Analysis of Life Insurance Lapses and Utility-Maximization of Shareholders' Expected Profit

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DECLARATION

Abstract

The problem of experiencing early terminations of life insurance contract has greatly affected insurers and yet, is one of the areas in the actuarial literature in which has received little attention. As a result of this, insurers guarantee a high yield, and sometimes offer high payouts (surrender and maturity values) in order to avoid surrenders. This makes the pricing of insurance contracts and also, the management of the corresponding asset portfolio difficult. Therefore, we have proposed methods or techniques to minimise the impact of surrenders on the life insurance company's fund. Particularly, we have looked at the impact of lapses on the performance (leading to profit/loss) of life insurance funds- a profit/loss model has been developed to be used by actuaries to determine the cost of the surrender option arising from the effects of financial and non-financial adverse selection in this regard. As a result, we have proposed techniques that will involve the policyholder in sharing the cost of surrender due to the option available to him, usually, at times which are favourable to him.

Further, numerical optimization routines and stochastic simulation techniques have been used to determine optimal strategic decisions that maximize the expected shareholders' profit. It links the approaches of utility theory and mean-variance analysis in obtaining numerical solutions (optimal values). In view of the fact that the life office could lose most of its prospective policyholders as a result of charging a higher premium (the proposed strategy), we have introduced a premium penalty model to take care of this effect.

In the case where there is a financial incentive to surrender, the optimal strategy is to impose a low premium loading for all values of r, which is different from the market loading and a low surrender penalty. By this strategy, the volume of business is expected to increase and so is the shareholders' expected profit. However, for the case where there is no financial incentive to surrender, the optimal strategy is to impose a high premium loading, not too close to the assumed market loading and charge a higher surrender penalty for relatively risk tolerant investors (for $r \geq \frac{1}{2}$). This was found to increase the corresponding shareholders' expected profit. Also, the results show that an optimal way of regulating the surrender basis is to change the surrender basis whenever the rate of investment return on assets rises by one and half percent or falls by a little above one percent. In view of the fact that the strategic decisions are considered in the context of utility theory, the results of the analysis have been shown to be similar to those of modern portfolio theory, as presented by Markovitz (1952). Finally, we have shown that the use of incorrect strategies can have an important effect on the shareholders' expected profit.

Chapter 1

INTRODUCTION

1.1 Background and Aims

The possibility of early surrender of life insurance policies is a 'systematic' risk for insurers and the consequences have greatly affected many life insurance companies in the U.K. and other parts of the world. The amount of benefits paid associated with this surrender amount in Britain to about several million pounds sterling, Scobbie et al (1969). Therefore, insurers are faced with the problems of either experiencing early terminations of life insurance contracts or have to guarantee a high yield in order to avoid these surrenders. This makes the pricing of insurance contracts and also, the management of the corresponding asset portfolio very difficult.

It is worth mentioning that *Lapse* occurs when premiums cease, before the policy "expiry" date is reached, without payment of a surrender value, for example, a term product at early durations. On the other hand, "surrender" is used to describe a discontinuation under which surrender values are available, Belth (1968). Further, from the insurance dictionary the actuarial word "*lapse*" in the context of life insurance is defined as the termination of a policy because of failure to pay a premium and lack of sufficient cash value to make up a premium loan.

The causes of lapses have been extensively studied for many decades by the insurance industry, and over the years several papers (for example, Belth (1968), Scobbie et al (1969), Thornber (1984), Le Grys (1987), Chung and Skipper (1987), Outreville (1990) and many more) have been published which investigate the factors affecting lapses. Some of these factors are mainly due to economic, financial, political and market performance. Other factors are due to mortality and change of geographical environment. For example, a policy is likely to be lapsed if the policyholder moves from one country to another. These factors will be reviewed in the subsequent sections of this chapter.

It is important to note that the research published in this area does not just centre on economic and market factors. But also, there are other approaches that have been used to analyse lapses some of which we will discuss later on in this thesis. For example, the area of selective lapsation, application of financial derivatives to manage interest rate and surrender option, adverse selection and withdrawals and many more have been looked at and reported in the literature as well. These concepts have a substantial influence on lapses and hence, the profitability of a company. So, it is important that we look into this area in great depth. Further, the risk level of an insured can also influence the lapse rate of a life company. For example, insureds with bad health are more likely to hold on to the policy than those with better health. Several authors, including Dukes and Macdonald (1980), Shapiro and Snyder (1981), Becker (1984), Macdonald (1997), and Jones (1998) have presented methods of looking at the impact of selective lapsation on mortality.

However, we have noticed that some of the methods used failed to address ways of providing lasting solutions to the consequences of lapses for life funds as opposed to exploring the factors affecting lapses. In other words, there has not been any published work on methods or techniques that could be used to minimise the impact of surrender on the life insurance company's fund. This is an area that mainly concerns life insurers. Therefore, this thesis will be looking at the impact of lapses on the performance (leading to profit/loss) of life insurance funds. We will consider surrenders due to financial and non-financial anti selection effects. Particularly, we will employ the methodology of Geman et al (1994), Macdonald (1997), and Jones (1998) to determine the profitability of a life company. We will look at these papers in detail later on in this chapter.

On this basis, we will be looking at strategic ways or approaches to minimise the life insurance company's losses due to surrender. In particular, we will design techniques or models that will involve the policyholder in sharing the cost of surrender due to the option available to him, usually, at times which are favourable to him. Numerical optimization techniques will be used to minimise the losses due to surrender if it occurs.

1.2 Overview/Structure of the Thesis

In the section that follows, a survey of published work on lapses and optimisation procedures is given. Although most of the papers have focused on the factors that affect lapse rate, we look at the concept of selective lapsation and numerical optimization techniques as well.

Chapter 2 discusses the effect of payouts on life assurance lapse rate. An empirical analysis was performed to find out if there is any relationship between lapse rate and relative payouts (surrender and maturity values relative to average market value). Financial data were available from the Department of Trade and Industry for the analysis. Statistical analysis, using rank correlation analysis, was performed on this data.

In Chapter 3, we propose a stochastic investment model corresponding to an office liability under investigation. In particular, we use the Wilkie (1995) gilt model at time t, with mean gilt yield replaced by our proposed surrender force of interest, δ_t^s. This is equal to the redemption yield in year t of an *(n-t)* year gilt. We also show how a model of δ_t^s can be fitted. Further, the main properties of this model are reviewed. Simulation results from the model are presented and analysed.

Chapter 4 introduces the general methodology used to derive the expected surrender profit/loss for non-participating life insurance policies. Particularly, we show how to develop a model of the expected surrender profit/loss in which there is no selection effect and another, where the effects of financial and non-financial adverse selection are incorporated. These models make it possible to investigate the impact of adverse selection effect on expected surrender profit/loss at time $t=n$. We describe the multiple state models used in the model of the expected surrender profit/loss. A method used to compute the transition probabilities to be used in the lapse model is also discussed. A sensitivity analysis of the results to economic factors will be performed as well. Chapter 4 attempts to lay the groundwork for building a model that will be used in Chapter 5.

Chapter 5 discusses or defines strategies by which the management of life insurance companies can provide its investors (shareholders) with acceptable profiles of returns from their investment. In other words, we discuss strategies that can be used to maximize the expected utility of shareholders' profit. Numerical optimisation will be described and then its application to the valuation of shareholders' expected profit will also be looked at. A sensitivity analysis to changes in model parameter values will be performed and the effects on profitability of using inappropriate strategies are discussed as well.

Finally, chapter 6 concludes by summarising the main arguments and by suggesting promising areas for future research.

1.3 Related Literature

The surrender of a policy (if it occurs) can have a tremendous effect on a company's profitability. Yet, very little has been researched and reported in the literature. Several authors have proposed different ways of analysing lapse rates, coupled with investigations of the effect of economic factors on lapse rate, some of which we will review shortly.

The earliest extensive discussion of methods for improving persistency/lapse rates was reported in 1914 in Record of American Institute of Actuaries (RAIA) journal. By 1921, a significant change in point of view about lapses being laid on the agent of the company had occurred. It was around this time that the effect of economic conditions on lapse rates was demonstrated. About 1925, measurable factors that are associated with lapse rates were discovered too. Even though there were substantial agreement on the factors causing high lapse rates, there was no information that gave useful clues to effective action. So, by the late thirties, the effect on lapse rates of income, occupation, sex, age, previous insurance, premium frequency, plan, and several other variables had been reported by a large number of studies. These variables were later on studied extensively by Richardson et al (1951). Also, there is historical evidence that economic recessions mostly affected the terminations of packages at durations subsequent to the second year.

Surprisingly, all of these analyses failed to measure the recognised interaction of various factors affecting lapse rates. Finally, a study by the Agency Management Association, AMA (1949) was able to provide a remedy to the above-mentioned deficiency. Since then there have been several papers on lapses/lapse analysis which are reviewed in this section.

For example, Richardson et al (1951) studied the causes of high lapse rates and the characteristics of business which has either high or low persistency. This study did not use any mathematical techniques in the calculation of lapse rate and also, did not perform any formal statistical analysis in their investigations. They conclude that a purely statistical approach is inadequate and that lapses are caused by human factors- "the reasons for which are not susceptible of statistical treatment".

Buck (1961) presents the result of a study of first year lapse rates and not-taken-up rates on a block of standard direct Ordinary policies recently paid for. The purpose of his studies were as follows:

- To identify attributes of policies and policyholders that influence lapse rates. These attributes were considered in the calculation of premiums and dividends.
- To determine the attributes of agents that influence lapse rates.
- And to demonstrate ways to improve the persistency of new business.

Then again, Buck did not perform any formal statistical analysis in his investigations and lapse rates are shown only by number of policies. He observed that the first year lapse rates decreased markedly with increasing amount of insurance and with increasing annual premium. Other factors that affect the first year lapse rate are sex and age of insured, and attributes of the agents- for example, newly appointed agents, length of service of agent, as the more experienced agents made a higher proportion of their sales to older policyholders of the company. However, age of the agent did not have a significant effect.

Further, Belth (1968) measured the effect of changes in lapse rates on the price of life insurance protection to policyholders. The effect of changes in lapse rate was compared with the effect of changes in mortality rates. In this analysis, lapse data relative to a number of companies (91 companies) were used to illustrate some of the differences among companies in their lapse experience. Also, the lapse rate formulas used by Belth were developed by the Life Insurance Agency Management Association (LIAMA) that conducts a continuing survey of the lapse experience of a number of companies that furnish data to the association; the annual publications of Bests' Life Insurance Reports; and the Institute of Life Insurance. The LIAMA, Best and the Institute of Life Insurance formulae are discussed in section 1.4.2 of this chapter. The conclusion of Belth's analysis is that lapse rates have a substantial effect on the price of life insurance protection to the policyholder, and that the use of expected lapse experience in the classification of life insurance applicants should be considered.

Also, Scobbie et al (1969) examined the level of withdrawal rates, the factors influencing these rates and some financial considerations. They analysed the office's withdrawal experience by the duration in force at date of withdrawal; occupation of proposer; purpose of assurance; originating branch office and agency connection. Note that Scobbie et al did not perform any formal statistical analysis in their investigations. The results of the analysis shows that withdrawal rates fall with increasing duration except during the third year where a significant increase was observed after exactly two complete years' premiums had been paid. Also, the analysis by occupation of the policyholder showed that withdrawal rates for manual workers (non-apprenticed and apprenticed trades) were much higher than those for managerial classes. Other conclusions drawn from the analysis include the following:

- The withdrawal rates for whole life without-profits had higher rates than other policies.

- The withdrawal rates for mortgage protection policies had poorer than the average experience.
- The withdrawal rates for endowment assurances, particularly with-profits assurance were observed to be lower than for all other classes of assurance. This is probably due to the savings element incorporated in this type of policy.
- The authors showed that poor withdrawal experience may be due to circumstances outside the control of the office such as a branch in an area being subjected to a temporary trade recession with associated high rates of unemployment.

Then, Crombie et al (1978), investigated the withdrawal experience of ordinary life business by using data supplied by seven Scottish life offices from 1972-1976. Their analysis indicate that four characteristics contribute ‘significantly’ to the variations in lapse rates. These are office, type of policy, age at entry, and duration of policy. It is worth mentioning that no formal statistical investigation was undertaken by the authors to qualify the term ‘significant’ as used in the discussion of the result of the analysis.

They observed that for with-profits endowment and term assurance, female lives experienced higher withdrawal rates (80%) than male lives at all durations and all ages at entry. Also, for with-profit and non-profit whole life assurance, female withdrawal rates were slightly higher (10-20%) than those for males. Further, it was observed that the withdrawal rates for with-profit endowment assurance increased as payment frequency increased. However, in the analysis of withdrawal rates by type of agent introducing the business, Crombie et al observed that insurance brokers were much more heavily involved in non-profit endowment and non-profit whole life assurance policy classes with heavy withdrawals. Again, estate agents had the heaviest overall withdrawal rates and chartered accountants had the highest rates overall, closely followed by the miscellaneous groups of other agent and no agent. Note that the estate agents were heavily involved in non-profit and term assurance, where term assurance was mainly being used as a cover for capital and interest repayment mortgages. Finally, middle level withdrawal rates were experienced by Banks, Building Societies, Solicitors and the office’s own staff.

Further, Haberman and Renshaw (1987) performed an extensive statistical analysis of life assurance lapses or withdrawal experience by applying the methodology of generalised linear models and a computer software package GLIM to lapse data considered earlier on by Crombie et al (1978). This improved the estimation of the underlying parameter values. One of the models fitted to the data was a linear model with normal error structure. The other model was a binary response model where the lapse data was treated as binary responses with the observed number of lapses in each category or policy characterization, modelled as a binary response variable. The discussion of these models is presented in section 1.4.2. The conclusions drawn from this analysis were similar to those of Crombie et al (1979) and include the following:

- Offices experienced similar patterns of lapses but to varying degrees of intensity.
- Lapse rates decreased with increasing age at entry.
- They observed a marked reduction in lapses for all types of policies at long durations in force.
- The non-profit whole life policies maintained decreasing pattern of lapses with increasing duration in force.
- There was a steady reduction in lapse rates with increasing age at entry for unit-linked. However, lapse rates were higher for non-profit policies than for the corresponding with-profit policies. Also, unit-linked policies had higher lapse rates than the other policy types at the youngest ages.

It is important to note that the Haberman and Renshaw (1987) models incorporated possible interactions between factors, while Crombie's approach could not deal with this aspect.

Sarma (1987) investigated the experience of lapses and surrenders of Rajkot Divisional Office of the Life Insurance Corporation of India by using a statistical based analysis. Particularly, analysis of variance was used in this case. Note that a preliminary analysis was performed initially, followed by an analysis of a three-way table. See section 1.4.2 for the model used for the three-way analysis.

The groupings recommended for the analysis by various factors were as follows:

1. Age (at date of investigation).
2. Duration in integral years. This is counted as at the beginning of the policy anniversary.
3. Plan of Assurance. That is, endowment, anticipated endowment and money back plans, whole-life policies, childrens' assurance and other plans.
4. Mode-wise (frequency of payment of premium) analysis (i.e., monthly, quarterly, half yearly and yearly).
5. Original policy term in groups (12 years or less, 13-15 years, 16-20 years, 21-25 years, 26-30 years and above 30 years).
6. Age at entry in 8 groups. (Age at 15-19 at entry, 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, and 50 and over).
7. Sum assured in Rupees (Rs) ranges: 10,000 or less, 10,001-25,000, 25,001-50,000, 50,001-100,000 and above 100,000.
8. Occupation code.
9. Rural and Urban classification.
10. Non-medical and medical.
11. Agent's status.

The following results were obtained from the preliminary analysis:

- The withdrawal rates decreased with duration. However, duration was not significant when duration and sum assured range were the factors for the two-way table.
- Mode was a significant factor. However, duration became significant but mode did not when the data were analysed by duration and mode.
- The plan of assurance was a significant factor along with duration in the two way table of plan and duration.
- The sum assured range was a significant factor when the data were analysed by sum assured range and age at entry, and also by sum assured range and duration.
- The original policy term was a significant factor when data were analysed by sum assured range and original term.
- Medical and non-medical was not a significant factor in this analysis.

- Age at entry was a significant factor when the data were analysed by plan and age at entry.

The conclusions drawn out from this analysis are as follows:

- The withdrawal rates increased for the younger age groups 15-19 and 20-29, but decreased with the later age.
- Durations 1 and 2 have an effect of increasing the rate. However, there is a decrease at later durations, the decrease being marked at durations 16 years and over.
- Both monthly and quarterly mode are associated with an increase in the rate (particularly, quarterly mode). Also, half-yearly and yearly modes are associated with a decrease in the rate.
- The effect of the interaction between age and duration is not significant. However, there was an increase in the withdrawal rates in the age group 15-19 and duration groups 3-8, and 9-15 years. In the case of age group 40 and over, there is an increase in the rate for durations up to 8 years.
- The interaction effect of age and mode was significant. Further, increases in the withdrawal rate arise in the case of age group 20-29 and modes (monthly, and quarterly) and also, for higher age groups for all modes except, monthly.

It is worth mentioning that Sarma's methods are inferior to Haberman and Renshaw (1987), but superior to Crombie et al (1979), in methodological and analytical terms.

Chung and Skipper (1987) examined the effect of interest rates on the cash and surrender values in order to determine whether policies with higher interest rates (insurer's currently advertised interest rates being credited on policies) generate higher values or not. The Spearman rank order correlation coefficient was used as part of a statistical analysis of surrender values and interest rates. Particularly, cash values and surrender values for durations of 1, 5, 15, and 20 years were compared with each insurer's currently advertised interest rate. Note that the values were based on a non-smoking male age 45 and the analyses were conducted using a level face amount of ($100,000).

Chung and Skipper (1987) observed no significant correlation between year one and five cash values and the credited interest rate. However, the interest rate becomes significant at durations 10, 15 and 20. Further, they observed a significant negative correlation between the level of surrender values and currently credited interest rate at duration one and a negative, but not significant, correlation at duration five. However, a significant positive relation exits at durations 10, 15, and 20. These suggest that policies with higher current interest rates do not necessarily generate higher surrender values for durations less than 10 years. Thus, the interest rate is a poor gauge if the person's planning horizon is less than 10 years. Over greater durations, higher interest rates tend to be associated with higher surrender values. However, it is important to note that even though there is a significant result at the longer durations, the dispersion of surrender values was large (determinant coefficients were quite small). This suggests that it is not always the case that policies with a higher interest rate usually provide higher surrender values in the long run and so the individual consumer should consider the large dispersion in surrender values in decision-making. Finally, Chung and Skipper suggested that "since current interest rates are not particularly reliable indicators of policy value, the prospective purchaser would be well-advised to place greater weight on the absolute level of projected surrender value accumulations (for a given outlay) rather than interest rates".

Outreville (1990) performed a statistical analysis on testing the emergency fund hypothesis (originally proposed by Linton (1932)) in relation to the annual average lapse rates for whole-life and ordinary life insurance policies. The emergency fund hypothesis, according to Outreville (1990), is that "cash values are utilized by policyholders as an emergency fund to be drawn upon in times of personal financial crisis", and that lapses should increase during recessions because some policyholders are unable to maintain premium payments for insurance coverage. The analysis was performed by using a regression equation which expresses lapse rate as a function of the variables: real transitory income per capita, real rate of return on alternative assets, price of insurance, unemployment rate and anticipated inflation rate. The results provided evidence in favour of the emergency fund hypothesis.

It is worth mentioning that tests of emergency fund hypothesis by Cummins (1975) proved unsuccessful for policy loans, and no direct estimation of lapse rates was provided. Further analysis on this hypothesis will be performed in chapter 2 of this thesis.

Another interesting area within the same context, but with different methodology from the ones mentioned so far, is an application of financial derivatives to manage interest rates and surrender options, authored by Albizzati and Geman (1994). They "addressed the surrender option pricing problem as the valuation of a contingent claim for the insurer, where the contingency is closely related to the level of interest rates, and directly priced by arbitrage the surrender option embedded in the life insurance policies". Further discussion on this investigation is given in section 1.4.3.

An important area which will be applied in this thesis is the area of selective lapsation. For example, Jones (1998) presents a model for examining the effect of various relationships between mortality rates and lapse rates on the mortality experience of a cohort of insured lives. In other words, he presents a model for individual mortality and its relation to lapsation. This is further discussed in detail in section 1.10. Similarly, Norberg (1988) presented a multiple-state model of temporary selection which provides an explanation for many selection phenomena.

1.4 Analysis of Lapse Risk Reported in the literature.

Now, in this introductory chapter we will look at the different ways of analysing lapse rates as reported in the literature. We shall divide our discussion into the following sections:

1. Causes/ factors affecting lapse rate (section 1.4.1)
2. Lapse Rate Models (section 1.4.2)
3. Valuation of Surrender Option in Life Insurance Policies (section 1.4.3)
4. Adverse Selection and Lapses (section 1.4.4)
5. Numerical Optimisation Techniques (section 1.4.5)

In what follows, we present a review of each of the above sections.

1.4.1 Causes/Factors Affecting Lapse Rate

The causes of lapse rate have been under study for many decades by the insurance industry and many researchers. Several papers have been published over the years on the factors affecting lapse rate and what follows is a review of some of these papers. Note that most of the factors listed are economic, financial or political in character.

i. An immediate need for cash to pay off outstanding debts, school fees, divorce, etc, may affect lapse rate.

ii. Various circumstances of the policyholder may also affect lapse rate-especially, for policyholders who are not fully familiar with the true nature of the financial contract they had purchased. So, with fluctuations in their personal circumstances occasioned by economic conditions, their financial resources may worsen. Therefore, such policyholders are likely to surrender or discontinued their policies in order to ease them of their financial difficulties, (Scobbie et al (1969)). Another example is that, an investor with a personal pension policy who unexpectedly changes employment to a non-contributory occupational pension scheme is most likely to lapse the former plan.

iii. The overselling or incorrect selling of business by sales organizations and agency will inevitably affect the level of lapse rates. That is, agency and sales agents are only interested in the quantity of business they issue since a high level of remuneration is related to the amount of business sold and so, they may overlook the policyholder's personal resources or circumstances and requirements (Thornber (1984), Le Grys (1987), SIB (1988)).

iv. Alterations to a mortgage can contribute to withdrawals since most life assurance contracts are effected in connection with house mortgages. Further, a policyholder may surrender the policy if the original purpose for setting up the policy is no longer relevant, for example, the policyholder may no longer have a mortgage.

v. Changes in either economic conditions or tax structure under which savings are encouraged could also have an effect on life assurance business as a policyholder may feel that the incentives to save are no longer there. Consequently, this could have an effect on withdrawal. Also, general economic conditions are a factor: Lapse rates are normally affected by the state of the economy, for example, lapses due to redundancy because of the inability to afford the premiums charged.

vi. The anticipated inflation rate and the real rate of return on alternative assets available on the market have an impact on lapses (Outreville (1990). Other factors could include the fact that surrenders are perceived as being more cost effective in the short term than taking a personal loan due to high interest rates, (Lamb (1989), Pipe (1990), Alexander (1991), Survey Research Associates (1992)). Finally, a change in interest rate (mainly a rise in interest rate) is another factor affecting the lapse rate (Chung and Skipper, 1987). Thus, policyholders may decide on early terminations of their existing policies and choose a higher yield savings alternative offered in the capital markets (e.g., money market funds), if the guaranteed return promised by the life insurers is not high enough compared to other forms of investment, mainly in the case of rise in interest rates, Albizzati and Geman (1994).

vii. Economic and political conditions may also affect a life insurance plan, thereby causing policyholders to lapse. For example, new pension legislation, new taxation rules or events, such as recession, may cause policyholders to lapse their policies. (SIB (1988)).

viii. Affordability and Suitability: An investor is more likely to stop paying premiums on a policy that is not affordable and does not meet his or her needs than one that is affordable and suitable. Further, an investor may stop paying premiums where the investor is sold a policy which costs more than he can afford.

ix. Changes in product design involve a loss to the policyholder and this may affect the lapse rate. Examples include the introduction of innovative product design (for example, for a switch from a personal pension plan to an FSAVC when the policyholder joins an occupational scheme), and the withdrawal of "old" products from the market, which involve a loss to the policyholder and may affect the lapse rate.

1.4.2 Lapse Rate Models

Having looked at the factors that affect lapse rates, we now consider some of the models that have been used in the analysis of lapse rates. The most common ones reported in the literature use the traditional way of measuring lapse rate. That is, the ratio of insurances going off the books due to termination of a policy by the insured (with or without surrender values) to a measure of the amount of the life insurance in force. However, in some of these examples, the approach used does not take into account the number of policies exposed to the risk of lapsing from the previous calendar years. But this is an important factor that could have an important effect on the results of any lapse analysis. Such a model is discussed in chapter 2. In what follows, we discuss some of the models reported in the literature.

A lapse model (for a 12-month period) to analyze the effect of changes in lapse rates on the price of life insurance protection to policyholders essentially defines a lapse rate for the first policy year in terms of the face amount of insurance. This measure, which was used by Belth (1968), was developed by the Life Insurance Agency Management Association (LIAMA), that conducts a continuing survey of the lapse experience of a number of companies that furnish data regularly to the Association. This was calculated by dividing the sum of lapses for each of the twelve months by the sum of the average monthly productions for the corresponding production periods (US and Canada 13-month Ordinary Lapse Survey, (1964)). A model of this kind does not consider 'exposed to risk' of lapsing.

Another lapse model similar to LIAMA was developed by Best (1967 and 1991). The model expressed the lapse rate as the sum of surrenders, expirations, lapses and net decreases with deductions made for amounts of ordinary business revived or increased. This expression is then divided by the sum of the previous year's amount of insurance issued, reinsurance assumed and the total in force at the end of the previous year. Note that the traditional approach was used here too.

Next, the formula for the voluntary termination rates compiled by the Institute of Life Insurance are described as follows:
"The termination rate is the ratio of the number of policies lapsed or surrendered (for cash, extended term, or reduced paid-up insurance), less reinstatements, to the mean number of policies in force". Life Insurance fact Book (1967).

Further, we consider a model that defines withdrawal rate of a policy at a given curtate duration (probabilistic model) as discussed by Sobbie et al (1969). This is given by :

$$R(a)=\frac{f(a)}{1-\sum_{b=0}^{b=a-1}f(b)} \qquad a=1,2,\dots\dots, \text{ and } R(0)=f(0).$$

where,
$R(a)$ is the withdrawal rate for curtate duration a and
$f(b)$ is the probability of withdrawal at curtate duration b, for $b=0,1,\dots,a-1$.

At the inception of a policy the probability of withdrawal occurring in a particular year at curtate duration t is calculated by dividing the number of policies terminating by lapse or surrender with t complete years premiums paid by the original population. The original population in this context was modelled to incorporate the number of new policies effected in the past calendar years. Actually, this was an approximation to the "original population" from which withdrawals could come and is given below. In this case, the assumptions made were as follows:

- The policy anniversaries are spread evenly over the calendar year.
- The distribution of policies by frequency of premium payment is 60% payable monthly, 25% payable quarterly, 5% payable half-yearly and 10% payable yearly.

- Policies terminating by withdrawal in a particular year are evenly spread over that year.
- The probability of withdrawal at exact duration $t+r$ $(0 \le r < 1)$ is constant for all values of r and is independent of the frequency of premium payment.
- The policies terminating by withdrawal are removed from the live file two months after the date of the first.

The formula adopted is given by:

$$0.4562E_t + 0.5396E_{t-1} + 0.0042E_{t-2}$$

where,

E_{t-a} represents the number of new policies effected in the calendar year $(t-a)$. For full explanation of the derivation of this formula see appendix 1 of Scobbie (1969).

An approach better able to describe the structure of data previously analysed by Scobbie et al (1979) was proposed by Haberman and Renshaw (1987), discussed earlier in section 1.3. We present in this section the models used. Here, the following covariates are investigated:

A- age at entry; 3 categories, i.e. $\begin{cases} i=1: early\,(15-29 years) \\ i=2: medium\,(30-39 years) \\ i=3: late\,(40-64 years) \end{cases}$

D- duration of policies; 3 categories, i.e., $\begin{cases} j=1: short\,(1-3 years) \\ j=2: medium\,(4-8 years) \\ j=3: long\,(9\, or\, more\, years) \end{cases}$

F- office, these are 7 denoted by k = 1,2,….,7

T- type of policies; 5 categories, i.e., $\begin{cases} \left.\begin{array}{l} l=1: with-profit \\ l=2: non-profit \end{array}\right\} endowment \\ \left.\begin{array}{l} l=3: with-profit \\ l=4: non-profit \end{array}\right\} whole-life \\ l=5: temporary \end{cases}$

Also, the number of lapses w_u, out of n_u exposures, for different u, are available for the analysis. The first model is a linear model with normal error structure. This is fitted to the lapse data (w_u, n_u).

The model is written as

$$Y_u = m_u + \varepsilon_u \quad \text{for each } u$$

where, $\varepsilon_u \sim N(0,\sigma^2)$ and $Y_u \sim N(m_u,\sigma^2)$ (since $E(\varepsilon_u)=0$ and so, $m_u = E(Y_u)$) for all u. According to Haberman and Renshaw (1987), the response variable is of the form

$$Y_u = \log\left(\frac{w_u}{n_u - w_u}\right)$$

and the structure of m_u, as

$$m_u = \alpha_i + \beta_j + \gamma_k + \delta_l$$

where the parameters $\alpha, \beta, \gamma, \delta$ relate to the covariates *A, D, F*, and *T* respectively. Further, by performing a test on the interactions between the various covariates, a significant result for the interaction term *D.T* is obtained. Therefore, the modified fitted model becomes

$$m_u = \mu + \alpha_i + \beta_j + \gamma_k + \delta_l + (\beta\delta)_{jl}$$

The parameters are estimated by using the maximum likelihood estimation method. See appendix A of Haberman and Renshaw (1987) for a full derivation of the parameter estimates.

Now, in the second model, the lapse data, (w_u, n_u) are treated as binary responses with the number of lapses in each unit u, modelled as a binomial response variable

$$w_u \sim Bin(n_u, p_u)$$

where p_u is lapse probabilities. Here, the structure of the model is given by

$$\log\left(\frac{p_u}{1-p_u}\right) = \mu + \alpha_i + \beta_j + \gamma_k + \delta_l + (\beta\delta)_{jl}$$

The estimates of the parameters are also estimated by using the maximum likelihood estimation method. See appendix B of Haberman and Renshaw (1987) for a full derivation of the parameter estimates.

In section, 1.3, we presented a review of the Sarma (1987) analysis. We now look at a model for describing how lapse rates vary within a population as discussed by Sarma.

In this work, withdrawal rates are calculated as the ratio of the number of withdrawals to total exposed to risk. In this case, withdrawals include lapses, foreclosures and surrenders and these are assumed to contribute one year to the total exposed to risk in the year of withdrawal. Note that exits other than withdrawals include deaths, maturities and transfers. These were assumed to contribute a fractional exposure.

In this analysis, θ_{ijk} is denoted as the value of the transformed variable corresponding to the observed ratio of withdrawals in age group *i*, duration *j* and mode *k*. Note that the transformed variable is given by $\log_e\left(\frac{Ratio}{1-Ratio}\right)$. The corresponding theoretical 'model' value μ_{ijk} can be expressed as:

$$\mu_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\beta\gamma)_{jk} + (\alpha\gamma)_{ik} + \varepsilon$$

where,

μ is the overall mean value of all groups

α_i is the addition for age group *i*

β_j is the addition for duration group *j*

γ_k is the addition for mode group *k*

$(\alpha\gamma)_{ij}$ is the addition due to interaction of age group *i* and duration *j*.

$(\beta\gamma)_{jk}$ is the addition due to interaction of duration group *j* and mode *k*.

$(\alpha\gamma)_{ik}$ is the addition due to interaction of age group *i* and mode *k*.

ε is the error term.

In order to minimise the error term, ε, we need to minimise the sum of squares expression $\sum(\theta_{ijk} - \mu_{ijk})^2$. This leads to the following parameter estimates:

$\mu = \overline{\theta}$ (mean values of θ_{ijk}).

$\alpha_i = \overline{\theta}_{i\bullet\bullet} - \overline{\theta}$, where $\overline{\theta}_{i\bullet\bullet}$ is the mean of the values of θ_{ijk} for age group *i* over duration groups and mode groups.

$\beta_j = \overline{\theta}_{\bullet j\bullet} - \overline{\theta}$, where $\overline{\theta}_{\bullet j\bullet}$ is the mean of the values for duration *j* over age groups and mode groups.

$\gamma_k = \bar{\theta}_{\bullet\bullet k} - \bar{\theta}$, where $\bar{\theta}_{\bullet\bullet k}$ is the mean of the values for mode k over age groups and duration groups.

$(\alpha\beta)_{ij\bullet} = \bar{\theta}_{ij} - \bar{\theta}_{i\bullet\bullet} - \bar{\theta}_{\bullet j\bullet} + \bar{\theta}$, where $\bar{\theta}_{ij}$ is mean of the values for age group i duration group j over all modes, and

$(\beta\gamma)_{\bullet jk}$ and $(\alpha\gamma)_{i\bullet k}$ have similar expressions as $(\alpha\beta)_{ij\bullet}$

Note that in the analysis of variance, the main factors are age, duration and mode and the interaction effects are age and duration, age and mode, and duration and mode.

A new development in calculating lapse rates was reported in the 1990's. In a study on lapse experience under lapse supported policies[1] carried out by the Canadian Institute of Actuaries (1996), lapse rates were determined monthly by sex, smoking status, policy size, issue age, premium frequency, calendar year, plan code, and for total company. In this investigation, there was no statistical analysis, and the model is descriptive and hence cannot be used for prediction purposes.

Given the methods and techniques used in the above papers for modelling lapse rates, we will use a lapse model that takes into account the risk factors associated with the business issued in the previous calendar years which the models reviewed above do not. In particular, we will allow for the number of policies exposed to the risk of lapsing from previous calendar years. We included this in our lapse rates calculation because the exposed to risk in the first year is based on new business figures in the current year and the previous year. Note that this type of model is dealt with in section 2.4.2.

From the papers reviewed so far we observed that most of the approaches used to analyse lapse rates are descriptive based, i.e., there were no statistical analyses involved. However, Haberman and Renshaw (1987) presented a statistical analysis of life assurance lapses that is better able to describe the structure of data previously analysed by Scobbie et al (1979). Also, Sarma (1987) used an approach similar to Haberman and Renshaw (1987) to analyse the lapse experience of Rajkot Divisional

[1] The reported experience is for policies whose lapse experience is not distorted by premium rate changes, not distorted by nonforfeiture values and is expected to be non-increasing over time.

Office of Life Insurance Corporation of India. From these we can conclude that Sarma's methods are inferior to Haberman and Renshaw (1987), but superior to Crombie et al (1979), in methodological and analytical terms.

1.4.3 Valuation of Surrender Option in Life Insurance Policies

1.4.3.1 Introduction

This is an interesting area within the context of surrender of a life policy. Albizzati and Geman (1994) address how to value the surrender option in the context of stochastic interest rate by using financial derivatives. In addition Albizzati and Geman consider a pool of homogeneous life insurance policies and use the fundamental averaging effect of the insurance mechanism in addressing problems of valuing surrender option. Although this thesis does not use financial derivatives in any of the analysis performed, we employ some of Albizzati and Geman's methodology, which will be reviewed shortly.

1.4.3.2 Valuation of Surrender Option.

The Surrender option is an exchange option, which gives the holder the right to exchange one security for another, Margrabe (1978). In valuing the surrender option, Albizzati and Geman (1994) considered a single premium deferred annuity contract. This contract is a tax-advantaged savings product offered by life insurers in France, so that, if the policy is held for eight years then interest income on it is tax free. However, there is a guaranteed minimum return paid to the policyholder. In addition the policyholder receives a cash surrender value on termination of the contract. In the case of early surrender, the tax rate on the policy depends on whether surrender occurs before or after four years. Thus, the tax rate at time t is represented by

$$x(t) = 0.381 \underset{t<4}{I} + 0.181 \underset{4\leq t<8}{I}$$

where I denotes an indicator function, with $I=1$ if the above inequality (e.g., $t<4$) is satisfied, and $I=0$ otherwise. It is important to note here that there is no penalty on surrender. By using the tax rate model Albizzati and Geman suggest that the payoff for a policyholder who terminates an insurance contract at time t (cash surrender value) is as follows:

$$K(t)=1+(e^{\lambda R(0,T)}-1)(1-x(t))$$

where $\lambda R(0,T)$ represents the return effectively paid to the policyholder (λ is a positive constant not greater than one and $R(0,T)$ is the yield on the assets). Further, Albizzati and Geman introduce a stochastic model of the dynamics of the term structure of interest rate which is assumed to be driven by a one-factor model with deterministic term structure of volatilities. For details of the modelling of the dynamics of term structure of interest rate, see Albizzati and Geman (1994). Finally, a closed form solution for the surrender option is derived and computed numerically under different interest rate volatilities.

Numerical values are calculated for the surrender (European) options with maturities, $t = 1,2,\ldots,7$ for different parameter values and the following observations are made.

i. This is a practical observation:- "At variance with a belief shared by a number of insurers preoccupied by this problem, the maximum value of the European option is observed for maturities reached two to four years after inception of the contract and not for maturities greater than four years. This means that "the tax effect is dominated by the time value of the option".

ii. "The surrender option on a policy has a greater value than the supremum of the values of the corresponding European option for different possible exercise dates".

iii. The price of the surrender option is sensitive to the volatility of interest rate and also, the value of λ.

iv. The price of the surrender option increases with the slope of the initial yield curve. However, it decreases with the level of the initial yield curve.

The conclusions drawn out from this study are as follows:

- the risk of experiencing early surrender of life insurance policies can be hedged by incorporating floating rate notes in the assets portfolio (interest rate caps that are available for 7- to 10-year maturities). According to Albizzati and Geman, "these caps should be tied to the interest rate index most closely related to the policy lapses that need to be hedged".
- In defining the insurer's marketing policy, the coverage cost or amount of benefit paid as a result of policy surrenders must be incorporated in the coefficient λ, which defines how much of the portfolio yield is allocated to the policyholder.

We will employ some of Albizzati and Geman's methodology in our analysis. These would be discussed in detailed in the subsequent chapters of the thesis.

1.4.4 Adverse Selection and Lapses

1.4.4.1 Introduction

In the discussion of the above review we realised that there is an important area that was not mentioned. This is the area of adverse selection and withdrawals. This concept has a substantial influence on lapses and hence, the profitability of the company. So, it is important that we look into this area in great depth. Further, the risk level of an insured can also influence the lapse rate of a life policy. For example, insureds with bad health are more likely to hold on to the policy than those with better health. Therefore, the risk level of the insured reflects the heterogeneity of the insured group. Hence, our lapse model should recognise the heterogeneity of the insured lives and include a random component that allows a substantial deterioration in the health of an individual. Several authors, including Dukes and Macdonald (1980), Shapiro and Snyder (1981), Becker (1984) and Jones (1998) have presented methods of looking at the impact of selective lapsation on mortality. We will look at some of these papers in detail and employ some of the methods used to determine the profitability of the company. What follows are definitions and the description of methods used in analysing selective lapsation.

1.4.4.2 Selection

Selection is defined as the operation of any variable factor, other than age, which tends to influence rates of mortality (Anderson and Dow, (1948)). In fact, selection is the process of subdividing data into more homogenous groups.

1.4.4.3 Adverse Selection

Suppose that an insurer offers coverage at the same price to insured with high and low loss probabilities. By this action, the high-risk insured would be prompted to buy larger policies than the low-risk insured and/or will cause some or all of the low-risk to remain uninsured. Thus, according to Cummins et al (1982) the tendency of high risks to be more likely to buy insurance or to buy larger amounts than low risks is known as *adverse selection.* We notice that adverse selection arises because of an "informational asymmetry", Cummins et al (1982), that is, applicants for insurance know their loss probabilities but companies either do not or are not permitted to use this knowledge even if they know.

We can illustrate adverse selection using a simple model given by Cummins et al (1982). They consider a market for one–year term life insurance in which there are high- and low-risk insureds with associated probability of death as θ_H and θ_L respectively. They assume an equal number of high- and low-risk insureds with "identical financial characteristics and identical utility functions". Further, they assume the loss amount to be Q for both groups. They expressed the demand for insurance by consumers in the two groups by the functions $Q_H(P)$ and $Q_L(P)$, where $Q_i(P)$ is the demand curve for risks in group *i,* and *p* is the premium per dollar of coverage. Note that as price declines, each group of risks demands more coverage. Thus, "the actuarial fair or expected value premiums per dollar of coverage are θ_H for the high-risk group and θ_L for the low risk group". Now, consider the graph of demand curves in figure 1.1.

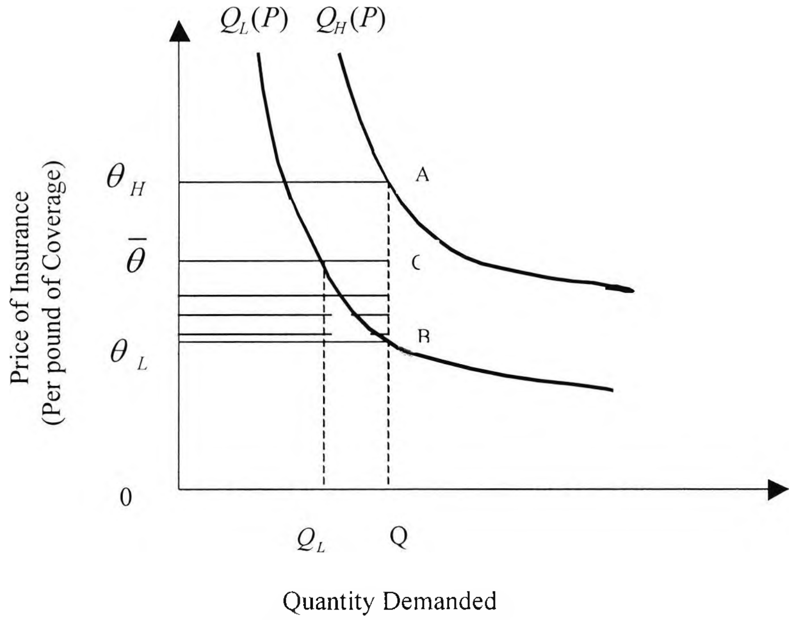

Figure1.1-Adverse Selection in a Simple Insurance Market

From the graph, we observe that the curve for the high-risk group lies to the right of that for the low-risk group. This shows that "high risks will purchase more insurance at every relevant premium rate because they are more likely to sustain losses". So, if companies can identify applicants who are high-risk and low-risk and offer coverage to each group at actuarial fair rates, then all consumers will demand coverage Q. However, because each group is charged the actuarially fair rate, premium revenues are produced in exactly the amounts needed to pay loss costs. This is given by the amount θ_H Q (area O θ_H AQ) for high risks and θ_L Q (area O θ_L BQ) for the low risks.

On the other hand, suppose that companies are unable to classify risks into high-and low-loss probability groups either because firms are unable to measure loss probabilities accurately before issuing coverage or because regulation or legislation does not permit them to classify. If high and low risks cannot be identified, the company will charge the average premium $\bar{\theta}$ to every one who applies for coverage. However, this will cause the low-risk consumers to cut back their demand for coverage to Q_L, whereas high-risk insureds will still purchase the full coverage amount Q, assuming that overinsuring is not permitted. This type of situation is known as adverse selection.

The effect of adverse selection on a company's profitability can be identified or studied by considering the following. From above the company thus collects premiums of $\bar{\theta}$ Q (area of rectangle 0 $\bar{\theta}$ CQ) from high-risk policyholders and has to pay expected losses of θ_H Q. This will result in a loss to the company, equal to the area θ_H AC $\bar{\theta}$. This loss is offset partially by overcharges to the low-risk insureds (area is denoted by straight lines). But the expected loss from each high-risk insured is larger than the gain from each low-risk insured. Hence, if even "none of the low-risks drop out of the market, the company will lose money and the plan will fail".

On the other hand, adverse selection behaviour in relation to lapsation is observed when an insured experiences a severe deterioration in health. For example, if an insured becomes terminally ill, all efforts will be made to ensure that his/her policy remains in force, Jones (1998).

1.4.4.4 Temporary Initial Selection

Consider a group of lives of the same age at the date of issue of a contract. As time goes by, some lives may fall sick or suffer accidents and so, the effect of medical underwriting tends to wear off with time. Eventually, this gets to a point where there is no difference between the health of this group of lives and any other group of lives of the same attained age, whose policies were issued at earlier ages. This effect of better than average health at the time of medical underwriting gradually wearing off with time since that event passes is called *temporary initial selection*.

Note that the process of checking that an applicant who is taking out new insurance contracts and having passed the medical examination tests (and so tends to be of better than average health) and is in reasonably good health is called *medical underwriting* (Subject A2 Core reading, 1996).

1.4.4.5 Heterogeneity

A population of individuals is homogeneous with respect to mortality if it consists of lives with similar characteristics, which affect an individual's mortality experience.

However, if such a population consists of lives with different characteristics, then it is heterogeneous. Heterogeneity in a population can be approached by subdividing the population into homogeneous groups. However, because individual lives are affected by a large amount of factors like sex, smoking, nature of employment, nutrition, environmental and lifestyle and the fact that every person is different from everyone else, any real sample of more than one life is likely to be heterogeneous and it will be impossible to produce a homogeneous population. Hence it is important that we consider heterogeneity.

The heterogeneity of insured lives has some interesting and important implications. Firstly, the expected lapse experience of any subset of an insured group depends on which lives are included in the subset. Secondly, because decisions on whether insured lives are to continue their life insurance policies are influenced by their perceived probabilities of death, the insured group at duration k after issue does not comprise a random sample of the insured group at issue. However, those whose health deteriorates drastically during the first k policy years are less likely to lapse their policies than those who remain healthy. This situation is another example of adverse selection. In this chapter we discuss the causes of heterogeneity in any population and the models of adverse selection as proposed in the literature.

1.4.5 Norberg's Model of Temporary Initial Selection

The Norberg (1988) model, which models the temporary initial selection effect and provides explanation of the selection phenomenon is discussed briefly in this section. This model is a simple Markov model in which the population is divided into "Insured and Not Insured" groups. Within each group, the individuals can be either "Insurable" or Uninsurable", with excess mortality among uninsurable individuals. Further, Norberg assumes that everyone in the population starts from the "Insurable and Not Insured" state but disallows transitions from the "Uninsurable, Not Insured" state into the corresponding insured state. In other words, he models the "declinature of uninsurable lives by the insurance company". Thus, in the Norberg model, adverse selection does not exist. Note that in this model the insured lives remain insured. But in practice, this is not so as a policy can be surrendered or lapsed. The model of the expected surrender profit/loss where the effect of financial and non-financial adverse selection are incorporated will be looked at later on in section 4.5.3. For details of the Norberg model and results, see Norberg (1988).

1.4.6 Macdonald's Model of Adverse Selection.

Macdonald (1997) considered the combined effect of underwriting and adverse selection among heterogeneous populations by using a simple Markov model. Although this discussion is in the context of genetic tests, the methodology will be applied later on to our investigations.

Following on from Cummins et al (1982), the three main elements that determine the extent of adverse selection in life insurance are:

i) The rate at which people buy life insurance. Thus, the higher the rate, the smaller the impact of a small proportion of adverse selectors.
ii) The extent to which people with a known risk factor, who are potential 'adverse selectors', are more likely to buy insurance.
iii) The extent to which "adverse selectors" insure their lives for higher amounts.

The Markov model shown in figure 1.2 represents the above elements. In the model a life is assumed to start at age x in the originating state (Not Tested, Not Insured). From there, a life can move between states as shown by the arrows, with probabilities governed by the transition intensities μ_{x+t}^{i01}, μ_{x+t}^{i02} and so on.

Thus, for small dt we define:

$\mu_{x+t}^{ijk}dt$ as the probability that a life in state ij at time $x+t$ moves to state ik during the next dt. Note that the intensities are expressed using a time unit of one year.

Figure 1.2 shows Macdonald's Markov model for the *ith* of *M* subgroups.

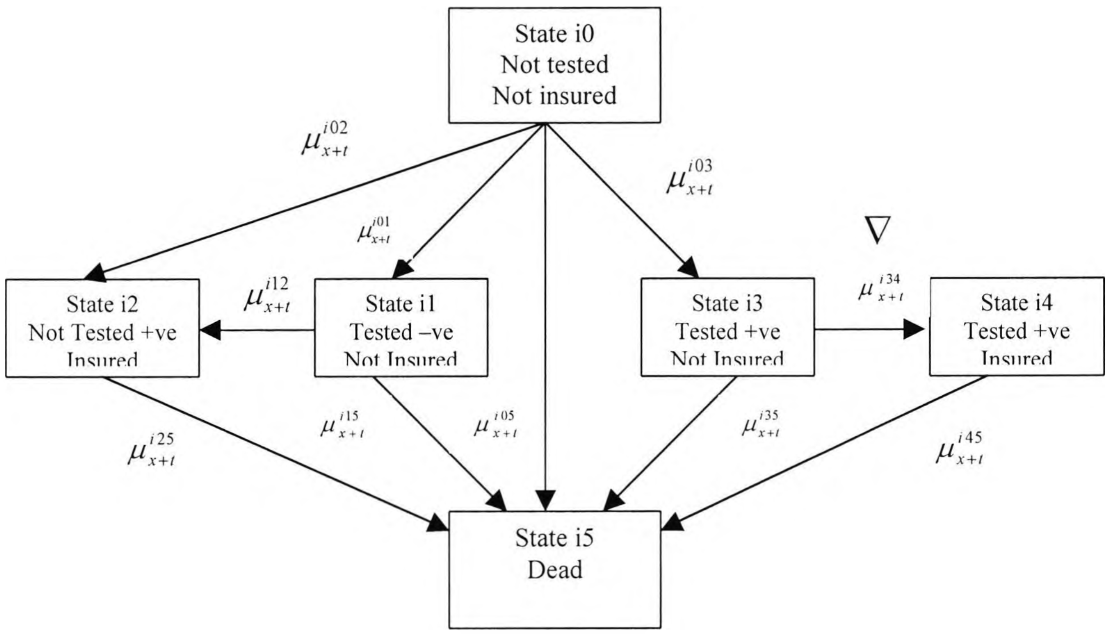

Figure 1.2- Macdonald's Markov Model for the ith of M subgroups

∇ -measure adverse selection

The main features of the model are as follows (these were quoted from Macdonald (1997)):

i) In the originating state a life is neither insured nor had any genetic test. From this state, a life can die, obtain insurance without taking a genetic test, or have a genetic test with a positive or negative result.

ii) "The rate of movement from the originating state and the "tested but negative" state into the insured state models the "normal" level of insurance against which adverse selection is measured".

iii) The rates of movement from the originating state into the two tested states model the extent of genetic testing. However, the difference between them models the likelihood of a genetic disorder being present.

iv) Finally, "the rate of movement from the "tested and positive" state into the corresponding insured state models the incentives of potential "adverse selectors" to insure themselves".

The heterogeneity of the population is represented by assuming that the population is divided into M subgroups, and that all members of a subgroup experience the same mortality relative to the average mortality of the population. For simplicity, suppose that the population is divided into two subgroups (i.e., $M=2$) with 1 as a "low mortality" subgroup and 2 as a "high mortality" subgroup. From figure 1.2, movement from the originating state i1 into state i2 models the 'normal' level of insurance. Also, movement from state i0 into state i1 or i3 models the extent of genetic testing. Finally, movement from state i3 into state i4 models the incentive of potential 'adverse selectors' to insure themselves.

Now, to model adverse selection, Macdonald varies the following characteristics of subgroup 2:

i) the rate of transfer to the insured state from the 'tested and positive' state, μ_{x+t}^{234} and

ii) the sums assured taken out by 'adverse selectors'.

Then Macdonald assumes the following:

a) The (average) force of mortality has Gompertz form based on the AM80 ultimate mortality table and is given by

$\mu_{x+t} = 0.00002072e^{0.103571(x+t)}$.

This forms the baseline for the analysis.

b) The rate of adverse selection in Group 2 to be $\mu_{x+t}^{234} = 0.25,\ 0.5$ or 1.0 (i.e. a high level of adverse selection).

c) Sums assured are one, two or four times as high as other lives; and

d) Lives who have tested positive are charged ordinary premium rates.

Using these assumptions, Macdonald computes the mean present value of losses as a percentage of baseline costs, with 'low' and 'high' incidence of genetic testing, assuming all sums assured are £1. In this case no adverse selection effect (i.e., $\mu_{x+t}^{234} = 0$) was considered. The results show that these ratios are small with the highest referring to short terms of insurance, whereas it is likely that 'adverse selectors' would prefer longer terms, including the more likely ages at death.

Later on, the case with adverse selection is taken into account. Thus, considering the fact that 'adverse selectors' tend to insure their lives for higher sums assured, Macdonald assumes the lives in group 2 who have tested positive apply for sums assured two or four times as high as other lives. Then he re-computes the mean present values of losses as a proportion of the baseline costs.

The results show that the losses exceed 20% of the baseline costs, only in the most extreme case (which require possibly unlikely circumstances to hold), and over short periods. Thus, the longer the time period considered, the greater would be the number of lives who buy insurance, and hence the larger is the pool of lives for spreading the cost of adverse selection. This suggests that "adverse selection with average sums assured might not, by itself, have a large effect on the ordinary class; the most costly aspect is likely to be higher than the average sums assured. This highlights the importance of limiting the sums assured which might be obtained without disclosure of known genetic information". Note that the above model of adverse selection is discussed within the context of modelling the ordinary rates class of business. In conclusion, Macdonald tentatively suggested that "if life insurance companies refrain from using (or are forbidden to use) the results of any genetic test in underwriting, additional mortality costs are likely to arise. However, if adverse selection does not extend to untypically large sums assured the magnitude of these costs is greatly reduced; large sums assured are the costliest aspect of adverse selection".

1.4.7 Jones's Model of Adverse Selection

Following Macdonald, Jones (1998) proposes a model that allows for population heterogeneity with respect to mortality in analysing the relationship between mortality rates and lapse rates. The force of mortality for an individual is assumed to be equal to the product of the frailty value and a function of age or time (this function can be thought of as the force of mortality of an individual with frailty one), so that those with higher frailty values have higher forces of mortality. It is worth mentioning that the application of frailty models to mortality has been extensively studied for many decades by several authors, including Beard (1959) and the important work of Vaupel et al (1979) which investigated the impact of heterogeneity in individual frailty on the dynamics of mortality.

Using the above frailty assumptions, Jones models anti-selective behaviour in lapsation by considering a cohort of insured lives of the same underwriting classification all of whom are issued policies at age a. Suppose that at issue, all insureds are in the healthy state. At any time thereafter an insured can move between the states as shown by the arrows in figure 1.3 which represents the appropriate multiple state model.

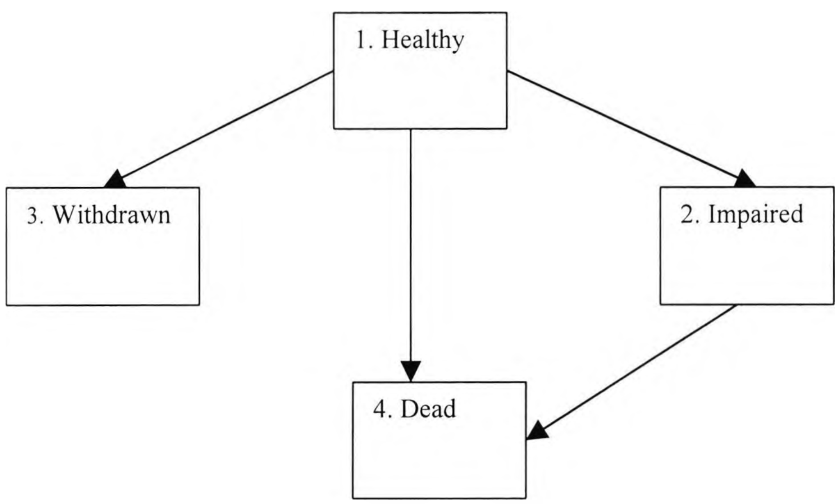

Figure 1.3: State transition diagram

Now, associated with each insured is a positive, continuous unobservable random variable Z, referred to as "risk level" of an insured. This allows us to reflect the heterogeneity in insured lives that exists. Let $\mu_{ij}(t,z)$ be the "force of transition" from state i to state j at time t after policy issue for an insured of risk level z.

The main features of the model are as follows:

i. In the originating state, a life is in the healthy state. From there an insured can lapse, thereby moving to the withdrawn state, or die, thereby moving to the dead state or get sick, thereby moving to the impaired state.

ii. If the insured moves to the impaired state before lapsing or dying, he/she is subject to a considerably elevated rate of mortality. Further, it is assumed that the insured is aware of this and therefore does not lapse while in this state. In other words, movement from the "impaired" state to the withdrawn state is not possible. Further, transition from the "impaired" state to "healthy" state (or recovery) is not allowed.

iii. Assume that $\mu_{24}(t,z)$ to be larger than $\mu_{14}(t,z)$, and $\mu_{23}(t,z)$=0 for all values of *t* and *z*.

Further assumed that $\mu_{14}(t,z)=z\mu_{14}(t)$, where $\mu_{14}(t)$ represents the force of mortality at time *t* for a healthy insured of risk level 1. Then insureds of other risk levels have forces of mortality proportional to $\mu_{14}(t)$. Similarly, assume that $\mu_{24}(t,z)=z\mu_{24}(t)$, $\mu_{12}(t,z)=z\mu_{12}(t)$ and $\mu_{13}(t,z)=z^{\gamma}\mu_{13}(t)$, $-\infty<\gamma<\infty$. The parameter γ allows us to specify the impact of the risk level on the force of lapsation while healthy. For example, if $\gamma=0$, then the force of lapsation is independent of the risk level. If $\gamma>0$, then insureds of a higher risk level are more likely to lapse than those of a lower risk level. On the hand if $\gamma<0$, then those of a higher risk level are less likely to lapse than those of a lower risk level.

Jones then assumed the form of the above functions to be:- $\mu_{14}(t)$ has a Gompertz form, given by $\mu_{14}(t)=B_1c_1^{35+t}$ (insured is age 35 at issue), where $B_1=0.00005$ and $c_1=1.075$. The values of B_1 and c_1 were chosen so that the resulting force is of an appropriate magnitude. Also, $\mu_{12}(t)=\mu_{14}(t)$ and $\mu_{24}(t)=10\mu_{14}(t)$. The force of lapsation for a standard individual has Makeham's form given by $\mu_{13}(t)=A_2+B_2c_2^t$, where $A_2=0.03$, $B_2=0.12$, and $c_2=0.7$ ($c_2<1$ in order to produce a function that decreases exponentially from 0.15 at $t=0$ towards 0.03 as *t* gets large). Finally, Jones assumed that the risk level *Z* has a gamma distribution with probability density function given by

$$f_z(z)=\frac{\alpha^{\alpha}}{\Gamma(\alpha)}z^{\alpha-1}e^{-\alpha z},\quad z>0.$$

Then *E[Z]=1* and the shape parameter, α, controls the degree of heterogeneity at policy issue. For example, with α=1, there is considerable heterogeneity. Using the above functions, Jones examined the behaviour of the cohort force of mortality by simply observing the ratio of this cohort (force of mortality) when Z has the above distribution at policy issue, to the Gompertz force, $\mu_{14}(t)$, for different values of γ (1,0, and −1) and for two choices of $\alpha\,(1\, and\, 4)$. Under these assumptions we can observe the effect of heterogeneity on the relationship between the risk level and the force of lapsation.

The following results were obtained when $\alpha = 1$.

i. When $\gamma = 1$, there is a higher force of lapsation associated with higher risk levels. So, the proportion of insureds in state 1 is lower at the higher risk levels and "this tends to elevate the cohort force of mortality".

ii. For $\gamma = 0$, we observe that the force of lapsation is independent of the risk level and is smaller relative to the force of mortality at higher risk levels. "This produces a larger proportion in state 1 at the higher risk levels, which tends to reduce the cohort force".

iii. Finally, for $\gamma = -1$, the force of lapsation is inversely related to the risk level. Like (ii) above, this produces a larger proportion in state 1 at the higher risk levels, which tends to reduce the cohort force of mortality.

With $\alpha = 4$, the relationship between the risk level and the force of lapsation is of smaller magnitude.

We note that the model of Jones (1998) does not allow for transitions from the impaired to the healthy state and nor from the impaired to the withdrawn state. But the transition from state 2 to 1 (sick to healthy state) can affect the lapse rate, mortality rate, and consequently, the profitability of the company and so it is important that we allow for such transitions in our model.

In chapter 4 of this thesis, we will employ the frailty models of Jones (1998) and the adverse selection model of Macdonald (1997) to model the expected surrender profit/loss where the effect of financial and non-financial adverse selection are incorporated. This means that our model will allow for transitions from the impaired to the healthy state and also, from the impaired to the withdrawn state. It is worth noting that allowing for transitions from sick to healthy will complicate the model and also, make the computation of the transition probabilities difficult. In view of this we will adopt an alternative approach suggested by Jones (1994) to deal with this problem. This is discussed in section 4.5.3 of the thesis.

From the models described in this section we have seen that Norberg (1988) models the temporary initial selection effect where there is no possibility of a transition from the 'uninsurable' state into the 'insurable state' and so, adverse selection does not exist. However, Macdonald (1997) introduced a simple Markov model to measure the combined effect of underwriting and adverse selection among heterogeneous populations by assuming that insurers do not behave in the way described in the Norberg model. Also, the Macdonald model illustrates the possible extent of the costs of adverse selection in the context of genetic tests. Finally, Jones (1998) model allows for population heterogeneity with respect to mortality in analysing the relationship between mortality rates and lapse rates.

1.5 Optimisation Techniques and Lapse Rate

As mentioned already, this thesis will apply optimisation techniques to minimise a life office's expected losses due to excessive surrender. In other words, we will apply optimisation techniques (specifically, numerical optimisation) to maximise the expected utility of shareholders. There are many types of numerical optimisation routines. The appropriate one to use depends on the type of objective function, i.e. whether it is linear or non-linear, whether the problem is constrained or unconstrained, whether first and possibly second derivatives for the objective function are available, etc. Several types of optimisation routines have been suggested to tackle the type of classification under investigation. For example, Powell's method is used for an objective function that is non-linear and whose first derivative cannot be computed. Also, conjugate gradient and quasi-Newton methods are appropriate if the first derivatives can be computed.

In our context, an unconstrained non-linear optimisation without derivatives will be appropriate. Particularly, Powell's method described in Press et al (1992) and Conn (1997). We will discuss these optimisation techniques in detail and suggest why this is appropriate in the subsequent chapters. Various results using this technique have been discussed by Powell (1964), Box (1965), Fletcher (1965), Powell et al (1975), Callier (1977 and 1978), Press et al (1992), and Conn (1997).

Powell (1964) has described a method of solving a non-linear unconstrained minimization problem based on the use of conjugate directions. The main idea of this proposal is that the minimum of a positive-definite quadratic form can be found by performing at most n (number of variables) successive line searches along mutually conjugate directions. The same procedure may be applied to non-quadratic functions, by adding a new composite direction at the end of each cycle of n line searches. This algorithm has enjoyed a lot of interest amongst many researchers and practitioners. We will discuss this method again and its application to our work in Chapter 5 of this thesis.

Chapter 2

Effect of Life Insurance Payouts on Lapse Rate

2.1 Introduction

This chapter of the thesis looks at the effect of life insurance payouts on lapse rates in the U.K. This is to investigate whether surrendering policyholders are better off than those who maintain their policies until maturity. Further, we will investigate if payouts are a determining factor for lapsing. This information will be used to determine the profitability of a company in the subsequent chapters of the thesis. The motivation for carrying out the above investigations is from Outreville (1990) on testing the emergency fund hypothesis (originally proposed by Linton (1932)). The emergency fund hypothesis, according to Outreville (1990), is that "cash values are utilized by policyholders as an emergency fund to be drawn upon in times of personal financial crisis". Then lapses should increase during recessions because some policyholders are unable to maintain premium payments for insurance coverage. His results provide evidence in favour of this hypothesis. On this note, one can expect a higher lapse rate if higher payouts are paid on surrender. What follows is a description and discussion of our investigations performed either to support or refute this claim. In this context we look at the effect of payouts (surrender and maturity values) and relative payouts (payouts relative to average market values) on life assurance lapse rates in U.K. These are areas of major concern for life insurers since high early terminations have frequently resulted in heavy losses to many companies and are one of the major causes of dissatisfaction among policyholders within the life business, Richardson and Hartwell (1951). Also explored is the use of a statistical based approach to determine the relationship if any, between surrender and maturity payouts, as well as between lapse rates and relative payouts.

These investigations are carried out partly by statistical modelling and partly, through empirical analysis. This chapter is divided into two sections.

The first section discusses the relationship between maturity and surrender values offered by the U.K. life insurance companies. This part is to investigate whether surrendering policyholders are better or worse off than those who maintain their policies until maturity. A rank correlation analysis is used in this regard. Other variables like the return on assets are also considered in the analysis. This is important in measuring the performance of a company's investment portfolio. We have included this variable in order to establish whether or not companies with high return from their investment portfolio do offer high benefit pay out.

The second section discusses the effect of relative payouts on lapse rates. In this context, we will investigate the effect of maturity and surrender values, relative to the average market payouts, on lapse rates. Here too, a rank correlation analysis is performed to access any relationship that might exist between lapse rate and relative payouts. Also, graphs of the ratio of *SV:MV* over time are plotted to investigate whether or not companies are paying out more surrender values at the expense of maturity values or vice versa. The next section presents the empirical hypotheses, the methodology used for the analysis, the results and the discussion of our findings.

2.2 Empirical Hypotheses

Life Insurance lapse rates are hypothesized to be related to several factors discussed in chapter 1. Hypothesis testing clearly sets out what we expect to observe or happen from this study and so it is important that we mention the hypothesis to be tested before we discuss the methodology used to perform the statistical analysis. In what follows, we set out the initial hypotheses (what we intuitively expect to exist) that were tested in this study and also discuss how the variables such as surrender and maturity values, and yield on assets, are expected to relate to lapse rate. Notice that it is unlikely that all of the hypotheses will be true.

1. Surrender and Maturity Values

Surrender values are cash payment to policyholders on termination of their existing policies before maturity. In this analysis, surrender values are defined by year of payment and current values provided. Here, we hypothesize that policies with higher surrender values (denoted by *SV*) tend to have higher maturity values (denoted by *MV*) than the ones with lower surrender values, assuming both contracts are expected to cover the same duration before surrender. Specifically, we compare the surrender value of an m-year endowment that has been in force for n years (denoted by $m|n\ SV$) with the maturity value of an n-year policy (denoted by n-year *MV*). m in this case is equal to 15 and 25 and n = 5, and 10 years. In other words, surrendering policyholders are not better off than continuing policyholders since both policyholders will receive some amount of benefit either at maturity or on termination of the contract before maturity. Therefore, we expect a positive correlation between surrender and maturity values.

2. Maturity Values relative to average market maturity value and Lapse rates

Policyholders are always looking for companies that offer higher maturity benefit than the overall average maturity value offered on the market for the same policy type. On this note, we hypothesize that companies that issue policies with lower maturity values relative to the average market maturity value are expected to have a higher lapse rate. In other words, companies that offer high maturity values relative to the average market maturity value tend to have low lapse rates. So, we expect a negative correlation between maturity values (relative to average market value) and lapse rates.

3. Surrender Values relative to average market surrender value and Lapse rates

We anticipate that companies that issue policies with surrender values higher than the average market surrender value are likely to experience higher lapse rate. This is due to the fact that policyholders normally will utilize this offer and tend to withdraw money in times of personal financial crisis. Therefore, we expect a positive correlation between surrender values (relative to average market value) and lapse rates.

4. Yield on Assets and payouts

Higher investment returns facilitate insurers in meeting their obligations to policyholders. Here, we anticipate that companies with higher returns are likely to pay out higher benefits (*SV* and *MV*) than those with lower returns. Hence, we expect a positive correlation between the yield on assets and payouts (*MV* and *SV*).

5. Ratio of *SV* to *MV*

The ratio *SV/MV* is a measure of determining whether or not companies are paying out more in terms of surrender values than corresponding maturity values or vice versa. In this case we compared $m|n$ *SV* with n-year *MV*, where $m = 15$ and 25 and $n = 10$. Thus, we hypothesize that companies that experience a high ratio over time are more likely to pay out a higher surrender value relative to the corresponding maturity value.

2.3 Data and Sample Characteristics

2.3.1 Data

Data on variables such as surrender and maturity values, return on assets, and relative payouts (i.e., surrender and maturity values relative to average market values), were sought in order to investigate the effect of these variables if any, on lapse rates. The source of data used is "Planned Savings" and the period of magazine issues was from 1986 to 1994. Note that 1985 data for most companies was missing and so, we did not consider it in the analysis. However, we did use it to illustrate a point in the numerical examples of section 2.4.3 and 2.5.1 (since 1985 data on Scottish Mutual was complete). The total sample consists of 25 life assurance companies with the selected policy types being 25, 15 and 10 year with profit endowment. The 25 companies were chosen because they appeared in almost all of the data sources under investigation and provided complete information on the variables of interest for this research . Further, data on surrender values consisted of values corresponding to the number of years the policy had run prior to surrender. For example, we considered for the case of a 25-year endowment policy, the surrender values corresponding to a policy that had run for 20 years, 10 years and 5 years since the inception of the policy. Similarly, for a 15-year endowment, we considered the surrender values corresponding to a policy that had run for 10 years and 5 years since the inception of the policy. These payouts were available for a gross annual premium of £100 to a male aged 29 years, 11 months at outset. Note that age at entry was the same throughout. The data were merged with historical data from the Department of Trade and Industry (DTI) returns of life insurance companies for the analysis. Data on new business issued during the year, total surrenders, and the total premium value in force at the start of year in question, for the 25 life offices were also obtained from the (DTI) returns database compiled by "Thesys Information limited". It is worth mentioning that information used to determine the gross return on assets and the adjusted lapse rate were obtained from Forms 9, 40, 43 and, 56 of the DTI Returns. Further note that lapse rates were measured by "premium" terminations. This is due to the fact that systems dependent on policy numbers are unreliable as a measure of termination rates. The reason for this is the "practice known as clustering- whereby a policy is given several policy numbers", SIB (1988).

2.3.2 Limitations of the Data

As mentioned already, data on 25 companies was available for the analysis. We note that not all of these companies appear in all the surveys, thereby disrupting the trend in payouts and lapse rates.

2.3.3 Sample Characteristics

Summary statistics of all the variables under investigation were computed and shown in Appendix 2.1. Figures 2.1-2.6 show the plots of the mean, median, first and third quartile of payouts (*MVs* and *SVs*) over time. Note that for this sample, $m|n$ *SV* is as defined in section 2.2. i.e., the surrender value at duration n of an m-endowment policy.

Figure 2.1 shows a plot of the average *MV* of a 10-year endowment paid over years, 1986-1994. Also shown are the median and lower and upper quartile of *MVs*. From figure 2.1, we observe that the average *MVs* of a 10-year endowment of all companies under investigation were quite stable over the entire period of 1986-1990, usually between £2000 and £2200. However, between 1991 and 1994, most of the companies were offering lower maturity values, as revealed by the plot. The highest average *MV* occurs in 1990 (a value of £2200) whereas the lowest average *MV* occurs in 1994 with a value of £1800.

Also, figure 2.2 shows a plot of the average *MV* of a 25-year endowment paid over years, 1986-1994. The median, lower and upper quartile of the *MVs* are shown as well. From figure 2.2, we observe that the mean *MV* a 25-year endowment policy increases during the period of 1986-1992, usually between £10,000 and £16,000. This time, it became quite stable after 1992 (see figure 2.2). The highest average *MV* occurs in 1992 (a value of £16,197) whereas the lowest average *MV* occurs in 1986 with a value of £10,905.

Further, figure 2.3 and 2.4 shows a plot of average *SV* of a 15-year and 25-year endowment at duration 10years, paid over years, 1986-1994. Also shown are the median, lower and upper quartile of the *SVs*. From figure 2.3, observe that the mean surrender values, *SVs*, of a 15-year endowment policies, surrendered after 10 years, were quite stable over the entire years under investigation. With this policy, the highest paid average SV occurred in 1990 with a value of £1686 while the lowest average SV, in 1994 and the value was £1542 (see figure 2.3).

Also, the mean *SV* of 25-year endowment policies, surrendered after 10 years was quite stable and consistent over the entire years of 1986-1994. The highest paid average *SV* occurred in 1990 with a value of £1430 and the lowest average value of £1289, in 1994 (see figure 2.4).

Finally, figure 2.5 and 2.6 compare the mean *SV* of a 15-year and 25-year endowment at duration 10 with *MV* for a 10 year policy made over the years, 1986-1994. From figure 2.5 and 2.6, we observe that companies were offering relatively lower *SVs* for long term policies (e.g., 25-year endowment) than short term policies (e.g., 15 year policy). This is due to the fact that the *SV* tend to blend towards the *MV* as duration increases. For example, a 15-year endowment surrendered after 10 years is closer to maturity than a 25 year, surrendered after 10 years.

It is worth noting that in figure 2.5 and 2.6, more *MVs* were paid than *SVs* over the entire period of 1986-1994 for the 15 and 25-year policies, indicating that low amounts are usually paid out on surrender.

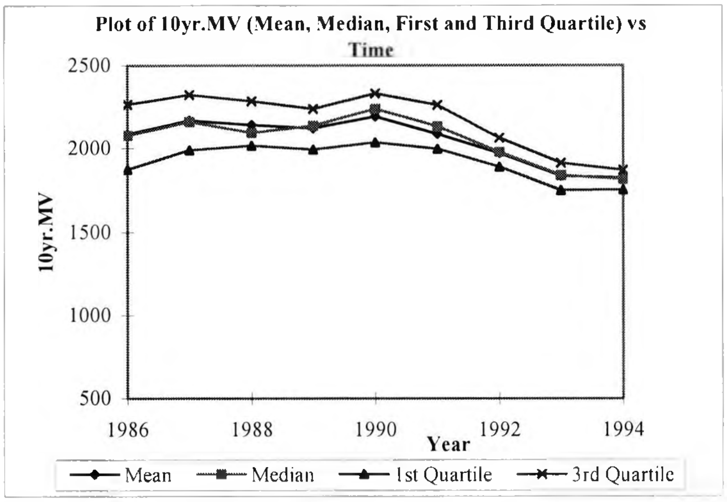

Figure 2.1: Plot of Mean, Median, First and Third Quartile of 10 yr. MV over time

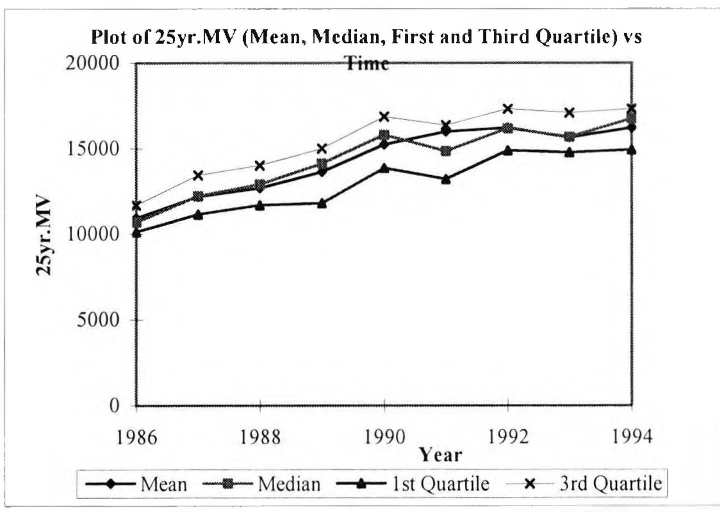

Figure 2.2: Plot of mean, Median, First and Third Quartile of 25 yr. MV over time

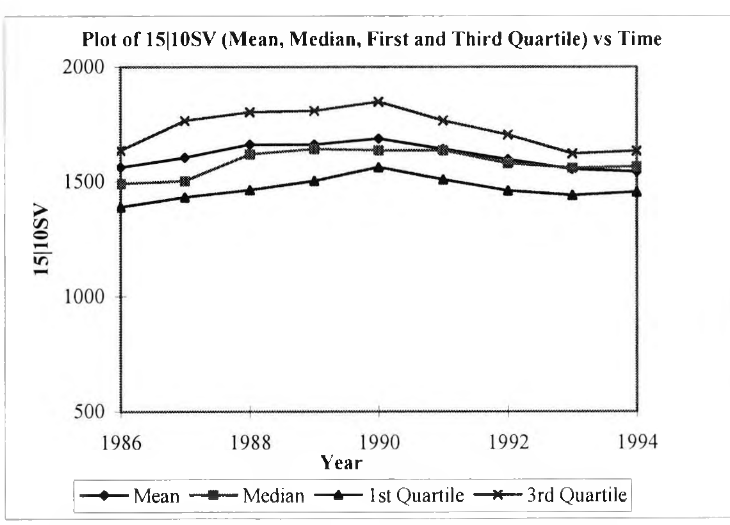

Figure 2.3: Plot of mean, Median, First and Third Quartile of 15|10SV over time

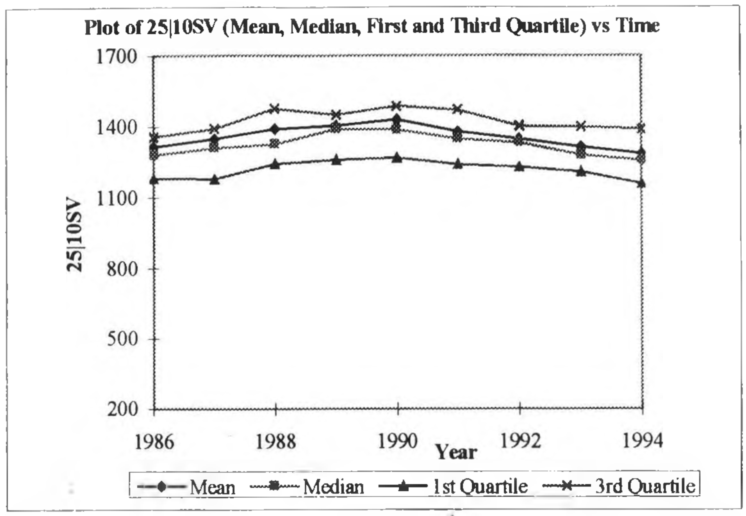

Figure 2.4: Plot of mean, Median, First and Third Quartile of 25|10SV over time.

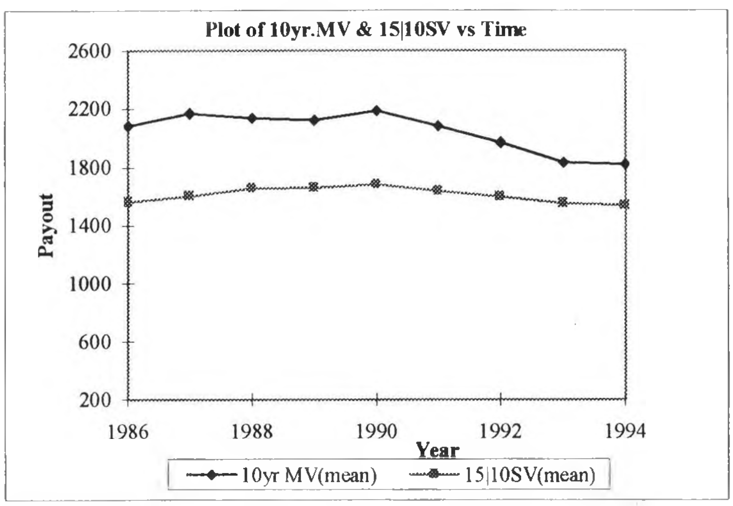

Figure 2.5: Plot of mean 10yr MV & of 15|10SV over time.

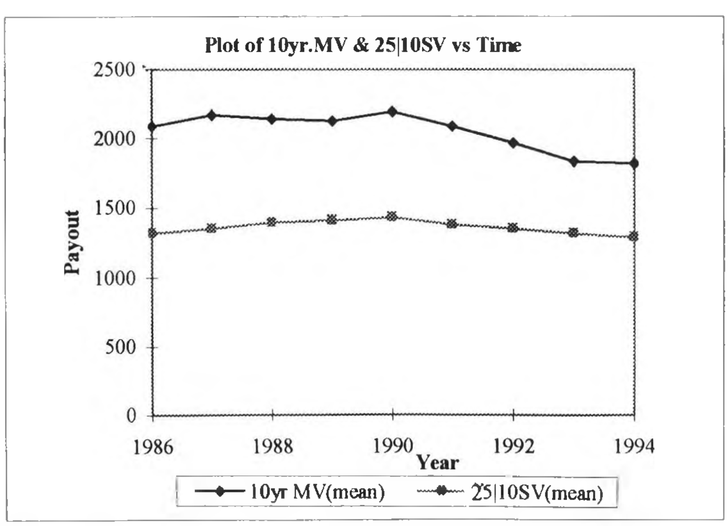

Figure 2.6: Plot of mean 10yr MV & of 25|10SV over time.

2.4 Determination of Adjusted Lapse Rate and Yield on Assets

2.4.1 Lapse Rate

The calculation of a lapse rate as reported in the literature mostly uses the ratio of insurance going off the books due to termination of a policy by the insured (with or without surrender values) to life insurance in force. For example, Hayward (1959) and Best's Insurance Management Report (1991) measured lapse rate by simply dividing the number of such lapses in a calendar year by the number of policies taken up in that year (volume of new business). Further, there are different kinds of approach that have been used to model lapse rate. Examples of such kinds are given in chapter 1. In these examples, the approach used does not take into account the number of policies exposed to the risk of lapsing from the previous calendar years. But this is an important factor that could have an effect on the results of any lapse analysis.

Therefore, our proposed model takes this factor into account and modifies the crude lapse rate to incorporate the number of policies exposed to the risk of lapsing in the year leading up to the *rth* policy anniversary (=curtate duration *r-1*). Note that the exposed to risk is defined as the number of those policies in force that could terminate within a 12 months period. For example, we calculate the exposed to risk of lapsing in the year leading up to the first policy anniversary (=curtate duration zero) in calendar year *y,* by using new business figures in the current and previous years. The information used in deriving the adjusted lapse rate can be obtained from the *DTI* returns. As mentioned in section 2.3, we measure lapse rate by "premium" termination since systems dependent on policy numbers are unreliable as a measure of termination rates. We derive a model for lapse rate as follows:

2.4.2 *The Lapse Model*

In this model, we consider a lapse rate in year y and define:

$X_1[y]$ = Year 1 lapse rate in calendar year y (Lapse rate in the first policy year i.e. between duration zero and one).

$X_2[y]$ = Year 2 lapse rate in calendar year y (Lapse rate in the second policy year i.e. between duration one and two).

$X_3[y]$ = Year 3 lapse rate in calendar year y (Lapse rate in the third policy year i.e. between duration two and three).

$E_1[y]$ = Exposed to risk of lapsing in year leading up to first policy anniversary (=curtate duration zero) in calendar year y.

$E_2[y]$ = Exposed to risk of lapsing in year leading up to second policy anniversary (=curtate duration one) in calendar year y.

$E_3[y]$ = Exposed to risk of lapsing in year leading up to third policy anniversary (=curtate duration two) in calendar year y.

Now, we define the number of crude lapses in a given year y as :

$$CL[y] = X_1[y] \cdot E_1[y] + X_2[y] \cdot E_2[y] + X_3[y] \cdot E_3[y] \qquad 2.1$$

Further, we express year 2 and year 3 lapse rate in terms of year one's lapse rate for simplicity, and assume that lapse rate in year two is one half year 1 lapse rate and that of year 3 is ¼ year 1 lapse rate (deduced from the empirical work of Butler (1994)).
This implies that:

$$X_2[y] = \frac{1}{2} \cdot X_1[y] \text{ for all } y;$$

$$X_3[y] = \frac{1}{4} \cdot X_1[y] \text{ for all } y; \qquad 2.2$$

Then again, we define :

$B[y,z]$= the business in force at start of year y, issued in year y-z, and

$NB[y]$= New business issued during year y.

If we assume that new business are issued uniformly over the year y, then, we obtain :

$$B[y,1] = NB[y-1] \cdot \left(1 - \frac{1}{2} \cdot X_1[y-1]\right) \qquad 2.3$$

$$B[y,2] = NB[y-2] \cdot \left(1 - \frac{1}{2} \cdot X_1[y-2]\right) \cdot \left(1 - \frac{1}{2} \cdot X_1[y-1]\right) \cdot \left(1 - \frac{1}{2} \cdot X_2[y-1]\right) \qquad 2.4$$

Now,

$$E_1[y] = \frac{1}{2} \cdot NB[y] + \frac{1}{2} \cdot B[y,1] \qquad 2.5$$

$$E_2[y] = \frac{1}{2} \cdot B[y,1] \cdot \left(1 - \frac{1}{2} \cdot X_1[y]\right) + \frac{1}{2} \cdot B[y,2] \qquad 2.6$$

$$E_3[y] = F[y] - E_1[y] - E_2[y] \qquad 2.7$$

where,

$F[y]$ = total premium in force at start of year y.

By substituting equations (2.2), (2.3) and (2.4) in equations 2.5-2.7, we obtain the following equations:

$$E_1[y] = \frac{1}{2} \cdot NB[y] + \frac{1}{2} \cdot NB[y-1] \cdot \left(1 - \frac{1}{2} \cdot X_1[y-1]\right) \qquad 2.5a$$

$$E_2[y] = \frac{1}{2} \cdot NB[y-1] \cdot \left(1 - \frac{1}{2} \cdot X_1[y-1]\right) \cdot \left(1 - \frac{1}{2} \cdot X_1[y]\right) +$$
$$\frac{1}{2} \cdot NB[y-2] \cdot \left(1 - \frac{1}{2} \cdot X_1[y-2]\right) \cdot \left(1 - \frac{1}{2} \cdot X_1[y-1]\right) \cdot \left(1 - \frac{1}{4} \cdot X_1[y-1]\right) \qquad 2.6a$$

$$E_3[y] = F[y] - E_1[y] - E_2[y] \qquad 2.7a$$

By putting equations 2.5a-2.7a in equation (2.1), we obtain the following quadratic equation:

$$(X_1[y])^2 \cdot \left\{\left(\frac{1}{16} \cdot NB[y-1]\right) \cdot \left(1-\frac{1}{2} \cdot X_1[y-1]\right)\right\} -$$

$$X_1[y] \cdot \left\{\begin{array}{l} \frac{3}{8} \cdot NB[y]+\frac{1}{2} \cdot NB[y-1] \cdot \left(1-\frac{1}{2} \cdot X_1[y-1]\right)+\frac{1}{4} \cdot F[y]+\frac{1}{8} \cdot NB[y-2] \cdot \\ \left(1-\frac{1}{2} \cdot X_1[y-2]\right) \cdot \left(1-\frac{1}{2} \cdot X_1[y-1]\right) \cdot \left(1-\frac{1}{4} \cdot X_1[y-1]\right) \end{array}\right\} +$$

$$CL[y] = 0$$

2.8

which by solving for $X_1[y]$ yielded

$$X_1[y] = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2a} \qquad 2.9$$

where

$$b = \left\{\begin{array}{l} \frac{3}{8} \cdot NB[y]+\frac{1}{2} \cdot NB[y-1] \cdot \left(1-\frac{1}{2} \cdot X_1[y-1]\right)+\frac{1}{4} \cdot F[y]+\frac{1}{8} \cdot NB[y-2] \cdot \\ \left(1-\frac{1}{2} \cdot X_1[y-2]\right) \cdot \left(1-\frac{1}{2} \cdot X_1[y-1]\right) \cdot \left(1-\frac{1}{4} \cdot X_1[y-1]\right) \end{array}\right\}$$

$$a = \left\{\left(\frac{1}{16} \cdot NB[y-1]\right) \cdot \left(1-\frac{1}{2} \cdot X_1[y-1]\right)\right\}$$

$$c = CL[y]$$

The items in the above equation can readily be found in Forms 43 and 56 of the *DTI* returns, with the exception of $X_1[y-1]$ and $X_1[y-2]$ (the previous year and a year immediately prior to previous year lapse rate) which cannot be determined directly from the information available. As a result of this we set the initial values of $X_1[y-1]$ and $X_1[y-2]$ to zero.

Therefore, equation (2.9) then becomes :

$$X_1[y] = \frac{\left(\frac{3}{8} \cdot NB[y]+\frac{1}{2} \cdot NB[y-1]+\frac{1}{4} \cdot F[y]+\frac{1}{8} \cdot NB[y-2]\right) \pm \left\{\left(\frac{3}{8} \cdot NB[y]+\frac{1}{2} \cdot NB[y-1]+\frac{1}{4} \cdot F[y]+\frac{1}{8} \cdot NB[y-2]\right)^2 - \frac{1}{4} \cdot NB[y] \cdot CL[y]\right\}^{\frac{1}{2}}}{\frac{1}{8} NB[y-1]}$$

2.10

From this equation, we can calculate $X_1[y]$ for all companies. Hence, $X_2[y]$ and $X_3[y]$ can be found easily from (2.2). In this case, since the available information obtained from the *DTI* returns is from 1985-1994, coupled with the fact that we had to set the initial values of the previous two years' lapse rate to zero for a particular calendar year, it was appropriate that the year y = 1987 be used. The next step is then to calculate X_1 [88] by using X_1 [87] and (X_1 [86]=0) as the new initial values. This process is repeated for 1989, 1990,.......,1994. The average value of all the X_i [y] values (where y is from1987-1994) for a particular company can then be used as our initial values for X_i [86] and X_i [85]. We then repeat the above process until no further change to X_1 [y] ensues (i.e. converges). Subsequently, we can follow the same procedure as outlined above to find the values of $X_i[88]$, $X_i[89]$,..... $X_i[94]$ for *i=1,2 and 3.*

2.4.3 Numerical Example

To illustrate the process described in section 2.4.2, we used the information given in the *DTI* Returns of Scottish Mutual Insurance Company for our calculations. Notice that we are using the year 1987 as the base year for reasons given in section 2.4.2. Thus, we desire to calculate the lapse rates for $X_i[87]$, for *i=1,2 and 3*. Subsequently, we can follow the same procedure as outlined in section 2.4.2 to find the values of $X_i[88]$, $X_i[89]$,..... $X_i[94]$ for *i=1,2 and 3*. In this case, we used data on new business, total surrenders, and total premium in force for the years, 1985,1986, and 1987 in computing year 1, 2, and 3+ adjusted lapse rate. We obtained the adjusted lapse rate for this company as follows:

Given :

NB[85] = 6160; NB[86] = 7536; NB[87] = 7467

Total Surrender in 1987, CL[87] = 2019

Total premium, F[87] = 45754

We calculate $X_1[87]$ as,

$$X_1[87]=\frac{\left(\frac{3}{8}\cdot NB[87]+\frac{1}{2}\cdot NB[86]+\frac{1}{4}\cdot F[87]+\frac{1}{8}\cdot NB[85]\right)\pm\left\{\left(\frac{3}{8}\cdot NB[87]+\frac{1}{2}\cdot NB[86]+\frac{1}{4}\cdot F[87]+\frac{1}{8}\cdot NB[85]\right)^2-\frac{1}{4}\cdot NB[87]\cdot CL[87]\right\}^{\frac{1}{2}}}{\frac{1}{8}NB[86]}$$

Thus,

$$X_1[87]=\frac{\left(\frac{3}{8}\cdot[7467]+\frac{1}{2}\cdot[7536]+\frac{1}{4}\cdot[45754]+\frac{1}{8}\cdot[6160]\right)\pm\left\{\left(\frac{3}{8}\cdot[7467]+\frac{1}{2}\cdot[7536]+\frac{1}{4}\cdot[45754]+\frac{1}{8}\cdot[6160]\right)^2-\frac{1}{4}\cdot[7467]\cdot[2019]\right\}^{\frac{1}{2}}}{\frac{1}{8}[7536]}$$

Hence,

$$X_1[87]=\frac{18776.625\pm 18675.99}{942}=0.1068.$$

Therefore, $X_1[87]= 0.1068$; $X_2[87]= 0.0534$; and $X_3[87]= 0.0267$.

Hence, the adjusted lapse rate for year 1 is 10.68%, for year 2 is 5.34% and for year 3+ is 2.67%.

2.5 Calculation of Gross Return on Assets

Having derived the adjusted lapse rate used in this analysis, we attempt to model the return on company's asset. This is also an important variable considered in our analysis. We model this return as follows.

Suppose that the values of a company asset portfolio at the beginning and end of the year are respectively A_{t-1} and A_t. Further, assume that claims occur on average, in the middle of the year. Then, the investment return on this portfolio can be expressed as

$$A_t = A_{t-1}(1+i_t)+(P_t - E_t - C_t)\times(1+i_t/2)-T \qquad 2.11$$

This implies,

$$i_t = \frac{(A_t + T) - [A_{t-1} + (P_t - E_t - C_t)]}{A_{t-1} + (P_t - E_t - C_t)/2} \qquad 2.12$$

where,

P_t = Premium received at start of year t.

E_t = Expenses incurred in year t.

C_t = Claims in policy year t.

T = Taxation.

i_t = gross return at time *t*.

From equation 2.12, we can determine the gross return on a company's investment at time *t*. However, for any particular company, we are interested in calculating the average yield over several years. This is to measure the investment performance of the company assets portfolio (office fund) over that period. This is described by McCutcheon and Scott (1986) as follows:

Suppose the period in question is from time t_0 to time t_n (t is measured in years) and that

$$t_0 < t_1 < t_2 < \dots\dots\dots < t_n$$

Then, the overall period $[t_0, t_n]$ can be divided into specified subintervals, $[t_0, t_1]$, $[t_1, t_2]$, ...$[t_{n-1}, t_n]$. Therefore, the average yield, I, for company j over the defined interval is given by

$$I_j = \{(1+i_1)^{t_1-t_0}(1+i_2)^{t_2-t_1}\dots\dots\dots\dots(1+i_n)^{t_n-t_{n-1}}\}^{\frac{1}{t_n-t_0}} - 1, \qquad j=1,2,\dots\dots\dots 25 \qquad 2.13$$

This equation can be used to determine the average return on the company's assets. All items in the above equation are readily available from the *DTI* returns (form 9 and 40). So, we can determine the average return for each office from 1986-1994 by first computing the gross return on assets from each year (i.e, 1985-1994). Then by using equation 2.13, we can calculate the average returns for company j, $j=1,2,\dots\dots,25$.

2.5.1 Numerical Example

In this example, we used information given in the *DTI* Returns of Scottish Mutual life Insurance Company for our calculations. In computing the return on assets we choose the year 1986. Subsequently, we can compute the returns for other calendar years using the same approach as 1986. We chose 1986 because of the fact that we had to consider the company asset portfolio at the beginning and end of the year in order to derive the return on asset. But since the available data is from 1985 onward, it was appropriate to use the year 1986. Now, to calculate the return on asset we use the following information which are given below :

P_{86} = 271,494	T_{86} =-763
C_{86} = 89,863	A_{85} = 894,572
E_{86} = 40,635	A_{86} =1,259,427

We obtained

$$i_{86} = \frac{(1{,}259{,}427 - 763) - [894{,}572 + (271{,}494 - 40{,}635 - 89{,}863)]}{[894{,}572 + (271{,}494 - 40{,}635 - 89{,}863) / 2]}$$

$= 0.2312$

By using the above procedure we obtain the following gross returns for the other years, 1987-1994:

$i_{87} = 0.00594$	$i_{90} = -0.04306$	$i_{93} = 0.2931$
$i_{88} = 0.1125$	$i_{91} = 0.1759$	$i_{94} = -0.0720$
$i_{89} = 0.1584$	$i_{92} = 0.2943$	

Hence, by using equation (2.13), we obtain

$I_1 = 0.1207$

Thus, the average return on Scottish Mutual assets from 1986-1994 is 12.07%.

The approach outlined in this example can be used to calculate the average returns on assets of the remaining companies used in this analysis.

2.6 Descriptive Analysis and Scatter Plots

2.6.1 Scatter Plots

Scatter plots were initially plotted to determine if any apparent correlation exists between lapse rate and the variables, *SV*, *MV*, yield on assets (denoted by *YA*), $X(i,t)$, $R(i,t)$, $R'(i,t)$ and $R''(i,t)$. We define these variables $X(i,t)$, $R(i,t)$, $R'(i,t)$ and $R''(i,t)$ as follows:

$$R(i,t) = \frac{MV(i,t)}{\overline{MV}(t)}$$

$$X(i,t) = \text{lapse rates of company } i \text{ in year } t$$

$$R'(i,t) = \frac{15|10SV(i,t)}{15|10\overline{SV}(t)}$$

$$R''(i,t) = \frac{25|10SV(i,t)}{25|10\overline{SV}(t)}$$

where

$M(i,t)$ = *MV* of company i at time t, $i = 1,2....25$.

$\overline{MV}(t)$ = mean *MV* of all companies at time t.

$15|10\ SV(i,t)$ = *SV* after 10 years of a 15-year endowment policy of company i at time t.

$15|10\overline{SV}(t)$ = mean *SV* of all companies at time t (*SV* of a 15-year endowment at duration 10)

$25|10SV(i,t)$ = *SV* at duration 10 years of a 25-year endowment policy of company i at time t.

$25|10\overline{SV}(t)$ = mean *SV* of all companies at time t (*SV* of a 25-year endowment at duration 10).

Thus, $R(i,t)$ is the ratio of the individual companies *MV* to the average market *MV* of all companies. Further, $R'(i,t)$ and $R''(i,t)$ are respectively, the ratio of $m|n$ *SV* and $m'|n$ *SV* to the corresponding average market surrender value of all companies, where $m = 15$, $m' = 25$ and $n = 10$ years in this case. The variable $R(i,t)$, measures the relative maturity payout offered by different companies against the average market value, whereas $R'(i,t)$ and $R''(i,t)$ measure the relative surrender payout offered by different companies against the average market value.

Figures 2.7-2.9 show the plots of n-year MV against $m|n\ SV$ of different companies in each calendar year. We also show in Figures 2.10-2.12, the plots of lapse rates against the variables $R(i,t)$, $R'(i,t)$ and $R''(i,t)$ of all the companies under investigation for the calendar year 1986-1994. Any significant trend (either positively or negatively) would indicate a stronger correlation between the variables under consideration or investigation. Consequently, further statistical tests, particularly a rank correlation analysis will be performed to confirm any significant correlation.

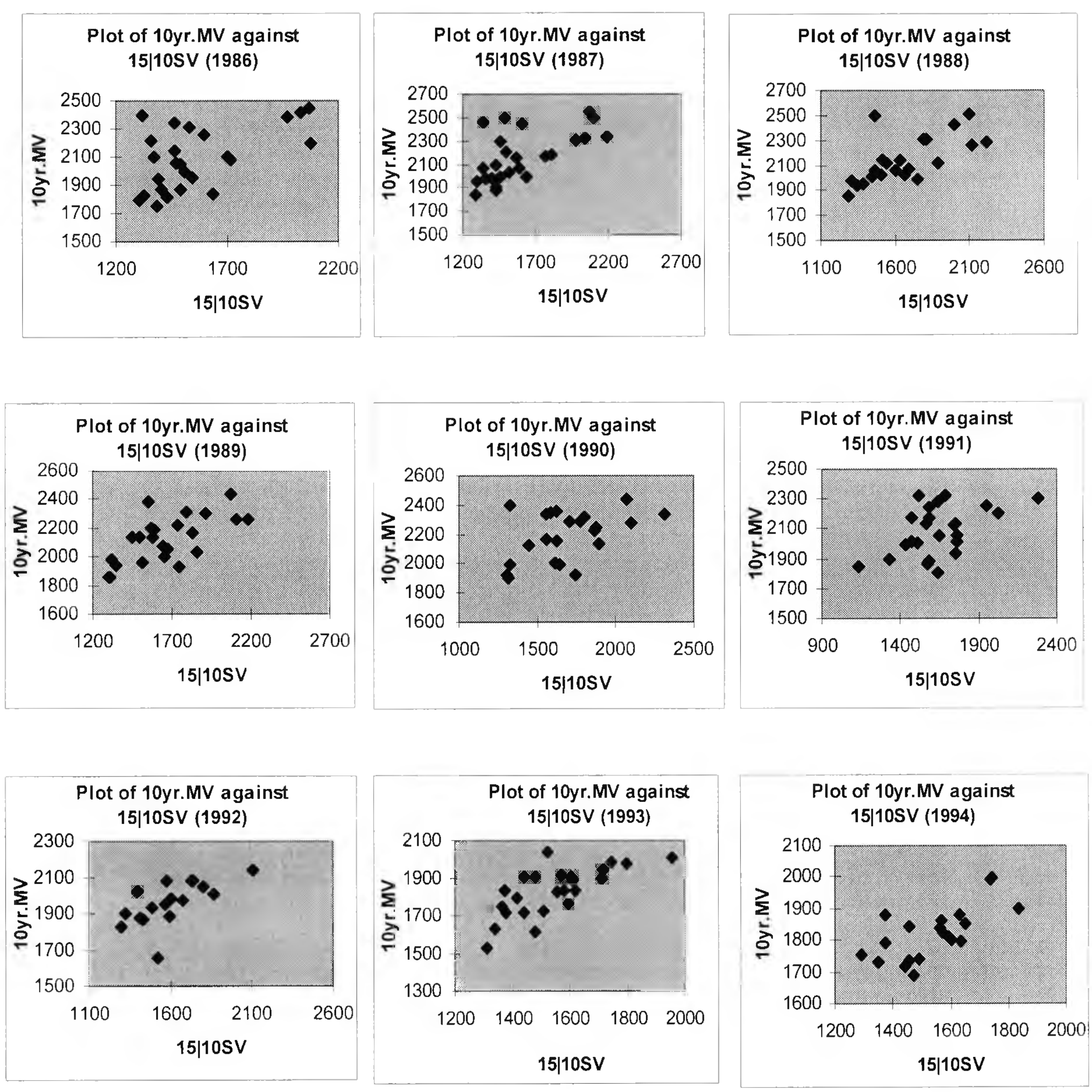

Figure 2.7: Plot of 10year MV against 15|10 SV

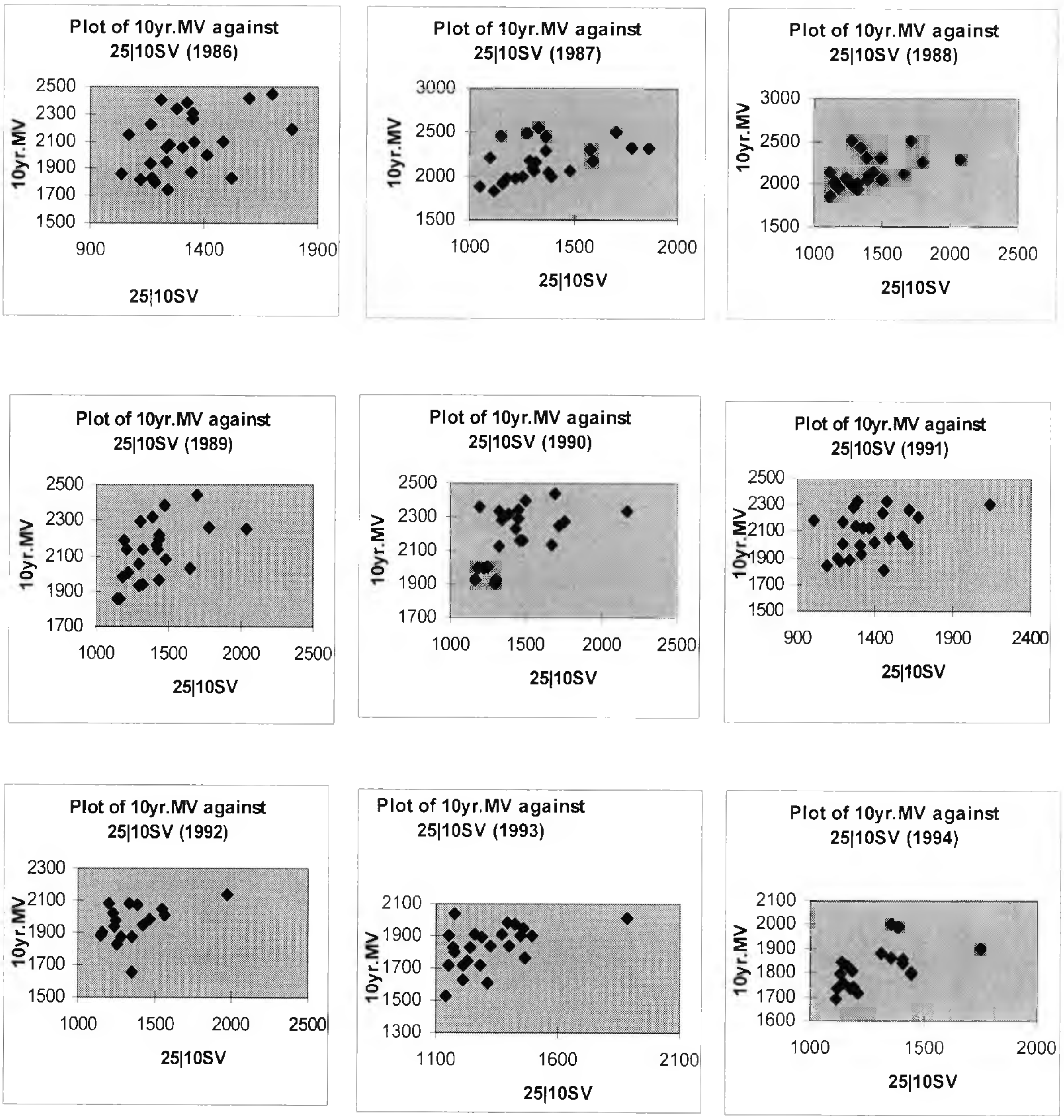

Figure 2.8: Plot of 10year MV against 25|10 SV

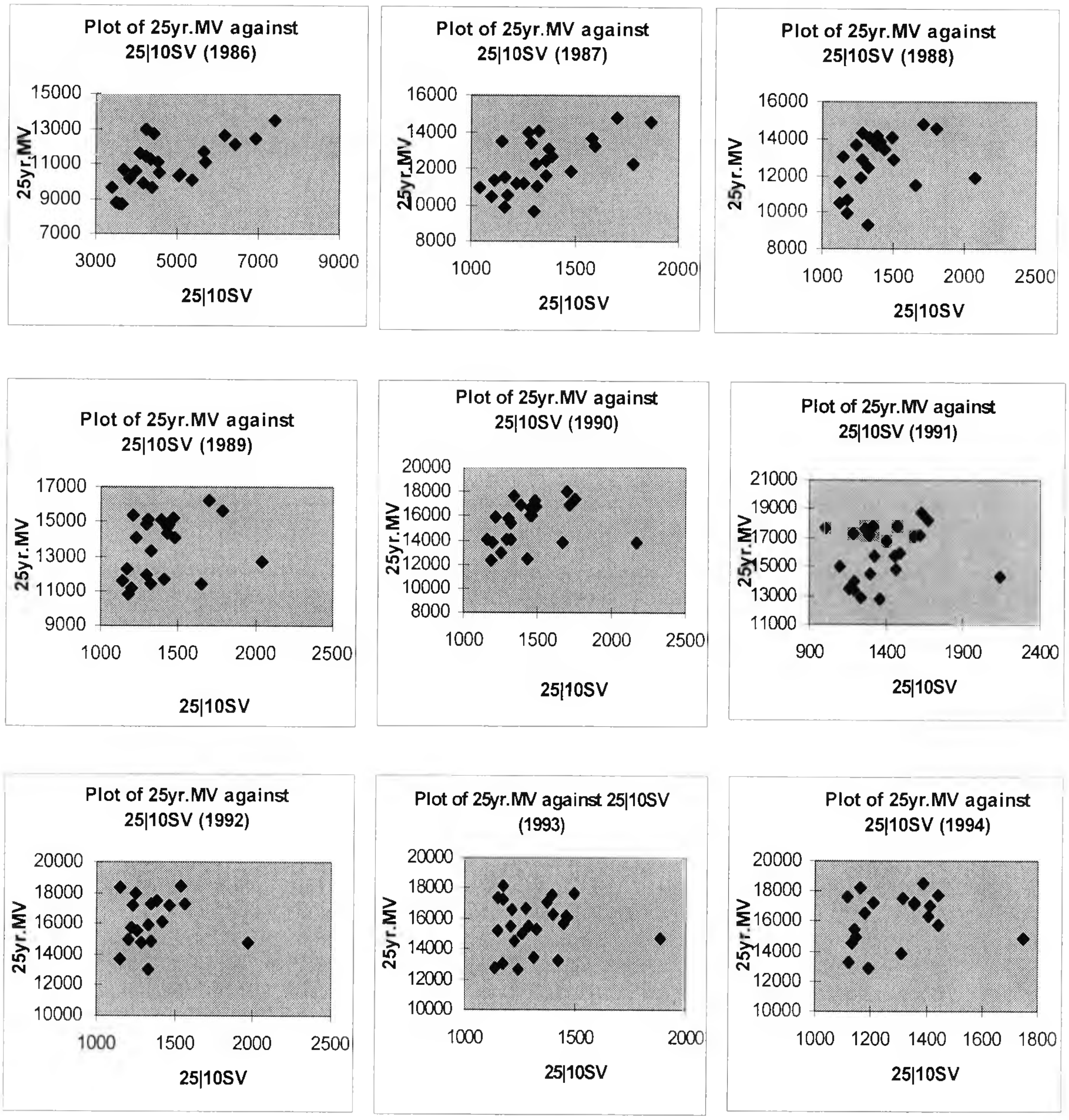

Figure 2.9: Plot of 25year MV against 25|10 SV

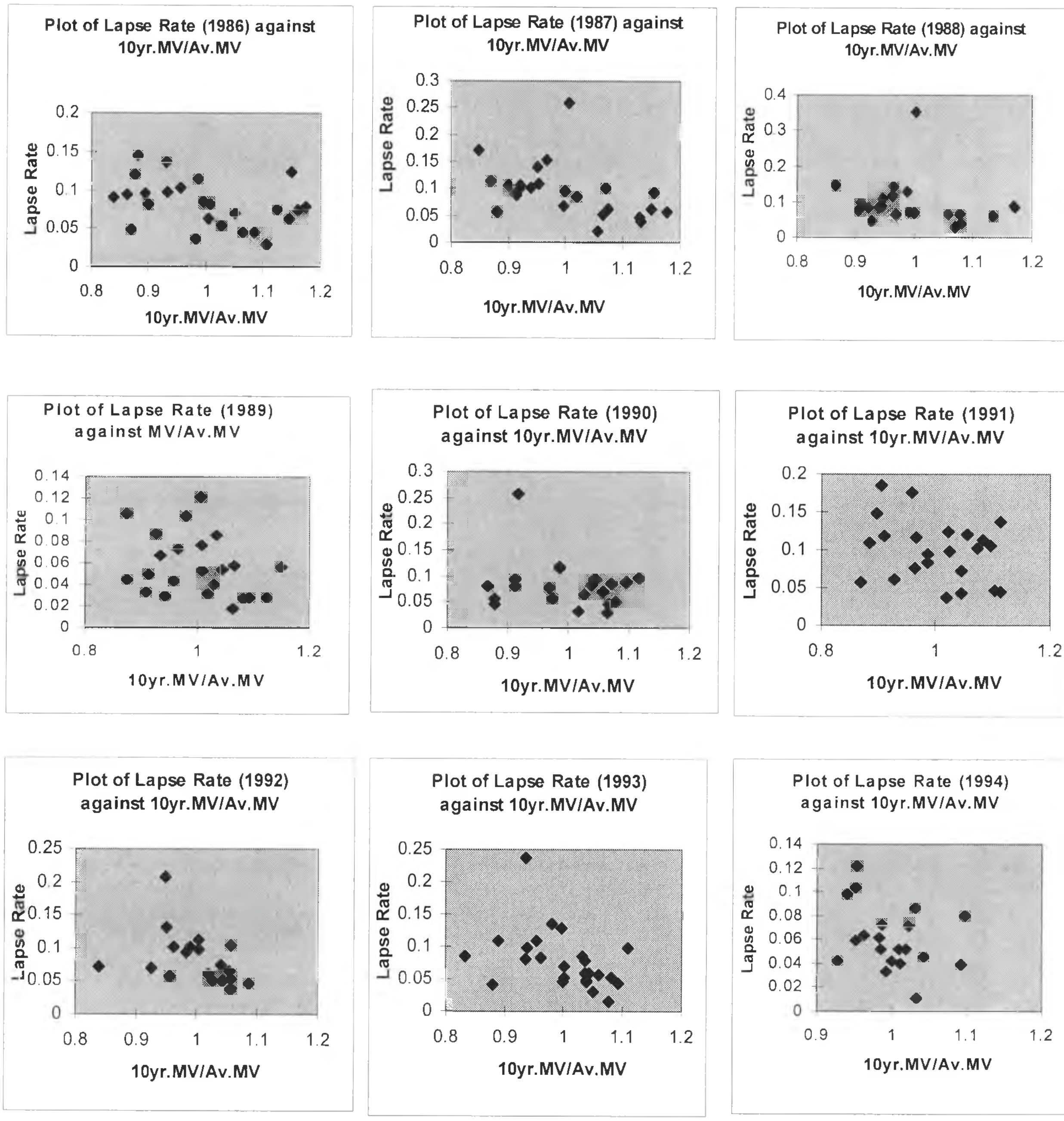

Figure 2.10: Plot of Lapse Rate against R(i,t) (10year MV/Av.MV)

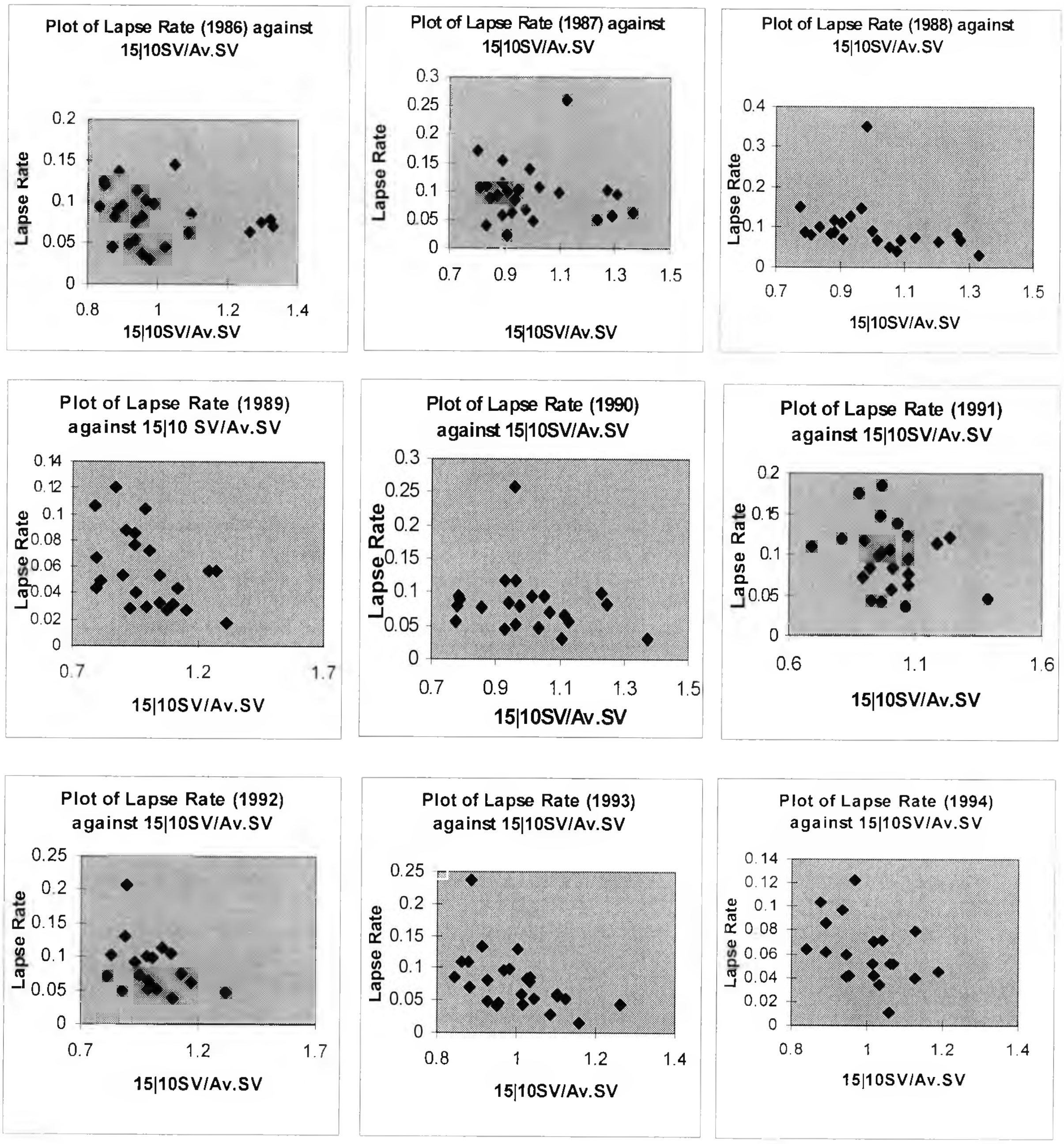

Figure 2.11: Plot of Lapse Rate against R'(i,t) (15|10 SV/Av.SV)

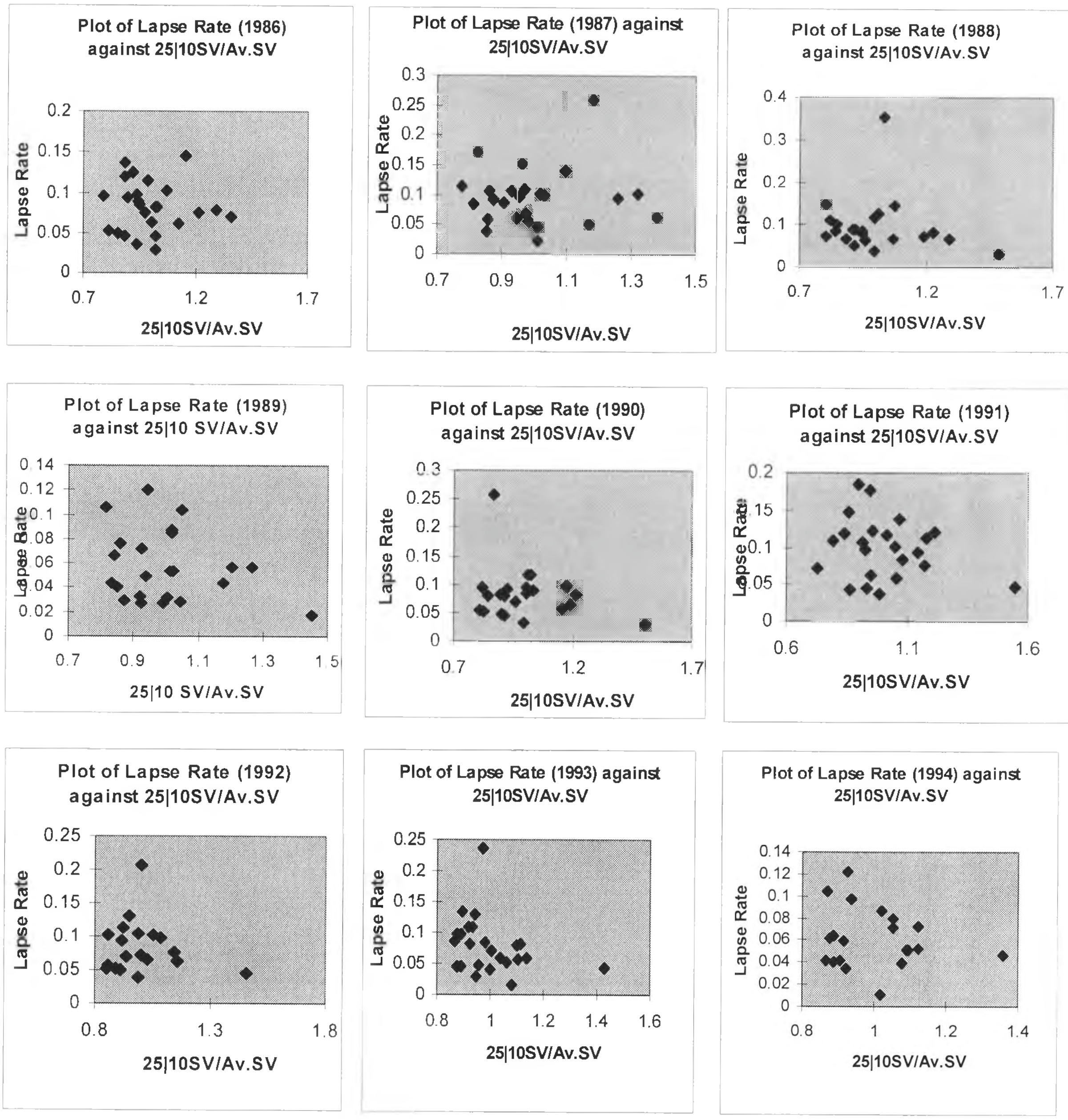

Figure 2.12: Plot of Lapse Rate against $R''(i,t)$ (25|10 SV/Av.SV)

From the scatter plots of Figures 2.7-2.9, we observe that the surrender values at duration 10 years, paid on 15 and 25-year with profit endowment appear to be positively related to maturity values of a 10-year endowment over the period of 1986-1994. Further, we observe that the *25|10 SV* also appears to be positively related to a *25-year MV* (see figure 2.9). This means that as *MV* of a contract increases, SV paid out also increases. Note that for both contracts, the policy had run for only 10 years and so the *SVs* paid were based on this duration.

On the other hand, we observe that the plots of lapse rate against relative payouts, $R(i,t)$, $R'(i,t)$ and $R''(i,t)$, (i.e., *SVs* and *MVs* relative to average market value) follow a pattern different from the one described in figures 2.7-2.9. This time, the plot of lapse rate is inversely related to $R(i,t)$ for some of the years under investigation. For example, in the year 1986, 1992 and 1993 (see figure 2.10). This means that as $R(i,t)$ increases, lapse rate decreases. In other words, companies with maturity values greater than the average market maturity value are likely to have lower lapse rates. This is not surprising as policyholders normally prefer policies with better value.

Also, the lapse rates appear to be inversely related to $R'(i,t)$ (not strong correlation) for some of the years under investigation (e.g., in years 1988 and 1993). see figure 2.11. This means that as $R'(i,t)$ increases, the lapse rate decreases.

In a similar way, the lapse rates appear to be inversely related to $R''(i,t)$ in the years 1988, 1989 and 1993 (see figure 2.12). This means that as $R''(i,t)$ increases, the lapse rate decreases.

The observations of figures 2.11 and 2.12 imply that companies with policies that pay surrender values higher than the average market surrender value are likely to have lower lapse rates. In other words, lower values of *SV* (relative to average surrender market value) are accompanied by higher lapse rates. This is not surprising since policyholders tend to surrender their policies when they perceive poor value for money. Note that further analysis will be performed to confirm or refute any significant correlation. This is discussed in the next section of this chapter.

Finally, there appear to be no obvious pattern or relationship between the variables, *MV* and *SV*, against yield on assets (not shown). This means that the correlation coefficient would probably be very close to zero.

2.6.2 Methodology

From the above discussions, we note that the scatter plots suggest that either there is a general association between the variables considered or no obvious pattern at all. However, to determine whether or not there is a true association or correlation, one needs to perform a statistical analysis. We will particularly look at any significant association by calculating the correlation coefficient between the lapse rate and the variables $R(i,t)$, $R'(i,t)$ and $R''(i,t)$. The methodology underlying this analysis is discussed below.

2.7 Rank Correlation Analysis

As in Chung and Skipper (1987), a rank correlation analysis is performed to find out if there is any relationship between the variables, surrender and maturity values of with profit endowment policies, on the basis of the payouts offered by the sample of companies under consideration. Also considered is the correlation between the lapse rate , $L(i,t)$, and the variables $R(i,t)$, $R'(i,t)$ and $R''(i,t)$. A rank correlation analysis is used in this case because it does not require any parametric or distributional assumptions and it is one of the most frequently used measures of association.

To determine the existence of any significant correlation between the above mentioned variables, we calculate the Spearman Rank correlation coefficient. Further, to calculate this coefficient we first need to rank the data in a sequence and then compare the rankings for each group. In this context we compare and calculate the correlation coefficient between the following pairs, (X_i, Y_i), (L_i, R_i), (L_i, R'_i), and (L_i, R''_i),

where,

X_i = rank of maturity values of company i, $i=1,2,...25$.

Y_i = rank of surrender values of company i, $i=1,2,...25$.

L_i = rank of lapse rate of company i, $i=1,2,....25$.

R_i = rank of $R(i,t)$.

R'_i = rank of $R'(i,t)$.

R''_i = rank of $R''(i,t)$.

Now, the Spearman rank correlation coefficient, r_s, can be calculated from the formula

$$r_s = 1 - \frac{6\sum_i d_i^2}{\eta(\eta^2 - 1)} \qquad 2.14$$

where,

r_s = Spearman's rank correlation coefficient.

η = number of pairs of ranked data.

d_i = difference between ranks for the two observations within a pair.

The value of r_s will vary between +1 and –1, which indicates perfect positive and negative correlation respectively. The closer the value of r_s is to either +1 or –1 shows how significant the result is. Thus, after calculating r_s, we test for the following null hypothesis:

$$H_o: \rho_s = 0$$

against the alternative hypothesis

$$H_1: \rho_s \neq 0,$$

where r_s is an estimate of ρ_s.

The value of r_s is compared with the critical values at 95% or 99% level for various values of η. In our context we compared the value of r_s with a table of critical values at 95% which is listed in table A-8 of Appendix A of Triola (1992). Since the number of data pairs in this analysis is less than 30, we referred to the critical values of Spearman's rank correlation coefficient. However, when the number of data pairs is greater than 30, the sampling distribution of r_s is approximately normal distribution with mean zero and variance $\frac{1}{\eta - 1}$.

It is important to note that in table 2.5 and 2.7, we performed a correlation analysis on the company's average return over the time period of [1986-year *(n-1)*] against the *SVs* paid to policyholders who surrendered the policy in year *n,* where *n* =1991, 1992,....1995. We chose the time interval to be from 1986 because the complete data set available starts from that year (1986). Further, we match the average returns from 1986 to year *(n-1)* against *SV* paid in year *n* because we deem it appropriate to compare *year (n-1)* returns to surrender benefits paid in year *n*. Note that the difference between table 2.5 and 2.7 is that, in table 2.5, the policies under investigation have been in force for 10 years, whereas in table 2.7, it has been in force for 5 years.

Further, in table 2.6, we performed a correlation analysis on the company's average return over the time period of [1986-year *(n-1)*] against the *MVs* paid to policyholders in policy year *n,* where *n* =1993 and 1994.

We present below a detailed results and discussion from this analysis.

2.7.1 Empirical Results of Rank Correlation Analysis and Discussions

The results of the rank correlation analysis are reported below:

Table 2.1

Rank Correlation Analysis of Life Assurance Surrender Values after n years of a 25 year with profit endowment vs. Maturity Values of a 25 year with profit endowment (1986-1994)

Type of Policy	Years	Number of Observations	Spearman's Rank Correlation Coefficient n (years)		
25 Year with Profit Endowment			20	10	5
	1986	25	0.6176**	0.5077**	-
	1987	25	0.6634**	0.6184**	0.2299
	1988	22	0.7008**	0.5051**	0.1451
	1989	23	0.6757**	0.4115*	0.2318
	1990	23	0.5623**	0.4468*	0.1844
	1991	24	0.5069**	0.1223	0.1354
	1992	19	0.4782*	0.1639	0.2174
	1993	25	0.5200**	0.1152	0.1624
	1994	20	0.4534*	0.2117	0.3163

** Significant at 1% Level

* Significant at 5% Level

Table 2.2

Rank Correlation Analysis of Life Assurance Surrender Values after n years of a 15 year with profit endowment vs. Maturity Values of a 15 year endowment (1986-1994)

Type of Policy	Years	Number of Observations	Spearman's Rank Correlation Coefficient n (years)	
15 Year with Profit Endowment			10	5
	1986	25	0.5459**	0.3666
	1987	25	0.6099**	0.4595*
	1988	22	0.6654**	0.1582
	1989	23	0.4243*	0.2864
	1990	23	0.4019*	0.297
	1991	24	0.5592**	0.4118*
	1992	19	0.6015**	0.3762
	1993	25	0.6595**	0.3946*
	1994	20	0.6443**	0.6679**

Table 2.3

Rank Correlation Analysis of Life Assurance Surrender Values after 10 years of a 15 year with profit endowment vs. Maturity Values of a 10 year with profit endowment (1986-1994)

Type of Policy	Years	Number of Observations	Spearman's Rank Correlation Coefficient Number of Years Policy has been in force
15 Year with Profit Endowment			10
	1986	25	0.4097*
	1987	25	0.6190**
	1988	22	0.7600**
	1989	23	0.5866**
	1990	23	0.3594
	1991	24	0.3708
	1992	19	0.6792**
	1993	25	0.7046**
	1994	20	0.6917**

Table 2.4

Rank Correlation Analysis of Life Assurance Surrender Values after 10 years of a 25 year with profit endowment vs. Maturity Values of a 10 year endowment (1986-1994)

Type of Policy	Years	Number of Observations	Spearman's Rank Correlation Coefficient Number of Years Policy has been in force
25 Year with Profit Endowment			10
	1986	25	0.3862*
	1987	25	0.5299**
	1988	22	0.5847**
	1989	23	0.5599**
	1990	23	0.5313*
	1991	24	0.2789
	1992	19	0.2099
	1993	25	0.4421*
	1994	20	0.5618**

Table 2.5: Rank Correlation Analysis of Life Assurance Average Yield vs. Surrender Values (1986-1994)

Type of Policy	Number of Years Policy Has been in Force	Year Policy Surrendered	Spearman's Rank Correlation Coefficient $[t_0 - t_n]$		
			[1986-1992] {20}	[1986-1993] {21}	[1986-1994] {19}
25 Year Endowment Policy	10	1993	0.0391		
	10	1994		0.2658	
	10	1995			0.3281
15 Year Endowment Policy	10	1993	0.0075		
	10	1994		0.1228	
	10	1995			-0.1

{ } = Number of observations.

[ ] = Overall investment period is from calendar year t_0 to t_n.

Table 2.6: Rank Correlation Analysis of Life Assurance Average Yield vs. Maturity Values (1986-1994)

Type of Policy	Policy Year	Spearman's Rank Correlation Coefficient $[t_0, t_n]$	
		[1986-1992] {20}	[1986-1993] {21}
25 Year Endowment Policy	1993	-0.3398	
	1994		0.0578

Table 2.7: Rank Correlation Analysis of Life Assurance Average Yield vs. Surrender Values (1986-1994)

Type of Policy	Number of Years Policy has been in Force	Year Policy Surrendered	Spearman's Rank Correlation Coefficient $[t_0, t_n]$				
			[1986-1990] {22}	[1987-1991] {20}	[1988-1992] {20}	[1989-1993] {21}	[1990-1994] {19}
25 Year Endowment Policy	5	1991	0.2377				
	5	1992		-0.1309			
	5	1993			-0.0677		
	5	1994				0.1293	
	5	1995					0.0105
15 Year Endowment Policy	5	1991	0.2763				
	5	1992		0.1573			
	5	1993			0.1474		
	5	1994				0.1812	
	5	1995					-0.0746

Table 2.8
Rank Correlation Analysis of Lapse Rate against R(i,t)

Type of Policy	Years	Number of observation	Spearman's rank correlation coefficient
With profit Endowment	1986	25	-0.4162*
	1987	25	-0.5392**
	1988	22	-0.3936
	1989	23	-0.3529
	1990	23	-0.1329
	1991	24	-0.2009
	1992	19	-0.5204*
	1993	25	-0.4977*
	1994	20	-0.3385

Table 2.9
Rank Correlation Analysis of Lapse Rate against R'(i,t)

Type of Policy	Years	Number of observation	Spearman's rank correlation coefficient
15 Year with profit Endowment	1986	25	-0.2738
	1987	25	-0.1869
	1988	22	-0.6329**
	1989	23	-0.4951*
	1990	23	-0.0826
	1991	24	-0.2052
	1992	19	-0.3281
	1993	25	-0.5769**
	1994	20	-0.4235

Table 2.10
Rank Correlation Analysis of Lapse Rate against R''(i,t)

Type of Policy	Years	Number of observation	Spearman's rank correlation coefficient
25 Year with profit Endowment	1986	25	-0.0773
	1987	25	-0.0315
	1988	22	-0.2282
	1989	23	-0.0974
	1990	23	-0.0712
	1991	24	-0.0878
	1992	19	-0.0702
	1993	25	-0.3685
	1994	20	-0.0173

* Significant at 5% Level
** Significant at 1% Level

From table 2.1, we observe a positive and significant correlation between *SVs* of a 25-year with profit endowment, which had run for 20 years and the corresponding *MV's* paid on this policy for the entire period of 1986-1994. This is not surprising since surrender values bases tend to "blend" towards maturity values as duration increases. However, interesting results were obtained at duration 10, which is 15 years from maturity. Thus, we observed a positive and significant correlation at this duration over the period of 1986-1990. This shows that in the 1980's (including 1990), companies that pay out higher *SVs* to surrendering policyholders also pay higher *MVs* to continuing policyholders. But we observe no significant correlation at duration 5, which is 20 years away from maturity. This is not surprising since there is normally high expense loading at the early durations of a life insurance policy, as observed by Chung and Skipper (1987).

However, we observe from table 2.2 significant results in some years (for e.g., 1987, 1991, and 1993-94) for the case of *15|5 SV* against a 15-year *MV*. This is probably due to the fact that this duration is not too far from maturity. We also observe significant results for the case of *15|10 SV* against a *15-year MV*.

Furthermore, table 2.3 and 2.4 present interesting results for the case where the durations of the policies being compared are the same. That is, *25|10 SV* against *10-year MV* and *15|10 SV* against *10-year MV*. In this case, we observe a positive and significant correlation between *15|10 SV* and *10-year MV* in the following years: 1986-1989 and 1992-94. See table 2.3. Also, we observe a positive and significant correlation between *25|10 SV* and *10-year MV* in the following years: 1986-1990 and 1993-94. See table 2.4. In both cases, the result is not surprising since surrender values tend to blend towards maturity values as duration increases.

A significant correlation between *MVs* and *SVs* suggests that, from a statistical standpoint, policies with higher *SVs* tend to have higher *MVs* than ones with lower *SVs*. This also suggests that higher *SVs* are not paid out at the expense of *MVs*, and so we can infer that surrendering policyholders are not better off than those who maintain their policies until maturity as hypothesized. However, this does not mean that subsidizing payouts by other means is completely ruled out. Rather, other factors like return on assets could be used in paying out benefits. Table (2.5-2.7) present results on this analysis (i.e., lapse rate against yield on assets).

In this analysis, we were expecting a positive and significant correlation between payouts and average yield on assets as hypothesized. But surprisingly, we observe no significant correlation between yields and payouts (see table 2.5-2.7). However, there is a positive correlation between surrender values and average yield on assets for both 15 and 25 year endowment at duration 10 (in this case, the average yield was calculated over the period 1986-1992, 1986-1993, 1986-1994). Note that there is a negative correlation for a 15-year endowment for which the period is 1986-1994 (see table 2.5). Also, the *r* values of both 15 and 25 year endowment at duration 5 and 10, calculated over the period 1986-1992, 1986-1993, 1988-1992, and 1990-1994 were close to zero (see table 2.5-2.7 again).

Further, there is some negative and positive correlation between surrender values and average yield on assets for both 15-year and 25-year endowment at duration 5 depending on the period used to calculate the average yield. Thus there is a negative correlation between surrender values and average yield on assets over the period of 1987-1991, 1988-1992 for 25-year endowment and at 1990-1994 for 15-year endowment. However, there is a positive correlation over the period of 1986-1990, 1988-1992 for both 15-year and 25-year endowment; over the period 1987-1991, 1988-1992 for a 15-year endowment and in 1990-1994 for a 25-year endowment (see table 2.7). Also, from table 2.6, we observe a negative but not significant correlation between the average return on assets over the period of 1986-1992 and the *MV's* paid in the policy year 1993 of a 25-year endowment policy.

Since there is no significant correlation between surrender/maturity values and average yield on assets for the type of policy under investigation (15 and 25-year endowment that has been in force for about 5 and 10 years), this suggests that a company's ability to pay out more benefits to policyholders does not depend primarily on their investment returns alone, but on other factors that need to be investigated. Regrettably, there was not enough information, such as economic variables, available to enable us to explore these factors. However, this could possibly be a future research topic of interest.

This result is similar to Chung and Skipper (1987) where a significant negative correlation between the level of surrender values and currently credited interest rate was observed at duration one and a negative correlation but not significant, was observed at duration five. However, a significant positive relation was found at durations 10, 15, and 20.

Finally, from table 2.8-2.10, we observe a negative and significant correlation between the lapse rate and $R(i,t)$ for 1986, 1987, 1992 and 1993. Also, we observe a similar one (negative and significant correlation) between the lapse rate and $R'(i,t)$ in 1988, 1989 and 1993. However, there are no significant results for the case of lapse rate versus $R''(i,t)$.

The results of table 2.8-2.10 suggest to some extent that lower *MVs* relative to the average market value are accompanied by higher lapse rate. This is not surprising because policyholders normally prefer policies with better value. Thus, policyholders are not likely to surrender if a higher maturity benefit (relative to the average market maturity value) is offered by the life office. This confirms our initial hypothesis that lower *MVs* relative to average market tend to be associated with higher lapse rates.

On the other hand, the above results could also mean that higher *SVs* relative to the average market value are not accompanied by high lapse rates- suggesting that policyholders do not surrender on the basis of the surrender value offered by the policy. This result does not confirm our initial hypothesis (3). So, we reject this hypothesis and can infer that policyholders do not necessarily surrender the policy when a higher surrender value is offered by the policy.

However, the results of tables 2.1-2.4 confirm hypothesis one. So we can infer that companies that pay out higher surrender values to surrendering policyholders tend to pay out higher maturity values.

The results of tables 2.5-2.7 do not confirm hypothesis four since we do not obtain any significant correlation between payouts and average yield on assets. Hence, we can infer that companies that offer higher *MV* or *SV* do not necessarily have a higher yield on their assets.

Further, the results of tables 2.8-2.10 also confirm hypothesis two. So we can infer that companies that offer higher maturity value relative to the average market maturity value tend to have lower lapse rates.

The next section discusses a graphical presentation of the behavior of companies' lapse rates over time.

2.8 Graphical presentation of the behavior of Companies Lapse Rate over time period (1986-1994).

We present in figure 2.13, a graph showing the behavior of the companies lapse rates over time period, 1986-1994. Also, figure 2.14 shows the graph of the mean lapse rate, as well as the median and quartiles (lower and upper quartile) lapse rate over time period, 1986-1994.

Figures 2.13 and 2.14, indicate that on average the lapse rate decreased from 1988 to 1989 and 1991-94, suggesting that companies were probably paying out higher *MV* or *SV* relative to average market *MV* or *SV* value. However, during some years, for example, 1989-91, the average lapse rate moved up either because companies were then paying out lower *MV* or *SV* relative to average *MV* or *SV* market value, or probably due to economic factors. Regrettably, these factors were not considered in this analysis due to lack of data on economic variables as mentioned before. We also observe that on average, most of the companies experienced a lapse rate of about 8-9% over the period of 1986-1994. However, a higher number of lapse rates were observed in 1987 and 1990/91, which was the time that the unemployment rate was high in the U.K.

Now, to determine if the changes as observed in figure 2.13 and 2.14 are actually real changes, but have not happened just by chance, a non-parametric test of homogeneity was performed. This was investigated by testing the hypothesis that lapse rates are the same over time (1986-1994), by using the Friedman's Test, described by Sokal et al (1981). Interestingly, the result shows that lapse rates differ significantly over time (and hence are not homogenous). See appendix 2.2 for detailed results of this analysis.

Further, figures 2.15 and 2.16 show the plot of the ratio of mean of *SV / MV* of all companies between 1986 and 1994. In particular, figure 2.15 shows the mean of *15|10 SV /10year MV*, whereas figure 2.16 shows the mean of *25|10 SV / 10-year MV*. Also shown is the median and quartiles of *SV/MV*. In these plots, our objective is to find out if companies were paying out more *SV* than *MV* or vice versa.

From figure 2.15, we observe a steadily upward trend from 1987 to 1993. This was then followed by a downward trend after 1993 (see figure 2.15). The increase in the ratio, *SV:MV* may well be due to falling bonus rates. Nevertheless, *SVs* did not fall so quickly. Also, figure 2.16 followed a trend similar to figure 2.15, except that there was a downward trend after 1993. That is, we observe a steadily upward trend from 1987 to 1993. Similarly, the increase in the ratio, *SV:MV* may well be due to falling bonus rates. Meanwhile, *SVs* did not fall so quickly. But the ratio values of figure 2.15 are higher than that of figure 2.16. This is probably due to the fact that the *SV* of a 15-year endowment at duration 10-years tend to blend towards the maturity values more than for a 25-year endowment at duration 10-years.

It is important to note that the Friedman's test was used again to determine if the ratio *SV:MV* actually differ significantly over time (1986-1994). This is to investigate whether or not companies were offering more *SV* to policyholders that surrender the policy as oppose to those who stayed on to receive maturity benefit. However, from figures 2.15 and 2.16, the ratio of *SV:MV* appears to differ slightly from 1986 to 1991 and so, the Friedman's test was then repeated on this variable, *SV:MV* over the stated period (1986-1991). See Appendix 2.2 for details of the results obtained.

Lastly, from the plots of figure 2.15 and 2.16, we observe that the ratio *SV:MV* increases over time, indicating that the a higher *SV* is paid out relative to the *MV* over the period. In other words, a lower *MV* is offered relative to *SV* over the period. This also confirms hypothesis five.

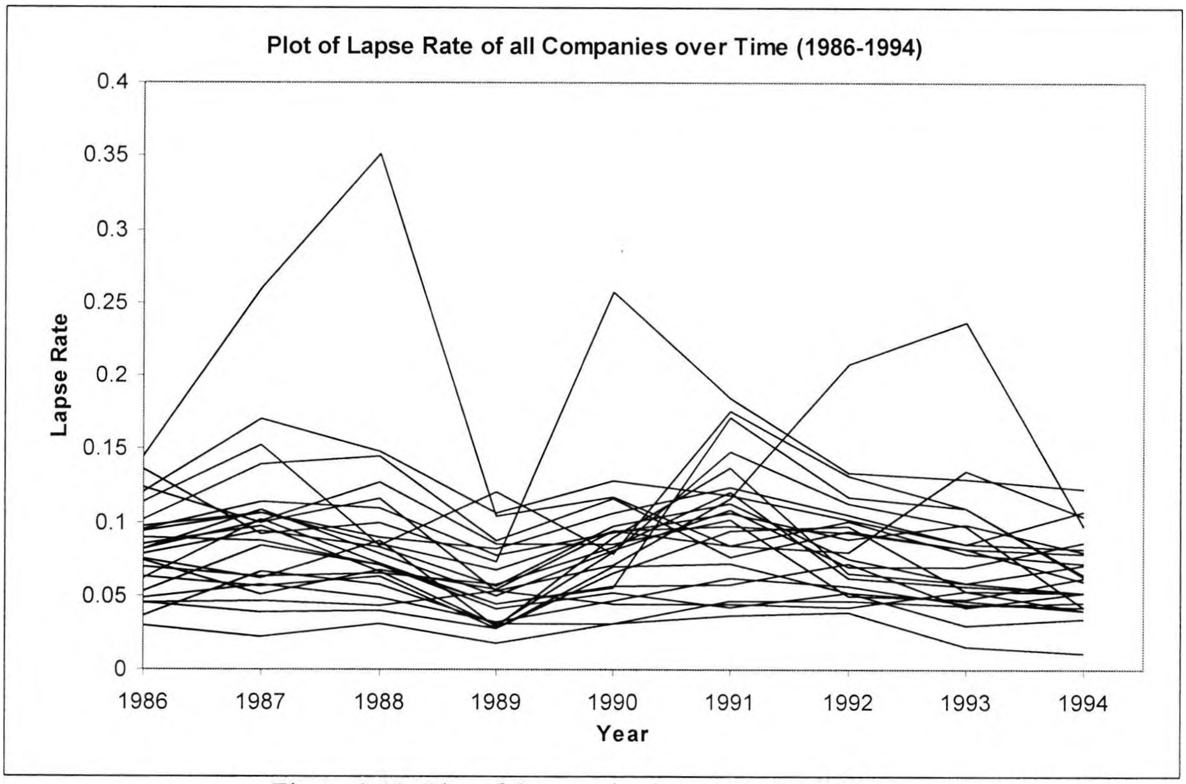

Figure 2.13: Plot of Companies lapse rate over time

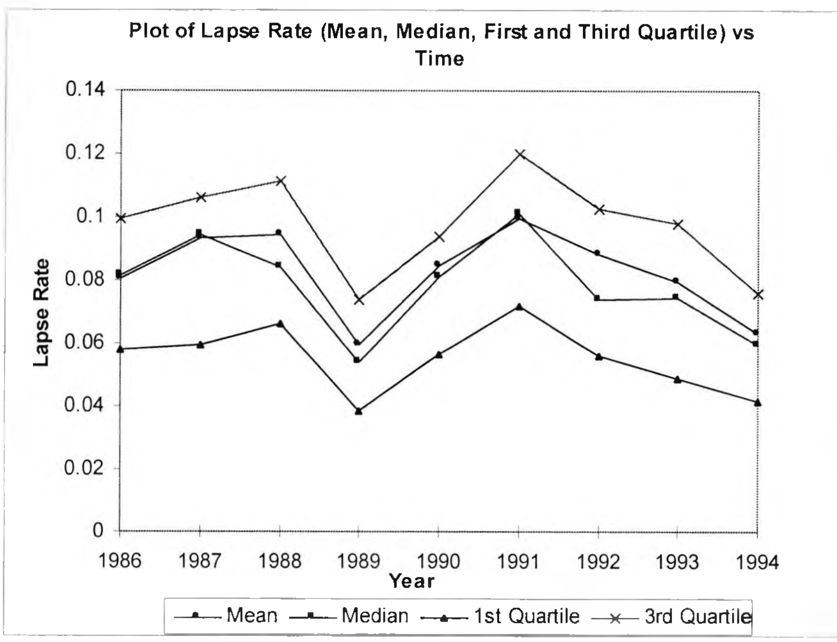

Figure 2.14: Plot of mean, median, first and third quartile lapse rate over time

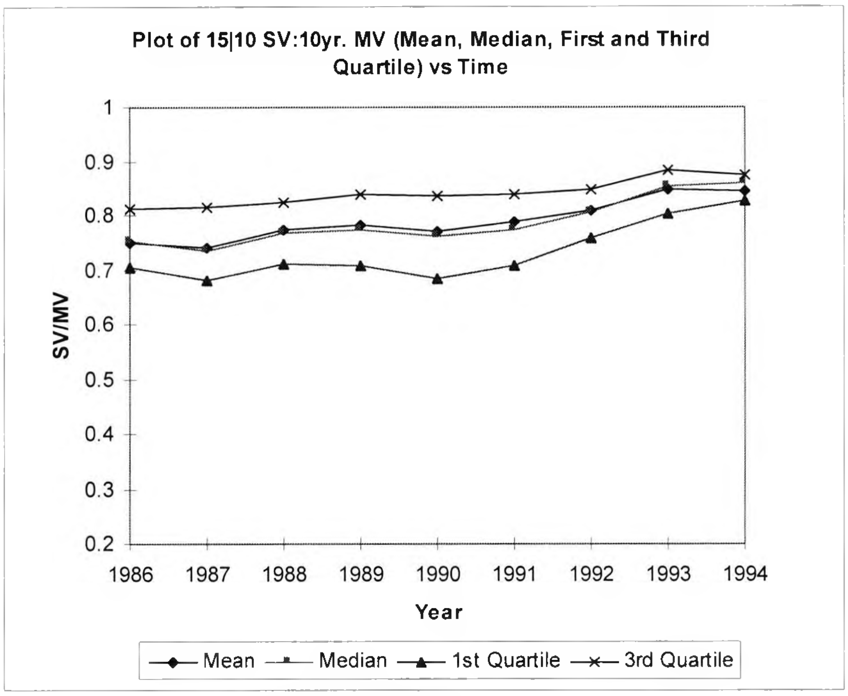

Figure 2.15: Plot of mean, median, first and third quartile of 15|10 SV:10yrMV over time

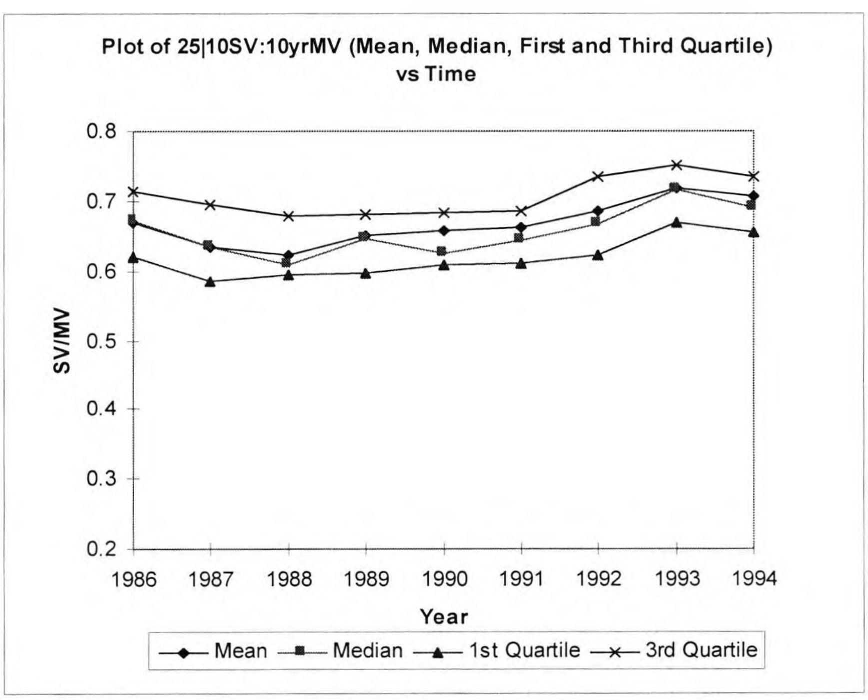

Figure 2.16: Plot of mean, median, first and third quartile 25|10 SV:10yr.MV over time

In this chapter we have proposed a model of lapse rate that takes into account the number of policies exposed to risk of lapsing in the year leading up to the *rth* policy anniversary (=curtate duration *r-1*). From the analysis performed we can conclude that payouts (*MV* or *SV*) do have an effect on lapse rate. That is, lower values of *MV* or *SV* (relative to the average market surrender or maturity value) are accompanied by a higher lapse rate. This implies that policyholders surrender the policy when they perceive poor value for money. We notice that this feature is particularly prevalent in the periods 1986-1987 and 1989-1991, - suggesting that it was very much in the public awareness at that time. Further, we observe that policies with higher *SVs* tend to have higher *MVs* than the ones with lower *SVs*. This implies that companies that pay out higher *SVs* to surrendering policyholders also pay higher *MVs* to continuing policyholders, indicating that the surrendering policyholders are not relatively better off than those that hold on to the policy until maturity. Lastly, we have shown that higher *SVs* relative to the average market value are not accompanied by high lapse rates- suggesting that policyholders do not surrender on the basis only of the surrender value offered by the policy.

The next chapter discusses the investment model used in this thesis.

Appendix 2

Appendix 2.1

Summary statistics of Life Assurance Maturity Values

Type of Policy	Years	Number of Obsevations	Mean	Standard Deviation	Median	First Quartile	Third Quartile
25 Year with Profit Endowment	1985	26	9872.5	1295.74	9648.5	8894.25	10975.25
	1986	25	10904.64	1331.6	10691	10097	11656
	1987	25	12186.28	1443.64	12219	11159	13405
	1988	22	12668.68	1578.53	12906	11670.5	14005
	1989	23	13593.78	1717.23	14115	11790.5	14985.5
	1990	23	15208.83	1890.72	15792	13850	16836
	1991	24	15964.46	1842.23	14830.5	13201	16355.5
	1992	19	16197.11	1578.62	16151	14884	17311.5
	1993	25	15623.19	1638.27	15669.44	14740.83	17060.83
	1994	20	16183.76	1657.96	16735.83	14912.78	17315.56

Summary statistics of Life Assurance Maturity Values

Type of Policy	Years	Number of Obsevations	Mean	Standard Deviation	Median	First Quartile	Third Quartile
10 Year with Profit Endowment							
	1986	25	2085.72	219.43	2078	1873	2263
	1987	25	2169.04	213.61	2160	1991	2322
	1988	22	2139.82	184.67	2093	2015.5	2282.25
	1989	23	2122.3	162.98	2138	1992.5	2237
	1990	23	2189.68	168.52	2237	2035.25	2328.5
	1991	24	2084.54	160.51	2134	1997.5	2260
	1992	19	1970.05	117.94	1979	1889.5	2064.5
	1993	25	1833.28	128 6	1836.94	1750	1912.5
	1994	20	1821.67	84.31	1815.56	1751.46	1867.57

Summary statistics of Life Assurance Adjusted Lapse Rate

Type of Policy	Years	Number of Obsevations	Mean	Standard Deviation	Median
25 and 10 Year with Profit Endowment	1986	25	0.0807	0.0334	0.0813
	1987	25	0.0936	0.0489	0.0943
	1988	22	0.0943	0.0625	0.084
	1989	23	0.0597	0.0306	0.0538
	1990	23	0.0845	0.0459	0.0808
	1991	24	0.0994	0.0401	0.1012
	1992	19	0.0883	0.0436	0.0737
	1993	25	0.0793	0.0445	0.0743
	1994	20	0.0598	0.0293	0.0596

Summary statistics of Life Assurance Surrender Values:
Policy has been in effect for 20 Years

Type of Policy	Years	Number of Obsevations	Mean	Standard Deviation	Median	First Quartile	Third Quartile
25 Year with Profit Endowment	1985	26	4510.19	971.79	4266.5	3890	5011
	1986	25	4732.76	1104.62	4373	3968	5380
	1987	25	5060.2	1291.58	4494	4128	6172
	1988	22	5566.59	1376.79	5639	4210.5	6696
	1989	23	5872.74	1413.99	6072	4586.5	7092
	1990	23	6263.13	1829.88	6539	4513.5	7380.5
	1991	24	6455.42	1752.71	6237.57	4521.38	7176.75
	1992	19	6748.21	1493.21	6890	5802.5	7691.5
	1993	25	6572.6	6827.78	5474.17	7612.78	1361.68
	1994	20	6926.17	7226.39	6400.97	7560.42	1209.05

Summary statistics of Life Assurance Surrender Values:
Policy has been in effect for 10 Years

Type of Policy	Years	Number of Obsevations	Mean	Standard Deviation	Median	First Quartile	Third Quartile
25 Year with Profit Endowment	1985	26	1307.08	166.2	1277.5	1211.25	1386
	1986	25	1316.32	185.31	1281	1183	1358
	1987	25	1348.48	213.59	1310	1179	1391
	1988	22	1392.27	238.92	1328	1244	1476
	1989	23	1405.13	216.86	1391	1261	1452.5
	1990	23	1430.83	241.34	1390	1271.5	1487
	1991	24	1380.21	236.67	1349	1243.5	1471.25
	1992	19	1349.37	193.22	1332	1231	1403.5
	1993	25	1316.72	161.84	1280.83	1210	1400.83
	1994	20	1288.71	160.29	1260.28	1160.49	1392.36

Summary statistics of Life Assurance Surrender Values:
Policy has been in effect for 10 Years

Type of Policy	Years	Number of Obsevations	Mean	Standard Deviation	Median	First Quartile	Third Quartile
15 Year with Profit Endowment	1985	26	1526.31	229.58	1467	1380.25	1607.75
	1986	25	1561.4	237.48	1491	1390	1637
	1987	25	1605.24	273.66	1504	1431	1765
	1988	22	1659.86	270.89	1618	1463.75	1801.75
	1989	23	1661.74	250.98	1640	1504	1808.5
	1990	23	1685.91	267.44	1634.5	1562	1848.25
	1991	24	1640.92	230.38	1634.5	1508.5	1766.5
	1992	19	1597.32	196.89	1579	1461.5	1702.5
	1993	25	1552.34	158.47	1558.89	1439.44	1620.28
	1994	20	1542.11	142.51	1566.25	1453.61	1633.19

Appendix 2.2

Nonparametric test of Homogeneity

This test was performed to determine if the changes as observed in the plots of lapse rate and payouts over time are actually real changes, or just happened by chance. Friedman's method, described by Sokal et al (1981) was used in this case. Note that in Friedman's method, the variates are ranked within each block (column in this case). Further note that in this analysis, the lapse rate and the ratio, SV:MV, of all companies are treated as blocks. The procedure is given below:

i) Assign ranks to items within blocks (columns). Note that if there are ties, treat them in the conventional manner by way of finding the average of ranks.

ii) Sum the ranks for each calendar year over the companies (rows in this case).

Define R_{ij} = sum of ranks over *j* for each *i*. The following test statistics is used :

$$X^2 = \left\{ \frac{12}{ab(a+1)} \sum_a \left(\sum_b R_{ij} \right)^2 \right\} - 3b(a+1)$$

where

a = treatments (i.e., years)

b = blocks (i.e., companies lapse rates and SV:MV)

The above test statistics follows a chi squared distribution with *(a-1)* degree of freedom. Therefore, we reject the under mentioned null hypothesis if $X^2 \rangle \chi^2[a-1]$.

Now, we consider the results of the following test:

1/

In this test, we are interested in the hypothesis that, lapse rate does not differ significantly over time.

The table below shows the lapse rates of a sample of Life companies from 1986-1994.

In this table, a=9 and b=14. Also shown are the corresponding ranks of each lapse rate over the time period, 1986-1994.

Lapse Rate

Company	cler	equitabl	friends	G.A Life	Nat Mutu	Norwich	Pearl	Refuge	Scot Am	Scot Life	Scot Mut	Scot Pro	Scot Wi	Standard
Year	Lapse Rate													
1986	0.0454	0.0296	0.0746	0.0488	0.114	0.0695	0.0822	0.0535	0.0745	0.0852	0.0813	0.1452	0.0632	0.0786
1987	0.0389	0.0219	0.0614	0.0577	0.1525	0.0626	0.1005	0.0843	0.0506	0.0967	0.1081	0.2595	0.0561	0.0943
1988	0.0393	0.0304	0.0872	0.049	0.084	0.0648	0.1269	0.071	0.0675	0.0667	0.0786	0.3512	0.0626	0.0826
1989	0.0278	0.0179	0.0286	0.0325	0.1207	0.0572	0.0856	0.0404	0.054	0.03	0.0495	0.1041	0.0271	0.0568
1990	0.0696	0.0308	0.0895	0.0462	0.0782	0.0824	0.0855	0.0521	0.0936	0.0797	0.0808	0.1167	0.0655	0.0977
1991	0.0719	0.0461	0.1369	0.0618	0.1762	0.1199	0.1012	0.0422	0.0976	0.1481	0.1086	0.0838	0.0938	0.1131
1992	0.0519	0.0463	0.0649	0.0569	0.131	0.0624	0.05	0.0522	0.0925	0.1132	0.0701	0.1011	0.0973	0.0751
1993	0.0296	0.0433	0.0582	0.0446	0.1092	0.0563	0.0476	0.046	0.0815	0.0968	0.0698	0.0787	0.0526	0.0588
1994	0.0339	0.0457	0.071	0.0418	0.0639	0.0515	0.0616	0.0403	0.0596	0.0417	0.0861	0.072	0.0515	0.0522

Table 1i.

Rank of Lapse rate

Company	cler	equitabl	friends	G.A Life	Nat Mutu	Norwich	Pearl	Refuge	Scot Am	Scot Life	Scot Mut	Scot Pro	Scot Wi	Standar	Rij	(Rij)^2
Year	Rank of Lapse Rate															
1986	6	3	6	5	5	7	4	7	5	5	6	7	6	5	77	5929
1987	4	2	3	8	8	5	7	9	1	6	8	8	4	7	80	6400
1988	5	4	7	6	3	6	9	8	4	3	4	9	5	6	79	6241
1989	1	1	1	1	6	3	6	2	2	1	1	5	1	2	33	1089
1990	8	5	8	4	2	8	5	5	8	4	5	6	7	8	83	6889
1991	9	8	9	9	9	9	8	3	9	9	9	3	8	9	111	12321
1992	7	9	4	7	7	4	2	6	7	8	3	4	9	4	81	6561
1993	2	6	2	3	4	2	1	4	6	7	2	2	3	3	47	2209
1994	3	7	5	2	1	1	3	1	3	2	7	1	2	1	39	1521
																49160

Table 1ii.

We calculate $\chi^2 = 48.19$.

From tables, $\chi^2(8)$ at 1% significant level is 20.09.

Hence, result is significant at 1% level of significance and so we reject the above hypothesis and conclude that the lapse rate differs significantly over time (1986-1994).

2/

In table 2i below, we are interested in testing the hypothesis that, the ratio 15|10SV:10yr.MV does not differ significantly over time (1986-1994). Thus, we have, a=9 and b=14.

15|10 SV:10yrMV

Company	Cler M&G	Equitable L	Friends Pr	G.A. Life A	Nat Mutual	Norwich U	Pearl Assu	RefugeAss	Scot Amica	ScotLifeAs	Scott Mutu	ScotProvA	Scottish W	Standard
Year	15|10 SV:10yrMV													
1986	0.611537	0.663053	0.624573	0.789967	0.714633	0.9466	0.796583	0.683302	0.83719	0.82435	0.653168	0.892098	0.826907	0.843954
1987	0.546455	0.637435	0.595753	0.751966	0.682403	0.939511	0.747544	0.680235	0.856834	0.813739	0.646518	0.828821	0.810737	0.839457
1988	0.773454	0.966798	0.585697	0.88256	0.717478	0.935897	0.731788	0.705882	0.783174	0.815154	0.694215	0.761083	0.823723	0.838658
1989	0.773748	0.968071	0.649476	0.904046	0.680263	0.931949	0.715914	0.726441	0.782785	0.826174	0.694014	0.788841	0.832317	0.847478
1990	0.776674	0.991427	0.554398	0.905925	0.680641	0.924362	0.676759	0.686997	0.773025	0.824737	0.692308	0.751274	0.836671	0.847319
1991	0.674632	0.990004	0.725118	0.907446	0.717718	0.919509	0.707328	0.728736	0.731959	0.842162	0.614092	0.805258	0.857004	0.862206
1992	0.795809	0.98271	0.760231	0.845377	0.7552	0.927825	0.693069	0.755887	0.769112	0.849114	0.711075	0.805955	0.811521	0.880429
1993	0.87545	0.977137	0.821496	0.862519	0.780159	0.882437	0.755283	0.778248	0.839514	0.87496	0.746862	0.843513	0.88352	0.901536
1994	0.880627	0.966193	0.84245	0.861801	0.737225	0.890021	0.765535	0.791139	0.840167	0.871196	0.731895	0.89117	0.850083	0.911765

Table 2i

Ranks of 15|10SV:10yr.MV (by Rows)

Company	Cler M&G	Equitable L	Friends Pr	G.A. Life A	Nat Mutual	Norwich U	Pearl Assu	RefugeAss	Scot Amic	ScotLifeAs	Scott Mutu	ScotProvA	Scottish W	Standard L	Rij	('Rij)^2
Year	Rank of 15|10 SV: 10yrMV															
1986	2	2	4	2	4	9	9	2	6	3	3	9	4	3	62	3844
1987	1	1	3	1	3	8	6	1	9	1	2	6	1	2	45	2025
1988	4	4	2	6	5	7	5	4	5	2	6	2	3	1	56	3136
1989	5	5	5	7	1	6	4	5	4	5	5	3	5	5	65	4225
1990	6	9	1	8	2	4	1	3	3	4	4	1	6	4	56	3136
1991	3	8	6	9	6	3	3	6	1	6	1	4	8	6	70	4900
1992	7	7	7	3	8	5	2	7	2	7	7	5	2	7	76	5776
1993	8	6	8	5	9	1	7	8	7	9	9	7	9	8	101	10201
1994	9	3	9	4	7	2	8	9	8	8	8	8	7	9	99	9801
																47044

Table 2ii

We calculate $\chi^2 = 28.08$.

From tables, $\chi^2(8)$ at 1% significant level is 20.08.

Hence, result is significant at 1% level of significance and so we reject the above hypothesis and conclude that the ratio 15|10SV:10yr.MV differs significantly over time (1986-1994).

3/

In table 3i, we are interested in testing the hypothesis that, the ratio 15|10SV:10yr.MV does not differ significantly over time (1986-1991). Here, a=6 and b=14.

15|10SV:10yr.MV

Company	Cler M&G	Equitable L	Friends Pr	G.A. Life A	Nat Mutual	Norwich U	Pearl Assu	RefugeAss	Scot Amic	ScotLifeAs	Scott Mutu	ScotProvA	Scottish W	Standard
Year	15|10 SV:10yrMV													
1986	0.611537	0.663053	0.624573	0.789967	0.714633	0.9466	0.796583	0.683302	0.83719	0.82435	0.653168	0.892098	0.826907	0.843954
1987	0.546455	0.637435	0.595753	0.751966	0.682403	0.939511	0.747544	0.680235	0.856834	0.813739	0.646518	0.828821	0.810737	0.839457
1988	0.773454	0.966798	0.585697	0.88256	0.717478	0.935897	0.731788	0.705882	0.783174	0.815154	0.694215	0.761083	0.823723	0.838658
1989	0.773748	0.968071	0.649476	0.904046	0.680263	0.931949	0.715914	0.726441	0.782785	0.826174	0.694014	0.788841	0.832317	0.847478
1990	0.776674	0.991427	0.554398	0.905925	0.680641	0.924362	0.676759	0.686997	0.773025	0.824737	0.692308	0.751274	0.836671	0.847319
1991	0.674632	0.990004	0.725118	0.907446	0.717718	0.919509	0.707328	0.728736	0.731959	0.842162	0.614092	0.805258	0.857004	0.862206

Table3i

Rank of 15|10SV:10yr.MV (by rows)

Company	Cler M&G	Equitable L	Friends Pr	G.A. Life A	Nat Mutual	Norwich U	Pearl Assu	RefugeAss	Scot Amic	ScotLifeAs	Scott Mutu	ScotProvA	Scottish W	Standard L	Rij	('Rij)^2
Year	Rank of 15|10 SV: 10yrMV															
1986	2	2	4	2	4	6	6	2	5	3	3	6	3	3	62	3844
1987	1	1	3	1	3	5	5	1	6	1	2	5	1	2	45	2025
1988	4	3	2	3	5	4	4	4	4	2	6	2	2	1	56	3136
1989	5	4	5	4	1	3	3	5	3	5	5	3	4	5	65	4225
1990	6	6	1	5	2	2	1	3	2	4	4	1	5	4	56	3136
1991	3	5	6	6	6	1	2	6	1	6	1	4	6	6	70	4900
																14708

Table 3ii

We calculate $\chi^2 = 6.16$.

From tables, $\chi^2(5)$ at 5% significant level is 11.07.

Hence, result is not significant at 5% level of significance and so we do not reject the above hypothesis and conclude that the ratio 15|10SV:10yr.MV does not differ significantly over time (1986-1991).

4/

In table 4, we are interested in testing the hypothesis that, the ratio 25|10SV:10yr.MV does not differ significantly over time (1986-1994).

Thus, in the table below, a=9 and b=14.

25|10SV:10yr.MV

Company	Cler M&G	Equitable L	Friends Pr	G.A. Life A	Nat Mutual	Norwich U	Pearl Assu	RefugeAss	Scot Amica	ScotLifeAs	Scott Mutu	ScotProvA	Scottish W	Standard
Year	25|10 SV:10yrMV													
1986	0.525462	0.584996	0.546502	0.619074	0.633447	0.815153	0.717565	0.498134	0.659917	0.601059	0.646975	0.829428	0.554484	0.694444
1987	0.469438	0.595113	0.511619	0.608285	0.620887	0.799657	0.676817	0.495703	0.682958	0.595205	0.633462	0.731069	0.520768	0.679712
1988	0.600086	0.905199	0.511786	0.644153	0.654419	0.794872	0.665563	0.52381	0.645707	0.596525	0.681818	0.670555	0.550247	0.683307
1989	0.600604	0.9051	0.616352	0.672718	0.620722	0.786125	0.653443	0.547575	0.646688	0.611888	0.682147	0.710438	0.567509	0.694547
1990	0.600432	0.930562	0.622759	0.677235	0.62082	0.770888	0.619616	0.506989	0.633348	0.610416	0.68019	0.677165	0.76502	0.69341
1991	0.463235	0.928727	0.635502	0.678387	0.652152	0.762165	0.648794	0.549425	0.60075	0.632959	0.594038	0.724927	0.768969	0.719539
1992	0.560429	0.919626	0.66779	0.613177	0.685333	0.777999	0.607426	0.578568	0.637913	0.632911	0.690789	0.728953	0.739768	0.75305
1993	0.64783	0.938756	0.71634	0.640516	0.701429	0.750429	0.660691	0.605839	0.70753	0.669087	0.720399	0.760374	0.763859	0.788442
1994	0.656629	0.922728	0.727083	0.639189	0.652745	0.758016	0.631644	0.61965	0.680493	0.661786	0.698936	0.803334	0.767266	0.805728

Table 4i

Rank of 25|10SV:10yr.MV (by rows)

Company	Cler M&G	Equitable L	Friends Pr	G.A. Life A	Nat Mutual	Norwich U	Pearl Assu	RefugeAss	Scot Amica	ScotLifeAs	Scott Mutu	ScotProvA	Scottish W	Standard L	Ri	('Ri)^2
Year	Rank of 25|10 SV : 10yrMV															
1986	3	1	3	3	4	9	9	2	6	3	3	9	3	4	62	3844
1987	2	2	1	1	3	8	8	1	8	1	2	6	1	1	45	2025
1988	5	4	2	6	7	7	7	4	4	2	5	1	2	2	58	3364
1989	7	3	4	7	1	6	5	5	5	5	6	3	4	5	66	4356
1990	6	8	5	8	2	4	2	3	2	4	4	2	7	3	60	3600
1991	1	7	6	9	5	3	4	6	1	7	1	4	9	6	69	4761
1992	4	5	7	2	8	5	1	7	3	6	7	5	5	7	72	5184
1993	8	9	8	5	9	1	6	8	9	9	9	7	6	8	102	10404
1994	9	6	9	4	6	2	3	9	7	8	8	8	8	9	96	9216
																46754

Table 4ii

We calculate $\chi^2 = 25.28$.

From tables, $\chi^2(8)$ at 1% significant level is 20.09.

Hence, result is significant at 1% level of significance and so we reject the above hypothesis and conclude that the ratio 25|10SV:10yr.MV differs significantly over time (1986-1994).

5/

In table 5, we are interested in testing the hypothesis that, the ratio 25|10SV:10yr.MV does not differ significantly over time (1986-1991). Here, a=6 and b=14.

25|10SV:10yr.MV

Company	Cler M&G	Equitable L	Friends Pr	G.A. Life A	Nat Mutual	Norwich U	Pearl Assu	RefugeAss	Scot Amica	ScotLifeAs	Scott Mutu	ScotProvA	Scottish W	Standard L
Year	25|10 SV:10yr.MV													
1986	0.525462	0.584996	0.546502	0.619074	0.633447	0.815153	0.717565	0.498134	0.659917	0.601059	0.646975	0.829428	0.554484	0.694444
1987	0.469438	0.595113	0.511619	0.608285	0.620887	0.799657	0.676817	0.495703	0.682958	0.595205	0.633462	0.731069	0.520768	0.679712
1988	0.600086	0.905199	0.511786	0.644153	0.654419	0.794872	0.665563	0.52381	0.645707	0.596525	0.681818	0.670555	0.550247	0.683307
1989	0.600604	0.9051	0.616352	0.672718	0.620722	0.786125	0.653443	0.547575	0.646688	0.611888	0.682147	0.710438	0.567509	0.694547
1990	0.600432	0.930562	0.622759	0.677235	0.62082	0.770888	0.619616	0.506989	0.633348	0.610416	0.68019	0.677165	0.76502	0.69341
1991	0.463235	0.928727	0.635502	0.678387	0.652152	0.762165	0.648794	0.549425	0.60075	0.632959	0.594038	0.724927	0.768969	0.719539

Table 5i

Rank of 25|10SV:10yr.MV (by rows)

Company	Cler M&G	Equitable L	Friends Pr	G.A. Life A	Nat Mutual	Norwich U	Pearl Assu	RefugeAss	Scot Amica	ScotLifeAs	Scott Mutu	ScotProvA	Scottish W	Standard L	Rij	('Rij)^2
Year	Rank of 25|10 SV:10yr.MV															
1986	3	1	3	2	4	6	6	2	5	3	3	6	3	4	51	2601
1987	2	2	1	1	3	5	5	1	6	1	2	5	1	1	36	1296
1988	4	4	2	3	6	4	4	4	3	2	5	1	2	2	46	2116
1989	6	3	4	4	1	3	3	5	4	5	6	3	4	5	56	3136
1990	5	6	5	5	2	2	1	3	2	4	4	2	5	3	49	2401
1991	1	5	6	6	5	1	2	6	1	6	1	4	6	6	56	3136
																14686

Table 5ii

We calculate $\chi^2 = 5.71$.

From tables, $\chi^2(5)$ at 5% significant level is 11.07.

Hence, result is not significant at 5% level of significance and so we do not reject the above hypothesis and conclude that the ratio 25|10SV:10yr.MV does not differ significantly over time (1986-1991).

From table 2-5, a significant result was obtained over the period of 1986-1994, which suggests that the ratio SV:MV were not the same over this period. This support the fact that companies were paying out more SV to surrendering policyholders as opposed to those that stayed on to receive maturity benefit. However, a non-significant result was obtained over the period of 1986-1991, which means that the ratio SV:MV was homogenous from 1986-1991 and non-homogenous afterwards. This again confirms that life offices' were paying higher SVs as MVs after 1991.

Chapter 3

3.1 The Investment Model

3.1.1 Introduction

In this section, we propose a stochastic investment model corresponding to the office's liability, to be defined in chapter 4. This model is similar to the model of index-linked gilts in Wilkie (1995), but with the mean value a function of the surrender force of interest. In particular, we use Wilkie's 1995 model of gilt yield, with mean gilt yield replaced by our proposed surrender force of interest, δ_t^s. This is equal to the redemption yield at 27/5/98 of a *t*-year gilt. With this feature, it can be used to meet the life office's liability when the insured can terminate the policy at any time *t*. A model of δ_t^s is obtained by fitting a non-linear regression equation via least squares to relevant financial data, to be discussed later on in this chapter. This model is realistic and relevant to our investigations, as well as the life office, faced with a possibility of early termination of life contracts.

A simulation-based approach will be used since the model under investigation follows a stochastic process. A description of the Wilkie model is given and also, measures taken to allow for some of its limitations are given as well. Further, we give a description of our proposed model. The simulation results produced by our model are then compared with those of Wilkie (1995) and subsequently analysed. A sensitivity analysis of the model to changes in the parameter values is presented as well.

3.1.2 Choice of Investment Model

Since this thesis is considering a conventional non-profit life policy that can be purchased in a lump sum, a suitable investment model corresponding to the life office's liability is the one mentioned in the introduction of section of 3.1.1. i.e., a model similar to Wilkie's 1995 model of gilt yield at time t, with mean gilt yield replaced by our proposed surrender force of interest, δ_t^s. We employed Wilkie's 1995 model because it remains the most widely used and accepted model in the actuarial profession in spite of the criticism it has attracted and also, because we desire to model a simple *AR(1)* process, independent of inflation for our investment model. Since it would be impractical to include in our model all the series suggested by Wilkie (1995), we will discuss only those that are relevant to our investigation, and also discuss our proposed model in the next section of this thesis.

3.2 Wilkie Stochastic Investment Model

The Wilkie model (originally proposed in 1986, and revised later on in 1995) is the principal stochastic investment model in use in the UK. Even though the model has attracted some criticism (for example, as reviewed by Kitts (1988,1990), Clarkson (1991), Geoghegan et al (1992), and Huber (1995)), little of this has been backed up with the suggestion of a meaningful alternative model. Nevertheless, Wright (1999), has recently suggested an alternative to the Wilkie model (see Wright (1999)). The proposed model (by Wilkie (1986)) discussed the following series:

i) force of price inflation,
ii) share dividend yield,
iii) force of share dividend growth,
iv) long-term interest rate (yield on $2\frac{1}{2}\%$ Consols).

The model was then extended by Wilkie (1995) to cover the following additional series:

v) force of salary growth,

vi) short-term interest rate,

vii) yield on long-dated, index-linked gilts,

viii) property rental yield,

ix) force of property rental growth.

Further, the model parameter values used in the original model have been updated based on the experience of the intervening 9 years.

As mentioned before, we will not review all the series listed above. Instead, we discuss in detail the gilt yield model (an *AR(1)* process, independent of inflation) since such a model is directly related to our investigations.

3.2.1 Model of Index-linked gilt Yield

The model proposed for index-linked gilt yield at time t, $R(t)$, published in Wilkie (1995) is as follows:

$$\ln R(t) = \ln RMU + RA.\left[\ln R(t-1) - \ln RMU\right] + RE(t) \qquad 3.1$$

where :

$R(t)$ is the yield on index-linked gilt at time t;

RMU is the mean gilt yield;

$RE(t)=RSD.RZ(t)$ is the random component of the gilt yield at time t; and

$RZ(t)$ is $N(0,1)$ white noise series.

This model implies that gilt yields follow an *AR(1)* process independent of inflational process with log-normally distributed error terms. *RMU* and *RSD* reflect the general level of the mean and standard deviation of gilt yields respectively. *RA* is a parameter, which controls the strength of the auto-regression. Thus, a higher value of *RA* implies that the series can be expected to move more slowly back towards the mean value over time, and vice versa.

The parameter values suggested by Wilkie (1995) are:

$RMU \quad = 0.04$

$RA \quad = 0.55$

$RSD \quad = 0.05$

Note that we are using the initial starting value used by Wilkie, which is

$R(0)=RMU=4\%$.

Wilkie shows that:

$$\ln R(t \to \infty) \sim N\left(\ln RMU, \frac{RSD^2}{1-RA^2}\right).$$

Therefore, the ultimate distribution of the gilt yield is log-normal with :

$$E[R(t \to \infty)] = \exp\left[\ln RMU + \frac{1}{2}\left(\frac{RSD^2}{1-RA^2}\right)\right]$$

$$= 4\%$$

and

$$V[R(t \to \infty)] = (E[R(t \to \infty)])^2 \left[\exp\left(\frac{RSD^2}{1-RA^2}\right) - 1\right]$$

$$=(0.24\%)^2.$$

It is worth mentioning that the simulations were obtained on the basis of the initial set of conditions provided (which we need in order to start any simulation). Thus, in this simulation, we use the 'neutral' starting conditions suggested by Wilkie (1995). This represents the long-term mean of the variables under investigations assuming that the random component is set equal to zero. Further, we generate values for the series $R(t)$, starting at time $t=0$ *for* $t=1$ to n ($n=20$ in this case) and perform many simulations (for example, 1000). We simulate independent unit pseudo-random variables for the white noise. It is worth noting that the randomization routine used in the program generates the same set of *20 x 1000* random numbers. This is to remove sampling error when comparing results.

Now, by using Wilkie's suggested parameter values, we plot 25 simulations of the future path of yield, $R(t)$, over a 20 year projection period.

Figure 3.1 shows the first 25 of the 1000 simulations of the gilt yield variable, $R(t)$.

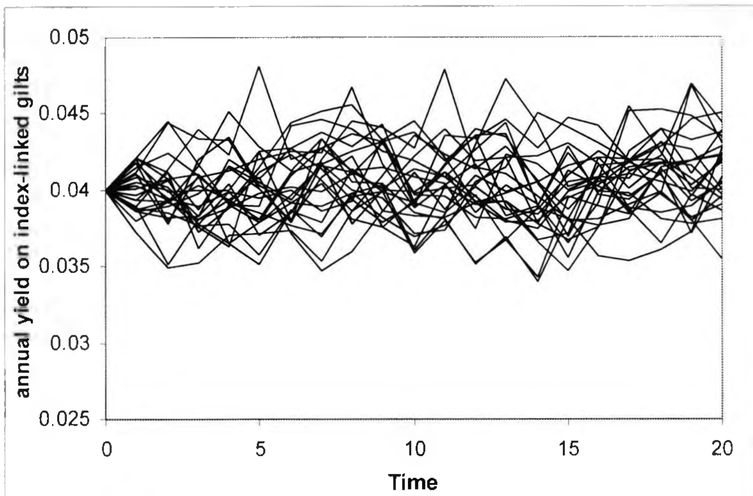

Figure 3.1-Twenty five simulations of the index-linked gilt yield model over 20 years using Wilkie (1995).

Figure 3.2 shows a plot of 10^{th}, 25^{th}, 50^{th}, 75^{th}, and 90^{th} percentiles of gilt yield distribution by using Wilkie's 1995 model. These percentiles are obtained by using the same 1000 simulations of future investment experience.

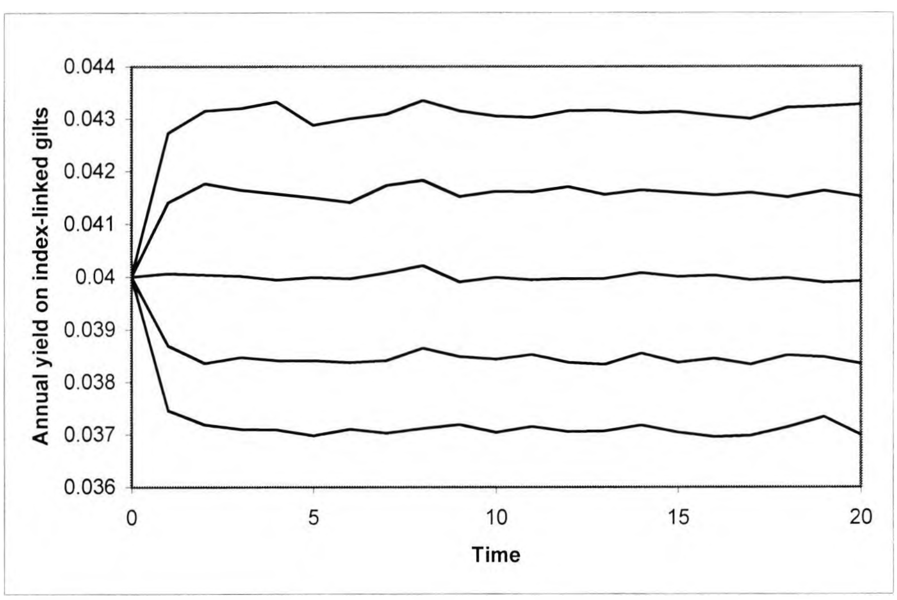

Figure 3.2-10^{th}, 25^{th}, 50^{th}, 75^{th} and 90^{th} percentiles of the index-linked gilt yield over 20 years using Wilkie (1995).

From figure 3.2, we observe that the equilibrium position is reached very quickly. Therefore, according to Wright (1998), the annual yields on gilts in consecutive years are (largely) independent.

Furthermore, figure 3.3 gives an empirical distribution of the shape of the gilt yield using the standard Wilkie(1995) model after the equilibrium position has been reached.

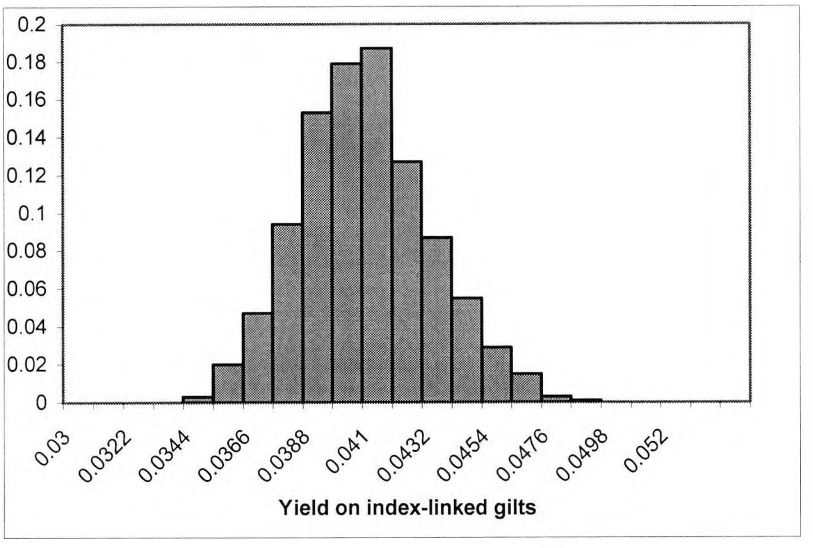

Figure 3.3: Empirical equilibrium distribution of the shape of index-linked gilt yield For standard Wilkie model.

From figure 3.3, we observed that the distribution appears to be positively skewed. This is clearly what would be expected for a log-normal random variable.

We assess the log normal fit on the simulated result by transforming the data using a natural logarithmic transformation. Thus, if the transformed distribution (i.e, *ln(R(t))* is normal, then it follows that *R(t)* has a log normal distribution. The empirical distribution of the shape of the transformed data from the standard Wilkie (1995) model is shown by figure 3.3a.

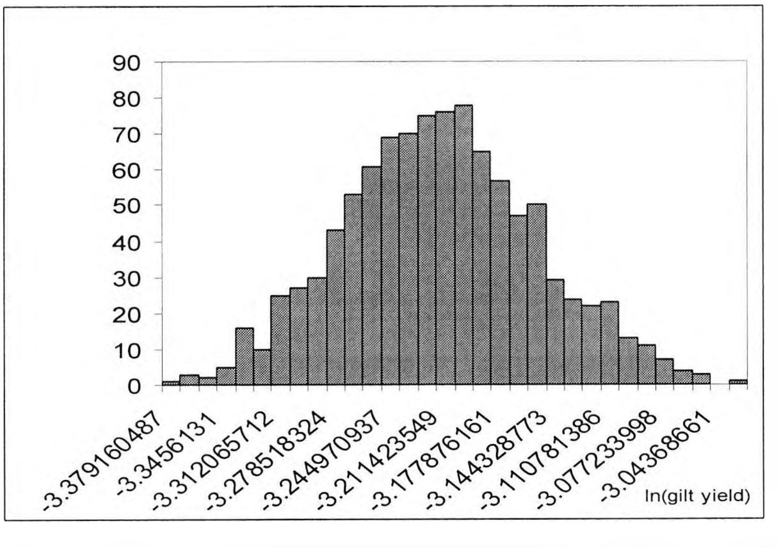

Figure 3.3a: Empirical equilibrium distribution of the shape of ln(gilt yield) for standard Wilkie model.

From figure 3.3a, we observe that the distribution appears to be normal.

Also, figure 3.3b shows a normal q-q plot of the transformed data. This is used to assess whether the data have a particular distribution, or whether two data sets have the same distribution. Thus, if the distributions are the same then the plot will be approximately a straight line, but the extreme points will have more variability than points towards the centre. However, a plot with a `U` shape means that one distribution is skewed relative to the other and finally, an `S` shape implies that one distribution has a longer tail than the other.

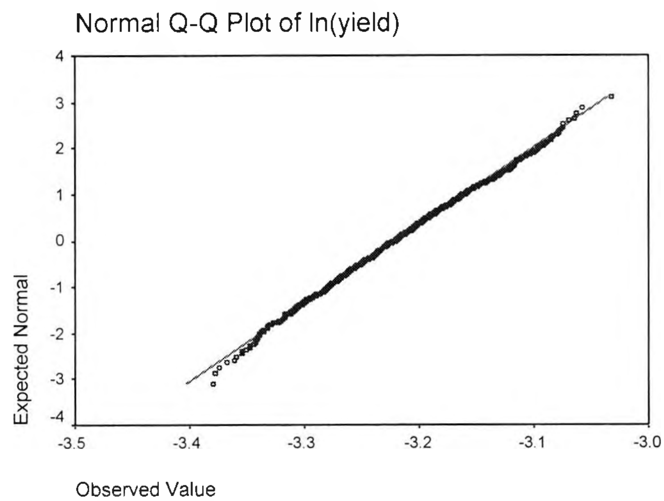

Figure 3.3b: Normal q-q plot of ln(gilt yield) for standard Wilkie model

From figure 3.3b, we observe that the plot is approximately a straight line indicating that the distribution appears to be normal. Hence, from figures 3.3a and 3.3b, we can say that *lnR(t)* is normal indicating that the distribution of *R(t)* at equilibrium is log normal.

It is worth mentioning that the results of section 3.2.1 are similar to those reported by Wilkie (1995).

3.3 A model of Force of Interest for projection year t.

3.3.1 Introduction

Before we discuss our stochastic investment model (gilt yield model) to be applied to the profit/loss model, it is important that we first look at a model of the gilt yield curve, which is the redemption yield at 27/5/98 of a t-year gilt considered as a function of surrender force of interest for projection year t, obtained by fitting a smooth curve to financial data (for example, the redemption yield of UK gilts plotted against redemption dates and published in the Financial Times (1998)). Such a model connects the gross redemption yields of gilt with different redemption dates. We seek a model of this kind because it is assumed that the investment decisions of investors are based on the expectations of the future level of interest rates for different time horizons. Further, the gilt yield with different/several redemption dates is appropriate for our studies, whereby surrender of a policy can be made at any time and the life office can redeem such a security to repay the policyholder.

We discuss below the types of term-structure models reported in the literature and a model of yield curve used in this investigation. We also look at different yield curves and attempt to investigate the effect of the shape of these curves on our investment model. These models are discussed below:

3.3.2 Types of Term-Structure Models

The models of term-structure of yield curve can be divided into three main categories. These are 'descriptive', 'equilibrium', and 'evolutionary' models. Fieldman et al. (1997) describes these models as follows:

- Descriptive models aim to describe the yield curve in terms of a mathematical equation, for example, a polynomial.
- Equilibrium models start with assumptions about economic variables, and then use these to drive the entire yield curve.
- Evolutionary models take the initial term structure as input, and then allow it to evolve into the future.

The actuarial approaches to modelling the gilt market have centred on descriptive models which are well suited for statistical analysis (e.g., yield indices). On the other hand, financial economics has tended to focus firstly on the equilibrium models, and then more recently on the evolutionary models. These models are used for the valuation of interest-rate contingent claims and other derivative products. On this note, we can say that a key factor to dictate the appropriate choice of a model is the purpose to which it is to be put.

Thus, in this thesis, we will use descriptive models to describe the yield curve under investigation. It is important to note that there are other approaches to describe the term-structure of interest rate as mentioned in the Bank of England Quarterly Bulletin, (1972), (1976), (1982), (1990), (1991) (these are descriptive models) and in the financial economics literature: see for example Panjer et al (1998).

3.3.3 Fitting of Yield Curve Model

The fitted model used in this thesis is similar to the FTSE Actuaries Government Securities (FTSEAGS) yield indices developed by Dobbie and Wilkie (1978) and later reformulated by Cairns (1997), which will be discussed later on in this section. That is, a model of redemption yield at 27/5/98 of a t-year gilt, which is obtained by fitting a non-linear regression via least squares to financial data. Such a model connects the gross redemption yields of gilts with different redemption dates.

In general, various techniques have been proposed in the literature for fitting yield curves. Examples of such techniques include the following:

- The use of piecewise polynomial functions- this is a consecutive series of polynomial functions which when joined together forms a curve.
- Cubic Spline technique- this connects each successive pair of nodes using cubic polynomials. A cubic spline is made up of natural and clamped cubic splines. For a natural cubic spline, the curve tends towards a straight line at the end points. (i.e., the second derivative of the function at the end points equals zero). Whereas for a clamped cubic spline, the first derivative of the function at the end points has specified values. (For a description of the cubic spline

techniques, see De Boor, (1978), McCutcheon, (1981), Burden and Faires (1985) and Booth et al (1993)).

Further, many authors have proposed models for the yield curve of gilt-edged stocks using different approaches. For example, Marshall (1954) and Pepper (1964) formulated a model for the yield curve defined by the gross redemption yields of stocks with different redemption dates or volatilities. Also, Burman and White (1972) formulated the adjustments required to incorporate the effect of coupon on the curves using the expectation hypothesis (i.e., investors' expectation of the future levels of interest rates), and introduced the notion of par yield curves to assist in judging the appropriate terms of new issues of gilt-edged securities. Their model has been used by the Bank of England for quite some time until it was superseded in 1990 by the modified model of Mastronikola (1991). Dobbie and Wilkie (1978) proposed a descriptive model of gross redemption yield (compounded semi-annually) for a bond which matures at time t. Their method splits the type of bond into high, medium and low-coupon bands and fits separate yield curves to each. A simple least-squares approach was used to estimate the parameters. However, this method has been identified as "susceptible to 'catastrophic' jumps when the least-squares fit jumps from one set of parameters to another set of quite different values". As a result of the findings of Dobbie and Wilkie (1978), Cairns (1997) reformulated their model in order to remove the risk of catastrophic changes. See Cairns (1997) for details of the yield model used. Also, Clarkson (1979) described the effect on the relationship between coupons and yields of the relative attractiveness of capital gains with coupon income. It is worth mentioning that our proposed yield curve model is similar to Dobbie and Wilkie and that of Cairns (1997), but approached differently.

In fact we could use a natural cubic spline to fit a curve passing through given data points so as to get a perfect fit. But this could result in achieving an erratic curve. In addition, the natural cubic spline is highly parameterised and too many parameters create problems with projections. As a result of this, the use of cubic splines is undesirable here. Therefore, to fit a model that will capture the underlying trend of the crude data but is not too erratic, we use a non-linear regression approach via least squares. This approach tries to estimate parameters to minimize the sum of squared differences between the response (i.e., the observed gross redemption gilt yields) and the prediction.

3.3.4 The Fitted Model

Now, a scatter plot of gross redemption yield against term to maturity (of financial data (UK Gilts) from the Financial Times for a typical day- May 27, 1998) follows a non-linear pattern. Therefore we propose a model of the form:

$$\delta_t^s = A + \exp(B \cdot t + C) + \varepsilon \qquad 3.2$$

where,

δ_t^s is the redemption yield at 27/5/98 of a *t*-year gilt.

A, B, C are parameters and ε, an error term is assumed Gaussian with constant variance.

Note that non-linear regression needs starting estimates for the parameters. These can be obtained from the plot of figure 3.4, which suggest that δ_t^s is near 0.0585 at *t* near *20.*

The model estimated parameters are given below.

Table 3.2: Model parameter values

Parameters	Value	Std Error	t-value	p-value
A	0.058500	0.0000865	675.0000	0.0001
B	-0.354991	0.0200896	-17.6704	0.0360
C	-4.420810	0.0384400	-115.005	0.0040

Residual standard error: 0.000256413 on 17 degrees of freedom.

By substituting the estimated parameter values (from the fitted model) in equation 3.2, we obtain the following model:

$$\delta_t^s = 0.0585 + \exp(-4.4208 - 0.35499 * t), \qquad 3.3$$

From the above estimates we observe that the parameter values are significant. Also, from figure 3.4, we observe that there are a number of departures from the curve from time 7 onwards, where the data show a series of waves and the fitted curve cuts through these. Nevertheless, since the curve captures the underlying trend of the crude data points and it is neither so erratic nor so regular that the trend is lost, we can say that the fitted curve is adequate. Thus, the fitted model is an exponential function of force of interest for projection year t and represents the mean function at time t, of our proposed stochastic investment model.

A scatter plot of the observations and a plot of our fitted model are shown in figure 3.4.

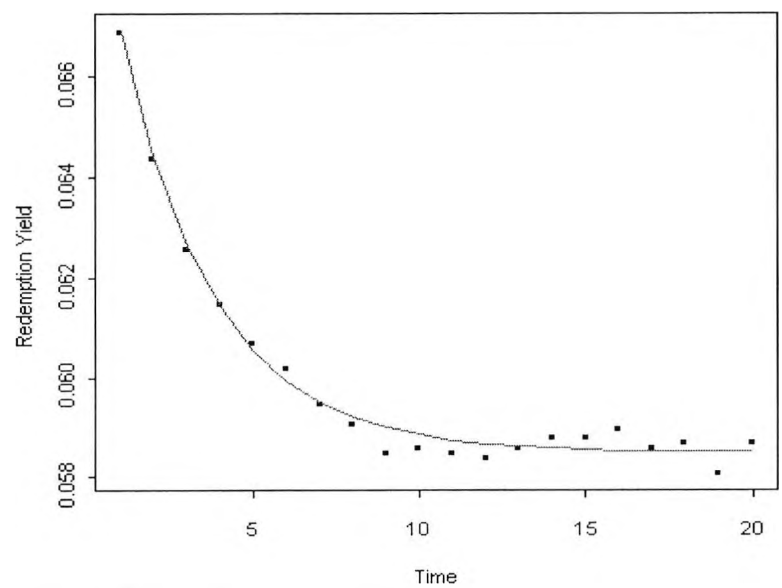

Figure 3.4- plot of fitted yield curve of redemption gilt yields on May 27, 1998 against time to redemption

We notice that Booth et al (1993) fitted a "best fit" spline to a large set of data points (gross redemption dates for British government bonds (gilt-edged securities) with a coupon rate between 7% and 13% on 8 January 1991)- the data points do not all lie precisely on the fitted curve. Further, Mastronikola (1991) modelled a par yield curve by using a smoothly spliced structure of cubic polynomial. This model was fitted to gilts data for each of 32 dates ranging from 1974 to 1989. The yield curve produced by Booth and that for March 31, 1989 and October 4, 1989 by Mastronikola are similar to our curve even though the methodology used is different. We chose these particular dates for comparison purposes.

3.3.5 Yield Curve with Positive gradient

We consider the effect of the shape of yield curves on our investment model. In particular, we look at the effect of yield curves with positive gradient, and also a flat yield curve, on our investment model. Figure 3.5 shows a new yield curve with positive slope. This curve was obtained by transforming the yield curve of figure 3.4, using the function defined by equation 3.4.

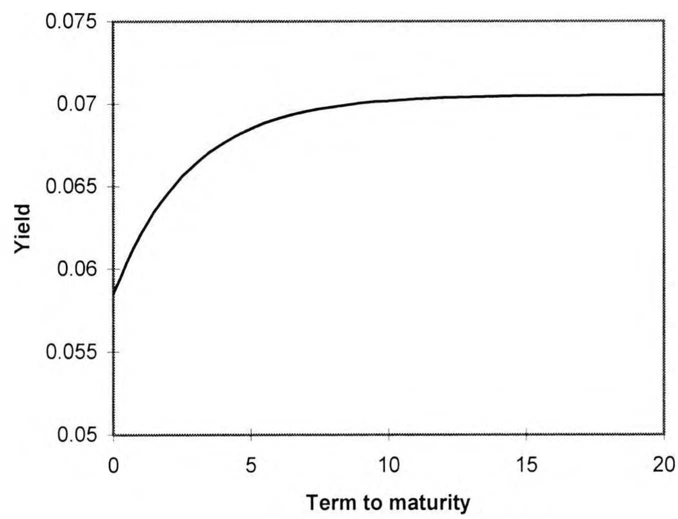

Figure 3.5 - Yield Curve of gilt yields (with Positive Gradient)

The modified model is given by:

$$\delta_t^{s'} = 0.0585 + \{\exp(-4.4208)\} \cdot \{1 - \exp(-0.35499t)\} \qquad 3.4$$

where,

$\delta_t^{s'}$ is the redemption yield at 27/5/98 of a t-year gilt (with positive gradient).

This model is also an exponential function of force of interest for projection year t and represents the mean function at time t, of our proposed stochastic investment model.

Finally, figure 3.6 shows a plot of flat yield curve, with constant value of 0.0585.

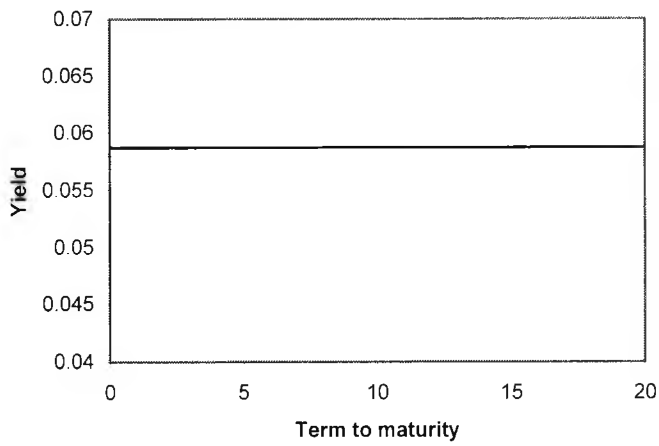

Figure 3.6 - Yield Curve of gilt yields (flat yield curve)

3.3.6 A model of Force of Interest for projection year t.

We now propose a model of a t-year gilt yield, δ_t, which is given by equation 3.5. This model is based on Wilkie (1995) model for *R(t)* in 3.2.1 with *RMU* replaced by the surrender force of interest δ_t^s. We have replaced *RMU* by the surrender force of interest because we believe that this is appropriate for modelling the life office's liability when the insured can terminate the policy at any time t, which is the case considered here.

Thus,

$$\ln \delta_t = \ln \delta_t^s + \alpha.\left[\ln \delta_{(t-1)} - \ln \delta_{(t-1)}^s\right] + \sigma.\varepsilon(t) \qquad 3.5$$

where

δ_t is the yield of a t-year gilt.

δ_t^s is the redemption yield at 27/5/98 of a t-year gilt. This is a function of the proposed yield curve model, as defined in section 3.3.4.

$\sigma.\varepsilon(t)$ is the random component of the gilt yield at time t.

α is the parameter controlling the strength of the auto-regression, as defined in section 3.2 and

$\varepsilon(t)$ is a *N(0,1)* white noise series.

This model says that the natural logarithm of a gilt yield follows a first order auto-regression process, with a mean, $\ln\delta_t^s$, and a parameter α, such that the expected yield in each year is equal to the mean at that time plus α times last years deviation from the mean at time t.

Parameter values:

The parameter values of model 3.5 are listed below. In view of the nature of our model, we believe that the following parameter values considered are reasonable.

$$\alpha = 0.9$$

$$\sigma = 0.05$$

$$\delta_{(0)} = 0.065.$$

$\delta_{(0)}$ is the 'neutral' starting condition. The standard deviation value σ is the same as suggested by Wilkie.

From the above model (equation 3.5), the mean function for the process $\ln\delta_t$ is given by:

$$E[\ln\delta_t] = \ln\delta_t^s + \alpha[E(\ln\delta_{(t-1)}) - \ln\delta_{(t-1)}^s]. \qquad 3.6$$

Note that in equation 3.6, we are using the limiting values for large n as an approximation. Hence, letting $t \to \infty$, we obtain :

$$E[\ln\delta_{(t\to\infty)}] = \frac{\ln\delta_{(t\to\infty)}^s - \alpha\ln\delta_{(t\to\infty)}^s}{1-\alpha} \qquad 3.7$$

$$= \ln\delta_{(t\to\infty)}^s$$

$$= \lim_{t\to\infty} \ln[0.0585 + \exp(-4.4238 - 0.4567.t)]$$

$$= \ln(0.0585)$$

$$= -2.8387.$$

Further, the variance function for the process $\ln\delta_t$ is given by :

$$V[\ln\delta_t] = \alpha^2.V[\ln\delta_{(t-1)}] + \sigma^2 \qquad 3.8$$

Letting $t \to \infty$, we obtain

$$V[\ln\delta_{(t\to\infty)}] = \frac{\sigma^2}{1-\alpha^2} \qquad 3.9$$

$$= 0.1147^2.$$

Now, if X is a random variable such that:

$$lnX \sim N(\mu, \sigma^2), \tag{3.10}$$

then, we can show that X has a log-normal distribution with:

$$E[X] = \exp(\mu + \tfrac{1}{2}\sigma^2) \tag{3.11}$$

$$V[X] = \{E[X]\}^2 . \left[\exp(\sigma^2) - 1\right]. \tag{3.12}$$

Therefore, we obtain

$$E[\delta_{(t\to\infty)}] = \exp\left[\ln \delta^s_{(t\to\infty)} + \frac{1}{2}\cdot\left(\frac{\sigma^2}{1-\alpha^2}\right)\right] \tag{3.13}$$

$$= 5.89\%.$$

$$V[\delta_{(t\to\infty)}] = \left(E[\delta_{(t\to\infty)}]\right)^2 . \left[\exp\left(\frac{\sigma^2}{1-\alpha^2}\right) - 1\right] \tag{3.14}$$

$$= (0.678\%)^2 .$$

Now, by using equation 3.5, we generate values for the series δ_t, starting at time $t=0$ *for* $t=1$ to n ($n=20$ in this case) and perform many simulations (for example, 1000). Also, by using the above parameter values, we plot 25 simulations of the future path of yield, δ_t, over a 20 year projection period. Figure 3.7 shows the 25 independent simulations of the future progress of gilt yield using equation 3.5.

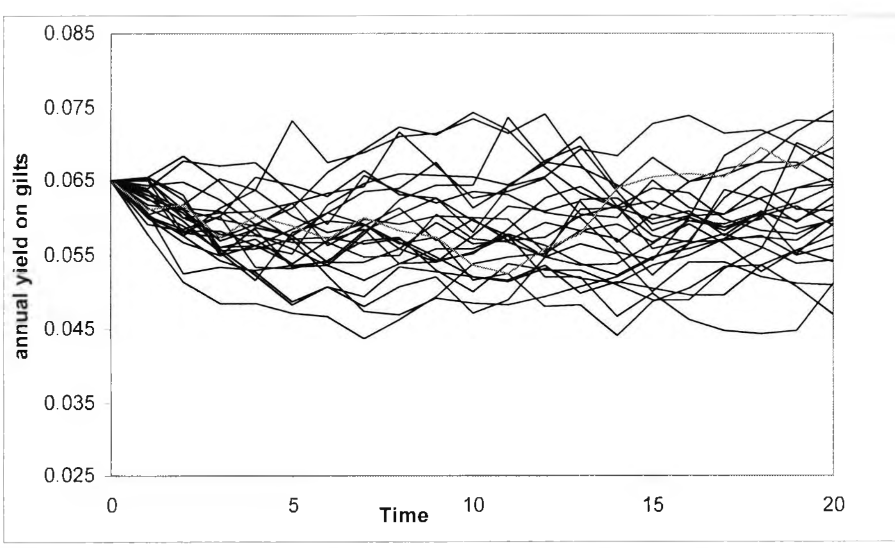

Figure 3.7-Twenty five simulations of a gilt model over 20 years using our proposed model

From above plot (figure 3.7), we observe that the process is erratic from year to year.

Further, figure 3.8 gives an empirical distribution of the shape of the probability distribution function (obtained by means of 1000 simulations) of the gilt yield (using our model) after equilibrium position has been reached.

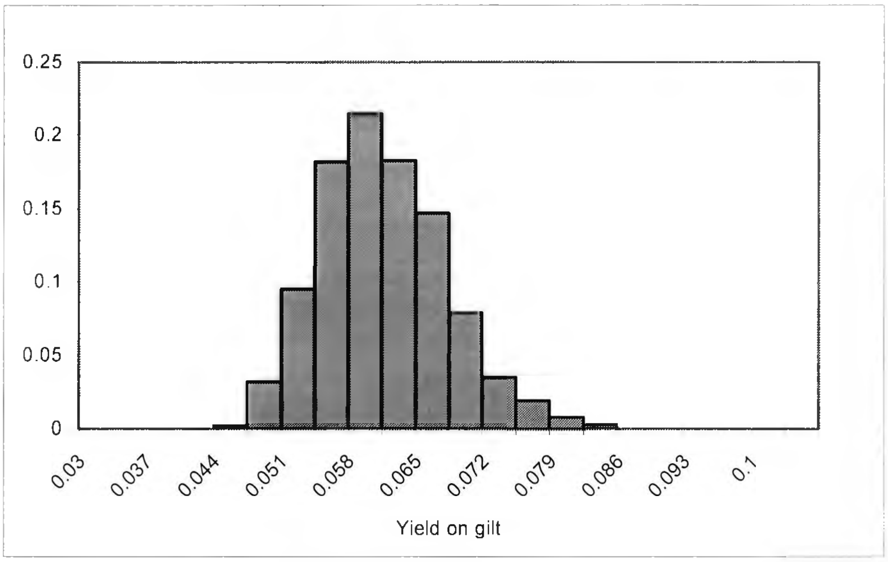

Figure 3.8: Empirical equilibrium distribution of the shape of gilt yield using our proposed model

From figure 3.8, we observed that the distribution is positively skewed, similar to the Wilkie (1995) model. This is what we would be expected for a log-normal random variable.

Further, we assess the log normal fit on the simulated result (using our proposed model) by transforming the data using a natural logarithmic transformation. Figure 3.8a gives an empirical distribution of the shape of the transformed data using our proposed model.

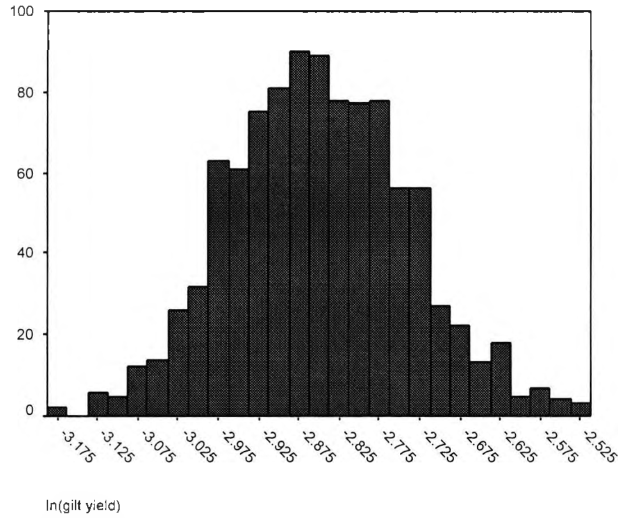

Figure 3.8a: Empirical equilibrium distribution of the shape of ln(gilt yield) using our proposed model.

From figure 3.8a, we observe that the distribution appears to be normal.

Also, figure 3.8b shows a q-q plot of the transformed data using our proposed model.

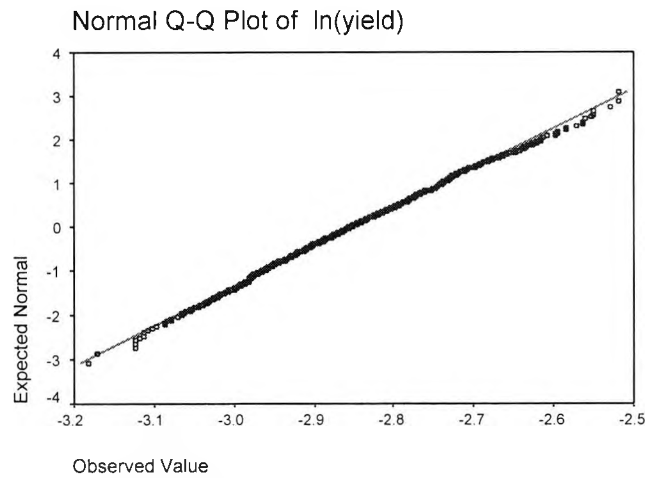

Figure 3.8b: Normal Q-Q plot of ln(gilt yield) using our proposed model

From figure 3.8b, we observe that the plot is approximately a straight line indicating that the distribution appears to be normal. Hence, from figure 3.8a and 3.8b, we can say that $ln\delta_t$ is normal indicating that the distribution of δ_t at equilibrium is log normal.

Finally, figure 3.9 shows a plot of the 5th, 10th, 25th, 50th, 75th, 90th and 99th percentiles of the distribution of the nominal annual yield on gilts in year t. These percentiles are obtained by using 1000 simulations of the future investment experience.

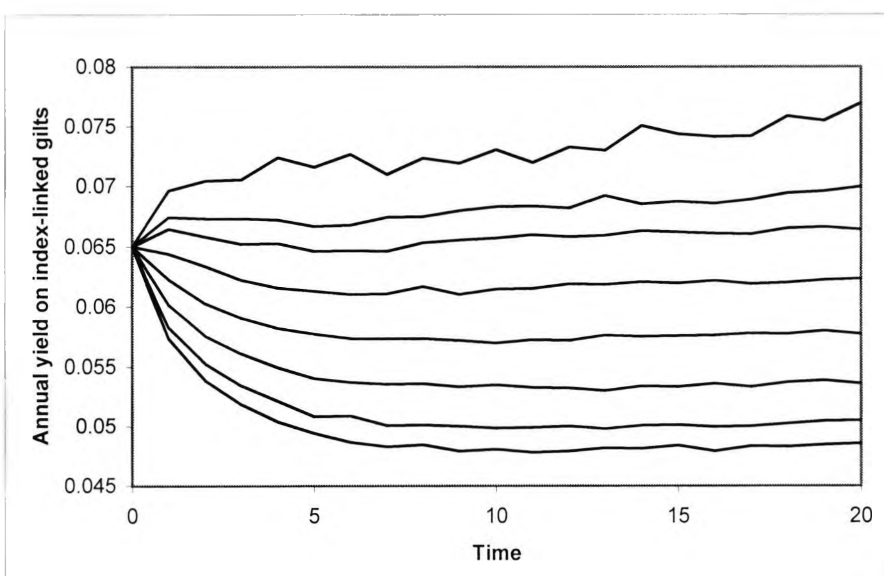

Figure 3.9- 5th, 10th, 25th, 50th, 75th and 90th percentiles of the gilt yield model over 20 years using our model

Like the percentiles from the Wilkie (1995) model of index-linked gilts, it can be seen that the equilibrium position is reached very quickly. This shows that the annual gilt yields achieved in consecutive years are (largely) independent.

3.3.7 Simulation Results

The expected yield from simulation, rounded to the nearest percent, to be realised at any future time t by using the gilt yield model (our model) is shown in table 3.2. Also shown, is the corresponding standard deviation of the process.

Term(years)	1	5	10	15	20
Mean(%)	6.2328	5.8273	5.7839	5.7830	5.8190
STD(%)	0.3158	0.5392	0.6099	0.6597	0.6523

Table 3.4: Results for gilt yield from 1,000 simulations using model 3.5

From table 3.4, we observe that the mean rate of return on the gilt is over 6% for term, n =1, dropping quickly to around 5.8% by term 5, and slowly further to a little over 5.7% for longer terms (i.e., term 10 and 15), and a little over 5.8% at term 20. The standard deviation is around 0.3% for term 1, but over 0.5% for terms 5, 10, and 15, reaching a maximum of 0.66% at term 15 and drops to 0.65% at term 20. Therefore, we can conclude that the mean returns are approximately constant (as the term increases) and also, there is less variability in the interest rate for shorter terms than in the longer terms.

We now looked at the effect of using different parameter values on the process. This is discussed below.

3.4 Effect of changing model parameter values on the process

Different parameter values were applied to our gilt yield model. This is to determine the effect on the expected yield in the long run as the model parameter values are changed. The model parameter values considered are discussed below. Note that apart from these values, all other model parameter values remained the same as before. In what follows, we calculate the expected value of gilt yield at expiry of the policy (i.e., term 20) and perform 1000 simulations as before.

Standard deviation effect

The effect of changing the standard deviation of the process on the expected gilt yield in the long run is looked at. That is, a standard deviation of 1%, 5%,10%, 15%, 20%, and 25% are considered. Table 3.5 shows the results obtained.

σ (%)	1	5	10	15	20	25
Mean(%)	5.7923	5.8191	5.9185	6.0962	6.3601	6.7214
STD(%)	0.1287	0.6522	1.3479	2.1273	3.0389	4.1435

Table 3:5 Results for gilt yield from 1,000 simulations using model 3.5 for different values of σ

We observe that as we increase the standard deviation of the process, the expected gilt yield in the long run increases. The mean increases because for a lognormal random variable the mean depends on σ, as shown by equation 3.13.

α - effect

The effect of α, the strength of auto-regression on the process is now looked at and the following results, shown at table 3.6, are obtained.

α (%)	50	60	70	80	90
Mean(%)	5.8531	5.8556	5.8582	5.8578	5.8191
STD(%)	0.3365	0.3655	0.4094	0.4848	0.6522

Table 3:6 - Results for gilt yield from 1,000 simulations using model 3.5 for different values of α

We know from section 3.2.1 that a higher value of α implies that gilt yields can be expected to move more slowly to the mean value and vice versa. Therefore, the results of table 3.6 shows that the mean rate of return on the gilt seems to reach a peak, which could be due to a random variation since the standard deviations are large.

On the basis of the above results, we re-ran the analysis with more simulation (for e.g., 5000) to check if the results are actually due to a random variation. The following results, shown at table 3.6a are obtained.

α (%)	50	60	70	80	90
Mean(%)	5.8553	5.8553	5.8555	5.8531	5.8134
STD(%)	0.3405	0.3687	0.4125	0.4886	0.6574

Table 3:6a - Results for gilt yield from 5,000 simulations using model 3.5 for different values of α

The results of table 3.6a shows that the mean rate of return on the gilt is level until α reaches values of over 0.80 and then it falls. This indicates that the peak in Table 3.6 is due to random variations.

Starting value effect

The effect of changes in the starting values of the process on the expected gilt yield in the long run was also looked at and the following results, shown at table 3.7, were obtained.

$\delta_{(0)}$ (%)	5	6	7	8	9	10
Mean(%)	5.6364	5.7628	5.8718	5.9679	6.0539	6.1319
STD(%)	0.6318	0.6459	0.6582	0.6689	0.6786	0.6873

Table 3:7: Results for gilt yield from 1,000 simulations using model 3.5 for different values of $\delta_{(0)}$

We observe that as we increase the model starting values the expected yield at term 20 increases as does the corresponding standard deviation. This is as expected.

Effect of shape of yield curve (yield curve with positive slope)

By using the yield curve of figure 3.5, we re-ran the simulation keeping the same parameter values as before. This is to determine the effect of the shape of yield curve (slope of curve) on the expected yield in the long run. Table 3.8 shows the results obtained.

Term(years)	1	5	10	15	20
Mean(%)	6.8418	7.3617	7.3625	7.2323	7.1742
STD(%)	0.3467	0.6812	0.7763	0.8250	0.8041

Table 3:8: Results for gilt yield from 1,000 simulations using model 3.5 with yield curve of positive slope as surrender force of interest.

Unlike the previous result where a yield curve of negative slope was considered, we observed that the mean rate of return on gilt yield steadily rises over 6.8% this time for term 1 until term 15, where it drops steadily thereafter. The standard deviation is around 0.3% for term 1, but over 0.6% for terms 5, 10, and reaches a maximum of about 0.83% at 15 and falls to about 0.80% at 20, which shows that there is less variability in interest rate for shorter terms than in the longer terms (the variability in interest rate increases as term increases). Thus, by comparing the results of tables 3.4 and 3.8 (where the shape of the yield curve is changed from one with a negative slope to a positive slope), we observe that the mean rate of gilt yield has increased and so has the standard deviation of the process. This means that the shape of yield curve has an effect on the expected yield in the long run.

Effect of flat yield curve on model

The simulation result obtained by using the flat yield curve of figure 3.6 in the investment model is shown table 3.9:

Term(years)	1	5	10	15	20
Mean(%)	6.4481	6.2971	6.1509	6.0204	5.9697
STD(%)	0.3267	0.5827	0.6486	0.6868	0.6691

Table 3:9: Results for gilt yield from 1,000 simulations using model 3.5 with flat yield curve as surrender force of interest.

The results from the above table exhibit a pattern, which is different from the other two yield curves considered. This time, we observed that the mean rate of return on gilts is over 6.4% for term 1. This drops steadily thereafter until the end of term. However, the standard deviation followed the same pattern as the other two cases. That is, the standard deviation is around 0.3% for term 1, but over 0.5% for terms 5, 10, and reaches a maximum of about 0.69% at 15 and falls to about 0.67% at 20, which shows that there is less variability in interest rate for shorter terms than in the longer terms (the variability in interest rate increases as term increases). It is worth mentioning that by comparing the results of tables 3.4 and 3.9 (where the shape of the yield curve is changed from one with a negative slope to a zero slope), we observe that the mean rate of return on gilts has increased and so does the standard deviation of the process. This means that the shape of yield curve has an effect on the expected yield in the long run.

In this chapter, we have considered an investment model that can be used to represent the life office's liability when the insured can terminate the policy at any time t. This model is similar to Wilkie (1995) gilt model (constructed from past experience), but with the mean value a function of the surrender force of interest. This feature makes the model more realistic for the long term than the random walk style of model established among financial economists and also, will appeal to the life office.

The result of the simulations obtained from our proposed investment model are similar to those of Wilkie (1995). Thus, we observe that the mean returns are approximately constant (as the term increases) and also, there is less variability in interest rates for shorter terms than for the longer terms. From the simulation results obtained, we can say that the shape of the surrender force of interest (shape of yield curve) has an effect on the expected yield in the long run. In addition, other variables like $\delta_{(0)}$, σ and α affect the expected yield in the long run.

We shall apply the investment model discussed in this chapter to modelling the expected present value of surrender profit/loss (surrender due to no selection effect and another due to financial and non-financial adverse selection effect). This is discussed in greater depth in the next chapter.

Chapter 4

4.1 Models of Surrender Profit/Loss of Non Participating Life Insurance Policies and Adverse Selection

4.1.1 Introduction

In this section, we shall consider non-participating (non-profit) 'conventional' life assurance business only. But the methodology can be extended to deal with problems involving conventional participating life contracts. In a traditional non-profit contract, the "contractual terms are agreed between the company and policyholder at the outset of the policy", Booth et al (1999). However, the terms cannot be changed thereafter. Under this contract, the policyholder agrees to pay a premium (could be single or annual), in return for a guaranteed benefit payable on the occurrence of the insured event(s) within a specified period of time, either for a whole life or for a limited period (term of the contract). Further, the operation of a non-profit policy within a life office is such that premiums paid by policyholders are accumulated in a fund of assets, expenses of the office are deducted from the fund and the policyholders' benefits are paid as claims occur. Therefore, the excess of fund value over expenses and claim payout constitutes profit, which is distributed to the providers of capital: namely, shareholders (or with-profit policyholders for a mutual company). We notice that the sale of a without-profit policy is considered by the life office as an "investment" on the part of the shareholders and/or with-profits policyholders. Similarly, the sale of a with-profit policy may be considered to be an "investment" on the part of the office's shareholders whereby future profits are shared between the with-profits policyholders and the shareholders.

Now, since the surrender of a life policy is at the option of the policyholder, it can greatly affect the profitability of the company if not properly controlled. For example, life office expects to make a loss if it pays for some policies surrender values greater than the corresponding accumulation of premiums less charges (asset shares). On the other hand, profits are expected to be made if the life office pays less than the corresponding asset shares. The consequences of the former risk to a life office (if it occurs) and the measures to be adopted to hedge such risks are what this chapter and the next one will attempt to address. In particular, we shall propose a model to determine the profitability of the company; that is, a model to compute the expected present value of surrender profit/loss at time $t = n$, where n is the term of the contract. Such a model will take into account the discontinuance risk (namely the extent to which the surrender value exceeds the policy's asset share at the same date) and the mortality risk. Further, we shall consider models of financial and non-financial adverse selection (for example, selective lapsation), similar to Macdonald (1997) and Jones (1998) as discussed in chapter one, and their impact on profitability of a life company. Note that non-financial adverse selection is a phenomenon related to mortality lapses. That is, non-healthy lives are less likely to lapse the policy for financial gains. On the other hand, the financial adverse selective effect relates to financial lapses, whereby the individual lives (healthy or non-healthy) are more likely to lapse for financial gains. In this context, it is worth noting that more surrenders are expected to occur among those policyholders who are offered high surrender values compared to those who are offered low surrender values. However, our results of chapter 2 does not support this.

The methodology used to compute the proposed model is simulation-based since the underlying process of the model can be stochastically described. In particular, the investment model used follows a stochastic process as described in chapter three. Thus, a Monte Carlo Simulation approach will be used to compute the expected surrender profit/loss at time $t = n$, (n=*20* years in this case). In what follows, we discuss the model of the office's expected profit/loss due to the options available to the policyholder- the most common of which is the right to terminate the contract for a cash surrender value.

4.2 Model of Expected Profit/Loss due to antiselective Lapsation

4.2.1 Introduction

In this model we assume (for simplicity) that surrender may occur only at the end of each policy year. Also, the model assumes that any member of the cohort has the same probability to surrender - : this follows the hypothesis that "lapses are nonselective". It is important to note that the model proposed in this section considers possible surrenders and policies still active at time t. This is similar to Albizzati and Geman (1994), but approached differently. Thus, the result from this model will serve as a preliminary analysis upon which we can build subsequent related models. Then we shall extend the model to take into account financial and non-financial adverse effects. This will be discussed in the next section. A sensitivity analysis of the proposed model to different factors will be discussed in section 4.7.3.

Like Albizzati and Geman (1994), we assume that the insurance policy considered can be purchased in a lump sum. Thus, we assume that a single premium, P_o, is paid at the inception of the contract. We further assume that the lifetime of this contract is $t=20$ years if there is neither early termination nor rollover. However, if a policyholder terminates the contract at any time t, a cash surrender value, SV, defined below is paid out. It is worth noting that there is normally a penalty, λ, charged on all policies surrendered. For simplicity, we will initially assume that there is none. This assumption will be relaxed later on to incorporate a surrender charge on policies surrendered. Further, the value of the policy at maturity, denoted by MV, is also defined below.

Now, corresponding to the above liability, the life insurer invests the single premium in a portfolio of bonds. Particularly, the life office buys a nominal amount of gilts at time $t=0$ with the single premium. Note that the investment model to use to match this liability is discussed in chapter 3. Suppose surrender occurs at time t, in which case the life office needs to sell assets sufficient enough to provide a cash sum of $SV(t)$. The corresponding amount of assets to be redeemed to pay for the surrender claim at time t (if it occurs) is denoted by $Am(t)$, and is also defined below.

4.2.2 Cash Surrender Value Calculation Basis

As mentioned before, the surrender of a policy constitutes a financial option, which may be exercised by the policyholder against an office's interests. Therefore, in setting up a scale of surrender values, the life office must make sure that the amounts which it pays do not normally exceed the corresponding asset shares it has earned. Thus, in ensuring that its surrender values come within the corresponding earned asset shares, a life office must adopt scales which offer payment that are equitable between the various classes of policyholders. A suitable method that can enable us to arrive at an equitable and practical scale of surrender values is the prospective approach and we have used this method in this case. This method is an alternative approach to Albizzati and Geman (1994), where the same force of interest was assumed for both MV and SV in the calculation of SV (reviewed in chapter 1). We adopt the prospective calculation method because, according to Lumsden (1987), it enables a high degree of equity between those policyholders who surrender and those who do not. It is worth noting that to calculate the prospective value of a policy, the life office discounts the expected future benefit and premium payments on reasonable assumptions, and the assumptions that we use are mentioned below.

Reasonable Assumptions

We based our calculation of surrender basis on the following reasonable assumptions:

i. Single premium, P_o, is assumed to be 1, for simplicity.

ii. Initial expenses, E_s, constitute 10% of premium.

iii. No renewal expenses are charged since the contract is a single premium policy.

iv. Interest rate applied is the redemption yield at 27/5/98 of a t-year gilt, δ_t^s. Also, interest applied to MV is the yield of a 20-year gilt, δ_n, (equal to 5.87%).

v. There is a surrender penalty, λ, charged on all surrendered policies. In the initial case, we have assumed there is no charge for simplicity.

vi. Mortality is assumed to be zero.

Based upon these assumptions, we therefore propose a model of cash surrender value at time t, denoted by $SV(t)$, valued prospectively, as follows:

$$SV(t) = (1-\lambda)\cdot MV \cdot e^{-(n-t-1)\delta_t^s} \qquad 4.1$$

Where

δ_t^s is the assumed surrender force of interest as defined in chapter 3.

MV is the value of policy at maturity. This is given by

$$MV = (P_o - E_s)e^{\delta_n n} \qquad 4.2$$

By using equations 4.1, 4.2 and the following parameter values: $P_o = 1$; $E_s = 10\%$; $\delta_n = 5.87\%$; $n=20$; $\lambda = 0\%$ and δ_t^s as defined in 3.3, we obtain a plot of cash surrender value received by a policyholder who terminates his or her contract at time t against policy years as in figure 4.1:

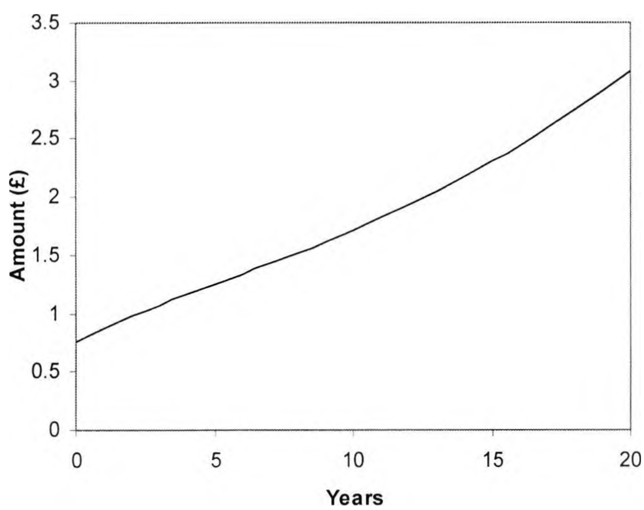

Figure 4.1: Plot of Cash Surrender value against Time (Years)

From the above plot we observe that a policyholder is expected to receive a surrender amount (prospective value of the policy) less than 1, during the early duration or early years of the contract. This is due to the high initial expenses associated with the contract. However, the net worth of the policy increases as time goes on, depending upon the office's future investment performance. In that case the surrendering policyholders are expected to receive a surrender amount at time t based on figure 4.1. It is worth mentioning that the numerical value of MV in this case is *2.9114*.

4.2.3 Lapse Model

Next, we propose a model of the lapse rate, *l(t)*, used in the profit model. We define *l(t)* as the probability of lapse in year t for a policy "alive" at start of year t. The proposed lapse rate, *l(t)*, is expressed as a deterministic function of time t. This is represented by a decreasing piecewise linear function which expresses the relationship between lapse rate and the policy year as shown in figure 4.2. Note that the proposed lapse rate follows the experience of past lapse rate, as analyzed by the Committee on the Expected Experience, Individual Insurance Section of the Canadian Institute of Actuaries, CIA (1996), coupled with the lapse analysis of chapter 2. In the former analysis, it is observed that lapse rate, as measured by calendar year decreases at time $t = 1$ until $t = 9$, where it appears to level off. The maximum lapse rate, *lmax,* at $t = 1$ is *9.0%* and the corresponding lapse rate where it appears to level off, *lmin,* at $t = 9$ is *1.4%*. However, in our case, we assume that *lmax=10%* and *lmin=6%*. This is due to the fact that from the lapse analysis of chapter 2, we observe a maximum average lapse rate of 10% and a minimum average lapse rate of 6%. Further, we assume that the lapse rate is expected to level off between time t_1 and t_2 at a value of *lmin*, similar to CIA (1996). From there, lapse rate is expected to decrease as duration increases until the expiry of the contract simply because it is unlikely for a policy to be surrended at these times, other than through death. The proposed form of *l(t)* is shown in figure 4.2:

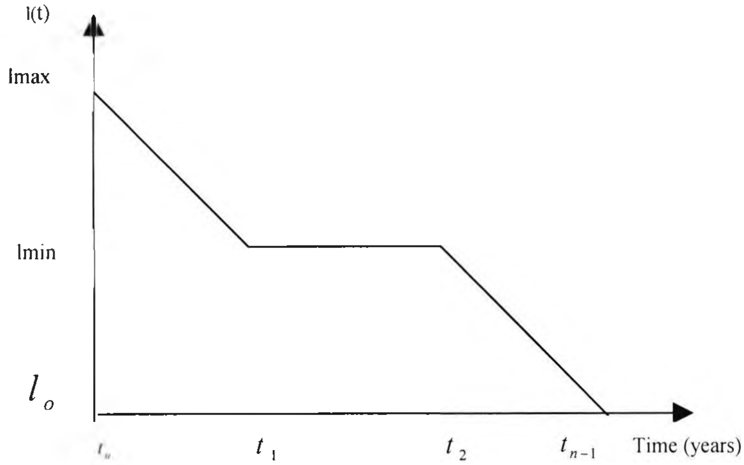

Figure 4.2: Graph of Lapse rate against Time (years)

or mathematically,

$$l(t) = \left\{ \frac{l_{\min} - l_{\max}}{t_1 - t_o} t + \frac{l_{\min} t_o - l_{\max} t_1}{t_o - t_1} \right\} \underset{t < t_1}{I} + l_{\min} \underset{t_1 \le t < t_2}{I} + \left\{ \frac{l_o - l_{\min}}{t_{n-1} - t_2} t + \frac{l_o t_2 - l_{\min} t_{n-1}}{t_2 - t_{n-1}} \right\} \underset{t \ge t_2}{I} \qquad 4.3$$

Where I denotes an indicator function, $I=1$ if the inequality (e.g. $t < t_1$) is satisfied and $I=0$ otherwise. As mentioned above, the values of *lmax* and *lmin* are respectively 0.10 and 0.06. Further, the corresponding t values are $t_o = 0$, $t_1 = 8$, $t_2 = 17$ and $t_{n-1} =$ 19. Note that $l(0)$ (or l_o) corresponds to a zero lapse rate (i.e., $l(0) = 0$).

Now, in what follows, we propose a model of expected surrender profit/loss at time $t = n$ and then discuss the simulation results obtained.

4.2.4 The Expected Surrender Profit Model

As mentioned earlier on, the contract under investigation is a conventional non profit policy which is purchased by a lump sum. Thus, a single premium is paid at the inception of the contract. In this context, the proposed expected present value of future profit/loss is the discounted value of the following terms at a constant force of interest δ_n:

Policy value at maturity (i.e., accumulated value of single premium less expenses at force of interest δ_n),

Less

Accumulated value of *SV(t)* to policyholders still alive at time *t*, at force of interest δ_t, multiplied by the probability *l(t)*.

Less

Maturity benefit paid to survival policyholders at time *t=n*.

From this we can compute the expected present value of surrender profit/loss at time *t=n*. This is given by:

$$S^n(0) = E\left\{\left((P_0 - E_s)e^{\delta_n n} - \sum_{t=0}^{n-1}\left(l(t)\cdot(ap)_t \cdot Am(t)\right) - (ap)_n \cdot MV(n)\right)e^{-\delta_n n}\right\} \qquad 4.4$$

Where,

$S^n(0)$ = Expected present value of surrender profit/loss at time $t= n$, valued at time zero.

l(t) = lapse rate at time *t*, a function of past lapse experience, as proposed above.

δ_t^s = Surrender basis force of interest at time *t*.

δ_t = Stochastic redemption yield of a *t*-year gilt.

E_s = Initial expenses.

MV(n) = Value of policy at maturity as defined before.

SV(t) = Cash surrender value at time *t* defined above.

$(ap)_t$ = Proportion of policies still alive at time *t*.

Am(t) = Nominal amount of gilts needed to be redeemed to cover the surrender value at time *t*. We may define this as the surrender value at time *t* per nominal price of gilt. In other words, the amount of nominal, *Am(t),* can be seen as the surrender value at time *t,* accumulated at stochastic force of interest, δ_t, to the end of the contract.

This is given by:

$$Am(t) = SV(t)e^{(n-t-1)\delta_t} \tag{4.5a}$$

By substituting *SV(t)* from equation 4.1, we obtain

$$Am(t) = \frac{(1-\lambda)\cdot MV\cdot e^{-(n-t-1)\delta_t^s}}{e^{-(n-t-1)\delta_t}} \tag{4.5b}$$

And

$$(ap)_t = \prod_{k=0}^{t-1}\{(1-l(k)\}. \tag{4.6}$$

Expressions for δ_t^s, δ_t are given in chapter 3.

By using the profit model and values of *lmax* and *lmin*, we generate values for the surrender profit model, starting at time *t=0 for t=1 to n* (*n=20* in this case) and perform the required number of simulations. The results obtained are presented in section 4.3.

4.3 Simulation Results

4.3.1 Model with no selection effect on Lapsation

The aim here is to determine the profitability of a company where no adverse selection effect is considered. In particular, the model neither considers the adverse mortality effect nor the financial adverse selection effect. Profitability is only dependent on the lapse rates and the proportion of policies still active in the pool of insurance policies. Note that the case of adverse selection will be considered later on in this chapter.

Another important feature that can affect the simulated results of the profit model is the relative difference between the force of interest as assumed in the surrender basis, denoted by δ_t^s, and the return on the stochastic investment model, denoted by δ_t. In other words, to access the profitability of a life company and the subsequent effect of adverse selective lapsation on the model, it is important to observe whether the force of interest as assumed in the surrender basis, δ_t^s, is paying out more or less than the return on the investment model, δ_t. We show below plots showing these effects. We also take note of the yield curve slope: whether it is positive or negative as this can affect the simulated results of the profit model. We look at the effect of slope of yield curve on the profit model.

We show in figure 4.3 and figure 4.4, graphs of yield curve of negative and positive slope respectively. For each figure, we show plots of δ_t^s, δ_t over time, where $\delta_{(0)}^s$ =0.071 and $\delta_{(0)}$ = 0.065 (initial starting values of the gilt yield model) in figure 4.3a and, $\delta_{(0)}^s$ = 0.071 and $\delta_{(0)}$ =0.07 in figure 4.3b. We denote the case where $\delta_{(0)}^s$ = 0.071 and $\delta_{(0)}$ = 0.065 by $\delta_t^s > \delta_t$ for most t as shown in figure 4.3a and for the case where $\delta_{(0)}^s$ = 0.071 and $\delta_{(0)}$ = 0.07, by $\delta_t^s < \delta_t$ for most t. See figure 4.3a and figure 4.3b for each case. We notice that figure 4.3a is the plot of surrender force of interest, δ_t^s, and the mean gilt yield at equilibrium position over each year, δ_t (i.e., plots of means from 1000 simulations). In this case, the initial starting value of the gilt yield model is 0.065 as stated above. However, figure 4.3b is generated in a similar way as figure 4.3a, with new initial starting condition as 0.07 this time. The initial starting value of 0.07 was chosen in order to obtain a curve where $\delta_t^s < \delta_t$.

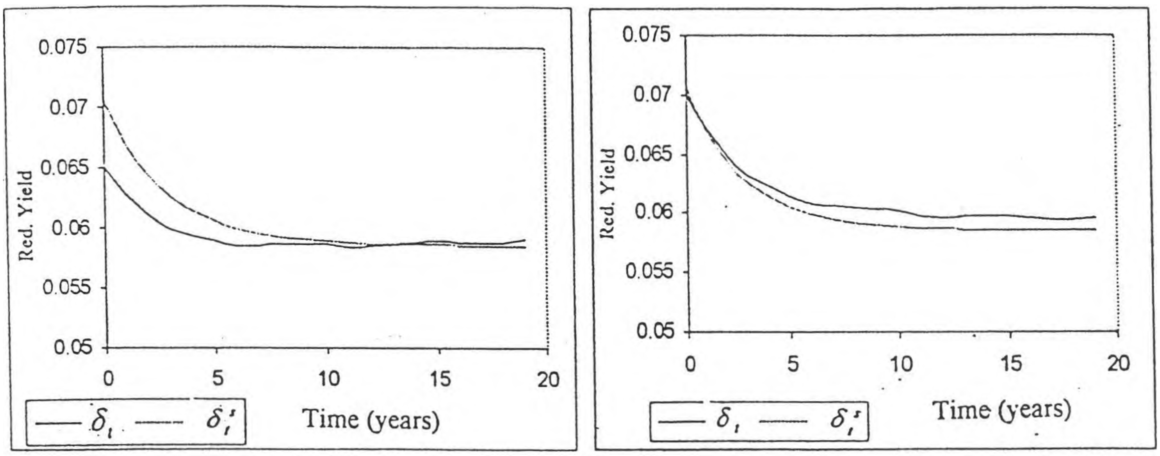

Figure 4.3a: $\delta_t^s > \delta_t$

Figure 4.3b: $\delta_t^s < \delta_t$

Figure 4.3: Plot of δ_t^s and δ_t against time for a yield curve of negative slope.

These plots show the extent on average of the adverse differences between the surrender value basis and the current redemption yield, illustrating the opportunity for loss that arises during the simulations.

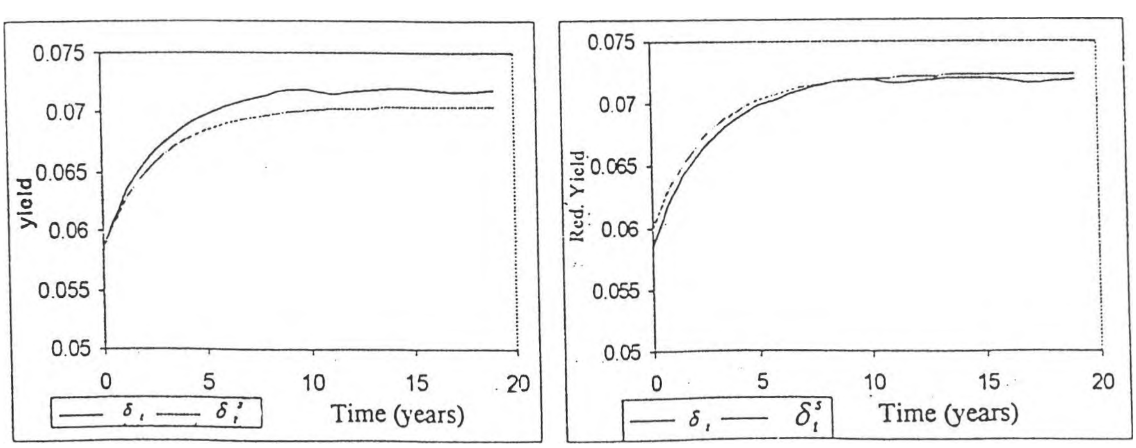

Figure 4.4a: $\delta_t^s < \delta_t$

Figure 4.4b: $\delta_t^s > \delta_t$

Figure 4.4: Plot of δ_t^s and δ_t against time for a yield curve of positive slope

Further, we show in figure 4.4 above, a plot of δ_t^s and δ_t over time for a yield curve of positive slope. This is obtained by inverting the yield curve of figure 4.3 (i.e., invert plot of δ_t^s against time). With this, we re run the simulations to obtain a mean simulated yield at equilibrium position, denoted by δ_t. In this case, an initial starting condition of 0.0585 was used. This is to obtain a curve where $\delta_t > \delta_t^s$. Similarly, figure 4.4b was obtained by multiplying δ_t^s by a scale factor of 1.025 and then re-running the simulations to obtain δ_t (same initial starting condition of 0.0585 was used for δ_t). This new shape (yield curve of positive slope) enables us to determine its effect on the profit model. Similarly, in figure 4.4, we denote the case where $\delta_{(0)}^s$ = 0.06 and $\delta_{(0)}$ = 0.0585 by $\delta_t^s > \delta_t$ for most t as shown in figure 4.4b and for the case where $\delta_{(0)}^s$ = 0.0585 and $\delta_{(0)}$ = 0.0585, by $\delta_t^s < \delta_t$ for most t as shown in figure 4.4a.

Now, what follows is a discussion of the simulation results of the surrender profit/loss model with no selection effect obtained by using the results in figures 4.3 and 4.4.

<u>Expected Present Value of Surrender Profit/Loss at t=20 years.</u>

In computing the expected present value of surrender profit/loss, the following model parameter values were used:

Term of contract	= 20years
Simulation	= 1000
Surrender penalty, λ	= 0%
Initial Expenses	= 10%
Standard deviation in the gilt model, σ	= 5%

lmax and *lmin* are respectively 10% and 6%.

The expected profit/loss obtained by using the above model parameter values are shown below (table 4.1). Note that we present results for both cases where $\delta_t^s > \delta_t$ and $\delta_t^s < \delta_t$ for most t. Further note that these parameter values were used as a baseline for these analysis. Unless otherwise stated, this will be used in all subsequent analyses. Also shown is the corresponding standard error of the process.

	σ =5%	
Slope of Yield curve :	Negative	
Difference in Payout :	$\delta_t^s > \delta_t$	$\delta_t^s < \delta_t$
E(Profit) :	0.028026	-0.0004049
Std error (profit) :	0.000803	0.0008731

Table 4.1: Result of expected present value of surrender profit/loss at t=20 years for yield curve of negative slope.

For the case where $\delta_t^s > \delta_t$, we observe a small profit when the model does not consider the adverse selection (financial and non-financial) effect on surrender. This is probably due to the fact that the model does not consider any anti-selective lapsation, coupled with the fact that the assumed surrender basis pays out more than the return on the investment model.

However, for the case where $\delta_t^s < \delta_t$, a small loss is observed. By taking into account the fact that the assumed surrender basis is paying less than the return on asset, this appears to be reasonable. That is, the no selection model in this case produces a small loss as expected. We notice that the standard error of profit increases when we changed the assumption of $\delta_t^s > \delta_t$ to $\delta_t^s < \delta_t$. This is probably due to the fact that we have introduced more variation in the stochastic model by increasing $\delta_{(0)}$ (but this effect would disappear as the term of the contract extends).

4.3.2 Effect of Relative payout on model.

From the results of the above analysis, we can determine the effect of changing the model assumption of $\delta_t^s > \delta_t$ to $\delta_t^s < \delta_t$ on the expected profit model. Note that a yield curve of negative slope is used. Thus, we observe that the expected profit decreases when we change the assumption of $\delta_t^s > \delta_t$ to $\delta_t^s < \delta_t$. This is actually reasonable, because we have increased the stochastic force of interest in the stochastic model and so we need to redeem a greater amount of assets to cover surrender value at t. Hence, a loss is observed when we changed $\delta_t^s > \delta_t$ to $\delta_t^s < \delta_t$. We also notice that the standard error of profit increases when we change the assumption of $\delta_t^s > \delta_t$ to $\delta_t^s < \delta_t$. This is probable due to the fact that we have introduced more variation in the stochastic model by increasing the value of $\delta_{(0)}$.

By repeating the analysis of table 4.1 for the case where a yield curve of positive slope is used, we obtain the following results presented in table 4.2. Note that the baseline model parameter values of table 4.1 were used here too.

Expected Present Value of Surrender Profit/Loss at t=20 years.

	σ =5%	
Slope of Yield curve :	Positive	
Difference in Payout :	$\delta_t^s > \delta_t$	$\delta_t^s < \delta_t$
E(Profit) :	0.006289	-0.0038928
Std error (profit) :	0.000957	0.0009613

Table 4.2: Result of expected present value of surrender profit/loss at t=20 years for yield curve of positive slope.

From table 4.2, we observe a result similar to the one described in table 4.1. However, there is a slight decrease in the expected profit and loss this time when the model assumption is changed from $\delta_t^s > \delta_t$ to $\delta_t^s < \delta_t$. Further, we observed that the profit and loss amount has decreased when a yield curve of positive slope is used, with the corresponding slightly change in the standard error of profit. This is probably due to the fact that by using a yield curve of positive slope, the stochastic force of interest in the stochastic model has increased than before. Therefore we need to redeem a greater amount of assets to cover surrender value at t. Hence, a reduced profit and loss is observed.

It is worth noting that from tables 4.1 and 4.2, we observe significant differences in mean value in relation to size of standard error of profit for the case where the model assumption is changed from $\delta_t^s > \delta_t$ to $\delta_t^s < \delta_t$, and also, for the case where a positive and negative slope of yield curve is used. This makes direct comparison possible.

4.4 Sensitivity of Expected Profit to Different factors

4.4.1 Effect of Yield Curve Slope

We look at the effect of the slope of yield curve (shape of yield curve) on the profit model by considering a yield curve of positive and negative slope as shown in figure 4.3 and 4.4. In this case, we consider the case where $\delta_t^s > \delta_t$ as assumed before, for consistency. Table 4.3 shows the results obtained.

Expected Present Value of Surrender Profit/Loss at t=20 years.

	σ =5%	
Difference in Payout :	$\delta_t^s > \delta_t$	
Slope of Yield curve :	Negative	Positive
E(Profit) :	0.028026	0.006289
Std error (profit) :	0.000803	0.000957

Table 4.3: Results of expected surrender profit/loss due to different shapes of yield curve.

From table 4.3, we observe that the expected profit decreases as the slope of the initial yield curve increases for $\delta_t^s > \delta_t$. The results is not surprising because the increase in the yield curve slope only affects the amount of assets that need to be sold to cover surrender values at time t. So, since the yield on assets has now increased, this implies that for a fixed lapse rate, the expected profit will decrease.

Also, the result is partly due to the fact that the assumed surrender basis is paying out more on surrender than the returns on investment model. Thus, the slope of the initial yield curve probably has some effect on expected profit.

Now, we re-ran the analysis of table 4.3 with more simulations, (i.e., 10,000) to determine its effect on the standard error of profit. The following results were obtained.

	σ =5%	
Difference in Payout :	$\delta_t^s > \delta_t$	
Slope of Yield curve :	Negative	Positive
E(Profit) :	0.028979	0.007382
Std error (profit) :	0.000246	0.000285

Table 4.3a : Results of expected surrender profit/loss by using 10,000 simulation.

From table 4.3a, we observe that the expected profit has slightly increased, and that the standard error of the profit has reduced by a small amount (which is reasonable).

4.4.2 Lapse Effect

Next, we consider the effect of lapse rates on the expected surrender profit at time $t=n$. In this case, we consider different values of *lmin* for fixed *lmax*. Note that the baseline yield curve is used here (i.e., negative slope), and also, $\delta_t^s > \delta_t$. Also note that the baseline model parameter values were used and standard deviation of the process is equal to 0.05. The following results as shown in the table 4.4 below were obtained.

<u>Expected Present Value of Surrender Profit/Loss at t=20 years.</u>

Lapse rate	*lmin*=0.05,*lmax*=0.1	*lmin*=0.06, *lmax*=0.1 (baseline results)	*lmin*=0.07, *lmax*=0.1
E(Profit)	0.027209	0.028026	0.028806
Std error (Profit)	0.000749	0.000803	0.000852

Table 4.4: Results of expected surrender profit/loss due to different lapse rate values

The lapse model is an important parameter in the computation of the expected surrender profit. Thus, from above table we observe that the expected surrender profit increases with increasing average lapse rate. Broadly speaking, increasing average lapse rate is normally accompanied by a decrease in expected profit since the company needs to sell more of its assets to pay for surrender benefits. Nevertheless, we observe that the expected profit in table 4.4 is probably influenced by the fact that the surrender basis pays more than the return on the investment model. So, in this case with $\delta_t^s > \delta_t$, it is not surprising that profits increase with increases in the average lapse rate. Further, we observe that the standard error of profit increases as we increase lapse rate.

4.4.3 Sensitivity to volatility of interest rates

The parameters involved in the gilt model which contributed to the volatility of the process are α (controls the strength of auto-regression) and σ (standard deviation in the gilt model). For fixed α, we consider different values of σ and determine its effect on the profit model. Table 4.5 shows the results obtained.

<u>Expected Present Value of Surrender Profit/Loss at t=20 years.</u>

	$\delta_t^s > \delta_t$		
Slope of Yield Curve:	Negative		
Stdev in Gilt Model, σ:	3%	5%	10%
E(Profit):	0.029794	0.028026	0.019795
Std error (Profit):	0.000499	0.000803	0.001671

Table 4.5: Result of changing the standard deviation, σ, of the stochastic yield model

From table 4.5 we observe that the expected surrender profit decreases with increasing volatility. The reason for this is that, increasing the volatility of the process implies that there is more uncertainty in the nominal amount function, and this affects the amount of assets that need to be redeemed to cover a surrender value at time *t*. Thus, for a fixed lapse rate, coupled with the fact that the surrender basis pays more than the return on assets, the expected profit is likely to decrease as observed. However, we observe that the option value increases as volatility increases, as in Albizzati and Geman (1994), since options are usually more valuable as σ increases. This is consistent with our results.

4.4.4 Sensitivity to α, strength of auto-regression (used in the investment model).

Following the discussion of sensitivity of σ, we look at the effect of α on profitability by considering different values of α for fixed σ. The table below shows the results obtained.

Expected Present Value of Surrender Profit/Loss at t=20 years.

	$\delta_t^s > \delta_t$		
Slope of Yield Curve:	Negative		
α effect:	85%	90%	95%
E(Profit):	0.024721	0.028026	0.032245
Std error (Profit):	0.000718	0.000803	0.000911

Table 4.5a : Results of expected surrender profit/loss due to different values of α.

We know from section 3.2.1 that a higher value of α implies that the gilt yield returns can be expected to move slowly back towards the mean value and vice versa. Therefore, from table 4.5a, we observe that as α increases, the expected profit steadily increases and so does the corresponding standard error. This is as expected since the standard deviation of the process increases as α approaches 1, (see equations 3.13 and 3.14).

For the above analysis studied so far, we have shown that a life office is expected to make a small profit under the assumptions set out in the model. Particularly, we have looked at the case where the model assumed no selection effect on surrender. We believe that the selection effects can affect profitability and so it is important that they are considered; therefore, we propose incorporating the mortality anti-selection effect as well as financial adverse selection effect in the model. This will be discussed in detailed in section 4.5 and subsequent sections. It is worth noting that this modified model is more realistic and more relevant to the life office. There may be a loss to the company as a result of adverse selection on surrender. Consequently, this loss can be charged to the policyholder for the right to surrender the policy at conditions favourable to him. This is to be discussed later on in chapter 5 of this thesis. In what follows we discuss in detail a model of adverse selection on surrender (both financial and non-financial) and its effect on the expected surrender profit/loss. A sensitivity analysis of the model parameter values is also performed.

4.5 Model of Adverse Selection on Surrender

4.5.1 Introduction

In this section we propose a model of financial and non-financial adverse selection on surrender, similar to that proposed by Albizzati and Geman (1994) and Jones (1998), and discuss its effect on the expected surrender profit/loss. In this context, we notice that a rational policyholder will exercise the option to surrender the policy if :

- the earned asset shares of the policy (accumulations of premiums less charges) fall below the surrender values (*SVs*) allowed.
- the surrender value of a policy exceeds the single premium of a new policy (market premium basis). Thus, an option will be exercised if the maturity value arising from initiating a new contract exceeds that of holding the existing contract till maturity. In other words, reinvesting the *SV* in another policy and getting a higher *MV* after *n-t* years than from the first policy.

Thus, if we let $D(t)$ be the ratio of the surrender value of a policy to the premium of a new policy, then a policyholder will exercise the option if $D(t)>1$. Throughout this chapter we may refer to $D(t)$ as a decision criterion. This is time dependent and it is similar to the one proposed by Albizzati and Geman (1994).

However, it is important to note that not all insureds may surrender the policy when $D(t)>1$, a feature which Albizzati and Geman did not consider. The risk level of the insured will influence their perception about when to exercise the option to surrender the policy and so it is important that we incorporate this feature when valuing the surrender profit/loss. Further note that the "risk level", a positive and continuous unobservable variable, represents the state of the individual. For example, unhealthy individuals are less likely to lapse their policy than are those who remain healthy (adverse selection). By incorporating this in our model, we could investigate the significance of the financial effect as well as the non-financial adverse selection effect on the model as opposed to just the financial effect in relation to $D(t)$, as investigated by Albizzati and Geman (1994). The adverse selection effect has already been discussed in chapter 1. Therefore, we shall model the expected surrender profit by allowing for the non-financial (e.g., mortality selection effect) effect as well as the financial anti-selection effect on surrender. Firstly, we present a lapse model which is a function of the risk level of the insured at time t and $D(t)$. This is based on Albizzati and Geman (1994) and Jones (1998).

But the risk level depends on the state of the individual, which can be described by considering a multiple state model as discussed below.

4.5.2 Multiple State Models for Life Contingencies

In this model, we consider a full endowment assurance contract (i.e., a contract with level death benefit equal to maturity benefit). Further, we consider a population comprising two risk-groups (healthy and unhealthy) so that there are possible transfers between them. Each group has different mortality and lapse rates. Now, we consider a cohort of insured lives with the same underwriting category, all of whom are issued policies at age x. Suppose that all insured are in a state called "healthy" at issue. At any time thereafter, an insured can experience the transitions shown in Figure 4.5:

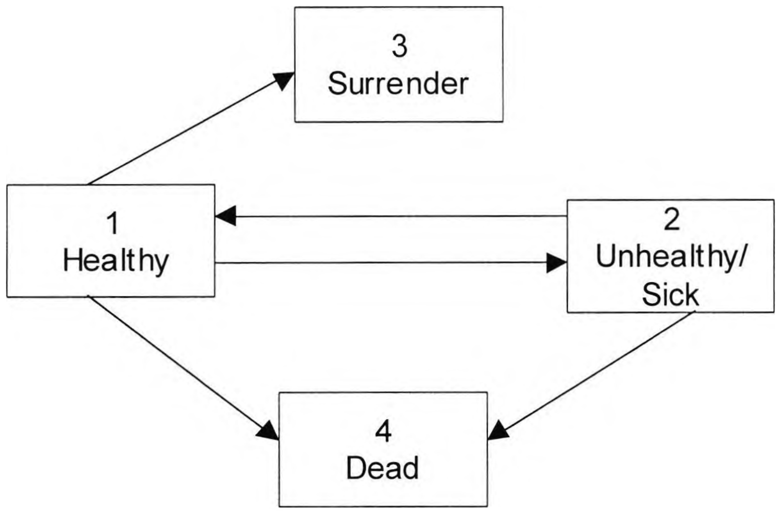

Figure 4.5: State transition diagram

The four states are labeled as follows: 1, healthy; 2, unhealthy (sick); 3, surrender; and 4, dead. The transitions are shown with arrows. Note that we have allowed for recovery from sickness (i.e., from sickness to healthy) in this model for reasons given in chapter 1. This transition was not allowed in Jones (1998) "for simplicity". Transitions from states 3 and 4 are taken to be not possible (i.e. absorbing states). It is assumed that healthy policyholders will lapse their policies for financial incentives, but unhealthy individuals will not lapse (adverse selection).

Now, let $\mu_{ij}(x+t)$ represent the transition intensities from state i to state j at an exact moment t, and these govern the rates of continuous transfer between states. Note that the force of transition is the instantaneous rate of transition from state i to state j at time t. Further, we denote the conditional probabilities associated with these transitions by $P_{ij}(x, x+t)$, which is the probability that the process is in state j at time $(x+t)$ given that it was in state i at time x.

In deriving an explicit expression for the conditional probability functions (see below), we assume that the transition intensities are constant (over each year of age of the policyholder) for simplicity and also in order to make the calculations tractable. This is presented below. Subsequently, we will apply the expressions for the conditional probability functions in the surrender profit/loss model.

4.5.3 Constant Forces of Transition

The assumption of a constant transition intensity is equivalent to the assumption that the time spent in each state is exponentially distributed and has no effect on the future length of time the policyholder will spend in that state. This means that a time-homogenous continuous time Markov process can be used to represent the transition between the states under consideration. Note that a process is time-homogenous if the transition probability $P_{ij}(x, x+t)$ depends only on t and not on x and $x+t$ individually (Haberman and Pitacco (1999)).

Define the transition probability function as

$$P_{ij}(x, x+t) = \Pr\{(x+t) = j | (x) = i\}, \qquad i, j \in \{1,2,3,4\}. \qquad 4.7$$

For $t \geq 0$, we define :

$$P_{ij}(t,t) = \delta_{ij}, \qquad 4.7a$$

where,

δ_{ij} denotes the Kronecker delta which is given by

$$\delta_{ij} = \begin{cases} 1 & if \quad i = j \\ 0 & otherwise \end{cases} \qquad 4.7b$$

We notice that the conditional transition probability satisfies the following properties.

$$\sum_{j=1}^{k} P_{ij}(x, x+t) = 1 \quad \text{for all } t \geq 0 \qquad 4.8a$$

and

$$0 \leq P_{ij}(x, x+t) \leq 1 \text{ for all } t \geq 0. \qquad 4.8b$$

Moreover, the transition probabilities satisfy the Chapman-Kolmogorov equation given by

$$P_{ij}(x, x+t+u) = \sum_{l=1}^{k} P_{il}(x, x+t) P_{lj}(x+t, x+t+u) \qquad i, j \in (1,2,\ldots,k) \qquad 4.9$$

This equation describes a path by which an individual in state i at time x must follow in order to get to state j at time $x+t+u$. Thus, he must visit some state l at an arbitrary intermediate time $x+t$.

The transition intensities are defined as:

$$\mu_{ij}(t) = \lim_{h \to 0} \frac{P_{ij}(t, t+h)}{h}, \qquad (i \neq j) \qquad 4.10a$$

These limits are assumed to exist for all t and all $i \neq j$. If the above Markov process is assumed to be time homogenous, then we have

$$\mu_{ij}(t) = \lim_{h \to 0} \frac{P_{ij}(h)}{h} \qquad 4.10b$$

Further, we assume that the transition intensities are constant functions i.e. $\mu_{ij}(t) = \mu_{ij}$. With this we can find explicit expressions for the conditional transition probability functions. Also, for each i, we define $\mu_{ii} = -\sum_{j:j \neq i} \mu_{ij}$ (note that summation is over the states j which are linked directly to state i). Thus, we have:

μ_{12} = force of incidence of sickness.

μ_{13} = force of incidence of lapsing by healthy individuals.

μ_{14} = force of mortality for healthy individuals.

μ_{21} = force of recovery from sickness.

μ_{23} = force of incidence of lapses by sick individuals.

μ_{24} = force of mortality for sick individuals.

We can now set up equations for the transition probabilities by using the Kolmogorov system of difference-differential equations and the constant transition intensity assumption mentioned above. The Kolmogorov systems of difference-differential equations are presented below.

Kolmogorov's Forward and Backward Equation

Now, the forces of transition and the transition probability functions are related by the Kolmogorov's forward and backward equations, which are

$$\frac{d}{dt}P_{ij}(x,x+t)=\sum_{l=1}^{k}P_{il}(x,x+t)\mu_{lj}(x+t), \qquad 4.11$$

and

$$\frac{d}{dx}P_{ij}(x,x+t)=-\sum_{l=1}^{k}\mu_{il}(x)P_{lj}(x,x+t) \qquad 4.12$$

respectively, with boundary conditions as given in equation 4.8b. We use the above Kolmogorov backward system of difference-differential equations to set up equations for the conditional transition probabilities (e.g., Jones (1994), Haberman (1995), and Rickayzen (1997)). Note that the backward system of equations is derived by partitioning the intervals *(0, t+h]* into two non-overlapping intervals *(0, h]* and *(h, t+h]*. By letting $h \to 0$, we obtained a set of difference-differential equations which in our case, are given by:

$$\left.\begin{aligned}
\frac{d}{dt}P_{11}(t) &= -(\mu_{12}+\mu_{13}+\mu_{14})P_{11}(t)+P_{12}(t)\mu_{21}\\
\frac{d}{dt}P_{12}(t) &= -(\mu_{21}+\mu_{23}+\mu_{24})P_{12}(t)+P_{11}(t)\mu_{12}\\
\frac{d}{dt}P_{13}(t) &= \mu_{13}P_{11}(t)+P_{12}(t)\mu_{23}\\
\frac{d}{dt}P_{14}(t) &= \mu_{14}P_{11}(t)+P_{12}(t)\mu_{24}\\
\frac{d}{dt}P_{21}(t) &= -(\mu_{12}+\mu_{13}+\mu_{14})P_{21}(t)+P_{22}(t)\mu_{21}\\
\frac{d}{dt}P_{22}(t) &= -(\mu_{21}+\mu_{23}+\mu_{24})P_{22}(t)+P_{21}(t)\mu_{21}\\
\frac{d}{dt}P_{23}(t) &= \mu_{13}P_{21}(t)+P_{22}(t)\mu_{23}\\
\frac{d}{dt}P_{24}(t) &= \mu_{14}P_{21}(t)+P_{22}(t)\mu_{24}
\end{aligned}\right\} \qquad 4.13$$

It is worth mentioning that in equation 4.13, we have collapsed the conditional transition probability notation from $P_{ij}(u,u+t)$ to $P_{ij}(t)$ as shown in the definition of transition intensities where we have assumed the Markov process to be time homogenous and assumed $\mu_{ij}(t)=\mu_{ij}$.

We follow the method outlined by Jones (1994) to solve this set of differential equations. That is, we first express the forces of transition probability functions in a matrix form. Thus, let Q be a $4x4$ matrix with (i,j) entry μ_{ij} and $P(t)$ a $4x4$ matrix with (i,j) entry $P_{ij}(t)$. Now, corresponding to Equations (4.11) and (4.12), the Kolmogorov differential equations can be written respectively as

$$P'(t) = P(t).Q \qquad 4.14a$$

$$P'(t) = Q.P(t) \qquad 4.14b$$

with boundary conditions P(0)=I, where I is identity matrix. Note that $P'(t)$ is the matrix entry $\frac{d}{dt}P_{ij}(t)$. The solution to Equations (4.14a) and (4.14b) is given by

$$P(t) = e^{Qt}. \qquad 4.15$$

However, if Q has distinct eigenvalues, d_1, d_2, d_3, d_4, then, according to Cox and Miller (1965),

$$Q = A \cdot D \cdot C \qquad 4.16a$$

where,

$$C = A^{-1}, \text{ and } D = diag(d_1, ..., d_4) \qquad 4.16b$$

and the $i'th$ column of A is the right-eigenvector associated with eigenvalue d_i $(i=1,....,4)$.

Furthermore,

$$P(t) = A \cdot diag(e^{d_1 t}, ..., e^{d_4 t}) \cdot C. \qquad 4.17$$

From equation (4.17), we can now write

$$P_{ij}(t) = \sum_{n=1}^{4} a_{in} c_{nj} \cdot e^{d_n t} \qquad 4.18$$

where a_{ij} and c_{ij} are the (i,j) entries of A and C, respectively.

Hence, the problem of finding the transition probabilities is now reduced to a problem of determining the eigenvalues and eigenvectors of the force of transition matrix Q. By using computer software such as Mathematica, we can obtained an expression for $P(t)$. Details of the formulation of $P(t)$ are shown in Appendix 4.1.

4.5.4 Parameter Values

Having obtained an expression for *P(t)*, we now consider how each parameter value has been chosen. We assume that the parameter values listed below are for insured lives aged 35.

For μ_{12}, (i.e., Healthy-Sick), we consider values of the sickness inception rate as described in *part C of CMIR 12 (1992)*. We used values for a deferred period of 4 weeks because we want to include all sickness rates, but exclude trivial sickness claims.

Also, for μ_{21}, (i.e., Sick $\rightarrow$Healthy), we consider the recovery rates described in section 3, *part B of CMIR (page 34)*. The rates as listed vary by duration of sickness (measured in weeks), thus requiring a semi-Markov presentation. However, for the purpose of our model, we considered a value of 2.0 at all ages, which seems reasonable, as observed in Rickayzen (1997).

Further, for μ_{14}, (Healthy$\rightarrow$Dead), we consider the mortality rates for male-permanent Assurance of 1979-82, duration 0 as shown in *Table E17 of CMIR 12 (1992)*. In this case we use mortality rates listed in the *CMIR* paper because our model is considering only healthy lives.

For μ_{24}, (Sick$\rightarrow$Dead), we consider mortality intensities described in section 6, *part B of CMIR 12*. We therefore use values of $\nu_{x+t,t}$ at various ages set out on page 39, where $\nu_{x+t,t}$ is the transition intensity from sick to dead at current age $x+t$ and duration of sickness t.

Finally, we have assumed a lapse rate of 6% (average lapse rate) as a base of our analysis (from the previous analyses discussed in chapter 2). Then, we can investigate the effect of lapses on profitability by varying lapse rate values whilst the other parameter values are fixed. It is important to note here that we have also assumed μ_{23}

to be zero as in Jones (1998). This is due to the fact that we have assumed that only healthy lives will lapse their policies.

4.5.5 Proportion of Sick and Healthy Lives

For the two risk-group population (healthy and unhealthy) under investigation, it is important to know the relative proportion of lives in each group at any particular age, x. This is useful in explaining the impact of selective lapsation on profitability. Thus, the proportion of sick and healthy lives between ages x and $x+t$ can be found by using the following models:

$$l^H_{x+t+1} = l^H_{x+t} \cdot P_{11}(x+t, x+t+1) + l^S_{x+t} \cdot P_{21}(x+t, x+t+1) \qquad 4.19$$

$$l^S_{x+t+1} = l^S_{x+t} \cdot P_{22}(x+t, x+t+1) + l^H_{x+t} \cdot P_{12}(x+t, x+t+1) \qquad 4.20$$

Where,

l^H_{x+t}, l^S_{x+t} are respectively the proportion of healthy and sick lives who are expected to survive to age $x+t$, and the transition probability, $P_{ij}(x, x+t)$ as defined (4.7). Note that the initial values are given by: $l^H_x = 1$ and $l^S_x = 0$ for x=*30* and t = *0,1,......(n-1).*

In the computation of P_{ij}'s defined in above equation (4.19 and 4.20), we have assumed the following parameter values of μ_{ij} for reasons as given before. That is, μ_{12}= 0.2059; μ_{13}=0.06; μ_{23}= 0; μ_{14}=0.0008; μ_{24}= 0.125. By using these values, we obtained plots of proportion of healthy and sick for different recovery rates, μ_{21}, as shown below. The particular values of μ_{21} considered are 2, 1, and 0.5 and the corresponding plots are as shown by figures 4.6a, 4.6b, and 4.6c respectively. We choose these values for comparison purposes.

Figure 4.6 below shows the plot of the proportions of sick and healthy lives at time t (from 0 to 20) for different values of μ_{21}.

Figure 4.6 : Plot of l^{H}_{x+t}, l^{S}_{x+t} against $\mu_{21} = 2$, 1, and 0.5.

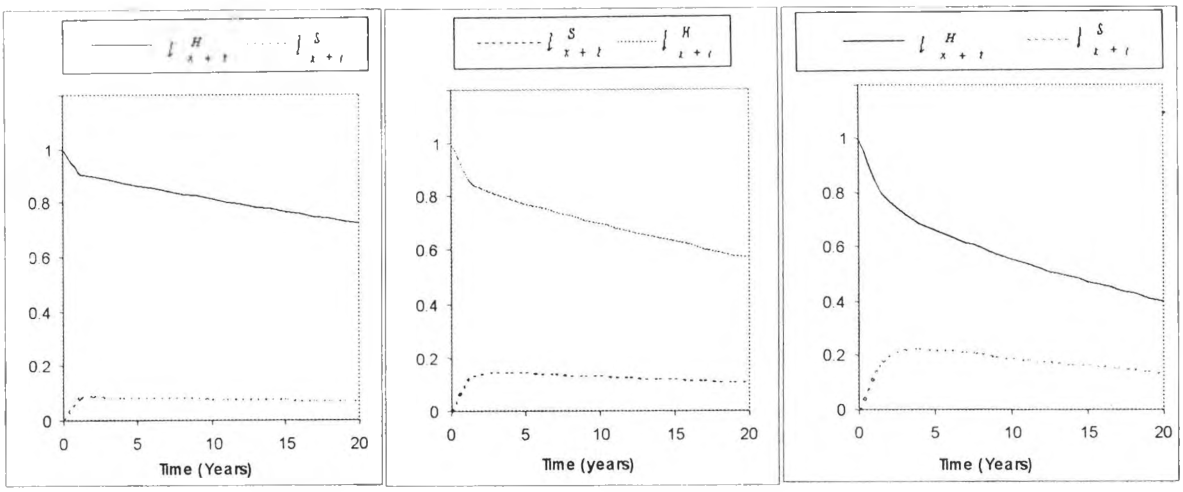

Figure 4.6a : $\mu_{21}=2$ Figure 4.6b: $\mu_{21}=1$ Figure 4.6c: $\mu_{21} = 0.5$

4.5.6 Model of Lapse Rate as a function of Decision Criterion, D(t)

Now, having chosen the multiple state model and the model parameter values, we propose a model of lapse rate, which in this case is a function of *D(t)* and the risk level of the insured. As mentioned before, *D(t)* is the ratio of the surrender value of the old contract to the single premium of the new contract.

Thus, an expression for *D(t)* is given by:

$$D(t) = \frac{(1-\lambda)\cdot \bar{A}_{x+t+1:\overline{n-t-1}|(\delta_t^S)}}{(1+\varphi)\bar{A}_{x+t+1:\overline{n-t-1}|(\delta_t)}} \tag{4.21}$$

Where,

φ = management fees of initiating a new contract.

λ = surrender penalty

δ_t^s, δ_t as defined before.

$\overline{A}_{x+t+1:\overline{n-t-1|}}$ is an endowment assurance payable continuously at the death of a policyholder or when he/she survives at the expiry of the policy (i.e. at time $t = n$). An expression for this term is given by:

$$\overline{A}_{x+t+1:\overline{n-t-1|}(\delta_t^s)} = 1 - \delta_t^s \cdot \overline{a}_{x+t+1:\overline{n-t-1|}} \quad 4.22a$$

But,

$$\overline{a}_{x+t+1:\overline{n-t-1|}} = \ddot{a}_{x+t+1:\overline{n-t-1|}} - \frac{1}{2}\left(1 - A_{x+t+1:\overline{n-t-1|}}^{\;\;\;\;\;1}\right) \quad 4.22b$$

and

$$\ddot{a}_{x+t+1:\overline{n-t-1|}} = \frac{1}{l_{x+t+1}} \sum_{j=0}^{n-t-1} l_{x+t+1+j} \cdot e^{-j\delta_t} \quad 4.22c$$

where,

$$l_{x+t} = l_{x+t}^H + l_{x+t}^S .$$

Also,

$$A_{x+t+1:\overline{n-t-1|}}^{\;\;\;\;\;1} = \frac{l_{x+n}}{l_{x+t+1}} \cdot e^{-(n-t-1)\delta_t} . \quad 4.22d$$

Hence,

$$\overline{A}_{x+t+1:\overline{n-t-1|}(\delta_t^s)} = 1 - \frac{\delta_t^s}{l_{x+t+1}} \cdot \left\{ \sum_{j=0}^{n-t-1} l_{x+t+1+j} \cdot e^{-j\delta_t} - \frac{1}{2}\left(l_{x+t+1} - l_{x+n} \cdot e^{-(n-t-1)\delta_t}\right) \right\} \quad 4.23$$

We assume that a rational policyholder will surrender the policy if $D(t)>1$. The above decision criterion is similar to that proposed by Albizzati and Geman (1994), but approached differently. Therefore, we propose a lapse rate model that can be expressed as a deterministic function of the decision criterion, $D(t)$, in a similar way to Albizzati and Geman (1994). Like Albizzati and Geman (1994), this function is a non-decreasing piecewise linear function, which expresses the relationship between the lapse rate and the policyholder's decision criterion. This is shown in figure 4.7.

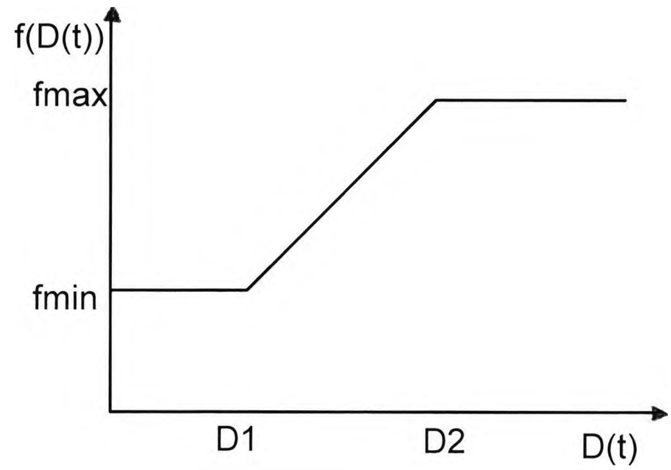

Figure 4.7: Lapse rate versus policyholders decision criterion, D(t)

Given the presentation in figure 4.7, we propose the following lapse model:

$$f(D(t)) = f_{\min} \underset{D(t)<D_1}{I} + \left\{ \frac{f_{\max}-f_{\min}}{D_2-D_1} D(t) + \frac{f_{\max}D_1 - f_{\min}D_2}{D_1-D_2} \right\} \underset{D_1 \le D(t)<D_2}{I} + f_{\max} \underset{D(t)\ge D_2}{I}, \qquad 4.24$$

Where, *fmax* and *fmin* are respectively the maximum and minimum lapse rate expected to observe or experience. In this case we choose *fmin=1*, which is the minimum level if policyholders behave 'irrationally' (i.e., a policyholder may decide to surrender the policy even if there is no financial incentive to do it). In this case, *D(t)* is less than *D1*. The region between *D1* and *D2* represents the lapse rate when the policyholder behaves 'rationally', i.e., a surrender due to financial incentives. Finally, we choose *fmax=6*, a maximum level which we deem to be reasonable. Furthermore, we assume *D1=1* and *D2=2* which satisfy the conditions set out in the above equation.

4.5.7 Model of Adverse Selection effect on Surrender

Now, by using the proposed lapse model we redefine the force of incidence of lapsing by healthy lives (μ^{13}) as a function of the above lapse model. This is to enable us to investigate the significant financial anti-selection effect on surrender. In this context, we assume that only healthy lives will lapse their policies for financial gains (as in figure 4.5). We assume that lapse will only take place if *D(t)>1*. Since unhealthy individuals are less likely to lapse even if *D(t)>1* (adverse effect), we have therefore assumed $\mu^{23}_{x+t} = 0$. With this structure, we can determine the impact of the adverse financial effect due to lapses on the expected profit/loss model.

We therefore propose an 'adverse' force of incidence of lapsing as follows:

$$\mu^{13\,*}_{x+t} = \mu^{13}_{x+t} \cdot f(D(t)) \qquad 4.25$$

Where, $\mu^{13\,*}_{x+t}$ represent force of incidence of lapsing due to selection effect; μ^{13}_{x+t} and *f(D(t))* as defined already and *f(D(t))* follows 4.24. Note that by the constant force of transition assumption μ^{13}_{x+t} is equal to μ^{13}. By using the proposed model we can re-compute the transition probability $P_{13}(x+t, x+t+1)$, by using equation 4.25 this time. Subsequently, this will be used in the expected surrender profit/loss model. We now present a model of expected surrender profit/loss in the context of adverse selection effect.

4.6 Model of Expected Surrender Profit/Loss due to selective effect

4.6.1 Introduction

This section presents a model for computing the expected surrender profit/loss of the insurer by allowing for the above selective effects. Firstly, we shall consider only the non-financial adverse selection effect (i.e., mortality adverse selection effect). This is to enable us to investigate the impact of adverse mortality selection effect on profitability. Then again we look at the impact of both financial and non-financial adverse selection effect on the company's profitability. Then we will be able to compare the relative effect of one type of selection with the other. Further, we perform a sensitivity analysis in respect of the model parameter values. Thus, we will also look at the effect of the yield curve slope on the profit model and also, examine the effect of volatility of the process on the model. Finally, we will consider the effect of α, strength of auto-regression, on profitability.

4.6.2 Model of non-financial (mortality) adverse effect

The proposed model in this context is similar to the one proposed in section 4.2. That is, the single premiums paid less the expenses of the office are accumulated in a fund. This time, suppose a policyholder dies or surrenders the policy at time t, then the office needs to sell assets sufficient enough to pay for death or surrender benefits at that time. Survival benefits are also paid out to policyholders that survive to the end of the contract. Therefore, the excess fund value over expenses and claims payout constitutes the expected surrender profit, which is distributed to providers of the capital.

Note that in this model we consider only the non-financial adverse effect. Further, we assume that lapses occur at the end of the calendar year and that all the assumptions of section 4.2 hold. We propose a model of the form:

$$S^n(0) = E\left\{\left((P_0 - E)e^{\delta_n n} - \sum_{t=0}^{n-1} lp(t)Am(t) - \sum_{t=0}^{n-1} d(t)\frac{MV}{e^{-(n-t-1)\delta_t}} - (l^H{}_{x+n} + l^S{}_{x+n})MV\right)e^{-\delta_n n}\right\} \quad 4.26$$

Where,

$S^n(0)$ = Expected present value of surrender profit/loss at time $t = n$.

lp(t), <u>lapse rate</u> at time t, $= l^H_{x+t} \cdot P_{13}(x+t, x+t+1) + l^S_{x+t} \cdot P_{23}(x+t, x+t+1)$ 4.27

d(t), <u>death rate</u> at time t, $= l^H_{x+t} \cdot P_{14}(x+t, x+t+1) + l^S_{x+t} \cdot P_{24}(x+t, x+t+1)$ 4.28

$MV = \dfrac{(P_o - E)}{\overline{A}_{x:\overline{n}|}}$ (maturity benefit valued at δ_n)

$Am(t) = \dfrac{(1-\lambda)\overline{A}_{x+t+1:\overline{n-t-1}|}}{e^{-(n-t-1)\delta_t}}$ (*Am(t)* is the amount of nominal to redeem to cover SV at time t).

λ = surrender penalty

δ_n = 5.87% (redemption yield of a 20 year gilt)

δ_t^s, δ_t as defined before, and

$\overline{A}_{x:\overline{n}|}$ as defined already.

The expression $\frac{MV}{e^{-(n-t-1)\delta_t}}$ is similar to equation 4.5a, i.e., the MV at time t, accumulated at stochastic force of interest, δ_t to the end of contract.

It is worth noting that *lp(t)* is the probability that a policyholder (healthy) aged x will lapse between ages $x+t$ and $x+t+1$, and *d(t),* probability that a policyholder (healthy) aged x will die between ages $x+t$ and $x+t+1$.

4.6.3 Model of combined adverse mortality and financial effect

Unlike the profit model of section 4.6.2 which considered only the non financial adverse selection effect, our new proposed model takes into account both the financial and non financial anti-selection effects. In other words, we now consider a model that can enable us to investigate the impact of financial anti-selection effect on the profit model. This modified model assumes that healthy individuals will surrender their policy due to financial incentives.

In this case the proposed model is given by equation 4.26, where the lapse model *lp(t)* this time incorporates the adverse selective model of equation 4.25. Here too, we assume that $\mu_{23} = 0$. By using this model, we can compute the present value of the expected surrender profit/loss at time $t = n$ and hence, can analyze the impact of selective lapsation on profitability. Stochastic simulation was used to compute the expected surrender profit/loss and the procedure is the same as used in section 4.2.

In this context, we anticipate a loss to the insurer due to financial anti-selection effect on surrender. In other words, we expect more healthy lives to surrender the policy for financial gain since unhealthy lives are less likely to lapse the policy for financial gains. This is the period of greatest financial loss on surrender to the insurer, as the company needs to sell more of its assets to pay for surrender benefits at those times. Consequently, this loss could be charged to the policyholder for the right to surrender the policy at times that are favourable to him (a strategy to minimize the rate of surrenders). This is discussed fully in the next chapter of this thesis.

What follows is a discussion of the results obtained from this analysis.

4.7 Discussion of Results

4.7.1 Model considers non-financial anti-selection effect.

This section examines the results obtained by using the expected surrender profit/loss model. Note that in the discussion of the results that follow, the model considers only the non-financial anti-selection effect on surrender. In order to measure this effect we have assumed that μ_{21} = 2. We set out below Table 4.6 showing the results obtained from this model. Further, it is worth noting that, in this analysis, the assumed surrender basis pays out more than the return on the investment model and so this may affect the result of the surrender profit model. In other words, the result of the surrender profit model is affected by the case where the parameter value $\delta^{s}_{(0)}$ =0.071 is greater than $\delta_{(0)}$ =0.065. The model parameter values given below are used as a baseline for this analysis and the expected profit or loss observed is expressed as a proportion of the premium paid. A yield curve of negative gradient is used. The proportion of sick and healthy lives in each population discussed in section 4.5.5 will be used throughout this analysis.

<u>Simulation Results (Model considered only non financial anti selection)</u>

The following parameter values were used:

Term of contract	= 20years
Simulation	= 1000
Surrender penalty, λ	= 0%
Initial Expenses	= 10%
Standard deviation in the gilt model, σ	= 10%

Model Parameter Values (baseline values):

$\mu_{12} = 0.2059$

$\mu_{21} = 2$

$\mu_{13} = 0.06$

$\mu_{23} = 0$

$\mu_{14} = 0.0008$

$\mu_{24} = 0.125$

The expected profit from the 1000 simulations, obtained by using the above model parameter values are given in Table 4.6. Also shown is the corresponding standard deviation of the profit.

Expected Present Value Surrender Profit/Loss at T=20years ($\mu_{21} = 2$)

Lapse rate	0%	2%	5%	8%	10%	12%	15%
E(profit)	0.00050	-0.08616	-0.21305	-0.33633	-0.41655	-0.49525	-0.61047
Std(profit)	0.02454	0.03149	0.04170	0.05163	0.05809	0.06443	0.07372

Table 4.6: Result of expected present value surrender profit/loss at T=20years ($\mu_{21} = 2$)

We observe a loss due to the adverse mortality effect. This increases as the lapse rate increases. The reason for this is that, as the lapse rate increases, more healthy lives leave the system, leading to higher average mortality amongst the continuing policyholders. This results in a loss as presented above. Note that even though the life office is expected to make a loss due to adverse mortality effect, this is more significant when there is a higher lapse rate than about 5%. Also, we notice that our model produces effectively a zero profit/loss at zero lapses as anticipated which can be attributed to the fact that most of the time, the assumed surrender basis (δ_t^s) offers payouts which are relatively higher than the return on our investment model (δ_t). We shall look at the case where $\delta_t^s < \delta_t$ and its effect on the expected surrender profit/loss model later in section 4.7.3. We notice that the standard deviation of profit/loss increases as we increase lapse rate. This is due to the fact that there is more variability in the system as more healthy lives leave the system.

Lastly, we would like to emphasis here that although the expected losses as observed are quite high for higher lapse rates, we are anticipating greater losses when the model considers financial anti-selection effect. Subsequently, we should be able to compare the relative effect of one type of selection to the other. We shall discuss this effect in section 4.8.

Recovery Rate effect (Effect of changing μ_{21}=2 to μ_{21}=1)

We also look at the effect of the recovery rate on profitability by reducing the value of μ_{21} from 2 to 1. In other words, we look at the impact on the profit model by assuming that the unhealthy individuals spend more time sick. With $\mu_{21}=1$ and with the same parameter values given above, we obtain the following results as set out in table 4.7.

Expected Present Value of Surrender Profit/Loss at T=20years (at $\mu_{21} = 1$)

Lapse rate	0%	2%	5%	8%	10%	12%	15%
E(profit)	0.00152	-0.07529	-0.18779	-0.29709	-0.36824	-0.43803	-0.54025
Std(profit)	0.03639	0.04254	0.05153	0.06027	0.06596	0.07155	0.07973

Table 4.7 : Result of the expected present value of surrender profit/loss at $\mu_{21}=1$

The results reveal that as the lapse rate increases, the expected loss due to the adverse selective effect also increases. This time the losses are relatively lower than the previous case (when the baseline parameter values were used). The reason for this is that lives in the system spend more time sick and therefore, are less likely to surrender their policies for financial gains. Nevertheless, the lower recovery rates lead to higher average mortality amongst the continuing policyholders. This produces loses as shown in table 4.7.

The next section of this chapter looks at the results of incorporating the financial anti selection effect in the profit model.

4.7.2 Model considers Financial anti selection effect as well as Mortality effect

So far the above results of the expected surrender profit/loss model discussed do not consider the financial anti-selection effect. We now look at the results of analysis when this effect (financial anti-selection effect) is included. It is possible that such an effect can lead to significant losses. Then again, we assume that μ_{21} = 2 in this case. Table 4.8 below shows the results of both financial and non-financial anti selection effect on the expected surrender profit/loss. Further, a sensitivity analysis of the results to changes in different factors is also presented.

Simulation Results

In this section the model parameter values are the same as in table 4.6. However, the following values were used.

The Term of contract	= 20years
Simulation	= 1000
Surrender penalty, λ	= 0%
Initial Expenses	= 10%
New Contract fees, φ	= 5%
Standard deviation in the gilt model, σ	= 10%

Note that the parameter values *D1, D2, fmax* and *fmin* are the same as defined in section 4.5.6.

Table 4.8 sets out the results of the analysis.

Expected Present Value Surrender Profit/Loss at t=20years (μ_{21} = 2)

Lapse rate	0%	2%	5%	8%	10%	12%	15%
E(profit)	0.00050	-0.10665	-0.26293	-0.41402	-0.51196	-0.60774	-0.74747
Std(profit)	0.02454	0.04691	0.07944	0.10979	0.12882	0.14691	0.17237

Table 4.8: Result of expected present value surrender profit/loss at t=20 years ($\mu_{21} = 2$)

In this case, we observe a relatively greater surrender loss due to financial anti selection effect. These losses increase as the lapse rate increases. The reason for this is that since there are financial incentives available to the policyholders, coupled with the fact that lives spend less time sick, lapse rates are expected to increase. This is also the time of greatest financial loss to the insurer, as the company needs to sell more of its assets to pay for surrender benefits at those times. Therefore, the life insurance company is expected to experience a greater loss than the previous case (non-financial anti selection effect). This implies that the adverse financial effect is more marked than the mortality effect in this case.

4.7.3 Sensitivity of Expected Profit to Different factors

We look at the effect of different factors on the profit model. This can be done by varying the variables of interest one at a time whilst the other parameter values are fixed. We therefore consider the following variables and discuss how sensitive they are to the profit model.

Recovery Rate effect (Effect of changing $\mu_{21} = 2$ to $\mu_{21} = 1$)

We look at the effect of recovery rate on profitability by reducing the value of μ_{21} from 2 to 1. In other words, we look at the impact on the profit model by assuming that the unhealthy individuals spend more time sick. With $\mu_{21} = 1$ and with the same parameter values given above (section 4.7.2), we obtain the following results as set out in table 4.9.

Expected Present Value of Surrender Profit/Loss at t=20years ($\mu_{21} = 1$)

Lapse rate	0%	2%	5%	8%	10%	12%	15%
E(profit)	0.00152	-0.08737	-0.21729	-0.34323	-0.42505	-0.50519	-0.62235
Std(profit)	0.03639	0.05024	0.07057	0.08992	0.10219	0.11399	0.13079

Table 4.9: Result of the expected present value of surrender profit/loss at μ_{21} =1

From above table, the relative amount of losses due to the adverse financial effect when the recovery rate is halved is not surprising. This is due to the fact that the unhealthy lives in the system spend more time sick and therefore, are less likely to surrender the policy in spite of the financial incentives available to them. This is consistent with the theory of selective lapsation.

Effect of changing the standard deviation, σ, of the Stochastic yield model

Further, we look at the effect of changing the standard deviation of the stochastic gilt yield model on the expected surrender profit/loss. Here, for fixed α (strength of auto-regression) we consider different values of σ and repeat the analysis of Table 4.8 (baseline analysis). We set out below, a table showing the effect of σ on the expected surrender profit/loss at time t = 20 years. Note that μ_{21} = 2 and μ_{13} = 0.05 are as assumed in base case result. The parameter values are the same as used in the base case.

Expected Present Value of Surrender Profit/Loss at time t=20year

Stdev of Yield Model	5%	10%	15%	20%	25%
E(profit)	-0.23532	-0.26293	-0.31854	-0.40844	-0.54068
Std(profit)	0.02346	0.07944	0.177235	0.331403	0.577626

Table 4.10 : Result of changing the standard deviation, σ, of the Stochastic yield model

We observe that as the standard deviation of the yield model increases, the expected loss increases as expected. The reason for this is that an increase in volatility implies that there is more uncertainty in the nominal amount function which in turn affects the amount of assets which need to be redeemed to cover the surrender value at time t. Therefore, for a fixed lapse rate, coupled with the fact that the surrender basis pays more on surrender than the return on assets, the expected loss is likely to increase as observed. This means that the profit/loss model is sensitive to the standard deviation of the gilt model.

Note that the standard deviation of profit increases as σ increases. The result is consistent with that of table 4.5 where the expected profit decreases as σ increases. However, we observe that the option value increases as volatility increases in the case of Albizzati and Geman (1994). This is consistent with our results.

Sensitivity to α, strength of auto-regression (used in the investment model).

Following the discussion of sensitivity of σ, we look at the effect of α on profitability by considering different values of α for fixed σ. Table 4.10a shows the results obtained.

Expected Present Value of Surrender Profit/Loss at t=20 years.

	$\delta_t^s > \delta_t$		
Slope of Yield Curve:	Negative		
α effect:	85%	90%	95%
E(Profit):	-0.26083	-0.26293	-0.26689
Std(Profit):	0.065193	0.079436	0.101569

Table 4.10a : Result of expected present value of surrender profit/loss for different value of α

We know from section 3.2.1 that the higher the value of α implies that the gilt yield returns can be expected to move more slowly towards the mean value and vice versa. Therefore, from table 4.10a, we observe that as α increases for fixed σ and μ_{13}, the expected loss steadily increases and so does the corresponding standard deviation. This is as expected since the standard deviation of the process increases as α approaches 1, (see equations 3.13 and 3.14).

Effect of Surrender penalty on Expected Profit/Loss

In practice, small surrender penalties are normally charged on policies that are surrendered because of competition from other companies, Albizzati and Geman (1994). These charges vary from one life office to other. Nevertheless, there is a maximum penalty that a life office can realistically charge. It is hoped that imposing a higher surrender penalty may reduce lapses. Such a strategy will be looked at in Chapter 5.

In this analysis, we look at the effect of surrender penalty on the expected surrender profit/loss by varying the values of λ, whilst the other model parameter values remained fixed. Table 4.11 shows the effect of λ on the expected surrender profit/loss.

Expected Present Value of Surrender Profit/Loss at Time t=20years

Surrender Penalty	0%	5%	10%	15%	20%	25%
E(profit)	-0.26293	-0.23899	-0.22085	-0.20559	-0.19196	-0.17911
Std(profit)	0.07944	0.06354	0.05254	0.04532	0.04092	0.03841

Table 4.11: Result of expected present value of surrender profit/loss for different values of λ

From table 4.11, we observe that as the surrender penalty increases, the expected loss decreases as anticipated. This means that increasing the surrender penalty may probably discourage policyholders from lapsing, and therefore reduce any expected losses. Note that a 0% surrender penalty means that there is no surrender penalty charge on lapsing. Further, we observe that as the surrender penalty increases, the standard deviation of profit decreases. This is probable due to the fact that as surrender penalty increases, there is less financial incentive to surrender and so, there is less variation in the *Am(t)* model.

Effect of New Contract fees on Expected Profit/Loss

For most life insurance contracts, there is a fee charged whenever a new contract is initiated. Broadly speaking, we believe that charging higher contract management fees may likely discourage policyholders from lapsing. Thus, in this section we look at the effect of new contract management fees, φ on the expected surrender profit/loss. We set out below, a table showing the effect of φ on the expected surrender profit/loss. It is also important to note that the baseline parameter values were used in order to observe effectively the φ effect.

Expected Present Value of Surrender Profit/Loss at t=20years

φ	1%	5%	10%	15%	20%	25%
E(profit)	-0.27916	-0.26293	-0.25253	-0.24709	-0.24398	-0.24217
Std(profit)	0.09239	0.07944	0.06677	0.05804	0.05209	0.04813

Table 4.12: Result of expected present value of surrender profit/loss for different values of φ

In this case, the expected loss due to surrender decreases when we increase the management fee of initiating a new contract, φ. As expected, increasing φ implies that the financial incentive available for lapsing has reduced. So, we expect the rate at which policyholders lapse their policies to decrease. This in turn implies that the expected loss due to surrender decreases as observed. The result of table 4.12 is consistent with that of table 4.11, where the expected loss decreases as λ increases. This is as expected since the decision criterion variable involves λ, φ and the ratio $(1-\lambda)/(1+\varphi)$.

Effect of Yield Curve Slope on Model

We look at the effect of the slope of the yield on the profitability of the company by considering a yield curve of positive and negative slope as shown in figures 4.3 and 4.4. In this case, we consider the case where $\delta_t^s > \delta_t$ as assumed before, for consistency. Table 4.13 shows the results obtained.

Expected Present Value of Surrender Profit/Loss at Time t=20years

Difference in Payout :	$\delta_t^s > \delta_t$	
Slope of Yield curve :	Negative	Positive
E(Profit) :	-0.26293	-0.31139
Std(profit) :	0.07944	0.11255

Table 4.13: Result of expected present value of surrender profit/loss for different shape of yield curve

We observe that the expected loss increases with the slope of initial yield curve. In other words, the expected profit decreases with the slope of the initial curve for $\delta_t^s > \delta_t$. The results are not surprising because the increase in the yield curve slope only affects the amount of assets that need to be sold to cover surrender values at time *t*. So, since the yield on assets has now increased, this implies that for a fixed lapse rate, the expected loss will increase. Also, the result is partly due to the fact that the assumed surrender basis is paying out more on surrender than the returns on the investment model. Therefore, the expected profit/loss is affected by the slope of initial yield curve.

Effect of Relative payout on model.

From the above analysis performed so far (from section 4.5 onwards), we notice that the profit/loss model assumed a surrender basis that pays out values which are higher than the return on the proposed investment model. (i.e., $\delta_t^s > \delta_t$). We therefore consider the case where the assumed surrender basis pays out values which are lower than the return on our investment model. (i.e., $\delta_t^s < \delta_t$) and the results obtained are shown below.

The above-mentioned scenario can enable us to determine the effect of changing the model assumption of $\delta^{s}_{(0)}$=0.071 and $\delta_{(0)}$=0.065 (i.e., $\delta^{s}_{t} > \delta_{t}$) to $\delta^{s}_{(0)}$=0.071 and $\delta_{(0)}$=0.07 ($\delta^{s}_{t} < \delta_{t}$) on the expected profit model. Table 4.14 shows the results obtained when the baseline model parameter values are used. In this case a yield curve of negative slope is used.

Expected Present Value of Surrender Profit/Loss (yield curve of negative slope)

Slope of Yield curve :	Negative	
Difference in Payout:	$\delta^{s}_{t} > \delta_{t}$	$\delta^{s}_{t} < \delta_{t}$
E(Profit) :	-0.26293	-0.29332
Std(profit) :	0.07944	0.09670

Table 4.14: Result of expected present value of surrender profit/loss when the assumption $\delta^{s}_{t} > \delta_{t}$ is changed to $\delta^{s}_{t} < \delta_{t}$ (for yield curve of negative slope)

As shown in table 4.14, we observe that the expected loss relatively increases when we change the assumption of $\delta^{s}_{t} > \delta_{t}$ to $\delta^{s}_{t} < \delta_{t}$. This is actually reasonable, because we have assumed in the model that there is financial incentive to surrender the policy, coupled with the fact that a yield curve of negative slope is used. This means that with increased stochastic force of interest in the stochastic model, we need to redeem a greater amount of assets to cover surrender value at t. Hence, a greater loss is observed.

We notice that the standard deviation of profit increases when we change the assumption of $\delta^{s}_{t} > \delta_{t}$ to $\delta^{s}_{t} < \delta_{t}$. This is probable due to the fact that we have introduced more variation in the stochastic model by increasing the value of $\delta_{(0)}$. This result is consistent with that of table 4.1, where the expected profit decreases as the assumption of table 4.14 changes.

By repeating the analysis of table 4.14 for the case where a yield curve of positive slope is used, we obtain the following results presented in table 4.15.

Expected Present Value of Surrender Profit/Loss (yield curve of positive slope)

Slope of Yield curve :	Positive	
Difference in Payout:	$\delta_t^s > \delta_t$	$\delta_t^s < \delta_t$
E(Profit) :	-0.31139	-0.31563
Std(profit) :	0.11255	0.11574

Table 4.15 : Result of expected present value of surrender profit/loss when the assumption $\delta_t^s > \delta_t$ is changed to $\delta_t^s < \delta_t$ (for yield curve of positive slope)

From table 4.15, we observe a result similar to the one described in table 4.14. However, there is a slight increase in the expected loss this time when the model assumption is changed from $\delta_t^s > \delta_t$ to $\delta_t^s < \delta_t$. Further, we observe that the loss amount has increased when a yield curve of positive slope is used, with the corresponding high value of standard deviation. This is probably due to the fact that by using a yield curve of positive slope, the stochastic force of interest in the stochastic model has increased. Therefore we need to redeem a greater amount of assets to cover surrender value at t. Hence, a greater loss is observed. Also, there is more variation in the stochastic model by increasing the value of $\delta_{(0)}$. This accounts for the higher standard deviation of expected profit.

4.8 Relative effect of one type of selection compared with another

From tables 4.7 and 4.8, we have shown the results of the expected profit/loss when the recovery rate is halved and also, when the financial anti-selection effect is introduced in the model. Now, in order to explore the effect of these additional factors on the proposed profit model, and also, to investigate which factor has the greatest effect, we compare the results of tables 4.7 and 4.8 against our baseline result of table 4.6. Particularly, we look at the difference in the expected profit/loss between the baseline values and that due to the financial effect and recovery rate effect.

Let the baseline expected loss be m_b and the baseline variance of loss be s_b^2.

Let the expected loss due to recovery rate effect (when $\mu_{21}=2$ is changed to $\mu_{21}=1$) be m_1 and the corresponding variance of loss be s_1^2.

Let the expected loss due to financial anti-selection effect be m_2 and the corresponding variance of loss be s_2^2.

By looking at the ratio of $\frac{s_1^2}{s_b^2}$ and $\frac{s_2^2}{s_b^2}$, we obtained the following results presented in tables 4.16 and 4.17.

Table 4.16 shows the difference in baseline expected surrender loss and that due to changes in recovery rate (i.e., when μ_{21}=2 is changed to μ_{21}=1). Also shown is the ratio of the corresponding variance of the process.

Difference in Expected Surrender Profit/Loss at time t=20years due to recovery rate (when $\mu_{21}=2$ was changed to $\mu_{21}=1$)

Lapse rate	0%	2%	5%	8%	10%	12%	15%
$m_1 - m_b$	0.00102	0.01087	0.02526	0.03924	0.04831	0.05722	0.07022
s_1^2/s_b^2	2.19895	1.82494	1.52703	1.36269	1.28931	1.23323	1.15916

Table 4.16: Result of expected present value of surrender profit/loss due to recovery rate effect

From table 4.16, we observe that the expected profit due to recovery rate effect increases as the lapse rate increases. As mentioned before in section 4.7, this could be attributed to the fact that the significant number of unhealthy lives in the system spend more time sick (due to μ_{21}=1) and therefore are less likely to surrender the policy for financial gains.

Next, we present in table 4.17 below, the difference in expected baseline surrender loss and that due to the introduction of financial anti-selection effect in the profit model. In other words, the difference between the expected surrender profit due to financial anti-selection effect and the baseline values is presented in table 4.17.

Difference in Expected Surrender Profit/Loss at time t=20years due to financial anti-selection effect

Lapse rate	0%	2%	5%	8%	10%	12%	15%
$m_2 - m_b$	0.00000	-0.02049	-0.04988	-0.07769	-0.09541	-0.11249	-0.13700
S_2^2 / S_b^2	0.03639	2.21914	3.62916	4.52190	4.91772	5.19908	5.46705

Table 4.17: Result of expected present value of surrender profit/loss due to financial adverse selection effect

From table 4.17 above, we observed that the expected loss due to financial anti selection effect increases as the lapse rate increases. This is due to the fact that the significant number of unhealthy lives in the system spend less time sick (due to μ_{21}=2) and so, these healthy lives are more likely to surrender the policy for financial gains which is offered by the policy.

Therefore, by comparing the two effects (recovery rate and financial anti selection effect), we note that, for the parameter values investigated, the financial anti-selection effect is more significant than the recovery rate effect. Hence we can say that the life office is likely to increase its expected losses due to financial incentives available to policyholders (lapse rate is expected to increase as well). On this note the life office needs to devise and adopt strategies to meet these expected losses due to the financial anti-selection effect. We will discuss 'strategies' to use in order to maximize the expected utility of shareholders' profit in chapter 5.

Appendix

Appendix 4.1

Let Q be a 4x4 matrix with *(i, j)* entry of μ_{ij}. Thus

$$Q=\begin{bmatrix} x & a & b & c \\ d & -y & 0 & f \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \text{A4.1}$$

Where $a=\mu_{12}$; $b=\mu_{13}$; $c=\mu_{14}$; $d=\mu_{21}$; $f=\mu_{23}$; $x=\mu_{12}+\mu_{13}+\mu_{14}$; and $y=\mu_{21}+\mu_{24}$

Notice that we have assumed that unhealthy lives will not lapse and so $\mu_{23}=0$.

The eigenvalues of Q is given by

$$\begin{bmatrix} 0 & 0 & \dfrac{-x-y-\sqrt{(x+y)^2-4(xy-ad)}}{2} & \dfrac{-x-y+\sqrt{(x+y)^2-4(xy-ad)}}{2} \end{bmatrix} \qquad \text{A4.2}$$

The eigenvectors corresponding to the eigenvalues Q is given by

$$E=\begin{bmatrix} -\frac{f}{d}-\frac{y(cd+fx)}{d(ad-xy)} & -\frac{(cd+fx)}{(ad-xy)} & 0 & 1 \\ -\frac{by}{(ad-xy)} & -\frac{bd}{(ad-xy)} & 1 & 0 \\ \frac{-(x-y+\sqrt{4ad+x^2-2xy+y^2})}{2d} & 1 & 0 & 0 \\ \frac{-(x-y-\sqrt{4ad+x^2-2xy+y^2})}{2d} & 1 & 0 & 0 \end{bmatrix}^T \qquad \text{A4.3}$$

Let $A=E^T$,

Where,

E^T is the transpose of *E*.

Further, we find the inverse of *A* which is given by

$$A^{-1}=\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ -\dfrac{d}{\sqrt{4ad+(x-y)^2}} & \dfrac{d\left(\frac{y-x+\sqrt{4ad+(x-y)^2}}{2d}\right)}{\sqrt{4ad+(x-y)^2}} & \dfrac{d\left(\frac{-bx-by+b\sqrt{4ad+(x-y)^2}}{2(ad-xy)}\right)}{\sqrt{4ad+(x-y)^2}} & \dfrac{d\left(\frac{-f}{d}-\frac{c\left(x+y-\sqrt{4ad+(x-y)^2}\right)}{2(ad-xy)}-\frac{fx\left(x+y-\sqrt{4ad+(x-y)^2}\right)}{2d(ad-xy)}\right)}{\sqrt{4ad+(x-y)^2}} \\ \dfrac{d}{\sqrt{4ad+(x-y)^2}} & \dfrac{d\left(\frac{x-y+\sqrt{4ad+(x-y)^2}}{2d}\right)}{\sqrt{4ad+(x-y)^2}} & \dfrac{d\left(\frac{bx+by+b\sqrt{4ad+(x-y)^2}}{2(ad-xy)}\right)}{\sqrt{4ad+(x-y)^2}} & \dfrac{d\left(\frac{f}{d}+\frac{c\left(x+y+\sqrt{4ad+(x-y)^2}\right)}{2(ad-xy)}+\frac{fx\left(x+y+\sqrt{4ad+(x-y)^2}\right)}{2d(ad-xy)}\right)}{\sqrt{4ad+(x-y)^2}} \end{bmatrix}$$

A4.4

But C= A^{-1}

Hence, $P(t) = A \cdot diag(e^{d_1 t}, \ldots\ldots, e^{d_4 t}) \cdot C$. A4.5

Therefore, we can now write

$$P_{ij}(t) = \sum_{n=1}^{4} a_{in} c_{nj} \cdot e^{d_n t} \qquad \text{A4.6}$$

Where a_{ij} and c_{ij} are the *(i,j)* entries of *A* and *C*, respectively.

Chapter 5

Utility-Maximisation of Shareholders' Expected Profit

5.1 Introduction

This section considers a typical U.K. proprietary life office that transacts conventional non-profit business, with surplus distributed to shareholders. Note that the same type of business was considered in chapter 4. There, we showed that the profitability of a company is mostly affected by adverse selection effects.

Ideally, the management of a life insurance companies aims to provide its investors, (policyholders or shareholders or both) with an acceptable profile of returns from their investment. One way by which this can be achieved is when the expected utility of shareholders' profit is maximized, while an acceptable level of solvency (an important aspect of the profile of returns to policyholders) and a suitable premium basis are maintained. Further, we can achieve this by developing strategies that would involve the policyholder in sharing the cost of surrender at times which are favourable to him. These strategies will be discussed shortly. There are other ways by which this can be achieved, as discussed by several authors. For example, Chadburn (1998) has examined the relative effects of a range of life insurance management strategies upon the profile of returns experienced by with-profits life insurance policyholders. Also, Ong (1995) and Booth et al (1997) have looked at optimal asset allocation strategies for a life office by using the utility maximization approach. Further, Kroll et al (1984) have compared the expected utility of the optimum portfolio for given utility functions with the expected utility of well-selected portfolios from the mean-variance efficient frontier.

In this chapter, we shall attempt to choose a blend of strategies that maximizes the expected utility of shareholders profit. A description of these strategies will be given in the next section. Numerical optimization procedures, similar to Ong (1995), have been used to produce all our results, and these are described in section 5.4. The results are analysed and a sensitivity analysis of the profit to changes in the model parameter values is also performed. Finally, the effect on profitability of using inappropriate strategies will be discussed.

5.2 Utility-Maximisation

5.2.1 Description of Optimal Strategic Decision-Making Process

In this section, we give a description of strategies used to maximize the expected utility of shareholders' profit. It is worth noting that the interest of life insurance company's investors (shareholders and policyholders) can be met (bearing in mind the possibility of lapses) by considering the four strategies described below.

The strategies are based on the insurance company's response to changes in interest rates. In other words these strategies are designed to control the frequency with which the surrender basis is changed. We propose these strategies based on the fact that a life office surrender basis sometimes pays out more than that which is provided by the investment model (returns) and vice versa. In what follows, we present the four strategies used in the maximization process.

Strategy 1 involves changing the surrender value basis whenever the difference between the average of the past three years' surrender basis interest assumptions and the average of the company's return on assets is greater than a decision variable, $d1$. That is, we change the surrender basis to a new basis if

$$\overline{\delta}_t^{sv(3)} - \overline{\delta}_t^{(3)} > d1$$

where

$\overline{\delta}_t^{(3)}$ is the average of the past three years' return on assets and

$\overline{\delta}_t^{sv(3)}$ is the average of the past three years' surrender basis. Thus,

$\overline{\delta}_t^{(3)} = \dfrac{\delta_t + \delta_{t-1} + \delta_{t-2}}{3}$ and $\overline{\delta}_t^{sv(3)} = \dfrac{\delta_t^{sv} + \delta_{t-1}^{sv} + \delta_{t-2}^{sv}}{3}$, for $t=3,4,\ldots,(n-1)$.

Note that *d1* presents the insurance company's response to a fall in interest rates. In other words, the company needs to change the surrender basis whenever the average of the past three years' stochastic force of interest corresponding to the return on assets is lower than the average of the past three years surrender basis less *d1*. Further note that we choose three years for the averaging period because we believe it is a reasonable period over which to smooth the asset returns and the surrender basis assumptions.

Strategy 2 involves the life office changing its surrender basis whenever the difference between the average of the past three years returns on the company's assets and the corresponding surrender assumptions for that period is greater than a decision variable *d2*, defined below. That is, change the surrender basis if

$\overline{\delta}_t^{(3)} - \overline{\delta}_t^{sv(3)} > d2$

Also note that *d2* is the company's response to a rise in interest rate. Thus, the value of *d1* and *d2* determines when to change the surrender basis on the basis of changes (rise or fall) in interest rates. For example, an optimal strategic way of regulating the surrender basis will be to change the basis if interest rates rise by *d1* or fall by *d2*.

Strategy 3 involves charging the policyholder for the right or option to surrender the policy during the most unfavourable economic conditions (i.e., impose a premium loading). However, there is a risk of undercharging or overcharging the policyholder for this right. Therefore, we have proposed a model for the premium penalty to allow for such a risk. This will be discussed in section 5.2.4.

Strategy 4 involves charging the policyholder who surrenders the policy with a higher surrender penalty. This is to discourage policyholders from lapsing. However, we feel that it will not be ethical to charge an excessive amount because of regulations and competition from other companies. Therefore, we include a constraint on the amount of the penalty.

Finally, the probability of insolvency must be controlled. Then, we shall choose a blend of these strategies that maximises the expected utility of shareholders' profit. These strategies are represented by the following decision variables outlined below:

d1 Change the surrender value basis to a new basis (defined below), whenever the difference between the average of the past three years surrender basis and the returns on company assets (past three years) is greater than the decision variable under consideration, named surrender basis1.

Note that the new basis is the old basis, adjusted by adding or subtracting the above difference.

d2 Change the surrender value basis to a new basis whenever the difference between the average of the past three years returns on assets and the corresponding surrender basis (of past three years) is greater than the decision variable under consideration, named surrender basis 2.

d3 Charge the policyholder (impose a premium loading) for the right or option to surrender the policy during the most unfavourable economic conditions.

d4 Charge the policyholder who surrenders the policy with a higher surrender penalty. This is to discourage frequent surrender of policies.

5.2.2 Formulation of Problem

In this section we shall denote *SP* as the present value of shareholders' surrender profit which is defined below, and **d**={d1,….,d4}, a vector of decision variables.

We write *SP* as:

$$SP = \left\{ \left((P_0^* - E)e^{\delta_n n} - \sum_{t=0}^{n-1} l(t)Am^*(t) - \sum_{t=0}^{n-1} d(t)\frac{MV}{e^{-(n-t-1)\delta_t}} - (l^H{}_{x+n} + l^S{}_{x+n})MV \right) e^{-\delta_n n} \right\}$$

where

$$P_0^* = P_0 + d_3$$

$$MV = \frac{\left(P_o^* - E\right)}{\overline{A}_{x:\overline{n}|}}$$

$$Am^*(t) = \frac{(1-d_4)\overline{A}_{x+t+1:\overline{n-t-1}|(d_1 \,\&\, d_2)}}{e^{-(n-t-1)\delta_t}}$$ (i.e., $\overline{A}_{x+t+1:\overline{n-t-1}|}$ is defined in a similar way to equation 4.22a–4.23 with δ_t^s adjusted by d_1 or d_2 and $Am^*(t)$ is the amount of nominal to redeem to cover *SV* at time *t*).

It is important to note that for the case where the model considers financial adverse selection effect, the 'adverse' force of incidence of lapsing is defined in a similar way to equation 4.25 where the decision criterion formula, *D(t)*, is re-defined as

$$D(t) = \frac{(1-d_4)\cdot\overline{A}_{x+t+1:\overline{n-t-1}|(d_1 \,\&\, d_2)}}{(1+\varphi)\overline{A}_{x+t+1:\overline{n-t-1}|(\delta_t)}}.$$

Now, our objective is to choose **d** that maximises the expected utility of *SP* subject to *m (m=4 in this case)* constraints of the form $g_k \le d_k \le h_k$, *k=1,2,...m,* where the lower and upper constraints g_k and h_k are constants. Thus, the form of the optimization problem in this case can be expressed as

$$\underset{d}{Max}\, E\{U(SP)\}$$

subject to the following constraints :

$$0 \le d_1 \le 0.02$$
$$0 \le d_2 \le 0.02$$
$$0 \le d_3 \le 0.50$$
$$0 \le d_4 \le 0.25$$

where *U(SP)* is the utility of the present value of the shareholders' profit.

The constraints, *d1* and *d2* were chosen to be less than 0.02 because on the basis of our surrender force of interest and stochastic models, we believe the rise or fall of interest rate (based on the value of force of interest) should not exceed 0.02 in order to trigger a change in surrender basis. Alternatively, we believe that if the difference between the average of the past three years surrender basis and that of the interest rate is less than the chosen constraint of 0.02, then there cannot be any change of basis irrespective of the movement of interest rate.

Further, the constraint of *d3* was chosen to lie within the range 0 to 0.5 because we believe that it is reasonable to charge an amount of up to 0.5 considering the fact that the single premium paid by the insured is assumed to be 1. However, we believe that a value of 0.8 would be too high to charge since this could lead to loss of business.

Finally, we chose *d4* to be 0.25 because we believe that it is a reasonable amount to charge, since imposing a higher penalty will probably lead to loss of business due to competition from other companies.

Clearly, the objective function is highly non-linear since the model of *SP* has a stochastic element in it. In this case the first (and possibly even second) derivatives of the objective function are unavailable.

Furthermore, when faced with a constrained optimization problem, the general aim according to Walsh (1979) is to reduce it to an unconstrained problem or to a sequence of unconstrained problems. In view of this we shall consider a transformation by which the above-defined constrained optimization can be reduced to a form in which no constraints explicitly appear so that the derivatives are not required. Such a transformation will be discussed in section 5.3.4. Subsequently, we will resort to Powell (1964), (1965), (1975)'s method of finding the minimum of a function which does not require the use of derivatives. This method has been described by Fletcher (1965), Nash (1979), Press (1992), Ong (1995), and Conn et al (1997). We will discuss this method (Powell (1965), (1975)) in section 5.3.3. Next we look at how the utility function was chosen.

5.2.3 *Choice of Utility function*

The utility function is an attempt to assign a numerical value to different levels of wealth allowing for an individual's preferences. Although utility functions may take many functional forms, there are in general some properties which most utility functions should satisfy in decisions involving wealth. That is, for a given level of wealth, the utility function, *U(.)* should be monotonically increasing. That is, $U'(.) > 0$, meaning that individuals do not prefer less wealth to more. According to Booth (1997), "any individual who did prefer less wealth to more could presumably find ways of disposing of wealth until this position was rectified." Also, the utility function, *U(.)* should be concave. That is, $U''(.) < 0$, which corresponds to investors being risk averse. In other words, the value that one puts on a given increment in wealth does not increase as one's level of wealth increases.

There are several forms of the utility function that have been proposed and their properties discussed in the literature, for example, quadratic utility, exponential, logarithmic and power utility functions. However, this section will look at only one example, (exponential utility function), which is relevant to our work. For further discussion of the properties of utility functions, see Booth (1995), (1997), Pratt (1964), Kroll (1984), Fishburn (1970), Ong (1995) and Gerber and Pafumi (1999).

Broadly speaking, the choice of utility function depends in part, on the function under investigation and also, whether its risk aversion properties are reasonable. Risk aversion was defined by Pratt (1964) as follows. For a given utility function *U(.)*, the requirement $U''(.) < 0$ ensures that an individual is risk averse. However, the extent to which an individual can see how risk aversion changes is by looking at the risk premium an investor requires for an actuarially neutral investment. Pratt (1964) showed that any risk premium is approximately proportional to:

$$\psi(.) = -\frac{U''(.)}{U'(.)},$$

where $\psi(.)$ is defined as the measure of risk aversion (absolute). Thus, the higher $\psi(.)$ becomes, the more averse the individual to risk and so lower risk tolerance. On the other hand, relative risk aversion was defined by Pratt to be $\rho(x) = x\psi(x)$ for wealth x. Note that according to Bernstein and Damodaran (1998), the risk premium is the expected rate of return in excess of the risk-free interest rate that an investor demands to compensate for the risks inherent in an investment. In other words, the risk premium is the reward for holding a risky asset or portfolio rather than the risk-free asset.

5.2.4 Properties of Utility Functions

We consider the risk aversion properties of some utility functions discussed by Pratt (1964), Ong (1995) and Booth (1997) to help us choose an appropriate utility function for our investigations. Thus, with a starting value of wealth x, the linear utility function is defined by:

$$U(x) = a + bx$$

With $\quad U' = b$ and $U'' = 0$

And so, $\quad \psi(x) = \frac{-U''(x)}{U'(x)} = 0$ for all values of x.

Therefore, by using Pratt's measure of risk aversion, the linear utility function has a constant (zero) risk aversion.

The quadratic utility function takes the form:

$$U(x) = a + bx + cx^2, \; c < 0, \; b > 0$$

We have $\quad U' = b + 2cx$ and $U'' = 2c$

And so, $$\psi(x) = -\frac{2c}{b + 2cx}$$

In order for the function to exhibit risk aversion it is necessary for $c < 0$. This measure of risk aversion increases until such a point that $b = -2cx$. According to Booth (1997), this is the point at which the quadratic utility function peaks and it is only meaningful over this range. Therefore, the quadratic utility function implies increasing risk aversion over all relevant values of the starting value of wealth. According to Ong (1995), the main disadvantage of using the quadratic utility function is that "the restricted range in which this function is meaningful and the property of increasing risk aversion make it less appealing, especially in comparison with the exponential, logarithmic or power utility functions".

The exponential utility function is of the form:

$$U(x) = -exp(-kx), \qquad k>0$$

with $$U'(x) = k\exp(-kx) \text{ and } U''(x) = -k^2\exp(-kx)$$

and therefore,

$$\psi(x) = k,$$

which is referred to as the constant absolute risk aversion property. This implies that the decision making process will depend on the amount invested and not initial level of wealth for a given investment. This is illustrated below.

Suppose an investor with an exponential utility function of wealth x invests an amount w at a random rate of return R, then the utility maximizing decision is to maximize:

$$E[U(x+wR)] = E[-exp(-k(x+wR))] = p.E[exp(-kwR)]$$

Where $p = -exp(-kx)$. Hence, according to Ong (1995), the optimal decisions based on maximizing $E[U(x+wR)]$ and $E[U(wR)]$ will be identical.

The logarithmic utility function is given by:

$$U(x) = ln(x)$$

with $$U'(x) = \frac{1}{x} \text{ and } U''(x) = -\frac{1}{x^2}$$

Clearly, this function will be valid only for positive real values of x.

Therefore,

$$\psi(x) = \frac{1}{x},$$

which displays a decreasing absolute risk aversion, though it has the property of constant relative risk aversion, defined in Pratt(1964) as:

$$\rho(x) = x\psi(x) = 1$$

This means that decisions will depend upon the proportion of wealth invested, and not on the starting level of wealth. Hence, according to Booth (1997), "investors having a logarithmic utility function, which invest in the same proportion of their wealth, would require the same risk premium for a risky investment and would have the same optimal strategy for the investment of a given proportion of wealth".

The power utility function is of the form:

$$U(x) = x^c$$

where $0 < c < 1$. This function also exhibits constant relative risk aversion.

Thus,

$$U'(x) = cx^{c-1} \text{and } U''(x) = c(c-1)x^{c-2}$$

And $$\psi(x) = -(c-1)x^{-1}$$

Therefore,

$$\rho(x) = 1 - c$$

which is less than one. This means that the power function is also less averse than the logarithmic function. If $c=1$, then it is simply a linear utility function and $\psi(x) = \rho(x) = 0$, which implies risk neutrality.

Throughout this chapter, the investor will be assumed to have an exponential utility function because of its desirable properties and for reasons of tractability. The use of the exponential function can enable us to compare a range of investor risk preferences.

We therefore define the utility of shareholders profit as follows:

$$U(SP) = -\exp\left(-\frac{N(l) \cdot SP}{r}\right) \qquad 5.1$$

where,

SP is the present value of shareholders profit at time $t = 20$, defined in section 5.2.2.

r is the reciprocal of the risk aversion parameter, *k*, which is a relative measure of risk tolerance. The higher is the value of *r*, the greater is the tolerance to risk. That means that for a given amount invested, an investor with a higher value of *r* would be expected to invest in a more risky portfolio. In our case the optimal strategic decision is that which maximizes

$E\left(-e^{-N(l)\cdot\frac{SP}{r}}\right)$ or minimizes $E\left(e^{-N(l)\cdot\frac{SP}{r}}\right)$. The latter optimization problem represents the conventional way of approaching a problem like this.

N(l) is the amount of business issued per loading *l*, and is introduced to take care of any bias caused by charging a higher premium penalty. This is discussed in the next section.

5.2.5 Model of Number of Business Issued at Office Loading

In view of the fact that a life office could lose most of its prospective policyholders as a result of charging a higher premium (in order to cover the cost of the policyholder's right to surrender the policy), the optimization result could be biased if no action is taken to compensate for this. As a result, we need to impose a premium penalty to take care of such bias. We present below a model of the premium penalty to allow for this effect. This model is similar to the hazard rate function of an exponential distribution. We choose this model because it is simple, but essential in this

investigation. According to Ross (1996), the hazard rate of a continuous random variable *X* having distribution function *F* and probability density function *f*, is defined by

$$\gamma(t) = \frac{f(t)}{\bar{F}(t)} = \frac{f(t)}{1-F(t)}$$

By using the above definition, we model the amount of business issued per office loading as follows:

Suppose the rate at which the number of policies, *N(l),* are issued at life office's loading *l*, relative to the market loading is $\gamma(l)$. Then

$$\frac{dN(l)}{dl} = -\gamma(l)N(l) \qquad 5.2$$

which implies that

$$N(l) = N_o.\exp\left\{-\int_{\beta}^{l}\gamma(s)ds\right\} \qquad 5.3$$

where $N(l)$ is the number of policies issued at office's loading *l,* and $N(\beta) = N_o$ is the initial number of policies at initial loading, β. We assume a hazard rate of the form

$$\gamma_o(l) = k_0.\left(\frac{l}{\beta}\right)^{b_o - 1} \qquad \text{for } l > \beta \qquad 5.4$$

which is a power function of order $(b_o - 1)$, with k_o representing the base rate of reduction in new business if $l > \beta$, and b_o, the shape parameter of the model.

Further, we assume that β is a loading factor at which the office initial loading equals the market initial loading. In this case, if the office loading were to become greater than the market loading, then the volume of new business would decrease. However, if the office loading is less than the market loading, then, the volume of new business will increase. By solving equation 5.3, we obtain the following equation

$$N(l) = N_o.\exp\left\{-\frac{k_o}{\beta^{b_o-1}b_o}\left(l^{b_o} - \beta^{b_o}\right)\right\} \qquad 5.5$$

Now, for $l < \beta$, we assume that the volume of business depends on the office's loading in relation to the market loading and therefore propose a hazard rate of the form:

$$\gamma_1(l) = a_1 + k_1.\left(\frac{\beta}{l + c_1}\right)^{b_1-1} \qquad 5.6$$

where

k_1 is the base rate of reduction in new business if $l < \beta$

c_1 is a constant parameter to remove singularity at zero.

a_1 is a constant.

By substituting this hazard rate function in equation 5.3 and solving it, we obtain the following equation:

$$N(l) = N_1.\exp\left\{a_1 \cdot (\beta - l) + \frac{k_1.\beta^{b_1-1}}{2 - b_1}\left[\left((\beta + c_1)^{2-b_1} - (l + c_1)^{2-b_1}\right)\right]\right\} \qquad 5.7$$

where $N_1 = N_o$ (initial number of policies at initial loading, β). Hence, by combining equations 5.5 and 5.7, we obtain the following model of the number of policies issued at office loading l.

$$N(l) = \begin{cases} N_o.\exp\left\{-\frac{k_o}{\beta^{b_o-1}b_o}\left(l^{b_o} - \beta^{b_o}\right)\right\} & \text{if } l > \beta \\ N_1.\exp\left\{a_1 \cdot (\beta - l) + \frac{k_1.\beta^{b_1-1}}{2 - b_1}\left[\left((\beta + c_1)^{2-b_1} - (l + c_1)^{2-b_1}\right)\right]\right\} & \text{if } l < \beta \end{cases} \qquad 5.8$$

Note that we have assumed N_o $=100\%$ if $\beta=20\%$. This value of beta is chosen because it represents a realistic value at which market loading equals office loading. The implication of this model is a discontinuity in the gradient of $N(l)$ at $l=\beta$.

It is worth mentioning that the value of l used in the utility model is the optimized value found by our optimization procedure; see section 5.3.2 for the optimization method used.

Also, in above equation, the following parameter values are used:

$N_o = 100\%; N_1 = 100\%; \beta = 0.2; b_o = 4; b_1 = 3/2; k_o = 3; k_1 = 1; a_1 = 0.2; c_1 = 0.001$

By using the above model (equation 5.8), we obtain the following plots:

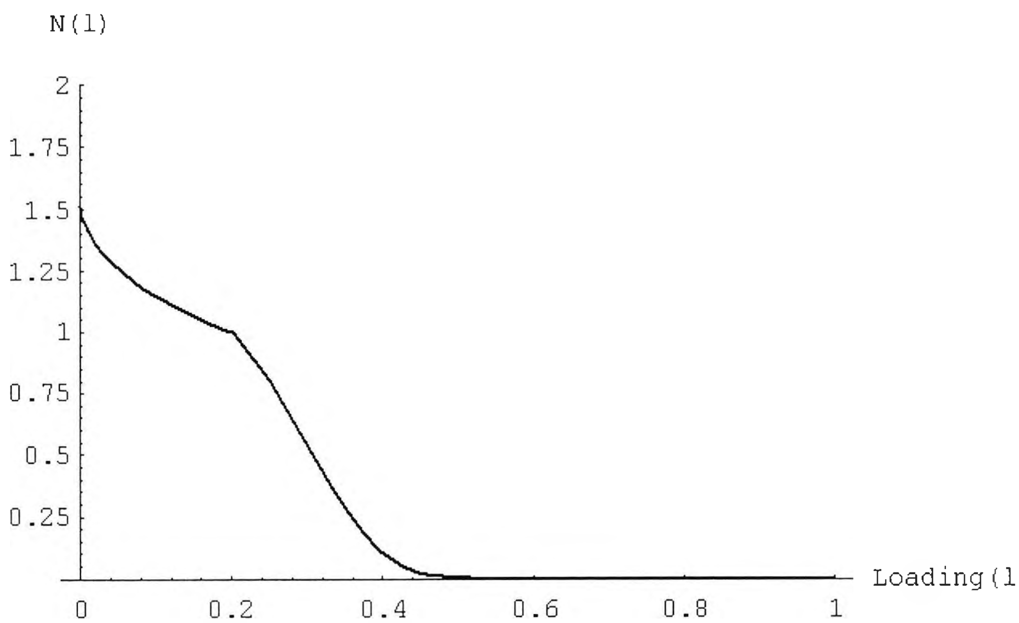

Figure 5.1: Plot of Number of business against premium loading for the case where $\beta=0.2$

In the above figure 5.1, the implication is that the company would have no new business left if l exceeded about 45%.

Figure 5.1a shows the plot of number of business against premium loading for the case where $\beta=0.3$.

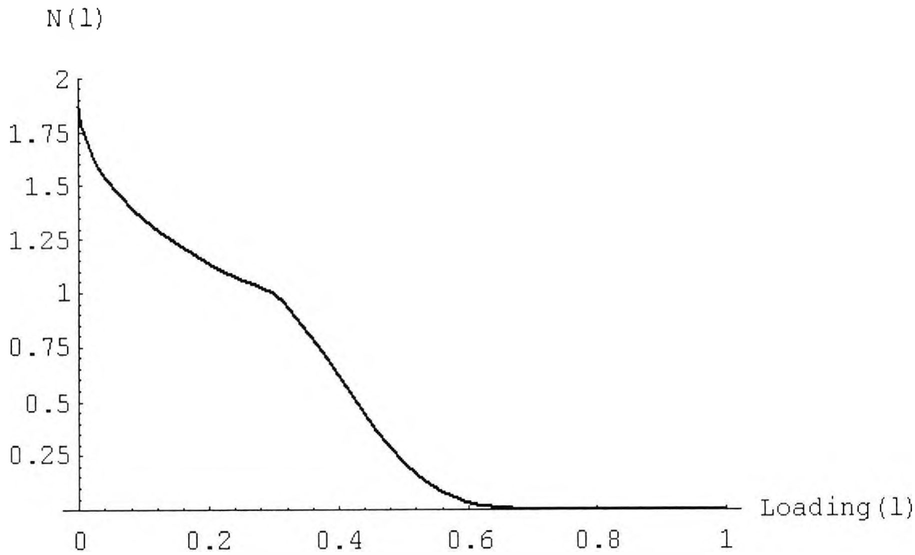

Figure 5.1a: Plot of Number of business against premium loading for the case where $\beta = 0.3$

In figure 5.1a, the implication is that the company would have no new business left if l exceeded about 65%.

Now, suppose that l is reduced to 0.1, then, by using equation (5.8) and the following parameter values:

$N_o = 100\%; \beta = 0.2; b_o = 5; b_1 = 2; k_o = 4; k_1 = 0.9995; c_0 = 0.009,$

we obtain the following plot as shown below.

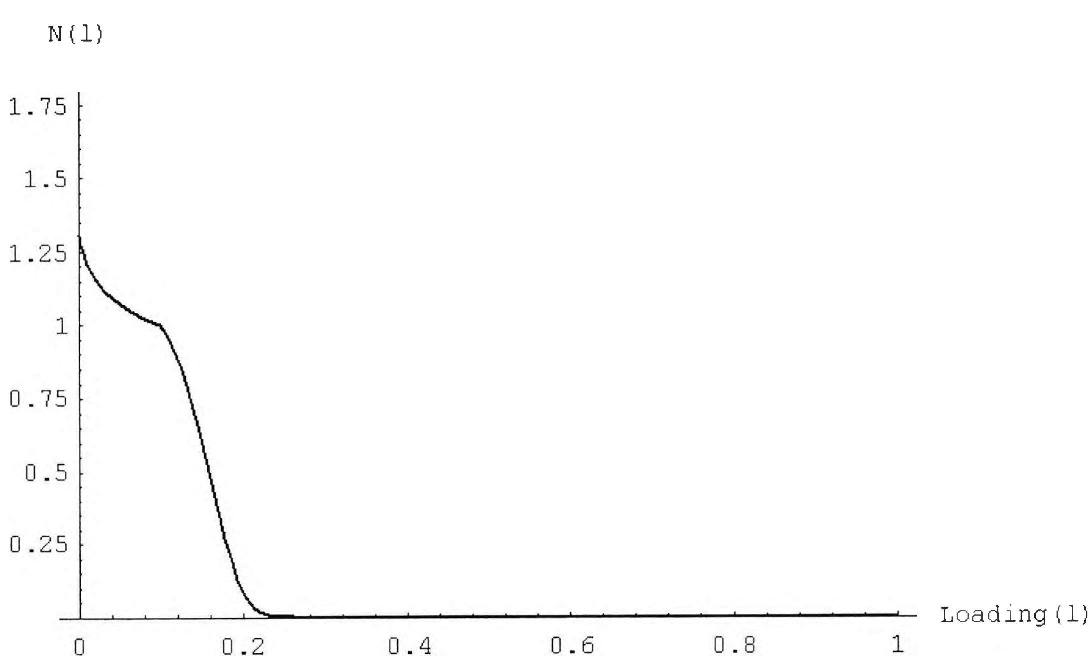

Figure 5.2: Plot of Number of business against premium loading for the case where $\beta = 0.1$.

We have decided on the value of $\beta = 0.1$ and 0.3 for comparison purposes and for the sensitivity analysis, which is reported later on in this chapter. From figure 5.2, the implication is that the company would have no new business left if β exceeded about 25%.

In the following sections, we discuss the optimization method used in the maximization process and the analysis of the results obtained.

5.3 Optimisation Method

5.3.1 Introduction

This section is concerned with a type of optimization problem in which the objective function is highly non-linear and the number of independent variables is small. Broadly speaking, the problem to consider is that of finding the maximum or minimum of a function of n variables, $f(x_1,......,x_n)$, say, where n may be any integer greater than zero. The derivatives of the objective function may or may not be available. In either case, there is a method or optimization routine that we can follow in order to solve such a problem, Walsh (1975). Further, the function may be unconstrained or subject to one or more constraints. But, the most obvious case is the general unconstrained optimization problem and the problem is merely to find values of **x**, which maximize or minimize $f(\mathbf{x})$. Even for a given constrained problem, techniques exist which makes it possible to convert or write down an equivalent unconstrained problem, Walsh (1975). This will be discussed later on in section 5.3.4.

In this chapter, we shall specifically consider a problem of minimizing a nonlinear objective function of a small number of independent variables (four in this case) when derivatives of the function are unavailable. The derivatives are unavailable because the objective function is a result of a large and complex computer simulation (arising from a stochastic element associated with it). Further note that since the objective function is subject to the same number of constraints as the independent variables, we have applied certain techniques in order to convert it into an equivalent unconstrained problem which will be discussed later on in section 5.3.4. However, transforming the variables is not necessarily the most numerically efficient way of addressing constrained problems as it increases the extent of non-linearity in the problem. Nevertheless, it had been noted by Box (1966) that "despite not being one-to-one, such transformations would still yield correct results as additional local optima would not be introduced as a consequence". In this case converting the problem to an unconstrained one appears to work well and is simpler than working with a more sophisticated algorithm for a linearly constrained problem.

Since the derivatives of the objective function are unavailable, we shall use Powell's (1965) method, described by Press (1992) and Sprott (1991), to find the optimized strategic decisions and an estimate of the objective function. Further, there are other optimization methods which have been applied to problems without constraints.
These include the following: the method of Fletcher and Reeves (1964), which uses the properties of conjugate directions and Barnes' (1965) method for solving sets of simultaneous non-linear equations. These methods are not appropriate for our problem - Fletcher and Reeves' (1964) method requires the first derivatives of the function to be computed whereas in the latter method, it is often not feasible to reformulate each optimization problem as the solution of a set of simultaneous equation. As a result of this we did not use these methods in our investigation. In what follows, we describe the numerical optimization routines used to optimize the strategic decision values and to obtain an estimate of the objective function.

5.3.2 Numerical Optimization

In view of the complexity of the model to be optimized, we use numerical optimization routines to obtain the expected utility maximization of the present value of shareholders profit. In this case, by simulating a number of scenarios (e.g. 500) from the profit model, an estimate of the objective function may be optimized using the minimization algorithms discussed below.

Furthermore, for each decision, the expected frequency of ruin (probability of ruin) is calculated as the proportion of the 500 in which the life office becomes technically insolvent at least once during the 20-year projection period.

The classification of numerical optimization routines depends on whether the objective function is linear or non-linear, whether the problem is constrained or unconstrained, and so on. As mentioned before, the current problem under investigation requires a non-linear optimization routine, since the objective function is non-linear in **d.** Further, by using a suitable transformation, we are able to transform the decision variables in the unconstrained case. Hence, it suffices to consider a problem of unconstrained non-linear optimization in this section. We discuss below some of the optimization routines used in this section.

5.3.2 Unidimensional Minimization

For one-dimensional minimization (line minimization) without calculation of the derivative, we bracket the minimum by using the golden section search method described in Press et al, (1992), and then the method of Brent (1973), which is discussed below.

The golden section search guarantees that an interval containing the minimum converges to this minimum. We employ this procedure, because it is more robust than the inverse parabolic interpolation used by Brent (1973) and the number of iterations does not have to be determined in advance. The process can then be terminated at any iteration by any criterion.

Brent's method is used to obtain a minimum by combining the above method (golden section search) with inverse parabolic interpolation. Inverse parabolic interpolation means that if the function is parabolic near to the minimum, then a parabola fitted through any three points will take us in a single leap to the minimum. Therefore, Brent's method uses the inverse parabolic interpolation when the function is well behaved and switches to the more robust golden section search when this fails.

5.3.3 *Multidimensional Minimization*

The algorithms for multidimensional minimization are all iterative-based processes. They work by an iterative process of deriving an appropriate search direction and minimizing the given function along this direction by using a unidimensional sub-algorithm as discussed above. Thus, if we denote $x_1, x_2, \ldots\ldots x_i$ as successive approximations to the minimum of a function, *f(x)*, and $\mathbf{P}_i$, as a search direction, then the *ith* iteration involves minimizing $f(\mathbf{x}_i + \lambda_i \mathbf{P}_i)$ with respect to the scalar, λ_i. Setting $\mathbf{x}_{i+1} = \mathbf{x}_i + \lambda_i \mathbf{P}_i$, the process is repeated until a tolerance level is satisfied.

There are many types of multidimensional minimization routines reported in the literature, for example, Powell's method (1964), (1975), and one described in Press et al (1992), which do not require the calculation of derivatives. Others include the conjugate gradient method and the quasi-Newton method, which do involve the use of derivatives in the determination of search directions. However, since for the current problem we do not require the use of derivatives in the minimization process, we shall use Powell's method, which is described below. For description of the other two methods, see Walsh (1979), Gill et al (1981), Scales (1985), and Beale (1988).

Powell's Method

Powell's described method of solving a non-linear unconstrained minimization problem is based on the use of conjugate directions. In order to define conjugate directions, we begin by supposing that F(**x**) is a positive definite quadratic function, whose second derivative matrix is **G**. The *N* nonzero directions $\mathbf{P}_d$ (*d=1,2,....,N*) are mutually conjugate if and only if the equations

$$\mathbf{P}_d^T \mathbf{G} \mathbf{P}_q = 0, \quad d \neq q,$$

hold, Powell (1975).

Thus, the main idea of Powell's method is that the minimum of a positive-definite quadratic form can be found by performing at most *N* (number of variables) successive line searches along mutually conjugate directions. That is, for a starting vector X_0, the function in question is successively minimized along *N* linearly independent directions $\mathbf{P}_1,......,\mathbf{P}_N$ (search vectors) to produce a new point $\mathbf{X}_N$. Suppose $\mathbf{P}_d$ is the search vector which causes the largest decrease in function value, then we replace the direction $\mathbf{P}_d$ by $\mathbf{P}_{d+1}$, $\mathbf{P}_{d+1}$ by $\mathbf{P}_{d+2}$ and so on until $\mathbf{P}_{N-1}=\mathbf{P}_N$. The next stage is to set $\mathbf{P}_N = \mathbf{X}_N - \mathbf{X}_0$. This process is repeated for the updated starting vector and direction vectors until the process converges. Note that the above procedure may be applied to non-quadratic functions, by adding a new composite direction at the end of each cycle of N line searches.

It is important to note that the conjugate direction methods avoid the drawbacks of other methods described by Walsh (1979), Gill et al (1981), Scales (1985), and Beale (1988) (for example, quasi-Newton method) because according to Powell, they do not require estimates of gradient vector, $\mathbf{g}_k$ at $\mathbf{X}_k$. However, it has some disadvantages e.g., it is sometimes "awkward to ensure that all the N nonzero directions $\mathbf{P}_d$ ($d=1,2,.....,N$) have good linear independence", Powell (1975).

5.3.4 Bound Constraints

As mentioned before, the constrained optimization problem defined here can be reduced to an unconstrained problem by transforming the independent variables and leaving the objective function unaltered. Powell's (1964) method has been used in this case. That is, a given independent variable, x_i, subject to constant lower and upper constraints, $g_i \le x_i \le h_i$, in the *x-space*, can be transformed to an unconstrained optimum in the *y-space* by using the following model suggested by Box (1966) and Powell (1964):

$$x_i = g_i + (h_i - g_i)\cdot \sin^2 y_i \,. \qquad 5.9$$

By using this transformation, we can eliminate the appearance of the inequality constraints from the optimization problem under investigation. According to Box (1966), the advantage of this transformation is that "they have been found to result in the correct solutions being obtained easily for problems for which alternative methods made only slow progress, or ceased to make any progress whatsoever once one or two constraints are reached, even when the current point was still a long way from the optimum".

By applying Powell's (1964) and Box's (1966) method to our problem, we obtained the following transformations:

$$\begin{aligned} d_1 &= 0.02 \cdot \sin^2 y_1 \\ d_2 &= 0.02 \cdot \sin^2 y_2 \\ d_3 &= 0.5 \cdot \sin^2 y_3 \\ d_4 &= 0.25 \cdot \sin^2 y_4 \end{aligned} \qquad 5.10$$

Thus, we have transformed from the constrained case in the *d-space* to an unconstrained case in *y-space*. Hence we can apply the optimization methods of solving for the minimum of an unconstrained problem which we have discussed already.

5.4 Optimisation Results

5.4.1 Results where there is no financial incentive on surrender

The aim here is to obtain optimal strategies that maximize the expected utility of shareholders' profit at time $t=20$ years valued at time zero, when different values of β are used (β is the loading factor at which the office's initial premium loading equals the market initial loading). We show below the optimization results for the cases where $\beta = 0.2$ and $\beta = 0.1$. The sensitivity analysis of the model parameter values will be looked at. Finally, we look at the effect of using incorrect strategies on the company's profitability. It is important to note that in the discussions to follow we have assumed that there is no financial incentive on surrender in the profit model. Further, the parameter values (base values) of the profit model used in chapter 4 will be used here too. Also note that in this section the assumed surrender basis is paying out more on surrender than returns on the investment model. Finally, the optimization results for $\beta = 0.2$ will serve as the base result.

5.4.2 *Optimization Results (where $\beta = 0.2$, $N_o = 100\%; N_1 = 100\%; b_o = 4;$ $b_1 = 3/2; k_o = 3; k_1 = 1; a_1 = 0.2; c_1 = 0.001$)*

The optimal strategic decision values that maximize the expected utility of present value of shareholders' profit for various values of *r* are given in the table below. Also shown are the expected present value of shareholders' profit and standard deviation of the profit.

r	Sbasis1 (d_1)	Sbasis2 (d_2)	Ploading (d_3)	Spenalty (d_4)	E(USP)	E(SP)	Std(SP)
8	0.01514	0.01138	0.22871	0.13037	-0.9873	0.1564	0.04115
4	0.01517	0.01139	0.22834	0.13031	-0.9748	0.1562	0.04073
2	0.01514	0.01133	0.22699	0.12964	-0.9489	0.1557	0.04033
1	0.01504	0.01110	0.22004	0.12648	-0.8675	0.1529	0.03971
0.5	0.01472	0.01058	0.20562	0.11958	-0.8114	0.1476	0.03697
0.25	0.01418	0.00986	0.18711	0.11038	-0.6499	0.1402	0.03692
0.125	0.01339	0.00872	0.15688	0.09559	-0.4112	0.1287	0.03309

Table 5.1. Optimal decisions that maximizes expected utility of shareholders' profit, with mean utility and standard deviation of shareholders' profit.

Here, we are using an arbitrarily chosen range of values of risk tolerance, *r*, in order to compare different strategies. We use optimal strategic decision values for the case where $r=1$ as our base values.

From table 5.1, we observe that for $r=1$ (base result), the optimal strategies for a company to maximize the expected utility of its shareholders profit is to charge a loading of 22.00% on the premium. Then impose a penalty of 12.65% on all policies surrendered. In addition, optimal ways of regulating the surrender basis according to above results would be to change the surrender basis whenever the difference between the average of the past three years return on assets and that of the past three years surrender basis is within 1.11% and 1.50%. In other words, it is optimal for companies to change the surrender basis if the interest rate on assets rises by 1.50% or falls by 1.11%.

Further, we observe that as the relative risk tolerance increases, the expected shareholders' profit also increases, which is intuitive since an investor with a higher value of r would be expected to invest in a more risky portfolio and so, expect to increase his profit for increasing values of r (shown in table 5.1). Also, we observe that as r increases the standard deviation of the shareholders' expected profit increases, which is intuitive. This result is similar to Ong (1995) but in a different context, -thus, Ong (1995) has observed that the means and standard deviations of the accumulated fund (an initial amount of 1 invested with no explicit liabilities involved) increase with increasing r values. It is also interesting to note that the above results are similar to those of modern portfolio theory, Markovitz (1952). That is, for a shareholder investing in a more risky portfolio is likely to receive higher expected shareholders' profit, accompanied by higher degree of uncertainty. That is, the risk tolerant investor is expected to receive a greater amount of return from the greater amount of risk incurred.

Finally, we note that the expected frequency of ruin is zero in all cases investigated.

We notice that the optimal values of d_1 and d_2 are close to zero which implies that the company would be changing the surrender value basis continuously. This is probably an expensive task to embark upon since there is a cost associated with changing the surrender value basis too frequently. Therefore, we expect this to reflect on the results of our expected profit model. However, this does not happen because our profit model does not incorporate or consider (for reasons of simplicity) the cost of changing the surrender value basis too frequently. This could be a useful area for future research as a possible extension to the thesis.

As a check that the optimization routine and the optimal decision values are efficient and precise, we re-ran the optimization analysis with varying constraints. That is, we vary the lower bound constraints by adding to it, 0.01, whilst the upper bound constraints remain fixed. Similarly, we add 0.01 to the upper bound constraints whilst the lower bound constraints remain fixed. We chose 0.01 for comparison purposes.

For the case where the lower bound constraints are changed, the following results as presented in table 5.1a are obtained.

r	Sbasis1 (d_1)	Sbasis2 (d_2)	Ploading (d_3)	Spenalty (d_4)	E(USP)	E(SP)	Std(SP)
8	0.01514	0.01138	0.22874	0.13038	-0.9873	0.1564	0.04114
4	0.01517	0.01138	0.22833	0.13030	-0.9748	0.1562	0.04072
2	0.01514	0.01133	0.22700	0.12965	-0.9501	0.1557	0.04032
1	0.01504	0.01110	0.22004	0.12648	-0.8675	0.1529	0.03971
0.5	0.01472	0.01058	0.20560	0.11957	-0.8114	0.1476	0.03698
0.25	0.01418	0.01026	0.18710	0.11038	-0.6502	0.1408	0.03694
0.125	0.01339	0.01015	0.15688	0.09560	-0.4119	0.1295	0.03315

Table 5.1a. Optimal decisions that maximizes expected utility of shareholders' profit, with mean utility and standard deviation of shareholders' profit for the case where the lower bound constraint is changed.

From table 5.1a, we observe that the results/values of the optimal decision variables, d_1-d_4 are similar to that of table 5.1. This shows that the results are reasonable and precise.

Further, for the case where the upper bound constraints are changed, we obtained the following results as shown by table 5.1b.

r	Sbasis1 (d_1)	Sbasis2 (d_2)	Ploading (d_3)	Spenalty (d_4)	E(USP)	E(SP)	Std(SP)
8	0.01514	0.01139	0.22878	0.13041	-0.9873	0.1564	0.04111
4	0.01516	0.01137	0.22800	0.13015	-0.9748	0.1561	0.04073
2	0.01514	0.01134	0.22704	0.12968	-0.9501	0.1557	0.04030
1	0.01504	0.01110	0.22004	0.12649	-0.8675	0.1529	0.03970
0.5	0.01472	0.01058	0.20558	0.11957	-0.8114	0.1476	0.03697
0.25	0.01418	0.00986	0.18706	0.11037	-0.6499	0.1402	0.03692
0.125	0.01339	0.00872	0.15688	0.09561	-0.4115	0.1287	0.03310

Table 5.1b. Optimal decisions that maximizes expected utility of shareholders' profit, with mean utility and standard deviation of shareholders' profit for the case where the upper bound constraint is changed.

Also, the results/values of the optimal decision variables, d_1-d_4 are similar to that of table 5.1, which shows that the results are reasonable and precise.

5.4.3 Optimization Results where $\beta = 0.1$

Now, for the case where $\beta = 0.1$, we obtained the following optimal strategic decision values for different values of r as shown in the table below. Also shown is the corresponding expected shareholders' profit and standard deviation for each value of r.

r	Sbasis1 (d_1)	Sbasis2 (d_2)	Ploading (d_3)	Spenalty (d_4)	E(USP)	E(SP)	Std(SP)
8	0.01025	0.00679	0.10237	0.04750	-0.9918	0.1060	0.06012
4	0.01063	0.00717	0.10891	0.05043	-0.9841	0.1055	0.05991
2	0.01029	0.00687	0.10440	0.04850	-0.9671	0.1022	0.05832
1	0.01052	0.00676	0.09379	0.04267	-0.9340	0.0994	0.05181
0.5	0.01023	0.00632	0.08209	0.03693	-0.8674	0.0945	0.04688
0.25	0.01018	0.00595	0.06703	0.02912	-0.7483	0.0884	0.03902
0.125	0.00986	0.00533	0.04810	0.01964	-0.5546	0.0809	0.03226

Table 5.2. Optimal decisions that maximizes expected utility of shareholders' profit, with mean utility and standard deviation of shareholders' profit.

From table 5.2, we observe that the expected present value of shareholders' profit increases as r increases. This is similar to the base result of table 5.1. However, we observe that the expected shareholders' profit in this case is lower than the previous results. This is due to the fact that we have imposed an optimal loading lower than or close to the market loading, β, and a lower surrender penalty, for $r \leq 1$ since β has been reduced. Thus, the shareholders' expected profit reduces as shown in table 5.2. Also, we observe that as r increases the standard deviation of the shareholders' profit increases as expected. This is similar to the results of table 5.1. It is worth mentioning that according to our volume of business model, if the company's loading is greater than the market loading, then we expect the volume of business to decrease. However, if the company's loading is lower than the market loading, then we expect the volume of business to increase.

5.4.4 Optimization Results where $\beta = 0.3$

For the case where $\beta = 0.3$, we obtained the following optimal strategic decision values for different values of r as shown in table 5.2a. Also shown is the corresponding expected shareholders' profit and standard deviation for each value of r.

r	Sbasis1 (d_1)	Sbasis2 (d_2)	Ploading (d_3)	Spenalty (d_4)	E(USP)	E(SP)	Std(SP)
8	0.01713	0.01641	0.38215	0.19984	-0.9859	0.2168	0.05718
4	0.01717	0.01638	0.38038	0.19919	-0.9719	0.2162	0.05722
2	0.01709	0.01615	0.37329	0.19602	-0.9438	0.2134	0.05782
1	0.01711	0.01581	0.36100	0.19096	-0.8871	0.2086	0.05731
0.5	0.01676	0.01471	0.32627	0.17551	-0.7728	0.1946	0.05349
0.25	0.01531	0.01183	0.24256	0.13661	-0.5688	0.1615	0.04518
0.125	0.01379	0.00902	0.16205	0.09888	-0.3119	0.1311	0.03190

Table 5.2a. Optimal decisions that maximizes expected utility of shareholders' profit, with mean utility and standard deviation of shareholders' profit.

From table 5.2a, we observe that the expected present value of shareholders' profit increases as r increases. This is similar to the base result of table 5.1. However, we observe that the expected shareholders' profit in this case is higher than the previous results (base result). This is due to the fact that we have imposed a loading higher than β and a higher surrender penalty for relatively risk tolerant investors (for $r \geq \frac{1}{2}$), since β has increased. Thus, the expected shareholders' profit increases as shown in table 5.2a. Also, we observe that as r increases the standard deviation of the shareholders' profit increases as expected. This is similar to results of table 5.1.

Beta effect (when $\beta = 0.2$ is changed to $\beta = 0.1$)

We have defined β as a loading factor by which the office initial loading is equal to the market loading (see section 5.2.5 for model of volume of business issued). Now, by comparing the above results (tables 5.1 and 5.2) we can assess the effect of varying β (changed from 0.2 to 0.1) on profitability.

Let the expected shareholders profit for which $\beta = 0.2$ (baseline results from table 5.1) be ϕ_1 and the corresponding variance (baseline) be s_1^2.

Let the expected shareholders' profit for which $\beta = 0.1$ (from table 5.2) be ϕ_2 and the corresponding variance be s_2^2.

By looking at the ratio of $\frac{s_1^2}{s_2^2}$, we obtained the following results as presented in tables 5.2b.

Table 5.2b shows the difference in expected shareholders' profit for which β=0.2 and β=0.1. Also shown is the ratio of the corresponding variance of the process.

<u>Difference in Expected Shareholders' Profit at time t=20years due to β (when β = 0.2 was changed to β = 0.1)</u>

r	0.125	0.25	0.5	1	2	4	8
$\phi_1 - \phi_2$ rel. difference	0.0478 (0.3714)	0.0518 (0.3695)	0.0531 (0.3598)	0.0535 (0.3499)	0.0535 (0.3436)	0.0507 (0.3246)	0.0504 (0.3223)
s_1^2 / s_2^2	1.05212	0.89526	0.62190	0.58745	0.47821	0.46220	0.46849

Table 5.2b : Results of Expected Shareholders' Profit due to β effect (when β = 0.2 was changed to β = 0.1)

From tables 5.1 and 5.2, we observe that for different values of *r* when β is changed from *0.2* to *0.1,* the expected shareholders' profit has reduced to the values shown in table 5.2b. That is, the percentage of reduction in shareholders' expected profit relatively decreases as *r* increases. The reason is similar to that discussed for Table 5.2. Further, the corresponding ratio of variance when β is changed from *0.2* to *0.1* shows that there is a difference in the variance of shareholders' expected profit for different *r*.

Beta effect (when $\beta = 0.2$ is changed to $\beta = 0.3$)

Following the discussion of the above beta effect (when β= 0.2 is changed to β= 0.1), we also look at the effect of β when β= 0.2 is changed to β= 0.3. Table 5.2c shows the results obtained.

Let the expected shareholders' profit for which β= 0.3 (from table 5.2a) be ϕ_3 and the corresponding variance be s_3^2.

By looking at the ratio of $\frac{s_3^2}{s_1^2}$, we obtained the following results as presented in tables 5.2c.

Table 5.2c shows the difference in expected shareholders' profit for which β=0.2 and β=0.3. Also shown is the ratio of the corresponding variance of the process.

<u>Difference in Expected Shareholders' Profit at time t=20years due to β (when β= 0.2 was changed to β= 0.3)</u>

r	0.125	0.25	0.5	1	2	4	8
$\phi_3 - \phi_1$ (rel. difference)	0.00240 (0.0183)	0.02130 (0.1319)	0.04700 (0.2415)	0.05570 (0.2670)	0.05770 (0.2704)	0.06000 (0.2775)	0.06040 (0.2786)
$\frac{s_3^2}{s_1^2}$	0.92937	1.49751	2.09337	2.08286	2.05542	1.97364	1.93085

Table 5.2c : Results of Expected Shareholders' Profit due to β effect (when β= 0.2 was changed to β= 0.3)

From tables 5.2c, we observe that for different values of *r* when β is changed from *0.2* to *0.3,* the corresponding changed in expected shareholders' profit increases. That is, the relative change in shareholders' expected profit increases as *r* increases. The reason is similar to that discussed for table 5.2a. Also, the corresponding ratio of variance when β is changed from *0.2* to *0.1* shows that there is a difference in the variance of shareholders' expected profit for different *r*.

5.4.5 Results where effect of financial incentive to surrender are included

So far the above results concerning the optimal decisions that maximize the expected shareholders' profit do not include the effect of the financial incentive to surrender in the model. We now look at results of the analysis when this effect is considered. Particularly, we have assumed in our model that there is an additional financial incentive to surrender. It is possible that such an effect can lead to significant profit to the shareholders since there is a penalty charged on surrender. We set out below a table showing the optimized strategic decision values for various values of *r*. Also shown are the expected shareholders' profit and standard deviation of the profit. As mentioned in chapter 4, we begin with the assumption that $\mu_{21} = 2$. Note that the table below shows the optimization results where $\beta = 0.2$.

r	Sbasis1 (d_1)	Sbasis2 (d_2)	Ploading (d_3)	Spenalty (d_4)	E(USP)	E(SP)	Std(SP)
8	0.01298	0.00821	0.14436	0.08925	-0.9739	0.22409	0.05264
4	0.01297	0.00818	0.14349	0.08887	-0.9484	0.22365	0.05207
2	0.01292	0.00812	0.14214	0.08815	-0.8994	0.22292	0.05130
1	0.01288	0.00809	0.14166	0.08784	-0.8092	0.22266	0.05091
0.5	0.01283	0.00802	0.13936	0.08675	-0.6558	0.22125	0.04962
0.25	0.01270	0.00778	0.13263	0.08359	-0.4319	0.21757	0.04539
0.125	0.01267	0.00768	0.12963	0.08223	-0.1937	0.21574	0.03979

Table 5.3. Optimal decisions that maximizes expected utility of shareholders' profit, with mean utility and standard deviation of shareholders' profit.

As with the results of table 5.1, we observe from table 5.3 that expected profit increases as *r* increases. This is due to the fact that more lapses are expected to occur since there is an additional financial incentive available to surrender. Moreover, since lapse rates are linked to the level of surrender penalty, it implies that more profit is expected on surrender. Therefore, the expected profit increases as shown in the above table. Further, we observe that as *r* increases the standard deviation of the shareholders' profit increases as expected. This is similar to the results of table 5.1.

An important feature worth noting from the above strategies and the corresponding relative risk tolerance in table 5.3 is that, *E(SP)* increases as the premium loading is increased, and relatively low values of d_1 and d_2 are optimal for all values of *r* examined. Further we note that the risk averse shareholders (low values of r) have a slight preference for more regular monitoring and changes in the surrender basis, while the risk tolerant shareholders have a slight preference for the upfront loading approach (which is clearly more risky, as the company has less ongoing or retrospective control) as the surrender value basis is only allowed to change less often.

5.5 Sensitivity Analysis of Model Parameter Values.

In this section we look at the effect of varying the model parameter values on the expected shareholders' profit. Thus, by using the base result (when $r=1$ and $\beta = 20\%$), we calculate the expected value of shareholders profit for different values of a particular decision variable, say μ_{21}, whilst the other optimal decision variables are kept constant. The following results are obtained.

μ_{21} Effect

By increasing the value of μ_{21}, we mean that the insured lives spend less time sick. This means that the time spent in the sick state is expected to be shorter, and so, the overall death rate decreases. Therefore, the amount of benefits paid on death are expected to be reduced, which subsequently would lead to an increase in profit. Thus, we look at the effect of μ_{21} on profitability by varying the transition intensity, μ_{21}, parameter. Table 5.4 shows the results obtained. (Note that the base recovery rate is 2.0).

μ_{21} Effect				
	μ_{21}	E[U(SP)]	E[(SP)]	Std[(SP)]
	0.5	-0.95137	0.05544	0.06477
	1.0	-0.90896	0.10356	0.05072
	1.5	-0.88395	0.13312	0.04264
$\beta = 0.2$	2.0	-0.86751	0.15297	0.03971
r =1	2.5	-0.85589	0.16739	0.03374
	3.0	-0.84726	0.17818	0.03103

Table 5.4. Results of E[U(SP)], E[SP], and Std[SP] for different values of μ_{21} parameter.

As expected, we observe from Table 5.4 that the expected shareholders' profit increases as μ_{21} increases.

μ_{13} Effect

The force of lapsation is a significant parameter in the computation of the expected surrender profit. Therefore, it is important that we look at its effect on company profitability by way of varying the parameter μ_{13}. Table 5.5 shows the results obtained.

μ_{13} Effect				
	μ_{13}	E[U(SP)]	E[(SP)]	Std[(SP)]
	0.01	-0.96721	0.03623	0.03205
	0.05	-0.86751	0.15297	0.03971
$\beta = 0.2$	0.10	-0.79432	0.24769	0.04116
r = 1	0.15	-0.74840	0.31159	0.04226
	0.20	-0.71439	0.36143	0.04524

Table 5.5. Result of E[U(SP)], E[SP], and Std[SP] for different values of μ_{13}

From Table 5.5, we observe that as μ_{13} increases, the expected shareholders' profit increases. Essentially, this is due to the fact that on average, lapses lead to profits since there is a surrender penalty imposed on the policy. It is worth noting that in the profit model, we have assumed that μ_{13} is linked to the level of surrender penalty imposed on the policy. (see section 4.5.7)

Life Office Initial Expenses Effect

Broadly speaking, a life insurance policy normally incurs higher expenses during the early stages than during the latter part of the policy. This means that changes in the office initial expenses may have some effect on the expected shareholders' profit. We therefore look at this effect by varying the initial expense parameter. The following results as shown in table 5.6 are obtained.

Initial Expenses Effect				
	Expenses	E[U(SP)]	E[(SP)]	Std[(SP)]
$\beta = 0.2$ r = 1	0.05	-0.85571	0.16779	0.03986
	0.10	-0.86751	0.15297	0.03971
	0.15	-0.87947	0.13833	0.03613
	0.20	-0.89160	0.12359	0.03486
	0.25	-0.90390	0.10886	0.03359
	0.30	-0.91638	0.09413	0.03233

Table 5.6. Result of E[U(SP)], E[SP], and Std[SP] for different values of initial expenses

From table 5.6, we observe that the expected shareholders profit decreases with increasing initial expenses as expected - the so call "front end" loading effect.

Effect of Yield Curve Slope on Model

We look at effect of the slope of yield curve on profitability of a company by considering a yield curve of positive and negative slope as discussed in chapter 4.

(See figures 4.3 and 4.4). In this case, we consider the case where $\delta_t^s > \delta_t$ as assumed in the profit model. Table 5.7 below shows the results obtained.

	σ =10%	
Difference in Payout :	$\delta_t^s > \delta_t$	
Slope of Yield curve :	Negative	Positive
E(SP) :	0.15297	0.11925
Std(profit) :	0.03971	0.04801

Table 5.7. Result of E[U(SP)], E[SP], and Std[SP] for slope of Yield Curve

We observe that the expected shareholders' profit decreases as the slope of initial yield curve increases for $\delta_t^s > \delta_t$. The results are not surprising because an increase in the slope of the yield curve only affects the amount of assets which need to be sold to cover the surrender values to be paid at time t. So, an increase in the yield on the assets implies that, for a fixed lapse rate, the expected profit will decrease. Also, the result is partly due to the fact that the assumed surrender basis is paying out more on surrender than the returns on the investment model ($\delta_t^s > \delta_t$). Therefore, the expected profit is affected by the slope of initial yield curve. It is important to note that this result is consistent with that of chapter 4, section 4.7.3 where the expected loss increases with the increase in the slope of the yield curve.

Effect of Relative payout on model.

From the above analysis performed so far, we note that the profit model assumes that the surrender basis that pays out values which are somewhat higher than the return on the proposed investment model. (i.e., $\delta_t^s > \delta_t$). We therefore consider the case where the assumed surrender bases rather pays out values which are somewhat lower than the return on our investment model (i.e., $\delta_t^s < \delta_t$). The results obtained are shown below.

The above-mentioned scenario can enable us to determine the effect of either $\delta_{(0)}^s$=0.071 and $\delta_{(0)}$=0.065 (i.e., $\delta_t^s > \delta_t$) or $\delta_{(0)}^s$=0.071 and $\delta_{(0)}$=0.07 ($\delta_t^s < \delta_t$) on the expected profit model. Table 5.8 shows the results obtained when both $\delta_t^s < \delta_t$ and $\delta_t^s > \delta_t$ are considered. In this case a yield curve of negative slope, similar to that of chapter 4 is used, for consistency.

Slope of Yield curve :	Negative	
Difference in Payout:	$\delta_t^s > \delta_t$	$\delta_t^s < \delta_t$
E(Profit) :	0.15297	0.13503
Std(profit) :	0.03971	0.04007

Table 5.8. Result of E[U(SP)], E[SP], and Std[SP] for different mode of payment

We observe that the expected profit decreases relatively when we change the assumption of $\delta_t^s > \delta_t$ to $\delta_t^s < \delta_t$. This is as expected because we have assumed in the model that there is financial incentive to surrender the policy, coupled with the fact that a yield curve of negative slope is used. This means that for an increased force of interest, we need to redeem a greater amount of assets to cover the surrender value at t. Hence a smaller expected profit is obtained.

We notice that the standard deviation of profit increases when we change the assumption of $\delta_t^s > \delta_t$ to $\delta_t^s < \delta_t$. This is due to the fact that we have introduced more variation in the stochastic model by increasing the value of $\delta_{(0)}$. It is worth mentioning that this result is also consistent with that of chapter 4 where the expected loss relatively increases when we consider the above assumptions of $\delta_t^s > \delta_t$ and $\delta_t^s < \delta_t$.

Effect of changing the standard deviation, σ, of the Stochastic yield model (when $\delta_t^s > \delta_t$)

Now, we look at the effect of varying the standard deviation parameter of the stochastic gilt yield model on the expected surrender profit/loss. Here, for fixed α (strength of auto-regression) we consider different values of σ. Table 5.9a below shows the results obtained. Note that $\delta_t^s > \delta_t$ in this case.

Volatility Effect				
	σ	E[U(SP)]	E[(SP)]	Std[(SP)]
	0.05	-0.86163	0.15996	0.01926
	0.10	-0.86751	0.15297	0.03971
$\beta = 0.2$	0.15	-0.87758	0.14170	0.05949
r =1	0.20	-0.89405	0.12412	0.09124
	0.25	-0.92342	0.09644	0.14568

Table 5.9a. Result of E[U(SP)], E[SP], and Std[SP] for different values of σ when $\delta_t^s > \delta_t$.

We observe that, as the standard deviation of the yield model increases, the expected profit decreases. The reason for this is that an increase in volatility implies that there is more uncertainty in the nominal amount function; which in turn affects the amount of assets which need to be redeemed to cover a surrender value at time t. Therefore, for a fixed lapse rate, coupled with the fact that the surrender basis pays more than the return on assets, the expected profit is likely to decrease as observed. This means that the results of the profit model are sensitive to the standard deviation of the gilt model.

Here too, the result is consistent with that of chapter 4 where the expected loss increases as standard deviation of the yield model increases. However, we observe that the option value increases as volatility increases in the case of Albizzati and Geman (1994). This is consistent with our results.

Effect of changing the standard deviation, σ, of the Stochastic yield model (when $\delta_t^s < \delta_t$)

Following the discussion of sensitivity of σ for $\delta_t^s > \delta_t$, we look at the effect of σ when $\delta_t^s < \delta_t$. Table 5.9b shows the results obtained.

Volatility Effect				
	σ	E[U(SP)]	E[(SP)]	Std[(SP)]
	0.05	-0.88678	0.15559	0.02109
	0.10	-0.89181	0.14876	0.04128
$\beta = 0.2$	0.15	-0.90103	0.13652	0.06626
r = 1	0.20	-0.91655	0.11691	0.10245
	0.25	-0.94484	0.08508	0.16527

Table 5.9b. Result of E[U(SP)], E[SP], and Std[SP] for different values of σ when $\delta_t^s < \delta_t$.

Like the results of table 5.9a, we observe that the expected present value of shareholders' profit decreases as the standard deviation in the gilt model increases. This is for reasons similar to those discussed for table 5.9a. However, we observe that the expected shareholders' profit in table 5.9a is somewhat greater than that obtained here whereas the corresponding standard deviation is less than that obtained here. This is probably due to the fact that the surrender basis this time pays less than the return on the assets. Further, since $\delta_t^s < \delta_t$, a greater amount of assets need to be sold to cover the surrender values at time t. Hence, the expected shareholders' profit decreases.

Effect of New Contract fees on Expected Profit/Loss

For most life insurance contracts, there is a fee charged whenever a new contract is initiated. Broadly speaking, we believe that charging higher contract management fees is likely to discourage policyholders from lapsing. Thus, we look at the effect of new contract management fees on expected surrender profit/loss by varying this parameter. The results are shown in table 5.10.

Management fee effect				
	φ	E[U(SP)]	E[(SP)]	Std[(SP)]
$\beta = 0.2$ r = 1	0.10	-0.80191	0.21047	0.03855
	0.15	-0.80494	0.20687	0.03836
	0.20	-0.80784	0.20345	0.03817
	0.25	-0.81061	0.20019	0.03799

Table 5.10. Result of E[U(SP)], E[SP], and Std[SP] for different values of new contract fees

In this case, the expected profit due to surrender decreases when we increase the management fee for initiating a new contract, φ. Increasing φ implies that the financial incentive available for lapsing has reduced. So, we expect the rate at which policyholders lapse their policies to decrease. This in turn implies that the expected profit due to surrender decreases as observed.

5.6 Effect of using inappropriate decision strategies on Expected Shareholders' Profit

We have shown in section 5.5 that the expected shareholders' profit is affected by changes in some of the model parameter values. Now, suppose that our initial assumptions used in the optimization procedure are different from the experience in the real world. Then in that case our strategies too will differ and the expected profit will also be affected.

In this section, we look at the extent to which the strategies have changed and how much profit/loss is gained/lost by using the wrong strategies. One way to achieve this is to re-optimize the optimization problem in question at the new assumptions considered below (supposed real world value). The difference in profit/loss amount due to the old and new assumptions would then show how much would be gained or lost. On the other hand, the difference in profit between the actual experience and that of using the optimal strategic values based on the new assumptions leads to a profit/loss on using the wrong decision. Note that the profit due to the actual experience is the profit obtained by re-running the simulation with our new assumption as a new variable while maintaining the optimal decision values obtained. The initial assumptions and the supposed real world values are assumed to be as follows:

- β changed from 20% to 10%. (β is a loading factor by which the office initial loading equals to the market loading)
- Recovery rate changed from 2 to 1
- Lapse rate changed from 5% to 10%
- New contract management fees changed from 5% to 10%

We shall consider the effect from each assumption separately. It is important to note that in the discussions to follow, we consider only the case $r = 1$. This is because we are interested in how the shareholders' expected profit is affected by using inappropriate strategies for the case where the shareholders have an intermediate level of risk aversion.

Results due to β effect

In this example, when we assume that $\beta = 0.2$, (used in the new business model) the expected profit is *0.1530* and the optimal decision parameters are as follows: for *r=1*, d_1 = *0.0150*; d_2 = *0.0111;* d_3 = *0.1264;* and d_4 = *0.2200.*

Now, suppose from experience, that the actual value of $\beta = 0.1$, for which the actual profit is *0.15485*. Note that the actual profit of using $\beta = 0.1$ is obtained by re-running the simulation with $\beta = 0.1$ as a new variable while maintaining the optimal decision values obtained. Therefore, there is a profit of *0.15485-0.1530* = *0.00185*. Further, if we had known $\beta = 0.1$, then we would have chosen the following strategies:

for *r=1*, d_1 = *0.0105*; d_2 = *0.0068;* d_3 =*0.0427*; and d_4 = *0.0938* (i.e., optimized value obtained by using $\beta = 0.1$) with expected profit equal *0.0994.* Hence, the expected loss arising from using the incorrect decision is *0.0994-0.15485* = *0.05545.*

Results due to μ_{13} effect

In this example, when we assume that μ_{13}=5%, the expected profit is *0.1530* and the optimal decision parameters are as follows: for *r=1*, d_1 = *0.0150*; and d_2 = *0.0111;* d_3 = *0.1265; and* d_4 = *0.2200.*

Suppose from experience, the actual value of μ_{13}=10%, for which the actual profit is *0.2477*. Therefore, there is a profit of *0.2477-0.1530* = *0.0947*.

Further, if we had known μ_{13}=10%, then we would have chosen the following strategies:

for *r=1*, d_1 = *0.0149*; d_2 = *0.0109;* d_3 = *0.1235*; and d_4 = *0.2138* (i.e., optimized values obtained by using μ_{13}=10%) with expected profit equal *0.2445.* Hence, the expected loss arising from using the wrong decision is *0.2445-0.2477* = *0.0032*.

Results due to μ_{21} effect

In this example, when we assume that μ_{21}= 2, the expected profit is *0.1530* and the optimal decision parameters are as follows: For *r=1*, d_1 = *0.0150*; d_2 = *0.0111;* d_3 = *0.1264;* and d_4 = *0.2200.*

Suppose from experience, the actual value of μ_{21} = 1, for which the actual profit is *0.1036*. Therefore, there is a loss of *0.1530-0.1036* = <u>*0.0494*</u>.

Further, if we had known μ_{21} =1, then we would have chosen the following strategies: for $r=1$, d_1 = *0.0162*; d_2 = *0.0133*; d_3 = *0.1559;* and d_4 = *0.2829.* (i.e., optimized values obtained by using μ_{21}=1) with expected profit equal *0.1236.* Hence, the expected profit arising from using the wrong decision is *0.1236-0.1036* = <u>*0.0200*</u>.

<u>Results due to new contract management fees, φ effect</u>

In this example, when we assume that, φ=5%, the expected profit is *0.2227* and the optimal decision parameters are as follows: for $r=1$, d_1 = *0.0129*; d_2 = *0.00809.* d_3 = *0.0878;* and d_4 = *0.1417.*

Suppose from experience, the actual value of φ=10%, for which the actual profit is *0.21047*. Therefore, there is a loss of *0.2227-0.2105* = <u>*0.0122*</u>.

Further, if we had known φ=10%, then we would have chosen the following strategies:

for $r=1$, d_1 = *0.0129*; d_2 = *0.00810*; d_3 = *0.0879*; and d_4 = *0.1469* (i.e., optimized values obtained by using μ_{13}=10%) with expected profit equal *0.21847* Hence, the expected profit arising from using the wrong decision is *0.21847-0.21047* = <u>*0.0080*</u>.

From the above results, we observe that making a wrong decision on the basis of a wrong assumption about the loading factor has an effect on the expected shareholders' profit. Thus, there is an expected loss of about 5% from using the incorrect decision in this case. This result is probably due to the fact that if a higher value of β is assumed instead of a relatively lower one, then we need to impose a higher loading than expected and so this could lead to higher expected profit than anticipated, as explained before.

Also, there is an expected loss arising from making a wrong decision based on the lapse rate assumption (i.e., assumed a lapse rate of 5% instead of 10%). Thus, there is an expected loss of about 0.3% from using the incorrect decision in this case. The result is probably due to the fact that a higher lapse rate could lead to a higher expected profit and vice versa since a surrender penalty is linked to lapse rate, as mentioned before.

On the other hand, there is a small expected profit arising from making a wrong decision based on the μ_{21} assumption (i.e., assumed a higher value of the recovery rate (μ_{21}=2) instead of a lower one (μ_{21}=1)). Thus, there is an expected profit of about 2% from using the incorrect decision in this case This is probably due to the fact that less benefit is expected to be paid on death since the individuals spend less time sick than expected.

Lastly, there is also a small expected profit on making a wrong decision based on the new contract management fees assumption (i.e., assuming a fee of 5% instead of 10%). Thus, there is an expected profit of about 0.8% from using the incorrect decision in this case. This is probably due to the fact that assuming a lower value of φ increases the financial incentive available for lapsing. Hence, we expect a profit from higher/more lapses than expected, since the surrender penalty is linked to lapse rate.

From above results, we can say that the expected shareholders' profit will be affected by using the incorrect strategies. The main conclusions are given in the next chapter.

Chapter 6

Conclusions and Future Work

6.1 Overview and Main Results

In this thesis, we have applied numerical optimization routines to determine optimal strategic decisions that maximize the shareholders' expected present value of profit. It links the approaches of utility theory and mean-variance analysis in obtaining numerical solutions (optimal values). In addition, we have developed a profit/loss model that can be used by actuaries to determine the cost of the surrender option arising from the effects of financial and non-financial adverse selection. Further, the strategies that have been developed can enable insurers to involve policyholders in sharing the above cost (due to surrender at times which are favourable to the policyholder).

In view of the fact that the strategic decisions are considered in the context of utility theory, the results of the analysis have been shown to be similar to those of modern portfolio theory, Markovitz (1952). The results are computed by using stochastic simulation techniques and numerical optimization routines. We have decided on this technique in order to show greater realism in the decision models used. Before we discuss the main results of this thesis, we shall briefly review the main concepts and results of the previous chapters.

In chapter 1, we have asserted that not all insureds will surrender the policy for financial gains. Rather, the risk level of the insured has a certain degree of influence over one's perception of when to exercise the option to surrender the existing policy.

Chapter 2 looked at the effect of life insurance payouts on lapse rates in the U.K. In this chapter, we have developed a model of lapse rate that takes into account the number of policies exposed to risk of lapsing in the year leading up to the *rth* policy anniversary (=curtate duration *r-1*). Statistical analysis of lapse rate and payout variables revealed that:

a) Companies that pay out higher surrender values (SV) to surrendering policyholders also pay higher maturity values (MV) to continuing policyholders, indicating that the surrendering policyholders are not relatively better off than those that hold on to the policy till maturity. Furthermore, analysis of the ratio of *SV:MV* over the years of 1986-1994 and 1986-1991 supports the hypothesis that companies were paying out more *SVs* to surrendering policyholders than to those that stayed on to receive maturity benefits.

b) Lower values of *MV* and *SV* (relative to average market value) are accompanied by higher lapse rates. Therefore, we can say that policyholders tend to surrender their policies when they perceive poor value for money.

c) Further, we have shown that higher *SVs* relative to the average market value are not accompanied by high lapse rate- suggesting that policyholders do not surrender on the basis of the surrender value offered by the policy.

d) Cash surrender values are utilized by policyholders as an "emergency fund" to be drawn from in times of personal financial crisis (as in Outreville (1990)).

In chapter 3, we have proposed a stochastic investment model corresponding to the office's liability. In particular, we have used the Wilkie (1995) model of gilts at time *t*, with mean gilt yield replaced by our proposed surrender force of interest, δ_t^s. This is equal to redemption yield at 27/5/98 of a *t*-year gilt. A model of δ_t^s was obtained by fitting a non-linear regression equation via least squares to a particular set of financial data for U.K gilts. The results of the simulations obtained from our proposed investment model are similar to Wilkie (1995). However it is important to note that there are other ways of describing the term structure as mentioned in Bank of England Quarterly Bulletin, (1990), (1991) and the financial economics literature: see for example Panjer et al (1998).

In chapter 4, we have developed models of surrender profit/loss for non-participating life insurance policies. Particularly, we have developed a model of the expected surrender profit/loss in which there is no selection effect and a second model where the effect of financial and non-financial adverse selection are incorporated. These models make it possible to investigate the impact of adverse selection on expected surrender profit/loss at time $t=n$. Thus, the following conclusions are reached.

a) A small expected profit is obtained when the model considers no selection effect and the assumed surrender basis pays more than the returns on investment (and vice versa).

b) For the case where the model considers the non-financial adverse selection effect on surrender, the results produce an expected loss to the insurer. The expected loss is more significant when there is a higher lapse rate than about 5%.

c) Interestingly, a relatively greater surrender loss is obtained when the profit model considers financial adverse selection effect (i.e., the model assumes that there is an additional financial incentive to surrender). This means that the life office is likely to increase its expected loss due to the financial adverse selection effect.

   Nevertheless, we observe that the risk level of the insured affects the profitability of the company in spite of the financial incentive available on surrender. For example, we observe that the expected loss decreases when we assume that the individuals or lives in the system spend more time in the sick state. In this case, those with a higher risk level probably do not surrender whether there is a financial incentive or not. Therefore, the risk level of an insured can significantly affect the propensity to lapse (as in Jones (1998) and Berger (1976)). This affects the profitability of the company as observed in this research.

d) From the profit/loss analysis performed, we also observe that the following factors affect the profitability of the company:- surrender penalty, new contract management fees, slope of yield curve, lapse rate and recovery rate.

In addition, we observe that whether we assume that the surrender basis pays out more than the returns on asset, $\delta_t^s > \delta_t$ or vice versa (i.e., $\delta_t^s < \delta_t$)) also affects the profitability of the company. That is, the expected profit/loss decreases/increases when we change the surrender basis assumption from $\delta_t^s > \delta_t$ to $\delta_t^s < \delta_t$.

Chapter 5 discusses ways by which the management of life insurance companies can provide its investors (shareholders) with acceptable profiles of returns from their investment. In other words, we have discussed strategies that can be used to maximize the expected utility of shareholders' profit. Sensitivity analyses of model parameter values are performed and the effect on profitability of using inappropriate strategies has been discussed as well.

In view of the fact that a life company's profitability is affected by financial and non financial adverse selection effects on surrender, we have developed strategies for the insurance company to involve the policyholders in sharing the cost due to surrender at times which are normally favourable to them. i.e., to impose a premium loading for this right and a surrender penalty on policies surrendered. These modifications are intended to reduce the number of lapses.

Further, we have proposed strategies to control the number of times the surrender basis is changed. These can reduce the cost involved arising from frequent changes of basis. The strategy is as follows:

Either

- change the surrender basis whenever the difference between the average of the past three years' basis and the returns on company's assets (past three years' returns) is greater than an optimal value found.

or

- change the surrender basis whenever the difference between the average of the past three years' returns on company's assets and the corresponding basis (past three years' returns) is greater than an optimal value found.

To reduce the risk of overcharging/undercharging the policyholder for the right to surrender the policy at times, which are favourable to him, we have introduced a premium penalty model. This model can be used by actuaries to determine the penalty (expected volume lost) of charging higher/more premium loading than the market loading and also, the expected volume gained if we charge a lower premium loading relative to the market loading. The following results or/and conclusions are reached:

a) From the simulation results produced by the profit/loss model, the optimal strategic decision values seem to be intuitive. That is, for the case where there is no financial incentive to surrender, the optimal strategy is to impose a high premium loading, not too close to the assumed market loading and charge a higher surrender penalty for relatively risk tolerant investors (for $r \geq \frac{1}{2}$). Therefore, this increases the corresponding shareholders' expected profit. See table 5.1.

b) In the case where there is a financial incentive to surrender, the optimal strategy is to impose a low premium loading which is different from the market loading, and a low surrender penalty for all values of r. By this strategy, the volume of business is expected to increase and so is the shareholders' expected profit. See table 5.3.

c) From the volume of new business per loading model, if the company's loading is greater than the market loading, β, then we expect the number of business to decrease. However, if the company's loading is lower than the market loading, then we expect the number of business to increase. On this note, we have shown that if the market loading is increased, it is optimal to impose a higher loading (greater than β) and a high surrender penalty for relatively risk tolerant investors. On the other hand, if β is decreased, it is optimal to impose a lower loading and surrender penalty for $r \leq 1$. See tables 5.2 and 5.2a.

d) In addition to a), b), and c), we observe that the risk averse shareholder (low values of r) has a slight preference for more regular monitoring and changes in the surrender basis. (since the surrender basis is only allowed to change less). However, the risk tolerant shareholders have a slight preference for the upfront loading approach (which is clearly more risky, as the company has less ongoing or retrospective control).

It is important to note that we have shown that the results of tables 5.1 and 5.3 are similar to those of modern portfolio theory, Markovitz (1952). That is, we have observed that as the relative risk tolerance increases, the expected shareholders' profit increases, which is intuitive since an investor with a higher value of r would be expected to invest in a more risky portfolio and so, expect to increase his profit for increasing values of r. Further, we observe that as r increases the standard deviation of the shareholders' expected profit increases, which is intuitive.

e) It is worth mentioning that the following model parameter variables affect the expected shareholders profit: lapse rate, recovery rate, initial expenses, slope of yield curve, and standard deviation of stochastic yield model (as in chapter 4).

f) The use of wrong strategies did have some effect on the shareholders' expected profit. Thus, there is a loss on making a wrong decision based on an overestimate of β (i.e., assumed a higher value of β (=0.2) in the model instead of a lower value (= 0.1)). This result is probably due to the fact that a higher value of β could lead to a higher expected profit and vice versa, as explained before.

g) Also, there is a loss on making a wrong decision based on an overestimate of the lapse rate assumption (i.e., assumed a lapse rate of 5% instead of 10%). The result is probably due to the fact that a higher lapse rate could lead to a higher expected profit and vice versa, as explained before.

h) However, there is a small profit on making a wrong decision based on an overestimate of the recovery rate assumption (i.e., assumed a high value of recovery rate (μ_{21}=2) instead of a lower one (μ_{21}=1)). This is probably due to the fact that less benefit is expected to be paid on death since the individuals spend less time sick than expected.

i) Lastly, there is a small profit on making a wrong decision based on an underestimate of the new contract management fees assumption (i.e., assumed a fee of 5% instead of 10%). This is probably due to the fact that assuming a lower value of φ increases the financial incentive available for lapsing. Hence, we expect a profit from higher/more lapses than expected since the surrender penalty is linked to lapse rate.

6.2 Future Research

The models used are complex in an attempt to match the important features of the real world. However, it would clearly be possible to relax many of the assumptions or augment many of the features of the models. This exercise would be a trade-off between complexity and adherence to the real world.

This section considers useful areas for future research as possible extensions to the thesis:

a) In the model of expected surrender profit/loss due to selective effect, we have assumed a constant force of transitions between all states. It would be more realistic to use force of transition functions which are piecewise constant (as discussed by Jones (1994) and Rickayzen (1997)).

b) The model of financial disincentive, *D(t)* (decision criterion) used in this thesis is a non-decreasing piecewise function. It would be possible to assume a smooth continuous function based on a hazard rate function.

c) Further, it would be possible to adapt the multiple state model to include transitions from the healthy state to a "long term sick" state from which it is assumed that recovery is impossible (as in Rickayzen (1997) or Haberman and Pitacco (1999)) and also, to apply the methodology used in this thesis to different types of insurance, including with profits business, annual premiums.

d) In addition, it would be possible to consider other utility functions (as discussed in chapter 5) and other changes to the model (for example, a different model for describing the term structure of interest rates) to the model.

e) It would be possible to use Jones' model of selective lapsation based on frailty.

f) Also, other asset models apart from the Wilkie model could be considered.

g) Finally, other performance criteria other than maximum expected utility could be considered.

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