Statistical Modelling of Excess Mortality of Medically Impaired Insured Lives

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TO MY PARENTS

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Declaration

Abstract

A complete framework for the statistical modelling of excess mortality within the actuarial context is described, based on the theory of generalised linear models. In this context, the measure of excess mortality considered is the standardised mortality ratio. The modelling framework allows model building using several explanatory factors. The statistical significance of explanatory factors can be tested and, furthermore, the effect of covariate interactions can be assessed. Residual analysis is considered as a means of model checking and allows systematic and isolated departures from the model to be identified. A convenient and practically expedient method of calculating model based confidence intervals for the mortality ratio is developed. Three particular model structures are considered (the multiplicative, additive and power structures) and a unified approach to modelling excess mortality is presented.

The modelling approach has appealing connections with the traditional actuarial approach to the measurement of excess mortality and to the numerical rating system used almost universally in life insurance underwriting. These connections are explored and it is proposed that the modelling approach offers a scientifically sound approach to life insurance underwriting.

The modelling approach is used to analyse an extensive data set, namely the Prudential Impaired Lives data set, and the results are compared with previous results based on that data set and also with underwriting manuals in current use.

PART I THE DATA SET and THEORY

Chapter 1 Introduction

1.1 Measuring Excess Mortality — A Summary

Excess mortality is said to occur when a group of individuals, all exhibiting a particular risk factor, experience mortality rates over and above standard mortality rates. Standard mortality rates are those pertaining to a group of individuals not exhibiting the risk factor, but who are in all other respects the same. Examples of possible types of risk factors are medical impairments, occupational hazards, hazardous pursuits and even geographical location of residence and ethnic origin. Often, combinations of these risk factors are of interest, for example, the effect of a particular disease within different ethnic communities, or the effect of combinations of related medical impairments.

The classical actuarial approach to the measurement of excess mortality is the comparison of actual with expected deaths for a group of individuals exhibiting the particular risk under consideration. Using exposed-to-risk theory, expected deaths are calculated based on a set of suitable standard mortality rates controlling as closely as possible for factors such as sex and age. If a group of individuals, homogeneous with regard to the risk under consideration, is followed over time, and the actual deaths observed and the expected deaths calculated, then the ratio

$$\frac{\text{Actual Deaths}}{\text{Expected Deaths}}$$

represents a measure of excess mortality and is called the *mortality ratio* (MR). Mortality ratios have been used by actuaries to measure excess mortality since the 18th Century, and the method is sometimes known as the "method of expected number of deaths" (see Keiding (1987)).

Although less frequently used, other methods of measuring excess mortality exist which may be more suitable in certain circumstances. These have been described thoroughly by Haberman

(1982 and 1988) and are not considered here any further.

The use of mortality ratios became the chosen method of reporting results from investigations into the mortality experience of impaired insured lives. The earliest of such studies was the 1903 Specialized Mortality Investigation which was conducted in North America. Many studies have been conducted in North America subsequently, the most recent being the 1983 Medical Impairment Study.

The first large scale British study was initiated in 1947 by the Prudential Assurance Company (see Clarke (1961)) and is currently still operative, over 45 years later. Results of interim analyses have been reported by Preston and Clarke (1966), Clarke (1979), and Leighton (1987). In all of these studies, mortality ratios were used to measure excess mortality.

A combined offices investigation in the UK was set in motion in 1982 by the Continuous Mortality Investigation Bureau. As yet, only preliminary results have been reported (*C.M.I.R.* 11 (1991)); it will be some years before sufficient data have accumulated to allow a comprehensive analysis.

Impaired lives investigations are carried out in order to measure the mortality experience within different classes of impairment with a view to developing sound premium rating structures, and also to monitor the experience over time so that any changes in the experience (adverse or favourable) may be incorporated in the rating structure as soon as possible.

The favoured method of assessing risks for life assurance is through the numerical rating system, introduced by Rogers and Hunter in 1919. According to the system, an estimated mortality ratio associated with a set of risk factors is derived by summing a series of debits and credits which relate to the levels of the risk factors. Using this estimated mortality ratio, a suitable premium for the risks under consideration can be calculated. Using the numerical rating system implicitly assumes an underlying additive linear model for the mortality ratio, although it is unclear how the credits and debits should be estimated from the data of

impaired lives investigations.

Modelling excess mortality of impaired lives was first attempted by Renshaw (1988) who used a variant of Cox's proportional hazards model (Cox (1972)) embedded within the generalised linear model framework. The result is that rating factors have a multiplicative effect on the mortality ratio. The modelling approach has appealing connections to the traditional approach but also has the advantages that the statistical significance of rating factors can be assessed and the interdependence of rating factors can be explored explicitly. Furthermore, complex models can be fitted involving several rating factors together, and their interactions.

1.2 The Thesis – Aims and Outline

The aims of this thesis are twofold. The first aim is to review the traditional approach to measuring excess mortality and the numerical rating system, and also to describe and develop further the modelling approach introduced by Renshaw. The second aim is to report some of the results of a thorough reanalysis of the Prudential impaired lives data set using the modelling approach.

During the time taken to conduct the research on which this thesis is based, interim progress reports were prepared periodically which briefly outlined any theoretical advances and gave full details of the analysis of the Prudential data (see England (1990), (1991) and (1992)). A more comprehensive treatment of the theoretical aspects is included in this thesis, although only a subset of the results is shown.

An additional interim progress report (see England and Verrall (1992)) was also prepared in which the Bayesian theory of dynamic generalised linear models (West *et al* (1985)) was applied to the excess mortality of diabetics. The theoretical aspects of that report are not included in this thesis, although a brief description of the results can be found in Chapter 21.

For presentation purposes, the material included in this thesis has been divided into two parts. Part I covers the theoretical aspects (Chapters 1 to 13), and the empirical results and related matters are covered in Part II (Chapters 14 to 25).

A description of the Prudential data set is provided in Chapter 2, and the classical actuarial approach to the measurement of excess mortality and the numerical rating system are outlined in Chapter 3. A brief introduction to modelling excess mortality is provided in Chapter 4 followed by a description of generalised linear models in Chapter 5. In Chapter 6, which is largely mathematical, modelling excess mortality is formally described, and the methodology is illustrated in Chapter 7. Theoretical developments are considered in the remainder of Part I. In particular, analysis of residuals is advocated as a tool for model checking and is described in Chapter 8. *Model based* confidence intervals for the mortality ratio are derived in Chapter 9; these have similarities to confidence intervals calculated using traditional methods. In Chapter 10, the multiplicative model is reformulated within the generalised linear model framework which paves the way for the introduction of two alternative models, the additive and power models, which are outlined in Chapters 11 and 12. The additive model may be perceived as providing a theoretical statistical basis for the numerical rating system, and the power model may be used to assess the optimal model and formally discriminate between the additive and multiplicative models. Chapter 13 briefly considers some of the difficulties of modelling excess mortality in practice.

In Part II of this thesis, results from a thorough analysis of eight of the impairments from the Prudential impaired lives data set are presented. Preliminary considerations regarding the format of the results and the reasons for including the eight impairments are presented in Chapter 14. In Chapter 15, a brief description of previous studies based on the Prudential impaired lives data set is given, together with reasons for differences in the results of the various studies. A discussion of the basis used for the calculation of expected deaths appears in Chapter 16. Results from the analysis of the eight impairments considered are given in Chapters 17 to 24. The final chapter, Chapter 25, is devoted to the conclusions.

It is hoped that the material presented will be of value to:

(i) reinsurance companies preparing and updating underwriting manuals

(ii) insurance companies wishing to analyse their impaired lives experiences

(iii) those individuals in the medical profession interested in the excess mortality of medically impaired lives

(iv) others in the actuarial profession.

Chapter 2 The Data Set

2.1 Introduction

In 1947, the Prudential Assurance Company decided to institute an enquiry into the mortality of medically impaired lives (see Clarke (1961) and Preston and Clarke (1966)). The venture was designed to be both medical and actuarial. The data for the investigation were drawn from holders of life assurance policies effected since July 1947 in the ordinary branch of the Prudential Assurance Company. Policies were included if the life assured exhibited one of a long list of impairments identified at proposal. Lives exhibiting two or more major impairments were excluded from the investigation, where an impairment is regarded as *major* if it would warrant a surcharge in its own right. It was not deemed practicable to extend the scope of the investigation so as to include every impairment encountered in the course of underwriting. For impairments which occur comparatively infrequently, sufficient data would not have accumulated to provide useful results. Accordingly, the rarer conditions have, in general, been excluded. However, at the outset, it was not possible to foretell the quantity of data which would be forthcoming and certain groups were included where experience has shown that the data have proved insufficient.

2.2 The Coding Scheme and Information Recorded

A detailed coding scheme for impairments included in the investigation was devised by the company's Principal Medical Officer at the time, T. W. Preston, M.D. The impairments considered were divided into nine broad categories, as follows:

1) Circulatory Impairments
2) Stomach and Intestines
3) Nervous Disorders, Head and Ear Impairments
4) Tuberculosis

5) Endocrine Group
6) Underweight and Overweight
7) Respiratory Group
8) Urinary Group
9) Tumours and Miscellaneous

Each major impairment group was further subdivided into its constituent impairments, giving approximately 50 different medical conditions. Full documentation of the classification by medical status is given in Appendix 1. Since 1947, some impairments have been dropped from the study which were originally included and some impairments have been added which were not. A few impairments have had their associated codings changed. A revision of the coding scheme was introduced on the 1st January 1980. The study is, in fact, ongoing, with data extending to the present day. However, this thesis considers only those policies effected between July 1947 and December 31st 1987. By the end of 1987, data were available concerning over 650,000 policies effected on impaired lives (where the impairment was present at the outset).

In the infancy of the investigation, data were stored on punched cards. For each policy included in the investigation, a separate card was punched which contained all the relevant information for that particular policy, viz:

1) Policy Number
2) Medical Bureau Code (plus subsidiary code)
3) Date of Entry (year and month)
4) Age at Entry (next birthday)
5) Date of Exit (year and month)
6) Age at Exit (next birthday)
7) Curtate Duration at Exit
8) Mode of Exit (still in force, withdrawal or death)
9) Cause of Death
10) Joint Life Marker
11) Sex

Rapid advances in computer technology since 1947 allow the data to be stored nowadays on magnetic tape, or even the internal memory of a desk-top computer. The complete data set can be accommodated in approximately 24 Megabytes of computer memory space.

Those involved in planning the study showed considerable foresight and adopted a detailed classification of impairments which was criticised in its day for being too detailed. To this criticism the powerful riposte was made that "once the cards have been punched and the data tabulated, groups can always be combined but they can never be further subdivided". It is only now, with sufficient data and the use of statistical software packages, that full advantage of the detailed classification can be made.

The Medical Bureau code consists of three digits for each impairment. The first digit indicates the generic nature of the impairment (e.g. 1 for circulatory impairments). Details of the particular condition are indicated by the second and third digits. In addition to the three digit code, a subsidiary code of a single letter was adopted to indicate such subsidiary details as family history or duration since the observation of symptoms. In previous studies based on the Prudential Impaired Lives data set, no attempt has been made to analyse data according to these subsidiary attributes. The subsidiary codes were considered in the analysis covered by this thesis; thus results of an analysis including the subsidiary codes appear for the first time.

2.3 Cause of Death

The cause of death was broken down and coded into 17 different causes, as follows:

1) Tuberculosis
2) Cancer of lung, bronchus, trachea
3) Other cancers
4) Leukaemia
5) Diabetes
6) Vascular lesions, cerebral haemorrhage, embolism
7) Coronary disease, angina

8) Hypertension, arteriosclerosis, other circulatory or heart disease
9) Influenza (merged with code 17 on account of small numbers)
10) Pneumonia
11) Bronchitis
12) Peptic ulcer
13) Nephritis
14) Motor accident
15) Other accident
16) Suicide
17) Other causes (unspecified)

An interesting feature of an investigation into the mortality of impaired lives is the discovery of which causes of death are particularly associated with specific impairments identified at entry. Such an analysis is of considerable interest from a medical aspect, although not of direct concern from the point of view of underwriting. A comprehensive analysis by cause of death was not the prime purpose of this thesis, hence its treatment has been limited.

2.4 Principal Exclusions from the Data Set

The investigation was designed to measure the mortality experienced within specified classes of impairment with a view to developing bases for assessing future ratings. Its purpose was not to test the underwriting decisions of the past. Consequently, the terms of acceptance were not taken into account, so that the experience contains both lives who were surcharged and those who were not.

Other information which would be of interest, but which is not available, concerns:

- Duration since onset of impairment (occasionally included within subsidiary codes)
- Sum Assured
- Type of Policy
- Experience of lives declined for insurance.

The exclusion of declined lives, although inevitable, is some cause for regret, since an investigation into the experience of lives declined for insurance could yield useful information. It may well be that life offices are declining certain classes of lives who could in fact be insurable, if suitably surcharged.

Chapter 3 The Classical Actuarial Approach

3.1 Measurement of Excess Mortality and History

As mentioned briefly in the introduction, the traditional actuarial method of measuring excess mortality is the comparison of actual with expected deaths for a group of individuals exhibiting the particular risk factor under consideration. Following the notation of Haberman (1982) and (1988), let d_t, q_t and E_t be the observed number of deaths, the mortality rate and the initial exposed-to-risk for the group under consideration for the interval of follow-up between durations t and $t+1$ (say, measured in years). Let q_t' be the standard mortality rate and define d_t', the expected number of deaths, as $d_t' = E_t\, q_t'$.

The Interval Mortality Ratio (*IMR*) for the interval t to $t+1$, denoted by k_t, is given by:

$$k_t = \frac{d_t}{d_t'} = \frac{q_t}{q_t'}$$

Clearly this is a comparison of actual with expected deaths and is the same, in principle, as the Standardised Mortality Ratio used widely by demographers, medical statisticians and the like. Notice also that the mortality ratio may be expressed equivalently as a ratio of mortality rates.

If k_t is greater than one, the mortality rates in the study group are higher than the standard rates and the group is said to exhibit *excess* mortality. If k_t is less than one, the mortality rates in the study group are lower than the standard rates. Often, the ratio is expressed as a percentage, where 100% represents standard mortality and is the benchmark against which levels of mortality in the study group are judged. Frequently mortality that is greater than standard is expressed as *extra* mortality, that is, the amount over and above 100%. For example, if in a study of individuals exhibiting a particular medical impairment, 75 individuals were observed to die and 50 would have been expected to die under standard death rates, the mortality ratio is 75/50 or 1.50. Expressed as a percentage, this is 150% which represents

extra mortality of $+50$.

Sometimes, when numbers of deaths (or expected deaths) are small, neighbouring intervals of follow-up may be combined over an n year period (say from duration 0 to n) to give:

$$k = \frac{\sum_{t=0}^{n-1} d_t}{\sum_{t=0}^{n-1} d'_t}$$

Notice this is a comparison of deaths observed in the n year period and deaths expected over the period had standard death rates applied. This ratio has been described by Haberman (1988) as the Annual Exposure Cumulative Mortality Ratio (denoted AE_n) and its properties considered by him. It is also known simply as the A/E ratio (see, for example, Clarke (1979)).

This comparison of actual with expected deaths has a long history which has been investigated by Keiding (1987). The earliest description of the method comes from an English actuary living in the 18th century called William Dale, who strove to highlight the inadequacies of the plans of the Laudable Society of Annuitants which provided annuities for its members and/or their widows. Dale calculated expected deaths based on Halley's Life Table. An extract from Dale (1777) reads:

> "The *real* mortality for seven years past in the Laudable Society, has been compared with the *expected* mortality by Dr. Halley's Table; the particulars are hereto annexed which will shew that but *few more than half* so many have hitherto died in the Society, as Breslaw mortality supposed would die."

Dale's calculations show that the observed mortality ratio was in fact 57%, showing that the mortality experience was considerably lighter than that expected, with serious consequenses for the writing of annuity business.

The method of expected number of deaths continued to be used throughout the 19th century. In particular, it was routinely used by Dr. William Farr, who is better known for conceiving the idea of the English Life Tables and himself producing Numbers 1, 2 and 3.

The use of mortality ratios became the method of choice of reporting results of studies investigating the mortality experience of impaired insured lives. The first large scale study was conducted in North America, based on the experience of various classes of lives between 1870 and 1899, and is known as the Specialized Mortality Investigation (1903). Classes of risks investigated included different countries of origin, different ethnic backgrounds, different occupations and different medical impairments. Mortality ratios were used in reporting the results of this study and inferences were made based on a comparison of actual with expected deaths.

North American actuaries and medical directors have long been in advance of their UK counterparts in investigating the mortality of impaired lives; many large scale investigations have been conducted in the US since the beginning of the century. In the UK, such investigations were seen to be too difficult to attempt. Borrowing from G. K. Chesterton, Steeds (1965) summarised the situation succinctly by saying "It is not the case that an investigation into the mortality of impaired lives has been tried and found wanting; it has been found hard and not tried".

The Prudential Impaired Lives study was the first large scale study to be established in the UK. The designers of the study were R. D. Clarke and T. W. Preston who showed considerable foresight and persisted courageously in the face of strong opposition, as the following extract from Perks (1952) illustrates:

> "Some comment seems to be desirable on the absence from this paper of any reference to the *causes of rating*, i.e. the impairments and other features which provide the basis for treating a life as sub-standard. The plain truth is that the futility from the life office's point of view of attempting to subdivide the data according to cause of rating was long ago recognized. The vast numbers of different causes of rating, the various degrees of severity, the varying periods for which an impairment may have existed or since it apparently cleared up, the varying treatments and the unlimited combinations of impairment that arise, would make an investigation of the combined experience of the offices difficult enough and of doubtful utility, even if the mortality of the first-class standard and of the sub-standard groups were not changing all the time. But

this continual flux, together with the need to subdivide at least by age and duration seems to make an investigation by cause of rating, even on the largest scale, a futile proceeding so far as life assurance underwriting is concerned. There is, however, a vast store of information in the papers of life offices which might well provide valuable information from a medical or sociological point of view. The Institute can and should take an active interest in disinterested research of this kind. It would be a mistake, however, and would seriously reduce the value of any investigation if its form and the analysis of the resulting statistics were misdirected to the out-moded purpose of facilitating life assurance underwriting."

Fortuitously, not everybody shared this pessimistic and myopic standpoint; the Prudential study remains operative and the usefulness of such studies has now been recognized. After much deliberation extending over several decades, a combined offices investigation was launched under the auspices of the Continuous Mortality Investigation Bureau. Although the design of a combined offices investigation was described in 1950 (see Springbett (1950)), the investigation actually started on the 1st January 1982 (*C.M.I.R.* 8 (1986)). Results of the first serious attempt at an analysis of the data were reported in *C.M.I.R.* 11 (1991), based on the combined experience of 20 contributing offices between 1983 and 1986. Mortality ratios have been used extensively to report results based on the Prudential study and on the *C.M.I.B.* study.

Although other measures of excess mortality exist and may be more informative in certain circumstances (see Haberman (1982) and (1988)), the use of mortality ratios is by far the most common.

3.2 Underwriting Practice: The Numerical Rating System

Inspired by the manner in which the results of the Specialized Mortality Investigation (1903) were presented as mortality ratios, Oscar H. Rogers and Arthur Hunter, Medical Director and Actuary respectively of the New York Life Insurance Company, devised a system of risk evaluation based on mortality ratios expressed as a percentage (see Rogers and Hunter (1919)).

This system is known as the *numerical rating system.* The principle of the system assumes that average mortality is represented as 100%, and each factor influencing mortality is expressed numerically in terms of percentage mortality. Debits or credits are allotted to each factor in multiples of five according to whether it has an unfavourable or favourable influence. The total sum is then computed and the result expressed with reference to the standard of 100%. Allowing Rogers and Hunter to explain the system in their own words, they propose that:

"Every risk for life insurance is found to be made up of the following factors:

1. Build (Weight in relation to Height),
2. Family Record,
3. Occupation,
4. Personal History,
5. Habits,
6. Physical Condition,
7. Habitat or Residence,
8. Moral Hazard,
9. Plan of Insurance Applied For.

The underlying principle in the numerical method of medical selection rests on the assumption that the average risk accepted by a company has a value of 100%, and that each one of the factors which make up a risk shall be expressed numerically in terms of 100% and that, by summation of them, or by some modification of their summation, the value of any risk shall be determined and expressed with relation to that standard. Everyone who passes judgment upon a risk carries out this process in his mind. . .

. . . Thus the reviewer carries on in his mind a process of addition and subtraction, or a modification of this process, according as each factor is favourable or unfavourable or negative, and his final judgment of the risk is the total of these various favourable and unfavourable impressions. The numerical method expresses each step in this mental process in terms of a definite standard and the final valuation of the risk, with comparatively few exceptions of material importance, is the sum of these various items. . .

. . . Wherever there is clear evidence that two factors are interdependent so that their addition is not sufficient or is distinctly too large, allowance is made for that interdependence. . .

. . . The valuation of all such cases, especially where the factors may be interrelated,

must always be tempered by the judgment of the medical expert."

Rogers and Hunter presented a full critique of the system and published the basic ratings in use at the time. Commenting on various objections which had been raised concerning the system, they stated:

> "We do not believe that the whole truth is included in the system as now practised or that it cannot be changed to advantage in many of its details. In fact we stand ready to modify our present views in the light of new evidence. . . We agree fully with the actuary of one of the large companies who employs the system when he says that the numerical system can be criticized freely from the theoretical standpoint but in practical use it is a powerful aid in the selection of risks for insurance.
>
> The old adage that "The proof of the pudding is in the eating of it," applies nowhere more forcefully than in the case of the numerical method of valuing lives for insurance."

Over 70 years later, the numerical rating system is now used almost universally for the underwriting of lives and has completely replaced older and more empirical methods of risk evaluation.

The magnitude of the debits and credits used in the numerical rating system is estimated using a combination of medico-statistical studies and perceived wisdom of the medical profession. The credits and debits have been compiled into underwriting manuals by the major life insurance and reinsurance companies around the world.

Once a mortality ratio has been obtained using the numerical rating system, the percentage extra mortality can be readily converted into an extra premium. Using a computer, it is quite feasible to process schedules for the main classes of business showing extra premiums corresponding to various levels of extra mortality by age. Alternatively, it is possible to create a rating schedule showing the number of years that must be added to an applicant's age for specific levels of extra mortality to arrive at the same premium as would be charged using the above method.

Particular difficulties in underwriting arise when two or more adverse factors that are interdependent occur in the same individual. In this case, the summation of the debits for each factor could produce a false impression of the rating required. When this situation arises, there are three possible variations of underwriting practice which can be adopted according to the nature of the factors under consideration, as follows:

1) The debit for one factor may be deemed to be wholly accounted for by the debit for the other and therefore should not be debited separately.

2) Only a fraction of the debit for a subsidiary interdependent factor need be added to the debit for the principal factor.

3) The sum of debits for certain factors may require the further addition of a fraction of that sum.

This problem concerning interdependence was noted by Rogers and Hunter and others describing the numerical rating system. More recently, Brackenridge (1985) states that:

> "Interdependence of impairments occurs more frequently than is generally realized. Failure to recognize the interrelationship between rateable features of family history, personal history and physical examination is one of the reasons why the aggregate of numerical ratings for multiple impairments often turns out to be inconsistent with good judgement. The proper weight to be given to interdependent impairments requires a wide experience of clinical medicine *and the mortality statistics of impaired lives.*" [author's emphasis]

One of the advantages of *modelling* excess mortality is that the interdependence of impairments (or rating factors) can be investigated explicitly. The final sentence in the above quote implies that Brackenridge disagrees with Perks as to the usefulness of impaired lives investigations.

One of the principal failings of the numerical rating system is that it does not cope well with

the situation where a risk decreases rapidly with duration, for example after major surgery. In this type of situation, it is common to defer the application for a period of time or to impose a temporary addition to the premium which reduces over time. An alternative is to use a diminishing debt (or *lien*), by which only a certain proportion of the sum assured is payable on death, but no addition is made to the premium. The debt is usually designed to reduce annually until it eventually runs off. Diminishing debts are seldom used nowadays; the preferred method of treatment is to use a temporary extra premium.

The numerical rating system was first described in this country briefly by MacLaren (1927), and more completely in two papers presented to the Institute of Actuaries Students' Society (see Wood (1932) and Chiles (1935)), although it is evident that the system was already well known. In these papers, both authors conclude by highlighting the advantages of the system while recognizing its limitations. In particular, C. F. Wood concluded with:

> "I should like to point out that it has never been suggested that the Numerical Rating System is perfect – if any better method can be invented it will surely supplant the Numerical System."

A. B. Chiles concluded with similar words:

> "It must not, therefore, be considered that the Numerical Rating Systems which are in use today will be retained permanently in their present form. The basis on which to develop sound underwriting practice has been established, but it may be many years before all the essential information has been obtained from the statistics and, even then, experience will change and necessitate changes in practice. The Numerical Rating System is in the process of evolution: it is not a *fait accompli*".

Despite these caveats, the numerical rating system has hardly changed since its introduction in 1919, and no attempt has been made at finding a theoretical statistical basis for the system. In Chapter 11, such a basis is proposed upon recognizing that the numerical rating system implicitly assumes an underlying linear model, with an additive structure, for the mortality ratio.

Chapter 4 An Introduction to Modelling Excess Mortality

4.1 What is a Model?

Much of the work of science is involved in understanding and experimenting with systems. Coupled with this is the observation of phenomena, and the recording of the observations as data. However, data have no meaning in themselves; they are only meaningful in relation to a conceptual *model* of the phenomenon studied, where a model is a representation of reality. Quoting John von Neumann (Gleick (1988)),

> "The sciences do not try to explain, they hardly even try to interpret; they mainly make models. By a model is meant a mathematical construct which, with the addition of certain verbal interpretations, describes observed phenomena. The justification of such a mathematical construct is solely and precisely that it is expected to work"

The key point of this definition provided by von Neumann is that a model is a *mathematical construct*. This mathematical construct is a formula involving parameters associated with variables considered to be important in understanding the system under scrutiny. The variables may be quantitative or qualitative.

Model building is concerned with fitting a mathematical construct to observed data. The aim of model building is to discover which variables are important in a model and how they interact which each other. It may not be necessary to include in the model all of the variables observed, in order to describe adequately the phenomenon under study. A model should be as simple as possible while still describing the salient features of the data.

No model will fit observed data exactly. In making observations, there is usually experimental error which is invariably assumed to be random. The role of model building is to separate the random component from the systematic component, and model the systematic component.

Statistical modelling implicitly makes allowance for the random nature of data by assuming a particular statistical distribution for the random component. The importance of a variable included in (or excluded from) the systematic component is then assessed according to its statistical significance. The number of variables included in a model is of utmost importance since simplicity and adherence to data are both crucial. A parsimonious model is needed containing as few parameters as possible, but as many as necessary. A statistical model is justified on the overall goodness-of-fit, which can be assessed using statistical measures and by an analysis of residuals.

Finally, it is important to realize that a variety of statistical models may be plausible, but it is usually necessary to choose an optimum model, given the data available. Even this optimum model will not fit the data exactly. A recognition of this has led eminent statisticians, such as Box, Shewhart, Nelder and Pregibon, to remark, somewhat cynically, that "All models are wrong, but some are useful".

4.2 Modelling Excess Mortality

It is well known that the mortality ratio for a particular risk depends on the levels of certain influential factors. This is the underlying tenet of the numerical rating system. The aim of *modelling* excess mortality is to impose a mathematical construct on the mortality ratio, where this mathematical construct involves those factors (and only those factors) deemed to influence the mortality ratio. Furthermore, it is desirable if such a mathematical construct can represent the interdependence between factors, where this interdependence is significant. This is achieved by expressing the mortality ratio as a regression type formula, or a transformation of a regression type formula, involving factors (or *covariates*) and their associated parameter estimates. In what follows, a factor is defined as a possibly influential variable, taking many levels. Factors may be qualitative, such as severity of a disease, or quantitative, such as levels of blood pressure. Where a factor is quantitative and measured on a continuous scale, the levels of the factor are determined by subdividing the range of possible values into sections,

which do not have to be of equal length.

Suppose an investigation into the excess mortality of hypertensives is being conducted, and it is believed that excess mortality is a function of blood pressure levels and weight. Let blood pressure be measured according to n levels, indexed by r, and let weight be measured according to m levels, indexed by s. Furthermore, let the mortality ratio, θ, be represented by a function h of a linear combination of parameter estimates associated with the various levels of blood pressure and weight. Denoting parameter estimates associated with the levels of blood pressure as α_r, and denoting parameter estimates associated with the weight levels as β_s, then:

$$\theta_{rs} = h(\mu + \alpha_r + \beta_s)$$

where θ_{rs} is the mortality ratio associated with blood pressure level r and weight level s, and μ is a constant. If it is believed that blood pressure and weight act interdependently, then an *interaction* term, $\alpha\beta_{rs}$, can be introduced, so that:

$$\theta_{rs} = h(\mu + \alpha_r + \beta_s + \alpha\beta_{rs})$$

Obviously, this can be extended to include many more factors and their possible interaction terms. The general situation can be written as:

$$\theta_j = h(\underline{\beta}'.\underline{z}_j)$$

where the mortality ratio is indexed loosely by the suffix j, $\underline{\beta}$ is a vector of parameter estimates and $\underline{z}_j$ is a vector of covariates.

Three particular functions, h, are proposed for modelling excess mortality, leading to three model structures; the multiplicative model, the additive model and the power model. This leads to the following relationships between the mortality ratio and the linear combination of parameter estimates:

$$\theta_j = exp(\underline{\beta}'.\underline{z}_j) \quad - \quad \text{multiplicative model}$$

$$\theta_j = (\underline{\beta}'.\underline{z}_j) \quad - \quad \text{additive model}$$

$$\theta_j = (\underline{\beta}'.\underline{z}_j)^{\frac{1}{\gamma}} \quad - \quad \text{power model}$$

Notice that, under the multiplicative model, the linear combination of parameter estimates act multiplicatively on the mortality ratio when exponentiated. Also notice that the additive model, essentially, is the model underlying the numerical rating system. That is, the mortality ratio is represented by a series of parameter estimates (which may be negative) added together. The parameter estimates are analogous to the debits and credits used in the numerical rating system. Interdependence of rating factors may be accommodated by including interaction terms.

The power model is, in fact, a family of models since the parameter γ may take any value. When $\gamma=1$, the additive model is obtained, whereas the multiplicative model is obtained in the limit as $\gamma \to 0$ (see McCullagh and Nelder (1989)). For values of γ between 0 and 1, the power model may be perceived as being in between the additive and multiplicative models. The value of the power giving the optimum fit need not lie in the range 0 to 1, however.

In the actuarial context, modelling excess mortality was first described by Renshaw (1988), who developed a multiplicative model for the mortality ratio. In his paper, Dr. Renshaw showed how the parameters in the multiplicative model could be estimated by treating the problem as a *generalised linear model* (*GLM*). Renshaw used part of the Prudential data set, the hypertensive subset, to illustrate the techniques, and concluded by saying:

> "I would suggest that the GLM approach outlined here could pave the way for a completely new, scientifically sound approach to life insurance underwriting. It offers a more dynamic means of model building than has hitherto been attempted in this field in which the relationship between individual factors and their interactions on excess mortality may be assessed . . . "

This paper by Renshaw made a significant contribution to the field of measuring excess mortality, and provides a very useful starting point for the material outlined in this thesis. It transpires that both the additive and power models can also be embedded within the generalised linear modelling framework. Moreover, the model proposed by Renshaw can be reformulated, such that a unified approach to modelling excess mortality can be developed.

Although the *application* as a generalised linear model is straightforward, the *formulation* of the problem is somewhat technical. It is helpful to have some understanding of generalised linear models before developing models of excess mortality within this framework, therefore the following chapter provides a brief introduction to generalised linear models.

Chapter 5 An Introduction to Generalised Linear Models

5.1 Description of a Generalised Linear Model

Generalised linear models are a restricted class of statistical models. Formalising the discussion from Chapter 4, the modelling process may be thought of as one in which there is a set of data $y_1, y_2, \ldots, y_n$ matched by a set of *theoretical* values $m_1, m_2, \ldots, m_n$. The y's are called the *response* or *dependent variable*. For a good model, the m's should be derived from a small set of parameters associated with a set of *explanatory variables*. Furthermore, the resulting set of m's should be close to the original data, the y's. The model fitting process involves two basic decisions:

(i) the choice of the relation between the m's and the underlying parameters of the model

(ii) the choice of a measure of discrepancy which defines the goodness-of-fit.

The first choice relates to the systematic component of the model, and the second is governed by assumptions made about the random component.

Recall, the classical linear model (see e.g. Cochran and Cox (1957)). The response is considered to be the sum of several systematic components and one random component. The random component is assumed to have a Normal distribution, thus:

$$y_i = \sum_{j=1}^{p} \beta_j x_{ij} + \epsilon_i \qquad \text{where } \epsilon_i \sim N(0, \sigma^2)$$

The x_{ij} may be the values of p covariables or they may be "dummy" variables indicating the presence or absence of an effect, or they may be a mixture of both. The ϵ_i represent independent variables which have a random, distorting effect on the observations. Thus the structure of y_i is represented as the *sum* of a *linear combination of systematic components* and *one random component*. In the classical case, $y_i = m_i + \epsilon_i$ where $m_i = \sum_{j=1}^{p} \beta_j x_{ij}$.

The classical linear model was *generalised* by Nelder and Wedderburn (1972) in two specific ways. Firstly, the distribution of y_i is no longer constrained to be Normal, but may be any member of the exponential family of distributions. This includes the Normal, binomial, Poisson, χ^2, gamma, negative binomial and inverse Gaussian amongst its members. Secondly, the mean is allowed to be functionally related to the linear combination of systematic components. That is:

$$\eta_i = g(m_i) \tag{5.1}$$

where $\eta_i = \sum_{j=1}^{p} \beta_j x_{ij}$ and is called the *linear predictor.*

The function g is called the *link function* and must be monotonic, so that its inverse h exists, and must be differentiable over its domain. Obviously, the classical linear model is reproduced when a Normal error structure is used together with the identity link function, so that $m_i = \eta_i$.

Any structure which satisfies the above criteria is termed a *generalised linear model* (*GLM*). A particular *GLM* can be identified by specifying the error distribution of the random component, the make-up of the linear predictor and the function linking the means to the linear predictors.

5.2 Estimation of the Linear Parameters

Nelder and Wedderburn (1972) provide the mathematics by which the parameters included in the linear predictor, for a particular model structure, are estimated using maximum likelihood techniques. Several computer software packages are available which include the facility to fit generalised linear models. One particular package, *GLIM* (Generalised Linear Interactive Modelling), was specially written with generalised linear models in mind (Baker, Clarke and Nelder (1987)) and is a powerful tool for fitting *GLM*s, obtaining the associated parameter

estimates, and carrying out other relevant calculations. *GLIM* was used to provide the results in Part II of this thesis.

Occasionally, it may be required to fit a model where some of the β_j parameters in the linear predictor η_i are known in advance. If a subset of the β_j parameters are fixed, the sum of their contributions to η_i is called an *offset* so that:

$$\eta_i = \mathit{offset} + \sum \beta_j x_{ij} \tag{5.2}$$

where the summation is over the terms for which the β_j are not fixed. In fitting such a model, the offset is first subtracted from the linear predictor and the result can then be regressed on the remaining covariates.

5.3 Goodness-of-Fit

A key question which needs to be addressed in model fitting is "How well do the data support the model?". Associated with this is the determination of the usefulness of an extra covariate included in the model, or conversely, the lack of fit induced by omitting that parameter.

The make-up of the linear predictor expresses the influence of the explanatory variables on the response. Interpretation of the model is aided if there are a small number of terms included in the model, although the trade-off between number of terms and goodness-of-fit should always be borne in mind. It is useful to distinguish special cases of the linear structure, as follows. If there are n observations and n parameters included in the model, then the maximum likelihood estimates of the m_i (the fitted values) are the observations themselves. This is known as the *saturated* or *full* model. The data are reproduced exactly, but without any model simplification. The other extreme is to propose one common value for the m_i, that is $m_i = m$ for all i. This is called the *null* model, and is the simplest type of model structure. In most cases, neither model will adequately represent the structure of the data, and a less extreme

model, involving a few key parameters, will need to be considered. The model under investigation, at any one time, is called the *current* model.

A statistical measure of the plausibility of a model is the *likelihood* of the model given the data. By comparing the likelihood of the current model (l_c) to the likelihood of the full model (l_f), a measure of the acceptability of the current model relative to the full model is obtained. Such ideas underlie the use of the statistic $S_{c,f}$, called the *scaled deviance* where:

$$S_{c,f} = -2\,log\left(\frac{l_c}{l_f}\right)$$

The exact mathematical form of the deviance depends on the distribution of the error structure assumed. The assessment of the goodness-of-fit of a *GLM* requires that the scaled deviance be matched against a theoretical distribution which represents its sampling distribution if the model were true. This sampling distribution can be obtained from general results from statistical inference. The following result is derived, for example, by Kendall and Stuart (1967), assuming certain regularity conditions: if l_1 and l_2 are the likelihoods for models 1 and 2 respectively, and if model 2 is nested in model 1 (i.e. the parameters in model 2 are a subset of those in model 1), then if model 2 were correct, $S(2,\ 1) = -2\,log(l_2/l_1)$ is distributed as χ^2 with $t_1 - t_2$ degrees of freedom, where t_i is the number of independent parameters estimated under model i. The distribution is exact for a Normal error structure with identity link, but only approximate for other error/link combinations.

The above result enables the usefulness of the linear parameters of the *GLM* to be assessed. If model 1 contains two sets of parameters $\beta_1, \ldots, \beta_t$ and $\beta_{t+1}, \ldots, \beta_p$ and model 2 contains only the second set $\beta_{t+1}, \ldots, \beta_p$, then $S(2,\ 1)$ is distributed as χ^2_t under model 2. Thus, a test of the hypothesis $\beta_1 = \beta_2 =, \ldots, = \beta_t = 0$ may be performed by comparing $S(2,\ 1)$ and the relevant percentage point of χ^2_t.

Consider the situation where model 1 is the saturated model (containing n parameters) and model 2 has r parameters. Then $S(2,\ 1)$ represents the lack of fit induced by estimating with r

parameters instead of n. Similarly, if a third model, model 3, is nested in model 2 and contains t parameters ($t<r$), it is possible to form $S(3, 1)$. Furthermore,

$$\begin{aligned} S(3, 1) - S(2, 1) &= -2\,log(l_3/l_1) + 2\,log(l_2/l_1) \\ &= -2\,log(l_3/l_2) \\ &= S(3, 2) \end{aligned}$$

which is the scaled deviance for the extra parameters between models 2 and 3, and since these models are nested, it is distributed as χ^2_{r-t} when model 3 is correct. This is an important result because it shows that the effect of a model simplification (from model 2 to 3) can be assessed by differencing the scaled deviances of each model compared to the saturated model, and referring this difference to the appropriate χ^2 distribution. In particular, if model 3 is the null model (which is nested within any more complex model since it is the simplest model structure) and model 2 is any other model, then $S(3, 2)$ may be referred to the appropriate χ^2 distribution to assess the goodness-of-fit relative to the null model.

It is possible to build up a table of differences of deviances for a sequence of nested models to assess the statistical significance of additional terms included in a model. This result is used extensively to assess the statistical significance of factors influencing excess mortality.

It is important to note that Baker *et al* (1987) state that, apart from the case of a Normal error/identity link, the scaled deviance is known to be distributed as χ^2 only asymptotically, and rather little is known about how good the asymptotic approximation is for small data sets. It seems that the approximation may be better for the *difference* of two deviances, which expresses the effect of adding a term to a model, than for an absolute deviance expressing the goodness-of-fit of a single model. No attempt should be made at interpreting the value of the absolute deviance, and the statistical significance of a factor assessed according to a difference in deviances can only be thought of as a guide.

5.4 Examination of the Data

An important part of the modelling process is to check the adequacy of a model graphically by plotting *residuals* against the fitted linear predictor. If a model is adequate, the resultant graph should be a scatterplot showing no obvious trend. Any noticeable pattern would be indicative of lack of fit, implying a systematic departure from the model (i.e. the model is incorrect). The possibility of the data containing gross errors should also be considered. Residual plots are very useful in detecting gross errors which identify themselves as isolated departures from the model. Thus, graphical inspection of residual plots is a powerful tool in the model fitting process. There are a variety of definitions of a *residual* and the most suitable when modelling excess mortality is considered in greater depth in Chapter 8.

Chapter 6 Modelling Excess Mortality

6.1 Introduction

In 1972, Sir David Cox presented his paper "Regression Models and Life Tables" to the Royal Statistical Society. That paper was concerned with the incorporation of regression type arguments into life-table analysis and was recognised at the time as a pioneering contribution to statistical theory. It is probably not an overstatement to say that Cox's paper revolutionised the field of medical statistics and the analysis and interpretation of epidemiological studies. Central to the method was the introduction of the *multiplicative hazards model* (also known as the *proportional hazards model* or just the *Cox model*)

$$\lambda(t, \underline{z}) = \lambda_0(t)\, exp(\underline{\beta}' \underline{z})$$

where

$\lambda(t, \underline{z})$ = instantaneous failure rate (*hazard rate*) at time t for an individual, or inanimate object, with associated vector of covariates $\underline{z}$

$\lambda_0(t)$ = *unknown* instantaneous failure rate at time t for an individual (or inanimate object) under "standard" conditions (known as the *base-line* hazard)

$\underline{\beta}$ = vector of unknown regression parameters

Thus, the $exp(\underline{\beta}' \underline{z})$ term may be percieved as a proportionality factor adjusting the base-line hazard to allow for the effects of the covariates $\underline{z}$. In actuarial terminology, the hazard rate is the force of mortality, time is age and failure is (typically) death (see Section 6.2).

The model proposed by Cox assumed that the base-line hazard was unknown, although it was also suggested that $\lambda_0(t)$ could be restricted qualitatively, for example assuming it to be a monotonic function or a step function (the latter being a suggestion of Professor T. W. Tukey). There has been a proliferation of applications of the method since 1972, principally in the field

of medical statistics, and usually assuming $\lambda_0(t)$ to be unknown. There have been relatively few applications in which the base-line hazard is assumed known at the outset. Specific cases include Breslow *et al* (1983), Berry (1983) and Hill *et al* (1985) who all used national population mortality rates as the known base-line hazard; in essence, they adopted Tukey's suggestion by imposing a step function on the base-line hazard.

Cox anticipated that "the applications are more likely to be in industrial reliability studies and in medical statistics than in actuarial science" although Professor Bernard Benjamin, in the discussion of Cox's paper, indirectly hinted at the potential of the methods for investigating the mortality of special groups, such as those with medical impairments, by suggesting a connection between such studies and reliability trials. Inspired by these comments and by the papers of Breslow *et al*, Berry, and Hill *et al*, Renshaw (1988) showed how the multiplicative hazards model could be used to model excess mortality of medically impaired insured lives. Renshaw also established a connection with traditional actuarial methods and illustrated the methodology with reference to the Prudential impaired lives data set.

The complete technical derivation is outlined below, starting with the definition of the hazard rate, and showing that this is identical to the force of mortality familiar to actuaries. After that, some useful results involving survival distributions are shown, which are needed when developing the survival likelihood. Having developed the survival likelihood, the multiplicative hazards model can be introduced and the generalised linear modelling analogue established. Finally, the connection with traditional actuarial methods can be explored.

6.2 The Hazard Rate and the Force of Mortality

Let the random variable T denote the lifetime (time of failure) of a living organism or an inanimate object (e.g. a light bulb). The relative instantaneous failure rate at time point t, denoted $\lambda(t)$, is called the ***hazard*** rate or ***intensity*** rate. Strictly,

$$\lambda_T(t) = \lim_{\delta t \to 0} \frac{\text{Prob}(t < T < t+\delta t \mid T > t)}{\delta t}$$

That is, the hazard rate is the limit, as δt tends to zero, of the probability of failure in the interval t to $t+\delta t$ conditional on survival to time t, divided by δt (see, for example, Elandt-Johnson and Johnson (1980)).

The force of mortality at exact age x is defined to be

$$\mu_x = -\frac{1}{l_x}\frac{dl_x}{dx}$$

where l_x is the number of lives at exact age x in a mortality study (see, for example, Neill (1986)). Now $\frac{dl_x}{dx}$ may be expressed as $\lim_{\delta x \to 0} \frac{l_{x+\delta x} - l_x}{\delta x}$ so that

$$\mu_x = -\frac{1}{l_x} \lim_{\delta x \to 0} \frac{l_{x+\delta x} - l_x}{\delta x}$$

$$= \lim_{\delta x \to 0} \frac{l_x - l_{x+\delta x}}{\delta x \, l_x}$$

$$= \lim_{\delta x \to 0} \frac{{}_{\delta x}q_x}{\delta x}$$

where ${}_{\delta x}q_x$ is the probability of death in the interval x to $x+\delta x$, conditional on survival to age x. Thus,

$$\mu_x = \lim_{\delta x \to 0} \frac{\text{Prob(death in } x \text{ to } x+\delta x \mid \text{survival to } x)}{\delta x}$$

This is identical to the hazard rate in which failure is death and lifetime, T, is age at death. Replacing the random variable T by X gives the result,

$$\mu_x \equiv \lambda(t)$$

Now consider the hazard rate of a study group with certain characteristics ($\underline{z}$) and denote this

hazard rate by $\lambda(t, \underline{z})$. If $\lambda^*(t)$ represents a ***known*** standard base-line hazard function, and if the multiplicative hazards model (Cox model) holds, then

$$\lambda(t, \underline{z}) = \lambda^*(t) \exp(\underline{\beta}' \underline{z}) \tag{6.1}$$

The proportionality factor, $\exp(\underline{\beta}' \underline{z})$, measures the effect of the characteristics $\underline{z}$ on the failure rate ***relative to the known standard.***

Obviously, if the proportionality factor, $\exp(\underline{\beta}' \underline{z})$, is greater than one, the failure rate in the study group is greater than the standard failure rate, and if $\exp(\underline{\beta}' \underline{z})$ is less than one, the failure rate in the study group is lower than the standard failure rate.

Suppose the study is investigating the excess mortality, relative to a known standard, of a group of individuals with some associated risk characteristics, such as severity of a disease, say. Then under the multiplicative hazards model,

$$\mu(x, \underline{z}) = \mu^*(x) \exp(\underline{\beta}' \underline{z}) \tag{6.2}$$

where $\mu(x, \underline{z})$ is the failure rate of the study group, $\mu^*(x)$ is from a known set of standard mortality rates (such as A67-70) and $\exp(\underline{\beta}' \underline{z})$ may be perceived as an excess mortality factor. Rearranging equation 6.2,

$$\exp(\underline{\beta}' \underline{z}) = \frac{\mu(x, \underline{z})}{\mu^*(x)}$$

which shows that the excess mortality factor may be expressed as the ratio of forces of mortality. This can be compared with the Interval Mortality Ratio, k_t, from Chapter 3, where $k_t = \frac{q_t}{q'_t}$, a ratio of mortality rates.

6.3 Useful Results from Survival Analysis

Let the random variable T denote age at death (lifetime). The *failure distribution*, $F(t)$, is defined to be the probability of death before some time t, thus:

$$F(t) = P(T \leq t)$$

The *survival function*, $S(t)$, is defined to be the probability of surviving to time t, thus:

$$S(t) = P(T > t) = 1 - F(t)$$

By definition, $t > 0$ and $S(0) = 1$ (i.e. $F(0) = 0$).

The *absolute instantaneous failure rate*, $f(t)$, is given by:

$$f(t) = \frac{dF(t)}{dt} = \lim_{\delta t \to 0} \frac{P(t < T < t+\delta t)}{\delta t}$$

and the *relative instantaneous failure rate*, $\lambda(t)$, is given by:

$$\lambda(t) = \lim_{\delta t \to 0} \frac{P(t < T < t+\delta t \mid T > t)}{\delta t}$$

Hence,
$$\lambda(t) = \frac{f(t)}{S(t)} = \frac{f(t)}{1-F(t)}$$

$$= \frac{-d\,log(1-F(t))}{dt}$$

$$\Rightarrow \quad \lambda(t) = \frac{-d\,log(S(t))}{dt}$$

Hence $-log\,S(t) = \int_0^t \lambda(u)\,du$ which gives $S(t) = exp\,(-\int_0^t \lambda(u)\,du)$.

Suppose an individual was known to be alive at time τ. Then the probability that the same individual survives to time t, where $t > \tau$, is given by:

$$P(T > t \mid T > \tau) = \frac{S(t)}{S(\tau)}$$

$$= exp(-\int_0^t \lambda(u)\,du + \int_0^\tau \lambda(u)\,du)$$

$$= exp\,(-\int_\tau^t \lambda(u)\,du) \qquad (6.3)$$

The probability density that the same individual then dies at time t is given by:

$$P(T > t \mid T > \tau) \lim_{\delta t \to 0} \frac{P(t < T < t+\delta t \mid T > t)}{\delta t} = \frac{S(t)}{S(\tau)}\,\lambda(t)$$

$$= \lambda(t)\ exp(-\int_\tau^t \lambda(u)\,du) \qquad (6.4)$$

6.4 The Survival Likelihood

Consider a sample of N individuals participating, at some time or other, in a mortality study. Let τ_i be the time at which individual i entered the study and let t_i be the time at which i was last observed. Denote by D the set of d individuals dying, and by $\overline{D}$ the set of $(N-d)$ individuals who were alive when last observed.

From equation 6.3, those individuals alive when last observed contribute to the survival likelihood an amount equal to

$$\frac{S(t_i, \underline{z}_i)}{S(\tau_i, \underline{z}_i)} = exp(-\int_{\tau_i}^{t_i} \lambda(u, \underline{z}_i)\,du)$$

The vector of covariates $\underline{z}$ is needed to provide additional insight by catering for the distinguishing characteristics of individual i within the study group.

From equation 6.4, those individuals observed to die contribute an amount

$$\frac{S(t_i, \underline{z}_i)}{S(\tau_i, \underline{z}_i)}\,\lambda(t_i, \underline{z}_i) \;=\; \lambda(t_i, \underline{z}_i)\; exp(-\int_{\tau_i}^{t_i} \lambda(u, \underline{z}_i)\, du)$$

Thus the survival likelihood, L, is given by:

$$L \;=\; \prod_{i \in \bar{D}} exp(-\int_{\tau_i}^{t_i} \lambda(u, \underline{z}_i)\, du) \prod_{i \in D} \lambda(t_i, \underline{z}_i)\; exp(-\int_{\tau_i}^{t_i} \lambda(u, \underline{z}_i)\, du)$$

Introducing an indicator, δ, where $\delta_i \;=\; 1$ for a death

$= \; 0$ if individual i was censored (through withdrawal or reaching the outer temporal limit of the study)

$$L \;=\; \prod_{i=1}^{N} \lambda(t_i, \underline{z}_i)^{\delta_i}\; exp(-\int_{\tau_i}^{t_i} \lambda(u, \underline{z}_i)\, du)$$

and the log likelihood is given by:

$$log\; L \;=\; \sum_{i=1}^{N} (\delta_i\, log\, \lambda(t_i, \underline{z}_i) \;-\; \int_{\tau_i}^{t_i} \lambda(u, \underline{z}_i)\, du) \qquad (6.5)$$

Equation 6.5 is important when deriving the generalised linear modelling analogue of the models of excess mortality considered in this thesis.

6.5 The Multiplicative Hazards Model and the *GLM* Analogue

Recall equation 6.1, the multiplicative hazards model with known base-line hazard,

$$\lambda(t, \underline{z}) \;=\; \lambda^*(t)\, exp(\underline{\beta}'\, \underline{z})$$

Trivially, the effect of introducing the multiplicative hazards model (6.1) into the log

likelihood function (6.5) is to give:

$$log\ L(\underline{\beta}) = \sum_{i=1}^{N}(\delta_i\, log(\lambda^*(t_i)\, exp(\underline{\beta}'\,\underline{z}_i)) - \int_{\tau_i}^{t_i}\lambda^*(u)\, exp(\underline{\beta}'\,\underline{z}_i)\, du)$$

$$= \sum_{i=1}^{N}\delta_i\, log\lambda^*(t_i) + \sum_{i=1}^{N}\delta_i(\underline{\beta}'\,\underline{z}_i) - \sum_{i=1}^{N} exp(\underline{\beta}'\,\underline{z}_i)\int_{\tau_i}^{t_i}\lambda^*(u)\, du$$

Partition the individuals i into j homogeneous cohorts by writing $i=(j,\ k)$. That is, after partitioning, individual i is reclassified as individual k from cohort j. In relation to an impaired lives investigation, the cohorts are subsets of the data classified according to age at entry, policy duration and severity of the disease, say.

Due to the partitioning, $\underline{z}_i = \underline{z}_{jk} = \underline{z}_j$ for all $k \in j$.

Therefore, $exp(\underline{\beta}'\,\underline{z}_i) = exp(\underline{\beta}'\,\underline{z}_{jk}) = exp(\underline{\beta}'\,\underline{z}_j)$ for all $k \in j$, giving:

$$log\ L(\underline{\beta}) = \sum_j\Big(\sum_k(\delta_{jk}\, log\lambda^*(t_{jk})) + (\underline{\beta}'\,\underline{z}_j)\sum_k\delta_{jk} - exp(\underline{\beta}'\,\underline{z}_j)\sum_k\int_{\tau_{jk}}^{t_{jk}}\lambda^*(u)\, du\Big)$$

$$= \sum_j\Big(\sum_k(\delta_{jk}\, log\lambda^*(t_{jk})) + d_j(\underline{\beta}'\,\underline{z}_j) - e_j\, exp(\underline{\beta}'\,\underline{z}_j)\Big)$$

where $d_j = \sum_k\delta_{jk}$ = actual number of deaths observed in cohort j (6.6)

$$e_j = \sum_k\int_{\tau_{jk}}^{t_{jk}}\lambda^*(u)\, du = \text{accumulated integrated base-line hazard in cohort } j \quad (6.7)$$

Furthermore,

$$log\ L(\underline{\beta}) = c + \sum_j(d_j(log\ e_j + \underline{\beta}'\,\underline{z}_j) - exp(log\ e_j + \underline{\beta}'\,\underline{z}_j))$$

where $c = \sum_j\Big(\sum_k(\delta_{jk}\, log\lambda^*(t_{jk}) - d_j\, log\, e_j\Big)$, being a constant independent of $\underline{\beta}$.

Writing $$\log m_j = \log e_j + \underline{\beta}' \underline{z}_j \tag{6.8}$$

gives $$m_j = e_j \exp(\underline{\beta}' \underline{z}_j) \tag{6.9}$$

and $$\log L(\underline{\beta}) = c + \sum_j (d_j \log m_j - m_j) \tag{6.10}$$

Equation 6.10 is the kernel of the log likelihood of independent Poisson variables $d_j \sim IPoi(m_j)$ where m_j is given by equation 6.9.

To compute the maximum likelihood estimates for the regression parameters $\underline{\beta}$ and various model structures $(\underline{\beta}' \underline{z})$, the generalised linear model analogue can be used based on independent Poisson variables $d_j \sim IPoi(m_j)$ with means

$$m_j = e_j \exp(\underline{\beta}' \underline{z}_j)$$

and log-link function

$$\eta_j = \log m_j = \log e_j + \underline{\beta}' \underline{z}_j \tag{6.11}$$

where e_j is given by equation 6.7.

It is trivial to verify that the log likelihood function of this *GLM* is identical to that quoted in equation 6.10. Comparison of equation 6.11 with 5.1 and 5.2 reveals that the function g in equation 5.1 is the logarithm of the mean, and the extra term, $\log e_j$, must be declared as an offset when model fitting. That is, the $\log e_j$ term may be perceived as part of the linear predictor $\underline{\beta}' \underline{z}$, being an extra term with known regression coefficient, having value equal to one.

6.6 The Accumulated Integrated Base-line Hazard

From equation 6.7, the accumulated integrated base-line hazard is the summation, over all

individuals k in cohort j, of the standard hazard function integrated between time of entry and time of exit for each individual. Thus

$$e_j = \sum_k \int_{\tau_{jk}}^{t_{jk}} \lambda^s(u)\, du = \text{accumulated integrated base-line hazard in cohort } j$$

In the actuarial context, the known standard hazard function is the force of mortality by age from a known standard table (such as A67-70 or AM80), and the integral is calculated as the summation of the force of mortality between age at entry and age at exit.

The accumulated integrated base-line hazard, e_j, can be interpreted as the expected number of deaths in cohort j, had standard mortality rates applied. This interpretation can be justified since the expected value of the accumulated integrated base-line hazard is equal to the expected number of deaths, had standard mortality rates applied. This was proved briefly by Berry (1983). A more complete proof is shown below, by considering the integrated hazard for the ith individual (where $i=(j, k)$).

Consider the complete follow-up case, i.e. there are no withdrawals or losses to follow up. This assumption has been made to simplify the presentation.

For an individual, define:

(i) entry to study at time 0 (this involves a change of origin)

(ii) maximum follow-up time T

(iii) death at time T' (which may be after time T; if so, unobserved)

(iv) x, an indicator random variable, where $x = 0$ for survival $(T' > T)$

$= 1$ for death $(T' \leq T)$

(v) $\lambda^*(t)$, the standard hazard function, where $\lambda^*(t) = \dfrac{f(t)}{1 - F(t)}$

where $f(t)$ is the absolute instantaneous failure rate, conditional on survival to time 0, and $F(t)$ is the failure distribution (see Section 6.3).

The integrated hazard for individual i observed up to time T' $(0 < T' \leq T)$ is given by:

$$e_i = \int_0^{T'} \lambda^*(t)\ dt$$

To calculate the expected value of e_i, it is necessary to consider the possibility of death before T, or after T. Suppose death occurs at time s, prior to T, then $T' = s$ with probability $f(s)$, where $0 < s < T$. If death occurs after the maximum follow-up time, T, then all that is known is that the individual survived to time T, with probability $1 - F(T)$. Therefore the expected value of the integrated hazard, $E[e_i]$, is given by

$$E[e_i] = \int_0^T f(s) \int_0^s \lambda^*(t)\ dt.\ ds \quad + \quad \Big(1 - F(T)\Big) \int_0^T \lambda^*(t)\ dt \tag{6.12}$$

The first part of equation 6.12 relates to the contribution to the expected value made by the possibility of death occurring at time s, integrated over all possible values of s from 0 to T. The second part is simply the contribution made by the possibility of survival to time T, the maximum follow-up time.

Integrating the first expression of equation 6.12 by parts gives:

$$E[e_i] = \left[F(s) \int_0^s \lambda^*(t)\ dt \right]_0^T - \int_0^T F(s).\ \lambda^*(s)\ ds \quad + \quad \int_0^T \lambda^*(t)\ dt \quad - \quad F(T) \int_0^T \lambda^*(t)\ dt$$

$$= F(T) \int_0^T \lambda^*(t)\ dt \quad + \quad \int_0^T \lambda^*(s)\ [1 - F(s)]\ ds \quad - \quad F(T) \int_0^T \lambda^*(t)\ dt$$

The first and last terms in the above expression cancel, giving:

$$E[e_i] = \int_0^T \lambda^*(s)\,[1-F(s)]\,ds$$

$$= \int_0^T f(s)\,ds$$

$$\Rightarrow \qquad E[e_i] = F(T)$$

The contribution that an individual makes to the expected number of deaths is 0 if the individual survives to T, with probability $1-F(T)$, and 1 if the individual dies before T, with probability $F(T)$. The expected number of deaths is $E[x]$ where

$$E[x] = 0.\ Prob(\text{survival to } T) + 1.\ Prob(\text{death before } T)$$

$$= 0.\ (1-F(T)) + 1.\ F(T)$$

$$\Rightarrow \qquad E[x] = F(T)$$

Therefore $E[e_i] = E[x]$ and the interpretation that the integrated hazard represents the expected number of deaths is justified. The *accumulated* integrated base-line hazard in cohort j can therefore be interpreted as the expected number of deaths in cohort j, had standard mortality rates applied.

6.7 The Connection with the Traditional Actuarial Approach

To establish a connection between the mortality factors $exp(\underline{\beta}' z)$ and the traditional actuarial mortality ratios, consider the equation

$$m_j = E[d_j] = e_j\ exp(\underline{\beta}' z_j)$$

which can be written as

$$exp(\underline{\beta}' \underline{z}_j) = \frac{m_j}{e_j}$$

Replacing m_j and $\underline{\beta}$ by their estimators $\hat{m}_j = d_j$ and $\hat{\underline{\beta}}$ gives

$$exp(\hat{\underline{\beta}}' \underline{z}_j) = \frac{d_j}{e_j} = \frac{\text{Actual Deaths}}{\text{Expected Deaths}} \tag{6.13}$$

The mortality factor, $exp(\underline{\beta}' \underline{z})$, is therefore equivalent to the traditional actuarial mortality ratio.

It should be noted that equation 6.13 is exact provided the specific covariate structures associated with single factor models or two (or more) fully interactive models are selected. If other models are selected, it is necessary to replace d_j by $E[d_j]$ in equation 6.13, where $E[d_j]$ represents the expected number of deaths under the model (i.e. the fitted values), to be distinguished from the expected number of deaths under standard mortality rates.

This completes the technical derivation of the multiplicative model derived by Renshaw, and shows how the mortality ratio can be modelled by imposing on it a mathematical construct, that construct being the exponential of a regression type formula.

6.8 Modelling Excess Mortality in Practice

The application of the methodology is straightforward. Given a sample of N individuals participating in a mortality study, the first step is to partition the individuals into relatively homogeneous cohorts indexed (loosely) by the suffix j. Obviously, it is necessary to ensure that each cohort contains sufficient data to make the construction of traditional mortality ratios meaningful. For each individual, the information needed for the intermediate calculations is age at entry (τ_i), age at exit (t_i) and mode of exit (δ_i). Given only this information, it is then

possible to calculate the observed number of deaths (d_j) and expected deaths (e_j) for each cohort. The values of d_j and e_j, together with a coding scheme to identify the covariate structure, can then be used as inputs to a statistical modelling package, such as *GLIM*, for final model fitting, where the parameter estimates $\underline{\hat{\beta}}$ are calculated for specific model structures. The statistical significance of covariates, and their possible interactions, included in (or excluded from) the model can be tested, and the overall goodness-of-fit assessed by reference to residual plots.

Chapter 7 Illustrating the Methodology

7.1 Introduction

Some of the results from Part II of this thesis shall be used to illustrate the methodology discussed so far. Results are taken from the analysis of male lives suffering from "Impairments of the Coronary Arteries". A more detailed analysis of this impairment (and an analysis of female lives) can be found in Chapter 17. This impairment has been chosen for illustration purposes because the amount of data is small and there are few explanatory factors available, resulting in straightforward analysis. Other impairments considered in Part II are more complex, in some cases considerably so.

Initially, consider the data set (see Chapter 2). Of the data available for each policyholder, the information needed is:

(1) Medical Impairment (including further sub-classification)

(2) Date of Entry

(3) Age Next Birthday at Entry

(4) Date of Exit

(5) Mode of Exit (withdrawal, death)

(6) Sex

Items 1 and 6 provide the necessary information to break the sample into fairly homogeneous cohorts. Item 3 provides the necessary values of τ_i and items 2, 3 and 4 together provide the values of t_i needed when calculating the accumulated integrated base-line hazard. The values of δ_i are provided by item 5.

The initial selection and sorting of data was achieved using the SPSS^X statistical software package. For both male lives and female lives separately, a subset of the full data set was

created which included only those lives identified as suffering from "Impairment of the Coronary Arteries". This includes the conditions Thrombosis, Occlusion, Ischaemia, Infarction and Angina.

To create the homogeneous cohorts, the data were partitioned according to:

(a) Age at entry taking four levels: (1) 16-39
(2) 40-49
(3) 50-59
(4) 60-79

(b) Whether or not "complications" are present taking two levels:

(1) without complications
(2) with complications

where "complications" is defined as subsequent chest pain on exertion.

For each of these subgroups, the number of deaths observed (d_j) and accumulated integrated base-line hazard (e_j) were calculated (using Fortran 77 programs specially written for this purpose), further subdividing by 3 duration groups (0-2 years, 2-5 years and 5-8 years). This gave a total of 24 cohorts (4×2×3).

Having partitioned the data according to the covariate classification chosen and calculated the number of deaths observed and accumulated integrated base-line hazard for each cross-classified cohort, the data were fed into the *GLIM* software package for model fitting and statistical analysis. The method of model fitting adopted was forward stepwise *i.e.* start with the simplest model (the *null* model) and include parameters one by one.

7.2 The Null Model

The null model is the simplest type of model structure in which the linear predictor is

represented by a single parameter, i.e. $(\underline{\beta}\ \underline{z}) = \mu$. Effectively, this is combining over all age at entry, duration and complications groups to give an overall mortality ratio equivalent to

$$\frac{\text{Total deaths in study group}}{\text{Total expected deaths}}$$

which is an estimate of the mortality ratio associated with impairments of the coronary arteries as a whole.

Fitting this model in *GLIM* returns the parameter estimate $\hat{\mu} = 0{\cdot}9076$. From equation 6.13, the mortality ratio is given by $exp\ (\underline{\hat{\beta}}'\underline{z})$ giving $e^{0{\cdot}9076} = 2{\cdot}48$

Thus the overall mortality ratio for life assurance policyholders with impairments of the coronary arteries at entry is 248% (extra mortality = +148%).

7.3 Main Effects Models

More information can be obtained by fitting models which allow for the factors of interest believed to influence excess mortality. These factors are called *main effects* to distinguish them from the *interaction* terms (which relate to the interdependence between factors). Consider models including the main effects fitted separately (*i.e.* look at age at entry, duration, complications separately).

7.3.1 Age at Entry

Denoting parameter estimates associated with the levels of age at entry as α, indexed by i, then the parametric representation of the linear predictor is:

$$\underline{\beta}'\underline{z}_i = \mu + \alpha_i \qquad i = 1, \ldots, 4$$

Fitting the age at entry main effect model in *GLIM* returns the parameter estimates:

$$\hat{\mu} = 2{\cdot}912 \qquad \hat{\alpha}_1 = 0 \qquad \hat{\alpha}_2 = -1{\cdot}236 \qquad \hat{\alpha}_3 = -1{\cdot}679 \qquad \hat{\alpha}_4 = -2{\cdot}471$$

For technical reasons, the first parameter estimate of any factor included in a model is assigned the value zero, thus $\hat{\alpha}_1 = 0$.

The mortality ratios are found by exponentiating the linear predictor, therefore calculating $exp(\underline{\hat{\beta}}' \underline{z}_i)$ for each i gives:

Age at Entry	Mortality Ratio
16 - 39	18·39
40 - 49	5·34
50 - 59	3·43
60 - 79	1·55

These results indicate that proposers for life assurance aged under 40 suffering from impairments of the coronary arteries constitute a substantial extra risk. Clearly, excess mortality decreases as age at entry increases.

7.3.2 Policy Duration

Denoting parameter estimates associated with the levels of policy duration as δ, indexed by j, then the parametric representation of the linear predictor is:

$$\underline{\beta}' \underline{z}_j = \mu + \delta_j \qquad j = 1, \ldots, 3$$

Fitting the policy duration main effect model in *GLIM* returns the parameter estimates:

$$\hat{\mu} = 1{\cdot}205 \qquad \hat{\delta}_1 = 0 \qquad \hat{\delta}_2 = -0{\cdot}4727 \qquad \hat{\delta}_3 = -0{\cdot}4073$$

Exponentiating the linear predictor for each value of j gives:

Duration	Mortality Ratio
0 - 2 yrs	3·34
2 - 5 yrs	2·08
5 - 8 yrs	2·22

These results show that the mortality ratio in the first two years after entry is higher than subsequently.

7.3.3 Complications

Denoting parameter estimates associated with the levels of complications as γ, indexed by k, then the parametric representation of the linear predictor is:

$$\underline{\beta}' \underline{z}_k = \mu + \delta_k \qquad k = 1, 2$$

Fitting the complications main effects model in *GLIM* returns the parameter estimates:

$$\hat{\mu} = 0{\cdot}7893 \qquad \hat{\gamma}_1 = 0 \qquad \hat{\gamma}_2 = 0{\cdot}2771$$

resulting in the following mortality ratios:

	Mortality Ratio
Without Complications	2·20
With Complications	2·90

As expected, there is a higher risk associated with the presence of complications.

7.3.4 Significance of Main Effects

The results according to main effects fitted separately could, of course, have been obtained using traditional actuarial methods. However, one of the advantages of the modelling approach is that it is now possible to assess the statistical significance of the main effects. That is, it is possible to answer such questions as "Is age at entry a significant rating factor?" and "What about the presence or absence of complications?". These questions are answered with recourse to the model deviances. The null model is a simpler model than the main effects models, and we know from Section 5.3 that the difference in deviances between the null model and the main effects models follows a χ^2 distribution.

Using the deviances provided by *GLIM* when fitting the particular models, a deviance table for the main effects models may be drawn up as shown below. The differences in model deviances are referred to the appropriate χ^2 distribution to assess the significance of main effects. The null model is denoted by H_0, the age at entry model by A, the policy duration model by D, and the complications model by C.

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev.	D of F	
H_0	116·56	23			
A	33·75	20	82·81	3	$\ll$ ·05%
D	102·27	21	14·29	2	·075%
C	110·95	22	5·61	1	1·75%

Analysis of the differences in model deviances indicates that all three main effects are highly statistically significant since the "tail area" is less than 5% in all cases. Strictly, the observed significance level represents the probability of observing such an extreme result if the alternative hypothesis (in this case, the null hypothesis) is true.

7.4 More Complex Models

Since all three main effects are significant, we may be interested in more complex models, looking at, say, age at entry and policy duration combined, or including all three factors together. We may also be interested in the effect of interdependence of rating factors, assessed by the inclusion of interaction terms.

7.4.1 Main Effects Fitted Together, No Interaction

Since all three rating factors are statistically significant, they will need to be included together in a model in order to assess, as accurately as possible, the rating required for a given combination of factor levels. The simplest type of model structure catering for all three rating factors is fitted by including the main effects together, but without interaction terms. The *GLIM* notation for this model is A+D+C, with parametric representation of the linear predictor given by:

$$\underline{\beta}' \underline{z}_{ijk} \quad = \quad \mu + \alpha_i + \delta_j + \gamma_k$$

The associated mortality ratios are found by exponentiating the linear predictor, thus:

$$exp\,(\underline{\beta}' \underline{z}_{ijk}) \quad = \quad exp(\mu)\; exp(\alpha_i)\; exp(\delta_j)\; exp(\gamma_k)$$

that is, the effects are *multiplicative*.

The mortality ratio of 18·39 for the age at entry group 16 to 39 (in the age at entry main effects model of section 7.3.1) was based on only 9 deaths. Therefore, it was decided to combine ages at entry 16 to 39 and 40 to 49 when considering more complex models, resulting in only three levels for the age at entry factor ($i=1, \ldots, 3$).

The parameter estimates obtained by fitting model A+D+C are as follows:

$$\hat{\mu} = 1{\cdot}910$$

$$\hat{\alpha}_1 = 0$$
$$\hat{\alpha}_2 = -0{\cdot}5668$$
$$\hat{\alpha}_3 = -1{\cdot}354$$

$$\hat{\delta}_1 = 0$$
$$\hat{\delta}_2 = -0{\cdot}4109$$
$$\hat{\delta}_3 = -0{\cdot}3354$$

$$\hat{\gamma}_1 = 0$$
$$\hat{\gamma}_2 = 0{\cdot}3359$$

The mortality ratios calculated for each combination of i, j and k are shown in Table 7.4.1.

Table 7.4.1 Mortality Ratios, Model A+D+C (Multiplicative Structure)

Without Complications

		Age at Entry		
		16 - 49	50 - 59	60 - 79
Duration	0 - 2	6·75	3·85	1·76
	2 - 5	4·46	2·54	1·16
	5 - 8	4·86	2·77	1·26

With Complications

		Age at Entry		
		16 - 49	50 - 59	60 - 79
Duration	0 - 2	9·45	5·39	2·46
	2 - 5	6·24	3·56	1·62
	5 - 8	6·80	3·88	1·77

A direct result of using the multiplicative model, without interaction terms is that there is an underlying pattern in the tables of mortality ratios. Close inspection reveals that:

(i) Entries under "with complications" are 1·4 times equivalent entries under "without complications".

(ii) Entries in the second row are 0·66 times entries in the first row, and entries in the third row are 0·72 times entries in the first row.

(iii) Entries in the second column are 0·57 times entries in the first column, and entries in the third column are 0·26 times entries in the first column.

There is no conflict between the results shown here and the results for main effects (*i.e.* figures are of the same order and changes are in the same direction). The advantage is that more information is conveyed using simple mathematical relationships. Furthermore, the 18 entries in the tables of mortality ratios were derived from just six parameter estimates.

Whereas results for the main effects models fitted separately can be reproduced using traditional methods, the above results cannot.

7.4.2 Interaction Terms

The significance of interdependence between rating factors can be assessed by fitting models including interaction terms. In *GLIM* notation, a model includes interaction terms if an asterisk appears between the symbols for model factors. For example, A*C+D represents a model including all three factors and the interaction between age at entry and the presence or absence of complications. In this example concerning Impairment of the Coronary Arteries, the models which need to be investigated are:

A*C+D with parametric representation $\underline{\beta}' \underline{z}_{ijk} = \mu + \alpha_i + \delta_j + \gamma_k + \alpha\gamma_{ik}$

C*D+A with parametric representation $\underline{\beta}' \underline{z}_{ijk} = \mu + \alpha_i + \delta_j + \gamma_k + \delta\gamma_{jk}$

A*D+C with parametric representation $\underline{\beta}' \underline{z}_{ijk} = \mu + \alpha_i + \delta_j + \gamma_k + \alpha\delta_{ij}$

These models can be fitted in *GLIM* and the difference in deviances between model A+D+C and these models referred to the appropriate χ^2 distribution to assess the statistical significance of the interaction terms, as shown in the following deviance table:

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev.	D of F	
A+D+C	23·647	18			
A*C+D	21·718	16	1·93	2	35%
C*D+A	21·973	16	1·67	2	45%
A*D+C	20·050	14	3·60	4	47%

The results indicate that none of the first order interaction terms are statistically significant.

The only other model which can be fitted is model A*D*C, the saturated model. This model reproduces the raw mortality ratios for each combination of i, j and k and will have a deviance of zero since it gives a perfect fit, but no model simplification. The saturated model is the only other model which can be obtained from traditional actuarial methods, but is unnecessarily complex since interaction terms are not statistically significant. This leaves the model A+D+C as the optimal model in that it is parsimonious and conveys the salient features of the data available.

Chapter 8 Model Checking using Residuals

8.1 Introduction

A *residual* is a measure of discrepancy between an observed value and a fitted value under a given model. Residuals can be used to explore the overall adequacy of fit of a model, and may also indicate the presence of anomalous values requiring further investigation. The model checking techniques which are advocated in this chapter are informal, that is, techniques which rely on the human mind and eye to detect a pattern. A successful model is deemed to be one which leaves a patternless set of residuals when plotted against the linear predictor, or against one of the covariates in the linear predictor; the justification being that if a pattern can be detected, a better model can be found.

There are a variety of definitions of a residual, the simplest being the *raw residual* which is just the difference between the observed value and the fitted value. That is, using the notation of Section 5.1:

$$\begin{aligned} \textit{raw residual} &= \textit{datum} - \textit{fitted value} \\ &= y_i - m_i \end{aligned} \tag{8.1}$$

The raw residual is often used in association with the Normal error distribution, in which case, for a satisfactory model, a histogram of residuals would be Normal with zero mean and variance σ^2. For generalised linear models, an extended definition of residuals is required, applicable to all the distributions that may replace the Normal, and which have the same properties as standard Normal residuals. Three definitions of residuals shall be considered; the *Pearson residual*, the *Standardised residual*, and the *Deviance residual*.

8.2 The Pearson Residual

The Pearson residual, r_p, is defined by

$$r_p = \frac{y - m}{\sqrt{V(m)}} \tag{8.2}$$

where $V(m)$ is known as the *variance function* (the dependence on the suffix i has been dropped for simplicity). The variance function is one of the terms in the parameterization of a distribution in terms of the exponential family (see McCullagh and Nelder (1989)). For the Normal distribution, $V(m)=1$, while for the Poisson distribution, $V(m)=m$. Thus, for the Normal distribution, the Pearson residual is exactly the same as the raw residual. For the Poisson distribution,

$$r_p = \frac{y - m}{\sqrt{m}} \qquad y \sim IPoi(m) \tag{8.3}$$

8.3 The Standardised Residual

The standardised residual, r_s, is the residual automatically provided by the modelling package *GLIM*, and is defined by

$$r_s = \frac{y - m}{\sqrt{\phi\ V(m)}} \tag{8.4}$$

where ϕ is known as the *scale parameter*, and again comes from the parameterization of a distribution in terms of the exponential family. For the Normal distribution, $\phi = \sigma^2$, therefore

$$r_s = \frac{y - m}{\sigma} \qquad y \sim IN(m, \sigma^2) \tag{8.5}$$

Using the Normal distribution, if a model is satisfactory, a histogram of standardised residuals should be Normal with zero mean and unit variance.

For the Poisson distribution, $\phi = 1$, therefore

$$r_s = \frac{y - m}{\sqrt{m}} \qquad y \sim IPoi(m) \tag{8.6}$$

giving $r_s = r_p$, i.e. the Standardised residual is identical to the Pearson residual.

A disadvantage of the Pearson and standardised residuals is that a histogram of these residuals for non-Normal distributions is often markedly skewed, making them altogether inappropriate as a model checking tool for anything other than the Normal distribution. They are therefore unsuitable for checking the models of excess mortality proposed in this thesis, which are all based on the Poisson distribution.

For illustration purposes, consider the analysis of male hypertensives using the multiplicative model (see Chapter 18). In particular, consider the model structure in which all main effects are fitted together, without interaction terms (model A+B+C+D+H). Figure 8.1 shows a histogram of standardised residuals and in Figure 8.2, a scatterplot of standardised residuals fitted against the linear predictor (having first removed the offset) is shown.

Figure 8.1 Histogram of Standardised Residuals, Male Hypertensives

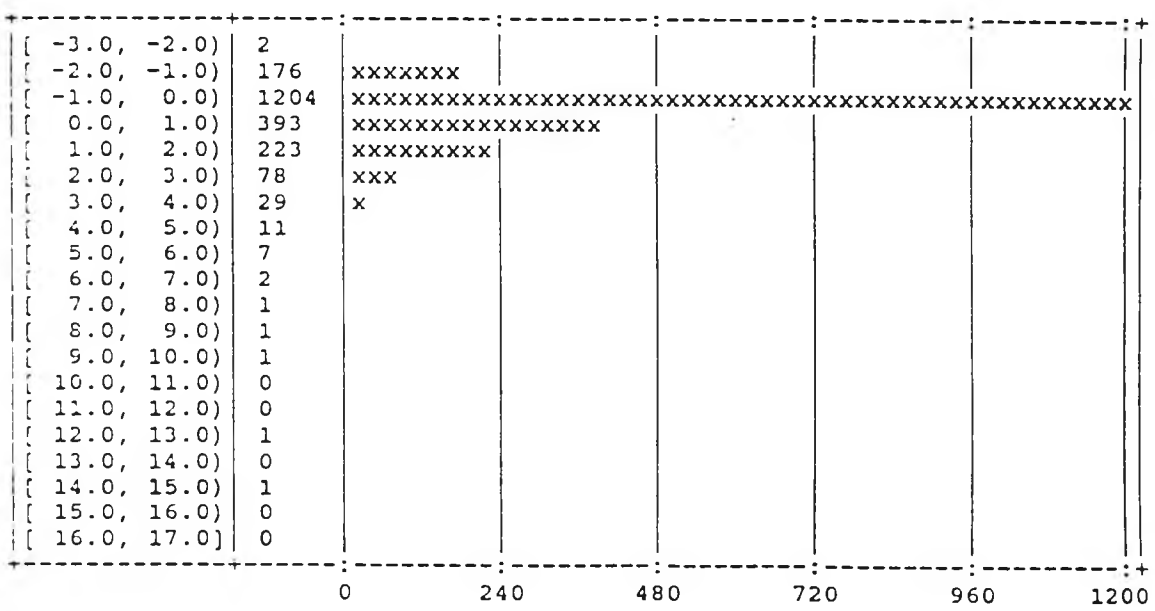
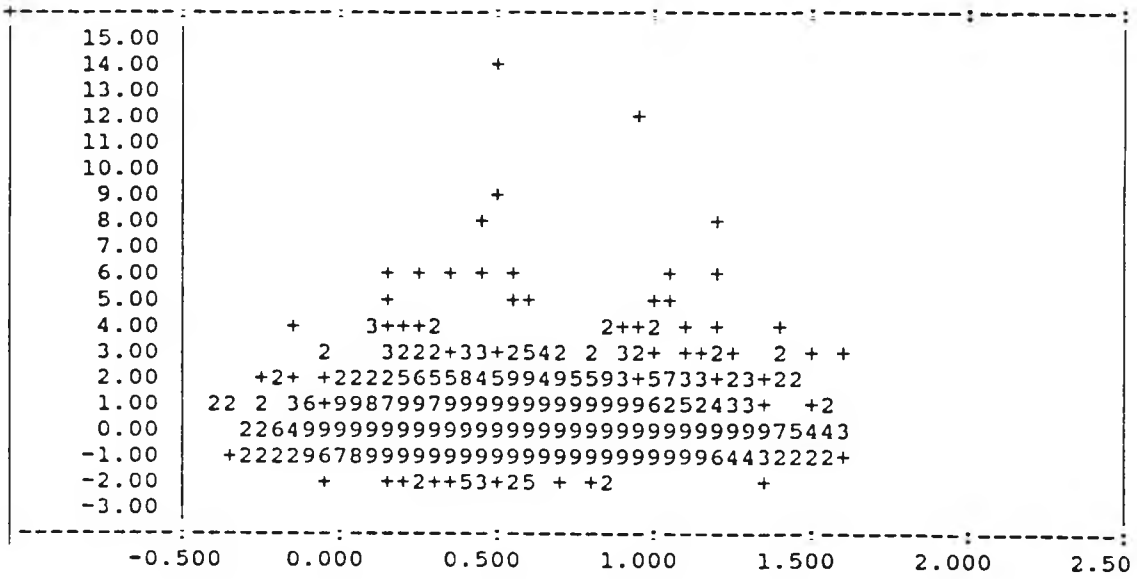


Figure 8.2 Standardised Residuals against Linear Predictor, Male Hypertensives



It can be seen that the histogram of standardised residuals is highly skewed, and would be rejected outright if this model was based on a Normal distribution. The scatterplot of standardised residuals also looks highly unsatisfactory, and seems to indicate the presence of many outliers (where the absolute value of the residuals is greater than 3, say).

8.4 The Deviance Residual

A type of residual which seems to perform well with all generalised linear models is the *deviance residual*, defined to be the signed square root of the contribution that each observation makes to the *unscaled deviance*. The unscaled deviance, $D_{c,f}$, is equal to the scaled deviance, $S_{c,f}$, multiplied by the scale parameter ϕ (the scaled deviance was introduced in Chapter 5). Thus,

$$D_{c,f} = \phi S_{c,f}$$

For the Normal distribution,
$$D_{c,f} = \sum_i (y_i - m_i)^2 \qquad (8.7)$$

and for the Poisson distribution
$$D_{c,f} = \sum_i 2\left(y_i \log\left(\frac{y_i}{m_i}\right) - (y_i - m_i)\right) \qquad (8.8)$$

For the Poisson distribution, the scaled deviance equals the unscaled deviance since the scale parameter equals one.

Let the contribution that each observation makes to the unscaled deviance be v_i, then

$$D_{c,f} = \sum_i v_i$$

The deviance residual, r_d, is the signed square root of the contribution that each observation makes to the deviance, therefore:

$$r_d = sign(y_i - m_i)\sqrt{v_i} \tag{8.9}$$

Using equation 8.7, the deviance residual for the Normal distribution is given by

$$r_d = y_i - m_i \qquad y \sim IN(m, \sigma^2) \tag{8.10}$$

For the Normal distribution, therefore, $r_d = r_p$ *i.e.* the deviance residual is exactly the same as the Pearson residual (which is exactly the same as the raw residual).

Using equation 8.8, the deviance residual for the Poisson distribution is given by

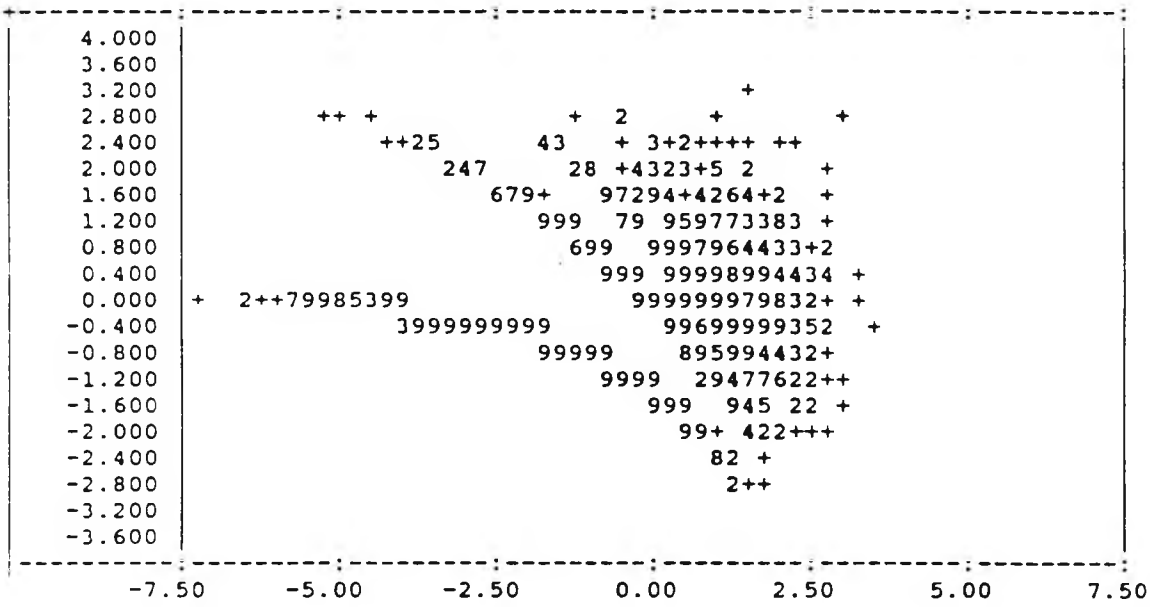
$$r_d = sign\,(y_i - m_i)\sqrt{2\left(y_i\, log\left(\frac{y_i}{m_i}\right) - (y_i - m_i)\right)} \qquad y \sim IPoi(m) \tag{8.11}$$

Notice that for both distributions, $\sum r_d^2 = D_{c,f}$.

If a model provides a satisfactory fit, a histogram of deviance residuals for any error distribution should be *approximately* Normal with zero mean and variance ϕ. Furthermore, a scatterplot of deviance residuals against linear predictor (less any offset declared) should be pattern free. That is, the scatter should be centred around zero and have a constant range.

Any observable pattern would be indicative of lack of fit. In such a case, a transformation of the data may be necessary, or account may need to be taken of factors other than those included in the current model. An example of a scatterplot of residuals showing an observable pattern is shown in Figure 8.3.

Figure 8.3 Scatterplot of Residuals Showing Noticeable Pattern



Outliers can also be detected by plotting deviance residuals and identify themselves as points on the graph very far from the rest. This may indicate something unusual about those particular points; they may be extreme values where the model simply does not apply, or the values may simply be wrong (*e.g.* incorrect recording of data). It is usual to assign outliers zero weight and refit the model, taking any necessary further action if the deviance and the parameter estimates change substantially.

Returning to the analysis of male hypertensives, it can be seen from Figures 8.4 and 8.5 that the histogram of deviance residuals and the scatterplot of deviance residuals against linear predictor are both highly satisfactory, indicating that this is an acceptable model. Deviance residuals are used exclusively in the results shown in Part II of this thesis.

Figure 8.4 Histogram of Deviance Residuals, Male Hypertensives

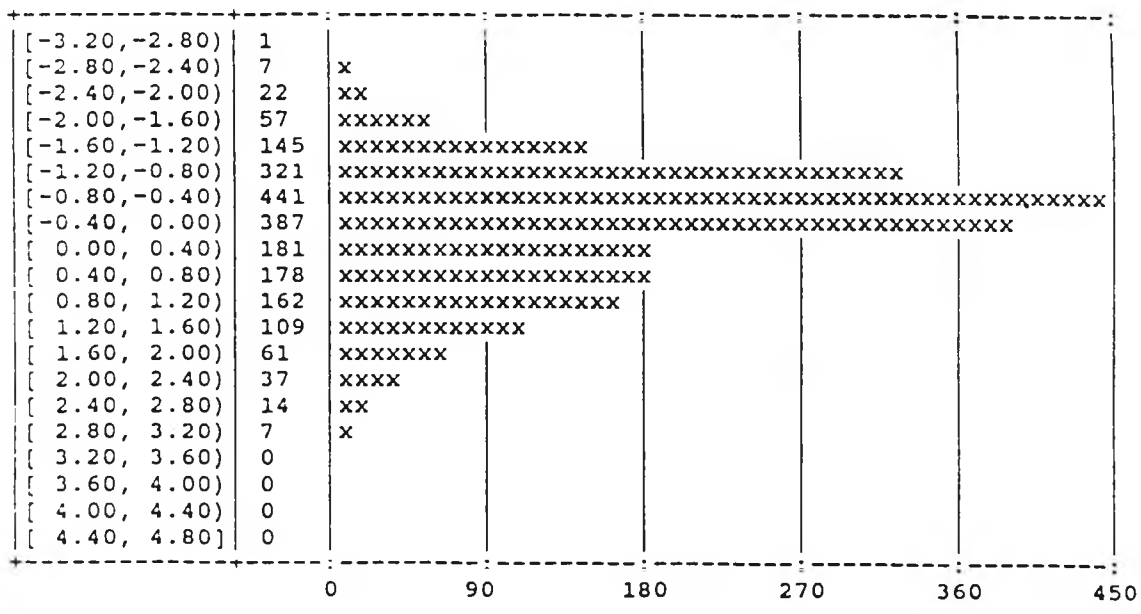
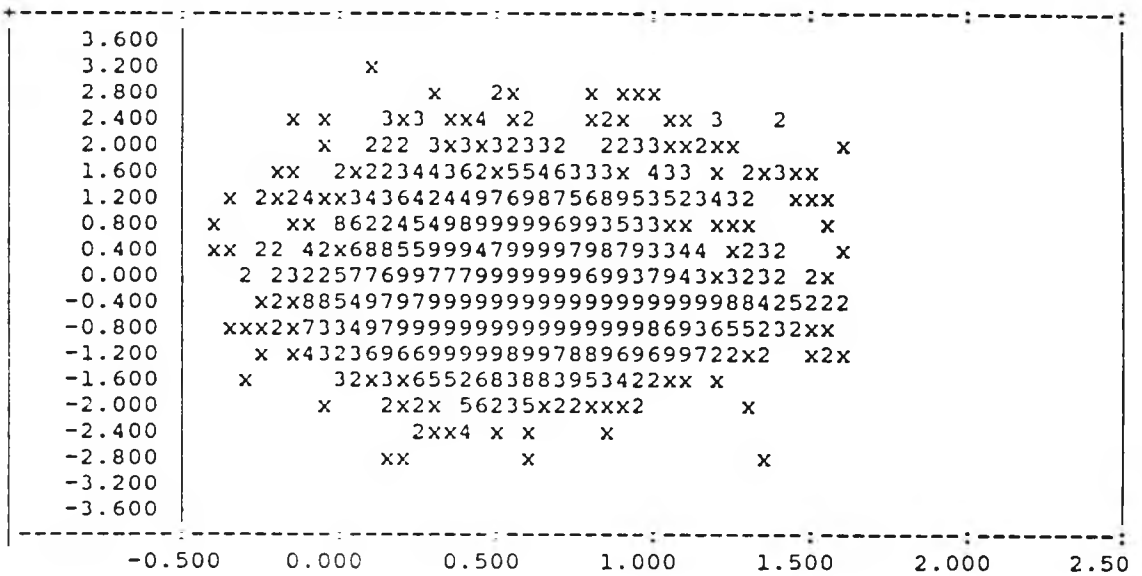


Figure 8.5 Deviance Residuals against Linear Predictor, Male Hypertensives



Chapter 9 Confidence Intervals for Mortality Ratios

9.1 The Traditional Approach

Consider measures of excess mortality presented in the form of estimates of mortality ratios. These estimates are sometimes based on small numbers of deaths and it is helpful to have some notion of the limits within which the "true" value of the estimate may lie. One practical approach is to adopt the concept of *confidence limits* that occurs in sampling theory. This requires making reasonable assumptions as to the distributional form of the random variable considered, such as the number of deaths or the mortality ratios based on it.

The traditional approach is to assume that deaths are observations of a Poisson random variable. If the observed number of deaths is d and the expected deaths is e, then an estimate of the mortality ratio, θ, is given by:

$$\hat{\theta} = \frac{d}{e}$$

Since deaths are assumed to be distributed as a Poisson random variable, the mean and variance of the number of deaths are both estimated by d. Therefore, an estimate of the variance of θ is given by:

$$Var(\theta) = Var(\frac{d}{e}) = \frac{1}{e^2} Var(d) = \frac{d}{e^2} \qquad (9.1)$$

assuming the expected number of deaths is constant. It is also possible to derive this formula using a maximum likelihood approach without the Poisson assumption (see Kilpatrick (1962) and Haberman (1988)). Yule (1934) is usually accredited with first proposing equation 9.1 for the variance of the mortality ratio, although, in fact, an equivalent formula was proposed by Westergaard some fifty years earlier (see Westergaard (1882) and Keiding (1987)).

When the number of deaths is "large", it is usual to use a Normal approximation. "Large" in

this context usually means "greater than 35" (Singer and Levinson (1976) and Batten (1978)), although "greater than 100" is also used (Lew and Gajewski (1990)). From equation 9.1, the standard error of the mortality ratio, $s.e.(\theta)$, is given by:

$$s.e.(\theta) \quad = \quad \sqrt{\frac{d}{e^2}} \quad = \quad \frac{\hat{\theta}}{\sqrt{d}}$$

Standard errors in this form were used in previous reports based on the Prudential Impaired Lives data set (Clarke (1961), Preston and Clarke (1966), Clarke (1979), Leighton (1987), Papaconstantinou (1988)).

Approximate 90% confidence limits, $C.L.(\theta)$, for the mortality ratio are given by:

$$C.L.(\theta) \quad = \quad (\hat{\theta} \pm 1{\cdot}645\, s.e.(\theta)) \quad = \quad \hat{\theta}(1 \pm \frac{1{\cdot}645}{\sqrt{d}})$$

and approximate 95% confidence limits are given by:

$$C.L.(\theta) \quad = \quad \hat{\theta}(1 \pm \frac{1{\cdot}96}{\sqrt{d}})$$

When the number of deaths is less than 35, the errors in the above formulae become appreciable. In particular, the limits are no longer symmetrical about the estimator $\hat{\theta}$ (although it is possible, but not common practice, to calculate non symmetrical confidence limits). A better estimate for the confidence limits can be obtained by using the exact Poisson distribution for the number of deaths. Table 9.1 shows the upper and lower limits at the 90% and 95% levels with respect to numbers of deaths up to 100. For example, suppose the observed number of deaths is 20 and 10 deaths were expected. The estimate of the mortality ratio is 200%. From Table 9.1, the 90% lower limit of the number of deaths is 13·3 and the upper limit is 29·1. The 90% confidence limits of the mortality ratio are therefore 133% and 291%. Using the Normal approximation, the 90% confidence limits would be 126% and 274%. The Poisson based confidence limits, together with the Normal approximation when the number of deaths is large, have been propounded by Singer and Levinson (1976), Batten

(1978), Papaconstantinou (1988), and Lew and Gajewski (1990).

Table 9.1. Confidence Limits based on Number of Observed Deaths

Deaths	95% Limits		90% Limits	
Observed	Lower	Upper	Lower	Upper
d	*LL*	*UL*	*LL*	*UL*
3	0·6	8·8	0·8	7·8
4	1·1	10·2	1·4	9·2
5	1·6	11·7	2·0	10·5
6	2·2	13·1	2·6	11·8
7	2·8	14·4	3·3	13·1
8	3·5	15·8	4·0	14·4
9	4·1	17·1	4·7	15·7
10	4·8	18·4	5·4	17·0
11	5·5	19·7	6·2	18·2
12	6·2	21·0	6·9	19·4
13	6·9	22·2	7·7	20·7
14	7·7	23·5	8·5	21·9
15	8·4	24·7	9·2	23·1
16	9·1	26·0	10·0	24·3
17	9·9	27·2	10·8	25·5
18	10·7	28·4	11·6	26·7
19	11·4	29·7	12·4	27·9
20	12·2	30·9	13·3	29·1
22	13·8	33·3	14·9	31·4
24	15·4	35·7	16·5	33·8
26	17·0	38·1	18·2	36·1
28	18·6	40·5	19·9	38·4
30	20·2	42·8	21·6	40·7
32	21·9	45·2	23·3	43·0
34	23·5	47·5	25·0	45·3
36	25·2	49·8	26·7	47·5
38	26·9	52·2	28·5	49·8
40	28·6	54·5	30·2	52·1
45	32·8	60·2	34·6	57·7
50	37·1	65·9	39·0	63·3
55	41·4	71·6	43·4	68·9
60	45·8	77·2	47·9	74·4
65	50·2	82·8	52·3	79·9
70	54·6	88·4	56·8	85·4
75	59·0	94·0	61·3	90·9
80	63·4	99·6	65·9	96·4
85	67·9	105·1	70·4	101·8
90	72·4	110·6	75·0	107·2
95	76·9	116·1	79·6	112·7
100	81·4	121·6	84·1	118·1

9.2 Model Based Confidence Intervals

In the modelling context, the problem is a little more complicated. The mortality ratio $\hat{\theta}$ is now represented by the exponential of a sum of parameter estimates, representing the effects of different levels of significant factors. That is:

$$\hat{\theta} = exp(\underline{\hat{\beta}}'\underline{z})$$

In *GLIM* terminology, $\underline{\hat{\beta}}'\underline{z}$ is called the linear predictor, and a logarithmic link is declared. Using *GLIM*, estimates of the β parameters are given together with their standard errors. The problem is to find a simple method of calculating an approximate confidence interval for $exp(\underline{\hat{\beta}}'\underline{z})$. Furthermore, it is desirable if this model based confidence interval is similar to the confidence interval calculated using traditional methods, where a comparison is possible (*i.e.* when main effects are fitted separately).

The simplest solution to this problem, which is easy to implement in practice, is to calculate an approximate confidence interval for the linear predictor, then transform back through the link function to create a confidence interval for θ (see Aitkin *et al* (1990)). Since the linear predictor is approximately Normally distributed, confidence limits for the linear predictor, $C.L.(lp)$, are given by:

$$C.L.(lp) = (\underline{\hat{\beta}}'\underline{z}) \pm 1{\cdot}645\ s.e.(lp) \qquad 90\%\ \text{limits}$$

or

$$C.L.(lp) = (\underline{\hat{\beta}}'\underline{z}) \pm 1{\cdot}96\ s.e.(lp) \qquad 95\%\ \text{limits}$$

where $s.e.(lp)$ is the standard error of the linear predictor. The standard error of the linear predictor is simply the square root of the variance of the linear predictor, which is obtainable directly from *GLIM*. Exponentiating the lower and upper limits obtained in this way gives the desired result; that is, *model based* confidence limits of the mortality ratios.

9.3 Implementation in *GLIM*

Implementation in *GLIM* is straightforward and only requires the use of three system vectors, *%lp*, *%os*, and *%vl*. *%lp* is a vector containing values of the fitted linear predictors, together with any offset (*%os*) declared in the model. *%vl* is a vector containing values of the variances of the linear predictors. *%vl* needs to be extracted after fitting a model by submitting the *GLIM* command **$*extract* %*vl* $**. Estimates of the model based mortality ratios are obtained using:

$$\hat{\theta} = exp(\%lp - \%os)$$

and model based confidence limits of the mortality ratios are obtained using:

$$C.L.(\theta) = exp(\%lp - \%os \pm 1{\cdot}645\sqrt{\%vl}) \qquad \text{90\% limits} \qquad (9.2)$$

or

$$C.L.(\theta) = exp(\%lp - \%os \pm 1{\cdot}96\sqrt{\%vl}) \qquad \text{95\% limits} \qquad (9.3)$$

The variances of the linear predictors are calculated automatically within *GLIM* using the design matrix, $\underline{z}$, and the variance/covariance matrix, V, using the relationship

$$\%vl = \underline{z}' V \underline{z}$$

9.4 Illustrating the Techniques

To illustrate the techniques, consider the male hypertensives analysed by Age at Entry only (see Chapter 18). Table 9.2 shows the observed mortality ratios by Age at Entry together with the number of deaths on which these results were based. Table 9.2 also shows lower and upper 95% confidence limits of the mortality ratio using traditional methods and the model based methods described here. As can be seen from Table 9.2, when the number of deaths is large,

the confidence limits given by the two methods are almost identical.

Table 9.2. Male Hypertensives by Age at Entry

Age at Entry	Mortality Ratio %	Number of Deaths	95% Confidence Limits Normal Approx. *LL*	*UL*	Model Based *LL*	*UL*
16 to 39	177	450	161	193	161	194
40 to 49	210	1029	197	223	198	224
50 to 59	139	1127	131	147	131	147
60 to 79	126	942	118	134	118	135

For a comparison when the number of deaths is small, consider the female diabetics, also analysed by Age at Entry only (see Chapter 21). Table 9.3 shows the observed mortality ratios by age at entry together with the number of deaths on which these results were based. Table 9.3 also shows the exact Poisson confidence limits, the model based confidence limits, and confidence limits based on the Normal approximation.

Table 9.3. Female Diabetics by Age at Entry

Age at Entry	Mortality Ratio %	Number of Deaths	95% Confidence Limits Normal Approx. *LL*	*UL*	Model Based *LL*	*UL*	Exact Poisson *LL*	*UL*
16 to 39	536	17	281	791	334	862	312	859
40 to 49	682	21	390	974	445	1046	422	1046
50 to 59	271	8	83	459	136	541	117	534
60 to 79	173	6	35	311	78	383	63	377

Inspecting Table 9.3, it can be seen that the model based confidence limits compare well with the exact Poisson confidence limits, particularly at the upper limits. The lower limits tend to be overestimated by the model based method, particularly when the number of deaths is below 10. However, one would hesitate before reporting results based on fewer than 10 deaths

anyway. Overall, the model based confidence limits perform extremely well. When the number of deaths is low, the standard symmetrical confidence limits based on the Normal approximation do not compare well, and show appreciable errors at both the lower and upper limits.

The power of the model based method of calculating confidence limits is particularly evident when more complex models are considered, including several factors together and their possible interactions. For any combination of parameter estimates forming a linear predictor, it is a simple matter to calculate confidence limits for the resultant model based mortality ratio within the *GLIM* software package, by straightforward application of equation 9.2 or 9.3.

For example, consider "Impairments of the Coronary Arteries". From Section 7.4.2, we know that the optimal model includes all three rating factors together, without interaction terms i.e. model A+D+C. Suppose we are interested in finding an approximate 95% confidence limit for the mortality ratio when age at entry is between 50 and 59, policy duration is between 2 and 5 years, and complications are present. From Section 7.4.1, the factor levels for this scenario are $i = j = k = 2$, giving a mortality ratio of 3·56. Straightforward application of equation 9.3 in *GLIM* gives the approximate confidence interval as (2·76, 4·57).

Thus the model based confidence limits are simple to calculate and practically expedient: the same techniques are used when the number of deaths is small or large, and when the fitted model is simple or complex.

Confidence limits for the mortality ratios have not been shown in Part II of this thesis because their inclusion would complicate the numerous tables and expand their size unduly. However, sufficient information is provided to enable confidence limits to be calculated for results from the main effects models using traditional methods.

Chapter 10 The Multiplicative Model Reformulated

In Chapter 6, the multiplicative model for the mortality ratio was derived by first developing the survival log likelihood (equation 6.5), then introducing the multiplicative hazards model (equation 6.1). Finally, after some algebraic manipulations, the generalised linear model analogue was developed. It is shown below that, starting from the same survival log likelihood, the multiplicative model may be reformulated using alternative algebraic manipulations.

From equation 6.5,

$$log\ L\ =\ \sum_{i=1}^{N}\left(\delta_i\, log\,\lambda(t_i,\ \underline{z}_i)\ -\ \int_{\tau_i}^{t_i}\lambda(u,\ \underline{z}_i)\,du\right)$$

Introducing the multiplicative hazards model

$$\lambda(t,\ \underline{z})\ =\ \lambda^*(t)\,exp(\underline{\beta}'\,\underline{z})$$

gives $$log\ L(\underline{\beta})\ =\ \sum_{i=1}^{N}(\delta_i\, log\,\lambda^*(t_i)\ +\ \delta_i(\underline{\beta}'\,\underline{z}_i)\ -\ \int_{\tau_i}^{t_i}\lambda^*(u)\,exp(\underline{\beta}'\,\underline{z}_i)\,du)$$

Partitioning individuals i into cohorts j by writing $i=(j,\ k)$ implies:

$$log\ L(\beta)\ =\ \sum_{j}\Big(\sum_{k}(\delta_{jk}\, log\,\lambda^*(t_{jk}))\ +\ d_j(\underline{\beta}'\,\underline{z}_j)\ -\ e_j\,exp(\underline{\beta}'\,\underline{z}_j)\Big)$$

where $d_j\ =\ \sum_k \delta_{jk}\ =$ actual deaths in cohort j

$e_j\ =\ \sum_k \int_{\tau_{jk}}^{t_{jk}}\lambda^*(s)\,ds\ =$ accumulated integrated base-line hazard in cohort j

$\underline{z}_i\ =\ \underline{z}_{jk}\ =\ \underline{z}_j$ for all $k \in j$

This gives $$log\ L(\beta) \ = \ c + \sum_j e_j\Big(\frac{d_j}{e_j} log(exp(\underline{\beta}'\underline{z}_j) - exp(\underline{\beta}'\underline{z}_j)\Big)$$

where c is a constant independent of β. Thus,

$$log\ L(\beta) \ = \ c + \sum_j e_j\Big(\frac{d_j}{e_j} log\, m_j - m_j)\Big) \tag{10.1}$$

where $$m_j \ = \ exp(\underline{\beta}'\underline{z}_j) \qquad i.e. \quad log\ m_j \ = \ \underline{\beta}'\underline{z}_j \tag{10.2}$$

Equation 10.1 is the kernel of the log likelihood of independent Poisson variables $\frac{d_j}{e_j} \sim IPoi(m_j)$ with weights equal to e_j and where m_j is given by equation 10.2.

Comparing equations 10.1 and 10.2 with 6.10, 6.8 and 6.9, it can be seen that, instead of modelling the number of deaths as observations of a Poisson random variable (as Renshaw suggested), it is possible to model the ratio of actual to expected deaths directly. Again a Poisson error structure is used together with the logarithmic link function. However, an offset is no longer needed; instead the expected deaths must be declared as weights (a facility allowed in *GLIM*). This means that there are two methods of analysis for the multiplicative model within the generalised linear modelling framework. Either:

(1) model the number of deaths, d_j, as the dependent variable with the logarithmic link function and offsets equal to the logarithm of the expected deaths, $log\ e_j$, or

(2) model the ratio of actual deaths to expected, $\frac{d_j}{e_j}$, as the dependent variable with the logarithmic link function and weights equal to the expected deaths, e_j.

It does not matter which method is used for model fitting since the resultant parameter estimates for the excess mortality factors are identical (although, occasionally, there may be insignificant differences in the fourth decimal places of the parameter estimates and their standard errors due to limits in the accuracy with which *GLIM* performs the necessary computations).

There are no advantages in using the alternative formulation rather than the formulation proposed by Renshaw, but it is useful since it provides the basis by which two alternative models, the *additive* model and the *power* model may be introduced. Using the additive or power models, it is not possible to derive the generalised linear model analogue using the offset facility; hence weights must be used. Reformulating the multiplicative model to use weights rather than offsets simply allows the modelling approach to be unified, using any of the three models.

Chapter 11 The Additive Model

11.1 Introduction

Models of excess mortality considered so far have been based on the multiplicative hazards model, with the result that the mortality ratio is modelled by a series of parameter estimates *multiplied* together. However, actuaries and underwriters are more familiar with the numerical rating system in which a series of credits and debits are *added* together to calculate approximate mortality ratios. Since its introduction, no attempt has been made to provide a theoretical statistical basis for the numerical rating system, by which the magnitude of the credits and debits can be estimated from data using statistical techniques. Instead, the credits and debits have been estimated using a combination of results from medico-statistical studies and wisdom of the medical profession discerned over time.

One of the principal shortcomings of the numerical rating system as it stands is that it is difficult to assess the combined effects of different impairments (and rating factors) occurring together (see Chapter 3). This problem has always been recognised and often commented on. For example, with regard to diabetics, Brackenridge (1985) states:

> "A key question that has never been satisfactorily answered from insurance statistics is: does the sum of the mortality for diabetes alone and for a complicating impairment alone represent the true expected mortality when they occur together? If not, what additional mortality may be expected when diabetes exists with different impairments?"

These questions concerning interdependence of impairments can only be answered with recourse to impaired lives investigations. If sufficient data can be accumulated, the modelling approach can be used with powerful effect to assess the statistical significance of rating factors and their interactions, and furthermore to quantify their effect.

11.2 Theoretical Derivation of the Additive Model

Outlined below is a method by which the mortality ratio can be modelled using an additive structure. This provides a theoretical statistical basis for the numerical rating system, and provides a way in which the questions raised by Brackenridge can be answered in a scientific manner. The derivation is similar to that outlined in the reformulation of the multiplicative model. Again, the starting point is the survival log likelihood (equation 6.5)

$$log\ L\ =\ \sum_{i=1}^{N}\left(\delta_i\, log\,\lambda(t_i,\ \underline{z}_i)\ -\ \int_{\tau_i}^{t_i}\lambda(u,\ \underline{z}_i)\,du\right)$$

Now, instead of introducing the multiplicative hazards function, a purely additive structure is imposed on the excess mortality factor. Thus:

$$\lambda(t,\ \underline{z})\ =\ \lambda^*(t)(\underline{\beta}'\,\underline{z}) \tag{11.1}$$

Model 11.1 is similar, in principle, to the "multiplicative with additive relative risk" models of Berry (1980), Thomas (1981) and Breslow (1985). In those models, the excess mortality factor, $\underline{\beta}'\,\underline{z}$, is simply reparameterised as $\underline{\beta}'\,\underline{z}\ =\ (1+\underline{\beta}'\,\underline{z})$. Moreover, in those models, an important distinction is that the hazard rate itself is being modelled, rather than the rate ratio.

Introducing equation 11.1 into the log likelihood function (equation 6.5) gives:

$$log\ L(\underline{\beta})\ =\ \sum_{i=1}^{N}\left(\delta_i\, log\,\lambda^*(t_i)\ +\ \delta_i\, log(\underline{\beta}'\,\underline{z}_i)\ -\ \int_{\tau_i}^{t_i}\lambda^*(u)(\underline{\beta}'\,\underline{z}_i)\,du\right)$$

Partitioning individuals i into cohorts j by writing $i=(j,\ k)$ implies:

$$log\ L(\beta)\ =\ \sum_{j}\left(\sum_{k}(\delta_{jk}\, log\,\lambda^*(t_{jk}))\ +\ d_j\, log(\underline{\beta}'\,\underline{z}_j)\ -\ e_j(\underline{\beta}'\,\underline{z}_j)\right)$$

where $\underline{z}_i\ =\ \underline{z}_{jk}\ =\ \underline{z}_j$ for all $k\ \in\ j$ and d_j and e_j are given by equations 6.6 and 6.7 respectively.

This gives $$log\ L(\beta) \ = \ c + \sum_j e_j\Big(\frac{d_j}{e_j} log(\underline{\beta}'\underline{z}_j) - (\underline{\beta}'\underline{z}_j)\Big)$$

where c is a constant independent of β. Thus,

$$log\ L(\beta) \ = \ c + \sum_j e_j\Big(\frac{d_j}{e_j} log\, m_j - m_j)\Big) \tag{11.2}$$

where $$m_j \ = \ (\underline{\beta}'\underline{z}_j) \tag{11.3}$$

Equation 11.2 is the kernel of the log likelihood of independent Poisson variables $\frac{d_j}{e_j} \sim IPoi(m_j)$ with weights equal to e_j and where m_j is given by equation 11.3.

Under the generalised linear model analogue, the ratio of actual deaths to expected, $\frac{d_j}{e_j}$, is modelled as the dependent variable with a Poisson error structure, the *identity* link function and weights equal to the expected deaths, e_j.

If this is compared with the reformulated multiplicative model, it can be seen that the only difference between the additive model and the multiplicative model (when modelling) is that a logarithmic link is declared for the multiplicative model and an identity link is declared for the additive model. Once the actual deaths and expected deaths have been calculated for each cohort indexed loosely by j, it is a simple matter to model the mortality ratio with a multiplicative or additive structure within the *GLIM* software package.

For the additive model, estimators of the mortality ratios are obtained directly from the linear predictor. There is no need to transform back through the link function since the identity link function is used. That is:

$$\hat{\theta} \ = \ \hat{\underline{\beta}}'\underline{z}$$

11.3 Illustrating the Methodology

To illustrate the methodology, consider Impairments of the Coronary Arteries, also used to illustrate the methodology of the multiplicative model in Chapter 7. Results for the null model and main effects models fitted separately using the additive model are identical to those given by the multiplicative model. The differences occur when two or more factors are modelled together, with or without their interactions.

Consider modelling Age at Entry (A), Policy Duration (D), and Complications (C) together, without interaction terms. This is equivalent to Model A+D+C in Chapter 7. The parameter estimates given by fitting this model using an additive structure are as follows:

$$\hat{\mu} = 6{\cdot}460$$

$$\hat{\alpha}_1 = 0$$
$$\hat{\alpha}_2 = -2{\cdot}675$$
$$\hat{\alpha}_3 = -4{\cdot}417$$

$$\hat{\delta}_1 = 0$$
$$\hat{\delta}_2 = -0{\cdot}906$$
$$\hat{\delta}_3 = -1{\cdot}021$$

$$\hat{\gamma}_1 = 0$$
$$\hat{\gamma}_2 = 0{\cdot}563$$

The complete set of mortality ratios given by fitting this model are shown in Table 11.1. Notice that the difference between the values in each row is constant. Also, the difference between values in each column is constant and the difference between values under "with complications" and "without complications" is constant. Therefore the complete table may be reproduced from just six parameter values.

Table 11.1 Mortality Ratios, Model A+D+C : Additive Structure

Without Complications

		Age at Entry		
		16 - 49	50 - 59	60 - 79
Duration	0 - 2	6·46	3·78	2·04
	2 - 5	5·56	2·88	1·14
	5 - 8	5·44	2·76	1·02

With Complications

		Age at Entry		
		16 - 49	50 - 59	60 - 79
Duration	0 - 2	7·02	4·34	2·60
	2 - 5	6·12	3·44	1·70
	5 - 8	6·00	3·32	1·58

To decide whether the multiplicative structure or additive structure gives the better fit, it is necessary to look at the deviances given by the two structures for the same model. From Section 7.4.2, the deviance for Model A+D+C using the multiplicative structure is 23·647. Using the additive structure, the deviance for the equivalent model is 27·985. Therefore, the multiplicative structure gives the better fit since it gives a lower deviance. Furthermore, fitting models including first order interaction terms with an additive structure indicates that there is a marginally statistically significant interaction between age at entry and the presence of complications. Using an additive structure, Model A*C+D would probably be accepted as the optimal model, with a resulting deviance for this model of 23·353. This model is more complicated than the optimal model using a multiplicative structure, for an insignificant saving in deviance. We can infer from this that the multiplicative structure is superior, in this case.

Chapter 12 The Power Model

12.1 Introduction

In general, there are no *a priori* reasons why the multiplicative structure or additive structure should provide the best fit for models of excess mortality. It is quite possible that neither structure fits very well. One method of formally discriminating between multiplicative and additive models is to embed them within a parametric family of models which contains both as special cases. This is achieved through the introduction of the family of models

$$\frac{\lambda(t, \underline{z})^{\gamma}}{\lambda^{*}(t)^{\gamma}} = (\underline{\beta}' \underline{z}) \tag{12.1}$$

which relates the rate ratio to the linear predictor by means of the power transform with exponent γ. The additive model corresponds to the case $\gamma = 1$, and the multiplicative model is obtained in the limit as γ tends to zero (see McCullagh and Nelder (1989)). The power model may be perceived as being in between the multiplicative and additive models for values of γ between 0 and 1 (the value of the power is not constrained to lie in the range 0 to 1, however). A similar approach has been proposed by Aranda-Ordaz (1983), Breslow (1985), Muirhead and Darby (1987), and Keiding *et al* (1990), although, in those papers, an important distinction is that the hazard rate itself is being modelled, rather than the rate ratio.

12.2 Theoretical Derivation of the Power Model

Like the multiplicative model and additive model, the starting point in the development of the generalised linear model analogue is the survival log likelihood given by equation 6.5:

$$log\ L = \sum_{i=1}^{N} (\delta_i \log \lambda(t_i, \underline{z}_i) - \int_{\tau_i}^{t_i} \lambda(s, \underline{z}_i)\, ds)$$

Rearranging 12.1, $\lambda(t, \underline{z}) = \lambda^*(t)(\underline{\beta}' \underline{z})^{\alpha}$ where $\alpha = \frac{1}{\gamma}$ (12.2)

The effect of introducing 12.2 into the survival log likelihood (6.5) is to yield:

$$log\ L(\beta) = \sum_{i=1}^{N} (\delta_i log \lambda^*(t_i) + \delta_i log(\underline{\beta}' \underline{z}_i)^{\alpha} - (\underline{\beta}' \underline{z}_i)^{\alpha} \int_{\tau_i}^{t_i} \lambda^*(s)\, ds)$$

Partitioning individuals i into cohorts j by writing $i=(j, k)$ implies:

$$log\ L(\beta) = \sum_{j} \Big(\sum_{k} (\delta_{jk} log \lambda^*(t_{jk})) + d_j log(\underline{\beta}' \underline{z}_j)^{\alpha} - e_j (\underline{\beta}' \underline{z}_j)^{\alpha} \Big)$$

where $\underline{z}_i = \underline{z}_{jk} = \underline{z}_j$ for all $k \in j$ and d_j and e_j are given by equations 6.6 and 6.7 respectively.

This gives $$log\ L(\beta) = c + \sum_{j} e_j \Big(\frac{d_j}{e_j} log(\underline{\beta}' \underline{z}_j)^{\alpha} - (\underline{\beta}' \underline{z}_j)^{\alpha} \Big)$$

where c is a constant independent of β. Thus,

$$log\ L(\beta) = c + \sum_{j} e_j \Big(\frac{d_j}{e_j} log\, m_j - m_j) \Big) \tag{12.3}$$

where $$m_j = (\underline{\beta}' \underline{z}_j)^{\alpha} \quad \text{i.e.} \quad m_j^{\gamma} = (\underline{\beta}' \underline{z}_j) \tag{12.4}$$

Equation 12.3 is the kernel of the log likelihood of independent Poisson variables $\frac{d_j}{e_j} \sim IPoi(m_j)$ with weights equal to e_j and where m_j is given by 12.4.

It should be noted that the derivation of the additive model is simply a special case of the derivation of the power model for the case where $\gamma=\alpha=1$.

Under the generalised linear model analogue, the power model is implemented by modelling the ratio of actual deaths to expected, $\frac{d_j}{e_j}$, as the dependent variable with a Poisson error structure, the *power* link function and weights equal to the expected deaths, e_j.

Obtaining the optimum value of the power, γ, is an iterative process. The model is fitted using a range of values of γ and the resultant model deviances are plotted against γ to obtain a *deviance profile*. By inspection, or using a search refining routine, the optimum value of γ is that which minimises the deviance. Having calculated the optimum value of the power for a given model, the model based mortality ratios are calculated by transforming back through the link function. That is:

$$\hat{\theta} = (\underline{\hat{\beta}}' \underline{z})^{\frac{1}{\gamma}}$$

12.3 Illustrating the Methodology

Consider again Impairments of the Coronary Arteries. Like the additive model, results for the null model and main effects models fitted separately using the power model are identical to those given by the multiplicative model, and differences occur when two or more factors are modelled together, with or without their interactions.

Again consider Model A+D+C (main effects fitted together, without interaction). Fitting the model for values of γ between -0.5 and $+0.5$ allows a deviance profile to be constructed, as shown in Figure 12.1. The optimal model is obtained when the value of the power is $-0{\cdot}288101$, since a model fitted with that value of the power gives the minimum deviance ($23{\cdot}290$), and leads to the following parameter estimates:

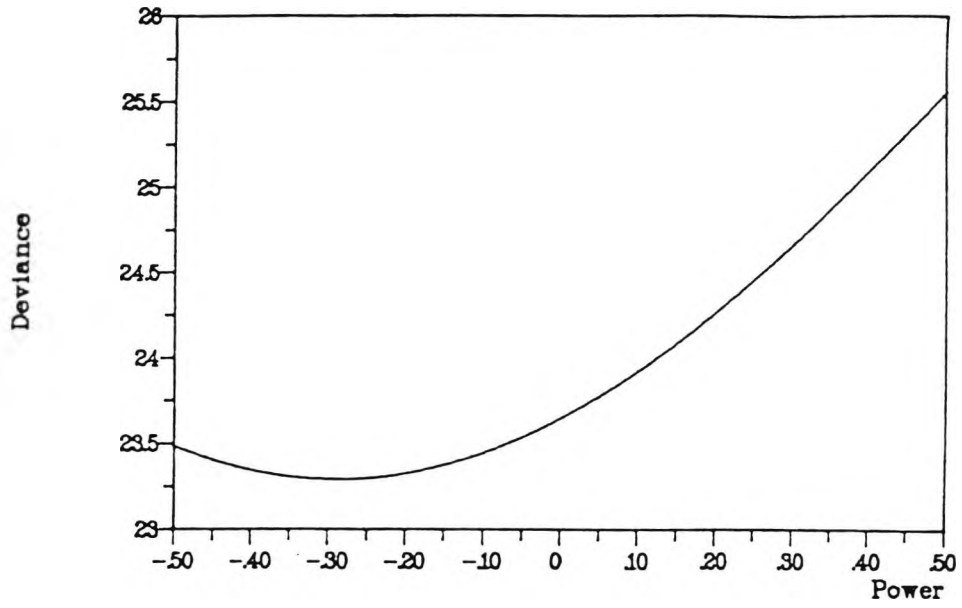
$$\hat{\mu} = 0{\cdot}5821$$

$$\begin{aligned} \hat{\alpha}_1 &= 0 \\ \hat{\alpha}_2 &= 0{\cdot}1005 \\ \hat{\alpha}_3 &= 0{\cdot}2793 \end{aligned}$$

$$\begin{aligned} \hat{\delta}_1 &= 0 \\ \hat{\delta}_2 &= 0{\cdot}0844 \\ \hat{\delta}_3 &= 0{\cdot}0616 \end{aligned}$$

$$\hat{\gamma}_1 = 0$$
$$\hat{\gamma}_2 = -0{\cdot}0733$$

Figure 12.1 Deviance Profile, Impairment Of the Coronary Arteries: Model A+D+C



The mortality ratios given by fitting this model are shown in Table 12.1. It should be remembered that this complete table can be reproduced from knowledge of the power, γ, and just six parameter estimates. It should also be noted that the mortality ratios shown here are very similar to the mortality ratios shown in Table 7.4.1 (using the multiplicative structure). Furthermore, the deviance given by the optimal power model (23·290) is very close to the deviance given by the multiplicative model (23·647). It would therefore seem reasonable, on grounds of simplicity, to be satisfied with the multiplicative structure as a reasonable model of excess mortality of male lives suffering from Impairment of the Coronary Arteries.

Table 12.1 Mortality Ratios, Model A+D+C : Power Structure

Without Complications

		Age at Entry		
		16 - 49	50 - 59	60 - 79
Duration	0 - 2	6·54	3·76	1·68
	2 - 5	4·09	2·51	1·21
	5 - 8	4·61	2·79	1·32

With Complications

		Age at Entry		
		16 - 49	50 - 59	60 - 79
Duration	0 - 2	10·44	5·58	2·29
	2 - 5	6·13	3·56	1·61
	5 - 8	7·02	4·00	1·76

Chapter 13 Modelling Excess Mortality in Practice

13.1 Modelling Excess Mortality — A Unified Approach

Whether the multiplicative, additive or power structure is being used to model excess mortality, it has been shown in Chapters 10, 11 and 12 that the implementation within the generalised linear model framework is essentially the same. In each case, the ratio of actual to expected deaths can be modelled using a Poisson error structure, with weights equal to the expected deaths. The different model structures are obtained simply by choosing between link functions; for the multiplicative model, choose the logarithmic link; for the additive structure, choose the identity link; and for the power structure, choose the power link. This provides a unified approach to modelling excess mortality.

13.2 Which Structure: Multiplicative, Additive or Power?

Although the additive structure has an appealing connection with the numerical rating system, substantial difficulties can be anticipated when using this structure to model excess mortality. This is because parameter estimates in the linear predictor may be positive or negative, and some combinations of parameter estimates for particular model structures could result in a negative linear predictor overall. This implies a negative mortality ratio, since the identity link function is used. This is obviously untenable, since mortality ratios must be greater than zero. There are internal checks within the *GLIM* system to prevent this from happening (when using the Poisson error structure with identity link), which often results in an error message saying that an inappropriate model has been chosen. The same problem can also appear when using the power structure, for the same reasons. Slightly more flexibility in modelling is allowed by setting up either model using the *OWN* directives within *GLIM*, although the system might still crash nonetheless.

Using the multiplicative model, there are no such problems because the logarithmic link ensures that the resultant mortality ratios are always greater than zero. The theoretical range of the linear predictor is always between plus and minus infinity, but the resultant mortality ratios from a fitted model are constrained to be positive, since they are calculated as the exponential of the linear predictor.

Provided that a satisfactory fit can be achieved, the multiplicative structure should be the first choice for modelling excess mortality, since it is the easiest to work with, and has reasonable properties. The additive and power structures are inherently flawed, for the reasons stated above, and should only be used if there are good *a priori* reasons for doing so, or if empirical evidence suggests that a much better fit is provided by using either of these alternative structures.

The multiplicative, additive and power models which are the basis of this thesis are intended to represent excess mortality in a way which is convenient, flexible, entirely empirical and has an appealing connection with traditional actuarial mortality ratios. It is not suggested that these are the *only* models that can be used to model excess mortality; a better model may well be suggested for a particular impairment in the light of prior knowledge. However, in general, it will not be possible to attach any physical interpretation to a particular fitted model in terms of disease aetiology or the controlling process causing "early" death.

The results shown in Part II of this thesis have all been prepared using the multiplicative structure to model excess mortality, as proposed by Renshaw (1988). All models including two or more rating factors fitted together were also fitted using the additive and power structures. In no case did either of the alternative structures improve the fit sufficiently to warrant its use; in most cases, the fit obtained using these alternatives was considerably worse, or it was not possible to achieve a fit at all.

PART II RESULTS

Chapter 14 Preliminary Considerations

14.1 Introduction

The modelling approach outlined in Part I of this thesis has been used to analyse the entire Prudential Impaired Lives data set. Full results can be found in England (1990, 1991 and 1992). Included in this thesis are the results from an analysis of just eight of the impairments; as such, the results shown here are only a subset of the complete results, but are more than sufficient to show that the theory has considerable practical use.

14.2 Presentation of Results

For each impairment, a consistent approach has been taken to the presentation of the results. The coding scheme used by the Prudential is presented first, including the subsidiary codes, followed by the classification adopted for analysis. Starting with male lives, summary statistics are presented showing total number of policies, total number of deaths, distribution of policies by age at entry and distribution of deaths by cause. The results follow next, starting with the null model (the overall mortality ratio), then showing results for the main effects models (mortality ratios by individual rating factors). After assessing the statistical significance of rating factors, more complex models may be considered, and interdependence between significant rating factors investigated. The analysis is then repeated for female lives.

Having analysed the data, a comparison with other studies is presented, firstly considering other studies based on the Prudential data set, then considering other studies with which a suitable comparison can be made. Finally, current underwriting practice is considered briefly, with reference to the underwriting manuals of three reinsurance companies (Mercantile and General, Munich Re and Swiss Re).

At each stage, comments on the results are brief, merely highlighting the important features without repeating in words what is given in the tables. Furthermore, little attempt has been made at interpreting the results; interpretation is left to medical practitioners, underwriters and others more qualified to perform such a task.

14.3 Classification for Analysis

The classification adopted for analysis is highly dependent on the amount of data available. Where possible, the full medical coding, including subsidiary codes, has been used to try to give as many medical factors as possible. In this respect, the subsidiary codes are particularly important; it is surprising that these have never been used in any analysis in the history of the Prudential data set. Results according to the subsidiary codes are, therefore, shown for the first time, and show the effect on excess mortality of such features as family history or time since the symptoms of a particular condition were observed.

Ratings given by the numerical system are often shown according to broad groupings of age at entry. Similar broad groupings of age at entry have therefore been used as an additional factor in modelling excess mortality. For each impairment, the effect of age at entry can be measured and its statistical significance assessed.

Where an impairment has been included since the start of the investigation, the data were further subdivided by calendar year of entry, giving another model factor. Results by calendar year of entry can show changes in excess mortality over time, although often these results show no general trend, and any fluctuations are therefore hard to interpret.

The effect of policy duration has also been included, where possible. The results can highlight when, during the life of a policy, the greatest excess risk occurs. Care must be taken in interpretation of the results by policy duration, however, since the results at the highest policy duration can only come from policies issued during the earliest calendar years of the study: a

strong calendar year effect might also appear within the results by policy duration.

For each factor, results must be interpreted with reference to the number of deaths on which the results were based. Little confidence can be attached to results based on very low numbers of deaths. Where the number of deaths is below 10, neighbouring groups have been combined. Even results based on 10 deaths might be considered unreliable; it might be better to group further until results are based on 20 or 30 deaths. Results given by individual model factors are shown together with the number of deaths on which the results were based, where the number of deaths are shown in parentheses (unless stated otherwise, any number shown in parentheses refers to numbers of deaths).

14.4 Cause of Death

For each impairment, total deaths are tabulated by cause of death, and also by percentage deaths by cause. To interpret these results, it is necessary to make a comparison with percentage deaths expected for each cause, had the impairment not been present. It is very difficult to find a suitable basis for this comparison, mainly due to the length of the investigation (which is 41 years for the data available); percentage deaths by cause in the general population changes over time. Since an analysis by cause of death, although interesting, was not the prime purpose of this study, the solution adopted by Papaconstantinou (1988) was also adopted here. That is, a comparison is made with the percentage deaths by cause for all impairments combined in the entire Prudential data set. This is not ideal since those impairments for which there is a large volume of data will have a greater influence on the basis used. Where a comparison is made for individual impairments, the comparison is shown graphically. Table 14.1 shows the percentage deaths by cause for all impairments combined in the entire Prudential data set, for males and females separately, together with the total deaths on which the results were based. It can be seen that the amount of data available for analysing female lives is considerably smaller than the amount available for male lives. Consequently, analysis of the female experience is often less detailed than for males.

Table 14.1 Percentage Deaths by Cause, All Impairments Combined

	Cause of Death	Percentage Deaths	
		Males	Females
1	Tuberculosis	0·2	0·4
2	Lung Cancer	7·1	3·9
3	Other Cancer	14·5	29·8
4	Leukaemia	1·1	0·9
5	Diabetes	0·3	0·7
6	Vascular Lesions	5·8	9·3
7	Ischaemic Heart Disease	34·4	15·2
8	Other Circulatory Diseases	7·3	6·6
9	Influenza	0·0	0·1
10	Pneumonia	0·9	1·0
11	Bronchitis	0·9	0·6
12	Peptic Ulcer	0·4	0·2
13	Nephritis	0·3	0·2
14	Motor Accident	1·5	0·9
15	Other Accident	2·4	2·7
16	Suicide	1·9	3·1
17	Miscellaneous (unspecified)	20·8	24·4
	Total Deaths	28762	2635

14.5 The Impairments Considered

The eight impairments considered in this thesis are:

1 Impairment of the Coronary Arteries
2 Hypertension
3 Epilepsy
4 Psycho-Neuroses
5 Diabetes Mellitus
6 Underweight
7 Overweight
8 Asthma

The full results are shown in Chapters 17 to 24, but a brief summary of the reasons for including the various impairments is outlined below.

Impairment of the coronary arteries is a relatively small subset of data, but is simple to analyse and gives good results, which is why it was used for illustration purposes in Part I. The results in Part II are more complete. The analysis of this impairment shows that the modelling approach can be used to good effect, even when the amount of data available is small.

The hypertensive subset is much larger and can be analysed using six rating factors. Use of the subsidiary codes allows family history of cardiovascular disease to be analysed for the first time. Overall, the results for this impairment are interesting, and show that overweight combined with hypertension does not have a statistically significant effect on excess mortality. An analysis of female hypertensives is also interesting and shows that excess mortality for female hypertensives is somewhat lower than excess mortality for males.

The analysis of epileptics is, again, based on a fairly small subset of data, but is interesting since it allows the effect of "duration since last episode" to be assessed. These results are shown for the first time, since this factor was included in the subsidiary code.

Psycho-neuroses also constitutes a large subset of data, and can be analysed using five rating factors. Not surprisingly, the most significant rating factor is severity of the condition. Perhaps the most interesting feature of this data set, however, does not involve modelling excess mortality, but is found in the analysis of cause of death. The levels of "accidents" and suicides are alarming, and a further breakdown of these causes by severity of the condition is somewhat shocking.

The subsidiary codes again proved interesting when analysing diabetics, since they allowed analysis of family history of diabetes. Overall, diabetics require fairly complex analysis due to the calendar year of entry effects. Interpretation of the results when modelling factors together

is difficult, and highlights the problems which can manifest themselves when fitted models are not simple.

Both the underweight and overweight subsets are interesting because of the large volume of data available for analysis. The overweight subset constitutes about $\frac{1}{6}$ of the entire data set. Again, interpretation and presentation of results becomes difficult when complex models are considered.

Yet again, the subsidiary codes become useful when analysing asthmatics, and allow results to be obtained by "duration since last attack".

Chapter 15 Previous Studies Based on the Prudential Data

15.1 Scope of Previous Studies

Several authors have reported results based on the Prudential Impaired Lives data set, mostly within the pages of the Journal of the Institute of Actuaries (together with lengthy discussions). A comparison of the various reports is informative and provides insight into the changes in excess mortality over the forty years that the investigation has been operative. To aid interpretation, it is worth considering differences in the scope of the studies and the approach adopted by the various authors. Table 15.1 shows the calendar years of experience used in each study and the impairments investigated.

Table 15.1 Studies Based on the Prudential Data

Author	Calendar Years of Study	Impairments investigated
Clarke (1961)	1947-58	All impairments
Preston and Clarke (1966)	1947-63	All impairments
Clarke (1979)	1964-73	All impairments
Leighton (1987)	1974-83	All Impairments
Papaconstantinou (1988)	1947-81	All impairments
Renshaw (1988)	1947-81	Hypertension only
Haberman and Renshaw (1990)	1947-81	Peptic Ulcer only

The studies by Clarke (1961), Preston and Clarke (1966), Clarke (1979), and Leighton (1987) form a series in which the authors use the same approach in their analyses. Traditional methods are used to produce standard mortality ratios (A/E ratios) only. The differences in the reports are a result of the calendar years of experience considered, and the control experience used in the calculation of expected deaths. Excess mortality of female lives has been commented on briefly where there were sufficient data to provide useful results.

Papaconstantinou used the entire data set (as available at the time) in his analysis, and again used conventional exposed-to-risk theory in the calculation of mortality rates. However, he used the data available to produce not only the familiar mortality ratios (A/E ratios) but also *Excess Death Rates*, and measures of excess mortality based on cumulative mortality. He considered all impairments for which there were more than 100 entrants.

The Prudential Impaired Lives data set was first used in statistical modelling of excess mortality by Renshaw, who adopted the multiplicative hazards approach. Renshaw used the same data set as Papaconstantinou (1947-81), and provided results for Hypertensives only. Subsequently, Haberman and Renshaw used the modelling approach to analyse the Peptic Ulcer subset, again using the modelling approach and data for the period 1947-81.

The modelling approach proposed by Renshaw was used to generate the results shown in this thesis, but includes additionally data for the period 1982-87. Thus, the results are based on the experience of policies effected between 1947 and 1987. Also included is an analysis involving the subsidiary codes.

15.2 Differences in Results

On the whole, the results of the various authors concur remarkably well. Where there are differences, these will be due to combinations of the following factors:

(i) the period under study. Obviously, results will differ purely and simply because they are based on different calendar years of experience.

(ii) the control experience used in the calculation of expected deaths. Again, it is obvious that this will affect the results obtained. However, it is less obvious that a small difference in the expected deaths may change the value of a mortality ratio significantly,

when expected deaths are low. This is because expected deaths appear in the denominator.

(iii) the method used for the calculation of expected deaths. Using traditional methods, expected deaths are given by an expression of the form Eq' (or $\sum Eq'$), that is, the exposed-to-risk multiplied by the standard mortality rate. Using the modelling approach, expected deaths are given by an expression of the form $\sum\int \lambda^s(t)\,dt$, that is, the accumulated integrated base-line hazard. Differences may arise in the values of expected deaths given by these methods. Again, when expected deaths are low, these differences may cause a significant change in the value of a mortality ratio, since expected deaths appear in the denominator.

(iv) errors. Whereas researchers should take great care to eliminate any sources of error, it is possible that errors occur which affect the results obtained, especially in a large scale study, such as an impaired lives investigation. Errors may be due to:

- incorrect recording of data
- mistakes in data manipulation
- programming mistakes
- incorrect calculation using results
- typographical errors in reports.

Usually, major errors are immediately noticeable but minor errors may pass undetected. A good example of an error found in the course of analysing the Prudential data is the discovery of 23 males recorded as suffering from fibroids of the uterus.

Chapter 16 The Basis For Expected Deaths

16.1 Introduction

Choosing a suitable control experience for the calculation of expected deaths is a difficult task. Reading the ensuing discussion of papers presented to the Institute of Actuaries concerning the mortality of impaired lives, it is clear that criticism often rests with the choice of control experience used.

The most appropriate basis for measuring the excess mortality experienced by the various classes of impaired lives would be the (graduated) experience of first class standard assured lives of comparable duration over the same period of time, insured by the same life office. Since this is not available, a suitable alternative must be found.

16.2 The Control Experience Used in Previous Studies

For the early investigations based on the Prudential data (Clarke (1961) and Preston and Clarke (1966)), considerable effort was expended in finding a control experience having the same "weighted mean point of time" and "weighted mean duration" as the impaired lives data. This was abandoned in later studies in favour of the A67-70 table (Clarke (1979)), and the A67-70 table with a one year age deduction (Leighton (1987)). Papaconstantinou (1988) used the entire data set as available at the time (1947-81) instead of restricted calendar years of exposure, and the problem of finding a suitable control experience became more acute. Papaconstantinou used a complex procedure for adjusting the A67-70 table (with a 2 year select period) to give a mortality table for each calendar year between 1947 and 1981. The modifications were based on approximate algebraic relationships proposed by the Continuous Mortality Investigation Bureau (*C.M.I.R* 3 (1978)), which related the A67-70 table to the mortality experience of specific quadrennia between 1949 and 1978. Linear relationships were

proposed of the form:

$$q^{(i)}_{[x]+n} = a^{(i)} q^{67-70}_{[x]+n} + b^{(i)} \qquad n=0, 1, 2 \qquad (16.1)$$

where a and b are constants and the superscript (i) refers to the period under consideration, and $q^{67-70}_{[x]+n}$ refers to the mortality rate at age $[x]+n$ from the A67-70 table.

By applying linear interpolation and extrapolation techniques on the constants $a^{(i)}$ and $b^{(i)}$, Papaconstantinou produced a set of constants $a^{(k)}$ and $b^{(k)}$, where the superscript (k) refers to individual years between 1947 and 1981. For female lives, Papaconstantinou used the rates produced for male lives with a 4 year age deduction.

A major problem of adjustments of the form shown in equation (16.1) is that the same adjustment is applied across all years of age x. This cannot be justified since mortality rates at older ages have not changed much over time, although mortality rates at younger ages have changed considerably.

Renshaw (1988), in his turn, felt that the method adopted by Papaconstantinou was unnecessarily detailed and condensed Papaconstantinou's rates by averaging over consecutive 5 yearly intervals, commencing with 1947-51 and ending with 1977-81. However, Renshaw used the modelling approach in his analysis, which necessitates the use of forces of mortality rather than initial rates of mortality, and transformed the condensed rates into forces of mortality using the relationship

$$\lambda^{*}(x, d, c) = -log(1-q(x, d, c))$$

where $\lambda^{*}(x, d, c)$ represents the standard force of mortality at age x, duration d, and calendar year interval c, and $q(x, d, c)$ represents the equivalent initial mortality rate.

Renshaw voiced concern over his choice of control experience after inspecting results for male

hypertensives by calendar year of entry, which showed a significant peak in calendar years of entry 1967 to 71. Renshaw suggested that a possible cause of this feature might be the particular choice of control experience, and envisaged further research into the influence of the "base-line hazard function" used as the basis for expected deaths.

16.3 The Control Experience Used in this Study

In the analysis covered in Part II of this thesis, it was decided, for male lives, to use forces of mortality from the A67-70 table (with 2 year select period) unmodified for all calendar years of entry. The period used in forming this table (1967 to 70) is roughly mid-way through the period for which the Prudential data are available (1947 to 87). The expected deaths calculated for the earlier part of the study will tend to be understated (resulting in an overstatement of the excess mortality). Similarly, the expected deaths calculated for the later part of the study will tend to be overstated (resulting in an understatement of the excess mortality). However, a comparison of the results of Renshaw with the results included in this thesis (see Chapter 18) reveals that the differences in bases for expected deaths used has, on the whole, made little difference to the figures for excess mortality. Nevertheless, the rather simple basis for expected deaths used here is not ideal. A better basis could be created by obtaining the Continuous Mortality Investigation Bureau data for whole life and endowment assurances, grouping into suitable intervals (quadrennia or quinquennia), and graduating to form a smooth set of rates for each time interval. However, this would involve a considerable amount of extra work.

For female lives, initially, the analysis was carried out twice, using both the A67-70 (2) select table with a 4 year age deduction, and the FA75-78 table. However, early results indicated that the A67-70 table with a 4 year age deduction for female lives is not a particularly good choice in this context. The Continuous Mortality Investigation Bureau investigated the mortality of female whole life and endowment assurance policyholders (see *C.M.I.R.* 5 (1981)) and reported "The expected deaths have been calculated according to the A1967-70 table with

a four year age deduction; this table was based on a male experience and it is believed the 4-year adjustment corresponds approximately to current practice". However, they express some doubt over the use of the A67-70 table adjusted in this way (with the same adjustment made across all years of age) concluding that "the A1967-70 table is of quite a different shape" (to a contemporaneous table for female lives). In the light of this, it was decided to use only the FA75-78 table as a basis for expected deaths for female lives.

For some impairments, the only data available is in respect of calendar years of entry 1980 to 1987, due to a change in classification on 1/1/1980. In those cases, forces of mortality from the AM80 and AF80 tables (see *C.M.I.R.* 10 (1991)) were used as the basis for expected deaths.

Chapter 17 Impairment of the Coronary Arteries

17.1 Prudential Impairment Codes and Classification for Analysis:

Age below 50 (at entry)

171	Number of coronary vessels involved unknown	- no complications.
172	Number of coronary vessels involved unknown	- with complications.
173	One coronary vessel involved	- no complications.
174	One coronary vessel involved	- with complications.
175	More than one coronary vessel involved	- no complications.
176	More than one coronary vessel involved	- with complications.

Age above 50 (at entry)

181 - 186 defined as for 171-176.

Subsidiary Codes:

A	Onset within 2 years	- no surgery.
B	Onset 2-4 yrs ago	- no surgery.
C	Onset 4-6 yrs ago	- no surgery.
D	Onset over 6 yrs ago	- no surgery.
E	Onset within 2 years	- surgery.
F	Onset 2-4 yrs ago	- surgery.
G	Onset 4-6 yrs ago	- surgery.
H	Onset over 6 yrs ago	- surgery.

Impairment of the coronary arteries includes: thrombosis, occlusion, ischaemia, infarction, angina. "Complications" involve subsequent angina, dyspnoea or chest pain on exertion. "Surgery" is usually coronary by-pass.

The sub-classification for this impairment group is comprehensive, giving plenty of scope for analysis. However, this classification was introduced on 1/1/80, replacing code 180 (impairment of coronary arteries), resulting in insufficient data available for extensive analysis.

Consequently, for analysis, the data were grouped by age of entry, policy duration, and whether complications are apparent or not, as follows:

A:	Age at Entry - 4 levels	1 - Age at Entry 16 to 39
		2 - Age at Entry 40 to 49
		3 - Age at Entry 50 to 59
		4 - Age at Entry 60 to 79
D:	Policy Duration - 3 levels	1 - 0 to 2 yrs
		2 - 2 to 5 yrs
		3 - 5 to 8 yrs
I:	Complications - 2 levels	1 - Without Complications
		2 - With Complications

17.2 Male Lives

17.2.1 Summary Statistics

Number of Entrants: 3307
Number of Deaths: 297

Table 17.1 Distribution by Age at Entry

Age at Entry	Number of Entrants	Percentage
16-39	170	5·1
40-49	893	27·0
50-59	1454	44·0
60-79	790	23·9
Total	3307	

Approximately $\frac{2}{3}$ of policyholders are aged over 50 at entry, with only 5% aged under 40 at entry.

Table 17.2 Distribution by Cause of Death

	Number of Deaths	Percentage
Lung Cancer	6	2·0
Other Cancer	24	8·1
Vascular Lesions	8	2·7
Ischaemic Heart Disease	178	59·9
Other Circulatory Diseases	11	3·7
Motor Accident	1	0·3
Miscellaneous (unspecified)	69	23·2
Total	297	

Not surprisingly, ischaemic heart disease is the predominant cause of death, taking almost 60% of total deaths. This compares with approximately 35% dying from this cause in the entire Prudential data set.

17.2.2 Results: The Overall Mortality Ratio

Taken as a group, the overall mortality ratio for male life assurance policyholders with impairments of the coronary arteries at entry was found to be 248% (extra mortality = +148%) based on 297 deaths.

17.2.3 Results: Main Effects Fitted Separately

Table 17.3 Mortality Ratios by Age at Entry (Factor A)

Age at Entry	MR %	
16 - 39	1839	(9)
40 - 49	534	(45)
50 - 59	343	(129)
60 - 79	155	(114)

Table 17.4 Mortality Ratios by Policy Duration (Factor D)

Duration	MR %	
0 - 2	334	(118)
2 - 5	208	(128)
5 - 8	222	(51)

Table 17.5 Mortality Ratios by "Complications" (Factor I)

	MR %	
Without Complications	220	(160)
With Complications	290	(137)

Clearly excess mortality decreases as age at entry increases (1839% for age at entry 16-39, 155% for age at entry 60-79). However the mortality ratio of 1839% for age at entry 16-39 is based on only 9 deaths. Consequently, these were grouped with age at entry 40-49, giving:

Table 17.6 Mortality Ratios by Age at Entry (grouped)

Age at Entry	MR %	
16 - 49	606	(54)
50 - 59	343	(129)
60 - 79	156	(114)

If a mortality ratio of over 500% is considered an uninsurable risk, then two distinct groups appear, those aged under 50 at entry who would be considered uninsurable and those aged over 50 at entry who would be insurable.

Looking at mortality ratios by duration, it can be seen that excess mortality drops after the first two years from over 300% to just over 200%.

For the first time, results can be shown with respect to whether complications are observed at entry. Where there are no complications, a mortality ratio of 220% is observed, which rises to 290% when there are additional complications.

17.2.4 Significance of Main Effects

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev.	D of F	
H_0	116·56	23			
A	42·41	21	74·15	2	$\ll$ ·05%
D	102·27	21	14·29	2	·075%
I	110·95	22	5·61	1	1·75%

Referring the differences in model deviances to the appropriate χ^2 distribution reveals that all the main effects tested are very highly significant (tail area less than 5% in all cases). This leads to the investigation of more complex models, starting with a model including all three significant main effects together.

17.2.5 Main Effects Fitted Together, No Interaction : Model A+I+D

Table 17.7 Mortality Ratios (%) given by Model A+I+D

Without Complications

		Age at Entry		
		16 - 49	50 - 59	60 - 79
Duration	0 - 2	675	385	176
	2 - 5	446	254	116
	5 - 8	486	277	126

With Complications

		Age at Entry		
		16 - 49	50 - 59	60 - 79
Duration	0 - 2	945	539	246
	2 - 5	624	356	162
	5 - 8	680	388	177

From Table 17.7, it can be observed that:

(i) entries under "with complications" are 1·4 times equivalent entries under "without complications";

(ii) entries in the second row are 0·66 times entries in the first row, and entries in the third row are 0·72 times entries in the first row;

(iii) entries in the second column are 0·57 times entries in the first column, and entries in the third column are 0·26 times entries in the first column.

This is a direct result of using the multiplicative model, without interaction terms. There is no conflict between the results shown here and the results for main effects (i.e. figures are of the same order and changes are in the same direction). The advantage is that more information is conveyed using simple mathematical relationships.

17.2.6 Models Including Interaction Terms

Models involving first order interaction terms were fitted, leading to the following deviance table:

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev.	D of F	
A+I+D	23·647	18			
A*I+D	21·718	16	1·93	2	35%
I*D+A	21·973	16	1·67	2	45%
A*D+I	20·050	14	3·60	4	47%

Referring differences in model deviances to the appropriate χ^2 distribution indicates that none of the first order interaction terms are statistically significant. This leaves the model A+D+I as the "best" model in that it is parsimonious and conveys the salient features of the data available.

17.3 Female Lives

17.3.1 Summary Statistics

Number of Entrants: 417

Number of Deaths: 29

Table 17.8 Distribution by Age at Entry

Age at Entry	Number of Entrants	Percentage
16-39	18	4·3
40-49	73	17·5
50-59	135	32·4
60-79	191	45·8
Total	417	

Almost half of the policyholders were aged over 60 at entry, with only $\frac{1}{5}$ below age 50.

Table 17.9 Distribution by Cause of Death

	Number of Deaths	Percentage
Lung Cancer	2	6·9
Other Cancer	1	3·4
Leukaemia	1	3·4
Vascular Lesions	4	13·8
Ischaemic Heart Disease	14	48·3
Other Circulatory Diseases	2	6·9
Miscellaneous (unspecified)	5	17·2
Total	29	

Ischaemic heart disease was the cause of death for almost half the total deaths, with circulatory diseases taking almost 70% of deaths altogether (none of the deaths came from policies taken out below age 50 at entry). In the entire Prudential data set, deaths from circulatory diseases

accounted for only 31% of total deaths.

17.3.2 Results: Overall Mortality Ratio

The overall mortality ratio for female lives was found to be 184%, based on 29 deaths. This is lower than the equivalent figure for male lives.

17.3.3 Results: Main Effects Fitted Separately

Due to the low number of deaths, it is only sensible to look at results for the main effects fitted separately, and even then after grouping the data.

Table 17.10 Mortality Ratios by Age at Entry (Factor A)

Age at Entry	MR %	
16 - 59	255	(6)
60 - 79	171	(23)

Table 17.11 Mortality Ratios by Policy Duration (Factor D)

Duration	MR %	
0 - 2 yrs	266	(13)
2 - 8 yrs	147	(16)

Table 17.12 Mortality Ratios by "Complications" (Factor I)

	MR %	
Without Complications	117	(12)
With Complications	308	(17)

Excess mortality is higher for policies taken out below age 60. Also, excess mortality is a lot higher for low policy durations. Excess mortality is low when there are no complications (MR of 117%) but rises substantially (to 308%) in the presence of complications.

17.3.4 Significance of Main Effects

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev.	D of F	
H_0	16·076	23			
A	15·384	22	0·692	1	45%
D	13·639	22	2·437	1	12·5%
I	9·358	22	6·718	1	< 1%

Referring differences in model deviances to a χ^2 distribution reveals that age at entry and policy duration are not statistically significant, and presence or absence of complications is highly statistically significant. There is, however, some evidence of higher mortality ratios at lower ages at entry and lower policy duration, although not strong enough to be statistically significant.

17.4 Comparison with Other Studies – Male Lives

Cardiovascular disorders are the principal cause of death in the UK and USA, particularly ischaemic heart disease. Consequently, many studies have been undertaken showing results for rates of mortality with which some sort of comparison can be attempted. However care must be taken since many of the results are from North American studies which are now at least 20 years out of date, and are frequently based on low numbers of deaths.

The Prudential impairment group named "Impairment of the Coronary Arteries" comprises a number of specific conditions, namely: Thrombosis, Occlusion, Ischaemia, Infarction, Angina

Brackenridge (1985) presents results for these impairments separately under the general title "Ischaemic Heart Disease" and Singer & Levinson (1976) present results under the title "Coronary Heart Disease".

17.4.1 Other Studies Based on the Prudential Impaired Lives Data Set

Since this impairment group was introduced in 1959 results have been reported for male lives by Clarke (1979), Leighton (1987) and Papaconstantinou (1988).

Clarke (1979) and Leighton (1987)

The study by Clarke was based on lives exposed-to-risk between 1964 and 1973, and the study by Leighton was based on lives exposed to risk over the next ten years, viz 1974-83.

Table 17.13 Mortality Ratios given by Clarke and Leighton

Policy Duration	Mortality Ratios	
	Clark (64-73)	Leighton (74-83)
0 - 1	297 (11)	589 (23)
1 - 2	346 (12)	472 (29)
2 and over, age at entry under 50	1146 (14)	877 (32)
2 and over, age at entry over 50	388 (44)	237 (159)
2 and over, all ages	462 (58)	269 (191)
All Durations Combined	411 (81)	300 (243)

(Number of deaths in parentheses)

The results from the studies by Clarke and Leighton are shown in Table 17.13. The study by Clarke seems to show excess mortality increasing with policy duration, whereas Leighton's study shows decreasing excess mortality with increasing policy duration. Both studies show substantially higher excess mortality for ages at entry below 50. The conclusion drawn by

Leighton was that extra mortality is of the order of +400%, substantially higher at young ages. For underwriting purposes he proposed a descending scale from +800 for age 40 at entry, to +300 for age 60 at entry. This would not seem unreasonable in the light of the results shown in this thesis.

Papaconstantinou

Papaconstantinou analysed all the data available for code 180 between 1959 and 1979 (Code 180 was replaced by codes 171-176 and 181-186 on 1/1/80). He noted a total number of 2688 entrants (male lives only). The overall mortality ratio was found to be 617% (higher than the two previous studies), with mortality ratios of 1489% below age 50 at entry and 458% above age 50 at entry. Results by age at entry and policy duration are shown in Tables 17.14 and 17.15 (with numbers of deaths in parentheses).

Table 17.14 Papaconstantinou: Mortality Ratios by Age at Entry

Age at Entry	MR %	
30 - 49	1489	(102)
50 - 59	562	(136)
60 - 79	270	(36)

Table 17.15 Papaconstantinou: Mortality Ratios by Policy Duration

Duration	MR %	
0-2	1104	(86)
2-10	543	(167)

These results give the highest mortality ratios of any study based on these data. In particular, Papaconstantinou's results by Policy Duration do not seem to agree with the results found by others. This may be due to the basis for expected deaths adopted by Papaconstantinou.

All studies based on the Prudential data show mortality ratios decreasing with increasing age at entry (with substantially higher excess mortality at young ages at entry) and most studies agree that excess mortality decreases with increasing policy duration.

17.4.2 The Lincoln National Life Insurance Company 1970

Of all the North American studies available, the study by Cochran and Buck (1970) is perhaps the best as a source for comparison purposes since it is based on the experience of life insurance policyholders insured by a single company and all suffering from conditions classified by the Prudential as impairments of the coronary arteries. Brackenridge quotes results from this source, as do Singer and Levinson (extensively). The study was based on the experience of 10,434 life insurance policyholders insured by the Lincoln National Life Insurance Company between 1947 and 1961, the end point of the investigation being the policy anniversary in 1969 (or prior termination date). The impairments considered were Infarction, Occlusion, Thrombosis, Angina, and Coronary Sclerosis. Cases diagnosed as suffering from definite coronary artery disease were divided into three groups designated A, B and C. Combining over these three groups gives a study population with impairments very similar to those included in the Prudential group "impairment of the coronary arteries", with an overall mortality ratio of 310% based on 346 deaths. Mortality ratios by age at entry, policy duration and time from attack to policy application are shown in Tables 17.16, 17.17 and 17.18 (with numbers of deaths in parentheses).

Table 17.16 Cochran & Buck: Mortality Ratios by Age at Entry

Age at Entry	MR %	
10 - 39	336	(24)
40 - 49	344	(154)
50 - 64	280	(168)

Table 17.17 Cochran & Buck: Mortality Ratios by Policy Duration

Duration	MR %	
0 - 3 yrs	457	(84)
3 - 5	360	(55)
5 - 10	250	(100)
> 10	282	(107)

Table 17.18 Cochran & Buck: Mortality Ratios by Interval from Attack to Policy Application

Interval	MR %	
0 - 2 yrs	253	(27)
2 - 5	332	(95)
5 - 10	321	(159)
> 10	295	(65)

The results show certain similarities with those of the Prudential (80-87) study, with mortality ratios falling with increasing age at entry and policy duration. A surprising feature is the low mortality ratio for ages at entry 10-39.

17.5 Comparison with Other Studies — Female Lives

It is generally regarded that ischaemic heart disease occurs less commonly in women than men, particularly at younger ages. However there is very little information available regarding the survival rates of women suffering from coronary heart disease and whether these differ from survival rates for men.

One study which has reported mortality ratios for men and women was based on members of the Health Insurance Plan of Greater New York (HIP) with diagnosis of first acute myocardical

infarction or angina pectoris (without previous infarction) made between 1961 and 1965 (see Singer & Levinson (1976), Frank *et al* (1973) and Weinblatt *et al* (1973)).

17.5.1 HIP Results

Total Coronary Heart Disease - Myocardial Infarction and Angina Pectoris - all ages combined (25-64): (Basis of expected mortality: cohorts of male and female HIP members free of CHD.)

	MR %
Males	385
Females	355

These results show higher excess mortality overall than the Prudential (80-87) study, although both studies seem to indicate that for this impairment, male lives are a slightly higher extra risk to a life assurance company than female lives.

17.6 Rating of "Impairment of the Coronary Arteries"

Because this impairment includes a variety of specific medical conditions, it is only possible to give a general description of current underwriting practice. Moreover, the underwriting manuals considered take different approaches to the rating of the various conditions. In general, for each condition, ratings are determined by time since last attack and age at entry. Applicants are likely to be postponed for at least six months since last attack, with high ratings initially reducing to low ratings steadily over as much as ten years. Ratings at low ages at entry are consideraby higher than those at high ages. For example, the basic ratings proposed by Swiss Re for "less than two years since last attack" are as follows:

Age at Entry:	34 or less	+400
	35 to 44	+275
	45 to 54	+175
	55 or more	+120

For "2 to 5 years since last attack", the ratings are approximately half those for "less than 2 years since last attack". For "more than 5 years since last attack", the ratings are approximately a quarter of those for "less than 2 years since last attack".

In general, although the ratings are quite high, they do not seem high enough at the younger ages at entry, when compared with the results from the Prudential study.

Current practice would seems to be to rate female lives in exactly the same way as male lives, with no adjustment to allow for differences between the sexes.

Chapter 18 Hypertension

18.1 Prudential Impairment Codes and Classification for Analysis:

Weight standard ±19%			Weight standard +20% or over			Blood Pressure (mm Hg)	
Age at Entry			Age at Entry				
<40	40-59	>59	<40	40-59	>59	Systolic	Diastolic
110	130	150	120	140	160	150-165	<95
111	131	151	121	141	161	>165	<95
112	132	152	122	142	162	150-165	95-105
113	133	153	123	143	163	>165	95-105
114	134	154	124	144	164	150-165	95-105
115	135	155	125	145	165	>165	>105
116	136	156	126	146	166	150-165	Uncertain
117	137	157	127	147	167	>165	Uncertain
118	138	158	128	148	168	<150	≥95

Subsidiary codes:

- A Family History - Good (coded E prior to 1.1.80)
- B Family History - Fair - one death below age 65 from vascular accident*
- C Family History - Poor - two or more deaths below age 65 from vascular accident*
- F Family History - Fair with two or more deaths from cardiovascular disease+
- G Family History - Poor - general tendency to early death
- H Family History - Poor - with two or more deaths from cardiovascular disease+

* introduced 1.1.80

+ deaths at age 70 and above disregarded.

This impairment group refers to essential (primary) hypertension only, and may include slight or moderate tachycardia or slight arteriosclerosis.

The hypertension impairment group has been in existence since the start of the Prudential impaired lives investigation in July 1947. Consequently there are plenty of data for extensive analysis. The classification of the subsidiary codes changed on 1/1/80 resulting in slight problems when analysing results concerning family history.

For analysis the data were sub-classified according to:

A: Age at Entry - 4 levels

1 - Age at Entry 16 to 39
2 - Age at Entry 40 to 49
3 - Age at Entry 50 to 59
4 - Age at Entry 60 to 79

B: Blood Pressure - 7 levels

		Diastolic Pressure (mm Hg)		
		<95	95-105	>105
Systolic Pressure (mm Hg)	<150		7	
	150-165	1	3	5
	>165	2	4	6

C: Calendar yr of Entry - 8 levels

1 - 47 to 51
2 - 52 to 56
3 - 57 to 61
4 - 62 to 66
5 - 67 to 71
6 - 72 to 76
7 - 77 to 81
8 - 82 to 86

D: Policy Duration - 6 levels

1 - 0 to 2 yrs
2 - 2 to 5 yrs
3 - 5 to 10 yrs
4 - 10 to 15 yrs
5 - 15 to 20 yrs
6 - 20 to 40 yrs

H: Family History - 2 levels

1 - Good
2 - Poor (one or more death from cardiovascular disease - deaths at age 70+ discarded).

W: Weight at Entry - 2 levels

1 - Standard ± 19%
2 - Standard + 20% or over

18.2 Hypertension – Some Considerations

18.2.1 Classification

It is customary to classify hypertension as ***primary*** (essential), constituting the vast majority, or ***secondary*** to a long list of diseases (some pathological process).

Brackenridge (1985) lists the possible causes of secondary hypertension as:

1) Renal causes.
2) Coarction of the aorta.
3) Phaeochromocytoma.
4) Cushing's syndrome, corticosteroid therapy.
5) Primary aldosteronism.
6) Polyarteritis nodosa.
7) Thyrotoxicosis.
8) Oral contraceptives.

Secondary hypertension is amenable to treatment to the extent that the underlying cause can be successfully treated. Hypertension, whatever the cause, leads to cardiovascular damage and the benefits of antihypertensive therapy in secondary hypertension are every bit as useful in reducing mortality as in primary hypertension.

In the Prudential study the hypertension group refers to primary hypertension only.

18.2.2 Criteria for Hypertension

As Lew and Singer (see Singer and Levinson (1976)) point out, "blood pressure may be considered *elevated* only in terms of some *normal* standard."

Brackenridge (1985) states that "the definition of normal blood pressure or range of normal

blood pressure to the underwriter would be that which in the group produces mortality ranging from 100% or below to not more than 125% of standard insurance acceptances.

The New York Heart Association (1955) propose that:

> "Any blood pressure combination up to and including 139/89 (139 mm Hg systolic and 89 mm Hg diastolic) is regarded as normotensive. Any combination including a systolic pressure of 160 and up, or a diastolic pressure of 95 and up, or both, is classified as definitely hypertensive. Any combination below 160/95 is classified as borderline hypertensive provided it is not within the normotensive limit."

Lew and Singer reported that the above definition "has been widely accepted" and used this definition themselves. Brackenridge, too, seems happy with the above definition:

> "Major statistical blood pressure studies which have been carried out in North America . . . indicate that normal blood pressure is 140/90 mm Hg or below, and that mortality gradually rises higher than 125 percent of average as blood pressure increases above this level, except in age groups over 50, when slightly higher levels of blood pressure still produce mortality within the average range."

Other studies have also tended to support the view that the initial level of blood pressure consistent with standard mortality under age 50 is close to 140/90 mm Hg.

Furthermore, it is generally accepted that blood pressure rises gradually as age increases, and increased levels in older age groups may still be compatible with average mortality. Also, significant differences in mortality with blood pressure level are observed in the "normal" or normotensive range.

18.3 Male Lives

18.3.1 Summary Statistics

Number of Entrants: 29919
Number of Deaths: 3548

Table 18.1 Distribution by Age at Entry

Age at Entry	Number of Entrants	Percentage
16-39	11399	38·1
40-49	8195	27·4
50-59	7419	24·8
60-79	2902	9·7
Total	29915	

Approximately $\frac{2}{3}$ of policyholders are aged below 50 at entry, $\frac{1}{3}$ above age 50 at entry.

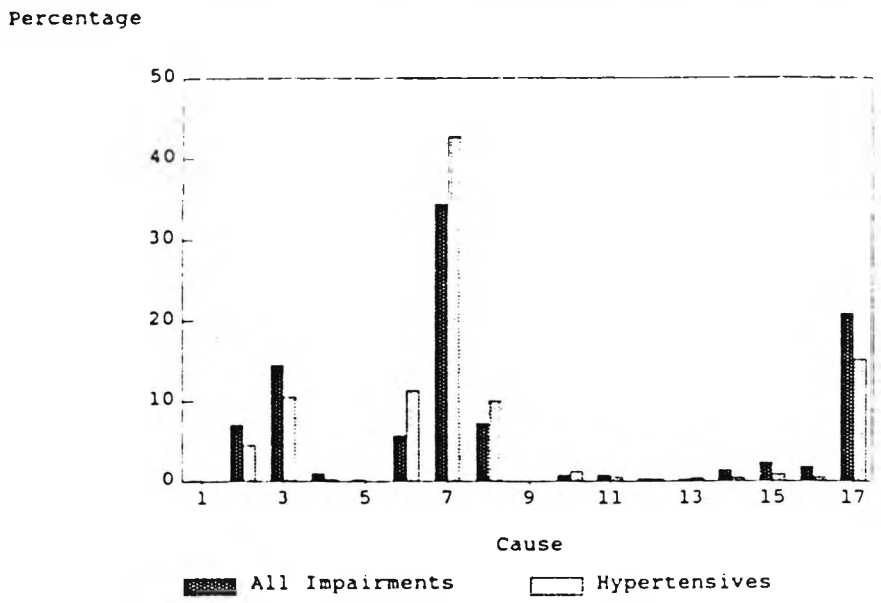
Table 18.2 Distribution by Cause of Death

	Number of Deaths	Percentage
1. Tuberculosis	2	0·1
2. Lung Cancer	164	4·6
3. Other Cancer	376	10·6
4. Leukaemia	11	0·3
5. Diabetes	3	0·1
6. Vascular Lesions	403	11·4
7. Ischaemic Heart Disease	1516	42·7
8. Other Circulatory Diseases	358	10·1
9. Influenza	3	0·1
10. Pneumonia	47	1·3
11. Bronchitis	23	0·6
12. Peptic Ulcer	13	0·4
13. Nephritis	18	0·5
14. Motor Accident	20	0·6
15. Other Accident	34	1·0
16. Suicide	21	0·6
17. Miscellaneous (unspecified)	536	15·1
Total	3548	

Circulatory diseases are the predominant cause of death, taking 64·2% of total deaths. This compares with only 47% of total deaths in the entire Prudential study coming from circulatory

diseases. A graph of percentage deaths by cause for the hypertension subset and all impairments in the entire Prudential data set is shown in Figure 18.1.

Figure 18.1 Percentage Deaths by Cause: Male Hypertensives



18.3.2 Results: The Overall Mortality Ratio

Taken as a group, the overall mortality ratio for male hypertensives was found to be 154% (based on 3548 deaths), using the A67-70 (2) table as a basis for expected deaths.

18.3.3 Results: Main Effects Fitted Separately

Table 18.3 Mortality Ratios by Age at Entry (Factor A)

Age at Entry	MR %	
16 - 39	177	(450)
40 - 49	210	(1029)
50 - 59	139	(1127)
60 - 79	126	(942)

Excess mortality is higher for ages at entry below age 50, as would be expected. However, a surprising feature here is the rise in excess mortality (from +77% to +110%) for the age at entry group 40 to 49 compared with age at entry 16 to 39.

Table 18.4 Mortality Ratios by Blood Pressure (Factor B)

		Diastolic Pressure (mm Hg)		
		<95	95-105	>105
Systolic Pressure (mm Hg)	<150		145 (591)	
	150-165	133 (1237)	166 (833)	180 (132)
	>165	159 (202)	185 (293)	257 (260)

As expected, the mortality ratios increase with increasing blood pressure (from top left to bottom right). The pressure levels shown here are consistent with the definitions of hypertension given earlier. None of the associated mortality ratios are below 125%, therefore we are quite clearly dealing with blood pressure levels outside the normal range (or normotensive range).

Table 18.5 Mortality Ratios by Weight Levels (Factor W)

	MR %	
Standard ± 19%	153	(2914)
Standard +20% or over	162	(634)

Although there is a slight increase in extra mortality associated with overweight, this increase is not as large as may have been expected.

Table 18.6 Mortality Ratios by Policy Duration (Factor D)

Duration	MR %	
0 - 2 yrs	150	(279)
2 - 5	135	(584)
5 - 10	164	(1120)
10 - 15	175	(765)
15 - 20	155	(443)
over 20 yrs	128	(357)

Excess mortality falls after the first two years duration then rises steadily to a peak at fifteen years duration, after which excess mortality falls off again.

Table 18.7 Mortality Ratios by Family History (Factor H)

	MR %	
Good	148	(2645)
Poor	177	(903)

These results clearly show a rise in excess mortality associated with a family history of cardiovascular disease. This is the first time results have been shown including family history.

Table 18.8 Mortality Ratios by Calendar Year of Entry (Factor C)

	MR %	
1947 - 51	157	(694)
1952 - 56	154	(842)
1957 - 61	156	(655)
1962 - 66	170	(639)
1967 - 71	186	(274)
1972 - 76	157	(167)
1977 - 81	105	(205)
1982 - 86	104	(72)

The mortality ratios for calendar years of entry 1947 to 1961 are surprisingly stable (approx. 155%). Beyond 1961 the mortality ratios rise, reaching a peak for calendar years of entry 1967 - 71. Beyond 1971 the mortality ratios fall until there is almost no excess mortality.

18.3.4 Significance of Main Effects

Referring differences in model deviances (see table below) to the appropriate χ^2 distribution reveals that all the main effects are highly significant with the exception of Weight, which is not significant (although there is some evidence of a higher mortality ratio with higher weight levels). Consequently, the weight factor can be dropped from subsequent model fitting.

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev.	D of F	
H_0	3615·7	3808			
A	3464·7	3805	151·0	3	$\ll$ ·05%
B	3509·6	3802	106·1	6	$\ll$ ·05%
C	3553·2	3801	62·5	7	$\ll$ ·05%
D	3575·3	3803	40·4	5	$\ll$ ·05%
H	3594·5	3807	21·2	1	$\ll$ ·05%
W	3614·0	3807	1·7	1	20%

18.3.5 More Complex Models

More complicated models (other than main effects fitted separately) may be fitted and the significance of interaction terms assessed. The results from the more important of these models shall be reported. In presenting the results, it is useful to think in terms of a parametric representation of the *GLIM* models.

	Factor	Parameter	
A	Age at Entry	α_i	$i = 1, \ldots, 4$
B	Blood Pressure at Entry	β_j	$j = 1, \ldots, 7$
C	Calendar Year of Entry	γ_k	$k = 1, \ldots, 8$
D	Policy Duration	δ_l	$l = 1, \ldots, 6$
H	Family History	ρ_m	$m = 1, 2$

An additional parameter, μ, is involved which is similar to the constant coefficient in conventional linear regression.

For example, Model A+B+C+D+H has parametric representation:

$$exp\,(\mu + \alpha_i + \beta_j + \gamma_k + \delta_l + \rho_m)$$

for the mortality factors. This equals:

$$exp\,(\mu).\ exp\,(\alpha_i).\ exp\,(\beta_j).\ exp\,(\gamma_k).\ exp\,(\delta_l).\ exp\,(\rho_m)$$

i.e. multiplicative effects.

18.3.5.1 Main Effects Fitted Together, No Interaction

Model : A+B+C+D+H

Parametric Representation : $exp\,(\mu + \alpha_i + \beta_j + \gamma_k + \delta_l + \rho_m)$

This model caters for all five (significant) factors simultaneously. Mortality ratios may be deduced from Table 18.9 by forming the product of relevant entries (multiply by 100 for the ratio as a percentage).

Table 18.9 Excess Mortality Factors: Model A+B+C+D+H

$exp(\mu) = 1{\cdot}95$

AGE AT ENTRY :	16-39	40-49	50-59	60-79
$exp(\alpha_i)$	1·00	1·09	0·70	0·65

PRESSURE :

$exp(\beta_j)$		Diastolic Pressure (mm Hg)		
		<95	95-105	>105
Systolic Pressure (mm Hg)	<150		0·96	
	150-165	1·00	1·18	1·22
	>165	1·35	1·47	1·95

CALENDAR YEAR :	47-51	52-56	57-61	62-66	67-71	72-76	77-81	82-86
OF ENTRY $exp(\gamma_k)$	1·00	0·98	0·94	0·94	0·99	0·81	0·60	0·63

DURATION :	0-2	2-5	5-10	10-15	15-20	>20
$exp(\delta_l)$	1·00	0·90	1·01	0·96	0·80	0·72

FAMILY HISTORY :	Good	Poor
$exp(\rho_m)$	1·00	1·20

Consider a hypothetical example: A man took out a whole life policy in 1977 at age 45. Upon medical examination his blood pressure was recorded as 155/100. From the proposal form it was found that his family history of cardiovascular disease could be classified as "Good". The policy has now been in existence for 10 years and an estimate of the excess mortality associated with this risk is required for the remainder of the policy.

Policy Duration	Mortality Ratio	Excess Mortality
10-15 yrs	$1{\cdot}95\times1{\cdot}09\times1{\cdot}18\times0{\cdot}60\times1{\cdot}00\times0{\cdot}96 = 1{\cdot}44$	+44 %
15-20 yrs	$1{\cdot}95\times1{\cdot}09\times1{\cdot}18\times0{\cdot}60\times1{\cdot}00\times0{\cdot}80 = 1{\cdot}20$	+20 %
>20 yrs	$1{\cdot}95\times1{\cdot}09\times1{\cdot}18\times0{\cdot}60\times1{\cdot}00\times0{\cdot}72 = 1{\cdot}08$	+8 %

18.3.5.2 Residual Plots

If a model provides a good fit, a histogram of deviance residuals should be approximately bell shaped (i.e. approximately Normal). Also, a scatter plot of deviance residuals against linear predictor should be pattern free. Any other patterns would be indicative of lack of fit.

Figure 18.2 Histogram of Residuals

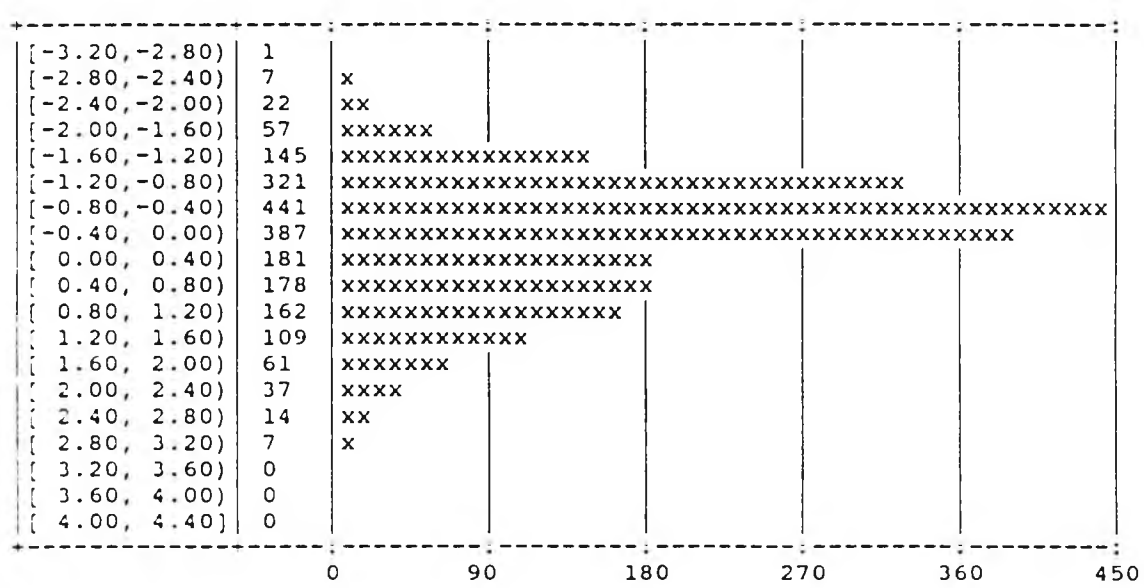
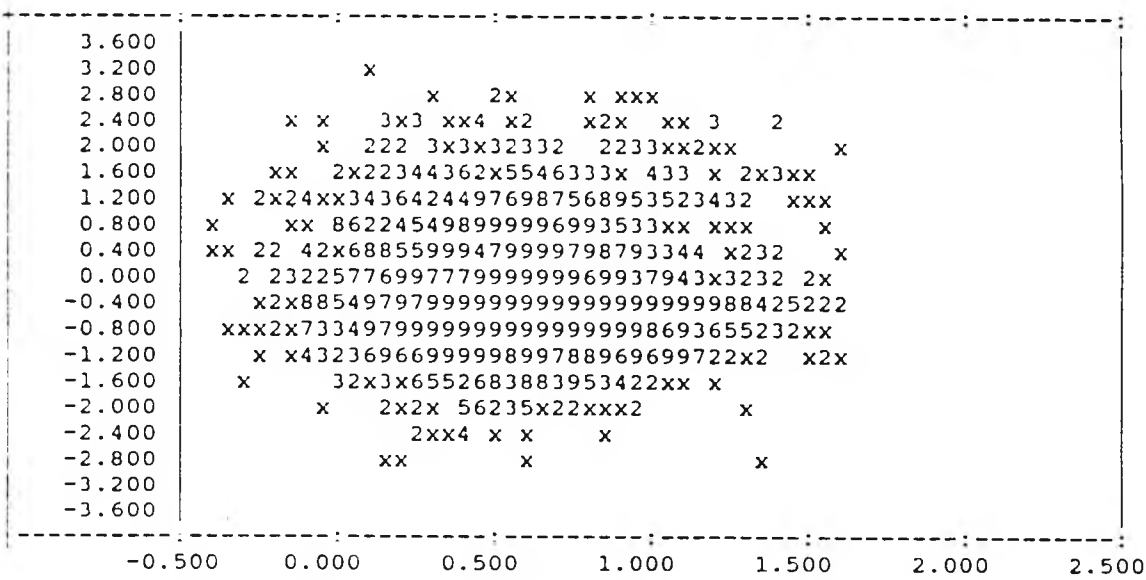


Figure 18.3 Residuals against Linear Predictor



The shape of the histogram of residuals for this model is highly satisfactory. It may be noticed that it is slightly skewed. This is to be expected when using a generalised linear model with a Poisson error structure.

The plot of residuals against linear predictor is also highly satisfactory.

18.3.5.3 First Order Interactions

Models including first order interaction terms were fitted and their deviances analysed to assess the significance of the first order interaction terms, leading to the following deviance table:

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev.	D of F	
A+B+C+D+H	2076·6	2107			
A*B+C+D+H	2051·9	2089	24·7	18	14 %
A*D+B+C+H	2053·5	2092	23·1	15	8 %
A*C+B+D+H	2051·6	2086	25·0	21	25 %
A*H+B+C+D	2071·7	2104	4·9	3	19 %
B*C+A+D+H	2044·2	2065	32·4	42	40 %
B*D+A+C+H	2029·2	2077	47·4	30	2 %
B*H+A+C+D	2068·7	2101	7·9	6	25 %
C*D+A+B+H	2059·9	2082	16·7	25	90 %
C*H+A+B+D	2072·9	2100	3·7	7	80 %
D*H+A+C+B	2073·2	2102	3·4	5	63 %

These results indicate that the interaction between Blood Pressure and Policy Duration is statistically significant. Although not shown in this thesis, the interaction between age at entry and policy duration may also be investigated, since it is marginally significant (see England (1990)).

The mortality factors given by fitting Model B*D+A+C+H are shown in Table 18.10.

Table 18.10 Excess Mortality Factors: Model B*D+A+C+H

Parametric Representation : $exp\,(\alpha_i)\ exp\,(\gamma_k)\ exp\,(\rho_m)\ \ exp\,(\mu + \beta_j + \delta_l + \beta\delta_{jl})$

AGE AT ENTRY :	16-39	40-49	50-59	60-79
$exp\,(\alpha_i)$	1·00	1·09	0·70	0·65

CALENDAR YEAR :	47-51	52-56	57-61	62-66	67-71	72-76	77-81	82-86
OF ENTRY $exp\,(\gamma_k)$	1·00	0·98	0·94	0·94	0·99	0·81	0·60	0·63

FAMILY HISTORY :	Good	Poor
$exp\,(\rho_m)$	1·00	1·20

$exp\,(\mu + \beta_j + \delta_l + \beta\delta_{jl})$

$l = 1$: Duration 0-2			$l = 2$: Duration 2-5			$l = 3$: Duration 5-10		
	— 1·73 —			— 1·68 —			— 1·69 —	
1·63	2·21	1·32	1·86	1·95	2·29	2·07	2·32	2·75
3·82	3·40	5·00	1·51	2·99	3·59	2·62	2·80	3·61

$l = 4$: Duration 10-15			$l = 5$: Duration 15-20			$l = 6$: Duration >20		
	— 1·99 —			— 1·70 —			— 1·23 —	
1·70	2·18	2·45	1·55	1·89	1·63	1·46	1·87	1·03
3·38	2·66	3·47	2·19	1·92	2·12	1·59	1·11	2·99

Again, mortality factors may be found by forming the product of relevant entries. Residual plots look very similar to those for the model A+B+C+D+H. The main source of the interaction may be highlighted by fitting the related model B*D (parametric representation : $exp\,(\mu + \beta_j + \delta_l + \beta\delta_{jl})$) and presenting the mortality ratios obtained in graphical form, as shown in Figures 18.4 and 18.5. If there were no interaction terms (or the interaction terms were not significant) the lines on the graph would not cross, therefore, the main source of interaction is the policy duration group 0 to 2 years.

Figure 18.4 Mortality Ratios: Model B*D, Policy Duration 0 to 10 years

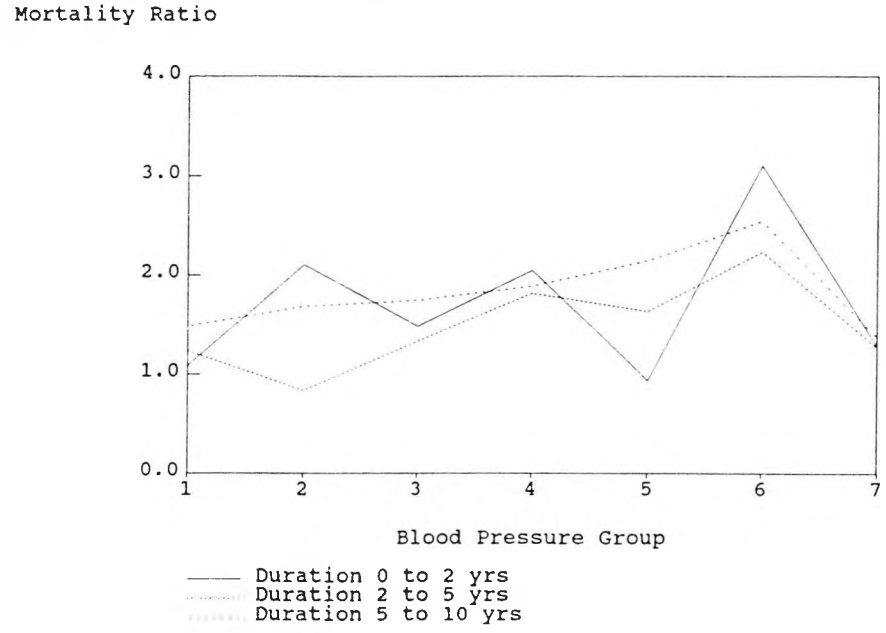
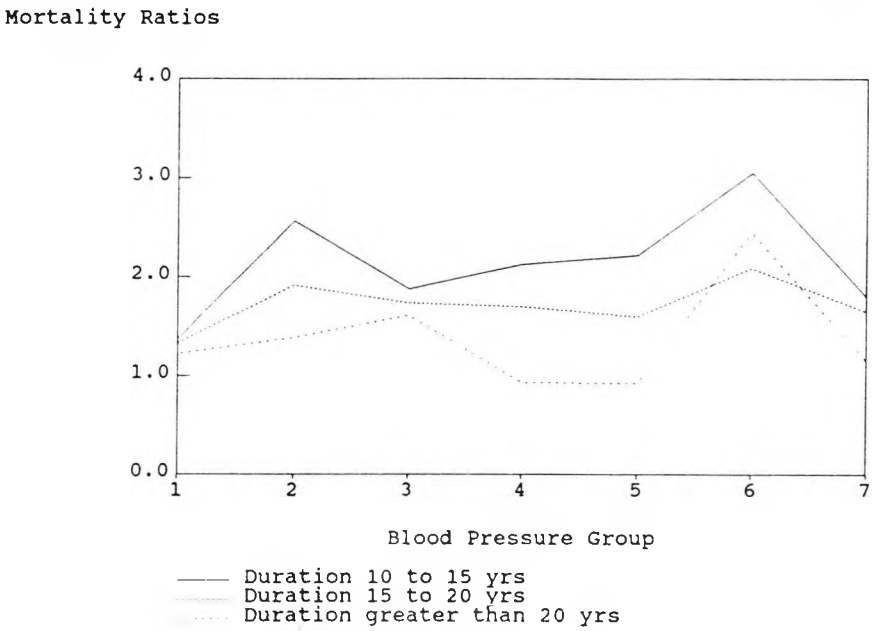


Figure 18.5 Mortality Ratios: Model B*D, Policy Duration over 10 years



18.3.5.4 Other Models

It is possible to fit other models including more than one interaction term in the same model, or higher order interaction terms. As Renshaw (1988) states, "Clearly more detailed models still involving complex interaction terms are open to scrutiny by this method . . . . provided the data are sufficiently numerous to render the exercise meaningful". Furthermore,

consideration needs to be given to interpretation of fitted models: the more complex the model, the harder it is to interpret.

Also, less complicated models may be fitted when it is necessary to compare results with other studies in which the classification of data is not as broad as in this study. However, it is certainly the case that the use of any model simpler than the main effects model fitted simultaneously (Model A+B+C+D+H) would constitute a loss of information.

18.4 Female Lives

For female lives, the analysis was carried out twice; once using the FA75-78 table as a basis for expected deaths, and once using the A67-70 table with a 4 year age deduction.

18.4.1 Summary Statistics

Number of Entrants: 5551

Number of Deaths: 375

Table 18.11 Distribution by Age at Entry

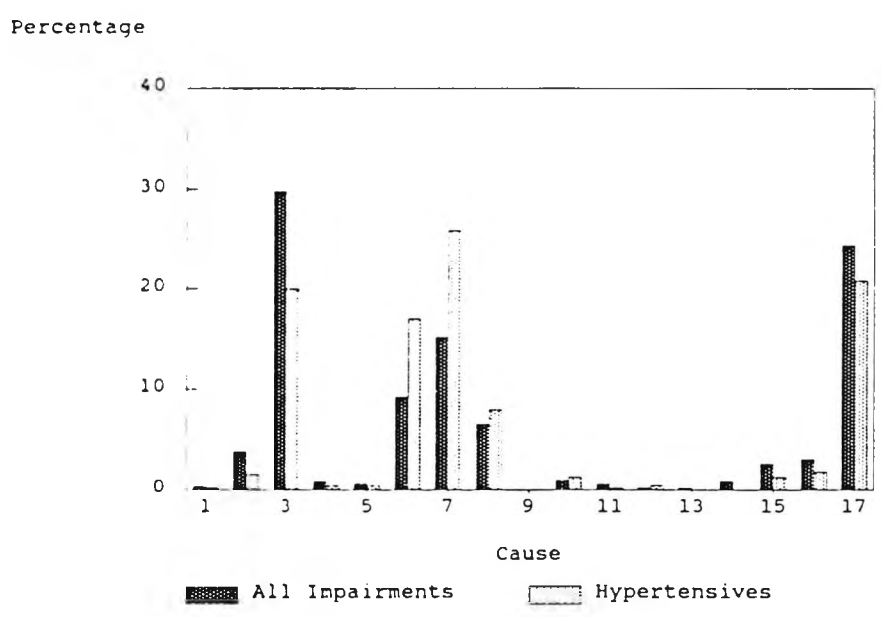
Age at Entry	Number of Entrants	Percentage
16-39	879	15·8
40-49	1500	27·0
50-59	1861	33·5
60-79	1311	23·6
Total	5551	

57% of policyholders were aged above 50 at entry, 43% below age 50 at entry.

Table 18.12 Distribution by Cause of Death

	Number of Deaths	Percentage
1. Tuberculosis	1	0·3
2. Lung Cancer	6	1·6
3. Other Cancer	75	20·0
4. Leukaemia	2	0·5
5. Diabetes	2	0·5
6. Vascular Lesions	64	17·1
7. Ischaemic Heart Disease	97	25·9
8. Other Circulatory Diseases	30	8·0
9. Influenza	0	0
10. Pneumonia	5	1·3
11. Bronchitis	1	0·3
12. Peptic Ulcer	2	0·5
13. Nephritis	0	0
14. Motor Accident	0	0
15. Other Accident	5	1·3
16. Suicide	7	1·9
17. Miscellaneous (unspecified)	78	20·8
Total	375	

Figure 18.6 Percentage Deaths by Cause: Female Hypertensives



Percentage deaths by cause are shown in Figure 18.6. Circulatory diseases (causes 6, 7 and 8) are the cause of death for 51% of total deaths. This compares with 31% of total deaths for all impairment groups combined over the entire Prudential data set.

18.4.2 Results: The Overall Mortality Ratio; Base-line Hazard – FA75-78

The overall mortality ratio for female hypertensives, using the FA75-78 table as a basis for expected deaths, was found to be 121% (based on 375 deaths observed).

18.4.3 Results: Main Effects Fitted Separately; Base-line Hazard – FA75-78

Table 18.13 Mortality Ratios by Age at Entry (Factor A)

Age at Entry	MR %	
16 - 39	168	(13)
40 - 49	173	(63)
50 - 59	123	(120)
60 - 79	107	(179)

Excess mortality is higher for ages at entry below 50. There is a slight rise in excess mortality (from +68% to +73%) between ages at entry 16 to 39 and 40 to 49. Excess mortality falls sharply above age 50 at entry. Although the mortality ratios themselves for female lives are different from those for male lives, there are similarities in the pattern of movements, in that initially the mortality ratio rises slightly to a maximum for the 40 to 49 age at entry group, then drops considerably as age at entry rises above 50.

Table 18.14 Mortality Ratios by Blood Pressure (Factor B)

		Diastolic Pressure (mm Hg)		
		<95	95-105	>105
Systolic Pressure (mm Hg)	<150		—— 118 (36) ——	
	150-165	111 (127)	126 (72)	183 (13)
	>165	118 (50)	130 (49)	144 (28)

There is some evidence of an increase in excess mortality with increasing blood pressure (from top left to bottom right), but it is not as convincing as for male lives. Only four entries show mortality ratios above 125%, the upper limit of the normotensive range.

Table 18.15 Mortality Ratios by Weight Levels (Factor W)

	MR %	
Standard ± 19%	122	(291)
Standard +20% or over	120	(84)

The mortality ratios for the two weight levels are very close indeed. These results show slightly lower excess mortality associated with overweight. Clearly this factor is not statistically significant.

Table 18.16 Mortality Ratios by Family History (Factor H)

	MR %	
Good	115	(275)
Poor	145	(100)

Like male lives, there is clearly a rise in excess mortality associated with a family history of

cardiovascular disease.

Table 18.17 Mortality Ratios by Policy Duration (Factor D)

Duration	MR %	
0 - 2 yrs	110	(45)
2 - 5	98	(82)
5 - 10	123	(130)
10 - 15	141	(60)
15 - 20	151	(30)
over 20 yrs	170	(28)

Excess mortality falls after the first two years duration then steadily increases as policy duration increases.

Table 18.18 Mortality Ratios by Calendar Year of Entry (Factor C)

	MR %	
1947 - 51	134	(53)
1952 - 56	141	(71)
1957 - 61	125	(61)
1962 - 66	155	(68)
1967 - 71	163	(24)
1972 - 76	84	(16)
1977 - 81	90	(60)
1982 - 86	85	(22)

Excess mortality hits a peak for years of entry 1967-71. In this respect the results are similar to those for male lives. After 1971, The mortality ratios drop substantially.

18.4.4 Significance of Main Effects

Referring differences in model deviances to the appropriate χ^2 distribution (see table below) reveals that age at entry and calendar year of entry are highly statistically significant, family history of cardiovascular disease is statistically significant and policy duration is marginally significant. A surprise here is that blood pressure levels are not statistically significant (although there is some evidence of higher mortality ratios associated with the higher blood pressure levels).

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev.	D of F	
H_0	919·2	1647			
A	907·8	1644	11·4	3	1 %
B	915·0	1641	4·2	6	65 %
C	899·5	1640	19·7	7	·75 %
D	909·0	1642	10·2	5	9 %
H	915·2	1646	4·0	1	4 %

18.4.5 Other Models

More complicated models may be fitted provided there are sufficient data available (in terms of numbers of deaths) to provide meaningful results. With this in mind, interaction terms were not considered, but results showing the significant main effects fitted simultaneously are given.

18.4.5.1 Main Effects Fitted Together, No Interaction

Model : A+C+D+H

Parametric Representation : $exp\,(\mu + \alpha_i + \gamma_k + \delta_l + \rho_m)$

The mortality ratios are found by forming the product of relevant mortality factors from Table 18.19.

Table 18.19 Excess Mortality Factors: Model A+C+D+H

$exp(\mu) = 1{\cdot}62$

AGE AT ENTRY :	16-39	40-49	50-59	60-79
$exp(\alpha_i)$	1·00	1·07	0·74	0·72

CALENDAR YEAR :	47-51	52-56	57-61	62-66	67-71	72-76	77-81	82-86
OF ENTRY $exp(\gamma_k)$	1·00	1·04	0·94	1·18	1·27	0·67	0·77	0·74

DURATION :	0-2	2-5	5-10	10-15	15-20	>20
$exp(\delta_l)$	1·00	0·87	0·98	0·98	1·01	1·30

FAMILY HISTORY :	Good	Poor
$exp(\rho_m)$	1·00	1·21

The plots of deviance residuals for this model are shown in Figures 18.7 and 18.8. These plots are not as satisfactory as those for male lives, although the range of values of residuals is reasonable (-3·20 to +3·20).

Figure 18.7 Histogram of Residuals

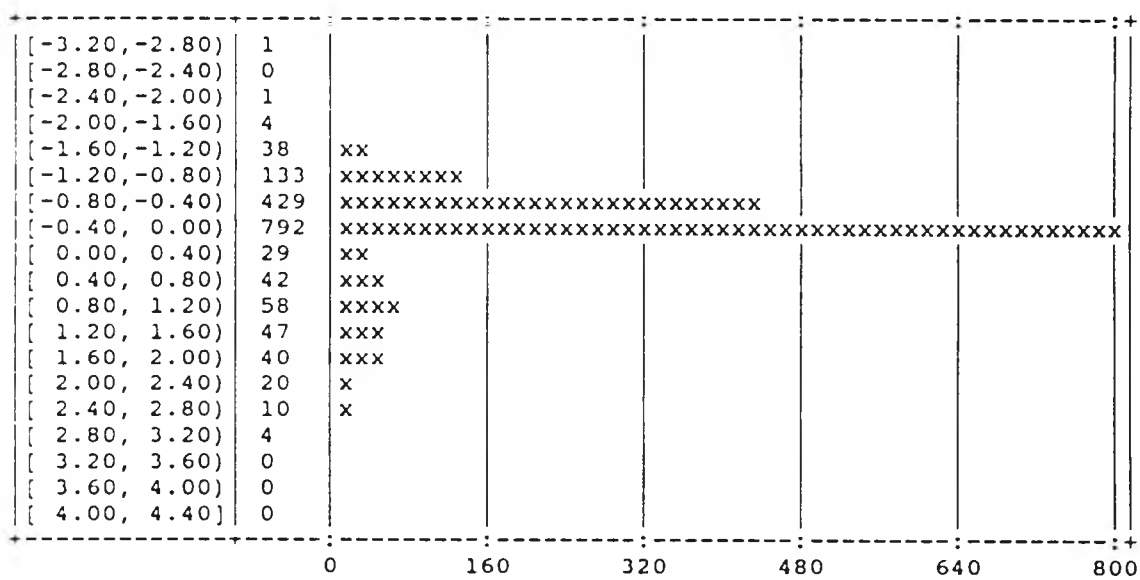
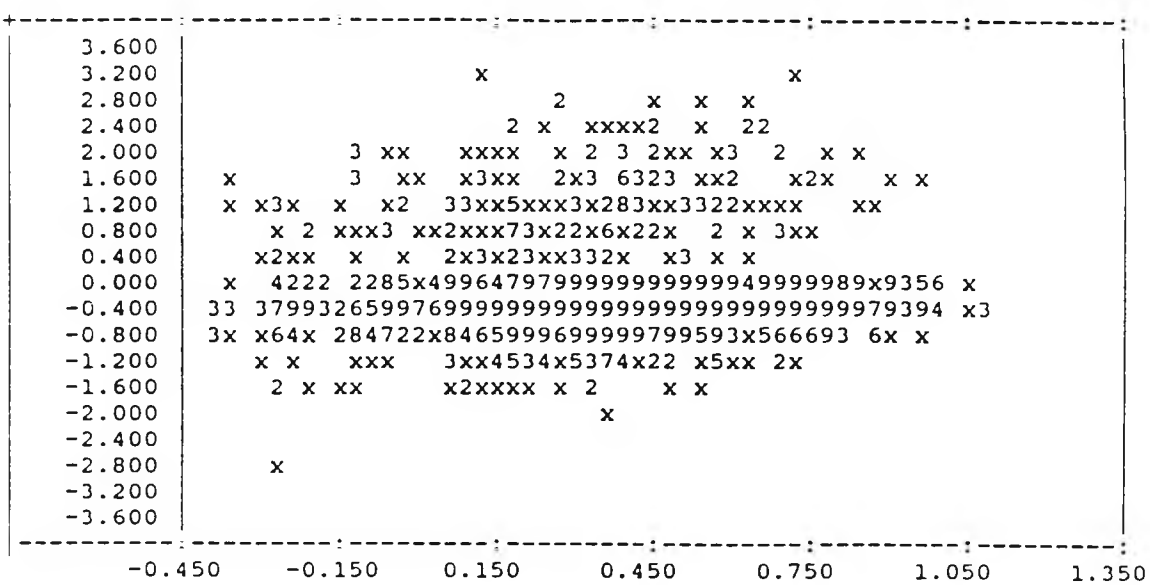


Figure 18.8 Residuals against Linear Predictor



18.4.6 Results: Base-line Hazard : A67-70 with a 4 year age deduction

The data for female hypertensives were also analysed using the A67-70 (2) select table with a 4 year age deduction. However, results indicate that the A67-70 table adjusted in this way is not a suitable basis for expected deaths for female lives, since they show an overall mortality ratio for female hypertensives which is lower than standard. Results are reported briefly for main effects fitted separately. More complex models are not considered.

The overall mortality ratio for female hypertensives was found to be 98% based on 375 deaths, using the A67-70 (2) table with a 4 year age deduction as a basis for expected deaths. Results for main effects are shown in Tables 18.20 to 18.24.

Table 18.20 Mortality Ratios by Age at Entry (Factor A)

Age at Entry	MR %	
16 - 39	161	(13)
40 - 49	153	(63)
50 - 59	98	(120)
60 - 79	84	(179)

Table 18.21 Mortality Ratios by Calendar Year of Entry (Factor C)

	MR %	
1947 - 51	106	(53)
1952 - 56	113	(71)
1957 - 61	99	(61)
1962 - 66	122	(68)
1967 - 71	130	(24)
1972 - 76	68	(16)
1977 - 81	72	(60)
1982 - 86	71	(22)

Table 18.22 Mortality Ratios by Blood Pressure (Factor B)

		Diastolic Pressure (mm Hg)		
		<95	95-105	>105
Systolic Pressure (mm Hg)	<150		96 (36)	
	150-165	88 (127)	101 (72)	149 (13)
	>165	93 (50)	104 (49)	116 (28)

Table 18.23 Mortality Ratios by Policy Duration (Factor D)

Duration	MR %	
0 - 2 yrs	97	(45)
2 - 5	78	(82)
5 - 10	97	(130)
10 - 15	110	(60)
15 - 20	121	(30)
over 20 yrs	146	(28)

Table 18.24 Mortality Ratios by Weight Levels (Factor W)

	MR %	
Standard ± 19%	97	(291)
Standard +20% or over	98	(84)

18.5 Comparison with Other Studies

Many studies have been carried out investigating mortality rates associated with blood pressure levels in North America, particularly amongst insured lives. Studies go back at least as far as 1925. Perhaps the most well known studies are the Blood Pressure Study 1939, and the Build and Blood Pressure studies of 1959 and 1979. Also, results from several studies have been reported based on the Prudential impaired lives data set since its inception in 1947.

18.5.1 Studies Based on the Prudential Impaired Lives Data Set

Authors reporting results for the hypertensive subset of the Prudential impaired lives data set are shown in Table 18.25, together with the overall mortality ratios and number of deaths on which the results were based.

Table 18.25 Prudential Impairment Studies of the Hypertensive Subset

Author	Calendar Yrs of Study	MR (%)	Number of Deaths
Clarke (1961)	1947-58	151	413
Preston & Clarke (1966)	1947-63	166	944
Clarke (1978)	1964-73	145	1080
Leighton (1987)	1974-83	143	798
Papaconstantinou (1988)	1947-81	145	3021
Renshaw (1988)	1947-81	154	3019
England (1990)	1947-86	154	3548

The mortality ratios found by the various authors compare well. In particular, the results of Renshaw and England are identical (although see section 18.5.2). This is perhaps surprising given the differences in the bases used for expected deaths (although the periods of investigation are very similar). Papaconstantinou and Renshaw used the same study group, but found different mortality ratios (145% and 154% respectively). This is probably due to the use of different standard experiences as the basis for expected deaths (Papaconstantinou modified the A67-70 select table to produce a different table for each calendar year 1947 to 1981. Renshaw used the modifications adopted by Papaconstantinou but averaged over quinquennia to produce a different table for each quinquennium 47-51, 52-56, 57-61, 62-66, 67-71, 72-76 and 77-81).

All studies observed higher mortality ratios at the younger ages at entry and all observed increasing excess mortality with increasing blood pressure.

Although hypertension may be caused (or exacerbated) by the policyholder being overweight, in no study has overweight in conjunction with hypertension been found to add substantially to the risk. This has been noted by all authors. For example:

> "Surprisingly, the experience among lives who were more than 20% above standard weight is no worse than in the standard group" - Clarke (1961).
>
> "...the overweight lives show only a slight excess mortality over lives of standard weight and in no case is the excess statistically significant" - Preston and Clarke (1966).
>
> "The presence of overweight in conjunction with hypertension has, on the whole, yielded results suggesting that overweight does not add materially to the risk" - Clarke (1978).
>
> "The recent experience reinforces the earlier suggestion that overweight in conjunction with hypertension does not appear to add materially to the risk." - Leighton (1987).

When presenting results for mortality ratios by blood pressure levels, Papaconstantinou and

Renshaw report results using the classification "below average", "average" and "above average" for systolic and diastolic pressure. This is misleading since the data refer to blood pressure levels in the hypertensive range which are, by definition, well above average.

None of the previous studies included family history of cardiovascular disease as a factor in the analysis.

18.5.2 A Comparison with the Results of Renshaw (1988)

A direct comparison with the results of Renshaw (1988) is possible, since the methodology used in this study is the same as that used by Renshaw. A comparison between the studies is useful since it highlights differences in the results caused by differences in the bases used in the calculation of expected deaths. Other differences between the studies are:

1) This study includes data available for the quinquennium 1982-86;

2) The factor "family history of cardiovascular disease" is now included.

The mortality ratios for main effects fitted separately which were quoted by Renshaw are shown in Table 18.26. It should be noted that the overall mortality ratio (152%) is lower than either of the results by weight levels. This should not be so, since the overall mortality ratio is a weighted average of mortality ratios by individual main effects, where the weights are the expected deaths. The expected deaths can be found (approximately) by dividing the number of deaths for each level of each main effect by the respective mortality ratio. A little arithmetic reveals that the overall mortality ratio should be 154%, and the quoted figure of 152% must be a typing error.

The results presented in Tables 18.3 to 18.8 for main effects compare favourably with those shown in Table 18.26. The most noticeable differences are for calendar year of entry. This is

to be expected given the choice of base-line hazard used. Using A67-70 select for all quinquennia will tend to overstate excess mortality in the earlier years and understate excess mortality in the later years.

Table 18.26 Renshaw (1988) : Mortality Ratios, Covariates Fitted Separately

Overall Mortality Ratio : 152 (3019)

Age at Entry :	16-39	40-49	50-59	60-79
	180 (353)	213 (882)	142 (992)	122 (792)

Blood Pressure :

Systolic Pressure (mm Hg)	Diastolic Pressure (mm Hg) <95	95-105	>105
<150		142 (508)	
150-165	132 (1087)	168 (726)	177 (104)
>165	163 (153)	201 (243)	284 (198)

Calendar Year of Entry :	47-51	52-56	57-61	62-66	67-71	72-76	77-81
	140 (681)	145 (803)	155 (615)	174 (555)	205 (217)	173 (96)	141 (52)

Policy Duration :	0-2	2-5	5-10	10-15	15-20	>20
	132 (223)	136 (480)	160 (1010)	172 (713)	155 (365)	149 (228)

Weight Levels :	Standard ± 19%	Standard + 20% or over
	153 (2508)	164 (511)

Further comparison is possible by looking at the main effects model fitted simultaneously. The excess mortality factors given by Renshaw after fitting Model A+B+C+D are shown in Table 18.27. The α_i, β_j, γ_k and δ_l are as before. Notice that $k = 1, \ldots, 7$ since the most recent quinquennium (82-86) is not included. Also, there are no results for family history. The

equivalent model has been fitted using the data set upon which results in this thesis are based, (i.e. drop the factor H and fit Model A+B+C+D) with excess mortality factors shown in Table 18.28.

Table 18.27 Renshaw: Excess Mortality Factors for Model A+B+C+D

$exp\ \mu = 1{\cdot}32$

i	1	2	3	4			
$exp\ \alpha_i$	1	1·14	0·75	0·67			
j	1	2	3	4	5	6	7
$exp\ \beta_j$	1	1·28	1·18	1·50	1·14	1·91	0·93
k	1	2	3	4	5	6	7
$exp\ \gamma_k$	1	1·03	1·07	1·14	1·31	1·14	1·06
l	1	2	3	4	5	6	
$exp\ \delta_l$	1	1·06	1·26	1·28	1·14	1·20	

Table 18.28 England: Excess Mortality Factors for Model A+B+C+D

$exp\ \mu = 2{\cdot}02$

i	1	2	3	4				
$exp\ \alpha_i$	1	1·10	0·71	0·66				
j	1	2	3	4	5	6	7	
$exp\ \beta_j$	1	1·34	1·18	1·47	1·22	1·94	0·96	
k	1	2	3	4	5	6	7	8
$exp\ \gamma_k$	1	0·98	0·94	0·94	0·97	0·79	0·59	0·61
l	1	2	3	4	5	6		
$exp\ \delta_l$	1	0·90	1·01	0·96	0·80	0·71		

The effect of changing the baseline hazard and ignoring family history is largely swallowed up by the "μ" parameter (a change from 1·32 to 2·02), but also affects the γ_k and δ_l terms. In the results of both Renshaw and England, the trends in γ_k and δ_l are consistent with the underlying trends in the results of the calendar year of entry and policy duration main effects fitted separately. It is interesting to note that the α_i and β_j shown by Renshaw and England are very similar; a change in the baseline hazard has affected these terms very little.

Fitting the more complex model B*D+A+C enables a comparison with Table 6.5 of Renshaw (1988), which includes the most significant interaction term (B.D). Excess mortality factors given by this model structure for the two studies are shown in Tables 18.29 and 18.30.

Table 18.29 Renshaw: Main Effects Plus Major Interaction Term — Model B*D+A+C

$exp\,(\mu + \beta_j + \delta_l + \beta\delta_{jl})$

$l = 1$: Duration 0-2			$l = 2$: Duration 2-5			$l = 3$: Duration 5-10		
	— 1·13 —			— 1·34 —			— 1·39 —	
1·15	1·65	0·90	1·51	1·46	1·70	1·74	1·98	2·19
2·67	2·06	2·97	1·07	2·58	2·75	2·12	2·39	3·04

$l = 4$: Duration 10-15			$l = 5$: Duration 15-20			$l = 6$: Duration >20		
	— 1·79 —			— 1·55 —			— 1·20 —	
1·52	1·98	2·09	1·48	1·86	1·30	1·72	2·09	0·80
2·69	2·43	3·45	1·92	2·01	1·75	1·75	1·37	4·91

Age at Entry :	16-39	40-49	50-59	60-79
$exp\,(\alpha_i)$	1·00	1·15	0·75	0·67

Calendar Year :	47-51	52-56	57-61	62-66	67-71	72-76	77-81
of Entry $exp\,(\gamma_k)$	1·00	1·04	1·07	1·14	1·30	1·14	1·02

Table 18.30 England: Model B*D+A+C — For Comparison with Renshaw Table 6.5

$exp\,(\mu + \beta_j + \delta_l + \beta\delta_{jl})$

$l = 1$: Duration 0-2			$l = 2$: Duration 2-5			$l = 3$: Duration 5-10		
	— 1·79 —			— 1·74 —			— 1·75 —	
1·69	2·30	1·37	1·92	2·02	2·38	2·14	2·41	2·86
3·93	3·53	5·19	1·55	3·10	3·73	2·68	2·90	3·75

$l = 4$: Duration 10-15			$l = 5$: Duration 15-20			$l = 6$: Duration >20		
	— 2·06 —			— 1·76 —			— 2·05 —	
1·76	2·26	2·54	1·61	1·95	1·69	1·51	1·93	1·04
3·43	2·74	3·62	2·22	1·98	2·19	1·62	1·16	3·02

Age at Entry :	16-39	40-49	50-59	60-79
$exp\,(\alpha_i)$	1·00	1·10	0·71	0·66

Calendar Year :	47-51	52-56	57-61	62-66	67-71	72-76	77-81	82-86
of Entry $exp\,(\gamma_k)$	1·00	0·98	0·94	0·95	0·97	0·78	0·58	0·61

The "α_i" terms are very similar, however comparison of the other terms is not so straightforward. The γ_k terms reported by Renshaw are a little higher than those of this study, but show the same general trend (*e.g.* there is a peak at $k = 5$). The differences can be accounted for by the differences in base-line hazards used. Comparison of the $exp(\mu+\beta_j+\delta_l+\beta\delta_{jl})$ terms becomes more straightforward if the $exp(\mu)$ term is removed and treated separately, as shown in Tables 18.31 and 18.32.

Removing the $exp(\mu)$ term reveals similarities in the results, particularly at low durations ($l =$ 1 and 2). Like the main effects model fitted simultaneously, there is a large change in the $exp(\mu)$ term (1·15 to 1·69), again indicating that it is this parameter that will be affected the most by a change in the base-line hazard used (provided the overall shape of the hazard function remains approximately the same).

Table 18.31 Renshaw: Interaction part of Model B*D+A+C

$exp(\mu) = 1{\cdot}15$

$exp\,(\beta_j + \delta_l + \beta\delta_{jl})$

$l = 1$: Duration 0-2			$l = 2$: Duration 2-5			$l = 3$: Duration 5-10		
	— 0·98 —			— 1·17 —			— 1·21 —	
1·00	1·43	0·78	1·31	1·27	1·48	1·51	1·72	1·90
2·32	1·79	2·58	0·93	2·24	2·39	1·84	2·08	2·64
$l = 4$: Duration 10-15			$l = 5$: Duration 15-20			$l = 6$: Duration >20		
	— 1·56 —			— 1·35 —			— 1·04 —	
1·32	1·72	1·82	1·29	1·62	1·13	1·50	1·82	0·70
2·34	2·11	3·00	1·67	1·75	1·52	1·52	1·19	4·27

Table 18.32 England: Interaction part of Model B*D+A+C

$exp(\mu) = 1{\cdot}69$

$exp\,(\beta_j + \delta_l + \beta\delta_{jl})$

$l = 1$: Duration 0-2			$l = 2$: Duration 2-5			$l = 3$: Duration 5-10		
	— 1·06 —			— 1·03 —			— 1·04 —	
1·00	1·36	0·81	1·14	1·20	1·41	1·27	1·43	1·69
2·33	2·09	3·08	0·92	1·84	2·21	1·59	1·72	2·22
$l = 4$: Duration 10-15			$l = 5$: Duration 15-20			$l = 6$: Duration >20		
	— 1·22 —			— 1·04 —			— 1·21 —	
1·04	1·34	1·51	0·95	1·16	1·00	0·89	1·14	0·62
2·03	1·62	2·15	1·31	1·17	1·30	0·96	0·69	1·79

As duration increases above $l = 2$, the values of $exp(\mu + \beta_j + \delta_l + \beta\delta_{jl})$ do not compare well. This is in some ways surprising given the similarities of the methods used. The

differences indicate that perhaps the model is too complicated given the data available (i.e. using the model B*D+A+C results in overparameterisation). A large number of terms in the model will give a small amount of data in some cross-classified cells. The problem will be particularly acute at the highest durations combined with the highest blood pressure levels. This is observed in Tables 18.31 and 18.32 (e.g. $j = 7$, $l = 6$ gives 4·27 versus 1·79). To alleviate this problem, the data could be re-classified by, say, combining over duration groups 10-15, 15-20 and >20 years to give one group, >10 years. However, a model fitted with this classification is not available from the results of Renshaw, therefore no comparison may be made to check the validity of the results of fitting such a model.

In the light of this, care must be taken when using the models including interaction terms. To be confident of the effect of an interaction, results need to be based on large numbers of deaths and the interaction terms must be very highly significant.

18.5.3 Female Lives: A Comparison with Other Studies Based on the Prudential Data Set

Preston and Clarke (1966) and Papaconstantinou (1988) have reported results for excess mortality of female hypertensives. A comparison with the results reported in this thesis is shown in Tables 18.33 and 18.34.

Table 18.33 Preston and Clarke (1966): Mortality Ratios for Female Hypertensives

Overall Mortality Ratio: Preston and Clarke 125%, England 121%

	Preston & Clarke		England	
Age at Entry	MR %	deaths	MR %	deaths
40 - 59	137	(50)	140	(183)
over 60	112	(40)	107	(179)

There is a remarkable degree of similarity between the results of Preston and Clarke and those of England.

Table 18.34 Papaconstantinou (1988): Mortality Ratios for Female Hypertensives

Overall Mortality Ratio = 89% (England 121%)

	Papaconstantinou		England	
Age at Entry	MR %	deaths	MR %	deaths
16 - 39	228	(10)	168	(13)
40 - 49	111	(45)	173	(63)
over 50	83	(215)	113	(299)
Duration	**MR %**	**deaths**	**MR %**	**deaths**
0-4 yrs	129	(26)	102	(127)
5-9 yrs	81	(51)	123	(130)
10-14 yrs	78	(97)	141	(60)
over 15 yrs	102	(96)	160	(58)

There is an apparent inconsistency between the deaths observed by Papaconstantinou and England at durations above 10 years. Since the data set used by Papaconstantinou is a subset of the data set upon which the results in this thesis are based, it is inconceivable that the number of deaths reported by Papaconstantinou could be higher than those reported by England. The results of England were double checked and found to be correct.

The Mortality Ratios observed by Papaconstantinou and England do not compare particularly well for female hypertensives.

18.5.4 Comparison with the Build and Blood Pressure Studies - North America

The ongoing build and blood pressure studies conducted in North America are a useful source for comparison with the results of the Prudential study. Results from the Blood Pressure Study 1939, the Build and Blood Pressure Study 1959, and the Blood Pressure study 1979 are shown in the form of mortality ratios in Tables 18.35 to 18.37. The values which may be compared with the Prudential results are shown within boxes.

Table 18.35 The Blood Pressure Study 1939

Systolic pressure (mm Hg)	Diastolic (fifth phase) pressure (mm Hg)						
	64–73	74–83	84–88	89–93	94–98	99–103	104–108
Ages 10–29							
108–117	†	†	126	•			
118–127	†	†	103	113	132	•	
128–132	101	100	101	107	160		
133–137	90	98	101	127	•		•
138–142	•	106	96	128	•	•	
143–147		•	•	•	•		
148–157	•	•	•	•	•	•	•
Ages 30–39							
108–117	†	†	73	•	•		
118–127	†	†	93	115	106		
128–132	93	95	107	125	117	•	•
133–137	86	119	120	153	160	•	•
138–142	123	107	121	142	198	284	•
143–147		•	152	144	205	•	•
148–157	•	•	•	193	246	308	•
Ages 40–49							
108–117	†	†	94	•	•		
118–127	†	†	86	103	122	•	
128–132	93	95	98	103	107	•	•
133–137	111	103	109	118	139	•	•
138–142	121	119	130	146	170	166	•
143–147	•	146	142	176	225	168	•
148–157	•	204	196	212	223	304	411
158–167	•	•	•	221	408	421	•
Age 50 and over							
108–117	†	†	123	•	•		
118–127	†	†	72	78	122		
128–132	83	85	93	92	98	•	•
133–137	84	91	96	97	100	95	
138–142	104	101	106	118	132	118	•
143–147	119	131	118	125	140	150	•
148–157	115	130	152	148	180	213	213
158–167	•	185	171	192	218	211	326

The blood pressure groups adopted in the various studies do not conform exactly with those of the Prudential study. The results of the 1939 study tend to be slightly higher than those of the Prudential study, but even so, they compare well. An interesting feature is the trend over age

groups. Mortality ratios seem to be slightly higher for the 40-49 age group than the 30-39 age group, but reduce above age 50. This trend is similar to the results shown earlier for the Prudential data set.

The results shown in Table 18.36 for the 1959 study tend to be slightly lower than the equivalent results from the Prudential study.

Table 18.36 The Build and Blood Pressure Study 1959

Diastolic (fifth phase) pressure (mm Hg)	Systolic pressure (mm Hg) 98–127	128–137	138–147	148–157	158–167
Ages 20–29					
83–87	110	110	130		
88–92	125	140	140		
Ages 30–39					
68–82		110	130	150	
83–87		120	170	230	
88–92	115	150	200		
93–97		225	to	250	
Ages 40–49					
68–82		105	140	155	
83–87		125	155	230	300+
88–92		140	185	205	285
93–97		135	190	335	
98–102		185	225	350+	
Ages 50–59					
68–82		105	140	145	
83–87		110	140	175	
88–92		110	155	200	
93–97		165	195	215	255
98–102			178	193	370
Ages 60–69					
68–87			125	160	
88–92			140	150	
93–97			165		

Table 18.37 The Blood Pressure Study 1979

Systolic pressure (mm Hg)	Mortality ratio (%)		Diastolic pressure (mm Hg)	Mortality ratio (%)	
	Men	Women		Men	Women
Under 108	71	83	Under 73	85	87
108–117	77	90	73–77	92	96
118–127	89	93	78–82	99	103
128–137	111	107	83–87	118	114
138–147	135	121	88–92	136	132
148–157	166	135	93–97	169	167
158–167	206	169	98–102	200	181
168–177	218	178	103–107	258	208
178–187	232	278	108–112	244	195

The results of the 1979 Blood Pressure study are particularly useful as a source for comparison. The mortality ratios shown in Table 18.37 refer to a study group of insured lives who took out policies between 1954 and 1972. The Prudential data allow values to be calculated which compare roughly to the classification of blood pressure levels shown in the boxes in Table 18.37, and are shown in Table 18.38.

Table 18.38 Prudential Results for Comparison with the Blood Pressure Study 1979

Systolic Pressure (mm Hg)	Mortality Ratio	
	Men	Women
< 150	145	118
150-165	146	119
> 165	196	128

Diastolic Pressure (mm Hg)	Mortality Ratio	
	Men	Women
< 95	136	113
≥ 95	170	130

The mortality ratios for male lives compare well. For female lives, the mortality ratios for the Prudential study are slightly lower than those for the 1979 Blood Pressure study.

18.6 Rating of Hypertensives

The underwriting manuals of three re-insurance companies were considered. In general, the ratings recommended by the companies are consistent with the results shown earlier from the Prudential study. All three companies quite correctly make allowance for the family history of cardiovascular disease. Only one company mentions reducing the ratings for female lives, although the results of the Prudential study indicate that this would be reasonable. Furthermore, there is little evidence from the reults of the Prudential study to justify making an addition for overweight.

The salient features of the method of underwriting hypertensives used by each company are reported below.

18.6.1 Munich Reinsurance Company

Ratings are by systolic and diastolic pressure, within 5 age at entry groups. The ratings are consistent with results from the Prudential study. Where there is a family history of three or more family deaths (parents, siblings) caused by cardiovascular-renal disease, an addition of +25 is made. When considering overweight, the rating for weight as an impairment alone is added, with an extra addition for the more severe weight cases. Female lives are not mentioned and are presumably rated in the same way as male lives. There is no mention of a standard life table to which the ratings apply.

18.6.2 Swiss Reinsurance Company

Ratings are by broad groupings of systolic and diastolic pressure, within 4 age at entry groups. The ratings are consistent with results from the Prudential study. For more than 1 case of coronary artery, cerebrovascular or peripheral artery disease below age 60 in the family history an addition is made which is proportional to the blood pressure/age rating. The presence of overweight is not mentioned. Female lives are not mentioned and, presumably, are therefore

rated in the same way as male lives. There is no mention of a standard life table to which the ratings apply.

18.6.3 Mercantile and General Reinsurance Company

Ratings are by narrow groupings of systolic and diastolic pressure, within 5 age groups. The ratings are consistent with results from the Prudential study. Where there is a family history of deaths from cardiovascular-renal disease under age 60 in parents or siblings, an addition of +25 is made. Where there are no cardiovascular-renal deaths under age 65 among parents or siblings, a credit of −25 is given. Any rating for obesity is added to the final rating for blood pressure. For female lives, $\frac{3}{4}$ of the ratings for male lives are used.

The resultant ratings are referred to a table which converts the excess mortality to an addition in years of age which applies to a standard life table, namely A67-70 (2) select for males with a one year deduction to age and A67-70 (2) select for females with a five year age deduction.

18.7 Concluding Remarks

Commenting on the rating of hypertensives, Brackenridge writes:

> "From the point of view of practical underwriting an applicant who is rateable on account of blood pressure is a rather worse risk if there is a family history of two or more deaths in parents or siblings under age 60 from cardiovascular-renal causes whether these were associated with hypertension or not, and an addition of +25 to +50 should be made to the blood pressure rating depending on age. Conversely, if the family history is excellent . . a credit of 15 can be allowed."

From the Prudential study for both male and female lives, a "poor" family history of cardiovascular disease (one or more deaths below age 70) results in an addition of +30 when compared with a "good" family history (treating family history as a main effect separately).

When family history is considered together with other factors (*e.g.* main effects fitted simultaneously) a "poor" family history results in a 20% higher rating than the equivalent rating for a "good" family history (for both male and females lives), *i.e.* a proportionate increase.

Regarding female lives, Brackenridge writes:

> "Women with hypertension seem to fare so much better than men as regards mortality that considerably more latitude can be exercised in underwriting female lives with elevated blood pressure. From the results of comparative studies of death rates in men and women suffering from hypertension, it would be reasonable to assume that expected mortality in women with hypertension lies between 65 and 70 per cent of that for men with equivalent levels of blood pressure".

This is not an unreasonable statement in the light of the results of the Prudential study.

Chapter 19 Epilepsy

19.1 Prudential Impairment Codes and Classification for Analysis:

Pre 1/1/80:

300	Petit Mal
301	Grand Mal (Idiopathic)

Post 1/1/80:

302	Petit Mal (excluding temporal lobe epilepsy)
303	Temporal Lobe Epilepsy (excluding grand mal)
304	Grand Mal (idiopathic or traumatic) - not more than 6 episodes per year
305	Grand Mal (idiopathic or traumatic) - 7 to 12 episodes per year
306	Grand Mal (idiopathic or traumatic) - more than 12 episodes per year

Subsidiary Codes:

A	Attacks within 2 years
B	No attack for 2-5 years
C	No attack for 5-10 years
D	No attack for over 10 years

The change in coding on 1/1/80 presents some difficulty in conducting a complete analysis. Although the new codings are more comprehensive, the time elapsed since the change has not proved long enough to enable much data to accumulate. In particular, there have been 0 deaths recorded from code 302 (petit mal), only 2 deaths from code 303 (temporal lobe) and 15 deaths from codes 304-306 combined (grand mal). In view of this, analysis has been restricted to grand mal cases only. The data for codes 304-306 were combined with the data for code 301 to give results for the entire period covered by the data set (1947-86).

For analysis data were sub-classified according to:

A:	Age at Entry	4 levels :	1 - 16 to 39	
			2 - 40 to 49	
			3 - 50 to 59	
			4 - 60 to 79	
D:	Policy Duration	6 levels :	1 - 0 to 2 years	4 - 10 to 15 years
			2 - 2 to 5 years	5 - 15 to 20 years
			3 - 5 to 10 years	6 - >20 years
C:	Calendar Yr of Entry:	8 levels :	1 - 47 to 51	5 - 67 to 71
			2 - 52 to 56	6 - 72 to 76
			3 - 57 to 61	7 - 77 to 81
			4 - 62 to 66	8 - 82 to 86
E:	Episodes:	2 levels :	1 - Episodes within 2 years	
			2 - No episodes for at least 2 years	

19.2 Male Lives

19.2.1 Summary Statistics

Number of Entrants: 6923

Number of Deaths: 206

Table 19.1 Distribution by Age at Entry

Age at Entry	Number of Entrants	Percentage
16-39	5880	84·9
40-49	817	11·8
50-59	196	2·8
60-79	30	0·4
Total	6923	

The percentage of entrants in the 16 to 39 age at entry group is very high, at 85%. Very few entrants were aged above 60 at entry.

Table 19.2 Distribution by Cause of Death

	Number	Percentage
Lung Cancer	10	4·9
Other Cancer	27	13·1
Vascular Lesions	8	3·9
Ischaemic Heart Disease	36	17·5
Other Circulatory Diseases	7	3·4
Pneumonia	1	0·5
Bronchitis	1	0·5
Motor Accident	8	3·9
Other Accident	14	6·8
Suicide	4	1·9
Other Causes (unspecified)	90	43·7
Total	206	

Deaths from "other causes" are noteworthy in that they are twice those observed in the entire impaired lives data set taken as a whole. As there is no specific coding for epilepsy in the cause of death classification, it is possible that some of the deaths within the "other causes" category are from epilepsy.

19.2.2 Results: The Overall Mortality Ratio

Using the A67-70 (2) experience as a basis for expected deaths, the overall mortality ratio was found to be 183%, based on 206 deaths.

19.2.3 Results: Main Effects Fitted Separately

Table 19.3 Mortality Ratios by Age at Entry (Factor A)

Age at Entry	MR %		
16 - 39	218	(127)	
40 - 49	165	(59)	
50 - 59	126	(18)	} 105 (20)
60 - 79	42	(2)	

The mortality ratios clearly decrease as age at entry increases, with excess mortality of +118% for the age at entry group 16 to 39 reducing to near standard mortality for age at entry above 50.

Table 19.4 Mortality Ratios by Duration Since Last Episode (Factor E)

	MR%	
Episodes within 2 years	240	(129)
No episodes for at least 2 years	130	(77)

Significant excess mortality is experienced by policyholders suffering attacks in the two years prior to application for life assurance.

Table 19.5 Mortality Ratios by Policy Duration (Factor D)

Duration	MR %	
0 - 2 yrs	253	(28)
2 - 5	240	(45)
5 - 10	208	(56)
10 - 15	188	(42)
15 - 20	96	(17)
over 20 yrs	111	(18)

These results show excess mortality decreasing as policy duration increases. Excess mortality is highest in the first 2 years that a policy is operative. Excess mortality is very low after 15 years duration.

Table 19.6 Mortality Ratios by Calendar Year of Entry (Factor C)

	MR %			
1947 - 51	170	(15)		
1952 - 56	198	(33)		
1957 - 61	196	(30)		
1962 - 66	172	(39)		
1967 - 71	147	(28)		
1972 - 76	218	(33)		
1977 - 81	163	(19)	}	184 (28)
1982 - 86	255	(9)		

There is no obvious pattern in the above figures. All show excess mortality to some degree.

19.2.4 Significance of Main Effects

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev.	D of F	
H_0	285·17	277			
A	273·35	275	11·82	2	0·30 %
D	264·32	272	20·85	5	0·10 %
C	282·18	271	2·99	6	81 %
E	266·55	276	18·62	1	0·001 %

Referring differences in model deviances to the appropriate χ^2 distribution reveals that age at entry, policy duration and duration since last episode were all found to be highly statistically significant. Calendar year of entry was not significant (statistically).

19.2.5 Other Models

The significant main effects were fitted together and models including first order interaction terms were fitted, leading to the following deviance table.

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev.	D of F	
A+D+E	226·08	269			
A*D+E	216·43	259	9·65	10	47%
A+D*E	217·68	264	8·4	5	13%
A*E+D	224·13	267	1·95	2	38%

These results show that none of the first order interaction terms were found to be statistically significant, leaving Model A+D+E as the optimal model.

19.2.6 Main Effects Fitted Together, No Interaction : Model A+D+E

The results from fitting this model may be shown in two forms. The first involves the parametric representation, with excess mortality factors shown in Table 19.7. Individual mortality ratios may be deduced by forming the product of relevant entries.

Table 19.7 Excess Mortality Factors given by Model A+D+E

Parametric Representation : $exp\,(\mu + \alpha_i + \delta_j + \zeta_k)$

$exp\,(\mu) = 4{\cdot}12$

Age at Entry :	16 to 39	40 to 49	50 to 79
$exp\,(\alpha_i)$	1·00	0·77	0·42

Policy Duration :	0 - 2	2 - 5	5 - 10	10 - 15	15 - 20	>20
$exp\,(\delta_j)$	1·00	1·00	0·88	0·73	0·33	0·37

Episodes :	Episodes within 2 yrs	No Episodes for at least 2 yrs
$exp\,(\zeta_k)$	1·00	0·53

Alternatively, a table showing the mortality ratios for all combinations of the various levels of A, D and E may be prepared, by multiplying together the relevant entries in Table 19.7. The results displayed in this alternative manner are shown in Table 19.8.

Table 19.8 Mortality Ratios given by Model A+D+E

Episodes Within 2 years :

	Policy Duration					
Age at Entry	0 - 2	2 - 5	5 - 10	10 - 15	15 - 20	>20
16 to 39	4·12	4·13	3·64	3·01	1·38	1·51
40 to 49	3·17	3·18	2·80	2·32	1·06	1·16
50 to 79	1·73	1·73	1·53	1·26	0·58	0·63

No Episodes for at least 2 years :

	Policy Duration					
Age at Entry	0 - 2	2 - 5	5 - 10	10 - 15	15 - 20	>20
16 to 39	2·19	2·19	1·93	1·60	0·73	0·80
40 to 49	1·68	1·69	1·49	1·23	0·57	0·62
50 to 79	0·92	0·92	0·81	0·67	0·31	0·34

This alternative form is the usual way of displaying mortality ratios, but it hides the implications of the underlying multiplicative model in which entries under the heading "No Episodes for at least 2 years" are a fixed multiple (0·53) of equivalent entries under the heading "Episodes within 2 years", entries in each column are a fixed multiple of entries in other columns, and entries in each row are a fixed multiple of entries in other rows. (It should be noted that this is a feature of the model structure chosen and the same feature was also observed in Section 17.2.5).

The parametric representation is the most efficient way of storing the results from fitting this model, but the usual way of presenting the results is in the form of Table 19.8 above.

Inspecting Table 19.8, it can be seen that some of the mortality ratios at the highest policy duration groups where there have been no episodes within the last two years are very low (implying mortality rates considerably lower than standard). The model clearly does not fit very well in these cases. A better fit (although more erratic) could be obtained by including the interaction between policy duration and duration since last episode, although that interaction was not found to be statistically significant.

19.2.7 Residual Plots

The residual plots for Model A+D+E are shown in Figures 19.1 and 19.2.

Figure 19.1 Histogram of Residuals

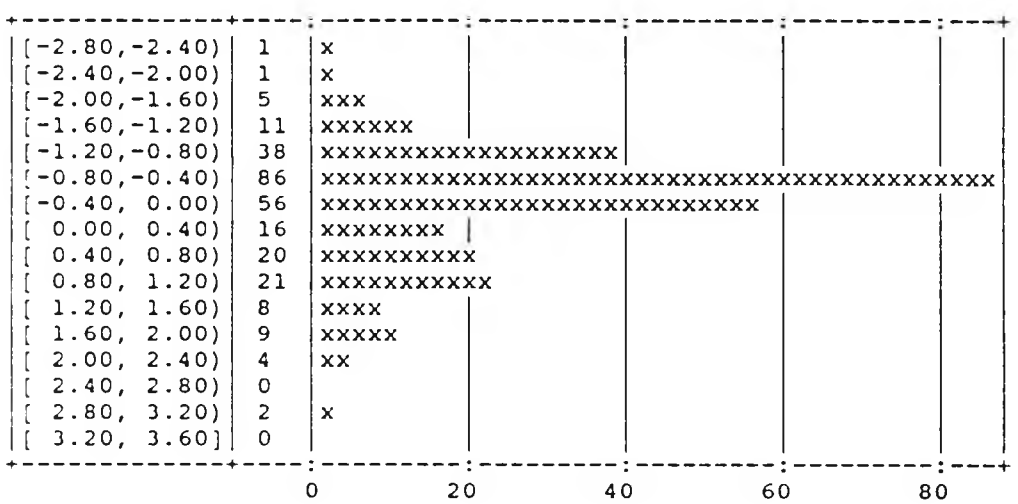
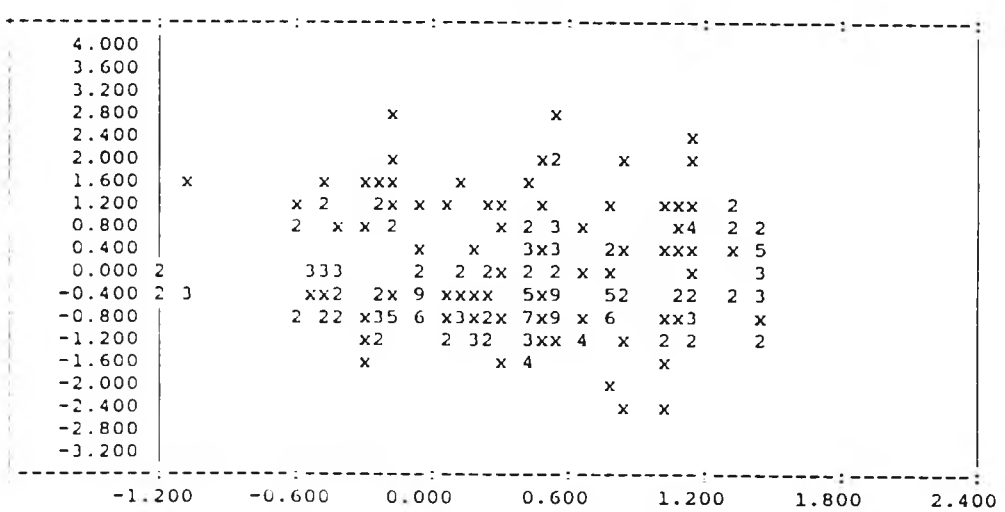


Figure 19.2 Residuals against Linear Predictor



The histogram of residuals is approximately Normal in shape and the plot of residuals against linear predictor shows a reasonable scatter. The range of residuals is a little wide however, indicating that this model is satisfactory rather than ideal.

19.3 Female Lives

19.3.1 Summary Statistics

Number of Entrants: 1493
Number of Deaths: 21

The amount of data available for female epileptics is small, which results in only basic analysis.

Table 19.9 Distribution by Cause of Death

	Number	Percentage
Cancer (excluding lung)	2	9·5
Vascular Lesions	3	14·3
Ischaemic Heart Disease	2	9·5
Other Circulatory Diseases	1	4·8
Pneumonia	1	4·8
Bronchitis	1	4·8
Accident (excluding motor)	2	9·5
Suicide	1	4·8
Other Causes (unspecified)	8	38·1
Total	21	

Because the number of deaths is low, very little can be inferred from the distribution of deaths by cause. However, like male lives, deaths from "other causes" are noteworthy as taking a high proportion of the total.

Table 19.10 Distribution by Age at Entry

Age at Entry	Number of Entrants	Percentage
16-39	1202	80·5
40-49	206	13·8
50-59	67	4·5
60-79	18	1·2
Total	1493	

Like male lives, the vast majority of policyholders were aged below 40 at entry, with very few aged above 50 at entry.

19.3.2 Results

The low number of deaths makes a full analysis difficult. Consequently, results by policy duration and calendar year of entry have not been reported. Furthermore, it is not meaningful to consider statistical significance of main effects.

The overall mortality ratio was found to be 209%, based on 21 lives and using the FA75-78 table in the calculation of expected deaths.

Table 19.11 Mortality Ratios by Age at Entry (Factor A)

Age at Entry	MR %		
16 - 39	325	(14)	
40 - 49	212	(6)	
50 - 59	50	(1)	122 (7)
60 - 79	-	(0)	

The mortality ratios appear to fall as age at entry rises, but these results are based on very low numbers of deaths.

Table 19.12 Mortality Ratios by Duration Since Last Episode (Factor E)

	MR%	
Episodes within 2 years	270	(12)
No episodes for at least 2 years	161	(9)

Mortality ratios are higher when there is a recent history of episodes of epilepsy.

19.4 Comparisons with other studies

Attention is focused on Grand Mal epilepsy only.

All the authors conducting large scale studies based on the Prudential Impaired Lives data set have reported results for epilepsy. The results for overall mortality ratios are summarised in Table 19.13 (male lives only).

Table 19.13 Prudential Impairment Studies of the Epileptic Subset

Author	Calendar Yrs of Study	MR (%)	Number of Deaths
Clarke (1961)	1947-58	225	9
Preston & Clarke (1966)	1947-63	292	25
Clarke (1979)	1964-73	213	57
Leighton (1987)	1974-83	139	69
Papaconstantinou* (1988)	1947-81	267	190
England (1991)	1947-86	183	206

* grand mal and petit mal combined

The results of the various authors seem to vary considerably, which is probably a reflection on the low numbers of deaths on which the studies were based. The study of Leighton (1987) shows results which are noticeably low.

The only other results provided for female lives are given by Papaconstantinou, who reported an overall Mortality Ratio of 184% based on 14 deaths (petit mal and grand mal combined). This compares with an overall mortality ratio of 209% based on 21 deaths given by England (grand mal only).

A very interesting source for comparison purposes is the 1983 Medical Impairment Study (see Lew and Gajewski 1990)) which considered grand mal epilepsy in insured lives in the United States of America. The study covered 32,542 policies issued from 1952 to 1976 to persons with grand mal epilepsy. It was observed that the age distribution of the impairment group was strongly skewed towards the younger ages, with only about 11% at ages 50-59 and about 2% at ages 60-69. A similar (but more pronounced) effect was noted in the Prudential study.

Tables 19.14, 19.15 and 19.16 below show the results of the 1983 Medical Impairment Study (M.I.S.), together with the results of the Prudential study. Attention is drawn to the remarkable similarity between the overall mortality ratios for male and female lives in the two studies.

Table 19.14 The 1983 M.I.S. : Overall Mortality Ratios

1983 M.I.S.			1991 Prudential		
Males	178	(385)	Males	183	(206)
Females	203	(61)	Females	209	(21)

Table 19.15 1983 M.I.S. : Mortality Ratios by Age at Entry (Male lives only)

1983 M.I.S.				1991 Prudential			
Age at Entry	MR%			Age at Entry	MR%		
15-39	206	(262)		16-39	218	(127)	
40-49	145	(94)		40-49	165	(59)	
50-59	120	(27)	} 118 (29)	50-59	126	(18)	} 105 (20)
60-61	100	(2)		60-69	42	(2)	

After grouping to give a reasonable number of deaths on which the results are based, the mortality ratios for the two studies by age at entry are again remarkably similar.

Table 19.16 1983 M.I.S. : Mortality Ratios by Policy Duration (Male lives only)

1983 M.I.S.			1991 Prudential		
Duration	MR%		Duration	MR%	
1-2 yrs	184	(57)	0-2	253	(28)
3-5	178	(81)	2-5	240	(45)
6-10	186	(120)	5-10	208	(56)
11-15	187	(87)	10-15	188	(42)
16-25	137	(40)	15-40	103	(35)

The results by policy duration are not similar. The results of the 1983 Medical Impairment Study seem to be stable for the first 15 years policy duration, with a sudden fall thereafter. The results of the Prudential study show a reducing trend with increasing policy duration.

Expected deaths in the 1983 Medical Impairment Study were calculated on the basis of the 1965-70 Basic Tables (modified); expected deaths in the 1991 Prudential Study were calculated on the basis of the A67-70 tables (males) and the FA75-78 tables (females).

19.5 Additional Considerations

With reference to the impairment "epilepsy", Brackenridge (1985) reports:

> "Despite the wide range of antiepileptic drugs available for treatment, mortality associated with epilepsy as a whole still remains substantially above standard, although it varies slightly according to the type and frequency of fits, and reduces with increasing duration since the last attack. Among epileptics, the causes of death which are above the average expected are epilepsy itself, accident and suicide, diseases of the heart and circulation, and cerebrovascular diseases.

The results of the Prudential study show that excess mortality reduces with increasing duration

since last attack. Although the data are now classified by type and frequency of fits, insufficient data have accumulated to give satisfactory results. The comments on cause of death above would seem to add weight to the possibility of excess deaths classified as "other causes" in the Prudential study being from epilepsy itself.

19.6 Rating of Epileptics

A brief summary of the ratings recommended by the three reinsurance companies considered is outlined below.

Swiss Re:

Grand Mal and Petit Mal are rated along the same lines, considering Epilepsy Present and History of Epilepsy separately. Present is defined as last attack of either grand mal or petit mal within 2 years, history of epilepsy defined to be at least 2 years since the last attack.

The recommendations are as follows:

	Epilepsy Present	History of Epilepsy
Age at Entry 44 or less	+100	+50
At at Entry 45 or more	+50	+25

These ratings are reduced or increased in the presence of favourable or adverse conditions (e.g. not more than 2 attacks per year; or more than very modest alcohol intake).

Mercantile and General:

For Grand Mal cases with onset within 6 months, the recommendation is to postpone. Thereafter, the following ratings are recommended:

Age at Entry:	Under 40	40 or over
Up to 12 major attacks/year	+50 to +150	+25 to +75
More than 12 major attacks/year	+150 up	+100 up

A proposer free from attacks for three years or more would be considered for acceptance as standard.

Munich Re:

Munich Re consider Grand Mal and Petit Mal separately, but make no distinction by age at entry, except where age at entry is less than 16 where the applicant is rated as severe. Munich Re rate according to number of attacks per year and time since last attack as follows:

Time Since Last Attack	Mild	Moderate	Severe
0-2 yrs	+100 to +150	+175 to +250	Decline
2-5 yrs	+75 to +100	+125 to +175	+150 to +225
5-10 yrs	+25 to +75	+25 to +125	+75 to +150
over 10 yrs	0	Usually 0	0 to +50

Mild = 1 to 6 attacks per year
Moderate = 7 to 12 attacks per year
Severe = over 12 attacks per year

The ratings recommended by the three companies shown here seem to be realistic, in the light of the Prudential results. In particular, the results of the Prudential study indicate that ratings should depend on age at entry and duration since last attack.

Fitting Model A+E (age at entry and time since last attack together, no interaction) from the

Prudential study provides a basis for rating (and comparison) which is based on real data. Mortality ratios, together with values of *extra* mortality, given by fitting this model are as follows:

Age at Entry	Episodes within 2 yrs	No episodes for at least 2 yrs
16 to 39	276 (+176)	155 (+55)
40 to 79	193 (+93)	108 (+8)

Ratings above standard are given in parentheses.

Chapter 20 Psycho-Neuroses

20.1 Prudential Impairment Codes and Classification for Analysis:

320	Psycho-neuroses: mild
321	Psycho-neuroses: moderate
322	Psycho-neuroses: severe
323	With features suggestive of psychosis

Subsidiary codes:

A	Symptoms within 2 years
B	Symptoms within 2-5 years
C	Symptoms within 5-10 years
D	No symptoms for at least 10 years

This impairment has been in existence since the start of the investigation, with classification codes unchanged throughout the study. There is a large number of entrants, giving plenty of scope for a full analysis.

For analysis, the data were classified according to:

S:	Severity	4 levels:	1 - mild	3 - severe
			2 - moderate	4 - features suggestive of psychosis

T:	Time since last attack	4 levels:	1 - within 2 yrs	3 - 5 to 10 yrs
			2 - 2 to 5 yrs	4 - more than 10 yrs

A:	Age at Entry	4 levels:	1 - 16 to 29	3 - 40 to 49
			2 - 30 to 39	4 - 50 to 79

D:	Policy Duration	6 levels:	1 - 0 to 2 yrs	4 - 10 to 15 yrs
			2 - 2 to 5 yrs	5 - 15 to 20 yrs
			3 - 5 to 10 yrs	6 - over 20 yrs

C:	Calendar Year of Entry	8 levels:	1 - 1947 to 51	5 - 1967 to 71
			2 - 1952 to 56	6 - 1972 to 76
			3 - 1957 to 61	7 - 1977 to 81
			4 - 1962 to 66	8 - 1982 to 86

20.2 Male Lives

20.2.1 Summary Statistics

Number of Entrants: 41941

Number of Deaths: 1310

The total number of entrants in this impairment group is large. Over half of these, however, were classified as mild.

Table 20.1 Distribution by Severity

Severity	Number	Percentage
Mild	24795	59·1
Moderate	12635	30·1
Severe	2277	5·4
With symptoms suggestive of psychosis	2234	5·3
	41941	

Table 20.2 Distribution by Age at Entry

Age at Entry	Number of Entrants	Percentage
16-29	13842	33·0
30-39	16037	38·2
40-49	8859	21·1
50-79	3203	7·6
Total	41941	

Over 70% of entrants were below age 40 at entry.

Table 20.3 Distribution by Cause of Death

	Number of Deaths	Percentage
1. Tuberculosis	-	-
2. Lung Cancer	91	6·9
3. Other Cancer	170	13·0
4. Leukaemia	14	1·1
5. Diabetes	1	0·1
6. Vascular Lesions	48	3·7
7. Ischaemic Heart Disease	426	32·5
8. Other Circulatory Diseases	60	4·6
9. Influenza	-	-
10. Pneumonia	8	0·6
11. Bronchitis	4	0·3
12. Peptic Ulcer	-	-
13. Nephritis	2	0·2
14. Motor Accident	22	1·7
15. Other Accident	49	3·7
16. Suicide	98	7·5
17. Other Causes	318	24·3
Total	1311	

Noteworthy are deaths from accident and suicide, which here account for 12·9% of total deaths. In the complete Prudential study, deaths from accident and suicide accounted for 5·8% of total deaths. It is likely that some deaths recorded as accidental were, in reality, suicides. Furthermore, if there is any doubt, deaths which appear to be suicides are often recorded by coroners as "Open Verdicts", which are usually then classified as "Other Accidents".

Graphs of percentage deaths by cause are shown in Figures 20.1 and 20.2. Figure 20.1 shows percentage deaths by cause from all severity groupings compared with all deaths in the entire Prudential study, and Figure 20.2 compares percentage deaths by cause for each severity classification.

Figure 20.1 Percentage Deaths by Cause : Psycho-Neuroses (Males)

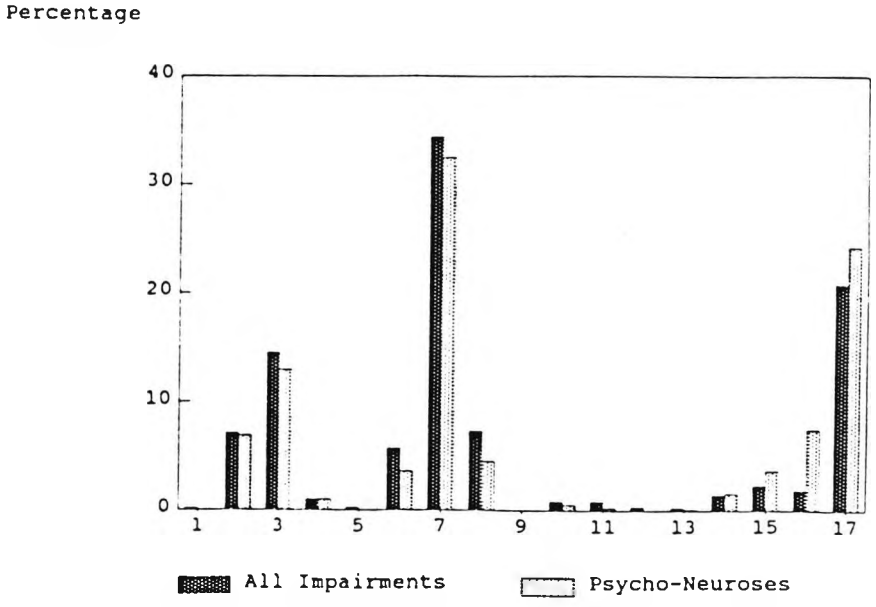
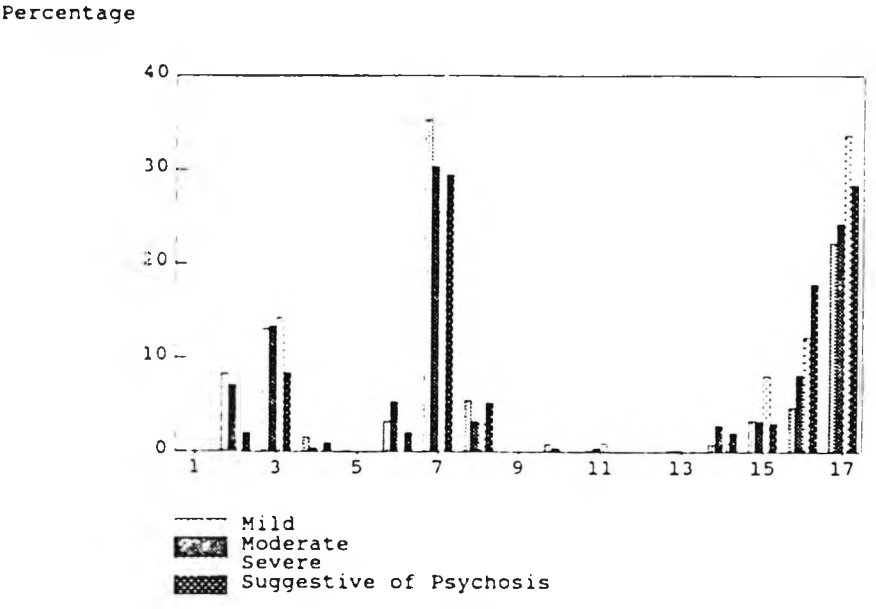


Figure 20.2 Percentage Deaths by Cause and Severity : Psycho-Neuroses (Males)



The graph of percentage deaths by each severity classfication is interesting when considering suicide as a cause of death (cause number 16). The graph clearly shows the percentage committing suicide rising with increasing severity of the condition. Where features are suggestive of psychosis, 18% of deaths were recorded as suicide (nearly one in five) which is somewhat shocking.

20.2.2 Results: The Overall Mortality Ratio

The overall mortality ratio was found to be 107%, based on 1310 deaths, showing very little excess mortality. The A67-70 (2) table was used as a basis for expected deaths.

20.2.3 Results: Main Effects Fitted Separately

Table 20.4 Mortality Ratios by Severity (Factor S)

	MR%	
Mild	99	(669)
Moderate	110	(448)
Severe	134	(98)
With Symptoms Suggestive of Psychosis	149	(95)

Excess mortality increases with severity, although the 'mild' classification shows no excess mortality at all.

Table 20.5 Mortality Ratios by Time Since Last Attack (Factor T)

	MR%	
Symptoms within 2 years	107	(723)
Symptoms within 2-5 years	102	(255)
Symptoms within 5-10 years	105	(178)
No symptoms for at least 10 years	118	(154)

Time since the last attack seems to make little difference to the observed mortality ratios.

Table 20.6 Mortality Ratios by Age at Entry (Factor A)

Age at Entry	MR %	
16 - 29	121	(123)
30 - 39	110	(361)
40 - 49	108	(507)
50 - 79	99	(319)

There is a tendency for excess mortality to fall as age at entry rises, although the difference between the highest and lowest age at entry groups is only 22 percentage points.

Table 20.7 Mortality Ratios by Policy Duration (Factor D)

Duration	MR %	
0 - 2 yrs	111	(105)
2 - 5	101	(190)
5 - 10	107	(335)
10 - 15	115	(303)
15 - 20	105	(199)
over 20 yrs	103	(178)

Policy duration seems to make very little difference to the excess mortality figures.

Table 20.8 Mortality Ratios by Calendar Year of Entry (Factor C)

	MR %	
1947 - 51	135	(147)
1952 - 56	115	(237)
1957 - 61	97	(191)
1962 - 66	117	(254)
1967 - 71	99	(199)
1972 - 76	104	(178)
1977 - 81	87	(92)
1982 - 86	65	(12)

The mortality ratios by calendar year of entry are fairly erratic, although there is a tendency for excess mortality to fall with increasing calendar year of entry. This, to some extent, will be exaggerated by the choice of the base-line hazard used. The mortality ratio of 65% for calendar years of entry 1982-1986 is particularly low, but is based on a low number of deaths (associated 95% confidence interval is [33%, 114%]).

20.2.4 Signficance of Main Effects

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev.	D of F	
H_0	1846·0	2508			
A	1841·8	2505	4·2	3	24 %
S	1827·5	2505	18·5	3	<·05 %
T	1843·9	2505	2·1	3	55 %
D	1843·4	2503	2·6	5	76%
C	1825·1	2501	20·9	7	<0·5 %

Referring differences in model deviances to the appropriate χ^2 distribution, as shown in the table above, reveals that severity and calendar year of entry are statistically significant. However, because the mortality ratios by calendar year of entry showed erratic behaviour rather than a clear trend over time, it was decided not to consider models including severity and calendar year of entry together. Furthermore, using the A67-70 table as a basis for expected deaths is likely to underestimate expected deaths in the early years of the study and overestimate expected deaths in the later years of the study. Consequently, the models S+C and S*C have not been investigated because the results would be misleading, especially where interest is focused on the more recent years.

However, the data from 1977-87 have been re-analysed using the AM80 table as a basis for expected deaths. Analysis was carried out considering severity, age at entry and policy duration.

20.3 Calendar Years of Entry 1977-87 : Male Lives

20.3.1 Results: The Overall Mortality Ratio

The overall mortality ratio was found to be 97%, based on 125 deaths, showing mortality very near standard. The AM80 table was used as a basis for expected deaths.

20.3.2 Results: Main Effects Fitted Separately

Table 20.9 Mortality Ratios by Age at Entry (Factor A)

Age at Entry	MR %	
16 - 29	99	(5)
30 - 39	99	(17)
40 - 49	76	(28)
50 - 79	108	(75)

There is little difference in the results by Age at Entry.

Table 20.10 Mortality Ratios by Severity (Factor S)

Severity	MR%	
Mild	74	(52)
Moderate	98	(45)
Severe	200	(12)
With Symptoms Suggestive of Psychosis	246	(16)

The mild and moderate categories show no excess mortality. The severe category and "with symptoms suggestive of psychosis" categories show excess mortality of approximately +100 and +150 respectively (although these results are based on low numbers of deaths).

Table 20.11 Mortality Ratios by Policy Duration (Factor D)

Duration	MR %	
0-2 yrs	72	(22)
2-5	115	(57)
5-11	95	(46)

The mortality ratio is lower in the first two years policy duration than subsequently.

20.3.3 Significance of Main Effects

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev.	D of F	
H_0	97·99	63			
A	95·30	60	2·69	3	44%
S	76·608	60	19·38	3	·02%
D	94·24	60	3·75	3	29%

Referring differences in model deviances to the appropriate χ^2 distribution indicates that the only statistically significant main effect is severity, which is highly statistically significant.

20.4 Female Lives

20.4.1 Summary Statistics

Number of Entrants: 13505
Number of Deaths: 171

Although the number of entrants for this impairment is large, there have been relatively few deaths, implying a high withdrawal rate. Like male lives, almost 60% of entrants were classified as mild, with very few classified as severe or with features suggestive of psychosis (see Table 20.12.

Table 20.12 Distribution by Severity

Severity	Number	Percentage
Mild	7973	59·0
Moderate	4368	32·3
Severe	622	4·6
With symptoms suggestive of psychosis	542	4·0
	13505	

Table 20.13 Distribution by Cause of Death

	Number of Deaths	Percentage
1. Tuberculosis	-	-
2. Lung Cancer	11	6·4
3. Other Cancer	50	29·2
4. Leukaemia	-	-
5. Diabetes	-	-
6. Vascular Lesions	7	4·1
7. Ischaemic Heart Disease	21	12·3
8. Other Circulatory Diseases	8	4·7
9. Influenza	-	-
10. Pneumonia	2	1·2
11. Bronchitis	2	1·2
12. Peptic Ulcer	-	-
13. Nephritis	-	-
14. Motor Accident	-	-
15. Other Accident	10	5·8
16. Suicide	16	9·4
17. Other Causes	44	25·7
Total	171	

Like male lives, deaths from suicide and accident are striking. They constitute 15·2 percent of overall deaths from this impairment compared with 6·7 percent of deaths from accident and suicide in the impaired lives study group as a whole. Figures 20.3 and 20.4 show graphs of

percentage deaths by cause, the first showing all severity groupings combined compared with all deaths in the study, the second comparing deaths by cause for each severity category.

Figure 20.3 Percentage Deaths by Cause : Psycho-Neuroses (Females)

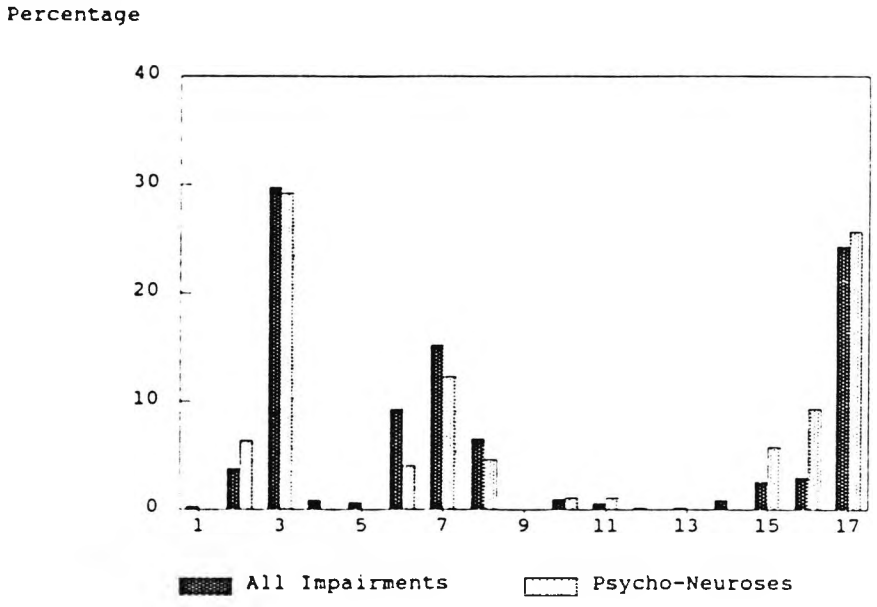
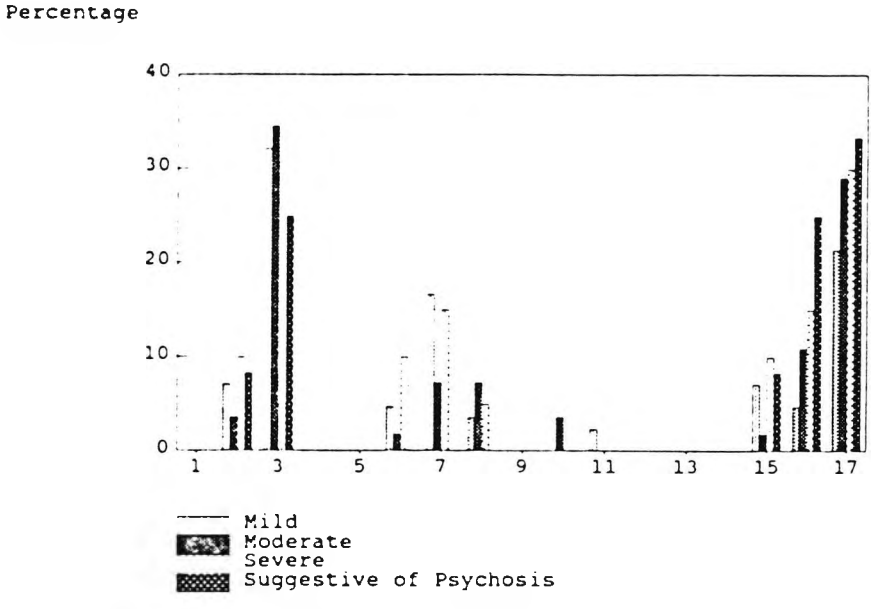


Figure 20.4 Percentage Deaths by Cause and Severity : Psycho-Neuroses (Females)



Again, it is clear that percentage deaths from suicide rise with increasing severity. Where symptoms are suggestive of psychosis, suicide accounted for 1/4 of total deaths.

Table 20.14 Distribution by Age at Entry

Age at Entry	Number of Entrants	Percentage
16-29	3497	25·9
30-39	5055	37·4
40-49	3439	25·5
50-79	1514	11·2
Total	13505	

A little over 60% of entrants were aged below 40 at entry.

20.4.2 Results: The Overall Mortality Ratio

The overall mortality ratio was found to be 99%, based on 171 deaths, showing no excess mortality. The FA75-78 table was used as a basis for expected deaths.

20.4.3 Results: Main Effects Fitted Separately

Table 20.15 Mortality Ratios by Severity (Factor S)

	MR%	
Mild	83	(84)
Moderate	104	(55)
Severe	203	(20)
With Symptoms Suggestive of Psychosis	134	(12)

The "mild" and "moderate" classifications show mortality near standard. The classifications "severe" and "with features suggestive of psychosis" show excess mortality, with the classification "severe" showing the highest level (+103).

Table 20.16 Mortality Ratios by Time Since Last Attack (Factor T)

	MR%		
Symptoms within 2 years	93	(117)	
Symptoms within 2-5 years	135	(34)	
Symptoms within 5-10 years	124	(17)	} 91 (20)
No symptoms for at least 10 years	36	(3)	

After grouping the last two levels to give an adequate number of deaths, time since last attack seems to make little difference to figures for excess mortality.

Table 20.17 Mortality Ratios by Age at Entry (Factor A)

Age at Entry	MR %	
16 - 29	98	(10)
30 - 39	110	(40)
40 - 49	113	(67)
50 - 79	81	(54)

The highest mortality ratio is from the 40 to 49 age at entry group, showing excess mortality of only +13.

Table 20.18 Mortality Ratios by Policy Duration (Factor D)

Duration	MR %		
0 - 2 yrs	113	(27)	
2 - 5	102	(46)	
5 - 10	84	(46)	
10 - 15	118	(29)	
15 - 20	104	(13)	} 96 (23)
over 20 yrs	87	(10)	

There is no obvious pattern in the results by policy duration.

Table 20.19 Mortality Ratios by Calendar Year of Entry (Factor C)

	MR %		
1947 - 51	60	(7)	} 112 (35)
1952 - 56	142	(28)	
1957 - 61	131	(16)	
1962 - 66	85	(15)	
1967 - 71	126	(27)	
1972 - 76	89	(28)	
1977 - 81	82	(39)	
1982 - 86	106	(11)	

There is no obvious pattern in the results by calendar year of entry.

20.4.4 Significance of Main Effects

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev.	D of F	
H_0	608·3	1784			
A	604·5	1782	3·8	2	15 %
S	596·2	1781	12·1	3	0·7 %
T	604·7	1782	3·6	2	17 %
D	605·6	1780	2·7	4	61 %
C	602·9	1778	5·4	6	49 %

Referring differences in model deviances to the appropriate χ^2 distribution reveals that the only statistically significant factor is "severity" of the condition.

20.5 Calendar Years of Entry 1977-87 : Female Lives

For completeness, the data for the period 1977-87 were analysed separately, using the AF80 table as a basis for expected deaths. During this period, there were 8144 entrants (over half classified as mild) and 59 deaths.

20.5.1 Results: The Overall Mortality Ratio and Main Effects Fitted Separately

The overall mortality ratio was found to be 93%. The results for the main effects were as follows:

Table 20.20 Mortality Ratios by Age at Entry (Factor A)

Age at Entry	MR %		
16 - 29	116	(3)	} 112 (14)
30 - 39	111	(11)	
40 - 49	100	(18)	
50 - 79	83	(27)	

Mortality ratios appear to fall as age at entry increases.

Table 20.21 Mortality Ratios by Severity (Factor S)

	MR%	
Mild	66	(23)
Moderate	124	(29)
Severe, or with features suggestive of Psychosis	145	(7)

As expected, the mortality ratios increase with severity, although the figure for the highest severity group was based on only 7 deaths.

Table 20.22 Mortality Ratios by Policy Duration

Duration	MR %	
0-2 yrs	90	(14)
2-5	104	(26)
5-11	84	(19)

Policy duration seems to make little difference to the figures for excess mortality.

20.5.2 Significance of Main Effects

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev.	D of F	
H_0	46·229	47			
S	39·856	45	6·373	2	4%
A	45·295	44	0·934	3	82%
D	45·70	45	0·529	2	77%

Referring differences in model deviances to the appropriate χ^2 distribution reveals that the only statistically significant main effect is "severity".

20.6 Comparison with Other Studies

The classification codes used by the Prudential for this impairment give no details of the clinical criteria used for subclassification into mild, moderate and severe psychoneurosis, which makes it extremely difficult to compare results with other studies which are not based on the Prudential data set, particularly studies conducted abroad. Consequently, comparison is restricted to those studies conducted using the Prudential data set only. Table 20.22 shows, for male lives, the mortality ratios together with number of deaths observed by each author reporting results based on the Prudential data, by severity classification.

Table 20.22 Prudential Impaired Lives Studies: Male Lives

Author	Severity								Overall	
	Mild		Moderate		Severe		Psychosis†			
Clarke (1961)	–		–		–		–		100	(51)
Preston and Clarke (1966)	127	(74)	140	(64)	––	138	(16)	––	133	(154)
Clarke* (1979)	94	(155)	116	(142)	101	(21)	139	(24)	105	(342)
Leighton* (1987)	91	(287)	99	(169)	160	(54)	144	(42)	101	(552)
Papaconstantinou (1988)	–		–		–		–		143	(1001)
England (1991)	99	(669)	110	(448)	134	(98)	149	(95)	107	(1310)

† strictly, with features suggestive of psychosis * duration 2 and over only

The overall mortality ratios show low excess mortality, although this hides the differences observed within severity groups, largely because of an uneven distribution of policies issued within each category (mild 60%, moderate 30%, others 10% approximately). It is clear from Table 20.22 that excess mortality rises with worsening severity. The mild classification shows no excess mortality and can therefore be accepted at standard rates. The most severe category, where features are suggestive of psychosis, shows excess mortality of approximately +50. Suicide and accidents account for most of the excess deaths.

For female lives, it is only possible to compare overall mortality ratios, as shown in Table 20.23.

Table 20.23 Prudential Impaired Lives Studies: Female Lives

Author	Publication Date	Calendar Yrs of Study	Overall MR%
Preston & Clarke	1966	1947-63	124 (18)
Papaconstantinou	1988	1947-81	114 (104)
England	1991	1947-86	99 (171)

Again, the overall mortality ratios are deceptive since they are heavily weighted towards the mild and moderate categories. The only results available showing mortality ratios by severity for female lives are those shown earlier in this thesis.

20.7 Rating of Neuroses

In "Medical Selection of Life Risks", Brackenridge (1985) recommends the following ratings:

Classification	Rating
Mild	+20 to +35
Moderate	+75
Severe	+100

Brackenridge adds that "where a neurosis has remained symptomless for five years or longer, standard rates should be considered provided there is no latent, severe personality disorder."

The results of this study indicate that these ratings are a little high, and certainly there is no evidence to suggest that time since the symptoms were last observed makes any difference to the ratings which should apply. Commenting on Neuroses, Brackenridge writes:

> "A neurosis is a disorder of personality and behaviour which . . . has its origins in childhood when it is nourished by a faulty home environment. It is a lifelong disorder."

The fact that it is a lifelong disorder would seem to indicate that time since the presence of symptoms should make no difference to observed mortality ratios.

A brief summary of the methods of rating this impairment used by the three reinsurance companies considered is outlined below.

Swiss Re:

Swiss Re take a fairly relaxed approach to underwriting this impairment and apply a rating of +25. This may be reduced to 0 or increased to +50 in the presence of favourable or adverse conditions. However this rating is only applied provided

(i) There are no signs of associated depression, personality disorder or psychosis, otherwise rate accordingly.

(ii) There is no alcohol abuse or drug dependence for the last 3 years, otherwise postpone.

(iii) There are manifestations within the last 3 years, otherwise disregard.

Munich Re:

Munich Re make distinctions between mild and severe classifications and whether under treatment or recovered. Where an applicant is under treatment and the condition is classified as mild, a rating of +50 upwards is recommended. An applicant classified as severe and under treatment would usually be declined.

Where an applicant has recovered, the distinction is made between cases where only one episode occurred and where episodes were recurrent, in which case a different rating is applied depending on the time elapsed since the last episode. The ratings applied are as follows:

One Episode:

Last Episode:	Mild	Severe
0-1 yrs	+25	+50 up
1-2 yrs	0	+25 to +50
2-3 yrs	0	+0 to +25

Recurrent Episodes:

Last Episode:	Mild	Severe
0-1 yrs	+50 up	Usually postpone
1-2 yrs	+25	+75 up
2-3 yrs	0	+50 to +75
3-4 yrs	0	+25 to +50
4-5 yrs	0	0 to +25

Mercantile and General:

Mercantile and General use classifications which can be stated as mild, moderate and severe psychoneurosis and rate accordingly as follows:

Mild:

Up to 3 attacks	+0
Recurrent attacks	Up to 2 per 1000 S/A

Moderate:

One attack, within 1 year	3 per 1000 S/A for 3 years
2nd year	3 per 1000 S/A for 2 years
Thereafter	+0
Recurrent attacks	3 per 1000 S/A

Severe:

One attack, within 1 year	5 per 1000 S/A for 3 years
2nd year	5 per 1000 S/A for 2 years
Thereafter	0
More than one attack,	
last within 1 year	+50 and 5 per 1000 S/A for 4 years
2nd year	+50 and 5 per 1000 S/A for 3 years
3rd year	+50 and 5 per 1000 S/A for 2 years
4th and 5th years	+50
Thereafter	0

This is a more complicated approach, involving an addition to the premium, possibly with a mortality rating up as well.

Results based on the Prudential data set provide no foundation for rating according to time elapsed since the symptoms were last observed. All three underwriting manuals studied apply a low rating where the impairment is classified as mild, which agrees with the findings of the Prudential study. The experience of the "Severe" and "with features suggestive of Psychosis" groups does not seem to indicate that these cases are uninsurable; automatically declining these groups could result in the loss of potentially profitable business.

Chapter 21 Diabetes Mellitus

21.1 Prudential Impairment Codes and Classification for Analysis:

Pre 1/1/80:

520	Age under 30,	weight standard ± 10%
521		weight more than 10% below standard
522		weight more than 10% above standard
523	Age 30 - 50,	weight standard ± 10%
524		weight more than 10% below standard
525		weight more than 10% above standard
526	Age over 50,	weight standard ± 10%
527		weight more than 10% below standard
528		weight more than 10% above standard

Subsidiary Codes:

E	Family History, no diabetes
F	Family History, one or more diabetes

Post 1/1/80:

570	Age under 30 at entry
571	Age 30 - 34 at entry
572	Age 35 - 39 at entry
573	Age 40 - 44 at entry
574	Age 45 - 49 at entry
575	Age 50 and over at entry

Subsidiary Codes:

A	Duration since diagnosis (or detection) 0 to 5 years
B	Duration since diagnosis (or detection) 5 to 10 years
C	Duration since diagnosis (or detection) 10 to 15 years
D	Duration since diagnosis (or detection) 15 to 20 years
E	Duration since diagnosis (or detection) 20 to 30 years
F	Duration since diagnosis (or detection) over 30 years

The coding for this impairment changed on 1/1/80, with the new codes having been brought into line with those for the condition "Uninvestigated Glycosuria". When enough data have accumulated, a comparison between these impairments will be possible. In the period between 1/1/80 and 31/12/87, there were 451 entrants, but no deaths recorded. Consequently, it was not possible to analyse the post 1980 experience, and attention has been restricted to those policies issued between 1947 and 1980.

For analysis, data were classified according to:

W: Weight : 3 levels
- 1 - Standard ± 10%
- 2 - More than 10% below standard
- 3 - More than 10% above standard

H: Family History : 2 levels
- 1 - no diabetes
- 2 - one or more diabetes

A: Age at Entry : 4 levels
- 1 - 16 to 29
- 2 - 30 to 39
- 3 - 40 to 49
- 4 - 50 to 79

D: Policy Duration : 6 levels
- 1 - 0 to 2 years
- 2 - 2 to 5 years
- 3 - 5 to 10 years
- 4 - 10 to 15 years
- 5 - 15 to 20 years
- 6 - over 20 years

C: Calendar Year of Entry: 7 levels
- 1 - 1947 to 51
- 2 - 1952 to 56
- 3 - 1957 to 61
- 4 - 1962 to 66
- 5 - 1967 to 71
- 6 - 1972 to 76
- 7 - 1977 to 79

21.2 Male Lives

21.2.1 Summary Statistics

Number of entrants: 5287

Number of deaths: 603

Table 21.1 Distribution By Age at Entry

Age at Entry	Number of Entrants	Percentage
16 to 29	1977	37·4
30 to 39	1431	27·1
40 to 49	1130	21·4
50 to 79	749	14·2
Total	5287	

Approximately $\frac{2}{3}$ of entrants were aged below 40 at entry, with only approximately $\frac{1}{7}$ aged 50 and above at entry.

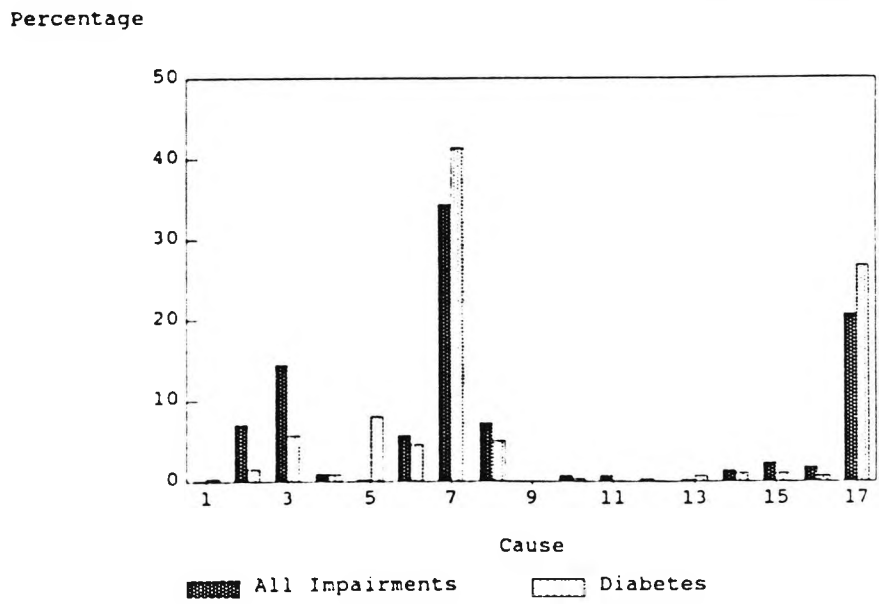
Table 21.2 Distribution by Cause of Death

	Number of Deaths	Percentage
1. Tuberculosis	3	0·5
2. Lung Cancer	10	1·7
3. Other Cancer	35	5·8
4. Leukaemia	6	1·0
5. Diabetes	49	8·1
6. Vascular Lesions	28	4·6
7. Ischaemic Heart Disease	250	41·5
8. Other Circulatory Diseases	31	5·1
9. Influenza	-	-
10. Pneumonia	3	0·5
11. Bronchitis	1	0·2
12. Peptic Ulcer	1	0·2
13. Nephritis	5	0·8
14. Motor Accident	7	1·2
15. Other Accident	7	1·2
16. Suicide	5	0·8
17. Miscellaneous (unspecified)	162	26·9
Total	603	

Figure 21.1 shows percentage deaths by cause for diabetics and all deaths in the entire Prudential data set. Noteworthy are deaths from diabetes and ischaemic heart disease. Deaths

from diabetes account for 8·1% of total deaths for diabetics which compares with 0·3% of deaths from diabetes in the entire data set. There is a higher incidence of deaths from ischaemic heart disease, but not from other circulatory diseases. Although only based on 5 deaths, Nephritis is also a noteworthy cause of death.

Figure 21.1 Distribution by Cause of Death : Diabetics (Males)



21.2.2 Results: The Overall Mortality Ratio

The overall mortality ratio was found to be 280% based on 603 deaths. The A67-70 (2) table was used as a basis for expected deaths.

21.2.3 Results: Main Effects Fitted Separately

Table 21.3 Mortality Ratios by Weight Levels (Factor W)

	MR%	
Weight more than 10% below standard	303	(124)
Weight standard ± 10%	272	(391)
Weight more than 10% above standard	289	(88)

The weight factor does not seem to have made much difference to the figures for excess mortality. The underweights fare slightly worse than the overweights.

Table 21.4 Mortality Ratios by Age at Entry (Factor A)

Age at Entry	MR %	
16 - 29	495	(83)
30 - 39	417	(146)
40 - 49	280	(198)
50 - 79	190	(176)

It is clear that excess mortality is far higher for the younger ages at entry and that the mortality ratios reduce dramatically as age at entry increases. Excess mortality is considerable for ages at entry below 40.

Table 21.5 Mortality Ratios by Policy Duration (Factor D)

Duration	MR %	
0 - 2 yrs	200	(34)
2 - 5	243	(97)
5 - 10	252	(183)
10 - 15	302	(138)
15 - 20	360	(97)
over 20 yrs	403	(54)

Clearly excess mortality rises as policy duration increases.

Table 21.6 Mortality Ratios by Family History (Factor H)

	MR %	
No Diabetes	281	(554)
One or more Diabetes	267	(49)

The family history of diabetes makes very little difference to the figures for excess mortality.

Table 21.7 Mortality Ratios by Calendar Year of Entry (Factor C)

	MR %	
1947 - 51	316	(49)
1952 - 56	286	(75)
1957 - 61	314	(101)
1962 - 66	328	(157)
1967 - 71	263	(119)
1972 - 76	249	(68)
1977 - 79	159	(34)

The mortality ratios were reasonably stable up until 1966. There has been an apparent drop in excess mortality from 1967 onwards.

21.2.4 Significance of Main Effects

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev.	D of F	
H_0	696·37	717			
W	695·20	715	1·17	2	56 %
H	696·30	716	0·07	1	79 %
A	622·67	714	73·70	3	$\ll$ ·05 %
D	675·19	712	21·18	5	0·08 %
C	676·00	711	20·37	6	0·25 %

Referring differences in model deviances to the appropriate χ^2 distribution reveals that the factors age at entry, policy duration and calendar year of entry are all highly statistically significant, leading to the investigation of more complex models.

21.2.5 Main Effects Fitted Together, No Interaction

The simplest model catering for all three significant main effects together is Model A+D+C with parametric representation for the mortality ratio given by $exp\ (\mu+\alpha_i+\delta_j+\gamma_k)$. This

model ignores interaction terms. Table 21.8 below shows the results from fitting this model. Mortality ratios may be deduced by forming the product of relevant entries.

Table 21.8 Excess Mortality Factors: Model A+D+C

$exp(\mu) = 4{\cdot}22$

Age at Entry :	16 to 29	30 to 39	40 to 49	50 to 79
$exp(\alpha_i)$	1·00	0·80	0·53	0·38

Policy Duration :	0 - 2	2 - 5	5 - 10	10 - 15	15 - 20	>20
$exp(\delta_j)$	1·00	1·31	1·35	1·34	1·37	1·24

Calendar Yr of Entry	47-51	52-56	57-61	62-66	67-71	72-76	77-79
$exp(\gamma_k)$	1·00	0·94	0·99	1·05	0·88	0·85	0·61

Mortality ratios produced from this model are consistent with the results from the main effects models fitted separately. That is, mortality ratios reduce as age at entry increases, rise as policy duration rises, and are reasonably stable until about 1967 and reduce thereafter.

21.2.6 Models Including First Order Interactions

The models A*D+C , A+D*C , and A*C+D were fitted to assess the significance of the first order interaction terms, leading to the following deviance table:

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev.	D of F	
A+D+C	607·55	703			
A*D+C	591·52	688	16·03	15	38 %
A+D*C	568·75	679	38·80	24	2·9 %
A*C+D	568·68	685	38·87	18	0·30 %

Referring differences in model deviances to the appropriate χ^2 distribution indicates that the

interactions between calendar year of entry and both age at entry and policy duration are statistically significant. These two interaction terms were included in the same model, which resulted in the fitting of Model C*(A+D).

21.2.7 Main Effects Fitted Together, with Interactions : Model C*(A+D)

This compound model caters for all three significant main effects and the statistically significant first order interaction terms. The associated parametric representation for the mortality ratios is $exp(\mu+\alpha_i+\delta_j+\gamma_k+\alpha\gamma_{ik}+\delta\gamma_{jk})$. The mortality ratios given by fitting this model are shown in Table 21.9.

Table 21.9 Mortality Ratios: Model C*(A+D)

Calendar Year of Entry : 1947-51

	Age at Entry			
Policy Duration	16-29	30-39	40-49	50-79
0 to 2 years	29·87	12·49	9·58	0·98
2 to 5	13·95	5·83	4·47	0·46
5 to 10	9·37	3·92	3·00	0·31
10 to 15	9·03	3·77	2·90	0·29
15 to 20	12·09	5·06	3·88	0·40
over 20 years	9·10	3·80	2·92	0·30

Calendar Year of Entry : 1952-56

	Age at Entry			
Policy Duration	16-29	30-39	40-49	50-79
0 to 2 years	9·16	6·37	3·18	3·88
2 to 5	5·52	3·84	1·91	2·34
5 to 10	7·71	5·36	2·67	3·27
10 to 15	5·60	3·89	1·94	2·37
15 to 20	4·44	3·09	1·54	1·88
over 20 years	4·20	2·92	1·46	1·78

Calendar Year of Entry : 1957-61

Policy Duration	Age at Entry 16-29	30-39	40-49	50-79
0 to 2 years	2·79	1·47	1·05	0·77
2 to 5	6·26	3·31	2·35	1·73
5 to 10	7·25	3·83	2·72	2·00
10 to 15	4·77	2·52	1·79	1·32
15 to 20	12·08	6·38	4·54	3·34
over 20 years	7·74	4·09	2·91	2·14

Calendar Year of Entry : 1962-66

Policy Duration	Age at Entry 16-29	30-39	40-49	50-79
0 to 2 years	2·42	2·64	1·88	1·72
2 to 5	4·63	5·07	3·61	3·30
5 to 10	3·65	3·99	2·84	2·60
10 to 15	4·49	4·91	3·49	3·20
15 to 20	3·86	4·22	3·01	2·75
over 20 years	3·83	4·19	2·98	2·73

Calendar Year of Entry : 1967-71

Policy Duration	Age at Entry 16-29	30-39	40-49	50-79
0 to 2 years	3·67	4·86	3·24	1·59
2 to 5	2·62	3·47	2·31	1·14
5 to 10	3·55	4·69	3·13	1·54
10 to 15	4·31	5·70	3·80	1·87
15 to 20	2·74	3·63	2·42	1·19

Calendar Year of Entry : 1972-76

Policy Duration	Age at Entry 16-29	30-39	40-49	50-79
0 to 2 years	1·41	1·26	0·77	0·75
2 to 5	4·34	3·88	2·38	2·30
5 to 10	4·33	3·88	2·38	2·29
10 to 15	4·40	3·94	2·41	2·33

Calendar Year of Entry : 1977-79

Policy Duration	Age at Entry			
	16-29	30-39	40-49	50-79
0 to 2 years	2·09	1·46	0·89	0·48
2 to 5	4·88	3·39	2·06	1·13
5 to 10	5·12	3·56	2·17	1·18

Within each calendar year of entry group, the values in each row are a fixed multiple of the values in the first row and the values in each column are a fixed multiple of the values in the first column. This is a result of the (A+D) part of the model. However, this multiplicative effect does not hold when comparing one calendar year of entry group with another, which is a result of the interactions with calendar year of entry.

Comparing the calendar year of entry groups, it can be seen that the trends by age at entry and duration are similar, but the values themselves are often markedly different. This is useful as a record of what was observed, but not very useful when it comes to forecasting. The usefulness of such a complex model as this is therefore open to question. The more recent calendar year of entry groups do not have entries for the highest duration groups, since there are no data available for certain cross classifications of calendar year of entry and policy duration.

Looking at the deviances of various models, the following table can be produced:

Model	Deviance	Differences	
H_0	696·37		
A	622·67	73·70	(3)
A+D+C	607·55	15·12	(11)
C*(A+D)	536·91	70·64	(42)

(degrees of freedom in parentheses)

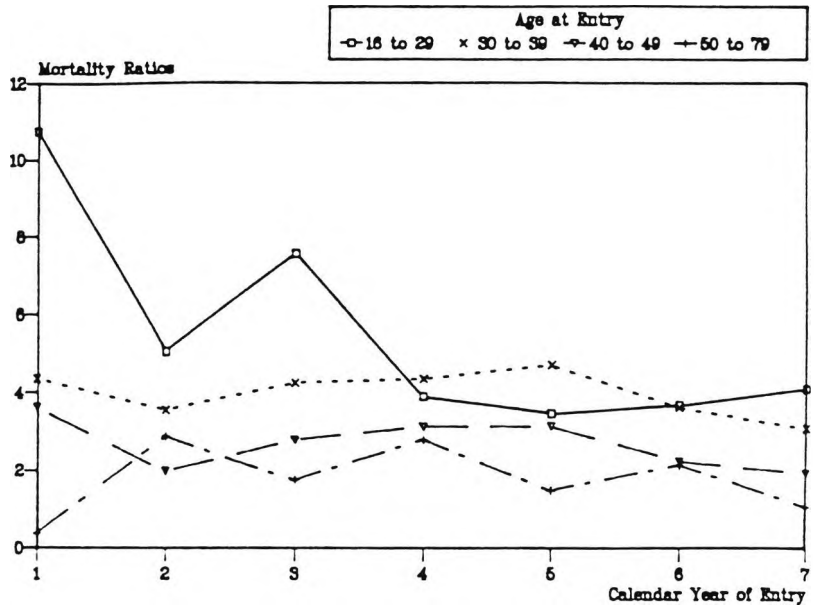
The inclusion of the age at entry main effect factor leads to a dramatic reduction in the deviance (73·7) for a small change in degrees of freedom. The further inclusion of policy duration and calendar year of entry does not affect the deviance so noticeably (a reduction of only 15·12), but the inclusion of significant first order interaction terms again results in a large drop in the deviance (although together with a large change in degrees of freedom). The most important factor to take into consideration is therefore age at entry.

21.3 Dynamic Generalised Linear Models

While analysing excess mortality of diabetics, the specific problems associated with calendar year of entry (and noted above) have led to the investigation of other methods of dealing with this model factor. One method which showed potential uses the Bayesian theory of dynamic generalised linear models propounded by West *et al* (1985), and used by Gamerman (1991) to estimate survival functions. Using the Bayesian approach, calendar year of entry is no longer included as a model factor. Instead a model is fitted which has the same structure within each time period (rather like model C*(A+D) in Section 21.2.7), but the parameters in the model for each time period are related sequentially to those in other time periods using a recursive prior-posterior analysis. The result is that the effect of the time period is partially smoothed out, where the analyst has control over the degree of smoothing. The details can be found in England and Verrall (1992), but a brief description of the results is shown below.

Consider modelling age at entry and calendar year of entry only. The results shown earlier indicate that age at entry and calendar year of entry are both statistically significant, and furthermore that the interaction between age at entry and calendar year of entry is statistically significant. Thus, mortality ratios may be modelled using model A*C, in which the mortality ratios by age at entry are, in effect, fitted separately in each calendar year of entry group. The results of fitting this model are shown graphically in Figure 21.2.

Figure 21.2 Mortality Ratios : Model A*C

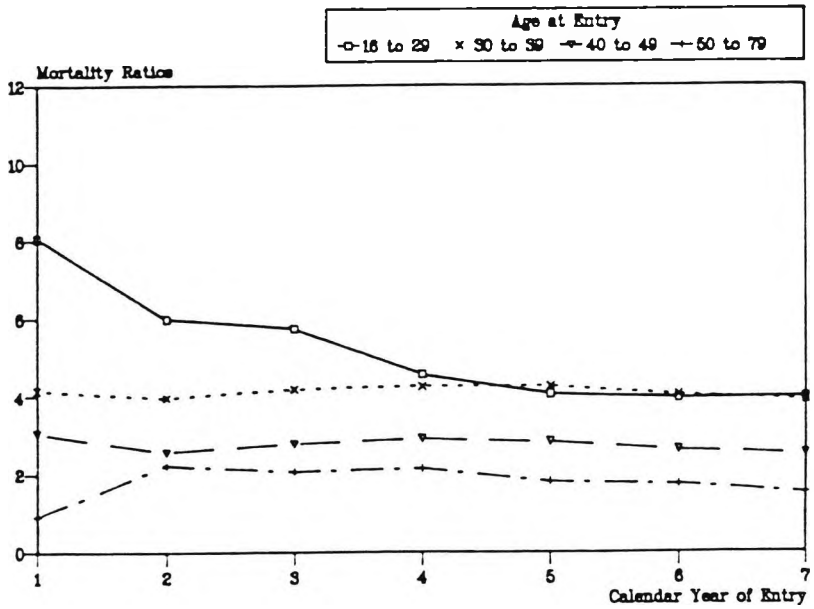


Focusing attention on changes in mortality ratios for each age at entry group, it can be seen that the mortality ratios move fairly erratically, whereas it would be reasonable to expect mortality ratios to change smoothly over time.

Using the Bayesian approach, models may be fitted in which, essentially, information is passed between time periods, where the amount of information passed controls the degree of smoothing. At one extreme, no information is passed between time periods and the results are exactly the same as those shown in Figure 21.2. At the other extreme, the mortality ratio for a particular time period is based on all the data available for all time periods. Effectively, this ignores calendar year of entry and the results are identical to those obtained by fitting age at entry separately (shown in Table 21.4). Somewhere between these extremes, it is possible to imagine a model in which mortality ratios are different in each calendar year of entry group, but change fairly smoothly.

The optimal model under the Bayesian approach is chosen with regard to the trade-off between smoothness and goodness-of-fit. Results from the optimal model for male diabetics, considering only the factor age at entry, are shown graphically in Figure 21.3. Notice that the mortality ratios show the same general trend as those in Figure 21.2, but with a greater degree of smoothness.

Figure 21.3 Mortality Ratios Using Dynamic Bayes Approach



Finally, it is worth pointing out that the inclusion of factors other than age at entry presents no additional difficulty in the modelling, only in the presentation of results.

21.4 Female Lives

21.4.1 Summary Statistics

Number of entrants: 576

Number of deaths: 52

There were far fewer female entrants than male entrants with this impairment. It was not possible to analyse the female lives by family history of diabetes or calendar year of entry, because of insufficient deaths. Results have been reported for weight, age at entry and policy duration only.

Table 21.10 Distribution By Age at Entry

Age at Entry	Number of Entrants	Percentage
16 to 39	398	69·1
40 to 49	107	18·6
50 to 59	50	8·7
60 to 79	21	3·6
Total	576	

Most of the entrants (~70%) were aged below 40 at entry.

Table 21.11 Distribution by Cause of Death

	Number of Deaths	Percentage
Tuberculosis	1	1·9
Cancer (excluding lung)	6	11·5
Diabetes	13	25·0
Vascular Lesions	1	1·9
Ischaemic Heart Disease	15	28·8
Other Circulatory Diseases	2	3·8
Pneumonia	1	1·9
Accident (excluding motor)	1	1·9
Miscellaneous (unspecified)	12	23·1
Total	52	

Comparing the percentage deaths by cause for female diabetics and all female deaths in the entire study shows that deaths from diabetes and ischaemic heart disease are noteworthy as being high, and deaths from cancer (excluding lung) are noteworthy as being low.

21.4.2 Results: The Overall Mortality Ratio

The overall mortality ratio was found to be 410% based on 52 deaths, using the FA75-78 table as a basis for expected deaths. This shows considerable excess mortality.

21.4.3 Results: Main Effects Fitted Separately

Table 21.12 Mortality Ratios by Weight Levels (Factor W)

	MR%	
Weight more than 10% below standard	263	(7)
Weight standard ± 10%	417	(35)
Weight more than 10% above standard	613	(10)

The low number of deaths in the below and above standard groups make it difficult to interpret these results.

Table 21.13 Mortality Ratios by Age at Entry (Factor A)

Age at Entry	MR %			
16 - 39	536	(17)		
40 - 49	682	(21)		
50 - 59	271	(8)	} 218	(14)
60 - 79	173	(6)		

There is a tendency for the mortality ratios to fall as age at entry rises. Again, these results are based on low numbers of deaths in each group.

Table 21.14 Mortality Ratios by Policy Duration (Factor D)

Duration	MR %			
0 - 2 yrs	337	(4)	} 299	(12)
2 - 5	283	(8)		
5 - 10	344	(18)		
10 - 15	665	(13)		
15 - 20	327	(3)	} 644	(22)
over 20 yrs	1101	(6)		

There is a tendency for the mortality ratios to rise as policy duration increases.

21.4.4 Significance of Main Effects (after grouping)

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev.	D of F	
H_0	70·519	65			
W	67·486	63	3·033	2	22%
A	57·79	63	12·729	2	0·17%
D	64·722	63	5·797	2	5·5%

Although the mortality ratios by weight category were very different, the low number of deaths result in this factor being non-significant statistically. Age at entry is highly statistically significant, and policy duration is marginally statistically significant. It is not meaningful to fit more complex models due to the low numbers of deaths.

21.5 Comparison with Other Studies

Diabetes mellitus has been recognised for a very long time (apparently since before the birth of Christ). Before the discovery of insulin in 1921, death would occur fairly rapidly after onset. The discovery of insulin was a major breakthrough and has improved survival significantly. Several studies have been conducted around the world (see Brackenridge (1985) and Lew and Gajewski (1991)) and the major findings are that:

1) excess mortality is highest for young ages at detection
2) female lives fare slightly worse than male lives with the condition
3) family history of diabetes makes no difference to the mortality rates of diabetics
4) weight levels make little difference to the mortality rates of diabetics
5) diabetics needing treatment with insulin have higher mortality ratios than those needing treatment by oral drugs, who fare slightly worse than those needing only a controlled diet.

For comparison with the results of this report, attention is focused on insured lives studies only.

21.5.1 Other Studies Based on the Prudential Data Set

Initially, consider overall mortality ratios, shown in Table 21.15

Table 21.15 Prudential Data Set: Diabetics – Overall Mortality Ratios

Author	Calendar Yrs of Study	Mortality Ratios	
		Males	Females
Clarke (1961)	1947-58	260 (27)	–
Preston and Clarke (1966)	1947-63	278 (71)	–
Clarke* (1979)	1964-73	240 (157)	–
Leighton* (1987)	1974-83	286 (264)	–
Papaconstantinou (1988)	1947-81	310 (481)	408 (40)
England (1991)	1947-86	280 (603)	410 (52)

* Duration 2 and over only

The results in the above table compare well, perhaps with the exception of Papaconstantinou's results for male lives, which are higher than any of the other results. For female lives, the two results given compare very well. For male lives, it is also possible to compare results by age at entry and weight levels. The table below shows results by age at entry:

Table 21.16 Prudential Data Set: Diabetics – Results by Age at Entry.

Author	Mortality Ratios by Age at Entry		
	under 30	30 to 50	over 50
Preston and Clarke (1966)	833 (10)	352 (37)	174 (24)
Clarke (1979)*	420 (19)	284 (94)	157 (44)
Leighton (1987)*	466 (30)	336 (180)	167 (54)
Papaconstantinou (1988)	1242 (70)	418 (269)	166 (142)
England (1991)	495 (83)	325 (344)	190 (176)

* Duration 2 and over only, and ages at entry above 30 only

The results shown in Table 21.16 compare very well with the exception of Papaconstantinou's results for ages at entry below 50. In particular, the mortality ratio of 1242 % at ages at entry below 30 is significantly higher than any other result. This reinforces earlier worries about the suitability of the basis for expected deaths used by Papaconstantinou. The results suggest excess mortality in the region of +400 for ages at entry below 30 reducing to +100 for ages at entry above 50.

Using the results of some of the authors, it is possible to give mortality ratios by weight levels at entry, shown in Table 21.17.

Table 21.17 Prudential Data Set: Diabetics — Results by Weight Levels

Author	Mortality Ratios by Weight at Entry		
	>10% below S	S ± 10%	>10% above S
Clarke (1979)*	223 (25)	222 (91)	245 (22)
Leighton (1987)*	265 (44)	269 (155)	297 (35)
England (1991)	303 (124)	272 (391)	289 (88)

S = Standard

* Results obtained using figures for ages at entry above 30 only

The results of England are a little higher than those of the other two authors since the figures are based on the entire age range, not just ages at entry above 30 (which experience lighter mortality). However, the interesting observation here is that weight levels do not make much difference to the observed mortality ratios.

21.5.2 The 1983 Medical Impairment Study

The 1983 Medical Impairment Study (M.I.S.) is based on the experience of insured lives in North America. The impairment "Diabetes" covered approximately 37,000 policies issued from 1952 through to 1976 at standard or substandard premium rates to persons with diabetes mellitus. Persons with other impairments were excluded. The basis for expected deaths was

the 1965-70 Basic Tables (modified). Table 21.18 shows overall mortality ratios for males and females for the 1983 M.I.S. and the Prudential results reported by England, and Table 21.19 shows a further breakdown by age at entry (male lives only).

Table 21.18 1983 M.I.S. and Prudential Study: Overall Mortality Ratios

Study	Overall Mortality Ratios	
	Males	Females
1983 M. I. S.	211 (1373)	288 (173)
England	280 (603)	410 (52)

Table 21.19 1983 M.I.S. and Prudential Study: Results by Age at Entry

1983 M. I. S.	
Age at Entry	MR %
15 - 39	313 (363)
40 - 49	200 (403)
50 - 59	195 (473)
60 - 69	149 (134)

England	
Age at Entry	MR %
16 - 39	442 (229)
40 - 49	280 (198)
50 - 59	218 (130)
60 - 79	138 (46)

Although the results of the 1983 M.I.S. are (on the whole) lighter, the trend by age at entry is very similar.

Also available in the 1983 M. I. S. are the results by treatment, shown briefly in Table 21.20 for males amd females treated orally and by insulin.

Table 21.20 1983 M.I.S.: Mortality Ratios by Treatment

1983 M. I. S.	Mortality Ratios %	
	Oral	Insulin
Males	198 (309)	376 (212)
Females	257 (43)	515 (41)

The results of the 1983 M.I.S. show that excess mortality is higher in diabetics who require treatment by insulin. Unfortunately, the Prudential data are not classified by treatment, so no comparison is possible. However the results of two British studies (see Shaw (1974), Greenhalgh and Rutter (1980), and Shenfield *et al* (1979)) also show that mortality ratios differ by treatment, with insulin dependent diabetics showing the highest excess mortality, followed by those treated orally, followed by those controlled by diet only. Of course, this is not a reflection on the efficacy of treatment types, since mild cases need only dietary control and severe cases need insulin.

Another interesting group within the 1983 Medical Impairment Study is the impairment "Family History of Diabetes". This impairment covered over 250,000 policies. The observed mortality ratios were 84% for males and 110% for females indicating that this impairment does not constitute an extra risk.

21.6 Rating of Diabetes Mellitus

The three underwriting manuals considered differ in their approach to the rating of this disorder. In general, all use the numerical rating system to apply a basic rating which is then modified using a series of credits and debits according to the presence or absence of a long list of additional risk factors.

Swiss Re:

The approach adopted by Swiss Re would appear to be the simplest of the three. The distinction is made between Type 1 Diabetes Mellitus (Insulin Dependent Diabetes Mellitus or IDDM) and Type 2 Diabetes Mellitus (Non Insulin Dependent Diabetes Mellitus or NIDDM). The basic rating is by age at entry only, with the basic rating raised or lowered according to adverse or favourable prognostic factors. Duration since diagnosis is not considered, neither are female lives. The basic ratings by age at entry are as follows:

Age at Entry	Type 1 (IDDM)	Type 2 (NIDDM)
34 or less	+300	+150
35 to 44	+200	+100
45 to 54	+130	+70
55 or over	+90	+40

The addition for insulin dependence is approximately twice that for non insulin dependence. Using the figures above implies that the mortality ratio for non insulin dependence is approximately two thirds that of insulin dependence.

Swiss Re also considers family history of diabetes as an impairment (provided there are no signs of diabetes mellitus in the applicant), which is given a rating of +25 provided diabetes is known in both parents, otherwise it is disregarded.

Munich Re:

The approach adopted by Munich Re is more complicated. The basic rating applied is by age at application and duration since diagnosis. However, type of treatment is taken into account as an additional factor. Female lives are not mentioned.

The basic ratings are as follows:

	Time Elapsed Since Diagnosis				
Age at Application	up to 5 yrs	6-10	11-15	16-20	over 20 yrs
15-19	200 up	IC, UD	UD	D	–
20-24	175	200	IC	D	D
25-29	150	175	200	IC, UD	UD
30-34	100	125	150	175	IC
35-39	75	100	125	150	175 up
40-49	50	75	100	125	150 up
50 up	25	50	75	100	125 up

IC = Individual Consideration UD = Usually Decline

The additional ratings for types of treatment are as follows:

Oral hypoglycaemic agent	Add 0
Insulin dosage (daily units)	
- up to 80	Add 0
- 81 to 100	Add 25 to 75
- over 100	Usually decline

A long list of other factors are considered for additional rating.

Mercantile and General:

The basic rating applied by Mercantile and General is by age at application and duration since diagnosis, considering Type 1 (IDDM) and Type 2 (NIDDM) diabetes mellitus separately. Female lives are not mentioned.

The basic ratings are as follows:

Type 1 — Insulin Dependent Diabetes Mellitus

	Time Elapsed Since Diagnosis			
Age at Application	less than 10 yrs	10-15	16-20	over 20 yrs
16-25	150	200	225	250 up
26-35	100	150	200	225 up
36-45	50	100	150	200 up
46-60	25	50	100	150 up
over 60	up to 25	25	50	75 up

Type 2 – Non Insulin Dependent Diabetes Mellitus

	Time Elapsed Since Diagnosis			
Age at Application	less than 10 yrs	10-15	16-20	over 20 yrs
16-25	100	125	150	175
26-35	50	100	125	150
36-45	25	50	75	100
46-60	up to 25	25	up to 50	up to 50
over 60	0	0	0	up to 25

Again, a long list of other factors are considered for an additional rating.

Although there is some evidence of a lighter mortality experience in more recent years, the results of the Prudential study indicate that the ratings applied by Munich Re and Mercantile and General appear to be a little on the light side. Evidence of statistical studies appears to indicate that treatment type is a significant factor, as is duration since diagnosis. Statistical studies also show that female lives suffer heavier mortality than male lives with this impairment, and this should be an underwriting consideration.

Chapter 22 Underweight

22.1 Prudential Impairment Codes and Classification for Analysis:

600, 610, 620	Weight 20-30% under standard, chest expansion satisfactory
601, 611, 621	Weight 20-30% under standard, chest expansion unsatisfactory
602, 612, 622	Weight 20-30% under standard, not medically examined
603, 613, 623	Weight more than 30% under standard, chest expansion satisfactory
604, 614, 624	Weight more than 30% under standard, chest expansion unsatisfactory
605, 615, 625	Weight more than 30% under standard, not medically examined

Subsidiary codes:

E	Family history good
F	Family history indifferent
G	Family history poor

The number of policies included under this classification is large - a little over 30,000 for males and females combined. However, 90% of these are classified as having a good family history, with approximately 1% with a poor family history. The number of deaths with a poor family history is very low, therefore the family history factor was not considered in the analysis.

For analysis, data were classified according to:

W: Weight at Entry - 2 levels:
- 1 - 20 to 30% below standard
- 3 - more than 30% below standard

E: Chest Expansion - 3 levels:
- 1 - Satisfactory
- 2 - Unsatisfactory
- 3 - Not medically examined

A: Age at Entry - 4 levels:
- 1 - 16 to 39
- 2 - 40 to 49
- 3 - 50 to 59
- 4 - 60 to 79

D: Policy Duration - 6 levels:
- 1 - 0 to 2 years
- 2 - 2 to 5 years
- 3 - 5 to 10 years
- 4 - 10 to 15 years
- 5 - 15 to 20 years
- 6 - over 20 years

C: Calendar Year of Entry - 8 levels:

1 - 1947 to 51	5 - 1967 to 71
2 - 1952 to 56	6 - 1972 to 76
3 - 1957 to 61	7 - 1977 to 81
4 - 1962 to 66	8 - 1982 to 86

22.2 Male Lives

22.2.1 Summary Statistics

Number of Entrants: 23561

Number of Deaths: 882

Table 22.1 Distribution By Weight

Weight at Entry	Number	Percentage
20 to 30% under standard	22843	97·0
More than 30% below standard	718	3·0
	23561	

The vast majority of policyholders were between 20% and 30% below standard weight at entry.

Table 22.2 Distribution By Chest Expansion

	Number	Percentage
Satisfactory	6902	29·3
Unsatisfactory	443	1·9
Not medically examined	16216	68·8
	23561	

Less than two percent were classified as having an unsatisfactory chest expansion, and almost 70% were not medically examined, presumably because there was no medical evidence to indicate that the applicant should be examined. It might seem reasonable, *a priori*, to expect the not medically examined group to experience excess mortality close to, or lower than, the satisfactory group.

Table 22.3 Distribution by Age at Entry

Age at Entry	Number	Percentage
16 to 39	16342	69·4
40 to 49	4833	20·5
50 to 59	2042	8·7
60 to 79	344	1·5
	23561	

Almost 70% of entrants were below age 40 at entry and only 10% above age 50 at entry.

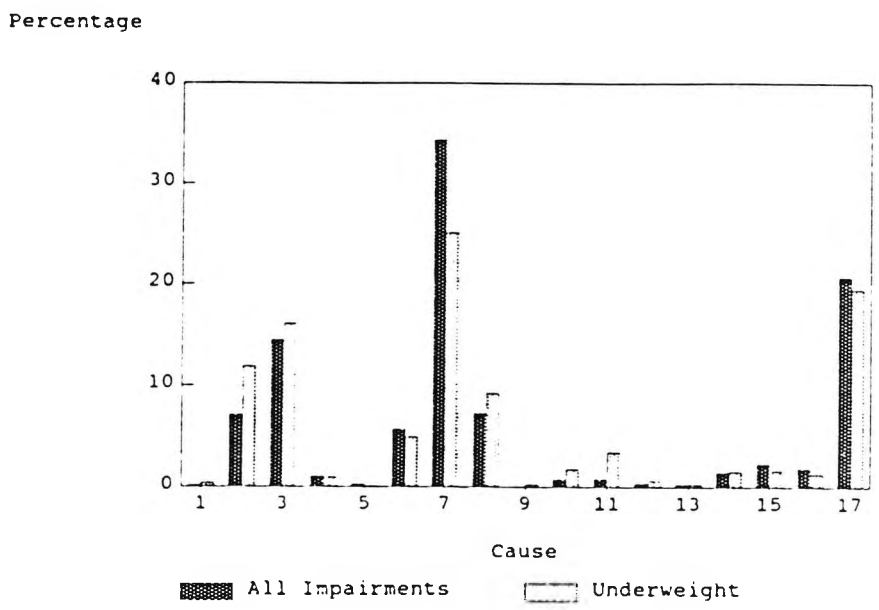
Table 22.4 Distribution by Cause of Death

	Number of Deaths	Percentage
1. Tuberculosis	4	0·5
2. Lung Cancer	105	11·9
3. Other Cancer	142	16·1
4. Leukaemia	9	1·0
5. Diabetes	1	0·1
6. Vascular Lesions	44	5·0
7. Ischaemic Heart Disease	222	25·2
8. Other Circulatory Diseases	82	9·3
9. Influenza	3	0·3
10. Pneumonia	16	1·8
11. Bronchitis	31	3·5
12. Peptic Ulcer	6	0·7
13. Nephritis	3	0·3
14. Motor Accident	14	1·6
15. Other Accident	15	1·7
16. Suicide	12	1·4
17. Other Causes	173	19·6
Total	882	

A comparison with percentage deaths by cause for all impairments combined is shown in Figure 22.1. Noteworthy are deaths from cancers, influenza, pneumonia and bronchitis.

Cancers account for 28% of deaths amongst people with this impairment compared with 21·6% for all deaths in the entire study. Deaths from influenza, pneumonia and bronchitis account for 1·8% of total deaths in the entire study and 5·6% here. Although this represents over 3 times as many deaths from these three conditions, the actual number dying is low. Deaths from Ischaemic Heart Disease are low at 25·2% compared with 34·4% of deaths from Ischaemic Heart Disease in the entire study.

Figure 22.1 Distribution by Cause of Death: Underweight (Males)



22.2.2 Results: The Overall Mortality Ratio

The overall mortality ratio was found to be 105% representing very low excess mortality. This was based on 882 deaths using the A67-70 (2) table as a basis for expected deaths.

22.2.3 Results: Main Effects Fitted Separately

Table 22.5 Mortality Ratios by Weight Levels at Entry (Factor W)

	MR%	
20-30% below standard	103	(845)
more than 30% below standard	159	(37)

The 20-30% below standard group showed mortality very near standard. Excess mortality of approximately +60% was experienced in the "more than 30% below standard" group.

Table 22.6 Mortality Ratios by Age at Entry (Factor A)

Age at Entry	MR %	
16 - 39	87	(166)
40 - 49	114	(333)
50 - 59	107	(269)
60 - 79	104	(114)

The age at entry group 40 to 49 shows the highest excess mortality at +14%, the other groups being not far from standard.

Table 22.7 Mortality Ratios by Policy Duration (Factor D)

Duration	MR %	
0 - 2 yrs	82	(48)
2 - 5	75	(91)
5 - 10	111	(238)
10 - 15	113	(209)
15 - 20	108	(153)
over 20 yrs	116	(143)

There is a tendency for the mortality ratios to rise as policy duration increases.

Table 22.8 Mortality Ratios by Chest Expansion (Factor E)

	MR%	
Satisfactory	110	(518)
Unsatisfactory	122	(39)
Not medically examined	95	(325)

As expected, excess mortality was higher where chest expansion was unsatisfactory than when it was satisfactory. The "not medically examined" category showed a lower mortality ratio than the other two groups, which would seem to justify the decision not to examine these applicants.

Table 22.9 Mortality Ratios by Calendar Year of Entry (Factor C)

	MR %			
1947 - 51	127	(186)		
1952 - 56	108	(218)		
1957 - 61	120	(193)		
1962 - 66	108	(161)		
1967 - 71	81	(65)		
1972 - 76	66	(28)		
1977 - 81	42	(21)	}	49 (31)
1982 - 86	76	(10)		

There is a tendency for the mortality ratios to fall with increasing calendar year of entry. The mortality ratios in the most recent years are very low. To some extent, these results will be due to the base-line hazard chosen (*i.e.* using A67-70 throughout).

22.2.4 Significance of Main Effects

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev	D of F	
Π_0	621·39	732			
W	615·60	731	5·79	1	1·6 %
E	615·81	730	5·58	2	6·1%
A	612·50	729	8·89	3	3·0 %
D	603·73	727	17·66	5	0·4 %
C	575·53	726	45·86	6	<<·001 %

Referring differences in model deviances to the appropriate χ^2 distribution reveals that all of

the main effects are statistically significant, with the exception of "chest expansion". This leads to the investigation of more complex models.

22.2.5 Interaction Terms

Models including the statistically significant main effects and first order interaction terms were fitted leading to the following deviance table:

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev	D of F	
W+A+D+C	556·55	717			
W*A+D+C	548·89	714	7·66	3	5·3 %
W*D+C+A	553·37	712	3·18	5	67·5 %
W*C+A+D	552·87	711	3·68	6	72·2 %
A*C+W+D	541·72	699	14·83	8	6·2 %
A*D+W+C	542·70	702	13·85	15	53·7 %
D*C+W+A	532·98	693	23·57	24	48·7 %

None of the first order interaction terms are significant at the 5% level. However, the weight/age interaction only just misses this criterion, therefore the results of fitting this model are also included below.

22.2.6 Main Effects Fitted Together, No Interaction

The simplest model catering for all significant main effects together is Model W+A+D+C with parametric representation for the mortality ratio given by $exp(\mu+\alpha_i+\beta_j+\delta_k+\gamma_l)$. In this model, interaction terms are ignored. The excess mortality factors given by fitting this model are shown in Table 22.10. Mortality ratios may be deduced by forming the product of relevant entries.

Table 22.10 Excess Mortality Factors: Model W+A+D+C

$exp\ (\mu) = 1{\cdot}01$

Weight at Entry :	20-30% below standard	more than 30% below standard
$exp\ (\beta_j)$	1·00	1·62

Age at Entry :	16 to 39	40 to 49	50 to 59	60 to 79
$exp\ (\alpha_i)$	1·00	1·23	1·16	1·13

Policy Duration :	0 - 2	2 - 5	5 - 10	10 - 15	15 - 20	>20
$exp\ (\delta_k)$	1·00	0·85	1·16	1·10	1·04	1·11

Calendar Yr of Entry	47-51	52-56	57-61	62-66	67-71	72-76	77-86
$exp\ (\gamma_l)$	1·00	0·85	0·95	0·86	0·65	0·54	0·43

Mortality ratios produced from this model are consistent with the results from the main effects models fitted separately.

22.2.7 Main Effects fitted together and Age/Weight Interaction Term

Model W+A+D+C+W.A includes the interaction between weight levels and age at entry. The associated parametric representation of the mortality ratio given by this model is $exp(\mu+\alpha_i+\beta_j+\delta_k+\gamma_l+\alpha\beta_{ji})$.

Mortality ratios may be deduced from Table 22.11 by forming the product of relevant entries. The age/weight interaction is interesting. With weight 20-30% below standard, the lowest mortality ratios are at the lowest ages at entry and the highest ages at entry. The opposite effect is shown where weight is more than 30% below standard, with the highest mortality ratios appearing for the lowest and highest age at entry groups.

Table 22.11 Excess Mortality Factors : Model W+A+D+C+W.A

$exp\ (\mu) = 0{\cdot}97$

$exp(\beta_j + \alpha_i + \alpha\beta_{ji})$:

		Weight at Entry	
		20-30% below S	more than 30% above S
Age at Entry	16 to 39	1·00	3·22
	40 to 49	1·30	1·34
	50 to 59	1·22	1·63
	60 to 79	1·16	2·63

Policy Duration :	0 - 2	2 - 5	5 - 10	10 - 15	15 - 20	>20
$exp\ (\delta_k)$	1·00	0·85	1·16	1·10	1·04	1·12

Calendar Yr of Entry	47-51	52-56	57-61	62-66	67-71	72-76	77-86
$exp\ (\gamma_l)$	1·00	0·85	0·95	0·87	0·65	0·53	0·42

Residual plots for both of these more complex models were found to be highly satisfactory.

22.3 Female Lives

22.3.1 Summary Statistics

Number of Entrants: 9576

Number of Deaths: 324

Table 22.12 Distribution by Weight

Weight at Entry	Number	Percentage
20 to 30% under standard	9238	96·5%
more than 30% below standard	338	3·5%
	9576	

Like male lives, the vast majority of policyholders were between 20% and 30% below standard weight at entry, with very few more than 30% standard weight at entry.

Table 22.13 Distribution by Chest Expansion

Chest Expansion	Number	Percentage
Satisfactory	1499	15·7
Unsatisfactory	105	1·1
Not Medically Examined	7972	83·2
	9576	

Again like male lives, very few female lives were classified as having an unsatisfactory chest expansion. The majority of lives classified as underweight were not medically examined.

Table 22.14 Distribution By Age at Entry

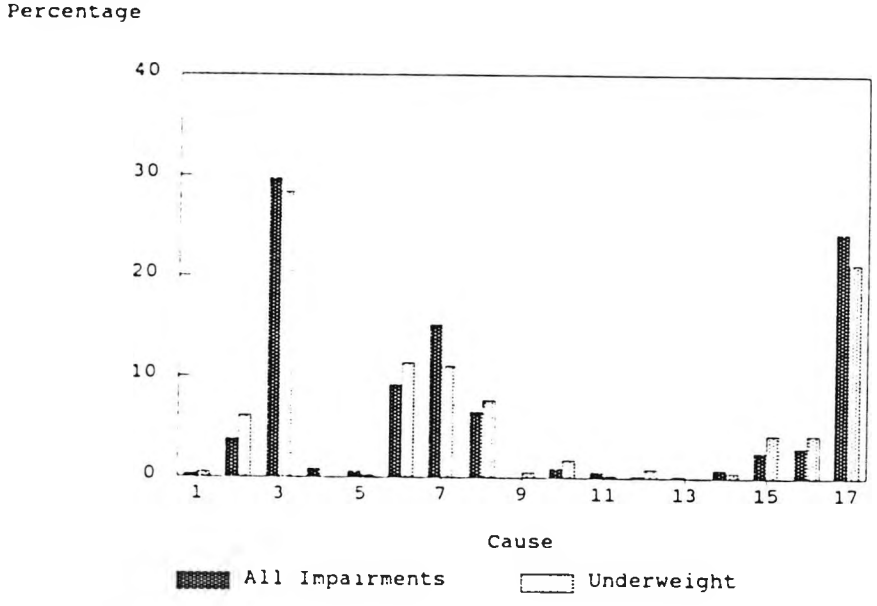
Age at Entry	Number of Entrants	Percentage
16-39	3988	41·6
40-49	3402	35·5
50-59	1851	19·3
60-79	335	3·5
Total	9576	

Over 75% of entrants were below age 50 at entry, with over 40% aged 16 to 39 at entry.

Table 22.15 Distribution by Cause of Death

	Number of Deaths	Percentage
1. Tuberculosis	2	0·6
2. Lung Cancer	20	6·2
3. Other Cancer	92	28·4
4. Leukaemia	-	-
5. Diabetes	1	0·3
6. Vascular Lesions	37	11·4
7. Ischaemic Heart Disease	36	11·1
8. Other Circulatory Diseases	25	7·7
9. Influenza	2	0·6
10. Pneumonia	6	1·9
11. Bronchitis	1	0·3
12. Peptic Ulcer	3	0·9
13. Nephritis	-	-
14. Motor Accident	2	0·6
15. Other Accident	14	4·3
16. Suicide	14	4·3
17. Other Causes	69	21·3
Total	324	

Figure 22.2 Distribution by Cause of Death: Underweight (Females)



Compared to percentage deaths in the study overall, none of the causes of death for the impairment underweight stand out as being drastically different. There is a slightly higher incidence of deaths from respiratory disorders (2·8% against 1·7%) and also a slightly higher incidence of deaths from accidents and suicide (9·2% against 6·7%).

22.3.2 Results: The Overall Mortality Ratio

The overall mortality ratio was found to be 112% showing little excess mortality. This was based on 324 deaths using the FA75-78 table as a basis for expected deaths.

22.3.3 Results: Main Effects Fitted Separately

Table 22.16 Mortality Ratios by Weight levels at Entry (Factor W)

	MR%	
20-30% below standard	110	(305)
more than 30% below standard	141	(19)

The 20-30% below standard group showed low excess mortality. Excess mortality was higher for the "more than 30% below standard" group, with excess mortality of +40%.

Table 22.17 Mortality Ratios by Chest Expansion (Factor E)

	MR%	
Satisfactory	102	(86)
Unsatisfactory	58	(3)
Not Medically Examined	117	(235)

Very little can be inferred from the result of the unsatisfactory group since it is based on only 3

deaths. The "satisfactory" group shows mortality very near standard, and the "not medically examined" group shows excess mortality of approximately +20%. It is perhaps a little surprising that the policyholders who were not medically examined show the highest mortality ratio.

Table 22.18 Mortality Ratios by Age at Entry (Factor A)

	MR %	
16 - 39	131	(55)
40 - 49	125	(145)
50 - 59	99	(94)
60 - 79	82	(30)

The mortality ratios clearly fall as age at entry rises.

Table 22.19 Mortality Ratios by Policy Duration (Factor D)

Duration	MR %	
0 - 2 yrs	121	(29)
2 - 5	124	(65)
5 - 10	92	(86)
10 - 15	120	(79)
15 - 20	114	(37)
over 20 yrs	120	(28)

Except for the mortality ratio of 92% at durations 5 to 10 years, there is little variation in the figures for excess mortality by policy duration.

Table 22.20 Mortality Ratios by Calendar Year of Entry (Factor C)

	MR %	
1947 - 51	114	(73)
1952 - 56	111	(90)
1957 - 61	112	(61)
1962 - 66	92	(30)
1967 - 71	135	(22)
1972 - 76	166	(16)
1977 - 86	99	(32)

The mortality ratios are fairly stable for the first 15 years calendar year of entry, becoming more erratic thereafter, possibly because of lower numbers of deaths.

22.3.4 Signficance of Main Effects

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev	D of F	
H_0	415·70	701			
W	414·13	700	1·57	1	21 %
E	412·23	699	3·47	2	18 %
A	407·11	698	8·59	3	3·5 %
D	410·37	696	5·33	5	38%
C	410·52	695	5·18	6	52 %

The only main effect found to be statistically significant was age at entry, so more complex models were not investigated.

22.4 Comparison with Other Studies

22.4.1 Previous Studies Based on the Prudential Data Set

All authors presenting results based on the entire data set give results for this impairment. Table 22.21 shows overall mortality ratios together with numbers of deaths, for male and female lives, given by the various authors.

Table 22.21 Prudential Data Set: Underweight — Overall Mortality Ratios

Author(s)	Calendar Yrs of Study	Overall MR%	
		Males	Females
Clarke (1961)	1947-58	95 (70)	–
Preston and Clarke (1966)	1947-63	109 (172)	112 (95)
Clarke (1979)*	1964-73	113 (315)	–
Leighton (1987)*	1974-83	96 (237)	–
Papconstantinou (1988)	1947-81	108 (753)	74 (277)
England (1992)	1947-86	105 (882)	112 (324)

* 20-30% below standard, entry ages over 30 only, durations 2 and over only.

The results of the various authors compare favourably and indicate a low level of excess mortality associated with this impairment. However, it should be noted that over 95% of policyholders were in the 20-30% below standard weight category. The results shown earlier indicate excess mortality of around +50 where weight is more than 30% below standard.

A further comparison can be made with the results of Papaconstantinou and, for completeness, with the results of the 1979 Build Study conducted in North America, shown in Table 22.22 by age at entry.

Table 22.22 Underweight: Mortality Ratios by Age at Entry (male lives)

	Papaconstantinou		England		1979 Build Study**		
16 to 39	159	(121)	87	(166)	15-39	97	(9,624)
40 to 49	114	(293)	114	(333)	40-49	100	(12,100)
50 to 79	93	(339)	106	(383)	50-69	117	(12,650)

**15-35% underweight

Although comparing well at ages above 40, the results given by Papaconstantinou and England are very different at ages below 40. The results of England, however, are more in line with the results of the American 1979 Build Study.

The results of Papaconstantinou also allow a comparison of mortality ratios by policy duration, shown in Table 22.23.

Table 22.23 Underweight: Mortality Ratios by Policy Duration

Duration	MR % Papaconstantinou		MR % England	
0-2 yrs	91	(37)	82	(48)
2-5 yrs	75	(82)	75	(91)
5-10 yrs	107	(222)	111	(238)
10-15 yrs	108	(192)	113	(209)
over 20 yrs	138	(220)	112	(296)

The results of the two studies compare favourably at all but the highest durations and indicate that excess mortality increases slightly with policy duration.

22.4.2 Other Studies and Further Considerations

Three North American studies considering underweight are the Build and Blood Pressure Study

(1959) [BPPS], the Build Study (1979) [BS] and the American Cancer Society Study (1979) [ACS] (see Brackenridge (1985)). The results of these studies by weight category and sex are shown in Table 22.24.

Table 22.24 Underweight: Mortality Ratios given by American Studies

Departure From Average Weight	Males			Females		
	BS	BBPS	ACS	BS	BBPS	ACS
over 20% below	105	90	110	110	99	100
10-20% below	94	95	100	97	95	95

These results show mortality ratios very close to standard for males and females. In fact, those who were moderately underweight experienced mortality which was lighter than standard. This fact has been known for some time. According to Brackenridge (1985):

> "Before mortality statistics relating to build were first published in the USA in 1903, underweights were considered to be much poorer insurance risks than overweights, mainly due to the high death rate from tuberculosis . . . In about the middle of this century, the emphasis began to shift mainly because of the elimination of tuberculosis as a significant factor in mortality, and underweight, far from being an adverse feature, gradually gave rise to a more favourable mortality experience in most age groups than average weight itself."

Van Hallie and Lew (see Lew and Gajewski (1990)) make the same observation:

> "The concept of desirable weight evolved as a consequence of mortality investigations conducted by life insurance companies since the turn of the century. These studies showed that, in general, persons whose weights were somewhat below average lived longer than those whose weights were close to average."

Certainly, the results of the Prudential study do not disagree with these statements. Leighton, in 1987, reported: "It is clear that a reasonable degree of underweight (up to 30%) in itself no

longer presents any reason for concern in underwriting".

22.5 Rating of Underweight

The underwriting manuals consulted differ slightly in their approach to the rating of this impairment. None consider the impairment alone to be serious provided that any serious underlying cause is excluded before acceptance. A summary of the ratings is outlined below.

Munich Re

Ratings are from a detailed table considering height and weight combinations. These may be translated approximately into a table showing percentage departure from average weight:

Departure from Average Weight	Rating
10% below	0
20% below	+10 to +20
30% below	+20 to +50

Swiss Re

Applicants are classified as moderately or severely underweight according to a detailed height and weight table, then rated by age at entry according to the following table:

Age at Entry	Rating
34 or less	+50
35 to 44	+40
45 to 54	+35
55 or more	+30

Mercantile and General

Ratings according to build are by a detailed height and weight table. For underweight, the rating is specified simply as "+25 up".

The ratings recommended by the three companies are all fairly low. This is entirely in agreement with the results of the Prudential study which indicate that this impairment is only a cause for concern where weight levels are well below standard.

Chapter 23 Overweight

23.1 Prudential Impairment Codes and Classification for Analysis:

650, 660, 670	Weight 20-30% over standard, girth satisfactory
651, 661, 671	Weight 20-30% over standard, girth unsatisfactory
652, 662, 672	Weight 20-30% over standard, not medically examined
653, 663, 673	Weight 30-40% over standard, girth satisfactory
654, 664, 674	Weight 30-40% over standard, girth unsatisfactory
655, 665, 675	Weight 30-40% over standard, not medcially examined
656, 666, 676	Weight over 40% above standard, girth satisfactory
657, 667, 677	Weight over 40% above standard, girth unsatisfactory
658, 668, 678	Weight over 40% above standard, not medically examined

Subsidiary Codes:

E	Family history good
F	Family history indifferent
G	Family history poor

The number of policies classified as overweight is extremely large — over 100,000 for males and females combined, which constitutes approximately 1/6 of the entire data set. However, most of these (approximately 85%) were also classified as having a good family history, with only 1% having a poor family history. Nonetheless, family history was still considered in the analysis.

For analysis, data were classified by:

W: Weight at Entry - 3 levels: 1 - Weight 20-30% over standard
2 - Weight 30-40% over standard
3 - Weight over 40% above standard

G: Girth - 3 levels: 1 - Satisfactory
2 - Unsatisfactory
3 - Not Medically Examined

H: Family History - 3 levels: 1 - Good
2 - Indifferent
3 - Poor

A: Age at Entry - 4 levels: 1 - 16 to 39
2 - 40 to 49
3 - 50 to 59
4 - 60 to 79

D: Policy Duration - 3 levels: 1 - 0 to 2 yrs
2 - 2 to 5 yrs
3 - 5 to 10 yrs
4 - 10 to 15 yrs
5 - 15 to 20 yrs
6 - over 20 yrs

C: Calendar Year of Entry - 8 levels: 1 - 1947 to 51
2 - 1952 to 56
3 - 1957 to 61
4 - 1962 to 66
5 - 1967 to 71
6 - 1972 to 76
7 - 1977 to 81
8 - 1982 to 86

23.2 Male Lives

23.2.1 Summary Statistics

Number of Entrants: 77685
Number of Deaths: 3012

Table 23.1 Distribution by Weight at Entry

Weight at Entry	Number	Percentage
20-30% over standard	51519	66·3
30-40% over standard	16880	21·7
more than 40% over standard	9286	12·0
	77685	

Two thirds of policyholders were in the 20-30% above standard category, approximately 1/5 in the 30-40% over standard category and approximately 1/8 in the more than 40% over standard

category.

Table 23.2 Distribution by Girth

Girth	Number	Percentage
Satisfactory	34912	44·9
Unsatisfactory	9637	12·4
Not Medically Examined	33136	42·7
	77685	

Only appoximately 1/8 of policyholders were considered as having an unsatisfactory girth. 45% were considered to be satisfactory and over 40% were not medically examined, presumably because there was no medical evidence to suggest that the applicant should be examined.

Table 23.3 Distribution by Family History

Family History	Number	Percentage
Good	67278	86·6
Indifferent	9558	12·3
Poor	854	1·1
	77685	

The vast majority (over 85%) were classified as having a good family history. Only 1% were classified as having a poor family history.

Table 23.4 Distribution By Age at Entry

Age at Entry	Number of Entrants	Percentage
16-39	63485	81·7
40-49	10273	13·2
50-59	3238	4·2
60-79	689	0·9
Total	77685	

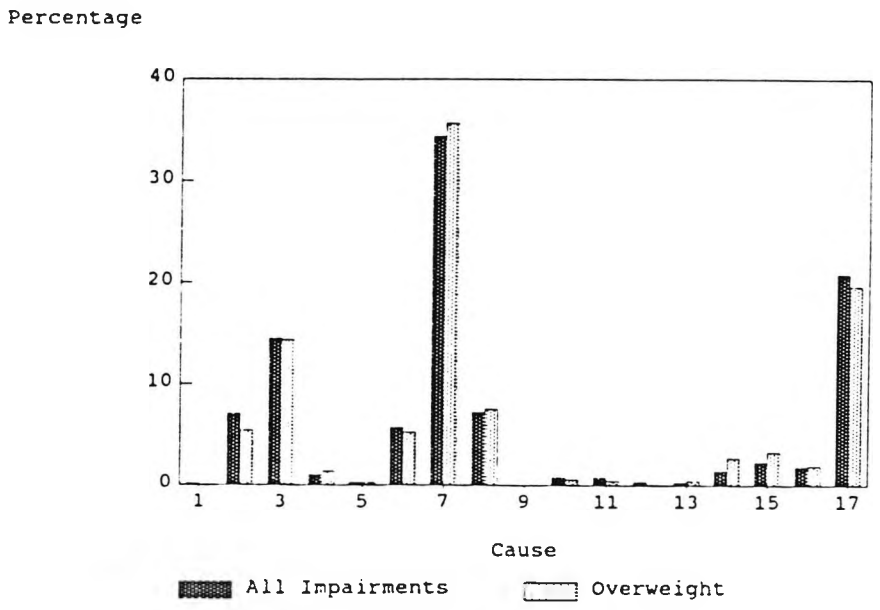
Most of the policyholders (over 80%) were in the 16 to 39 age at entry category. Only 5% were above age 50 at entry, and less than 1% aged 60 or above.

Table 23.5 Distribution by Cause of Death

	Number of Deaths	Percentage
1. Tuberculosis	-	-
2. Lung Cancer	166	5·5
3. Other Cancer	433	14·4
4. Leukaemia	44	1·5
5. Diabetes	10	0·3
6. Vascular Lesions	160	5·3
7. Ischaemic Heart Disease	1075	35·7
8. Other Circulatory Diseases	230	7·6
9. Influenza	-	-
10. Pneumonia	20	0·7
11. Bronchitis	16	0·5
12. Peptic Ulcer	4	0·1
13. Nephritis	15	0·5
14. Motor Accident	84	2·8
15. Other Accident	102	3·4
16. Suicide	61	2·0
17. Other Causes	592	19·7
Total	3012	

Male overweights contribute a little over 10% to the total deaths in the entire study. A comparison with percentage deaths by cause for the entire Prudential data set is shown graphically in Figure 23.1. The percentage deaths by cause for overweights are very similar to the percentage deaths by cause in the entire study. This may be explained partly by the fact that this impairment group contributes such a large number of deaths to the total, and, as such, the comparison is not particularly good. Perhaps accidents are noteworthy here, taking 6·2% of total deaths compared with 3·9% in the entire study.

Figure 23.1 Distribution by Cause of Death: Overweight (Males)



23.2.2 Results: The Overall Mortality Ratio

The overall mortality ratio was found to be 125% based on 3012 deaths. The A67-70 (2) table was used as a basis for expected deaths.

23.2.3 Results: Main Effects Fitted Separately

Table 23.6 Mortality Ratios by Weight Levels at Entry (Factor W)

	MR%	
20-30% over standard	122	(2135)
30-40% over standard	126	(610)
more than 40% over standard	148	(267)

There is little difference between the 20-30% above standard group and the 30-40% above standard group, with excess mortality at approximately +25. This rises to near +50 when weight at entry is more than 40% above standard.

Table 23.7 Mortality Ratios by Girth (Factor G)

	MR%	
Satisfactory	118	(1452)
Unsatisfactory	127	(702)
Not Medically Examined	139	(858)

The difference between the mortality ratios for the satisfactory and unsatisfactory groups is not large. The highest mortality ratio was experienced for the not medically examined group, showing that the office has been selected against by not performing a medical examination.

Table 23.8 Mortality Ratios by Policy Duration (Factor D)

Duration	MR %	
0 - 2 yrs	118	(174)
2 - 5	121	(348)
5 - 10	124	(639)
10 - 15	134	(660)
15 - 20	133	(578)
over 20 yrs	117	(613)

There is little variability in the mortality ratios by policy duration.

Table 23.9 Mortality Ratios by Age at Entry (Factor A)

Age at Entry	MR %	
16 - 39	132	(1357)
40 - 49	131	(956)
50 - 59	111	(472)
60 - 79	103	(227)

The mortality ratios for ages at entry 16 to 39 and 40 to 49 are very similar. Above age 50 at entry, there is a tendency for the mortality ratios to fall until mortality is very near standard above age 60 at entry.

Table 23.10 Mortality Ratios by Family History (Factor H)

	MR %	
Good	120	(2370)
Indifferent	145	(577)
Poor	164	(65)

Clearly excess mortality rises as the family history deteriorates.

Table 23.11 Mortality Ratios by Calendar Year of Entry (Factor C)

	MR %			
1947 - 51	137	(511)		
1952 - 56	130	(887)		
1957 - 61	126	(697)		
1962 - 66	120	(456)		
1967 - 71	114	(246)		
1972 - 76	106	(109)		
1977 - 86	113	(90)	} 105	(106)
1982 - 86	74	(16)		

There is a tendency for the mortality ratios to fall with increasing time. This, to some extent, will be amplified by the choice of the base-line hazard, ie A67-70 throughout.

23.2.4 Significance of Main Effects

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev	D of F	
H_0	2443·5	3055			
W	2435·2	3053	8·3	2	1·6 %
G	2428·9	3053	14·6	2	·07 %
H	2423·6	3053	19·9	2	·005 %
A	2421·0	3052	22·5	3	·008 %
D	2434·7	3050	8·8	5	12 %
C	2428·3	3049	17·9	7	1·9 %

Referring differences in model deviances to the appropriate χ^2 distribution (shown overleaf) reveals that all of the main effects, with the exception of policy duration, are highly statistically significant, leading to the investigation of more complex models.

23.2.5 Main Effects Fitted Together, No Interaction

The simplest model catering for all significant main effects together is Model W+G+H+A+C with parametric representation for the mortality ratio given by $exp(\mu+\alpha_i+\gamma_j+\lambda_k+\omega_l+\rho_m)$. In this model, interaction terms are ignored. The results from fitting this model are shown in Table 23.12. Mortality ratios may be deduced by forming the product of relevant entries.

Table 23.12 Excess Mortality Factors: Model W+G+H+A+C

$exp(\mu) = 1{\cdot}28$

Weight	20-30% over S		30-40% over S		more than 40% over S		
$exp(\omega_l)$:	1·00		1·07		1·27		
Girth	Satisfactory		Unsatisfactory		N.M.E.		
$exp(\lambda_k)$:	1·00		1·10		1·20		
Family History	Good		Indifferent		Poor		
$exp(\rho_m)$:	1·00		1·24		1·43		
Age at Entry	16-39		40-49		50-59		60-79
$exp(\alpha_i)$:	1·00		0·99		0·83		0·78
Calendar Year of Entry	47-51	52-56	57-61	62-66	67-71	72-76	77-86
$exp(\gamma_j)$:	1·00	0·93	0·89	0·84	0·80	0·75	0·73

Trends in the movement of factors are in the same direction as trends for main effects fitted separately. Residual plots for this model were highly satisfactory (not shown).

23.2.6 First Order Interactions

Models including first order interaction terms were also fitted, and differences in deviances referred to the appropriate χ^2 distribution to assess the statistical significance of the interaction terms, as shown in the following table:

Model	Deviance	D of F	Differences		Observed Sig
			Deviance	D o F	Level
W+G+H+A+C	2353·8	3040			
W*G+H+A+C	2344·2	3036	9·6	4	4·7 %
W*H+A+C+G	2349·3	3036	4·5	4	34 %
W*A+C+G+H	2344·4	3034	9·4	6	15 %
W*C+G+H+A	2342·7	3028	11·1	12	52 %
G*H+A+C+W	2349·6	3036	4·2	4	38 %
G*A+C+W+H	2331·8	3034	22·0	6	·13 %
G*C+W+H+A	2336·9	3028	16·9	12	15 %
H*A+C+W+G	2348·7	3034	5·1	6	53 %
H*C+W+G+A	2328·8	3028	25·0	12	1·5 %
A*C+W+G+H	2330·4	3022	23·4	18	18 %

Interactions between Weight and Girth, between Girth and Age at Entry and between Family History and Calendar Year of Entry are statistically significant, leading to the investigation of models including interaction terms. For all of the models involving interaction terms, plots of residuals were satisfactory, but have not been shown here.

23.2.7 Main Effects plus Weight/Girth Interaction: Model W*G+H+A+C

This model has parametric representation $exp(\mu+\alpha_i+\gamma_j+\rho_m+\lambda_k+\omega_l+\lambda\omega_{kl})$ for the mortality

ratios. The excess mortality factors for this model are shown in Table 20.13. Mortality ratios may be deduced by forming the product of relevant entries.

It is not easy to interpret the weight/girth figures, which is probably due to the fact that a large number of policyholders were not medically examined, and that most of the lives were classified with weight between 20% and 30% above standard. It is not known how many of the "not medically examined" policyholders would have fallen in the satisfactory or unsatisfactory groups at each weight level, had they been examined. It is interesting to note that the figures for the other factors are very similar to the equivalent figures in the main effects, no interaction model.

Table 23.13 Excess Mortality Factors: Model W*G+H+A+C

$exp(\mu) = 1{\cdot}28$

		Weight (S = Standard)		
$exp(\lambda_k+\omega_l+\lambda\omega_{kl})$		20-30% over S	30-40% over S	more than 40% over S
Girth	Satisfactory	1·00	1·14	1·13
	Unsatisfactory	1·11	1·05	1·61
	N.M.E.	1·21	1·32	0·98

Family History :	Good	Indifferent	Poor
$exp(\rho_m)$	1·00	1·24	1·44

Age at Entry :	16-39	40-49	50-59	60-79
$exp(\alpha_i)$	1·00	0·99	0·84	0·78

Calendar Year	47-51	52-56	57-61	62-66	67-71	72-76	77-86
of Entry $exp(\gamma_j)$:	1·00	0·93	0·89	0·84	0·80	0·75	0·73

23.2.8 Main Effects plus Girth/Age at Entry Interaction: Model G*A+C+W+H

This model has parametric representation $exp(\mu+\alpha_i+\gamma_j+\lambda_k+\omega_l+\rho_m+\alpha\lambda_{ik})$ for the mortality ratio. Excess mortality factors given by fitting this model are shown in Table 23.14. Overall,

there is a tendency for the mortality ratios to fall as age at entry rises. However, the results with regard to girth show no discernable pattern (which is the cause of the significance of the interaction). The problems associated with the classification of girth noted in Section 23.2.7 will also affect this model. Classifying a factor (such as girth) qualitatively in this way is far from ideal from an analysis point of view and when comparing results with other studies. Again, the figures for the other factors are very similar to the equivalent figures in the main effects, no interaction model.

Table 23.14 Excess Mortality Factors: Model G*A+C+W+H

$exp(\mu) = 1{\cdot}30$

Weight :	20-30% over S	30-40% over S	over 40% above S
$exp(\omega_l)$	1·00	1·06	1·24

		Age at Entry			
$exp(\alpha_i+\lambda_k+\alpha\lambda_{ik})$		16 to 39	40 to 49	50 to 59	60 to 79
Girth	Satisfactory	1·00	0·96	0·78	0·88
	Unsatisfactory	1·33	1·04	0·85	0·76
	N.M.E.	1·10	1·25	1·29	0·77

Family History :	Good	Indifferent	Poor
$exp(\rho_m)$	1·00	1·24	1·42

Calendar Year	47-51	52-56	57-61	62-66	67-71	72-76	77-86
of Entry $exp(\gamma_j)$:	1·00	0·93	0·89	0·84	0·80	0·76	0·74

23.2.9 Main Effects plus Family History/Calendar Year of Entry Interaction: Model H*C+W+G+A

This model has parametric representation $exp(\mu+\alpha_i+\gamma_i+\lambda_k+\omega_l+\rho_m+\gamma\rho_{jm})$ for the mortality ratio. Excess mortality factors from fitting this model are shown in Table 23.15. Again, mortality ratios may be deduced by forming the product of relevant entries. In general, the

mortality ratios increase as family history deteriorates. For family history classified as "good", the mortality ratios fall with increasing calendar year of entry. This trend does not appear in the "indifferent" and "poor" family history categories. In particular, when family history is classified as "poor", the mortality ratios fall and then rise with increasing calendar time. This result has no apparent interpretation, but is probably a result of having only 64 deaths in total from the "poor" family history classification.

Table 23.15 Excess Mortality Factors: Model H*C+W+G+A

$exp(\mu) = 1{\cdot}29$

Weight	20-30% over S	30-40% over S	more than 40% over S
$exp(\omega_l)$	1·00	1·07	1·27

Girth	Satisfactory	Unsatisfactory	NME
$exp(\lambda_k)$	1·00	1·10	1·19

Age at Entry	16-39	40-49	50-59	60-79
$exp(\alpha_i)$	1·00	0·99	0·83	0·78

$exp(\gamma_j+\rho_m+\gamma\rho_{jm})$		Family History		
		Good	Indifferent	Poor
	47-51	1·00	1·15	1·47
	52-56	0·94	1·11	1·23
Calendar	57-61	0·91	0·99	0·90
Year of	62-66	0·84	1·02	0·80
Entry	67-71	0·71	1·27	1·83
	72-76	0·72	1·03	1·74
	77-86	0·64	1·25	3·51

23.2.10 Other Models

Models which are even more complex could be investigated, for example models involving

second order interaction terms. However, this leads to serious difficulties in presenting and interpreting the results. Furthermore, results become less meaningful with insufficient numbers of deaths. For these reasons, models involving more than one first order interaction term were not considered.

It is also possible to fit less complex models which might be useful when making comparisons with other studies.

23.3 Female Lives

23.3.1 Summary Statistics

Number of Entrants: 23103
Number of Deaths: 234

This is the largest single impairment group for female lives. However, there were insufficient data to consider analysis including the family history factor, since over 85% were classified as "good".

Table 23.16 Distribution by Weight

Weight at Entry	Number	Percentage
20-30% over standard	11653	50·4
30-40% over standard	5319	23·0
more than 40% over standard	6131	26·5
	23103	

Approximately half of the entrants were classified as 20-30% over standard, and approximately a quarter each were classified as 30-40% over standard and more than 40% over standard.

Table 23.17 Distribution by Age at Entry

Age at Entry	Number of Entrants	Percentage
16-39	18635	80·7
40-49	3105	13·4
50-59	1140	4·9
60-79	223	1·0
Total	23103	

Over 80% of entrants were aged between 16 and 39 at entry. However, only approximately $\frac{1}{3}$ of deaths came from this age at entry group.

Table 23.18 Distribution by Cause of Death

Cause	Number of Deaths	Percentage
1. Tuberculosis	-	-
2. Lung Cancer	12	5·1
3. Other Cancer	68	29·1
4. Leukaemia	5	2·1
5. Diabetes	2	0·9
6. Vascular Lesions	17	7·3
7. Ischaemic Heart Disease	39	16·7
8. Other Circulatory Diseases	14	6·0
9. Influenza	-	-
10. Pneumonia	3	1·3
11. Bronchitis	2	0·9
12. Peptic Ulcer	-	-
13. Nephritis	2	0·9
14. Motor Accident	5	2·1
15. Other Accident	4	1·7
16. Suicide	7	3·0
17. Other Causes	54	23·1
Total	234	

A comparison with percentage deaths by cause for all impairments combined is shown in

Figure 23.2. The percentage deaths by cause are very similar to the percentages by cause in the entire data set. Like male lives, deaths from the overweight category contribute almost 10% to the total female deaths in the entire study.

Figure 23.2 Distribution by Cause of Death: Overweight (Females)

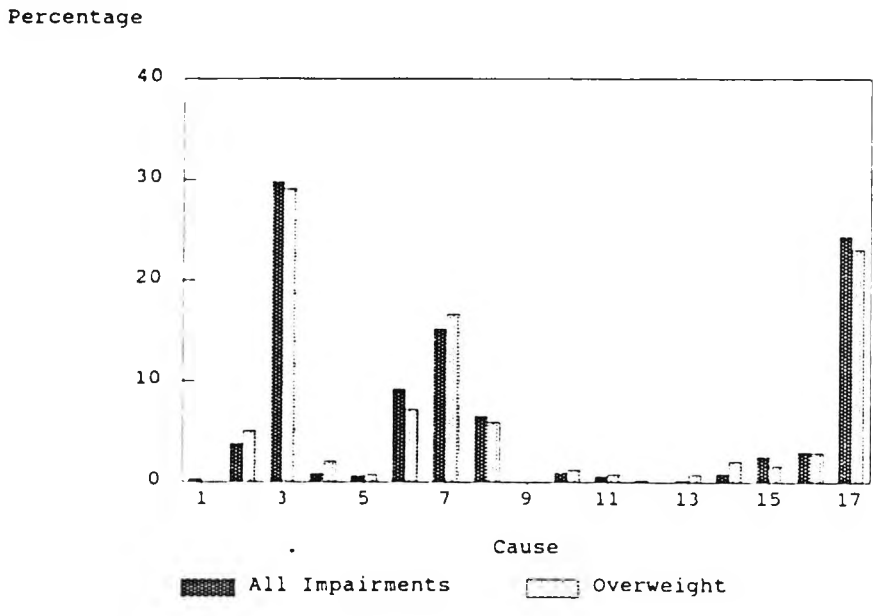


Table 23.19 Distribution by Girth

Girth	Number	Percentage
Satisfactory	9169	39·7
Unsatisfactory	1367	5·9
Not Medically Examined	12567	54·4
	23103	

Over 50% were not medically examined, and less than 6% were classified as "unsatisfactory".

23.3.2 Results: The Overall Mortality Ratio

The overall mortality ratio was found to be 99% based on 234 lives. This shows no excess mortality. Expected deaths were calculated using the FA75-78 table as a basis for expected deaths.

23.3.3 Results: Main Effects Fitted Separately

Table 23.20 Mortality Ratios by Weight Levels at Entry (Factor W)

	MR%	
20-30% over standard	93	(125)
30-40% over standard	107	(58)
more than 40% over standard	106	(51)

Slight excess mortality is shown where weight is more than 30% above standard.

Table 23.21 Mortality Ratios by Girth (Factor G)

	MR%	
Satisfactory	105	(139)
Unsatisfactory	100	(20)
Not medically examined	89	(75)

There is not much difference between the satisfactory and unsatisfactory groups. The lowest mortality ratio was recorded where applicants were not medically examined.

Table 23.22 Mortality Ratios by Age at Entry (Factor A)

Age at Entry	MR %	
16 - 39	98	(87)
40 - 49	106	(73)
50 - 59	88	(48)
60 - 79	103	(26)

The range of the mortality ratios by age at entry is narrow (less than 20 percentage points). There is no noticeable trend in the movements of the mortality ratios.

Table 23.23 Mortality Ratios by Policy Duration (Factor D)

Duration	MR %	
0 - 2 yrs	56	(15)
2 - 5	83	(39)
5 - 10	108	(73)
10 - 15	101	(45)
15 - 20	138	(39)
over 20 yrs	101	(23)

The mortality ratio is very low in the first two years since entry. After that, the mortality ratio rises until about 10 years duration, after which it is reasonably stable.

Table 23.24 Mortality Ratios by Calendar Year of Entry (Factor C)

	MR %	
1947 - 51	114	(43)
1952 - 56	95	(45)
1957 - 61	117	(46)
1962 - 66	91	(31)
1967 - 71	149	(30)
1972 - 76	98	(14)
1977 - 86	57	(25)

Although the mortality ratios show a wide range, there is no noticeable trend in the mortality ratios by calendar year of entry.

23.3.4 Significance of Main Effects and First Order Interation Term

Referring differences in model deviances to the appropriate χ^2 distribution (as shown overleaf) reveals that policy duration and calendar year of entry are statistically significant, although their interaction is not. This led to investigation of the model D+C, although it could be argued that this is not particularly useful given the erratic behaviour of the results by calendar year of entry.

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev	D of F	
H_0	665·9	1188			
W	664·8	1186	1·1	2	58 %
G	664·5	1186	1·4	2	50 %
A	664·9	1185	1·0	3	80 %
D	654·2	1183	11·7	5	3·8 %
C	650·1	1182	15·8	6	1·5 %
D+C	644·1	1177			
D*C	615·0	1153	29·1	24	22 %

23.3.5 Main Effects Fitted Together, No Interaction: Model D+C

This is the simplest model which may be fitted including both policy duration and calendar year of entry, with multiplicative relationships between rows and between columns. The mortality ratios given by fitting this model are shown in Table 23.25.

Table 23.25 Mortality Ratios: Model D+C

		Policy Duration (years)					
		0-2	2-5	5-10	10-15	15-20	over 20
	1947-51	0·73	1·03	1·20	1·05	1·46	1·08
	1952-56	0·61	0·86	1·00	0·88	1·22	0·90
Calendar	1957-61	0·75	1·06	1·24	1·08	1·50	1·11
Year of	1962-66	0·58	0·81	0·95	0·83	1·15	0·85
	1967-71	0·97	1·36	1·59	1·39	1·93	1·42
	1972-76	0·66	0·94	1·10	0·96	1·33	0·98
	1977-86	0·43	0·60	0·71	0·62	0·86	0·63

Fitting this model, it is possible to produce results for certain cross classifications of policy duration and calendar year of entry for which there is no data at present. For example,

calendar year of entry 1977 to 86 and duration over 20 years would need data from deaths recorded after 1997 at the earliest. The results, of course, follow the same pattern as the results in other calendar year of entry groups for which data are available. Since the results by calendar year of entry show a wide range, but without a noticeable trend, the usefulness of this model is limited.

23.4 Comparison with Other Studies

All authors studying the entire Prudential data set have reported results for this impairment. Table 23.26 shows the results by weight category and also shows the overall mortality ratios reported by each author. Due to very low numbers of deaths attributed to certain impairment codes, results available from the earlier studies are incomplete. The omissions are not serious, however, and figures shown in the table below have been derived from the data available (number of deaths in parentheses).

Table 23.26 Prudential Data Set: Overweight

Author(s)	Calendar Years of of Study	Weight levels at entry (S=standard) 20-30% over S	30-40% over S	more than 40% over S	Overall Mortality Ratio
Clarke (1961)	1947-58	118 (162)	110 (36)	187 (23)	121 (221)
Preston & Clarke (1966)	1947-63	134 (411)	152 (111)	184 (42)	141 (564)
Clarke (1979)*	1964-73	117 (742)	122 (187)	133 (66)	119 (995)
Leighton (1987)*	1974-83	117 (743)	115 (208)	144 (88)	118 (1039)
Papaconstantinou (1988)	1947-81	132 (1804)	149 (488)	171 (207)	138 (2499)
England (1992)	1947-86	122 (2135)	126 (610)	148 (267)	125 (3012)

* Duration 2 and over

The results of the various studies compare favourably. It can be seen clearly that excess mortality rises as weight levels rise, from about +20 for weight levels 20-30% above standard to about +50 upwards for weight levels above 40% over standard.

Results were presented for female lives by only three of the authors, as follows:

Author(s)	Overall MR%	
Preston and Clarke	101	(49)
Papaconstantinou	84	(190)
England	99	(234)

These results indicate that excess mortality for female lives who are overweight is negligible.

For male lives, a further comparison can be made with the results of Papaconstantinou and, for completeness, with the results of the 1979 Build Study conducted in North America, shown in Table 23.27 by age at entry.

Table 23.27 Overweight: Mortality Ratios by Age at Entry (male lives)

	Papaconstantinou		England			1979 Build Study*	
16 to 39	218	(1063)	132	(1357)	15-39	142	(1892)
40 to 49	123	(820)	131	(956)	40-49	136	(2013)
50 to 79	94	(616)	108	(699)	50-69	126	(1490)

* 25%-65% overweight

The results by age at entry clearly show excess mortality falling as age at entry rises. This effect is more pronounced when looking at the results of Papaconstantinou, which do not compare well with the results of England. However, the results of England compare well with the results of the 1979 Build Study

For completeness, it is worth comparing results by weight category of the three major investigations into Build in North America. These are the Build and Blood Pressure Study (1959) [BBPS], the Build Study (1979) [BS], and the American Cancer Society Study (1979) [ACS]. For male lives, the results of these studies are shown in Table 23.28, together with the results of England (1992). The results of the various studies compare extremely well.

Table 23.28 Overweight: Mortality Ratios given by American Studies

Weight levels at entry	Study			
	BBPS	BS	ACS	England (1992)
20-30% above standard	125	120	121	122
30-40% above standard	142	133	137	126
40-50% above standard	167	150	162	148

23.5 Rating of Overweight

Each of the underwriting manuals considered uses a detailed build table in the process of rating for this impairment. The most detailed table is found in the underwriting manual of Munich Re, the least detailed in the manual of Mercantile and General. The more detailed tables are, however, not so easy to use. Swiss Re seem to offer the most pragmatic approach to the rating of this impairment. An outline of the approaches adopted by the three companies considered is shown below.

Mercantile and General and Munich Re

For a given height, the ratings proposed by Mercantile and General depend on the weight level and age at entry. There are different ratings for ages at entry up to 55 and over 55. It is not easy to convert these ratings into a format classified by "percent overweight" for comparison purposes, since desirable weight levels change with age.

Munich Re show detailed tables of average weight for heights at different ages for males and females separately. Munich Re also show weight levels for degrees of overweight, again classified by age and height. These tables provide help in converting the ratings suggested by Mercantile and General into ratings by "percent overweight". A summary of the ratings recommended by Mercantile and General and Munich Re is shown in Table 23.29.

Table 23.29 Approximate Ratings for Overweight: Mercantile & General and Munich Re

% overweight	Rating Guideline	
	Mercantile & General	Munich Re
20	+10	+10
40	+20	+40
60	+75	+100

The ratings proposed by Munich Re are a little higher than those proposed by Mercantile and General. The results of the Prudential study imply a slightly lower rating at the highest levels of overweight, perhaps in the order of +50.

Mercantile and General make no reference to adapting these ratings according to girth measurements or family history, whereas Munich Re recommend an addition for unsatisfactory girth and poor family history.

Swiss Re

Swiss Re use a table showing height and weight to classify individuals as Mild, Moderate or Marked overweight. Rating is then simply according to a severity and age classification as shown in Table 23.30.

Table 23.30 Approximate Ratings for Overweight: Swiss Re

	Severity		
Age at Entry	Mild	Moderate	Marked
34 or less	+50	+100	+150
35 to 44	+35	+70	+100
45 to 54	+20	+40	+70
55 or more	+15	+30	+40

Mild corresponds approximately to 30-40% overweight, Moderate corresponds approximately to 40-60% overweight, and Marked to 60%+.

Debits and Credits are awarded according to girth classification and family history.

The approach adopted by Swiss Re is simple to apply, although the ratings imply mortality ratios which are higher than those given by the Prudential data.

Chapter 24 Asthma

24.1 Prudential Impairment Codes and Classification for Analysis:

Pre 1/1/1980:

701, 711, 721 Bronchial Asthma

Post 1/1/1980:

705, 715, 725	Mild asthma
706, 716, 726	Moderate asthma
707, 717, 727	Severe asthma

Subsidiary codes:

A	Symptoms within 3 years
B	Symptoms within 3-6 years
C	Symptoms within 6-10 years
D	Symptoms over 10 years ago

The change in coding introduced on 1/1/1980 presents some difficulty in the analysis of asthmatics. Codes 701, 711 and 721 were replaced by codes 705-707, 715-717 and 725-727 on 1/1/80 to incorporate a severity classification. However, insufficient time has elapsed for enough data to accumulate to conduct a full analysis of the most recent experience. The analysis was therefore conducted twice; once using all the data and once using only the data collected since 1/1/1980.

For analysis, data were classified according to:

T:	Time since presence of symptoms:	4 levels	1 - within 3 years
			2 - 3 to 6 years ago
			3 - 6 to 10 years ago
			4 - over 10 years ago

A:	Age at Entry	4 levels :	1 - 16 to 29	3 - 40 to 49
			2 - 30 to 39	4 - 50 to 59

D:	Policy Duration	6 levels :	1 - 0 to 2 years	4 - 10 to 15 years
			2 - 2 to 5 years	5 - 15 to 20 years
			3 - 5 to 10 years	6 - over 20 years
C:	Calendar Yr of Entry:	8 levels :	1 - 47 to 51	5 - 67 to 71
			2 - 52 to 56	6 - 72 to 76
			3 - 57 to 61	7 - 77 to 81
			4 - 62 to 66	8 - 82 to 86

24.2 Male Lives 1947 to 1986

24.2.1 Summary Statistics

Number of Entrants: 31661

Number of Deaths: 662

Table 24.1 Distribution by Time since Last Attack

Time Since Last Attack	Number	Percentage
Within 3 years	22688	71·7
3 to 6 years ago	2377	7·5
6 to 10 years ago	2142	6·8
over 10 years ago	4454	14·1
Total	31661	

The majority of policyholders (over 70%) were classified as having an attack within 3 years.

Table 24.2 Distribution by Age at Entry

Age at Entry	Number of Entrants	Percentage
16-29	18036	57·0
30-39	9013	28·5
40-49	3413	10·8
50-79	1199	3·8
Total	31661	

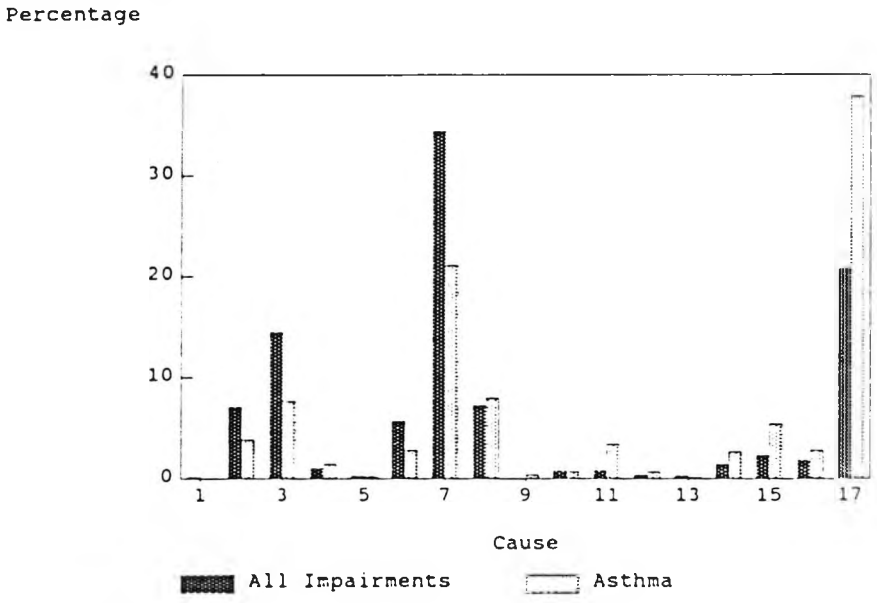
Almost 60% of entrants were aged below 30 at entry. Only 4% were aged 50 and above at entry.

Table 24.3 Distribution by Cause of Death

		Number of Deaths	Percentage
1	Tuberculosis	-	-
2	Lung Cancer	26	3·9
3	Other Cancer	51	7·7
4	Leukaemia	10	1·5
5	Diabetes	2	0·3
6	Vascular Lesions	19	2·9
7	Ischaemic Heart Disease	140	21·1
8	Other Circulatory Diseases	53	8·0
9	Influenza	3	0·5
10	Pneumonia	5	0·8
11	Bronchitis	23	3·5
12	Peptic Ulcer	5	0·8
13	Nephritis	1	0·2
14	Motor Accident	18	2·7
15	Other Accident	36	5·4
16	Suicide	19	2·9
17	Miscellaneous (unspecified)	251	37·9
	Total	662	

The distribution by cause of death for asthmatics is very different from the distribution by cause of death for the entire study. A graphical comparison is shown in Figure 24.1. Deaths from cancer account for 11·6% of deaths for asthmatics compared to 21·6% in the study overall. Also deaths from circulatory diseases account for only 32% of deaths compared to 47·5% in the entire study. Deaths from accidents and suicide are noticeably high, accounting for 11% of deaths for asthmatics compared to only 5·8% in the entire study. Particularly noticeable, however, are deaths from "Other causes (unspecified)" which here account for 37·9% of deaths compared to only 20·8% in the entire study.

Figure 24.1 Distribution by Cause of Death: Asthmatics (Males)



24.2.2 Results: The Overall Mortality Ratio

The overall mortality ratio was found to be 117% based on 662 deaths using the A67-70 table as a basis for expected deaths. This represents low excess mortality overall.

24.2.3 Results: Main Effects Fitted Separately

Table 24.4 Mortality Ratios by Time Since Last Attack (Factor T)

	MR%	
Within 3 years	134	(507)
3 to 6 years ago	59	(29)
6 to 10 years ago	86	(38)
No symptoms for at least 10 years	94	(88)

Most of the deaths were recorded by the group where the last attack occurred within 3 years of taking out the policy. This group showed the highest mortality ratio at 134%, subsequently dropping to mortality below standard. Combining the last three groups gives a mortality ratio of 83% where the last attack occurred more than 3 years ago.

Table 24.5 Mortality Ratios by By Age at Entry (Factor A)

Age at Entry	MR %	
16 - 29	114	(160)
30 - 39	116	(195)
40 - 49	121	(185)
50 - 79	119	(122)

There is very little variation in the observed mortality ratios by age at entry.

Table 24.6 Mortality Ratios by Policy Duration (Factor D)

Duration	MR %	
0 - 2 yrs	194	(101)
2 - 5	122	(107)
5 - 10	119	(150)
10 - 15	121	(132)
15 - 20	95	(87)
over 20 yrs	87	(85)

The highest mortality ratio is observed in the first 2 years policy duration, with excess mortality of +94. From 2 to 15 years, excess mortality is fairly stable at around +20, reducing to below standard mortality at durations above 15 years.

Table 24.7 Mortality Ratios by Calendar Year of Entry (Factor C)

	MR %	
1947 - 51	123	(24)
1952 - 56	129	(131)
1957 - 61	137	(154)
1962 - 66	117	(127)
1967 - 71	102	(92)
1972 - 76	101	(66)
1977 - 81	92	(45)
1982 - 86	136	(23)

With the exception of the mortality ratio of 136% for calendar years of entry 1982 to 86, there has been a tendency for excess mortality to fall over calendar time. There was a particularly sharp drop between the pre '67 and post '67 experiences.

24.2.4 Significance of Main Effects

Model	Deviance	Degrees of Freedom	Differences		Observed
			Dev	D of F	Sig Level
H_0	623·6	653			
T	588·79	650	34·81	3	<·01 %
A	623·23	650	0·37	3	94 %
D	589·15	648	34·45	5	<·01 %
C	612·15	646	11·45	7	12 %

Referring differences in model deviances to the appropriate χ^2 distribution (as shown above) reveals that time since last attack and policy duration are highly statistically significant, leading to models including both of these factors together.

Time since last attack was grouped into "within 3 years" and "at least 3 years" and the models T+D and T*D fitted to assess the significance of the interaction between time since last attack and policy duration. The interaction term was not found to be statistically significant (see table below), leaving model T+D as the most parsimonious.

Model	Deviance	Degrees of Freedom	Differences		Observed
			Dev	D of F	Sig Level
T+D	565·62	647			
T*D	561·98	642	3·64	5	60%

24.2.5 Significant Main Effects Fitted Together, No Interaction: Model T+D

The mortality ratios, expressed as a percentage, are shown in the body of Table 24.8. Where the last attack was within 3 years, there is significant excess mortality (+112) in the first two

years policy duration, reducing to standard only after 20 years. Where the last attack was at least 3 years ago, excess mortality was only experienced in the first two years since entry.

Table 24.8 Mortality Ratios given by Model T+D

		Time Since Last Attack	
		Within 3 years	At least 3 years ago
Policy Duration	0-2 years	212	137
	2-5	134	87
	5-10	133	86
	10-15	138	89
	15-20	111	72
	over 20 years ago	102	66

24.3 Female Lives 1947 to 1986

24.3.1 Summary Statistics

Number of Entrants: 5050

Number of Deaths: 68

Table 24.9 Distribution by Age at Entry

Age at Entry	Number of Entrants	Percentage
16-29	2400	47·5
30-39	1477	29·2
40-49	799	15·8
50-79	374	7·4
Total	5050	

Roughly half of the entrants were aged below 30, and half aged 30 and above at entry.

Table 24.10 Distribution by Cause of Death

	Number	Percentage
Cancer (excluding lung)	9	13·2
Leukaemia	1	1·5
Vascular Lesions	2	2·9
Ischaemic Heart Disease	6	8·8
Other Circulatory Diseases	7	10·3
Bronchitis	5	7·4
Accident (excluding motor)	2	2·9
Other Causes (unspecified)	36	52·9
Total	68	

Noteworthy are deaths from circulatory diseases, which here account for only 22% of deaths, compared with 30% in the study overall. Also, deaths from "other causes (unspecified)" are noteworthy since they account for 52·9% of deaths for asthmatics, compared to only 23·1% in the study overall.

For female lives, the number of deaths for this impairment is low. Therefore, it was possible to analyse the data by age at entry and policy duration only. The vast majority of lives were classified as "last attack within 3 years".

24.3.2 Results: The Overall Mortality Ratio

The overall mortality ratio was found to be 168% based on 68 deaths, using the FA75-78 table as a basis for expected deaths. This shows excess mortality higher than for male lives.

24.3.3 Results: Main Effects Fitted Separately

Table 24.11 Mortality Ratios by Age at Entry (Factor A)

Age at Entry	MR %	
16 - 29	251	(15)
30 - 39	266	(23)
40 - 49	151	(18)
50 - 79	85	(12)

Although based on low numbers of deaths, there is a clear tendency for the mortality ratios to fall with increasing age at entry. There is a significant difference between ages at entry below 40, and ages at entry 40 and over.

Table 24.12 Mortality Ratios by Policy Duration (Factor D)

	MR%	
0 to 5 years	163	(29)
5 to 10 years	162	(20)
over 10 years	182	(19)

There is little variation in the results by policy duration.

24.3.4 Significance of Main Effects

Model	Deviance	Degrees of Freedom	Differences		Observed
			Dev	D of F	Sig Level
H_0	163·31	158			
A	149·79	155	13·52	3	0·4 %
D	163·14	156	0·17	2	92 %

Referring differences in model deviances to their appropriate χ^2 distribution (as above) reveals

that Age at Entry is highly statistically significant, and that policy duration is not at all significant. Therefore, there is no justification for fitting models including both of these factors together.

24.4 Male Lives 1980 to 1987

24.4.1 Summary Statistics

Number of entrants: 8038
Number of deaths: 48

Due to the low number of deaths, it was not possible to analyse the post 1980 experience by time since last attack.

Table 24.13 Distribution by Cause of Death

Cause	Number of Deaths	Percentage
Lung Cancer	2	4·2
Other Cancer	5	10·4
Leukaemia	1	2·1
Ischaemic Heart Disease	13	27·1
Other Circulatory Diseases	1	2·1
Bronchitis	1	2·1
Accident(excluding motor)	3	6·3
Suicide	1	2·1
Other Causes (unspecified)	21	43·8
Total	48	

Deaths from circulatory diseases account for only 29·2% of deaths here compared to 47·5% in the study overall. Also, deaths from "other causes" are noteworthy, accounting for 43·8% of deaths here compared to only 20·8% in the study overall. These observations are similar to those considering all asthmatics for the period 1947 to 1986.

Table 24.14 Distribution by Age at Entry

Age at Entry	Number of Entrants	Percentage
16-29	3247	40·4
30-39	2759	34·3
40-49	1374	17·1
50-79	658	8·2
Total	8038	

Almost $\frac{3}{4}$ of entrants were aged below 40 at entry. Only 8% were aged 50 and above at entry.

24.4.2 Results: The Overall Mortality Ratio

The overall mortality ratio was found to be 120% based on 48 deaths using the AM80 table as a basis for expected deaths. This is very similar to the overall mortality ratio of 117% found for all male asthmatics in the period 1947 to 1986.

24.4.3 Results: Main Effects Fitted Separately

Table 24.15 Mortality Ratios by Age at Entry (Factor A)

Age at Entry	MR %	
16 - 39	133	(16)
40 - 49	101	(10)
50 - 79	122	(22)

There is some evidence of a higher mortality ratio for ages at entry below 40, although the results are based on low numbers of deaths.

Table 24.16 Mortality Ratios by Policy Duration (Factor D)

Duration	MR %	
0-2 yrs	149	(24)
2-8 yrs	100	(24)

There is some evidence of a higher mortality ratio in the first two years policy duration.

Table 24.17 Mortality Ratios by Severity (Factor S)

	MR %			
Mild	87	(22)		
Moderate	152	(19)	} 176	(26)
Severe	298	(7)		

There is evidence of excess mortality increasing with severity. However, the result of 298% for the severe category is based on 7 deaths only. Combining with the moderate category gives a mortality ratio of 176%, which is a lot higher than the mortality ratio of 87% for the mild category. The mild category shows no excess mortality.

24.4.4 Significance of Main Effects

Model	Deviance	Degrees of Freedom	Differences		Tail Area
			Dev	D of F	
H_0	38·405	35			
A	37·911	33	0·496	2	78%
D	36·571	34	1·834	1	17·6%
S	32·589	34	5·816	1	1·6%

Although there is some evidence of a lower mortality ratio for policy durations above 2 years, this is not statistically significant when referring differences in model deviances to the appropriate χ^2 distribution (as shown above). The only statistifcally significant main effect is severity.

24.5 Female Lives 1980 to 1987

24.5.1 Summary Statistics

Number of Entrants: 2974

Number of Deaths: 19

Because of the low number of deaths, it is only possible to assess extra mortality for the whole group together.

Table 24.18 Distribution by Cause of Death

	Number of Deaths	Percentage
Cancer (excluding lung)	2	10·5
Leukaemia	1	5·3
Vascular Lesions	1	5·3
Ischaemic Heart Disease	3	15·8
Bronchitis	2	10·5
Other Causes	10	52·6
Total	19	

The low numbers of deaths makes it very difficult to comment on these results. Perhaps it should be noted, however, that over half the deaths were classified as attributable to "other causes (unspecified)".

24.5.2 Results: The Overall Mortality Ratio

The overall mortality ratio was found to be 173% based on 19 deaths using the AF80 table as a basis for expected deaths. This is very similar to the result of 168% found for all female asthmatics in the period 1947 to 1986.

24.6 Comparison with Other Studies

All previous studies based on the Prudential data have considered this impairment. However, only the study by England used the subsidiary codes as part of the analysis. Overall mortality ratios given by the various studies are shown in the Table 24.19 (for male lives only).

Table 24.19 Prudential Impaired Lives Studies

Author	Calendar Yrs of Study	MR (%)	Number of Deaths
Clarke (1961)	1947-58	211	(32)
Preston & Clarke (1966)	1947-63	206	(83)
Clarke* (1979)	1964-73	133	(185)
Leighton* (1987)	1974-83	93	(233)
Papaconstantinou (1988)	1947-81	210	(515)
England (1992)	1947-86	117	(662)

* durations 2 and over only

There are considerable differences in the results of the various studies. The results of Preston and Clarke (1966), Clarke (1979) and Leighton (1987) form a series over time when considering the calendar years of study. The studies show a dramatic improvement in the mortality experience between 1947 and 1983. The results of Papaconstantinou and England do not agree which is surprising since they refer to very similar study periods. However, the result of Papaconstantinou is higher than any of the results of the three previous studies. Since the period of study considered by Papaconstantinou (1947 to 81) is approximately the same as the period of study considered by the previous three authors combined (1947 to 1983), the result of Papaconstantinou is surprising, since it should be (approximately) a weighted average of the previous three results..

It is possible to estimate the overall mortality ratio for the period 1947 to 1983 from the results of Preston and Clarke, Clarke, and Leighton. Since the mortality ratio is derived from the ratio of actual to expected deaths, it is possible to calculate the expected number of deaths

upon which each of the results is based. Thus:

	MR%	Deaths	Expected Deaths
Preston and Clarke	206	83	40·29
Clarke	133	185	137·59
Leighton	93	233	250·54

The total number of deaths over the period 1947 to 1983 was 501 and the total expected deaths was 428·42, giving an estimated mortality ratio for the period 1947 to 1983 of 117%. Reassuringly, this is identical to the result given by England.

A further comparison can be made with the results of the 1983 Medical Impairment Study which considered insured lives in the United States. The study considered 125,377 policies issued from 1952 to 1976 at standard or substandard rates to persons with a history or findings of asthma. The overall mortality ratio, for male lives, was 118% based on 1,945 deaths, which is extremely close to the 117% found by the Prudential study. For female lives, the overall mortality ratio given by the 1983 M.I.S. was 183% which compares well with the overall mortality ratio of 168% given by the Prudential study.

24.7 Rating of Asthma

All of the underwriting manuals considered make the distinction between asthma being present and asthma being in the history of the applicant. Age of the applicant is not considered by any of the underwriting manuals. This is entirely in agreement with the results of the Prudential study. The ratings recommended are summarised below.

Munich Re

Munich Re distinguish between occasional and frequent attacks. The ratings may be summarised briefly as follows:

Occasional	Present	0 to +50
	In history	Usually 0
Frequent	Present	+75 up
	In history: within last 2 years	up to +50
	over 2 years ago	Usually 0

Mercantile and General

Where there are symptoms within the last two years, Mercantile and General rate by severity, as follows:

Symptoms within 2 years	Mild	0 to +25
	Moderate	+50 to +100
	Severe	+125 to +175
No symptoms for at least 2 years		0

Swiss Re

If there have been no symptoms for at least two years, no rating is applied. If symptoms are present (i.e. within 2 years), a rating of +75 is applied. This rating is decreased or increased in the presence of favourable or unfavourable additional features.

In the light of the results of the Prudential study, the ratings recommended by the three companies considered here seem cautious, but not unreasonable. In a few years, when sufficient data accumulate, it will be interesting to see results by severity from the Prudential study.

Chapter 25 Conclusions

25.1 Theoretical Considerations

In this thesis, the problem of statistical modelling of excess mortality of impaired insured lives has been considered from a theoretical and practical point of view. The theory brings together traditional actuarial practice, statistical modelling, and theoretical work used primarily in the field of medical statistics. The theory is easy to implement in practice and has appealing connections with the traditional actuarial approach to the measurement of excess mortality, but also offers many additional benefits which accrue from the modelling framework employed. The techniques associated with generalised linear models can be used to good effect to allow a more thorough and scientifically sound treatment of the measurement of excess mortality than is available using only traditional methods.

When *modelling* excess mortality, it is possible to measure excess mortality in a systematic manner including any number of influential factors together simultaneously. It is also possible to assess which of these factors has a statistically significant effect on excess mortality, and hence, which should be taken into consideration when underwriting. Furthermore, it is possible to explore directly the effect of interactions which determine the level of interdependence between rating factors. Interdependence of rating factors has always posed a problem using traditional methods.

By the inclusion or exclusion of model factors and their interactions, it is possible to arrive at an optimal model of excess mortality for a particular impairment, given the data available. The suitability of this optimal model can be checked by analysing residuals, which will indicate an overall lack of fit or isolated departures from the model requiring further investigation. If necessary, after fitting a model, it is also possible to calculate confidence intervals for mortality ratios given by the model. For model factors fitted individually, these confidence intervals are similar to those given using traditional methods. They can be calculated quickly and easily

and the same methodology can be used regardless of whether the model is simple or complex.

Three model structures have been proposed for modelling excess mortality: the multiplicative model, the additive model, and the power model. All three models can be embedded within the generalised linear model framework, and models can be fitted with the same degree of ease requiring only the calculation of the actual deaths and expected deaths. A particular model structure is chosen simply by the choice of *link function* within the *GLIM* software package. Thus, a unified approach to modelling excess mortality has been presented.

The multiplicative model has particularly pleasing properties and should be the first choice for modelling excess mortality, provided a satisfactory fit can be achieved. The additive model has an appealing connection with the numerical rating system, although substantial difficulties can be anticipated when using this structure to model excess mortality. In general, there will be no *a priori* reasons why either the multiplicative or additive model should provide the better fit, and the power model can be used to discriminate between the two. The power model is, in fact, a family of models including the multiplicative and additive models as special cases, and can be used to find an optimal model which is neither multiplicative nor additive. Like the additive model, difficulties can be anticipated when using the power structure to model excess mortality.

It is worth repeating that, usually, it will not be possible to attach any physical interpretation to a particular fitted model, and that the models proposed are intended to represent excess mortality in a way which has an appealing connection with traditional actuarial mortality ratios, and which is convenient, flexible and entirely empirical.

The theory of dynamic generalised linear models mentioned in Chapter 21 is an innovation which takes modelling excess mortality one step further still. Using a Bayesian approach, models of excess mortality can be created which change smoothly over time, which could be useful if it is necessary to forecast excess mortality beyond the end of an investigation.

25.2 Practical Considerations

The practical aspects of modelling excess mortality have been illustrated by a re-analysis of some of the impairments included in the Prudential impaired lives data set. A thorough investigation involves basic data analysis as well as modelling with factors fitted individually and together, and the assessment of the statistical significance of factors and their interactions. It is also important that results are compared fully with those available from previous studies and with results available from other impaired lives investigations (such as those conducted in North America).

On the whole, the results obtained are highly informative and are consistent with the results based on other studies. A comparison with the ratings suggested by the underwriting manuals considered reveals that, on the whole, the ratings recommended are not in disagreement with the mortality experienced by impaired lives insured by the Prudential Life Assurance Company.

The results in relation to the subsidiary codes appear for the first time, since the subsidiary codes have been ignored in previous studies based on the Prudential data set. These results are often useful and show the importance of scrutinizing the data available and not simply copying what has been done in the past.

Male lives and female lives have been analysed separately and the results, which are often different, not only justify this approach, but deem it essential. There is, however, a considerably smaller amount of data available for female lives which often results in only a simple analysis.

The main findings for the eight impairments considered in this thesis are summarised below. It should be noted that when considering "significant main effects", it is not suggested that these are the only significant rating factors for a particular impairment, but that these are the

only ones for which data are available.

For impairments of the coronary arteries, the significant main effects are age at entry, policy duration and the presence or absence of complications. Excess mortality is considerably higher at the younger ages at entry, and circulatory diseases account for the majority of the excess deaths. Female lives appear to be a slightly lower extra risk than male lives, and this should be an underwriting consideration. A comparison with current underwriting practice appears to indicate that ratings are a little on the low side at the youngest ages at entry.

There was a large amount of data for hypertensives, and the results indicate that the significant main effects are blood pressure level, family history of cardiovascular disease, age at entry, policy duration and calendar year of entry. It should also be noted that overweight in conjunction with hypertension does not add materially to the risk. Therefore, the practice of increasing the rating for hypertension further still to allow for weight level is not wholly justifiable. The results also indicate that female lives suffering from hypertension are a lower extra risk than equivalent male lives.

For epilepsy, the significant main effects are age at entry, policy duration, and duration since last episode. The results by duration since last episode are important since they appear for the first time, being derived from the subsidiary code. Overall, the results are very similar to those from North American studies.

For psycho-neuroses, the most significant rating factor is severity of the condition. It appears that only the most severe cases exhibit excess mortality, and that excess deaths are predominantly from "accidents" and suicide. There is no evidence to suggest that duration since the last attack affects excess mortality.

Concerning diabetes mellitus, the significant main effects are age at entry, policy duration and calendar year of entry. There is no evidence to suggest that weight levels or family history of diabetes affects excess mortality. Overall, the levels of excess mortality for this impairment are

fairly high, with female lives faring slightly worse than male lives. Results from other studies indicate that duration since detection and severity of the condition (which is reflected by the treatment regimen) are also significant rating factors. Current underwriting practice for this impairment is complicated.

For underweight lives, the significant main effects are weight levels at entry, age at entry, policy duration and calendar year of entry. Overall, low levels of excess mortality are associated with this impairment, with only the more severe cases causing concern. Male lives and female lives appear to experience similar levels of excess mortality. Current underwriting practice takes a relaxed approach to this impairment, provided any underlying cause is excluded before acceptance.

Overweight lives constitute the largest single impairment group within the Prudential impaired lives data set. The significant main effects are weight at entry, girth at entry, family history, age at entry and calendar year of entry. The results showed statistically significant interactions between weight and girth, girth and age at entry, and family history and calendar year of entry, although it is difficult to interpret these interactions. For female lives, excess mortality is negligible. Current underwriting practice for this impairment seems reasonable.

For asthmatics, the significant main effects are time since last attack and policy duration. There is no evidence to suggest that age at entry is a significant rating factor. Time since last attack is the most significant rating factor with excess mortality mostly restricted to cases where the last attack was within the three years prior to application for insurance. These results again show the importance of an analysis including the subsidiary code. Female asthmatics appear to experience higher excess mortality than the equivalent male lives. Current underwriting practice seems to ignore age at entry, but takes into account whether asthma is present or in the history of the applicant. This is entirely in agreement with the results of the Prudential study.

25.3 Limitations of Impaired Lives Investigations

One of the limitations of an impaired lives investigation involving modelling excess mortality is that a large amount of data is needed to provide good results, especially when complex models are considered. It is necessary to leave an investigation running for many years before sufficient data accumulate to provide useful results. Furthermore, the only factors which can be included in a model are those for which data have been recorded, which means that it is important to design the study carefully from the outset. Of course, these limitations are also common to impaired lives investigations analysed using traditional methods.

Another limitation when modelling excess mortality is that the results of complex models involving interaction terms may be hard to present in a reasonable form, and even harder to interpret.

Despite these limitations, it is hoped that anyone conducting an impaired lives investigation in the future will consider using the modelling approach advocated in this thesis to analyse the experience rather than traditional methods, since it is possible to reproduce the results obtained using traditional mortality ratios as well as providing a more thorough analysis.

25.4 Further Research

Concerning further research on the theoretical side, there is scope for investigating the inclusion of continuous variables in the models of excess mortality. In this thesis, only models involving factors have been considered, where a factor is a (possibly) influential variable taking many levels. Where factors are quantitative and show a clear trend (which need not be linear), it is possible that excess mortality may be described better by regarding the variable as a continuous covariate. This, however, requires that data have been recorded in a suitable manner.

There is also scope for further research into the dynamic models of excess mortality. It is possible that erratic changes in excess mortality by age at entry or policy duration could also be smoothed using dynamic models in the same way that excess mortality by calendar year of entry was smoothed when analysing diabetics. Further research into the behaviour of these models when several model factors are included together is also needed.

Apart from the mortality ratio, another measure of excess mortality which is sometimes used (although not commonly) is the *excess death rate per* 1000 *per year* (EDR), which is based on the difference between observed and expected mortality rates rather than the ratio of the rates. Using the notation of Section 3.1:

$$EDR = 1000\times\left(\frac{d_t - d_t'}{E_t}\right)$$

$$= 1000\times(q_t - q_t')$$

It might be possible to extend this to include a model of the excess death rate. For example, using the notation of Section 6.2, it might be possible to consider models of the form

$$\lambda(t, \underline{z}) = \lambda^*(t) + (\underline{\beta}' \underline{z})$$

That is,

$$(\underline{\beta}' \underline{z}) = \lambda(t, \underline{z}) - \lambda^*(t)$$

In this case, the mortality factor $(\underline{\beta}' \underline{z})$ may be perceived as analagous to the excess death rate, ignoring the factor of 1000.

On the practical side, further research should be planned to provide updates of results based on the continuing experience of the Prudential data set. It will be interesting to see the development of the experience of impairments which were introduced on 1/1/1980, or which had their associated codings changed on that date.

If new investigations are planned in the future, it would be useful to include additionally such information as terms of acceptance, sum assured, type of policy, and duration since onset of the impairment in order to understand and explain more precisely the excess mortality experience.

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Appendix 1 Prudential Impairment Codes

1. CIRCULATORY IMPAIRMENTS

Cerebrovascular Disorders

105	Spontaneous sub-arachnoid haemorrhage - treated conservatively
106	Spontaneous sub-arachnoid haemorrhage - treated surgically
107	Cerebrovascular accident (stroke, or cerebral haemorrhage) - age at entry under 40
108	as above, age at entry 40-60
109	as above, age at entry over 60

Subsidiary codes:

A	Episode within last 2 years
B	Episode 2-4 yrs ago
C	Episode 4-6 yrs ago
D	Episode over 6 yrs ago

Hypertension

Weight standard ±19% Age at Entry			Weight standard +20% or over Age at Entry			Blood Pressure (mm Hg)	
<40	40-59	>59	<40	40-59	>59	Systolic	Diastolic
110	130	150	120	140	160	150-165	<95
111	131	151	121	141	161	>165	<95
112	132	152	122	142	162	150-165	95-105
113	133	153	123	143	163	>165	95-105
114	134	154	124	144	164	150-165	95-105
115	135	155	125	145	165	>165	>105
116	136	156	126	146	166	150-165	Uncertain
117	137	157	127	147	167	>165	Uncertain
118	138	158	128	148	168	<150	≥95

Subsidiary codes:

A	Family History -	Good (coded E prior to 1.1.80)
B	Family History -	Fair - one death below age 65 from vascular accident*
C	Family History -	Poor - two or more deaths below age 65 from vascular accident*
F	Family History -	Fair with two or more deaths from cardiovascular disease+
G	Family History -	Poor - general tendency to early death
H	Family History -	Poor - with two or more deaths from cardiovascular disease+

* introduced 1.1.80
+ deaths at age 70 and above disregarded.

This impairment group refers to essential (primary) hypertension only, and may include slight or moderate tachycardia or slight arteriosclerosis.

Impairment of the Coronary Arteries

Age below 50 (at entry)

171	Number of coronary vessels involved unknown	- no complications.
172	Number of coronary vessels involved unknown	- with complications.
173	One coronary vessel involved	- no complications.

174	One coronary vessel involved	- with complications.
175	More than one coronary vessel involved	- no complications.
176	More than one coronary vessel involved	- with complications.

Age above 50 (at entry)

181 - 186 defined as for 171-176.

Subsidiary Codes:

A	Onset within 2 years	- no surgery.
B	Onset 2-4 yrs ago	- no surgery.
C	Onset 4-6 yrs ago	- no surgery.
D	Onset over 6 yrs ago	- no surgery.
E	Onset within 2 years	- surgery.
F	Onset 2-4 yrs ago	- surgery.
G	Onset 4-6 yrs ago	- surgery.
H	Onset over 6 yrs ago	- surgery.

Rheumatic Heart Disease

187	Symptomless, no treatment or restrictions at work.
188	Requiring drugs or work restrictions.

Subsidiary Codes:

A	Valvotomy.
B	Valve Replacement.
C	Unoperated.

Non-Rheumatic Mitral Incompetence

189 Non-rheumatic mitral incompetence (prolapsing cusp)

Subsidiary codes: as for Rheumatic Heart Disease

2. STOMACH and INTESTINES

Dyspepsia, Gastritis

222 Dyspepsia - chronic or prolonged attacks, ulcer excluded by tests
223 Dyspepsia - suggestive of ulcer but not proven (duodenitis etc)
224 Dyspepsia - brief attack - apparently of no serious significance, no special investigations
225 Dyspepsia - brief attack - apparently of no serious significance, investigations carried out with negative result

Subsidiary Codes:

A	Symptoms within 2 years
B	Symptoms 2-4 years ago
C	Symptoms 4-6 years ago
D	Symptoms over 6 years ago

Cholecystitis

230 Without stones, no operation
231 With stones, no operation
232 With stones, cholecystotomy
233 With stones, cholecystectomy

Subsidiary codes:

A Symptoms within 2 years
B Symptoms 2-4 years ago
C Symptoms 4-6 years ago
D Symptoms over 6 years ago.

Peptic Ulcer (gastric or duodenal)

243 Symptoms or medication within 3 years - apparently resolved
244 No symptoms or medication for 3 years - resolved
245 Infrequent symptoms, occasional medication
246 Frequent or chronic symptoms, frequent or continuous medication

Subsidiary Codes

A Surgery performed - no history of perforation
B No surgical treatment - no history of perforation
C Surgery performed - history of perforation
D No surgical treatment - no history of perforation

Intestinal Disorders

290 Gluten Intolerance

Subsidiary Codes for 290

A Diet Continuing
B Not on diet - apparently resolved

291 Ulcerative Colitis: Symptoms within 3 years but apparently resolved
292 Ulcerative Colitis: No Symptoms for 3 years, apparently resolved
293 Ulcerative Colitis: Mild Symptoms, intermittent medication
294 Ulcerative Colitis: Moderate to severe symptoms, continuous medication
295 Crohn's Disease: Symptoms within 3 years but apparently resolved
296 Crohn's Disease: No symptoms for 3 years, apparently resolved
297 Crohn's Disease: Mild symptoms intermittent medication
298 Crohn's Disease: Moderate to severe symptoms, continuous medication

Subsidiary Codes for 291-298:

A No surgical treatment
B Partial colectomy
C Procto colectomy

3. NERVOUS DISORDERS, HEAD AND EAR IMPAIRMENTS

Epilepsy

Pre 1/1/80:

300	Petit Mal
301	Grand Mal (Idiopathic)

Post 1/1/80:

302	Petit Mal (excluding temporal lobe epilepsy)
303	Temporal Lobe Epilepsy (excluding grand mal)
304	Grand Mal (idiopathic or traumatic) - not more than 6 episodes per year
305	Grand Mal (idiopathic or traumatic) - 7 to 12 episodes per year
306	Grand Mal (idiopathic or traumatic) - more than 12 episodes per year

Subsidiary Codes:

A	Attacks within 2 years
B	No attack for 2-5 years
C	No attack for 5-10 years
D	No attack for over 10 years

Head Injury

319	Severe head injury with prolonged unconsciousness (no epilepsy)

Psycho Neuroses

320	Psycho-neuroses: mild
321	Psycho-neuroses: moderate
322	Psycho-neuroses: severe
323	With features suggestive of psychosis

Attempted Suicide

324	Attempted Suicide, one attempt only
325	Attempted Suicide, more than one attempt

Migraine

330	Migraine	mild
331	Migraine	severe

Attacks of Unconsciousness

340	Attacks of unconsciousness, of uncertain origin.

Disseminated Sclerosis

352	Onset before age 25 : abnormal physical signs at entry - nil
353	Onset before age 25 : abnormal physical signs at entry - mild
354	Onset before age 25 : abnormal physical signs at entry - moderate

355 Onset after age 25 : abnormal physical signs at entry - nil
356 Onset after age 25 : abnormal physical signs at entry - mild
357 Onset after age 25 : abnormal physical signs at entry - moderate

Retrobulbar Neuritis

358 Retrobulbar Neuritis

Chronic Suppurative Otitis Media

Pre 1/1/80

360 Chronic Suppurative Otitis Media : without operation
361 Chronic Suppurative Otitis Media : with mastoidectomy

Post 1/1/80

363 Chronic Suppurative Otitis Media

Subsidiary codes for 319-363:

A Symptoms within 2 years
B Symptoms within 2-5 years
C Symptoms within 5-10 years
D No symptoms for at least 10 years

Alcohol Abuse

370 Alcoholism : Total Abstinence without relapse
371 Alcoholism : Total Abstinence with one minor relapse
372 Alcoholism : Total Abstinence with one or more major relapse
373 Heavy Drinking : 8 to 14 single drinks per day
373 Heavy Drinking : more than 14 single drinks per day

Subsidiary codes for 370 - 372 :

A Period since total abstinence 1 - 2 years
B Period since total abstinence 2 - 3 years
C Period since total abstinence 3 - 5 years
D Period since total abstinence over 5 years

Subsidiary codes for 373 - 374 :

A Currently drinking as coded
B Total abstinence, or moderated to an acceptable level within 2 years
C Total abstinence, or moderated to an acceptable level over 2 years

Alcoholism is defined as alcohol dependence interfering with work and social life, so as to require medical attention. Heavy drinking is defined as an average daily consumption of 8 or more single drinks taken consistently over a number of years.

Alcohol Table:

A single drink = 10 grams (or millilitres) of pure alcohol

Beer, 1 pint = 2 single drinks
Spirits, 1 double = 2 single drinks ; One bottle of spirits = 24 single drinks
Wine, 2 glasses = 2 single drinks

4. TUBERCULOSIS

Pulmonary Tuberculosis

Pre 1/1/80 Pulmonary Tuberculosis (not treated by special method of collapse)

440	Mild cases (discovered by mass radiography, etc, never sputum positive):	Type 1
441	Mild cases	Type 2
442	Mild cases	Type 3
443	Mild cases	Type 4
444	Moderate or severe cases	Type 1
445	Moderate or severe cases	Type 2
446	Moderate or severe cases	Type 3
447	Moderate or severe cases	Type 4

Subsidiary Codes:

A	within 3 years
B	3-6 years ago
C	6-10 years ago
D	Over 10 years ago

Post 1/1/80

470	Mild pulmonary tuberculosis
471	Moderate to severe pulmonary tuberculosis

Subsidiary Codes:

A	Treatment within 3 years
B	Treatment ceased 3 or more years ago

5. ENDOCRINE GROUP

Glycosuria

Pre 1/1/80: S = Standard
BSTT = Blood Sugar Tolerance Test

50-	Glycosuria (not proved to be Diabetes Mellitus); no BSTT
500	One test only, weight : S ± 10%
501	One test only, weight : S − 10%
502	One test only, weight : S + 10%
503	Inconstant, two or more tests, weight : S ± 10%
504	Inconstant, two or more tests, weight : S − 10%
505	Inconstant, two or more tests, weight : S + 10%
506	Persistent, two or more tests, weight : S ± 10%
507	Persistent, two or more tests, weight : S − 10%
508	Persistent, two or more tests, weight : S + 10%

51- Glycosuria (not proved to be Diabetes Mellitus); BSTT
510 to 518, as for 500 to 508

519 Proved Renal Glycosuria

Post 1/1/80:

553	Proved Renal Glycosuria
56-	Uninvestigated Glycosuria
560	Age under 30 at entry
561	Age 30 - 34 at entry
562	Age 35 - 39 at entry
563	Age 40 - 44 at entry
564	Age 45 - 49 at entry
565	Age 50 and over at entry

Subsidiary Codes for 500-519:

E	Family History, no diabetes
F	Family History, one or more diabetes

Subsidiary Codes for 553- 565

A	Duration since diagnosis (or detection)	0 to 5 years
B	Duration since diagnosis (or detection)	5 to 10 years
C	Duration since diagnosis (or detection)	10 to 15 years
D	Duration since diagnosis (or detection)	15 to 20 years
E	Duration since diagnosis (or detection)	20 to 30 years
F	Duration since diagnosis (or detection)	over 30 years

Diabetes Mellitus

Pre 1/1/80:

520	Age under 30,	weight standard ± 10%
521		weight more than 10% below standard
522		weight more than 10% above standard
523	Age 30 - 50,	weight standard ± 10%
524		weight more than 10% below standard
525		weight more than 10% above standard
526	Age over 50,	weight standard ± 10%
527		weight more than 10% below standard
528		weight more than 10% above standard

Subsidiary Codes:

E	Family History, no diabetes
F	Family History, one or more diabetes

Post 1/1/80:

570	Age under 30 at entry
571	Age 30 - 34 at entry
572	Age 35 - 39 at entry
573	Age 40 - 44 at entry
574	Age 45 - 49 at entry
575	Age 50 and over at entry

Subsidiary Codes:

A	Duration since diagnosis (or detection)	0 to 5 years
B	Duration since diagnosis (or detection)	5 to 10 years
C	Duration since diagnosis (or detection)	10 to 15 years
D	Duration since diagnosis (or detection)	15 to 20 years

E Duration since diagnosis (or detection) 20 to 30 years
F Duration since diagnosis (or detection) over 30 years

Thyroid Dysfunction

Pre 1/1/80

54- Goitre (simple or unclassified)
540 Goitre (simple or unclassified): no operation
541 operation
542 Thyroid Adenoma, etc : no operation
543 operation
544 Myxoedema

55- Goitre (exophthalmic) (including toxic goitre, hyperthyroidism, Grave's disease, thyrotoxicosis)
550 No special treatment
551 Surgical treatment
552 Other special treatment

Post 1/1/80

580 Thyrotoxicosis - under current treatment
581 Thyrotoxicosis - treatment ceased, apparently euthyroid
582 Myxoedema - idiopathic
583 Myxoedema - post thyrotoxic

Subsidiary codes for range 540-552 and 580-583:

A within 3 years
B 3 to 6 years
C 6 to 10 years
D over 10 years

Adrenal or Pituitory Failure

590 Adrenal Failure — on successful replacement therapy
591 Pituitory Failure — on successful replacement therapy

Subsidiary Codes:

A Diagnosed before age 20 — idiopathic
B Diagnosed before age 20 — other cases
C Diagnosed after age 20 — idiopathic
D Diagnosed after age 20 — other cases

6. UNDERWEIGHT and OVERWEIGHT

Underweight

600, 610, 620 Weight 20-30% under standard, chest expansion satisfactory
601, 611, 621 Weight 20-30% under standard, chest expansion unsatisfactory
602, 612, 622 Weight 20-30% under standard, not medically examined

603, 613, 623 Weight more than 30% under standard, chest expansion satisfactory
604, 614, 624 Weight more than 30% under standard, chest expansion unsatisfactory
605, 615, 625 Weight more than 30% under standard, not medically examined

Subsidiary codes:

E	Family history good
F	Family history indifferent
G	Family history poor

Overweight

650, 660, 670	Weight 20-30% over standard, girth satisfactory
651, 661, 671	Weight 20-30% over standard, girth unsatisfactory
652, 662, 672	Weight 20-30% over standard, not medically examined
653, 663, 673	Weight 30-40% over standard, girth satisfactory
654, 664, 674	Weight 30-40% over standard, girth unsatisfactory
655, 665, 675	Weight 30-40% over standard, not medcially examined
656, 666, 676	Weight over 40% above standard, girth satisfactory
657, 667, 677	Weight over 40% above standard, girth unsatisfactory
658, 668, 678	Weight over 40% above standard, not medically examined

Subsidiary Codes

E	Family history good
F	Family history indifferent
G	Family history poor

7. RESPIRATORY GROUP

Hay Fever

700, 710, 720	Hay Fever, simple

Chronic Bronchitis

702, 712, 722	Chronic Bronchitis, without emphysema
703, 713, 723	Chronic Bronchitis, with emphysema

Emphysema

704, 714, 724	Emphysema without bronchitis

Asthma

Pre 1/1/1980:

701, 711, 721	Bronchial Asthma

Post 1/1/1980:

705, 715, 725	Mild asthma
706, 716, 726	Moderate asthma
707, 717, 727	Severe asthma

Bronchitis

730	Recurrent Acute Bronchitis

Bronchiectasis

740	Bronchiectasis, no signs at entry
741	Bronchiectasis, slight to moderate signs at entry
742	Bronchiectasis, severe signs at entry

Subsidiary Codes for 700-742

A	Symptoms within 3 years
B	Symptoms within 3-6 years
C	Symptoms within 6-10 years
D	No symptoms for at least 10 years

8. URINARY GROUP

Urinary Calculus

800	Urinary Calculus, no operation
801	Urinary Calculus, voided or removed per urethrum
802	Urinary Calculus, removed by nephrotomy
803	Urinary Calculus, removed by nephrectomy
804	Renal colic (indefinite origin)

Cystitis/Pyelitis

820	Cystitis, without calculus
821	Pyelitis, without calculus

Pyuria/Haematuria

830	Pyuria of obscure origin
831	Haematuria of obscure origin or unclassified

Albuminuria

840	Orthostatic Albuminuria, ages under 30
841	Orthostatic Albuminuria, ages 30 and over
842	Other forms of Albuminuria
843	History of Nephritis

Other Renal Disorders

850, 860, 870	Hydronephrosis, no operation
852, 861, 871	Nephrectomy for trauma
853, 862, 872	Nephrectomy for conditions other than trauma, calculus or hydronephrosis
854, 863, 873	Hydronephrosis with nephrectomy
855, 864, 874	Hydronephrosis with operation other than nephrectomy

Codes 860-864 (unilateral) and 870-874 (bilateral) replaced codes 850, 852-855 on 1/1/80.

Subsidiary Codes for 800-874

A	Symptoms within 3 years
B	Symptoms 3-6 years ago
C	Symptoms 6-10 years ago
D	Symptoms over 10 years ago

Miscellaneous Disorders

879 Nephrotic Syndrome
880 Ileal Conduit (apparently uncomplicated)
890 Renal failure - treated by dialysis
891 Renal failure - treated by transplant

Subsidiary Codes for 880-891

A Operation (or dialysis commenced) within 2 years
B Operation (or dialysis commenced) 2-5 years ago
C Operation (or dialysis commenced) over 5 years ago

9. TUMOURS and MISCELLANEOUS

The Breast

933 Malignant Breast Tumour

Pre 1/1/80:

932 Non-malignant breast tumour : confirmed
934 Non-malignant breast tumour : unconfirmed

Post 1/1/80:

935 All non-malignant breast conditions (confirmed or unconfirmed)

Female Genital Organs

940 Uterus, fibroids (confirmed)

Malignant Tumours Excluding Breast

Pre 1/1/80:

903 The skin and superficial tissue, including external genitalia
— malignant tumours (other than rodent ulcer)
912 Lips, mouth and salivary glands, malignant tumours
952 Testicle, malignant tumours
962 Miscellaneous tumours, not previously classified, malignant tumours

Post 1/1/80:

964 All malignant tumours other than breast tumours

Subsidiary Codes for 932-964

A Tumour present at time of proposal
B Tumour removed by operation, radiotherapy to within 5 years
C Tumour removed by operation, radiotherapy to within 5 to 10 years
D Tumour removed by operation, radiotherapy to over 10 years