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EVOLUTION OF SYSTEM, MODELLING, AND CONTROL CONCEPTS IN ANCIENT GREECE

By

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To Prof. Nicos Karcanias

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DECLARATION

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ABSTRACT

The aim of this thesis is to investigate the historical origins of system, modelling, and automatic control concepts as well as to follow their development. An attempt is made to place the early formation and evolution of these concepts within the framework of their current understanding. Our research focuses on ancient Greece and involves searching through the primary sources and the literature on archaeological findings related to system, modelling and control ideas.

Nowadays, the application fields of system, modelling, and control concepts are mostly associated with complex, industrial or business processes and problems. However, the emergence of the system and modelling concepts is to be found in the early scientific and technological human thought, when the under consideration issues are the creation and the structure of the world, the natural processes and phenomena, or the functions of human body. On the other hand, the mythical intention of constructing automatic machines that imitate the living beings at the beginning, and the realisation of them in the Hellenistic times, give birth to the concept of feedback and control. The study of the primitive appearance of system, modelling, and control concepts and their evolution from the Mythical period to the Hellenistic era serves the purposes of: a) the in depth understanding of these concepts, b) placing them in a proper historical setup, c) highlighting aspects of the current advanced system theories, modelling methodologies and control methods from a historical perspective, and d) expanding their application in more and more complicated problems.

Chapter 1

INTRODUCTION

1. INTRODUCTION

1.1 Objectives and Structure of the Thesis

The aim of this thesis is to investigate the origins of the System, Modelling, and Control concepts and their early appearance in the first steps of scientific and technical thought. The intention is to contribute to a modern account of these concepts and to show the continuity in their development. In the first place, the study of the origins of such complicated concepts presents problems and difficulties. The written sources are limited and of fragmentary nature, the information is ambiguous, the cultural, intellectual, and historical background is different, and the under formation concepts do not appear in a recognisable form but have to be determined gradually. A certain amount of conjecture unavoidably enters into our attempt. Conversely, our study of the origins presents remarkable advantages. The above-mentioned concepts appear in a general form and bear the substance of the phenomena, free of details and specialisations.

Although the study of development of Control and Automation notions, as well as of early technology has been the subject of a number of previous investigations[1], examining control, within the framework of development of modelling and systems concepts, is a task that has not been attempted before. The current developments in the field of Systems and Control suggests that a proper study of the field requires the understanding of a plethora of issues, i.e., the evolution of systems, the different forms of modelling, the formulation of mathematics and physics, the emergence of mechanisms and machines, and the early appearance of control and automation ideas. The latter is the culmination of all previous issues and this integrated view on the development of the field forms the distinguishing feature of this research. The aim is to provide a framework that covers most aspects of the study rather than going into considerable depth on specialised issues. The sources for this study are diverse and include philosophy, mathematics, history, literature, medicine, archaeological evidence and so on. This comes naturally due to the fact that System, Modelling and Control ideas spread through different aspects of society beyond science and technology.

[1] [Mayr, 1, 1970 & 2, 1972], [Drachmann, 1, 1948 & 2, 1963]

The focus of interest is on the study of the early Greek texts[2], not because we underestimate the other great civilisations, but because it was in ancient Greece that the transition from the experience to the science occurred. The first known written sources are the Homeric Epics of *Iliad* and *Odyssey* (8^{th} century B.C.) and constitute the framework of the Greek Mythology, which is consequently the starting-point of our research. Following, we present the Presocratic philosophers (7^{th} and 6^{th} B.C. centuries), the so-called physicists, who became the founders of philosophy and science and formed a physical account of the cosmos; we then proceed to the Classical period (5^{th} and the 4^{th} B.C.), which is characterized by a conceptual, mathematical, and geometrical account of the world, and conclude our journey at the Hellenistic era (4^{th} to the 1^{st} B.C.) of the quantitative, arithmetical, and mechanical conception of the universe. However, the quotation of evidence follows a thematic rather than a historic order.

The concepts of System, Modelling, and Control are closely interrelated. Modelling and control processes arise out of the need to study, understand, and manipulate a system. In other words, the integrated process of knowledge and dominance over a system involves the process of simulating and modelling the system, i.e., creation of a model that has the same features and behaviour with the system, as well as the process of controlling it. Understanding that the organisation of processes and objects leads to behaviours that are different than those of the particular elements, has been a notion that has taken some time to evolve. The instigating aspect of this realisation has been the observation of manmade processes in the first instance. The modelling of processes and phenomena, on the other hand, has emerged early on and expresses the cumulative effort to store the different forms of achieved knowledge. Conceptualisation and conceptual modelling appear first and then formal forms of modelling in terms of mathematics, physical laws of describing and explaining natural phenomena follow. The translation of conceptualisation of processes together with the understanding of behaviours of the basic processes has led to the first technological

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[2] Note that in many cases the evidence the ancient writers provide is indirect. This must be read with caution because we run a risk of a superficial or unscientific interpretation. Therefore, throughout the thesis, wherever it is considered of vital importance, we cite the texts and the documents of written sources, as they have survived through the ages, since they are our only source of information about certain theories, technologies, and inventions.

achievements (mechanisms, machines, constructions). The latter were considerably assisted by the developments in the fields of mathematics - both conceptually and computationally. The role of mathematics in the development of modelling concepts and tools forms a major part of our investigation. We view the evolution of automation and control mechanisms as the culmination of a body of knowledge that is based on the understanding of processes, the ability to quantify their behaviour at a local and global level, and on the use of the notion of a manmade system aiming at achieving a certain goal.

Presumably, the concepts of modelling and control bring to our mind complicated manmade systems, for example, business or industrial processes, automatic machines, robots, and so on, which should work safely, under specific prescriptions, necessities that presuppose advanced modelling methodologies and control mechanisms. However, when investigating the origins of system, modelling and control, we do not meet such systems or advanced ways of modelling and controlling them.

The primary issue of the early man is to understand and interpret the rational order of the universe, and to observe and study the natural phenomena and living beings. The universe, the natural phenomena, or the living beings and the human body are unknown physical systems for him. Although the early researchers and philosophers were not aware of the system concept itself, neither were they aiming towards proper definitions of a system, nor the establishment of a concrete systemic theory, they managed to lay down in an indirect way the basic steps in the development of system theory and set the foundations of the system concept. More precisely, they express some of the main features of the system and form its concept by asking questions about the composition, the primary elements, and the main parts of the under investigation physical system, distinguishing it from its environment and realising that this is characterised by dynamical processes. Similarly, though the ancient scientists are not concerned about the modelling concept or the modelling methodologies, their efforts, as for example when trying to explain the planetary motion, leads them to make use of something that represents the unknown, or in advanced cases, of some kind of mechanical models. On other occasions, when the issue under consideration is related to philosophical aspects of thought, they introduce conceptual models, as for example the theory of eternal ideas that play the role of the models of the sensible world and reality. Consequently, the concepts of model and modelling emerge.

The concept of control, on the other hand, in its general meaning indicates the intention of man to dominate over his environment. After the first steps of understanding, interpreting, and modelling the universe, the following phase is to 'control' it. Or in other words, after the first steps of observing and studying the natural phenomena or the living beings the next step is to imitate them. Additionally, the control concept is related to the higher intention of man to create machines, which work on their own. After having invented tools that build up his strength and power and devices that work by means of the human or animal power, making life even easier, man intends to create machines that work by themselves, machines that approach the living beings' operation, machines that have 'soul' or internal energy, i.e., automatic machines. Automatic machines would be meaningless if they worked uncontrolled. This is besides, the fundamental difference between physical systems and the manmade ones resulted as products of technology: In the physical systems, external events and phenomena may act on the system and excite response. However, such inputs are rarely under the control of humans and thus they are merely appearing as disturbances. The distinguishing feature of manmade systems is that external inputs may be assigned independently and thus, there is potential for controlling the system behaviour by appropriately selecting the inputs in an open, or closed-loop (feedback) way. Therefore, the ability of the early engineers to construct automatic machines results in the concepts of feedback and control.

Along with the purpose of detecting the emergence of the concepts of system, modelling and control, this research work intends to follow the evolution of these concepts throughout the ages. In particular, Part I of the thesis consisting of nine chapters concerns the evolution of system and modelling concepts, whereas the three chapters of Part II present the evolution of control concept and control mechanisms. Such segregation is considered to be necessary because the origins of the former concepts need to be investigated in the realm of philosophy and science, whereas the origins of control have to be found in the field of technology. The lining up of chapters is made mainly by a thematic rather than a chronological point of view. Therefore, the structure of the thesis is the following:

Primarily, Chapter 2 provides a review of the fundamental issues and concepts in the area of Systems, Modelling, and Control. The intention here is to remain at the level of concepts and to express the current thinking and understanding of them. Such a review is proven necessary in order to evaluate the historical evolution of systems, models, and control ideas, to place the early developments within the current level of understanding, development and achievements, and to verify that the way the ancient philosophers perceived these concepts is not very different from the contemporary understanding; hence, it does constitute the basis of the most modern theories.

Chapter 3 covers the mythical period from Orphic Rhapsodies to Homer and Hesiod. The fundamental issue of early man to understand and explain the universe and the at that time lack of science leads to the generation of Greek myths, which however reveal the conception that the world system is a dynamic process characterised by evolution and time while at the same time, indicate the first efforts to approach the unknown creation and structure of the world by means of models.

The following period of the Presocratic philosophers and the development of the physical conception of the world is the subject matter of Chapter 4. The rejection of any metaphysical speculation, the introduction of abstract thinking, the formation of opposite qualities bearing the concepts of contradiction, are steps in human thought closely related to the conceptual modelling. The search of primary, eternal elements out of which arises the world, the functional relations between elements and qualities, dynamical aspects such as time evolution, motion and change, and the establishment of the first physical laws, pave the way to the science of physics and subsequently, to a physical approach to the concept of modelling. Simultaneously, the consideration that the unknown system of the world consists of basic elements or parts, which have dynamical relations among them, i.e., it is characterised by a dynamical behaviour, reveals early notions about the concept of system. The early notions of the system as they appear in the ancient Greek sources, in the Hippocratic holistic perception of the world or of the human body as general systems, in the first known definition of system by Kallicratides, and in the system of Logic of Aristotle, are the main themes of Chapter 5.

The mythical and presocratic periods' scattered evidence of the origins and evolution of modelling concept acquires a proper realisation in the Classical times. The platonic theory of Ideas, the distinction of types of models, the description of the creation of conceptual models, and the algorithmic method of dichotomy that aims at the definition of specific topics, lead to advanced conceptual models, relations, and methodologies that are the subject of Chapter 6.

In addition to theoretical and philosophical approach for the interpretation and understanding of the world, the philosophers and scientists are occupied with many other topics related to mathematics, astronomy, and mechanics. Their remarkable contributions to the field of mathematics, such as the concept of number related to measurement and the development of geometry (classification and properties of shapes), the correspondence between numbers and musical sounds, and the advanced theory of proportion and analogy, indicate the first steps that lead to the mathematical modelling, which are cited in Chapter 7. The exact numerical approximation of commensurable and incommensurable qualities, by means of the Euclidean anthyphaeretic method, or the synthesis of arithmetical and geometrical algorithms for the computation of irrational numbers by Archimedes, give the notions of numerical modelling and are examined in Chapter 8.

Chapter 9 concerns the progress of modelling concept in the field of mechanics and astronomy. The elementary mechanical models that present the world by means of rotating wheels or the heavenly bodies as bowls filled with fire, the improved astronomical models of Aristotle and Eudoxus, the construction of planetaria, and the exact quantitative mechanism of Antikythera are only some examples of the mechanical and astronomical modelling evolution.

Part II concerns the evolution of the concept of control and follows similarly the same crucial periods of the development of Greek thought. However, since the concept under consideration is closely related to the evolution of the technological thought, special attention is paid to the technological achievements of these periods.

Therefore, our passing through the mythical period brings to light not only the generation of mythical models, but also these parts of Greek myths that are related to the intention of constructing automatic and self-controlling machines. The mythical automata, though they are not real technological achievements, constitute evidence of the human vision and technological intention of having 'clever' machines that work on their own. Chapter 10 examines mythical automata from such a point of view, and quotes the references of automatic constructions in Babylon and Egypt. Chapter 11 describes the necessary theoretical framework that enables the construction of automatic mechanisms in the Hellenistic period. In particular, the frequent appearance of terms 'automaton', or 'automatic' in ancient Greek sources, the platonic introduction of 'cybernetics' as the art of controlling a ship or an army, the Socratic maieutic method as a closed loop control system, constitute the base on which the science of automation and control later on flourished. The culmination of the mythical dream and the theoretical approach to automatic machines ends in Chapter 12 with the construction of automata, the realisation of machines that not only do they have the ability of self-motion, but also of self-controlling and constitute examples either of open loop or of closed loop control systems.

Finally, Chapter 13 is a synoptic chapter that provides the opportunity to summarise the main findings in the schematic form of a table, where the early philosophers and scientists appear according to their contribution to the conceptual, physical, and mathematical modelling, as well as to the concepts of systems and control. In addition, this chapter outlines the further work and research issues that may extend and complement the present research.

Chapter 2

CURRENT THINKING ON SYSTEMS, MODELLING AND CONTROL

2. CURRENT THINKING ON SYSTEMS, MODELLING AND CONTROL

2.1 Introduction

Examining the evolution of Systems, Modelling and Control concepts and notions involves an assessment of the early developments, with respect to our current understanding of them. For the purposes of this thesis, we achieve this by presenting an overview of the fundamentals from a conceptual viewpoint that avoids the use of mathematics and puts the emphasis on the concepts. This summary does not aspire to be a rigorous introduction, but rather a conceptual presentation; the rich control and systems bibliography[3] provides a rigorous and detailed discussion of the relevant concepts.

The distinguishing feature of this thesis is that it places the evolution of Control and Automation concepts within a wider framework of developments of notions of systems, measurement, and modelling, which in turn are affected by developments in mathematics, physics, astronomy, philosophy, and so on. The latter provide the main sources for information, whereas myths, art and other forms of human activity contain also additional valuable evidence. Setting up the framework for the work in this thesis involves examination of issues, such as:

- The notion of a system and its fundamentals
- The act of systems modelling
- The fundamentals of control

which are considered subsequently in a descriptive, conceptual way, rather than a formal mathematical that is beyond the needs of this thesis. As such, this chapter provides the description of the key concepts, the evolution of which is considered in the rest of the thesis.

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[3] [Checkalnd, 1, 1981], [Klir, 1, 1967], [Goguen, 1, 1970], and [Kailath, 1, 1980], [D'Azzo *et al.*, 1, 1966 & 2, 1995], [Kuo, 1, 1991]

2.2 The Notions of a System

The study of physical and human related activities involves the notion of system in an essential way. The term 'system', as applied to general analysis, was originated as recognition that meaningful investigation of a particular phenomenon can often be achieved by explicitly accounting for its composite nature (interconnection of more basic elements in a certain) and its environment. The notion of the system is fundamental and it is formally defined as:

Definition (2.1): A *system* is an, organisation, interconnection of objects, which is embedded in a given environment. (Karcanias, 1, 1994)

The above definition is very general and has as fundamental elements:

- Objects
- Connectivities – Relations
- Environment

These elements may be symbolically represented as:

SYSTEM	$\Longleftrightarrow$	**OBJECTS + RELATIONSHIPS +** **BOUNDARIES – ENVIRONM ENT**

The above symbolic representation has to be seen in a general functional composition way, rather than a mere union, addition. In simple terms, it means that understanding whatever is associated with a system requires the understanding of whatever is linked to objects, the effect of relations and structure in shaping knowledge based objects, and finally the role of environment in defining stimuli and determining the nature of parameter, variables linked to objects. It is the interaction between the three basic elements that makes a system different than the simple addition of its constituent parts.

Remark (2.1): The concept of a system is referred to the level of reality, i.e., we consider it as a physical or manmade object in our sphere of reality. This observation is essential, since later on we shall examine the notion of system model.

2.2.1 Objects and their Classification

The basic components of the system notion are defined below in some more detail.

Definition (2.2): An ***object*** is a general unit (abstract or physical), which is defined in terms of its attributes and the possible relations between them. (Karcanias, 1, 1994)

Thus, we consider here objects to be the most primitive concept and we allow objects to be almost anything. By not restricting the definition to any particular class we allow the freedom to consider systems from any domain. An object is defined in terms of its attributes, which are parameters, variables associated with it. As such knowledge derived from observations defines these particular attributes and the relationships between them. The relationships between the attributes may be functional, linguistic, structural, and so on, and express knowledge that stems from past history of the object, or the environment in which it has been operating. Thus, it is the collection of observations and the possible relations between them (derived from past knowledge), which define an object.

There are different types of objects; they may be ***atomic*** or ***composite***, ***individual*** or ***relational***, and ***determinate*** or ***indeterminate***. The consideration of objects, which are ***atomic***, implies our inability, or lack of desire to decompose them to simpler elements. The emergence of objects, which are themselves interconnections of other more basic objects, leads to the notion of ***composite*** objects. Composite objects are associated with a topological organisation of more primitive elements and they are themselves systems. ***Individual*** objects are simply characterised by their attributes, whereas ***relational*** objects involve both attributes and their relations, if such relations exist. Object with well defined attributes and possibly relations between them called ***determinate***; otherwise, they are said to be ***indeterminate***. Defining some deeper aspects and classification of objects requires some definition of the notion of environment for an object, which is given below.

Definition (2.3.): For a given object, we define its ***environment*** as the space of objects, signals, events, structures, which are considered topologically external to the object, and linked to the object in terms of relations with its structure and attributes. (Karcanias, 1, 1994)

The essence of the definition is that for every given object we may identify a boundary around it that includes all structure and attributes associated with the object. Of course, the considered object may be related to other objects and attributes, which however are considered external since they imply crossing of the boundary. The existence of the object's environment is synonymous with the definition of the object and the general topology of the system is manifested as the crossings of internal-external attributes, which in turn express the connectivities of the object to its environment. We may represent the above as:

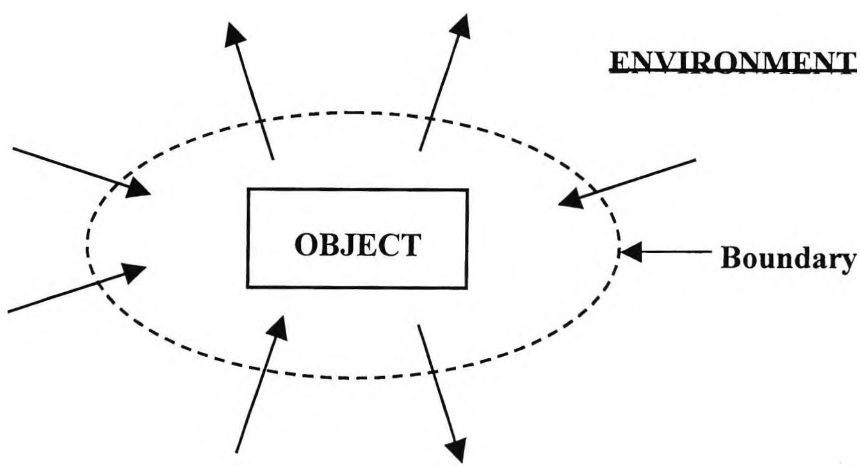

Figure 2.1: Object – Environment Relation

In the above diagram, lines crossing the boundary indicate the embedding of the object to its environment. The object will be called ***embedded***, if there is at least a connection to its environment; otherwise, i.e., when there is no connection, then the object is called ***free***. Free objects may be studied on their own without considering any environment. The family of embedded objects may be further classified by assigning direction to the line connectivities and thus, considering arrows. The assignment of direction is equivalent to introduction of 'causality' in the traditional way (cause and effect). In the above diagram, arrows directed from the environment to the object are referred to as ***object inputs***, or ***stimuli***, and those with direction from the object to the environment are called ***object outputs***, or ***influences***. Stimuli and influences are attributes generated respectively in the environment, object boundary. The linking of objects to their environment provides a further classification of them. An object is called ***autonomous***, when it has no inputs, or stimuli from the environment, and it is called ***non-active***, when there are no outputs, or influences. For autonomous objects the evolution of their attributes is not affected by external stimuli, but only from internal ones (i.e., initial conditions). An object that is non-active does not generate stimuli for objects in its environment. Objects are referred to as ***non-autonomous***, or ***forced***, when they have inputs; in this case the environment plays a crucial role in determining the evolution of the object variables. Similarly, the object will be called ***active***, when there are outputs, or influences; in this case the object generates stimuli for objects in its environment. Such classification stems from topological considerations and some basic understanding of causality, or notion of flow.

Objects may be in two distinct modes. An object is said to be in ***static mode***, when its attributes are not changing as time changes. The object will be said to be in a ***dynamic mode***, when its attributes are changing as a function of time, or events. Note that the term dynamic refers to phenomena that produce time-changing patterns, the characteristic of the pattern at one time being interrelated with those of other times. The term is nearly synonymous with the ***time-evolution***, or ***pattern of change***. Thus, 'dynamic' refers to unfolding of events in a continuing evolutionary process. The deeper characterisation of objects uses properties of their attributes, which are formally defined below:

Definition (2.4): An ***attribute*** for an object is an assignable, and possibly identifiable, measurable characteristic of the object. (Karcanias, 1, 1994)

In the current context, assignability implies ability to associate a label, tag with it, whereas identifiability implies capability to understand its relationships with other attributes and its contribution to the structuring of the object. Measurability means ability to quantify the correspondence between the attribute and a set of values, or functions of time. Attributes, which are determined on an object in the static mode, will be referred to as ***parameters***, whereas those defined only on the dynamic mode will be called ***variables***. Note that such a classification may depend on the different stages of the lifecycle of the object. Determining the parameters of the object may be the outcome of direct observations, a priori knowledge, or the result of experiments at some time in past.

Consider an object in the free mode and the set of nontrivial relations defined on it. All variables entering nontrivial relations will be referred to as ***implicit states***. A subset of this set, which is crucial in defining the object, is introduced below:

Definition (2.5): Any subset of the implicit states with the properties that they are independent (in some specific sense) and which describe completely the set of nontrivial relations, will be called the set of ***states*** of the object. (Karcanias, 1, 1994)

The notion of the state is fundamental for dynamic objects and it is defined on the free mode, as well as the embedded mode. Defining the notion of the state permits complete knowledge of all attributes defined on the object, and provides ways for describing the time, event evolution of all object attributes. The cardinality of the state set provides a measure of complexity of the object and it is referred to as ***dimension*** of the object.

For embedded objects described as in figure 2.1, the totality of variables associated with the object may be classified as shown below:

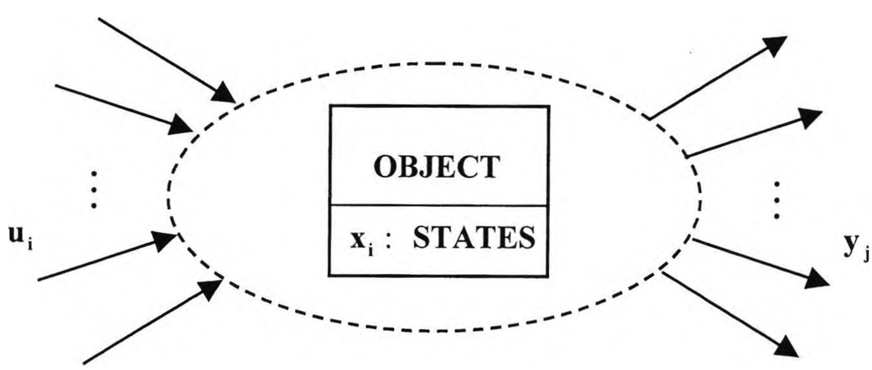

Figure 2.2: Object and Fundamental Variables

In figure 2.2 we have: x_i are the elements of the state set, u_i expresses the stimuli (originating from the environment), and y_i the influences of the object to its environment. We may arrange all such variables in the form of vectors and this is represented by: $\underline{x}=[\ldots,x_i,\ \ldots]^t$, $\underline{u}=[\ldots,u_i,\ \ldots]^t$, $\underline{y}=[\ldots,y_j,\ \ldots]^t$; $\underline{x}$, $\underline{u}$, $\underline{y}$ are referred to correspondingly as ***state-, input-, output-vectors***; the vector $\underline{\xi}=[\underline{y}^t,\underline{x}^t,\underline{u}^t]^t$ made up from the three sub-vectors is referred to as the ***composite vector*** of the object.

Objects for which the full set of relations contains at least one nontrivial element will be called ***relational***. In terms of the composite vector $\underline{\xi}$ the nontrivial relations may be expressed in the functional representation form as:

$$\underline{h}:\{\underline{\xi}\}\rightarrow\{\underline{\xi}\} \tag{2.1}$$

In this relation $\{\underline{\xi}\}$ denotes the set of all values of the composite vector for the complete lifecycle of the object; $\{\underline{\xi}\}$ will be usually referred to as the ***composite space*** of the object. A description like that of (2.1) does not make a distinction between the input, output, state components of $\underline{\xi}$ and thus we say that lacks orientation; for this reason the description in (2.1) will be referred to as ***non-oriented relational*** description. For relational objects where $\underline{\xi}$ may be partitioned in the $\underline{\xi}=[\underline{y}^t,\underline{x}^t,\underline{u}^t]^t$ form, where $\underline{x}$, $\underline{u}$, $\underline{y}$ correspond to state, input, output vectors it may be possible to express the relation in (2.1) in the form:

$$\underline{f}:\{\underline{x}\}\times\{\underline{u}\}\rightarrow\{\underline{x}\},\ \ \underline{r}:\{\underline{x}\}\times\{\underline{u}\}\rightarrow\{\underline{y}\} \tag{2.2}$$

Whenever we define such relational descriptions, we say that the specific partitioning of $\underline{\xi}$ introduces an orientation and (2.2) introduces an ***oriented relational*** description. Note that $\{\underline{x}\},\{\underline{u}\},\{\underline{y}\}$ denote the sets of all values of the $\underline{x}$, $\underline{u}$, $\underline{y}$ vectors for the complete lifecycle of the object and shall be referred to in turn as ***state-, input-, output-spaces*** associated with the given orientation of the object. The orientation of the object may be interpreted as a partitioning of $\underline{\xi}$, such as $\underline{h}$ produces a corresponding pair of relations $(\underline{f},\underline{r})$, which define (2.2). For an object having an oriented relational description as in (2.2) it may be possible to have a relationship:

$$\underline{g}:\{\underline{u}\}\rightarrow\{\underline{y}\} \tag{2.3}$$

This relationship links in a direct way inputs (stimuli, causes) to outputs (influences, effects); whenever such relations may be established, the object will be called ***input-output relational***.

We may represent (2.3) as:

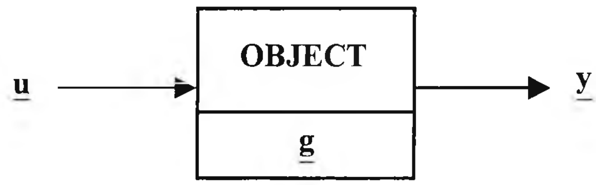

Figure 2.3: Input-Output Relational Object

2.2.2 Interconnection, Composite Objects and System Structure

The concept of a system implies the organisation of objects in a specific way and in general terms this is what describes the notion of a composition structure. Oriented objects, in the form described by figure 2.2, interact in a certain manner via a given topology of stimuli and influences and this produces composite objects exhibiting properties, which in general are different to those of constituent objects. Composition of objects has two main aspects:

1. Definition of an object based, local interconnection structure
2. Rules for interconnecting objects

The definition of an orientation, as described in the previous section involves a partitioning of the composite vector $\underline{\xi}$ as:

$$\underline{\xi} = [\underline{y}^t; \underline{x}^t; \underline{u}^t]^t = [\underline{\tilde{y}}^t; \underline{w}^t; \underline{x}^t; \underline{\tilde{u}}^t; \underline{v}^t]^t \tag{2.4}$$

In the above partitioning we refine the definition of $\underline{y}$, $\underline{u}$ vectors by partitioning them into components $(\underline{\tilde{y}}, \underline{w})$ and $(\underline{\tilde{u}}, \underline{v})$, where the constituent elements have the following interpretation:

- $\underline{\tilde{y}}$ is the sub-vector of $\underline{y}$, the entries of which can be physically measured
- $\underline{w}$ is the sub-vector of $\underline{y}$, the entries of which become inputs to other objects
- $\underline{\tilde{u}}$ is the sub-vector of $\underline{u}$ that can be arbitrarily assigned
- $\underline{v}$ is the sub-vector of the input $\underline{u}$, whose elements are generated as outputs by the objects in the neighbourhood and thus, their values are determined by the overall system

Note that in the above definition $\underline{u}$, $\underline{y}$ may have repeated components since $\underline{\tilde{u}}$ and $\underline{v}$, $\underline{\tilde{y}}$ and $\underline{w}$ may have common elements. In terms of the above classification, figure 2.2 may be refined as shown below:

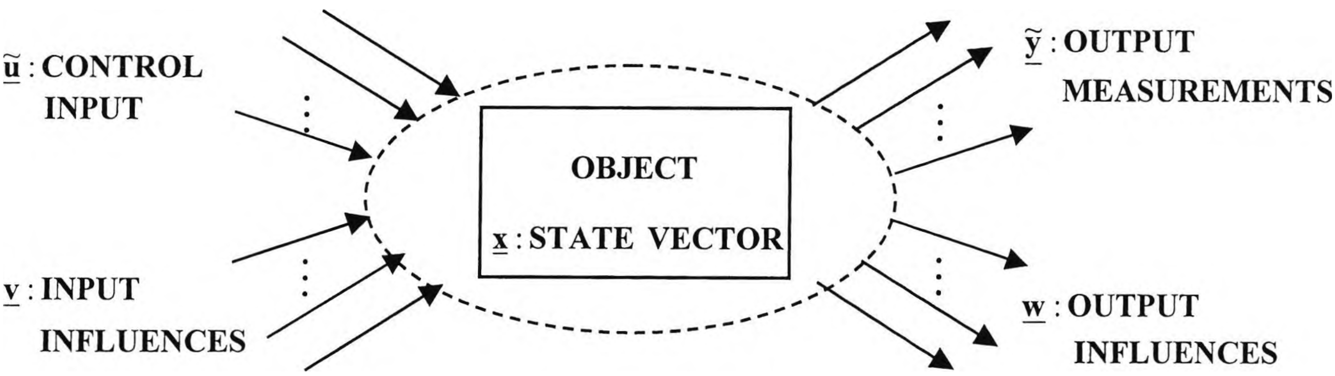

Figure 2.4: Object oriented and its embedding into the Environment locally

The vectors $\underline{u}$, $\underline{y}$ represent the local control and output vectors, and $\underline{v}$, $\underline{w}$ are the vectors expressing the connection of the object to the system through the interconnection topology. For a given object B the above diagram indicates the embedding within the system and the general system environment and this is represented in figure 2.5 (a refinement of figure 2.4), where:

- ***Internal System Environment***: refers to the space within the system boundaries, where interactions between objects take place.
- ***General System Environment***: refers to the environment of the overall system, defined by the external space with respect to system boundaries and which corresponds to all objects of the system.

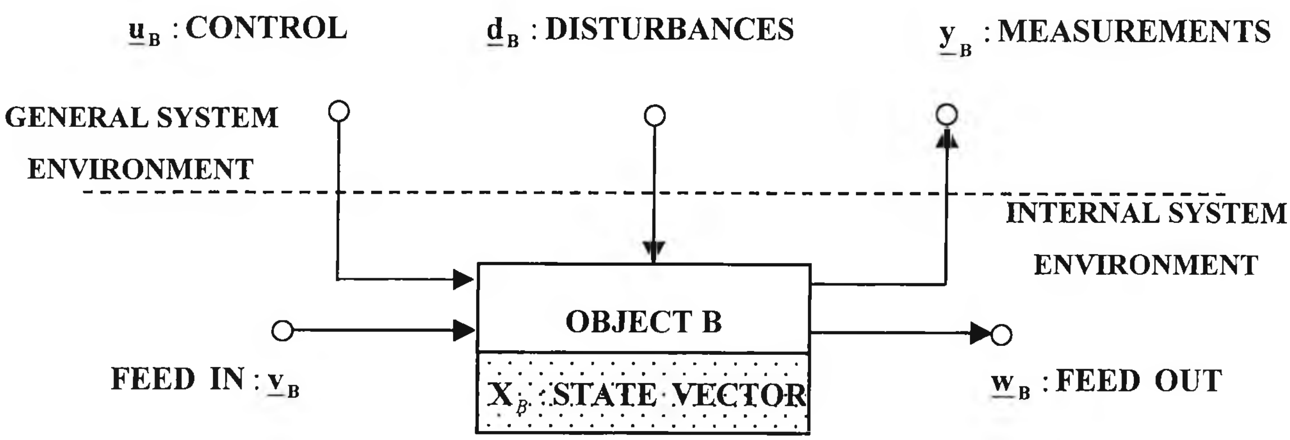

Figure 2.5: Embedding of Object within a system and its Environment

In the above diagram we introduce also the vector $\underline{\tilde{d}}_B$ to denote all variables influencing B object, the values of which are pre-assigned by some mechanism in the environment; this vector is referred as a ***local disturbance***.

The vectors $\underline{u}_B$, $\underline{y}_B$, $\underline{d}_B$ are elements of the ***global control measurement*** and ***disturbance structure*** of the system, whereas the pair $(\underline{v}_B, \underline{w}_B)$ defines the available variables that enter into relations with other objects within a given interconnection topology. This interconnection structure is the second fundamental ingredient of the system notion and it is defined below.

Definition (2.6): If $\{B_i, i \in \underline{\rho}\}$ is the set of objects of the system and $(\underline{v}_i, \underline{w}_i)$, $i \in \underline{\rho}$ the set of local interconnection vectors, then $(\underline{v}, \underline{w})$ is the pair of aggregate ***input-, output-interconnection vectors***, where $\underline{v} = \underline{v}_1 \oplus \ldots \oplus \underline{v}_\rho$, $\underline{w} = \underline{w}_1 \oplus \ldots \oplus \underline{w}_\rho$, and V, W denote the sets of values of such vectors. An ***interconnection structure*** is any map, relation F, such as that $F : W \to V$.

Clearly, the nature of objects and their associated variables defines the nature of the interconnection structure. Such structure may be ***fixed***, that is not changing within the system lifecycle, and they may be ***evolving***, that is they may be changing within the system lifecycle. Typical forms of interconnection structures are those defined by graphs. If $\{B_i, i \in \underline{\rho}\}$ is the set of objects contained in the system, then

$B_a = \{B_1;...;B_\rho\}$ is defined as the ***aggregate of the system objects*** and it is simply the listing of the objects. The notion of the system is different than that of B_a, since the system expresses the action of structure on B_a and this may be denoted symbolically as an operation *, that is:

$$S = B_a * F \tag{2.5}$$

If $(\underline{u}_i, \underline{d}_i, \underline{y}_i, \underline{v}_i, \underline{w}_i)$ is the ordered set of vectors associated with every object B_i and denote by $\underline{u} = \underline{u}_1 \oplus ... \oplus \underline{u}_\rho$, $\underline{d} = \underline{d}_1 \oplus ... \oplus \underline{d}_\rho$, $\underline{y} = \underline{y}_1 \oplus ... \oplus \underline{y}_\rho$, $\underline{v} = \underline{v}_1 \oplus ... \oplus \underline{v}_\rho$, $\underline{w} = \underline{w}_1 \oplus ... \oplus \underline{w}_\rho$ the direct some vectors, then the notion of the system expressed as in (2.5) may be represented diagrammatically as it is shown in figure 2.6, where block-diagram notation is used to denote the action of the interconnection map F on the aggregate of $\{B_i, \ i \in \underline{\rho}\}$, or system aggregate B_a.

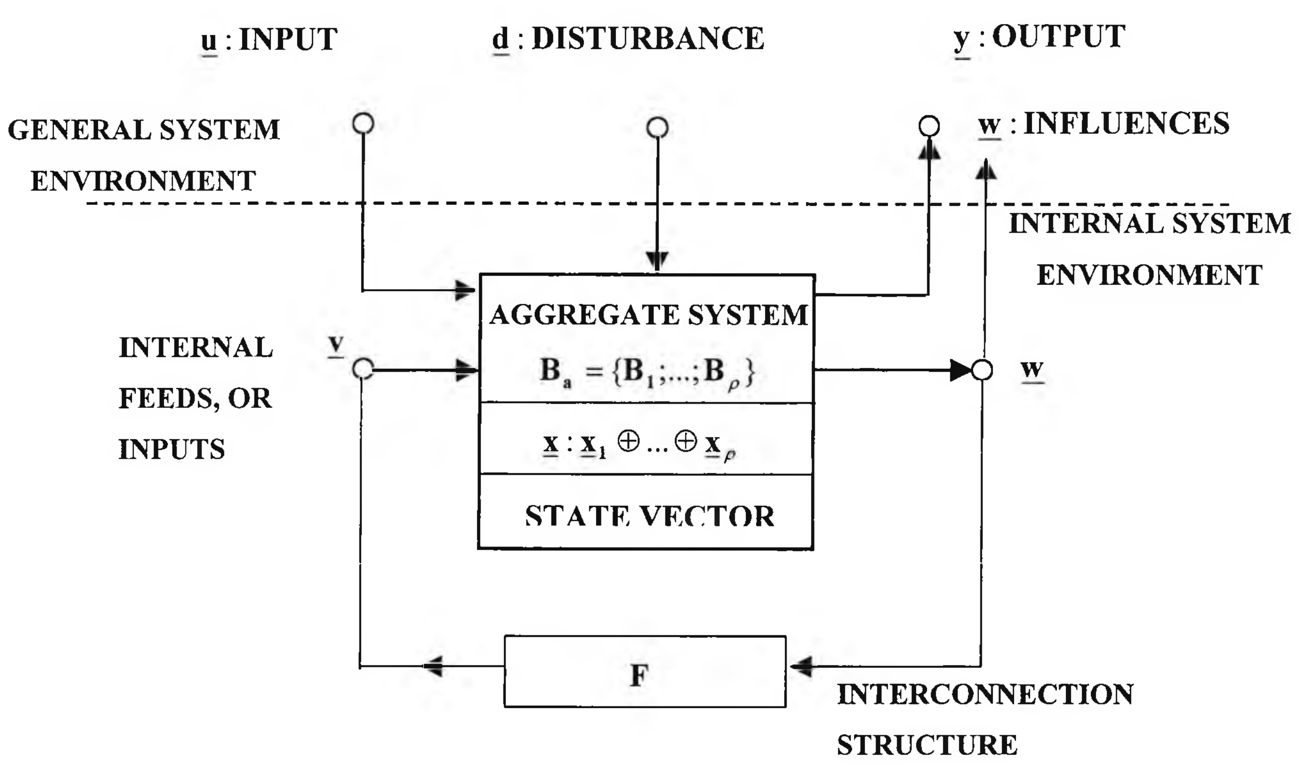

Figure 2.6: Overview of a structured, interconnected system

The nature of the system we are dealing with determines the specific properties and nature of F and leads to some classification of systems. The map F, or the system is called ***natural***, if F is the result of a physical process, which cannot be affected by human intervention. If the map F is the result of human construction aiming at a system serving a given goal, then it will be called ***design*** map. There exist systems, natural or designed, where there is some ability to alert F without affecting the overall functionality of the system; we shall refer to such systems as ***flexible*** and for such systems redesigning F becomes an important issue.

2.2.3 The System and its Environment

Composite systems may be combined according to some rule defined by an overall interconnection topology F' and along the same lines used for composition of objects, and illustrated as in figure 2.6. We shall assume now that the elements of the system are objects B_i, or systems themselves S_i, and that $S_a = \{S_1;...;S_\rho\}$ is the aggregate and F the interconnection structure. Referring to figure 2.6 we may identify a number of important vectors associated with the system. These vectors may evolve in time, or as result of events and the set of all their values are referred to as spaces. Thus, we define the following spaces:

- $\underline{x}$: State vector and X, space of all values of $\underline{x}$ is the system ***state space***.
- $\underline{u}$: Input vector and U, space of all values of $\underline{u}$ is the system ***input space***.
- $\underline{y}$: Output vector, or vector of measurements and Y, space of all values of $\underline{y}$ is the ***output space*** of the system.
- $\underline{d}$: Disturbance vector and D, space of all values of $\underline{d}$ is the ***disturbance space***.
- $\underline{w}$: Influences vector and W, space of all values of $\underline{w}$ is the ***influence space***.
- $\underline{v}$: Internal input vector and V, space of all values of $\underline{v}$ is the ***internal input space***.

The vector $\underline{x}$ represents the total knowledge on the internal system mechanism and its components are variables that may be identified down to subsystems, or object level. The nature of properties of the state vector, and thus also the state space X are products of interaction of the corresponding state vector of aggregate system and the interconnection topology. There are classes of systems, where there is a relation or map

r that is defined: $r : X \rightarrow X$; this expresses the time evolution of the state vector and it is referred to as ***internal relations map***. The properties of r are shaped by the corresponding maps defined on the subsystem level and the interconnection topology. The vector $\underline{u}$ represents the set of all external variables, which affect at least one object, or subsystem and the values of which can be arbitrarily assigned. The vector $\underline{y}$ represents all measurements that can be performed on the objects, or subsystems. The spaces U and Y are linked to the internal state space X. The coupling of U to X and X to Y is expressed by relations, maps that are denoted by g, h respectively, and referred to as ***input-, measurement-maps*** correspondingly. The nature and properties of g, h express part of the interaction of internal mechanism to another environment and manifest the desire of the system designer to control, influence the systems behaviour, as well as measure it.

The interaction of internal mechanism to the environment has also two other signals, event components. The external vector of disturbances $\underline{d}$ consists of inputs, the values of which cannot be arbitrarily assigned, but determined by some independent mechanism. The coupling of disturbances to the internal mechanism expresses relations denoted by δ and referred to as ***disturbance map***. Note that the disturbances generated by known processes express the embedding of the given system in a wider context of interconnected systems and they will be referred to as ***loading disturbances***.

The variables that are measured express the knowledge extracted from the system. There exist however variables, which may be measured, or not measured, and which affect the systems, or their nature is critical for the system itself. Such variables are linked to the internal mechanism through some map γ, referred to as the ***influences map*** and express the system influences variables. The vector of internal feeds, or internal inputs $\underline{v}$ contains internal variables expressing the internal influences between objects, or subsystems and their monitoring may be of importance. The linking of $\underline{v}$ to the internal mechanism (states) is manifested by a map β, which is referred to as the ***connectivities map***, and is affected by the interconnection structure and the subsystems influence map.

The vectors, spaces and maps described above provide the signals based dimension of the notion of the system, which is complementary to the structural dimension based on the subsystems, objects and interconnection structure. For a large number of systems, especially those linked to human activities or those that result from human design, an additional dimension to the notion of system appears, which has to do with some purpose, or goal associated with the system operation. The imposition of a purpose, goal on a system may be formally expressed as a set of externally imposed rules, which represent objectives, performance indices, constraints, operational instructions represented as a set $\Omega = \{\omega_{i,}\ i \in \underline{\mu}\}$ and referred to as the ***operational set*** of the system. This represents higher-level functionalities, which affect the system behaviour, but not in a direct signal, or event way. The functionality of the system, as this is represented by higher-level goals, crucially depends on the nature of Ω. This set may contain rules, which affect the behaviour of individual processes, may alert the topology of interconnection to guarantee an alternative operational scheme, or may change the objectives, goals of the system operation. This set is linked to the lifecycle aspects of the system and its elements and their functionality are defined at a higher level. In general, Ω may be seen as the goal setting governor of the system, which introduces the lifecycle aspects, initiates alternative operational modes, goals and stimulates needs for changing control strategies in response to new requirements. Ω may even redesign the system when drastic changes are required. The issue of redesign, initiated by demands of complying to the new criteria contained in Ω, which are activated at some point, may involve drastic changes on the system affecting sub-processes, the local measurement-control structure and possibly the interconnection structure. An overview of this alternative viewpoint of the system is given in figure 2.7.

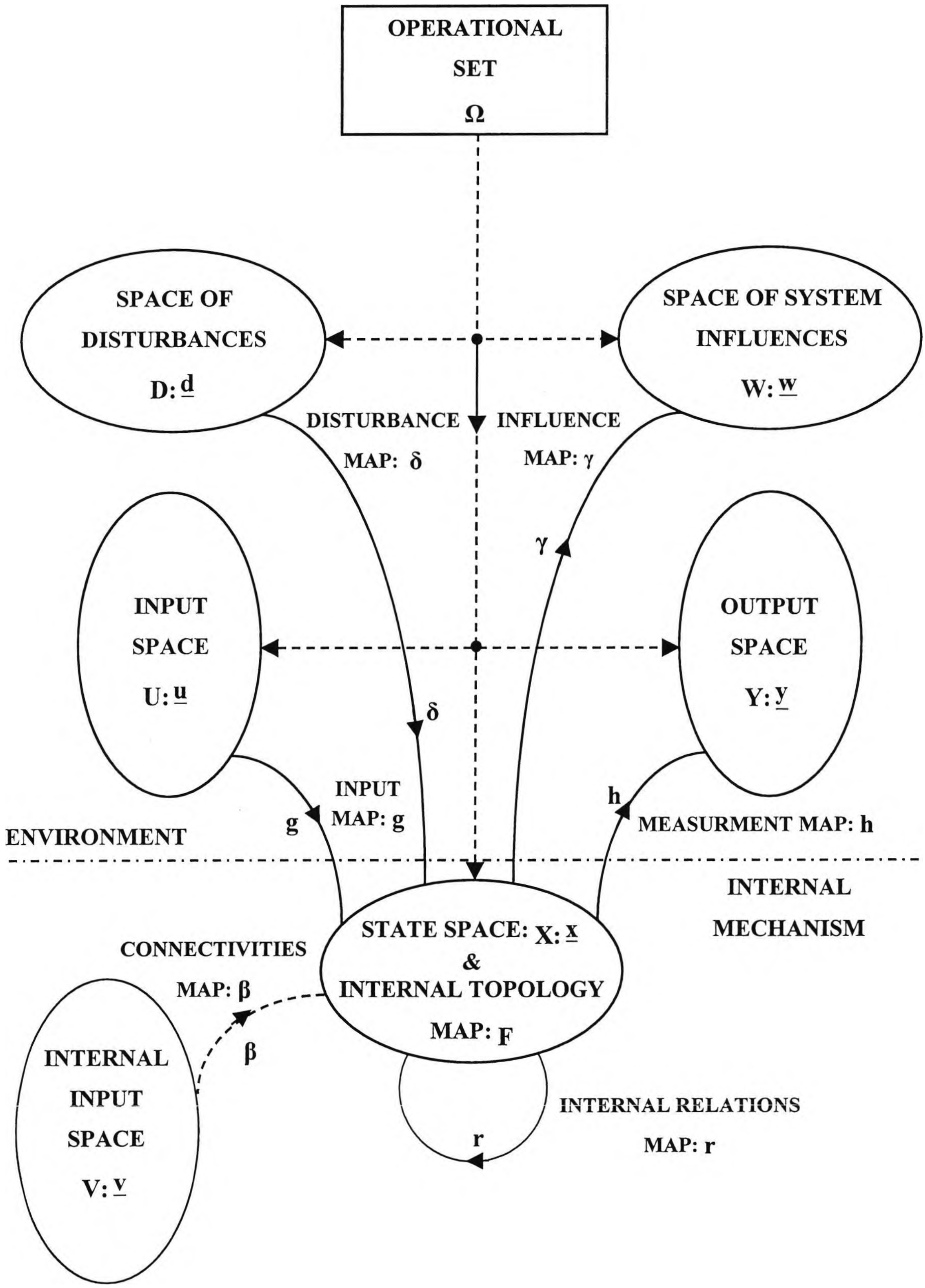

Figure 2.7: System Maps Overview

2.3 System Behaviour, Dynamics and System Properties

Figure 2.7 provides a description of the system as a set of spaces characterizing the range of values of important vectors and the relationships between them expressed in terms of maps functions such as:

Input-State relationship: $g : U \to X$

State-State relationship: $r : X \to X$

State-output relationship: $h : X \to Y$

Disturbance-State relationship: $\delta : D \to X$

State-Influence relationship: $\gamma : X \to W$

Such a description of the system is of conceptual nature, since the exact nature of the relationships, maps and the corresponding spaces, sets is not specified. The diagram also introduces same elementary topology, since it defines the spaces between which the maps are defined; the latter together with the internal topology of the interconnections, as defined by the interconnection map F, provides the fundamental structural information of the system and introduces its ***basic structure***. A richer subclass of the general family of systems may be introduced by considering the notion of behaviour of the fundamental system vectors, which in turn allows the study of fundamental system properties.

We consider now systems for which at least one of its components, or subsystems may be set in a dynamic mode as a result of initial conditions, or the value of external vector (inputs, disturbances); Such systems are called ***dynamic***. For dynamic systems, the set of all possible values of the variables obtained under a given initial value is referred to as behavior and expresses the time-event evolution of values of the dynamic variables involved. If $\underline{\xi}$ is the composite or implicit vector associated with the system, then its behaviour is referred to as ***implicit behaviour*** or ***implicit trajectory***. For relational systems (their definition follows along similar lines to those of objects) there exist relations ω defined by:

$$\omega : \{\underline{\xi}\} \to \{\underline{\xi}\}, \quad or \quad \omega(\underline{\xi}) = 0 \tag{2.6}$$

This relationship is satisfied by all implicit behaviours that may be associated with the system. An orientation on the relational system is associated with the partitioning of $\underline{\xi}$ vector into:

$$\underline{\xi} = [\underline{y}^t, \underline{x}^t, \underline{u}^t]^t \tag{2.7}$$

In (2.7) $\underline{x}$, $\underline{u}$, $\underline{y}$ are the vectors interpreted in the standard way. For families of systems the partitioning of $\underline{\xi}$ as in (2.7) may lead to a reduction of (2.6) into the form:

$$\underline{\tilde{g}}(\underline{x},\ \underline{u}) = 0 \tag{2.8}$$

$$\underline{y} = \tilde{h}(\underline{x},\ \underline{u}) \tag{2.9}$$

Systems for which such reductions are possible will be called ***solvable*** and for them the notion of ***state***, ***input***, and ***output trajectory*** are introduced as partitioning of the implicit trajectory. If conditions (2.8) and (2.9) can be solved and produce the expressions:

$$\underline{x} = \rho(\underline{u},\ \underline{x}(0)) \tag{2.10}$$

$$\underline{y} = \sigma(\underline{u},\ \underline{x}(0)) \tag{2.11}$$

If the relations ρ, σ are uniquely defined, and $\underline{x}(0)$ denotes the value of $\underline{x}$ at same initial time, then the solvable orientation will be called ***regular***. The relation ρ will be called ***state transition*** and σ ***output transition***. In the following, we shall consider systems for which there exists at least a regular solvable orientation.

Definition (2.7): A system S with a regular solvable partitioning will be called ***dynamic***, if relations (2.10), (2.11) contain explicitly $\underline{x}(0)$; otherwise if relations (2.10), (2.11) are independent of initial conditions, the system will be referred to as ***static***.

The meaning of a dynamic system is that the time evolution of the trajectories $\underline{x}$, $\underline{y}$ is depended not only on the external excitation force $\underline{u}$, but also on the past history of the system state, as this is expressed by $\underline{x}(0)$. On the other hand, static objects or systems characterised by relations of the (2.10), (2.11) type, are independent of $\underline{x}(0)$ and thus they are of instantaneous nature. With dynamical systems a crucial notion that emerges is that of the equilibrium, which is defined below:

Definition (2.8): For a dynamical system S, a pair of constant vectors $\underline{x}^*, \underline{y}^*$ defines a ***static equilibrium***, if for $\underline{x}(0) = \underline{x}^*$ and $\underline{u} = \underline{u}^*$, then:

$$\underline{x}^* = \rho(\underline{u}^*, \underline{x}^*) \tag{2.12}$$

Furthermore, if $\underline{u}^* = \underline{0}$, the equilibrium is called ***free*** and if $\underline{u}^* \neq \underline{0}$, it will be called ***forced***.

The above definition characterises the equilibria as the fixed points of the state transition. This implies that the equilibria correspond to pairs $\underline{x}^*, \underline{u}^*$, which when they are considered as inputs to state transition result in no movement at all. Definition (2.8) may be extended to that of a ***dynamic equilibrium***, when we allow $\underline{u}^*$ to become a trajectory rather than a constant input. In this case the dynamic equilibrium characterises a forced trajectory of the system that results in a nominal operation corresponding to a fixed initial condition and given input trajectory.

In the following, the trajectory that results from zero input, i.e., $\underline{u} = \underline{0}$, will be called ***free motion***, whereas that corresponding to $\underline{u} \neq \underline{0}$ will be referred to as ***forced motion***. An important qualitative property of trajectories with respect to equilibria is that of stability, which characterises the behaviour of trajectories with respect to an equilibrium point. Introducing this notion requires some topology on the spaces of signals $\underline{X}$, $\underline{U}$, $\underline{Y}$ in terms of some metric. Without attempting to define the specifics of such topologies the following notions are important:

a) Boundedness of a general vector
b) Region of a given point $\underline{x}^*$ with radius R, $\Omega(\underline{x}^*, R)$

The nature of the system determines that of the corresponding variables and thus the nature of signal spaces. Boundedness is reduces to a distance problem for each of the variables with a domain of values, which may be a general set (signals, sequences, and so on). Similarly, defining regions for a point requires the definition of a distance function. System, for which a metric topology may be defined on their signal spaces, will be referred to as ***metric systems***.

Definition (2.9): For a metric object B or system S, with a static equilibrium point $\underline{x}^*$ we consider two spheres centred at $\underline{x}^*$ with radii $r,\ R,\ \Omega(\underline{x}^*, r),\ \Omega(\underline{x}^*, R)$ such that $r < R$. We may classify equilibria in the following way:

I. $\underline{x}_B^*$ will be called ***state bounded***, if for any $\underline{x}(0) \in \Omega(\underline{x}^*, r)$ the free trajectory $\underline{x} = \rho(\underline{0}, \underline{x}(0)) \in \Omega(\underline{x}^*, R)$ for all time.

II. $\underline{x}^*$ will be called ***asymptotically stable***, if it is state bounded and $\underline{x} = \rho(\underline{0}, \underline{x}(0)) \to \underline{x}^*$ as $t \to \infty$

III. $\underline{x}^*$ will be called ***unstable***, if no matter how small r is selected, there exist at least one $\underline{x}(0) \in \Omega(\underline{x}^*, r)$ such that for some time $t > \tau$ the free trajectory $\underline{x} = \rho(\underline{0}, \underline{x}(0))$ escapes $\Omega(\underline{x}^*, R)$.

The above notions are expressions of the standard definitions of internal stability of dynamical systems expressing notions of Lyapunov stability, instability. In this more general setup, however, the selection of appropriate metric topology is crucial in defining the notions. Such topologies have to be natural and be linked to the specific characteristics of the object under consideration. The definition above may readily extended to a characterisation of stability of forced motion when $\underline{u}$ is a fixed input and we consider variations in the initial conditions $\underline{x}(0)$. An alternative notion of stability based on the input-output properties is defined below:

Definition (2.10): For a metric object B, or system S, we may define alternative notions of stability as shown below:

I. The system is ***Bounded-Input, Bounded-Output stable***, or simply ***BIBO-stable***, if for all bounded inputs $\underline{u}$ and for $\underline{x}(0) = \underline{0}$, the forced output trajectory $\underline{y} = \sigma(\underline{u}, \underline{0})$ is bounded.

II. The system is ***totally stable***, or simply ***T-stable***, if for any bounded input $\underline{u}$ and any $\underline{x}(0)$ bounded (within a given set), the state and output trajectories $\rho(\underline{u},\ \underline{x}(0)),\ \sigma(\underline{u},\ \underline{x}(0))$ are bounded.

The characterisation of such properties depends on the nature of the system and the selected metric topology. Different types of criteria may be derived for classes of models representing families systems. The notions of stability have been presented in an abstract way aiming to cover all families of metric objects. Characterising such properties in terms of criteria requires use of models for the different types of systems and it is beyond the scope of this work.

For families of solvable partitioning of (2.10), (2.11) type, it may be possible to eliminate $\underline{x}$ from (2.11) and derive a uniquely defined relationship between $\underline{y}$ and $\underline{u}$, that does not involve $\underline{x}$. Then (2.10), (2.11) may be represented in an equivalent manner as:

$$r(\underline{x},\ \underline{u}) = 0 \tag{2.13}$$

$$\underline{y} = \Phi(\underline{u}) \tag{2.14}$$

and this description has been referred to as ***strongly oriented*** in the input-output sense and Φ is defined as a ***transfer relation***. The description (2.14) on its own does not necessarily provide a complete representation of the object behaviour. We may classify such descriptions in the following way:

Definition (2.11): A strongly oriented description will be called ***complete***, if there is a procedure of reconstructing the relationship (2.13) from (2.14). Otherwise, it will be called ***incomplete***.

Completeness, thus refers to ability to recover a relationship between internal variables, states, from input-output or transfer relationships. Assessment of presence of such property requires use of specific features of the particular objects. For complete objects Φ is adequate to describe the object in the input-output sense. Objects for which the initial state $\underline{x}(0) = \underline{0}$ are referred as ***relaxed***. An important feature of the dynamic behaviour that can be discussed in terms of the transfer relation is defined below:

Definition (2.12): Consider the strongly oriented object, or system represented by the transfer relation Φ. It will be called ***causal*** or ***non-anticipatory***, if the output of the system at time t does not depend on the input applied after time

t; it depends only on the input applied before and at time t. Otherwise, it will be called ***non-causal***, or ***anticipatory***.

Causality, in short, implies that the past affects the future, but not conversely. Hence, if a relaxed object is causal, its transfer function can be written as:

$$\underline{y}(t) = \Phi(\underline{u}(-\infty, t)), \quad t \in (-\infty, \infty) \tag{2.15}$$

The output of a non-causal system depends not only on the past input, but also on the future value of the input. This implies that a non-causal system is able to predict the input that will be applied in the future. For real physical systems, this is impossible. However, for processes involving human operators, or some form of intelligence, non-causality may be naturally observed property.

Two important properties related to the family of state, output and input trajectories, in relation to the spaces X, Y and U, are those expressing ability to transfer the object state between two points of X by some appropriate input, and the ability to reconstruct the initial state of the object by knowledge of the input and output trajectory. These properties are defined below:

Definition (2.13): Consider a dynamic object B, or system S with state, output trajectory families as in (2.10), (2.11) defined for all possible inputs $\underline{u}$. We define:

I. The object, or system as ***reachable***, if given any two points $\underline{x}_1, \underline{x}_2 \in X$ there exists an input trajectory $\underline{u} \in U$ such that $\underline{x}_2 = \rho(\underline{u},\ \underline{x}(0) = \underline{x}_1)$, and this occurs in finite time. If there exists a pair $(\underline{x}_1, \underline{x}_2)$, for which this property does not hold true, then the system will be called ***non-reachable***.

II. The object, or system is ***reconstructable*** if knowledge of the input and output trajectories over a finite time allows the reconstruction of the state trajectory and thus, also the initial state of the system; otherwise, the system is called ***unreconstructable***.

The characterisation of these properties in terms of specific criteria requires the consideration of particular classes represented by specific families of models.

2.4 Systems Modelling

2.4.1 The Modelling Problem

So far we have considered the notion of the system, its behaviour and properties. We make the implicit assumption that the notion of the system is identified with some reality, and such reality may be understood, known to various degrees. The process of obtaining a description, or representation of this reality is the art of modelling and the final product is a model.

Definition (2.14): A ***model*** of a given system is an object, or concept that is used to represent the system. It is reality scaled down and converted to a form we can comprehend. A ***mathematical model*** is a model whose parts are mathematical concepts, such as diagrams, constants, variables, functions, inequalities, and so on.

Development of models for systems is a complex process performed by the modeller, the person developing the model, and it is affected by factors, such as:

- Knowledge, information, and data available on the system.
- The use, purpose of the model.
- The nature, domain of the system.
- The skills, approach, philosophy adopted by the modeller.

Acquiring knowledge on a system is a complex process that involves measurement, interpretation of data, development of useful information and finally transforming information into knowledge. Each of the above steps is complex and involves processes, which are not always well defined. It also involves ability on behalf of the modeller to visualise, which requires imagination and intuition. Intuition is something difficult to define and even more to impart by formal instruction; it usually amounts to an innate ability to make judicious guesses and judgments in the absence of adequate supporting evidence. The overall process of acquiring knowledge is a "projection process", where large sets lead to the development of smaller but richer sets in a finite number of steps. Knowledge may also exist due to past experience and can be expressed in different forms such as:

a) System structure as this may be manifested by topology of interconnections, structure of signal spaces and maps.
b) Variables of importance, parameters and their values.
c) Physical laws describing the relationship between variables.
d) Constraints imposed on system operation.
e) Properties observed of system dynamic behaviour.

The use, or purpose of the model is a factor that affects its nature and properties. Models used for controlling the system have in general different properties and features to those used for predicting the evolution of system properties, or values of key variables in the future. The purpose of the model thus, introduces some bias and it may be necessary to use alternative models for different uses of the system. This bias is frequently the result of emphasising the role of certain variables and parameters at the expense of others as a result of achieving the given goal and keeping model complexity within manageable limits.

The nature or domain of the system has a crucial effect on the nature of the model. In fact, the nature of measurements, the ability to have controlled experiments, past knowledge, relationships between components and variables, ability to decompose problem to smaller problems are affected by the nature of the system. The system domain also affects the type of statements, which express some knowledge. Physical systems have laws stemming from physics, chemistry, and biology, and usually well identifiable structure, which allows the development of certain families of models. This is not the case with the family of so-called 'soft' systems, where it is difficult to develop laws along similar lines to those in physics.

The development of the model as an act of extracting knowledge from the system and then representing it, is an act that goes through the modeller and it is thus, affected by his or her skills and philosophical approach. Describing the skills requires by the modeller involves quantifiable aspects that can be taught and aspects acquired by experience, as well as general understanding of wider fields and capabilities of the individual. Visualisation and intuition cannot possibly be taught, but one can be helped in making inspired guesses by developing a conceptual, mental framework. An integral part in the development of such a framework is the approach adopted in the pursuit of scientific truth. The basic approaches, which underpin the modelling problem, are (Vemuri, 1978):

- Leibnizian approach
- Lockean approach
- Kantian approach
- Hegelian approach

The Leibnizian approach usually attributed to Leibniz, is based on the premise that ***truth is analytic***. Therefore, a system can be defined completely by a formal, or symbolic procedure. Within this approach, one attempts to reduce any problem to a formal mathematical or symbolic representation. Formal models derived within this approach include the laws of physics and mathematical programming models. The emphasis in constructing such models is in exploring the formal structure and the associated properties of the system. The area of physical modelling is predominately affected by this approach. The types of problems, which are well suited to the Leibnizian approach, are those involving a simple and well-defined structure and problems in which the underlying assumptions are clearly defined.

The Lockean approach, due to the English empirical philosopher Locke, is based on the assumption that ***truth is experimental***. This implies that a model of a system is necessarily empirical and that the validity of a model does not rest upon any prior assumptions except those associated with the data set. If the Leibnizian approach is considered as a deductive process, the Lockean approach is an inductive process. Statistical techniques are representative examples of the Lockean approach.

The Kantian approach, produced by the German philosopher Kant, is based on the assumption that ***truth is synthetic***. This implies that experimental data and the theoretical base are inseparable. Thus, crucial to this approach is that theories cannot be built without the experimental evidence and data cannot be collected unless a theory tells what data to collect. A distinguishing feature of this approach is that it requires one to examine at least two different representations or models before selecting one of them. Examples of the Kantian approach are abundant in a modern technological world and include normative forecasting, program planning, cost effectiveness analysis, and so on, amongst those from the 'soft systems' area.

The Hegelian approach is derived from the dialectical idealism of the German philosopher Hegel and it is the culmination of the work of Socrates and Plato. Central to Hegel's philosophy is the precept that any system can be visualised as a set of logical categories, and these logical categories generate their own opposites. The implication is that ***truth is conflictual***, and the process of determining is complex and depends upon the existence of a thesis and an antithesis. The union of these opposites leads to a more adequate grasp of the nature of things until finally all possible points of view with all their seeming conflicts become the constituents of one comprehensive system. An interesting application of the Hegelian approach can be found in the management games, used by some companies.

The construction of models uses the theories developed within the different branches of science. The strength of science is largely measured by its capability in making intelligent observations and predictions, as well as in providing concepts and tools, which underpin technology and applications. Theories in science are developed using Leibnizian, Lockean, Kantian, or Hegelian approaches. A theory usually deals with aspects of reality that are not immediately evident to the senses, but which describe certain forms of behaviour observed in a system. In the physical sciences, when a theory is tested and found to describe adequately the aspect of reality with which it deals, it is called a ***law***. In the behavioural sciences, where the corresponding systems are referred to as 'soft', since it is difficult to make positive statements, the probabilities are suitable to provide more appropriate means for stating results of observations. Models are constructed using theories, but there are fundamental differences between theories and models. If a model in question is physical, then it allows one to play around with theory in a rather concrete way. A model is a representation of a system in a convenient form, so that conjectures made about the performance of the system can be tested. A model is meant to imply a manifestation of the interpretation that a modeller gives to observed facts. Facts remain unchanged, but models change.

An overview of the modelling problem is described in figure 2.8, which clearly represents the model as a projection of the system (leaving in the sphere of reality), through the modeller in a knowledge space, domain.

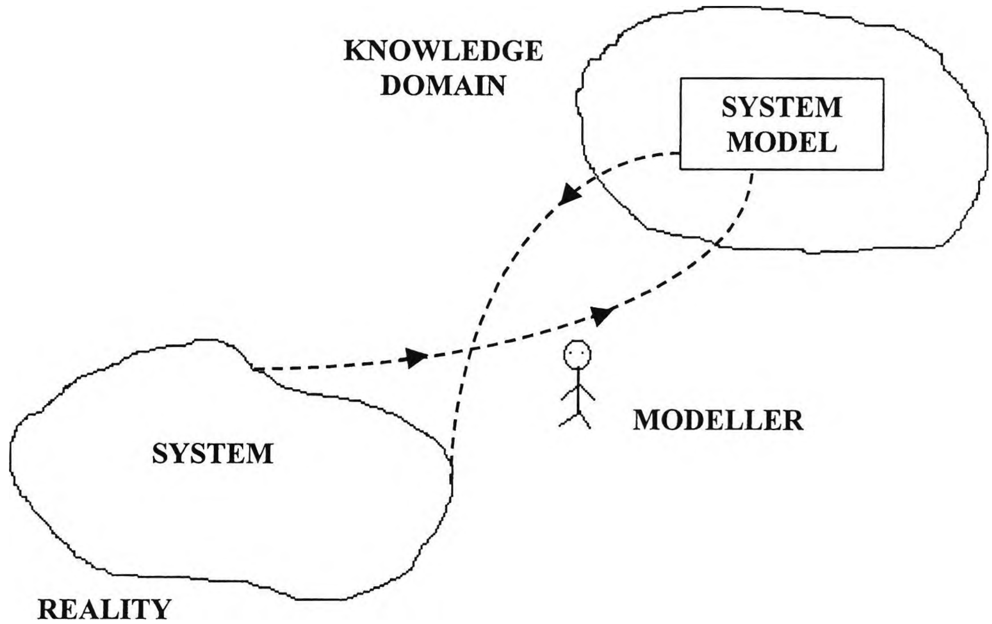

Figure 2.8: The problem of modelling: Extraction and knowledge representation

A system model is also a system, but it belongs in the general set up where all knowledge is reposed. There is a clear distinction between the model and the system, since the model is a perception of the system and the two belong to different domains. A system may have a large number of models of variable accuracy and complexity.

2.4.2 Classification of Models

Models can be categorised into a variety of classes and this classification can be based on different criteria. Two major classes are:

- Descriptive models
- Prescriptive models

The first class of models, as their name implies, are those, which attempt to describe an observed regularity without necessarily seeking to provide an explanation for the observations made. Such models represent the first stage of rationalization, generalisations and theory building and are expressed, in general, in a native natural language. The major advantage of descriptive models is that the cost for predictions is extremely low; their major disadvantage is that the method of prediction is usually internal and cannot be communicated easily. Prescriptive models, on the other hand, are normative. Normative models imply the establishment of standards or correctness and thus, a normative model by its nature is more suitable for predictive purposes. The transition between descriptive and prescriptive models is not abrupt, but passes through the notion of the conceptual model. We may describe this transition as follows:

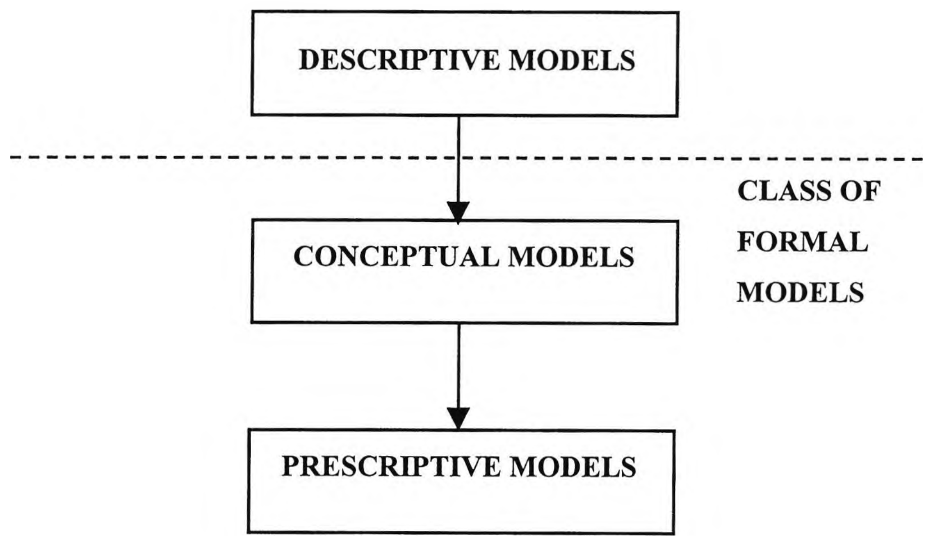

Figure 2.9: General Classes of Models

A ***conceptual model*** is derived from descriptive models and it is the simplest formal model of the system that provides information on the general system structure and on the sub-processes, as well as on the variables associated with them. Such models express topology, structure of signal spaces and data structures, but do not provide information on behavioural aspects. As such, a conceptual model may be considered as the simplest of the prescriptive models (fundamental topology and data), or as the progenitor of the subsequent prescriptive models, which contain the information provided by the conceptual model. The notion of ***formal model*** is linked to prescriptive models employing mathematical concepts and tools to describe not only the structure, but also behavioural properties. Formal models are concerned with the abstraction, structure and representation of aspects of the observed system in a way that enables the reasoning about the system. A classification of formal models is given in figure 2.10:

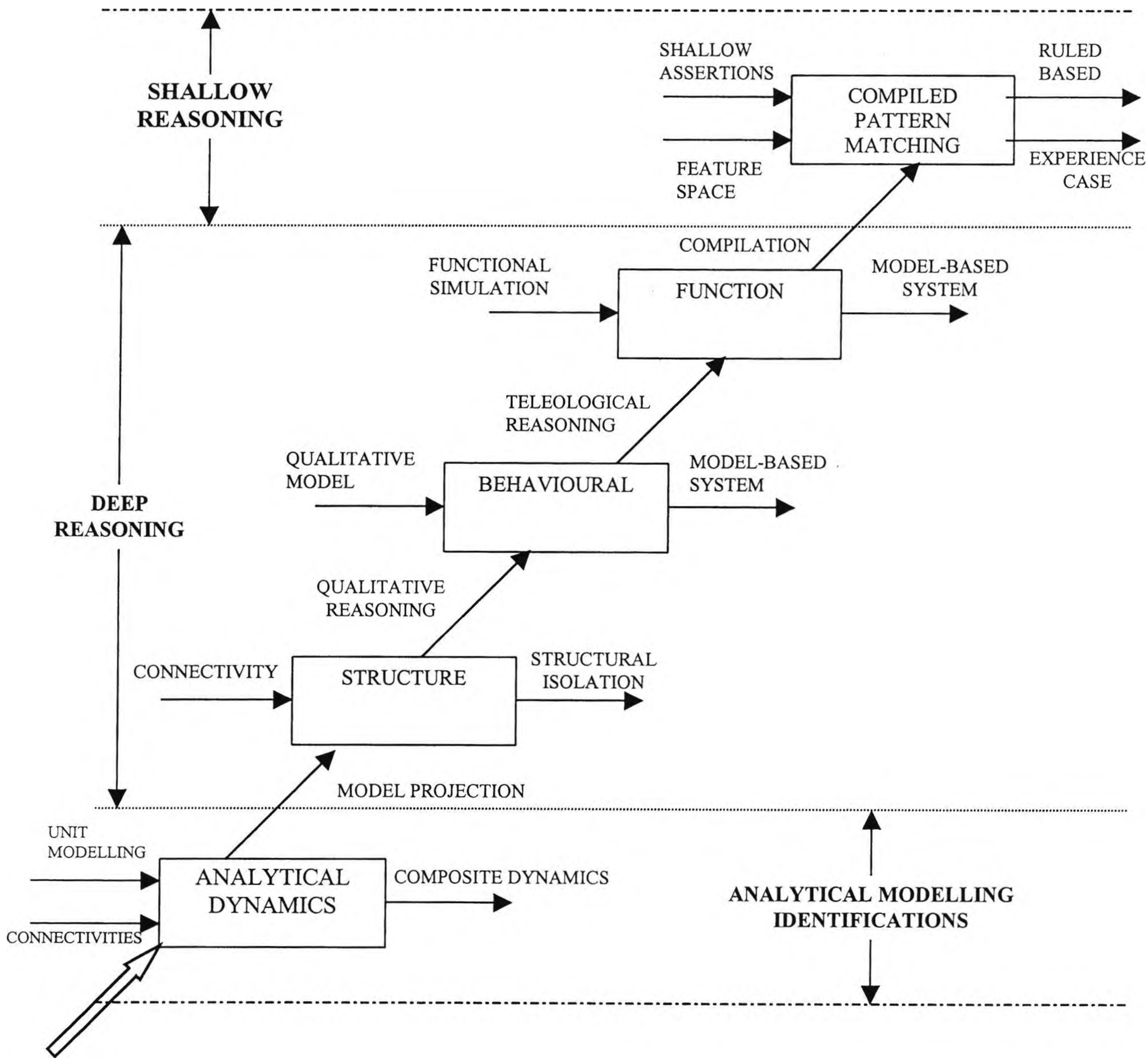

Figure 2.10: Nesting of formal Modelling

This diagram classifies the different types of diagnostic reasoning and represents five levels of knowledge representation and families of formal models. These are analytical, dynamical, structural, behavioural, functional and pattern matching. From each knowledge representation we are able to derive the next higher level of representation. The additional layer introduced here is the first corresponding to analytical dynamics and contains a family of models of detailed dynamical description; such models are based on identification, or detailed modelling of basic sub-processes. The layers above correspond to families of reasoning (deep and shallow) type models. This scheme may be used as a basis for diagnostic analysis, decision, management system, as long as the analytical tools that can be based on such models are fully explored.

Another major classification of models concerns those models referred to as 'black' and 'white'. White models are based on physical, chemical and/or biochemical principles and their development requires a lot of process insight and knowledge of physical/chemical relationships. Such models can be applied to a wide range of conditions, contain a small number of parameters and are especially useful in the design process, when experimental data are not available. Black models, on the other hand, are based on standard relationships between input and output variables, contain many parameters, require little knowledge of the process, and are easy to formulate; however, such models require appropriate process data, generated by suitable experimentation on the real system; such models are only valid for the range where data are available. Black models can be turned into grey ones, if some internal structure is assumed to be known; similarly, white models may be turned to grey models, if certain parameters are not known and their determination is left to some identification of parameters.

Models of systems are used for the analysis and design and provide the framework for developing solutions. The development of reliable models is an iterative process that involves identification of parameters, testing of assumptions and validation of results.

2.5 Control Concepts and Principles

The behaviour of system under external inputs and disturbances has to fulfil certain objectives, which are set externally by the designer, or user of the system. To achieve these objectives, actions are required and this is the task for the Control Design. For the system described below diagrammatically in figure 2.11 the control problem is defined as follows:

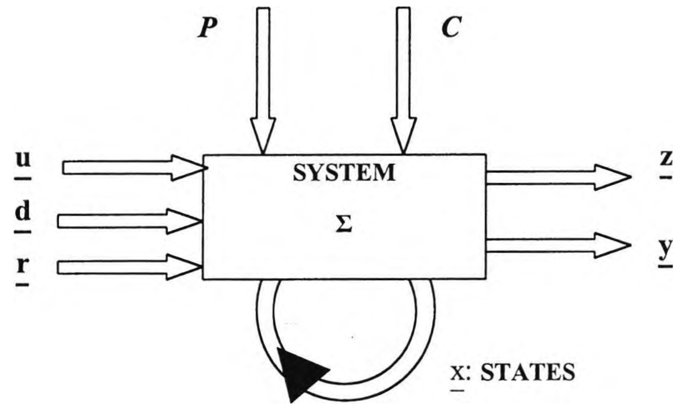

Figure 2.11: Open loop control configuration

Open loop control system: Given a set of objectives P, representing performance indices, criteria, a set of process constraints C, a disturbance vector $\underline{d}$, a reference or target vector $\underline{r}$, design a control input vector $\underline{u}$ such that the controlled variable vector $\underline{z}$ follows the specified reference vector $\underline{r}$, whereas the states, inputs and measured variables satisfy the constraints C and the performance indices P are optimised in some sense.

The above is clearly a decision-making problem and selecting $\underline{u}$ amongst all possible inputs has to take into account the understanding we have about the system model (used in the selection process) and requires exact knowledge of the disturbances over the lifecycle of operations of the system. The model and disturbance uncertainties make the above open loop configuration rather unrealistic, and leads to the following configuration, referred to as closed loop, which involves the use of feedback.

Closed loop control system: Given the set of objectives P, constraints C, disturbance vector $\underline{d}$, and target vector $\underline{r}$, the design of the required control input $\underline{u}$ is to be achieved by using the alternative feedback configuration shown below:

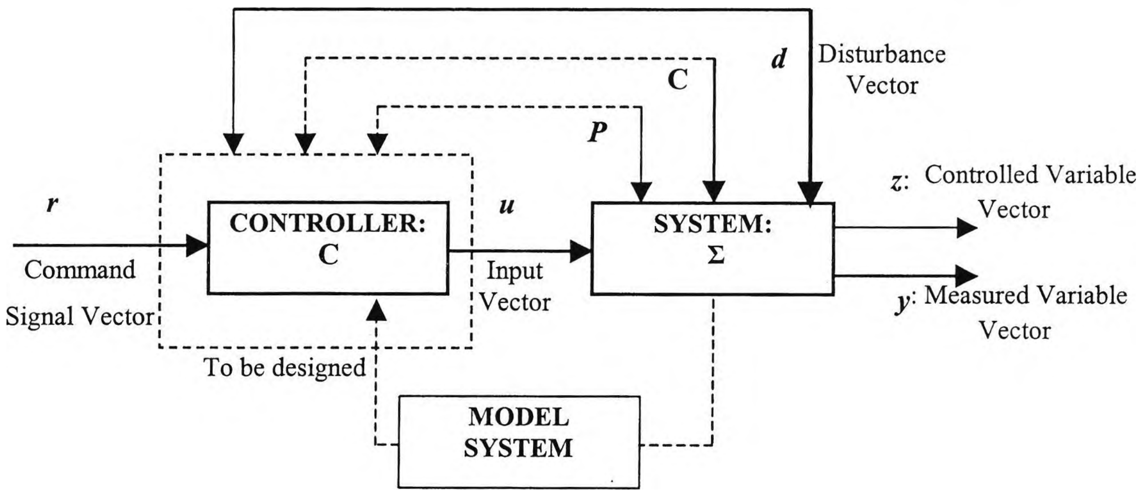

Figure 2.12: Closed loop, or feedback configuration

In this figure, the desired input is produced as the output of a new dynamical system, defined as the ***controller***. The controller is using as inputs the command signal $\underline{r}$ and the real measurement of the command variable vector. The controller acts on the error signal and this generates the required input vector $\underline{u}$.

The design of the controller involves the use of all available information about the system and measurements to provide all available information on the variables associated with the system. In particular this involves:

- Providing the controller with a model of the system.
- Providing information on performance indices and constraints characterising performance operation.
- Measuring all disturbances that can be measured, which affect the system and provide all information regarding unmeasured disturbances.
- Use of the feedback principle to generate the required control input.

The use of feedback, that is measurement of required response and comparison with ideal response, enables the overcoming of difficulties associated with model uncertainty and lack of knowledge on the lifecycle behaviour of disturbances. In this closed loop configuration the design task now becomes a procedure for synthesis-design of a new system, the controller. Systems Theory provides the analysis of concepts and tools, whereas Control Theory deals with the solvability of certain types of control problems, which are posed in the context of specific types of models used to represent system behaviour. Control Design uses both Systems and Control Theory and deals with the derivation and implementation of control schemes, which achieve the overall control objectives.

The great advantage of the closed-loop or feedback configuration is its ability to handle model and signal uncertainty. Moreover, it achieves the basic objectives by the continuous effort to minimise the error between command signal (output) and reference signal (desirable input). The design of feedback system is a complex problem and it is equivalent to the construction of a new dynamic system that will achieve the reproduction of the desirable control signal that will activate the system as an input.

The control problem has many dynamic objectives, which relate to the performance of outputs, as well as internal variables, states, and plant inputs. These may summarised as:

- Stability of the closed loop system.
- Ability to track asymptotically the required reference signals.
- Ability to reject asymptotically at the system outputs the effects of measured and unmeasured disturbances.
- Ability to reproduce as close as possible an output by a given reference input.
- Capability of achieving the above under model uncertainty.
- Achieving all above, satisfying operational constraints and optimising system performance.

The above is a number of requirements for the design of the control system, controller. Very frequently, there is conflict between the different requirements and thus control design has in its centre the problem of resolving this fundamental 'design dilemma'. The above problems emerge indifferent variations due to the nature of the system, the type of available models, the dimensionality of the system in term of dimensions of respective vectors and the centralised-decentralised nature of the information and control structure.

The problem of control design is in many respects dual to that of measurement. Measurement of the system variables for monitoring, and/or control purposes may not be a straightforward process. This may require some additional design tasks. The key problems in measurement are:

a) Measurement, Actuation of physical variables.

b) Estimation problem.

c) System aspects of measurement and actuation.

The first of the above problems is the standard problem of measurement, or action upon physical variables, known as ***traditional instrumentation*** (for technological processes). Amongst the measurement problems with a clear system dimension are those of the b) and c) type. In particular:

Estimation problem: Given a system, which is driven by a control input vector $\underline{u}$, and measures all physically possible output variables $\underline{y}$ through a measurement system. This measurement system produces measurements $\underline{z}$, which are defined by $\underline{y}$, but they may be corrupted by disturbances, or noise signal vectors $\underline{d}$. The problem of estimating the true system state then becomes a design problem, where a new dynamical system is to be constructed, called the ***estimator***, which produces estimates of the true states in an asymptotic sense. The estimator is provided with information about the system model, measured outputs $\underline{z}$ and driving control signal $\underline{u}$ and produces the estimated state vector $\hat{x}$. The overall configuration is denoted in the following diagram:

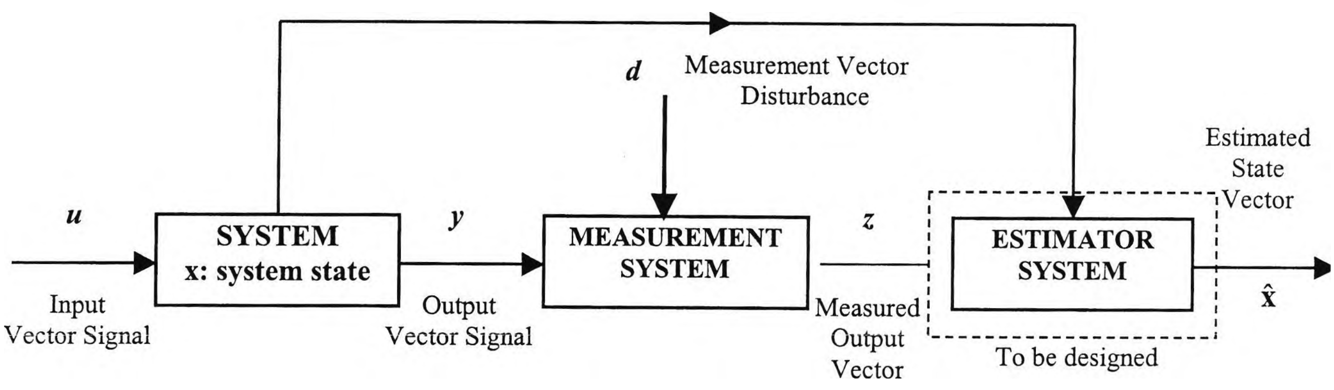

Figure 2.13: Diagram of the state estimation problem

State estimation is part of the overall problem of measurement and diagnostics, where certain aspects of the internal behaviour of the system have to be estimated, or quantitative functional characteristics have to be computed (for instance quality parameters, critical values for behaviour of other processes, and so on). There is a classical duality between the control and estimation problem, which is linked to the duality between Reachability and Reconstructability on a given system.

Global instrumentation problem: The selection of measurement and actuation variables for as system is a design task that precedes the control design. So far the traditional techniques based on heuristics, physical instrumentation are used for such selection of input and output schemes. However, this stage of design that deals with the selection of systems, sets of measurements and actuation variables has a significant effect on the shaping of the final model properties and is referred to as ***global instrumentation process*** (Karcanias, 4, 2001). In fact, the effect of global instrumentation is to modify the system (interconnection of basic subsystems). This process is reminiscent of an evolutionary process (Karcanias, 1, 1994) and the emerging design task is the control of the process aiming at solutions, final systems of which the standard control problem is not difficult to solve. Such tasks are of a generalised control type character (control of system model evolution) and are part of the integrated system design (Karcanias, 2, 1995).

A more general configuration representing the control, estimation, and modelling tasks that also includes the higher level strategies (embedding of a system and control structure in a hierarchy of decision making functions) is described below and denotes the overall supervisory and control problem.

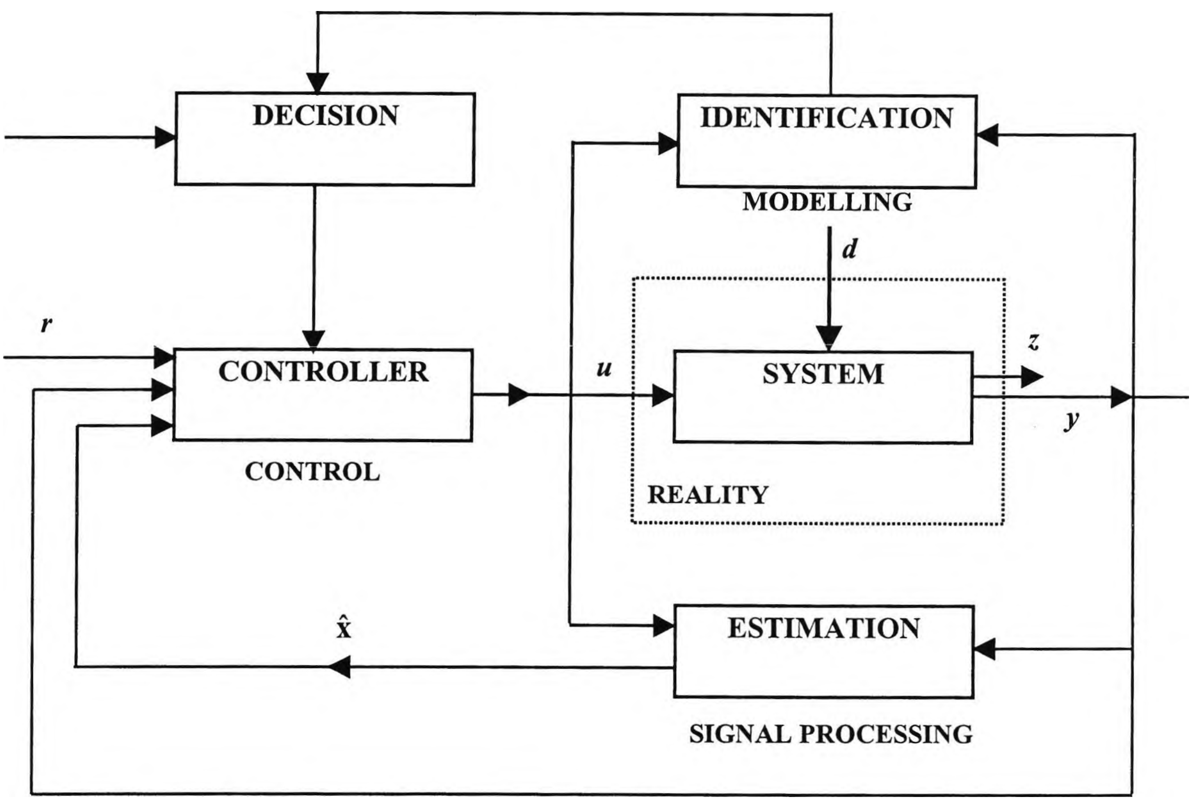

Figure 2.14: General Control, Modelling, Estimation and Coordination Configuration

The traditional elements in this configuration are the tasks of ***identification*** of models of the system and the ***decision making*** process. Identification of models is linked to model derivation based on structural and input, output behavioural information and it is a task within the overall modelling exercise. The decision making block is the aggregate representation of the overall higher level originating goal, objectives driving the control action. The controller in this extended configuration is now provided also with inputs from decision and coordination, as well as real and estimated variables behaviour. The above scheme is typical for engineering type problems, but provided with appropriate descriptions for the key blocks, also defines a general set up for decision, measurement, diagnostics and control of general processes.

2.6 Conclusion

This chapter provides a theoretical framework on the current thinking of the concepts of system, modelling, and control. It summarises the basic terminology of the notion of the system and its fundamentals, the conceptual and philosophical dimension of systems modelling, as well as the major control concepts. Such a framework is considered as necessary, so as going back and looking for the origins of the system, modelling, and control ideas, we will be able to point out and evaluate their emergence and development.

The following chapters present our passing through the centuries and the theories of the ancient Greek philosophers and attempt to draw the line that connect the early appearance and evolution of these concept with the current thinking.

PART ONE:
THE EVOLUTION OF THE CONCEPTS OF SYSTEM AND MODELLING

Chapter 3

MYTHS AND CONCEPTS OF MODELLING

PART ONE: THE EVOLUTION OF THE CONCEPTS OF SYSTEM AND MODELLING

3. MYTHS AND CONCEPTS OF MODELLING

3.1 Introduction

In this chapter, we are concerned with the mythological period (8th century B.C.) of Greek historical period, where we find the first extant written sources. Though these are expressed in the language and through the persons of myth, they reflect the level of social organisation and practical technique. They are the earliest forms of formal education, which is the inculcation of a set of explicit beliefs about the world.

Particularly, the Greek myths assemble elements not only of Greek wisdom and technique, but also of the other Mediterranean people, their achievements, their inventions, and their knowledge, and provide the early forms and evidence for the development of Greek science. After all, the occurrence of Greek myths does not happen accidentally, but only when an early necessity of world's explanation emerges. A necessity, whose scientific answer is given a century later by the Presocratic philosophers. However, the fundamental issue of Presocratics to specify the basic element or elements, that gave rise to the world, appears similarly all over the Greek myth. In the early literary monuments of the Orphic Rhapsodies[4], the Hesiodic Theogony, and the Homeric poems, in addition to the search of the basic elements, the general structure of the present world, as well as the sequence of events are of wider interest. Even though all these topics are covered by literary and mythical narrations, and the world appears as a world of anthropomorphic deities, a more critical inspection reveals on the one hand, a peculiar historicity, and on the other, symbols, analogies, and models related to the modelling concept. The world comes into sight as a great,

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[4] The so-called **Orphic Rhapsodies** of which many fragments survive (Kern, frr. 59-235), mostly through quotation in Neoplatonic works, are a late compilation of hexameter verses of varying date of composition. The Derveni papyrus shows that some derive from the fifth or even the sixth century B.C. Nevertheless, no other author, before the full Christian period, seems to have heard of most of them and it seems highly probable that their elaboration into an Orphic Iliad was not taken in hand until the third or fourth century A.D. (Kirk *et al*., 1983).

unknown system and the myth itself as the explanation or the model of it, whereas the natural elements or the human values take the form of the mythical gods or heroes. The mythical symbols or models concern either the creation or the structure of the world. The former are mostly associated with cosmogony, with the dynamic evolution and the genesis of the world, whereas the latter provide a physical explanation of the world. This chapter will cover both types of models.

3.2 Dynamical Models of the World's Creation

3.2.1 Orphics Cosmogonies

The Orphic Theogonies are poems, which give an account of the history of the gods and the origin of the world. There seem to have been several Orphic Theogonies, but most have not survived. The Rhapsodic Theogony, which is also called the Sacred Discourse in twenty-four Rhapsodies, or just the Rhapsodies, does survive in fragments. The Rhapsodies consist of 24 parts and constitute a complex tale of mythology and theogony. In the early 6th century A.C., the Neoplatonist Damascius, last head of the Academy of Plato, detected four separate Orphic Theogonies, which follow:

(I) Derivation from Night

A first orphic account of cosmogony is the derivation of world from Night:

Damascius *De principiis* 124 (DK I B 12)[5]: "The theology ascribed to Orpheus in Eudemus the Peripatetic kept silence about the whole intelligible realm... but he made the origin from **Night**..." (Kirk *et al.*, 1983)

(II) Derivation from Chronos

Another interesting orphic account of cosmogony ascribes the origin of world to Chronos (Time):

Damascius *De principiis* 123 (DK I B 12): "In these Orphic Rhapsodies, then, as they are known, this is the theology concerned with the intelligible; which the

[5] DK (abbreviation) Die fragmente der Vorsokratiker, 5th to 7th eds., by H. Diels, ed. with additions by W. Kranz. (The 6th and 7th eds. are photographic reprints, 1951-2 and 1954, and the 5th, with Nachträge by Kranz).

philosophers, too, expound, putting **Chronos** in place of the one origin of all..." (Kirk *et al.*, 1983)

Chronos causes the creation of two new concepts, of Aither and Chaos, and the world acquires the figurative form of the egg. In Orphic Rhapsodies, the egg appears as a symbol of the world. The substance of this cosmogony is given schematically:

$$\textbf{Chronos} \longrightarrow \left\{ \begin{array}{l} \text{Aither} \\ \\ \text{Chaos} \end{array} \right. \longrightarrow \text{Egg}$$

Figure 3.1: Derivation from Chronos (Kirk *et al.*, 1983)

(III) Derivation from Water and Matter

Another dynamic account of cosmogony determines Water and Matter as the first cosmic elements of world's creation.

Damascius *De principiis* 123 (DK I B 13): " The Orphic Theology which is said to be according to Hieronymus and Hellanicus (if indeed he is not the same man) is as follows: **water existed from the beginning, he says, and matter**, from which earth was solidified ..." (Kirk *et al.*, 1983)

A summary of the full description of this cosmogony is given in the following figure:

$$\left. \begin{array}{l} \textbf{Water} \\ \\ \textbf{Matter} \longrightarrow \text{Earth} \end{array} \right\} \textbf{Chronos} \longrightarrow \left\{ \begin{array}{l} \text{Aither} \\ \textbf{Chaos} \\ \text{Erebus} \end{array} \right\} \longrightarrow \textbf{Egg} \longrightarrow \textbf{God}$$

Figure 3.2: Derivation from Water and Matter (Kirk *et al.*, 1983)

(IV) Derivation from Air and Night

Lastly, another cascade process for cosmogony begins with Air and Night and proceeds progressively to Tartaros, Titans, Egg, and Offspring.

Damascius *De principiis* 124 (DK 3 B 5): "Epimenides posited two first principles, **Air and Night** [...] from which Tartaros was produced [...] from all of which two Titans were produced [...] from whose mutual mingling an egg came into being [...] from which, again, other offspring came forth." (Kirk *et al.*, 1983)

Air
Night } → **Tartaros** → **Titans** → **Egg** → **Offspring**

Figure 3.3: Derivation from Air and Night (Kirk *et al.*, 1983)

In terms of system representation, this progressive process may be depicted as a block diagram, whose subsystems constitute the particular procedures of cosmogony itself.

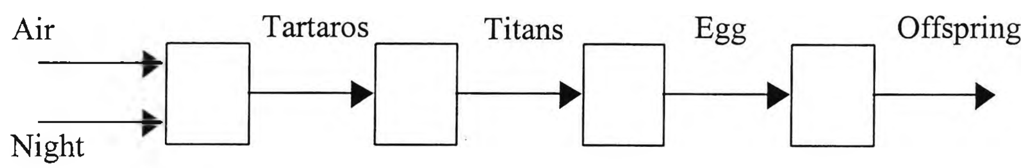

Figure 3.4: Block diagram for the derivation from Air and Night

These above-mentioned passages lay emphasis on the first principles from which the world emerged. Particular interest is given to the sequence or better to the cascade process of world's formation. Even though they do not present a complete model of the world itself, what is noteworthy is the appearance of the egg as an element of cosmogony, and thus of a first model of the world. The next fragment shows that the egg was considered to be a resemblance of the celestial sphere:

Achilles Isag. 4 (DK I B 12, Kern Fr. 70): "The arrangement which we have assigned to the celestial sphere the Orphics say is similar to that in eggs: for the relation which the shell has in the egg, the outer heaven has in the universe, and as the Aither depends in a circle from the outer heaven, so does the membrane from the shell." (Kirk *et al.*, 1983)

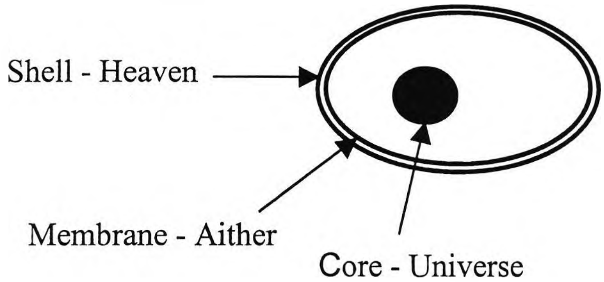

Figure 3.5: Egg as a model of the celestial sphere

A familiar element of genesis at that time is the egg. The form, the structure and the shape of it are similar to the world itself. The egg, as the above figure shows, is a model of the celestial sphere, where the shell is the heaven, the membrane is the aither and the core is the universe. This subjective approach of the world, no matter how simple it may appear, it presents an ellipsoid, egg-shaped universe with centre and sheath, which is close to the contemporary perception of the world.

Additionally, the egg could be considered not only as a model of the universe, but also as a model of its creation. The concept of egg on the one side, and the form itself of it on the other, are closely related to the genesis and creation. Otherwise, the shape of egg (the fact that it is not absolutely round) shows the process of its creation. And the process of world's creation is simulated in orphic cosmogonies with the birth of birds and animals.

3.2.2 Homeric Cosmogony

A simple conception of the nature of the world, which is traced in scattered references in Homer, is roughly as follows.

Around the flat disc of earth, on the horizon, runs the river Oceanus, encircling the earth and flowing back into itself from this all other waters take their rise, that is, the waters of Oceanus pass through subterranean channels and appear as the springs and sources of other rivers:

> *Iliad,* XXI, 194: "Him (Zeus) not even Lord Acheloos equal, nor the great might of deep-flowing Oceanus, from whom, indeed, all rivers and all sea and all springs and deep wells flow" (Kirk *et al.*, 1983)

Oceanus, which is unique and unexpected, is the source or the origin of all things:

> *Iliad,* XIV, 200: (Hera speaks) "For I am going to see the limits of fertile earth, Oceanus begetter of gods and mother Tethys..." and

> *Iliad,* XIV, 244: (Hypnos speaks) "Another of the everlasting gods would I easily send to sleep, even the streams of river Oceanus who is the begetter of all..."(Kirk *et al.*, 1983)

Thales formulates a similar concept, about two centuries later, i.e., Water is the origin of world's creation. Homer is close to this physical explanation of the world as perceived by the Presocratic philosophers. However, he occupies himself mostly with the structure of the universe and gives a descriptive representation of it in his Iliad.

3.2.3 Hesiodic Cosmogony

Homeric poems (middle of the 8th century) and the works of Hesiod (end of the 8th century) are the earliest literary monuments, where one can find fundamental conceptions of the creation of the world. Two major poetic works are attributed to Hesiod: *Works and Days* (which includes, among other things, a manual of farming) and the *Theogony*, which recounts the origin of the gods and the world. In *Theogony*, we find a genius model of the world's creation, from primeval **chaos** to the nature, the gods, and the humans. Hesiod's first principle is chaos from which the world arises. We encounter here the time evolution and the process of world's formation.

> Hesiod, *Theogony*, 116: "Verily first of all did **Chaos** come into being, and then broad-bosomed Gaia [earth], a firm seat of all things for ever, and misty Tartaros in a recess of broad-wayed earth, and Eros, who is fairest among immortal gods, looser of limbs, and subdues in their breasts the mind and thoughtful counsel of all gods and all men. Out of Chaos, Erebus and black Night came into being; and from Night, again, came Aither and Day, whom she conceived and bore after mingling in love with Erebus. And Earth first of all brought forth starry Uranus [heaven], equal to herself, to cover her completely round about, to be a firm seat for the blessed gods forever. Then she brought forth tall Mountains, lovely haunts of the divine Nymphs who dwell in the woody mountains. She also gave birth to the unharvested sea, seething with its swell, Pontos, without delightful love; and then having lain with Uranus she bore deep-eddying Oceanus, and Koios and Krios and Hyperion and Iapetos..." (Kirk *et al.*, 1983)

The Hesiodic description evolves in a complicated genealogical tree of the generation of all the elements of the world, natural, divine, and human. In the following diagram we present indicatively some of its steps:

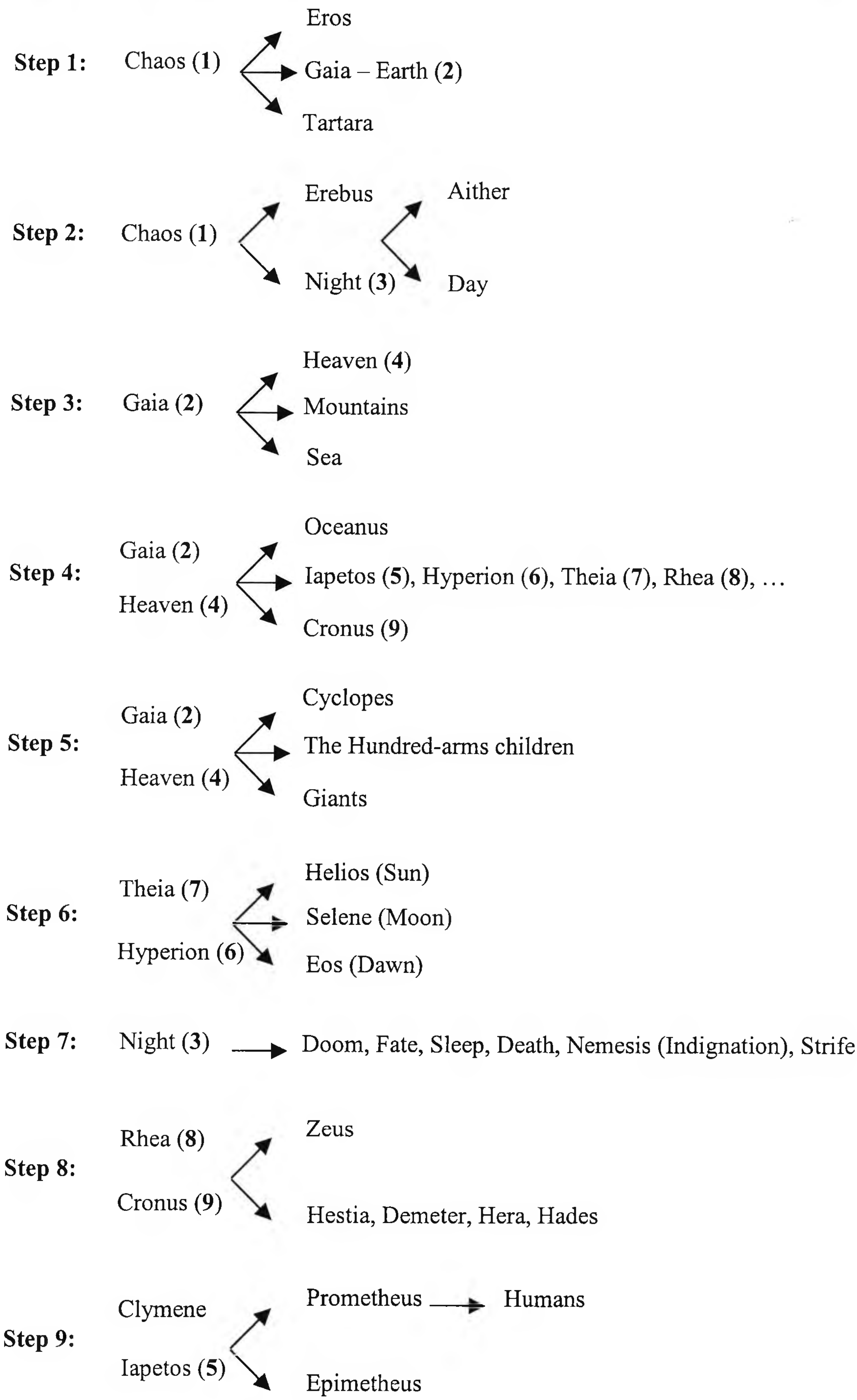

Figure 3.6: Time evolution and process of world's formation according to Hesiod

In Hesiod's cosmogony we have a dynamic process of the world's creation. His first principle, Chaos, has a double meaning: a) it means the void, the empty space, the yawning gap and b) it comes from the Greek word *σχίσμα* that means schism, breaking off. According to Hesiod, the world arises from Chaos, i.e., from the empty space, the void after the occurrence of a schism, i.e., a breaking off, a crack (we call it today "big bang")[6], out of which arises the Gaia (earth) and the Eros, in a sense of the matter and the energy, respectively. (Schlagel, 1995) summarises the general features of Hesiod's way of constructing the world in the following points: a) the things originated by 'separating apart from' or 'differentiating from' an original gap or chasm, and b) Eros, the sexual intercourse, is the main model of generation.

The Hesiodic model of cosmogony is of particular interest, firstly because of its physical explanation, according to which the opposite elements of matter and energy come from the schism of void, and secondly because of its conceptual consideration, according to which a complicated schema of time progress and sequential development of the phenomenon of cosmogony is formed.

We could summarise the main points of these three mythical cosmogonies in the following lines: a) clarification of phenomena, b) classification and hierarchy, c) time succession, d) primary cause, and e) process of creation. In the first stage, the Orphics specify the origins of the world and introduce the concept of egg as a model of the universe and its creation. In the second stage of Homer, the poet introduces a physical conception for the origin of the world, similar to that of the Presocratics. And in the third stage, Hesiod originates a complicated dynamical model of world's generation and analyses the process and the time evolution of it.

3.3 Mythical Models of the World's Structure

3.3.1 Myths and Gods as Symbols

Myth and Science are two concepts opposite, contradictory, and incompatible. Myth is the symbol of imagination and poetic rising, whereas science is the symbol of

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[6] By 1980 scientists believed that the "big bang" theory was the most likely explanation for the origin of the universe. This theory holds that all matter in the Universe existed in a cosmic egg (smaller than the size of a modern hydrogen atom) that exploded, forming the Universe.

order, knowledge, and truth. Is it probable that these concepts coexist? Could someone derive scientific information from myths? Is there any historicity in myths and particularly in Greek myths?

Greek myths provide evidence, especially in the branch of cosmogony, that the later scientific theories, explaining the creation of the world, are not a sudden jump but a continuous development, and on the other hand, they indicate the intentional rationalisation of the process of world's creation inherent in the mythological descriptions. The double characteristic of myths to collect, classify, and register all the significant past achievements and inventions, as well as to create and form new symbols and new theoretical inventions and predictions, is also a manifesting characteristic of science. The order and the method of myths, and the intention of poets to comprehend and express by means of their texts the natural and the human world, give rise to the, at that time, unformed notion of science.

The mythical epics of Homer and Hesiod, the most ancient written texts, are the primary sources of Greek myth that put together the preceding verbal tradition. The myth writers and poets not only did they create the Greek gods and formulate the genealogy, but also formed the History collecting the previous experience, knowledge, and legends. They became the great teachers who educated the Greeks. It is worth mentioning that Homeric poems are still one of the modules of Greek schools.

Greek myths derived their stories from real historical facts such as the first racial wars that in myths were described by the battles of the Titans (Titanomachy), or the later city-wars that were symbolised by the Trojan War, or the great nautical campaigns that were expressed by Odysseus. The fact that myths describe true events but in a different way than history does, constitutes a crucial point that has to be taken under serious consideration before someone considers myths only as literary achievements of human mind. From another point of view, closer to our research, myths could be considered as representation of actual incidents, as images of something that exists or happens, as models of real events that includes not only the events themselves but also the man who creates these models, i.e., the modeller.

In addition, myths included the human efforts to interpret natural phenomena, inexplicable at that time, and the human intentions to comprehend and to describe the whole world, human and physical. For example, something that was of great respect, or something that was beyond human mind, that was unknown and could not be explained with the existing knowledge, was being ascribed to gods and their wills. The Greek gods appeared as symbols of the natural elements (e.g., Zeus-Heaven, Poseidon-Sea), of all virtues, of beauty (Venus), of power (Zeus), of wisdom (Athena), of art (Apollo-Music), of all mechanical arts (Hephaestus), of knowledge, of science, and of human business (e.g., Artemis-hunting, Mars-war). Another indicative example of symbolising something real by means of a symbol is the Homeric description of Achilles' shield, which is a symbol of both the human and the physical world.

These first attempts to represent something abstract or tangible by means of a symbol, to form an analogy between a real thing and a symbol, or even to model the whole world on a shield, form a reference to the conceptual modelling activities. This process that human mind followed at a time, when there were neither physics nor scientific astronomy to explain natural phenomena, nor generally any science to answer all the questions and queries, when the 'system' was absolutely unknown, was twofold: a) the first activity was to understand the problem by using descriptions expressed in natural language and b) the second activity was that of abstraction and symbolism. The latter substantial activity leads to the abstraction, symbolism, and modelling of the real system.

3.3.2 The Homeric Explanation of the World Structure

In his poems, Homer often attempts to interpret the physical structure of world and to present in a figurative way the morphology of earth, as well as the order of the heavenly bodies.

- **The form of heaven**

According to him, the heaven is a solid hemisphere like bowl and the earth is a flat circular disc; over the flat earth is the vault of heaven, like a sort of dome exactly covering it.

Odyssey, 15, 329: "[...] whose wantonness and violence reach the iron heaven" (Murray, 1919)

Iliad, XVII, 425: " [...] and the iron din went up through the unresting [void] air to the brazen heaven..."

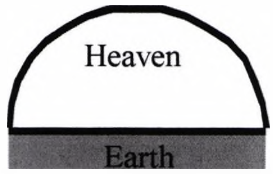

Figure 3.7: Heaven as a hemispherical dome over the earth

- **The symmetry of Heaven and Tartaros**

Below the earth is Tartaros, covered by the earth and forming a sort of vault symmetrical with the heaven; Hades is supposed to be beneath the surface of the earth, as far from the height of the heaven above as from the depth of Tartaros below, i.e., presumably in the hollow of the earth's disc.

> *Iliad*, VIII 13: " [...] I shall take and hurl him into murky Tartaros, far, far away, where is the deepest gulf beneath the earth, the gates whereof are of iron and the threshold of bronze, as far beneath Hades as heaven is above earth ..." (Murray, 1924)

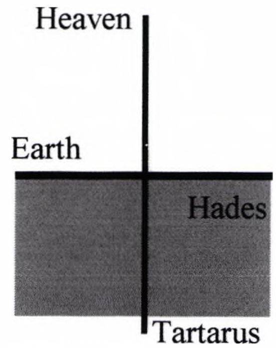

Figure 3.8: The symmetrical order of heaven, earth, Hades, and Tartaros

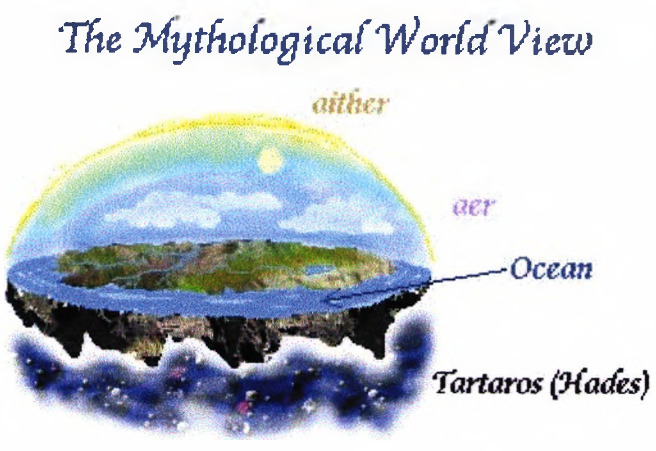

Figure 3.9: The mythological worldview[7]

[7] http://www.perseus.tufts.edu/GreekScience/Students/Ellen/EarlyGkAstronomy.html#RTFToC2

3.3.3 Achilles' Shield as Model of Natural and Human World

In *Iliad*, the poet Homer describes the god of technology, the famous Hephaestus to construct Achilles' weapons. Among the many other significant works of art, such as the automatic tripods and the self-adjusting blowers, Hephaestus also creates the shield of Achilles. This shield is not only a piece of art, an aesthetical, technical, and philosophical creation, but also a model of the whole world. Even though the whole Homeric description of shield is worth to be read by anyone, we choose to quote here only these parts that demonstrate the shield as a model of the world.

Initially, Homer gives the technical details of shield's construction:

Iliad, XVIII, 483-607: "First fashioned Hephaestus a shield, great and sturdy,
adorning it cunningly in every part. And round about it set a bright rim,
threefold and glittering, and therefrom made fast a silver baldric.
Five were the layers of the shield itself. And on it he wrought many
curious devices with cunning skill."

And the description begins with the images appearing on the shield. First of all and in the middle of the shield, Homer places the earth, the sky, the sea, the sun, and the moon, i.e., the perceptible elements of the natural world:

"Therein he (Hephaestus) wrought the earth, therein the heavens therein the sea,
and the unwearied sun, and the moon at the full."

The description ends with the location of the Ocean at the outermost part of earth. The end of shield's description coincides with the end of the natural world, which is the great Ocean:

(607): "Therein he set also the great might of the river Oceanus, around the uttermost
rim of the strongly-wrought shield."

Up to this point the Homeric mythical perception of the physical world results in the next models:

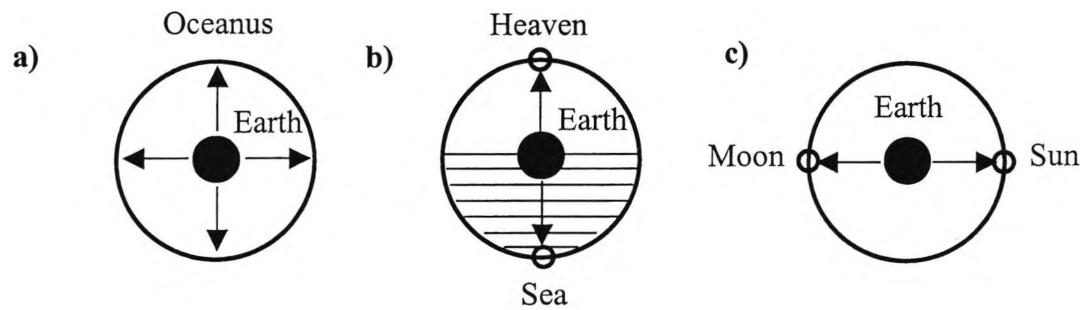

Figure 3.10: Homeric models of the earth and its surroundings
a) Map of the earth, b) Morphology of the earth, c) Planetary system

Following, Homer places on Achilles' shield the fixed stars; Polar Bear in the north, Orion in the south, Pleiades in the east, and Hyads in the west:

"And therein all the constellations wherewith heaven is crowned,
the Pleiades, and the Hyads and the mighty Orion,
and the Bear, that men call also the Wain,
that circleth ever in her place, and watcheth Orion,
and alone hath no part in the baths of Ocean."

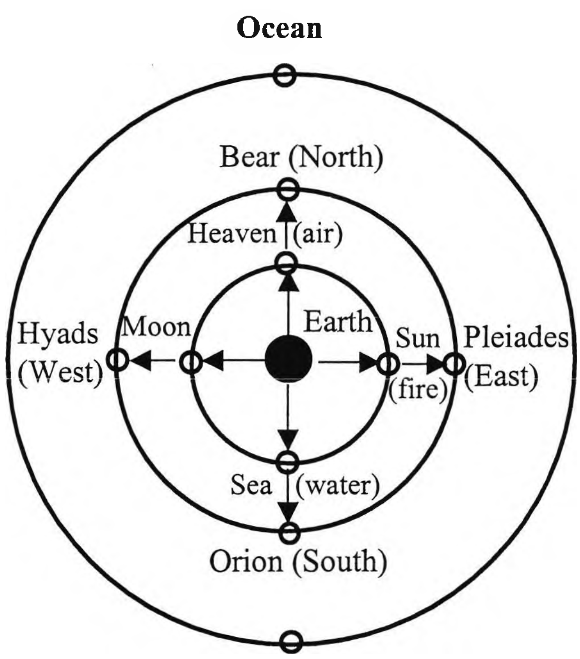

Figure 3.11: Homeric models of the heavenly vault

After the description of the natural world, Homer places on the shield elements related to the human world. Firstly come the social elements, the human cities; one city in peace, where the social functions of marriage, market, and law court appear, and a second city in war that includes the functions of the armies, the siege, and the battle.

"Therein fashioned he also two cities of mortal men exceeding fair.
In the one there were marriages and feastings [...]
the folk were gathered in the place of assembly; for there a strife had arisen...
and each was fain to win the issue on the word of a daysman [...]
But around the other city lay in leaguer two hosts of warriors gleaming in armour [...]
The wall were their dear wives and little children guarding,
as they stood thereon, and therewithal the men that were holden of old age [...]
Then set they their battle in array and fought beside the riverbanks,
and were ever smiting one another with bronze-tipped spears.
And amid them Strife and Tumult joined in the fray, and deadly Fate..."

Afterwards, the description of the human agricultural works, such as the ploughing, the reaping, and the grape-harvest, and the stock-breeding works, such as the pasture of bulls and sheep, follows. At the end, the dance and the rest of the hard work appear:

"Therein he set also soft fallow-land, rich tilth and wide,
that was three times ploughed. Ploughers full many therein
were wheeling their yokes and driving them this way and that [...]
Therein he set also a king's demesne-land,
wherein labourers were reaping, bearing sharp sickles in their hands [...]
Therein he set also a vineyard heavily laden with clusters,
a vineyard fair and wrought of gold. Black were the grapes,
and the vines were set up throughout on silver poles [...]
And therein he wrought a herd of straight-horned kine,
the kine were fashioned of gold and tin,
and with lowing hasted they forth from byre to pasture [...]
Therein also the famed god of the two strong arms wrought a pasture in a fair dell,
a great pasture of white-fleeced sheep, and folds, and roofed huts, and pens [...]
Therein furthermore the famed god of the two strong arms
cunningly wrought a dancing-floor [...]
There were youths dancing and maidens of the price of many cattle,
holding their hands upon the wrists one of the other..." (Murray, 1924)

Even if someone maintains that such a shield has never been constructed and it is only a literary invention, it would be an omission not to see that this poetic description hides a logical structure, which enables the classification of the poetic pictures, and moreover indicates how the man, at that period, perceives the world. This classification of the natural elements leads to a land-planning arrangement of the whole world, as shown in the following figure:

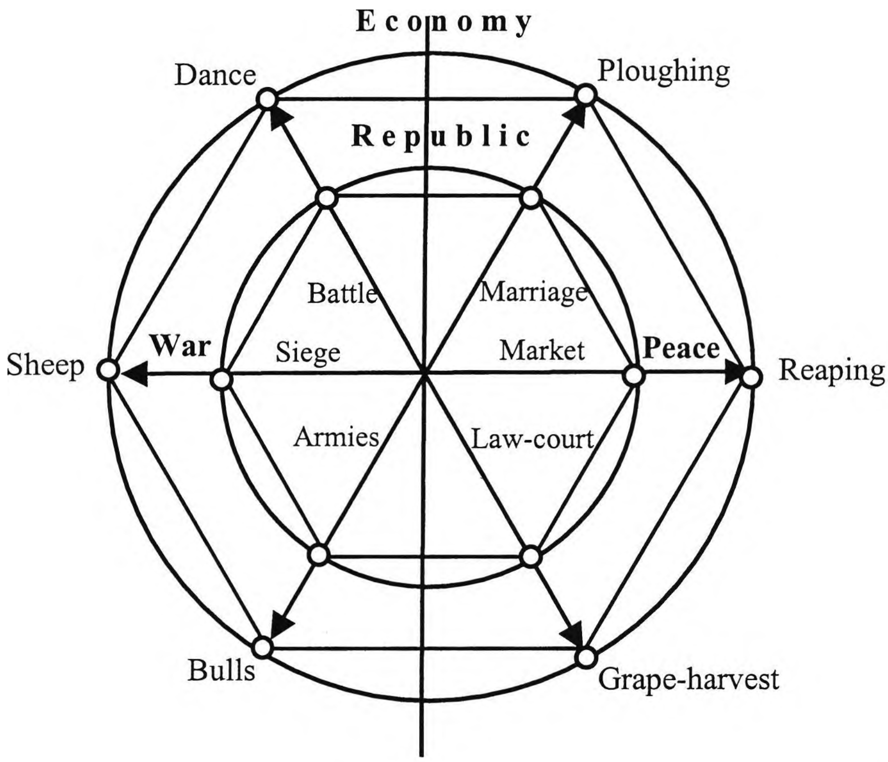

Figure 3.12: Combinational Homeric model of social and economic human life

Therefore, the whole description of the shield's construction can be considered as a model of the natural world together with the human cosmos. Man is on the earth from where he looks out to the sun, the moon, the heaven, and the sea, which form the sensible world. He places therefore the earth in the centre and above it is the heaven, beneath it is the sea, and left and right of it are the moon and the sun, respectively. He is also able to observe the stars that help him to orientate himself. So he places them on an outer homocentric circle. Bear points out the North and is placed above the earth, Orion underneath it and Pleiades and Hyads on the left and right of it. All together constitute the heaven vault. Moreover, he has the sense that somewhere around him is the Oceanus, the end of the world, to whom he gives the outermost circle.

This is the first unity, which has to do with the natural world. To the second unity belong the human cities; on the one hand a peaceful city, whereas on the other, a city under war. The third unity regards the human activities such as farming, cattle, ploughing and harvest. All these unities are enclosed by the last one that is the marginal outermost wreath, the Oceanus. The following figure gives a notion of this account:

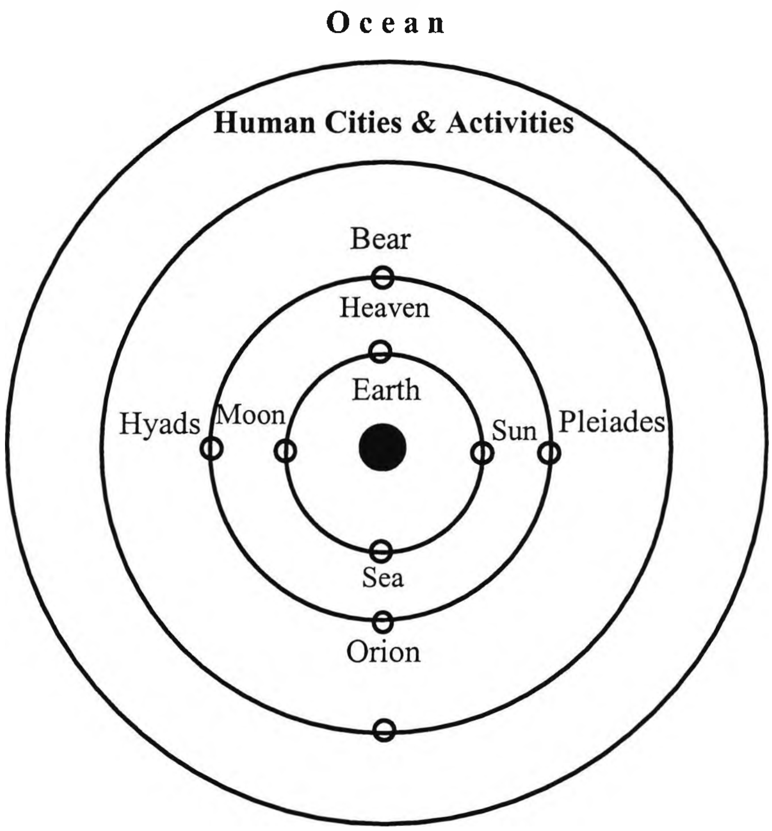

Figure 3.13: Homeric Shield as model of natural and human world

What is this shield and what does it symbolise? It is obvious that it is not just a simple weapon. It is a condensed expression not only of the Homeric poetry, but also of the technological and scientific knowledge of that era, and especially of the sciences of nature and man, such as cosmology and philosophy, astronomy and social sciences, mathematics and dialectic. It is remarkable that the elements appearing on the shield are opposite: heaven–sea, sun–moon, republic–economy, peace–war, nature–man, natural world–human world. The choice of such opposite elements introduces the concept of contradiction. On the whole, the introduction of the contradiction concept is a considerable discovery of a cell, an element, a primary model of a relation, a function, or generally of the substance of things. This concept characterizes the ancient Greek philosophical thought from Homer to Aristotle, and their efforts to describe, interpret, and model dynamical and complex systems such as the world, the human societies and the life itself.

The main features of the above model are: a) Systematisation and classification of the present principles and elements of the world, b) Choice, distinction and differentiation among the substantial elements, which are included in the model, c) Land-planning setting of principles and elements and detection of the sequence among them, d) Determination of contradictions and consequently assessment of the relations between them.

Other noteworthy features of Homeric shield are: a) In the extensive description of his hero's shield, Homer attempts to determine the frame into which all the human acts take place, and especially the very facts of the tragic epic. In a way, the epic of Iliad appears as a circumstance, a detail of the shield's depictions. Thus, it seems as if the whole is enclosed in the particular, the system in the model. b) The description and structure of Achilles' shield constitute an introductory form of the Method of Dichotomy of Plato. The latter feature will be elaborated in chapter 6, after the introduction of platonic method.

3.4 Conclusion

Greek myths come into being only when human thought is mature enough, so as to attempt a physical and scientific account of world's structure and creation. They are written in order to describe things not as they happen but as they could have happened. They interlace the imagination with the historical truth, and the imaginary symbols with real events. On the one hand, they constitute symbols of real things, and on the other, they invent symbols, such as the gods or the Homeric shield in order to explain unknown phenomena or to represent the human activities and the world. In other words, we could say that they 'act' both as a model and as the modeller himself.

The necessity of understanding and interpreting the world urges the early researchers to study the world, in the way that a scientist of today studies an unknown system. Or reversely, the way the ancients handled their 'unknown system' establishes an approaching method of creating a model that can describe the behaviour and the structure of the system, a method that constitutes the starting point of the following development of modelling. The contribution of myths to the concept of modelling concerns in particular the branch of conceptual modelling, since myths constitute early forms of conceptual modelling that are based on analogies.

Even though the development of conceptual modelling has been influenced by the fields of Databases, Artificial Intelligence, Programming Language, and Software Engineering, it involves a deeper, much more philosophical analysis of an entity, rather than the restricted elaboration of a computerised information system. In the case that the conceptual modelling is regarded in terms of a philosophical approach, and not just as a modern methodology developed for the construction of databases, its origin can be found in the very first attempts of conceiving the world. And these attempts, expressed in myths by the early description of the form of heaven, the morphology of the earth, the structure of planetary system, the process of world's creation as a dynamic process, as well as the conception, the form, and the content of Homeric shield, and by Hesiod's creation of a complete, unified, and reasonable picture of the universe, lead to the first steps of conceptualisation that in turn sets the foundations of the notion of conceptual modelling.

Furthermore, in the mythological period, the human mind poses the fundamental issue of determining the primary elements of which the world consists. The Homeric Oceanus, the Hesiodic Chaos, and the Orphic Night, Matter, Chronos, or Water as the primary elements, and the first models of the structure of the world, such as the Orphic egg, give a notion of physical modelling, which however acquires a more precise form by the presocratic philosophers a century later.

Chapter 4

EARLY PHYSICS
PHYSICAL APPROACH TO MODELLING

4. EARLY PHYSICS – PHYSICAL APPROACH TO MODELLING

4.1 Introduction

The ideas considered so far, whatever their occasional stirrings of scientific interest, have been bound up with the whole background of gods and myths, and the shape and development of the world are seen primarily in their terms. In the ensuing years (6^{th} century B.C.), the Greek philosophy makes its appearance. New philosophical modes of thought take place first in Ionia in the works of Thales, Anaximander, Anaximenes, and Heraclitus, and then in the Greek cities in south Italy, in the works of Pythagoras and Empedocles. The object of study of these thinkers is *physis* – nature in its widest sense – and thus they become the *physikoi* – physicists. They are convinced that the world is an ordered system that yields to rational investigation. Therefore, they initiate a serious, critical inquiry into the nature of this system. They ask about its ingredients, composition, structure, and operation. They ask about its shape and location and speculate about its origin. They want to know whether it is made of one element or many and what the essence of these elements is. Some of them have a physical worldview such as Thales, Anaximenes and Empedocles; and others such as Anaxagoras, Leucippus and Democritus a mechanistic one.

Parallel to these endeavours to account for the stuff of the world, in terms of elements abstracted from experience, for example water (Thales), air (Anaximenes), and fire (Heraclitus), they also attempt to explain the processes of the world. New conceptualisations and theories arise either in terms of occurrences abstracted from common experiences, such as 'separating off' (Anaximander), 'Love and Strife' (Empedocles), 'Nous or Mind' (Anaxagoras), and mechanical impact (Democritus), or in terms of mechanical explanations, such as the wheels of Anaximander or the rings of Parmenides, as we will explore in chapter 9.

In this chapter, we will not restrict ourselves to this physical or material worldview the Ionian philosophers have formulated. We will also examine another view of nature, that of Aristotle. Aristotle rejects the emphasis the materialists put on matter and the Platonists on form. He compounds Matter and Form and considers both equally important for the understanding of the basic character of nature. Aristotle considers that the world is real, tangible, and perceptible by the senses. He considers that even more primitive than the four basic elements (water, air, earth, fire) are the so-called sensible qualities (cold, hot, moist, dry), which contribute not only to the creation of the world, but also to the creation of the basic elements themselves. His perspective results in the formulation of the theory of the sensible qualities, as well as the theory of the four types of causation. Later on, in the Hellenistic period, the consideration of nature, the explanation of its structure and processes lead to the introduction of physical laws and the development of the science of Physics. Archimedes' contribution to the field of physics and the theoretical framework, that allows the evolution of physical modelling by means of his laws on lever and hydrostatics, is of great importance.

The historical evolution of human thought, the steps it undertook in order to understand the world, its passage from mythical assumptions that attribute to gods' will any unanswered questions, to the physical elements and qualities as the origin of world, to the conceptualisation of eternal forces as the primary cause of world's change, motion and operation, and to the development of physical laws, may be paralleled, from our perspective, with the pass to physical and conceptual modelling approach. This chapter covers the nature of both physical and conceptual worldview, and aims at specifying the corresponding modelling approaches.

4.2 Physical Models of the World – The Fundamental Elements

Regarding the matter of the independence and rationality of the thought of the presocratic philosophers in comparison to the mythical conception of the world, as well as to the mythopoetic writings of Homer and Hesiod, there are remarkable different opinions. For example, (Cornford, 1957) is convinced of the continuity in thought and of the extensive influence of the earlier mythopoetic traditions on the thought of the Milesian philosophers (Thales, Anaximander, Anaximenes) in particular, and of the Presocratics in general, whereas (de Santillana, 1961) characteristically says of Thales: "It would be very wrong to imagine only medicine men behind him, or sad mythographers like Hesiod [...]. What we discern in their background are not priests and prophets, but legislators, engineers, and explorers." On the other hand, the fact that each Olympian god has his own proper domain, as well as the relations between them, is regarded by (Wilbur *et al.*, 1979) as bearing a resemblance to the view developed by the Presocratic philosophers that there are four elements, which interact between each other under certain principles.

The main objective of presocratic philosophy is to find out the origins of the nature, the fundamental element or elements of the world. Especially, the system of ideas produced by the Milesians, Thales, Anaximander, and Anaximenes about the nature of the universe can be recognised as a new beginning. The Presocratics, in general, give various answers to the central problem of the ultimate element, which contain none of the personification or deification we saw in Homer and Hesiod. They look for something permanent, persisting through the chaos of apparent changes; and they think that it could be found by asking the question: "what is the world made of?"

The answers to such a question are found within the natural elements. The properties and characteristics of the fundamental natural elements in connection with the observation, and sometimes with the experiment and the technical experience, provide explanations about the processes of nature and answers to the question "how does the world work?" Consequently, the presocratic approach to the fundamental issue of understanding, investigation, and explaining the creation and operation of the world by means of the natural elements has an influence on the development of physical modelling processes. On the other hand, the conceptualisation of the basic processes or the notion of force for explaining movement and phenomena gives some evidence of the development of conceptual modelling.

4.2.1 The Water by Thales of Miletus

Thales of Miletus (*ca.* 624-547 B.C.), the first Greek philosopher and the founder of scientific thought, is the one who formulates the principle that all things have a common physical origin. He is interested in almost everything, investigating almost all areas of knowledge, philosophy, history, science, mathematics, engineering, geography, and politics. In the question "what is the world made of?" he answers that the world is made of water. He considers **Water** as the fundamental substance of which everything is made and consists. At first sight, such a theory does not seem satisfactory. May be it carries overtones of the mythical conception that Oceanus comes first in the order of the things and surrounds the whole universe, such as in the world model of Achilles' Shield.

On the other hand, may be Thales distinguishes water from the other basic elements, after simple observation that gives much evidence of water's ubiquity. The obvious changes of water into its three states, solid (snow or ice), liquid (water), and gaseous (steam), the easiness to connect clouds, fogs, dew, rain, hail, with the water of sea and river, as well as the fact that the seed of everything is wet, or in other words the fact that life and water are interlinked, constitute only some of the reasons, which led Thales to the conclusion that if there is an original, ubiquitous, and life-giving substance, water is the best guess.

Our knowledge of Thales' beliefs depends virtually completely on Aristotle:

> Aristotle, *Metaphysics* A3, 983b 6: " [...] there must be some natural substance, either one or more than one, from which the other things come-into-being, while it is preserved. Over the number, however, and the form of this kind of principle they do not all agree; but Thales, the founder of this type of philosophy, says that it is water (and therefore declared that the earth is on water), perhaps taking this supposition from seeing the nature of all things to be moist, and the warm itself coming-to-be from this and living by this (that from which they come-to-be being the principle of all things) - taking the supposition both from this and from the seeds of all things having a moist nature, water being the natural principle of moist things." (Kirk *et al.*, 1983)

Thales formulates a cosmological model, where the universe is full of water and the earth as a flat disk or a wooden board, floats on water. This theory constitutes also the explanation for earthquakes in contrast with Hesiod's statement that earthquakes are caused by the god of Poseidon. Aristotle reports Thales' cosmological sophistication as follows:

> Aristotle, *De caelo* B13, 294 a 28: "Others say that the earth rests on water. For this is the most ancient account we have received, which they say was given by Thales the Milesian, that it stays in place through floating like a log or some other such thing (for none of these rests by nature on air, but on water) - as though the same argument did not apply to the water supporting the earth as to the earth itself." (Kirk *et al.*, 1983)

Thales also introduces a dynamical conception of his cosmological model. According to him, all the physical creatures come into being by the constant transformation of water, which is perceived not as dead matter but as an active element that bears energy. This energy, which is inherent in the concept of motion, is called 'soul'. The 'soul' is a perpetually moving and self-moving nature. Aristotle quotes

Thales' opinion that the magnet has a soul in it because it moves the iron. Thus, the universe is full of 'souls', of energy sources that cause the universe to be moved.

> Aristotle, *De Anima (On the Soul)*, 405a 20-22: "Thales, too, to judge from what is recorded about him, seems to have held soul to be a motive force, since he said that the magnet has a soul in it because it moves the iron[8]." (McKeon, 1941)

If this tradition is right, which means that Thales knows the properties of the lodestone, he might be considered as the founder of the magnetism.

Thales' 'water', as the fundamental element, constitutes not only the constituent element of world's structure, but also the element of its evolution and function.

4.2.2 The Infinity by Anaximander

Other Milesians of the sixth century seem to have given different answers to the question of the underlying reality of the universe. Anaximander (611-547 B.C.), a pupil of Thales, is ascribed with holding the theory that the first principle (i.e., material cause) and element of existing things is the **Apeiron** (infinity) and he is the first to introduce this name for the first principle. He considers that this world is only one of a number of possible worlds that have been separated from the apeiron.

> Hippolytus, *Ref.* I, 6, 1-2, (DK 12 A11): "Anaximander, son of Praxiades, of Miletus: [...] he said that the principle and element of existing things was the **apeiron**, being the first to use this name of the material principle. (In addition to this he said that motion was eternal, in which it results that the heavens come into being.) [...] He said that the material principle of existing things was some nature coming under the heading of the apeiron, from which come into being the heavens and the world in them." (Kirk *et* al., 1983)

Anaximander, in order to account for the endless transformation of phenomena in the world and also for the first principle of all things, turns to an inexhaustible and endless entity, usually translated as infinite, or indefinite, or boundless, the so-called apeiron, from which emerges a seed that gives rise to the cosmos:

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[8] De Anima, Translated by J. A. Smith (http://ccat.sas.upenn.edu/jod/texts/aristotle.soul.html)

Simplicius in *Physics* 24, 21: "It is clear that he [Anaximander], seeing the changing of the four elements into each other, thought it right to make none of these the substratum, but something else beside these..." (Kirk *et* al., 1983)

Anaximander's apeiron is the matter from which the worlds arise and to which they return – analogous to the water from which all things originated. But water is visibly present in the world. Anaximander takes the leap to regard a concept as the ultimate element rather than an item of experience, something that is not only invisible, but also difficult to be defined. He regards that such a principle, which is first, fundamental and the source of all things, cannot be derived directly from experience but from an idea, from a concept of what such a principle has to be like. By such a viewpoint, we could say that Anaximander introduces a material interpretation of infinity.

However, for the explanation of natural phenomena and meteorological events, such as the wind, the evaporation from the sea, or the condensed masses of vapour that form the clouds, Anaximander resorts to the experience, or to visible and tangible physical elements:

a) Hippolytus *Ref.* I, 6, 7: "Winds occur when the finest vapours of the air are separated off and when they are set in motion by congregation; rain occurs from the exhalation that issues upwards from the things beneath the sun, and lightning whenever wind breaks out and cleaves the clouds."

b) Aetius III, 3, 1-2: "(On thunder, lightning, thunderbolts, whirlwinds and typhoons.) Anaximander says that all these things occur as a result of wind: for whenever it is shut up in a thick cloud and then bursts out forcibly, through its fineness and lightness, then the bursting makes the noise, while the rift against the blackness of the cloud makes the flash."

d) Aristotle, *Meteol.* B 1, 353 b 6: "For first of all the whole area round the earth is moist, but being dried by the sun the part that is exhaled makes winds and turnings of the sun and moon, they say, while that which is left is sea; therefore they think that the sea is actually becoming less through being dried up, and that some time it will end up by all being dry [...]" (Kirk *et al.*, 1983)

The step of crucial importance in Anaximander's cosmogony is the introduction of the concept of opposed substances. The interaction of opposites is the basic idea in Heraclitus' philosophy and recurs also in Empedocles' and Anaxagoras' speculation.

Anaximander ends up in the theory of opposites, by observing the main seasonal changes, in which heat and drought in summer alternate with cold and rain in winter. Therefore, the main opposites in his cosmogony are the hot and the cold substance, i.e., flame or fire and mist or air. Hot and cold, and the associated dryness and moisture, form an additional pair of cosmological opposites, most notably involved in the large-scale changes in the natural world:

Simplicius in *Physics* 24, 21: " [...] and he produces coming-to-be not through the alteration of the element, but by the separation off of the opposites through the eternal motion." (Kirk *et* al., 1983)

In the previously quoted passage, Anaximander introduces the notion of eternal motion, which is responsible for the creation of heavens. Even though no explanation is given regarding the nature or the origin of this eternal motion, it is evident that it is the required cause for the ceaseless transformations of the world. In addition, it indicates that even at that early period of science, the first researchers were awake to the time dependency of phenomena and to the essentiality of motion concept in expressing state evolution and behaviour.

In relation to the Hesiodic concept of chaos, the 'infinity' of Anaximander has a physical as well as a mathematical meaning. It means the unlimited, the endless space, i.e., the infinite structure of the world system, and on the other, the infinite, perpetual, uninterrupted time, i.e., the continually progressive function of the world.

4.2.3 The Air by Anaximenes

The last of the Milesians, Anaximenes (*ca.* 585-528 B.C.), asserts **Air** as the fundamental element in the world. His theory is revealed by:

a) Diogenes Laërtius, II, 3: "Anaximenes son of Eurystratus, of Miletus, was a pupil of Anaximander; some say he was also a pupil of Parmenides. He said that the material principle was air and the infinite..."

b) Theophrastus *ap. Simplicium in Phys.* 24, 26: "Anaximenes son of Eurystratus, of Miletus, a companion of Anaximander, also says, like him, that the underlying nature is one and infinite, but not undefined as Anaximander said but definite, for he identifies it as air; and it differs in its substantial nature by rarity and density. Being made finer it becomes wind, then cloud, then (when thickening still more) water, then earth, then stones; and the rest come into being from these" and

c) Hippolytus *Ref.* I, 7, 1: "Anaximenes [...] said that infinite air was the principle, from which the things that are becoming, and that are, and that shall be, and gods and things divine, all come into being, and the rest from its products." (Kirk *et* al., 1983)

In the first fragment, it is stated that the material principle is air and the infinite, while in the ones by Theophrastus and Hippolytus, it is said that the underlying nature, which is the air, is the only one and it is infinite. Most likely, Theophrastus and Hippolytus describe air as infinite, because as Anaximenes says, it surrounds all things and alters its appearance according to how much there is of it in a particular place. It gets harder and heavier accordingly as more of it is packed into a given space. Rarefaction and condensation are his key words. Rarefied Air is Fire. Condensed Air becomes first Water and then Earth. He also considers rarefaction to be accompanied by heat and condensation by cold:

Plutarch *de prim. frig.* 7, 947 f (DK 13 B 1): " [...] or as Anaximenes thought of old, let us leave neither the cold nor the hot as belonging to substance, but as common dispositions of matter that supervene on changes; for he says that matter which is compressed and condensed is cold, while that which is fine and 'relaxed' (using this very word) is hot. Therefore, he said, the dictum is not an unreasonable one, that man releases both warmth and cold from his mouth: for the breath is chilled by being compressed and condensed with the lips, but when the mouth is loosened the breath escapes and becomes warm through its rarity..." (Kirk *et al.*, 1983)

Anaximenes' system is characterized by progressive clarification, refinement, and extension of the concepts that appear in previous theories. His explanation is clear and fit to the cyclical transformation of nature, which reappears in Timaeus by Plato. According to (Schlagel, 1995), this type of explanation allows for functional correlations among the variables, motion, density, and temperature, with the possibility of quantifying these correlations. For example, an increase in motion causes a decrease in density and an increase in temperature, while a decrease in motion causes an increase in density and a decrease in temperature.

Anaximenes' interest in the quantitative values of air, or in other words in how the qualitative changes could be reduced to quantitative relations, gives a notion of qualitative reasoning (see also section 9.2), where the most important information of a quantity is whether it is increasing, decreasing, or remaining the same. The decrease of air (rarefaction) causes the production of fire and the increase of it (condensation) causes the generation of water and earth. He supports his theory by using empirical examples as the one of breath that illustrates how variations of density can result in differences in temperature.

In brief, one could say that the 'infinite air' of Anaximenes gives a physical interpretation to the infinity by Anaximander, i.e., it associates the concept of the physical element of air with the mathematical concept of infinity. In addition, Anaximenes' further analysis of the evolution of the world and of the creation of the natural elements by means of rarefaction and condensation of air, gives a physical interpretation to the steps of this evolution.

4.2.4 The Fire by Heraclitus

The next philosopher in succession, who tried to give an account for the world's creation and function, is Heraclitus (540-480 B.C.) of Ephesus. Heraclitus' work *Περί Φύσεως* (On Nature), in three books, does not survive as a continuous whole. What we have instead is a collection of more than 100 independent fragments, most of which are citations by authors from the period of 100-300 A.D. Notwithstanding their sporadic presentation and transmission, they comprise a philosophy that is clearly focused upon a determinate set of ideas.

Down to his time, the philosophers had viewed the world as a huge edifice (Popper, 1962) of which the material things were the building material. They considered philosophy or physics as the investigation of nature; as the investigation of the original element or material out of which this edifice, the world, has been built. Heraclitus introduces an absolutely innovative view, a dynamical conception of the world. He considers the world not as an edifice, not as a stable structure, but as a colossal process, as a totality of all events, changes, and facts. Process and change are the basic principles of things. His doctrine has been summed up in the phrase: "Everything flows." He does not believe in an evolution of the world out of a primary simple state. It "is, was, and ever will be" what it is now. He tries to answer the questions: What continuously changes while ever remaining the same (Schlagel, 1995)? How could one embody the abstract concept of continuous change in a more concrete image? In answering these questions only one image is appropriate, that of **Fire**. Therefore, he introduces Fire as the prime element, because it is so active and transforms everything. Fire is for him not the element out of which the world has been created, i.e., as water was for Thales, but the image, the symbol for the changing world. His viewpoint is revealing by:

Fr. 90, Plutarch *de E.* 8, 388 D: "All things are an equal exchange for fire and fire for all things, as goods are for gold and gold for goods" and

Fr. 30, *Clement Strom.* V, 104, 1: "This world-order [the same of all] did none of gods or men make, but it always was and is and shall be: an everliving fire, kindling in measures and going out in measures." (Kirk *et al.*, 1983)

Heraclitus compares the changing world to a process that never rests, the process of combustion. The world is an ever-living fire, where the 'measures' of fuel kindled and the 'measures' of fire extinguished in smoke or vapour remain constant. It is impossible for fire to consume its nourishment without at the same time giving back what it has consumed already. It is a process of eternal 'exchange' like that of gold for goods and goods for gold. This unceasing process of flux expresses dynamic evolution and related to energy.

According to Heraclitus, the process of combustion with the three states of fuel, fire, and vapour or smoke, corresponds to the continually change of fire into water and then into earth, and the change back of earth to water and water again to fire. The world, therefore, arises from fire, and in alternating periods is resolved again into fire, to form itself anew out of this element. The division of unified things into a multiplicity of opposing phenomena, i.e., the path to earth, is the 'Downward path', and is the consequence of a war or strife, whereas the path that leads back to unity, i.e., the path to fire, is the 'Upward Path', and it is the consequence of Harmony and peace. Nature is constantly dividing and uniting herself, so that the multiplicity of opposites does not destroy the unity of the whole.

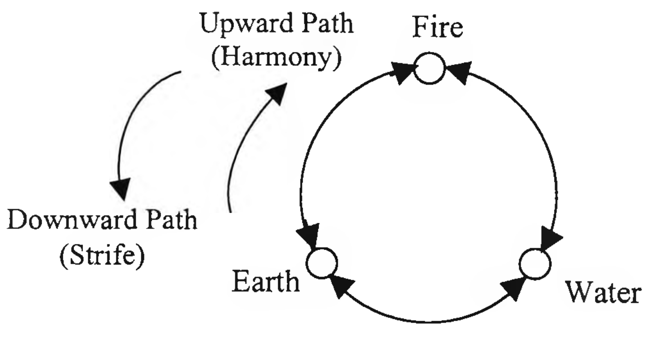

Figure 4.1: The eternal transformation of the natural elements by Heraclitus.

Strife and harmony appear as opposite forces into things, where the former moves things to the Downward Path and the latter to the Upward Path. The existence of matter in any particular state is the result of a balance between these opposing forces. One force is gaining on the other, gradually and alternatively. In this way, Heraclitus introduces the so-called *Opposite Tension doctrine*. At the age he lives in, this doctrine is so novel that it is not easy for him to express it. This difficulty makes the use of a series of symbols and comparisons unavoidable. Some of them are quoted below:

a) Fr. 61, Hippolytus *Ref.* IX, 10, 5: "Sea is the most pure and the most polluted water; for fishes it is drinkable and salutary, but for men it is undrinkable and deleterious."

b) Fr. 60, Hippolytus *Ref.* IX, 10, 4: "The path up and down is one and the same."

c) Fr. III, Stobaeus *Anth.* III, 1, 177: "Disease makes health pleasant and good, hunger satiety, weariness rest." and

d) Fr. 88, Ps.-Plutarch *Cons. ad Apoll.* 10, 106 E: "And as the same thing there exists in us living and dead and the waking and the sleeping and young and old; for these things having changed round are those, and those having changed round are these." (Kirk *et al.*, 1983)

Thus, Heraclitus attempts to answer not only the question of what the world is made of, but also the vital question of what the process of world's evolution is made of, or how and why this process takes place. The concept of contradiction emerges as the functional element that explains the 'how' and 'why' questions of world's process and function.

All things consist of opposites under an internal tension. Even if a thing seems phenomenally composed and harmonic, and no motion is visible, in fact this harmony is dynamical. It is a harmony of many lively and opposite movements in the inner part of the body, which counteract between each other and therefore they are imperceptible. Harmony, not in the meaning that it has in music by Pythagoras, but as a matching and construction of a complicated whole, is the product of the unity between opposites. This idea of unity of opposites expresses notions of equilibrium. This concept of dynamical harmony is closely related to the concept of feedback and the function of a closed loop control system with constant input (regulation), as we will see in chapter 11.

On the other side, war or strife is the above all creative, general, and determinative force that provides the essential conflict between the opposites, so as the unity and the harmony of the changing process to be achieved. In other words, the harmony and the unity have as presupposition the 'of poles apart' concept, that of the conflict. This viewpoint is an additionally evidence to the existence of opposites. Heraclitus' explanation of nature in terms of opposites and exchanges of energy, points out the modern qualitative reasoning technique. Correspondingly, what is of great importance in Heraclitean approach to the physical system of cosmos are the qualitative changes occurred herein.

With *Opposite Tension doctrine* Heraclitus is established as the founder of dialectic method, which is taken up later on in the work of Plato. His expression that 'everything arises from one and one from everything', as well as the aforesaid passages bear the seeds of the Dichotomy Method of Plato, which is the subject of chapter 6. In these statements the basic law of Dichotomy Method that 'one is divided into two' is absolutely unequivocal.

4.2.5 The Four Elements of Empedocles

The interpretation of the world in the sense of finding out the fundamental physical elements proceeds in the person of Empedocles of Acragas (490-430 B.C.). He is distinguished not only as a philosopher, but also for his knowledge of natural history and medicine, and as a poet and statesman. He undertakes a synthetic, combinational conception of the world. The first step towards this direction is the rejection of monism of his predecessors. In the large existing fragment of his didactic poem *De natura* (Pedersen, 1993), he sets four qualitatively different primary elements as the origin of the world, the so-called four roots, of which all the substances known by experience are conceived to be composed.

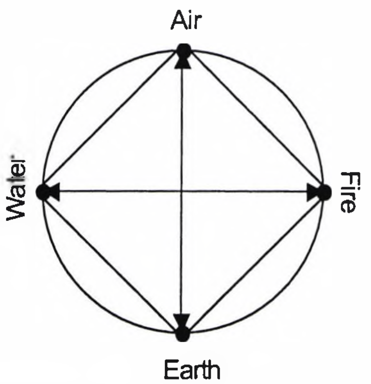

Figure 4.2: The four roots of Empedocles.

According to (Dijksterhuis, 1, 1964), they bear the names of four substances that reveal the characteristic differences between bodies: the solid Earth, the liquid Water, the gaseous Air, and the glowing, consuming Fire. Thus, the four roots of Empedocles hold the three physical states of solid, liquid, and gaseous, and the element of fire as energy. They also hold the basic opposites, Water-Fire, Air-Earth, i.e., two pairs of opposite that constitute the tetrad.

Aristotle *Metaphysics* A4, 985a 31-33 (DK 31 A 37): "Moreover, he was the first to make the material 'elements' four." (Kirk *et al.*, 1983)

In his *Tetrasomia*, or *Doctrine of the Four Elements*, Empedocles described these elements not only as physical manifestations or material substances, but also as spiritual essences. He associates these elements with four gods and goddesses - air with Zeus, earth with Hera, fire with Hades, and water with Nestis (believed to be Persephone). Nestis is the source of moisture and water, but over the other three there is disagreement. Theophrastus, for example, seems to have identified the 'shining Zeus' as fire, Hera as air, and Aidoneus (i.e., Hades) as earth:

Fr. 6, Aetius I, 3, 20: "Hear first the four roots of all things: shining Zeus, life-bringing Hera, Aidoneus and Nestis who with her tears waters mortal springs." (Kirk *et al.*, 1983)

This characterization of four elements as gods is presumably designed, on the one hand so as to indicate what is sound in traditional conceptions of divinity and on the other, to claim for them powers and properties, as yet undefined, which make them worthy of awe. On the contrary, we could say that Empedocles gives a corporeal explanation of the gods, ascribing to them physical substance, i.e., he identifies them with the primary material elements.

However, these four distinct ingredients alone cannot explain motion and change. Therefore, Empedocles introduces two additional, immaterial principles: **Love** and **Strife**, which induce the four roots to congregate and separate. The two forces, as an equivalent to the Tension of Heraclitus or to the 'soul' of Thales, set the elements in motion. Love tends to draw the four elements into a mixture and Strife to separate them again. Given the four roots of things plus the two motive forces, there is no beginning or ending in time, but an eternal cyclical process of combing and separating according to whether the effects of love or strife are predominant. When love reigns, all the elements tend to be blended in a silent, unmoving mixture since there is no strife. During the reign of love, strife begins to reassert its influence, the motion and the process of separation recommence. Between the two phases, when either love or strife reigns, there are transitional stages consisting of various combinations of the elements, and it is during these periods that the formation of the physical universe takes place, as well as the generation of organic and plant life. The generation of plants and animals follows also the double process of the coming together of things under the influence of love, and of their separation under the influence of strife. At this stage, Empedocles describes the shape of the universe as a sphere. Additionally, he is the one who returns to the orphic egg-model of the world, by regarding a cosmological model, where the heavens form an egg-shaped surface made of crystal, the fixed stars being attached to it, but the planets are free (Sarton, 1, 1993). (Sambursky, 1956) relates the Empedocles' controlling cosmological conceptions of Love and Strife to the modern scientific conceptions of the attraction and repulsion of forces.

Along with the doctrine of the four roots as a physical model of the world structure, and the relations among the roots expressed by the concepts of Love and Strife, as the simplest form of parts of the operation of the world, bearing the notion of conceptual modelling, Empedocles contributes to the development of modelling by introducing the concept of analogy and the process of reasoning by analogy. He investigates the invisible air we breathe and tries to explain the process of respiration. He notes that all living organisms inhale and exhale through tubes, the so-called today capillary tubes, which may be different from arteries and veins. Inhalation occurs when the blood recedes from the surface drawing the air in, whereas exhalation occurs when the blood returns forcing the air out. In order to illustrate this mechanism of the respiratory process that cannot be directly observed, Empedocles uses reasoning by analogy, comparing this process to the process of filling a porous vessel with water. He uses, therefore a water clock, which is called *clepsydra*[9] and shows that, if the open end of the clepsydra is thrust under water while a finger is held over the hole in the tip of the cone, the contained air prevents the water from entering the clepsydra (Schlagel, 1995).

[9] Clepsydra consists of a hollow cylinder, open at one end and terminating at the other in a cone with a small aperture at the tip. It is used to measure time by filling it with water and letting the water escape through the small hole at the tip of the cone. Like the sand in an hour-glass, the water runs through in a measured interval of time. Clepsydra will be examined in details in chapter 12.

Conversely, the full clock, though it turns upside-down, cannot empty itself as long as a finger is kept over the hole. The pressure of the air keeps the water in.

> Aristotle, *De Respiratione*, 473b9: " [...] Then, when the fluid blood rushes away thence, the bubbling air rushes in with violent surge; and when the blood leaps up, the air is breathed out again, just as when a girl plays with a clepsydra [a vessel perforated at the bottom] of gleaming brass. When she puts the mouth of the pipe against her shapely hand and dips it into the fluid mass of shining water, no liquid enters the vessel, but the bulk of the air within, pressing upon the frequent perforations, holds it back until she uncovers the dense stream; but then, as the air yields, an equal bulk of water enters..." (Schlagel, 1995)

Empedocles with his experiment proves the corporeality of air. Prior to Empedocles, air has not been distinguished from empty space; nevertheless, he manages to prove that the viewless air is something that occupies space and exerts power. Out of this fundamental discovery of the corporeality of air arises a widespread interest in the principles and applications of pneumatics, which results in complicated closed loop control systems of the Alexandrian period, as we will examine later on.

Empedocles' reference to the clepsydra to depict the process of respiration, or in other words the use of such an analogy does not suggest the setting up of an experimental situation that confirms or disconfirms his theory. Whereas the aim of an experiment is to test a theory, the purpose of analogical reasoning is to use a familiar process to illustrate the mechanism of an unknown process based on similarities between the two (Schlagel, 1995). The procedure of analogical reasoning, i.e., the use of a model that is analogous with the under examination unknown system, is the main principle on which the concept of modelling is based.

4.2.6 The Opposite Qualities by Anaxagoras

Some similarities with Empedocles' approximation are noticed in Anaxagoras' cosmology. Anaxagoras of Clazomenae (*ca.* 500-428 B.C.) is considered as the first teacher of natural philosophy, as the forerunner of Plato and Aristotle. According to him, there is neither coming into being nor ceasing to be, but there are only compositions and decompositions. He is, like Empedocles and the atomists after him, a dualist; and his dualism is, in a sense, a dualism of Mind or *Nous* and Matter. This dualism appears as an attempt to combine matter and energy, so that to describe the process of evolution and genesis. Anaxagoras is the first to suggest Mind as the primary cause of physical changes. The universe is originally 'a chaos' of innumerable seeds to which Mind gives order and form. He holds that everything is infinitely divisible and that even the smallest portion of matter contains some of each element. Mind is a substance that enters into the composition of living things, and distinguishes them from dead matter. According to him, the world begins with a vortex set up in a portion of the mixed mass in which "all things are together", by his *Nous* (Mind):

> Fr. 13, Simplicius in *Physics* 300, 31: "And when Mind initiated motion, from all that was moved Mind was separated, and as much as Mind moved was all divided off; and as things moved and were divided off, the rotation greatly increased the process of dividing." (Kirk *et al.*, 1983)

Mind is the source of all motion. We have again the concept of continuous change and motion, as in Heraclitus thinking, which related to energy. Most likely, Mind, in Anaxagoras' philosophy, has the meaning of the necessary energy in any change, transformation of one thing to another and motion. It causes a rotation, which is gradually spreading throughout the world, and forces the lightest things to go to the circumference, and the heaviest to fall towards the centre. This rotation is also directly responsible for the separation, which in turn, leads to cosmogony. Mind, having initiated the rotation, remains alone ultimately responsible; but at the same time, as it is evident from the statement at the end of the above passage, once the original motion has been imparted, purely **mechanical factors** begin to operate and the agency of Mind itself becomes less direct (Kirk *et al.*, 1983). (Schlagel, 1995) considers the physical force of Mind as the initiator and the controller of the mechanical rotation causing the separating off process.

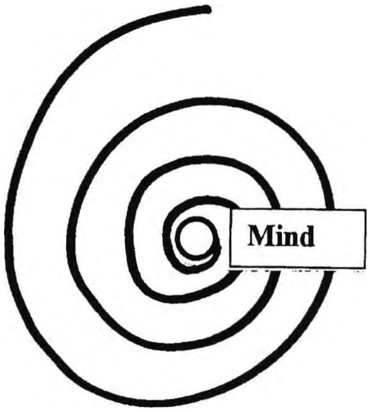

Figure 4.3: The widening spiral rotation of the world by Anaxagoras.

Having analysed the role of Mind in the creation of the world, Anaxagoras goes on to introduce the process of separation and the formation of opposite qualities:

Fr. 12, Simplicius in *Physics* 164, 24 and 156, 13: " [...] Mind controlled also the whole rotation, so that it began to rotate in the beginning. And it began to rotate first from a small area, but it now rotates over a wider and will rotate over a wider area still. And all things that were to be – those that were and those that are now and those that shall be – Mind arranged them all, including this rotation in which are now rotating the stars, the sun and moon, the air and the aither that are being separated off. And this rotation caused the separating off. And the dense is separated off from the rare, the hot from the cold, the bright from the dark and the dry from the moist..." (Kirk *et al.*, 1983)

The idea of opposite qualities, which are inherent in a unity but are separated off because of Mind's rotation, constitutes the basis of Anaxagoras' cosmology. The initial unity, the so-called Nous, rotates as a machine. During this rotation the dense is separated off from the rare, the hot from the cold, the bright from the dark and the dry from the moist. Then the cold, the dense, the dark, and the moist come together and solidify in the form of Earth, whereas the hot, the rare, the bright, and the dry become Aither, Heaven, which encircle the Earth.

Fr. 2, Simplicius in *Physics* 155, 31: "For air and aither are being separated off from the surrounding mass, which is infinite in number."

Fr. 15, *ibid.* 179, 3: "The dense and the moist and the cold and the dark came together here, where the earth now is, while the rare and the hot and the dry and the bright went outwards to the further part of the aither."

Fr. 16, *ibid.* 179, 8 and 155, 21: "From these things, as they are separated off, the earth is solidified; for water is separated off from the clouds, earth from water, and from earth stones are solidified by the cold; and stones tend to move outwards more than water." (Kirk *et al.*, 1983)

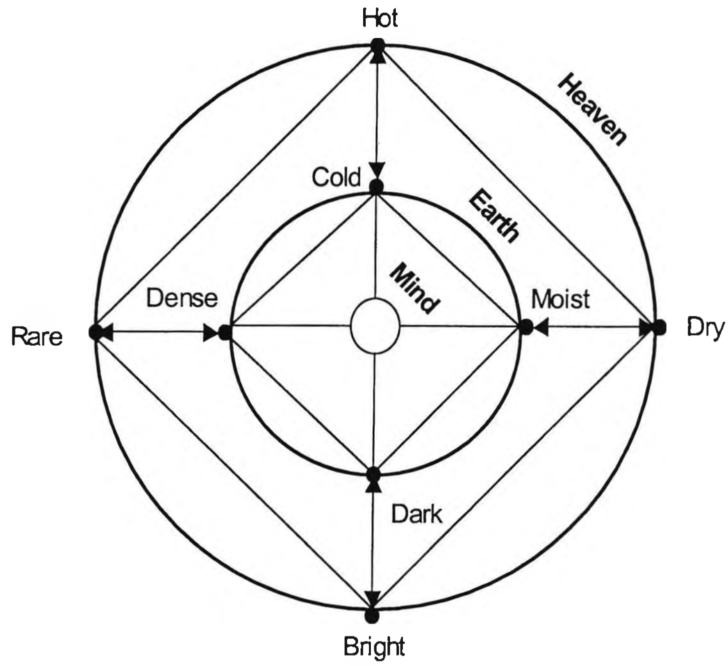

Figure 4.4: Anaxagoras' cosmology

This concept of Nous that puts things in motion, in modern cosmogony is equivalent of the gravitational field that has initiated motion and rotation, which are then taken over by the physical laws. Physics, in terms of motion, enters in Anaxagoras' thinking in a more specific way. By integrating the previous work and by realizing the need for movers to set things in motion, Anaxagoras introduces Mind or Nous as the prime mover. In addition, Mind bears the concept of controller. A complicated function of a complex system (e.g., the world), which is characterized by rotation and contradiction and secures the continual dynamical balance of opposite qualities (hot-cold, dark-bright, and so on), could not be different from having in its kernel a powerful controller. Besides, the main difference between the living, animate beings and the lifeless things is that the former possess exactly this Mind-controller.

In addition, Anaxagoras occupies himself with the process of the formation of sea, rivers, sun, moon, and stars, and gives an approximately true explanation of the Nile floods. He describes the sun, the moon, and the stars as red-hot stones; the moon – whose phases he explains nearly correctly – is similar to the earth; the heavenly bodies, he holds, are fragments of the original mass hurled out centrifugally by the rotation of the cosmos.

Hippolytus *Ref.* I, 8, 3-10 (DK 59 A 42): " [...] The sun, the moon and all the stars are red-hot stones, which the rotation of the aither carries round with it. Beneath the stars are certain bodies, invisible to us, which are carried round with the sun and moon. We do not feel the heat of the stars because they are so far from the earth; moreover, they are not as hot as the sun because they occupy a colder region. The moon is beneath the sun and nearer to us. The sun exceeds the Peloponnese in size. The moon has not any light of its own but derives it from the sun. The stars in their revolution pass beneath the earth. Eclipses of the moon are due to its being screened by the earth, or, sometimes, by the bodies beneath the moon; those of the sun to screening by the moon when it is new [...]" (Kirk *et al.*, 1983)

It is noteworthy to mention here Anaxagoras' epoch-making discovery that the moon does not shine by its own light but it receives its light from the sun. His assertion that the sun and the moon are material bodies, rather than deities, was the reason that he was prosecuted and exiled from Athens. This situation recalls the unfortunate prosecution of Galileo many years later.

4.2.7 The Microcosm by Leucippus and Democritus

Leucippus of Miletus (480-400 B.C.) and Democritus of Abdera (460-370 B.C.), the so-called atomists, add another perspective to the up to this point physical explanation of the world. These two of the most brilliant physical theorists of all times remain relatively unknown. Concerning Leucippus very little is known of his writings, while the little that has survived of the fifty-two separate works of Democritus consists mainly of ethical aphorisms. However, it is generally agreed that the atomic theory has originated from Leucippus, whereas Democritus has developed its implications and worked it out in greater detail.

The main concept of atomism is that the world consists of an infinite number of atoms and of kinds of atoms moving randomly in an infinite void. The indivisible atoms are homogeneous as regards quality, but heterogeneous as regards shape and arrangement. Their motions and collisions, as well as the possibility of infinite variations in their spatial positions and configurations, account for the great diversity of substances and the complex phenomena we experience. The explanation that Leucippus and Democritus give about the formation of the world, which consists of many other smaller worlds, out of vortices or whirlpools of atoms is:

> Diogenes Laërtius IX, 31 (DK 67 A I): "Leucippus holds that the whole is infinite [...] part of it is full and part void [...] Hence arise innumerable worlds, and are resolved again into these elements. The worlds come into being as follows: many bodies of all sorts of shapes move 'by abscission from the infinite' into a great void; they come together there and produce a single whirl, in which, colliding with one another and revolving in all manner of ways, they begin to separate apart, like to like. But when their multitude prevents them from rotating any longer in equilibrium, those that are fine go out towards the surrounding void as if sifted, while the rest 'abide together' and, becoming entangled, unite their motions and make a first spherical structure. This structure stands apart like a 'membrane', which contains in itself all kinds of bodies; and as they whirl around owing to the resistance of the middle, the surrounding membrane becomes thin, while contiguous atoms keep flowing together owing to contact with the whirl. So the earth came into being, the atoms that had been borne to the middle abiding together there. Again, the containing membrane is itself increased, owing to the attraction of bodies outside; as it moves around in the whirl it takes in anything it touches. Some of these bodies that get entangled form a structure that is at first moist and muddy, but as they revolve with the whirl of the whole they dry out and then ignite to form the substance of the heavenly bodies." (Kirk *et al.*, 1983)

This passage (formally attributed to Leucippus, but no doubt representing the general views of Democritus also) gives the account of the formation of worlds in two stages. The first stage is when a large collection of atoms becomes isolated in a large patch of void, and the second stage is when they form a whirl or vortex. The vortex-action causes similar atoms to tend towards similar. The bigger atoms are concentrated in the centre, whereas the smaller are moving outwards. Atomists are the first to formulate such a peculiar idea of infinite number of worlds. However, it is not necessary for these worlds to be alike. Their creation is such a randomly procedure that it is possible one world to have sun, or moon, or water, whereas another one not. For example, the lack of atoms that move outwards could cause the creation of a world without heavenly bodies.

Summarising, Leucippus and Democritus assume that all matter is made up of atoms, which are tiny imperishable units that have fixed properties, such as their lack of qualitative differentiation, i.e., they do not carry differences such as taste or colour, but only differences of a spatial character, such as size and shape, and their indivisibility, and though they remain unchanging they can move about in space or void. In addition, the fact that they can be combined together in various ways results in the creation of macrocosm. The atomists are the first to consider similarities between macrocosm and microcosm, and maybe, we dare to say, they are the first to seek for a common theory for the rationalisation of these two worlds. Besides, they attribute all change in the world to the rearrangement of atoms in void. By such an explanation, they escape somehow from the dilemma about the nature of change and the 'contrary' between Heraclitus and Parmenides, i.e., in the doctrine of Leucippus and Democritus, as against the state flux of Heraclitus, the relative stability of being is postulated, and as against the permanence of Parmenides (for more details see chapter 9) the reality of motion.

If we consider the 'why' questions of an event, we may mean either two things. We may mean: "What purpose did this event serve?" or "what earlier circumstances caused this event?" In the first case we obtain a teleological explanation or in other words, an explanation by final cause, whereas in the second case we are looking for a mechanistic explanation. The mechanistic question is the one that leads to scientific knowledge.

The atomists ask the mechanistic question, and give a mechanistic answer. According to their answer, the world and its various parts derive from the mechanical sorting of atoms in the primeval vortex. They consider the reality as a lifeless piece of machinery, in which everything that occurs is the necessary outcome of inert, material atoms moving according to their nature. No mind and no divinity intrude into this world. Life itself is reduced to the motions of atoms (Lindberg, 1992). In comparison to Empedocles and Anaxagoras, who believe that forces are necessary to bring about motion of the primary substance – the former Love and Strife, the later Mind – the atomists, like their early predecessors, transfer motion to the primary substance itself.

Democritus says that there is an infinite number of worlds since the motion has no beginning and the mass of the atoms and empty space have no limits:

> Hippolytus *Ref.* I, 13, 2 (DK 68 A 40): " [...] he (Democritus) spoke as if the things that are were in constant motion in the void; and there are innumerable worlds, which differ in size. In some worlds there is no sun and moon, in others they are larger than in our world, and in others more numerous. The intervals between the worlds are unequal; in some parts there are more worlds, in others fewer; some are increasing, some at their height, some decreasing; in some parts they are arising, in

others failing. They are destroyed by collision one with another. There are some worlds devoid of living creatures or plants or any moisture." (Kirk *et al.*, 1983)

The world, according to the atomists, consists of infinite in number, indivisible, and moving matter that in turn creates infinite number of worlds. However meagre these first principles of a physical theory may seem, it is important that a beginning is made at all with such a theory, and that the great thought of the unity of the world is conceived.

4.3 Physics in Aristotle

Aristotle (384-322 B.C.), the greatest collector of all existing knowledge in all fields of science, continues the Ionian tradition of research of nature. He has devoted much time to the discussion of physical questions, which are applied mainly to the terrestrial part of the world. He has also occupied himself with astronomical questions, to the moon and beyond, that are related to the other part of the world, the celestial. Although, in many cases, astronomy is mixed with physics, in this chapter we will try to detect Aristotle's physical conception and we will postpone the exploration of his astronomical viewpoint to chapter 9.

In order to avoid confusion, one should keep in mind that the recent conception of physics is absolutely different from that of the ancient and medieval times. Particularly, let us see some indicative points of view of recent researchers on what Aristotle's physics is. (Sarton, 1, 1993) considers that the focal subject in Aristotle's physics is the theory of motion or of change, while according to H. Leisegang[10], Aristotle's physics is simply a total contemplation of the nature through the perspective of the four types of causation, as Aristotle himself discerns them. (Düring, 1994), on the other hand, formulates a broader opinion. He confutes Leisegang's view as absolutely wrong, and asserts that the theme of Aristotle's physics is the physical processes, the physical things and the relevant to them concepts of genesis, change, and motion.

The main points in Aristotle's cosmology are: a) the description of the various types of natural changes or motions, both in the terrestrial and celestial region of the.

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[10] Düring, I., Aristotle, Μορφωτικό Ίδρυμα Εθνικής Τραπέζης, Αθήνα, 1994, *Footnote 41*: H. Leisegang, article *'Physic', RE* XX, 1040.

world, b) the analysis of the different kinds of basic substances or elements, and c) the explanation of the essential causes of these motions.

4.3.1 Types of Change & Sensible Qualities by Aristotle

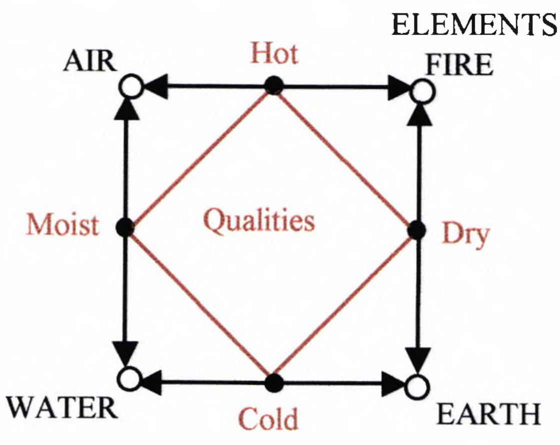
While the whole trend until now is towards a dynamic world of continuous mutual transformation of material elements, most philosophers of later times tend to concentrate more on considering the elements of the world static, fixed and unalterable part of the structure of the universe. Principally, Aristotle denies the possibility of a beginning and maintains the idea that the universe must be eternal. This eternal universe according to him is a great sphere, which is divided into an upper (the celestial) and a lower (the terrestrial or sublunary) region, by the spherical shell, in which the moon is situated.

As we have already seen, Anaximander introduces the principle of the opposite substances, Empedocles the theory of the four roots, and Anaxagoras the theory of movers. Aristotle combines these theories, accepts the four elements, at least to account for the changes that occur in the terrestrial region, but in addition he recognizes that everything in the world is characterized by an inherent tension of motion and change. The terrestrial region, which consists of the four elements, is characterized by birth, death, and alteration. Aristotle distinguishes four types of change or motion: a) local motion, that is a motion of an object from one place to another, e.g., the different positions of sun or moon, b) generation or destruction, where generation is the motion from a lesser to a higher degree of perfection, e.g., birth, and destruction is the opposite motion, from a higher to a lower form, such as death, c) alteration, where the substance of an object remains the same even if it receives, for example, another shape or colour, and d) the increase or decrease of quantity or magnitude, e.g., a child grows or an older person shrinks in size.

As, according to Aristotle, everything in nature has both the material and the formal aspect, he connects all change and motion as well as the conception of the primary elements with his hylemorphism (for details see next paragraph). All change and motion involve the following principles: a) the matter remains the same throughout the change and is the subject of the changing attribute, such as the sun or the child, and b) the form, is the definable quality or structure that the matter loses or acquires, such as sun's initial and new position. The material of each thing has the potentiality, if acted on by an appropriate cause, to replace an initial quality by a new contrary one.

However, even in substantial changes, such as the burning of a tree, there is the 'prime matter' or the basic matter of the universe that still persists, although its identity is not perceptible. Because this basic matter of the universe always has existed and will exist, nothing absolutely comes to be or ceases to be. And this is Aristotle's law of the conservation of matter.

Having formulated the viewpoint that all physical changes occur by the interaction of the four primary elements – earth, water, air, and fire, Aristotle tries to determine the prime matter and the pair of contrary qualities out of which these elements are composed. He states that the material bodies are characterized in a greater or a lesser degree by four fundamental qualities or properties, the so-called sensible or elementary qualities, which are hotness, dryness, coldness, and moisture, and the elements are raised by combination of these qualities. He assigns to each element a combination of two of these qualities or properties. Four binary combinations can be made: hot and dry, hot and moist, cold and moist, cold and dry. The assignment of one property or of three of them to each element is impossible, because in the former case the mutual changes of elements cannot be explained, and in the latter the two of the properties will be opposites and thus incompatible in the same element. Each element and its two non-opposite sensible qualities is shown in the following figure:



Cold + Dry = EARTH
Cold + Moist = WATER
Hot +Moist = AIR
Hot + Dry = FIRE

Figure 4.5: Aristotelian elements and sensible qualities.

Other qualities in things, such as 'heavy-light', 'hard-soft', and so on, are derived from these more basic qualities. Heavy, for example, is a further property of Earth and Water due to their tendency to move towards the centre of the world, whereas light is an additional characteristic of Air and Fire in virtue of their motion towards the limit of the universe. Therefore, Aristotle calls light everything that moves upwards and heavy everything that moves downwards, where 'up' is the outer limit of the universe and 'down' is the centre of the universe.

De Caleo (On the Heavens), IV, ch. 1, 308a28-31: "By absolutely light, then, we mean that which moves upwards or to the extremity, and by absolutely heavy that which moves downward or to the centre." (Schlagel, 1995)

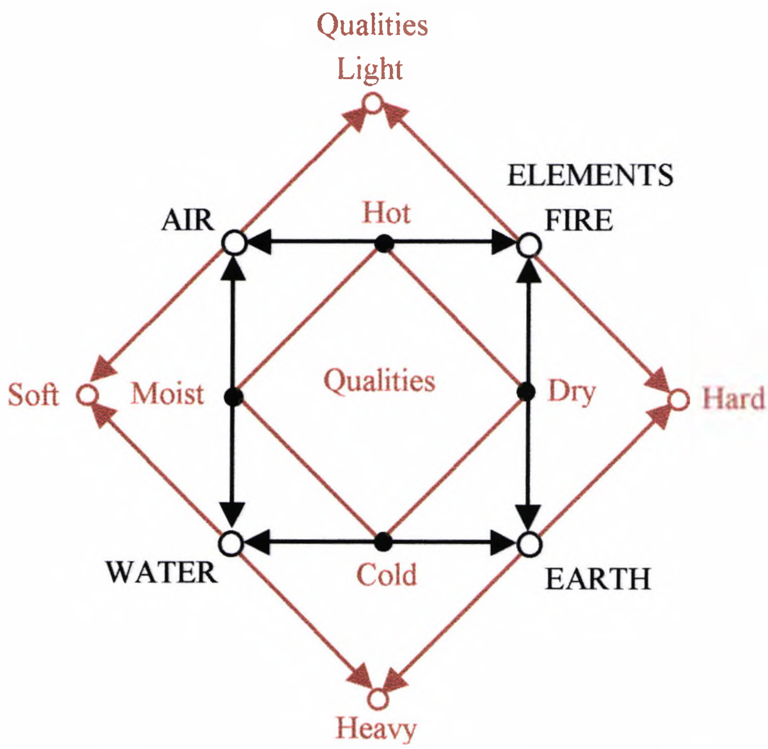


Figure 4.6: Aristotelian successive interaction between qualities and elements.

The principle of 'heavy-light' or of 'upwards-downwards' leads to a dual or binary system of the terrestrial elements. Earth is heavier than water and fire is lighter than air. The forms upward and downward of the terrestrial motions are rectilinear, i.e., the elements tend to move along straight lines, earth downwards (from the periphery of the universe to the centre of it), fire upwards (from the centre of the universe to the border of it), water and air in between the centre and the periphery of the world.

Consequently, earth and water descend towards the centre of the universe because both elements are heavy. But because earth is heavier, its material is concentrated in the centre (geocentric conception of universe), with water in a concentric spherical shell outside it. On the other hand, the nature of air and fire is to ascend toward the periphery of the terrestrial region because both are light. But because of its greater levity, fire occupies the outermost region, with air in a concentric sphere inside it. The Aristotelian elements form a set of concentric spheres: fire on the outside, followed by air and water, and earth at the centre, as it is shown in the following figure:

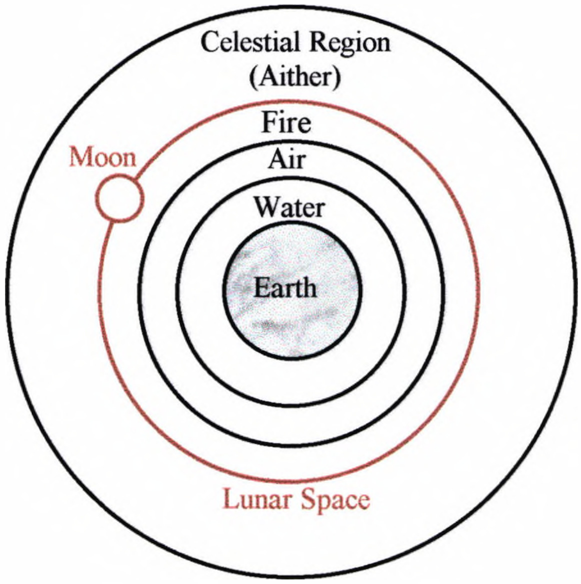


Figure 4.7: Aristotelian terrestrial concentric spheres (Lindberg, 1992)

Concerning the changeless celestial region, which will be elaborated in chapter 9, we just mention that therein the bodies move eternally in rectilinear, circular, or mixed motions, and they are not made out of the terrestrial elements (earth, water, air, fire) but of another substance, which is the incorruptible fifth element, the Aither. According to the Aristotelian principle, there are two types of natural motion: the uniform motion of the celestial bodies in circular paths and the rectilinear motion of the earthly bodies. The rectilinear motion is considered as contrary to circular motion. It has a beginning and an end, as well as the possibility of reversible direction, whereas the circular motion has no beginning and no end, and therefore no contrary form. Circular motion is regarded as the primary motion because the perfect is prior to imperfect, and the circle is a perfect thing[11]. The perfection and uniqueness of circular motion lead to the introduction of the fifth, weightless, and even finer than fire element of Aither.

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[11] Aristotle, *De Caleo*, I, ch. 2, 269a19-20: "For if the natural motion is upward, it will be fire or air, and if downward, water or earth. Further, this circular motion is necessarily primary. For the perfect is naturally prior to the imperfect, and the circle is a perfect thing." (Translated by J. L. Stocks, http://classics.mit.edu/Aristotle/heavens.1.i.html)

Consequently, Aristotle introduces a physical model of natural elements and sensible qualities in the form of a geometrical algorithm, and brings together two different things: the material elements related to the structure of the system and the qualities, the properties that constitute the element of system's function. The application of these properties in the terrestrial and the celestial area result in the invention of a spherical model of the world.

4.3.2 Types of Causation by Aristotle

Aristotle considers everything in nature to be constituted of two essentially different elements: Form and Matter, the external and the internal quality, respectively. He gives to these terms another meaning different of what they mean for us. An object is said to belong to classes as a result of its form, but it is said to be individual as a result of matter. Matter is in a sense the principle of individuation, whereas form gives object its essential character. It is by its form that we recognize it for what it is. This doctrine of considering things to be constituted by form and matter is called hylemorphism, from the Greek words *ὑλη* (substance or matter) and *μορφή* (form).

Between the two aspects of matter and form there is a dynamic relationship, which is the process of Potentiality and Actuality, the process of passing from the undetermined or unfinished into the determined or finished. What does it mean? As stated by Aristotle in his work, matter has the potentiality of form, while it is the form that actualises the matter. Or in other words, the form exists actually, whereas the matter exists potentially (Schlagel, 1995):

> *On Generation and Corruption*, I, 319a 27-29: "For if a substantial thing comes-to-be, it is clear that there will 'be' (not actually, but potentially) a substance, out of which its coming-to-be will proceed and into which the thing that is passing-away will necessarily change[12]."

[12] De Generatione et Corruptione, Translated by H. H. Joachim (http://classics.mit.edu/Aristotle/gener_corr.1.i.html)

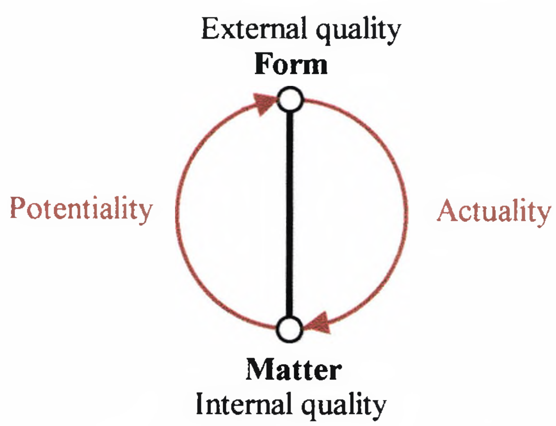

Figure 4.8: The material elements, Form and Matter, and their opposite relations, Potentiality and Actuality by Aristotle

Bronze (matter), for example, has the potentiality for becoming a statue (form), whereas the shape of the statue is the actualisation of the substance of the bronze. Says (Taylor, 1916), the individual when finally determined by the form is the Actuality of which the undeveloped matter was the Potentiality. The conception of the world involved in these contrasts of Matter–Form, Potential–Actual, finds its fullest expression in Aristotle's theory of the four causes of the production of things.

According to Aristotle, as it is shown by his own words in the following passages, the only way to get knowledge of a thing is to grasp the 'why' of it, which is the primary cause of it.

> *Posterior Analytics*, 71β 9-12: "We suppose ourselves to possess unqualified scientific knowledge of a thing, as opposed to knowing it in the accidental way in which the sophist knows, when we think that we know the cause on which the fact depends, as the cause of that fact and of no other, and, further, that the fact could not be other than it is."
>
> *Posterior Analytics*, 94 α 20: "We think we have scientific knowledge when we know the cause, and there are four causes: (1) the definable form [the so-called material cause], (2) an antecedent which necessitates a consequent [the so-called formal cause], (3) the efficient cause, (4) the final cause[13]." (Ross, 1971, vol. 1)

Aristotle refers to the four causes also in his work of *Physics*:

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[13] Analytica Posteriora, Translated by G. R. G. Mure, M.A., (http://classics.mit.edu/Aristotle/posterior.html)

> *Physics*, 194b 15-195a: "Now that we have established these distinctions, we must proceed to consider causes, their character and number. Knowledge is the object of our inquiry, and men do not think they know a thing till they have grasped the 'why' of (which is to grasp its primary cause). So clearly we too must do this as regards both coming to be and passing away and every kind of physical change, in order that, knowing their principles, we may try to refer to these principles each of our problems. In one sense, then, (1) that out of which a thing comes to be and which persists, is called 'cause', e.g., the bronze of the statue, the silver of the bowl, and the genera of which the bronze and the silver are species. In another sense (2) the form or the archetype, i.e., the statement of the essence, and its genera, are called 'causes' (e.g., of the octave the relation of 2:1, and generally number), and the parts in the definition. Again (3) the primary source of the change or coming to rest; e.g., the man who gave advice is a cause, the father is cause of the child, and generally what makes of what is made and what causes change of what is changed. Again (4) in the sense of end or 'that for the sake of which' a thing is done, e.g., health is the cause of walking about. ('Why is he walking about?' we say. 'To be healthy', and, having said that, we think we have assigned the cause.) The same is true also of all the intermediate steps, which are brought about through the action of something else as means towards the end, e.g., reduction of flesh, purging, drugs, or surgical instruments are means towards health. All these things are 'for the sake of' the end, though they differ from one another in that some are activities, others instruments[14]." (McKeon, 1941)

Therefore, in order to investigate the world more deeply, Aristotle introduces four types of causation, i.e., four explanatory conditions and factors that are involved in things that exist in the terrestrial world. The first is the material factor or 'that out of which a thing comes to be and which persists', the second is the formal cause or 'the statement of the essence', the third is the efficient cause or 'the primary source of the change or rest', and the forth is the final cause or 'that of the sake of which' something is done. The material cause is the material necessary to produce the phenomenon; the formal cause is the form introduced into this matter so that an independent substance emerges; the efficient cause is the force necessary to unite matter and form; and the final cause is the purpose of the whole process. For example, a bed is a bed, because it

[14] Physica, Translated by R. P. Hardie and R. K. Gaye, (http://classics.mit.edu/Aristotle/physics.2.ii.html)

is made of wood (material cause) in a given shape (formal cause) by a carpenter (efficient cause) for the purpose of providing sleep (final cause). Aristotle supposes the same scheme to be valid for the knowledge of the natural world. The material cause is closely related to matter, whereas the formal, the efficient, and the final causes are three different aspects of form itself. Aristotle says that these three causes often converge on one thing, which is form[15]. So these three causes also refer to by Aristotle as the primary cause. To grasp the primary cause is to grasp the why of a thing.

Moreover Aristotle criticises the early philosophers that recognized only the first kind of cause, the Matter of a thing, as the main principle of everything. He distinguishes only Anaxagoras, who recognizes the need of an efficient cause, by asserting that parallel to the universal material should be some other cause to account for the transformations of material. In his work of *Physics* B 4-8, Aristotle introduces two other efficient causes: *τύχη*, chance, and *αυτόματον*, spontaneity, not as the sufficiently worthy causes for the order in the universe, but as capable of producing any effect. In other words, *τύχη* and *αυτόματον* are considered as co-ordinate agents with Mind in producing the phenomena of the Universe. For Aristotle, a spontaneous event is an event that might have happened because of a particular reason, or because of an external cause. For example a stone that falls and strikes a man spontaneously, it might have been the weapon of his enemy or it just rolls off the cliff. In the second case the stone does not drop in order to strike the man, though it appears so. On the other hand, an event by chance, is also an event of apparent purposefulness, but it is restricted to human activities.

Therefore, Aristotle considers that any thing, any system, and so the world, has an external quality, the so-called Form, and an internal, the so-called Matter. They are connected between them by the opposite relations of Potentiality and Actuality. He also defines four types of causation, the material, the formal, the efficient, and the final cause, sorting in this way the particular qualities of things.

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[15] Aristotle, *Physics* II.7, 198 a 25: "Now, the causes being four, it is the business of the physicist to know about them all, and if he refers his problems back to all of them, he will assign the 'why' in the way proper to his science-the matter, the form, the mover, 'that for the sake of which'. The last three often coincide; for the 'what' and 'that for the sake of which' are one, while the primary source of motion is the same in species as these (for man generates man), and so too, in general, are all things which cause movement by being themselves moved" (Physica, Translated by R. P. Hardie and R. K. Gaye, (http://classics.mit.edu/Aristotle/physics.2.ii.html)

4.4 Archimedes and Physics

The earliest theories of the Ionian school, where the nature is of vital importance, are closely associated with keen observation of nature and with attempts to explain the phenomena on the basis of physical or mathematical laws. The explicit formulation of laws of mathematical form has been made by the time of Pythagoras, as we will see in chapter 7. On the other hand, a considerable development of laws in the field of physics is taking place in the Alexandrian period and especially in the work of Archimedes.

Archimedes (286-212 B.C.), a native of Syracuse, made discoveries and inventions in the fields of geometry, arithmetic, physics, and engineering. We shall consider his mathematical and mechanical inventions later. At this point, we will examine his contribution to physics. Amongst his physical discoveries, the laws of hydrostatics, such as the law of buoyancy, and the laws of lever are the most important. The former is based on the use of a dimensional physical quantity, the density or specific gravity, whereas the latter on pure proportion, similarly to the law of vibrating strings by Pythagoras (chapter 7). According to (Sambursky, 1956), both natural laws are static, i.e., they are laws in which time does not appear.

It is useful to list here the titles of his writings that have survived, indicating the nature and the range of his physical, mathematical, and mechanical investigations:

- *On the Equilibrium of Planes*, Books I, and II, in which he describes the laws of levers and of equilibrium.
- *The Quadrature of the Parabola.*
- *On the Sphere and Cylinder,* wherein among other things he estimates the surface of a sphere and the volume of a cylinder.
- *On Spirals,* wherein he demonstrates the properties of spirals.
- *On Conoids and Spheroids,* wherein he determines by means of a method similar to integration, the volumes and the areas of segments of various geometrical figures.
- *On floating Bodies*, Books I, and II, in which he describes the Archimedean Principle, as we will see later on.
- *The Measurement of a Circle,* wherein he approximates the value of π.
- *The Sand Reckoner* (*Arenarius*), wherein he establishes a complex system of notation.

- *The Method*, a letter to Eratosthenes, wherein he partly reveals the secrets of his discoveries. In it, Archimedes distinguishes between the method of discovery and the method of deductive proof of theorems.
- *A Book of Lemmas*, translated from the Arabic, which contains elementary propositions related to some of his lost works.

a. The Leverage Law

Archimedes' treatise *On the Equilibrium of Planes*, in two books, develops the principle of the lever and determines the centre of gravity of various figures, such as parallelogram, triangle, and parallel trapezium, in a throughout geometrical treatment. In Book II, he occupies himself with the finding of centres of gravity of parabolic segments. The lever, the simplest of devices giving a mechanical advantage, is essentially a rigid bar turning on a fixed pivot. The central principle of lever is that two forces, which hold it in equilibrium, are inversely proportional to their distances from the pivot (Hull, 1959). The theory of levers is based on the following postulates:

Archimedes, *On the Equilibrium of Planes*, Book I, Postulate 1: "Equal weights at equal distances are in equilibrium, and equal weights at unequal distances are not in equilibrium but incline towards the weight which is at the greater distance"

Postulate 2: "If, when weights at certain distances are in equilibrium, something be added to one of the weights, they are not in equilibrium but incline towards that weight to which the addition was made"

Propositions 6, 7: "Two magnitudes, whether commensurable [Proposition 6] or incommensurable [Proposition 7], balance at distances reciprocally proportional to the magnitudes" (Cohen *et al.*, 1966)

Based on the principle of the lever, Archimedes solved the problem of moving a given weight, however large, by a given force, however small. He explained this absurdity of moving a heavy body by a small force by means of the circle's properties: "greater circles overcome lesser ones when they revolve about the same centre" (Sarton, 2, 1993). He noticed that the nature of the circle appertains the nature of the lever. For in the lever, both force and burden move through an arc of a circle, i.e., the circumference of the circle suits to the essential character of the lever. Therefore, the longer the arm of a lever, the lesser the need for a force to be operated on it.

According to Plutarch[16] (D. L. Simms, *Archimedes the Engineer*, in the book of Hollister-Short *et al.*, 1995) Archimedes wrote to the King of Syracuse Hieron II, who was both a relative and a friend of his, about this possibility of moving some weight by a tiny force. His legendary boast to Hieron is often recalled: "Give me another world (a point of support, a fulcrum) to stand on, and I shall move the earth". When Hieron invited him to put his theorem into practice, Archimedes chose for his demonstration a three-masted fully laden ship, which had been beached by many men. He seated himself some distance away and with no great effort he drew the ship towards him. He had devised for this experiment a combination of pulleys, the so-called *polyspaston*, i.e., a tackle with a large number of sheaves in each of the two pulley blocks. Apart from Plutarch's record, there are a few other references[17], such as that of Athenaeus[18]. According to Athenaeus, Archimedes managed to move the ship not by the use of a complex system of pulleys but by the use of a windlass:

> Athenaeus, *Deipnosophists (Δειπνοσοφιστής, i.e., Philosophers at Dinner)*, 5.203-209, ed. C.B. Gulick (Cambridge, 1928): "Archimedes the mechanician alone was able to launch the ship with the aid of a few persons... by the construction of the windlass he was able to launch the ship of so great proportions in the water. Archimedes was the first to invent the construction of the windlass." (D. L. Simms, *Archimedes the Engineer*, in the book of Hollister-Short *et al.*, 1995)

The laws of levers are the basis of operation, for example, of elevators and steam cranes.

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[16] Plutarch's *Parallel Lives*, part trans. I. Scott-Kilvert as *Makers of Rome*: *Marcellus*, 14 (Harmondsworth, 1965), 14.7-9, 99

[17] Proclus, *Commentary on the First Book of Euclid's Elements*, 3, 63-4, trans. intro. notes J. R. Morrow (Princeton, 1970), 51; Oribasios, *Collectii medicarum reliquiae*, 44.22, ed. J. J. Reader (Leipzig, 1933), 4 (D.L. Simms, Hollister-Short *et al.*, 1995); A. Holm, *Geschichte Siciliens im Alterthum*, III (Leipzig, 1898), pp. 39-41; A. Favaro, *Archimede* (Roma, 1923), p. 24 (Dijksterhuis, 2, 1987)

[18] Athenaeus of Naucratis lived about 200 A.D., first in Alexandria, later in Rome. His work of *Deipnosophists* contains fragments from a great many ancient writers on all sorts of subjects (Dijksterhuis, 2, 1987)

b. The Buoyancy Law

In the treatise *On Floating Bodies*, in two books, Archimedes lays the foundation of the science of hydrostatics. In Book I, he proves the so-called Archimedean Principle or Principle of floatation, according to which a body wholly or partly immersed in a fluid loses an amount of weight equal to that of the fluid displaced. Book II investigates the condition of stability of a right segment of a paraboloid of revolution floating in a fluid. Concerning the Archimedes' principle we have:

> Proposition 5: "Any solid lighter than a fluid will, if placed in the fluid, be so far immersed that the weight of the solid will be equal to the weight of the fluid displaced..."
>
> Proposition 6: "If a solid lighter than the fluid be forcibly immersed in it, the solid will be driven upwards by a force equal to the difference between its weight and the weight of the fluid displaced..." (Cohen *et al.*, 1966)

The Archimedes' Principle could be formulated as follows: The weight of the fluid that a solid displaces is equal to the buoyant force, i.e., the upward thrust, that the fluid exerts on the solid. Consequently, for the solid to float the buoyant force must be equal to the weight of the solid. The buoyant force is: $F_B = mg$, where m is the mass of fluid displaced and g is the gravitational acceleration. Another form of Archimedes' Principle is: $F_B = \rho_{fluid} g V_{solid}$, where ρ is the density of the fluid. This is due to the law of buoyancy that large iron ships sail on water or gas-filled balloons travel through the air.

Archimedes discovered this theory when "on getting into a tub observed that the more his body sank into it the more water run out over the tub[19]" (Cohen *et al.*, 1966). Based on this principle, he managed to determine the specific gravity of bodies and to solve the problem of King Hieron's crown. Hieron requested Archimedes to consider whether or not the golden crown that he had requested to be made for him, had been adulterated with a mass of silver. Archimedes detected the mixing of silver with the gold by weighting in water the crown itself, as well as two masses of the same weight as the crown, one of gold, the other of silver. By comparing the relative amounts of water

[19] Vitruvius, *On Architecture* IX, Introduction 9-12, translation of M. H. Morgan

displaced by the same weight of the gold, the silver, and the mixture of the two he found the specific gravity of the three.

The physical laws of Archimedes constitute elements of the essential theoretical and practical framework, upon which flourishes the development of physical modelling.

4.5 Conclusion

The second stage of human thought is the transition from the mythical to the presocratic physical interpretation of the world. Even though in many cases the original works of the Presocratics have survived either only in fragments, or only in the reports of later writers, what is obvious throughout their thinking is that they reject any metaphysical speculation that characterized the whole mythical period, they pose fundamental questions, observe the nature, search for principles, relations and general laws that govern the natural phenomena, and they introduce theories concerning the primary elements, the structure of the world, and the detailed analysis of the world's phenomena and situations. All these result in physical and conceptual models of the world' creation and function. Presocratic philosophers look for an eternal, inexhaustible, underlying substance, out of which the elements of the world as well as the opposite quantities arise and arrive at generalisations, such as all things are water or air, and so on. The four elements, to which they conclude, are the primary physical elements, which correspond to the three conditions of nature: solids (earth), liquids (water), and gases (air), whereas the fourth, the fire, corresponds to the energy, while the opposite quantities, dominating the doctrines of Heraclitus, Anaxagoras, or Empedocles, could be considered as derivatives of these four elements. However, there are quite a few cases, where the origin, the source of all things is derived from a concept or from more than one element or from the qualitative differences and quantitative proportions of the elements providing the first integration and understanding of interaction between fundamental elements. Examples of this are: the Infinite by Anaximander, the four roots of Empedocles, the dualism of Mind and matter of Anaxagoras, the theory of the infinite numbers of atoms that move randomly in an infinite void by the atomists, and the sensible qualities of Aristotle and their interactions.

Along with these physical models related to the composition and structure of the world, there is the issue of the operations, the dynamic behaviours and the processes of the world. The conceptualisation of concepts such as time evolution, motion and change unavoidably partake. The early physicists detect the initial force (prime mover) that causes the motion and the changes happen in the world and result again in the formation of physical models adequate to explain these changes. Their theories constitute the foundation stone for physics and other modern theories. For example, Anaximander's cosmological opposites of hot-cold and dry-moist and their separation off process are responsible for the changes occur in the natural world; Anaximenes' rarefaction and condensation concepts, as an attempt to reduce the qualitative changes of air to quantitative relations, reveals similarities with the contemporary qualitative reasoning process; Empedocles' forces of Love and Strife cause the congregation or separation of the four roots; Anaxagoras' force of Mind rotates and causes the separation off of the opposite qualities that in turn result in the creation of the earth and the heaven; Heraclitus' opposite forces of Strife and Harmony, into things and qualities, act as the prime mover; the four types of causation of Aristotle explain all the changes, and so on. Heraclitus' declaration that everything flows and Aristotle's doctrine of the inherent motion and change that characterise all things, i.e., the conceptualisation of dynamical phenomena, lead to the concept of time and to the future efforts to measure and simulate time. This in turn, results in the construction of sundials and water clocks as models of the relative motion of sun and the flow of water, respectively, as explored in chapter 12.

The physical interpretation of the world results in models either of the structure or of the function of the system under examination. The structure is analysed into primary elements or parts, whereas the function into qualities, relations, contradictions, causes, and results. These efforts, of understanding what the causes and what the effects are, by observing both in temporal and spatial order, aim at the acquisition of knowledge. This is a subject that prevails into the whole period of antiquity, either directly, by introducing the way of getting knowledge of a thing (Aristotle), or as the result of observations and scientific researches. The modelling of processes and phenomena, on the other, expresses the cumulative effort to organise, store and represent the different forms of achieved knowledge. The characteristic of knowledge acquisition, structuring and representation, dominant in the thought of the Presocratics and Aristotle, refers to conceptual modelling. The logic behind this statement is that, in modern terms, the conceptual modelling is defined as a process that involves: a) the stage of knowledge acquisition, where the knowledge is being extracted by the comprehensive description of the investigated system, b) the stage of knowledge structuring, which concerns the selection and the classification of the acquired knowledge and of all the relevant to the problem concepts that are best capable to describe the structure and the behaviour of the system, and c) the stage of knowledge representation, which leads to the depiction of the main entities of the problem, their attributes and their relations. These stages lead to the construction of a functional schema, which is one of the main concepts of conceptual modelling. This conceptual schema serves a predefined set of goals and can be used as a reference for the entire development process of a system. A more indicative reference to the conceptual modelling will be given in chapter 6, where we will explore Plato's philosophy about models, modelling, and conceptual schemas and algorithms.

Chapter 5

EARLY NOTIONS OF THE SYSTEM

5. EARLY NOTIONS OF THE SYSTEM

5.1 Introduction

Up to this point, an analysis of some cosmogonic and cosmological aspects of the world associated with the concept of modelling has been provided. The focus has been laid on the formation of mythical, cosmological, or physical models of the world. Their citation follows the chronological order of their appearance. In this chapter, we will stand at the antipode of these models, at the system itself. Even though the word 'system' appears frequently in ancient Greek sources, only one proper definition of what system is can be found in the words of the Pythagorean Kallicratides. However, the way the ancient philosophers approach the unknown to them systems, such as the world, the natural phenomena, or the human body, with the intention to understand, explain, and/or to model them, bears the early notion of the system.

In addition, we will look at Hippocrates' holistic approach to the concept of system, which is formed by his naturalistic approach to medicine and was in sharp contrast to the religious views that preceded him. His method is to ignore all the gods and to hold that all diseases are natural phenomena governed by natural laws. Characteristically, he quotes in a treatise on *Epidemic Diseases* "Nature is the healer of all disease. Let foods be your Medicine and your Medicine your Foods." Only if a man lives a life in accordance with the laws of Nature, health, harmony, and balance does he manage to preserve himself. He considers the human body as a general system composed of fours humours; black bile, yellow bile, phlegm, and blood, and stresses the importance of observation, diagnosis, and treatment in order to create a diagnostic model.

Following, Aristotle's invention of Logic, as the result of his efforts to build up a rational and transmissible system of thought is examined. Interested in every area of human knowledge about the world, Aristotle aimed at unifying all of them in a coherent system of thought by developing a common methodology that would serve equally well as the procedure for learning about any discipline. This formal system of thought makes a new realm of thought possible, an ability to answer questions of logical consequence and proof. The treatises related to the system of thought analyse and place things in a hierarchical way, as for example, the ten categories of predications, the syllogism and its correct figures, and the demonstration by deduction or induction. As (Jonathan Lear, 1988) has put it, "Aristotle's primary goal is not to offer a practical guide to argumentation but to study the properties of inferential systems themselves."

5.2 The Concept of 'System' in Ancient Greek Sources

In ancient Greek written sources, the word 'system' appears frequently and the meaning of it varies accordingly when referring to philosophy, to music, or to medicine. Let us cite some indicative instances:

- *Σύστημα*: System, a whole composed of many parts

  Plato, *Epinomis*, 991e: " [...] all geometric constructions, all systems of numbers, all duly constituted melodic progressions, the single ordered scheme of all celestial revolutions[20] ..." (Hamilton *et al.*, 1969)

- In philosophy: synthesis, epopee (Aristotle, *Poeitika*, 18,13)
- In politics: organised government, constitution, and polity (Plato, *Laws*, 686b)
- In music: harmony, system of musical intervals

  Plato, *Philebus*, 17d: "[Socrates speaks...] when you have learned what sounds are high and what are low, and the number and nature of the intervals and their limits or proportions, and the **systems** compounded out of them ... you have technical skill" (Jowett, 1964, vol. 3)

- In poetry: the connection of many lyrics in a whole
- In medicine: the 'whole' of body

  Hippocrates, *Aphorisms*, 15: "When the throat is diseased, or tubercles form on the body, attention must paid to the secretions; for if they be bilious, the disease affects the **general system**; but if they resemble those of a healthy person, it is safe to give nourishing food[21]."

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[20] *Epinomis*, Translated by A.E. Taylor

[21] Translated by Francis Adams (http://classics.mit.edu//Hippocrates/aphorisms.html)

Hippocrates, *The Book of Prognostics*, 12: "But you must not allow yourself to be deceived if such urine be passed while the bladder is diseased; for then it is a symptom of the state, not of the **general system**, but of a particular viscus[22]."

The Greek word *σύστημα* (system) comes from the ancient verb '*συνίστημι*' that means:

- Establish, unite, combine, connect, compose, form, construct something solid, solidify, and thicken.
- Maintain order, form party, engage, interweave, be related, become, be composed, happen, exist, and be coherent.

These references come to us mostly by Plato, Aristotle, and Hippocrates. However, even the Presocratic philosophers with their opinion that the unknown world, i.e., the system they have to look into, is an ordered system that yields to rational investigation, contribute to the initially formation of the concept of system. They lay down the primary conditions that must be fulfilled: the under examination system has to have a rational structure, to follow specific rules, and in general to be logically explainable. They ask about the ingredients, the composition, the structure, and the operation of the system. They want to know whether the system is made of one element or many, the essence of these elements, the behaviour, and the processes that take place so as these elements to result in the creation of it. Regarding the creation and the structure of the world system they ask questions, such as:

- What is the nature of the system?
- What are the basic, fundamental, primary elements of system's creation?
- In which parts could the system be analysed?

Regarding, on the other hand, the behaviour and the function of the system they ask:

- How is the system being generated and what is the prime cause for this generation?

[22] Translated by Francis Adams (http://classics.mit.edu//Hippocrates/prognost.html)

- How does the system work?
- How does it progress?

More precisely, Thales is the first to introduce a system analysis by investigating the basic elements of a system, and Anaximander considers the system (world) as a dynamical, time-dependant unity with ceaseless transformations, consisting of opposite qualities, such as, hot-cold, dry-moist, and introduces the notion of the required cause, the input of the system, the, as he calls, eternal motion, which provokes the system and is responsible for all the changes. Anaximenes goes further and tries to find the functional correlations among its variables of motion, density, and temperature. His system follows a continually cyclical transformation of rarefaction and condensation of air. For Heraclitus, the system is a colossal process of events, changes, and facts, and he focuses his view mostly on the dynamical evolution of it. Empedocles, Anaxagoras, and the atomists Leucippus and Democritus, regard the system as a complicated whole. In particular, Anaxagoras speaks of compositions and decompositions that characterise the world and rejects the possibility of coming into being or ceasing to be. In this way, he introduces the concept of composite systems that can be split into objects (decomposition) interacting between each other or united again (composition) forming other systems. This notion of combining together in various ways the smallest parts of world, the atoms, results in the formation of macrocosm according to Leucippus and Democritus. The theory of the atomists bears additionally the concept of the time evolution since at the first stage a large collection of atoms becomes isolated in a large patch of void, and at the second they form a whirl or vortex, out of which the system of world arose.

Later on, Plato aiming at the proper definition of specific concepts, sees that things can naturally be compounded into one and divided into many and introduces a method that goes on from the general to the particular and vice-versa and results in the desired definition (chapter 6). The contributing point to the evolution of the concept of system is the conception that the whole, i.e., the unified system, consists of partial components that interrelate with each other, so as to form the general.

5.3 The First Definition of System

The first definition of system comes from the ancient times. The Lakonian, and Pythagorean Kallicratides, in his work *«Περί οίκων ευδαιμονίας»* (On the Happiness of Family), defines what 'system' is and explains it in terms of three examples: a) the system of dance in the singing societies, b) the system of the crew in a ship, and c) the system of the family, where the people are next of kin (have between them kindred relationships). This definition appears in the Doric dialect in the Anthology of J. Stobaei[23] (Stobaei, J., Economicos, 16, 485):

> Any system consists of contrary and dissimilar elements, which unite under one optimum and return to the common purpose[24].

[Example 1: The dance]

Any particular dance constitutes a system in the singing societies. This system has a common purpose and ends up in a common result that is the harmony, the concordance of sound and motion.

[Example 2: The crew]

In the ships, the system of the crew is composed of contrary and dissimilar elements that unite in one optimum, which is the captain, and return to the common purpose, which is the good sailing.

[Example 3: The family (the household)]

The family exists as a system in societies formed by relatives, because it is composed of different parts that unite in something optimum, the head of the family, and return to the common target, which is the alike thinking.

According to Kallicratides, a system exists only in a specific whole (e.g., a society) and is characterized of three things:

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[23] Ιωάννου Στοβαίου Ανθολόγιον, Joannis Stobaei Florilegium. Ad Manuscriptorum Fidem Emendavit et Supplevit, Thomas Gaisford, A. M., Vol. III, OXONII, E Typographeo Clarendoniano, MDCCCXXII (1822).

[24] Translated from ancient Greek by the author

1. The particular and different elements, i.e., the parts that determine its material substance (for example, the dancers of the dance, the crew in the ship, the members of a family).
2. The optimum element that unites all the others, or in other words, the element that composes, co-ordinates, and instructs the particular elements, secures their unity, and decides on the structure of the system (for example, the music of a dance, the captain in the ship, the head of the family)
3. The desired common purpose, the target that governs the system and specifies its behaviour (for example, the concordance of sound and motion in a dance, the good sailing and the right direction of a ship, the harmony in a family)

Thus, a complicated system consists of its elements *A*, *B*, *C*, *D*, *E*, its structure, i.e., the way these elements connect between them, and it is depicted by the relations *a*, *b*, *c*, and so on, and of the desired target *r*, the so-called reference input in modern terminology, and whose fulfilment depends on the good function of the system.

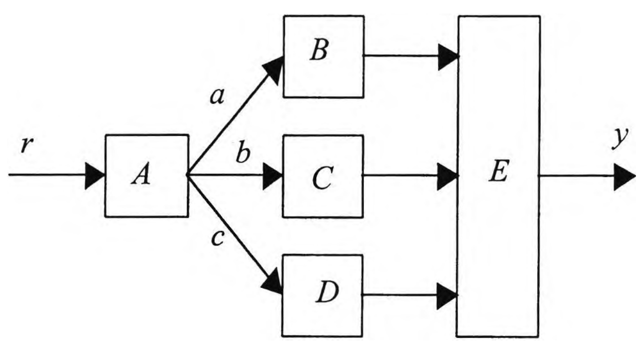

Figure 5.1: Analysis of a system by Kallicratides

A brief definition of the system according to Kallicratides' own version could be the following:

A system is defined by means of its elements, structure, and the target, which specifies system's function.

However, the system as it is described by Kallicratides could be considered not as a simple open system, but as a closed loop control system bearing the following characteristics[25]:

1. The system consists of opposite parts, i.e., it contains the concept of contradiction
2. It unites to the one optimum, which in modern terms called the controller that aims at the optimisation, and
3. It returns to the common target, i.e., it holds the potentiality of return, or in other words it embodies feedback, which results in the common objective and ensures the desired balance and harmony.

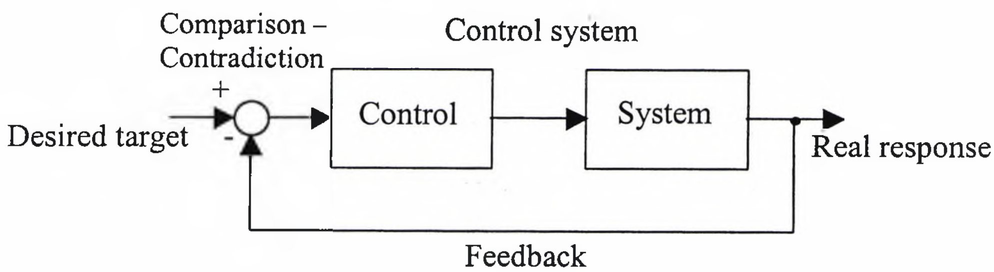

Figure 5.2: Control system by Kallicratides

A particular definition of closed loop control system according to Kallicratides could be the following:

A closed loop control system is defined by means of its contradictory elements, of the optimal control it aims, and of the feedback principle that secures the desired target.

In the following lines Kallicratides interprets and analyses further the system of the household (Stobaei, J., Economicos, 16, 485, 10-20):

In simple words, every family or household is similar to the musical triangular instrument of psaltery that has three sides: (*η εξάρτησις*) the equipment [especially of musical arrangement], (*η συναρμογή*) the connection or the relevance, and (*η αφή και χρήσις*) the feel and the use of music. [In the case of the family] the equipment (*εξάρτησις*) is the composition of all the parts that form the whole system

[25] The suggestions of open and closed loop interpretation of Kallicratides work are introduced by the author.

> of the kindred society. Two of these parts, the man and his possession, or the governed and the used, are first and maximum. In the case of the living beings the first and maximum parts are the soul and the body. The former is the part that governs and uses, whereas the latter is the part that is under command and use. And on the one hand, the life is the acquired organ of human being, whereas on the other, the body is the inherent and relative organ of soul. [Similarly, in the case of the family] some of the people, who are members of the family, are relatives and others are known. Relatives are those that have relationships by blood, whereas known are those that have relationships by marriage [...] regarding the parts [the first and the maximum] of the possession they are divided into the essential and the free. Essential is anything that serves the needs of life, whereas free is anything that concerns the way of life and the embellishment of man...

According to Kallicratides, in the example of family, the systems are composed of:

1. The particular elements (*εξάρτησις*)
2. The way they connect to each other (*συναρμογή*)
3. The relation that characterizes them (*αφή και χρήση*)

The elements of the system are divided into these that govern, i.e., the controller, and into those that are under control. Undoubtedly, into Kallicratides' definition of system, the concept of control appears.

5.4 The Holistic Approach to the Concept of System by Hippocrates

The notion of system, which is widespread in the Greek bibliography and which occupies the thought of the early philosophers so as to try to define it, acquires an integrated form in the field of medicine on account of the physician Hippocrates of Cos (*ca.* 460-377 B.C.). On the one hand, he attempts to transform Greek medicine to a rational system separated from philosophy, religion, and mysticism, and on the other, he regards the human body as the general system, in which the particular organs and their operations fit harmonically. His method is to ignore all the gods and to hold that all diseases are natural phenomena governed by natural laws. The 'Father of Medicine', as he is often described, gathers in his face all the works on all sorts of matters connected with medicine up to the 5th century B.C. The *'Hippocratic*

Collection' or *'Hippocratic Corpus'*, about eighty-seven works, free of magical tendencies or supernatural causes, is included in the earliest Greek medical writings. They deal with almost every branch of medical art and science and include both technical and non-technical discussions. Obviously, not all of these treatises that in some way come to be associated with the name of Hippocrates, are writings by him himself.

The basic principle, which is established by Hippocrates and which is also associated with the notion of system, is the precise observation. The motto of Hippocratic School, according to (Heidel, 1941) might be the words: *Epidemics,* VI, ii, 12: "Nothing at random; overlook nothing." According to Hippocrates there is a clear differentiation between speculation and guesses and exact knowledge obtained from observation. His own words for that are: "To know is one thing, merely to believe one knows is another. To know is science, but merely to believe one knows is ignorance."

The very characteristic of Hippocratic method is that it deals with the individual but it aims at a total unified picture of a diseased state. The construction of such a picture is based on the diagnosis resulting from careful inquiry and examination of all the factors regarded as significant. Some of these factors are: what discomfort or pain the patient feels and where it is located, when and under what circumstances it is first experienced, the previous condition of the patient as well as his constitution, character and habits.

The Hippocratics, by their rich experience, are able to determine what is and what is not significant, to observe certain indications that are not typical, to compare each case under consideration with others bringing out the differences and the similarities, and to classify what marks are typical or essential. The observation of similarities and differences of cases results in generalisations, whereas the whole procedure results in the description and definition of a disease as an entity with certain character. From data that presents differences and similarities arise inferences that lead to the increase of knowledge. Even though the Hippocratics are mainly practical men and they do not discuss in a theoretical way how and with what restrictions the inferences can be justified, they seek to reduce to a minimum the chances of error.

The Hippocratic method, we could say, encloses the notion of 'cycle', the main concept on which the following development of feedback and control theory has been based. The whole procedure the Hippocratics follow is a cycle. The clinical records of previous cases are necessary for the creation of a new diagnosis, and each diagnosis they end up with is used as prognosis to a new case. The process of taking advantage of the results of experiments so that to make the right decision, is a circular process. This circular process of creating diagnostic models, where each diagnosis is used as the essential knowledge in order to recognise, and eventually foretell different stages that occur in every disease, is similar to the feedback process, where the output of a closed loop control system 'returns' so as to regulate the input of the system (chapter 12).

5.4.1 Concept of Analogy

Hippocratics evolve their system of medicine by taking advantage of the previous doctrines of the Presocratic philosophers (e.g., the four fundamental elements) and of the Pythagoreans (e.g., the significance of number 4, see chapter 7), as well as, of the concept of analogy. The Presocratics Anaximander and Heraclitus develop the idea that the man is subject to the same law as the universe, e.g., the fire by Heraclitus that directs the cosmic events also operates in human soul as reason – logos. The Hippocratics on the other hand, believe in an analogy between the cosmos and the microcosm. In their writings, the relation of microcosm and macrocosm is a matter of imitation, of similarity or correspondence (*μίμησις* in Greek). This imitation may be turned either way, i.e., the human bodies imitate the cosmos as that the parts of the cosmos imitate human organs (Burkert, 1972). Therefore, they assert that the four elements, earth, air, water, and fire, form the basis not only of all things but also of the human body. The nature of the four elements suggests four qualities, the so-called by Aristotle sensible qualities: dry, cold, hot, and moist. The combination of these qualities, in groups of two, constitutes the four elements as it is shown in figure 4.5. Applied to the system of medicine the four qualities become the four fluids or humours of the human body, i.e., hot + moist = blood, cold + moist = phlegm, hot + dry = yellow bile, and cold + dry = black bile.

When a person is healthy, i.e., in a normal state, the fluids exist in his body in harmonious proportions. What a physician has to do in a case of a disease, i.e., in an abnormal condition, is to restore the disturbed harmony in the relation of the elements and humours.

Except the relationship of the four humours to the four elements there is also a link, an analogy between the humours and the seasons, which in turn have been matched with the four Ages of Man: boyhood, youth, manhood, and old age.

The following analogies arise:

Elements	*Humour*	*Qualities*	*Season*	*Age of Man*
Air	Blood	Hot +Moist	Spring	Boyhood
Fire	Yellow Bile	Hot + Dry	Summer	Youth
Earth	Black Bile	Cold + Dry	Autumn	Manhood
Water	Phlegm	Cold + Moist	Winter	Old Age

Figure 5.3: The Hippocrates' analogies

The notion of the four elements and of the bodily humours, as crucial elements in health and disease, was already familiar by the time of the Presocratics philosophers. However, the bringing together in a theory that was to be known as Humoralism occurred by Hippocrates and his work of *Nature of Man*:

> "The human body contains blood, phlegm, yellow bile and black bile. These are the things that make up its constitution and cause its pains and health. Health is primarily that state in which these constituent substances are in the correct proportion to each other, both in strength and quantity, and are well mixed. Pain occurs when one of the substances presents either a deficiency or an excess, or is separated in the body and not mixed with the others." (Lindberg, 1992)

In the book of (Jackson, 1986) we find a passage[26] that characteristically says: "The notion of humours as such comes from empirical medicine. The notion of the tetrad, the definition of health as the equilibrium of the different parts, and the definition of the sickness as the disturbance of the equilibrium are Pythagorean contributions (which were taken up by Empedocles). The notion that in the course of the seasons each of the four substances in turn gains the ascendancy seems to be purely Empedoclean. But the credit for combining all these notions in one system, and

[26] Raymond Klibansky, Erwin Panofsky, and Fritz Saxl, *Saturn and Melancholy: Studies in the History of Natural Philosophy, Religion, and Art* (New York: Basic Books, 1964), p.8.

thereby creating the doctrine of humoralism, which was to dominate the future, is no doubt to the writer of the *Nature of Man*, i.e., to Hippocrates."

One of the works included in the Hippocratic collection, *De Arte*, which appears not to have been written by a physician but most likely by the sophist Protagoras, deals with the art of medicine and in particular with the process of inferring from the visible and known to the invisible and unknown.

> *De Arte,* x f. (VI, 16ff., Littré[27]): " [...] what escapes the sight of the eyes is mastered by the sight of the mind, for what the physician cannot see with the eye nor learn by hearing, he pursues with reasoning. He must be careful however to proceed calmly and deliberately rather than with rashness and violence."

This process of reasoning from the visible and known to the hidden and unknown is mostly based on the concept of analogy. It is indicated not only by the aforesaid analogies between the four elements of the world and the four humours of human body, but also by the similarities derived from familiar facts and processes. Hippocratics, being aware of the natural phenomena and the way they have been explained by the predecessor philosophers, try to give answers to the unknown operations of human's body. In many cases, analogies are drawn from familiar instruments, e.g., the eye is likened to a lantern and the ear to trumpet, or from the domain of physics and mechanics, e.g., there is a similarity between the tube for drawing liquid from a container and the cupping glass. They compare the human embryo and its development to the growth of a plant from a seed. The process of respiration, that is for them a form of nutrition, is related to the familiar fact of evaporation, i.e., the conversion of water into air.

The Hippocratics derive their theories either from reasoning by analogy, or from experiments made in the ordinary routine of life or in the practice of industrial arts. For example, they mention two experiments that have to do with evaporation, by means of which they come to the conclusion that different waters have different weights. The detailed description of their experiments is beyond the scope of this thesis.

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[27] The only complete translation into a modern language (French) of the whole of the Greek text of the *Hippocratic Collection* is Emile Littré, Oeuvres complètes d' Hippocrate in 10 volumes (Paris, 1839-61, reprinted 1961).

5.5 Aristotle's System of Logic

Aristotle is a very systematic philosopher in the sense that ideas developed in one area of investigation often find applications in other areas (Robinson, 1995). In his work of *Organon* (Greek word for 'tool'), he analyses his conceptions of the nature of scientific explanation and of the methods for establishing scientific principles, or in other words, in this particular work, he introduces the logical tools for scientific work. Says (Thurston, 1899), "Aristotle introduced the scientific logic, founded the earliest school of scientific investigation and experimental research, and brought about the foundation the first real university, in the true sense of that word. He founded the modern scientific method, which consists in the observation and interrogation of nature, collecting facts and noting phenomena and all their visible and sensible relations, until these facts and phenomena, having been collected in sufficient number and in sufficient close relation, some evident sequence or formal connection can be discovered among them, and this, formulated, is enunciated as a law of nature, a foundation stone of the science under investigation."

From the time of Andronikus of Rhodes (*ca.* 60 B.C.), the earliest known editor of Aristotle's works, and downwards, it is considered that the logical treatises stand first among the written and the printed works of Aristotle. It is because the logical treatises are not so much a part of philosophy as an instrument, the use of which must be acquired by anyone before the comprehension of philosophy (Grote, 1872). Says (Heath, D., 1877): "Aristotle's Logic is itself meant to be an *organon*, a tool, for reconciling and bringing into active co-operation the Sciences of the Laws of Thinking and the Laws of Nature."

According to Aristotle the kernel of scientific inquiry and explanation is something he calls Demonstration. Each object or phenomenon, in addition to certain individual characteristics, possesses also some necessary properties, which cause it to be the kind of thing it is. The scientific knowledge is closely related to the ability to demonstrate that a necessary property is inherent in an object because the object belongs to a specific species, which is characterized by that essential property. In other words, the concept of scientific inquiry is based on a method of proof, on a procedure of demonstrating by deductive or syllogistic reasoning that certain conclusions follow certain premises. Aristotle's standpoint that an event is explained if we specify its cause is expressed now by his view that the demonstration is the cause of a conclusion, the mean to exhibit the reason for the conclusion being what it is.

Let us see in more detail what Demonstration is and how Aristotle examines a system by analysing its logical structure and hierarchy. *Organon* consists of six treatises that deal either with what we call Logic nowadays, or with the use of Logic in science and in dialectic.

In the first treatise, the so-called *Categories*, he gives a fundamental classification of words (1a 1-15), i.e., of the simplest units of language, according to the kinds of things they refer to. The categories he comes up with are:

1. *Substance* - a particular thing that has no contrary (e.g., a lion)
2. *Quantity* or *how much* (e.g., two feet long)
3. *Quality* or *what manner* of (e.g., cold), i.e., by virtue of quality people or things are said to be of one or another kind
4. *Relation* (e.g., double)
5. *Place* or *where* (e.g., at park)
6. *Time* or *when* (e.g., tomorrow)
7. *Position* or *in what posture* (e.g., is sitting)
8. *Having* (e.g., is armed)
9. *Acting* or *activity* (e.g., cuts)
10. *Being acted upon* or *passivity* (e.g., is cut).

Aristotle is not altogether exempt from the Pythagorean and the Platonic tradition, which ascribes to number 10 a peculiar virtue and perfection. Therefore, he considers that number 10 is the suitable number for a complete list of general categories as heads of information.

The next higher level of complexity, which is the subject of the second treatise, the so-called *On Interpretation*, consists of the simplest sentences. A sentence can be constructed by two terms – not any two, but an appropriate pair, where each term has significance by its own – such as 'All M is P', or 'All P is M'.

The topic of the third and the forth treatises, the so-called *Prior Analytics* and *Posterior Analytics*, respectively, or simply *Analytics*, is again a higher level of complexity, this of Arguments or Syllogisms. More precisely, the former discusses the Syllogism generally, whereas the latter deals with Demonstration. To form a valid syllogism, there must be two propositions including three terms and no more, 'S is M', 'M is P', so "S is P'. For example:

Socrates is a human Being.

All human beings are mortal.

Therefore, Socrates is mortal.

The two terms S and P, 'Socrates' and 'mortal', which appear as Subject and Predicate of the conclusion, are called the minor and the major term or extreme, respectively. The third term M, 'human being', appears in both premises but not in the conclusion and called the middle term. This middle term is of crucial importance, because it is the cause of the deduction. Only through the term 'human being' the deduction is possible. This logical rule in system terminology concerns the cascade connection of two systems:

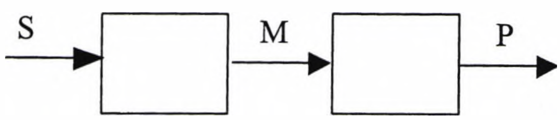


Figure 5.4: The Aristotelian syllogism as a net of systems

This sort of logical inference is generally represented with a line under the premises in place of the 'therefore'.

For example: All M is P,
S is M
―――――
S is P.

Such inferences, according to (Patzig, 1968), and the traditional Logic, occur not only in the above figure, but also in three more figures, which are the following:

I.	All M is P,	**II.**	All P is M,	**III.**	All M is P,	**IV.**	All P is M,
	S is M		S is M		M is S		M is S
	S is P.		S is P.		S is P.		S is P.

It is obvious in the above figures that the middle term M appears in both premises but does not appear in the conclusion. If M stands chiastically, we have the case of the figure IV and I. Aristotle does not make any distinction between the first and the fourth figure, so the figures IV and I of the traditional Logic constitute the first figure of the Aristotelian logic (*Prior analytics*, A 4, 25b32-37). If M stands at the end of both premises, we have the figure II (*Prior analytics*, A 5, 26b34-39), and if at the beginning, we have the figure III (*Prior analytics*, A 6, 28a10-15).

Obviously, there are numerous syllogisms that arise by the arrangement of categorical terms and words. Some arrangements result in valid syllogisms; others do not. The above-mentioned four figures constitute valid syllogisms.

The next topic of higher complexity than the syllogism is the demonstration. A valid syllogism is a demonstration. However, not every valid syllogism is a demonstration. The distinction is based on the character of demonstration's premises. In order a valid syllogism to constitute a demonstration, i.e., to serve the purpose of a true scientific explanation, the premises of the syllogism must be of a certain sort. In the above example, one proves that Socrates is mortal by demonstrating that Socrates is a human being and all human beings possess the essential property of mortality.

Conclusively, Aristotle:

1. Composes the logical structure and hierarchy of the system of thought he examines. We could say that by means of his Logic he undertakes a digital analysis of the system.
2. Distinguishes the categories out of which the structure and the function of the system are made.
3. Formulates the basic rules of logic, the syllogisms, which characterize the relations among the particular elements of the system.

This analysis of Aristotle enables the depiction of a system or process in the form of a logical digital algorithm, which in modern terminology acquires the form of a digital program.

5.6 Conclusion

The contribution of the Presocratic philosophers to the development of science is accompanied by a contribution to the emergence of the concept of system, as well as to the introduction of the basic characteristics of a system. Primarily, the way these early scientists approach the unknown system of the world ascribes to the system concept the notion of order and rationality. In addition, Presocratic system-analysis by searching the basic elements or parts of the under consideration system, the introduction of the notion of dynamical behaviour, the examination of the functional relations among the system's variables, and the conception of a system as a colossal process of events and changes credit the system with some of its fundamental features. Thus, in Presocratics' thought, we find the origin of the concept of system, as well as some prominent steps of its evolution, nevertheless in an indirect way since their subject of study is not the very concept of system. However, there is a direct, precise definition of the system itself that comes to us by the Pythagorean Kallicratides. According to him, a system is defined by the different elements, structure, and the objective related to its operation, whereas a control system is defined by its contrary elements, the optimal control, and the feedback principle for ensuring the good operation of it. The latter definition that bears the concept of control and ascribes to the system a specific target, contributes to the evolution of the control concept, which will be elaborated in chapter 11.

The further evolution of system concept is attributed to Hippocrates. He introduces the notion of the general system, to which all the particular functions belong. In his theory, he ascribes to the system a further feature to those of the Presocratic, that of totality. He conceives the whole world and the human body as general systems composed of particular functions in equilibrium. Disturbance of the equilibrium in one of these functions results in disease, whereas the re-establishment of the equilibrium is healing. Of wider interest in Hippocratic speculation, is the development of the analogies between different physical phenomena or systems, contributing in this way to the concept of analogy, which in turn is closely related to the development of modelling.

Last but not least, Aristotle adds an additional characteristic to the system, that of hierarchy. Hierarchy is the dominant feature in the system of thought he develops. He distinguishes the levers of complexity in his system of Logic, by going from categories, to sentences, then to syllogisms, and finally to valid syllogisms, i.e., demonstrations. The following of the steps of hierarchy, that Aristotle has determined, and the arrival at the last level of demonstration are equivalent to the specification of the cause or the reason that explains an event or a conclusion of the system of thought.

Chapter 6

CONCEPTUAL MODELS, RELATIONS, AND METHODOLOGY

6. CONCEPTUAL MODELS, RELATIONS, AND METHODOLOGY

6.1 Introduction

Conceptual modelling, as the process of building the mental framework, precedes the other types of modelling. In the period of Homer and Hesiod, this is expressed as a process of creating symbols, e.g., the Gods as symbols of the human virtues, whereas in the age of Presocratic philosophers, the conceptualisation of fundamental notions and the knowledge acquisition and organisation contribute to the evolution of conceptual modelling. However, a direct theoretical analysis of the process of modelling, the types of models, as well as a representative expression of conceptual modelling by means of a reasoning methodology, the so-called Method of Dichotomy, are illustrated in Plato's work.

Concerning the issue of reality and its perception, Plato (427-348/47 B.C.) confutes the materialism of the Presocratics and maintains the omnipresence of design in universe. He is convinced that all things of the world – material or immaterial – are copies (models) of ideal prototypes (systems). In other words, he believes that behind the world of the senses - the world of imperfect and changeable phenomena - is the world of the perfect and immutable Ideas, where the models of all the sensible things and phenomena are found. He is the first to define what model (*ομοίωμα*) is and to analyse the types of models and the general process of modelling from a theoretical perspective.

Furthermore, he is able to see things that can naturally be compounded into one and divided into many and as a result he presents a method based on the principle of dichotomy, on the 'taking apart' or 'dividing' process (*διαίρεσις*), which is followed by the 'bringing-together' process (*συναγωγή*). Similarly to the modern conceptual modelling process, Plato commences his methodology by introducing the overall goal of the problem and carries on by identifying the relevant entities and their relations through a series of questions. In this way, he introduces a conceptual algorithm that leads to the definition of any specific concept. Before the elaboration and application of this algorithm, for example, on the definition of 'Sophist', Plato begins by taking up some lesser thing, e.g., the definition of 'angler', which is used as a model of the greater. He knows that if great things have to be properly worked out, one ought to practice on small and easier things before attacking the greatest. The way he deals with the great things (categories) by means of smaller (species) points to the modelling process, where the study of an unknown or complicated system is effectively achieved by means of a model that simulates the behaviour of it.

6.2 The Concept of Modelling in the Ancient Greek Sources

Notions that are associated with the concept of modelling are found in many ancient texts. Let us first site some of them:

- *Όμοιος*: similar, resembling something but not the same, alike (in Latin *similis*)
- *Ομοιότητα*: similarity, likeness, resemblance
- *Ομοίωμα*: model or image, remembrance, copy, or representation of something that is similar to the original

  Plato, *Phaedrus* 250a, b: "[...] souls do not easily recall the things of the other world; [...] only few retain an adequate remembrance (*ομοίωμα*) of them..." (Jowett, 1964, vol. 3)

- *Μιμέομαι*: become or be constructed absolutely similar, imitate
- *Είδωλον*: idol, image, model, phantasm

  Plato, *Laws* 959b: " [...] when we are dead, the bodies of the dead are quite rightly said to be our shades or images..." (Jowett, 1964, vol.4)

  Shadow, reflection

  Plato, *Sophist* 266d: "[...] and other products of human creation are also twofold and go in pairs; there is the thing [with which the art of making the thing is concerned], and the image [with which imitation is concerned]..." (Jowett, 1964, vol. 3)

  Plato, *Phaedo* 66b: "[...] when real philosophers consider all things, they will be led to make a reflection (*είδωλον*)..." (Jowett, 1964, vol. 1)

- *Παράδειγμα*: model or image (Eupalinus of Megara – 6th century B.C. – uses this word so as to describe the model he designed before the construction of the homonym aqueduct in Samos)
- *Ανάλογον*: proportionate or consistent to a proportion, in relation to something else, similar

  Plato, *Timaeus*, 69b: " [...] when all things were in disorder God created in each thing in relation to itself, and in all thing in relation to each other..." (Jowett, 1964, vol. 3)

- *Αναλογία*: proportion, equality of ratios (Plato, *Timaeus* 31C, 32C)

  Rate, similarity of relations

  Plato, *Statesman* 257b: "[...] When you rated sophist, statesman, and philosopher at the same value, though they are farther apart in worth than your mathematical proportion can express[28]...

The above-mentioned references obviously concern the general framework of the concepts of model and modelling. Notions, such as analogy and similarity are the paradigm under which the art of modelling is developed. However, besides these mediate allusions, there are also many immediate mentions and descriptions related to the art of modelling itself, as will be illustrated in the next section.

6.3 Plato's Theoretical Approach to Conceptual Modelling

6.3.1 The Art of Making Models

The art of modelling is being posed by the Greek philosophical thought in three realms:

- The first one has to do with the Being (*είναι*), i.e., with the structure, the form, and the constitution. It answers the question "what is the world made of?"
- The second one has to do with the Becoming (*γίγνεσθαι*), i.e., with the dynamic, and the operation. It answers the question "how does the world work?"

[28] This text is based on the following book(s): Plato. Plato in Twelve Volumes, Vol. 4, 9, 12 translated by H. N. Fowler, Cambridge, MA, Harvard University Press, London, William Heinemann Ltd. 1977, 1925, 1921, respectively

- The third one has to do with the Reason (*αίτιον*), i.e., with the primitive archetype or the way by which the model has been constructed, and answers the question 'why'.

Out of these three unities only the first one finds an integrated mathematical formulation in ancient times. The numbers, the geometry, and the stereometry enable the construction of models of the world's structure. The lack of integral and differential calculus does not permit the formation of mathematical models, which could explain systems' dynamical behaviour. This fact has as a result the need for philosophical interpretation of the dynamical behaviour of systems to become more urgent. The third category, which is related to the construction of models, occupies Plato in his works of *Republic* and *Timaeus*. Let us first cite Plato's references that reflect his view on this subject and then remark upon them:

> *Timaeus,* 28C-29A: " [...] we must go back to this question about the world: After which of the two models did its builder frame it – after that which is always in the same unchanging state, or after that which has come to be? Now if this world is good and its maker is good, clearly he looked to the eternal (model) [...] Everyone, then, must see that he looked to the eternal (model) [...] Again, these things being so, our world must necessarily be a likeness of something (*εικόνα κάποιου άλλου*)..." (Cornford, 1977)

The nature of Plato's eternal model is as follows:

> *Timaeus,* 30C: " [...] we have now to state what follows next: What was the living creature in whose likeness he framed the world? We must not suppose that it was any creature that ranks only as a species; for no copy of that which is incomplete can ever be good. Let us rather say that the world is like, above all things, to that Living Creature of which all other living creatures, severally and in their families, are parts. For that embraces and contains within itself all the intelligible living creatures, just as this world contains ourselves and all other creatures that have been formed as things visible." (Cornford, 1977)

Plato's view about the world as a likeness, a model of something eternal, is quite interesting. Up to now, we came across cases where the natural world was the unknown system that had to be specified. And the specification of its structure or behaviour was often undertaken indirectly through a model, conceptual, physical, or mathematical. Plato now adds a further conception, i.e., the visible world, the system, is portrayed as a living creature made after the likeness of an eternal intelligible model. Therefore, although in many areas of study the consideration of an unknown system requires a model, the latter is the everlasting concept that exists long before the system itself. And even more, the creation and existence of the system is based on the imitation of this model. Plato introduces in this way a circular relationship between the system and the model where it is trivial to recognise which comes first. But the process of modelling and the verification of a model is exactly this, i.e., a continuous transformation from the real system to its model and vice-versa.

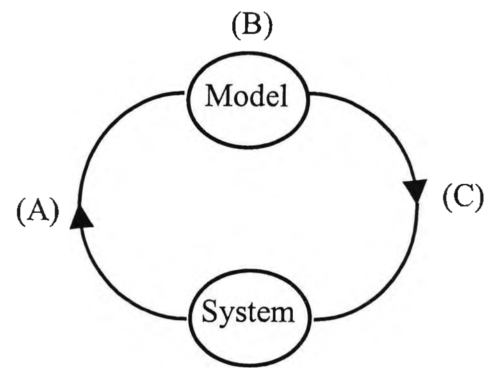


Figure 6.1: The process of modelling

The stage from the system to the model (A) concerns the observation, the experience, and the continuous attempt of search and verification of the real life system, the stage (B) concerns the synthesis, the planning and the operation of the model, and the stage from the model to the system (C) concerns the verification, the comparison and the application.

According to Plato, world's creation as a process of copying the real will be good, if the divine craftsman copies an eternal model. The eternal model is that one that determines the principles, the specifications, the conditions, and the demands that the under construction system, the world, must fulfil. We could say that this is the model "0", the primitive, the fundamental.

In *Republic* Plato introduces his distinction between the different types of models.

Republic X, 596a-602a: "Whenever a number of individuals have a common name, we assume them to have also a corresponding idea or form [...] Let us take any common instance; there are beds and tables in the world [...] But there are only two ideas or forms of them – one the idea of a bed, the other of a table [...] And the maker of either of them makes a bed or he makes a table for our use, in accordance with the idea [...] but no artificer makes the ideas themselves [...] And the painter too is, as I conceive, just such another – a creator of appearances [...] But then I suppose you will say that what he creates is untrue. And yet there is a sense in which the painter also creates a bed?

Yes, he said, but not a real bed [...]

Well then, here are three beds: one existing in nature, which is made by God [...] There is another which is the work of the carpenter [...] And the work of the painter is a third [...] Beds, then, are of three kinds, and there are three artists who superintend them: God, the maker of the bed, and the painter [...]

Of the painter we say that he will paint reins, and he will paint a bit [...] and the worker in leather and brass will make them [...] but does the painter know the right form of the bit and reins? Nay, hardly even the workers in brass and leather who make them; only the horseman who knows how to use them – he knows their right form [...]

There are three arts which are concerned with all things: one which uses, another which makes, a third which imitates them [...] And the excellence or beauty or truth of every structure, animate or inanimate, and of every action of man, is relative to the use for which nature or the artist has intended them [...] Then the user of them must have the greatest experience of them, and he must indicate to the maker the good or bad qualities which develop themselves in use; for example, the flute-player will tell the flute-maker which of his flutes is satisfactory to the performer; he will tell him how he ought to make them, and the other will attend to his instructions [...] The one knows and therefore speaks with authority about the goodness and badness of flutes, while the other, confiding in him, will do what he is told by him [...] The instrument is the same, but about the excellence or badness of it the maker will only attain to a correct belief; and this he will gain from him who knows, by talking to him and being compelled to hear what he has to say, whereas the user will have knowledge..." (Jowett, 1964, vol. 2)

In this passage, Plato reflects on the relationship between the actual bed constructed by a carpenter and the idea or definition of a bed in carpenter's mind. For him the good type of craftsman is the carpenter who makes the actual bed, taking for his model 'the real bed' – a form that he does not create or invent, but which exists in the nature of things. Conversely the bad type is a painter who takes for his model the carpenter's actual bed, i.e., a generated thing, and produces something that itself is not real. It is only an image of an image.

He distinguishes three types of models and three types of arts or applications:

a) The primitive or the fundamental model, which is the one made by god

b) That one made by the craftsman, whose target is to construct a functional model

c) That made by the imitator, who intends to create a sensible and aesthetically good model, but without any functionality.

Correspondingly, there are three different aspects of modelling:

a) The viewpoint of the user, the person who knows the application, the practical attribution, and the utility. He is the only who has the experience and can indicate the good or bad qualities to the maker.

b) The viewpoint of the maker, who knows how to make a functional thing but only under the instructions either of the "god", i.e., of the principles that govern the under construction thing or of the user. He is in the middle.

c) The last point of view is that of the imitator, and that is connected to the art. The imitator is out of the circle, which is formed by the user and the maker. He makes his own model without having in mind its application and without being based on the divine archetype. His archetype is that one, which is perceptible by the man. He considers the human peculiarities, the human "imperfections" and depicts the world according to them. For example, it is human characteristic to perceive the nearby object big whereas the faraway object small, i.e., it is human characteristic the perspective way of vision. Thus, a perspective schema of an object is the model not only of the object itself but also of the way that man sees the object. A characteristic example is that of the ancient Greek pillars. Their form (wider downwards and narrower upwards) is a model of the way that man perceives a huge pillar and in controversy, they give the impression that they are bigger than in reality.

The types of model described by Plato as well as the types of models formulated by our understanding nowadays, are represented in the following figure:

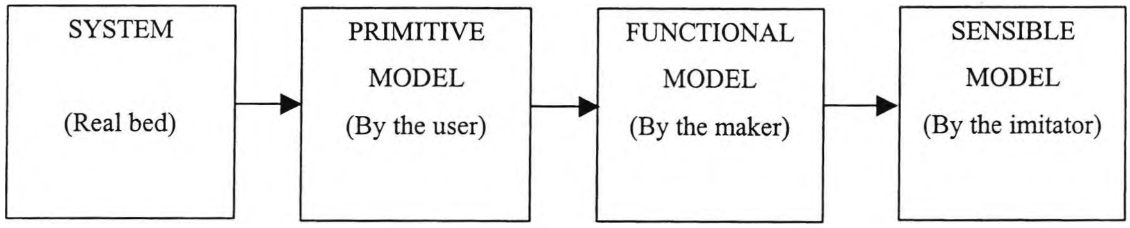


Figure 6.2: Types of model by Plato

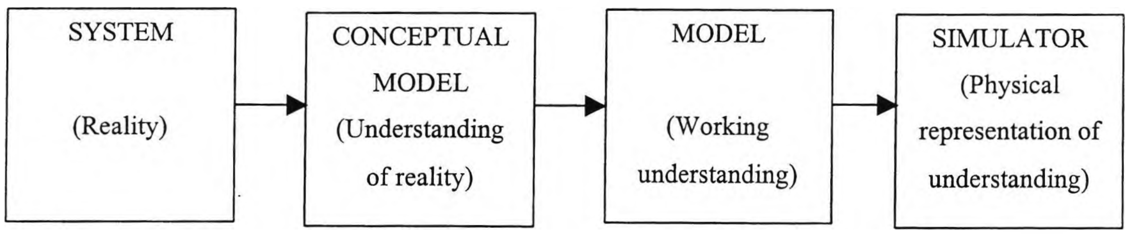


Figure 6.3: Contemporary types of model

It is obvious by these two figures that there is an absolute correspondence between the platonic models and models of today.

6.3.2 The Allegory of the Cave

In many cases, the process of modelling relates to the way the human mind works and more precisely to its operation of simplification. Mind functions in a way so as to acquire an overview of the basic characteristic of a domain. Correspondingly, in the process of modelling the domain is represented either by a set of equations, or by a picture or a diagram and the system is viewed from a certain scale and under certain limits and approximations. The system is considered as one identical whole, where its various features are considered as aspects of this whole. When isolation, unity, clarity, and homogeneity of perspective and viewpoint are reached, an adequate model is formed. This model represents either parts of the system and their connections, or states of the system and their connections, or simultaneously parts and states and their connections.

This art of representing a system, an object, a phenomenon, a plant, and generally something real or natural, by means of model construction, depends not only on the modelling process but also on other remarkable parameters. It depends first and foremost on a very subjective factor, i.e., the designer of the model himself including his scientific background and knowledge. In addition, the purpose of modelling, whether it is a theoretical interpretation or a practical application, the previous knowledge and experience, the material conditions, and the historical and social factors play an important role as well. Because of all these parameters, even if the system or the phenomenon we would like to model is constant and unchangeable, it is possible to have numerous models of it. Each time that one of these factors changes it results to another different model. Any model of today could be considered out-of-date the day after due to a change in one of the above parameters.

Let us see, how Plato by means of an allegory perceives all these factors and parameters involves in the process of modelling. In the aforementioned passages of *Timaeus* and *Republic*, Plato introduces clearly the notion of model as the approximation or projection of reality. In the known passage of the 'allegory of the Cave', he gives an indicative description on the way we understand the reality, which is interrelated to the process of creating conceptual models and in general to the process of modelling:

> *Republic,* 514a-518b: " [...] Human beings housed in a underground cave, which has a long entrance open towards the light and as wide as the interior of the cave; here they have been from their childhood, and have their legs and necks chained, so that they cannot move and can only see before them, being prevented by the chains from turning round their heads. Above and behind them a fire is blazing at a distance, and between the fire and the prisoners there is a raised way; and you will see, if you look, a low wall built along the way [...] men passing along the wall carrying all sorts of vessels, and statues and figures of animals made of wood and stone and various materials, which appear over the wall [...] Some of them are talking, others silent [...] and they see only their own shadows, or the shadows of one another, which the fire throws on the opposite wall of the cave [...] how could they see anything but the shadows if they were never allowed to move their heads? And of the objects that are being carried in like manner they would only see the shadows [...] and if they were able to converse with one another, would they not suppose that they were naming what was actually before them? [...] And suppose further that the prison had an echo which came from the other side, would they not be sure to fancy when one of the passers-by spoke that the voice which they heard came from the passing shadow? [...] To them, I said, the truth would be literally nothing but the shadows of the images ... " (Jowett, 1964, vol. 2)

By this allegory Plato studies the process of creating a model of the real world and the errors that may be contained in this model. We can infer the following:

1. According to Plato the shadow is an interesting model of a real object. The process of modelling is described as a process of depicting something, e.g., the depiction of object's shadow on a wall or a screen.
2. The model is the true image of the real system. It may be contain errors that have to do with the way of depicting the real system.
3. The accuracy of the model depends firstly on the light that lights the real object, which is either the sun or the fire in the cave. It means that it depends on the point of view of the system's examination or on the kind of modelling process.
4. It also depends on the ability to comprehend parts of the system or the whole of it. The one-sided viewpoint of modelling process is expressed allegorically by the disability of the prisoners to move their legs and necks.
5. Another factor is the experience and the existent knowledge. These are symbolised here by the fact that the prisoners born in this cave never have seen the real world.

Thus, the process of modelling according to Plato has the next parameters:

- Representation of real system
- Determination of the factors that affect modelling
- Determination of the viewpoint
- Securing multi-sided facing of the problem
- Essential knowledge and experience.

6.3.3 Method of Dichotomy or Division

Many dialogues of Plato's works have as their subject matter the definition of a specific concept. According to Plato, the parley of a theme demands in the first instance its definition, which in turn means to collect all its *disiecta membra*, partial components, into a uniform whole, a unified Idea. Based on the concept of dichotomy, on the principle 'the one is divided into two', Plato introduces a particular form of his dialectic method, i.e., the Dichotomous Method. The real dialectical person is the one who knows to divide properly an idea or a concept that is under investigation. In many works, and in particular in the dialog of *Sophist*, which could be claimed to be the last step of Plato's dialectic method and the preamble of the Theory of Ideas, we can find applications of the method of dichotomy. The objective purpose of *Sophist* is the nature of negation, the production of knowledge, the mental process and the methodology that leads to the establishment of Science (epistemology). In addition, the effort to define properly the notion of Sophist is undertaken by means of the method of dichotomy. Before the overall formulation and application of the method, Plato quotes a simple example, or in other words, he creates a model – *angler's definition* – by means of which he develops the process and the steps of his method. The method uses as a criterion a property that has to be defined, and which is successively approximated by starting from bigger well-defined categories and then by a 'guided' division converging to the property. As a result, a genealogical tree of all the concepts related to basic subject is created. These concepts are classified in successive levels, by means of an inductive reasoning process, a logical algorithm that goes from the general to the particular. The platonic method may be regarded as a dynamical process that goes either from the general to the particular, from the cause to the result (analysis), or from the particular to the general, from the result to the cause (synthesis).

- **Example 1: Angler's definition, *Sophist,* 218d-220e**

Assume that the 'angler' is the parameter X that Plato intends to define. Before beginning the division process, he determines that the angler is a craftsman, i.e., a man possessing some skill, some art. This is the primary category from where the division begins: Category **A: Man.** The next category **B** arises by dichotomy. Man is divided into: **b_1: man not having an art**, and **b_2: man having an art**. Decision: Angler is a man having an art. Category **C**: Arts are divided into: **c_1: productive or creative art**, the art of constructing or producing, and **c_2: acquisitive art**, the art of learning and cognition. Decision: The art of angling belongs to the acquisitive art. Category **D**: The acquisitive art is subdivided into: **d_1: of exchange**, that is effected by gifts, hire, purchase, and **d_2: of conquest**, which takes by force of word or deed. Decision: The art of the angler is that of the conquest. Category **E**: The conquest is subdivided into: **e_1: the open or visible force** (fighting), and **e_2: secret or hidden force** (hunting). Decision: The art of the angler is that of hunting. Category **F**: Hunting is subdivided into: **f_1: of living prey or animal hunting,** and **f_2: of lifeless prey.** Decision: The art of the angler is that of the animal hunting. Category **G**: Living prey is subdivided into: **g_1: land-animal hunting**, and **g_2: water-animal hunting** (hunting of swimming animals, i.e., fishing). Decision: The art of the angler is that of water-animal hunting. Category **H**: Water-animal hunting is subdivided into: **h_1: capture with enclosures** (takes them when they are in nets), and **h_2: capture by striking** (takes them by a blow). Decision: The art of the angler is that of capture by striking. Category **J**: Capture by striking is subdivided into: **j_1: by the light of a fire** (i.e., fishing by torchlight), and **j_2: in daytime** (i.e., hunting with sharp points, barbing). Decision: The art of the angler is of barbing. The model that Plato creates by means of the angler's example is shown in the following figure:

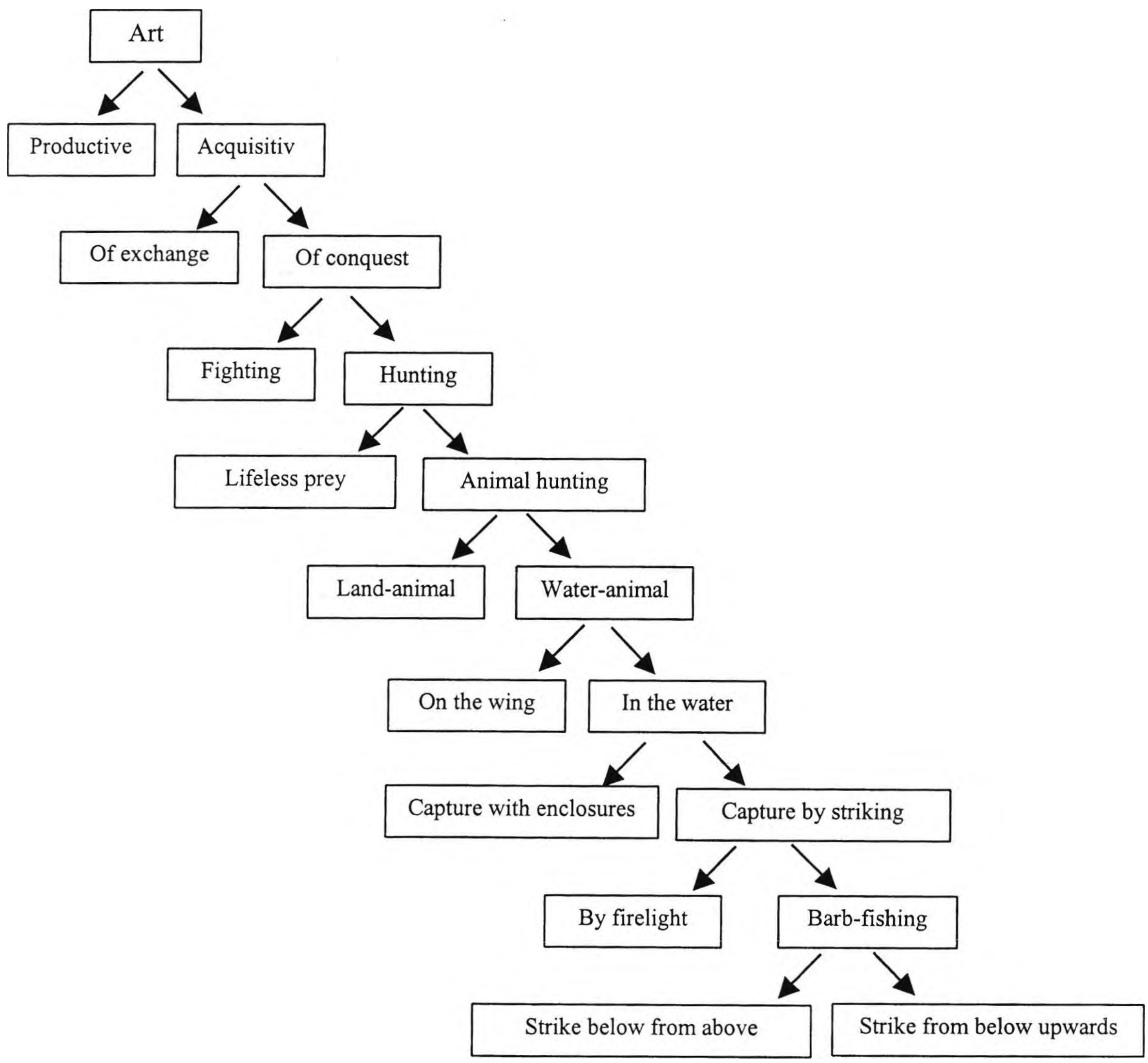


Figure 6.4: Platonic model of the Dichotomy Method

According to this Platonic method, one keeps on the procedure by leaving each time the left side of the division unexamined (b_1, c_1, d_1, e_1, f_1, g_1, h_1), and by elaborating only the right sub-category, i.e., by dividing the right side in smaller and smaller sub-categories to some extent where one right sub-category agrees with the initial parameter X. The definition of X results by 'bringing together' the successive characteristics of the right subdivisions (b_2, c_2, d_2, e_2, f_2, g_2, h_2). Thus, the 'compounding' process leads to the following definition:

> **Definition**: One half of the entire art was acquisitive, and, of this, half was conquest or taking by force, and, of this, half was hunting, and half of hunting was hunting animals, and half of this was hunting water animals, and the lower part of this was fishing, and, of this, half was striking; and a part of striking was fishing with a barb, and one half of this again, being the kind which strikes with a hook and draws the fish from below upwards, and this is the art which we have been seeking, and which from the nature of the operation is denoted angling or drawing up.

A second example of this method concerns the definition of the sophistic art and in particular of the Sophist, the professor of wisdom. The procedure begins as before, i.e., by determining the primary category and then proceeds in making the proper divisions.

- **Example 2: First Sophist's definition, *Sophist,* 221a-223b**

Category A: Man. Category **B**: Man is divided into: **b_1: man not having an art**, and **b_2: man having an art**. Decision: Sophist is a man having an art. Category **C**: Arts are divided into: **c_1: productive or creative art**, the of constructing or producing, and **c_2: acquisitive art**, the art of learning and cognition. Decision: The art of Sophist is acquisitive art. Category **D**: The acquisitive art is subdivided into: **d_1: of exchange**, that is effected by gifts, hire, purchase, and **d_2: of conquest**, which takes by force of word or deed. Decision: The art of the Sophist is that of the conquest. Category **E**: The conquest is subdivided into: **e_1: the open force** (fighting), and **e_2: secret force** (hunting). Decision: The art of the Sophist is that of hunting. Category **F**: Hunting is subdivided into: **f_1: of living prey,** and **f_2: of lifeless prey.** Decision: The art of the sophist is that of the living prey. Category **G**: Living prey is subdivided into: **g_1: land hunting,** and **g_2: water hunting**. Decision: The art of the sophist is that of the land hunting. Category **H**: Land hunting is subdivided into: **h_1: hunting of tame animals** (hunting of men), and **h_2: hunting of wild animals**. Decision: The art of the Sophist is that of tame animals hunting. Category **J**: Tame animals hunting is subdivided into: **j_1: hunting with violence,** and **j_2: hunting by the art of persuasion**. Decision: The art of the Sophist is of capture persuasion. Category **K**: Art of persuasion is subdivided into:

k_1: **private**, and k_2: **public**. Decision: The art of the Sophist is private. Category L The private part is subdivided into: l_1: **receives hire**, and l_2: **brings gifts**. Decision: Sophist receives hire. Category M: Hire is subdivided into: m_1: **for the sake of virtue demands money**, the **Sophistic**, and m_2: **possessing flattery.**

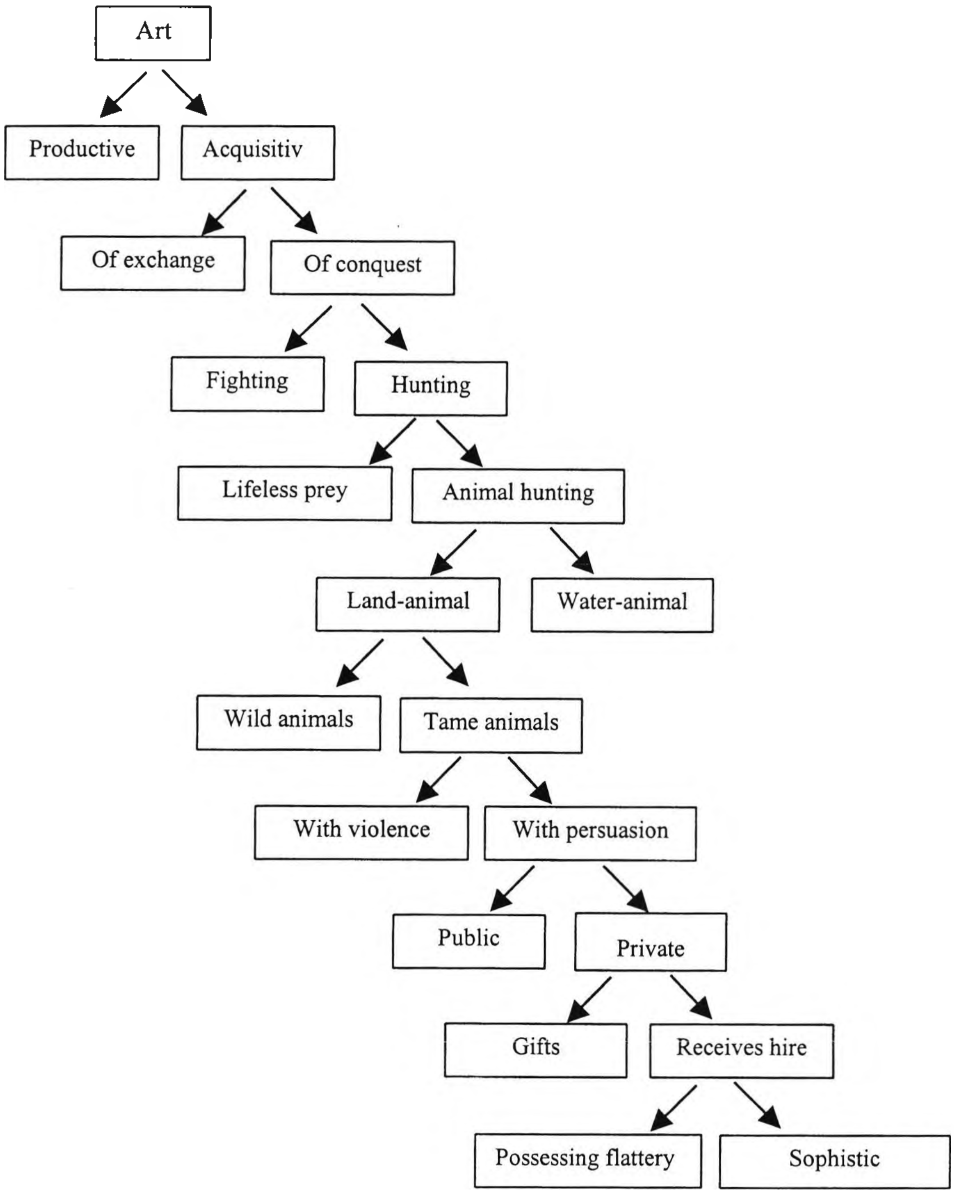


Figure 6.5: Dichotomy Method for the 1st definition of the Sophist

Therefore, the definition of the sophistic art, i.e., the sophistry, arises by the following algorithm (figure 6.5):

Plato, *Sophist,* 223b: " [...] his art may be traced as a branch of the appropriative, acquisitive family – which hunts animals – living – land – tame animals; which hunts man,

– privately – for hire – taking money in exchange – having the semblance of education; and this is termed sophistry, and is a hunt after young men of wealth and rank – such is the conclusion." (Jowett, 1964, vol. 3)

The third example of the Dichotomy method deals with another definition of the sophist, i.e., with another procedure or path that ends in the same result.

- **Example 3: Second Sophist's definition:** ***Sophist,*** **223c-224e**

Categories **A**, **B**, **C**, and **D** are the same. Category **E**: The art of exchange is subdivided into: $\mathbf{e_1}$**: of giving**, and $\mathbf{e_2}$**: of selling**. Decision: The art of the Sophist is that of selling. Category **F**: Selling is subdivided into: $\mathbf{f_1}$**: sale of a man's own productions**, and $\mathbf{f_2}$**: exchange of the works of others**. Decision: The art of the Sophist is exchange of the work of others.

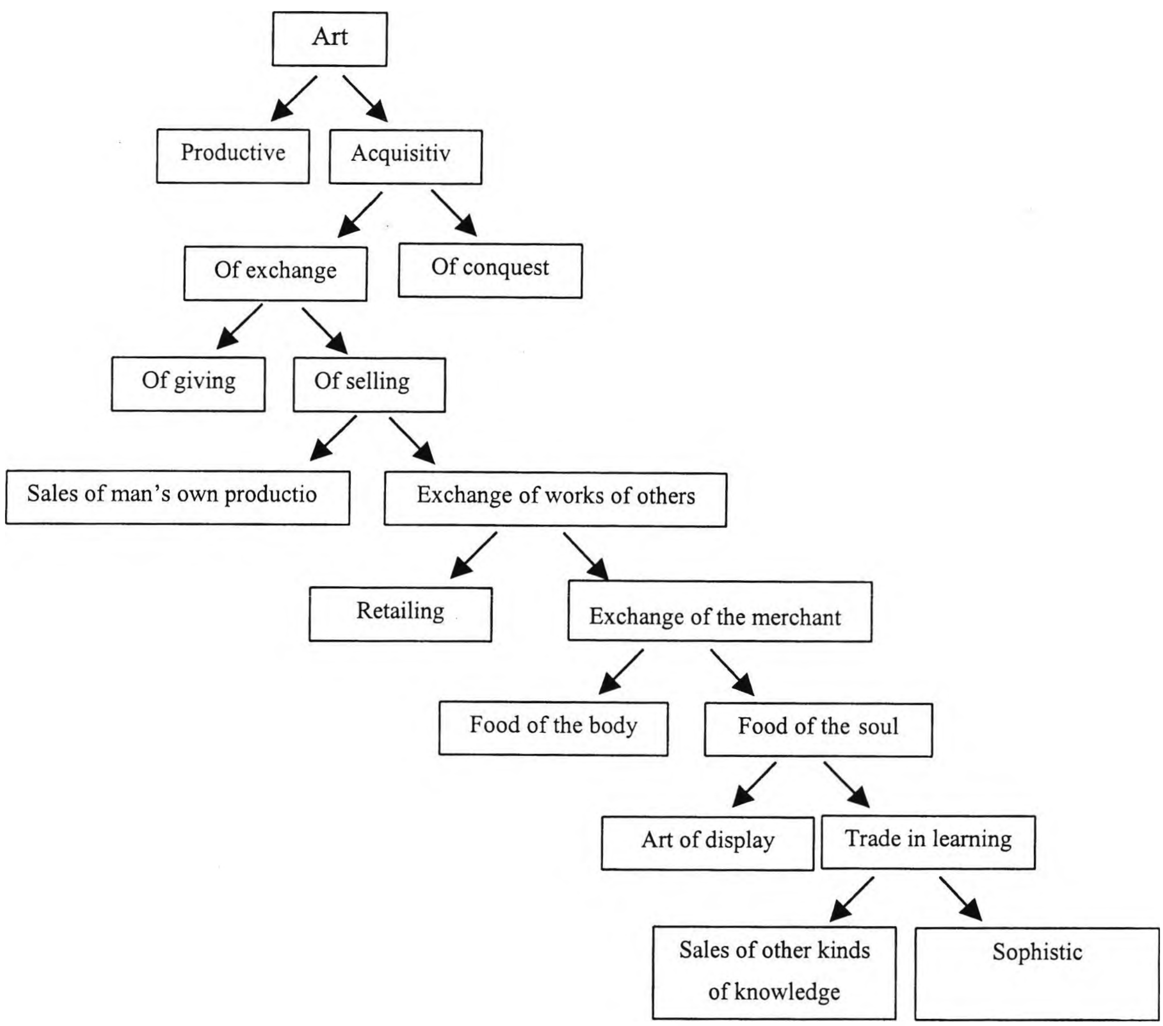


Figure 6.6: Dichotomy Method for the 2nd definition of the Sophist

Category **G**: Exchange of the work of others is subdivided into: **g_1: retailing,** and **g_2: exchange of the merchant**. Decision: The art of the Sophist is of exchange of the merchant. Category **H**: Exchange of the merchant is subdivided into: **h_1: food for the body**, and **h_2: food of the soul**. Decision: The art of the Sophist is of the food of the soul. Category **J**: The food of the soul is subdivided into: **j_1: the art of display**, and **j_2: trade in learning**. Decision: Sophist's art is of trade in learning. Category **K**: Trade in learning is subdivided into: **k_1: sale of the knowledge of virtue (sophistic)**, and **k_2: sale of other kinds of knowledge.**

We derive the second definition of the sophistic art from the following algorithm (figure 6.6):

> Plato, *Sophist,* 224e: " [...] Then that part of the acquisitive art which exchanges, and if exchange which either sells a man's own productions or retails hose of others, as the case may be, and in either way sells the knowledge of virtue, you would again term sophistry?" (Jowett, 1964, vol. 3)

The forth example of the platonic method has to do again with the definition of the sophist.

- **Example 4: Third Sophist's definition: *Sophist,* 225a-226a**

Categories **A**, **B**, and **C** are the same. Category **D**: The acquisitive art is subdivided into: **d_1: of exchange**, that is effected by gifts, hire, purchase, and **d_2: of fighting.** Decision: The art of the Sophist is that of the fighting. Category **E**: The art of fighting is subdivided into: **e_1: the competitive,** and **e_2: the pugnacious**. Decision: The art of the Sophist is that of pugnacious. Category **F**: Pugnacious is subdivided into: **f_1: violent**, and **f_2: controversy**. Decision: The art of the Sophist is of controversy. Category **G**: Controversy is subdivided into: **g_1: forensic controversy,** and **g_2: disputation**. Decision: The art of the Sophist is of disputation. Category **H**: Disputation is subdivided into: **h_1: random discussion about contracts**, and **h_2: argumentation,** by rules of art to dispute. Decision: The art of the Sophist is of argumentation. Category **J**: Argumentation is subdivided into: **j_1: one sort wastes money**, and **j_2: the other makes money.** Decision: Sophist's art is of making money.

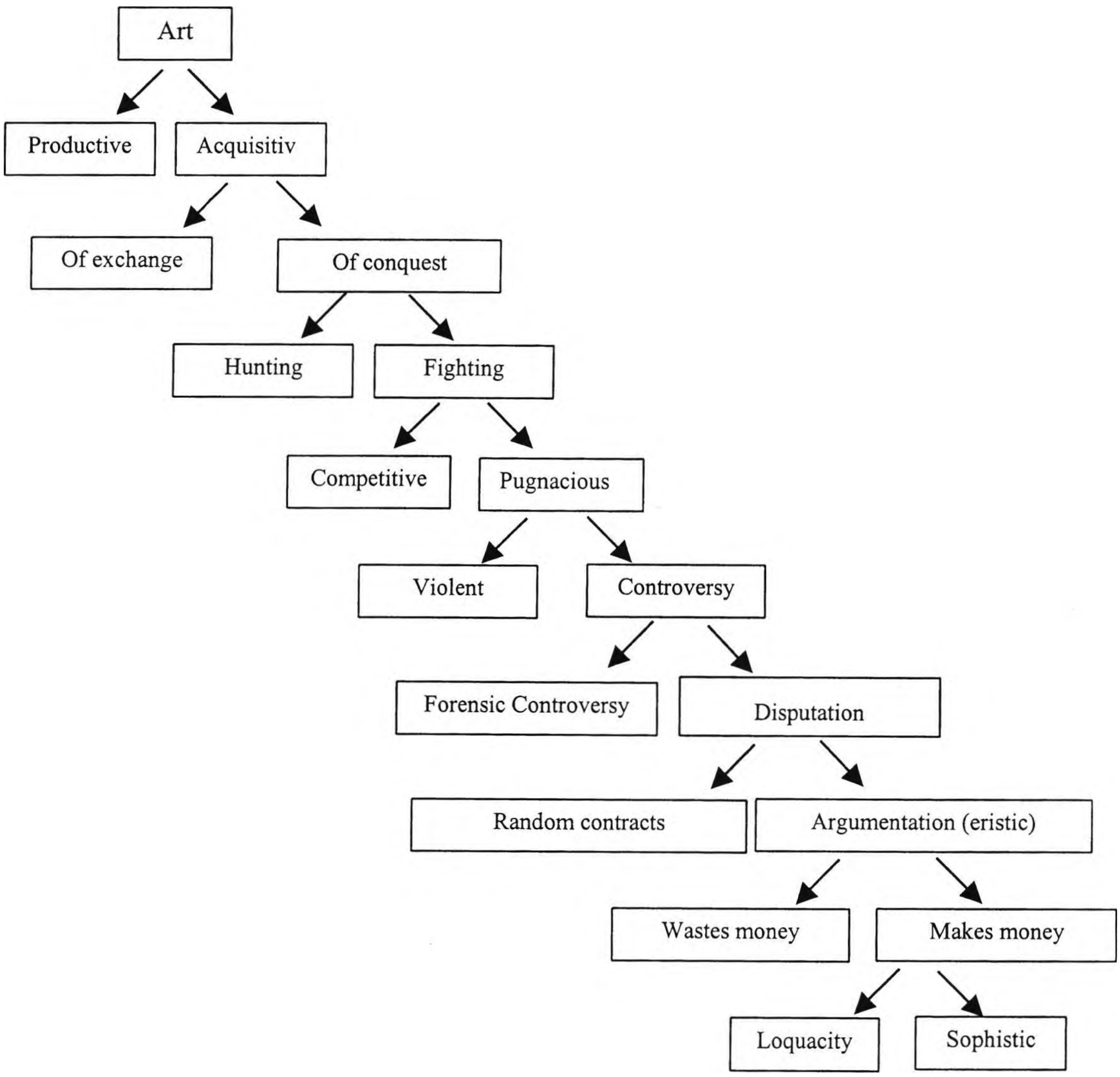
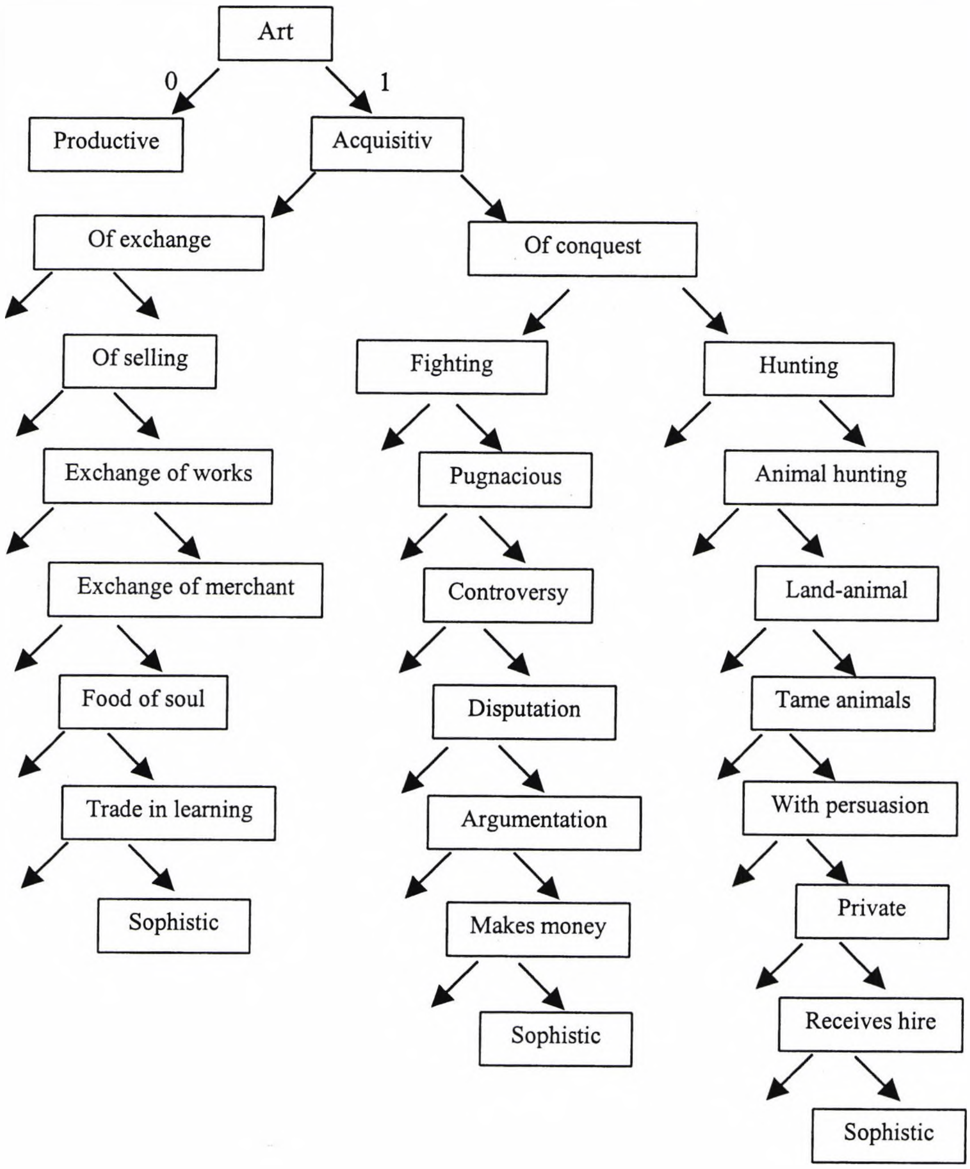


Figure 6.7: Dichotomy Method for the 3rd definition of the Sophist

And the third definition of the sophist is (figure 6.7):

Plato, *Sophist,* 226a: "[...] he is the moneymaking species of the Eristic, disputatious, controversial, pugnacious, combative, acquisitive family, according to this latest turn of the argument." (Jowett, 1964, vol. 3)

The combination of the three different definitions of the Sophist are shown in the following figure:



Sophistry = **(1 0 1 1 1 1 1 1)** = **(1 1 0 1 1 1 1 1 1)** = **(1 1 1 1 1 1 1 1 1 1)**

Figure 6.8: The definitions of Sophistic

The method of Plato creates a dual system that allows the correspondence of a complicate dual number to each level of the dichotomic range. In the above figure we use the symbols of '0' and '1' for the left and the right subdivision, respectively.

Plato applies this method in order to distinguish the numbers. He considers that numbers are produced by a successive dichotomic process and gives the name of 'eidetic' number. It is shown below:

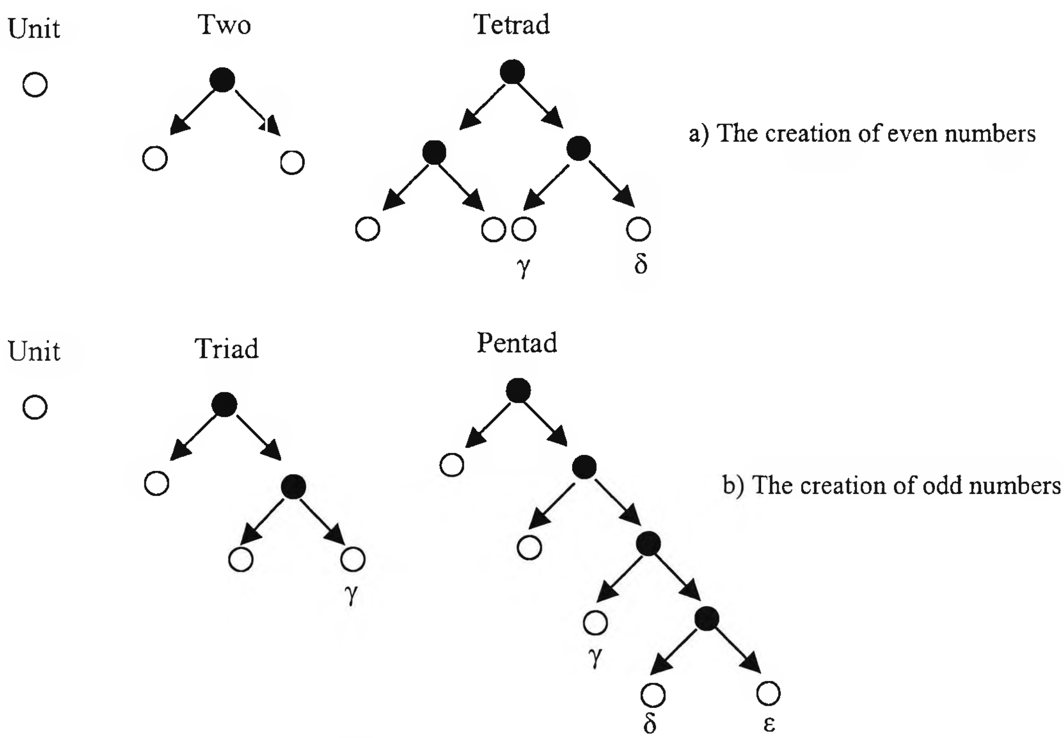


Figure 6.9: Eidetic numbers of Plato

The each time considered concept, idea, or category consists of the following two components: the limited and the unlimited. By examining further any unified concept, category, it is discovered that it contains other concepts, sub-categories, or species. And we continue this procedure of examining the new concepts and finding the species they contain, as long as new species arise. Only if the procedure cannot be repeated because of the lack of new species, we leave 'things to go to the infinite, to the unlimited' (Taylor, 1976). By this method, we ascertain that each category or species is simultaneously one and unlimited many, as well as how many it is. For example, the saying that the animal concept constitutes a category and there is an indefinite number of animals is not adequate; we should undertake a rational division that will conclude how many and which the animals are.

If we try to depict schematically (figure 6.10) and in an integrated form the platonic method we will result in a type of a genealogical tree that leads from the general category (genus) to the particular species. The binary numbers (0, 1) indicate the left and the right subcategories respectively, and their combinations the particular species. Thus, each species at any category level can be determined by the group of the previous decisions that lead to it, or simpler by a binomial number.

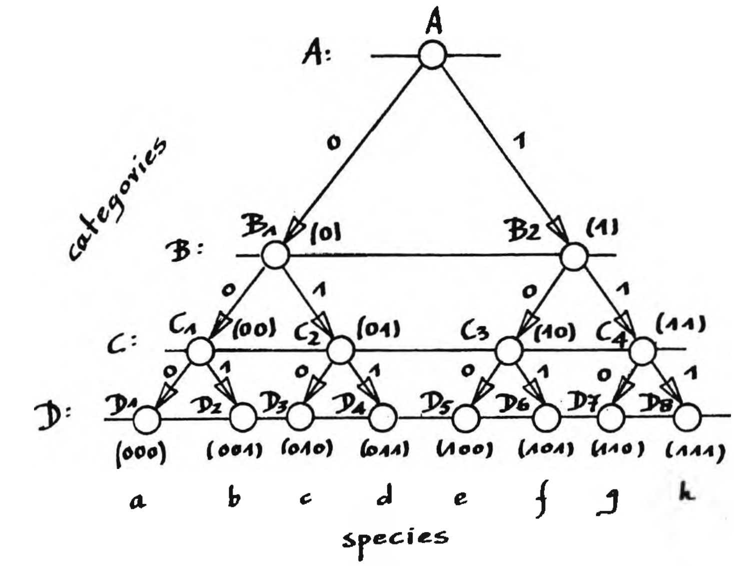


Figure 6.10: The genealogical tree of the Dichotomy Method, starting from the primary category A (genus) and ending to the final category D (species)

A Cartesian and a Polar form of the division method are shown below:

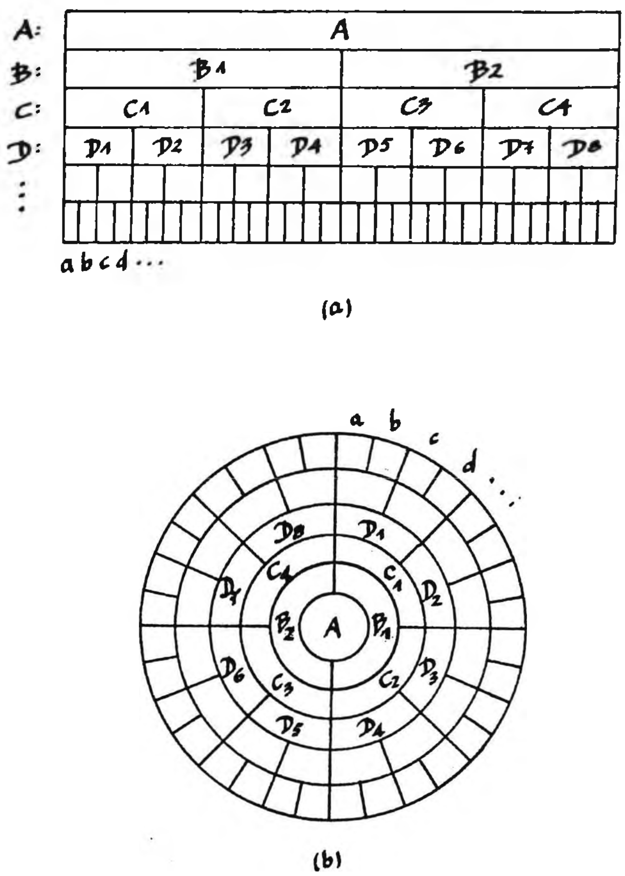


Figure 6.11: Cartesian and polar form of the Dichotomy Method

6.3.4 Dichotomy Method & Achilles' Shield

In his epic of Iliad, Homer dedicates almost one whole rhapsody in the construction of Achilles' shield by Hephaestus. The technical descriptions that decorate the poetic verses and unite the poetry with the technology represent the shield as an existent work of unrivalled art and technique. The shield itself depicts the whole world, natural and human, since on it not only the earth, the heavenly bodies, and the fixed stars appear, but also the human cities and activities. Therefore, as it has already been represented in chapter 3, it constitutes a complete model of the universe. A model that combines and presents by means of a masterly logic and land-planning arrangement, on the one hand, the geocentric structure of our solar system, and on the other, the anthropocentric form of the Greek archaic society with its cities and human activities. In the Homeric shield, it is possible to find the fundamental principles of the dialectic thought, the origins of the dynamic dichotomy method that constitute the foundation of the platonic philosophy.

Thus, the Homeric shield, as it is shown in the figure 3.12, can be considered as the polar form of the platonic dichotomy method and in general as the precursor of the philosophical and scientific ideas of the classical period. Moreover, if we depict the shield's elements in the genealogical tree form (figure 6.12), we may assume that the structure and the description of the shield follow the main steps of the dichotomy method, and probably they have an influence on the platonic thought.

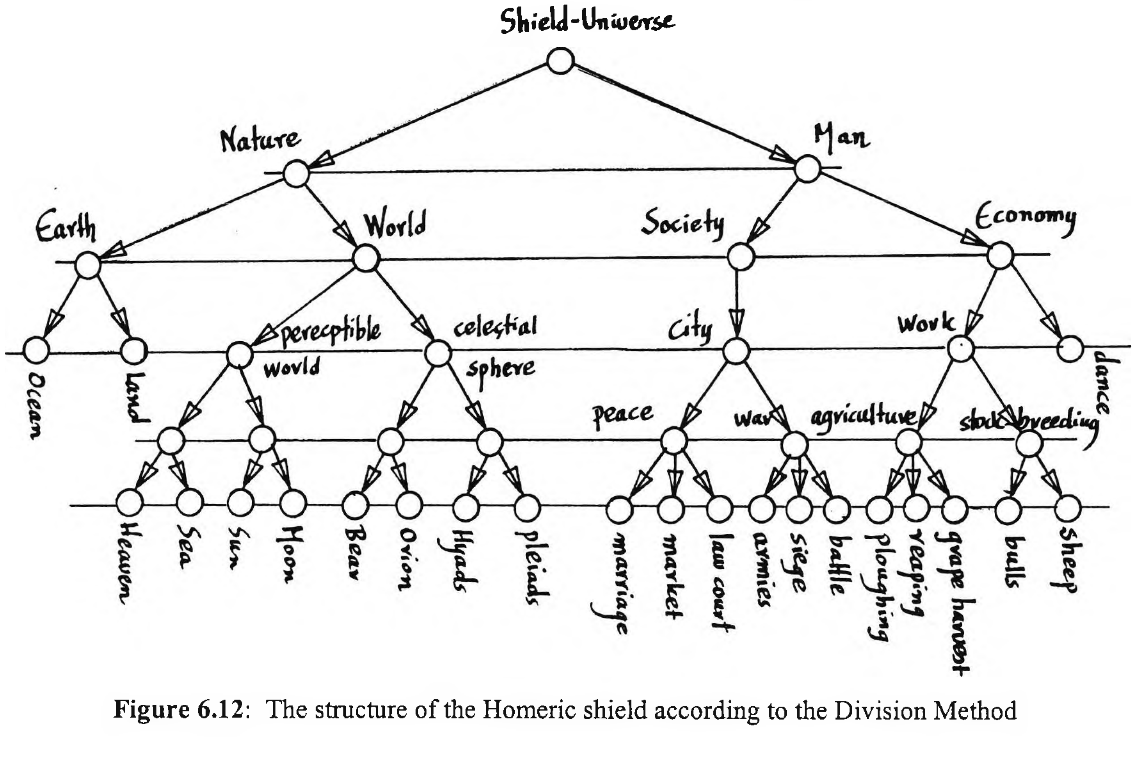


Figure 6.12: The structure of the Homeric shield according to the Division Method

6.3.5 Aristotle's Logic and Plato's Division Method

One could argue that Aristotle's Syllogistic Method is an improved version of the Dichotomy Method of Plato. According to Aristotle, the Platonic Logical Division constitutes only a mere fragment of the syllogistic procedure, and those who employ it are ignorant of Syllogism. However, Aristotle has based his ideas on the Logical Division method in order to develop his syllogistic method. The latter is confirmed in his work of:

Prior Analytics I 31(46a 31-33): "it is easy to see that division into classes [Plato's method] is a small part of our syllogistic method..." (Ross, 1971, vol. 1)

Continuing his view on the platonic method, Aristotle says that Plato uses the method of division so as to determine the substance of a particular concept or subject and to classify it within the logical hierarchy. He criticises the platonic method because it tries to demonstrate – what never can be demonstrated – the essential constitution of the Subject: *Prior Analytics* I 31(46a 34): "the division method by Plato is a weak syllogism..." Instead of selecting a middle term, as the Syllogism requires, more universal than the Subject but less universal than the Predicate, it inverts the proper order, and takes for its middle term the highest universal. What really requires to be demonstrated, it never demonstrates but assumes (Grote, 1872).

For example, for the determination of the substance of man, the platonic method begins by laying down that man is an animal, and that every animal is either mortal or immortal, and that it can be either logical or non-logical, and so on, till the point where the process of division reaches the initial subject, which is 'man'.

Prior Analytics I 31 (46b1-12): "Division [Plato's Method] has a contrary intention: for it takes the universal as middle. Let animal be the term signified by A, mortal by B, and immortal by C, and let man, whose definition is to be got, be signified by D. The man, who divides, assumes that every animal is either mortal or immortal: i.e., whatever is A is all either B or C. Again, always dividing, he lays it down that man is an animal, so he assumes A of D as belonging to it. Now the true conclusion is that every D is either B or C, consequently man must be either mortal or immortal, but it is not necessary that man should be a mortal animal[29]..." (Ross, 1971, vol. 1)

[29] Analytica Priora, Translated by A. J. Jenkinson, M.A.

In the above-mentioned example, the most universal term, animal, is selected as middle or as medium of proof; while after all, the conclusion demonstrated is not that man is mortal, but that man is either mortal or immortal. The position that man is mortal is assumed but not proved (*Prior Analytics* I 31 (46b1-12)). On the other hand, the syllogistic method has exactly the opposite aim, i.e., to prove that man is a mortal animal and not to assume it.

Despite the criticism, Aristotle's syllogistic method has been established upon the basic mechanism of the platonic division method. The mechanism of division that classifies in a logical way the sub-divisions of a general subject constitutes the critical methodological starting point of a syllogism.

6.4 Conclusion

Conceptual modelling, in modern terms, is defined as a process that involves knowledge acquisition, structuring, and representation, and leads to the construction of a functional schema that serves a predefined set of goals. The main issues in conceptual modelling are: Specification of goals and objectives that usually are given as external inputs to the problem, specification of the relevant framework of the problem, definition and representation of entities and their relations, leading to the conceptual schema of the problem, verification or validation of the right schema, with respect to the predefined goals and objectives, and modification of derived schema in order to satisfy the objectives.

In this chapter, we find the concept and the main issues of conceptual modelling, as well as the fundamental elements that contributed to the evolution of modelling through Plato's philosophy on the process of modelling, the types of models, and the conceptual algorithm derived by the method of dichotomy. Plato occupies himself with the process of knowledge and develops a theory of 'two' worlds', the world of ideal prototypes and the copy, the model of this world, i.e., the real world. Even though Plato ignores that even this ideal world arises through human mind and the reality the mind experiences, the focal point in his theory, is the perception and description of the process of modelling, in other words, the process of the projection of a richer and more complicated world (system) to another simpler and more perceptible world (model), but in a diverse way. The real world is, on the one hand, the unknown system that is examined through models that simulate its structure and behaviour, and on the other, the model of another eternal world, according to Plato. This simultaneous consideration of a system as the model of other systems also refers to a hierarchy in a complicated whole composed of many sub-systems, where the relation of a system to the lower sub-system is a relation of the system with its model.

The noteworthy part of Plato's theory, directly connected with our investigation of the origins of modelling, is also his classification of the types of models: primitive, functional, and sensible model or, in modern terms, the conceptual model of understanding the reality, the working model, and the physical representation of this understanding by the simulator.

Following, Plato introduces the Dichotomy or Division method as a process, a tool that allows the definition of complex notions in terms of fundamental concepts, and encloses the main issues of conceptual modelling methodology. First and foremost, Plato specifies the objective of applying his method, which is the definition of a specific topic. He tries to find the general category, to which the investigated concept belongs, i.e., specification of the general framework. The next step is to divide the general category into sub-categories and species, i.e., definition of the entities and their relations. As a result, he develops an algorithmic process, where the intended definition arises by means of finite successive steps and decisions. More precisely, the under consideration definition is given by the combination of the decisions that have been made in any single step. Finally a conceptual schema arises, a net or a tree of transition, either from the general to the particular or from the particular to the general, and constitutes a topological model of the general category, its sub-categories, and the species. The platonic method bears similarities with the top-down, bottom-up modelling approaches that can be combined, depending on the type of the system.

In the centre of the Platonic method is the effort to reduce the whole into simpler parts. This process can be interpreted as an effort to reduce complexity of objects by division. Between the general (whole) and the particular (part), a scale of successive notions, which approach gradually the species, comes into being. In this way, a two-dimensional table of notions arises, which includes the species and their categories ordered and numbered by a binary numeration.

Chapter 7

THE GEOMETRICAL AND MATHEMATICAL APPROACH TO MODELLING

7. THE GEOMETRICAL AND MATHEMATICAL APPROACH OF MODELLING

7.1 Introduction

According to (Walter, 1984), "mathematical modelling is an attempt to describe some part of the real world in mathematical terms. It is an endeavour as old as antiquity but as modern as tomorrow's newspaper. Mathematical models have been built in the physical, biological, and social sciences. The building blocks have been taken from calculus, algebra, geometry, and nearly every other field within mathematics. A mathematical model is a model whose parts are mathematical concepts, such as constants, variables, functions, equations, inequalities, and so on."

If we were to consider the chronological order of the evolution of the building blocks contributing to the genesis and development of mathematical modelling, we would start with Thales and his generic types of proposition and concept of analogy. However, we will not be persistent to the chronological consideration of the fundamental steps of the early mathematical thought. We will study the Pythagoreans and their great progress in the theory of numbers firstly, and then we will describe the evolution of the concept of analogy from Thales to Heron of Alexandria. The Pythagorean consideration that numbers dominate the whole universe results in the formation of a mathematical view of nature. For Pythagoreans the "real" is the mathematical harmony that is present in nature. They discover the wonderful progressions in the notes of the musical scale by finding the relation between the length of a string and the pitch of its vibrating note. They extend these relations to the distances the heavenly bodies have from the earth - the centre of the world. In this way, they also work out the concept of analogy, a crucial concept for the construction of models and the development of modelling.

Other milestones of early mathematical thought contributing to the building of mathematical modelling are related to geometry, measurement, and calculus. The conception of the basic geometrical figures as models of physical things, the process of measurement, of correspondence between numbers and magnitudes, as well as the early calculus by Archimedes are only some of the milestones, which will be elaborated in this chapter.

7.2 Mathematical Conception of the World

7.2.1 The Concept of Number

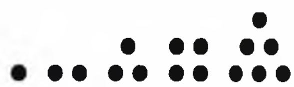
Shortly after the Ionian philosophers and their physical and mechanical models of the cosmos, the so-called Italian School of Pythagoras (582-500 B.C.) and his followers hit upon a different idea and technique. Convinced that pure mathematics rather than mechanics held the key to science, their "models" were mathematical rather than mechanical (Brumbaugh, 1966).

The Pythagoreans, in opposition to the Ionians, do not try to describe the universe in terms of the behaviour of certain material elements and physical processes but they describe it exclusively in terms of numbers. In other words, whereas the Ionians ask, "What is the world made of?" the Pythagoreans ask, "What is its structure?" i.e., the former see some kind of matter to be the basic world substance, and the latter see number or form as the first principle. Both schools make a definite distinction between matter and form until Aristotle, who introduces a close relation between matter and form, as we saw in chapter 4. Pythagoreans' assertion that numbers are the substance of all things and causes of the reality is a remarkable feat of intellectual abstraction. Aristotle declares their doctrine:

> Aristotle, *Metaphysics* A5, 985b 25 - 986a 5: "Contemporaneously with these philosophers and before them, the so-called Pythagoreans, who were the first to take up mathematics, not only advanced this study, but also having been brought up in it they thought its principles were the principles of all things. Since of these principles numbers are by nature the first, and in numbers they seemed to see many resemblances to the things that exist and come into being-more than in fire and earth and water (such and such a modification of numbers being justice, another being soul and reason, another being opportunity-and similarly almost all other things being numerically expressible); since, again, they saw that the modifications and the ratios of the musical scales were expressible in numbers; since, then, all other things seemed in their **whole nature to be modelled on numbers**, and numbers seemed to be the first things in the whole of nature, they supposed the elements of numbers to be the elements of all things, and the whole heaven to be a musical scale and a number[30]." (McKeon, 1941)

[30] Metaphysica, Translated by W. D. Ross

In mathematics, the Pythagoreans make very great progress, particularly in the theory of numbers and in geometry of areas and solids. These early mathematicians imagine numbers as functions of space, which results in a close connection between number and size. More precisely, they depict numbers as dots in sand or as pebbles, i.e., picture the numbers as having characteristic designs and classify them according to the geometrical figures made by the arrangements of the dots or pebbles. In general, the Pythagorean dot is used to provide a visual pattern for representing numbers. For example, the numbers 1, 2, 3, 4, 5 according to the dot pattern are:



This method is at once more abstract and more powerful; for the schema illustrates relations between numbers and leads to the discovery of new types of numbers, the so-called figured numbers[31], such as the triangular numbers 1, 3, 6, 10, ... because the corresponding dots could be arranged as triangles, and the square numbers 1, 4, 9, 16, ... because the corresponding dots could be arranged as squares.

Figure 7.1: Triangular and square numbers

The numbers either as geometrical depictions by the Pythagoreans, triangular and square, odd and even, male and female or as numerals, as symbols that at the beginning were expressed in terms of the Greek alphabet[32] α, β, γ, δ, ε, $\sigma\tau$, and so on, constitute quantitative models of the physical magnitudes and arise by the practical process of measurement.

[31] For a complete description of figured numbers see: Nicomachus of Gerasa (*ca.* 1st century A.D.), *Introduction to Arithmetic* II. 8-12. Translation of M. L. D'Ooge, New York, 1926, (Cohen *et al.*, 1966)

[32] The way of writing numbers by using letters of the alphabet is the Ionic or Alexandrian system. This system is the most common one in Alexandrian Greek mathematics and is found in Ptolemy's *Almagest* (Kline, 1972)

The Pythagoreans' doctrine that all things are numbers, has as a result specific numbers to be connected with certain qualities and conditions, i.e., specific numbers possess a direct emotional and special meaning. In this way, numbers are not simply symbols of quantitative relations, but they are the substance of the world.

Below are some examples of the emotional meaning of Pythagorean numbers:

- '*1*' represents *Intelligence*, being always motionless
- '*2*' is *Opinion* because it is oscillating and mobile
- '*4*' and '*9*' are *Justice*, since they are the first two numbers obtained by multiplying the first even and the first odd number by themselves
- '*5*' symbolises *Marriage* because it joins the first even to the first odd number, i.e., $2+3$. This is the 'male' marriage number
- '*6*', which is the result of the multiplication 2×3, is the "female" marriage numbers
- '*7*' represents *Time*, the "right time", since the period of seven days is so important in human affairs (Hutten, 1962).

According to another interpretation:

- '*1*' is identified with *Reason*
- '*2*' is identified with *Opinion* – a wavering fellow is Two; he does not know his own opinion
- '*4*' is identified with *Justice*, steadfast and square
- '*5*' suggests *Marriage*; the union of the first even with the first genuine odd number '*7*'suggests the maiden *goddess Athena* "because seven alone within the decade has neither factors nor product" (Turnbull, 1929).

According to a third interpretation:

- '*Unit or One*' symbolises Mind, Aither, and Energy
- '*Dual or Two*' is Matter from 'water' and 'earth'
- '*Triad*' is 'Time as deity', that is past, present, and future
- '*A sequence of four*' represents Area, Space, i.e., the "order of the world"
- '*A group of five*' symbolises the five elements of which the world consists: earth, water, air, fire, aither and the corresponding polyhedrons: cube, icosahedron, octahedron, tetrahedron, and dodecahedron

- '*Half a dozen*' is the six kinds of living beings: gods, demons, heroes, men, animals, and plants
- '*A group of seven*' represents the seven known planets: Mercury, Venus, Mars, Jupiter, Saturn, Moon, and Sun
- '*A group of eight*' represents the eight heavenly spheres and the eight musical sounds of scale and harmony of the world
- '*A group of nine*' symbolizes the nine cosmic spaces of the firmament
- '*Decade*' symbolises the universe, the whole world (Georgakopoulos, 1995).

Along with the emotional meaning Pythagoreans give to numbers, they also relate them to geometry. They discuss the properties of the dot pattern, schema, and in this way they transfer the emotional meaning of numbers to a higher level. These 'figured' numbers are:

- '*1*' is the point,
- '*2*' is the line,
- '*3*' is the triangle, and
- '*4*' is the tetrahedron, and so on.

Another Pythagorean correspondence is that one according to the minimum number of points or dots necessary to define a line, a surface, or a solid:

- '*1*' and '*2*' as above
- '*3*' is the surface, and
- '*4*' is the solid.

We find similar correspondences later on in the work of Euclid. He studies the relations between numbers and plane geometrical figures and a representative example of his relations is given in the drawings of Leonardo da Vinci (figure 7.2).

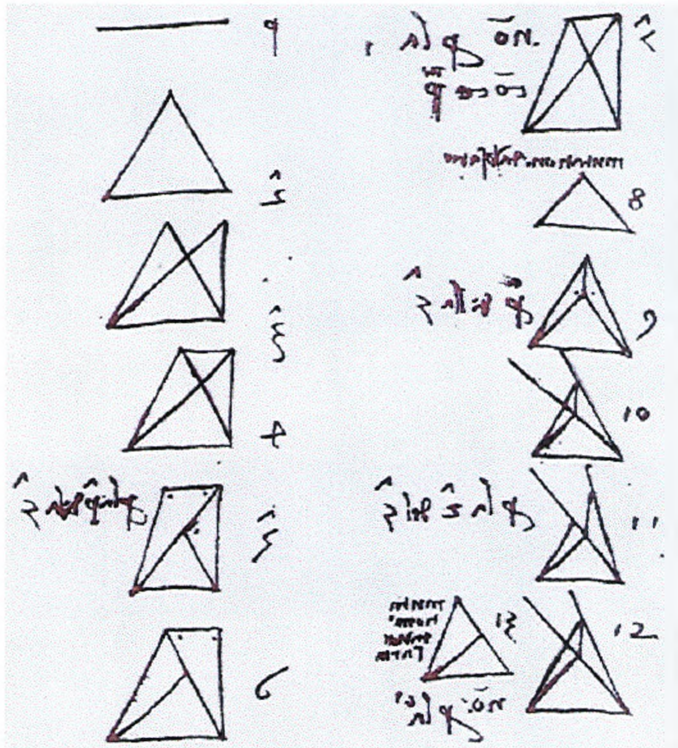


Figure 7.2: Leonardo's studies on the Euclidean relations between numbers and geometrical figures (Codex Atlanticus, Reti, *et al.*, 1974)

Additionally, the Pythagoreans introduce ten pairs of contracting opposites or principles, the so-called table of opposites, which in addition to the mathematical terms, such as limit and unlimited, odd and even, one and plurality, square and oblong, includes also pairs, such as male and female, good and evil. Aristotle describes it as follows:

> Aristotle, *Metaphysics* A5, 986a 25-30: " [...] there are ten principles, which they [the Pythagoreans] arrange in two columns of cognates-limit and unlimited, odd and even, one and plurality, right and left, male and female, resting and moving, straight and curved, light and darkness, good and bad, square and oblong[33]..." (McKeon, 1941)

Therefore, according to the Pythagoreans, the numbers acquire their meaning according to the subject, the physical magnitude, or the concept they symbolise.

[33] Metaphysica, Translated by W. D. Ross

7.2.2 Harmony

Pythagoreans see in numbers the key to understand the universe. They regard numbers as a fundamental aspect of reality, and mathematics as a basic tool for investigating this reality. Out of their One, Two, Three, and Four they could really build a world. This arithmetical progression, which is called the first tetractys (*Τετρακτύς*), adds up to the perfect number 10: $1+2+3+4=10$. As we have already seen, number 10 is a highly revered symbol by the Pythagoreans. The numbers (1, 2, 3, 4, 8, 9, 27) constitute respectively the second Pythagorean tetractys. They are ordered in a triangular form, where the numbers on the one side after 1 being successive powers of 2 and those on the other side successive powers of 3.

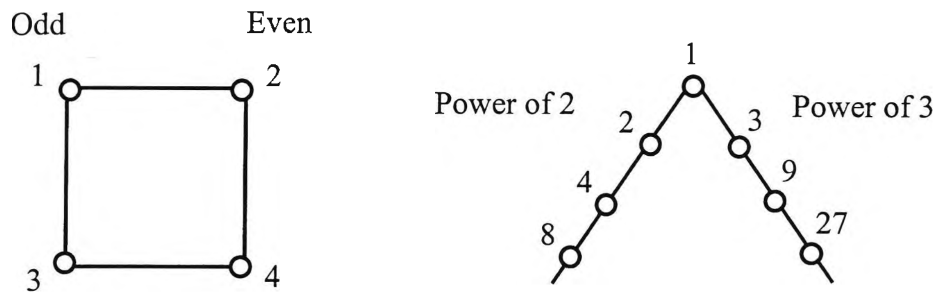


Figure 7.3: The first and second Pythagorean tetractys

Pythagoras uses the first tetractys in developing Harmony. Using a monochord, the so-called *canon*[34], an ancient instrument of one string stretched over a sounding board with a movable bridge set on a graduated scale, he studies the correlation between the ratios of the lengths of a vibrating string and the pitch of the produced sounds.

He proves that:

a) The ratio 1:2 corresponds to two sounds, from which the one has double pitch of the other, and they are produced by alternatively percussion on the whole string and the half of it (octave).

[34] Boethius, *De Institutione Musica*, I 10-11: "[...] the monochord was called *canon* not merely from the wooden ruler by which we measure the length of strings corresponding to a given tone, but because it forms for this type of investigation so definite and precise a standard that no inquirer can be deceived by dubious evidence." (Cohen *et al.*, 1966)

b) The ratio 2:3 corresponds to two sounds, which are produced by alternatively percussion on the whole string and the 2/3 of it, and the interval between them is the interval of fifth (quint)

c) The ratio 3:4 corresponds to two sounds, which are produced by alternatively percussion on the whole string and the 3/4 of it, and the interval between them is the interval of forth (quart)

These intervals (octave, fifth, and forth), that correspond to the first three harmonics of a sound (the second, the third, and the forth harmonic) and are expressed by simple arithmetical proportions (1:2, 2:3, 3:4), are defined by Pythagoras as the "perfect concordant". Moving forward in his study, Pythagoras discovers that the successive setting of intervals of fifth – the so-called cycle of fifths – gives seven different musical sounds (notes), which form, in the compass of an octave, a scale of five equal intervals (tones) and two smaller (semi-tones). In this way, the first known musical scale, the so-called Pythagorean scale, arises: $1, \frac{8}{9}, \frac{64}{81}, \frac{3}{4}, \frac{2}{3}, \frac{16}{27}, \frac{128}{243}, \frac{1}{2}$. Strings, whose lengths are divided according to these numerical proportions, produce the notes of the octave (figure 7.4).

Pythagoras does not restrict himself only to these musical observations. Furthermore, he ascertains that:

a) The numbers 1, 2, 3, 4, and so on correspond to the harmonics of sounds and define musical intervals and string lengths, which produce melodic sounds agreeable to the human ear. Thus, there is a correspondence between numbers, melodic sounds, and string lengths or in other words, there is a mathematical model of the natural phenomenon of the vibrating string.

b) The relationship between numbers and sounds defines the harmony. It is harmony that joins mathematics and music together. The musical scale, like the number series, allows division through harmony; concords and fractions correspond to one another. According to Pythagoras, harmony is inherent in the natural world and it is perceptible by human being.

c) Harmony characterizes the whole universe. According to Pythagoras, harmony is applied to nature by means of his theory of the planetary motion and the harmony of the spheres. He maintains that the numerical ratios, which are responsible for the musical notes correspond to the distances of the heavenly bodies, the planets, from the centre of the world, the earth. Thus, Pythagoras creates the first quantitative, mathematical model of the world. An evidence for that comes to us by Hippolytus: Hippolytus, III-1: "Pythagoras maintained that the universe sings and is constructed in accordance with harmony" (Sarton, 1, 1993)

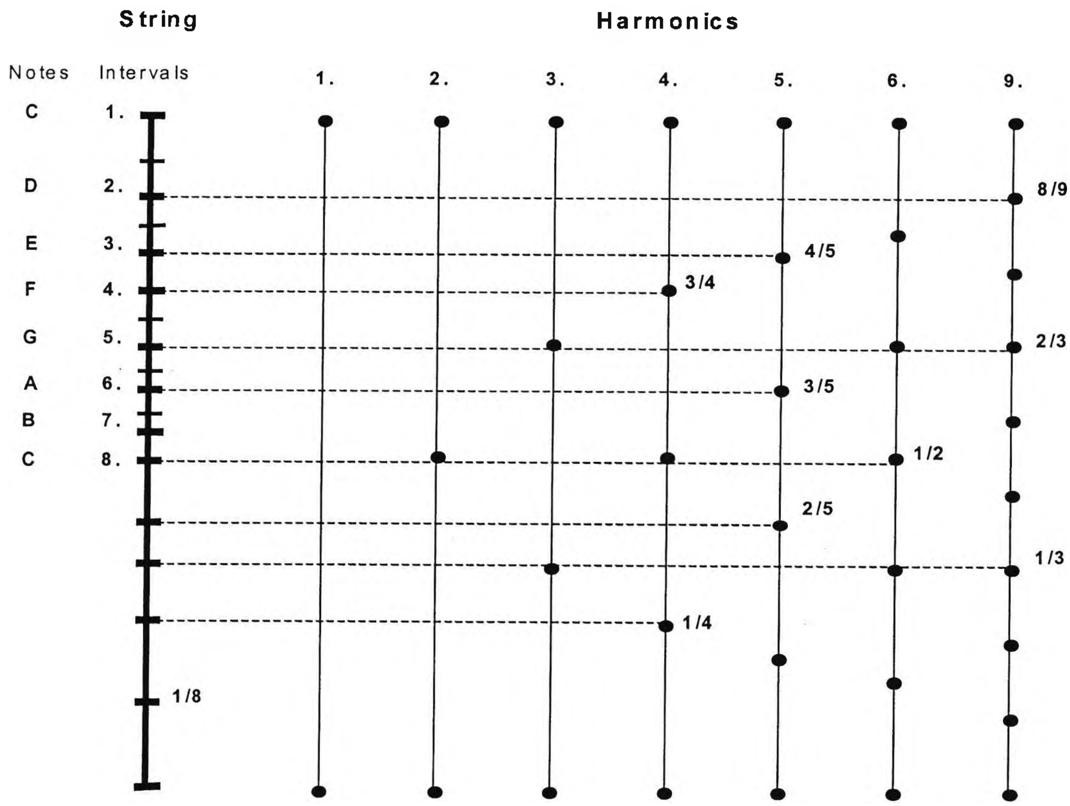


Figure 7.4: The relation between the harmonics and the length of the vibrating string

Even though at various time various systems are put forward by the Pythagoreans, the general principle underlying these systems is the attempt to link planetary motions and distances with the notes of the musical scale. The Pythagorean system of the harmony of the spheres is given by the following passage:

Alexander of Aphrodisias (leader of the Peripatetic School of Aristotle in the early part of the 3rd century A.D.), *Commentary on Aristotle's Metaphysics,* p. 542a5-18: "They [the Pythagoreans] said that the bodies that revolve round the centre have their distances in proportion, and some revolve more quickly, others more slowly, the sound which they make during this motion being deep in the case of the slower, and high in the case of the quicker; these sounds then, depending on the ratio of the distances, are such that their combined effect is harmonious [...] Thus, the distance of the sun from the earth being, say, double the distance of the moon, that of Aphrodite triple, and that of Hermes quadruple, they considered that there was some arithmetical ratio in the case of the other planets as well, and that the movement of the heavens is harmonious. They said that those bodies move most quickly which move at the greatest distance, that those bodies move most slowly which are at the least distance, and that bodies at intermediate distances move at speeds corresponding to the sizes of their orbits[35]." (Cohen *et al.*, 1966)

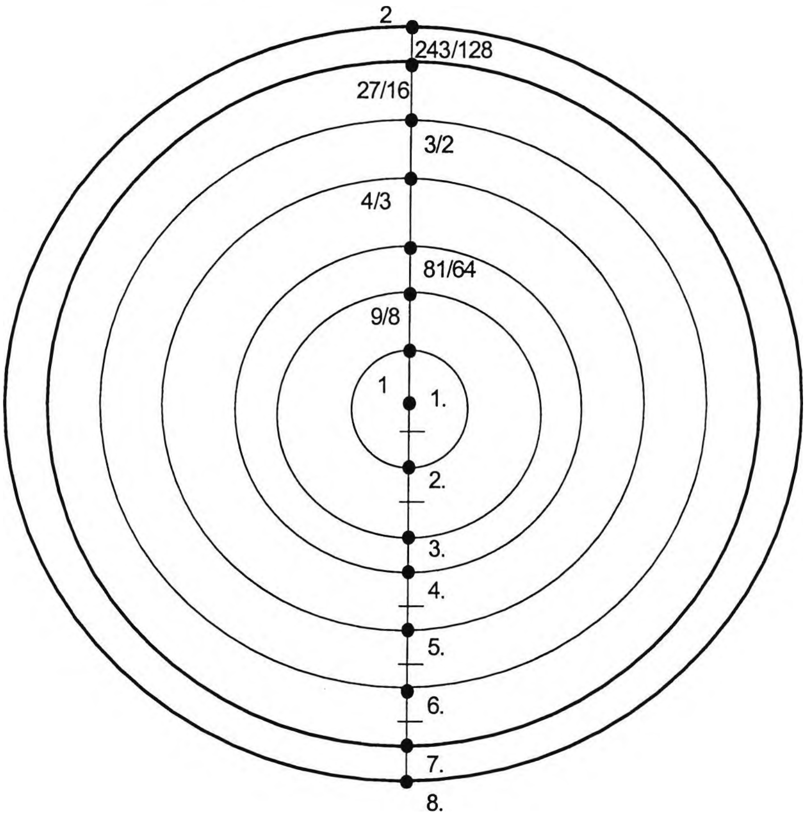


Figure 7.5: The Pythagoras' model of the universe. The relation between the distances of the planets from the earth and the numerical ratios of the musical notes

Aristotle in the next fragment mentions the theory that the movement of the stars produces a harmony, not as a theory of Pythagoras, but as the general theory of some thinkers:

Aristotle, *De caelo,* B 9, 290b12 (DK 58 B 35): " [...] Some thinkers suppose that the motion of [heavenly] bodies of that size must produce a noise, since on our earth the motion of bodies far inferior in size and in speed of movement has that

[35] Translation of T. L. Heath, Greek Astronomy

effect. Also, when the sun and the moon, they say, and all the stars, so great in number and in size, are moving with so rapid a motion, how should they not produce a sound immensely great? Starting from this argument and from the hypothesis that their speeds, as measured by their distances, are in the same ratios as musical concordances, they assert that the sound given forth by the circular movement of the stars is a harmony…" (Kirk *et al.*, 1983)

Thus, Pythagoras based on his experiments infers world's structure and forms a model of the world, in which the distances between the heavenly bodies and the centre of the world are defined by specific mathematical relations. Moreover, he considers the intervals of the vibrating string as a physical model of the universe and the mathematical ratios, from which these intervals are specified, as a mathematical model of the world.

After Pythagoras, Aristoxenos (*ca.* 360 B.C) of Tarentum, astronomer and musician, determines in his work *Αρμονικά στοιχεία (Harmonic Elements)* that the criterion of musical harmony is the subjective criterion of hearing instead of that one of numbers. He simplifies the numerical proportions of Pythagoras' scale by introducing the interval of third (terz), which corresponds to the fifth harmonic and in the ratio 4:5 of the length of the string. The new musical scale of Aristoxenos corresponds to the next mathematical proportions: $1, \frac{8}{9}, \frac{4}{5}, \frac{3}{4}, \frac{2}{3}, \frac{3}{5}, \frac{8}{15}, \frac{1}{2}$. The table below shows the relation of Pythagoras' scale with Aristoxenos' scale and with the contemporary logarithmic one.

Notes	Intervals	Pythagoras' scale	Aristoxenos' scale	Logarithmic scale
C	1.	1	1	1
D	2.	8/9=0,8889	8/9=0,8889	0,8909
E	3.	64/81=0,7901	4/5=0,8000	0,7937
F	4.	3/4=0,7500	3/4=0,7500	0,7492
G	5.	2/3=0,6667	2/3=0,6667	0,6674
A	6.	16/27=0,5926	3/5=0,6000	0,5946
B	7.	128/243=0,5267	8/15=0,5333	0,5297
C	8.	1/2	1/2	1/2

Later on, Plato, in his dialogue *Timaeus*, where he mostly describes the formation of the world by the divine craftsman (the Demiurge), considers that this world except of its body it also consists of soul. The world Soul is marked off into divisions, corresponding to the intervals of the musical scale.

Timaeus, 36E: " [...] a soul has part in reason and harmony..."

The Demiurge begins the creation of the world Soul by dividing the entire unity of soul-stuff (Sameness, Difference, and Existence) into parts measured by the numbers that constitute the two geometrical proportions (1, 2, 4, 8 and 1, 3, 9, 27) of the second Pythagorean tetractys:

> *Timaeus,* 35B-C: "And having made a unity of the three, again he divided this whole into as many parts as was fitting, each part being a blend of Sameness, Difference, and Existence. And he began the division in this way. First he took one portion (1) from the whole, and next a portion (2) double of this; the third (3) half as much again as the second, and three times the first; the fourth (4) double of the second; the fifth (9) three times the third; the sixth (8) eight times the first; and the seventh (27) twenty-seven times the first." (Cornford, 1977)

The platonic philosopher Theon of Smyrna[36] (2nd century A.D.) in his work *On the Tetractys and the Decade*, enumerates ten tetractys (sets of four things), where these four numbers are supposed to symbolise different magnitudes or concepts (some show Platonic influence):

1. Numbers	1, 2, 3, 4
2. Magnitudes	Point, line, surface (triangle), solid (tetrahedron)
3. Simple Bodies	Fire, air, water, earth
4. Figures of Simple Bodies	Tetrahedron, octahedron, icosahedron, cube
5. Living Things	Seed, growth in length, in breadth, in thickness
6. Societies	Man, village, city, nation
7. Faculties	Reason, knowledge, opinion, sensation
8. Parts of Living Creatures	Body, and the three parts of soul

36 Theon of Smyrna, *«Περί των κατά το μαθηματικόν χρησίμων, εις την Πλάτωνος ανάγνωσιν»*, ed. Dupuis, J., Paris, 1892

9. Seasons of the year	Spring, summer, autumn, winter
10. Ages	Infancy, youth, manhood, old age

All these ten tetractys are interpretations of the first tetractys, 1, 2, 3, 4 and there are ten of them because (10) is the perfect number. According to Theon, the second tetractys is formed from the first by multiplication; and in order to accommodate both the odd and the even numbers, it consists of two tetractys: 1, 2, 4, 8 and 1, 3, 9, 27.

Similarly to the Pythagoreans, Plato correlates the seven terms of the series 1, 2, 3, 4, 8, 9, 27 ('the double and triple intervals') with the distances of the seven planets either between each other or from the earth.

> *Timaeus,* 36D: " [...] but the inner revolution he split in six places into seven unequal circles, severally corresponding with the double and triple intervals, of each of which there were three in number ..." (Cornford, 1977)

By summarising, we could say that the numbers came into being by the practical process of measurement and experiment (e.g., the string of Pythagoras), became symbols of concepts, and constituted quantitative models of the world. In addition, Pythagoreans paved the way to the development of the Science of Mathematics.

7.3 Geometry and Measurement as Modelling Processes

The science of Mathematics of all the parts of philosophy was the earliest to be discovered. The Pythagoreans considered all mathematical science to be divided into four parts: arithmetic, geometry, music, and astronomy. The earliest extant reference to this fourfold division of mathematics is by Archytas of Tarentum, a Pythagorean philosopher of the 4th century B.C.:

> "They have given us clear knowledge about the speed of the stars, and their risings and settings, about geometry and numbers and spheric[37] and, not least, music. For these studies seem to be sisters[38]. "(Cohen *et al.*, 1966)

[37] The term 'spheric' in this connection refers to spherical geometry in its special relation to the circular motion of the heavenly bodies (Cohen *et al.*, 1966)

[38] Diels, *Fragmente der Vorsokratiker,* I. 331. 5-8

Arithmetic and music study quantity, whereas geometry and astronomy study magnitude (Rouse Ball, 1960). Of quantity one kind, the 'absolute', is viewed in regard to its character by itself (e.g., even or odd), having no relation to anything else, and the other, the 'relative', is considered in regard to its relations to another quantity (e.g., double or small). It is declared in the work *Introduction to Arithmetic* of the Pythagorean Nicomachus of Gerasa (*ca.* 50-120 A.D.): "Of quantity one part is studied by itself, namely that which has no sort of relation to another, and the other as having some sort of relation to another and capable of being thought of only in its relations to another." (I, 3 – Hoche, 5, 13, ff.) Similarly, a part of magnitude, the 'stable', is in a state of rest and stability, and another part, the 'moving', in motion and revolution. According to Theon of Smyrna, the platonic philosopher and mathematician, arithmetic deals with the first kind of quantity, i.e., the numbers, music with the relations between quantities, geometry with magnitude at rest, and astronomy with magnitude in motion. Arithmetic and geometry are connected, because magnitudes can be measured and can therefore be represented by numbers (Turnbull, 1929).

It is an oft-repeated story since the antiquity that the origins of Greek geometry are to be found in Egyptian land measurement:

> Proclus Diadochus, *Commentary on Euclid's Elements I*, pp. 64.7-70.18: "[...] It [geometry] owed its discovery to the practice of land measurements. For the Egyptians had to perform such measurements because the overflow of the Nile would cause the boundary of each person's land to disappear..." (Cohen *et al.*, 1966)

Besides, the etymological meaning of the Greek word *Γεωμετρία* (geometry) is exactly the land measurement. This word consists of two terms: a) *γη* – land, earth, i.e., the natural world, which is the one-, two-, and three-dimensional space, where the corresponding geometrical figures constitute models of the natural bodies, b) *μέτρηση* – measurement, i.e., the technical and mathematical process of comparison of two magnitudes. This process of comparing a magnitude with a measure (*μέτρο*) is a process of simulation and corresponds to the natural magnitude a number. It is a process, which is accomplished by two instruments: the ruler and the compass. The comparison presupposes the definition of the relationship of 'greater-less' and it is determined as an analogy between magnitudes or numbers. The distinction between the concept of magnitude and the concept of number, firstly, came into being by the Pythagoreans and later when Euclid defined them. The 'magnitude' expresses the quantity and is a symbol of everything that can be measured, e.g., of a line, an area or a volume, whereas 'numbers' are the result of measurement, the quantitative symbol of an analogy.

At the period that geometry is merely an empirical knowledge, and before its scientific formulation, the conception of the basic geometrical figures takes place. At that period, man studies and observes the world in order to understand it. This observation makes him aware of the first geometrical figures, which are represented as models of the physical things. For example, the observation of the straight sea horizon, and the circular disks of sun and moon, has as a result the creation of the geometrical figures of line and circle respectively. Sometimes these geometrical figures are not only models of physical things, but also symbols of conceptual elements. For example, the line is the symbol of genesis, evolution, and death because it has beginning, middle, and end, and the circle that has no beginning and end symbolises the everlasting, the immortal, and the divine.

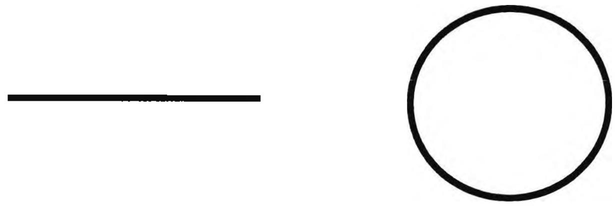


Figure 7.6: Line and circle as models of horizon and sun

More precisely, these two primary tools of geometry, line and circle, have been created through the following simulation processes:

Line:

1. Rectilinear physical system, e.g., horizon
2. Comprehension of the main property of it, i.e., line is the shortest way between two points
3. Construction of an instrument as the technical model of this property, i.e., the ruler
4. Geometrical abstract model, which is the rectilinear segment or line

Circle:

1. Circular physical system, e.g., sun disk
2. Comprehension of the main property of it, i.e., all the peripheral points of a circle have the same distance from its centre

3. Construction of an instrument as the technical model of this property, i.e., the compass
4. Geometrical abstract model, which is the circle

In general, the following schema arises:

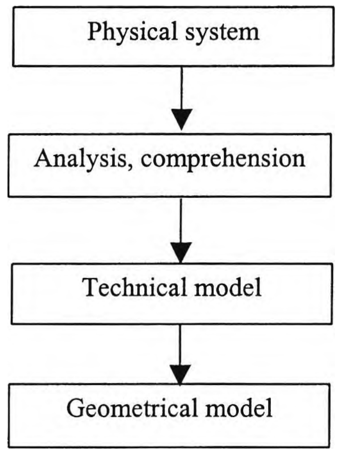


Figure 7.7: The steps of geometrical modelling approach

Analysis, technical construction, and mathematical modelling are the necessary steps that the human mind follows in order to comprehend a physical system. Even though one could claim that step 2 goes after step 4, and actually the formal scientific definition of line or circle comes many years later, the conception of line and circle as geometrical models could not have happened without the human mind being absolutely familiar with the basic property of line or circle, i.e., without being able to construct instruments that reproduce these properties.

The formulation of the first geometrical figures, as described previously, refers to the process of reasoning, which is called deductive. Deductive reasoning process starts by observing the very general properties, which take the form of definitions, postulates or axioms, and goes on by deriving from them logical statements that concern things or circumstances, which could occur in particular. The process of deduction, which characterizes the mathematical reasoning, has found an almost complete realisation in geometry, and for this reason the logical structure of geometry has been the model for all the exact sciences.

Parallel to these two basic geometrical figures, the introduction of the triangle, square, polygon, and polyhedron takes place, which similarly constitute models of the natural bodies. They are the Pythagoreans, who manipulate equilateral triangles and squares in three dimensions and result in the development of solid geometry and of the four 'regular solids', which are figures with all their sides and angles equal. These four are: a) the regular 4-sided pyramid or *tetrahedron*, b) the 6-sided *cube*, c) the 8-sided *octahedron*, and d) the 20-sided *icosahedron*. They are taken to represent the four elements of the physical world: the cube bounded by six squares is associated with earth; the tetrahedron bounded by four equilateral triangles with fire; the octahedron (8 triangles) with air; and the icosahedron (20 triangles) with water. Following, they are discovered the 5-sided plane figures or the regular pentagons. Pentagons could be built into a fifth regular solid, the 12-sided *dodecahedron*. It is taken to represent the universe. These five possible regular solids were studied later on by Plato and became known as the 'Platonic bodies'.

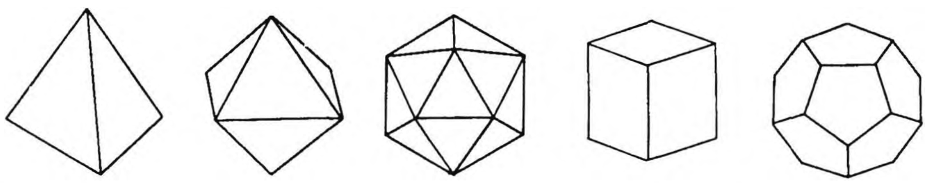


Figure 7.8: The five regular solids: Tetrahedron, octahedron, icosahedron, cube, and dodecahedron.

It is noteworthy, that while in the plane there is an infinite number of regular polygons, such as for example, the polygons that can be inscribed in and circumscribed about a circle, in the space, there are only five regular polyhedrons. Plato's explanation for this phenomenon, under the influence of Empedocles and his four roots (earth, water, air, and fire), is that since there are only five basic elements, it could not be possible for the regular bodies to be more. He associates each of the above figures with these elements.

> Plato, *Timaeus,* 55E-56A: "Let us next distribute the figures whose formation we have now described, among fire, earth, water and air. To earth let us assign the cubical figure; for of the four kinds earth is the most immobile and the most plastic of bodies [...] and of the remainder the least mobile to water, the most mobile to fire, and the intermediate figure to air. Again, we shall assign the smallest body to fire, the largest to water, and the intermediate to air; and again the body with the sharpest angles to fire, the next to air, the third to water. Now, taking all these figures, the one with the fewest faces (pyramid) must be the most mobile, since it has the sharpest cutting edges and the sharpest points in every direction, and moreover the lightest, as being composed of the smallest number of similar parts; the second (octahedron) must stand second in these respects, the third (icosahedron), third. Hence, in accordance

with genuine reasoning as well as probability, among the solid figures we have constructed, we may take the pyramid as the element or seed of fire; the second in order of generation (octahedron) as that of air; the third (icosahedron) as that of water." (Cornford, 1977)

He assigns the tetrahedron to fire, because the tetrahedron is the smallest, sharpest, and most mobile of the regular solids, and fire is the most penetrating of elements. He assigns the cube to earth because cube is the most stable of the regular solids. He assigns the octahedron to air, and the icosahedron to water. Finally, Plato finds a function for the dodecahedron (the regular solid closest to the sphere) by identifying it with the cosmos as a whole. Thus, the geometrical figures become geometrical models of the natural elements.

Leonardo da Vinvi studies the platonic solids and gives the following figures:

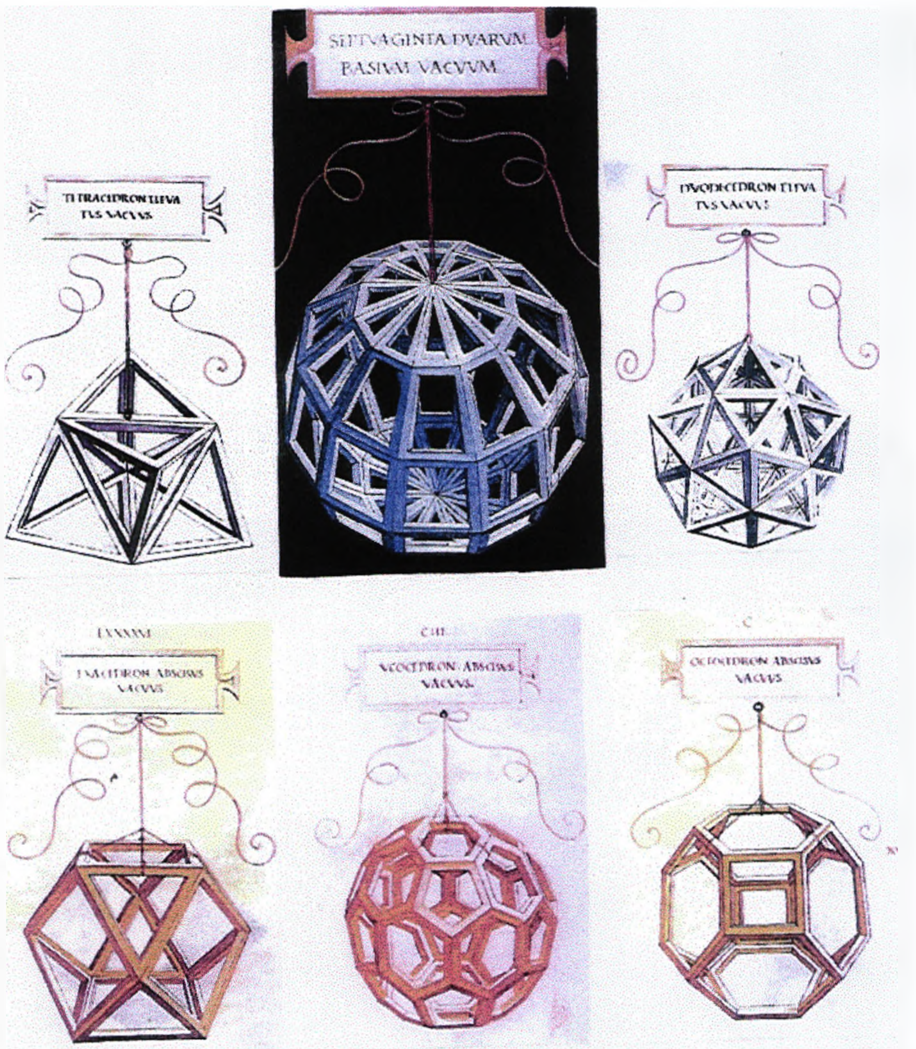


Figure 7.9: Leonardo's studies on the platonic solids (De divina proportione, Reti, *et al.*, 1974).

7.3.1 Thales and the Concept of Analogy

The art of modelling, as well as all the arts of design from their very beginning presuppose the principle of similitude. The effort to investigate the origins of formal modelling and the primary attempts to represent and examine a real system by means of a model, leads to this part of mathematics that has to do with the concept of analogy and the theory of proportion, as they have been formulated in the period from Thales to Heron.

Thales of Miletus is considered to be the 'father' of geometry, and the first mathematician. He deserves such a title because he formulates the proofs for a set of geometrical propositions and the solutions of elementary geometrical problems. Some of the propositions, which have been ascribed to him, are: a) a circle is bisected by any diameter, b) the angles of the base of an isosceles triangle are equal (Euclid I, 15), c) the angle in a semicircle is a right angle (Euclid III, 31), and d) the sides about equal angles in similar triangles are proportional. This last proposition is of wider interest because is related to the concept of analogy and consequently to the concept of modelling, as we shall see further down.

Let us firstly lay emphasis on the fact that these propositions do not concern a particular circle or triangle but they are applicable to any circle or triangle. Thales introduces the notion of the proof as a generic truth independent of numerical evidence. He wants to create generic types of propositions, which could be applicable in the whole class of circles, in the infinite number of circles in the world. This is a completely original aspiration. In order to realise it, he follows the steps that are described in the previous section, by means of which it is possible to create a 'Circle', i.e., the constructible, geometrical model of circle that represents all the potential circles. If a proposition is true for this model, it is secure that this is also true for any other circle.

In this effort to conceive such a geometrical model, the very concept of modelling is inherent. What else is this process of creating models than to secure that the behaviour of any real system is absolutely the same with the behaviour of its model? Instead of trying to study numerous real systems, would it not be more convenient to produce a model of them and study only the behaviour of it?

Thales also is ascribed with two applications of measurements. He finds how to calculate the distance of a ship at sea from observations taken at two points on land, and how to estimate the height of a pyramid from the length of its shadow. Both of these calculations are based on the aforementioned last proposition that two similar triangles have their sides in the same proportions even though their absolute magnitudes may be extremely different. The following figure gives a schematic representation of this proposition.

For example, assume that the unknown height of the pyramid is AB and its shadow is AO. In order to measure the height AB, and by being familiar with the property that the height of an object is analogous to the length of its shadow, he constructs a technical model of the object, in the case of the pyramid let say that this model is a staff, whose height is A_1B_1 and its shadow is A_1O.

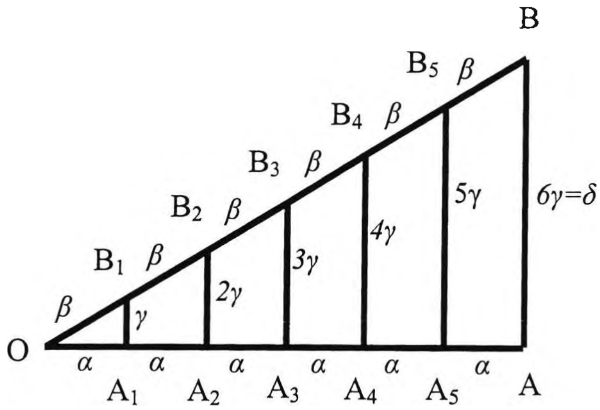


Figure 7.10: Geometrical presentation of the concept of analogy

By the geometrical interpretation of the proposition it arises that the triangles OAB and OA_1B_1 are proportional, and thus the arithmetical proposition of their sides is: $\frac{AB}{AO} = \frac{A_1B_1}{A_1O}$.

By this relationship and since the magnitudes AO, A_1B_1, A_1O are known, he measures the height AB: $AB = \frac{A_1B_1}{A_1O} \cdot AO$.

This process is shown in the next figure:

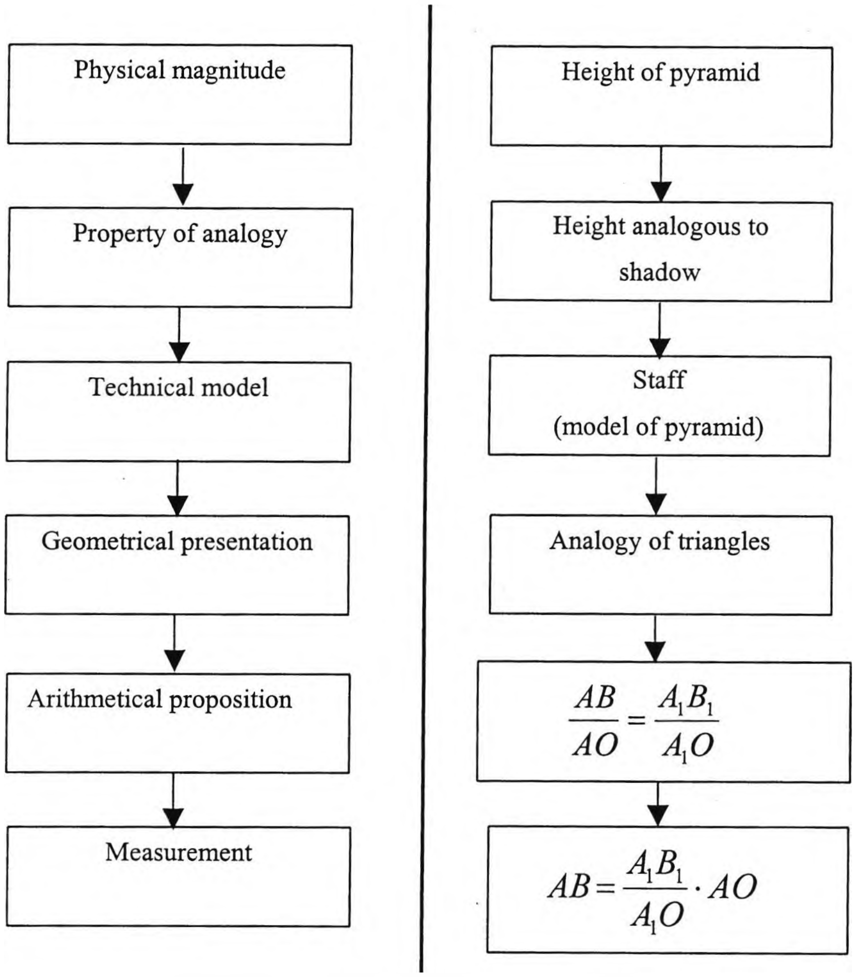


Figure 7.11: Measurement as modelling process

Aristarchus of Samos (310-230 B.C.) introduces a similar process of measurement in his treatise *On the Sizes and Distances of the Sun and Moon.* In this treatise, he outlines the plan for measuring the distances and sizes of celestial bodies. More precisely, he tries to estimate the diameter and the volume of Sun, as well as the solar and the lunar distance from the Earth. Even though his numerical results are pure, the fact remains that thanks to him it was made possible to measure the sizes and distances of the Sun and Moon; the actual number are less important than that possibility (Sarton, 2, 1993).

He measures the angle α that separates the two lines of sight, B and C, when the moon is at quarter-phase and half-full (figure 7.12). A and C intersect at right angles and therefore the ratio of B to C can be calculated. The disadvantages of Aristarchus' method are, on the one hand, the fact that the exact moment the moon's disk is half illuminated is not accurately determined, and on the other, the fact that even a small error in the measurement of angle α leads to a very large error in the ratio of B to C.

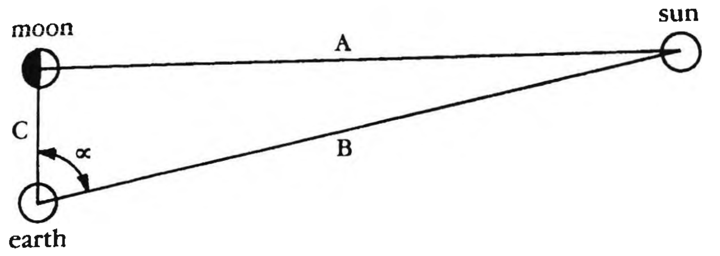


Figure 7.12: Measurement of solar and lunar distances from the earth by Aristarchus (Lindberg, 1992)

For example, Aristarchus' value of 87° (true value is 89° 52') led him to a large deviation of the actual value of this ratio, i.e., he figured the earth-to-sun distance to be about 20 times the earth-to-moon distance and the correct ratio is about 400:1. Therefore, Aristarchus model was so sensitive to errors in the data upon which it was based. In modern term, we could say that it was a non-robust model, because it was not relatively immune to errors in the input data, and as a result the accuracy of the observations affects the accuracy of any conclusions obtained from the model.

In addition, Aristarchus calculates the diameter of Sun seven times the diameter of the earth, whereas the actual value is about 300 times the diameter of the earth, and the Sun's volume 300 times the volume of the earth, whereas the actual value is 1.300.000 times the earth's volume. Although there is a large deviation from tha actual values, the superior mass of the sun is obvious and probably it is this realisation that brought Aristarchus to the discovery of the heliocentric system, as will see in chapter 9.

Aristarchus' method, though undetermined by mis-measurements, was logically and mathematically sound and remarkable, and moreover an anticipation to trigonometry. The physical magnitudes of solar and lunar distances from the earth are measured by means of the geometrical presentation of figure 7.12, where the ratio B: C is analogous to the actual ratio of earth-to-sun: earth-to-moon, by taking advantage of the analogy of the triangle of figure 7.12 and the actual triangle the three celestial bodies constitute.

On the generalisation of this concept of proportionality or analogy, the so important concept in Greek mathematics, the concept of modelling is based. In the case of systems entirely determined by n dimensions, as the triangles by their sides, one system is analogous to the other, or a model of the other, provided that the relations between their dimensions remain the same, even if their values change.

7.3.2 Geometrical Analogy by Heron of Alexandria

Heron, the Alexandrian engineer (*ca.* 1st century B.C.), describes in his work *On the Dioptra*, a topographical instrument, the so-called dioptra (diopter), for surveying and levelling, and for the determination of angles in both terrestrial and astronomical problems. He uses dioptra and creates a geometrical model for the opening of a rectilinear tunnel through a mountain, digging simultaneously from both ends.

According to Heron, we consider line ABΓΔ as the base of the mountain and B and Δ as the openings through which the required tunnel is built (figure 7.13).

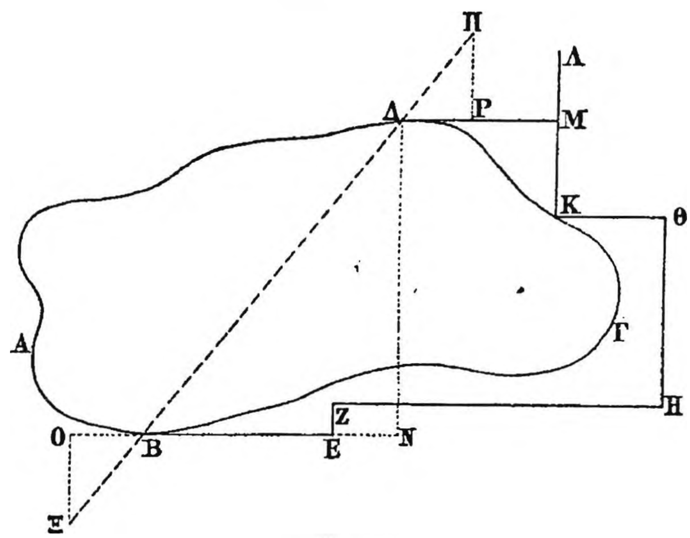


Figure 7.13: Geometrical model for a mountain rectilinear tunnel according to Heron (Shöne, 1903)

By making use of dioptra, it is drawn on the ground the crooked line BEZHΘKMΔ, where EZ is perpendicular to BE, ZH perpendicular τo EZ, HΘ perpendicular to ZH, and so on. BE is supposed to be extended to N and ΔN is perpendicular to BN. The lines ΔN and BN is possible to be computed because they are: ΔN= EZ+HΘ+KM and BN=BE+ZH-ΘK-MΔ. Let us consider, for example, that BN=5ΔN. If the line BΔ is drawn and extended to the point Ξ, and ΞO is drawn perpendicular to BE, and if similarly the line BΔ is extended to the point Π, and ΠP perpendicular to ΔM, then by means of the analogy between similar triangles we have: BO=5OΞ and ΔP=5PΠ. Thus, if we take a point, e.g., O, on BE, and draw OΞ at right angles to BO and make OΞ=1/5OB, then BΞ, if produced, will pass through Δ. Similarly, if we construct the triangular ΔΠP so as ΠP=1/5ΔP, then ΔΠ will, if produced pass through B. Therefore, we commence the tunnel operations along lines ΞB and ΠΔ and proceed by setting our direction line along these determined lines. If the tunnel is dug according to this description, the workers starting simultaneously from the opposite ends will meet (Heron, *On the Dioptra*, 15, Shöne, 1903).

Most likely, Heron tries to describe the method that some time before 500 B.C., the architect and engineer Eupalinus of Megara used in order to tunnel a mountain in Samos, as part of a new water supply system for the city. This tunnel is described by Herodotus and rediscovered in 1882. The tunnel operation was conducted from both openings of the tunnel and the workers met in the middle with remarkably accuracy. According to the method of Heron, the direction of the unknown line BΔ from the openings B, Δ is determined by constructing triangles, such as BOΞ and ΔΠΡ, which are similar and analogous between them and constitute models of the big, unknown triangle BΔN. The archeological findings of the tunnel of Eupalinus reveal tracings that correspond to the geometrical construction of such analogous triangles, as well as the word ΠΑΡΑΔΕΙΓΜΑ, which means image or model (see also chapter 6).

7.4 Archimedes and Early Calculus

Archimedes has been called the greatest mathematician of antiquity. Along with his interest in the field of physics (chapter 4) and mechanics (chapter 9), he also occupied himself with abstruse mathematical problems. For him every kind of art that applies itself to practical, daily needs is ignoble[39]. Along with the other mathematical discoveries of Archimedes (chapter 8), he contributes remarkably to the development of calculus and algebra, which in turn contribute to the formation of the building blocks of mathematical modelling.

We will start representing Archimedes' achievements in the fields of arithmetic and geometry with his method of expressing very large numbers, as it is described in his work of *Sand Reckoner* or *Arenarius*. The first sentences of this work are as follows:

> "There are some, King Gelon [son of Hieron II], who think that the number of the sand is infinite in multitude: and I mean by the sand not only that which exists about Syracuse and the rest of Sicily but also that which is found in every region whether inhabited or uninhabited. And again, there are some who, without regarding it as

[39] Plutarch, *Life of Marcellus*, 17.3-4

infinite, yet think that no number has been named which is great enough to exceed its multitude." (Turnbull, 1929)

In this work, Archimedes develops a system of notation and manages to express a number, which, in the current notation, would require 80 thousand million million ciphers. He asks: "How many grains of sand could the whole universe hold?" The answer to such a question demands firstly the calculation of the size of the universe and then the calculation of how many grains are contained in a unit of space. Provided that the necessary words for numbers are available, it is easy to calculate how many grains holds the whole universe. More precisely, he first calculates how many grains of sand would measure the diameter of a poppy seed. He carries out his arithmetical reductions from poppy seed to finger-breadth and then to stadium and so on (Turnbull, 1929). He finally shows that the amount of sand grains in the world is finite, an axiom often called as the Axiom of Archimedes. This axiom is of great importance because of its influence on the arithmeticians of the following centuries.

Another subject of study of Archimedes is the algebraic solution of equations, as it is shown in his famous Cattle Problem. Though the highest development in this branch of Greek mathematics is represented in the work *Arithmetica* of Diophantus (3^{rd} century A.D.), where he gives a collection of problems leading to equations determinate and indeterminate, an early type of indeterminate analysis is involved in the Cattle Problem of Archimedes. This problem leads to no determinate solution, for it involves eight unknown quantities connected by seven equations:

> Archimedes, *Opera* II. 528-532 (Heiberg): "Compute the number of cattle of the Sun [...] divided into four herds by differences in the color of their skin – one milk-white, the second sleek and dark-skinned, the third tawny-colored, and the fourth dappled. In each herd there was a great multitude of bulls, and there were these ratios. The number of white bulls... was equal to one-half plus one-third the number of dark-skinned, in addition to all the tawny-colored; the dark-skinned bulls were equal to one-fourth plus one-fifth the number of dappled, in addition to all the tawny-colored. The number of dappled bulls was equal to one-sixth plus one-seventh the white, in addition to all the tawny-colored. Now for the cows there were these conditions: the number of white cows was exactly equal to one-third plus one-fourth of the whole dark-skinned herd; the number of dark-skinned cows, again, was equal to one-fourth plus one-fifth of the whole dappled herd, bulls included; the number of

dappled cows was exactly equal to one-fifth plus one-sixth of the whole tawny-colored herd as it went to pasture; and the number of tawny-colored cows was equal to one-sixth plus one-seventh of the whole white herd [...] When the white bulls were mingled with the dark-sinned, their measure in length and depth was equal as they stood unmoved... and when the tawny-colored bulls were joined with the dappled ones they stood in perfect triangular form... " (Cohen *et al.*, 1966)

Let as consider that X, Y, Z, W represent the number of white, dark-skinned, tawny, and dappled bulls, respectively, and x, y, z, w the number of cows of the respective colours. According to the problem we have:

$$
\begin{aligned}
X &= (1/2 + 1/3)Y + Z, \\
Y &= (1/4 + 1/5)W + Z, \\
W &= (1/6 + 1/7)X + Z, \\
x &= (1/3 + 1/4)(Y + y), \\
y &= (1/4 + 1/5)(W + w), \\
w &= (1/5 + 1/6)(Z + z), \\
z &= (1/6 + 1/7)(X + x).
\end{aligned}
$$

In addition: $X + Y =$ a square number and $Z + W =$ a triangular number

Archimedes restricts himself only to show what equations are involved in this problem; for the complete solution of it, according to (Taton, 1963), would have taken up 744 pages, each containing 2.600 numbers!

Another invention of Archimedes is the integral calculus (Downs, 1982). More precisely, in his work *Method* he evaluates the ratio of the volumes of the spheroid, the obtuse-angled conoid, and the right-angled conoid to those of cones by means of a mechanical procedure (Cohen, 1966, p.69), whereas *On Conoids and Spheroids* by means of a purely geometrical procedure. In these procedures, he uses a form of integration: in the former he inscribes and circumscribes the volumes of these bodies with two series of cylinders, and in the latter he combines a Pythagorean algorithm based on figure numbers with Eudoxus' method of exhaustion (Taton, 1963).

His chief interest was in pure geometry, and he regarded his discovery of the ratio of the volume and the surfaces of a cylinder to these of a sphere inscribed on it as his greatest achievement (Downs, 1982). This ratio is equal to 3:2 and the proof is given in his treatise *On the Sphere and Cylinder* and also in his *Method* (Sarton, 2, 1993). Cicero reports[40] that he attached so much value to this finding that he ordered the diagram relative to it to be engraved on his tombstone (Downs, 1982).

Similar calculations, such as the ratio of the circumference of the circle to its diameter, i.e., number π, or the geometric approximation of number $\sqrt{3}$, are explored in chapter 8.

7.5 Conclusion

While the mythical and the following scientific period of the presocratic philosophers refer to the qualitative interpretation of the world, the mathematical period of Pythagoras and Plato contains elements of the quantitative interpretation of the world. As we will see in chapter 9, the latter is accomplished in the following period of the exact mechanical models of Archimedes and the Antikythera mechanism. The first steps for the mathematical depiction of a system are: a) the geometrical symbolisms and the definition of the concept of analogy, b) the definition of numbers and their ratios as a result of the comparison and measurement processes of the physical magnitudes, and c) the development of methods of approximation and algorithms of finite or infinite steps for the representation of irrational numbers (chapter 8).

More precisely, practice and experience, such as the process of measurement, take a theoretical form in terms of the first mathematical symbols of numbers that represent the quantitative models of the natural magnitudes. Particularly, the Pythagoreans consider the numbers as conceptual symbols that model the world, its properties and phenomena. The relationships and the comparisons between them result in the creation of 'ratio' or 'proportion', of the concept of analogy, the so important concept for the

[40] Cicero, *Tusculanarum disputationum*, v. 23; English translation of the relevant text in Sarton' G. *Appreciation of ancient and medieval science during the Renaissance (1450-1600)*, Philadelphia: University of Pennsylvania, 1955, p. 214.

process of modelling. This analogy in turn results in the harmony. Experimental methods are developed (e.g., string) and the musical scales lead to Pythagoreans' quantitative models of the world. Geometry, on the other, from a merely experimental method evolves to an exact theoretical science, which, by following the steps of analysis and technical implementation, leads to the creation of geometrical models that correspond to various physical systems. The analogy of geometrical magnitudes allows Thales to transform the process of measurement to a modelling process. And the whole world due to Pythagoras and Plato acquires the geometrical regular solids, the polyhedrons, on which the mathematical account of the world is based.

Chapter 8

NUMERICAL MODELLING AND APPROXIMATION: THE METHOD OF ANTHYPHAERESIS

8. NUMERICAL MODELLING AND APPROXIMATION: THE METHOD OF ANTHYPHAERESIS

8.1 Introduction

Continuing the investigation of the building blocks that give rise to the concept of mathematical modelling, and in addition to those described in the previous chapter, we will see here the development of methods of approximation and algorithms of finite or infinite steps for the representation of irrational numbers, which mainly contribute to the exact numerical mathematical modelling.

A simple example of an exact numerical model is the ratio of two numbers, which arises from the geometrical concept of analogy. The ratio of two numbers expresses the relationship between two magnitudes A and B. The discovery of irrational ratios and the need to suggest ways to approximate them has imposed the philosophical and mathematical thought of that time to go a little bit further. This progress in the field of philosophy is accomplished through Plato and his method of dichotomy, as already explained in chapter 6, and through Zeno and his contribution to the concept of infinity, which will be elaborated in this chapter. In the field of mathematics, on the other hand, Euclid's method of anthyphaeresis introduces the fundamental notions of creating sequences that approximate irrational numbers, and Archimedes applies the anthyphaeretic method not only on magnitudes, but also on the areas of particular figures, resulting in the evaluation of $\sqrt{3}$ or π. The common factor in all these approaches is the concept of algorithm, which in the anthyphaeretic method takes a more precisely geometrical and numerical form and leads to the estimation of irrational numbers, by means of unlimited approximations.

8.2 The Ratio as an Early Numerical Model

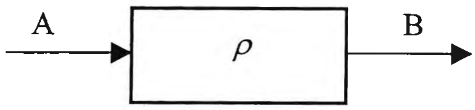
The arithmetised part of mathematics is characterized by the use of the concept of number. Mathematics in which the idea of number is extended beyond the numerical symbol to include fractional quantities, introduces new kinds of number by manipulating ratios.

Many cultures have developed their own versions of arithmetised mathematics. A great part of Babylonian mathematics deals with numbers that can be written as common fractions with denominators containing powers of two, three, and five. In the second century B.C., after the time of the merging of Greek geometrical and Babylonian arithmetical astronomy, the arithmetisation of geometry started. By choosing an assigned *unit* line, all lines become endowed with a numerical length. This unit line defines a *unit* square, whence all two-dimensional figures are assigned numerical areas. Similarly all three-dimensional figures are assigned numerical "volume".

Egyptian mathematics is also arithmetised, while Ptolemaic astronomy and mathematics of Heron and Diophantus are a profound mixture of Greek geometrical and Babylonian arithmetised method. Western mathematics since the sixteenth century has been dominated by the arithmetised point of view, and this culminated in the developments of the nineteenth and twentieth centuries.

Early Greek Mathematics seems to be initially non-arithmetised and not to have at its disposal the recent convenient algebraic notation. Actually, there is no evidence, before the time of Heron and Diophantus, for any conception of common fraction $\frac{p}{q}$ and their manipulations such as, for example $\frac{p}{q} x \frac{r}{s} = \frac{pr}{qs}$. Diophantus was the first Greek mathematician who recognised fractions as numbers. Fractions, being "numerical quantities", belong to the arithmetised part of mathematics and the non-arithmetised approach may often be signalled by the use of the alternative terminology of ratio or logos. The mathematicians of the classical period speak only of a ratio of integers, not of parts of a whole and the ratios were used only in proportions (Kline, 1972). The two words, ratio and proportion, *logos* and *analogon* in Greek, refer to different kinds of mathematical entities, thought they are frequently conflated and used as if they were interchangeable. The manifold meaning of the Greek word *logos* as "discourse", "reason", "argument", "inference", "logic or thought" presumably explains why these two concepts came to take such an enormous importance in Greek thought. The Latin *ratio* preserved much of the total meaning, but in our language it ended up designating numerical proportion alone. We could say in modern terms that a ratio is the function of two or possibly more variables whereas a proportion is a condition that may or may not hold between four objects (Fowler, 1999).

The ratio or logos ρ of two magnitudes A, B represents the relationship between A and B or in general the process of transition of the one magnitude to the other $B = \rho \cdot A$. It constitutes the mathematical model or the transfer function of a simple, linear, not dynamic proportional system.



$$\rho = \frac{B}{A}$$

Figure 8.1: Ratio as the transfer function of a system

The Pythagoreans work out the concept of analogy and theory of proportion with regard to numbers and introduce one more function, the one of number as natural law, as ratio and mathematical formula. They distinguish three types of ratios or three sorts of means: the arithmetic, the geometric, and the harmonic[41]. The arithmetic mean b of two numbers a and c is also known as the average of them. In other words the three terms a, b, and c are in arithmetic proportion when they increase by an equal amount as we go from one to another, i.e., the third exceeds the second by as much as the second exceeds the first: $a-b=b-c$ and the arithmetic mean is $b=\frac{a+c}{2}$ (e.g., 2, 4, 6). In this proportion, the ratio of the two larger terms is smaller, while the ratio of the two smaller terms is larger (i.e., $\frac{6}{4}<\frac{4}{2}$).

The geometric mean is being called proportion. Three terms are in geometric proportion when the first term is to the second as the second to the third, i.e., the three terms are in continuous proportion, $\frac{a}{b}=\frac{b}{c}$ and the geometric mean is $b^2 = a \cdot c$. The ratio of the two larger terms is equal to that of the two smaller.

Harmonic proportion is that in which the terms are such that if the first exceeds the second by a certain part of the first, the second will exceed the third by the same part of the third. For example, in the case of numbers 6, 4, 3, the number 4 is the harmonic

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[41] See Archytas, Frag. 2 (Diels, *Fragmente der Vorsokratiker,* I. 334.16-335.13), Cohen *et al.*, 1966)

mean of 6 and 3 since: $6=4+2,\ where\, 2=\frac{1}{3}\ of\, 6$, and $4=3+1,\ where\, 1=\frac{1}{3}\ of\, 3$. In this proportion the ratio of the two larger terms is larger, while the ratio of the two smaller is smaller (i.e., $\frac{6}{4}>\frac{4}{3}$).

These are the simplest types of ratios that are applicable only to commensurable numbers and magnitudes. The following invention of another type of ratio accomplished by Euclid, the so-called anthyphaeretic ratio, which will be explored later on, contributed to the complete mathematical approximation not only of commensurable but also of incommensurable quantities.

Summarising, the logos gradually develops from the philosophical concept of the relation of two things to the geometrical analogy of two magnitudes and finally to the numerical ratio of two quantities.

8.3 The Phenomenon of Incommensurability

The arithmetical consideration of the logos of two magnitudes results in a difficult mathematical problem. For the arithmetical logos of two magnitudes A and B can be either perfect or complete if the result is an integer, i.e., the less magnitude A measures B or A is part of B ($B=\alpha\cdot A$, where α: integer), or non-perfect or incomplete if the result is not an integer, i.e., magnitude A does not measure B ($B=\alpha\cdot A+\upsilon$, where the remainder $\upsilon<A$ and $\upsilon=B-\alpha\cdot A$). Non-perfect or incomplete logos lead to a new kind of number, the irrational one.

The first who faced the problem of irrational numbers were the Pythagoreans. We have already seen that numbers, and in particular integers, ruled the universe of the Pythagoreans. In the discovery of the harmonics of sounds and more precisely in the correspondence between numbers, melodic sounds, and string lengths the Pythagoreans saw a verification of their number philosophy. The finding out of 'golden' triangles, the sides of which were in the ratio 3:4:5, or 5:12:13, or 8:15:17, and the contemplation of them led to the great discovery of the so-called Pythagorean theorem. Therein, they saw the inherent union between arithmetic and geometry, an additional confirmation of their dictum.

But the triumph was short-lived. Indeed one of the immediate consequences of the theorem was the discovery of the irrational number $\sqrt{2}$, which was made geometrically by comparing the diagonal of a square with the length of one side and finding that it cannot be expressed as the ratio of two integers. This discovery caused a great consternation to the Pythagoreans. It was a revised outlook on matters of geometry. They had felt that given any two straight-line segments such as the side and the diagonal of a square there would be some integers a and b so that the ratio of the lengths of these segments would be $\frac{a}{b}$. The absence of such a ratio *(logos)* was declared by the name 'irrational' or '*alogon*', which is given to 'ratios' that are not rational numbers.

The phenomenon of magnitudes that could not be expressed as a ratio of integer numbers disturbed the up to that point perfect concordance between arithmetical and geometrical things. How can a number dominate the universe when it falls to account for even the most immediate aspect of the universe, geometry?

At this point, we will cite selectively some passages that refer to the way the first mathematicians and philosophers had perceived the phenomenon of irrationality and to their efforts to deal with it.

8.3.1 The Pythagorean Theorem

The arithmetical correlation of the plane figures with linear straight segments introduces the problem of the irrational numbers. It is clearly shown in the one of the most important mathematical discoveries of Pythagoras, in the so-called Pythagorean theorem. Its geometrical proof is presented in the following figure:

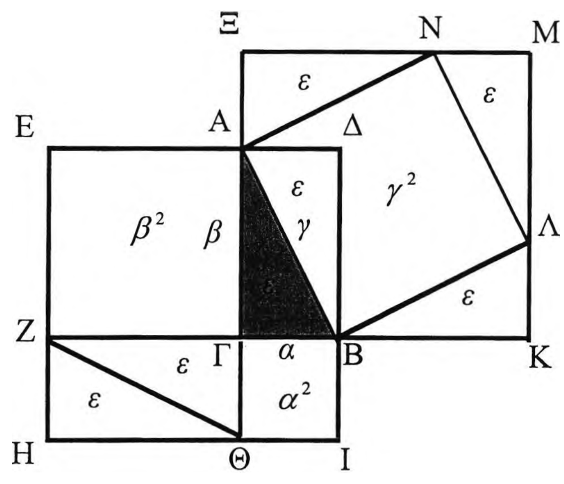


Figure 8.2: Geometrical proof of Pythagorean theorem

The squares EHIΔ and ΓKMΞ are equal so long as their sides are equal $(\alpha + \beta)$.

So
$$\alpha^2 + \beta^2 + 4\varepsilon = \gamma^2 + 4\varepsilon$$
$$or \quad \alpha^2 + \beta^2 = \gamma^2$$

By this theorem Pythagoras achieves the following:

1. He discovers an algebraic mathematical relation that combines all the sides of a right triangle. In other words, he creates an algebraic mathematical model that characterizes any right triangle. Simultaneously, this relation constitutes an algebraic model of the geometrical Pythagorean proof.
2. He corresponds, for example, to the rectilinear segments not only numbers but also their algebraic symbols (e.g., α, β). He does the same in the case of two-dimensional figures such as the squares of these segments. Following he correlates the one-dimensional and the two-dimensional magnitudes in a simple mathematical relation. But in this way he introduces the problem of irrational numbers, such as $\sqrt{2}$. This problem constitutes the subject of many researches afterwards.

8.3.2 References to Incommensurability by Plato

We have already mentioned the distinction Plato makes between the imperfect world of the sensible phenomena and their eternal models, the perfect Forms or Ideas. Mathematics occupies for him the intermediate place between these two worlds (Taton, 1963). The words 'Let none but geometers enter here' inscribed over the portals of his Academy give a notion of the much importance he attaches to mathematics. He was aware of all the problems that were of interest to the mathematicians of his day. Along with all the mathematical passages in his work, there are many occasions on which he refers to the theory of the irrational numbers, the discoveries of Theodorus and the contributions of Theaetetus to this subject.

> Plato, *Theaetetus*, 147d-148b: "Theodorus [of Cyrene, a pupil of Protagoras and a teacher of Plato] was writing out for us something about roots, such as the sides of squares three or five feet in area, showing that they are incommensurable by the unit: he took the other examples up to seventeen, but there for some reason he stopped. Now as there are innumerable such roots, the notion occurred to us of attempting to find some common description that can be applied to them all [...]

We divided all numbers into two classes: those which are made up of equal factors multiplying into one another, which we compared to square figures and called square or equilateral numbers; – that was one class [...]
The intermediate numbers, such as three and five, and every other number which is made up of unequal factors, either of a greater multiplied by a less, or of a less multiplied by a greater, and, when regarded as a figure, is contained in unequal sides; – all these we compared to oblong figures, and called them oblong numbers [...]
The lines, or sides, which have for their squares the equilateral plane numbers, were called by us lengths; and the lines whose squares are equal to the oblong numbers, were called powers or roots; the reason of this latter name being, that they are commensurable with the former not in linear measurement, but in the area of their squares. And a similar distinction was made among solids." (Jowett, 1964, vol. 3)

This passage is the first unequivocal and explicit reference to the phenomenon of incommensurability. It is presented mostly as a source of interesting and fruitful problems rather than a phenomenon that posed itself any fundamental conceptual difficulty for mathematicians. Plato here tells us about Theodorus of Cyrene (465-398 B.C.), who had been able to prove that the first seventeen numbers, except those of 1, 4, 9, and 16, are "incommensurable with one". Even though Theodorus had been dealt separately with every square he had been unable to establish a general rule. Theaetetus (417-369 B.C.) in his turn embarked on it, and undertook to set up a general theory of irrationality. The numbers, which are made up of equal factors multiplying into one another $(A = a \times a)$, are called "square or equilateral", (e.g., $1(1\times 1),\ 4(2\times 2),\ 9(3\times 3)$), whereas the numbers that are made up of unequal factors $(A = a \times \beta,\ \beta > a\ or\ \beta < a)$ are called "oblong" numbers, (e.g., $2(1\times 2),\ 3(1\times 3),\ 5(1\times 5),\ 6(2\times 3)$).

The square root of equilateral numbers is named 'length', which is an integer, e.g., $\sqrt{\alpha^2} = \alpha,\ \sqrt{1^2} = 1,\ \sqrt{2^2} = 2$, and so on, and of oblong numbers is called 'power or root', e.g., $\sqrt{\alpha x \beta},\ \beta <> \alpha$. The area of the square $AB\Gamma\Delta$ is $E = \alpha^2$, whereas the area of the square $AEZ\Gamma$ is $2E = 2\alpha^2$, where α is length, δ is power, and the ratio $\delta : \alpha$ is the irrational number $\sqrt{2}$.

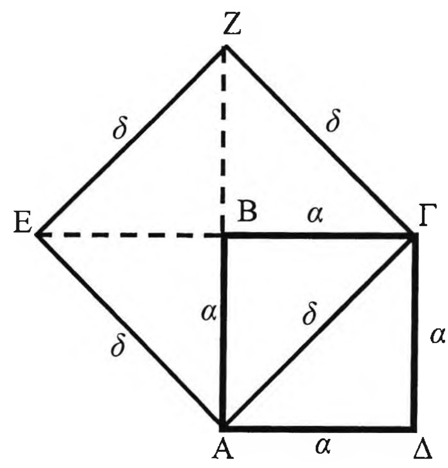
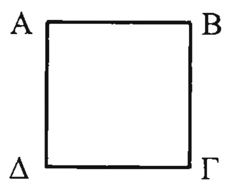


Figure 8.3: Geometrical representation of $\sqrt{2}$

Thus, by using geometrical figures and geometrical relations instead of arithmetical, problems, which were not susceptible of arithmetical solutions, could be satisfactorily solved. An indicative example, which involves the use of geometrical figures to circumvent an irrational number, is referred to one of Plato's dialogues, *Meno*.

Meno, 82a-85d: " [...]

(Socrates depicts on the ground a square. Let's say the $AB\Gamma\Delta$. If the sides AB and BΓ are two feet long then the area of $AB\Gamma\Delta$ is twice two feet $2\times 2 = 4$)



[...] *Socrates*: Now could one draw another figure double the size of this, but similar, that is, with all its sides equal like this one?

Boy: Yes.

Socrates: How many feet will its area be?

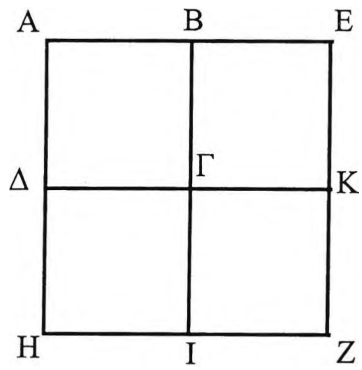
Boy: Eight.

Socrates: Now then, try to tell me how long each of its sides will be. The present figure has a side of two feet. What will be the side of the double-sized one?

Boy: It will be double, Socrates, obviously ...

Socrates: [...] You say that the side of double length produces the double-sized figure? [...]

(Duplication of side AB gives AE and if we depict four lines equal to AE we have the square AEZH :



The square AEZH contains four squares (ΑΒΓΔ, ΒΓΚΕ, ΓΚΖΙ, ΔΓΙΗ). Each of them has the same area as the square ΑΒΓΔ, i.e., four feet. The boy, under the instructions of Socrates tries to find how long are the sides of a square, whose area is 8 feet. He thought that the desired squared would have twice the side of square ΑΒΓΔ since its area is twice the area of ΑΒΓΔ. But if we take a side of 4 feet long the area of square AEZH that arises is 16 feet and not 8).

Socrates: But does it contain these four squares, each equal to the original four-feet one?

Boy: Yes.

Socrates: How big is it then? Won't it be four-times as big?

Boy: Of course.

Socrates: And is four-times the same as twice?

Boy: Of course not.

Socrates: But how much is it?

Boy: Fourfold.

Socrates: So doubling the side has given us not a double but a fourfold figure? [...]

Socrates: Then how big is the side of the eight-feet figure? This one has given us four-times the original area, hasn't it?

Boy: Yes.

Socrates: And a side half the length gave us a square of four feet?

Boy: Yes.

Socrates: Good. And isn't a square of eight feet double this one and half that?

Boy: Yes.

Socrates: Will it not have a side greater than this one but less than that?

Boy: I think it will.

Socrates: Right. Always answer what you think. Now tell me. Was not this side two feet long, and this one four?

Boy: Yes.

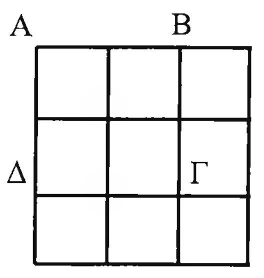
Socrates: Then the side of the eight-feet figure must be longer than two feet but shorter than four?

Boy: It must

Socrates: Try to say how long you think it is.

Boy: Three feet.

(Since the sides of the desired square of 8 feet area are longer than 2 feet and less than 4, the boy came to the conclusion that its side should be 3 feet long. This square arises if we take half of AB and add it to AB. Again it is a wrong conclusion since the area of the above square is 9 feet and not 8)



Socrates: If it is three feet this way and three that, will the whole area be three-times three feet?

Boy: It looks like it.

Socrates: And that is how many?

Boy: Nine.

Socrates: Whereas the square double our first square had to be how many?

Boy: Eight.

Socrates: But we haven't yet got the square of eight feet even from a three-feet side?

Boy: No.

Socrates: Then what length will give it? Try to tell us exactly. If you don't want to count it up, just show us on the diagram [...]

Socrates: [...] Tell me, boy, is not this our square of four feet? You understand?

Boy: Yes.

Socrates: Now we can add another equal to it like this?

Boy: Yes.

Socrates: And a third here, equal to each of the others?

Boy: Yes.

Socrates: And then we can fill in this one in the corner?

Boy: Yes.

Socrates: Then here we have four equal squares?

Boy: Yes.

Socrates: And how many times the size of the first square is the whole?

Boy: Four-times.

Socrates: And we want one double the size. You remember?

Boy: Yes.

Socrates: Now does this line going from corner to corner cut each of these squares in half?

Boy: Yes.

Socrates: And these are four equal lines enclosing this area?

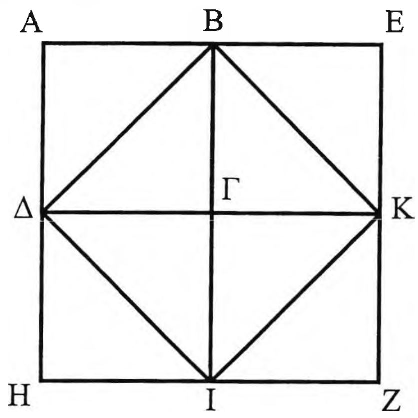
Boy: They are.

Socrates: Now think. How big is this area?

Boy: I don't understand.

Socrates: Here are four squares. Has not each line cut off the inner half of each of them?

(According to the above description, the resultant square is shown in the following figure. The diagonals ΔB, BK, KI and $I\Delta$ form the square ΔBKI, whose area is twice the area of $AB\Gamma\Delta$, i.e., 8 feet.



Thus, the side of a square whose area is double that of the one with sides two feet long has for its length an irrational number – $\sqrt{8}$ – which can be represented by the diagonal of the initial square)

Boy: Yes.

Socrates: And how many such halves are there in this figure?

Boy: Four.

Socrates: And how many in this one?

Boy: Two.

Socrates: And what is the relation of four to two?

Boy: Double.

Socrates: How big is this figure then?

Boy: Eight feet.

Socrates: On what base?

Boy: This one.

Socrates: This line which goes from corner to corner of the square of four feet?

Boy: Yes

Socrates: The technical name for it is 'diagonal'; so if we use that name, it is your personal opinion that the square on the diagonal of the original square is double its area[42]...."

In this outstanding dialogue, Socrates leads the boy not only to the conception of the nature of the irrational numbers, but also to an approximate algorithmic process for their calculation. Another approximate algorithmic method for the calculation of $\sqrt{2}$ is given in the Republic of Plato:

Republic, 546 b-d: "But to the knowledge of human fecundity and sterility all the wisdom and education of your rulers will not attain; the laws which regulate them will not be discovered by an intelligence which is alloyed with sense, but will escape them, and they will bring children into the world when they ought not. Now that which is of divine birth has a period which is contained in a perfect number, but the period of human birth is comprehended in a number in which first increments by involution and evolution (*or* squared and cubed) obtaining three intervals and four terms of like and unlike, waxing and waning numbers, make all the terms commensurable and agreeable to one another. The base of these (3) with a third added (4) when combined with five (20) and raised to the third power furnishes two harmonies; the first a square which is a hundred times as great ($400 = 4 \times 100$), and the other a figure having one side equal to the former, but oblong, consisting of a hundred numbers squared upon rational diameters of a square, i.e., omitting fractions, the side of which is five

[42] Meno: This text is based on the following book(s): Plato. Plato in Twelve Volumes, Vol. 2 translated by W.R.M. Lamb, Cambridge, MA, Harvard University Press; London, William Heinemann Ltd. 1962 & 1967

> ($7 \times 7 = 49 \times 100 = 4900$), each of them being less by one (than the perfect square which includes the fractions, sc. 50) or less by two perfect squares of irrational diameters (of a square the side of which is five = $50 + 50 = 100$); and a hundred cubes of three ($27 \times 100 = 2700 + 4900 + 400 = 8000$). Now this number represents a geometrical figure, which has control over the good and evil of births. For when your guardians are ignorant of the law of births, and unite bride and bridegroom out of season, the children will not be goodly or fortunate. And though only the best of them will be appointed by their predecessors, still they will be unworthy to hold their fathers' places, and when they come into power as guardians, they will soon be found to fail in taking care of us, the Muses, first by under-valuing music; which neglect will soon extend to gymnastic; and hence the young men of your State will be less cultivated." (Jowett, 1964, vol. 2)

In this passage, Plato enters the construction of the nuptial number, the legendary platonic number. This passage prompts the later descriptions of side and diagonal numbers in the commentaries (they will be quoted later on) of Theon of Smyrna and Proclus, where they describe a method of arriving at approximations to the value of $\sqrt{2}$ by approaching its value alternatively from the side of the too little and from that of the too great. Though the earliest mention of this method is in Theon, Plato probably was aware of this method since he knew that $\frac{7}{5}$ is an approximation to the value of $\sqrt{2}$.

The above description of the platonic number could be considered as an arithmetic model of human's fecundity periods.

The distinction between the commensurable and incommensurable numbers is undertaken also in the Platonic works of the Laws:

> *Laws,* 819d-820b: "*Athenian:* The next step of the teachers is to clear away, by lessons in weights and measures, a certain kind of ignorance, both absurd and disgraceful, which is naturally inherent in all men touching lines, surfaces and solids.
>
> *Clinias:* What ignorance do you mean, and of what kind is it?
>
> [...]
>
> *Athenian:* [...] you know what a line is? [...] And surface? [...] And do you know that these are two things, and that the third thing, next to these, is the solid?
>
> *Clinias:* I do.

Athenian: Do you not, then, believe that all these are commensurable one with another? [...] And you believe, I suppose, that line is really commensurable with line, surface with surface, and solid with solid?
Clinias: Absolutely.
Athenian: But supposing that some of them are neither absolutely nor moderately commensurable, some being commensurable and some not, whereas you regard them all as commensurable [...] Again, as regards the relation of line and surface to solid, or of surface and line to each other – do not all we Greeks imagine that these are somehow commensurable with one another?
Clinias: Most certainly.
Athenian: But if they cannot be thus measured by any way or means, while, as I said, all we Greeks imagine that they can, are we not right in being ashamed for them all, and saying to them, "O most noble Greeks, this is one of those 'necessary' things which we said it is disgraceful not to know, although there is nothing very grand in knowing such things[43]."

There are many other occasions in Plato's work on which he refers to irrationals, such as *Hippias Major* 306B, *Epinomis* 990C-991A, and so on, but it could be excessive to analyse all of them in details. The number of all these references is the clear evidence of Plato's constant attempts to surmount the problem of incommensurability, which was previously considered as an insurmountable obstacle.

The phenomenon of incommensurability and especially the incommensurability of the diagonal is also one of Aristotle's favourite mathematical illustrations, which is cited more than thirty times in his bulky work. For an analytical quotation of all ancient references to this phenomenon see Fowler, 1999, pp. 290-297.

8.4 The Concept of Infinite

So far, it is noticeable that the ancient Greek thinkers were trying to interpret the theoretical mathematical conceptions, the numbers and their ratios by using geometrical figures and methods. For them 'comprehensible' was what could be depicted by figures

[43] This text is based on the following book(s): Plato in Twelve Volumes, Vols.10 & 11, translated by R.G. Bury, Cambridge, MA, Harvard University Press; London, William Heinemann Ltd. 1967 & 1968.

and constructed by instruments, such as the ruler and the compass. In particular, whatever that could be measured. Magnitudes, figures, and numbers correspond between each other. The logical mathematical proofs are the consequence of the experimental geometrical methods.

The phenomenon of incommensurable magnitudes, of magnitudes that did not have any common measure or could not be constructed by the ruler and compass disturbed the pure geometrical interpretation of the world. What is $\sqrt{2}$? How can it be measured? The necessity to deal with such problems that could not be solved by pure geometry resulted in a new method of successive approximations, which in mathematics has been expressed by regressive algorithms, and in the distinction between finite and infinite steps of algorithms closely related to the concept of infinity.

Zeno of Elea, pupil of Parmenides, elaborated the concept of infinity in order to reverse the conventional geometrical logic and his efforts ended up to his paradoxes that there is not any motion; there is not any mass. Here are some examples.

8.4.1 Paradoxes of Zeno

The problem of the infinite, like the closely related problem of irrationals, grew up on Greek soil. In the fifth century B.C., beside Miletus and Tarentum, another important place where Greeks were engaged in laying a scientific foundation for the study of mathematics was Elea, which was founded in Sicily by Xenophanes (*ca.* 570-480 B.C.). The members of the Eleatic school, Parmenides (*ca.* 540-480 B.C.), Zeno (*ca.* 490-425 B.C), and Melissus (*ca.* 500-440 B.C.), who followed Xenophanes, were famous for the difficulties they raised in connection with questions that required the use of infinite series, such the well-known paradox of Achilles and tortoise.

What is the source of the concept of infinity, this faith in the inexhaustibility of the counting process? We know that any attempt on our part to exhaust number by counting would only end in our own exhaustion. The essential character of infinity is clearly described in this fragment due to Anaxagoras: *Fr. 3* "Among the small there is no smallest, but always something smaller. For what is cannot cease to be no matter how far it is being subdivided". Around infinity have grown up all the paradoxes of mathematics: from the arguments of Zeno to the antinomies of Kant and Cantor. "The Arguments of Zeno of Elea have, in one form or another," says Bertrand Russell, 1961, "afforded grounds for almost all the theories of space and time and infinity which have been constructed from his days to our own."

The first two Arguments of Zeno as recorded by Aristotle in his *Physics 239b5-20* are:

- The first Argument: *Dichotomy*

  "The first is the one on the non-existence of motion, on the ground that that which is in locomotion must arrive at the half-way stage before it arrives at the goal..."
- The second Argument: *Achilles and the Tortoise*

  "The second is the so-called Achilles, and it amounts to this, that in a race the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead..." (Wilbur et *al.*, 1979)

These arguments deal with the dichotomy of space and time. The infinite divisibility of space, in the first argument, and of time in the second argument, is the first step towards the geometrisation of mechanics. For endowing time with the attribute of infinite divisibility is equivalent to representing time as a geometrical line, to identifying duration with extension. In the first argument, Zeno says that the runner before reaching his goal, must reach the midpoint of the course, and it takes him a finite time to achieve this. He must also reach the midpoint of the remaining distance, and this too will take a finite time. Now what has been said once can always be repeated. There is an infinite number of stages in his traversing of the racecourse, and each one of these stages requires a finite stage. But the sum of an infinite number of finite integers is infinite. The runner, therefore, will never attain his goal.

The notion that what has been said or done can always be repeated, e.g., in the case of a rational number α the application of repetition gives the repeating sequence: α, a, a, ... constitutes the very concept of infinite and of infinite processes since the prototype of all infinite processes is the repetition. The two classical problems, the radicals and the evaluation of π, gave the impetus for the development of an important infinite process: the continued fractions.

An infinite process is defined as a set of operations generating an infinite sequence. The simplest type of sequence, that one of great historical and theoretical importance, is the geometrical sequence. Having selected any number for the first term and any other number for ratio, we proceed from term to term by multiplication through the ratio, e.g., $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, ..., \frac{1}{3^n}, ...$ The series generated by a geometrical sequence is called a geometrical progression. The geometrical sequence in the first argument of Zeno is $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, ...$ It generates the geometrical progression $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, ...$

The second Zero argument also involves a geometrical progression.

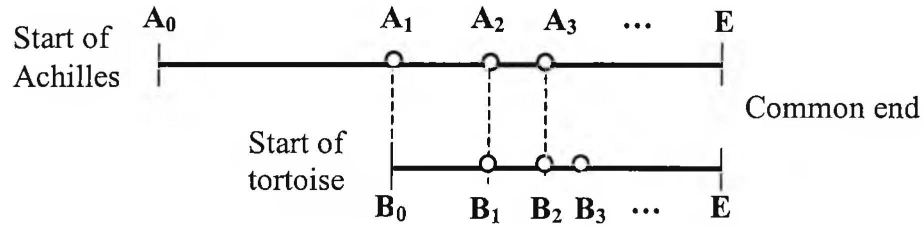


Figure 8.4: Zeno's paradox of Achilles and tortoise

Let assume that Achilles has to cover the total distance A_0E, whereas the tortoise starts from the point B_0 and has to cover the shorter distance B_0E. Regardless how fast Achilles runs, when he arrives at the starting point B_0 of the tortoise, she will have proceed to the point B_1. When Achilles arrives at the point B_1 the tortoise will be at the position B_2 and so on. Therefore, it seems that the tortoise will always precede. This paradox is due to the ignorance of the fact that: The sum of an infinite number of terms may be finite.

Zeno's arguments found their mathematical interpretation into Euclid's theorems. In particular in Book X, Euclid quotes an infinite process of successive approximations, of a 'geometrical algorithm', which allows the comparison of two unequal magnitudes:

- Two unequal magnitudes being set out, if from the greater there be subtracted a magnitude greater than its half, and from that which is left a magnitude greater than its half, and if this process be repeated continually, there will be left some magnitude which will be less than the lesser magnitude set out. (*Elem.* X, Prop. I)

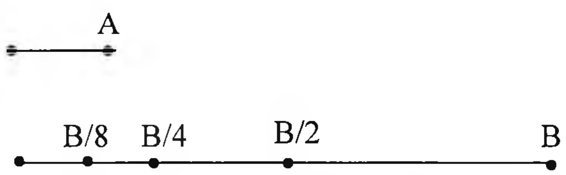


Figure 8.5: Approximation by dichotomy

According to this proposition, the less magnitude A can be approximated by successive bisections of the greater magnitude B.

The complete mathematical approximation of rational and irrational numbers either by the arithmetical or by the geometrical method is succeeded by the method of reciprocal subtraction or of anthyphaeresis by Euclid.

8.5 Euclid and the Method of Anthyphaeresis

Euclid of Alexandria (*ca.* 325-265 B.C.) is the one who undertakes the scientific formulation of geometry. In his *Elements*, he lays the foundations for the relations between geometrical figures and numbers. He also studies the problem of irrational numbers by taking advantage of the concept of dichotomy, of sequence, and of mathematical algorithm that are introduced by Plato in a theoretical point of view. At this point, we will cite the Euclidean definitions related to the dichotomy between numbers and magnitudes, as well as his definitions about their ratios and proportions.

8.5.1 The Euclidean Definitions about Magnitudes, Numbers and Ratios

Magnitudes

- A magnitude is a part of a magnitude, the less of the greater, when it measures the greater.
- The greater is a multiple of the less when it is measured by the less.
- A ratio is a sort of relation in respect of size between two magnitudes of the same kind.
- Magnitudes are said to have a ratio to one another, which are capable, when multiplied, of exceeding one another.
- Magnitudes are said to be in the same ratio, the first to the second and the third to the fourth, when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order.
- Let magnitudes, which have the same ratio be called proportional. (*Elem.* V, def. 1-6)

Numbers

- A unit is that by virtue of which, each of the things that exist is called one.
- A number is a multitude composed of units.
- A number is a part of a number, the less of the greater, when it measures the greater;
- But parts when it does not measure it.
- The greater number is a multiple of the less when it is measured by the less.
- An even number is that which is divisible into two equal parts.
- An odd number is that which is not divisible into two equal parts, or that which diners by an unit from an even number. (*Elem.* VII, def. 1-7)
- Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is to the forth. (*Elem.* VII, Def. 20)

Continuing, Euclid subdivides numbers into rational and irrationals and magnitudes into commensurable and incommensurable. The definitions of these concepts are:

Rational numbers

- Numbers prime to one another are those, which are measured by a unit alone as a common measure.
- A composite number is that which is measured by some number.
- Numbers composite to one another are those which are measured by some number as a common measure.
- A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced. (*Elem.* VII, def. 12-15)

The ratio of two prime numbers is called a rational number: $\rho = \frac{B}{A}$. Euclid defines the irrationality or incommensurability as follows:

Rational-irrational numbers, commensurable-incommensurable magnitudes

- Those magnitudes are said to be commensurable which are measured by the same measure, and those incommensurable which cannot have any common measure.

- Straight lines are commensurable in square when the squares on them are measured by the same area, and incommensurable in square when the squares on them cannot possibly have any area as a common measure.
- With these hypotheses, it is proved that there exist straight lines infinite in multitude which are commensurable and incommensurable respectively, some in length only, and others in square also, with an assigned straight line. Let then the assigned straight line be called rational, and those straight lines which are commensurable with it, whether in length and in square or in square only, rational, but those, which are incommensurable with, it irrational.
- And let the square on the assigned straight line be called rational and those areas which are commensurable with it rational, but those which are incommensurable with it irrational, and the straight lines which produce them irrational, that is, in case the areas are squares, the sides themselves, but in case they are any other rectilinear figures, the straight lines on which are described squares equal to them. (*Elem.* X, def. 1-4)

By defining afterwards the greatest common measure *(Prop. 3, 4)*, he assigns the dichotomy between commensurable and incommensurable magnitudes as follows:

- Commensurable magnitudes have to one another the ratio, which a number has to a number.
- If two magnitudes have to one another the ratio, which a number has to a number, the magnitudes will be commensurable.
- Incommensurable magnitudes have not to one another the ratio, which a number has to a number.
- If two magnitudes have not to one another the ratio, which a number has to a number, the magnitudes will be incommensurable. (*Elem.* X, prop. 5-8) (Heath, T., 2, 1956)

Additionally, Euclid formulates a theorem, which specifies the criterion of incommensurability:

"If, when the less of two unequal magnitudes is continually subtracted in turn from the greater, that which is left never measures the one before it the magnitudes will be incommensurable." (*Elem.* X, Prop. II) (Heath, T., 2, 1956)

8.5.2 The Anthyphaeretic Ratio

Euclid makes allusions to the method of anthyphaeresis or antanaeresis firstly in order to define the prime numbers (*Elem.* VII, Prop. I): "Two unequal numbers being set out, and the less being continually subtracted in turn from the greater (*ανθυφαιρουμένου δε αεί του ελάσσονος από του μείζονος*), if the number which is left never measures the one before it until a unit is left, the original numbers will be prime to one another" (Heath, T., 1, 1949), and secondly in the above-mentioned passage, in order to define incommensurability (*Elem.* X, Prop. II). Aristotle's celebrated passage that constitutes a great evidence on how early Greek mathematicians handled ratio or proportion, also refers to the method of anthyphaeresis.

> *Topics,* 158b 29-35: " [...] it appears also in mathematics that the difficulty in using a figure it sometimes due to a defect in definition; e.g., in proving that the line cuts the plane parallel to one side divides similarly both the line which it cuts and the area; whereas if the definition be given, the fact asserted becomes immediately clear: for the areas have the same [antanaeresis] fraction subtracted from them as have the sides: and this is the definition of the 'same ratio'..." (Ross, 1971, vol. 1)

Thus, as said by Aristotle, things are on the same ratio to one another when they have the same *ανταναίρεσις*. The etymology of the word is as follows: *υφαιρεώ* means to 'take away', to 'filch', *αναιρέω* to 'take away' or 'abolish'; and the prefix *αντι-* indicates that the 'taking away' or 'subtraction' from one magnitude answers to, or alternates with, a 'taking away' or 'subtraction' from another (Heath, T., 1, 1949). Let us give a more analytic description of what it does mean.

According to the anthyphaeretic method: given two numbers or two lines we count a) how many times the second line can be subtracted from the first line, b) how many times the remainder can be subtracted from second line, c) how many times the remainder of second line can be subtracted from the previous remainder, and so on, and this gives a string of numbers, possibly infinite, that characterize the relationship of size between the two things, and which is called 'anthyphaeretic ratio'. Since it is based on a process of repeated subtraction of one thing from another its ancient name of *antanaeresis* or *anthyphaeresis* can be translated as 'reciprocal subtraction'. The modern name of the whole process and the associated mathematical objects is 'Euclidean algorithm' or 'continued fractions', though the Euclidean algorithm is now generally construed as a division process and the continued fractions are now handled using the real numbers and a sophisticated generalisation of fractions.

It is noteworthy here that the systematic way of calculating gear ratios, such as those found in the Antikythera Mechanism, uses anthyphaeresis[44] (Fowler, 1999).

Let us elaborate the anthyphaeretic method.

The simple anthyphaeresis is the combination of the operation of subtraction and inversion, the reciprocal subtraction or the reciprocal retraction (*antanaeresis*). Thus, for two magnitudes A and B, where B>A, the anthyphaeresis is:

$$anth(A,B) = \frac{1}{B-A}$$

The reciprocal subtraction arises by the process of comparing and measuring two magnitudes. If A measures α times B, then $B = \alpha \cdot A$ and the ratio $\rho = \frac{B}{A} = \alpha$. But if A does not measure B, i.e., A measures B a times and the remainder is υ, then:

$$B = \alpha \cdot A + \upsilon,\ \upsilon < A \qquad (1)$$

Since B>A, the ratio

$$\rho = \frac{B}{A} \qquad (2)$$

will be greater than 1, i.e., $\rho > 1$ and by the relations (1) and (2) we have:

$$\rho = \alpha + \frac{1}{\rho_1} \qquad (3)$$

where $\rho_1 = \frac{A}{\upsilon} > 1$, is the comparison, the ratio of the initially less magnitude A to the first remainder υ. By the relation (3) we calculate ρ_1, which is the first anthyphaeresis of ratio ρ:

$$\rho_1 = \frac{1}{\rho - \alpha} \qquad (4)$$

The repetition of this process has as a result the following algorithm:

Step 1: For magnitudes A, B, if B>A and $B = \alpha \cdot A + \upsilon$, $\upsilon < A$ then

[44] See Price, 1975, p.58: "In this planetarium Archimedes would have used, perhaps for the first time, sets of gears arranged to mesh in parallel planes, and he would have been led to the rather elegant number manipulation which is necessary to get a set of correct ratios for turning the various planetary markers."

$$\frac{B}{A}=\alpha+\frac{\upsilon}{A} \quad or \quad \rho=\alpha+\frac{1}{\rho_1}$$

$$where \quad \rho=\frac{B}{A}>1, \quad \rho_1=\frac{A}{\upsilon}>1$$

Step 2: Comparison of the magnitudes A and υ, A>υ

$$A=\alpha_1\upsilon+\upsilon_1, \ \upsilon_1<\upsilon$$

$$or \quad \frac{A}{\upsilon}=\alpha_1+\frac{\upsilon_1}{\upsilon}, \quad \rho_1=\alpha_1+\frac{1}{\rho_2}$$

$$where \quad \rho_2=\frac{\upsilon}{\upsilon_1}>1$$

Step 3: Comparison of the magnitudes υ and υ_1,

$$\upsilon=\alpha_2\upsilon_1+\upsilon_2, \ \upsilon_2<\upsilon_1$$

$$or \quad \frac{\upsilon}{\upsilon_1}=\alpha_2+\frac{\upsilon_2}{\upsilon_1}, \quad \rho_2=\alpha_2+\frac{1}{\rho_3}$$

$$where \quad \rho_3=\frac{\upsilon_1}{\upsilon_2}>1 \quad etc.$$

The relationships of the successive anthyphaeresis, of the successive comparison between the less and the greater or more precisely between the less remainder and the greater preceding remainder form the next algorithm:

$$\rho=\alpha+\frac{1}{\rho_1}$$

$$\rho_1=\alpha_1+\frac{1}{\rho_2}$$

$$\rho_2=\alpha_2+\frac{1}{\rho_3}$$

...

$$\rho_\nu=\alpha_\nu+\frac{1}{\rho_{\nu+1}}$$

...

Where

$\rho,\rho_1,\rho_2,...,\rho_\nu,\rho_{\nu+1}$ are ratios greater than 1

and

$\alpha,\alpha_1,...,\alpha_\nu,...$ integers

that constitute the distributive measures of ratio ρ

This algorithm leads to the concise relation that determines/measures a ratio ρ (rational or irrational) by a group of integers ($\alpha,\alpha_1,...,\alpha_\nu,...$)

$$\rho = \alpha + \cfrac{1}{\alpha_1 + \cfrac{1}{\alpha_2 + \dots \atop \cdots \atop \alpha_\nu + \cfrac{1}{\alpha_{\nu+1} + \dots}}}$$

This relationship, which is the result of the anthyphaeretic algorithm, analyses the ratio ρ into the distributive anthyphaeretic integer measures $\alpha, \alpha_1, \dots, \alpha_\nu, \dots$ and allows the symbolism:

$$\rho = [\alpha, \alpha_1, \alpha_2, \dots, \alpha_\nu, \dots]$$

The two Euclidean proportions about anthyphaeresis can be formulated as follows:

1. The ratio ρ is rational if the anthyphaeretic algorithm consists of finite steps ν so that $\rho_{\nu+1} = 1$.
2. The ratio ρ is irrational if the anthyphaeretic algorithm is infinite.

Therefore, the irrational numbers are related to the concept of infinity and arise by an anthyphaeretic process of infinite steps.

Let us quote some examples on the anthyphaeretic algorithm:

Example 1: Numerical form of the anthyphaeretic algorithm of a rational ratio

Consider the unequal magnitudes A=3 and B=5. The ratio $\rho = \frac{B}{A} = \frac{5}{3}$ is analysed as follows:

Step 1: $\frac{5}{3} = 1 + \frac{2}{3}$, $\alpha=1$ and $\upsilon=2$

Step 2: $\frac{3}{2} = 1 + \frac{1}{2}$, $\alpha_1=1$ and $\upsilon_1=1$

Step 3: $\frac{2}{1} = 1 + 1$, $\alpha_2=1$ and $\upsilon_2=1$

Step 4: $\frac{1}{1} = 1 + 0$, $\alpha_3=1$ and $\upsilon_3=0$

So $\frac{5}{3}=[1,\ 1,\ 1,\ 1]$ or $\frac{5}{3}=1+\cfrac{1}{1+\cfrac{1}{1+1}}$

The rational number $\frac{5}{3}$ results after four anthyphaeretic steps.

Example 2: Numerical and geometrical forms of the anthyphaeretic algorithm of a rational ratio

Consider two rectilinear segments OA and OB that correspond to the numbers A=5 and B=8, which are commensurable and prime between each other. The ratio $\rho=\frac{B}{A}=\frac{8}{5}$ arises by a finite anthyphaeretic algorithm. Similarly to the example 1, numerically the algorithm is:

Step 1: $\frac{8}{5}=1+\frac{3}{5}$, α=1 and υ=3

Step 2: $\frac{5}{3}=1+\frac{2}{3}$, α_1=1 and υ_1=2

Step 3: $\frac{3}{2}=1+\frac{1}{2}$, α_2 =1 and υ_2=1

Step 4: $\frac{2}{1}=1+1$, α_3=1 and υ_3=1

Step 5: $\frac{1}{1}=1+0$, α_3=1 and υ_3=0

So $\frac{8}{5}=[1,\ 1,\ 1,\ 1,\ 1]$ or $\frac{8}{5}=1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+1}}}$

It is possible to reach the same result geometrically by means of a successive comparison process between the less and the greater. This process uses the compass as the comparison instrument and gives the linear geometrical form of the algorithm, which is shown in the following figure. The two under comparison rectilinear segments are commensurable so there is a common measure between them and the anthyphaeretic steps are finite.

Step 1: The comparison between the segments OA=A and OB=B gives the integer quotient $\alpha=1$ and the remainder $BA_1=\upsilon$.

Step 2: The remainder $BA_1=\upsilon$ after the comparison with the segment OA=A gives the integer quotient $\alpha_1=1$ and the remainder $OB_1=\upsilon_1$.

Step 3: The remainder $OB_1=\upsilon_1$ after the comparison with the segment $BA_1=B_1A_1=\upsilon$ gives the integer quotient $\alpha_2=1$ and the remainder $A_1B_2=\upsilon_2$.

Step 4: The remainder $A_1B_2=\upsilon_2$ after the comparison with the segment $OB_1=B_1B_2=\upsilon_1$ gives the integer quotient $\alpha_3=1$ and the remainder $A_2B_1=\upsilon_3$.

Step 5: The remainder $A_2B_1=\upsilon_3$ after the comparison with the segment $A_1B_2=A_2B_2=\upsilon_2$ gives the integer quotient $\alpha_4=1$ and the remainder is 0.

Thus the ratio $\rho=\dfrac{B}{A}=[1,\ 1,\ 1,\ 1,\ 1]=1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+1}}}=\dfrac{8}{5}$

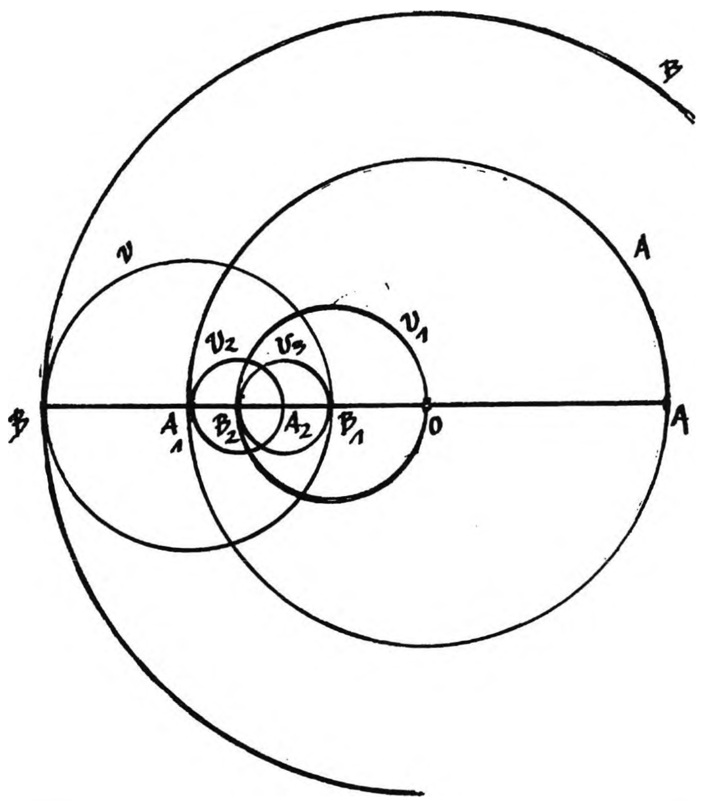


Figure 8.6: Linear geometrical anthyphaeretic algorithm of the comparison of segments OA, OB

The same procedure can be accomplished by using a plane geometrical algorithm, which approximates successively a rectangular by different squares.

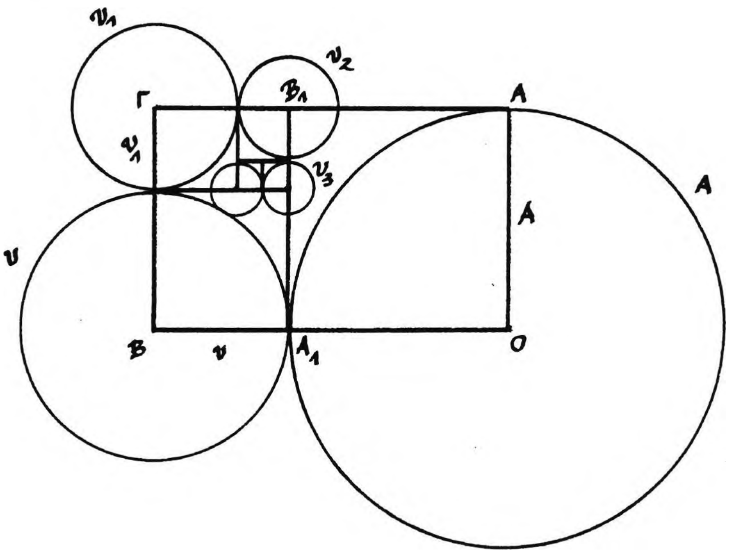


Figure 8.7: Plane geometrical anthyphaeretic algorithm of the comparison of segments OA, OB

The geometrical anthyphaeretic algorithms approximated in five finite steps the rational number $\frac{8}{5}$.

8.6 Infinite Anthyphaeretic Algorithms

In contrast to the rational numbers, which can be expressed by means of a finite anthyphaeretic steps, the irrational numbers demand infinite steps for their exact computation. Euclid and Archimedes invent such anthyphaeretic algorithms of infinite steps for the computation of the irrational numbers of $\sqrt{2},\ \sqrt{3},\ \pi$.

8.6.1 The Anthyphaeretic Algorithm of the Irrational Number $\sqrt{2}$

The irrational ratio $\sqrt{2}:1$ is defined as the ratio of the sides of two squares whose areas is double the one of the other, i.e., $\frac{E_\beta}{E_\alpha}=\frac{\beta^2}{\alpha^2}=2$. Therefore $\frac{\beta}{\alpha}=\sqrt{2}$. In figure 8.8 it is shown that the ratio $\sqrt{2}:1$ is the logos of the diagonal of one square to its side, because the square of the diagonal is double of the square of the side.

The computation of $\sqrt{2}$ could be realised geometrically by comparing the side of a square with its diagonal, as we have described in the example 2 (case of linear geometrical form).

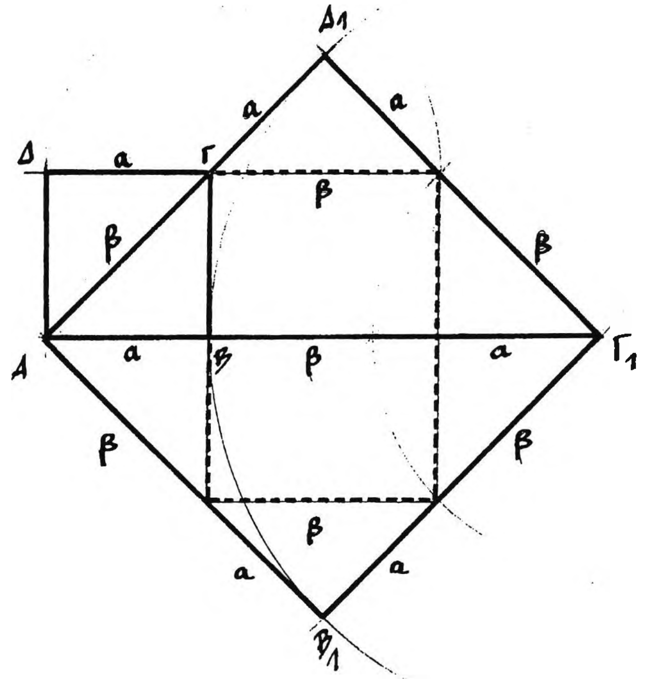


Figure 8.8: Geometrical interpretation of the irrational number $\sqrt{2}$

Another geometrical interpretation of $\sqrt{2}$ is possible by applying relations of proportion between two squares. Consider the square $AB\Gamma\Delta$, whose sides are α and the diagonal is β. We are looking for the ratio $\rho = \dfrac{\beta}{\alpha} = \sqrt{2}$. We construct another greater square, the $AB_1\Gamma_1\Delta_1$, whose side is $AB_1 = A\Delta_1 = \beta_1 = \alpha + \beta$. Geometrically it is shown that the diagonal of it is $A\Gamma_1 = \alpha_1 = 2\alpha + \beta$.

The ratio between the diagonal and the side of this square is $\rho_1 = \dfrac{\beta_1}{\alpha_1} = \dfrac{2\alpha+\beta}{\alpha+\beta}$.

Because of the analogy between the squares we have:

$$\rho_1 = \rho = \frac{\beta}{\alpha} = 1 + \frac{\alpha}{\alpha+\beta} = 1 + \frac{1}{1+\dfrac{\beta}{\alpha}} = 1 + \frac{1}{1+\rho}$$

The relation $\rho = 1 + \dfrac{1}{1+\rho}$ is of the form $x = f(x)$, i.e., the unknown x can be computed by a function f of its own. This kind of relation introduces an induction or a feedback, which results in an infinite process.

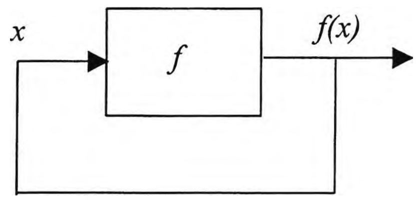


Figure 8.9: The infinite process of a feedback relation $x = f(x)$

The relation $x = f(x)$ entails the relation $x = f(f(x))$ and so on:

$$x = f(f(f...f(x))) \quad or \quad x = f^{\kappa}(x)$$

This relation leads to an infinite algorithm.

In our example, the relation $\rho = 1 + \frac{1}{1+\rho}$ leads to the infinite anthyphaeretic relation, which approaches the exact value of the irrational number $\sqrt{2}$ after an infinite number of steps:

$$\rho = \sqrt{2} = 1 + \cfrac{1}{1+1+\cfrac{1}{1+1+\cfrac{1}{1+1+...}}} = 1 + \cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+...}}}, \quad i.e. \ \rho = [1,\ 2,\ 2,\ 2,...]$$

The geometrical representation of this infinite anthyphaeretic process for the computation of $\sqrt{2}$ is given in the next figure.

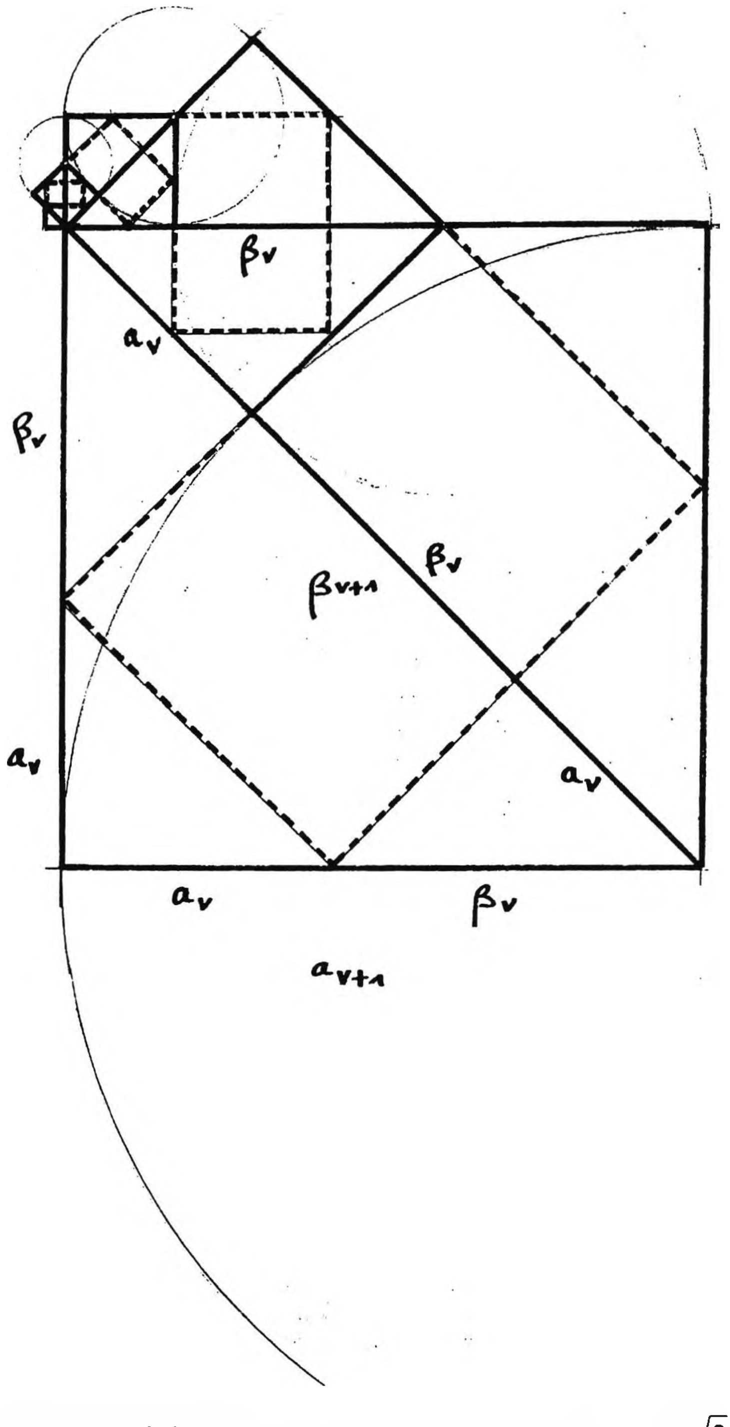


Figure 8.10: Infinite geometrical process of the irrational number $\sqrt{2}$

The algorithm of the ratios $\rho_v = \dfrac{\beta_v}{\alpha_v}$ of the diagonals β_v and the sides α_v of the recursive squares is respectively:

$$\rho = 1 + \frac{1}{1+\rho_1}$$

$$\rho_1 = 1 + \frac{1}{1+\rho_2}$$

...

$$\rho_v = 1 + \frac{1}{1+\rho_{v+1}}$$

...

The successive arithmetic approximations of the irrational number $\sqrt{2}$ are given in the following table.

v=0	$\rho_0 = [1] = 1$
v=1	$\rho_1 = [1,\ 2] = 1 + \frac{1}{2} = \frac{3}{2} = 1.5$
v=2	$\rho_2 = [1,\ 2,\ 2] = 1 + \cfrac{1}{2+\cfrac{1}{2}} = \frac{7}{5} = 1.4$
v=3	$\rho_3 = \frac{17}{12} = 1.41666$
v=4	$\rho_4 = \frac{41}{29} = 1.4137913$
v=5	$\rho_5 = \frac{99}{70} = 1.4142857$
v=6	$\rho_6 = \frac{239}{169} = 1.4142011$
v=7	$\rho_7 = \frac{577}{408} = 1.4142156$
...	...
	$\sqrt{2} = 1.4142136$

It is obvious that the approximations of the anthyphaeretic algorithm converge to the value of $\sqrt{2}$ in a helical form, where the odd approximations are greater, whereas the even are less than the final value.

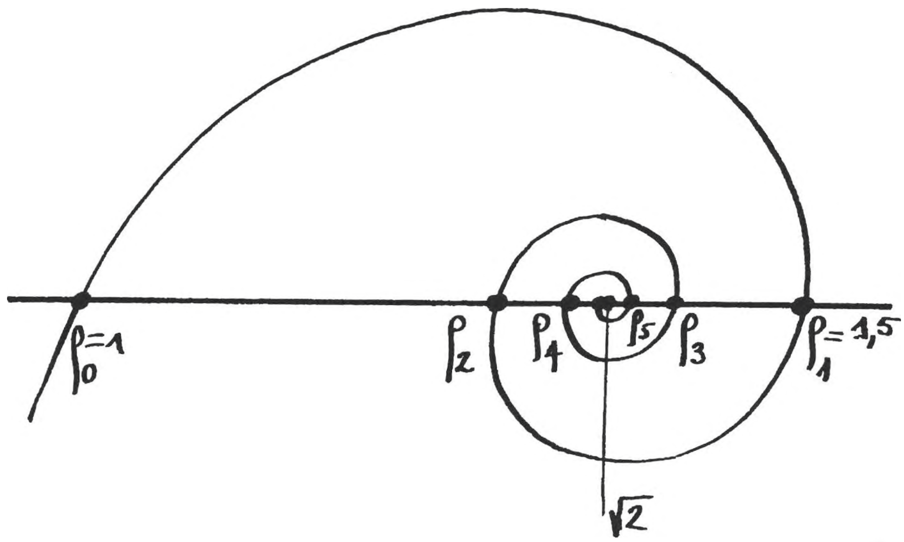


Figure 8.11: The infinite helix for the approximation of the irrational number $\sqrt{2}$

This is due to the following general property of anthyphaeretic ratios:

Let us consider the anthyphaeretic ratios

$$\rho = [\alpha_0,\ \alpha_1,\ \alpha_2,\ldots] \quad and \quad \rho' = [\alpha_0',\ \alpha_1',\ \alpha_2',\ldots]$$

It is $\rho > \rho'$ if $\alpha_0,\ \alpha_2,\ \alpha_4,\ldots > \alpha_0',\ \alpha_2',\ \alpha_4',\ldots$

or $\alpha_1,\ \alpha_3,\ \alpha_5,\ldots < \alpha_1',\ \alpha_3',\ \alpha_5',\ldots$

Thus, an anthyphaeretic ratio is greater than another one if the even measures of the first one are greater than that of the second whereas the odd measures are less.

Examples

1. [2, 3, 4] < [3, 3, 4] because 2 < 3
2. [2, 3, 4] > [2, 4, 4] because 3 < 4
3. [2] < [2, 4] < [2, 3, 4] < [2, 3] < [3]

Side and diagonal numbers for the computation of $\sqrt{2}$

Theon of Smyrna derives the same algorithms, by computing the successive ratios $\rho_\nu = \dfrac{\beta_\nu}{\alpha_\nu}$ of the diagonals β_ν and the sides α_ν of the infinite geometrical approximation of $\sqrt{2}$ (see figure 8.10). The side and the diagonal numbers are computed by the relations:

$$\alpha_{v+1} = \alpha_v + \beta_v, \quad \beta_{v+1} = 2\alpha_v + \beta_v$$

The algorithm starts with initial values $\alpha_0=1$ and $\beta_0=1$, i.e., the initial square is tiny and has its side equal to its diagonal.

Theon of Smyrna (2nd century A.C.), ed. Hiller, pp.42-5: "Just as numbers potentially contain triangular, square, and pentagonal ratios *(logoi),* and ones corresponding to the remaining figures, so also we can find side and diagonal ratios *(logoi)* appearing in numbers in accordance with the generative principles *(logoi);* for it is from these that the figures acquire balance. Therefore since the unit, according to the supreme generative principle *(logos),* is the starting-point of all the figures, so also in the unit will be found the ratio *(logos)* of the diagonal to the side. For instance, two units are set out, of which we set one to be a diagonal and the other a side, since the unit, as the beginning of all things, must have it in its capacity to be both side and diagonal. Now there are added to the side a diagonal and to the diagonal two sides, for as great as is the square on the side, taken twice, [so great is] the square on the diagonal taken once. The diagonal therefore became the greater and the side became the less. Now in the case of the first side and diagonal the square on the unit diagonal will be less by a unit than twice the square on the unit side; for units are equal, and 1 is less by a unit than twice 1. Let us add to the side a diagonal, that is, to the unit let us add a unit; therefore the [second] side will be two units. To the diagonal let us now add two sides, that is, to the unit let us add two units; the [second] diagonal will therefore be three units. Now the square on the side of two units will be 4, while the square on the diagonal of three units will be 9; and 9 is greater by a unit than twice the square on the side 2.

Again, let us add to the side 2 the diagonal of three units; the [third] side will be 5. To the diagonal of three units let us add two sides, that is, twice 2: there will be 7. Now the square from the side 5 will be 25, while that from the diagonal 7 will be 49; and 49 is less by a unit than twice 25. Again, if you add to the side 5 the diagonal 7, there will be 12. And if to the diagonal 7 you add twice the side 5, there will be 17. And the square of 17 is greater by a unit than twice the square of 12. When the addition goes on in the same way in sequence, the proportion will alternate; the square on the diagonal will be now greater by a unit, now less by a unit, than twice the square on the side; and such sides and diagonals are both expressible *(rhetos).*

The squares on the diagonals, alternating one by one, are now greater by a unit than double the squares on the sides, now less than double by a unit, and the alternation is regular. All the squares on the diagonals will therefore become double the squares on the sides, equality being produced by the alternation of excess and deficiency by the same unit, regularly distributed among them; with the result that in their totality they do not fall short of nor exceed the double. For what falls short in the square on the preceding diagonal exceeds in the next one." (Fowler, 1999)

This quotation sets out a column of side-roots and a corresponding column of diagonal-roots. The first number in each column is 1. Each subsequent side number is formed by adding to the previous side-number the corresponding diagonal-number, each subsequent diagonal-number by adding to the previous side-number diagonal-number twice the corresponding side-number. Thus we have:

Side-numbers	Diagonal-numbers
α_v	β_v
1	1
2	3
5	7
12	17
29	41
70	99

The ratios $\frac{1}{1}, \frac{3}{2}, \frac{7}{5}, \frac{17}{12}, \frac{41}{29}, \frac{99}{70}$ are successively closer approximations to the value of $\sqrt{2}$, alternately less and greater than it.

The anthyphaeretic method is also applied for the calculation of the irrational number $\sqrt{3}$, as well as for the so-called problem of 'squaring the circle'.

8.6.2 The Anthyphaeretic Algorithm of the Irrational Number $\sqrt{3}$

The ratio $\sqrt{3}:1$ is defined as the ratio of the sides of two squares, whose areas have ratio 3, i.e., $\frac{E_\beta}{E_\alpha} = \frac{\beta^2}{\alpha^2} = 3$. Therefore $\frac{\beta}{\alpha} = \sqrt{3}$. Geometrically the irrational number $\sqrt{3}$ arises from the logos of the short diameter of a regular hexagon to its side.

Thus let us consider the hexagon $AB\Gamma\Delta EZ$, which side is α and its short diameter is β, as it is shown in the following figure. By applying the Pythagorean theorem on the triangular $A\Gamma\Delta$ we have:

$$(A\Gamma)^2 = (A\Delta)^2 - (\Gamma\Delta)^2, \quad or \quad \beta^2 = 4\alpha^2 - \alpha^2 = 3\alpha^2 \quad so \quad \beta = \sqrt{3}\alpha$$

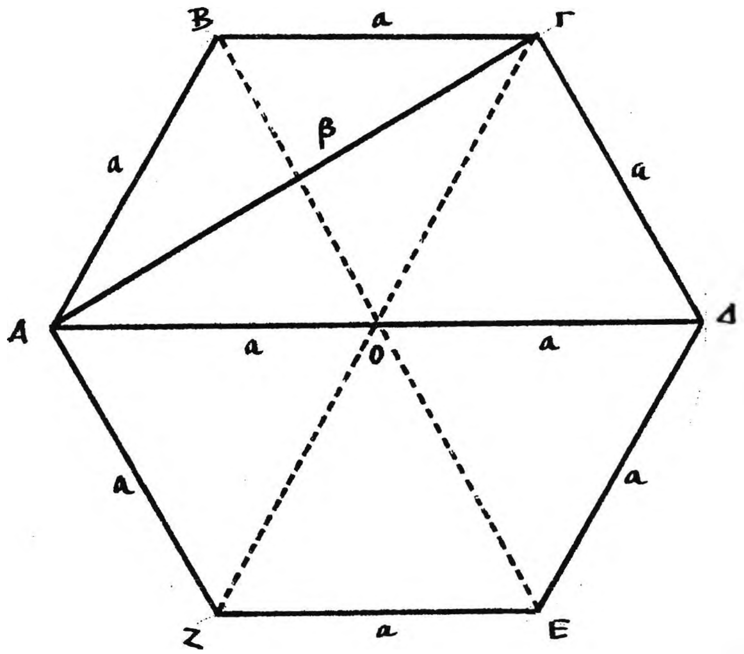


Figure 8.12: The geometrical interpretation of the irrational number $\sqrt{3}$

The anthyphaeretic method gives: $\sqrt{3}=[1, 1, 2, 1, 2, 1,...]$ and the successive approximations of $\sqrt{3}$ are shown in the next table:

ν=0	$\rho_0 = [1] = 1$
ν=1	$\rho_1 = [1, 1] = 1 + \frac{1}{1} = 2$
ν=2	$\rho_2 = [1, 1, 2] = 1 + \cfrac{1}{1+\cfrac{1}{2}} = \frac{5}{3} = 1.66$
ν=3	$\rho_3 = [1, 1, 2, 1] = 1.75$
ν=4	$\rho_4 = [1, 1, 2, 1, 2] = 1.727$
ν=5	$\rho_5 = [1, 1, 2, 1, 2, 1] = 1.733$
...	...

$$\sqrt{3}=1.7320508$$

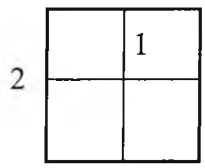
The helical approximation is $\rho_0 < \rho_2 < \rho_4 < \rho < \rho_5 < \rho_3 < \rho_1$

8.6.3 Archimedes and the Evaluation of $\sqrt{3}$

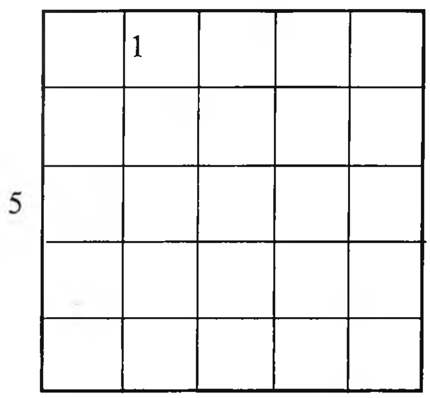
Archimedes extends the linear Euclidean anthyphaeretic method and develops algorithms for calculating the irrational number $\sqrt{3}$, as well as π, by approximating successively the upper and the low limits of both numbers.

Particularly in the case of $\sqrt{3}$ he calculates its value as follows:

If we consider the following squares:



We have: $2^2 = 3 \times 1^2 + 1$, i.e., $2^2 > 3 \times 1^2$, i.e., $\sqrt{3} < \frac{2}{1}$



And: $5^2 = 3 \times 3^2 - 2$, i.e., $5^2 < 3 \times 3^2$, i.e., $\sqrt{3} > \frac{5}{3}$

Similarly, $7^2 = 3 \times 4^2 + 1$, and $19^2 = 3 \times 11^2 - 2$, therefore, $\frac{19}{11} < \sqrt{3} < \frac{7}{4}$

$26^2 = 3 \times 15^2 + 1$, and $71^2 = 3 \times 41^2 - 2$, therefore, $\frac{71}{41} < \sqrt{3} < \frac{26}{15}$

Archimedes, by such a process, applies the anthyphaeretic method on the areas of particular figures and not on magnitudes as before. He continues and finally approximates the desired value by means of the following irrational numbers:

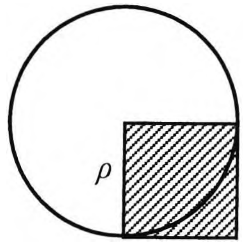
$$\frac{5}{3} < \frac{19}{11} < \frac{71}{41} < \frac{265}{153} < \frac{989}{571} < \frac{3691}{2131} < \sqrt{3} < \frac{1351}{780} < \frac{362}{209} < \frac{97}{56} < \frac{26}{15} < \frac{7}{4} < \frac{2}{1}$$

(Wilson, 1995)

i.e., $\sqrt{3} = 1{,}7320508$

8.6.4 Archimedes and the Evaluation of π

Before the exploration of Archimedes' method for the calculation of π, let us consider a circle and a square of radius and side ρ, respectively. If the area of the circle is E_κ and of the square E_τ we have:



$$\frac{E_\kappa}{E_\tau} = \frac{E_\kappa}{\rho^2} = \pi$$

Archimedes, who was always interested in measurements, also dealt with the incommensurable number π and gave its value approximately in various ways. In his work *The Measurement of a Circle*, Archimedes provides its anthyphaeretic computation. This work of Archimedes contains three propositions. According to the Proposition 1, the area of a circle is the same as that of a right-angled triangle whose sides are equal respectively to the radius ρ and the circumference π_κ of the circle:

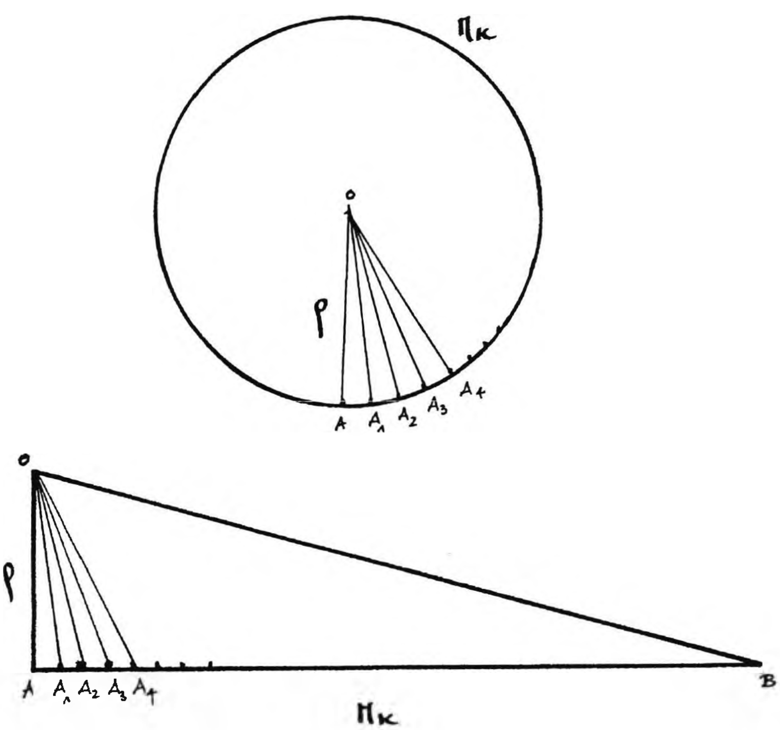


Figure 8.13: The approximate calculation of the circumference of the circle.

Archimedes proves this proposition and comes up with the above-mentioned constant ratio by dividing the circle and the triangle into small triangles of equal areas between each other.

From the conclusion that the area $E_{\kappa} = \pi_{\kappa} \cdot \frac{\rho}{2}$ and the relationship $\frac{E_{\kappa}}{\rho^2} = \pi$ or $E_{\kappa} = \pi\rho^2$, i.e., the number π is equal to ratio of the area of a circle to the square power of its radius, we have: $\pi_{\kappa} = \frac{2E_{\kappa}}{\rho} = \frac{2 \cdot \pi\rho^2}{\rho} = 2\pi\rho$

The irrational number π is equal to the ratio of the circumference to the diameter of a circle: $\pi = \frac{\pi_{\kappa}}{2\rho} = \frac{\pi_{\kappa}}{d}$.

This ratio is constant for all circles and is denoted by the symbol π after the 18th century A.D. The Babylonians and the Egyptians were aware of the existence and the significance of the constant π. They had also found its approximate value. Attempts to solve this problem fill the annuals of mathematics since the days of Pythagoras. Anaxagoras and other pre-Alexandrian men, through their attempts to solve the problem of 'squaring the circle', had also some bearing on π, since this problem is equivalent to the determination of π. For the area of a circle of unit radius is equal to π square units, and if the number π could be expressed rationally the whole question would be reduced to the construction of a square of a given area, i.e., to construction of number π.

Archimedes was already aware, as he declares at the end of his postulates in *Sphere and Cylinder I*, that:

> "If a polygon be inscribed in a circle, it is clear that the perimeter of the inscribed polygon is less than the circumference of the circle; for each of the sides of the polygon is less than the arc of the circle cut off by it" (Fowler, 1999)

In the Proposition 1 of the same treatise, he proves that the perimeter of a circumscribed polygon is greater than the circumference. In this way, he establishes the theoretical results about approximating the circumference of the circle by inscribing and circumscribing polygons. The actual calculation though is carried out in the work of *Measurement of the Circle*:

Proposition 3: "The circumference of any circle is greater than three times the diameter, and the excess is less than a seventh part of the diameter but more than ten seventy-first [parts of the diameter], i.e., $3\frac{10}{71} < \pi < 3\frac{1}{7}$." (Cohen *et al.*, 1966)

Let us see how Archimedes obtains such a result. He uses *the method of exhaustions*, invented by Eudoxus. The basis of this method is the notion that two variable magnitudes will approach a state of equality if their difference could be made deliberately small. In modern terms, it is a method of infinite processes, and infinite processes have for formulation the idea of limit. Thus, in the case of the circle, Archimedes 'traps' the circumference of it between two sets of regular polygons of an increasing number of sides, of which one set is circumscribed to the circle and the other is inscribed in it. He starts with hexagons and keeps on doubling the number until polygons of 96 sides are reached:

If the radius of the circle is ρ, then the side a_{in} of the inscribed polygon is also ρ and the height of it is $h_{in} = \rho\frac{\sqrt{3}}{2}$, therefore the area of it is $E_{in} = 6\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2}\cdot\rho^2 = \frac{3\sqrt{3}}{2}\cdot\rho^2$

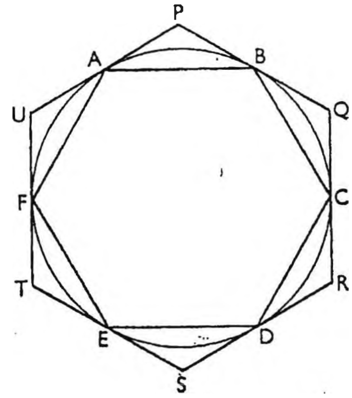


Figure 8.14: Approximation of circle's circumference by regular polygons (Hull, 1959).

Similarly the side of the circumscribed polygon is $a_{cir} = \frac{2\sqrt{3}}{3}\cdot\rho$, the height is $h_{cir} = \rho$, and therefore the area of it is $E_{cir} = 6\cdot\frac{1}{2}\cdot\frac{2\sqrt{3}}{3}\cdot\rho^2 = 2\sqrt{3}\cdot\rho^2$

According to the above-mentioned postulate $\frac{3\sqrt{3}}{2}\cdot\rho^2 < E_\kappa < 2\sqrt{3}\rho^2$, i.e., $\frac{3\sqrt{3}}{2} < \pi < 2\sqrt{3}$.

As the sides of the circumscribed and inscribed polygons become more and more and knowing already the value of $\sqrt{3}$, Archimedes calculates the ratio of the circumference of a circle to its diameter, i.e., π, with unprecedented accuracy. The successive perimeters of the inscribed polygons form one sequence, and those of the circumscribed form another. The circumference of the inner polygon grows longer and that of the outer polygon grows shorter, whereas, as Archimedes assumes, the circumference of the circle lies between these two perimeters. If the process were continued indefinitely, the two sequences would converge towards the same limit: the length of the circumference. If the diameter of this latter is unity, the common limit is π.

From the anthyphaeretic point of view (Fowler, 1999), the calculation shows that the ratio of circumference to diameter is less than three-times, seven-times, and greater than three-times followed by the ratio of seventy-one to ten (taken, as always, the greater to the less): $[3,\ 7,\ 10] < \pi < [3,\ 7]$

According to the anthyphaeretic steps, as they have been described previously, if we compare geometrically the diameter 2ρ with the circumference π_κ of the circle of figure 8.13 for example, by means of a compass, we will find that the diameter measures the circumference 3 times and there is a remainder υ_1. Following, we find that this remainder measures the diameter 7 times and there is a remainder υ_2, which in comparison with the first one gives 15, and so on. The expansion of these comparisons gives that $\pi=[3,\ 7,\ 15,\ 1,\ 292,\ 1,\ 1,\ 1,\ldots]$ and the successive approximations of it are:

v=0 $\pi_0 = [3] = 3$

v=1 $\pi_1 = [3,\ 7] = 3 + \frac{1}{7} = \frac{22}{7} = 3.142857$

v=2 $\pi_2 = [3,\ 7,\ 15] = 3 + \cfrac{1}{7 + \cfrac{1}{15}} = \frac{333}{106} = 3.1415094$

v=3 $\pi_3 = [3,\ 7,\ 15,\ 1] = \frac{355}{113} = 3.1415929$

v=4 $\pi_4 = [3,\ 7,\ 15,\ 1,\ 292] = \frac{103993}{33102} = 3.141592$

... ...

$\pi = 3.1415927$

The helical approximation is $\pi_0 < \pi_2 < \pi_4 < \pi < \pi_5 < \pi_1$

Summarising, Archimedes in both calculation of $\sqrt{3}$ and π uses the method of anthyphaeresis but he applies it on plane figures, whose areas are evaluated by the method of exhaustion. He results in anthyphaeretic algorithms of infinite steps that concern in the case of $\sqrt{3}$ squares, whereas in the case of π regular polygons. In each step there is either a positive or a negative remainder, which geometrically is depicted by means of inscribed or circumscribed figures. Moreover, Archimedes makes use of the Pythagorean theorem for calculating the areas of right-angled triangles and besides the geometric anthyphaeretic conception he introduces an algebraic one.

8.7 Conclusion

The discovery of numbers that are not integers, i.e., the discovery of irrational numbers, and in general the necessity to deal with such problems that could not be solved by pure geometry results in new methods of successive approximations, that were based on the theoretical elaboration of concepts such as sequence, algorithm, and infinity. This theoretical framework that was achieved by Plato's method of Dichotomy and by Zeno's paradoxes that afforded grounds for almost all the theories of space, time and infinity, paves the way to the Euclidean algorithmic method of anthyphaeresis and the Archimedean synthesis of arithmetical and geometrical algorithms, which in turn contribute to the exact mathematical approximation not only of commensurable, but also of incommensurable quantities, such as $\sqrt{2}$, $\sqrt{3}$, and π, and give the notions of numerical modelling. The method of anthyphaeresis is composed of finite or infinite algorithmic steps, depending on whether the estimated number is rational or irrational, respectively. The exact value of the rational or irrational number is approximated step by step. This notion of approximation refers also to the process of modelling. What else is a model than an approximation of a real life system or reality? Or at what else a reliable modelling methodology aims than at the constant evaluation and correction of the created model, so as to approximate the real system satisfactorily?

The method of anthyphaeresis similarly to the platonic method of dichotomy is an algorithmic method that involves processes, where the solution of a problem or the determination of a target is analysed in a number of successive decisions or relations. This dynamical process in the case of the platonic method results in a net or a tree of transition, a topological model of the general category and its sub-categories. The anthyphaeretic method, on the other hand, examines the relationship of two numbers or magnitudes from a mathematical point of view and results in a mathematical model of their ratio, which is an algorithm of convergent and recursive mathematical relations. In this case, it is even possible to create a geometrical model that allows the exact determination of rational or irrational numbers only by the use of the ruler and the compass, as well as the construction of convergent geometrical figures of helical form.

The central point of anthyphaeresis is the effort to ensure the geometric construction even for the irrational numbers, by means of an exact geometric or arithmetic infinite process. Such a process may be thought of as the arithmetical modelling of systems in terms of infinite sequences. The method of anthyphaeresis allows the calculation of an unknown irrational number x with successive approximations, in the same way as the output of a closed loop control system approximates successively the constant value of the reference input. As a result, a closed loop mathematical model is created, which is characterized by the successive relations $x = f(x)$, $x = f(f(x))$, and so on, and simulates the approximation process.

Chapter 9

MECHANICAL REALISATION OF COSMOLOGICAL MODELS AND EARLY MECHANISMS

9. MECHANICAL REALISATION OF COSMOLOGICAL MODELS AND EARLY MECHANISMS

9.1 Introduction

Up to this point, the discussion has focused on three characteristic ways of looking at nature and the basic elements of the universe. The first one, the mythical view during the period of the 8th century B.C. and before, is characterized by the *anthropomorphism,* i.e., the attempt to treat nature as human in form and function, e.g., Heaven – Zeus, Sea – Poseidon, and so on. The second worldview is that of the Presocratic philosophers, which could be called the physical or the material view. In this period of the physical philosophers (7th & 6th century B.C.), the dominant undertaking is that of the natural explanation of the world. The third way of looking at nature was that of ranging from the physical to the mathematical view. The main exponents of this view were the Pythagoreans, who almost made number itself the basic element of the universe. This view attained its earliest systematic expression in Plato, and its exact geometric and arithmetic form in Euclid and Archimedes.

However, the mathematical analysis and comprehension of a physical system and the development of its laws and properties (e.g., the properties of the circle) lead to the construction of a technical model, of a mechanism, which allows the reproduction of these properties (e.g., the compass). Therefore, besides the physical and mathematical conception of the world come the mechanical view and the mechanical cosmological models, which complement the foregoing worldviews and are the subject matter of this chapter. This latter approach derives its elements from Mechanics and Mechanical Engineering. World's structure and operation are compared to machines and technology[45] plays a significant role. We may talk for a *mechanomorphism*, which is expressed by the wheel model of Anaximander, the rings model of Parmenides, and the whorls model of Plato.

More precisely, this chapter cites Plato's mechanical model of the universe, as a system of concentric spheres centred on the earth, his attempts to explain the irregular

[45] According to (Klemm, 1959), Technology is *Machination*, an artful method, and the word is derived from the Greek word '*μηχανεύομαι*', i.e., 'I contrive a deception'.

motions of the planets, the improved cosmological model of Eudoxus, as well as, Aristotle's cosmological and astronomical aspects, and his efforts to interpret the planetary motions. Aristotle's viewpoint is that of seeking for a mechanical explanation or for the leading cause or reason (as we have already seen in chapter 4), in order to study the nature. According to (Thurston, 1899), the modern engineering, and all our marvellous progress in the material, spiritual, and intellectual development of the world, are traceable to their origin with Aristotle. Aristotle is the one who laid the foundation upon which the modern engineering was built by Archimedes and Heron and their successors, as for example Leonardo Da Vinci.

This chapter also includes the work of Archimedes, the inventions he made and particularly his construction of a mechanical celestial model, the so-called planetarium, that in turn leads to an exact quantitative model of the universe, such as the mechanism of Antikythera.

9.2 The Wheels of Anaximander

In the first place, Anaximander is the one, who introduces the use of models into science. A star map, a map of the world, and a marvellous model of concentric stovepipe wheels or rings of fire that explain planetary motion and represent world as a system in rotation, are among them. This work by Anaximander seems to be the first attempt **to understand the universe with the aid of a mechanical model.** Anaximander combines both the mechanical ingenuity with the scientific insight. These are obvious in his construction of the first map of the universe. From this map it can be inferred that Anaximander's universe has in the centre the earth, which is surrounded by the Oceanus and in the centre of the earth is the Mediterranean Sea, *Μεσόγειος* in Greek, which translates 'in the middle of the earth' *(στο μέσον της γης)*.

Herodotus (v. 49ff) gives a lively description of the impression made in Sparta by the invention of Anaximander to produce the first map, when Aristagoras displayed it to King Cleomenes, "a tablet of bronze on which the whole circle of the earth and every sea and river was engraved". (Heiberg, 1922)

Anaximander's working model of the earth or of the world is shown below:

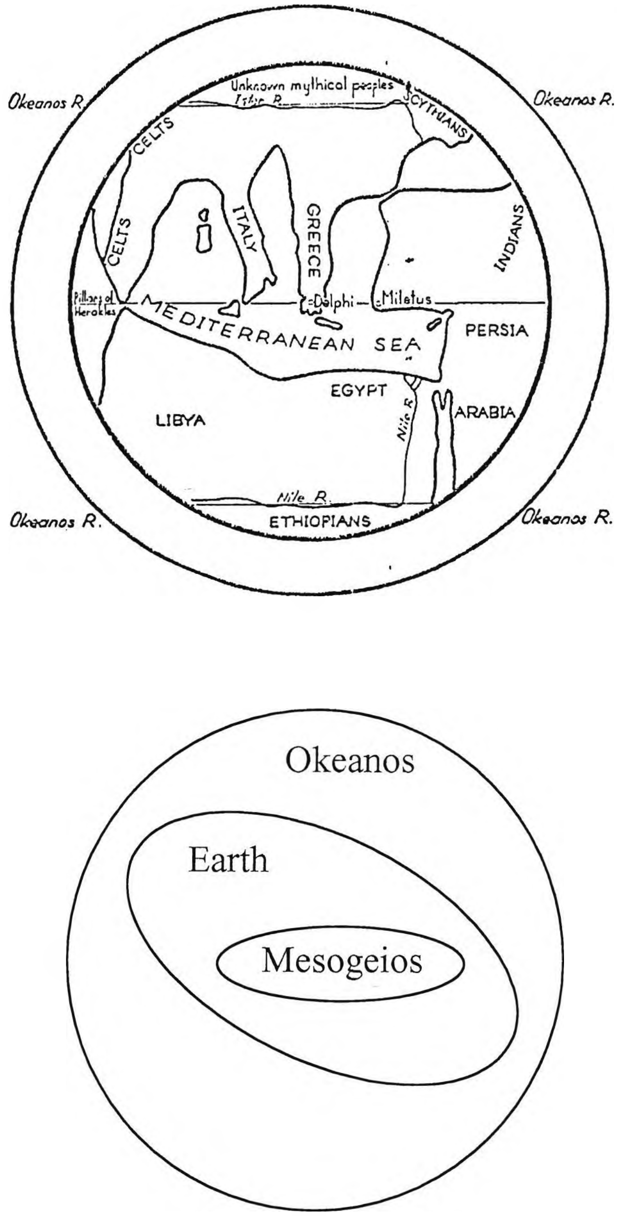


Figure 9.1: The first map of the world, drawn by Anaximander

In addition, Anaximander gives the first astronomical model, however simple and homely it may seem, it is the ancestor of the modern planetarium. He asserts the earth to be a disk in the centre of the world, surrounded by rings or wheels of hollow pipe. This disk of earth is a cylinder, which height is equal to the one third of its diameter. Each pipe around the earth, is made of a hard shell and it is full of fire, which is kept inside, except at certain openings, which are what we see as the sun, moon, and planets. The whole system has a daily revolution and in addition each wheel has a proper motion of its own (figure 9.2). In addition, he assigns dimensions to the size of sun (according to him, the sun's opening is the same size as the circumference of the earth) and a mathematical ratio between the diameter of the wheels of fire of the sun and moon and the diameter of the earth. More precisely the wheel of sun is 27 or 28 earth diameters, whereas the wheel of moon is 18 or 19. What is of primary significance here is Anaximander's attempt to fit astronomical phenomena into a mechanical model based on some mathematical ratios.

A schematic representation of Anaximander's model is given in the following figure.

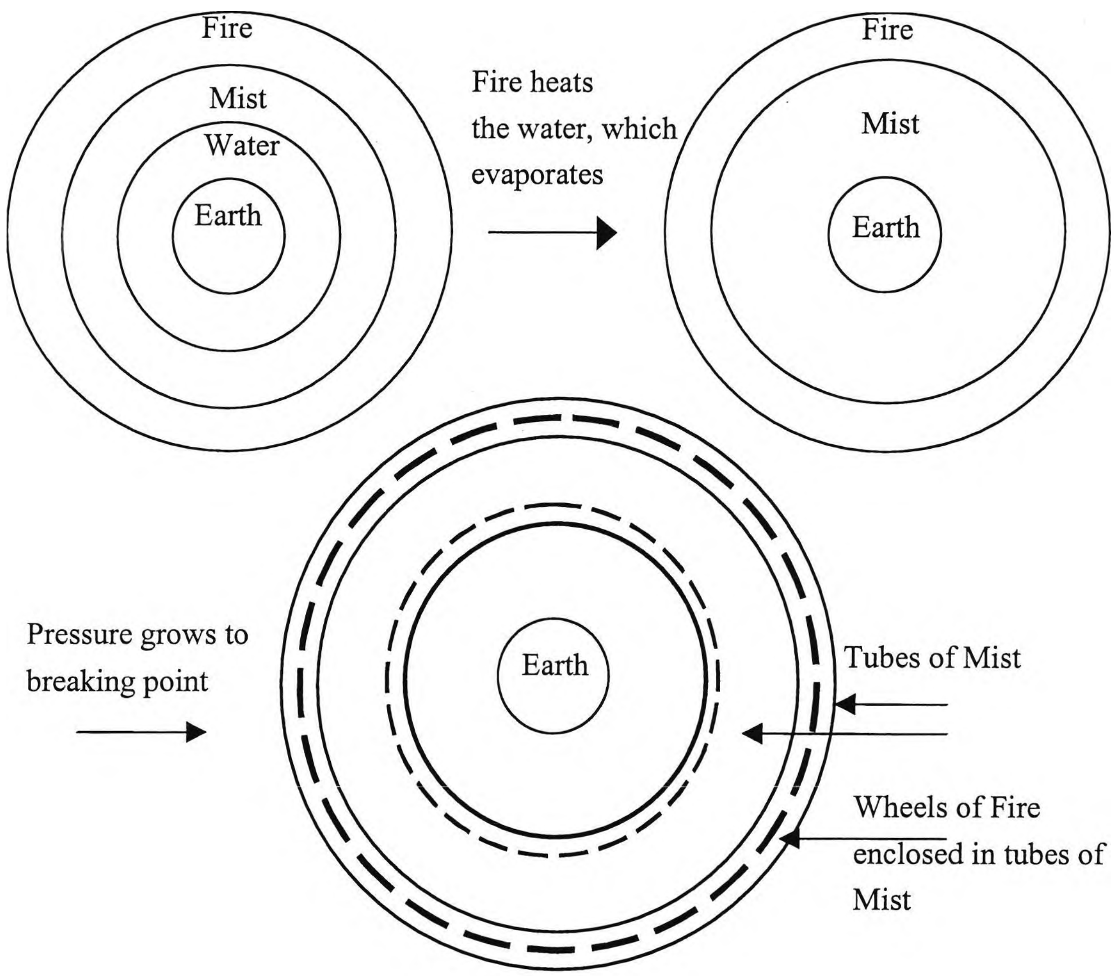


Figure 9.2: Anaximander's model of the universe

(Farrington, 1949) gives an analytical description of Anaximander's mechanical account of the universe: " [...] the four elements, of which the world is made, lay in a more stratified form: earth, which is the heaviest, at the centre, water covering it, mist above the water, fire embracing all. The fire, heating the water, caused it to evaporate, making the dry land appear, but increasing the volume of mist. The pressure grew to breaking point. The fiery integument of the universe burst and took the form of wheels of fire enclosed in tubes of mist circling round earth and sea. That is the working model of the universe. The heavenly bodies we see circling above our heads are holes in the tubes through which the enclosed fire glows. An eclipse is a closing, or partial closing, of a hole. Gone are the tubes of fire of Anaximander, which seem primitive in one aspect, but which attempted to supply a mechanical model of the universe."

Regarding Anaximander's model of the world, (De Santillana, 1961) points to the model of a 'winnower's sieve', a model used later by Presocratics and Plato. More precisely he says: "In that world-eddy, what is mixed in the uniform boundless must come to separate out; and the familiar image that came to Anaximander's mind must have been the Winnower's sieve, rotating and shaken, where the heavy grain remains in the middle and the chaff wanders toward the rim. For he said that the contraries 'separated out', earth going to the centre and fire to the outside, water and mist remaining in between." It is unlikely that we shall ever know which of these interpretations is correct.

In contemporary terms, the model of Anaximander could be seen, however in a primitive form, as having similarities with the qualitative reasoning technique. In the following lines, we will give a brief description of the main characteristic of this modelling technique: Qualitative reasoning or naïve physics is an alternative, far simpler physics, which helps to understand how humans model the behaviour of a physical system or how they reason the functioning of a process (Cellier, 1991). The main goal of qualitative reasoning is to provide a theoretical background for understanding the behaviour of physical systems. An additional scope is to predict their future behaviour and to explain how they achieve the predicted behaviour. It is based on the analysis of the structure of a physical system and its division into simple physical situations. Each physical situation is regarded as a physical device or machine, which consists of materials, components, and conduits. All of them contribute to the behaviour of the whole device. It is possible for the components to change the characteristic of the materials, whereas the conduits just transfer material from one component to the other without changing any characteristic of the material. The figural depiction of this qualitative reasoning process is:

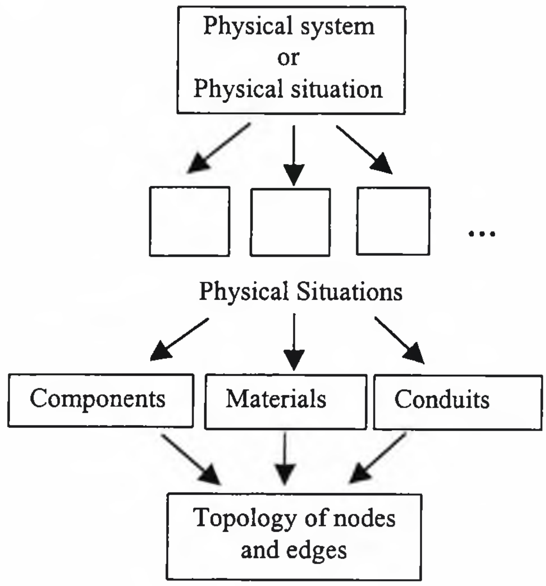


Figure 9.3: Qualitative reasoning process

The completion of this analysis, i.e., the determination of the simplest ingredients that constitute a physical situation, makes it possible to specify the structure and the behaviour of this physical situation. On the one hand, the physical structure is represented by a topology of system's ingredients, where nodes represent the components, and edges represent the conduits, and on the other, the behaviour is described by variables that take only a small number of values. These qualitatively described values sometimes cause loss of information, but it is assumed that this does not affect the whole procedure, since the most important information of a quantity is whether it is increasing, decreasing, or remaining the same. These three stages are described by +, -, and 0 respectively. The further elaboration of qualitative reasoning, i.e., the qualitative differential equations, the subdivision of components into regions, or the ways for predicting the behaviour of a device, are on the one hand, beyond the scope of this thesis, and on the other, if there are same similarities that refer to the Anaximander's model, they are found in the outline of qualitative reasoning, rather than in the detailed analysis of it.

Let as return to Anaximander's model. Initially, by means of this model, Anaximander aims at explaining a physical system, i.e., universe, which is considered as a whole composed of simple situations and elements that in turn appear as devices or machines working in a particular way. Anaximander's specification of world's elements and devises, as well as in their placing into the universe, could be compared to the components, materials, and conduits of the qualitative reasoning method. More precisely the fire, which changes the form of water, and the volume of mist could be regarded as a component of the world, the changeable water and mist as materials of the world, and the formed tubes that transfer materials (e.g., mist) as the conduits of the system. In addition, the consideration of the qualitative changes of the elements, e.g., the increase of the volume of mist, gives also reference to qualitative reasoning.

9.3 The Bowls of Heraclitus

Heraclitus with his archetypal form of matter, the Fire, asserts that the **heavenly bodies are bowls filled with fire**, and an eclipse occurs when the open side of a bowl turns from us:

> Diogenes Laërtius IX, 9-10 (DK 22 A I): "He does not reveal the nature of the surrounding; it contains, however, bowls turned with their hollow side towards us, in which the bright exhalations are collected and form flames, which are the heavenly bodies. Brightest and hottest is the flame of the sun.... And sun and moon are eclipsed when the bowls turn upwards; and the monthly phases of the moon occur as its bowl is gradually turned." (Kirk *et al.*, 1983)

Heraclitus here does not use an empirical sophisticated model in order to find explanations of astronomical phenomena, but an uncomplicated mechanical model probably based on the popular myth of the sun being carried around the river Oceanus in a golden bowl. According to this model, the heavenly bodies are described as bowls of fire nourished by exhalations from the sea. This bowl-model enables Heraclitus to explain eclipses and the phases of moon as due to the different turnings of the bowls.

9.4 The Rings of Parmenides

Parmenides of Elea, unlike the philosophers already discussed, takes the world not as the point of departure for his philosophical quests but as the speculative framework itself. He does not offer answers to the questions posed by the former Presocratics and occasionally denies what they assert. In particular, he is opposite to Heraclitus' doctrine of change and motion, i.e., for him movement is impossible, and the whole of reality consists of a single, motionless and unchanging substance.

The one and only work of him, a poem, is divided into three parts, 'Prologue', 'The Way of Truth', and 'The way of Opinion or the Way of Seeming'. The 'Way of Truth' concerns the logical principle governing the use of concepts, such as 'being and not being', 'change', 'motion', and so on. He argues herein that everything is what it is, so that it cannot become what it is not, i.e., change is incompatible with being and only the permanent aspects of the world can be considered truly real. On the other hand, in the 'Way of Opinion' Parmenides provides a complementary view of the world, a more empirical cosmogony and another mechanical model incorporating many of the doctrines of the previous philosophers: the heavens are divided into rings of fire similar to Anaximander; in the centre of the rings of fire a goddess steers all holding the keys of Justice and Necessity. This goddess here plays the same role as the Eros in Hesiodic cosmogony, who also controls mating and generation. Also, he reintroduces the sensible contraries, light and darkness, rare and dense and describes the world as a manifestation of these opposites. The following fragments give a precise description of Parmenides' model of the world:

> Aetius II, 7, 1 (DK 28 A 37): "Parmenides said that there were **rings wound one around the other**, one formed of the rare, the other of the dense; and that there were others between these compounded of light and darkness. That which surrounds them all like a wall is, he says, by nature solid; beneath it is a fiery ring; and likewise what lies in the middle of them all is solid; and around it is again a fiery ring. The middlemost of the mixed rings is the primary cause of movement and of coming into being for them all, and he calls it the goddess that steers (*κυβερνήτης*) all, the holder of the keys, Justice and Necessity. The air, he says, is separated off from the earth, vaporized owing to the earth's stronger compression; the sun is an exhalation of fire, and so is the circle of the Milky Way. The moon is compounded of both air and fire. Aither is outermost, surrounding all; next comes the fiery thing that we call the sky; and last comes the region of the earth." (Kirk *et al.*, 1983)

In addition, Simplicius mentions:

Fr. 12, Simplicius in *Physics* 39, 14 and 31, 13: "The narrower rings are filled with unmixed fire, those next to them with night, but into them a share of flame is injected; and in the midst of them is the goddess who steers all things..." (Kirk *et al.*, 1983)

It is worth mentioning that Parmenides, in this mechanical, rings-composed model of the world, refers to the notion of control. He perceives the need of a 'controlling' mechanism that steers and controls everything in the world system (see also section 11.3.3).

The mechanical viewpoint of Parmenides is followed by others more detailed descriptions. We have already mentioned, in chapter 4, Leucippus and Democritus and their conception that the reality is a lifeless piece of machinery and the world and its various parts result from the mechanical sorting of atoms in the primeval vortex. Further down, we will examine the Pythagorean mechanical conception and Plato's mechanical model of the world. However, only during the Hellenistic period we will see such models to be constructed, as a matter of fact.

9.5 Pythagorean Universe

The Pythagoreans, in opposition to the Ionians, do not describe the universe in terms of the behaviour of certain material elements and physical processes, but in terms of numbers. We have already discussed their doctrine in chapter 7. At this point we are mostly concerned with the planetary system of the Pythagoreans. The idea that the earth is a sphere is probably as old as Pythagoras. Most likely the observations that the surface of the sea is not flat but curved, or the fact that as a distant ship approaches one first notices its top and gradually the rest of, it may have suggested the spherical shape of the earth. The dogma of spherical perfection spreads out to all the celestial bodies, which are of spherical shape and move along circular paths. Moreover, the perfection that number 10 carries, since it is the summary of the four first numbers 1, 2, 3, and 4 that are of wider respect, should characterizes also the whole universe. In the following passage by Aristotle, the importance this number has for them is declared:

Aristotle, *Metaphysics* A5, 986a 8-13: " [...] as the number 10 is thought to be perfect and to compromise the whole nature of numbers, they [the Pythagoreans] say that the bodies, which move through the heavens are ten, but as the visible bodies are only nine, to meet this they invent a tenth – the 'counter-earth'[46]..." (McKeon, 1941)

Thus, in order to be consistent to their doctrine, they find no difficulty to invent a tenth heavenly body, the so-called counter-earth, in order for their number theory to be verified. In the centre of their planetary system, fire instead of the earth is placed. Earth moves in circular orbit around the central fire:

Aristotle, *De caelo,* II 13, 293a18 (DK 58 B 37): "Most people – all, in fact, who regard the whole heaven as finite - say that the earth lies at the centre of the universe. But the Italian philosophers known as Pythagoreans take the contrary view. At the centre, they say, is fire, and the earth is one of the stars, creating night and day by its circular motion about the centre. They further construct another earth in opposition to ours to which they give the name counter-earth... Their view is that the most precious place befits the most precious thing: but fire, they say, is more precious than earth, and the limit than the intermediate, and the circumference and the centre are limits. Reasoning on this basis they take the view that it is not earth that lies at the centre of the sphere, but rather fire..." (Cohen *et al.*, 1966)

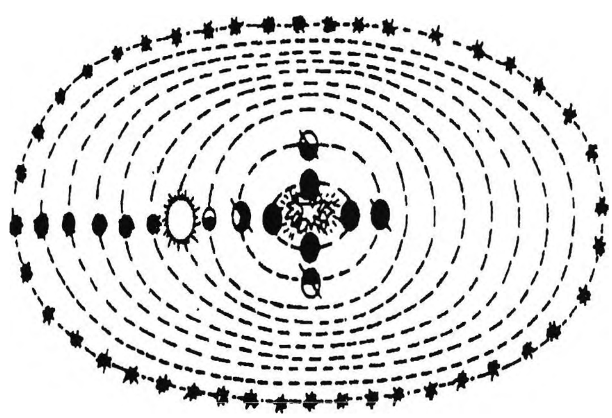


Figure 9.4: The Pythagorean Universe: (the earth and the 'counter-earth' are represented in four positions, Alic, 1992)

A variation of the Pythagorean universe is given by Aetius, who attributes the planetary system to the Pythagorean Philolaus of Tarentum (480-400 B.C.):

Aetius II, 7, 7 (DK 44 a 16): "Philolaus places fire around the centre of the universe, and calls it the 'hearth of the world', the 'house of Zeus', 'mother of the gods', 'altar, bond and measure of nature'. Then again there is another fire enveloping

[46] Metaphysica, Translated by W. D. Ross

the universe at the circumference. But he says that the centre is by nature primary, and around the centre ten divine bodies dance – first the sphere of the fixed stars, then the five planets, next the sun, then the moon, then the earth, then the counter-earth, and finally the fire of the 'hearth', which has its station around the centre." (Kirk *et al.*, 1983)

The mechanical viewpoint of the Pythagoreans takes a more precise form in Plato.

9.6 Plato's Cosmology

Cosmology occupies in Plato's work a considerable position. Although Plato's cosmological and astronomical system remains throughout fundamentally the same, the successive presentations of it at different periods of his life show different stages of development. Let us take the platonic dialogues in a chronological order.

In *Phaedo* Plato states his own view on the form of the earth. Similar to the Pythagoreans he considers earth to be spherical:

108C – 109A: " [...] My persuasion as to the form of the earth and the regions within it is [...] if the earth, being a sphere, is in the middle of the heaven, it has no need aither of air or of any other such force to keep it from falling, but that the uniformity of the substance of the heaven in all its parts and the equilibrium of the earth itself suffice to hold it; for a thing in equilibrium in the middle of any uniform substance will not have cause to incline more or less in any direction, but will remain as it is, without such inclination." (Heath, T., 4, 1981)

In *Republic* Plato describes a more complete system of the world. Even though it is blended with the myth of Er, the son of Armenius, and what Er has seen during the twelve days that his soul was wandering "into the heaven", the system could be regarded as **a mechanical model of the world**, which explains the motion of all heavenly spheres by means of mechanical elements such as the spindle and its pivot, and the whorl:

Book X, 616b – 617d: " [...] and on the fourth day they arrived at a point from which they saw extended from above through the whole heaven and earth a straight light, like a pillar, most like to the rainbow, but brighter and purer [...] for this light it is which binds the heaven together, **holding together the whole revolving firmament** as the undergirths hold together, triremes; and from the extremities they saw extended **the Spindle** of Necessity by which all the revolutions are kept up. The shaft and hook thereof are made of adamant, and the **whorl** is partly of adamant and partly of other substances. Now the whorl is after this fashion. Its shape is like that we use; but from what he said we must conceive of it as if we had one great whorl, hollow and scooped out through and through, **into which was inserted another whorl** of the same kind but smaller, nicely fitting it, like those boxes which fit into one another; and into this again we must suppose **a third whorl fitted, into this a fourth, and after that four more.** For the **whorls are altogether eight in number,** set one within another, **showing their rims above as circles** and forming about the shaft a continuous surface as of one whorl; while the shaft is driven right through the middle of the eighth whorl. The first and outermost whorl has the circle of its rim the broadest, that of the sixth is second in breadth, that of the fourth is third, that of the eighth is fourth, that of the seventh is fifth, that of the fifth is sixth, that of the third is seventh, and that of the second is eighth. And the circle of the greatest is of many colours, that of the seventh is brightest, that of the eighth has its colour from the seventh which shines upon it, that of the second and fifth are like each other and yellowier than those aforesaid, the third is the whitest in colour, the fourth is pale red, and the sixth is the second in whiteness. The **Spindle turns round as a whole with one motion**, and within the whole as it revolves the seven inner circles revolve slowly in the opposite sense to the whole, and of these the eighth goes the most swiftly, second in speed and all together go the seventh and sixth and fifth, third in the speed of its counter-revolution the fourth appears to move, fourth in speed comes the third, and fifth the second. And the whole Spindle turns in the lap of Necessity." (Heath, T., 4, 1981)

The meaning that is possibly given to the term 'Necessity' is that of the physical law that is seen as governing the things and human actions. There is an analogy between the rings of Parmenides' mechanical model and the platonic whorls here. (Heath, T., 4, 1981) quotes that the concentric whorls are **pure mechanism** and their astronomical equivalent is obvious. Even though Plato does not mention the heavenly spheres by name – he merely speaks of the 'first' or 'outermost', the 'second' and so forth down to the 'eight', he gives their attributes concerning colour, brightness, and 'borrowed light', so that anyone with even an elementary knowledge of astronomy can infer that: the outermost whorl (the first) represents the sphere of the fixed stars, the second whorl (reckoning from the outside) carries the planet Saturn, the third Jupiter, the fourth Mars, the fifth Mercury, the sixth Venus, the seventh the sun, and the eighth the moon. The earth, as always in Plato, is at rest in the centre of the system. The rim of the innermost whorl (the eighth) is the orbit of the moon. The outer rim of the next whorl (the seventh) is the orbit of the sun, and so on.

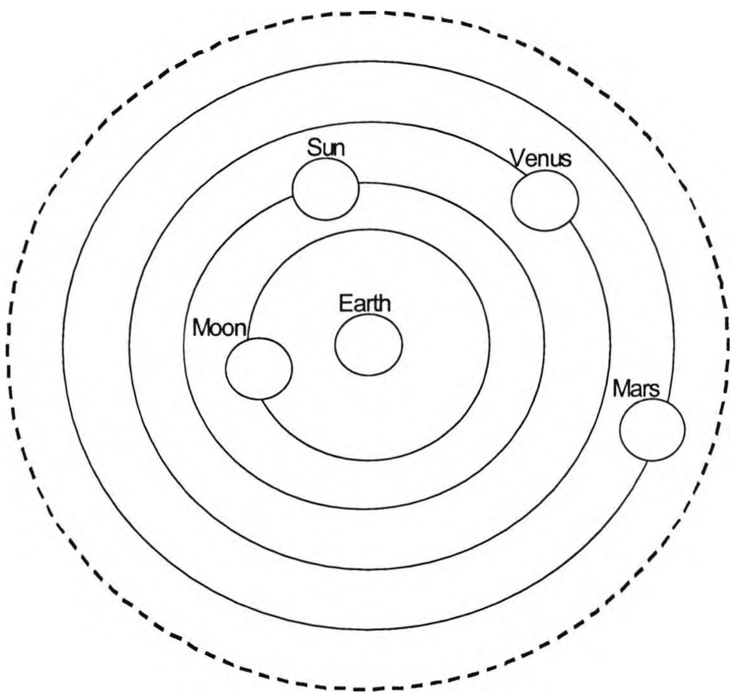


Figure 9.5: Plato's planetary system

The fact that the whorls have between each other different breadth indicates the difference between the distances from the earth of the planets carried by the whorls. The fact also that the eighth whorl revolves faster than the others suggests the motion of the fixed stars, which is the quickest rotation and takes place once in about 24 hours.

In *Timaeus*, the devoted work to the survey of the natural world, we find the most comprehensive statement of Plato's cosmological, astronomical and physical views. Plato depicts the world, as the handiwork of a divine craftsman, the Demiurge. The Demiurge is modelling the universe from the pre-existing chaos that was filled with un-formed material. He imposes order according to a given rational plan and by being based on geometrical principles he attributes to the universe the simplest and most perfect form – the spherical.

The dialogue includes an account of the creation and the structure of the universe, a detailed description of the motions of the stars and the planets, as well as a geometric reconstruction of the physical elements, as we saw in chapter 7. The main points of the platonic description are:

1) The earth is located in the centre of the closed spherical universe and the fixed stars in its periphery. The whole universe moves uniformly around the earth:

> *Timaeus,* 40A-C: " [...] and after the likeness of the universe he (Demiurge) gave them (heavenly gods) [...] distributing them all round the heaven, to be in very truth an adornment (cosmos) for it, embroidered over the whole. And he assigned to each two motions: one uniform in the same place, as each always thinks the same thoughts about the same things; the other a forward motion, as each is subjected to the revolution of the Same and uniform. But in respect of the other five motions he made each motionless and still, in order that each might be as perfect as possible. For this reason came into being all the fixed stars, living beings divine and everlasting, which abide for ever revolving uniformly upon themselves [...] And Earth he designed to be at once our nurse and, as she winds round the axis that stretches right through, the guardian and maker of night and day, first and most venerable of all the gods that are within the heaven." (Cornford, 1977)

2) There are seven heavenly bodies, the planets, which move autonomously and do not follow the motion of the fixed stars. Plato names the Sun, the Moon, the Venus (Morning Star), and the star of Mercury (Hermes):

> *Timaeus,* 38C-D: " [...] in order that Time might be brought into being, Sun and Moon and five other stars – 'wanderers', as they are called – were made to define and preserve the numbers of Time. Having made a body for each of them, the god set them in the circuits in which the revolution of the Different was moving – in seven circuits seven bodies: the Moon in the circle nearest the Earth; the Sun in the second above the Earth; the Morning Star and the one called sacred to Hermes in circles revolving so as, in point of speed, to run their race with the Sun, but possessing the power contrary to his; whereby the Sun and the star of Hermes and the Morning Star alike overtake and are overtaken by one another." (Cornford, 1977)

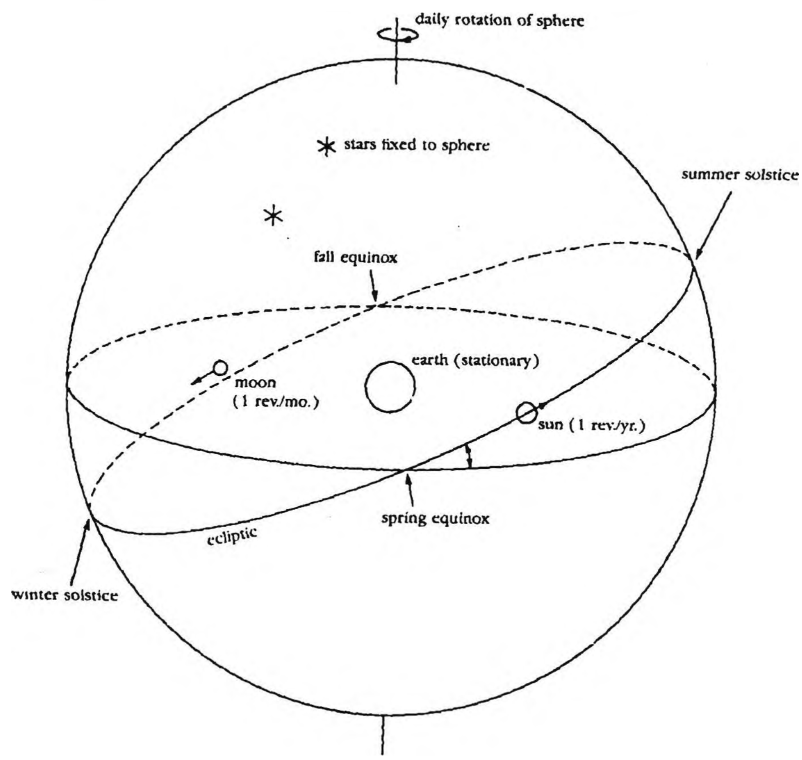


Figure 9.6: The celestial sphere according to Plato (Lindberg, 1992)

3) There are two regular revolutions that take place simultaneously. The one is that of the fixed stars – the so-called movement of the 'Same' – and the other is that of the seven planets – the so-called movement of the 'Other' or 'Different'. The revolution of the planets is opposed and inclined in comparison to the revolution of the fixed stars. Therefore, each planet executes simultaneously two opposite regular revolutions, i.e., the revolution of the fixed stars and its own revolution around the inclined ecliptic. The correlation of these revolutions results in a helical motion that is responsible for the irregularities of planetary motion.

> *Timaeus,* 36B-D: "Next he [the Demiurge] cleft the structure so formed lengthwise into two halves and, laying them across one another, middle upon middle in the shape of the letter X, he bent them in a circle and joined them, making them meet themselves and each other at a point opposite to that of their original contact; and he comprehended them in that motion which revolves uniformly and in the same place, and one of the circles he made *exterior and one interior.* The exterior movement he named the movement of the Same, the interior the movement of the Other. The revolution of the circle of the Same he made to follow the side (of a rectangle) towards the right hand, that of the circle of the Other he made to follow the diagonal and towards the left hand, and he gave the mastery to the revolution of the Same and

uniform, for he left that single and undivided; but the inner circle he cleft, by six divisions, into seven unequal circles in the proportion severally of the double and triple intervals, each being three in number; and he appointed that the circles should move in opposite senses, three at the same speed, and the other four differing in speed from the three and among themselves, yet moving in a due ratio." (Heath, T., 4, 1981)

The cutting of the circle of 'Different' into seven concentric circles produces seven orbits in a similar way as the eight whorls in the Myth of Er give eight orbits. The 'double and triple intervals' are the two series of 1, 2, 4, 8 and 1, 3, 9, 27, i.e., 2 and 3 together with their respective squares and cubes, which should be related to the distances of the seven planets either from the earth or from each other.

4) Plato's astronomical description closes with the remark that, without a visible model, all the complicated movements cannot be described.

Timaeus, 40C-D: "To describe the evolutions in the dance of these same gods, their juxtapositions, the counter-revolutions of their circles relatively to one another, and their advances; to tell which of the gods come into line with one another at their conjunctions, and which in opposition, and in what order they pass in front of or behind one another, and at what periods of time they are severally hidden from our sight and again reappearing send to men who cannot calculate panic fears and signs of things to come – to describe all this without **visible models** (*μιμήματα*) of these same would be labour spent in vain." (Cornford, 1977)

(Cornford, 1977) claims that Plato should have in front of his eyes a mechanic construction in order to describe in such detail the heaven structure. Analytically he says: "That the Academy possessed an armillary sphere[47] may be inferred from Timaeus later remark (40C) that the intricate movements of the planets cannot be explained without a visible model. Plato probably had it before him as he wrote. Theon tells us that he had himself made a 'sphere' to illustrate the Spindle of necessity in the Myth of Er"

[47] Armillary sphere: a skeleton sphere made of great circles adjusted around the same centre and graduated in degrees; one of the circle might be in the plane of the equator, and the other, perpendicular to it, would turn around the axis of the world.

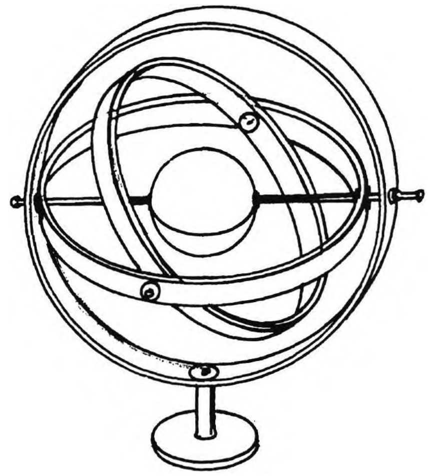


Figure 9.7: The potential visible mechanical model of Plato

(Brumbaugh, 1966) mentions the same assertion. He says that "in crucial passages Plato does write as though he was visualizing mechanical models: of metal bands, for the planetary orbits; of scales, for calculating relative planetary positions; of some celestial globe, turning "on a fine pivot", capable of reversing its direction; of a colourful mythical view of the universe in cross section."

9.6.1 Eudoxus – Improvements on Plato's Astronomical System

Plato describes many features of the cosmos. He proposes a spherical earth, defines various circles on the celestial sphere, and marks the paths of the sun, moon, and other planets. He knows the circuits of sun and moon, once a year and once a month respectively. He also knows the paths of circuits of Mercury, Venus, Mars, Jupiter, and Saturn. He also understands the irregularities of planetary motion and knows, however in a theoretical base, that these irregularities could be explained by the compounding of uniform circular motions. However, the practical speculation of the problem of planetary irregularities is ascribed to a contemporary of Plato, Eudoxus of Cnidus (390-337 B.C.)

Eudoxus creates a geometrical model, the so-called "two sphere model" in order to represent the stellar and planetary phenomena. He conceives the earth and the heavens as a pair of concentric spheres. The one is the terrestrial sphere, which is fixed in the centre, and the other is the celestial sphere, which rotates daily about a vertical axis. To the latter, the stars are affixed and along its surface move the sun, the moon, and the remaining five planets. The daily rotation of the celestial sphere explains the daily rising and setting of all of the celestial bodies. Thus, on the surface of the celestial sphere, while it is going through its daily rotation around the earth, occur all motions of sun, moon, and the remaining planets.

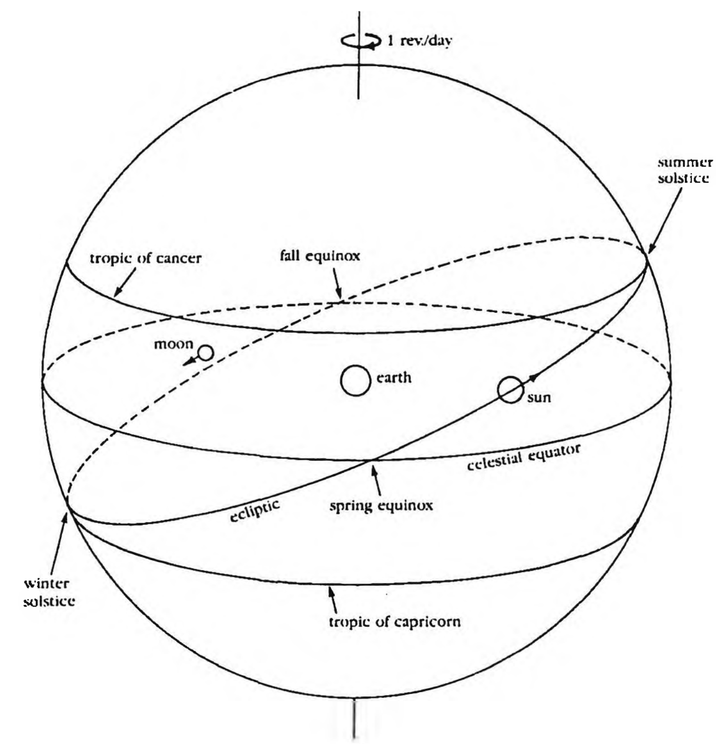


Figure 9.8: The Eudoxian "two-sphere model" of the cosmos (Lindberg, 1992)

The resulting motion, observed from the fixed earth, is a combination of the irregular motion of the planets around the ecliptic and the uniform daily rotation of the celestial sphere. Eudoxus, like Plato, knows that in order to bring order to this "complexity" in heavens he must treat each irregular planetary motion as a composite of series of simple uniform circular movements. To succeed in his aim, Eudoxus assigns to each planet a set of nested concentric spheres, the so-called deferent spheres, and to each sphere one component of the complex planetary motion.

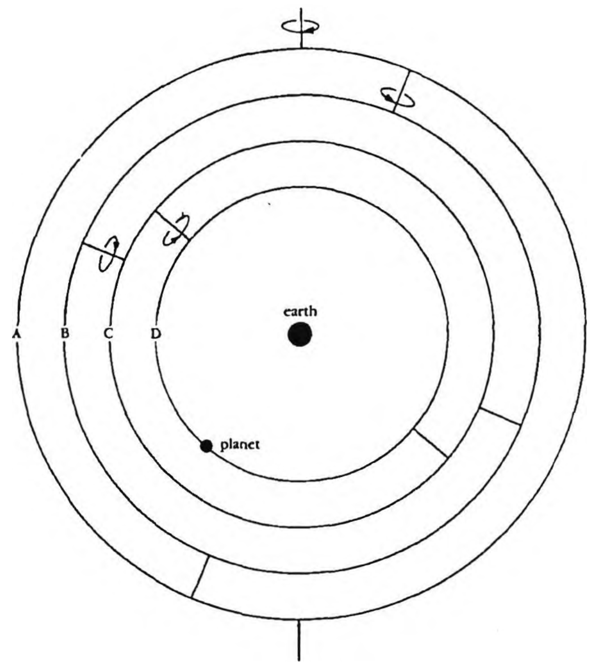


Figure 9.9: The Eudoxian spheres for each of the planets (Lindberg, 1992)

To the outermost sphere, Eudoxus assigns the daily rising and setting of the planet, to the second sphere in the series, which rotates uniformly about its axis but in the opposite direction, he assigns the slow west-to-east motion of planet around the ecliptic, whereas to the two inner spheres, which explain the changes in speed and latitude, he assigns the retrograde motion of each planet. In the case of the sun and moon, which do not undergo retrograde motion, only three spheres apiece is enough. The Eudoxian geometrical model of the planetary motions is improved in certain respects by Callippus of Cyzicus (*ca.* 370-310 B.C.), a contemporary of Aristotle, and constitutes the base for the construction of a potential mechanical model of the world, with many homocentric spheres or rings, which most likely, these ancient engineers had at their disposal. Aristotle adopts the Eudoxian model and elaborates it into a working physical system.

9.7 Aristotle's Cosmology

Aristotle continues the tradition of studying astronomy with the aid of geometrical models. He transforms the purely geometrical and theoretical astronomical system of his predecessors into a mechanical system of spheres and spherical shells, in actual contact with each other. He assumes that all the sets of spheres form part of one continuous system of spheres instead of separate sets of spheres, i.e., set of spheres for each planet.

His theory is given quite clearly in the next passage:

Metaphysics, Λ8, 1073b 45 - 1074a 15: "But it is necessary, if the phenomena are to be produced by all the spheres acting in combination, to assume in the case of each of the planets other spheres fewer by one; these latter spheres are those which unroll, or react on, the others in such a way as to replace the first sphere of the next lower planet in the same position [as if the spheres assigned to the respective planets above it did not exist], for only in this way is it possible for a combined system to produce the motion of the planets. Now the **deferent spheres** are, first, eight [for Saturn and Jupiter], then twenty-five more [for the sun, the moon, and the three other planets]; and of these only the last set [of five] which carry the planet placed lowest [the moon] do not require any reacting spheres. Thus the **reacting spheres** for the first two bodies will be six, and for the next four will be sixteen; and the total number of spheres, including the deferent spheres and those which react on them, will be fifty-five. If, however, we choose not to add to the sun and moon the [additional deferent] spheres we mentioned, the total number of the spheres will be forty-seven. So much for the number of the spheres." (Heath, T., 4, 1981)

According to this description, the deferent and the reacting spheres for each planet are:

Planets	Deferent spheres	Reacting spheres
Saturn	4	3
Jupiter	4	3
Mars	5	4
Mercury	5	4
Venus	5	4
Sun	5	4
Moon	5	0
	Total: 33	Total: 22

As we have already seen, Eudoxus treats each complex planetary motion as a composite of a series of simple uniform circular movements. He does this by assigning to each planet a series of concentric spheres (deferent spheres), and to each sphere one component of the complex planetary motion. Eudoxus probably has viewed these spheres not as material bodies but as theoretical and imaginary aids to understanding the motions geometrically, in the same way, as we consider the lines of longitude and latitude that we draw on a map. Aristotle, on the other hand, conceives the spheres as actually existent and crystalline because they are invisible, and being real they are able to transmit their motion from one sphere to the next. He makes all the sets to comprise one continuous and interconnected system. It means that the different spheres of each planet interconnect between each other and if all seven planets with its set of spheres were nested in concentric fashion, the innermost sphere of one planet (say Saturn) would transmit its motion to the outermost sphere of the planet just below it in the series (Jupiter). This fact, with the additional effect of Jupiter's own spheres, implies a complexity, which is encountered by Aristotle by inserting a set of reacting spheres between the innermost sphere of the one planet (Saturn) and the outermost sphere of the next planet (Jupiter). Similarly, he inserts a set of reacting spheres between the primary spheres belonging to every other pair of adjacent planets.

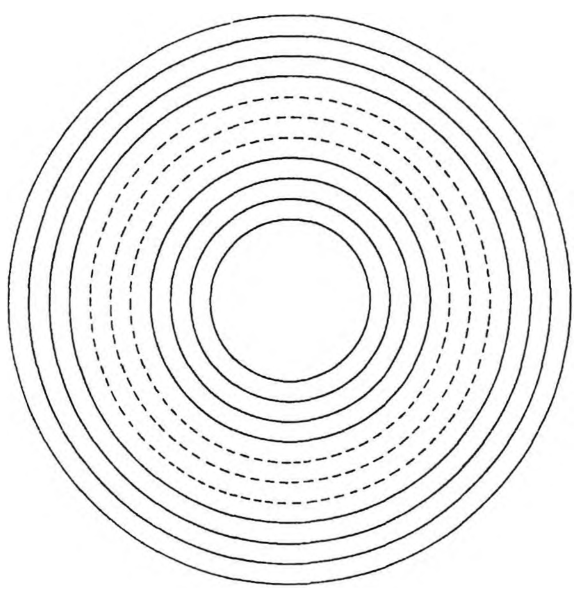


Figure 9.10: Aristotelian nested spheres (Lindberg, 1992)

In this figure, the four external solid lines represent the deferent spheres for Saturn and Jupiter (four spheres apiece). The three spheres between these two sets are the reacting spheres. A simplified version of the complex Aristotelian model is given in the next manuscript of the 12th century A.D.

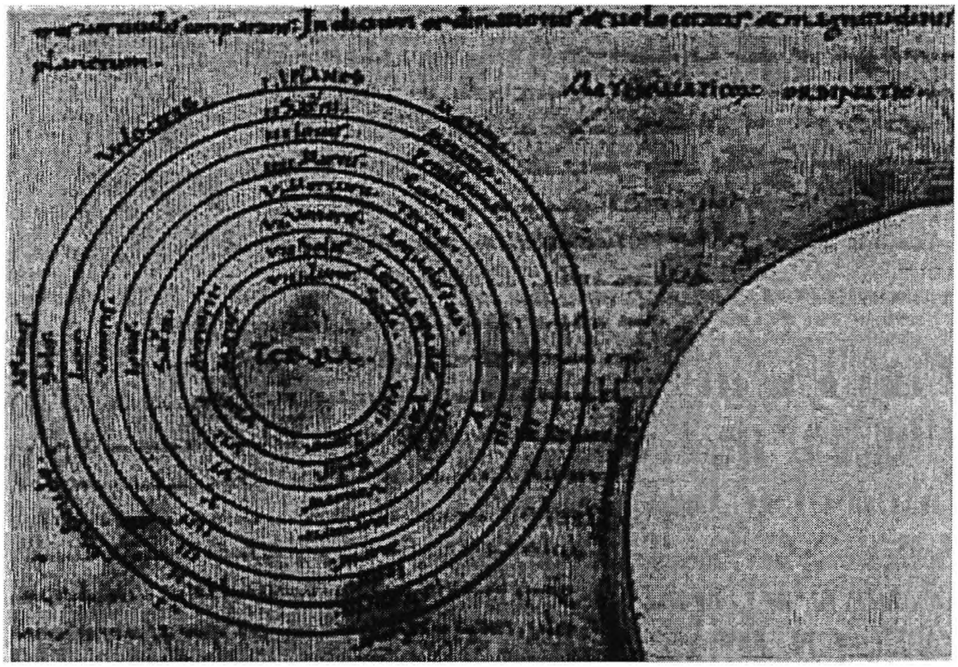


Figure 9.11: The simplified Aristotelian cosmology.
Paris, Bibliotheque Nationale, M S Lat. 6280, fol. 20r (12th c.)

The Aristotelian fifty-five planetary spheres, plus the sphere of fixed stars constitute an enormous complicated piece of celestial machinery. The above-described models of concentric spheres continue to dominate the popular picture of the cosmos throughout the period from Aristotle to Copernicus (1473-1543 A.D.). As for example, Claudius Ptolemy (108-168 A.D.) fully believes in the Aristotelian model of solar system and only makes some refinements upon it to account for the detailed motion of the celestial sphere. He formulates a series of mathematical models, which have the same aim, as those of Eudoxus, i.e., to discover the combination of uniform circular motions that would explain the observed variations in speed and direction of the planets. The mathematical techniques Ptolemy employs are vastly different. He uses circles instead of spheres. He formulates models, the so-called eccentric models, for simple cases of non-uniform motion, such as that of the sun around the ecliptic and for more complicated cases, the so-called epicycle on deferent models. Ptolemy's models are capable of making accurate quantitative forecasts of future planetary positions and they remain in effect until Copernicus creates the astronomical models of our own days.

The up to the time of Aristotle geocentric conception of the universe is confuted when Aristarchus of Samos (310-230 B.C.) proposes a heliocentric universe. Aristarchus, both a mathematician and astronomer, makes a series of astronomical observations and introduces a revolutionary model, where the sun and not the earth is the centre of the planetary motion. His theory ascribes to earth a movement of orbital rotation about the sun once a year, and a movement of axial rotation every twenty-four hours, as well as to the other planets movements in orbits around the sun. This theory is known because of a record found in one of Archimedes' works, "*The Sand-Reckoner*" and by a reference of Plutarch in his book "*Of the face in the Disc of the Moon*". Copernicus also records within his published works that he is aware that before him it was Aristarchus who developed the sun-centred solar system. However, Aristarchus' contemporaries did not accept his philosophy, because Aristotle's geocentric astronomical model (see also chapter 4) had strong followers and the introduction of Ptolemy's similar proposal soon afterwards made Aristotle's view even stronger.

9.8 Archimedes as a Mechanical Engineer

At the beginning of the Hellenistic period and shortly after the campaign of Alexander the Great, a significant leap in the scientific and technical thought occurs. The theory orientates itself towards practice and application in order to serve needs of war, as well as of peaceful life. Correspondingly, the philosophical, theoretical, and mathematical thoughts are expanded in the area of Mechanics, Pneumatics, Statics, and of other technical sciences. In parallel, the speculative and mathematical models are materialised and mechanical models, i.e., machines and gadgets are constructed, which not only do they depict the world, but also simulate its operation, the motion of heavenly bodies, and the time evolution of phenomena according to the physical law.

The pioneer of the effort to turn theoretical thought to its practical application, is Archimedes of Syracuse (287-212 B.C.). Even though, he is primarily a mathematician and he does not think much of his practical inventions, he exploits his mathematical knowledge and constructs engines (compound pulleys, odometer, catapult, screw, and so on), searches the natural world, discovers new physical laws, such as the law of hydrostatics (see chapter 4), and invents, if not the first, at least the most perfect model of the world, the so-called orrery or planetarium.

9.8.1 Mechanical Constructions of Archimedes

The focal point of the thesis concerns the investigation of those early philosophical, mathematical, and technological achievements that give rise to the development of the concepts of system, modelling, and control. However, in many cases it is necessary to list either the theoretical attainments or the practical inventions of antiquity, which though do not connect directly to the under investigation concepts, they contribute to an integrated consideration of the era or of the philosophers and scientists we examine. Therefore at this point, we will list Archimedes' achievements in the field of mechanics, aiming at giving the total framework of his genius as a physician, mathematician, and engineer, or in general as a scientist.

- **Archimedean crew or cochlias**

The water screw or cochlias usually referred as Archimedean screw because most likely the inventor of it was Archimedes when he was in Egypt (Diodorus Siculus 1.34.2, 5.37.3). This device is used to lift water for purposes of irrigation, to keep mines free from water, or to clear the holds of ships. It consists of a pipe, open at both ends, wound about a shaft. The shaft is inclined to the vertical at an angle and as it is rotated, the water travels up the pipe and empties at the higher end (Hull, 1959).

- **Military engines and catapults** for protecting Syracuse when it was under attack by the Romans under the command of Marcellus.

Among them there were lenses, which by taking advantage of the focusing properties of the concave mirror concentrate the sun's rays and set fire to the Roman ships at sea, the so-called burning mirrors, catapults that, through holes made in the walls, shoot arrows with great force and for considerable distances, or consisting of long moveable poles projecting beyond the walls hurl heavy rocks upon the enemies ships, and huge cranes that grapple the prows, lift the ships into the air, and let them fall again.

According to Plutarch, when Archimedes proved to the King Hieron his assertion of moving a given weight with a tiny force, by moving a merchantman (chapter 4), the King was deeply impressed and asked Archimedes to construct for him a number of engines designed both for attack and defence. Therefore:

Plutarch, *Life of Marcellus*, 14-19[48]: "When the Romans first attacked by sea and land, the Syracusans were struck dumb with terror and believed that nothing could resist the onslaught of such powerful forces. But presently Archimedes brought his

[48] Plutarch, *Life of Marcellus*, trans. I. Scott-Kilvert, *Makers of Rome: Nine lives by Plutarch*, Penguin, 1965

engines to bear and launched a tremendous barrage against the Roman army. This consisted of a variety of missiles, including a great volley of stones, which descended upon their target with an incredible noise and velocity. There was no protection against this artillery, and the soldiers were knocked down in swathes and their ranks thrown into confusion. At the same time huge beams were run out from the walls so as to project over the Roman ships: some of them were then sunk by great weights dropped from above, while others were seized at the bows by iron claws or by beaks like those of cranes, hauled into the air by means of counterweights until they stood upright upon their sterns, and then allowed to plunge to the bottom, or else they were spun round by means of windlasses situated inside the city and dashed against the steep cliffs and rocks which jutted out under the walls, with great loss of life to the crews. Often there would be seen the terrifying spectacle of a ship being lifted clean out of the water into the air and whirled about as it hung there, until every man had been shaken out of the hull and thrown in different directions, after which it would be dashed down empty upon the walls…" (Fauvel *et al.*, 1987)

9.8.2 Archimedes' Planetarium

Pappus of Alexandria (3rd century A.D.) in his work *Collections* (110, 24) refers to Archimedes and to the practical and theoretical inventions he came up with during his life. Pappus states[49] that the only mechanical book Archimedes wrote was on the construction of spheres, *Περί Σφαιροποιΐας* (On Sphere Making), which is lost; this described the construction of spheres that imitate the motions of the sun, the moon, and the five planets. Pappus also states that Archimedes is the only one to invent the art of making planetaria, i.e., pictures or models (*εικόνα*) of heaven. The formation of planetaria is considered as an art similar to the art of poetry and music. Martianus Capella (5th century A.D.) in his work *De nuptiis Philogiae et mercurii* (vol. II 212, ed. Ad. Dick, Lipsiae, 1825) mentions that the most significant achievements are the art of poetry, which is represented by Linus, Homer, and the poet of Mantua Virgil, the music, with its representative Orpheus and Aristoxenos, and the

[49] Pappus, *Mathematical Collection*, 8.3 "…Carpus of Antioch has written somewhere that Archimedes composed only a single book concerned with the mechanical arts, *On the Construction of an Orrery*, and that he did not think it worthwhile to write about his other inventions" (Humphrey *et al.*, 1998)

construction of planetaria, such as that of Archimedes and Plato. Following, we will give in translation the relative to the Archimedes' planetarium texts.

(I) From theory to practice and application

Plutarch (*ca.* 45-125 A.D.), *Plutarch's Lives, Marcellus* XIV, ed. A. H. Clough: " [...] These machines he (Archimedes) had designed and contrived, not as matters of any importance, but as mere amusements in geometry; in compliance with King Hieron's desire and request, some little time before, that he should reduce to practice some part of his admirable speculation in science, and by accommodating the theoretic truth to sensation and ordinary use, bring it more within the appreciation of the people in general. Eudoxus and Archytas had been the first originators of this far-famed and highly-prized art of mechanics, which they employed as an elegant illustration of geometrical truths, and as means of sustaining experimentally, to the satisfaction of the senses, conclusions too intricate for proof by words and diagrams. As, for example, so solve the problem, so often required in constructing geometrical figures, given the two extremes, to find the two mean lines of a proportion, both these mathematicians had recourse to the aid of instruments, adapting to their purpose certain curves and sections of lines..."

(II) Construction of mechanical models

Plutarch (*ca.* 45-125 A.D.), *Plutarch's Lives, Marcellus* XVIII, ed. A. H. Clough: " [...] others again relate that, as Archimedes was carrying to Marcellus mathematical instruments, dials, spheres, and angles, by which the magnitude of the sun might be measured to the sight, some soldiers seeing him, and thinking that he carried gold in a vessel, slew him..."

(III) The form and the function of planetarium

The following passages are evidence of the existence of an astronomical model, a planetarium made by Archimedes, which composes in exemplary way the perfect imitation and simulation of the cyclical revolution of heaven and planets.

- Proclus Diadochus (410-485 A.D.), *A commentary on the first book of Euclid's Elements,* p. 41,3, ed. G. Morrow: " [...] Under mechanics also falls the science of equilibrium in general and the study of the so-called centre of gravity, as well as the art of making spheres vated by Archimedes, and in general the art concerned with the moving of material things..."

- Firmicus, Julius (4th century A.D.), in his work *Mathesis* (vI 30. 26, ed. W. Kroll, F. Shutsch, and K. Ziefler, Lipsiae, 1913) mentions that Archimedes the Syracusian, who with his mechanical inventions often vanquished the Roman army, is the one who has constructed a sphere, which represents the revolutions of all heavenly bodies. This exemplary construction is a remarkable imitation of nature.
- Lactantius (3rd century A.D.), *Divinarum Institutionum,* Pars. I, lib. II 5, Samuel Brandt, Prapae – Vindobonae – Lipsiae, 1890: "Could Archimedes the Sicilian have devised from hollow brass a likeness and figure of the world, in which he so arranged the sun and moon that they should effect unequal motions and those like to the celestial changes for each day, as it were, and display or exhibit, not only the risings and setting's of the sun and the waxings and wanings of the moon, but even the unequal courses of revolutions and the wanderings of the stars as that sphere turned, and yet God Himself be unable to fashion and accomplish what the skill of a man could simulate by imitation?" (Price, 1975)
- In a poem by Claudius, Zeus is praised for the construction of mechanical models that represent faithfully the world he has created. The poem gives in a mythological way the notion of an occult mechanism (hidden influence), in the inner part of the planetarium that is responsible for the different routes of the star, as well as for the whole motion of it. Here it is mentioned that this planetarium by the Syracusian engineer is made of glass. Elsewhere, it is mentioned that it is made of brass. Claudius Claudianus (4th century A.D.), *Carminum corpusc.* LI (LXVIII), ed. M. Platnauer [Loeb], vol. II, 1963: "When Jove looked down and saw the heavens figured in a sphere of glass he laughed and said to the other gods: Has the power of mortal effort gone so far? Is my handiwork now mimicked in a fragile globe? An old man of Syracuse has imitated on earth the laws of the heavens, the order of nature, and the ordinances of the gods. Some hidden influence within the sphere directs the various courses of the stars and actuates the lifelike mass with definite motions. A false zodiac runs through a year of its own, and a toy moon waxes and wanes month by month. Now bold invention rejoices to make its own heaven revolve and sets the stars in motion by human wit..." (Price, 1975)
- In Archimedes' planetarium, according to Cicero (106-43 B.C.), the revolutions of the heavenly sphere are simulated in a more perfect way than they occur in nature itself. Cicero, *De natura deorum* II. XXX. 88, ed. Rackham [Loeb], 1961: "they think (thinkers) more highly of the achievement of Archimedes in making a model of the

revolutions of the firmament than of that of nature in creating them, although the perfection of the original shows a craftsmanship many times as great as does the counterfeit." (Price, 1975)

- Sextus Empiricus (2nd century A.D.) in his work *adv. Mathem.* (IX 115, vol. II, ed. Mutschmann, Lipsiae, 1924) expresses his admiration for the automatically moving mechanisms and the surprise that someone experiences when he observes Archimedes' planetarium, where the sun, the moon, and the other planets are all in motion. According to Sextus, even more significant than the planetarium itself is the craftsman, who has constructed it and the forces or mechanisms that set all the heavenly bodies in motion. A possible explanation about the way that every sphere or heavenly body of this model is set in motion will be given later.
- The earth, which is spherical, takes the centre of the planetarium: Ovidius (43 B.C.-17 A.D.), *Fastorum VI,* ed. F. Bömer – C. Winter [Loeb], 1957: " [...] There stands a globe hung by Syracusian art in closed air, a small image of the vast vault of heaven, and the earth is equally distant from the top and bottom. That is brought about by its round shape..." (Price, 1975)
- The function of Archimedes' astronomical model is described by Cicero:

  a) *Tusculan Disputations* I. XXV. 63, ed. J. E. King [Loeb], 1960: "For when Archimedes fastened on a globe the movements of moon, sun and five wandering stars, he, just like Plato's God who built the world in the *Timaeus,* made one revolution of the sphere control several movements utterly unlike in slowness and speed. Now if in this world of ours phenomena cannot take place without the act of God, neither could Archimedes have reproduced the same movements upon a globe without divine genius", and

  b) *De Re Publica* I. XIV. (21-22), ed. C. W. Ceyes [Loeb], 1961: " [...] But this kind of globe, he said, on which were delineated the motions of the sun and moon and of those five stars which are called wanderers, or, as we might say, rovers, (i.e., the five planets) contained more than could be shown on the solid globe and the invention of Archimedes deserved special admiration because he had thought out a way to represent accurately by a single device for turning the globe those various and divergent movements with their different rates of speed. And when Gallus moved the globe, it was actually true that the moon was always as many revolutions behind the sun on the bronze contrivance as would agree with the number of days it was behind it in the sky. Thus, the same eclipse of the sun happened on the globe as it would actually happen, and the moon came to the point where the shadow of the earth was at the very time when the sun [...] out of the region [...] (At this point eight pages of the Latin manuscript are missing)" (Price, 1975)

Conclusively, Archimedes' mechanical model of the world is a transparent (glassy according to Claudius Claudianus) globe, on which the fixed stars are fastened, in the inside of it there are spheres or rings that correspond to the sun, moon, and the five planets, and in the centre of it the earth is placed. These rings are able to move revolving around the earth and their movements are of unequal angular speeds. The motion is transmitted through a mechanical system, which most likely consists of a set of gears meshing in parallel planes to give the correct mean periodic rotations to the seven celestial bodies. Even if the device of Archimedes were considered to be the simplest possible one, it would still be impossible for another mechanism of strings or pulleys or anything else to give so appropriately the behaviour of the interlocking regular cycles that constitute the main corpus of astronomical theory at the time of Archimedes (Price, 1975).

(Brumbaugh, 1966) describes the planetarium of Archimedes as 'a new self-operating model of the cosmos.' The whole mechanism is set in motion by an external manually operated lever that is connected with an initial motive gear. The relations of motion transmissions are similar to the gears' diameters and reproduce the unequal angular speeds of heavenly bodies as a matter of reality. This model is therefore a functional and dynamical model of the planetary system, able to simulate the relative position of the earth and planets, and the possible coincidence of constellations in every moment of the calendar year; for Cicero says that the moon is always as many revolutions behind the sun on the bronze contrivance as would correspond to the number of days that the moon is behind the sun in the sky. The same eclipse of the sun happened on the globe as it would actually happen.

Such ways of transmitting motion through the use of gearing mechanisms are credited presumably in the works of, the contemporary to Archimedes, Alexandrian engineers, Ktesibios (*ca.* 250 B.C.), Philon (*ca.* 200 B.C.), and certainly in Heron of Alexandria (1st century B.C. or 1st century A.D.).

Heron reconstructs and improves engines of Archimedes, such as the odometer, and describes gearing mechanisms of unequal speed motions in his work of *Mechanics*. It is worth noticing that the principles embodied, as for example in the odometer, are common with those in the transmission system of a modern automobile.

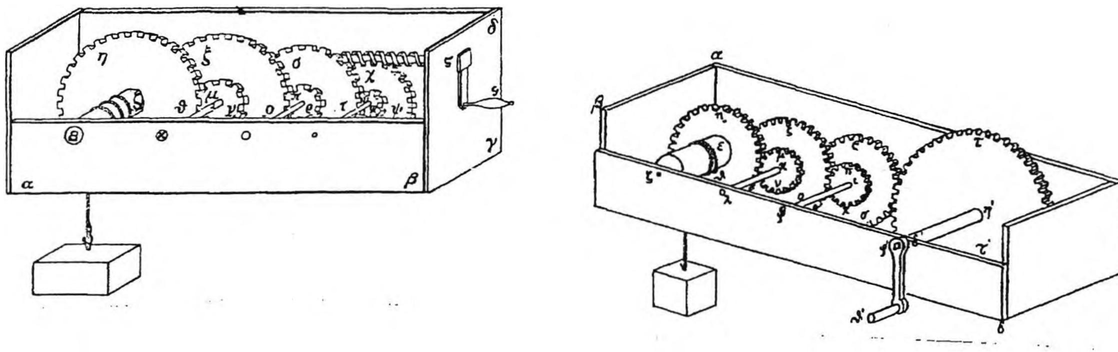


Figure 9.12: Gearing mechanisms by Heron. The unequal diameters provide unequal angular speeds. (Heron, *Mechanics*, ed. W. Schmidt, vol. V., Lipsiae, 1900)

Historical evidences that represent the continuation of such astronomical gearing systems are found firstly in the mechanism of Antikythera and many centuries later in the Arabic world. Typical are those of a solar gearing system by Al-Biruni (1000 A.D.) and of a calendar astrolabe by Muhammad ben Abi Bark of Isfahan (1222 A.D.)

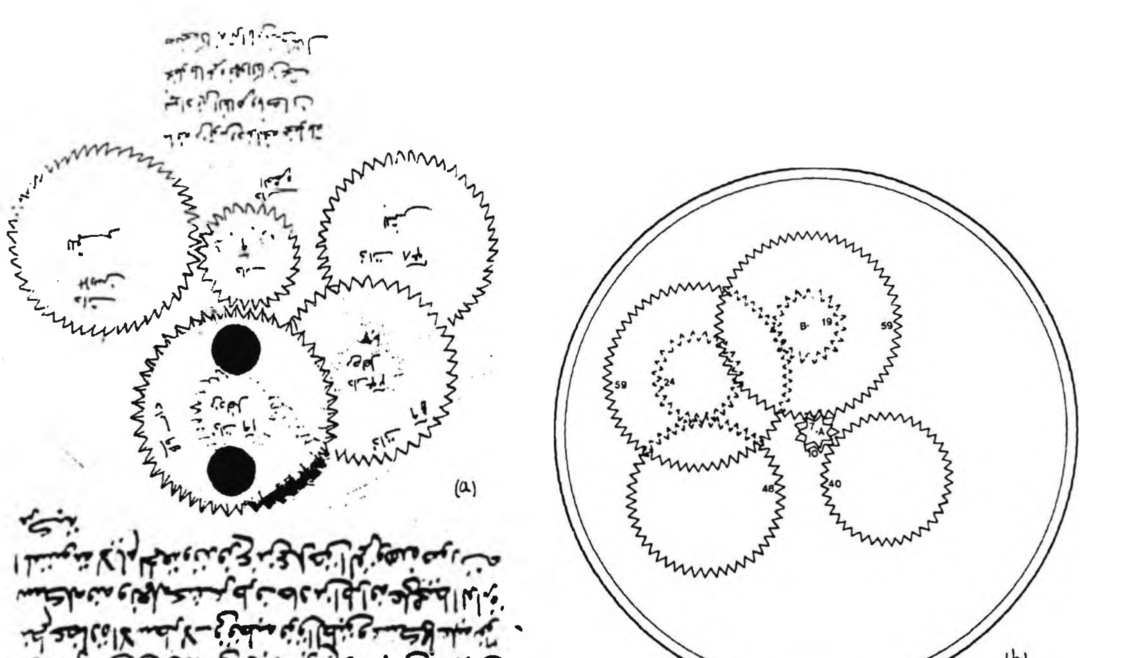


Figure 9.13: Solar mechanism of gears by Al-Biruni
Manuscript of 14th century, British Library Collection (MS5593)
Modern diagram, J.V. Field, M.T. Wright, 1985

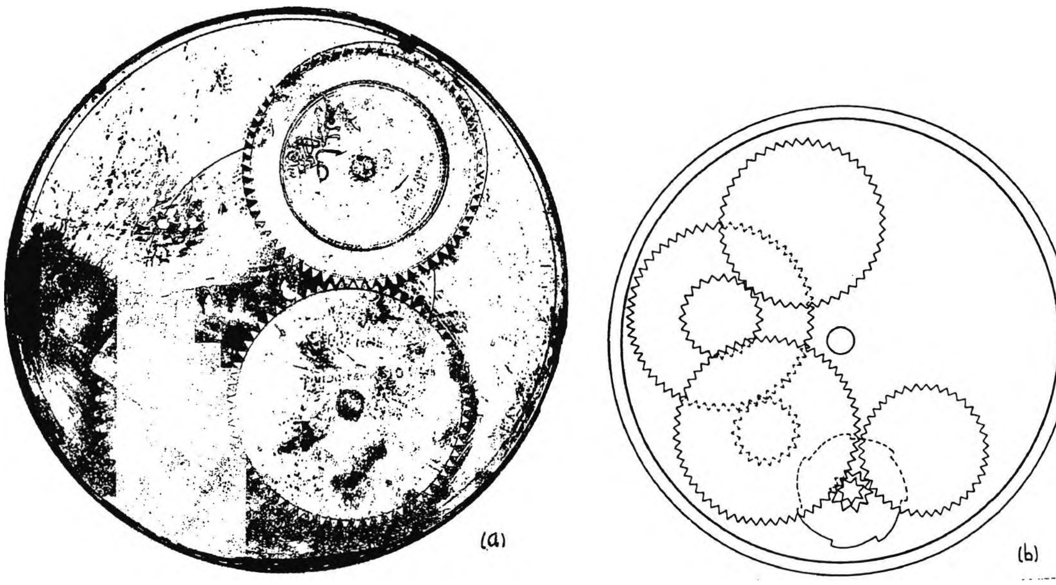


Figure 9.14: Calendar astrolabe by M. ben Abi Bark
Collection of the Museum of History and Science of Oxford (CCL5)
Modern diagram by J.V. Field and M.T. Wright, 1985

9.9 The Antikythera Mechanism

The Antikythera mechanism (*ca.* 80 B.C.), now exhibited in the national Museum of Athens, is the oldest and most complex surviving scientific instrument of antiquity. The mechanism fragments are discovered shortly after 1900 near the Greek island of Antikythera and reconstructed by Dr. Price. It is a singular astronomical or calendar calculating device, a mechanical model of the universe, a mechanical analogical computer with quantitative exact magnitudes, involving a very sophisticated arrangement of more than thirty gear wheels, which is constructed according to the Archimedean tradition of planetarium construction.

(Dr. Price, 1975), who elaborated and reconstructed the mechanism says: "The mechanism can now be identified as a calendar Sun and Moon computing mechanism which may have been made about 87 B.C. and used for a couple of years during which time it had several repairs. It was perhaps made by a mechanic associated with the school of Posidonios on the island of Rhodes, and may have been wrecked while being shipped to Rome about the time that Cicero was visiting that school *ca.* 78 B.C. The design of the mechanism seems to be very much in the tradition that began with the design of planetarium devices by Archimedes. It was continued through the Rhodian activity, transmitted to Islam where similar geared devices were produced, and finally flowered in the European Middle Ages with the tradition of great astronomical clocks and related mechanical devices that were crucial for the Scientific and Industrial Revolutions. Perhaps the most spectacular aspect of the mechanism is that it incorporates the very sophisticated device of a differential gear assembly for taking the difference between two rotations, and one must now suppose that such complex gearing is more typical of the level of Greco-Roman mechanical proficiency than has been thought on the basis of merely textual evidence. Thus, this singular artefact, the oldest existing relic of scientific technology, and the only complicated mechanical device we have from antiquity quite changes our ideas about the Greeks and makes visible a more continuous historical evolution of one of the most important main lines that lead to our modern civilisation."

The detailed examination either of the operation or of the reconstruction of the mechanism is out of our field of research. We will only cite some figures of it that give the main idea of how this astrolabe looks like. The four main fragments of the mechanism form physical joins, which show that they were part of a single mass:

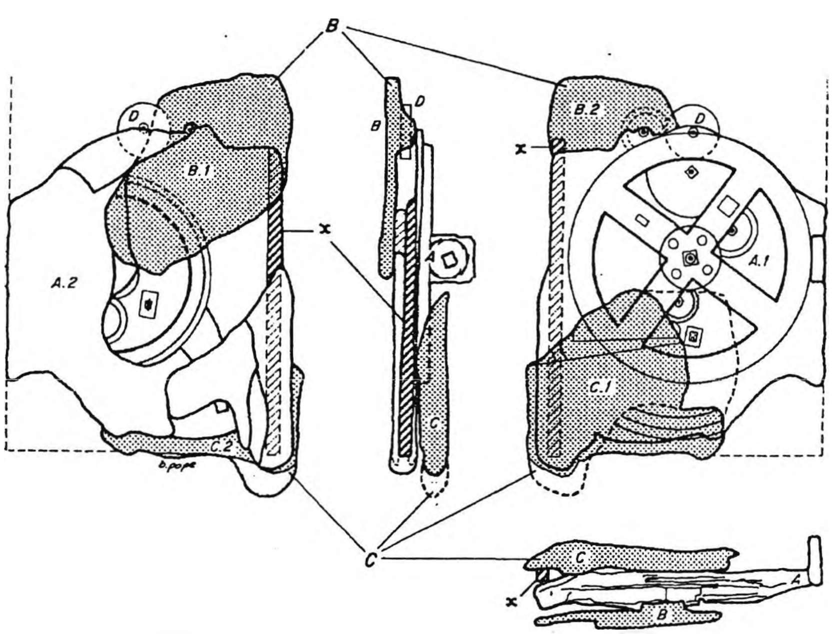


Figure 9.15: Schematic diagram showing the four main fragments (Dr. Price, 1975)

The front and the back of main fragment and the general plan of the thirty gear-wheels of Antikythera mechanism are shown below:

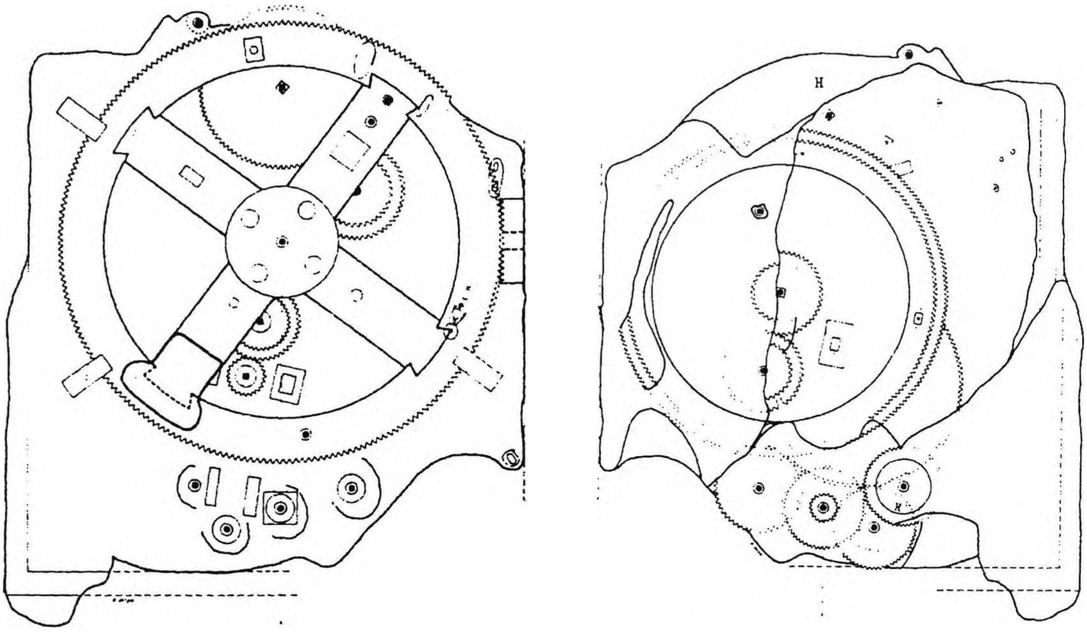


Figure 9.16: Schematic diagram of front and back of main fragments (Dr. Price, 1975)

The Antikythera mechanism had a complex net of gears, which allowed the exact simulation of the sun, moon, planets, and fixed stars' positions in relation with the earth.

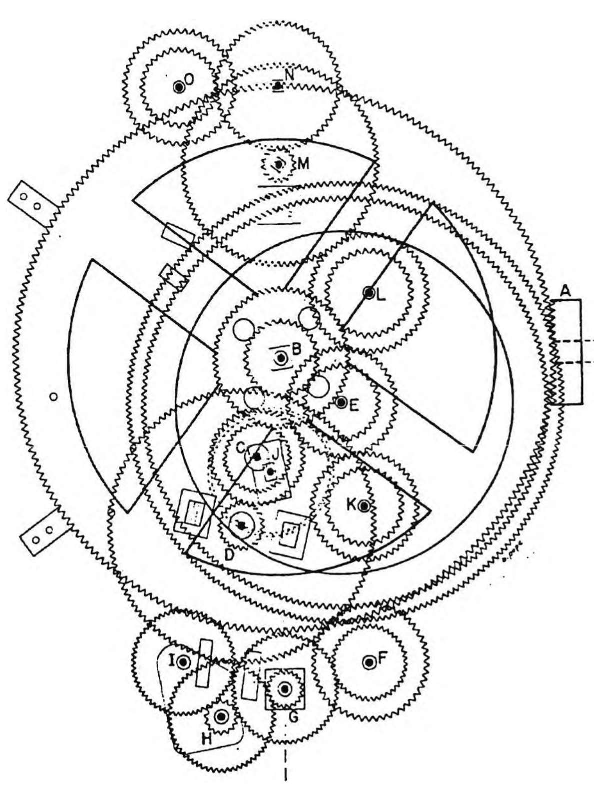


Figure 9.17: General plan of all gearing, composite diagram (Dr. Price, 1975)

This mechanism is an invaluable archaeological finding of the most complicated mechanical model of the world that has been found till nowadays.

9.10 Conclusion

Apart from the conceptual, physical and mathematical interpretation of an unknown system, there is also the mechanical description and realisation. The early philosophers do not content themselves with the theoretical interpretation of natural phenomena or world but they proceed to detailed descriptions of mechanical models and, in many cases, to the construction of complicated mechanisms that simulate the planetary motions. They realise that the construction of a scale model is an important step in order to understand, to describe, and even to 'control' the world.

More precisely, the development of technical thought, observation, and experiment in the presocratic period results in the use of simple 'machines' for the simulation of the operation of the world, such as the models of Anaximander and Parmenides. In the mechanical model of Anaximander, we find an early form of the contemporary qualitative reasoning process. It is remarkable also that the different parts of Anaximander's or Parmenides' models are compared to parts of real mechanical systems, such as wheels, or tubes, or rings. This comparison constitutes an approach to the notion of analogy. The concept of analogy, which has already been examined from a mathematical perspective by Pythagoras, Plato, and Euclid, finds its practical application in the construction of the mechanical models of the world. The efforts of representing the world by means of mechanical models continue not only in the age of Plato and Aristotle, but also during the following centuries.

Plato gives analytical descriptions of the motion of the earth, the planets, and the other heavenly bodies. In some cases, he uses elements of mechanics that result in mechanical models of the universe. His cosmological descriptions are so vivid, that force contemporary researchers to claim that he most likely had in front of his eyes a real mechanical construction, such as the armillary sphere. Although, Plato is aware of the irregularities of planetary motions, he encounters this problem only from a theoretical perspective. His theoretical solution of the compounding of uniform circular motions found its application in the 'two-sphere' geometrical model of Eudoxus.

In comparison to the imaginative and tender-minded Plato, who as a mathematician has a priori conceptions of the universe, the experimental and tough-minded Aristotle is a physician, who assumes and foretells as little as possible, but observes, takes notes, induces and deduces. Aristotle's explanation of the problem of planetary motions had as a result the system of the concentric spheres to be converted from a purely geometrical to a mechanical structure. Difficulties inherent in it led to the rise of systems involving epicycles and eccentric circles, such as the one of Ptolemy. Aristotle develops the theory of reacting spheres, i.e., additional spheres inherent between the innermost sphere of a planet and the outermost sphere of the next planet, so as to counteract the in between influence. This theory results in an integrated astronomical system.

Last but not least, the turn of the theoretical thought to its practical application and the realisation of conceptual models in a mechanistic way are integrated by Archimedes' achievements in the field of mechanics. Along with the numerous engines, such as the screw and the war machines, Archimedes constructs a planetarium, which is able to simulate the relative position of the earth and planets and the possible coincidence of constellations in every moment of the calendar year, and is characterised by the unequal speeds of the bodies. Although this planetarium is a quite exact quantitative simulation of the universe, the most perfect in construction and accuracy model of the universe is the mechanism of Antikythera, which completes the up to then efforts to create quantitative, mechanical models of the universe with the maximum possible accuracy.

PART TWO:
THE EVOLUTION OF THE CONCEPTS OF FEEDBACK AND CONTROL

Chapter 10

THE MYTHICAL INTENTION OF MAKING AUTOMATA

PART TWO: THE EVOLUTION OF THE CONCEPTS OF FEEDBACK AND CONTROL

10. THE MYTHICAL INTENTION OF MAKING AUTOMATA

10.1 Introduction

Up to this point, the discussion has mainly laid on the evolution of the concepts of system and modelling. It is time now to pay attention to the concept of control. Although the origins of the former have been investigated in the realm of philosophy and science, i.e., in the theoretical understanding of the rational order of the universe, the origins of the latter are mostly found in the field of technology, i.e., in the attempt of humans to 'control' and master the natural environment. But not only there: the Mind-controller of Anaxagoras, the 'contradiction' of Heraclitus, or the 'cybernetist' of Plato are examples of the early theoretical consideration of the problem of control.

Quite often, the Greeks are considered as theoretical thinkers without interest in practical application. Other occasions, such as in the view of Robert Hahn[50], it is asserted that "even the origin, if not the development of Greek science in the Classical period, owes more to technological and engineering concerns" and that in the face of Thales we meet a great engineer, whose "thinking was nurtured by a community engaged in projects of engineering, where the technological expertise played a crucial role in providing the detailed theoretical data". We will formulate an opinion somewhere between those extreme aspects by following again the crucial periods of the evolution of Greek science and technology. Evidence of early visions of technology and concepts of Control and Automation will be found even in the mythical period. We will examine the philosophical interpretation of the concepts of contradiction, feedback, and control in the classical era. Last but not least, we will explore the evolution of these concepts in the subsequent Hellenistic period, where many technological innovations have been created and put into use. In the works of Archimedes, Ktesibios, and Hero we will find important inventors that put their discoveries into practice.

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[50] Hahn, R., *"What did Thales want to be when he grew-up? Or re-appraising the roles of Engineering and Technology on the origin of early Greek Philosophy/Science"*, essay in the book of B. P. Hendley (see references)

In particular, in this chapter, we will point to the amazing moving statues and automatic machines of the mythical period. The mythical automata, though they are referred as the work of the gods and most likely were never materialised, bear the germ of the technological intention to construct machines, which not only do they move by themselves, but also control this motion, so as to result in the purpose they have been constructed for. They follow the anthropocentric conception of the first mythical models and express the intention of man to construct machines that imitate the life itself.

10.2 Mythical Automata

The ancient Greek written sources include many but scattered descriptions of technological findings and visions, of admirable technical constructions and inventions, among them descriptions of automata, namely machines moving by themselves, with internal energy, like living beings. Such descriptions are also found in abundance within the Greek ancient myth, the verses of the epics, and the mythical traditions. They appear either as poetic account of remarkable technological human inventions, or as imaginary technical achievements attributed to the gods. Homer mentions in his epics, in the middle of the 8^{th} century B.C., such mythical automatic machines, and in the same way he created the Greek gods, he is the first to introduce the technical term *'automaton'*. Three centuries later, *ca.* 450 B.C., Herodotus in his Histories describes the automata of other great civilisations of Mediterranean.

The word 'automaton' (*αυτόματο*) appears for the first time in the Homeric epic of Iliad. It constitutes simultaneously a poetical and a technological invention. Poetical, because in the ancient Greek poetry the concept of 'anthropomorphism', i.e., of ascribing human attributes to nature and natural phenomena, was dominant. Thus, the construction of automotive machines, of machines that looked like living beings, that were equipped with 'soul – *ψυχή*', strength and human abilities, was at the beginning no more than a mythical poetical invention, which however progressed in a technical vision and a materialised aspiration.

In fact, the historical evolution of technical thought took place in three stages. The first stage concerns the invention of 'tools', i.e., of those elements that expand human strength and range, such as the bludgeon, the javelin, and the sickle. The second stage has to do with the invention of 'machines' that move themselves at a task, by means of external, natural energy, such as the bow, the carriage, and the ship. And the last technological stage is related to the self-motivating machines, to the 'automata' that can manage complex tasks on their own, by means of internal energy, and are able to control and adjust their operation. Some examples of such automata are the automatic ships of Egyptians described by Herodotus and the automatic theatres of Heron from Alexandria.

The background of this technological evolution is detectable in the ancient Greek Myth. In its innumerable technological accounts we discover a) the first inventions and tools, the early technological tradition, b) the highly developed technology of the contemporary to the poets period, in the technological achievements of great technicians or technological civilisations, and c) the technological visions, such as the automata, which are ascribed to the technician God Hephaestus. In this point of view, the Greek myths and especially the Homeric epics condense the technological experience of the past, classify the inventions and the technical ideas of the present, and lead to the new ages of the future.

10.2.1 The Automata of Iliad

Iliad is the poetic description of a war. It combines the horridness of war into an epic poem filled with art, illustrious descriptions, and a myriad of wonderful literary images. It is the first written source of ancient Greece and describes the collision between the army of Achaeans and the defenders of Troy in the Aeolian Asia Minor. This collision is personified in the face of the two heroes: Achilles and Hector. This collision takes place under the exhortation of Gods and in some cases by their own participation. The technological accounts in Homeric Iliad are polemic, heroic, godlike, and in these descriptions, we identify some of the early technological notions:

a. The automatic gates of heaven

When Hera, the queenly goddess, daughter of great Cronus, decided to side with the Greeks during the Trojan War,

> E 720-733: "[she] went to and fro harnessing the horses of golden frontlets. And Hera quickly put to the car on either side the curved wheels of bronze, eight-spoked, about the iron axle-tree. Of these the felloe verily is of gold imperishable, and there over are tires of bronze fitted, a marvel to behold; and the naves are of silver, revolving on this

side and on that; and the body is plaited tight with gold and silver thongs, and two rims there are that run about it. From the body stood forth the pole of silver, and on the end thereof she bound the fair golden yoke, and cast thereon the fair golden breast-straps" (Murray, 1924).

After this admirable description of the divine chariot, Hera stepped upon the flaming car, touched the horses with the lash swiftly, and then

E 749-752: " [...] self-bidden (Homer uses the word *αυτόματα / automatically, by themselves*) groaned upon their hinges the gates of heaven which the Hours had in their keeping, to whom are entrusted great heaven and Olympus, whether to throw open the thick cloud or shut it to" (Murray, 1924).

Homer uses here for the first time the word 'αυτόματα'. The fantastic poetic picture of the automatic gates of heaven, which open on their own, does not have the precision or the completeness that appears in the description of the divine car. However, it introduces the concept of automatic motion. Possibly, it expresses a vague technical intention, an imaginary technical vision and paves the way for the later detailed descriptions of automatic machines that are ascribed to the great craftsman of Olympus, Hephaestus.

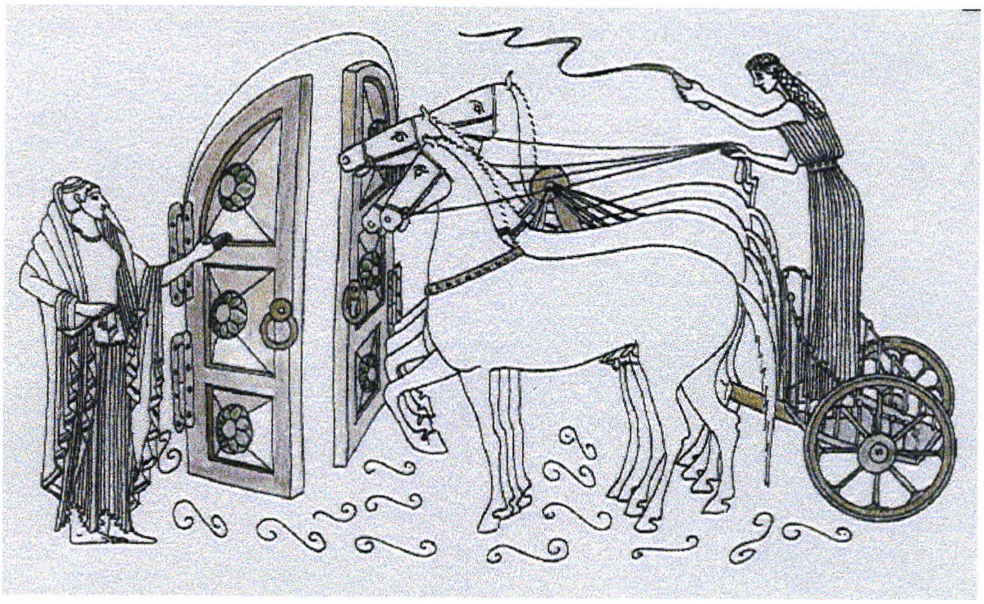


Figure 10.1: Hera and the automatic gates of heaven, (Kalligeropoulos, 1999)

b. The automatic tripods

In Rhapsody Σ (book18 of Iliad), which is called *Oplopoiea*, Homer summarises the most important automata made by Hephaestus. According to the Homeric myth, when Achilles decided to take part in Achaeans' War, he went to his mother, Thetis, and asked her to find the famous Hephaestus and beg him to make new weapons for her son.

> Σ 142-144: "[Thetis speaks] I will get me to high Olympus to the house of Hephaestus, the famed craftsman, if so be he will give to my son glorious shining armour" (Murray, 1924).

Thus, Thetis goes to Olympus and meets Hephaestus at his bronze mansion, where

> Σ 372-377: "she found him sweating with toil as he moved to and fro about his bellows in eager haste; for he was fashioning tripods, twenty in all, to stand around the wall of his well-built hall, and golden wheels had he set beneath the base of each that **of themselves they might enter the gathering of the gods** [*αυτόματα* by Homer] at his wish and again return to his house, a wonder to behold" (Murray, 1924).

The tripods were valuable vessels, which could be offered as special gifts in exceptional cases. These vessels were either static or moving, and could be useful either in offering water or wine to the guests, or as ritual vessels in religious ceremonies. In Homeric description, Hephaestus was making twenty tripods and was setting wheels beneath their base, so that they could be able to move even by themselves. This attribute of self-motion, that Heron of Alexandria constructs and describes centuries later by his moving automata, appears in myth as a need, as something possible to happen. It is represented by the poet and ascribed to the great craftsman.

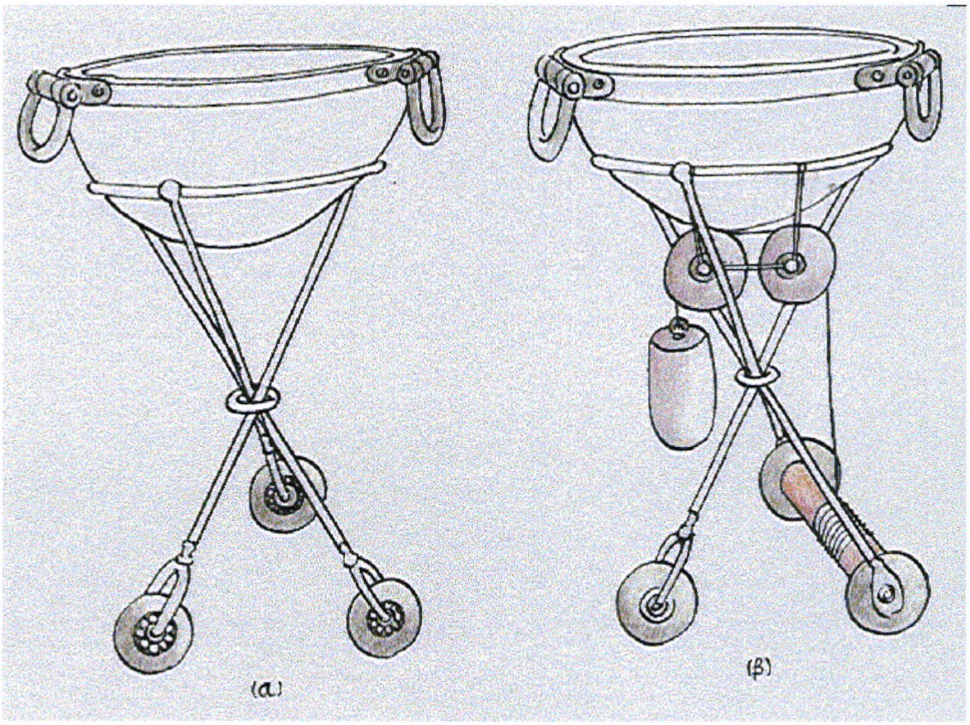


Figure 10.2: Mobile and self-moving tripods, [Kalligeropoulos, 1999]

c. The adaptive bellows

Later on, Homer presents Hephaestus to work on his bellows:

Σ 468-477: "So saying he left her there and went unto his bellows, and he turned these toward the fire and bade them work. And the bellows, twenty in all, blew upon the melting-vats, sending forth a ready blast of every force, now to further him as he laboured hard, and again in whatsoever way Hephaestus might wish and his work go on. And on the fire he put stubborn bronze and tin and precious gold and silver; and thereafter he set on the anvil-block a great anvil, and took in one hand a massive hammer, and in the other took he the tongs" (Murray, 1924).

The bellows, which were used by the mortals, were represented on ancient pots as huge, manually operated machines by one or two men. The use of bellows was necessary for the increase of the temperature of the furnaces, so that metals would melt. Hephaestus, being alone and without any help in his divine workroom, had at his disposal twenty bellows that could work automatically under his command. Moreover, these bellows were adaptive; he needed only to instruct them to start and they began automatically to operate, faster or more slowly, as the work required.

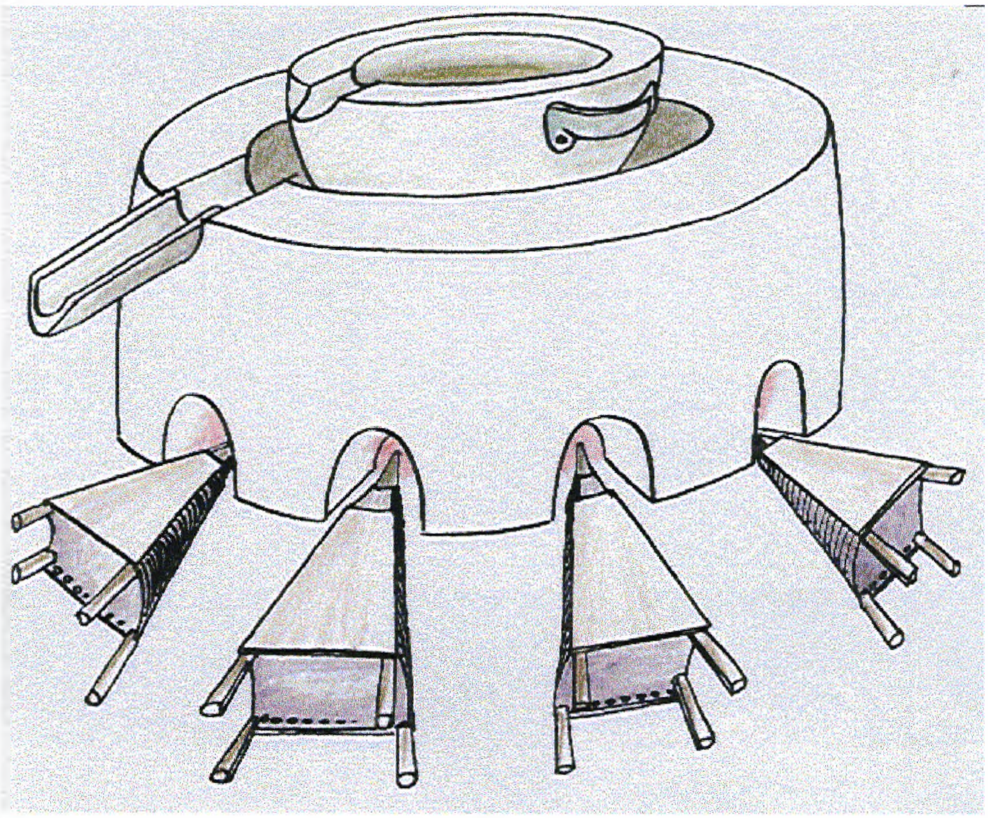


Figure 10.3: Bellows around a furnace, (Kalligeropoulos, 1999)

This is an ingenious conception, wish and desire of any metal worker, a need inherent in the requirement of work. Even the poem itself leads to such a technical vision. Since Hephaestus works on his own, it could be impossible for him to create all his marvellous works without possessing automatic, regulated bellows. How could he construct automatic tripods that serve the Gods, without being able to create something similar for his workroom? And could the divine workroom be different from a common one, in something else than the technology, the inventiveness, and the originality?

d. Female robots

Homer completes the technical vision of his era, by describing Hephaestus to create two manlike robots.

Σ 410-422: "He [Hephaestus] spake, and from the anvil rose, a huge, panting bulk, halting the while, but beneath him his slender legs moved nimbly. The bellows he set away from the fire, and gathered all the tools wherewith he wrought into a silver chest; and with a sponge wiped his face and his two hands withal, and his mighty neck and shaggy breast, and put upon him a tunic, and grasped a stout staff, and went forth halting; but there moved swiftly to support their lord handmaidens wrought of gold in the semblance of living maids. In them is understanding in their hearts, and in them speech and strength, and they know cunning handiwork by gift of the immortal gods. These busily moved to support their lord, and he, limping nigh to where Thetis was, sat him down upon a shining chair..." (Murray, 1924)

The fabulous achievements of the technologist god are completed by the construction of two manlike machines, two robots, which have sense, speech, strength, and are able to assist and accompany the cripple Hephaestus wherever he goes. Poet's imagination creates animate machines. Since the technologist god was able to make machines that were self-moving and even self-adaptive, why could not he create machines that were like living beings?

The myth of manlike machines reappears, during the return of the Argonauts from the ancient Kolchis, in the form of a copper hydraulic giant, the so-called Talos, who guarded Crete against intruders. The Argonauts could only land on Crete after Talos was destroyed through the intervention of Medea.

e. Talos, the copper giant

Plato regards Talos as the law keeper of Crete, who took his appellation from the bronze law-tables, which he was carrying (Plato, *Minos*, 320c).

Sophocles, in the fragment 161 of Daedalus, with the title *The End of Talos*, describes Talos as a bronze robot and explains his function as follows:

"Talos had in his ankle a syringe [σύριγγα] covered by a membrane."

The mythmakers Apollonius Rhodius and Apollodorus describe, in their work *Argonautica*, the ancient robot of Talos in more technical details. This copper giant was also credited to Hephaestus and could move its members by means of a hydraulic mechanism. This mechanism consisted of a long pipe, a vein full of a liquid similar to the mercury, the so-called ichor (*ιχώρ).* This vein was extended from the neck to the ankles of Talos and a membrane or a nail at the end of vein was keeping the ichor inside. Fire inside the robot was giving its internal energy. Apollodorus describes Talos as follows:

Apollodorus, *Library and Epitome*, 1, 9, 26: "Putting to sea from there, they (the Argonauts) were hindered from touching at Crete by Talos. Some say that he was a man of the Brazen Race, others that he was given to Minos by Hephaestus; he was a brazen man, but some say that he was a bull. He had a single vein extending from his neck to his ankles, and a bronze nail was rammed home at the end of the vein. This Talos kept guard, running round the island thrice every day; wherefore, when he saw the Argo standing inshore, he pelted it as usual with stones. His death was brought about by the wiles of Medea, whether, as some say, she drove him mad by drugs, or, as others say, she promised to make him immortal and then drew out the nail, so that all the ichor gushed out and he died. But some say that Poeas shot him dead in the ankle." (Frazer, 1921)

The noteworthy characteristic in this description is the invention of the liquid that runs throughout the body of Talos, the ichor. In combination with the fire, it acts as the necessary internal energy that moves Talos. Even though at that time the automata were only verbally expressed ideas, the mythmakers did not content themselves only with the theoretical intention of having automata. They went further to the practical details, conceiving as the main characteristic of an automatic machine, its internal energy, its 'soul', and looked for such energy sources to ensure the automatic motion and operation.

10.2.2 The Automata of Odyssey and Herodotus' Histories

If Iliad is the epic of war, Odyssey is the epic of the art of the sea. If Iliad is a hymn to the vigour and beauty of Achilles, Odyssey is a hymn to the versatility and inventiveness of ingenious Odysseus. And, if in Iliad the technical achievements are attributed to gods, in Odyssey, they are considered as technical achievements of man, as human works either of eponymous or anonymous craftsmen, and even of far and developed civilisations, such that of Phaeacians.

a. The palace and the dogs of Alcinous

In the far-distant Mediterranean city Scheria lived the Phaeacians and their king Alcinous. The way Homer describes the palace of Alcinous is far away from the descriptions of the Olympic gods' palaces. It was all made from metal, bronze doorsteps and walls, golden doors and silver jambs. In front of the royal palace, as the metallic guards, two doglike robots were standing, also works of Hephaestus to Alcinous:

VII 91-94: "On either side of the door there stood gold and silver dogs, which Hephaestus had fashioned with cunning skill to guard the palace of great-hearted Alcinous; immortal were they and ageless all their days" (Murray, 1919).

b. The automatic ships of Phaeacians

When Odysseus arrives at the mythical island of Phaeacians, he enters the palace of king Alcinous, admires the golden and silver doglike guards of Hephaestus, and hears from king Alcinous himself the following words concerning the shipbuilding art of Phaeacians:

VI 262-272: "But when we are about to enter the city, around which runs a lofty wall, a fair harbour lies on either side of the city and the entrance is narrow, and curved ships are drawn up along the road, for they all have stations for their ships, each man one for himself. There, too, is their place of assembly about the fair temple of Poseidon, fitted with huge stones set deep in the earth. Here the men are busied with the tackle of their black ships, with cables and sails, and here they shape the thin oar-blades. For the Phaeacians care not for bow or quiver, but for masts and oars of ships, and for the shapely ships, rejoicing in which they cross over the grey sea" (Murray, 1919).

It is obvious from the above quotation that these seamen had at their disposal a highly developed technology in shipbuilding. Following, Homer describes that their ships were automatic. These ships knew how to travel, to get theirs bearings, and to follow the desired destination without rudders or captains, just on their own. The king Alcinous offered such a ship to his honoured guest Odysseus:

VIII 555-563: "And tell me your country, your people, and your city, that our ships may convey you thither, **discerning the course by their wits**. For the Phaeacians have no pilots, nor steering-oars such as other ships have, but their ships of themselves understand the thoughts and minds of men, and they know the cities and rich fields of all peoples, and most swiftly do they cross over the gulf of the sea, hidden in mist and cloud, nor ever have they fear of harm or ruin" (Murray, 1919).

According to the above passage, these ships were designed with reason, or they were the ships with the designed reason, or the ships with artificial intelligence. However, these, with artificial-reason-constructed ships were created by humans and not from the gods. The fact that these ships knew all the countries and could drive you wherever, provided that the final target was known, implies the absolute knowledge of geography, of all the seaways, and possibly the detailed mapping of the then known world. The idea that they did not need captains or steering-oars hides the desire, the intention, the vision, and even the invention or the reputation of navigation organs, such as astrolabes or machines that allow the automatic piloting and the control of the route of the ship. It also implies the knowledge of astronomy, mathematics, and mechanics. Taking also into consideration that these ships did not have any fear of harm or ruin presupposes a great constructive skill, inventiveness, and particular shipbuilding expertise. These mythical ships of shipbuilders of Scheria have even in primitive form their historical ancestors.

10.2.3 The Automatic Egyptian Ships by Herodotus

The myth of the automatic ships of Phaeacians is realised in the historical descriptions of Herodotus. In the second book of his *Histories,* he describes analytically the construction of ships specialized in the sailing of Nile:

> II 96: "The boats in which they carry cargo are made of the acacia, which is most like the lotus of Cyrene in form, and its sap is gum. Of this tree they cut logs of four feet long and lay them like courses of bricks, and build the boat by fastening these four feet logs to long and close-set stakes; and having done so, they set crossbeams athwart and on the logs. They use no ribs. They caulk the seams within with papyrus. There is one rudder, passing through a hole in the boat's keel. The mast is of acacia-wood and the sails of papyrus" (Godley, 1920).

The noteworthy characteristic of these ships is that although they had a rudder and sails, by means of which the captain could control their course, they were additionally equipped with an ingenious mechanism of stabilization. This mechanism could be considered as an automatic piloting system or in other words as the control system of their course. The way this mechanism was eliminating the undesired movements and keeping the ship course stable is shown in the following description of Herodotus:

> II 96: "These boats cannot move upstream unless a brisk breeze continues; they are towed from the bank; but downstream they are managed thus: they have a raft made of tamarisk wood, fastened together with matting of reeds, and a pierced stone of about two talents' weight; the raft is let go to float down ahead of the boat, connected to it by

a rope, and the stone is connected by a rope to the after part of the boat. So, driven by the current, the raft floats swiftly and tows the "baris" (which is the name of these boats,) and the stone dragging behind on the river bottom keeps the boat's course straight. There are many of these boats; some are of many thousand talents' burden" (Godley, 1920).

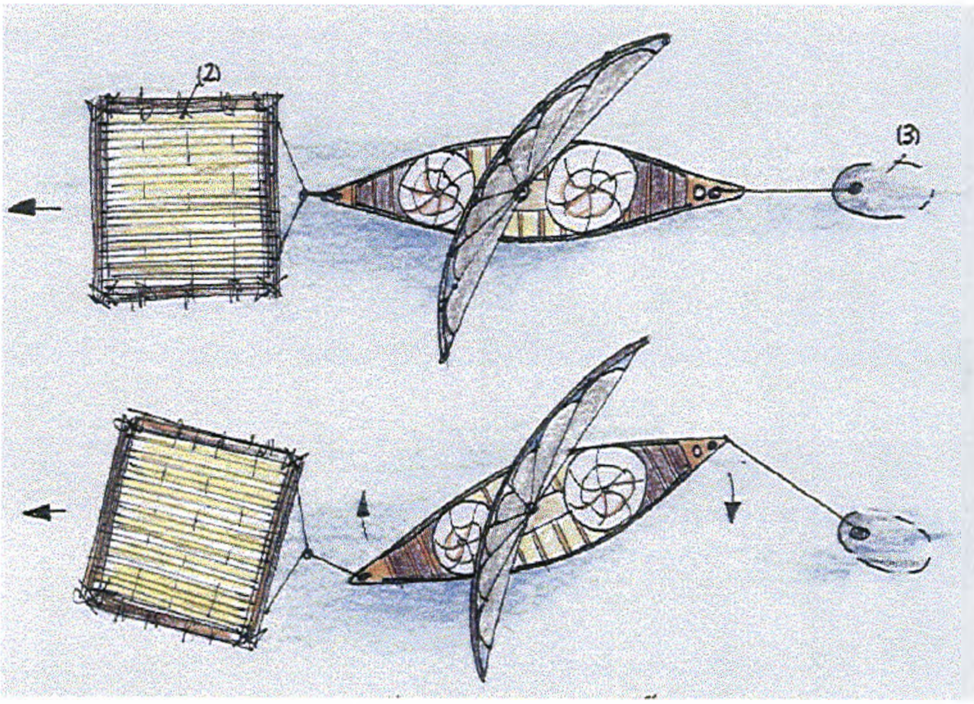


Figure 10.4: The control mechanism for the regulation of course, (Kalligeropoulos, 1999)

10.2.4 The Automata by Daedalus and Archytas

The dawn of the construction of the first historical automata in the ancient Greek technology is not clarified. It is said that Daedalus, a person between myth and history, constructed the first automobile machines. However, the first historical automaton is ascribed to the engineer Archytas.

a. The 'robot' by Daedalus

Daedalus, who is supposed to be the earliest sculptor (8^{th} or 7^{th} century B.C.), is credited with creations that could move on their own accord. They were so life-like that as Plato says they had to be prevented from running away. The following description of the moving statues of Daedalus explains also what kind of internal energy they had at their disposal, in order to be able to move by themselves:

Aristotle, *On the Soul*, 1.3.406b: "Some also say that the soul moves the body in which it is present just as it moves itself; for example Democritus says that Daedalus made his 'wooden Aphrodite' move by pouring mercury in it" (Humphrey *et al.*, 1998).

Daedalus' name is closely connected with the aspiration of man to fly like a bird. It is said that when Daedalus was taken prisoner, he made wings for himself and for his son Icarus, which were attached to their bodies with wax. Flying is another goal of early technological thought and dreaming. Even though it was never realized in antiquity, there is a reference of a mechanical bird by Archytas that could fly like a real one.

b. The mechanical dove by Archytas

The work of Archytas of Tarentum (*ca.* 428-350 B.C.), who belonged to the Pythagorean School and was a contemporary of Plato, is known only through fragments and references in the works of others. He advanced the study of three-dimensional geometry and found a remarkable solution to the problem of doubling the cube. He was the first to apply geometry to mechanics. He wrote on arithmetic and musical theory and is reputed to have designed and constructed a wooden dove capable of flying mechanically, perhaps by means of weights and compressed air:

Aulus Gellius, Attic Nights X. 12.8: "Many well-known Greeks and the philosopher Favorinus, a very assiduous antiquarian, have definitely asserted that Archytas constructed a wooden model of a dove according to certain mechanical principles, and that the dove actually flew, so delicately balanced was it with weights and propelled by a current of air enclosed and concealed within it. Indeed, in a matter so incredible it is preferable to quote Favorinus' actual language: "Archytas of Tarentum, who among other things was a mechanic, constructed a flying dove of wood...." (Cohen *et al.*, 1966)

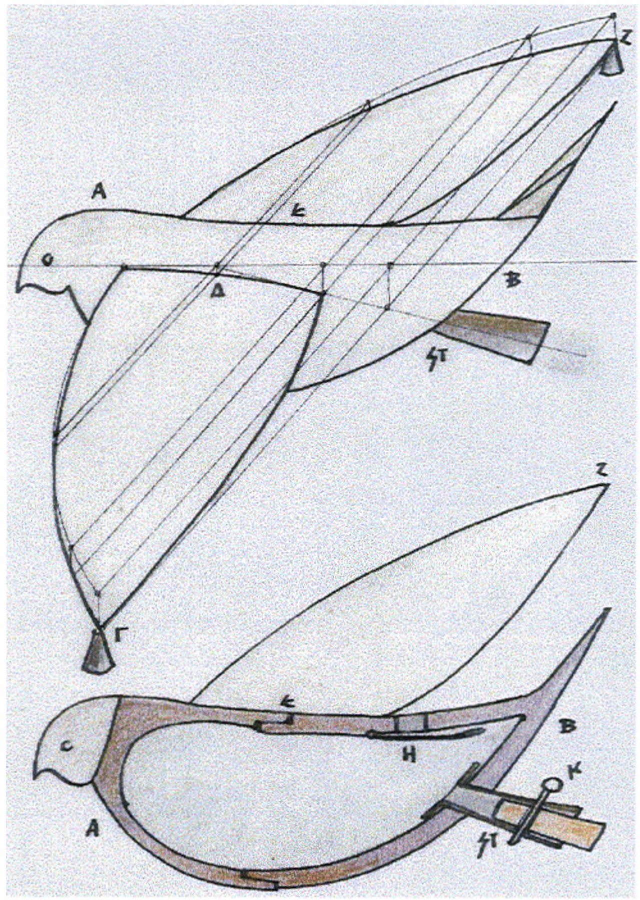


Figure 10.5: The flying dove of Archytas (Kalligeropoulos, 1999)

Aulus Gellius reports at third hand this story about the flying bird of Archytas, though he does not believe it. Nevertheless, the above description is another evidence of early man's efforts to construct a flying machine. This machine was able to fly by itself by using as its motive energy the expansion of the compressed air enclosed within it. It should have at its disposal a spout that was serving as the outlet of the compressed air. The direction of the spout was regulating the direction of the flight. In terms of control theory, we could say that it was an open loop control system, where the flying bird was the system and the spout the controller of it. This automatic bird of Archytas may be said to be the first known historical Greek automaton. For this pneumatic system, we dispose survived descriptions regarding its construction and operation.

10.3 Conclusion

Once tools had become relatively common, the next step was the conception of machines that move by themselves at a task. This conception was initially expressed as a human dream, as a vision that was described in literature from the time of Homer and onwards, until the experts in machinery of the Hellenistic period created automatic machines that could manage complex tasks.

In the frame of ancient Greek myth, there are many references to self-moving constructions, which imitate and simulate the operations of the life beings and are described by the Homeric word 'automata'. These mythical automata form a technological intention that is usually ascribed to gods, and in addition, they bear two fundamental features that in the following centuries constitute subjects of scientific and technological research. The first one is the ability to move on their own, i.e., to be equipped with internal energy, and the other is that their motion follows a pre-described plan, as for example the tripods of Hephaestus or the ships of Phaeacians, so as to result in the specific target, i.e., they have the ability of regulating and controlling their own function. The latter is of great importance.

What is the use of an automatic machine if it works uncontrolled? How does it imitate the living beings without bearing their ability of self-control? What is the use of the dove of Archytas? Being an open loop mechanism without the principle of feedback, it falls after a few meters without serving any specific purpose. And how is it possible to arrive to the advanced technological achievements of closed loop control mechanisms without passing firstly from the theoretical and even the mythical conception of them? The theoretical framework of the development of feedback and control concepts, and more specifically the construction of machines that have at their disposal these two characteristics, is the subject of the following chapters.

Chapter 11

THE THEORETICAL DEVELOPMENTS ON THE CONCEPTS OF FEEDBACK AND CONTROL

11. THE THEORETICAL DEVELOPMENTS ON THE CONCEPTS OF FEEDBACK AND CONTROL

11.1 Introduction

From the early times, man endeavours to understand the world he lives in. He collects information about the bewildering variety of natural phenomena around him, draws conclusions about their relations, and constructs theories to explain them. By means of counting, measuring, and reasoning he attempts to examine, evaluate, and 'control' the world (Trask, 1971). His desire to simulate nature and domesticate natural forces leads to the production of the first complex machines, which are the automata. As it is already shown, even the ancient Greek poets and mythographers expressed in their writings the desire to construct automatic machines. However, the realisation of such machines requires the exact theoretical analysis and understanding of the problem. How could a machine move by itself? What kind of internal energy could cause its motion? The answer to these questions follows, in the same way we have already mentioned, from the mythical speculation of the world to its philosophical interpretation. The internal source of energy for enabling a machine to move on its own had to be found into the fundamental physical elements of water, air, fire, and earth. The nature and the properties of these elements had to be thoroughly studied.

The presocratic and the classic philosophers engaged themselves - in a theoretical perspective - with the relation between the 'cause and the effect' (*αίτιον* and *αιτιατόν*), which characterizes any system in general. In particular, they studied the causes of motion, having as pattern the living beings, which are, nevertheless, characterized not only by motion, but also by the capability to control this motion and operation. A capability that allows them to accomplish the desired result, either by means of a command (*κέλευσμα*), i.e., functioning as open loop control systems, or by means of an internal programmed operation (*προαίσθηση*), i.e., functioning as closed loop control systems. In the latter case, the control systems, living or artificial, should know and measure the result of their actions, compare it with the desired one, and determine, in terms of this comparison, their behaviour. However, this act of comparing the cause with the effect constitutes a contradiction and creates a circle. Therefore, the theoretical examination of the control process passes through the contradiction concept and the development of the dialectic logic introduced by Heraclitus and elaborated by Socrates and Plato.

11.2 The concept of Self-motion and Automaton

In ancient Greece, there is a close relationship between philosophy, science, and technology. Philosophy interprets the achievements of technology, whereas technology accomplishes the visions of philosophy and the anticipations of science. For example, in order for the mythical intention of constructing automata to be materialized, the philosophers and engineers have to work complementary. Heron of Alexandria in the introduction of his *Pneumatics* characteristically says that the study of the Pneumatics was carried out by the old philosophers and engineers with great attention; the former examined them from a rational point of view, whereas the latter by using the senses and the experiment.

In the two worlds of philosophy and science on the one hand, and technology on the other, or of thought and action, or of mind and matter, the machine constitutes a unique link. It participates in both directions and affects philosophers as much as engineers in an exceptional way. The poets dream the invention of automatic machines that accomplish complex tasks, and the philosophers formulate the theoretical background that paves the way for the following construction of automata by the engineers. The concept of automatic machines is quite widespread, not only in the Greek myths, but also in the classical years, since Aristotle, for example, refers to self-motion and automaton frequently, while the study of the material elements by the Presocratics paves the way for the development of the necessary energy sources.

11.2.1 The Fundamental Elements as Sources of Energy

The main difference between automata (*αυτόματα*) and simple machines (*μηχανές*) is that although the latter operate by means of an external energy source, e.g., human or animal power, the automata have at their disposal an internal source of energy, a 'soul', as an inseparable element of them. In Greek thought, the 'soul' or '*psyche*' is not only the vital principle that gives things life, but also the inner power of self-motion. Thales of Miletus, for example, says that the magnet has a 'soul' because it moves the iron. Similarly a mechanism could move by itself, and hence imitate the behaviour of living things, only if the mechanic provided it with a 'soul', i.e., with an internal source of power. Therefore, the first step in the interpretation and construction of automata is the interpretation and construction of this internal source of power. Notions of potential internal source of energy that can cause the self-motion and self-operation of a machine are found even in the descriptions of the mythical automata. We have already seen the inner power of ichor in the case of giant Talos or the role of quicksilver in the wooden Venus of Daedalus.

However, the foundation of other sources of energy, such as the air-, water-, and steam-power, that are used basically from the Hellenistic time onwards, could be ascribed to the Presocratic philosophers. They attribute the creation of the world to the four material elements of earth, water, air, and fire. Even though, they do not consider them as potential energy sources in the construction of automatic machines, they realise their dynamics, so as to choose these elements and not others. It is not accidentally, that they consider them as 'animate' elements equipped with 'soul' (*ψυχή*), with energy that allows them to transform and to cause motion. This conception of the 'animate' material world constitutes the foundation of the technical solution to the problem of the self-motion of automata.

Let us see some examples:

- **Air energy**

We have already cited the self-moving dove of Archytas, which could fly by using as motive power the energy of the compressed air. The study of air as well as of its properties has as a result the foundation of the Pneumatics during the Hellenistic period.

- **Fire energy**

In many respects the ability to use fire can be regarded as the physical 'tool' in the early technological development of mankind. Heron of Alexandria uses in his Pneumatics the fire as an element that causes the expansion of air, which in turn compresses and sets in motion a liquid.

Heron, *Pneumatics*, A 12: "To construct an altar such that, when fire is raised on it, figures at the side shall offer libations..." (Cohen *et al.*, 1966)

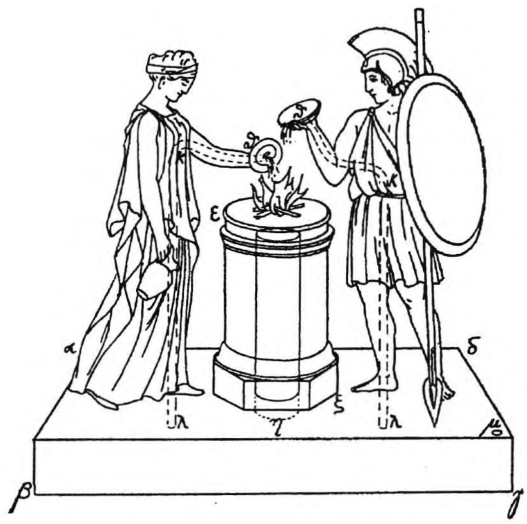
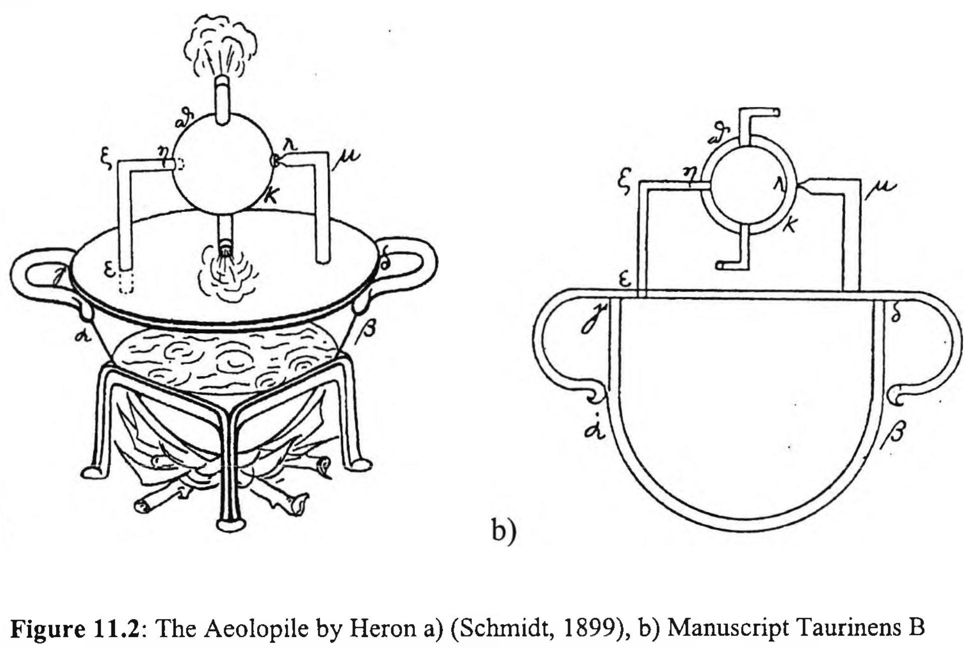


Figure 11.1: Heron's libations at an altar produced by fire (Schmidt, 1899)

In the case of Aeolopile, fire causes the creation of high-pressure steam, which in turn is responsible for the revolving motion of the sphere, as Heron describes in his Pneumatics, B 40: "Place a cauldron over a fire: a ball shall revolve on a pivot..." (Cohen *et al.*, 1966)



a) b)

Figure 11.2: The Aeolopile by Heron a) (Schmidt, 1899), b) Manuscript Taurinens B

- **Water energy**

The engineers of antiquity taking into account the properties of water and fluids constructed hydraulic control system for the regulation of the level or the flow of water. Heron in his Pneumatics describes hydraulic siphons that keep the height of water constant (Heron, A 1, 3, 13).

- **Falling weight energy**

The engineers of Hellenistic period, parallel to the above mentioned sources of energy that are based on the physical elements of the Presocratics, took also advantage of the fourth element, the earth and the internal power of 'it', i.e., the internal power of matter, the gravity. The potential energy of a falling mass because of its weight or the energy that stores a mass because of its spring or elasticity is used as the motive power of automata. For example, in the case of the moving automaton by Heron the fall of a weight connected with the motive axle of the automaton enables the forward motion of the mechanism.

However, the concept of internal power has earlier origins.

Thales introduces a dynamical conception of his cosmological model, according to which all the physical creatures come into being by the constant transformation of water. Water is for him an active element. It bears energy and 'soul', properties closely related to the concept of motion. According to him, the universe is full of 'souls', of energy sources that cause the universe to move (Aristotle, *De Anima (On the Soul)*, 405a20-22, see chapter 4)

Anaximander also occupies himself with the cause of the motion. He introduces the notion of eternal motion, which is responsible for the creation of heavens (Simplicius in *Physics* 24, 21, see chapter 4).

Empedocles introduces two additional, immaterial principles: Love and Strife, which as an equivalent to the Tension of Heraclitus or to the 'soul' of Thales, set the elements in motion.

Finally, **Heraclitus** introduces the concept of contradiction as the cause of motion and life. According to him, all things consist of opposites under an internal tension. In every system a dynamical harmony dominates, an equilibrium based on the unity of the opposites. This concept of the dynamic equilibrium is closely related to the concept of feedback and the function of a closed loop control system.

11.2.2 The Terms of 'Automaton' and 'Automatic' in Ancient Greek Sources

We have already seen that the words 'automaton' or 'automatic' emerge in the most ancient written epics, in the Homeric *Iliad* and *Odyssey*. Not only Homer, but also many other poets, writers, or philosophers make use of the word 'automatic' as an attribute to:

- Lifeless beings that have the ability of self-motion or self-action, i.e., automatic machines, such as the automatic gates of heaven (*Iliad*, E 749-752), the automatic tripods (*Iliad*, Σ 372-377), or the automatic irrigation systems of Egypt (Herodotus, *Histories*, II 149), Babylon (I 185), and Persia (III 117), the 'neurospasta' Egyptian statues, i.e., marionette, dolls with jointed arms discovered in Ancient Egyptian tombs (II 48), and the Egyptian ships that regulate their route by means of an automatic control system (II 96). Some of these examples of automatic machines have been represented in details in the previous chapter.
- Human beings that act by their own will, e.g., "Zeus the Deliverer is here; he came of his own accord[51]" (Aristophanes, *Plutus*, 1190)
- Natural activities, e.g., "Of themselves (automatically) diseases come upon men continually by day and by night[52]" (Hesiod, *Works and Days*), or the 'automatic' flow of a river: "the river rises of itself, waters the fields, and then sinks back again[53]" (Herodotus, *Histories*, II 14)
- Incidents that happen without any external influence, e.g., "… but the reason for the story of the spontaneous life (*αυτομάτου περί βίου*) of mankind is as follows[54] …" (Plato, *The Statesman*, 271E)

Aristotle refers frequently to the automata of the Classical age. His references constitute an additional source and proof of the existence of automotive machines long before the Hellenistic period and the writing of *Automatopoietice* (The art of making Automata) by Heron of Alexandria. Regarding the automata as admirable but incomprehensible achievements, Aristotle writes:

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[51] This text is based on the following book: Aristophanes, Wealth (Plutus), The Complete Greek Drama, vol. 2. Eugene O'Neill, Jr. New York, Random House, 1938.

[52] This text is based on the following book: The Homeric Hymns and Homerica with an English Translation by Hugh G. Evelyn-White. Works and Days. Cambridge, MA.,Harvard University Press, London, William Heinemann Ltd., 1914.

[53] This text is based on the following book: Herodotus, with an English translation by A. D. Godley (see references)

[54] This text is based on the following book: Plato, VIII The Statesman trans. by H.N. Fowler, Harvard University Press, London, 1925.

Metaphysics, 983a 15: "For all men begin by wondering that things are as they are when the cause has not been investigated, as in the case of marionettes [automata in ancient Greek text - moving by themselves]..." (Apostle, 1966)

Aristotle, by examining the cause of the movement and the regeneration of the sperm or semen, distinguishes the internal from the external motive energy, and compares the automotive live sperm with a mechanic automaton. According to him, the molecules of the sperm, when they are idle, have an internal motive power, on account of which they move. It is possible for one (sperm) to transfer the movement to the other, in the same way as the automata move, i.e., without being touched by anyone, but being pushed (primary) by an external power.

On the Generation of Animals, Book II, 734b 8-13: " [...] It is possible, then, that A [semen] should move B, and B move C; that, in fact, the case should be the same as with the automatic machines shown as curiosities. For the parts of such machines while at rest have a sort of potentiality of motion in them [internal motive energy], and when any external force puts the first of them in motion, immediately the next is moved in actuality. As, then, in these automatic machines the external force moves the part in a certain sense (not by touching any part at the moment, but by having touched one previously)..." (Ross, 1972, vol. 5)

In addition, Aristotle, in order to explain the movements of animals, uses as an example the automatic puppets and gives an analytically description of their movements that are similar to those of the animals.

On the Motion of Animals, 701b 2-16: "The movements of animals may be compared with those of automatic puppets, which are set going on the occasion of a tiny movement; the levers are released, and strike the twisted strings against one another; or with the toy wagon. For the child mounts on it and moves it straight forward, and then again it is moved in a circle owing to its wheels being of unequal diameter (the smaller acts like a centre on the same principle as the cylinders). Animals have parts of a similar kind, their organs, the sinewy tendons to wit and the bones; the bones are like the wooden levers in the automaton, and the iron; the tendons are like the strings, for when these are tightened or leased movement begins. However, in the automata and the toy wagon there is no change of quality, though if the inner wheels became smaller and greater by turns there would be the same circular movement set up. In an animal the same part has the power of becoming now larger and now smaller, and changing its form, as the parts increase by warmth and again contract by cold and change their quality…" (Ross, 1972, vol. 5)

In any case that an unknown system has to be studied and explained by means of an example or model, this model has to be simpler, more familiar, and more comprehensible than the unknown system. Although in the above-mentioned quotation of *Metaphysics* (983a 15) Aristotle considers the cause of movement of the automaton unintelligible, he uses exactly this example of automatic machines, in order to explain the movements of animals, the movements of living beings, the life itself. It means that in comparison to the miracle of life, the automatic machines are more familiar and more understandable. They are created either as imitations of the living beings or as models that explain the life itself.

Another significant reference of Aristotle is in his *Politics*, where he foresees the application of automatic machines either in the manufacture or in the daily human life. He considers that such a technological revolution would be also a sign of a social revolution, where there is no need of assistants or slaves, because of the existence of automatic tools that operate on their own.

Politics, 1253b 20: "… if every tool could perform its own work when ordered (*κελευσθέν* / by an external command), or by seeing what to do in advance (*προαισθανόμενον* / by an internal programming, having a predetermined internal function, a presentiment), like the statues of Daedalus in the story, or the tripods of Hephaestus which the poet says 'enter self-moved the company divine,' if thus shuttles wove and quills played harps of themselves (automatically), master-craftsmen would have no need of assistants and masters no need of slaves[55]."

Aristotle divides tools or machines or automata into these that operate under an external command (*κέλευσμα*), and those that have at their disposal an internal programming (*προαίσθηση*). Probably by the second category, Aristotle means automatic machines that are also able to control their operation by themselves.

[55] This text is based on the following book: Aristotle. Aristotle in 23 Volumes, Vol. 21, translated by H. Rackham, Cambridge, MA, Harvard University Press; London, William Heinemann Ltd., 1944

As it is shown in chapter 2, in contemporary terms, we make a distinction between the open and the closed loop systems, i.e., systems that embody or not feedback, return of part of the output to the input.

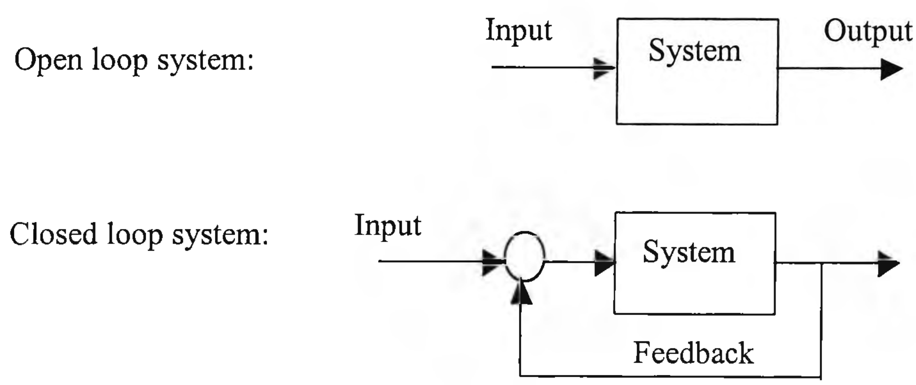

Figure 11.3: Open and Closed loop systems

The closed loop systems, the systems with feedback, constitute the principles of automatic control. They make their appearance in the field of technology during the Hellenistic age, when the engineers and the automata makers introduce the concept of closed loop system of automatic control in their treatises on the art of making automata. The exploration of the art of making automata, as well as examples of the first closed loop systems will be postponed to the next chapter. However the concept of cycle, of closed loop, of feedback, of contradiction has its origins in the ancient Greek dialectic thought of the presocratic and classic philosophers.

11.3 The Theoretical Background of the Feedback Control Systems

The self-motion is only the one side of the technological intention to study and finally to construct automatic machines. The other side is the self-control of the mechanical devices, which is closely related to the feedback principle.

(Porter, A., 1969) points out that a common characteristic of all machines or systems, that embody feedback, is that without feedback, the behaviour of the system would be more uncertain in the sense of being less predictable. The way feedback tends to minimise uncertainty is the following: Man has an innate urge to minimise the uncertainty of his environment. He wants to know how? Why? He usually finds out answers by carrying out experiments and he uses the results to make decisions. The decisions lead to actions and the effects of actions give rise to feedback paths, which in turn provide information concerning the success of the action. And so this process of the inverse relation between effect and cause, output and input guaranties the stability, harmony, equilibrium, and success.

Therefore, the attempts of the first philosophers to find the answers to all their how- or why-questions, or in other words to minimise the uncertainty of their environment, are closely related to the feedback principle. Their task of organising and structuring knowledge depends on the assembling of appropriate information and data. In addition, this process of taking advantage of the results of experiments so that to make the right decision, which in turn leads to actions, and the effects of action lead to feedback that provides information, is a circular process. If we would like to express it schematically it will have the form of the circle, a form that characterizes the closed loop systems, i.e., the systems with feedback. From this perspective, we can find similarities with the dialectic thought that characterizes the period from the Presocratic philosophers to the Classical years. In the words of Heraclitus or in the obstetric method of Socrates, we will see the mutual relationship between the cause and the effect, a relationship that might be depicted schematically as a circle.

11.3.1 The "Cycle" in the Dialectic Thought

In the evolution of ancient Greek thought, there are many instances where the pure straight logic is turned over, e.g., the known paradox of Epimenides, where by combining two reasonable statements, he results in the rejection of logic or in the circular self-refutation. More precisely: statement 1: "I am a Cretan", statement 2: "All the Cretans are liars". Therefore, if he is a Cretan, he lies, which means that he is not a Cretan and vice-versa. Epimenides forms with his statements a contradictory closed loop, which refutes continually itself.

Heraclitus afterwards, as the father of Dialectic, introduces the concept of contradiction by formulating propositions, such as those that have been introduced in chapter 4. The thought that change is cyclical is central in these propositions. For example, he is referring to the continuous cyclical process by speaking of the path up and down as being the one (*Fr. 60, Hippolytus Ref. IX, 10, 4*), or he stresses the continuous transformation of world processes, wherein fire is 'extinguished' to form the sea and the sea the earth, which in their turn are 'exchanged' back into fire. Whatever lives, lives by the destruction of something else:

Fr. 76: "Fire lives the death of air, and air of fire; water lives the death of earth, earth that of water." (Guthrie, 1997)

He creates cosmological cycles, where change involves opposites and oppositions operate as part of a unity so as to produce harmony. An indicative example of the unity and harmony of contrary concepts is given in the next passage, where life and death, waking and sleeping, young and old, are the one and the same. There is a constant transformation of the one into the other:

Fr. 88, Ps.-Plutarch Cons. ad Apoll. 10, 106 E: "And what is in us is the same thing: living and dead, awake and sleeping, as well as young and old; for the latter having changed becomes the former, and this again having changed becomes the latter." (Wilbur et *al.*, 1979)

The notion of cycle is also apparent in the Hippocratic process of creating diagnostic models, as it is shown in chapter 5. According to it, the clinical records of previous cases are necessary for the creation of a new diagnosis. And each diagnosis they end up is used as prognosis to a new case, i.e., as the essential knowledge in order to recognise, and eventually foretell different stages that occur in every disease.

Aristotle also formulates an interesting viewpoint about a finite universe with the properties of a closed loop system.

(Düring, 1966) in his remarkable work on Aristotle gives the following analysis of Aristotle's view of universe. Aristotle in his efforts to answer the question if the universe is limited or unlimited, finite or infinite, analyses the concept of Infinity. According to him, if something is characterized as infinite, it must be real, i.e., it must be simultaneously limited and finite, so as to be recognisable. Something infinite must exist within something finite. Concerning a real thing, what is infinite, is its possibility to be divided to infinity. For example, in the case of real numbers, the possibility of extending the sequence of numbers to infinity is infinite. Regarding the initial question about the universe, Aristotle ends up to the theory that the universe is the total summary of our knowledge about it, i.e., it is the whole or the total of the beings. By such a theory, the universe appears as a closed loop system, where all the particular things are connected into a whole. Only under this consideration, the concepts of infinite, motion, or time cease to be meaningless. Only into a closed loop system, the word infinite can characterize some of the processes that occur within it. Therefore, the universe is a closed loop system, complete, perfect, determinate, and finite, whereas the processes taking place in it are infinite.

However, Socrates is the one who applies masterly the dialectic method in his dialogues and develops his so-called obstetric (*μαιευτική*) or *maieutic* method.

11.3.2 The Socratic Maieutic Method

If those who have forwarded the advance of mankind are concerned to be of two types, of those, who with their thoughts or discoveries have made the world different, such as Galileo or Newton, and of those whose potent influence is individual and personal, and must be met in their works or writings, Socrates belongs to the second type.

Socrates (469-399 B.C.) is the son of a working sculptor and a midwife. Socrates, the teacher of Plato, lives on as a character in the dialogues of Plato. His early studies have been in natural science – physics, astronomy, and geography. According to Xenophon, a disciple of Socrates:

> " [...] He (Socrates) did not even discuss that topic so favoured by other talkers, 'the Nature of the Universe': and avoided speculation on the so-called 'Cosmos' of the Professors, how it works, and on the laws that govern the phenomena of the heavens[56] ..." (Sarton, 1, 1993)

But even if he had expressed an interest in such questions as the origin of the world, this would have happened during his early years. Afterwards, he turns to the elements of human life. He writes nothing himself, because for him the true approach to knowledge is not through books or lectures, but through conversation, discussion, question and answer. He makes a great contribution to the technique of thought, which

[56] Xenophon, Memorabilia, I, I, 10. Translation by E. C. Marchant, Loeb Classical Library, 1923

Aristotle describes as the discovery of inductive reasoning[57] and of general or universal definitions:

Metaphysics, M 3, 1078b 27: " [...] two things may be fairly ascribed to Socrates: inductive reasoning and universal definition. Both of these are associated with the starting-point of scientific knowledge[58]." (McKeon, 1941)

By this, Aristotle means that Socrates is the first man who systematically tries to get behind particular examples of e.g., justice or goodness to a general definition of what justice or goodness is. He does it inductively, by taking instances of just or good actions and trying to ascertain the elements common to all instances in each case. He begins by searching for a general principle, it takes it as a hypothesis, then examines whether it fits the facts or not, and retains or rejects it accordingly.

(Heidel, 1941) reports that Myson, one of the Seven Sages, used to say that one should not investigate facts by the light of arguments but arguments by the light of facts. According to Diogenes Laërtius, Socrates was following exactly this method:

Diogenes Laërtius I cviii Cf., Plutarch, Moralia, 75f., Diogenes Laërtius, II, xxix: "He (Socrates) had the skill to draw his arguments from facts." (Heidel, 1941)

Socrates has devoted his whole life to the task of exciting his leading idea as extensively and as vividly as possibly in others. (Otto Apelt, 1912) gives a good description of the Socratic method. Socrates' objective depends on the interlocutors he has in front of him. In the case of young people, he intends to lead them to the search of the right judgement, whereas in the case of the sophists and orators he aims at making them to change their mind. In both cases, he has an explicit purpose, a desired, determined 'output'. In the works of Plato, Socrates appears to apply the method of dialogue in order to succeed in his purpose. Socratic method is summarised into the terms: split, division, dichotomy, on the one hand, and duplication on the other.

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[57] The scientific investigation, which leads from the particular to the general and is based on observations and experience, is a method that in modern terms is called inductive reasoning. By following the inductive reasoning method, we discover a property of a certain class by repeating the observation or tests as many time as feasible. Then it may happen that a definite tendency will manifest itself throughout our observation and experimentation. This tendency is the accepted as a property of the class.

[58] Metaphysica, Translated by W. D. Ross

Socrates splits himself into two. The one part of him knows in advance how the discussion is going to end, i.e., he knows in advance the result, the 'output', whereas the other part travels the entire dialectic path along with his interlocutor. On the other hand, his interlocutors have no idea where he is leading them. During the dialogue with him, they have the chance to realise that he constantly demands total agreement from them. At the beginning, Socrates agrees with his partner's position, and gradually makes him admit all the consequences of his position, so as to lead him to recognise that his initial position is contradictory. Through this process, the interlocutor is cut in two as well: there is the interlocutor, as he was before the conversation with Socrates, and on the other hand, the interlocutor, who has identified himself with Socrates, in the course of their constant mutual accord.

According to (Kierkegaard, 1962), Socratic method is characterised by the master-disciple relationship, i.e., "to be a teacher does not mean simply to affirm that such a thing is so, or to deliver a lecture, and so on. To be a teacher in the right sense is to be a learner. Instruction begins when you, the teacher, learn from the learner, put yourself in his place so that you may understand what he understands and in the way he understands it". Socrates, being exactly this type of teacher, pretends that he wants to learn something from his interlocutor, pretends that he wants to be a learner. In fact, however, even though Socrates appears to identify himself with the interlocutor, at the end of the discussion, the interlocutor identifies himself with Socrates, or in other words he has been Socrates himself.

Socrates, as it is obvious in the Platonic works, applies his method in any dialogue upon any subject. We have already cited a passage of the Platonic work *Meno* (82a-85d) as a reference to the phenomenon of incommensurability. This quotation constitutes also an indicative example of the method of Socrates. In this discussion, Socrates tries to prove that we do not learn new things, but any knowledge pre-exists in our mind as a memory, even if we do not know it. The only thing we have to do is to recollect it step by step. In this example, Socrates makes the appropriate questions to a young boy who has no knowledge on geometry, or at least he thinks so. The only thing that the boy knows is the Greek language. As the dialogue proceeds, it is shown that the boy is able to answer the questions, and as a result Socrates proves his initial position.

At the beginning of the discussion, Socrates depicts on the ground a square whose sides are two feet long and wants the boy to find the area of it. If he had asked "what is the area of this square?" the ignorant of geometry boy would not have been able to answer. Therefore, he asks: "if in one direction the space was of two feet, and in other direction of one foot, the whole would be of two feet taken once?" In such a question the boy knows the answer and in the next question of "since this side is also of two feet, there are twice two feet" the boy also knows the answer and finally he is able to estimate that twice two feet is four. By such a procedure, i.e., by asking the proper questions Socrates manages to prove his initial suggestion that we always possess the knowledge and the acquisition of it has to do with the recollection and not with the teaching or the learning of it.

Summarising, in Socratic method that so masterly is described by Plato, Socrates himself 'controls' his interlocutor and 'drives' him at the desired result by determining each time his next question by means of the foreseen answer. From the control theory perspective, we could relate Socrates' method to a closed loop control system. This comparison is based on the following reasons:

- In a control system there is a desired output, a specific target that the system has to achieve. Similarly, in the Socratic method the result of the dialogue is pre-determined in Socrates' mind from the beginning.
- In the synthesis of a control system, the main question that has to be answered is "what is the appropriate input, or what kind of adjustments need to be done so as to result in the desired output?" Socrates has in his mind such questions in order to lead his interlocutor where he wants.
- Any closed loop control system has a controller. In the Socratic method, Socrates plays the role of the controller that at any time compares the desired output with the input and makes the appropriate corrections.
- Even the enigmatic declaration of Socrates: "I only know one thing: that is that I do not know anything", i.e., it is true only this that encloses the rejection of it itself, hides the notion of cycle, which is the main characteristic of the closed loop control systems.

Schematically, we have:

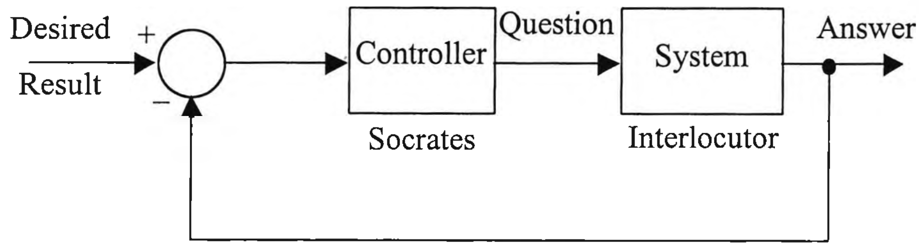

Figure 11.4: The dialectic Socratic method

11.3.3 The Term of 'Cybernetics' in Ancient Greek Sources

The idea of control is at the root of cybernetics, or in other words, the word of 'cybernetics' is the word for the study of controlling mechanisms. Any human action is controlled either because we choose to do something and make the appropriate moves in order to do it, or because, for our safety, we make some automatic or reflex action, like pulling away from something hot. In the word of (Nobert Wiener, 1948) the founder of Cybernetics, "we have decided to call the entire field of control and communication theory, whether in the machine or in the animal, by the name 'cybernetics', which we form from the Greek *κυβερνήτης* or steersman." Even though it is generally assumed that cybernetics is a recently invented subject, its roots were established many centuries ago.

Anaximander's conception of the world may be the earliest reference related to the concept of cybernetics – *κυβερνητική*. His world is a purposeful one. Therefore, he chooses Apeiron as the fundamental element of the world, because this 'boundless element' is the beginning of all things but has no beginning itself; it is both divine and immortal, and surrounds and steers all things:

> Aristotle, *Physics,* 203b7-9, 11-14 "But there cannot be a source of the infinite or limitless, for that would be a limit for it. Further, as it is a beginning, it is both uncreatable and indestructible... but it is this, which is held to be the principle of other things, and to encompass all and to steer all – *κυβερνών* – as those assert who do not recognise, alongside the infinite, other causes, such as Mind or Friendship. Further they identify it with the Divine, for it is 'deathless and imperishable' as Anaximander says, with the majority of the physicists." (Wilbur et *al.*, 1979)

We could assert that in Anaximander's purposeful world there is the need of a 'controlling' mechanism that creates, steers, and controls everything, which is the Infinite or Apeiron.

There is a similarly reference that has already been cited in chapter 9[59], in the passages of Aetius and Simplicius, where they describe the mechanical model of Parmenides. This model is composed of mixed rings and the "middlemost of them is the primary cause of movement and of coming into being for them all, and he (Parmenides) calls it the goddess that steers (*κυβερνήτης*) all..."

This primary ring is the 'cybernetist', i.e., it is the part that controls everything, the controller of the system in modern terms.

Plato is the next to point out the concept of cybernetics. He introduces the term 'cybernetics' as the art of controlling a ship, a chariot, an army, or a whole city. In his words:

Gorgias, 511D: "the art of cybernetics (piloting – *κυβερνητική*), saves not only our lives but also our bodies and our goods from extreme perils[60]..." and

Republic, 488D: " [...] the true cybernetist (pilot, shipmaster – *κυβερνήτης*) must give his attention to the time of the year, the seasons, the sky, the winds, the stars, and all that pertains to his art if he is to be a true ruler of a ship, and that he does not believe that there is any art or science of seizing the helm[61]..."

Plato, in his work of Phaedrus, describes mind as the pilot of the soul:

Phaedrus, 247c: "there abides the very being with which true knowledge is concerned; the colourless, formless, intangible essence, visible only to mind, the cybernetist (*κυβερνήτης*) of the soul..." (Jowett, 1964, vol. 3)

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[59] Section 9.4 (The rings of Parmenides), passages: Aetius II, 7, 1 (DK 28 A 37) & Fr. 12, Simplicius in *Physics* 39, 14 and 31, 13

[60] This text is based on the following books: Plato. Plato in Twelve Volumes, Vol. 2 & 3 translated by W.R.M. Lamb, Cambridge, MA, Harvard University Press; London, William Heinemann Ltd., 1962 & 1967, respectively.

[61] This text is based on the following books: Plato. Plato in Twelve Volumes, Vol. 5 & 6 translated by Paul Shorey, Cambridge, MA, Harvard University Press; London, William Heinemann Ltd. 1969.

Aristotle is also occupied with the example of the cybernetist, the shipmaster. He distinguishes the particular elements of control in live and lifeless tools:

Politics, 1253b: " [...] for the particular arts it would be necessary for the proper tools to be forthcoming if their work is to be accomplished, so also the manager of a household must have his tools, and of tools some are lifeless and others living (for example, for a helmsman the rudder is a lifeless tool and the lookout man a live tool - for an assistant in the arts belongs to the class of tools), so also an article of property is a tool for the purpose of life, and property generally is a collection of tools, and a slave is a live article of property[62]".

Thus, the control of the course of a ship is a closed loop system that consists not only of live but also of lifeless tools. The cybernetist, the helmsman, is the controller, who manipulates the lifeless tool of rudder, whereas the boatswain – the lookout man – acts as the feedback of the system that localizes the route of the ship and compares it with the desired route. It is shown in the following figure:

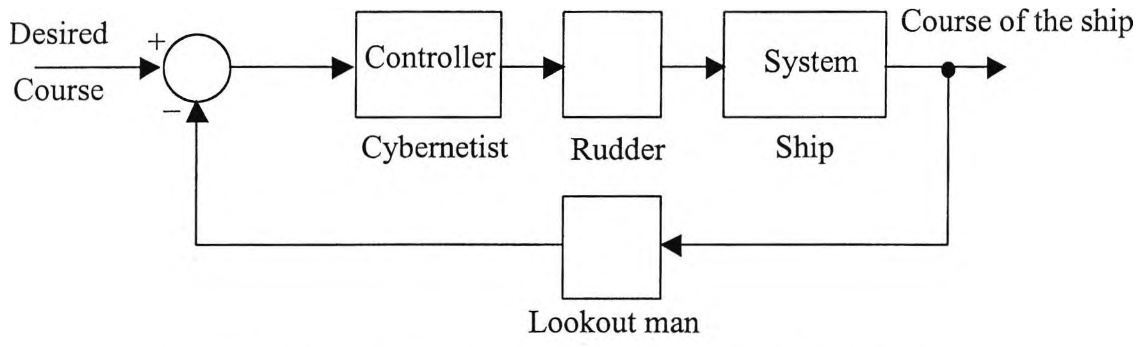


Figure 11.5: The Cybernetics or the art of controlling the ship course

According to (Porter, A., 1969) "fundamental in all feedback control systems is a capability to recognising change. Recognition, in the sense of detection of change is accompanied by a capability of measuring the magnitude of the change, and a capability of feeding back this information to a control centre where steps can be initiated to take the appropriate correcting action. In order to control a given process or system, we must consider a basic question: What quantities must be measured?"

[62] This text is based on the following book(s): Aristotle. Aristotle in 23 Volumes, Vol. 21, translated by H. Rackham, Cambridge, MA, Harvard University Press; London, William Heinemann Ltd. 1944.

The founder of actual measurements in astronomy, based upon comprehensive observation and the collection of experimental data, is Hipparchus of Nicaea (190-125 B.C.). His approach to science is essentially modern in that he collects data from accurate observations and then forms his theories to fit the observed facts. Hipparchus follows the movements of sun, moon, and planets by angular measurement, and deduced from his results improved values for the length of the year, the inclination of the ecliptic, and the eccentricity of the moon's orbit. He recognises that the distances of the sun and the moon from the earth are variable. He also originates the idea of specifying a point on the earth's surface by longitude and latitude. Hipparchus is the founder of empirical science and the inventor of trigonometry. He emphasises on precise measurements and by using the crudest of instruments, he classifies over one thousand stars according to brightness. Moreover, by means of his measurements he creates celestial charts that provide the early navigators with the means for laying and keeping a course. (Porter, A., 1969) regards Hipparchus as perhaps the first cybernetist.

11.4 Conclusion

The realisation and construction of automatic machines would not be possible without, in the first place, the visualisation of them expressed in myth by the desire "how good it would be to have automatic machines!" and secondly without the conceptual and theoretical framework created later on by the Presocratics and the Classical philosophers. The introduction and the study of the four fundamental elements by the Presocratics ensure the necessary internal energy, which in turn ensures the ability of self-motion that characterises the automatic machines. On the other hand, the development of the concepts of contradiction, dialectic, and feedback lays the foundation for the second prerequisite of automatic machines, i.e., the ability of controlling and regulating their operation. In particular, the introduction of the art of controlling a ship, a city, or an army lays the foundation for the science of Cybernetics. As for example, the desired route of a ship is achieved by means of lifeless tools, such as the rudder that regulates the direction of the ship, and of live tools, such as the lookout man, who measures and ascertains the ship position, and produces the feedback action.

In addition, the technique of precise measurements, the theoretical approach to the notion of automaton, the distinction between external commanded and internal programmed systems, the description of a fully automatised society, the Socratic method, and even the Democratic constitution of ancient Athens, where the governed citizens decide by themselves about the laws that will govern them, are some examples that pave the way to the following advanced technological achievements and the establishment of the control theory.

Chapter 12

THE CONSTRUCTION OF AUTOMATA: THE FIRST CLOSED LOOP CONTROL SYSTEMS

12. THE CONSTRUCTION OF AUTOMATA: THE FIRST CLOSED LOOP CONTROL SYSTEMS

12.1 Introduction

As we have already seen, the human vision of constructing tools or machines that work by themselves is expressed in the Greek literature and philosophy from the time of Homer onward. However, only in the Hellenistic period this impulse takes a more practical form, when the experts in machinery construct devices able to work and to execute complex movements on their own. First of all, the Syracusian Archimedes, mathematician and engineer, who studied in Alexandria, is the one who opens the way to the applied mechanics and constructs a plethora of machines, such as the endless screw, the planetarium, the hydraulic clock, complicated engines of war, and so on. Thus, he opens the way to the Alexandrian school of engineering, in which three famous mechanics flourished: 1) Ktesibios, who is considered as the inventor of the first feedback device, that is the float valve in his water clock, 2) Philon of Byzantium, who wrote a handbook, the so-called *Mechanical Syntaxis*, which determined the program of study for the new engineers, and 3) Heron of Alexandria, whose work is a landmark in the history of technology, not only because did he collect the up to those times known accomplishments, but also because his inventions paved the way to a new technological era.

These engineers undertake the task to realise the vision of automobile machines that are independent from human interference. In order to accomplish this task, they have to take advantage of the available energy sources, to study the properties of liquids and gases, to develop the science of Pneumatics, to invent programming methods for the movements, and to establish the at that time innovative branch of science: the *Automatopoietice*, i.e., the art of making automata. In addition, they have to collect and classify the previous experience of the construction of automata, to apply the theoretical knowledge of the preceding philosophers, and mainly to answer the question: How independent and automobile machines could be constructed? The keyword for answering this question was the principle of feedback. In this chapter, we will mainly focus our interest on the primary technical inventions concerning this art during the Hellenistic period.

12.2 The Contemporary Notions of Feedback and Control Systems

12.2.1 The Concept of Feedback Control

Feedback control is the basic mechanism by which systems, whether mechanical, electrical, or biological, maintain their equilibrium or homeostasis. In the higher life forms, the conditions under which life can continue are quite narrow. A change in body temperature by a degree is generally a sign of illness. The homeostasis of the body is maintained through mechanisms equipped with feedback control (Wiener, 1948).

(Lewis, 1992) defines feedback control as the use of difference signals, determined by comparing the actual values of system variables to their desired values, as a means of controlling a system.

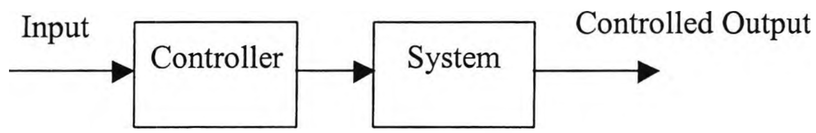
The phases of the development of mankind that affected the progress of feedback are: a) the Hellenistic period with the engineers of Alexandria, b) the Industrial Revolution in Europe, which was marked by the invention of advanced grain mills, furnaces, boilers, and the steam engine, c) the development of the telephone and mass communications, and the First and Second World Wars, during which the development of feedback control systems became a matter of survival, and d) the beginning of the space and computer age (Lewis, 1992). We will focus our interest on the first phase, where the need for the accurate determination of time and the attempt to construct automatic machines constitute the primary motivation for feedback control.

12.2.3 Open and Closed Loop Control Systems

In addition to chapter 2, we will cite here a quick reference to open and closed loop control systems and the relevant block diagrams. We have already seen that we call ***open loop system*** the system that communicates with its environment, i.e., it receives by its environment a stimulant, a question, a cause, the so-called input, and in response it gives a result, an answer, the so-called output. On the other hand, we call ***closed loop system*** the system that communicates with the environment not only by the direct way from the input to the output, but also by means of a reverse way of communication, from the output to the input, from the result to the cause, the so-called feedback.

In addition, we call ***open loop control system*** the system that is able to react upon its output, to control it by means of a controller. The process of such a control is called automation. Similarly, we call ***closed loop control system*** the system that not only does it control its output by using a controller, but also uses the controlled output so as to regulate its input by means of the feedback principle. Such a process is called automatic or dynamic control of the system. The block diagram of open and closed loop control systems are shown in the following figure:

Open loop control system:



Closed loop control system:

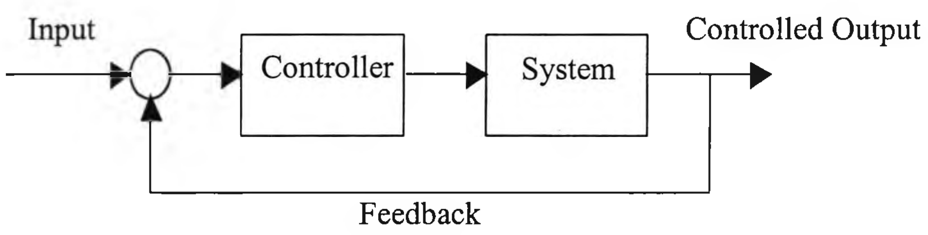


Figure 12.1: Open and closed loop control systems

The first closed loop control system in the history of modern technology is Watt's automatic steam engine, 1796. It can maintain the speed of rotation of the engine at a constant desired value, by means of a feedback system, the so-called centrifugal governor, even though there are changes in load or steam pressure. It accomplishes this by sensing the actual speed and adjusting the steam inlet valve of the engine accordingly (Mayr, 1970). The input of the system is the steam pressure, the controlled output is the speed of rotation or the angular velocity of the engine, and the controller is the inlet valve. A pair of centrifugal pendulums that measure the speed of rotation composes the feedback mechanism. The feedback signal is compared with the reference input and the result is expressed in the position of the valve.

However, the stage for the machine era has been set centuries earlier with the ingenious mechanic, hydraulic or pneumatic machines described in the writings of Ktesibios, Philon, and Heron. The clepsydra for example, followed by the hydraulic clock, may be said to have been the first automatic realisation of which practical application was made.

12.3 The Alexandrian Engineers

The historical evolution of ancient Greek automata culminates in the period of the Alexandrian engineers. A number of mechanical inventions were produced in Alexandria from the 2nd century B.C. onwards, especially after the ingenious Ktesibios had shown the many uses of water, the natural pressure of the air, and the pneumatic principles.

We will first cite some general and mostly biographical information of the three important engineers of Alexandrian era and afterwards their achievements in the area of automation.

12.3.1 Ktesibios

Ktesibios is now placed within the interval 300-230 B.C. (Usher, 1988). He was the son of a barber and has constructed in his fathers' shop an automatic mirror that could be raised or lowered to any height required, as well as produce sounds of music. It was supplied with a lump of lead that was acting as a counterweight. This lead counterweight was concealed in a pipe, and as it was moving rapidly through the pipe, it was producing a squeaking noise. It occurred to Ktesibios that a musical instrument could be built on this basis. Thus, the water organ, in which air was forced through different organ pipes, not by means of a falling weight but by the weight of water, is ascribed to Ktesibios.

> Vitruvius, *On Architecture*, IX 7-8, translation of M.H. Morgan, (Cambridge, Mass., 1914): " […] wishing to hang a mirror in his father's shop in such a way that, on being lowered and raised again, its weight should be raised by means of a concealed cord, he employed the following mechanical contrivance. Under the roof-beam he fixed a wooden channel in which he arranged a block of pulleys. He carried the cord along the channel to the corner, where he set up some small piping. Into this a leaden ball, attached to the cord, was made to descend. As the weight fell into the narrow limits of the pipe, it naturally compressed the enclosed air, and as its fall was rapid, it forced the mass of compressed air through the outlet into the open air, thus producing a distinct sound by the concussion. Hence, Ktesibios, observing that sounds and tones were produced by the contact between the free air and that which was forced from the pipe, made use of this principle in the construction of the first water organ…" (Cohen *et al.*, 1966)

He is also credited with having invented the force pump and the air-powered catapult. However, the most prominent invention of Ktesibios is the water clock, which according to Diels'[63] interpretation, is the earliest feedback device on record. We will see an analytical description of it in the following section.

12.3.2 Philon of Byzantium

It is now generally accepted that Philon of Byzantium followed Ktesibios in the interval of a single generation[64], i.e., he lived in the second half of the 3rd century B.C. It is not known where he spent his life. In his work of *Belopoieka*[65] he hints of having been in touch with persons who had known Ktesibios directly; in other cases researchers, such as (Usher, 1988) consider him as a pupil of Ktesibios. He is credited with being the author of the most important technical handbook of Hellenistic antiquity, the so-called *Μηχανική Σύνταξις* (Mechanical Syntaxis), i.e., a compendium of mechanical sciences. This manual contained nine books, from which the *Pneumatics* is the only that has survived via the Arabic, and indeed in two versions[66]. In his Pneumatics, Philon includes some of the most important applications of pneumatic systems, such as siphons, intriguing closed loop control systems for regulating liquid levels, pneumatic mechanisms of singing birds, as well as various types of pumps. An analytical description of the titles of Philon's work, which reveal not only the new Hellenistic conception of modern science, but also the content of the technical studies in Alexandria will be given in section 12.6 that concerns the early control curriculum in the University of Alexandria.

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[63] Diels, Hermann, *Antike Technik*, 3rd edition, Leipzig, 1924.

[64] Literature on Philon: A. G. Drachmann, *Ktesibios, Philon, and Heron,* Copenhagen, 1948. K. Orinski, O. Neugebauer, A.G. Drachmann, *Philon von Byzanz*, Pauly's Realencyclopädie 20.1, 1941

[65] Philon, *Belopoieka*, Greek and German, trans. H. Diels and E. Schramm, *Abhandlungen der preussischen der Wissenschaften*, 1918:Phil-hist. Klasse Nr. 16, 51.17, Berlin, 1919.

[66] 1) Philon, *Pneumatica*, Arabic version (Le livre des appareils pneumatiques et des machines hydrauliques), Arabic and French, trans. Carra De Vaux, Paris, 1902. 2) Philon, *Pneumatica*, Latin version (Liber Philonis de ingeniis spiritualibus), Latin and German, trans. W. Schmidt, Leipzig, 1899.

12.3.3 Heron of Alexandria

Heron of Alexandria remains a questionable figure. It is not clear if he lived during the 1st century B.C. or in the 1st century A.D. He is one of the few ancient writers on technical issues whose work was saved almost in its entirety. His works, even thought they were inspired by a no utilitarian delight in machines, they constitute a historical force that has affected not only technology but also philosophy, science, literature, or in short, our culture at large. The structure of his work follows the structure of the *Mechanical Syntaxis* of Philon. However, it is more integrated with new theoretical studies and technical innovation on the following subjects:

1. *Metrica I, II,* and *III,* which gives methods of measurement. More precisely, in Book I, Heron deals with areas of triangles, quadrilaterals, regular polygons of between 3 and 12 sides, surfaces of cones, cylinders, prisms, pyramids, spheres, and so on. In Book II, Heron considers the measurement of volumes of various three dimensional figures, such as spheres, cylinders, cones, prisms, pyramids, and so on, and in Book III, he deals with dividing areas and volumes according to a given ratio.
2. *Geometria* seems to be a different version of the first chapter of the *Metrica* based entirely on examples.
3. *Stereometrica* measures three-dimensional objects and it is at least in part a version of the second chapter of the *Metrica* based again on examples.
4. *On the Dioptra* Heron deals with theodolites and surveying. It contains a chapter on astronomy giving a method to find the distance between Alexandria and Rome. This method is based on the differences between local times, at which an eclipse of the moon is observed at each city.
5. *Catoptrica* deals with mirrors, reflectors, and elements of optics.
6. *Mechanica* in three books written for architects. Book I, examines how to construct three-dimensional shapes in a given proportion to a given shape. It also examines the theory of motion, certain problems of Statics, and the theory of the balance. In Book II, Heron discusses lifting heavy objects with a lever, a pulley, a wedge, or a screw. There is a discussion on centres of gravity of plane figures. Book III, examines methods of transporting objects by such means as sledges, the use of cranes, and looks at wine presses.

7. *Belopoieca* describes how to construct engines of war and including the oldest manuscript drawings. It has some similarities with the corresponding work of Philon.
8. *The Pneumatica* in two books *A* and *B*, studies mechanical devices worked by air, steam or water pressure. These books, with 43 chapters the first and 37 the second, contain descriptions of over 100 mechanisms. Further down, we will examine some of them in more detail.
9. *Automatopoietice* describes the art of constructing automatic theatres worked by strings, drums, and weights with.

Some of his remarkable contrivances included in the above-mentioned works are cogs, screws, screw-threads, cylinders, pistons, weights, compressed air and steam mechanisms with precisely worked tubes, and flap-valves. In addition, Heron describes jugs from where wine or water could be poured at will, spheres from which hot or cold water flowed, fountains, mechanical birds that moved and whistled, trumpets that sounded on the opening of a door, and so on. More precisely, in his *Pneumatics*, which is the oldest Greek writing that has been saved in its original form, Heron collects systematically the previous as well as the contemporary machines that have been set into motion by compressed air, steam or water. These machines were the earliest mechanisms to reproduce the sounds of living things and destined for the decoration of public areas and the aesthetic pleasure of their viewers. Along with many other astonishing things (80 in total), Heron includes in his Pneumatics (a complete list of Heron's automata is given in appendix 2):

- The automatic libations at an altar produced by fire (A 12).
- The hot-air apparatus that opened and shut temple doors automatically at the right moment, on the lighting and extinguishing of the altar fire (A 38).
- A public automatic fountain, where birds are singing and an owl is rotating by means of hydraulic siphons, mechanical systems of motion transmission, and pneumatic methods for the production of sounds (A 16).
- The hydraulic or water organ, the so-called *Hydraulis*, similar to the water organ of Ktesibios, which had an automatic manual or wind-driven piston mechanism (A 42, 43).

- The automatic fire-engine that worked like the modern two-stroke internal combustion engines (A 28).
- A primitive turbine driven by a steam jet, the so-called sphere of Aeolus or Aeolopile (B 11).
- An automatic coin-operated Holy Water dispenser (A 21).

On the other hand, in the work of *Automatopoietice*, Heron describes automatic mechanical systems capable of performing programmed movements. The main topic of it is the form and the art of construction of automatic theatres, which are divided into mobile and stationary automata. *Automatopoietice* has survived completely in at least 39 manuscripts, a fact that indicates the great interest Heron's students showed in his work. It was saved during the Roman and Byzantine years and valued by the Arab and European engineers of the Middle Ages. It has been translated into Arabic, Italian, French, and German and has become the basis for the reconstruction of the ancient theatres.

In these theatres, which were works of art and also high form of technology, little figures made music, danced and acted pieces, thunder rolled and lighting flashed, the sea raged, and ships were shattered on the rocks. More precisely, the mobile automata had the form of temples with figures on them, such as Dionysus or Nike that could turn, and Bacchantes that could dance to the sounds of drums and cymbals. They also had altars, where fires glared, flowers crowned the temple, and wine or milk flowed at regular intervals. All these happened automatically in a logical sequence of movements, which corresponded to the relevant myth, and at the end of the performance everything returned to its initial position (*Autom.*, 1.2).

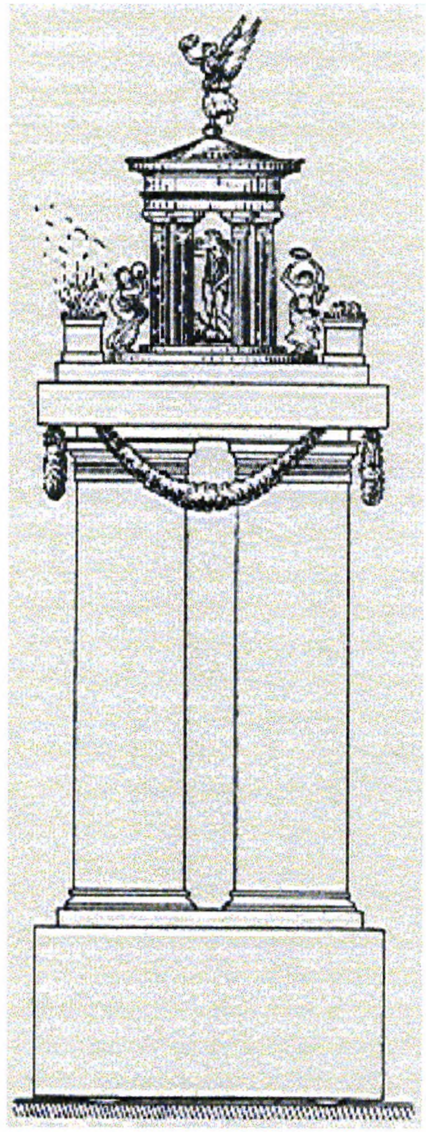

Figure 12.2: The mobile theatre of Heron (Usher, 1988)

On the other hand, the stationary automata may be said to have the form of the modern television. On a small pillar, a theatre stage is placed, the doors of which are capable of opening and closing by their own, and each time new figures are presented until the performance is over. And these figures may appear to be moving if the myth requires so, or appearing and disappearing from the stage, ships is possible to be moving in fleet order, dolphins to be swimming, fires may be lit, and sounds of thunder may be heard (*Autom.* 1.3). It is worth noticing the fact that the linking of automata to the myth is also shown by the theme the stationary theatres present. The themes are usually taken by the ancient Greek mythology, such as the history of Nauplius and Aias who return from the Trojan War, the Dionysian ceremonies and sacrifices on the altar of Dionysus, the figure of Hercules shooting arrows, and so on.

Heron, after describing the form, the geometry, and the movements of his automatic theatres, explains analytically the construction, as well as the operation issues. He has to face two functional problems: a) the motive mechanism and b) the programming of the movements. Let us examine the solution of these problem in particular examples.

12.4 Examples of Open Loop Control Systems

12.4.1 The Motive Mechanism in the Mobile Automaton of Heron

Heron in his *Automatopoietice* describes the motive mechanism of his mobile automaton. He takes advantage of the dynamic energy of a falling weight, and thus provokes all the complicated movements of automaton by the vertical fall of a leaden motive weight. A rope connects the weight with the rotating axle, which in turn is connected with the wheels or the other motive parts of the automaton.

More precisely, there is a box containing two compartments, of which the upper is filled with sand or something of the same sort. On the top of sand, the leaden weight connected to the wheel axle by a rope running over pulleys is placed. Between the two compartments there is a dividing wall with a small aperture (*Autom.* 9.5). When the aperture is opened, the sand is pouring gradually into the lower compartment, the weight sinks and pulls the rope, so that the wheel axle is turned and the theatre starts moving.

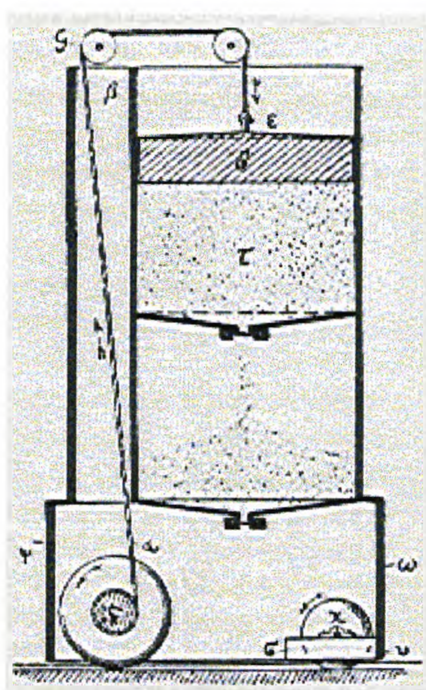

Figure 12.3: The motive mechanism of mobile automaton (Usher, 1988)

If the theatre is to move forward and then return the way it came, the rope is wound first in one and then in the other direction. If the theatre is to remain standing on the stage, the rope has to be loose. The length of the coil decides how long the automaton will stay unmoved. This is the motive mechanism of the mobile automaton of Heron, that is an open loop control system.

By using the dynamic energy of the leaden weight, which plays the role of the required internal motive power, the automaton and its motive parts move. The programming of the movements, as well as their control is achieved by using three different kinds of thread-windings around the wheel axle (*Autom.* 6.3). The first one is a clockwise winding, the other a counter-clockwise, and the third one is a loose winding that cause forward movement, backward movement, and immobility, respectively. In this way, Heron creates a trinitarian programming system. In digital logic terms, the three conditions could correspond to +1, -1, and 0.

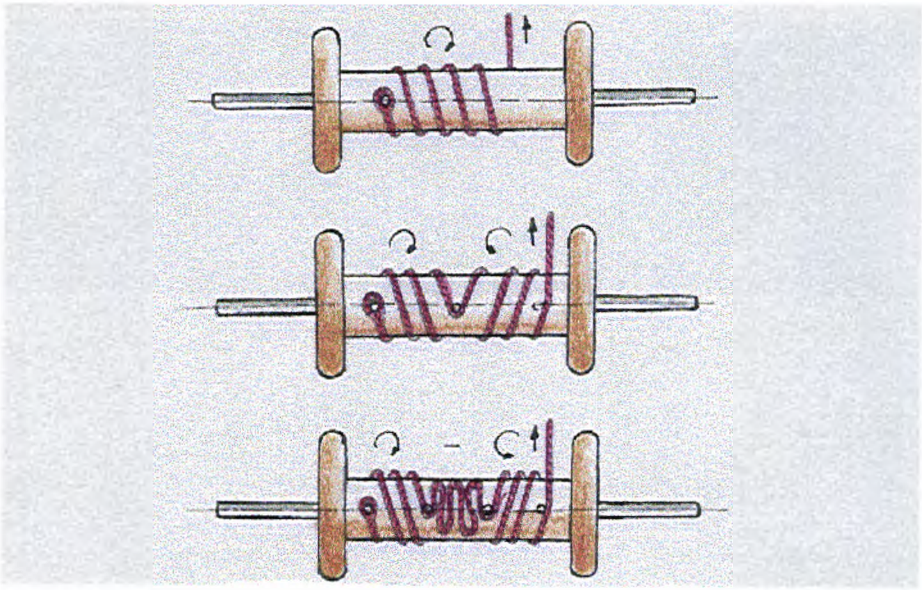

Figure 12.4: The programming of the movements of the mobile automaton (Kalligeropoulos, 1999)

Therefore, the automaton is placed in its initial position and without any interference it moves to another position either forward or backward. However, this is only one of the programmed movements that take place during the performance. The performance is staged by three-dimensional moving figures and a combination of equipments, such as the altar or the temple. As Heron goes along, he indicates that there can be a number of variations, but the outline of what happens is as follows (figure 12.2): Dionysus stands before an altar with a cup in hand and a panther at his feet. A circle of Bacchantes stands around the temple and a Nike surmounts the roof of it.

There is a sound of drums and cymbals and at once Dionysus turns towards the altar, a fire lights on the altar, milk or water gushes out from Dionysus' wand and wine flows from his cup, the area around the four pillars of the base is wreathed with flowers, the Bacchantes dance in a circle around the temple, and the Nike revolves (*Autom.* 4.2). Heron explains analytically every single mechanism: for motion, for the automatic lighting of fires, for the synchronised flow of milk and wine, for the rotation of figures, for the production of sounds, and for the crowning of temple with flowers. All these mechanisms are particular open loop control systems, extremely complicated in their construction, with a maze of pipes, cords, or pulleys and with controllers of various types, such as siphons, mechanical switches, and valves, which are housed in the hollow columns and in hidden top and bottom compartments of the stage set.

Forerunners of these complicated open loop control systems were the simple manually operated hydraulic systems named 'clepsydras', which evolved later in closed loop control systems.

12.4.2 The Clepsydra

The study of dynamical systems, of systems that are characterised by motion and change, led to the need of measuring time. The first dynamical phenomenon that constituted a pattern for time measurement and simulation was the relative motion of sun. A model of this motion was the motion of the shadow of a pointer, gnomon (*γνώμονας*) fixed on a slab or a spherical surface. The tracing of hours on the surface, in 12 equal intervals between the sunrise and sunset, led to the construction of sundials. The second dynamical phenomenon, pattern for the simulation of time, was the flow of water and led to the construction of hydraulic or water clocks. The precursor of the water clocks was the clepsydra, to which Empedocles also refers (chapter 4), and this was followed by water clocks of the type of clepsydra (two of them were found in the Market of Athens and the Amphiaraeion of Attica) that had at their disposal a tank with a spout, a pointer, and a table on which the twelve daily hours were traced. The disadvantage of these water clocks was the non-linear tracing of hours due to the non-linear relation of water outflow and water level in the tank. The water clocks that were constructed by the Alexandrian engineers, Ktesibios, Archimedes, Philon and Heron, had at their disposal a mechanism for regulating the water flow and level and secured the uniform motion of the pointer and consequently the linear tracing of hours on the table, as we will see below. Years later (Middle age), the water clocks were replaced by mechanical ones.

Regarding clepsydra, it is a hollow vessel, such as AB, that at the top has a tube, such as CD, that is open at the upper extremity, and in the lower part is pierced with numerous small holes. If someone plunges the vessel into water, the water enters through the holes, and if afterwards plugs the aperture C with his thumb it is possible to take an amount of water without being noticed.

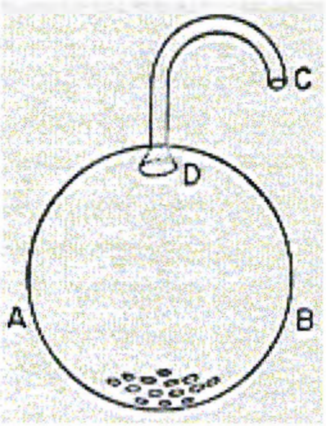

Figure 12.5: Clepsydra (Cohen *et al.*, 1966)

The Greek word for clepsydra (*κλεψύδρα*) indicates something that 'steals water' (*κλέβει ύδωρ*). It is an open loop system, where the control of water flow is achieved by means of the thumb. However, the main use of clepsydra was that of measuring time, either for domestic or for public use, in the courts or in the assembly of the city, in an attempt to measure the duration of speeches.

Aristotle, *The Athenian Constitution*, 67, 2: " [...] Four cases are taken in each of the categories defined in the law, and the litigants swear to confine their speeches to the point at issue... Water clocks are provided, having small supply-tubes, into which the water is poured by which the length of the pleadings is regulated [...] The official chosen by lot to superintend the water-clock places his hand on the supply tube whenever the clerk is about to read a resolution or law or affidavit or treaty[67]..."

Clepsydra concerns therefore, an open loop control system, where the controller of water flow is the official who places his hand on the supply tube whenever it is required. Another use of clepsydra was the agricultural, for regulating the watering

[67] The *Athenian Constitution* by Aristotle, translated by Sir Frederic G. Kenyon http://classics.mit.edu/Aristotle/athenian_const.html

time of fields. Pliny the Elder (23-79 A.D.), in his work of *Natural History*, in Book XVIII, which is on agriculture and agricultural techniques, refers to this use of clepsydra (18, 188)[68].

Last but not least, comes the military use of clepsydra, which involves measuring the watches for soldiers on guard. Aeneas Tacticus (mid-fourth century B.C.), a Greek general and military writer, who studies the problem of preserving the security of a city, gives a description of military nightly clepsydra (Fragment 22, 24)[69]. The night was split into four watches, vigils, three hours each. However, the fact that the antique hours limited differently throughout the year[70] had as a result the adjustment of clocks, so as to follow the changing duration of hours. Therefore, the military clepsydra had been constructed in its biggest size so as to correspond to the longest night of the year. Its adaptation to the changing duration of night vigils was achievable by the fact that one lined the inner surface of the clepsydra with more or less wax (Fragment 48)[71].

However, the clepsydra as a contrivance of measuring time has by technical perspective two weak points: a) the controller of water flow was the man and b) the flow of water was not uniform since it was depended on the level of water inside the clepsydra. The discovery that the outflow of a liquid from an orifice varies as the square root of the height of the liquid above the orifice was made in the 17th century. There are, though, passages in ancient bibliography indicating that Archimedes or the Alexandrian engineers were aware of the principle that the flow of a liquid through a hole pierced in the bottom of a vessel depends on the level of liquid inside the vessel, i.e., they knew that it is greater at its commencement and less as the contents of the vessel are reduced. As a result, they introduced the hydraulic siphons as mechanisms for securing a uniform flow, for regulating and controlling the liquid level and consequently the liquid flow. The siphon constitutes a simple closed loop control system and will be examined in the following section.

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[68] Latin text of the work of Pliny the Elder *Natural History* (Book XVIII, 188: See LOEB LIBRARY) http://www.ukans.edu/history/index/europe/ancient_rome/L/Roman/Texts/Pliny_the_Elder/18*.html

[69] Diels, H., 1965, pp.195, note 1.

[70] The ancient time was not divided from midday to midday into 24 equal hours as today, but the day was divided from sunrise to sunset into 12 hours and the night from sunset to sunrise into twelve watches; in this way the hours were long during summer, while the watches were short, and vice versa during winter (Drachmann, 1948).

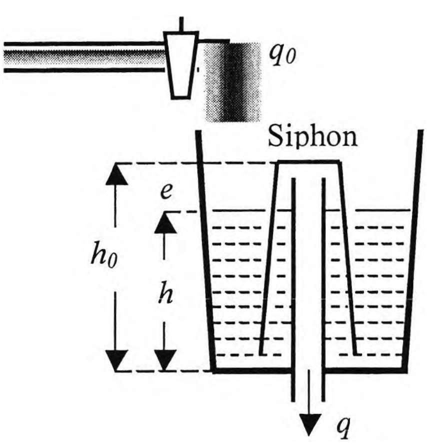
[71] Humphrey *et al.*, 1998, pp.520, (11.10).

12.5 Examples of Closed Loop Control Systems

12.5.1 The Hydraulic Siphon

During the Hellenistic period, various phenomena of pressure of liquids and gases were perceived and known in a practical way, and apparatus, such as the siphon or the suction pump, had been devised. The treatises on pneumatics of Ktesibios, Philon and Heron represent devises, such as altar figures pouring libations, water-clocks, cups with a constant level, and singing birds, based upon the siphon. The siphon as element of the pneumatic instruments appears in every possible form and combination; the curved or bent siphon, the enclosed siphon, and the straight siphon.

The siphon and especially the straight siphon operates as a hydraulic switch, which controls the level h of the liquid into a vessel so as not to go beyond a specific level h_0 that is equal to the height of the siphon. The constant flow q_0 has as a result the linear increase of the level h of the water into the vessel. If the surface of the water reaches the height h_0 of the siphon, the leg of the siphon fills with water and the water pours out rapidly with a flow $q>q_0$.



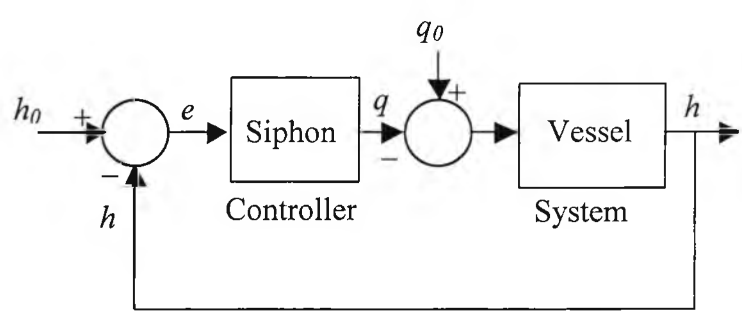


Figure 12.6: Block diagram of a vessel with straight siphon

The siphon constitutes here the controller of the closed loop control system. This system regulates the water level and finds its main application in the water clocks.

12.5.2 The Alarm Clock of Plato

The hydraulic clocks of the ancient Greeks were mainly daily clocks. Plato is said to have made first a night water clock used as an alarm clock.

Athenaeus, *Philosophers at Dinner (Deipnosophists)*, 4.174c: "But it is said that Plato provided a small notion of its construction by having made a clock for use at night that was similar to a water organ, although it was a very large water clock." (Humphrey *et al.*, 1998)

The working of it has been explained convincingly by (H. Diels, 1965). From a large clepsydra AA that lasts longer than six hours, the water drips into a container BB with a straight siphon Σ. The level of the water rises gradually and when it reaches the height of the siphon, in a definite time period, e.g., in six hours, it is emptied all at once, through the pipe Π, into another container ΓΓ. The air in ΓΓ is compressed and driven out through a whistle P that produces a sharp sound. Therefore, in six hours or in any predetermined time period the alarm clock calls. The mechanism for regulating the level of water into vessel BB is a closed loop control system, similar to that of the siphon.

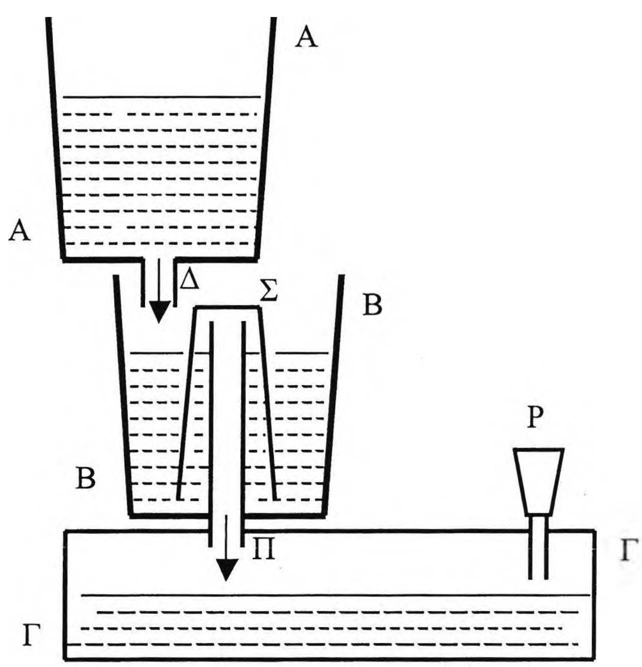


Figure 12.7: The alarm clock of Plato

Along with the siphons, many other mechanisms have been invented for regulating the liquid level in the hydraulic clocks.

12.5.3 Automatic Control of Fluid Level or Flow by Ktesibios

(Mayr, 1970) credits Ktesibios with the first known feedback device – a float valve used to create a steady drip of water into a cylindrical vessel, enabling the construction of an accurate water clock. The water clock of Ktesibios is described as follows[72]: Water flows into the vessel KLMN through the small opening at E and raises the float P, which holds a bar and pointer that marks a position on the drum TUSV. From this position it is possible to read the hour. The special significance of Ktesibios' water clock is that the rate of water flow from the primary reservoir is made uniform by means of a float valve that operates on the feedback principle. This self-regulating device is shown in figure 12.8 and 12.9 as the small tank BCDE served by a pipe ending in a conical cavity. The float G fits exactly to this cavity. If the flow of water through the tube A is more rapidly than the flow at E, the water in the tank BCDE rises until the float G is raised into the opening of the pipe; the water is shut off until the loss from BCDE allows the float to sink and admit more water (Usher, 1988).

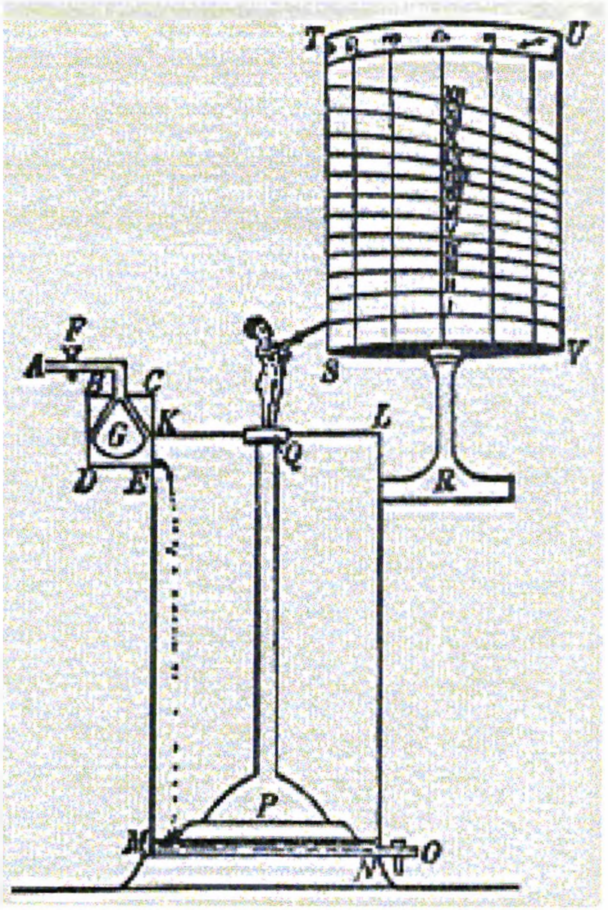
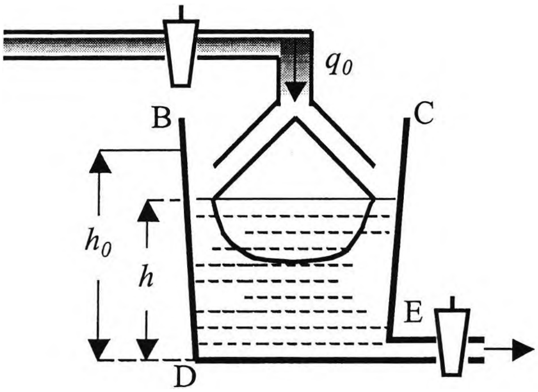


Figure 12.8: The water clock of Ktesibios (Diels, 1965)

Therefore, with this outstanding invention of Ktesibios, the rate of water dripping from the tank BCDE into the vessel KLMN remains constant, keeping constant the

[72] Vitruvius, *On Architecture*, 9.8.2-7 (Humphrey *et al.*, 1998)

velocity of bar PQ and the pointer. The block diagram of this closed loop control system is shown in the figure below:



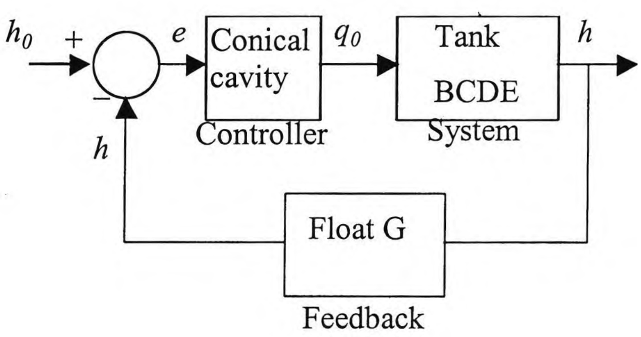


Figure 12.9: Regulation of water level or flow in Ktesibios' water clock

Ktesibios' float valve is a true self-regulating device that operates on the feedback principle. As the water pressure in the tank BCDE drops over time, the inflow to the valve tends to lessen, causing the float to start to drop, which in turn opens the inlet further and increases the inflow. Thus, the tendency for the inflow to decrease over time is automatically compensated for by the action of the valve. The height of the water in the valve tends to remain constant, and the resulting water clock is able linearly to trace the flow of time. (Richardson, 1991)

Archimedes is also credited with having constructed a similar hydraulic clock, though there are many controversial views that assert that it is an invention of the Alexandrian engineers[73]. The description of the motive mechanism of the clock has as follows: It consists of three vessels: a) a copper vessel a filled with water, wherein the float b connected to the drum E is placed, and this in turn is connected to the motive wheel z by means of a rope, b) a bigger container k underneath a, in which the water

[73] The baron Carra de Vaux reports in 1891 an Arabic manuscripts, the "*Kitab Arshimidas fi' amal al binkamat*" (*Journal Asiatique* 1891:8, ser. 17:287. 599), and Wiedemann, E., and Hauser, F., publish in 1918 a German translation of it, the so-called "*Uhr des Archimedes*", (Wiedemann *et al.*, 1, 1918), where it is given the description of a hydraulic clock attributed to Archimedes. On the other hand, Drachmann asserts that this clock described in the Arabic manuscript is nothing more than a variant of the Clock of Gaza, more recent than Archimedes, and relates the regulator of it with the flow regulator of Heron (Drachmann, 1, 1948)

from vessel a runs and c) a vessel T above a, wherein the drum E and other mechanisms are placed. Drum E is programmed so as to realise a full return per day, i.e., from the sunrise to sunset, when the float b completes its vertical displacement into vessel a (figure below).

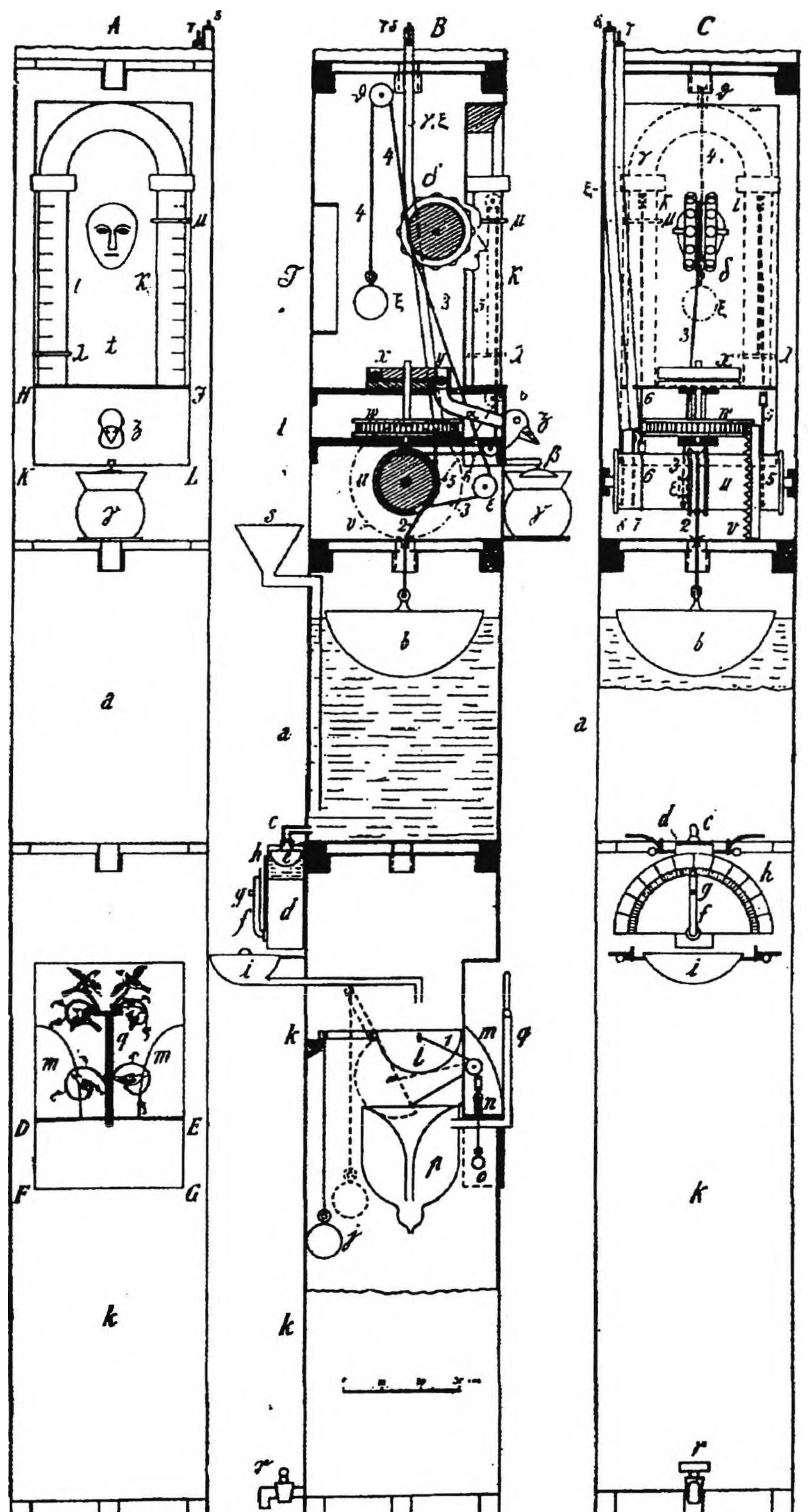


Figure 12.10: The water clock of Archimedes (Wiedemann $et\ al.$, 1918)

The uniform displacement of float *b* into vessel a is ensured by a mechanism similar to that of Ktesibios' clock (figure 12.9): a small tank *d* served by a pipe ending in a conical cavity contains float *l* that fits exactly to this cavity. The float is raised or sank into the tank so as to shut off the water flow or to leave more water to come into the tank (figure 12.11).

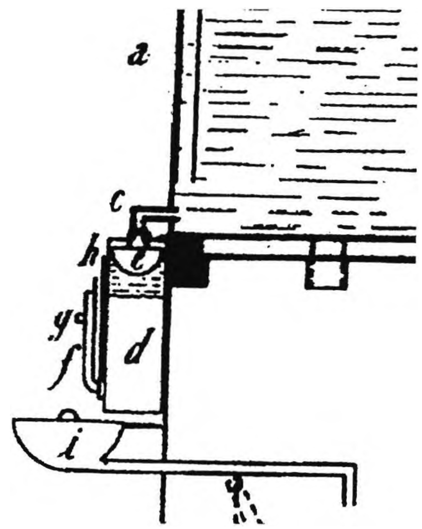


Figure 12.11: The mechanism for the regulation of water flow (detail, Wiedemann *et al.*, 1918)

Similarly, this mechanism constitutes a closed loop control system that secures a uniform, i.e., linear liquid flow.

Another mechanism for regulating the liquid level is given by Philon.

12.5.4 Automatic Control of Fluid Level or Flow by Philon

Philon's work *On Pneumatics* has been transmitted by the Arabic in two versions, one published in 1902 in French by the baron Carra de Vaux; the other, a fragmentary Latin translation from a lost Arabic copy was translated into German by W. Schmidt in 1899 (footnote 66). In chapter 17 of the 'Arabic' version (Carra de Vaux) Philon describes a float regulator to control the fluid level (figure 12.12):

"Let there be made an oblong base of wood aa. Then there is placed at one end of the base an oblong, round piece of wood $\beta\beta$; in this piece of wood there must be a container marked ε, and on this piece of wood a jar $\gamma\gamma$. And on the other end of the base a cup $\delta\delta$ is placed, and a container $\delta\varepsilon$ that is joined to the cup from beneath. A canal θ reaches both containers ε and $\delta\varepsilon$. Inside the jar there must be a pipe ζ coming from its bottom, and one end of it must reach towards the neck of the jar, and the other end of it must reach through the round wood into the canal in it; and there is on the mouth of the jar a cover, quite tight, well fitted, closing it securely, so that no trace of air can get in. In the jar there is another pipe that is marked κ. If the jar is filled wine, the pipe κ is open so that a little of this wine will flow into the cup, but next it stops and nothing of it will flow unless someone takes some of this wine. But if it is taken, there will flow again as much as was taken, and this will happen every time someone does that until the fluid in the jar is exhausted." (Drachmann, 1948)

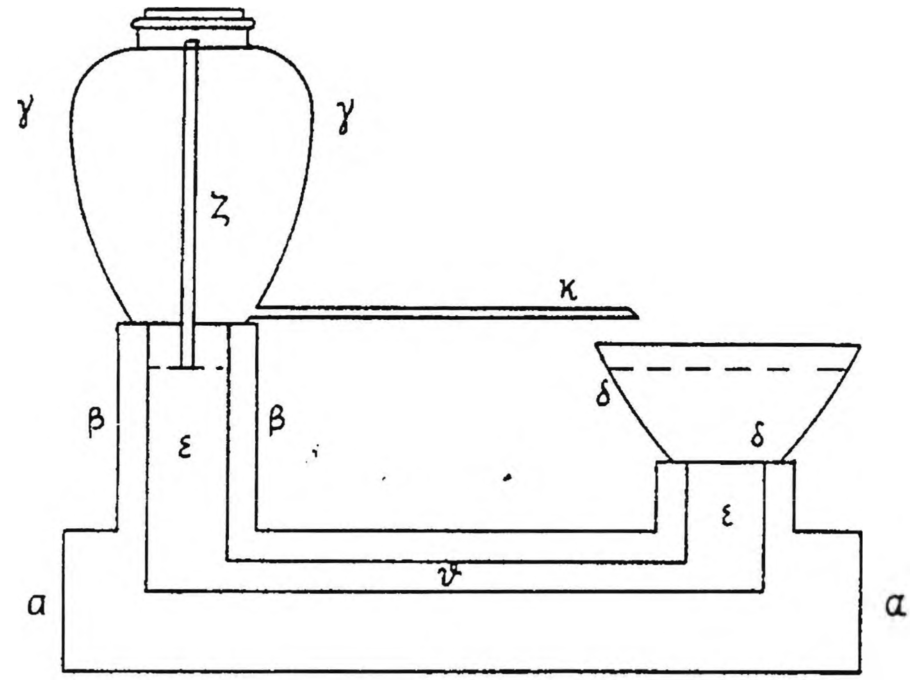
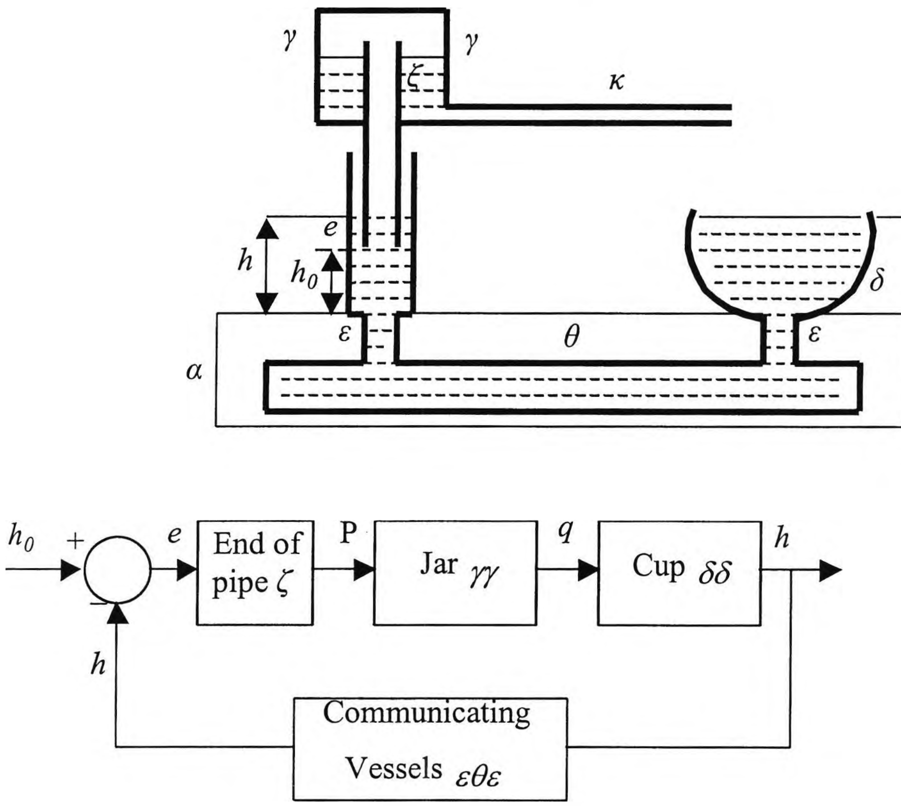


Figure 12.12: Regulation of fluid level by Philon (Drachmann, 1948)

This system of Philon concerns a closed loop control system for the regulation of liquid level. The system is at its equilibrium point if the wine level in cup $\delta\delta$ keeps closed the end of the pipe ζ that reaches into the container ε. In this case the pressure P that is created in the empty space of the jar $\gamma\gamma$, does not allow the wine to flow into the cup through the orifice of pipe κ. If someone takes some wine from cup $\delta\delta$, the end of the pipe ζ will be released, the pressure in the jar $\gamma\gamma$ will be re-established, and wine "as much as was taken" will flow through the pipe κ to the cup $\delta\delta$.

The block diagram is shown in the figure below:



Where:

h_0: desired level

h: actual level

q: flow from pipe κ into the cup $\delta\delta$

e: error

P: pressure in jar

Figure 12.13: Block diagram of regulation of fluid flow by Philon

Philon's level control system is also applied in the example of the oil lamp of Philon (Arabic version, chapter 20, Latin version, chapter 15).

12.5.5 The Oil Lamp of Philon

Philon shows in his Pneumatics an oil lamp with constant level. The oil is stored in a container above the lamp and gradually fills it up, when it is released by an air pipe being uncovered. In the Latin version it is described as follows:

"Similarly make another vessel abc which (being a sphere) consists only of one surface. On two sides it shall be provided with the outflow tubes cd and be, and with a pipe going down vertically into the vessel ghz (the belly of the lamp), which is fastened hermetically to both vessels at l and m. This is the pipe $klmn$. Certain parts of the vessel ghz located under the tubes cd, be, each under the appropriate one, may be extended on the outside in the fashion of night lamps. These are gt, sz. If one fills the vessel abc with water below the level n, then the liquid will flow through the tube cd to sz and through the opposite one be to gt on both sides into the vessel ghz, until it reaches the mouth of the pipe lk (within the lamp). When this mouth is closed (by the liquid), the outflow at d and e will stop. Assuming, for example, that the liquid in the vessel abc is oil, a wick or paper is placed into the oil of the vessel ghz. According to the quantity of oil consumed by burning in ghz, oil will gradually flow downward from abc through d and e. This process is of the same kind and has the same meaning." (Mayr, 1970)

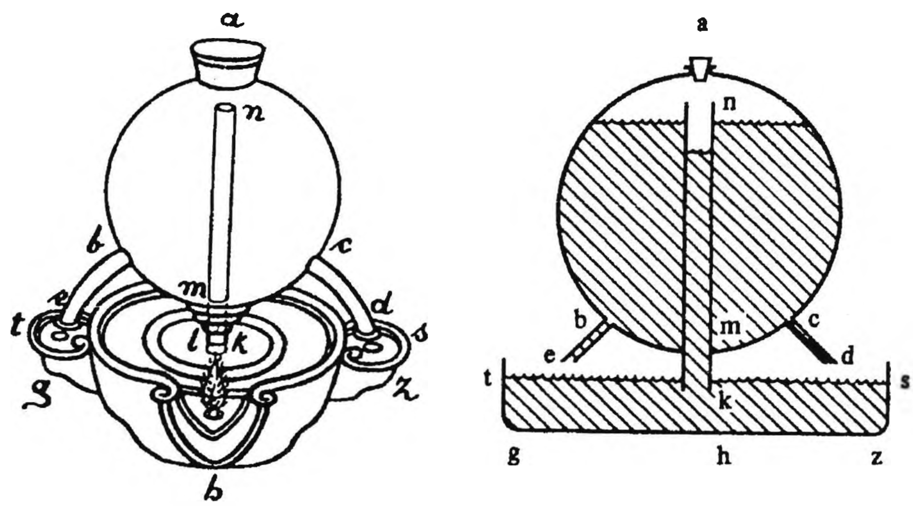


Figure 12.14: Philon's oil lamp with constant level (Mayr, 1970)

The filling of oil is achieved by a closed loop control system similar to that one examined in the previous example.

Finally, Heron invents a third hydraulic-mechanic system for the regulation of the liquid level.

12.5.6 Automatic Control of Fluid Level or Flow by Heron

Heron in his *Pneumatics* (B 31) describes a device for regulating the fluid flow, which can be regarded as a variation of Ktesibios' float valve. However, it is significantly improved in technical details and the control of the flow is achieved by means of a mechanical system.

Heron, *Pneumatics* B 31: "Let there be a vessel containing wine and provided with a spout, underneath which a goblet is placed: whatever quantity of wine is taken from the goblet, as much shall flow into it from the spout. Let $\alpha\beta$ (figure 12.15) be the vessel of wine, and $\gamma\delta$ the spout, to which are attached the valve $\varepsilon\xi$, and the rods $\eta\theta$, $\kappa\lambda$, κo, $\lambda\mu$; and beneath the spout place the cup π. To the rod κo fix a small basin (float) ρ contained in the vessel $\sigma\tau$, and let a tube $\upsilon\phi$, connect the vessels $\sigma\tau$ and π. When these arrangements are complete, if the vessels $\sigma\tau$ and π are empty, the basin a will fall to the bottom of $\sigma\tau$, and open the spout $\gamma\delta$. A stream will flow from $\gamma\delta$ into both the vessels $\sigma\tau$ and π, so that the basin will rise and shut the spout again, until we remove more liquid from the goblet. This result will happen as often as we remove liquid from π[74]."

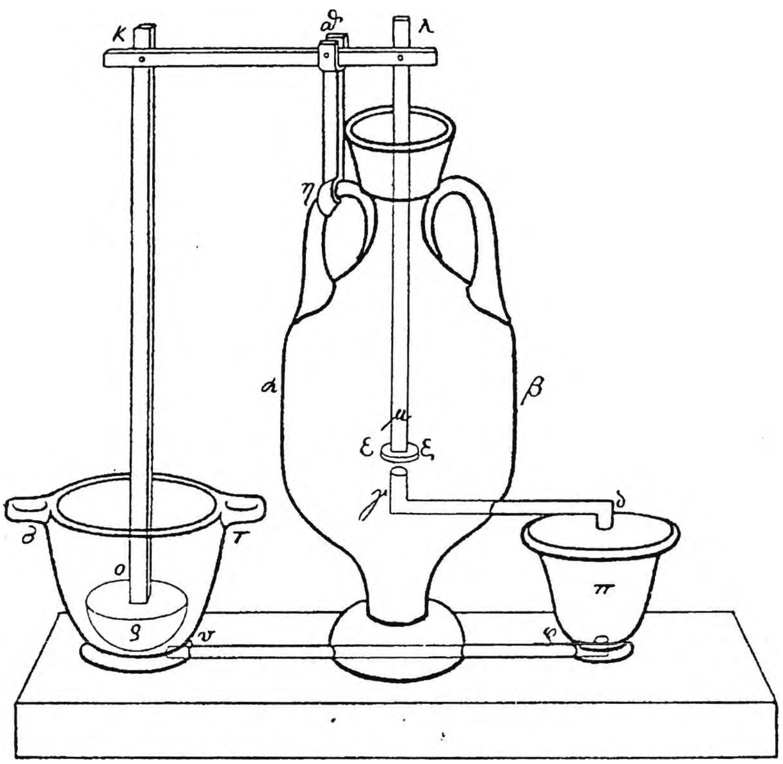
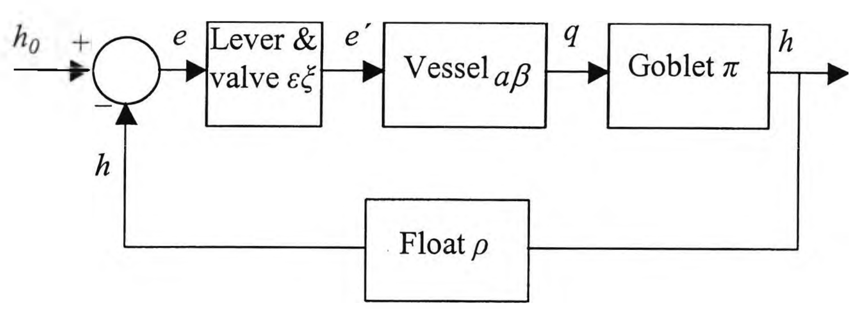


Figure 12.15: Regulation of fluid flow by Heron (Schmidt, 1899)

The block diagram below shows again an ingenious closed loop control system:



Where:

h_0: desired level

h: actual level

q: flow

e: error

e': valve opening

Figure 12.16: Block diagram of regulation of fluid flow by Heron

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[74] Hero of Alexandria from the original Greek translated for and edited by Bennet Woodcroft, Taylor Walton and Maberly, London, 1851.

Additional examples of closed loop systems with hydraulic or mechanical feedback can be found in Heron's Pneumatics. Some of them are: the automatic control of liquid level (A 19), the automatic control of fluid flow (A 20), and the automatic control of weight (B 30).

Thus, the dream of the mythical period, the theoretical approach of the classical age and the following Hellenistic scientific intention of constructing machines that work on their own, have at their disposal the ability of self-control, and accomplish the great leap of auto-motion, is already succeeded.

12.6 Early Control Curriculum in Alexandria

As it is already reported, Philon of Byzantium is credited with being the author of the most important technical handbook of Hellenistic antiquity, the so-called *Μηχανική Σύνταξις* (Mechanical Syntaxis), i.e., a compendium of mechanical sciences. This work became the foundation, the syllabus, for the training of the new engineers, and its chapters determined the branches of applied technical sciences of the Alexandrian years. This manual contained nine books, from which the *Pneumatics* is the only that has survived. We know, however, the titles of these books:

1. Introduction on applied and approximate mathematics.
2. *Mochlica*, i.e., *On the lever*, the relevant theory about levers and of Statics.
3. *Limenopoieca*, i.e., *On the building of seaports*, the art of constructing ports and elements of structural science and architecture.
4. *Belopoieca*, i.e., *On catapults*, the theory of shooting, kinetics and the construction of ballistic weapons.
5. *Pneumatics*, the theory concerning the properties of gases, steam and vacuum, and the construction of controlled pneumatic and hydraulic machines.
6. *Automatopoietice*, i.e., *On automatic theatres*, the art of making automata or the technology of automata, which summarizes all the above knowledge and applies it to the construction of automatic machines.

The three final books concern applications of war machines:

7. *On the building of fortresses.*
8. *On besieging and defending towns.*
9. *On stratagems.*

The text of Books 4, 5, 7 and 8 has survived, while the rest has been lost.

Another evidence of the program the students follow during their studies in the School of Alexandria, comes to us by Pappus:

> Pappus, *Mathematical Collection*, VIII 1-5: "... the mechanicians of Heron's school tell us the science of mechanics consists of a theoretical and a practical part. The theoretical part includes geometry, arithmetic, astronomy, and physics, while the practical part consists of metal-working, architecture, carpentry, painting, and the manual activities connected with these arts. One who has had instruction from boyhood in the aforesaid theoretical branches, and has attained skill in the practical arts mentioned, and possesses a quick intelligence, will be, they say, the ablest inventor of mechanical devices and the most competent master-builder. But since it is not generally possible for a person to master so many mathematical branches and at the same time to learn all the aforesaid arts, they advise a person who is desirous of engaging in mechanical work to make use of those special arts which he has mastered for the particular ends for which they are useful.
>
> The most important of the mechanical arts from the point of view of practical utility are the following. (1) The art of the *manganarii*, known also, among the ancients, as mechanicians. With their machines they need only a small force to overcome the natural tendency of large weights and lift them to a height. (2) The art of the makers of engines of war, who are also called mechanicians. They design catapults to fling missiles of stone and iron and the like a considerable distance. (3) The art of the contrivers of machines, properly so-called. For example, they build water-lifting machines by which water is more easily raised from a great depth. (4) The art of those who contrive marvelous devices. They too are called mechanicians by the ancients. Sometimes they employ air pressure, as does Heron in his *Pneumatics*; sometimes ropes and cables to simulate the motions of living things, e.g., Heron in his works on *Automata* and *Balances*; and sometimes they use objects floating on water, e.g., Archimedes in his work *On Floating Bodies*, or water clocks, e.g., Heron in his treatise on that subject, which is evidently connected with the theory of the sun dial. (5) The art of the sphere makers, who are also considered mechanicians. They construct a model of the heavens [and operate it] with the help of the uniform circular motion of water..." (Cohen *et al.*, 1966)

According to this passage, a student in the School of Alexandria who aspires to become a mechanician, has to follow, on the one hand, theoretical courses concerning the fields of geometry, arithmetic, astronomy, and physics, and on the other, courses concerning the mechanical arts but from the perspective of practical utility. The above-mentioned technical handbook of Philon, including the most of the mechanical arts, undoubtedly constitutes the essential syllabus for the training of the new mechanicians. We could infer, for example, that the first book *On the lever*, concerns the mechanical art of *manganarii*, i.e., it is possible to move a large weight with a small force by using the lever machine, *Belopoieca* or *On besieging and defending towns* concerns the theory of constructing engines of war and protecting a city under war and are suitable for the second mechanical art that Pappus describes, whereas *Pneumatics* or *Automatopoietice* provide the necessary knowledge for creating automatic machines, an additional mechanical art, which according to Pappus the nascent engineers have to study.

12.7 Conclusion

The first complex machines produced by man are the automata. Through them man attempts to simulate nature and, in general, to imitate the life itself. Automata constitute the first step in the realisation of man's dream to fly through the air like a bird, swim the sea like a fish, and to become in general the ruler of all nature. How? By creating automobile, independent, and self-controlled machines. The forerunners of the extremely elaborate automatic machines and advanced control theory we experience nowadays are the methodical study of the art of making automata in the Hellenistic times, as well as the numerous examples of open or closed loop control mechanisms represented in this chapter.

One might ask whether automatic machines such as those mentioned here have been actually been constructed or simply described in the survived ancient texts. Even though there is no archaeological evidence to that effect, there are two indisputable archaeological findings of the 1st century B.C. that show the remarkably high level of practical precision in the mechanics of Antiquity. The first one is the Antikythera mechanism that has been elaborated in details in chapter 9, and is exhibited in the Archaeological Museum of Athens. The other one is the Hydraulis of Heron that can be seen in the Archaeological Museum of Dion in central Greece.

Chapter 13

CONCLUSION

13. CONCLUSION

13.1 Conclusion

This thesis has examined the origins and the evolution of system, modelling, and control concepts, within the context of developments in the Greek world. For this purpose, the study of the works of the most important Greek poets, philosophers, mathematicians, and engineers (Appendix 1), either through their own writings or through researches, books, and articles by the later scientists, was necessary. Their contribution to the emergence and the developments of the concepts of system, conceptual, physical, and mathematical modelling, as well as of control is summarised below:

- The under examination concepts of system, modelling, and control have their origins in the ancient Greek thought, from myths to philosophy, mathematics, science, and technical achievements.
- The main characteristic of this thought is the search of the fundamental elements that compose a phenomenon, the basic causes that bring it on, the primary functions that characterise it.
- Conceptual modelling precedes the other types of modelling and is the result of the creation of symbols of the categories, virtues, properties and operations of the human and the natural world. Conceptual modelling expresses the first attempt to describe the understanding of the world and in particular, the systems existing in this.
- The physical conception of the world follows and the fundamental question of 'what is the world made of' is raised. The primary elements that constitute the world are either physical, or conceptual, or mathematical. However, they all emerge out of the physical reality and aim at the interpretation of this reality. Such an approach manifests an understanding that systems are compositions, interactions of simpler objects, i.e., systems under certain rules of interconnection.
- The intention for classification and the search of the primary do not stop at the elements that make up the world, but proceed to the search of the primary operations of the world, such as the operation of contradiction. This expresses the

effort to understand the rules of interaction and behaviour, which lead to complete types of behaviour associated with systems.

- The harmony and the strife of opposites constitute the basic feature of the dynamical behaviour of systems. The interaction between opposite elements, functions and qualities results in the motion, the evolution and the flow of things, and manifests the roots of dynamic behaviour of systems.
- The law that the one is divided into two allows the transition from the general to the particular and reversely. The method of dichotomy leads to the creation of a dynamic genealogical tree of concepts and species. The transition from a category (genus) to species demands a binary (digital) decision. The genus and the species can be categorised by means of this binary system. This expresses the effort to acquire deeper understanding of the classification of objects and properties, and provides foundations for a scientific conception of the world and its parts.
- The successive process of approximating a species by means of a sequence of binary decision leads to the development of a conceptual algorithm, similar to that of the mathematical modelling. Notions of models as approximations of reality are introduced and the mathematical algorithm emerges as tool for developing such models.
- The early mathematical models are geometric and relate closely to the process of measurement. Fundamental concept for the early mathematical modelling process is the concept of ratio and analogy, which are of conceptual nature despite their quantitative dimension.
- Geometrical models analogous to the physical systems are invented. The analogy among geometrical figures, numbers and magnitudes, e.g., sounds, introduces the concept of correspondence between different types of systems and the use of simpler systems to describe overall behaviour of more complex systems.
- The notion of a model and its validation (checking the reality) is formally introduced. The truth is verified when besides the qualitative come the quantitative criteria. The numbers as the result of analogy, comparison, and measurement, constitute infallible criterion of truth.
- Numbers in ancient Greece are the expression of the geometrical and physical analogies and their properties express the laws, which underpin the functioning of physical phenomena and system behaviour. The arithmetical ratio has the meaning

of the relationship between two magnitudes and constitutes the first form of mathematical correspondence that leads to modelling.

- The method of anthyphaeresis allows the exact approximation of rational or irrational numerical ratios by means of an algorithmic process of finite or infinite steps. This algorithmic process can be either geometrical or arithmetical and constitutes a dynamical process of the mathematical approximation of the desired numerical ratio. The notion of approximation is formally introduced and mathematical tools for working out approximations are defined.
- The exact quantitative consideration of systems leads to the construction of mechanical models that claim accurate simulation of original behaviour. Complicated mechanical models of the universe materialise the early conceptual interpretation of the operation of the world. The exact sciences move further from the stage of interpretation to the stage of application. The constructed models are not only mechanical, but also hydraulic and pneumatic.
- The notion of system, as this emerges in nature, society, human body, and human activities, is defined and its basic elements (objects, subsystems, and interrelationships between them) are specified. The significance of the system, as far as its potential to generate behaviour different than that of its elements, is identified.
- The significance of the fundamentals of systems is manifested with the development of the holistic approach in medicine, where notions of complexity, responses to different causes are identified as results of interacting system behaviour.
- Along with the question 'what is the system made of', the asking of the relation of the system to its environment, of the inner operation of the system, of its purpose and its governor that leads it on, arise. As a result, the concept of system is expanded and is linked closely to the concept of a control system.
- The mechanical analogies and simulation of human functions lead to the introduction of automata. The construction of automotive mechanisms demands the working-out of two problems: a) the problem of the internal energy that secures the auto-motion and b) the problem of the internal operation that secures the control, the programming of the movements.

- From a philosophical point of view, the control systems are distinguished into systems that work under an external command and systems that have an internal logic and programming at their disposal.
- The methodical study of the art of making automata is brought to a completion during the Hellenistic times and leads to the creation of extremely elaborate automatic machines as well as closed loop control systems with 'internal logic' that is materialised by mechanical or hydraulic feedback.
- The significance of automata and the art of automation is recognised by the shaping of a training program for early engineers in the School of Alexandria having as subject the development and design of automata.

We cite below a concise table with the findings of the whole research that constitutes a quick reference of the evolution of system, modelling and control concepts from the mythical to the Hellenistic period. It is built with respect to the philosophers and scientists who have conduced towards the emergence of these concepts. In particular, the early steps of science, philosophy, and technology, come along with the first notions of system theory, the development of conceptual, physical and mathematical modelling, as well as with the conceptualisation of automatic machines that work by means of internal energy and have the ability of controlling and regulating their operation.

Table 1. A summary of the development of modelling, system and control concepts

	Conceptual Modelling	Physical Modelling	Mathematical Modelling	Mechanical Modelling	Systems	Control
HOMER 8th B.C.	- Gods as symbols of natural elements & virtues - Achilles' shield as model of human and natural world	- Oceanus as origin of all things - First models of the world structure				- Mythical automata
HESIOD 700B.C.	- Gods as symbols and Theogony as evolutionary process	- Cosmogony as a dynamical process - Chaos as the origin of matter and energy				
ORPHICS	- Early conceptualisation of universe as a system	- Night, Chronos, Water, Matter, Air as primary elements - The egg as model of the universe				
THALES 624-547 B.C.	- Introduction of abstract thought, proof of results, logic in establishment of facts - Concept of analogy or logos	- Water as primary active element - The flat disk of the earth floating on water as a cosmological model	- Creation of generic types of propositions - The analogy of geometrical figures as a mathematical modelling process - Measurement as modelling process		- Introduction of system-analysis by searching for the basic elements of a system	
ANAXIMANDER 611-547 B.C.	- Introduction of the Infinity concept - Acceptance of the materialist model of the world	- The concept of Infinity as primary element - The notion of eternal motion as a dynamical aspect -Early form of qualitative reasoning process		- Mechanical model of the world consisting of rotating and flaming wheels	- Introduction of the dynamical behaviour as part of eternal motion	- Introduction of the Infinity as the 'governor', controller of the universe

	Conceptual Modelling	Physical Modelling	Mathematical Modelling	Mechanical Modelling	Systems	Control
ANAXI-MENES 585-528 B.C.	- Acceptance of the Materialists' conceptual model of the world and evolution to the characterisation of its elements	-The air as primary element - Functional relations between the natural magnitudes - Quantitative examination of phenomena	- Examination of the functional correlations among the variables of a system - Identification of variables associated with elements, which are interacting			
PYTHAGORAS 582-500 B.C.	- The concept of number as origin of the natural world Correspondence between numbers and concepts	- Fire-centric cosmological model and invention of a tenth heavenly body, the counter-earth - Relations between planetary distances and ratios of notes	- Classification of categories of numbers - Harmony as a correspondence between rational numbers and sounds - Relations between plane and linear geometrical figures - Introduction of irrational numbers	- Use of a one-string musical instrument (monochord) as a mechanical model that determines the quantitative relations between the lengths of the string and the pitch of the sound	- First definition of the system by the Pythagorean Kallicratides	
HERACLITUS 540-480 B.C.	- Concept of contradiction (e.g., Strife and Harmony) - Dynamical conception of the world (Everything flows)	- Fire as a model of the changing world, flows and dynamics		- Mechanical model of the world, where the heavenly bodies are represented as bowls filled with fire	- System as a colossal process of events, changes and progressive facts - Introduction of the concept of a dynamic equilibrium that dominates every system	- The concept of contradiction as theoretical background for feedback control
PARME-NIDES 540-480 B.C.				- Mechanical model of the world consisting of rings		
ANAXA-GORAS 500-470 B.C.	- Introduction of the concept of Mind as the primary cause of physical changes	- Separation and formation of opposite qualities			- System of matter as a complicated whole -Introduction of duality	- Mind as the 'governor', controller of the world

	Conceptual Modelling	Physical Modelling	Mathematical Modelling	Mechanical Modelling	Systems	Control
EMPE-DOCLES 490-430 B.C.	- Concepts of Love and Strife that set the primary elements in motion - Introduction of experimentation as a procedure to enhance knowledge	- Introduction of the four primary elements (Water, air, fire, earth) as the origins of the world -Experimentation with elements in search for properties	- Reasoning by analogy	-Use of analogues in describing the process of respiration		
ZENO 490-425 B.C.	- Introduction of paradoxes related to the concept of infinity -Contribution to scientific method		- Infinite series and the problem of their convergence			
S O C R A T E S 469-399 B.C.	- Introduction of inductive reasoning method for formulation of general definitions - Use of syllogisms with feedback ("The only thing I know is that I don't know anything")					- Maieutic (Obstetric) method as a closed loop system
DEMO-CRITUS 460-370 B.C.	-Materialistic concept of physical world	- Atomic theory for the explanation of the world structure - Similarities between macro- and microcosm		-Early notions of movement of elementary particles of matter	-Composite nature of system as interacting elements	
HIPPO-CRATES 460-377 B.C.	-Conceptualisation of human body as a complex interactive system of simpler processes and phenomena	- Analogy between physical elements, humours in the human body, and qualities			- Holistic approach to the concept of a system - Ascription of the feature of totality to a system - Definition of notion of state of human body	- Notion of 'cycle' in the circular process of creating diagnostic models

	Conceptual Modelling	Physical Modelling	Mathematical Modelling	Mechanical Modelling	Systems	Control
ARCHY-TAS 428-350 B.C.			- Definition of the arithmetic, geometric, and harmonic proportion - Mathematical study of the harmonic scales	- Foundation of scientific Mechanics		- Construction of a flying automatic dove, as an open loop control system
PLATO 427-348 B.C.	- Definition of model and distinction of model types - Theory of Ideas: the eternal ideas as conceptual models of reality - Process of creating a conceptual model - Use of the Method of Dichotomy for creating genealogical trees of related concepts -Scientific method	- Cosmological model of the planetary system	- Platonic regular solids - Algorithmic method of Dichotomy as a process of discrete decisions - Theoretical study of irrational numbers	- Creation of a world's mechanical model consisting of whorls	- Definition of system as a whole composed of many parts	- Introduction of cybernetics as the art of controlling a system (Ship, army, city) - Construction of an alarm clock as a closed loop control system
EUDO-XUS 390-337 B.C.		- Model of the stellar and planetary phenomena	- Geometrical model for the irregular planetary motions - Method of exhaustion			
ARISTOTLE 384-322 B.C.	- Introduction of the Logic system - Classification of types of causation - Relation between matter and form - Classification of types of change - Scientific methodology	- Introduction of the sensible qualities - Interaction between qualities and elements - Astronomical system consisting of 55 spheres	- Definition of the first mathematical principles	- Solution of mechanical problems such as the transmission of motion from one circle to another - Classification of types of mechanical behaviours, causations	- Logic as a system of thought -Ascription of the feature of hierarchy to a system	- Explanation of the motions of automata - Distinction between the open loop (external commanded) and closed loop (internal programmed) control systems - Imagination of a fully automatised society without slaves

	Conceptual Modelling	Physical Modelling	Mathematical Modelling	Mechanical Modelling	Systems	Control
E U C L I D 325-265 B.C.			- Foundation of scientific geometry - Anthyphaeretic method as a geometric and arithmetic algorithm for the exact evaluation of rational and irrational numbers			
ARISTAR-CHUS 310-230 B.C.	- Concept of analogy in measurement	- Creation of the first heliocentric astronomical system		- Development of measurement methodologies for estimating distances between planetary objects		
K T E S I B I O S 300-230 B.C.		- Study of hydraulic and pneumatics systems		- Construction of mechanical mechanisms such as the "automatic" mirror		- Construction of a hydraulic music instrument equipped with a control mechanism for the regulation of the air pressure - Construction of a complicated water-clock equipped with a mechanism for regulating the water flow
ARCHI-MEDES 283-212 B.C.		- Invention of static and hydrostatic laws -Use of laws of optics - Mechanics laws - Specific weights of materials	- Synthesis of geometrical and arithmetical algorithms for the computation of irrational numbers -Discovery of calculus	- Invention of simple machines as the lever and the screw - Construction of a mechanical celestial model (planetarium) - War machines		- Construction of a water clock equipped with a regulator for controlling the water flow

	Conceptual Modelling	Physical Modelling	Mathematical Modelling	Mechanical Modelling	Systems	Control
P H I L O N 2nd c. B.C.		- Study of Pneumatic and Hydraulic systems				- Invention of a devise for regulating the water level with a closed loop control mechanism - Self-regulated oil lamp
H E R O N 1st c. B.C. or 1st c. A.D.		- Study of Pneumatic, Hydraulic, and Mechanical systems	- Study on geometrical and stereometric measurements - Use of geometrical analogy for mechanical constructions	- Classification of the five simple machines as elements of mechanical systems - Development of gear mechanisms with quantitative accuracy - Construction of pneumatic, hydraulic, and mechanical automatic machines		- Invention of open and closed control mechanisms and automatic theatres

The work in this thesis has dealt with a rather long period, within which the evolution of systems, modelling and control ideas has been considered. Such a study will certainly benefit from a more in depth examination of the primary sources, which could provide additional evidence and also explain the evolution of ideas and the contribution from one to the other. The influence of civilisations preceding of that of the Greeks is worth examining. The need for such an investigation is motivated by the fact that although the notion of automata appears in the myths and Homer's poems, a large number of years are required for such concepts to re-emerge and materialise. It seems that many of the cosmological models and notions of mythical automata are memories from the distant past and effects of other civilisations. A similar issue arises by the fact that even though the ancient Greek philosophers and scientists approached, at least theoretically, the construction of automatic machines and the properties of high-pressure steam (example of Aeolopile) setting the foundations for the steam-engine, such a great number of years was required for the Industrial Revolution to happen. It is thus interesting to expand the geographical borders from the area of ancient Greece and to include other great civilisations that have contributed to the evolution of science and technology, as well as to extend the chronological borders and to follow and investigate the development of these concepts in the next centuries till the present years.

Further work envisaged could be the use of the current overview as a framework that would enabled the development of a more detailed and in-depth account along the lines of the issues raised in the above table.

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APPENDICES

Appendix 1: Chronological Order of Philosophers and Scientists

Homer	Ionia	Middle of 8^{th} c. B.C.
Hesiod	Askra of Boeotia	*ca.* 700 B.C.
Thales	Miletus	*ca.* 624-547 B.C.
Anaximander	Miletus	*ca.* 611-547 B.C.
Anaximenes	Miletus	*ca.* 585-528 B.C.
Pythagoras	Samos	582-500 B.C.
Xenophanes	Colophon	*ca.* 570-480 B.C.
Heraclitus	Ephesus	540-480 B.C.
Parmenides	Elea	*ca.* 540-480 B.C.
Anaxagoras	Clazomenae	*ca.* 500-470 B.C.
Melissus	Samos	*ca.* 500-440 B.C.
Empedocles	Acragas	490-430 B.C.
Zeno	Elea	*ca.* 490-425 B.C.
Leucippus	Miletus	480-400 B.C.
Philolaus	Tarentum or Croton	*ca.* 480-400 B.C.
Socrates	Athens	469-399 B.C.
Theodorus	Cyrene	465-398 B.C.
Democritus	Abdera	460-370 B.C.
Hippocrates	Cos	460-377 B.C.
Theaetetus	Athens	417-369 B.C.
Archytas	Tarentum or Taras	*ca.* 428-350 B.C.
Plato	Athens	427-348/47 B.C.

Eudoxus	Cnidus	390-337 B.C.
Aristoxenos	Tarentum	*ca.* 360 B.C
Callippus	Cyzicus	*ca.* 370-310 B.C.
Theophrastus	Lesbos	*ca.* 371-287 B.C.
Aristotle	Stagira	384-322 B.C.
Euclid	Alexandria	*ca.* 325-265 B.C.
Aristarchus	Samos	*ca.* 310-230 B.C.
Ktesibios	Byzantium	300-230 B.C.
Archimedes	Syracuse	287-212 B.C.
Philo	Byzantium	Second half of 3^{rd} c. B.C.
Hipparchus	Nicaea	*ca.* 190-125 B.C.
Heron	Alexandria	1^{st} c. B.C. or 1^{st} c. A.D.
Vitruvius	Formia of Latio	End of 1^{st} c. B.C.
Cicero	Arpinum	106-43 B.C.
Plutarch	Chaeronia of Boeotia	*ca.* 45-125 A.D.
Theon	Smyrna	*ca.* 70-135 A.D.
Claudius Ptolemy	Egypt	*ca.* 108-168 A.D.
Diogenes Laërtius	Laërte of Cilicia	*ca.* 200
Pappus	Alexandria	*ca.* 290-350 A.D.
Simplicius	Cilicia	*ca.* 500-533 A.D.
Leonardo da Vinci	Anchiano of Florence	1452-1519 A.D.

Appendix 2: List of Automata in Heron' *Pneumatics* and *Automatopoietice*

Pneumatics, Book A

- **Hydraulic siphons**
  1. The bent siphon (2, 13)
  2. The straight siphon (3, 13)
  3. A siphon of uniform flow (4)
  4. A siphon or partly uniform and partly non uniform flow (5)
  5. A co-axial siphon with vessel (6)
- **Devices of extensive use**
  6. Clepsydra (7)
  7. Two-compartment clepsydra offering hot and cold water (8)
  8. Drinking horn with two fluids (water and wine) divided into three parts offers water, or wine, or wine mix with water (18, 22)
  9. Automatic fountain (10)
  10. Automatic libations at an altar produced by fire (figure 11.1) (12)
  11. Unanimous bowls: two craters placed on a common base and having inside a curved siphon ending in an outlet. One of them is filled with wine and the other is empty. Water is poured into the latter that runs through the pipe and makes the wine of the former to flow out through the siphon, at the same time as the water flows also out (14, 23)
- **Devices producing sounds**
  12. Singing bird automaton (15)
  13. Whistling bird and owl automaton (16)
  14. Automatic trumpeting doorbell for a temple (17)
- **Hydraulic automata**
  15. Constant level bowls equipped with float regulator, which automatically are filled with as much liquid as was taken (19, 20)
  16. Automatic (coin-operated) Holy Water dispenser (21)
  17. The automatic fire-engine (28)
  18. Drinking animals (29, 30, 31)

19. Egyptian automata with wheels offering Holy Water: a wheel (found in Egyptian temples) is turned when the worshippers come in the temple and holy water flows out of the wheels as it turns (32)
20. Automatic vessel offering different kinds of wines: in this complicated vessel a number of quests can pour each his own wine, and then get it out again (33)
21. Oil-amp with self-trimming wick (34)
22. Satyr holding a bag of wine fills a basin without it to be overflowed (37)
23. Self-opening temple doors (38, 39)
24. Hercules shooting off an arrow at a snake that hisses (41)
25. Automatic water organ (42)
26. Windmill-powered water organ (43)

Pneumatics, Book B

1. The fair vessel (*dikaiometer*) that always pours out a certain measure of water at a time (1)
2. When a fire is lighted in front of a temple, figures start dancing (3)
3. Singing birds with intermittent voices (4, 5)
4. Spheres are suspended above heated air (steam) (6)
5. Descriptive model of the world (7)
6. Thermoscope (λιβάς), from which drips water when the sun shines on it (8)
7. Aeolopile, where fire causes the creation of high-pressure steam, which in turn is responsible for the revolving motion of the sphere (11)
8. Oil-lamp automatically filled with oil (22, 23)
9. Container giving out a certain measure of wine, by hanging on a weight (27, 30)
10. Automatic mechanism for regulating the fluid flow by means of a float and a lever mechanical system (figure 2.15) (31)
11. In front of a temple there is a casket with a turning wheel and a bird on it: when the wheel is turned the voice of the bird is sound (32)
12. Jar with self-regulated flow by means of a siphon and a mechanical lever
13. Bath stove (miliarium) heats water by blowing air on the coals so as to regulate the outlet of hot water (34, 35)

Automatopoietice, I, Mobile automata

1. The programming of the movements of the automaton by system of windings (figure 12.4) (6)
2. The movement of the automaton in a rectangular parallelogram (9)
3. Complex helical movements of the automaton (11)
4. The mechanism of the lighting of fires on the altars (12)
5. The hydraulic mechanism for the liquid flow and the mechanism for the rotation of idols (13)
6. The mechanism for the production of sounds (14)
7. The mechanism for the wreathing of temple with flowers (15)
8. The dance of the Bacchantes (16)

Automatopoietice, II, Stationary automata

9. The mechanism for the opening and the closing of the theatres doors at fixed intervals (23)
10. The mechanism for the movements of *Danaoi*, who work with saws and hammers (24)
11. The mechanism for the change of the scenery (25)
12. The mechanism for the coasting of the ships (26)
13. The mechanism for the movement of the dolphins (27)
14. The mechanism for the lighting of the torch (28)
15. The mechanism for Athena as *deus ex machina* (29)
16. The fall of the thunder and the disappearance of Ajax (30)

(*Herons von Alexandria, Druckwerke und Automatentheater, vol.1,* Schmidt, W., Leipzig, 1899 & *Ktesibios, Philon and Heron, A study on ancient Pneumatics*, Drachmann, A. G., Kopenhagen, 1948)

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