tag{31}$$

However, (31) has to be met for every $\delta(t)$; thus making it infeasible. In order to make the problem feasible we express the uncertainty in the inequality constraints. To this end, we introduce piecewise affine control rules as presented in [11]. We now define the decision variable of the water discharge of every hydroelectric power plant $i \in \mathscr{N}$ (pumping rate of the pumped storage hydro plants $k \in \mathscr{K}$) at time t to consist of a deterministic component $q_i^d(t)$ ($w_k^d(t)$) and another term that depends on the uncertain error:

$$q_i(t) = q_i^d(t) + a_{q_i}(t)\delta(t), \forall i \in \mathscr{N}, \delta(t) \in \Delta_t, \forall t \in \mathscr{T}, \tag{32}$$

$$w_k(t) = w_k^d(t) + a_{w_k}(t)\delta(t), \forall k \in \mathscr{K}, \delta(t) \in \Delta_t, \forall t \in \mathscr{T}, \tag{33}$$

where

$$\sum_{i\in\mathscr{N}} a_{q_i}(t) - \sum_{k\in} a_{w_i}(t) = 1, \forall t \in \mathscr{T}. \tag{34}$$

The stochastic terms imply that if an uncertain error is realized, it is allocated to the water discharge rates, and pumping rates (if pumped storage hydro is at a pumping interval) of hydroelectric power plants according to the coefficients $a_{q_i}(t)$ and $a_{w_k}(t)$, adjusting their set-points $q_i^d(t)$, $w_k^d(t)$.

Based on this formulation (31) now becomes

$$\begin{aligned}&\textstyle\sum_{i\in\mathscr{N}}(\alpha_i+\beta_i h_i(t)+\gamma_i q_i^d(t))\\&-\textstyle\sum_{k\in\mathscr{K}}(\alpha_k+\beta_k h_k(t)+\gamma_k w_k^d(t))\\&=\overline{\Delta P}_L(t), t\in\mathscr{T},\end{aligned}\tag{35}$$

i.e., we moved the uncertainty from the equality constraint to the inequality constraints. In particular, the uncertainty sources are introduced in the inequality constraints that include $q_i(t)$ and $w_k(t)$. However, moving the uncertainty to $q_i(t)$ and $w_k(t)$ has as a result to express the spillage effects in terms of piecewise affine control rules due to (6)-(8), (14). In this regard, we have

$$s_i(t) = s_i^d(t) + a_{s_i}(t)\delta(t), \forall i \in \mathscr{N}, \delta(t) \in \Delta_t, t \in \mathscr{T}, \tag{36}$$

$$a_{q_i}(t) + a_{s_i}(t) = 1, \forall i \in \tilde{\mathscr{N}}, t \in \mathscr{T}, \tag{37}$$

$$a_{q_k}(t) + a_{s_k}(t) - a_{w_k}(t) = 1, \forall k \in \mathscr{K}, t \in \mathscr{T}. \tag{38}$$

Now, (6)-(8), (14) become

$$\begin{aligned}V_1(t) &= V_1(t-1) + (r_1(t) - q_1^d(t) - s_1^d(t))\Delta t, &(39)\\ V_2(t) &= V_2(t-1) + (r_2(t) + q_1^d(t-\tau_1) + s_1^d(t-\tau_1)\\ &\quad -q_2^d(t) - s_2^d(t))\Delta t, &(40)\\ &\vdots\\ V_N(t) &= V_N(t-1) + (r_{N-1}(t) + q_{N-1}^d(t-\tau_{N-1})\\ &\quad +s_{N-1}^d(t-\tau_{N-1}) - q_N^d(t) - s_N^d(t))\Delta t, &(41)\end{aligned}$$

and

$$\begin{aligned}V_k(t) &= V_k(t-1) + (r_k(t) + q_{k-1}^d(t-\tau_{k-1})\\ &\quad +s_{k-1}^d(t-\tau_{k-1}) + w_k^d(t) - q_k^d(t)\\ &\quad -s_k^d(t))\Delta t.\end{aligned}\tag{42}$$

We also have the constraints for the additional decision variables

$$-1 \le a_{q_i}(t) \le 1, \forall i \in \tilde{\mathscr{N}}, t \in \mathscr{T}, \tag{43}$$

$$-1 \le a_{s_i}(t) \le 1, \forall i \in \mathscr{N}, t \in \mathscr{T}, \tag{44}$$

$$-u_{q_k}(t) \le a_{q_k}(t) \le u_{q_k}(t), \forall k \in \mathscr{K}, t \in \mathscr{T}, \tag{45}$$

$$-u_{w_k}(t) \le a_{w_k}(t) \le u_{w_k}(t), \forall k \in \mathscr{K}, t \in \mathscr{T}. \tag{46}$$

We include (45) and (46) to ensure that $a_{q_k}(t)$ ($a_{w_k}(t)$) will be zero when the pumped storage hydro plant is at a pumping (generating) mode.

The stochastic optimization problem may now be written as

$$\begin{aligned}&\max_{\substack{h_i(t),s_i(t),q_i(t),w_k(t),\\ a_{q_i}(t),a_{s_i}(t),a_{w_k}(t),\\ u_{q_k}(t),u_{w_k}(t),V_i(t)}} \sum_{t\in\mathscr{T}}\sum_{i\in\mathscr{N}} h_i(t) - \overline{M}\sum_{t\in\mathscr{T}}\sum_{i\in\mathscr{N}} s_i(t)\\ &\quad\text{subject to } (4),(9),(19)-(21),(34),(35),(37)-(46),\\ &\qquad q_i^{\mathrm{m}} \le q_i^d(t) + a_{q_i}(t)\delta(t) \le q_i^{\mathrm{M}},\\ &\qquad \forall i \in \tilde{\mathscr{N}}, \delta(t)\in\Delta_t, t\in\mathscr{T}\\ &\qquad P_i^m \le \alpha_i + \beta_i h_i(t) + \gamma_i(q_i^d(t) + a_{q_i}(t)\delta(t))\\ &\qquad \le P_i^M, \forall i \in \tilde{\mathscr{N}}, \delta(t)\in\Delta_t, t\in\mathscr{T}.\\ &\qquad u_{q_k}(t)P_{q_k}^m \le \alpha_k + \beta_k h_k(t) + \gamma_k(q_k^d(t)\\ &\qquad + a_{q_k}(t)\delta(t)) \le u_{q_k}(t)P_{q_k}^M, \forall k\in\mathscr{K},\\ &\qquad \delta(t)\in\Delta_t, t\in\mathscr{T}\\ &\qquad u_{w_k}(t)P_{w_k}^m \le \alpha_k + \beta_k h_k(t) + \gamma_k(w_k^d(t)\\ &\qquad + a_{w_k}(t)\delta(t)) \le u_{w_k}(t)P_{w_k}^M, \forall k\in\mathscr{K},\\ &\qquad \delta(t)\in\Delta_t, t\in\mathscr{T}\\ &\qquad u_{q_k}(t)q_k^m \le q_k^d(t) + a_{q_k}(t)\delta(t)\\ &\qquad \le u_{q_k}(t)q_k^M, \forall k\in\mathscr{K}, \delta(t)\in\Delta_t, t\in\mathscr{T},\\ &\qquad u_{w_k}(t)w_k^m \le w_k^d(t) + a_{w_k}(t)\delta(t)\\ &\qquad \le u_{w_k}(t)w_k^M, \forall k\in\mathscr{K}, \delta(t)\in\Delta_t, t\in\mathscr{T}.\end{aligned}\tag{47}$$

B. Equivalent Tractable Reformulation

The optimization problem given in (47) cannot be solved directly because some constraints apply for all $\delta(t) \in \Delta_t, t \in \mathscr{T}$; thus, the intersection of an infinite number of constraints. In this regard, we recast (47) into a tractable problem [12]. To make this reformulation more clear, we first go through a simple inequality constraint, i.e., $q_i^m \le q_i^d(t) + a_{q_i}(t)\delta(t) \le q_i^M$ and follow this procedure for all of the constraints that contain $\delta(t)$. For the upper bound, we have that:

$$\begin{aligned}&q_i^d(t) + a_{q_i}(t)\delta(t) \le q_i^M \overset{(30)}{\Leftrightarrow}\\ &q_i^d(t) + |a_{q_i}(t)|\theta(t)\overline{\Delta P}_L(t) \le q_i^M \Leftrightarrow\\ &-\tfrac{q_i^M - q_i^d(t)}{\theta(t)\overline{\Delta P}_L(t)} \le a_{q_i}(t) \le \tfrac{q_i^M - q_i^d(t)}{\theta(t)\overline{\Delta P}_L(t)}.\end{aligned}\tag{48}$$

In the same vein, for the lower bound we have that:

$$\begin{aligned}&q_i^d(t) + a_{q_i}(t)\delta(t) \ge q_i^m \overset{(30)}{\Leftrightarrow}\\ &q_i^d(t) - |a_{q_i}(t)|\theta(t)\overline{\Delta P}_L(t) \ge q_i^m \Leftrightarrow\\ &-\tfrac{q_i^d(t) - q_i^m}{\theta(t)\overline{\Delta P}_L(t)} \le a_{q_i}(t) \le \tfrac{q_i^d(t) - q_i^m}{\theta(t)\overline{\Delta P}_L(t)}.\end{aligned}\tag{49}$$

The resulting tractable mixed integer linear programming may be written as:

$$
\begin{aligned}
\max_{\substack{h_i(t),s_i(t),q_i(t),w_k(t),\\ a_{q_i}(t),a_{s_i}(t),a_{w_k}(t),\\ u_{q_k}(t),u_{w_k}(t),V_i(t)}} \quad & \sum_{t\in\mathscr{T}}\sum_{i\in\mathscr{N}} h_i(t) - \overline{M}\sum_{t\in\mathscr{T}}\sum_{i\in\mathscr{N}} s_i(t)\\
\text{subject to } & (4), (9), (19)-(21), (34), (35), (37)-(46)\\
& q_i^{\mathrm{m}} \leq q_i^d(t) - |a_{q_i}(t)|\theta(t)\overline{\Delta P}_L(t),\\
& \forall i \in \tilde{\mathscr{N}}, t\in\mathscr{T}\\
& q_i^d(t) + |a_{q_i}(t)|\theta(t)\overline{\Delta P}_L(t)\\
& \leq q_i^{\mathrm{M}},\ \forall i \in \tilde{\mathscr{N}}, t\in\mathscr{T}\\
& P_i^m \leq \alpha_i + \beta_i h_i(t) + \gamma_i(q_i^d(t)\\
& - |a_{q_i}(t)|\theta(t)\overline{\Delta P}_L(t)), \forall i \in \tilde{\mathscr{N}}, t\in\mathscr{T}.\\
& \alpha_i + \beta_i h_i(t) + \gamma_i(q_i^d(t)\\
& + |a_{q_i}(t)|\theta(t)\overline{\Delta P}_L(t)) \leq P_i^M,\\
& \forall i \in \tilde{\mathscr{N}}, t\in\mathscr{T}.\\
& u_{q_k}(t)P_{q_k}^m \leq \alpha_k + \beta_k h_k(t) + \gamma_k(q_k^d(t)\\
& - |a_{q_k}(t)|\theta(t)\overline{\Delta P}_L(t)), \forall k\in\mathscr{K}, t\in\mathscr{T}\\
& \alpha_k + \beta_k h_k(t) + \gamma_k(q_k^d(t)\\
& + |a_{q_k}(t)|\theta(t)\overline{\Delta P}_L(t)) \leq u_{q_k}(t)P_{q_k}^M,\\
& \forall k\in\mathscr{K}, t\in\mathscr{T}\\
& u_{w_k}(t)P_{w_k}^m \leq \alpha_k + \beta_k h_k(t) + \gamma_k(w_k^d(t)\\
& - |a_{w_k}(t)|\theta(t)\overline{\Delta P}_L(t)), \forall k\in\mathscr{K}, t\in\mathscr{T}\\
& \alpha_k + \beta_k h_k(t) + \gamma_k(w_k^d(t)\\
& + |a_{w_k}(t)|\theta(t)\overline{\Delta P}_L(t)) \leq u_{w_k}(t)P_{w_k}^M,\\
& \forall k\in\mathscr{K}, t\in\mathscr{T}\\
& u_{q_k}(t)q_k^m \leq q_k^d(t)\\
& - |a_{q_k}(t)|\theta(t)\overline{\Delta P}_L(t), \forall k\in\mathscr{K}, t\in\mathscr{T},\\
& q_k^d(t) + |a_{q_k}(t)|\theta(t)\overline{\Delta P}_L(t), \forall k\in\mathscr{K},\\
& t\in\mathscr{T},\\
& u_{w_k}(t)w_k^m \leq w_k^d(t) - |a_{w_k}(t)|\theta(t)\overline{\Delta P}_L(t),\\
& \forall k\in\mathscr{K}, t\in\mathscr{T},\\
& w_k^d(t) + |a_{w_k}(t)|\theta(t)\overline{\Delta P}_L(t) \leq u_{w_k}(t)w_k^M,\\
& \forall k\in\mathscr{K}, t\in\mathscr{T}. \qquad (50)
\end{aligned}
$$

V. Numerical Results

In this section, we illustrate the robust optimal dispatch of a pumped storage hydroelectric system with a cascade that contains two hydroelectric power plants, which is a subsystem of the Seven Forks system in Kenya [13]. This system even artificial is driven through data of an actual system. The system considered has one pumped storage hydro unit and a downstream hydroelectric power plant, i.e., $\mathscr{N} = \{1, 2\}$, $\mathscr{K} = \{1\}$, and $\tilde{\mathscr{N}} = \{2\}$. We assume that the cascade is working with a solar generation plant. The time horizon we wish to schedule the system operation is over one day, i.e., $\mathscr{T} = \{1, 2 \ldots, 24\}$. In this section, we will validate the results of the robust optimization with Monte Carlo simulations; and quantify the "cost" of uncertainty.

Reservoir	1	2
P_i^M [MW]	225	72
V_i^M [Mm^3]	21	10
V_i(start) [Mm^3]	12	6.7
h_i^M [m]	140	40
h_i^m [m]	131	31
q_i^m [m^3/s]	0	10
q_i^M [m^3/s]	189	265.68
w_i^M [m^3/s]	-	265.68

TABLE I: Hydroelectric system cascade data.

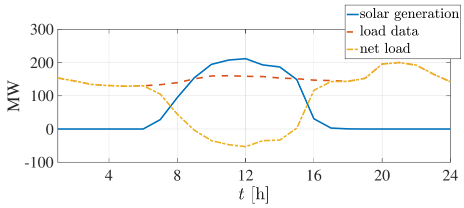


Fig. 1: Load, solar output and net load that system needs to meet.

A. System Description

The constraints of the system in terms of power output, live volume, head, and ramping characteristics are shown in Table I. The minimum power output, live volume, and pumping rate for all reservoirs are zero, i.e., $P_i^m = 0$, $V_i^m = 0$ for $i = 1, 2$, $w_1^m = 0$. The turbine generators efficiencies are $\eta_1 = 0.92$, and $\eta_2 = 0.89$. The time delays between the dams are considered to be zero; thus $\tau_1 = \tau_2 = 0$. The inflow to the system is considered to be constant for the first reservoir $r_1(t) = 50$ m^3/s; and to the second reservoir $r_2(t) = 0$, $\forall t \in \mathscr{T}$. The starting volume for each reservoir is given in Table I.

We assume that the cascade operates together with solar generation of 250 MW capacity. The solar generation and load data are depicted in Fig. 1.

B. Uncertainty modelling

We define the operation of the system for one day so that the net load is met. For these inputs, each of the hydroelectric power systems participates as shown in Fig. 2 which are the outcomes of (29). We notice that at the time where the net load is negative the pumped storage hydro plant is pumping the water as expected. In order to achieve maximum system efficiency, the first hydroelectric power station works to meet the load until the second hydro plant reaches its maximum volume and starts generating power at hour 18.

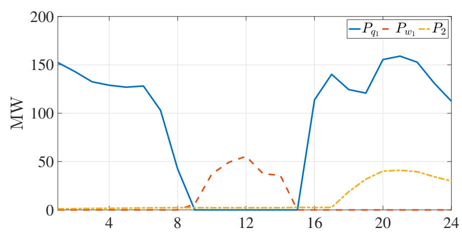


Fig. 2: Generating and pumping intervals for the hydroelectric system.

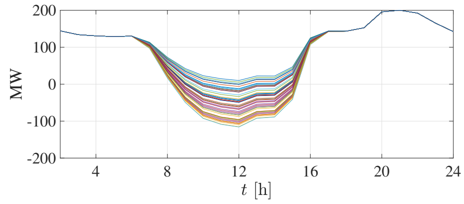


Fig. 3: Sample paths of net load for a 24-hour period.

However, the actual output of the solar generation may be different than that forecasted. In Fig. 3, the output of various sample paths of the net load for forecast error of the solar output up to 30% are depicted. We run the optimal dispatch for the pumped storage hydro cascade as described in (29) for sample paths of the net load for forecast errors 10-30%. Some representative results are depicted in Fig. 4. It may be seen in Fig. 4b that for higher forecast errors the value of the hourly head levels changes considerably for different sample paths. This is a result of different scheduling decisions based on the net load.

C. Influence of Uncertainty on Objective Function

In order to quantify how the uncertainty levels influence the value of the objective function, i.e., the head levels of the system, and the dispatch decisions we use the robust optimization formulation presented in (29). First, we need to build a reference case against which we will be comparing the robust optimization results. To this end, we run 500 experiments, i.e., Monte Carlo simulations, where the net load at period t was drawn at random, according to the uniform distribution on the segment $[(1-\theta)P_s, (1+\theta)P_s]$ where P_s is the solar output and θ is the "uncertainty level" characteristic for the experiment. We calculate the mean and standard deviation of the objective function, i.e., $\sum_{t=1}^{24}\sum_{i=1}^{2} h_i(t) - \sum_{t=1}^{24}\sum_{i=1}^{2} \overline{M}s_i(t)$, with $M = 10^8$. This mean value of the objective function for the different θ's, when all the solar generation output were known to us in advance, is found by using (29) to determine the optimal solution and is referred to as the "ideal" case.

We solve the robust optimization problem given in (50) to determine the influence of uncertainty θ on the head levels of the system. To this end, we test the optimal solution of the equivalent tractable robust reformulation of (50) with the "ideal" case for uncertainty levels of $5-15\%$. The results are summarised in Table II. As expected, the less is the uncertainty, the closer is the objective function to the ideal ones.

VI. Conclusion

In this paper, we formulated the optimal dispatch of a pumped storage hydro system by maximising the energy per cubic meter of water in the system by taking into account uncertainty. We incorporated the uncertainty sources into a robust variant of the dispatch problem. We used tools from robust optimization to reformulate the original intractable problem to an amenable form while preserving immunisation against uncertainty. In the case study, we validated the results of the robust optimization with Monte Carlo simulations and quantified the "cost" of uncertainty with a realistic system.

References

[1] A. Beluco, P. K. de Souza, and A. Krenzinger, "A method to evaluate the effect of complementarity in time between hydro and solar energy on the performance of hybrid hydro pv generating plants," *Renewable Energy*, vol. 45, no. Supplement C, pp. 24 – 30, 2012.

[2] ——, "A dimensionless index evaluating the time complementarity between solar and hydraulic energies," *Renewable Energy*, vol. 33, no. 10, pp. 2157 – 2165, 2008.

[3] H. I. Skjelbred, "Finding seasonal strategies for hydro reservoir scheduling under uncertainty," in *International Conference on Renewable Energy Research and Applications (ICRERA)*, Oct. 2013, pp. 579–583.

[4] R. E. Oviedo-Sanabria and R. A. Gonzlez-Fernndez, "Short-term operation planning of the itaipu hydroelectric plant considering uncertainties," in *Power Systems Computation Conference (PSCC)*, Jun. 2016, pp. 1–6.

[5] A. Hamann, G. Hug, and S. Rosinski, "Real-time optimization of the mid-columbia hydropower system," *IEEE Transactions on Power Systems*, vol. 32, no. 1, pp. 157–165, Jan. 2017.

[6] A. C. TalitaDal Santo, "Hydroelectric unit commitment for power plants composed of distinct groups of generating units," *Electric Power Systems Research*, vol. 137, pp. 16–25, Aug. 2016.

[7] J. G.-G. Manuel Chazarra, Juan Ignacio Prez-Daz, "Optimal joint energy and secondary regulation reserve hourly scheduling of variable speed pumped storage hydropower plants," *IEEE Transactions on Power Systems*, vol. 33, pp. 103–115, Jan. 2018.

[8] D. Apostolopoulou and M. McCulloch, "Optimal short-term operation of a cascade hydro-solar hybrid system," *under review in IEEE Transactions on Sustainable Energy*.

[9] Y. Degeilh and G. Gross, "Stochastic simulation of utility-scale storage resources in power systems with integrated renewable resources," *IEEE Transactions on Power Systems*, vol. 30, no. 3, pp. 1424–1434, May 2015.

[10] D. Apostolopoulou and M. McCulloch, "Cascade hydroelectric power system model and its application to an optimal dispatch design," in *IREP' 2017 - 10th Bulk Power Systems Dynamics and Control Symposium*, Aug. 2017, pp. 1–6.

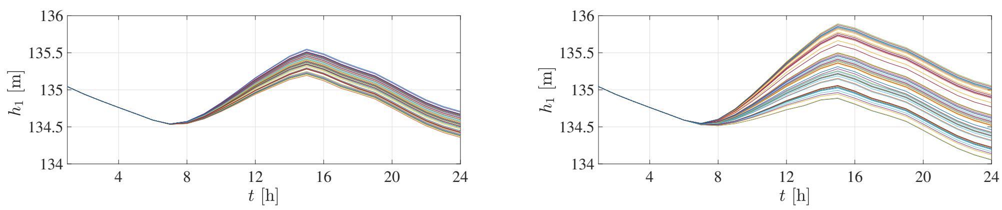
[11] K. Margellos and S. Oren, "Capacity controlled demand side management: A stochastic pricing analysis," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 706–717, Jan. 2016.

[12] A. Ben-Tal, A. Goryashko, E. Guslitzer, and A. Nemirovski, "Adjustable robust solutions of uncertain linear programs," *Mathematical Programming*, vol. 99, no. 2, pp. 351–376, Mar. 2004.

[13] M. Mbuthia, "Hydroelectric system modelling for cascaded reservoir-type power stations in the lower tana river (seven forks scheme) in kenya," in *3rd AFRICON Conference*, Sep. 1992, pp. 413–416.

Uncertainty	Tractable robust reformulation	Ideal case		Price of robustness
		Mean	Std	
5%	4091.8		1.0293	0.07 %
10%	4085.6	4,094.5	2.0357	0.2 %
15%	4077.2		2.9376	0.4 %

TABLE II: Head levels vs. uncertainty level.



(a) $\theta = 10\%$.

(b) $\theta = 30\%$.

Fig. 4: Hourly head levels of pumped storage hydro plant for different uncertainty levels θ for a one-day period.