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Buyer-Supplier Currency Exchange Rate Flexibility Contracts in Global Supply Chains

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Abstract

This paper analyzes a decentralized global supply chain under a newsvendor setting, where a supplier delivers a certain quantity of a single product to a buyer in accordance with the terms of a mutually agreed upon contract. This contract is signed prior to the delivery of the product and subsequent payment, thus, exposing the supply chain to the risk of currency exchange rate fluctuations. We propose two types of currency exchange rate flexibility contracts to explore the characteristics of exchange rate risk mitigation policies for the buyer and the supplier. Furthermore, we investigate the effects of the contract structures on the optimal order quantity, as well as the expected profits of both supply chain members. Our results show that the optimal order quantity of the buyer decreases when the wholesale price is uncertain due to exchange rate volatility. Also, both our proposed contracts tend to improve the expected profits of both the buyer and the supplier, when the payment is made in the supplier's currency, indicating the desirability of adopting such contractual agreements from the perspective of both parties. On the other hand, when the payment is made in the buyer's currency, our suggested contracts do not yield such win-win scenarios. Finally, we examine the effectiveness of availing the services of a local vendor, which is capable of satisfying any demand in excess of the quantity ordered from the foreign source with short notice, in order to mitigate the risks associated with an overseas order.

Keywords: Supply Chain Management, Currency Exchange Rate, Contracts, Newsvendor Model, Risk Sharing

1 Introduction

Global sourcing has become an effective way of achieving cost reduction in supply chains via the procurement of products from low-cost countries. Such offshore outsourcing may provide firms with considerable cost savings, but exposes their supply chains to a variety of risks associated with foreign exchange rate variation, production or transportation disruption, quality problems, supplier default, etc. Among these risks, the exchange rate fluctuation risk is consistently considered to be on the list of top concerns of supply chain managers (Liu and Nagurney, 2011). A survey of 500 supply chain executives responsible for supply chain risk management, conducted by the *Economist* magazine, shows

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that demand and exchange rate uncertainties are the two top-most concerns facing their supply chains (The Economist Intelligence Unit Report, 2009). More recently, a Business Continuity Institute (BCI) report lists exchange rate volatility as one of the top ten causes of disruption in global supply chains (Alcantara, 2014). There are various ways of dealing with exchange rate volatility from transaction exposure in the global market. Some global companies in developed countries have the opportunity to use a financial hedging mechanism by paying a fee for a financial contract with a third party financial institution. Such financial contracts are discussed in Carter and Vickery (1988). A recent review of models for supply chain risk mitigation reveals that reputation, credit, exchange rate and information risks, among other types of supply chain risks, have received limited attention due to a lack of proper modeling techniques (Rajagopal et al., 2017).

This paper focuses primarily on managing the operational risk due to currency exchange rate fluctuation in a global supply chain. We consider a decentralized supply chain, operating in a dynamic global environment involving a supplier and a buyer, facing uncertain demand, as well as payment uncertainty due to currency exchange rate variation. The main purpose of this paper is to design “exchange rate flexibility” contracts with the aim of mitigating supply chain risk by establishing a win-win policy for both the buyer and the supplier, should the exchange rate rise or fall at the time of payment. Subsequently, we investigate the effects of the chosen contract parameters on the optimal ordering policy and expected profits of the supply chain members in a newsvendor setting. In our modeling scenario, the supplier’s quoted wholesale price, w , is exogenous (i.e. it has already been pre-determined by the buyer and supplier at the outset, based on current market factors). However, the actual realized wholesale price is dependent upon the prevailing currency exchange rate at the time of payment, thus making it uncertain for either or both parties. After the wholesale price and the contract parameters have been established, the buyer determines its optimal order quantity, q . Suppose the current exchange rate (Buyer/Supplier) is x_o units in the supplier’s currency per unit of the buyer’s currency at the time of signing the contract. Since the actual payment by the buyer is made upon the fulfillment of the order at a later date, the future realization of the exchange rate, X , is a random variable which may be different from x_o . One of the parties stands to lose or gain based on the value of X relative to x_o . Thus, globalization has introduced an additional factor of complexity for decision making in supply chain networks. In short, currency exchange rate fluctuation appears to be a major risk of global sourcing contracts, which depends on the direction of the exchange rate movement from contract inception to the time of actual payment. As a result, a buyer can end up paying substantially more or less than the original contract price (Kouvelis, 1999).

In practice, a buyer and a supplier can either agree to equally share the change in price due to currency exchange rate fluctuation or use currency adjustment contract clauses that allow payment, as long as the exchange rate is within an agreed upon permissible band and renegotiating of the supply contract price should the realized exchange rate move outside the set band (Trent and Roberts, 2009; Monczka et al., 2015). Consider the following example: Svenska Cellulosa Aktiebolaget (SCA) entered into a transitional supply agreement with Procter & Gamble (P&G) in 2012, during an equipment replacement project at two of its manufacturing plants, for obtaining supplies of paper towels and tissues over a period of several months. The contract agreement allowed for price adjustments resulting from the cost of raw materials (i.e. pulp) and the volume of products supplied, but did not provide for

changes in the Pound Sterling/Euro exchange rate. The term of the agreement stated that the prices of all goods sold by P&G (supplier) to Svenska (buyer) was to be invoiced in Euros but the payment for all products supplied from the P&G UK plant would be settled in Pound Sterling. At the time of payment, the exchange rate between the Sterling and the Euro had declined (i.e., the Euro strengthened against the Sterling). The exchange rate to be applied for the payment became a legal dispute between both parties, because Svenska used a fixed exchange rate in its cost budgeting for the supply contract. The court ruled in favor of P&G and Svenska ended up paying more for the products in Pounds Sterling and also incurred additional litigation fees. A well-designed currency exchange contingent contract could have prevented such an occurrence and the supply chain contract payment dispute that arose due to the exchange rate volatility risk. Foreign exchange exposure usually has a major impact on the profitability, cash flows, and market value of a firm in a global economy (Arcelus et al., 2013). Profitability and cash flow problems can cause a supplier to go out of business, or render it unable to fulfill its contractual obligations. This might have a significant impact on a buyer's operations, such as disruptions resulting from supplier's cancellation of contract.

Existing works on supply chain contracts tend to focus on improving supply chain efficiency, but generally do not consider the issues of risk arising from price volatility due to currency exchange rate uncertainty, encountered by many supply chains in the global economy. A major aim of this study is to address this deficiency in the current literature. In this work, we propose two types of exchange rate flexibility contracts, viz: (i) Bounded exchange rate contract and (ii) Proportional Exchange rate contract. We examine the effects of both contract structures on the optimal order quantity of the buyer, as well as the expected profits of the buyer and the supplier. Our results indicate that both of the suggested contract structures seem to have some appeal to both the buyer and the supplier. We believe that these exchange rate flexibility contracts can provide formal frameworks for dealing with the risk resulting from exchange rate fluctuations.

The rest of this paper is organized as follows: In section 2, we review the related literature. The subsequent section provides the description of the models for our proposed contracts structures and presents the details of the analyses. The numerical examples illustrating our main results are reported in section 4. The final section provides some concluding remarks and managerial implications of this work.

2 Related Literature

The literature relevant to this study can be classified into two broad categories, viz: supply chain contracts and operational decisions under exchange rate uncertainty. Most of the existing works on supply chain contract studies tend to emphasize supply chain coordination mechanisms. These include buyback contracts, revenue-sharing contracts, quantity-flexibility contracts, sales-rebate contracts and quantity-discount contracts. Such coordination is not the focus of this study. We refer readers interested in supply chain coordinating contracts in a newsvendor setting to Cachon (2003). Tsay et al. (1999) provides the first comprehensive review of supply chain contracts with the inclusion of papers from other fields, which are considered foundational to this stream of supply chain management research. In this review, the objectives of these contracts are referred to as risk-sharing, since they provide mechanisms for the buyer and supplier to share the risks arising from various sources of uncertainty,

such as market demand, selling price, process yield, product quality, delivery time, etc. However, these authors' classification scheme does not capture exchange rate or supply chain contracts with exchange rate as a global element. Other factors of uncertainty include tariffs/duties, non-tariff trade barriers, corporate income tax, transportation time, inventory related costs and worker skill availability (Meixell and Gargeya, 2005).

According to Kim and Park (2014), there are two types of contracts usually found in practice for dealing with currency exchange rate volatility. One of these transfers the currency risk to a third party by purchasing a financial contract. The other rearranges the currency risk between the supply chain members involved, without any payment to an external party or agent. Carter and Vickery (1988) provide an excellent overview of different currency risk hedging mechanisms in global sourcing. Their empirical study shows that more than 50% of responding firms use some form of risk sharing contracts with their suppliers.

Several previous researchers have studied operational decisions under exchange rate uncertainty. Liu (2015) identify three approaches in the existing literature for dealing with exchange rate related risks in global supply chains: operational hedging, financial hedging and contract design. Some of the major efforts in addressing operational decisions under exchange rate uncertainty include works by Kogut and Kulatilaka (1994), Huchzermeier and Cohen (1996), Dasu and Li (1997), Kouvelis and Gutierrez (1997), Kouvelis (1999), Ding et al. (2007), Liu and Nagurney (2011), and more recently, Dong et al. (2014). The major difference between Dong et al. (2014) and Huchzermeier and Cohen (1996) is the consideration of competitive exposure. Kouvelis and Gutierrez (1997) were among the first to study the newsvendor problem towards developing an understanding of the implications of a global environment on the decision making of a firm.

Kazaz et al. (2005) explore the impact of currency exchange rate risk on the choice of production policies when the allocation decision can be postponed. These authors model the production planning problem as a two-stage recourse allocation program, by considering both production hedging and allocation hedging strategies. Their results provide insights on the trade-offs in production planning and allocation decisions, from an enterprise perspective. Another study by Wahab et al. (2011) considers a two-level international supply chain in an economic order quantity (EOQ) setting, to determine the optimal production-shipment policy for items with imperfect quality, under exchange rate uncertainty, while taking into account carbon emissions costs. The exchange rate examined in this study involve the US Dollar vs. the Thailand Baht. Their results show that as the exchange rate expressed in Baht increases (i.e. as the US Dollar strengthens), the buyer tends to purchase larger quantities due to decreasing total supply chain cost, leading to increased number of shipments and smaller shipment sizes.

In another paper, Liu and Nagurney (2011) examine the impacts of competition and exchange rate uncertainty on a firms' optimal pricing, production, and outsourcing decisions. They examine the behavior of firms with different risk attitudes and outline the effects of their decisions on profits and risks. This study develops a variational inequality model and utilize the classical mean-variance framework to model the decision-makers' behaviors and use simulation experiments to study the impacts of competition intensity and foreign exchange risk on supply chain firms with different risk attitudes. An interesting result of this study is that as exchange rate variability increases, the outsourcing activities

tend to decrease for the risk-averse firm, while they are always non-decreasing for the risk-neutral firm. Furthermore, for the risk-neutral firm, outsourcing activities tend to increase when the exchange rate fluctuation risk is “low to medium”. Also, the expected profits of risk-averse firms will always decrease with increasing exchange rate variability. The expected profits of risk-neutral firms tend to decrease if exchange rate uncertainty is relatively low and tend to increase if exchange rate uncertainty is high. As the exchange rate volatility increases, the average profits of risk-neutral firms will first increase and then become stable, while the expected profits of risk-averse firms will always decline.

Lee and Ren (2011) study a simple periodic-review stochastic inventory model to investigate the benefits of vendor managed inventory(VMI) systems, stemming from economies of scale in production and delivery in a global environment, under exchange rate uncertainty and large fixed costs of delivery. An exact, dynamic stochastic inventory model is developed for this problem and the exchange rate is modeled as a Markovian transition process with a known transition probability matrix. The supplier’s infinite horizon problem is solved using Howard’s (1960) policy-iteration method. The results suggest that the supplier is better off only when its fixed cost of production/delivery is much larger than the retailer’s fixed ordering cost. Also, VMI always yields lower supply chain total costs and higher supply chain cost reductions, compared to the case without VMI. They also contend that a state-dependent (s, S) policy is optimal for the supplier under a VMI arrangement. An earlier study by Gavirneni (2004) shows that an order up-to level policy is optimal for an inventory control problem, where the purchasing cost changes due to the influence of exchange rate variation and the conditions under which the optimal order-up-to levels are monotonically ordered are determined.

Hammami et al. (2014) consider a supplier selection problem in a global environment in the presence of uncertain fluctuations in currency exchange rates and price discounts. The problem is modeled as a mixed integer scenario-based stochastic programming problem, with the objective of minimizing the expected total system cost (i.e. the sum of purchasing price along with inventory, transportation, and supplier management costs). This model takes into account inventory costs with currency exchange rate risk and shows that the exchange rate affects the supplier selection decision in a global context. Also, a managerial insight gained in this work is that a firm can achieve significant cost savings by considering possible fluctuations in exchange rates during the supplier selection process. This result is consistent with the findings of Kouvelis (1999).

A global sourcing strategy often implies a decentralized supply chain system, since both the buyer and the seller tend to make independent decisions to optimize their respective individual operations. Kim and Park (2014) study a decentralized supply chain consisting of risk averse divisions of a multinational firm, whose total profits are affected by exchange rate fluctuations, using the mean-variance utility model. They conclude that a risk-sharing contract has a higher potential to achieve channel coordination. Also for risk-transfer, there exists a condition for a risk premium of the option such that it is optimal for the retailer to enter into a financial contract as long as the option is not costly.

Most of the above mentioned studies consider transnational supply chains with currency exchange rate uncertainty; but do not deal with risk-mitigating supply chain contracts. Also, most of the supply contracts research focus on demand uncertainty, while paying little attention to uncertain wholesale price (Tang, 2006). To the best of our knowledge, our work is the first paper to design supply chain contracts by explicitly considering currency exchange rate variability. Li and Kouvelis (1999) study sup-

ply contracts for deterministic demand in an environment of uncertain prices (caused by the exchange rate). Also, Kim and Park’s (2014) paper on risk sharing contracts assume deterministic demand. Our work differs from the above mentioned studies in that we consider a situation where the buyer experiences demand uncertainty and the realized wholesale price is uncertain due to potential exchange rate fluctuations.

3 The Models

As mentioned earlier, we consider a two-echelon global supply chain consisting of a buyer and a supplier dealing with a single product, in a newsvendor framework. The buyer is located in country B and the supplier is located in country S and operate under different currency regimes. We denote the currency denomination of the buyer as B and that of the supplier as S. To convert the buyer’s currency to the supplier’s currency, there exists a random exchange rate, X , i.e. 1 unit of the buyer’s currency is equivalent to X units of the supplier’s currency, and x_o is the current spot exchange rate, at the time of signing a purchase contract. The supplier’s marginal cost of producing a unit of the product is c , expressed in the supplier’s currency. The buyer’s cost of purchasing a unit of the product, as agreed upon by both parties, is w in the supplier’s currency or w_b in the buyer’s currency, and sells at a fixed price, p , in the latter’s currency, (w and p are both exogenous). We assume that $w > c$ and $p > w_b$. The buyer faces a stochastic demand, D , for the product over the selling season, with a probability density function $f(\xi)$ and a cumulative distribution function $F(\xi)$. The demand and the random exchange rate are assumed to be independent. In the case of a stock-out, a unit shortage or penalty cost, s , will be incurred by the buyer for unmet demand and the unit salvage value of any leftover unsold items is v . Since there is a time lag between the signing of the contract and actual payment, the exchange rate may change over this interval. The exchange rate can either go up or down and both supply chain members do not know what the exact value of X will be at the time of payment. They can jointly forecast the exchange rate which has a known probability density function g and the cumulative distribution function G with a mean of μ . Therefore, at the time of payment the buyer has to pay the purchase price in accordance with the contractual agreement.

3.1 Proposed Exchange Rate Flexibility Contracts

In this study, we propose two types of exchange rate flexibility contracts, namely: (i) Bounded exchange rate contract and (ii) Proportional exchange rate contract. In the first contract type, both supply chain members agree to set allowable bounds defined by two parameters, α and β , on the mean of the currency exchange rate. The second contract type specifies that the buyer and supplier agree to share the change in the currency exchange rate in either direction. In this contract, any change in the currency exchange rate from the mean at the time of signing the supply contract will be split between the buyer and the supplier in adjusting the wholesale price, based on an agreed upon percentage share denoted by a parameter $\phi \in \{\phi_u, \phi_d\}$. It is important to note that ϕ_u represents the agreed upon percentage share when the supplier’s currency depreciates (exchange rate increases in value) while ϕ_d represents the agreed upon percentage share when the supplier’s currency appreciates (exchange rate declines in value). In these contracts, both the buyer and supplier enter into a mutual agreement on the wholesale

price as a function of the value of the exchange rate, to settle the transaction payment. In the next section, we provide a detailed description of the bounded exchange rate contract while the details of the proportional exchange rate contract is presented in section 3.3.

3.2 Bounded Exchange Rate Contract

This contract type stipulates that both supply chain members agree to set allowable bounds defined by the two parameters α and β on the mean of the currency exchange rate at the time of signing the contract. Within these bounds, the wholesale price remains constant for one supply chain party while the other party either gains or loses depending on the realized spot exchange rate value at the time of payment. However, if the realized currency exchange rate falls outside the upper or the lower bound, the wholesale price payment is determined by either of the stipulated bounded values.

The contract scenario in which the unit wholesale price and the resulting payment transaction are in the buyer's currency is referred to as contract type 1a. On the other hand, the case in which the payment is made in the supplier's currency is referred to as contract type 1b. The definition of these contracts are outlined below and are illustrated in Figures 1 a and b.

Contract Type 1a: Let μ be the mean of the exchange rate distribution and w_b be the wholesale price of the product (in the buyer's currency), at the time of signing the purchase contract. Both parties agree to bounds defined by two parameters α and β on the mean exchange rate, with the upper bound of $\mu(1 + \alpha)$ and the lower bound of $\mu(1 - \beta)$ to be applicable when the actual payment occurs at a later date. Within the bounds, the supplier receives $w = w_b X$; where X is the spot currency exchange rate at the time of payment. Also, if the realized exchange rate is above the upper bound or below the lower bound, the actual unit price received is bounded by $w_b \mu(1 + \alpha)$ or $w_b \mu(1 - \beta)$, regardless of the actual value of X (a random variable) at the time of the payment transaction.

Contract Type 1b: Is similar to type 1a above, except that the payment is settled in the supplier's currency. Therefore, the buyer pays, $w_b = \frac{w}{X}$, in its own currency within the exchange rate bounds; where X is the spot currency exchange rate. Outside the bounds, the actual unit price paid is converted by $\frac{w}{\mu(1+\alpha)}$ or $\frac{w}{\mu(1-\beta)}$, irrespective of the realized exchange rate at the time of the payment transaction.

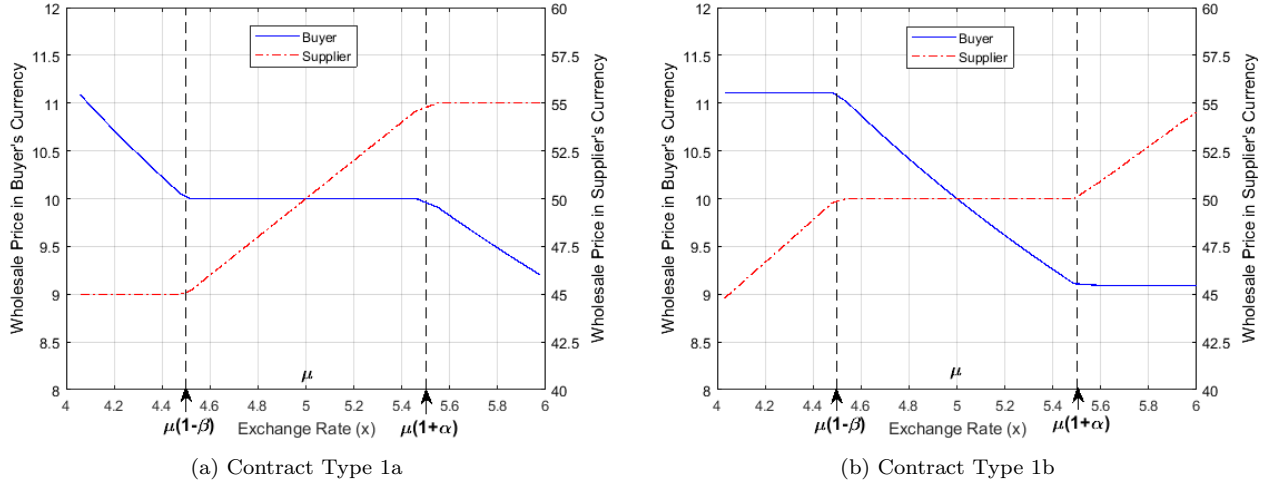


Figure 1: The Wholesale Price as a Function of the Exchange Rate with $\alpha = \beta = 0.1$

Figure 1 illustrates the characteristics of contract type 1. Suppose, the wholesale price $w_b = 10$ (in the buyer's currency) and the mean exchange rate, $\mu = 5$, (i.e. the wholesale price, $w = 50$, in the supplier's currency) at the time of signing the supply contract. Both the buyer and the supplier agree to set contract parameters: $\alpha = 0.1$ and $\beta = 0.1$. Therefore, the exchange rate used for determining the payment has an upper bound, $\mu(1 + \alpha) = 5.5$, and a lower bound, $\mu(1 - \beta) = 4.5$.

In this contract, the buyer pays a fixed wholesale price in its own currency within the contract set bounds. The supplier, on the other hand, faces the currency exchange rate risk. It can either benefit from its currency depreciation or suffer from its appreciation. However, outside the bounds, the supplier's gain (loss) from increasing (decreasing) currency exchange rate value is limited by the set bounds. Below the lower bound, the buyer sacrifices by paying more per unit selling price and above the upper bound, the buyer enjoys cost savings from the adjusted wholesale price. Also, there exist two special cases in this contract. In the first case, when $\alpha = \beta = 0$, the supplier receives a constant wholesale price, while the buyer faces an uncertain wholesale price. In the second case, when α becomes large and β tends to 1, the supplier's wholesale price received is uncertain due to the spot market currency exchange rate and the buyer now pays a fixed wholesale price in its own currency.

Figure 1b shows contract type 1b which is the direct opposite of contract type 1a. This contract structure is the same as contract type 1a, but the main difference is that the buyer becomes the exchange rate risk bearer. Now, the supplier receives a fixed wholesale price in its own currency within the bounds, while the buyer's wholesale price payment varies due to exchange rate fluctuations. However, outside the bounds, a similar policy as in contract type 1a is applied. Here, when $\alpha = \beta = 0$, the supplier receives varying wholesale price, while the buyer pays fixed wholesale price. Also, when α becomes large and β tends to 1, the supplier's wholesale price remains fixed, while the buyer faces uncertain wholesale price due to exchange rate volatility.

3.2.1 Analysis of Contract Type 1

This section presents the analysis of contract type 1 of the two proposed exchange rate contracts for risk mitigation in a global supply chain. It should be noted that these contracts are not supply chain

coordination mechanisms but are focused mainly on supply chain risk mitigation. Consider a contract with parameters α and β , which define the allowable deviation from the mean of the exchange rate distribution, μ . Contractually agreeing to the bounds on the currency exchange rate will, thus, limit loss/gain to the supplier. The detailed analysis of contract type 1a, where the payment is settled in the buyer's currency, show that there are no incentives for both the buyer and the supplier to agree to such a contract (see Appendix A.1). Therefore, in the next section, we present the analysis of contract type 1b.

3.2.1.1 Contract Type 1b: Buyer's Profit Analysis

Under this contract structure, payment is made in the supplier's currency. Let $R_b(q, D)$ be the buyer's revenue from an order quantity, q , with stochastic demand, D . At the end of the selling season, the revenue for the buyer is

$$R_b(q, D) = \begin{cases} pD + v(q - D), & \text{if } q \geq D, \\ pq - s(D - q), & \text{if } q < D. \end{cases} \quad (1)$$

In equation (1) above, $(q - D)$ units is the unsold left over amount at the end of the selling season salvaged for a revenue of v per unit, when the order quantity, q , is more than the realized demand. On the other hand, $(D - q)$ units represent lost sales which cost the buyer s per unit, if the realized demand is more than the order quantity, q .

Similarly, $C_b^{1b}(q, X)$ represents the cost of order quantity, q , given random exchange rate, X . The cost of the buyer as a result of ordering q units, comprises of three different cases: when the realized exchange rate is (i) greater than the set upper bound, (ii) within the lower and the upper bounds, and (iii) below the lower bound. The buyer's cost function in its own currency can be expressed as

$$C_b^{1b}(q, X) = \begin{cases} \frac{w}{\mu(1+\alpha)}q, & \text{if } X > \mu(1 + \alpha), \\ \frac{w}{X}q, & \text{if } \mu(1 - \beta) \leq X \leq \mu(1 + \alpha), \\ \frac{w}{\mu(1-\beta)}q, & \text{if } X < \mu(1 - \beta). \end{cases} \quad (2)$$

Hence, the expected profit of the buyer is

$$\Pi_b^{1b}(q) = E_D[R_b(q, D)] - E_X[C_b^{1b}(q, X)], \quad (3)$$

where $E_D[\cdot]$ and $E_X[\cdot]$ are the expectation operators under the demand and exchange rates, respectively. Equation (3) above can be rewritten as

$$\begin{aligned} \Pi_b^{1b}(q) = \int_0^q [p\xi + v(q - \xi)]f(\xi)d\xi + \int_q^\infty [pq - s(\xi - q)]f(\xi)d\xi - wq \left\{ \int_0^{\mu(1-\beta)} \frac{1}{\mu(1-\beta)}g(x)dx \right. \\ \left. + \int_{\mu(1-\beta)}^{\mu(1+\alpha)} \frac{1}{x}g(x)dx + \int_{\mu(1+\alpha)}^\infty \frac{1}{\mu(1+\alpha)}g(x)dx \right\}. \end{aligned} \quad (4)$$

The first order optimality conditions are necessary and sufficient, in order to obtain the optimal order quantity that maximizes the buyer's expected profit (4), yielding the following results. It is easy to

show that equation (4) is concave in q (see appendix A.2).

Proposition 1: *The optimal order quantity, q^* , of the buyer is*

$$q^* = F^{-1} \left(\frac{p - wx_b^{*1b}(\alpha, \beta) + s}{p - v + s} \right), \quad (5)$$

where

$$x_b^{*1b}(\alpha, \beta) = E_X \left[\frac{1}{X} (A^{-1} \mathbb{1}_{A>1} + \mathbb{1}_{A \leq 1 \leq B} + B^{-1} \mathbb{1}_{B<1}) \right], \quad A = \frac{\mu(1-\beta)}{X}, \quad \text{and} \quad B = \frac{\mu(1+\alpha)}{X}.$$

For proof of Proposition 1 and all other subsequent results, please refer to the appendices. Note that this result is similar to the one obtained for the classical newsvendor model except that the contract parameters (α, β) influence the optimal order quantity, q^* . When $\alpha = \beta = 0$, the buyer faces no uncertainty in the wholesale price due to the exchange rate fluctuation and the optimal order quantity, q^* , becomes the newsvendor quantity at a constant wholesale price. However, when α is large and β is close to 1, the buyer's optimal order quantity, q^* , becomes smaller than the newsvendor quantity under no exchange rate uncertainty. This is because the buyer faces uncertainty in the wholesale price, when there is no contract. Proposition 2 below shows the effects of the contract parameters (α, β) on the optimal order quantity of the buyer.

Proposition 2: *The buyer's optimal order quantity, q^* , increases as the allowable change for the exchange rate upper bound, α , increases and decreases as the allowable change for the exchange rate lower bound, β , increases .*

From Proposition 1, it is clear that an increase (a decrease) in the wholesale price will cause a decrease (an increase) in the optimal order quantity. Thus, an increasing α leads to a decreasing wholesale price and an increasing β results in an increasing wholesale price.

Next, we show the effects of changes in the contract type 1b parameters (α, β) on the expected profit of the buyer. From equation(4), the expected profit of the buyer at the optimal order quantity, q^* , can be expressed as

$$\Pi_b^{1b}(q^*) = (p - v) \int_0^{q^*} \xi f(\xi) d\xi - s \int_{q^*}^{\infty} \xi f(\xi) d\xi + (p - v + s) q^* \int_{q^*}^{\infty} f(\xi) d\xi + [v - wx_b^{*1b}(\alpha, \beta)] q^*. \quad (6)$$

From equation (6) above, it can be shown easily that Corollary 1 results directly from Proposition 2.

Corollary 1: *The buyer's expected profit increases as the allowable change for exchange rate upper bound, α , increases and decreases as the allowable change for exchange rate lower bound, β , increases.*

Since the optimal order quantity in (5) maximizes the expected profit of the buyer, α and β have the same directional impact on the expected profit as the optimal order quantity.

3.2.1.2 Contract Type 1b: Supplier's Profit Analysis

We now examine the effects of changes in the contract parameters, α and β , on the expected profit of the supplier. As before, the supplier's realized revenue comprises of three different cases, corresponding

to the buyer's cost in the previous section. The supplier's revenue function in its own currency is

$$R_s^{1b}(q^*, X) = \begin{cases} w \frac{X}{\mu(1+\alpha)} q^*, & \text{if } X > \mu(1+\alpha), \\ wq^*, & \text{if } \mu(1-\beta) \leq X \leq \mu(1+\alpha), \\ w \frac{X}{\mu(1-\beta)} q^*, & \text{if } X < \mu(1-\beta). \end{cases} \quad (7)$$

Then, the supplier's expected profit in its own currency is given by

$$\Pi_s^{1b} = E_X[R_s^{1b}(q^*, X)] - cq^*. \quad (8)$$

Thus, the expected profit of the supplier can be rewritten as

$$\Pi_s^{1b} = wq^* \left(\int_0^{\mu(1-\beta)} \frac{x}{\mu(1-\beta)} g(x) dx + \int_{\mu(1-\beta)}^{\mu(1+\alpha)} g(x) dx + \int_{\mu(1+\alpha)}^{\infty} \frac{x}{\mu(1+\alpha)} g(x) dx \right) - cq^*. \quad (9)$$

Simplifying equation (9) above, the expected profit of the supplier becomes

$$\Pi_s^{1b} = (wx_s^{*1b}(\alpha, \beta) - c) q^*,$$

where,

$$x_s^{*1b}(\alpha, \beta) = E_X[A^{-1} \mathbb{1}_{A>1} + \mathbb{1}_{A \leq 1 \leq B} + B^{-1} \mathbb{1}_{B < 1}], \quad A = \frac{\mu(1-\beta)}{X}, \quad \text{and} \quad B = \frac{\mu(1+\alpha)}{X}.$$

Proposition 3: *The supplier's expected profit decreases with increasing α and increases with increasing β .*

Proposition 3 summarizes the effects of α and β on the expected profit of the supplier. As $x_s^{*1b}(\alpha, \beta)$ decreases with α , the effective wholesale price the supplier receives decreases, thus, decreasing its expected profit. Similarly, $x_s^{*1b}(\alpha, \beta)$ increases as β increases, then the supplier's effective wholesale price increases, leading to a rise in its expected profit.

Note: Contract Type 1a is the case when the payment transaction is made in the buyer's currency. The analysis of this scenario shows that contract type 1a is indeed the inverse case of contract 1b. The detailed analysis can be found in Appendix A.1 and the result obtained leads to Proposition 4.

Proposition 4: *The results of the impacts of α and β obtained from contract type 1a (i.e. when payment is made in the buyer's currency) are opposite of all the results obtained in contract type 1b (i.e. when payment is made in the supplier's currency).*

Proposition 4 implies that when α is large and β is close to 1, the optimal order quantity, q^* , becomes the newsvendor quantity when there is no contract, because the buyer faces no uncertainty in the wholesale price. On the other hand, when $\alpha = \beta = 0$, the buyer faces uncertainty in the wholesale price due to the exchange rate fluctuation and the optimal order quantity, q^* , becomes smaller than the newsvendor quantity under no exchange rate uncertainty.

3.2.1.3 Symmetric Bounds

In order to examine the impact of the gap between the upper and the lower bound of the exchange rate on the buyer's order quantity and the expected profits of the buyer and the supplier, we set the value of α equal to β (denoted by γ , i.e. $\gamma = \alpha = \beta$). In contract type 1b, the tighter the gap, the less the uncertainty in wholesale price experienced by the buyer. For contract type 1b, the results obtained can be summarized by Propositions 5 and 6.

Proposition 5: *As the gap between the upper and the lower bound of the exchange rate increases (i.e. increasing γ value), the optimal order quantity, q^* , of the buyer decreases.*

As stated earlier, there is an inverse relationship between the wholesale price and the optimal order quantity. Based on the contract structure, the buyer's effective wholesale price in its own currency raises with γ . So, an increase in the γ value results in an increase in the wholesale price, thus, the optimal order quantity decreases.

Proposition 6: *When the contract is restricted to symmetric bounds, it is optimal to choose $\gamma = 0$ (i.e. $\alpha = \beta = 0$).*

This implies that an increasing γ value results in lower expected profits for both the buyer and the supplier. The decrease in the buyer's expected profit can be deduced directly from Proposition 5. Also, the expected profit of the supplier decreases as a result of the decrease in the order quantity from the buyer, since the expected wholesale price seems to remain unchanged.

Note: A similar result for contract type 1a follows from Proposition 4. In this case, the optimal order quantity of the buyer increases as the gap between the lower and the upper bound increases, leading to an increase in the expected profit of both the buyer and the supplier. A numerical example illustrating these results is presented in Table 1 in Section 4.

3.3 Proportional Exchange Rate Contract

We now outline and analyze contract type 2, in which both the supply chain parties agree to share the currency exchange risk. This contract can be designed in such a way that any change in the exchange rate from the mean exchange rate at the time of signing the purchase agreement will be split between the buyer and the supplier. This means that the two supply chain parties agree to share the exchange rate gain or loss by adjusting the wholesale price of the product according to an agreed upon percentage share. Thus, both supply chain members can benefit from an increasing exchange rate value or jointly suffer from a decreasing currency exchange rate value, depending on their exchange rate risk share. The buyer's percentage share is ϕ , while $(1 - \phi)$ is the supplier's share from the gain or loss on the wholesale price. For clarity, the buyer's percentage share when the exchange rate value is increasing is denoted by ϕ_u and ϕ_d represents the case when the exchange rate value is declining. We define this contract as follows:

Contract Type 2: At the time of signing the purchase contract, let μ be the mean exchange rate from the exchange rate distribution and w be the stipulated unit price of the product in the supplier's

currency. Therefore, the buyer's expected wholesale price payment is $w_b = \frac{w}{\mu}$. Both parties agree to a share of $\phi \in \{\phi_u, \phi_d\}$ for the buyer and $(1 - \phi)$ for the supplier, for any change in the exchange rate from the mean at the time of signing the supply contract. At the time of payment, the the buyer's actual wholesale price payment is $w\{\frac{\phi}{X} + \frac{(1-\phi)}{\mu}\}$.

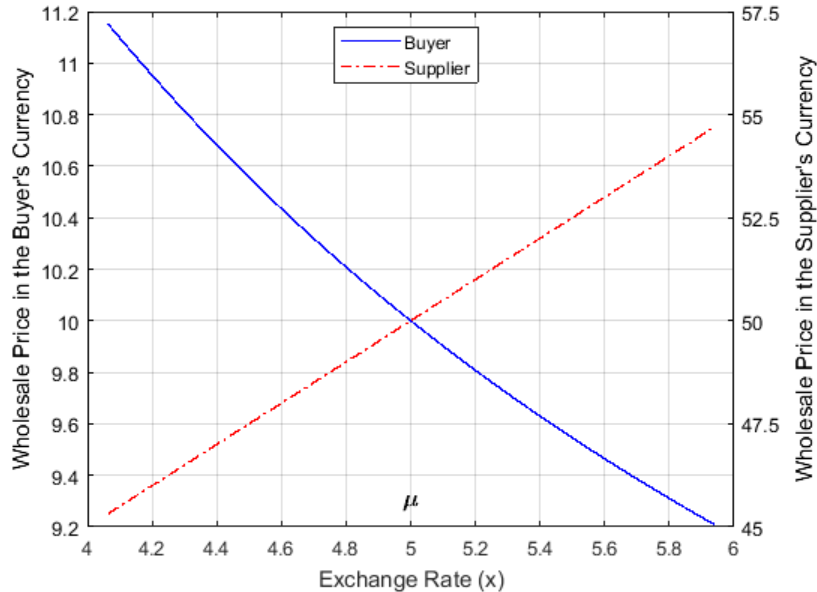


Figure 2: The Wholesale Price as a Function of the Exchange Rate with $\phi_u = \phi_d = 0.5$

Figure 2 illustrates the behavior of the wholesale price when the impact of any change in the exchange rate on the wholesale price is split equally between both the buyer and the supplier. In this type of contract, when $\phi = 1$, the buyer assumes all the exchange rate risk, making its wholesale price uncertain, as illustrated in Fig. 3a (i.e. the supplier receives a constant wholesale price in its own currency). This case is equivalent to the case when α is very large and β equal to 1 in contract type 1b. However, when $\phi = 0$, the supplier is exposed to the entire exchange rate risk as shown in Fig. 3b. Similarly, this is equivalent to the setting both α , and β to zero in contract type 1b.

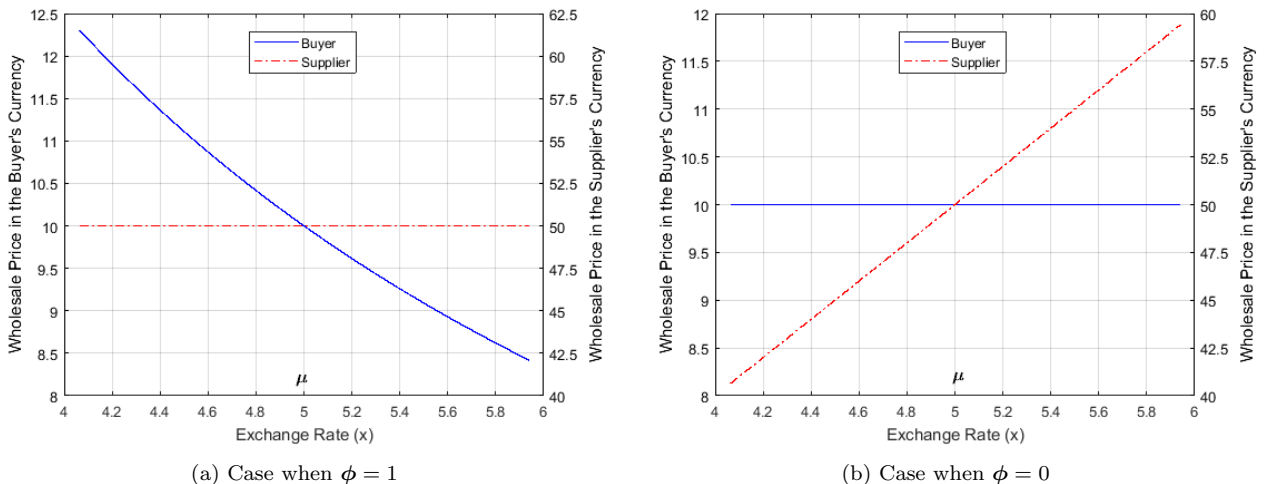


Figure 3: The Wholesale Price as a Function of the Exchange Rate at the extreme values

For example, suppose a U.S. firm (Buyer) contracts to purchase a product from a foreign firm (Supplier) at a unit wholesale price of 50.00 (in the supplier's currency). As before, let us assume that the mean exchange rate is $\$1 = 5$, the product cost to the US firm is expected to be \$10.00. Since the transaction payment will occur at a later date (say after 3 months), the exchange rate can either depreciate or appreciate with uncertainty in the exchange rate.

Assume that the buyer attempts to avoid the exchange rate risk by paying the wholesale price in its own currency. If the exchange rate in 3 months turns out to be $\$1 = 6$, the foreign firm's realized unit wholesale price is 60.00. The US firm misses the potential cost savings resulting from the foreign supplier's currency depreciation. On the other hand, if the exchange rate falls (e.g. $\$1 = 4$), the foreign firm will realize a unit wholesale price of 40.00 resulting in potential profit loss while the US firm has nothing to lose from the foreign supplier's currency appreciation. But, under this exchange rate sharing contract with a 50-50 (i.e. $\phi_u = \phi_d = 0.5$) share agreement between the US firm and the foreign firm, the product will cost the U.S. firm \$9.16 and the foreign firm will realize 55.00 when the foreign supplier's currency weakens. However, when the foreign supplier's currency strengthens, the US firm pays \$11.25 per unit as opposed to \$10.00, and the foreign firm realizes 45.00 instead of 40.00 per unit.

Now consider the case where the foreign firm (Supplier) receives the wholesale price in its own currency. If the exchange rate in 3 months is $\$1 = 6$, the US firm will pay \$8.33/unit achieving a gain through cost savings while the foreign firm B loses the extra profit as a result of this exchange rate variation. Alternatively, when the exchange rate falls to $\$1 = 4$, the US firm will pay \$12.50 per unit, resulting in a potential profit loss while the foreign firm has nothing to lose from its currency appreciation. Nevertheless, sharing the exchange rate at a 50 - 50 (i.e. $\phi = 0.5$) share yield the aforementioned results which is better than not sharing the exchange rate risk at all.

3.3.1 Contract Type 2: Buyer's Profit Analysis

Here, the cost of the buyer as a result of ordering q units, comprises of two different cases: when the realized exchange rate is (i) greater than the mean of the currency exchange rate at the time of signing the supply contract, or (ii) less than the mean of the currency exchange rate at the time of signing the supply contract. We denote $C_{b2}(q, X)$ as the purchase cost of quantity q , given random exchange rate, X . Thus, the buyer's cost function in its own currency is

$$C_{b2}(q, X) = \begin{cases} w\left\{\frac{\phi_u}{X} + \frac{(1-\phi_u)}{\mu}\right\}q, & \text{if } X \geq \mu, \\ w\left\{\frac{\phi_d}{X} + \frac{(1-\phi_d)}{\mu}\right\}q, & \text{if } X < \mu. \end{cases} \quad (10)$$

The cost function in (10) above captures the wholesale price gain or loss, resulting from the sharing of the currency exchange rate fluctuations between the buyer and the supplier. The buyer's revenue remains the same as in equation (1). Therefore, the expected buyer's profit function for this contract type is

$$\Pi_{b2}(q) = E_D[R_b(q, D)] - E_X[C_{b2}(q, X)]. \quad (11)$$

From (1) and (10), the expected buyer's profit can be expressed as

$$\begin{aligned} \Pi_{b2}(q) = & \int_0^q [p\xi + v(q - \xi)]f(\xi)d\xi + \int_q^\infty [pq - s(\xi - q)]f(\xi)d\xi - wq \left\{ \int_0^\mu \left[\frac{\phi_d}{x} + \frac{(1 - \phi_d)}{\mu} \right] g(x)dx \right. \\ & \left. + \int_\mu^\infty \left[\frac{\phi_u}{x} + \frac{(1 - \phi_u)}{\mu} \right] g(x)dx \right\}. \end{aligned} \quad (12)$$

The optimal order quantity that maximizes the buyer's expected profit can be determined from the first order optimality conditions with respect to q . Proposition 7 states the results.

Proposition 7: *For the proportional exchange rate contract, the optimal order quantity, q^* , of the buyer is*

$$q^* = F^{-1} \left(\frac{p - wx_{b2}^*(\phi_d, \phi_u) + s}{p - v + s} \right), \quad (13)$$

where

$$x_{b2}^*(\phi_d, \phi_u) = E_X \left[\left\{ \frac{\phi_d H}{\mu} + \frac{(1 - \phi_d)}{\mu} \right\} \mathbb{1}_{H > 1} + \left\{ \frac{\phi_u H}{\mu} + \frac{(1 - \phi_u)}{\mu} \right\} \mathbb{1}_{H \leq 1} \right] \quad \text{and} \quad H = \frac{\mu}{X}.$$

From (13), it is clear that the optimal order quantity depends on the buyer's share of the exchange rate gain or loss. Proposition 8 states the effect of risk sharing contract parameters, (ϕ_d, ϕ_u) on the optimal order quantity.

Proposition 8: *The buyer's optimal order quantity, q^* , increases as its share under an increasing exchange rate, ϕ_u , increases and decreases as its share under a decreasing exchange rate, ϕ_d , increases.*

Proposition 8 indicates that the buyer benefits from the depreciation of foreign supplier's currency, since the wholesale price becomes cheaper as its percentage share of the exchange rate risk increases under an increasing exchange rate situation. This increases the buyer's optimal order quantity, which in turn translates to higher profit for the buyer. On the other hand, when the buyer's percentage share of the exchange rate risk increases under a decreasing exchange rate situation, the reverse is the case.

Now, we examine the sensitivity of the expected profit of the buyer to the contract parameters ϕ_u and ϕ_d . From equation (12), the expected profit of the buyer at the optimal order quantity is

$$\Pi_{b2}(q^*) = (p - v) \int_0^{q^*} \xi f(\xi) d\xi - s \int_{q^*}^\infty \xi f(\xi) d\xi + (p - v + s) q^* \int_q^\infty f(\xi) d\xi + [v - wx_{b2}^*(\phi_d, \phi_u)] q^*. \quad (14)$$

It can be seen from equation (14) that Corollary 2 follows directly from Proposition 6.

Corollary 2: *The buyer's expected profit increases as its share under an increasing exchange rate, ϕ_u , increases and decreases as its share under a decreasing exchange rate, ϕ_d , increases.*

3.3.2 Contract Type 2: Supplier's Profit Analysis

Now, we analyze the impact of the exchange rate risk share on the expected profit of the supplier under increasing and decreasing exchange rate situations. The supplier's realized revenue under the exchange

rate risk contract in its own currency is given by

$$R_{s2}(q^*, X) = \begin{cases} w\{\phi_u + \frac{X(1-\phi_u)}{\mu}\}q^*, & \text{if } X \geq \mu, \\ w\{\phi_d + \frac{X(1-\phi_d)}{\mu}\}q^*, & \text{if } X < \mu. \end{cases} \quad (15)$$

As in the case for contract type 1, the supplier's expected profit is

$$\begin{aligned} \Pi_{s2} &= E_X[R_{s2}(q^*, X)] - cq^* \\ &= wq^* \left[\int_0^\mu \left\{ \phi_d + \frac{X(1-\phi_d)}{\mu} \right\} g(x) dx + \int_\mu^\infty \left\{ \phi_u + \frac{X(1-\phi_u)}{\mu} \right\} g(x) dx \right] - cq^*. \end{aligned} \quad (16)$$

Thus, the expected profit of the supplier can be expressed as

$$\Pi_{s2} = (wx_{s2}^*(\phi_d, \phi_u) - c)q^*,$$

where,

$$x_{s2}^*(\phi_d, \phi_u) = E_X \left[\left\{ \phi_d + \frac{(1-\phi_d)}{H} \right\} \mathbb{1}_{H>1} + \left\{ \phi_u + \frac{(1-\phi_u)}{H} \right\} \mathbb{1}_{H \leq 1} \right] \text{ and } H = \frac{\mu}{X}.$$

Proposition 9: *The expected profit of the supplier decreases as the buyer's share under an increasing exchange rate, ϕ_u , increases and increases as the buyer's share under a decreasing exchange rate, ϕ_d , increases.*

Under an increasing exchange rate scenario, the effective wholesale price received by the supplier declines as the buyer's percentage share of the exchange rate risk (ϕ_u) increases. This results in a lower supplier's expected profit. Similarly, under a decreasing exchange rate scenario, the effective wholesale price received by the supplier and its expected profit increases as the buyer's percentage share of the exchange rate risk (ϕ_d) increases. Section 4 provides a more detailed discussion of this Proposition, using an example.

3.3.3 Symmetric Contract Parameters

We investigate the effect of setting equal values for ϕ_d and ϕ_u (depicted by ϕ) on the buyer's order quantity decision, and the expected profits of the buyer and the supplier. The larger ϕ becomes, the more is the uncertainty experienced in the wholesale price for the buyer, due to the exchange rate variability. If, however, ϕ is less than 0.5, the buyer has a lesser exchange rate risk share than the supplier. On the other hand, when ϕ is greater than 0.5, the buyer is exposed to a greater exchange rate risk than the supplier. From our analysis, the resulting relationship between contract parameter, ϕ , the optimal order quantity, expected buyer's profit and the supplier's profit is presented in Proposition 10.

Proposition 10: *The optimal order quantity, q^* , and the expected profits of the buyer and supplier decrease as the buyer's share of exchange rate risk, ϕ , increases.*

Proposition 10 indicates that when the buyer assumes an equal percentage share of the exchange rate risk under both the increasing and decreasing exchange rate scenarios, its optimal order quantity

declines as its percentage share increases. The effective wholesale price becomes more costly for the buyer as a result of the increased uncertainty due to exchange rate variations. One would expect an increase in the expected profit of the supplier but interestingly there is also a decline in the expected profit of the supplier. This is because the effective wholesale price received by the supplier remains constant and its expected profit decline is only due to a decrease in the buyer's order quantity.

3.4 Model Extension with a Local Backup Supplier

We now consider a scenario where a buyer has the option of satisfying all its demand using two suppliers (a local or domestic supplier and a foreign supplier). This local supplier serves as an emergency procurement source for the product to prevent lost sales. Thus, when the realized demand exceeds the quantity obtained from an overseas supplier (i.e. the order quantity q), the remaining shortfall ($D - q$), is procured from the local supplier albeit at a higher cost, w_h . Now, the revenue function of the buyer from Equation (1) can be expressed as

$$R_b(q, D) = \begin{cases} pD + v(q - D), & \text{if } q \geq D, \\ pq + (p - w_h)(D - q), & \text{if } q < D. \end{cases} \quad (17)$$

Since our focus is on contract type 1b, where the payment to the foreign vendor is made in the supplier's currency, combining Equations (17) and (2), the buyer's expected profit is given by

$$\begin{aligned} \Pi_b^{2s}(q) = & \int_0^q [p\xi + v(q - \xi)]f(\xi)d\xi + \int_q^\infty [p\xi - w_h(\xi - q)]f(\xi)d\xi - wq \left\{ \int_0^{\mu(1-\beta)} \frac{1}{\mu(1-\beta)} g(x)dx \right. \\ & \left. + \int_{\mu(1-\beta)}^{\mu(1+\alpha)} \frac{1}{x} g(x)dx + \int_{\mu(1+\alpha)}^\infty \frac{1}{\mu(1+\alpha)} g(x)dx \right\}. \end{aligned} \quad (18)$$

Note that (18) is structurally similar to equation (4) which was utilized for developing Proposition 1. In a similar vein, equation (18) leads to Proposition 11 indicating the buyer's optimal order quantity from the foreign supplier that maximizes its expected profit.

Proposition 11: *for the two suppliers case, the buyer's optimal order quantity, q^* , is*

$$q^* = F^{-1} \left(\frac{w_h - wx_b^{*1b}(\alpha, \beta)}{w_h - v} \right), \quad (19)$$

where

$$x_b^{*1b}(\alpha, \beta) = E_X \left[\frac{1}{X} (A^{-1} \mathbb{1}_{A>1} + \mathbb{1}_{A \leq 1 \leq B} + B^{-1} \mathbb{1}_{B < 1}) \right], \quad A = \frac{\mu(1-\beta)}{X}, \quad \text{and} \quad B = \frac{\mu(1+\alpha)}{X}.$$

Along similar lines, the results for the proportional exchange rate contract (Contract type 2) in the case of an overseas vendor in conjunction with a backup domestic supplier are summarized in Proposition 12.

Proposition 12: *The optimal order quantity, q^* , of the buyer is*

$$q^* = F^{-1} \left(\frac{w_h - wx_{b2}^*(\phi_d, \phi_u)}{w_h - v} \right), \quad (20)$$

where

$$x_{b2}^*(\phi_d, \phi_u) = E_X \left[\left\{ \frac{\phi_d H}{\mu} + \frac{(1 - \phi_d)}{\mu} \right\} \mathbb{1}_{H > 1} + \left\{ \frac{\phi_u H}{\mu} + \frac{(1 - \phi_u)}{\mu} \right\} \mathbb{1}_{H \leq 1} \right] \text{ and } H = \frac{\mu}{X}.$$

Comparing the results stated in Proposition 1 with those indicated by Proposition 11, it is clear that the buyer will tend to order a smaller quantity from the foreign source in the presence of a backup domestic vendor, than it would when there is only a single foreign supplier. Similar conclusions can be arrived at for contract type 2, by comparing Propositions 7 and 12.

Comments: We have considered in addition, the case of two suppliers in two different foreign countries where all the buyer's demand has to be satisfied. Our results show that the buyer will always order the total required order quantity from the supplier with the lower wholesale price. The second supplier will be utilized only when the first supplier has insufficient capacity to meet the entire demand. This result is not surprising and provides support for Theorem 1 in Burke et al. (2007) under a reliability index value of 1.

4 Numerical Example and Sensitivity Analyses

This section presents a numerical example along with some sensitivity analyses, to illustrate the potential benefits of our proposed exchange rate flexibility contracts and develop some insights that cannot be readily observed in analytical results. Initially, we assume that $X \sim U(a, b)$ where $0 < a < b$ (similar to the premise adopted by Kim and Park (2014)) for simplicity and tractability of analyses. Subsequently, we utilize triangular distributions to model exchange rate variability, in order to examine the effects of skewness and asymmetric contract parameter values. Our aim is to shed more light on the performance characteristics of these contracts, to gain some useful managerial insights.

4.1 Uniform Exchange Rate Distribution

4.1.1 Results for Single Foreign Supplier

The parameter values used are: $D \sim U(20, 40)$, $p = B10$, $s = 0$, $v = B5$, $w_b = B7$, $\mu = 5$ (i.e. $B1 = S5$ at the time of signing the contract), $X \sim U(4, 6)$, $c = S15$. For contract type 1: $0 \leq \alpha \leq 0.2$, and $0 \leq \beta \leq 0.2$, while in contract type 2: $0 \leq \phi_u \leq 1$ and $0 \leq \phi_d \leq 1$. With the assumption of a uniform distribution of X , contract type 1b, where payment is made in the supplier's currency, the parameter $x_b^{*1b}(\alpha, \beta)$ for determining the optimal order quantity of the buyer in (5) can be expressed as

$$x_b^{*1b}(\alpha, \beta) = \frac{1}{b - a} \left(\frac{b}{\mu(1 + \alpha)} + \ln \left(\frac{1 + \alpha}{1 - \beta} \right) - \frac{a}{\mu(1 - \beta)} \right). \quad (21)$$

and $x_s^{*1b}(\alpha, \beta)$ in the supplier's expected profit function (9) simplifies to

$$x_s^{*1b}(\alpha, \beta) = \frac{1}{b - a} \left(\frac{b^2}{2\mu(1 + \alpha)} - \frac{a^2}{2\mu(1 - \beta)} + \frac{\mu(\alpha + \beta)}{2} \right). \quad (22)$$

Similarly, for contract type 1a where payment is made in the buyer's currency, we obtain the following results:

$$x_b^*(\alpha, \beta) = \frac{\mu(1-\beta)\{\ln(\mu(1-\beta)) - \ln a\} + \mu(\alpha + \beta) + \mu(1+\alpha)\{\ln b - \ln(\mu(1+\alpha))\}}{b-a}, \quad (23)$$

and

$$x_s^*(\alpha, \beta) = \frac{1}{b-a} \left(\frac{\mu^2(1-\beta)^2}{2} - a\mu(1-\beta) + b\mu(1+\alpha) - \frac{\mu^2(1+\alpha)^2}{2} \right). \quad (24)$$

Also, for contract type 2, $x_{b2}^*(\phi_d, \phi_u)$, the buyer's optimal order quantity is obtained from (13), where

$$x_{b2}^*(\phi_d, \phi_u) = \frac{\phi_d\mu(\ln \mu - \ln a) + (1-\phi_d)(\mu-a) + \phi_u\mu(\ln b - \ln \mu) + (1-\phi_u)(b-\mu)}{\mu(b-a)}, \quad (25)$$

while $x_{s2}^*(\phi_d, \phi_u)$ in the supplier's expected profit function is given by

$$x_{s2}^*(\phi_d, \phi_u) = \frac{\mu-a}{b-a} \left((1-\phi_d)\frac{\mu+a}{2\mu} + \phi_d \right) + \frac{b-\mu}{b-a} \left((1-\phi_u)\frac{b+\mu}{2\mu} + \phi_u \right). \quad (26)$$

These results stemming from the simplifying assumption that $X \sim U(a, b)$ are utilized in our computational analyses, using the symmetric contract parameters scenario for the expected profits of the buyer and the supplier are presented in Table 1.

Table 1: Contracts' effects on the expected profit of the supply chain members

Contract	Contract Parameters	Buyer's Expected Profit in its own Currency	Supplier's Expected Profit in its own Currency
Type1a	$\alpha = \beta = 0.00$	83.71	632.35
	$\alpha = \beta = 0.05$	84.53	635.15
	$\alpha = \beta = 0.10$	85.26	637.60
	$\alpha = \beta = 0.15$	85.79	639.34
	$\alpha = \beta = 0.20$	86.00	640.00
Type1b	$\alpha = \beta = 0.00$	86.00	640.00
	$\alpha = \beta = 0.05$	85.65	635.65
	$\alpha = \beta = 0.10$	84.88	633.38
	$\alpha = \beta = 0.15$	84.09	632.48
	$\alpha = \beta = 0.20$	83.71	632.35
Type 2	$\phi_u = \phi_d = 0.00$	86.00	640.00
	$\phi_u = \phi_d = 0.25$	85.43	638.09
	$\phi_u = \phi_d = 0.50$	84.86	636.17
	$\phi_u = \phi_d = 0.75$	84.30	634.26
	$\phi_u = \phi_d = 1.00$	83.71	632.35

We find that when the payment is made in the supplier's currency, contract type 1b and contract type 2 results in a win-win policy for both the buyer and the supplier as shown in Table 1. Under

contract type 1b, when the payment is made in the supplier's currency, only the buyer is exposed to the exchange rate related risk, when there is no contractual agreement (i.e $\alpha = 0.2$ and $\beta = 0.2$). The adoption of such a contract, improves the expected profits of both the supply chain members, as depicted in Table 1. For example, under contract type 1b with $\alpha = 0.1$ and $\beta = 0.1$, there is an increase in the buyer's expected profit from 83.71 to 84.88 and the supplier's expected profit increases from 632.35 to 633.38. Under both payment scenarios, any sudden adverse change in the currency exchange rate is compensated by the other supply chain party, if such a contract is adopted. The exchange rate bounded contract with payment in the supplier's currency (contract type 1b) above, support Propositions 5 and 6. As mentioned earlier, $\alpha \rightarrow \infty$ and $\beta = 1$ for contract type 1 translates to the case of no contract. Therefore, in Table 1, $\alpha = \beta = 0.2$ is tantamount to no contractual agreement. As we move from no contract to a more tightly bound contract, for contract type 1a, the expected profits tend to decline for both the supply chain parties. On the other hand, for contract type 1b, the expected profits of both the buyer and the supplier increases, making it a desirable contract.

Similarly, $\phi_u = \phi_d = 0$ in the exchange rate risk sharing contract (contract type 2) represents the case of no contract where the payment is made in the buyer's currency (i.e. the supplier bears all the exchange rate risk). On the other hand, $\phi_u = \phi_d = 1$ indicates no contract and payment is made in the supplier's currency (the buyer bears the entire exchange rate risk). Thus, it can be deduced from Table 1 that contract type 2 exhibits similar effects on the expected profits of both the buyer and the supplier, when payment is made in the supplier's currency. Contract type 2 can be desirable for both parties, if the payment is made in the supplier's currency.

Generally, contract type 1 limits the loss of the affected party in case of a sudden unfavorable movement of the exchange rate, while contract type 2 allows both supply chain parties to share the consequences of both favorable and unfavorable movements of the exchange rate. When the payment is made in the supplier's currency, both parties stand to gain by adopting either of the two suggested contract types. These types of contracts with payments made in the buyer's currency, do not seem to be beneficial for either the supplier or the buyer, and, hence, cannot be recommended. Also, incorporating a formal structure such as the exchange rate flexibility contracts proposed in this study, at the outset of an international supply contract eliminates the time, effort and costs associated with possible contract re-negotiation, if resorted to, and/or the resolution of disputes resulting in substantial litigation costs to both parties. In addition, other supply chain risks, such as the supplier going bankrupt or the cancellation of a contract that has not been executed, can be substantially reduced.

According to a Western Union white paper (2013), the main drawback of paying in dollars when a US-based company is dealing with a foreign supplier is that dollars are converted to the supplier's local currency at a local bank's prescribed rate, without the flexibility for the supplier to negotiate. Typical currency exchange premiums charged by these local banks can be up to 10% in some parts of the world. Therefore, a US buyer may be favorably disposed to adopting our suggested contract type 1b, by making payment in the supplier's local currency, without the conversion cost burden. In general, both of the suggested contract structures appear to have some appeal in terms of risk mitigation and/or cost avoidance from the perspective of the buyer as well as the supplier.

4.1.2 Results with Backup Local Supplier

Next, we present the results of our numerical experiments for the case involving a primary lower cost foreign source and a higher cost local or domestic backup supplier, for contract types 1b and 2, in Tables 2 and 3, respectively. The parameter representing the wholesale price of the local supplier is $w_h = B9.5$. These results can be compared with those pertaining to the case of a single foreign supplier, presented in Table 1 earlier. Note that under this scenario, the buyer satisfies the entire market demand, since the local supplier is used to avoid any potential shortage.

Table 2: Effects of 2 Suppliers under Contract Type 1b

Parameters		Order	Expected Profit	
α	β	Quantity	Buyer	Supplier
0.00	0.00	31.11	87.78	622.22
0.05	0.05	31.05	87.45	617.86
0.10	0.10	30.90	86.74	616.18
0.15	0.15	30.75	86.02	614.06
0.20	0.20	30.69	85.69	613.72

Table 3: Effects of 2 Suppliers under Contract Type 2

Parameters		Order	Expected Profit	
ϕ_u	ϕ_d	Quantity	Buyer	Supplier
0.00	0.00	31.11	87.78	622.22
0.25	0.25	31.00	87.25	620.10
0.50	0.50	30.90	86.73	617.97
0.75	0.75	30.70	86.21	615.85
1.00	1.00	30.69	85.69	613.72

Comparing Table 1 with Tables 2 and 3, we can readily observe that despite a reduction in the buyer's order size to the foreign supplier, its expected profit tends to increase. A reduction in the buyer's order quantity increases its expected shortage, which is procured from the local supplier, still contributing towards an increase in the former's expected profit, since $p > w_h$. At the same time, the local supplier gains from the larger expected shortage. This, however, leads to a decline in the expected profit of the foreign supplier under both contracts 1b and 2. This is not surprising in view of the presence of a local backup source, which, in effect, eliminates the buyer's lost sales.

When the variability in the exchange rate increases, the expected profits of both the buyer and the supplier decline, due to larger reductions in the order quantity to the foreign supplier, as shown in Tables 4 and 5.

Table 4: Effects of Exchange Rate Variability under Contract Type 1b for 2 Suppliers Case

Parameters $\alpha = \beta$	$X \sim U(3, 7)$			$X \sim U(2, 8)$		
	Order Quantity	Expected Profit Buyer Supplier		Order Quantity	Expected Profit Buyer Supplier	
0.00	31.11	87.78	622.22	31.11	87.78	622.22
0.05	31.04	87.42	612.45	31.04	87.41	607.02
0.10	30.85	86.48	604.72	30.83	86.40	593.93
0.15	30.57	85.16	598.67	30.51	84.88	582.64
0.20	30.25	83.64	593.98	30.11	82.98	572.84

Table 5: Effects of Exchange Rate Variability under Contract Type 2 for 2 Suppliers Case

Parameters $\phi_d = \phi_u$	$X \sim U(3, 7)$			$X \sim U(2, 8)$		
	Order Quantity	Expected Profit Buyer Supplier		Order Quantity	Expected Profit Buyer Supplier	
0.00	31.11	87.78	622.22	31.11	87.78	622.22
0.25	30.65	85.53	613.03	29.90	81.63	598.07
0.50	30.19	83.37	603.83	28.70	71.15	573.92
0.75	29.73	80.10	594.63	27.49	61.32	549.77
1.00	29.27	76.07	585.43	26.28	52.14	525.63

With higher currency exchange rate uncertainty under both contract structures, the local supplier tends to benefit from a greater need to make up for the buyer's shortfall, when the realized demand exceeds the order quantity from the foreign supplier. Note that this paper does not address the issue of an order splitting mechanism between the foreign and the local supplier. Therefore, we cannot state that the local supplier is favored against its foreign counterpart due to increasing exchange rate variability, apart from the fact that it stands to gain from larger shortages at the buyer's end.

4.2 Skewness in Exchange Rate Distribution

In this section, we model the exchange rate variability via a set of triangular distributions, to investigate the effects of skewness on our proposed contracts. Three cases involving the triangular distribution are considered for depicting different scenarios as follows: i) Left-skewed, with parameters: $a = 3.5, c = 5.5$, and $b = 6$; ii) Symmetric, with parameters: $a = 4, c = 5$, and $b = 6$; and iii) Right-skewed, with parameters: $a = 4, c = 4.5$, and $b = 6.5$. To focus our attention on the skewness effect, we set the mean of the exchange rate distribution for the three cases of the triangular distributions and other problem parameters the same as presented in the case of the uniform distribution assumption. Tables 6 and 7 show the results of our numerical analyses with different currency exchange rate distributions under asymmetric contract parameters for type 1b and 2 contracts.

Table 6: Effect of Skewness in Exchange Rate Distribution on the Expected Profits of the Supply Chain Members for Contract Type 1b

Parameter		Left Skewed		Symmetric		Right Skewed	
α	β	Π_b^{1b}	Π_s^{1b}	Π_b^{1b}	Π_s^{1b}	Π_b^{1b}	Π_s^{1b}
0.00	0.00	77.89	639.80	77.90	639.80	77.84	639.60
	0.05	73.48	648.33	73.43	649.12	72.41	650.74
	0.10	70.21	653.70	70.92	653.77	68.66	657.60
	0.15	68.06	657.41	69.91	655.34	67.07	660.09
	0.20	66.83	658.61	69.76	655.56	66.83	660.45
0.05	0.00	83.00	623.30	82.06	626.94	81.91	626.14
	0.05	78.50	633.05	77.53	637.36	76.40	638.71
	0.10	75.17	639.45	74.98	642.61	72.59	645.96
	0.15	72.97	643.13	73.95	644.14	70.98	648.48
	0.20	71.72	644.36	73.79	644.38	70.73	648.67
0.10	0.00	85.93	613.52	84.01	619.91	84.41	617.15
	0.05	81.39	623.55	79.44	630.64	78.85	630.02
	0.10	78.02	630.49	76.88	636.66	75.01	638.22
	0.15	75.81	634.20	75.84	638.27	73.38	640.91
	0.20	74.53	636.38	75.68	638.52	73.13	641.33
0.15	0.00	87.00	609.65	84.66	617.89	85.80	612.31
	0.05	82.44	620.05	80.08	628.15	80.22	624.70
	0.10	79.06	626.72	77.51	634.42	76.35	633.69
	0.15	76.83	630.91	76.47	636.45	74.72	636.67
	0.20	75.56	632.74	76.31	636.71	74.46	637.10
0.20	0.00	87.14	609.17	84.75	617.36	86.45	609.68
	0.05	82.58	619.40	80.17	628.02	80.85	622.79
	0.10	79.20	626.09	77.60	634.10	76.98	631.29
	0.15	76.97	630.49	76.56	636.14	75.34	634.30
	0.20	75.69	632.12	76.40	636.40	75.09	634.74

It can be deduced that the results shown in Table 6 provide support for Corollary 1 and Proposition 3 (which pertain to symmetric exchange rate distributions), regarding the expected profits of the supply chain members, except for symmetrical contract parameters under skewed exchange rate distribution scenarios. Nevertheless, regardless of skewness, the buyer would prefer a higher α and a lower β , whereas the supplier would prefer a lower α and a higher β , when contract type 1b is in effect.

Table 7: Effect of Skewness in Exchange Rate Distribution on the Expected Profits of the Supply Chain Members for Contract Type 2

Parameter		Left Skewed		Symmetric		Right Skewed	
ϕ_u	ϕ_d	Π_{b2}	Π_{s2}	Π_{b2}	Π_{s2}	Π_{b2}	Π_{s2}
0.00	0.00	77.89	639.80	77.90	639.80	77.84	639.60
	0.25	74.93	645.45	75.84	644.01	75.04	645.36
	0.50	72.00	649.59	73.80	648.28	72.27	650.45
	0.75	69.10	654.68	71.77	651.79	69.54	655.26
	1.00	66.24	659.51	69.76	655.56	66.83	660.45
0.25	0.00	80.18	631.92	79.60	633.87	80.03	631.42
	0.25	77.19	638.20	77.53	639.00	77.21	638.20
	0.50	74.23	643.41	75.47	643.20	74.42	643.93
	0.75	71.31	647.93	73.43	647.05	71.66	648.58
	1.00	68.42	652.99	71.40	650.74	68.93	654.20
0.50	0.00	82.48	624.81	81.31	628.61	82.24	624.62
	0.25	79.47	630.35	79.22	633.28	79.40	630.44
	0.50	76.49	636.60	77.15	638.20	76.58	636.80
	0.75	73.54	641.37	75.10	642.39	73.80	642.30
	1.00	70.63	646.26	73.06	646.43	71.04	647.12
0.75	0.00	84.80	616.91	83.02	623.05	84.46	616.44
	0.25	81.76	623.27	80.93	628.03	81.60	623.08
	0.50	78.76	628.98	78.84	632.49	78.76	629.07
	0.75	75.79	634.60	76.78	637.40	75.96	635.00
	1.00	72.85	639.54	74.72	641.58	73.18	640.67
1.00	0.00	87.14	609.17	84.75	617.36	86.71	608.63
	0.25	84.08	615.21	82.64	622.47	83.82	614.94
	0.50	81.05	621.54	80.55	627.25	80.96	621.54
	0.75	78.05	627.61	78.47	631.70	78.13	628.09
	1.00	75.09	633.20	76.40	636.40	75.33	633.60

Table 7, showing the results of the expected profits of the buyer and the supplier under contract type 2, provide support for Corollary 2, as well as Propositions 9 and 10, with or without skewness in the exchange rate distribution.

Next, we examine some intermediate contract cases with a deterministic exchange rate with no risk sharing between the buyer and the supplier, i.e. no exchange rate contract. In this case, the optimal order quantity of the buyer is 32 units, resulting in an expected profit of 86, and an expected profit of 640 for the supplier, in their respective currencies. These outcomes, in effect, stem from the solution to the classical newsvendor problem. With the introduction of uncertainty in the exchange rate, the buyer's order quantity is less than 32 units, leading to a decline in its own expected profit as well as that of the supplier. As discussed earlier, for contract type 1b, when α is large and β is close to 1 (similar to the case when ϕ_u and ϕ_d are both equal to 1 for contract type 2), the buyer's optimal order quantity,

q^* , becomes smaller than the newsvendor quantity under no exchange rate uncertainty. Nevertheless, when both supply chain parties enter into either a bounded or a proportional exchange rate contract, their expected profits improve compared to the case when there is exchange rate uncertainty, but no contractual agreement. For example, Table 6 shows that under contract type 1b, with $\alpha = \beta = 0.05$, the expected profits of the buyer under various exchange rate distribution skewness scenarios are 78.50 (left skewed), 77.53 (symmetric), 76.40 (right skewed). The corresponding expected supplier profits, in its own currency are 633.05, 637.36 and 638.71, respectively. Thus, the expected profits of both the parties improve, approaching the case of no uncertainty and no risk sharing, when contract type 1b is in effect. Similarly, from Table 7, under contract type 2 with $\phi_u = \phi_d = 0.5$, the expected profits of the buyer under the different skewness environments are 76.49 (left skewed), 77.15 (symmetric), 76.58 (right skewed). The corresponding expected profits for the supplier are 636.60, 638.20 and 636.80, respectively. For both parties, their expected profits are higher compared to the case of no contract under exchange rate uncertainty (i.e. $\phi_u = \phi_d = 1$), under different exchange rate distribution skewness conditions.

5 Conclusion and Managerial Implications

In this paper, we study a decentralized international supply chain under a newsvendor framework, in which a supplier produces and delivers a single product to a buyer, facing both stochastic demand and uncertainty in the converted wholesale price, resulting from random exchange rate fluctuations. We propose two exchange rate flexibility contract types, that can mitigate the effects of exchange rate fluctuations on the expected profits for both the supplier and the buyer. Also, we examine the effects of exchange rate distribution skewness on the expected profit of both the parties and investigate the effects of a local backup supplier together with a foreign supplier. We observe that with the addition of a local backup source, the buyer's expected profit tends to increase at the expense of the foreign supplier. Our results appear to be consistent with those obtained by related earlier research (Liu and Nagurney, 2011). For our model with a single foreign supplier, we find that the buyer tends to decrease its optimal order quantity when faced with wholesale price uncertainty, and both contract type 1b and contract type 2 improve the expected profits of both the supply chain parties, when the payment is made in the supplier's currency. Under our suggested contract type 2, both supply chain members have the potential for benefiting from each other in this setting, compared to not sharing the exchange rate risk (i.e. without such a contract). Both the contract types designed in this study appear to be desirable for both parties, if payment is made in the supplier's currency. In contrast, when payment is made in the buyer's currency, neither parties stands to gain from the proposed contract structures. Thus, other types of contracts need to be designed when payment is made in the buyer's currency. These exchange rate flexibility contracts eliminate the need for re-negotiation of contracts, should the exchange rate fall outside the agreed upon range or bound. Thus, such contracts provide formal frameworks for dealing with the risk resulting from exchange rate fluctuations in global supply chains.

Global supply chains tend to be decentralized by nature and are often difficult to manage for firms engaging in global outsourcing or procurement. Currency exchange rate fluctuation is one of the key characteristics of such supply chains and cannot be avoided. This study makes significant

contributions to the existing literature by extending the current body of knowledge on supply chain contracts to the realm of overall supply chain risk mitigation. It is important to explicitly state how exchange rate fluctuations are to be treated during the settlement of an international supply contract. The insights gained from this study can guide the decisions of practitioners, in terms of what elements to emphasize, vis-a-vis those that need adequate attention when entering into trans-national supply contracts. In organizations, functional departments such as finance and accounting, tend to make decisions based mostly on costs without careful consideration of the affected operational decisions. In reality, all these functional areas are interrelated and must work together to achieve supply chain success and gain competitive advantage in the global economy. Implementing these kinds of exchange rate flexibility contracts can help firms mitigate significant supply chain risks arising from exchange rate uncertainty, as we have seen in the case of P&G and Svenska (Petter and Leyland, 2012). Also, cost savings may be achieved because both parties can avoid the extra costs of re-negotiation or litigation that may arise from disputes concerning the exchange rate to be applied in the payment of eventual transaction settlements.

Our study assumes that the buyer faces a classical newsvendor problem. Thus, it appears that one future extension of this study would be to consider the risk attitudes of both the buyer and supplier in the supply contract. In our current setting, payment in the buyer's currency is not suitable for the contracts proposed here, because there are no incentives for both the supply chain parties to participate in such contracts. Thus, future research can explore another type of contract with a risk-sharing mechanism where $\delta\%$ of the price is settled in one currency and the remainder $(1 - \delta)\%$ in the other currency. Furthermore, a scenario where the payment is made using an acceptable standard currency, different from both the buyer's and the supplier's currencies, can be investigated. It is hoped that our suggestions and results will be helpful for future researchers in exploring these and other important issues in this area of research.

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Appendix A

A.1 Contract Type 1a

A.1.1 Buyer's Analysis

The buyer's expected profit in its own currency can be mathematically expressed as

$$\Pi_b(q) = E_D[R_b(q, D)] - E_X[C_b(q, X)].$$

Here, the cost function of the buyer is

$$C_b(q, X) = \begin{cases} w_b \frac{\mu(1+\alpha)}{X} q, & \text{if } X > \mu(1 + \alpha), \\ w_b q, & \text{if } \mu(1 - \beta) \leq X \leq \mu(1 + \alpha), \\ w_b \frac{\mu(1-\beta)}{X} q, & \text{if } X < \mu(1 - \beta). \end{cases} \quad (\text{A.1})$$

From equations (1) and (A.1), the expected profit of the buyer can be rewritten as

$$\begin{aligned} \Pi_b(q) = & \int_0^q [p\xi + v(q - \xi)]f(\xi)d\xi + \int_q^\infty [pq - s(\xi - q)]f(\xi)d\xi - w_b q \left\{ \int_0^{\mu(1-\beta)} \frac{\mu(1-\beta)}{x} g(x)dx \right. \\ & \left. + \int_{\mu(1-\beta)}^{\mu(1+\alpha)} g(x)dx + \int_{\mu(1+\alpha)}^\infty \frac{\mu(1+\alpha)}{x} g(x)dx \right\}. \end{aligned}$$

The first order optimality condition of the expected profit with respect to q is

$$\frac{d[\Pi_b(q)]}{dq} = \frac{dE_D[R_b(q, D)]}{dq} - \frac{dE_X[C_b(q, X)]}{dq}.$$

Applying Leibniz rule to the buyer's revenue function

$$\begin{aligned} \frac{dE_D[R_b(q, D)]}{dq} &= (p - v)f(q) + sqf(q) + (p + s - v) \int_q^\infty f(\xi)d\xi - (p + s - v)qf(q) + v \\ &= (p + s - v) \int_q^\infty f(\xi)d\xi + v, \\ \frac{dE_X[C_b(q, X)]}{dq} &= w_b \left\{ \int_0^{\mu(1-\beta)} \frac{\mu(1-\beta)}{x} g(x)dx + \int_{\mu(1-\beta)}^{\mu(1+\alpha)} g(x)dx + \int_{\mu(1+\alpha)}^\infty \frac{\mu(1+\alpha)}{x} g(x)dx \right\}. \end{aligned} \quad (\text{A.2})$$

Let

$$x_b^*(\alpha, \beta) = E_X[A\mathbb{1}_{A>1} + \mathbb{1}_{A\leq 1\leq B} + B\mathbb{1}_{B<1}], \quad A = \frac{\mu(1-\beta)}{X}, \quad \text{and} \quad B = \frac{\mu(1+\alpha)}{X}. \quad (\text{A.3})$$

Since

$$\begin{aligned} \int_q^\infty f(\xi)d\xi &= 1 - F(q), \\ \frac{d[\Pi_b(q)]}{dq} &= p + s - (p + s - v)F(q) - w_b x_b^*(\alpha, \beta) = 0 \\ F(q) &= \frac{p - w_b x_b^*(\alpha, \beta) + s}{p - v + s}, \end{aligned} \quad (\text{A.4})$$

In general terms, we obtain the following result depicting the buyer's optimal order quantity:

$$q^* = F^{-1} \left(\frac{p - w_b x_b^*(\alpha, \beta) + s}{p - v + s} \right).$$

In addition, it can easily be shown that the buyer's expected profit is concave in q , since the second derivative is negative. Thus, the second derivative obtained from Equation A.4 is stated below.

$$\frac{d^2[\Pi_b(q)]}{dq^2} = -(p + s - v)f(q) < 0.$$

Proposition 4 Proof: As $x_b^*(\alpha, \beta)$ increases as α increases, w_b increases resulting in decreasing optimal order quantity. Therefore, taking the derivative of Equation (A.3) with respect to α , we have

$$\frac{\partial x_b^*(\alpha, \beta)}{\partial \alpha} = \mu E_X \left[\frac{1}{X} \right] \mathbb{1}_{B < 1} > 0.$$

Similarly, as $x_b^*(\alpha, \beta)$ decreases as β increases, w_b decreases causing optimal order quantity to increase. Therefore, taking the derivative of equation (A.3) with respect to β . we have

$$\frac{\partial x_b^*(\alpha, \beta)}{\partial \beta} = -\mu E_X \left[\frac{1}{X} \right] \mathbb{1}_{A > 1} < 0.$$

A.1.2 Supplier's Analysis

The supplier's expected profit in its own currency is expressed as:

$$\Pi_s = E_X[R_s(q^*, X)] - cq^*.$$

where,

$$R_s(q^*, X) = \begin{cases} w_b \mu(1 + \alpha)q^*, & \text{If } X > \mu(1 + \alpha), \\ w_b X q^*, & \text{If } \mu(1 - \beta) \leq X \leq \mu(1 + \alpha), \\ w_b \mu(1 - \beta)q^*, & \text{If } X < \mu(1 - \beta). \end{cases}$$

This expected profit of the supplier can be rewritten as

$$\Pi_s = w_b q^* \left(\int_0^{\mu(1-\beta)} \mu(1-\beta)g(x)dx + \int_{\mu(1-\beta)}^{\mu(1+\alpha)} xg(x)dx + \int_{\mu(1+\alpha)}^{\infty} \mu(1+\alpha)g(x)dx \right) - cq^*,$$

Let,

$$x_s^*(\alpha, \beta) = E_X[X(A\mathbb{1}_{A > 1} + \mathbb{1}_{A \leq 1 \leq B} + B\mathbb{1}_{B < 1})], \quad A = \frac{\mu(1-\beta)}{X}, \quad \text{and} \quad B = \frac{\mu(1+\alpha)}{X} \quad (\text{A.5})$$

$$\Pi_s = (w_b x_s^*(\alpha, \beta) - c)q^*.$$

Proposition 4 Proof: As $x_s^*(\alpha, \beta)$ increases as α increases, w realized by the supplier will increase. Therefore, taking the first derivative of equation (A.5) with respect to α . we have

$$\frac{\partial x_s^*(\alpha, \beta)}{\partial \alpha} = \mu E_X[1] \mathbb{1}_{B < 1} > 0.$$

Similarly, if $x_s^*(\alpha, \beta)$ decreases as β increases, w realized by the supplier will decrease and the expected supplier's profit decreases. Once again, taking the first derivative of equation (A.5) with respect to β . we have

$$\frac{\partial x_s^*(\alpha, \beta)}{\partial \beta} = -\mu E_X[1] \mathbb{1}_{A > 1} < 0.$$

A.2 Contract Type 1b

A.2.1 Buyer's Analysis

The buyer's expected profit in its own currency is

$$\Pi_b^{1b}(q) = E_D[R_b(q, D)] - E_X[C_b^{1b}(q, X)].$$

Substituting the expected buyer's revenue from equation (1) and the buyer's cost function in equation (2), the expected profit of the buyer can be expressed as

$$\begin{aligned} \Pi_b^{1b}(q) = & \int_0^q [p\xi + v(q - \xi)]f(\xi)d\xi + \int_q^\infty [pq - s(\xi - q)]f(\xi)d\xi - wq \left\{ \int_0^{\mu(1-\beta)} \frac{1}{\mu(1-\beta)} g(x)dx \right. \\ & \left. + \int_{\mu(1-\beta)}^{\mu(1+\alpha)} \frac{1}{x} g(x)dx + \int_{\mu(1+\alpha)}^\infty \frac{1}{\mu(1+\alpha)} g(x)dx \right\}. \end{aligned}$$

Taking the first order condition of the expected profit with respect to q , we obtain

$$\frac{d[\Pi_b^{1b}(q)]}{dq} = \frac{dE_D[R_b(q, D)]}{dq} - \frac{dE_X[C_b^{1b}(q, X)]}{dq}.$$

From equation (A.2), the derivative of buyer's revenue function with respect to q is

$$\frac{dE_D[R_b(q, D)]}{dq} = p + s - (p + s - v)F(q),$$

and

$$\frac{dE_X[C_b^{1b}(q, X)]}{dq} = w \left\{ \int_0^{\mu(1-\beta)} \frac{1}{\mu(1-\beta)} g(x)dx + \int_{\mu(1-\beta)}^{\mu(1+\alpha)} \frac{1}{x} g(x)dx + \int_{\mu(1+\alpha)}^\infty \frac{1}{\mu(1+\alpha)} g(x)dx \right\}.$$

Let

$$x_b^{*1b}(\alpha, \beta) = E_X \left[\frac{1}{X} (A^{-1} \mathbb{1}_{A > 1} + \mathbb{1}_{A \leq 1 \leq B} + B^{-1} \mathbb{1}_{B < 1}) \right], \quad A = \frac{\mu(1-\beta)}{X}, \quad \text{and} \quad B = \frac{\mu(1+\alpha)}{X}. \quad (\text{A.6})$$

This leads to

$$p - wx_b^{*1b}(\alpha, \beta) + s = (p - v + s)F(q).$$

Then, we obtain the buyer's optimal order quantity as

$$q^* = F^{-1} \left(\frac{p - wx_b^{*1b}(\alpha, \beta) + s}{p - v + s} \right).$$

Similarly, as in the case of payment in the buyer's currency, we show that the buyer's expected profit is concave in q .

$$\frac{d^2[\Pi_b^{1b}(q)]}{dq^2} = -(p + s - v)f(q) < 0.$$

Proposition 2: *The optimal order quantity, q^* , of the buyer increases as the contract parameter α increases and decreases as the contract parameter β increases.*

Proof: If $x_b^{*1b}(\alpha, \beta)$ decreases as α increases, optimal order quantity will increase. Therefore, taking the derivative of equation (A.6) with respect to α . we have

$$\frac{\partial x_b^{*1b}(\alpha, \beta)}{\partial \alpha} = -\frac{1}{\mu(1+\alpha)^2} E_X[1] \mathbb{1}_{B < 1} < 0.$$

Similarly, if $x_b^{*1b}(\alpha, \beta)$ increases as β increases, the optimal order quantity will decrease. Therefore, taking derivative of equation (A.6) with respect to β . we show that

$$\frac{\partial x_b^{*1b}(\alpha, \beta)}{\partial \beta} = \frac{1}{\mu(1-\beta)^2} E_X[1] \mathbb{1}_{A > 1} > 0.$$

A.2.2 Supplier's Analysis

The supplier's expected profit in its own currency is given by

$$\Pi_s^{1b} = E_X[R_s^{1b}(q^*, X)] - cq^*,$$

where, the supplier's revenue function in its own currency is defined as

$$R_s^{1b}(q^*, X) = \begin{cases} w \frac{X}{\mu(1+\alpha)} q^*, & \text{if } X > \mu(1+\alpha), \\ wq^*, & \text{if } \mu(1-\beta) \leq X \leq \mu(1+\alpha), \\ w \frac{X}{\mu(1-\beta)} q^*, & \text{if } X < \mu(1-\beta). \end{cases}$$

Thus, the expected profit of the supplier can be expressed as

$$\Pi_s^{1b} = wq^* \left(\int_0^{\mu(1-\beta)} \frac{x}{\mu(1-\beta)} g(x) dx + \int_{\mu(1-\beta)}^{\mu(1+\alpha)} g(x) dx + \int_{\mu(1+\alpha)}^{\infty} \frac{x}{\mu(1+\alpha)} g(x) dx \right) - cq^*.$$

By defining

$$x_s^{*1b}(\alpha, \beta) = E_X[A^{-1} \mathbb{1}_{A > 1} + \mathbb{1}_{A \leq 1 \leq B} + B^{-1} \mathbb{1}_{B < 1}], \quad A = \frac{\mu(1-\beta)}{X}, \quad \text{and} \quad B = \frac{\mu(1+\alpha)}{X}. \quad (\text{A.7})$$

Therefore, the expected profit of the supplier's become

$$\Pi_s^{1b} = (wx_s^{*1b}(\alpha, \beta) - c) q^*.$$

Proposition 3: *The expected profit of the supplier decreases with increasing α and increases with increasing β .*

Proof: As before, if $x_s^{*1b}(\alpha, \beta)$ decreases as α increases, w received by the supplier will decrease. Therefore, taking the derivative of equation (A. 7) with respect to α . we have

$$\frac{\partial x_s^{*1b}(\alpha, \beta)}{\partial \alpha} = -\frac{1}{\mu(1+\alpha)^2} E_X[X] \mathbb{1}_{B<1} < 0.$$

Similarly, if $x_s^{*1b}(\alpha, \beta)$ increases as β increases, w received by the supplier will increase and the overall profit increases. Therefore, taking derivative of equation (A. 7) with respect to β . we have

$$\frac{\partial x_s^{*1b}(\alpha, \beta)}{\partial \beta} = \frac{1}{\mu(1-\beta)^2} E_X[X] \mathbb{1}_{A>1} > 0.$$

A.2.3 Proofs of Propositions 5 and 6

Proposition 5 Proof: If $x_b^{*1b}(\gamma)$ decreases (increases) the buyer's expected wholesale price decreases (increases). Thus, the buyer's optimal order quantity and expected profit increases:

$$x_b^{*1b}(\gamma) = E_X \left[\frac{1}{X} (A^{-1} \mathbb{1}_{A>1} + \mathbb{1}_{A \leq 1 \leq B} + B^{-1} \mathbb{1}_{B<1}) \right], \quad A = \frac{\mu(1-\gamma)}{X}, \quad \text{and} \quad B = \frac{\mu(1+\gamma)}{X}.$$

$$\frac{dx_b^{*1b}(\gamma)}{d\gamma} = \frac{1}{\mu(1-\gamma)^2} E_X[1] \mathbb{1}_{A>1} - \frac{1}{\mu(1+\gamma)^2} E_X[1] \mathbb{1}_{B<1} > 0$$

Condition:

$$\frac{1}{\mu(1-\gamma)^2} E_X[1] \mathbb{1}_{A>1} > \frac{1}{\mu(1+\gamma)^2} E_X[1] \mathbb{1}_{B<1}, \quad \text{so that} \quad \frac{dx_b^{*1b}(\gamma)}{d\gamma} > 0. \quad (\text{A.8})$$

Once the condition in A.8 is satisfied, the wholesale price increases, then the buyer orders a smaller quantity, thereby reducing its expected profit. Also, the expected profit of the supplier declines as a result of a reduced order quantity, when the realized expected wholesale price remains unchanged.

Proposition 6 Proof: To show that $x_s^{*1b}(\gamma)$ remains unchanged.

$$x_s^{*1b}(\gamma) = E_X[A^{-1} \mathbb{1}_{A>1} + \mathbb{1}_{A \leq 1 \leq B} + B^{-1} \mathbb{1}_{B<1}], \quad A = \frac{\mu(1-\gamma)}{X}, \quad \text{and} \quad B = \frac{\mu(1+\gamma)}{X}.$$

$$\frac{dx_s^{*1b}(\gamma)}{d\gamma} = \frac{1}{\mu(1-\gamma)^2} E_X[X] \mathbb{1}_{A>1} - \frac{1}{\mu(1+\gamma)^2} E_X[X] \mathbb{1}_{B<1}. \quad (\text{A.9})$$

From $\frac{dx_s^{*1b}(\gamma)}{d\gamma}$, $\frac{1}{\mu(1-\gamma)^2} E_X[X] \mathbb{1}_{A>1} = \frac{1}{\mu(1+\gamma)^2} E_X[X] \mathbb{1}_{B<1}$ must be true for the expected wholesale price realization of the supplier to remain the same. Therefore, the expected profit of the supplier only decreases as a result of a reduced order quantity from the buyer.

To show that contract type 1a results provide support for Proposition 4, we can show that $x_b^*(\gamma)$

decreases with γ as shown below:

Proposition 4 Proof: As stated earlier, if $x_b^*(\gamma)$ decreases (increases) the buyer's expected wholesale price decreases (increases). Thus, we can easily show that $x_b^*(\gamma)$ decreases as γ increases.

$$x_b^*(\gamma) = E_X[A\mathbb{1}_{A>1} + \mathbb{1}_{A\leq 1\leq B} + B\mathbb{1}_{B<1}], \quad A = \frac{\mu(1-\gamma)}{X}, \quad \text{and} \quad B = \frac{\mu(1+\gamma)}{X}$$

$$\frac{dx_b^*(\gamma)}{d\gamma} = \mu \left[E_X \left[\frac{1}{X} \right] \mathbb{1}_{B<1} - E_X \left[\frac{1}{X} \right] \mathbb{1}_{A>1} \right] < 0.$$

Condition:

$$E_X \left[\frac{1}{X} \right] \mathbb{1}_{A>1} > E_X \left[\frac{1}{X} \right] \mathbb{1}_{B<1}.$$

Consequently, we show that the expected profit of the supplier increases as a result of the increase in the order quantity from the buyer, since the expected wholesale price remains unchanged.

Proposition 4 Proof: If $x_s^*(\gamma)$ decreases (increases) the supplier's expected wholesale price decreases (increases), i.e.

$$x_s^*(\gamma) = E_X[X(A\mathbb{1}_{A>1} + \mathbb{1}_{A\leq 1\leq B} + B\mathbb{1}_{B<1})], \quad A = \frac{\mu(1-\gamma)}{X}, \quad \text{and} \quad B = \frac{\mu(1+\gamma)}{X}.$$

$$\frac{dx_s^*(\gamma)}{d\gamma} = \mu [E_X[1]\mathbb{1}_{B<1} - E_X[1]\mathbb{1}_{A>1}] = 0.$$

Since $\frac{dx_s^*(\gamma)}{d\gamma} = 0$ at the stationary point, it is easy to see that the expected wholesale price to remain unchanged.

Appendix B

B.1 Proportional Exchange Rate Contract

B.1.1 Buyer's Analysis

The buyer's expected profit in its currency is given by

$$\Pi_{b2}(q) = E_D[R_b(q, D)] - E_X[C_{b2}(q, X)].$$

From equations (1) and (10), the expected profit above becomes

$$\begin{aligned} \Pi_{b2}(q) = & \int_0^q [p\xi + v(q-\xi)]f(\xi)d\xi + \int_q^\infty [pq - s(\xi-q)]f(\xi)d\xi - wq \left\{ \int_0^\mu \left[\frac{\phi_d}{x} + \frac{(1-\phi_d)}{\mu} \right] g(x)dx \right. \\ & \left. + \int_\mu^\infty \left[\frac{\phi_u}{x} + \frac{(1-\phi_u)}{\mu} \right] g(x)dx \right\}. \end{aligned}$$

Let

$$x_{b2}^*(\phi_d, \phi_u) = E_X \left[\left\{ \frac{\phi_d H}{\mu} + \frac{(1-\phi_d)}{\mu} \right\} \mathbb{1}_{H>1} + \left\{ \frac{\phi_u H}{\mu} + \frac{(1-\phi_u)}{\mu} \right\} \mathbb{1}_{H\leq 1} \right] \text{ and } H = \frac{\mu}{X}. \quad (\text{B.1})$$

$$\Pi_{b2}(q) = \int_0^q [p\xi + v(q-\xi)]f(\xi)d\xi + \int_q^\infty [pq - s(\xi-q)]f(\xi)d\xi - wx_{b2}^*(\phi_d, \phi_u)$$

The first order optimality condition of the expected profit with respect to q is

$$\frac{d[\Pi_{b2}(q)]}{dq} = \frac{dE_D[R_b(q, D)]}{dq} - \frac{dE_X[C_{b2}(q, X)]}{dq}.$$

$$\frac{d\Pi_{b2}(q)}{dq} = p + s - (p - v + s)F(q) - wx_{b2}^*(\phi_d, \phi_u) = 0.$$

Generally, we obtain the buyer's optimal order quantity as

$$q^* = F^{-1} \left(\frac{p - wx_{b2}^*(\phi_d, \phi_u) + s}{p - v + s} \right).$$

Proposition 8 Proof: Taking the first derivatives of equation (B.1) above with respect to ϕ_u , and ϕ_d , we obtain

$$\frac{\partial x_{b2}^*(\phi_d, \phi_u)}{\partial \phi_u} = \frac{1}{\mu} E_X [H - 1] \mathbb{1}_{H\leq 1} < 0,$$

and

$$\frac{\partial x_{b2}^*(\phi_d, \phi_u)}{\partial \phi_d} = \frac{1}{\mu} E_X [H - 1] \mathbb{1}_{H>1} > 0.$$

B.1.2 Supplier's Analysis

The supplier's expected profit in its own currency is given by

$$\begin{aligned} \Pi_{s2} &= E_X [R_{s2}(q^*, X)] - cq^* \\ &= wq^* \left[\int_0^\mu \left\{ \phi_d + \frac{X(1-\phi_d)}{\mu} \right\} g(x) dx + \int_\mu^\infty \left\{ \phi_u + \frac{X(1-\phi_u)}{\mu} \right\} g(x) dx \right] - cq^*. \end{aligned}$$

Let

$$x_{s2}^*(\phi_d, \phi_u) = E_X \left[\left\{ \phi_d + \frac{(1-\phi_d)}{H} \right\} \mathbb{1}_{H>1} + \left\{ \phi_u + \frac{(1-\phi_u)}{H} \right\} \mathbb{1}_{H\leq 1} \right] \text{ and } H = \frac{\mu}{X}. \quad (\text{B.2})$$

Therefore,

$$\Pi_{s2} = (wx_{s2}^*(\phi_d, \phi_u) - c)q^*.$$

Proposition 9 Proof: Taking the first partial derivatives of equation (B.2) above with respect to ϕ_u and ϕ_d , we show that

$$\frac{\partial x_{s2}^*(\phi_d, \phi_u)}{\partial \phi_u} = E_X \left[1 - \frac{1}{H} \right] \mathbb{1}_{H\leq 1} < 0,$$

and

$$\frac{\partial x_{s2}^*(\phi_d, \phi_u)}{\partial \phi_d} = E_X \left[1 - \frac{1}{H} \right] \mathbb{1}_{H>1} > 0.$$

Proposition 10 Proof: If $x_{b2}^*(\phi)$ increases (decreases), the buyer's expected wholesale price increases

(decreases). Thus, the buyer's optimal order quantity and expected profit increases, i.e.

$$x_{b2}^*(\phi) = E_X \left[\left\{ \frac{\phi H}{\mu} + \frac{(1-\phi)}{\mu} \right\} \mathbb{1}_{H>1} + \left\{ \frac{\phi H}{\mu} + \frac{(1-\phi)}{\mu} \right\} \mathbb{1}_{H\leq 1} \right] \quad \text{and} \quad H = \frac{\mu}{X}.$$

$$\frac{dx_{b2}^*(\phi)}{d\phi} = E_X \left[\frac{1}{X} \right] \mathbb{1}_{H\leq 1} + E_X \left[\frac{1}{X} \right] \mathbb{1}_{H>1} - 2 > 0.$$

It can be shown that the decline in the expected profit of the supplier results from a decreasing order quantity, i.e.

$$x_{s2}^*(\phi) = E_X \left[\left\{ \phi + \frac{(1-\phi)}{H} \right\} \mathbb{1}_{H>1} + \left\{ \phi + \frac{(1-\phi)}{H} \right\} \mathbb{1}_{H\leq 1} \right] \quad \text{and} \quad H = \frac{\mu}{X}.$$

$$\frac{dx_{s2}^*(\phi)}{d\phi} = \frac{2}{\mu} - E_X[X] \mathbb{1}_{H\leq 1} - E_X[X] \mathbb{1}_{H>1}.$$

Therefore, once $\frac{2}{E_X[X] \mathbb{1}_{H\leq 1} + E_X[X] \mathbb{1}_{H>1}} = \mu$, the expected wholesale price realization remains constant for the supplier.