Essays on networks, subjective survival beliefs and annuity markets

A dissertation submitted for the Degree of Doctor of Philosophy

PhD candidate: Seung Yeon Jeong

Supervisors: Professor Steven Haberman

Dr Iqbal Owadally

Dr Douglas Wright

Faculty of Actuarial Science and Insurance

Bayes Business School

City, University of London

17 May 2023

Contents

List of Tables

List of Figures

Acknowledgement

First and foremost, I would like to express my deepest and most sincere gratitude to my research supervisors, Dr Iqbal Owadally, Professor Steven Haberman, and Dr Douglas Wright, in Actuarial Science, Bayes Business School, City, University of London, for giving me the opportunity to start research at a brilliant school and guiding me throughout the years of my PhD. They have given me the path to follow and have taught me the methodologies and theories for my research. It was a great privilege to have an opportunity to study with their guidance. I am extremely grateful for everything they have offered me, as I know it would not have been possible to start and finish this work without their support.

I am so grateful to my wife, who has supported me for the entire period of my study. She provided me with the best support during her pregnancy. I am very thankful to my parents for their support in educating me until now. I express my thanks to my parents-in-law, brother and brothers-in-law for supporting me in many ways.

Declaration

I grant powers of discretion to the Director of Library Services to allow the thesis to be copied in whole or in part without further reference to myself. This permission covers only single copies made for study purposes, subject to normal conditions of acknowledgement.

Abstract

People have been shown empirically to have subjective survival probabilities, which deviate from the objective survival probabilities derived from mortality data. Subjective survival beliefs affect many decision-making processes, including annuity purchases. We build an agent-based model to test if simple behaviour rules can re-create patterns provided by the Survey of Consumer Finance data. By building the model, we test whether agents are updating survival beliefs based on observations and if they are using simple behaviour rules. We further develop the agent-based model to simulate the annuity market with the optimism index that we introduce. We use a newly developed optimism index to formulate subjective survival probabilities and use the subjective survival probability to solve a life-cycle model of savings and consumption. Our life-cycle model finds contrasting results to a published model. Our model suggests that subjective mortality may not be a cause of the anomalies in savings and consumption behaviour, including annuity purchases. Finally, we build an agent-based model that creates a realistic network based on node characteristics, in particular agent's age. We find that the realistic network is constructed even without the desired degree of nodes. The last model simulates an annuity market after pension reform in 2015. The simulation result suggests that individuals' preference for an annuity is low as 5%.

Chapter 1

Network influence on subjective survival beliefs

1.1 Abstract

The subjective survival beliefs of an individual provide a foundation for many decision-making problems. However, studies on the reason behind subjective survival belief formation are uncommon. In this chapter, we offer an agent-based model where agents obtain survival subjective beliefs while surviving, giving birth and observing the deaths of others in their network. The model provides a simple rule for agents to calculate survival expectations, averaging the observed ages at death while removing any observed age at death when the individual survives past this age. We demonstrate such a simple rule can recreate responses from surveys of subjective survival expectations when the appropriate network size is used.

1.2 Introduction

Simon (1955) lists an individual's constraints when making a decision: limited and unreliable information, the agent's limited ability to evaluate available information, and time constraints. This problem arises when people think about their lifespan. Individuals estimate their likely lifespan based on their limited observations and use the estimates to make other decisions.

Studies of subjective survival beliefs illustrate some common patterns. Firstly, younger people tend to be pessimistic when they evaluate their survival probabilities, and older people tend to be optimistic. In other words, older people overestimate the chance of survival, while younger people underestimate the probability of survival (Hamermesh 1985, Hurd & McGarry 1995, Elder 2013, O'Dea et al. 2018). Secondly, it shows that the subjective mortality perceptions of people are affected by the actual ages of death of family members. Hamermesh (1985), Hurd & McGarry (1995), Gan et al. (2005), Salm (2010) and O'Dea et al. (2018) find that family members' age of death is a significant factor affecting subjective mortality beliefs.

Therefore, we propose an agent-based model (ABM) where agents give birth and die while updating individuals' subjective survival beliefs based on the simulated deaths of others within a network. The purpose of the model is to reproduce certain aspects of subjective survival expectations of individuals, which is an answer to the question of how old individuals think they will live in the Survey of Consumer Finance. The Survey of Consumer Finance also includes information on families' financial status and demographic characteristics. We propose a way of calculating subjective survival expectations that is simple and reasonable. Agents use the average age at death they observe in their network while removing any observed age at death when the individual survives past this age. We discover such simple rules lead to a realistic reproduction of survey data when the size and structure of the network are appropriate.

1.3 Literature review

1.3.1 Subjective survival beliefs

People exhibit psychological biases not only in making economic decisions involving risks but also in those involving health and safety issues. For example, Lichtenstein et al. (1978) show that individuals over-estimate small fatality risks while under-estimating substantial fatality risks. Sunstein (2002) illustrates how ordinary people misunderstand dangers to their lives which can create problems for them and why their perception of risk differs from the objective measures. In an early study of risk perception, Slovic (1987) shows that the risk perception of ordinary people, such as women voters, college students and club members, differs significantly from the risk perception of experts. This suggests ordinary people are influenced to a more significant extent by their own experience in contrast to experts using broader and more accurate data.

Similarly, Hamermesh (1985) shows that people have different abilities to estimate their lifespan. It is arguable whether people have limited cognitive abilities to measure survival probability accurately. Hurd & McGarry (1995), Smith, Taylor, Sloan, Johnson & Desvousges (2001), Gan et al. (2005) show that people can make a reasonably accurate estimation of survival based on their health behaviours, which are actions of individuals affecting their health, such as smoking, substance use, diet, sleep and so on. Furthermore, Smith, Taylor & Sloan (2001), Hurd & McGarry (2002) present evidence that subjective survival beliefs can serve as predictors of actual mortality. On the other hand, Elder (2013), Ludwig & Zimper (2013), Peracchi & Perotti (2011), Bissonnette et al. (2017), O'Dea et al. (2018) document biases in subjective survival beliefs compared to the objective survival probabilities.

Using the data of the Health Retirement Study and the Asset and Health dynamics among the oldest old, the study of Gan et al. (2005) show various risk factors are used by individuals in their estimation of their future lifespan. However, the risk factors include some measures which cannot be seen as objective as health behaviours. For instance, their result shows that people who have parents who died early are more pessimistic, males are more optimistic, and older people are more optimistic. O'Dea et al. (2018) also find similar results. The authors identify the relationship between stated survival probabilities and risk factors, including smoking, alcohol consumption, health conditions and the age of parents' deaths. Patterns of subjective survival beliefs which these factors cannot explain were described as follows; people underestimate their survival probability when they are younger, while older people in their 80s and beyond overestimate their survival probability. Similar behavioural biases have been presented from the cumulative prospect theory Tversky & Kahneman (1992), which suggests people tend to under-weight the probability of average events while over-weighting the probability of unlikely events.

There could be other behavioural biases that underlie subjective survival perception. Biases could be driven by motivational factors, as people only see or learn what they want to see or learn. Kastenbaum (2000) argued that motivational biases play an essential role in forming survival beliefs. The behaviour of older people expressing more optimistic survival views could be driven by confirmatory or motivational biases, as people do not want to think about the notion of their own death. Grevenbrock et al. (2021) find that a confirmatory bias is essential for forming survival beliefs at all ages. However, they also find that cognitive weakness is an increasingly important contributor to overestimating survival chances over one's life cycle.

1.3.2 Network

The influence of peers is one of the crucial topics in behavioural sciences. Sacerdote (2001) asserts the importance of social interaction in the economy, despite difficulties in measuring them. A handful of empirical literature documents that peers can influence economic decisions when interacting. Duflo & Saez (2002) show peers affect retirement saving. Hong et al. (2004) document stock market participation influenced by peers. Bizjak et al. (2009) present corporate compensation and merger practices that peers can affect. Ahern et al. (2013) demonstrate peers affecting entrepreneurial risk. Lerner & Malmendier (2013) suggest neighbours can influence general economic attitudes such as risk aversion.

As suggested, the decision-making process of people is influenced by their peers. However, it is unclear whether peers directly affect the decision-making process or whether they influence others by giving information indirectly. We postulate that people use essential information obtained by peers when making a decision: including estimation of survival expectations. For example, people can use observations from their family members, friends, and neighbours' death age to estimate survival expectations. In other words, individuals use samples to evaluate the information and selecting the sample from the population is directly linked to their network. Therefore, we postulate that networks could play an important role in determining the survival expectation of individuals. In this paper, we suggest that underestimation or overestimation of subjective survival beliefs can be driven by the size and the structure of the network from which individuals learn about mortality.

1.3.3 Agent-based model

An ABM has agents who follow simple but heterogeneous behavioural rules and who interact within the network implicit in the model. As a result of the agents' interactions, feedback arises, and the feedback can lead to unexpected and complex phenomena. Bonabeau (2002) summarises three major benefits of an ABM over other modelling techniques.

First, an ABM can capture emergent phenomena. Emergent phenomena arise from the agents' interactions, resulting in system-wide properties that go beyond the properties of the model's components. An example is a traffic jam, where many agents are involved and cause a force in the opposite direction to the direction in which cars are moving. Emergent phenomena can be difficult to predict and can sometimes be counterintuitive. An ABM can be used to study emergent phenomena when individual behaviour is nonlinear and heterogeneous, when individual behaviour exhibits temporal correlations and when including learning and adaptation.

Second, an ABM describes a system naturally. An ABM can describe and simulate a system composed of many heterogeneous, boundedly rational agents. Using bottom-up behavioural rules can be more natural than using a mathematical description of top-down dynamics to model a system. Therefore, an ABM is suitable for individuals exhibiting behaviours that cannot be explained with mathematical description because of agents' heterogeneity and randomness in behaviour. In other words, an ABM can describe a system when a description of activities is easier and more natural than processes to explain the system.

Finally, an ABM is flexible because it is straightforward to change the quality and quantity of agents. An ABM provides a natural framework to incorporate the complexity of agents. A model designer can assign agents various features, such as rationality, learning ability, and interaction rules. One may use an ABM when there are unknown parameters, which requires some fine-tuning using simulations to find suitable parameter values.

We aim to study the link between survival expectation and network topology. Survival expectation from the SCF is compared with the survival expectation formed by an ABM simulation. An ABM is constructed with a simple and realistic mechanism. The model can change the network's degree distribution by altering the parameter values. Therefore, we are able to test under what circumstances a simulation can best replicate the survey data. We give the details of survey characteristics, an explanation of an ABM, and the model description throughout the section. The detailed model descriptions are in the following order; fertility assumption, mortality assumption, network creation, subjective survival beliefs and parameters.

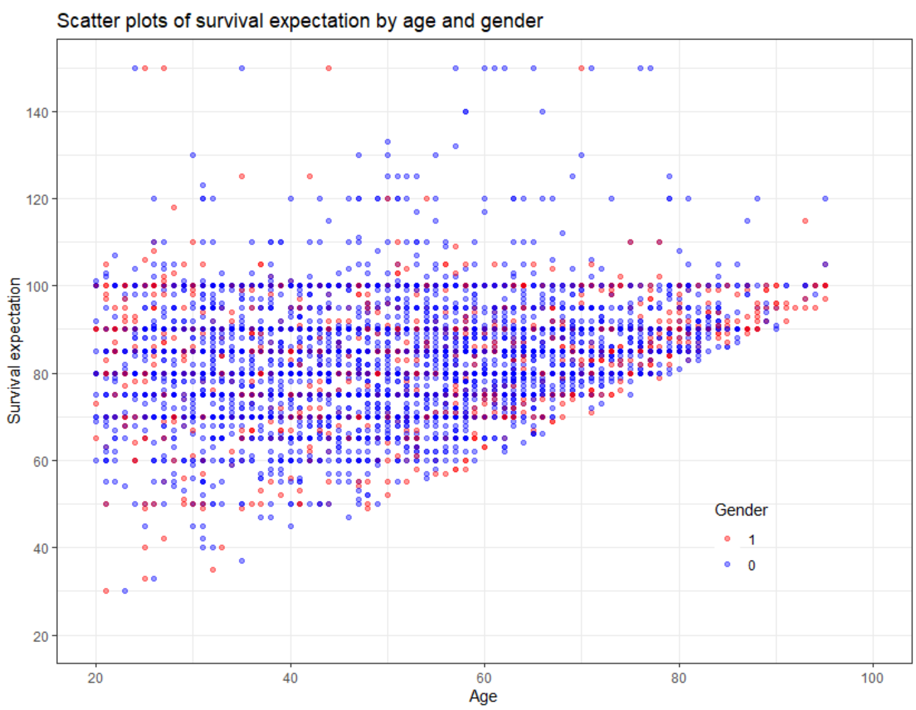


Figure 1.1: Scatter plot of survival expectation by age and gender (Source: The SCF, 2010)

1.4 Data

Longitudinal panel studies such as the U.S. Health and Retirement Study(HRS), the English Longitudinal Study of Ageing (ELSA)or The Survey of Health, Ageing and Retirement in Europe (SHARE). Typically, these surveys ask individuals to estimate their probability of surviving the next 10 or 15 years. Therefore, these surveys are frequently used to study subjective survival beliefs. Hurd & McGarry (1995), Elder (2013) and Salm (2010) use the HRS data and O'Dea & Sturrock (2021) use the ELSA data. On the other hand, some studies use data from the Survey of Consumer Finance (SCF) Puri & Robinson (2007) and Heimer et al. (2019), where the survey asks survival expectations of the respondents.

The SCF is a triennial cross-sectional survey of U.S. families sponsored by the United States Federal Reserve Board in cooperation with the U.S. Treasury Department. The survey data include information on families' financial status and demographic characteristics. One of the survey questions asks individuals to what age they think they will live. We refer to the answer to this question as a survival expectation. Figure 1.1 shows a scatter plot of individuals' age and survival expectations from the study in 2010 SCF [1]. We choose the 2010 survey to simulate an annuity market for at least ten years until 2020. The scatter plot displays two conspicuous features. First, the leading diagonal boundary where age equals survival expectation suggests that people think they will survive at least to the current age. Secondly, the visible horizontal lines at specific survival expectations on the plot suggest that many respondents might present reporting behaviour such as rounding at ages that are multiples of 5 years.

The data in the SCF consists of 4,988 men and 1,494 women. The mean age of respondents is 50.73, and the mean age of survival expectation is 83.4.

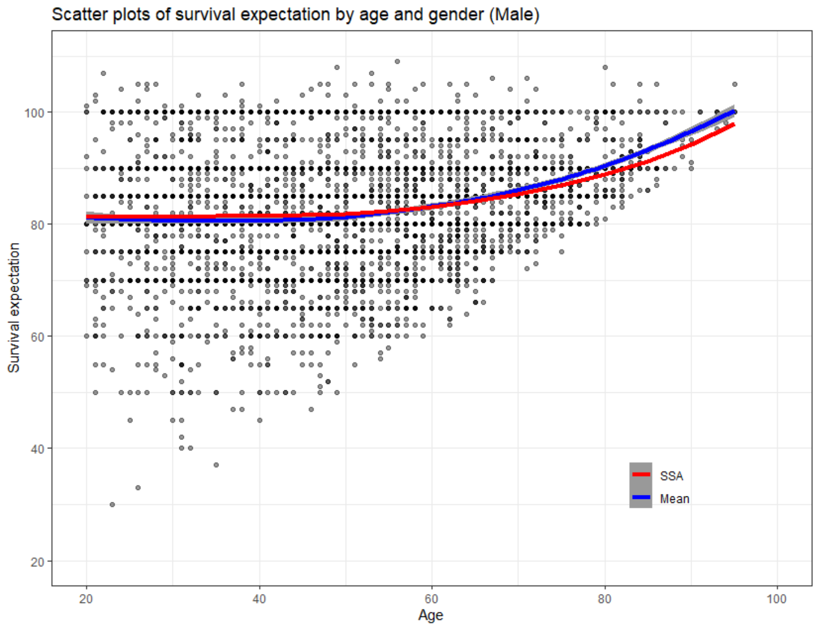
[1] https://www.federalreserve.gov/econres/scf_2010.html

The data was adjusted as follows, 16 agents under age 20 were removed as the size of this group was small to make a good comparison. 102 agents whose survival expectation is over 110 are also removed because agents may choose the maximum available age. It is difficult to assume that the respondents are making a reasonable estimation based on their observation rather than simply giving the maximum value possible. Table 1.1 shows the characteristics of the respondents. Heimer et al. (2019) compare the

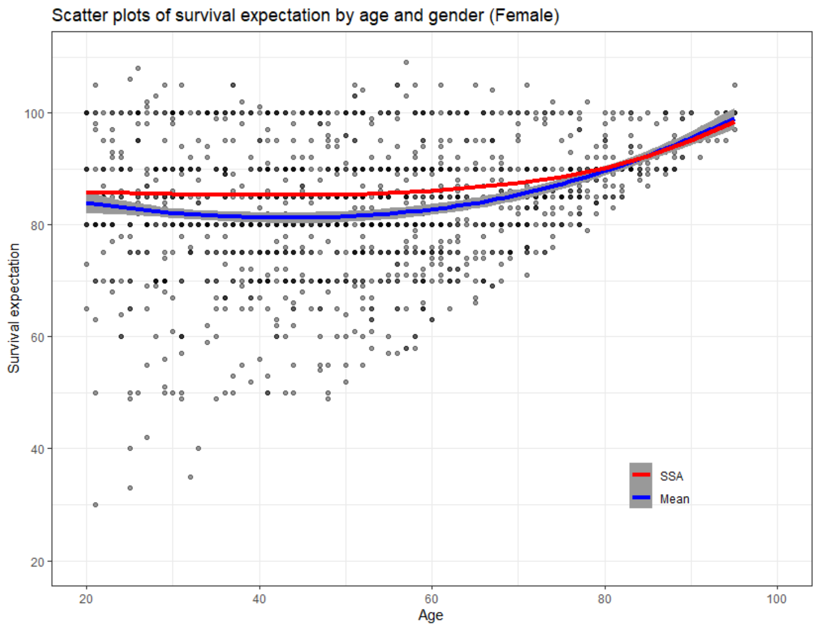
Table 1.1: Characteristics of the survey respondents with and without removal. (Source: The SCF, 2010)

Statistic	N	Mean	St. Dev.	Min	Max
Gender	6,482	0.230	0.421	0	1
Age	6,482	50.69	15.88	18	95
Survival Expectation	6,482	83.40	11.97	30	150
Gender(adjusted)	6,364	0.231	0.421	0	1
Age(adjusted)	6,364	50.73	15.80	20	95
Survival Expectation(adjusted)	6,364	82.78	10.87	30	109

projected longevity estimates calculated using actuarial probabilities for the Social Security Administration (SSA) to the subjective survival beliefs in the 2010 wave of the SCF. They show that respondents in the 2010 wave have subjective beliefs similar to other studies; underestimation of survival probability at younger ages and overestimation at older ages. Figure 1.2 also uses the same data as their study. We choose the SCF because of its large sample and long history of continuity. The first SCF survey was in 1983 with 3,824 families, with a survey conducted every three years since then. Moreover, the SCF includes information such as age, survival expectation and life insurance holdings. Subjective survival beliefs from the SCF are compared with the simulated result from an ABM, where agents obtain subjective survival beliefs from simply observing deaths around them.



Male



(a) Female

Figure 1.2: Subjective and objective survival expectation at different ages and for different gender, Mean: smoothed average of subjective survival expectation by age and SSA: SSA projected longevity estimates. (Source: Heimer et al.,2019)

1.5 Model description

We develop a model where agents observe other agents' death and subsequently update their subjective survival beliefs. In this model, all agents give birth and die with age-specific probabilities. Each agent builds a family and learns survival expectations from the ages of death of connected agents. Therefore, realistic mortality and fertility assumptions are one of the key features of the models.

This model, at first, will consider cases where only female agents are tracked and recorded since it is easier to model mothers giving birth as a function of the age variation is better defined than for fathers. Second, O'Dea et al. (2018) shows that female family members' death age has a greater effect than male family members' death age on the subjective survival belief of female agents, while the subjective survival beliefs of male agents are more affected than female family members' death age by male family members' death age. Therefore, it would be more complicated to include different gender when it is possible that gender affects the formation of survival expectation. Finally, including other elements of family-building processes, such as marriage, is complicated as it involves variables other than age.

An ABM is suitable in the context of subjective survival belief formation because this involves heterogeneous agents who learn and adapt their risk perceptions within the network. Also, interacting to learn subjective survival beliefs from peers could be considered a natural human thought process. Therefore, the process can naturally explain the economic phenomena without using loss aversion and risk preference parameters which are arbitrarily determined. Finally, creating agents who learn survival beliefs from the network has the potential for extensions. Other factors, such as wealth, socio-economic status, health, and education, can be incorporated, and different network topologies can be tested.

1.5.1 Fertility

For the fertility probability, the Hadwiger model (Hadwiger 1940) with calibrated parameters by Peristera & Kostaki (2007) was employed. As the ABM that we build assumes female agents as representatives of households, the probability of giving birth in the model is halved from the Hadwiger fertility model. Therefore, the fertility probability at age x equals

$$P_f(x) = \frac{1}{2}\frac{fg}{h}\left(\frac{h}{x}\right)^{\frac{3}{2}}\exp\left[-g^2\left(\frac{h}{x}+\frac{x}{h}-2\right)\right]. \tag{1.1}$$

We assume the fertility model to be time-invariant. Changing mortality rates over time can affect subjective survival beliefs over time. However, changing the fertility rate only adds complexity to the model. The 1996 Swedish fertility rate from Peristera & Kostaki (2007) was selected as it maintains a stationary population over time. Figure 1.3 shows the probability distribution of the Hadwiger model with $f = 0.91, g = 3.85$ and $h = 29.78$.

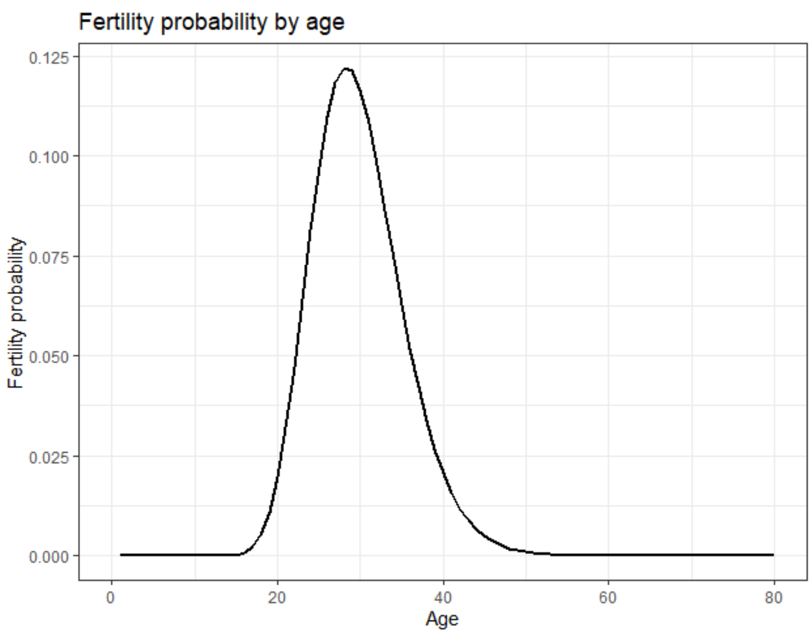


Figure 1.3: Fertility probability with $f = 0.91, g = 3.85$ and $h = 29.78$. (Source: Peristera and Kostaki, 2007)

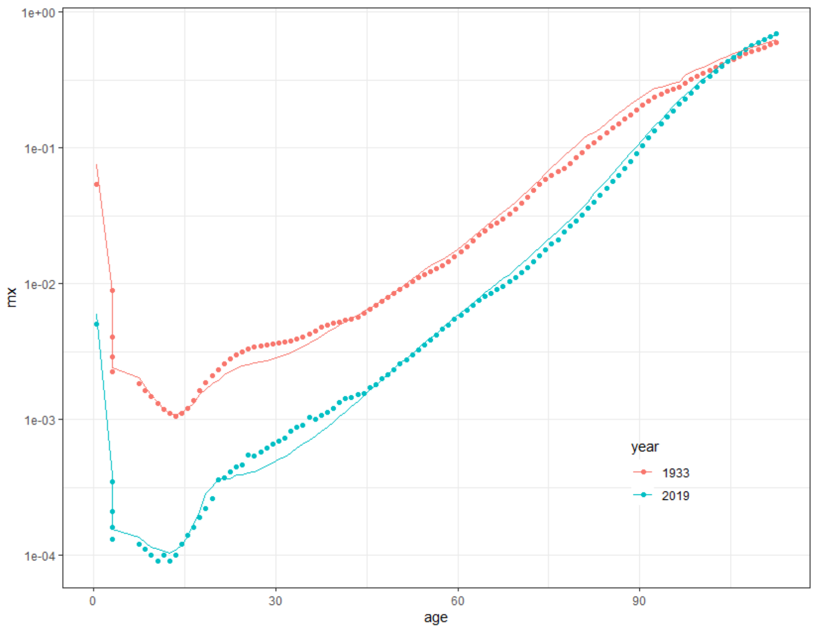
1.5.2 Mortality

Mortality data for the US from 1933 to 2019 were collected from Human Mortality Database[2]. Mortality rates before 1933 are unavailable from the database, so we use the Lee-Carter stochastic mortality model Lee & Carter (1992) to create mortality rates before 1933. We choose the Lee-Carter model as the Lee-Carter model is simple in structure and widely used by demographers, statisticians and actuaries (Booth & Tickle 2008). The Lee-Carter model comprises a time series vector which captures mortality trends over time and age-specific vectors. Therefore, the mortality rate for each age in each year can be estimated. Figure 1.4a illustrates mortality estimations using the Lee-Carter model and actual mortality rates for all ages in 1933 and 2019. The simulation needs at least 110 years to ensure that every agent alive has complete observations of death age. For instance, agents born in 1990 cannot observe the death of agents whose age is over 57 when the simulation starts from the year 1933. Every agent alive in 2019 can observe death ages from 0 to 109 when the simulation starts in 1900. Therefore, complete mortality tables for 1900 - 1933 are created using the Lee-Carter model. The Lee-Carter model predicts the age-period surface of log-mortality rates in terms of two vectors of age dimension and time dimension. According to the Lee-Carter model, log-mortality rates $logm_{xt}$ at age x and time t is:

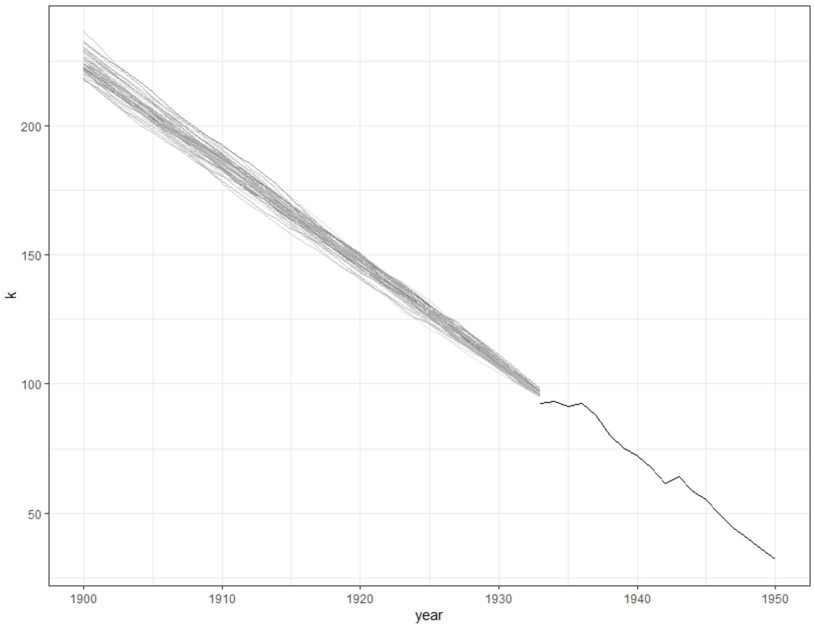
$$logm_{xt} = a_x + b_x k_t + \epsilon_{xt}, \tag{1.2}$$

where the vector a along the age dimension can be interpreted as an average age profile of mortality, the vector k along the time dimension is mortality changes over time, and the vector b along the age dimension reflects changes in age group along with the changes in k. Finally, the error term ϵ_{xt} shows age-period effects that the model does not capture.

[2] https://www.mortality.org/



(a) The Lee-Carter for 1933 and 2019



(b) k and simulated k for 1900 - 1950

Figure 1.4: The Lee-Carter model

The vector k has 87 values from 92.42 to -49.41 from 1933 to 2019. The values for 1933 to 1950 were selected to find k for 1909 to 1932 using a type of back calculation. Drift for k between 1933 and 1950 is -3.83 each year. We use a simple linear model instead of stochastic simulations as in Figure 1.4b to find k for 1909 and 1933.

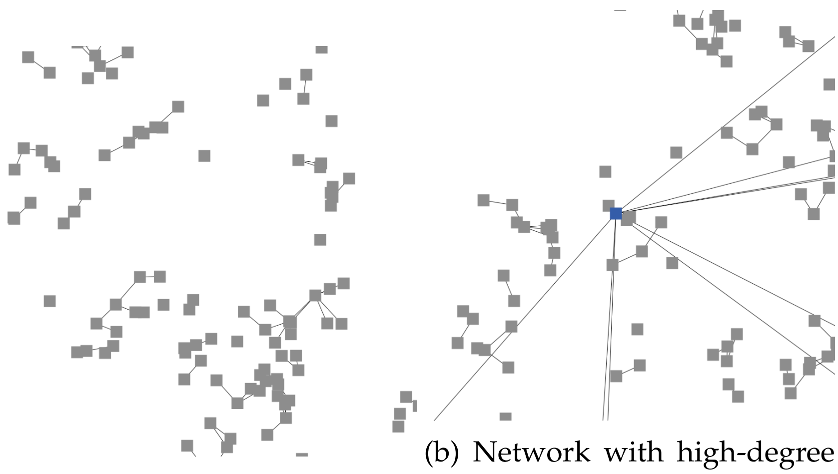
The agents in the model are interconnected through a network. The structure of interconnectedness is key for the model. In this model, we assume agents update survival expectations when observing the death of others. Therefore, the structure of interconnectedness is defined by the number of agents and the age of others to whom agents are connected. In other words, subjective survival belief formation is deeply related to the number of agents and the age of others to whom agents are connected. We assume that the subjective survival belief of an individual is determined by the number of agents connected to the agents and the age of the connected agents. Considering connected agents as a sample, the sampling method will always affect subjective survival belief estimation.

1.5.3 Network

The network links are constructed between family members and other random agents. We assume relationships among agents are represented on a two-dimensional plane with X and Y axis where X, Y coordinates represent either the social or geographical location of an agent. The model's agents connect to family members and other agents whose coordinates are close to themselves. Specifically, they create networks with non-family members based on the Erdős–Rényi(ER) (Erdős & Rényi 1960) model. In the ER model, $G(n, M)$ model gives graphs with n nodes M edges and $G(n, p)$ model gives graphs with n nodes and the probability of including each edge p. The probability of generating each graph with n nodes and M edges is given as:

$$p^M(1-p)^{\binom{n}{2}-M} \tag{1.3}$$

We use the Block Two-Level Erdos-Renyi (BTER) model by Seshadhri et al. (2012). The model has two layers of network construction: a small network within a community with the first ER model and a larger network outside of a community with the Chung-Lu (CL) model (Chung & Lu 2002), which is a weighted ER model. In our model, the smaller community is



(a) Network formation

(b) Network with high-degree agents

Figure 1.5: Agents and network links illustration on two-dimensional plane

constructed with the ER model first, and selected agents make connections with a large number of agents outside of the community. We assume the community members for an agent are agents within a radius of the agent. Furthermore, it was assumed that the probability of the connection among community members is 1. This assumption makes the model simpler and computation faster. The second layer of the CL model is replaced with the ER model for computational efficiency.

The small community mostly represents family and very close friends. Family members are connected as they are born, and agents connect with every other agent within a specified radius. For example, first, a daughter is assigned a coordinate randomly close to her mother. When a mother agent has coordinates $X = p$ and $Y = q$, the daughter agent would have X coordinate as $p + u_1$ and Y coordinate as $q + u_2$, where u_i and u_2 is a random variable, $u_1, u_2 \sim U(-0.5, 0.5)$. Thus, newly born agents will spread around the mother agent as in Figure 1.5a.

Second, agents make connections to other agents who are not family members based on the ER model. We refer to these agents as network neighbours. We assume the probability of the first ER model is 1 for simplicity while changing the number of nodes based on the distance from the given coordinates. In this case, the only possible number of edges M equals to $\binom{n}{2}$ from Equation 1.3. This equals a simple random graph with n nodes and some edges with $M = \frac{n(n-1)}{2}$.

The presence of the high-degree agents makes a more realistic model. This is because the right-skewness of distribution is one of the key characteristics of a real network. High-degree agents can exemplify famous figures such as celebrities. It is also possible to consider them as a representative value of a cohort. We assume all high-degree agents provide an age at death. Death ages provided by the high-degree agents can be unevenly distributed when high-degree agents are randomly selected from survivors and deceased. Therefore, all high-degree agents are assumed to be dead. Finally, we assume one high-degree agent is selected from a cohort. We test the range or size of a cohort and the probability with which high-degree agents connect to others based on the ER model.

The model assumes agents learn their subjective survival beliefs by observing the death of their network neighbours and the death ages provided by high-degree agents. Agents are assumed to calculate their expected death age from death ages that they personally observe within their network. One of the two conditions to update survival belief is that the death year of an observed dead agent has to be later than the birth year of the agent who is updating the belief. Moreover, we assume the observed death age has to be greater than the observing agent's age. In other words, when updating their subjective survival beliefs, agents will exclude death ages lower than their current age from the sample of observed death ages. The diagonal shape of Figure 1.1 suggests most people do not think they will die immediately.

The subjective survival expectation of an agent is the average death age of his neighbours whose ages at death are at least the agent's age. The subjective survival expectation e_{x_i} of an agent i at age x is

$$e_{x_i} = \frac{1}{w}\left(\sum_j tx_j\right), \tag{1.4}$$

where tx_j is the death age of selected neighbour j, and w is the number of neighbours j whose death ages have to be greater than the agent's age.

1.6 Model calibration and validation

Simulated survival expectations and surveyed survival expectations are compared to show whether they are significantly different when several parameter values are introduced. We first estimate a combination of parameter values using the mean squared deviation and validate the combination employing the two-dimensional Kolmogorov-Smirnov test.

There are three network creation parameters: δ, π, and κ. The parameter δ determines the radius of a circle centred around an agent within which the agent can create a link in the network. For example, when an agent i and j on a plane with coordinates X_i, Y_i and X_j, Y_j create network when

$$\sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2} \leq \delta. \tag{1.5}$$

π determines the probability of the second ER model for high-degree agents. When the network is created by simulation, a high-degree agent creates a connection with another agent if a uniformly distributed random variate between 0 and 1 is less than π.

κ determines the cohort of the high-degree agents. For example, when $\kappa = 10$, the ages of high-degree agents are multiples of 10 from 20 to 100. An illustration of how the distance parameter and probability parameter affect the distribution of survival expectation is shown in Figure 1.6. Larger δ and larger π lead to smaller dispersion in survival expectation.

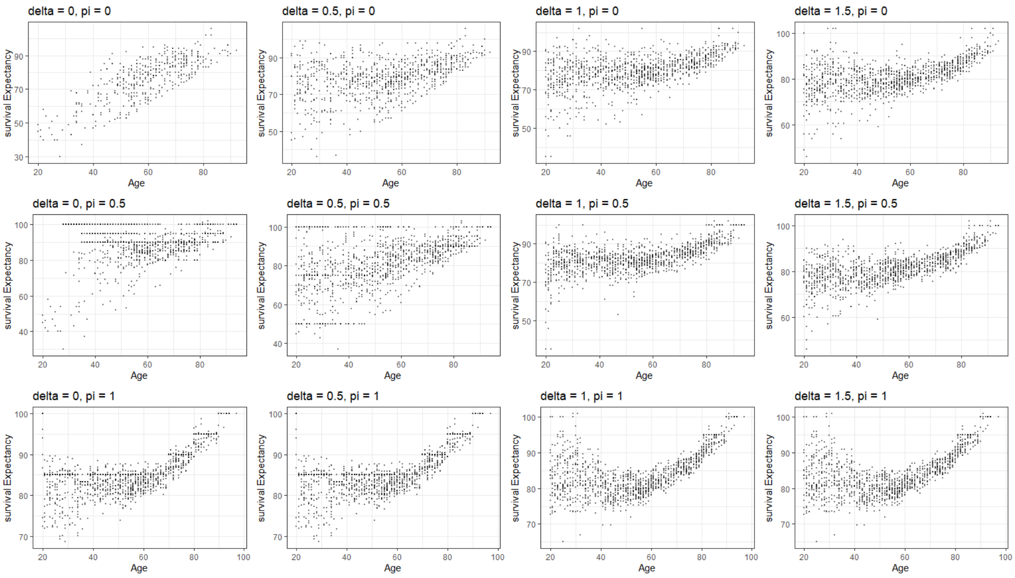


Figure 1.6: Parameter values π and δ

We run several simulations with different parameter values 100 times each to find a combination of parameter values which reproduces the survival expectation of the SCF survey respondents. We first estimate the combination with the mean squared deviation of the simulated result and the data. Then, a squared difference of the proportion of agents at each age x for each subjective survival expectation e_x from 100 simulations accumulated is calculated as in Equation 1.6.

$$\text{MSD} = \sum_{x=20}^{100} \Bigg(\sum_{e_x=20}^{120} \big(\frac{I(e_x)}{I(x)} - \frac{J(e_x)}{J(x)}\big)^2 \Bigg). \tag{1.6}$$

$I(x)$ is the number of agents from data whose age is x, and $I(e_x)$ denotes the number of agents from data whose survival expectation age at x is e_x. Similarly, $J(x)$ is the number of simulated agents with age x, and $J(e_x)$ is the number of simulated agents with survival expectation e_x at age x. Our strategy involves finding a combination of parameter values which minimizes the mean squared difference (MSD), starting from a coarse grid of parameter values and refining the grid successively. The coarse grid uses the parameter values in Equation 1.7.

$$\delta \in \{0, 0.25, 0.5, 1\}, \pi \in \{0, 0.25, 0.5, 1\}, \kappa \in \{1, 5, 10\}. \tag{1.7}$$

Table 1.2 suggests that the ten smallest MSD values are derived when π is between 0 and 0.25. Therefore, we create the second grid, which is finer than the first grid as in Equation 1.8. Finally, we make the grid even finer and test the combination in Equation 1.9

Table 1.2: 10 smallest MSD values from the first grid

MSD	δ	π	κ
11.1784	0.00	0.25	5
11.2997	0.25	0.25	5
11.9719	0.50	0.25	5
12.1117	0.50	0.00	1
12.1117	0.50	0.00	5
12.1117	0.50	0.00	10
12.8319	0.25	0.25	10
12.8340	1.00	0.00	1
12.8340	1.00	0.00	5
12.8340	1.00	0.00	10

$$\delta \in \{0.05, 0.1, 0.15, 0.2\}, \pi \in \{0.05, 0.1, 0.15, 0.2\}, \kappa \in \{5\}. \tag{1.8}$$

Table 1.3: 10 smallest MSD values from the second grid

MSD	δ	π	κ
9.87833	0.20	0.20	5
10.05988	0.20	0.10	5
10.06937	0.20	0.15	5
10.67628	0.20	0.05	5
10.74190	0.05	0.15	5
10.87702	0.15	0.20	5
10.91202	0.10	0.15	5
10.94591	0.05	0.20	5
11.19107	0.15	0.15	5
11.19196	0.05	0.10	5

$$\begin{aligned} &\delta \in \{0.12, 0.14, 0.16, 0.18, 0.2, 0.22, 0.24\}, \\ &\pi \in \{0.12, 0.14, 0.16, 0.18, 0.2, 0.22, 0.24\}, \kappa \in \{5\}. \end{aligned} \tag{1.9}$$

Table 1.4: 5 smallest MSD values from the third grid

MSD	δ	π	κ
9.87534	0.12	0.14	5
9.87833	0.20	0.20	5
9.90828	0.20	0.18	5
9.96288	0.20	0.22	5
10.00447	0.20	0.14	5

Table 1.4 shows that our best estimates of the parameters, which yield the smallest MSD, are $\delta = 0.12, \pi = 0.14$ and $\kappa = 5$. The second validation uses the two-dimensional Kolmogorov-Smirnov test(the Fasano–Franceschini test) developed by Fasano & Franceschini (1987) on the best estimates of the parameters that yield the smallest MSD results from Table 1.4. According to Ness-Cohn & Braun (2021), the two-dimensional two-sample Kolmogorov-Smirnov test could be used to evaluate the null hypothesis that two random samples of any dimension were drawn from the same underlying probability distribution.

Table 1.5: Test statistics of the Fasano–Franceschini test

MSD	δ	π	κ	D-statistics	P-value
9.87534	0.12	0.14	5	2	0.059
9.87833	0.20	0.20	5	20.857	0.208

Our best estimation of the combinations derives the smallest MSD value when $\delta = 0.12, \pi = 0.14$ and$\kappa = 5$. The Fasano–Franceschini test suggests that the simulated result from the combination cannot reject the null hypothesis when the critical point is higher than 5 per cent. In other words, the null hypothesis can be rejected at the significance level of 10 per cent. Therefore, we test the second-best combination of $\delta = 0.2, \pi = 0.2$ and$\kappa = 5$, as the MSD value is fairly close to the best combination. We validate that there is no evidence to reject the null hypothesis of equal distribution for the second-best combination of parameter values. We further validate the combination of parameter values visually in our next section.

1.7 Results

The simulation starts in the year 1900 with 2,000 agents and finishes in the year 2009. All agents in this simulation are assumed to be female, as explained earlier. We present the results of 100 simulations combined from the previous section to present a result in this section. Figure 1.7 illustrates the scatter plot of survival expectation from the 100 simulations with parameter values $\delta = 0.2, \pi = 0.2$ and$\kappa = 5$. The figure illustrates the diagonal boundary and horizontal lines at multiples of 5 years old.

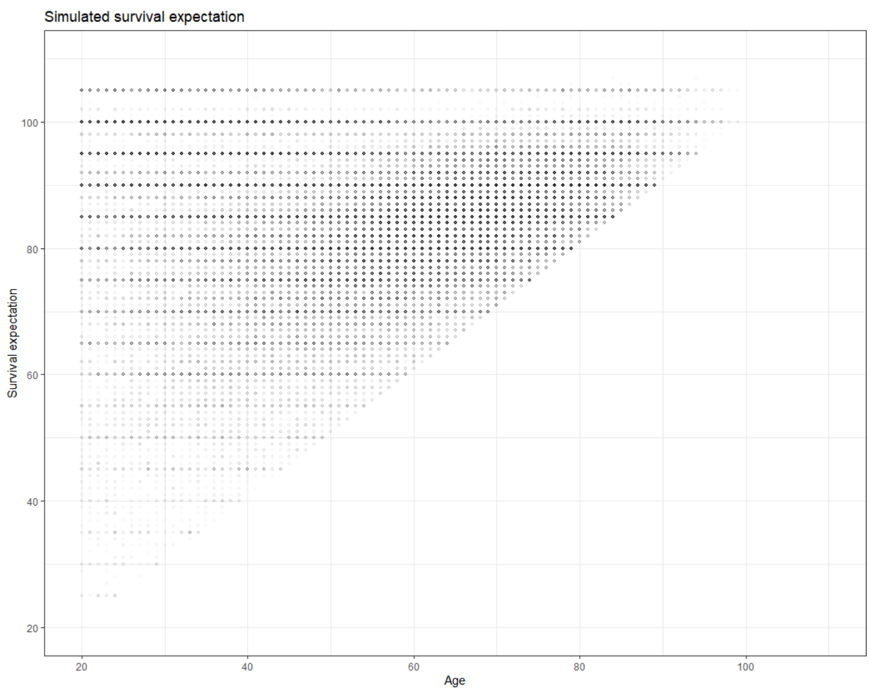
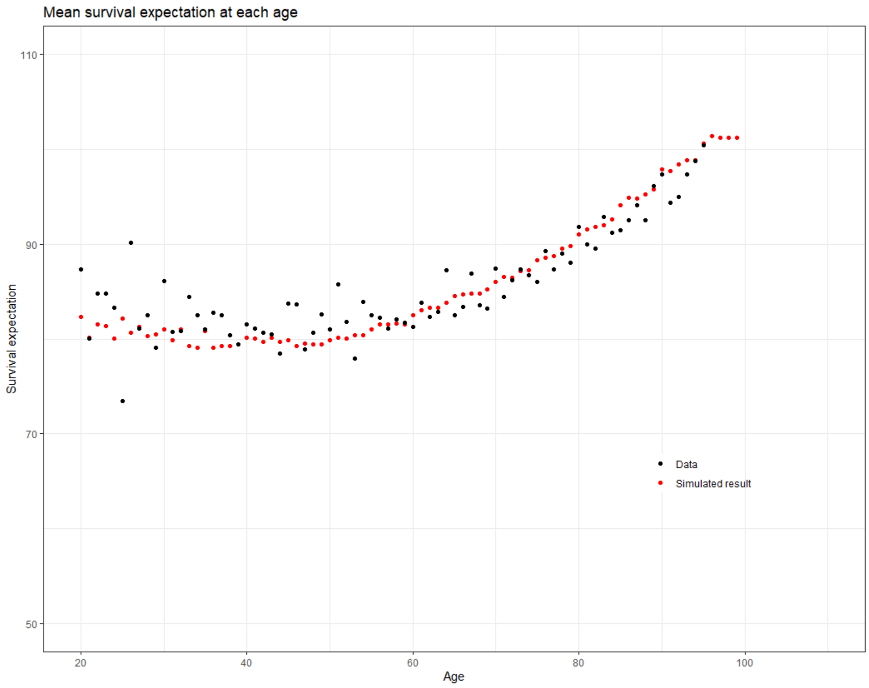


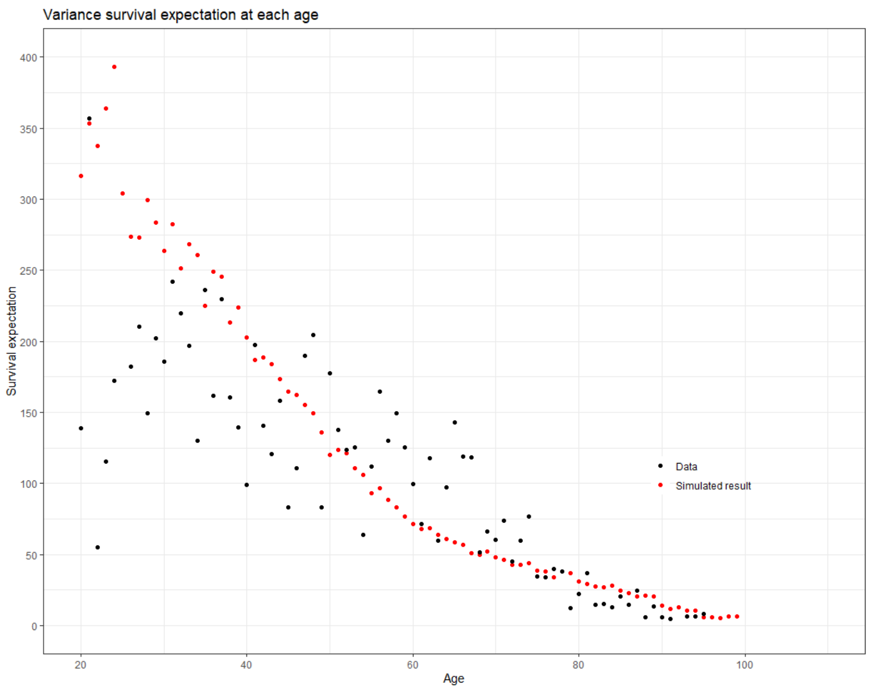
Figure 1.7: Scatter plot of survival expectation from 100 simulations

Figure 1.8 shows means and variances of survival expectations at each age for both survey data and simulation. Figure 1.8a suggests survival expectations at each age of simulated result are close to survival expectations of survey respondents. The result is unsurprising, as the MSD values suggest the difference between the simulated result and the data are not large. On the other hand, the plot of variances, Figure 1.8b, is interesting. The data and the simulated result show a decrease in variance over age. The decrease in variance suggests agents have a smaller range of death ages, and this reduces the variance of survival expectations.

Figure 1.9 illustrates the distributions of survival expectations for all



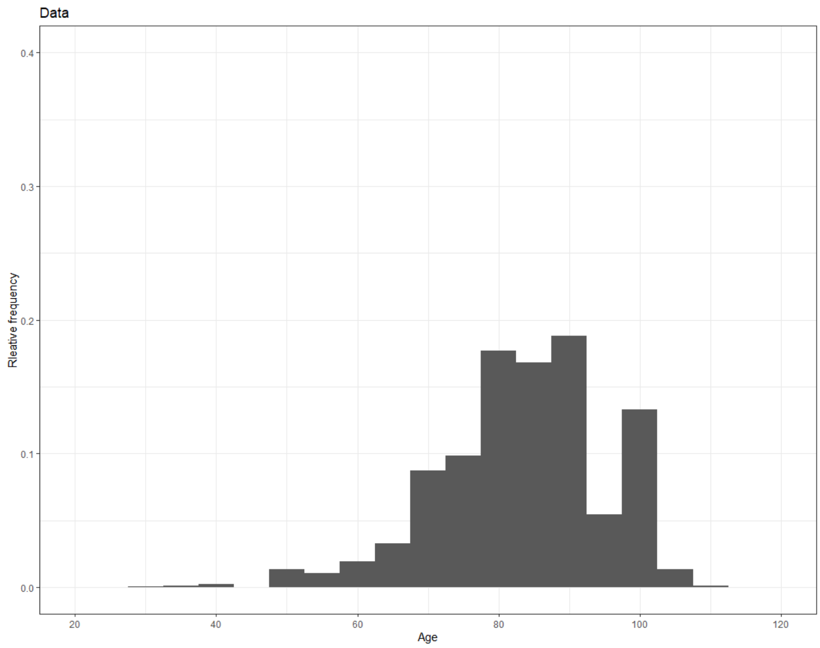
(a) Mean of survival expectation at each age



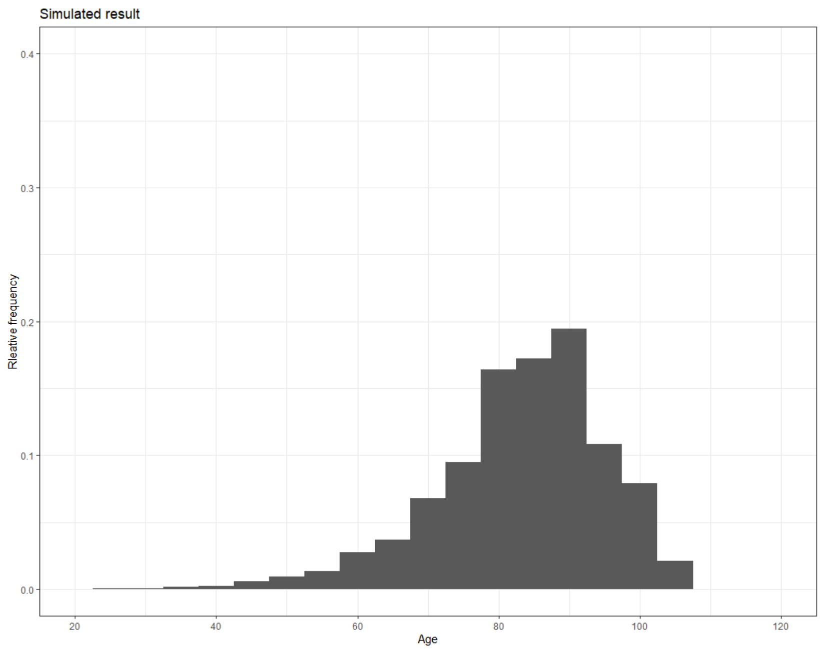
(b) Variance of survival expectation at each age

Figure 1.8: Comparison of mean and variance from simulation and data.

agents regardless of age from the simulation and the data. The distribution of survival expectation in Figure 1.9 shows that the distribution derived



(a) Histogram from the data



(b) Histogram from the simulation

Figure 1.9: Histograms of survival expectation at all ages.

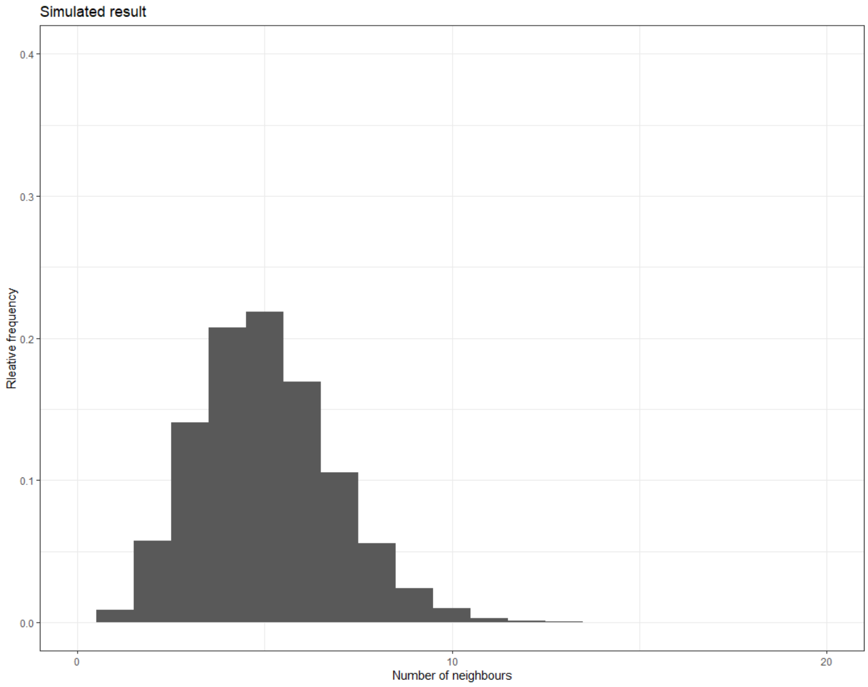
from the simulation is close to the distribution from the survey data. The histogram of the simulation exhibits patterns which can be observed in the histogram of the data: such as left skewness and mode around 80 to 90 years old. Finally, Figure 1.10 demonstrates the distribution of the number of network neighbours of simulated agents. Figure 1.10a shows the total number of network neighbours agents have, and Figure 1.10b illustrates the number of dead network neighbours observed. The modal number of network neighbours per agent is five in Figure 1.10a. This is smaller than the number of edges in real networks. For example, according to Stanford Network Analysis Project[3] (SNAP) data, nodes from Facebook have 21.84 edges on average. However, a smaller number of network neighbours or the number of death observations does not necessarily mean agents have smaller networks. It could suggest that people are updating subjective survival expectations using observations from a small but influential group of network neighbours.

The simulation result has shown that the survival expectation of people can be induced by observations of the death age of others, and our model replicates the survival expectation from the SCF data. The network size required to reproduce realistic survival expectations appears smaller than the real social network. But this could suggest people are selective in choosing samples when updating important information.

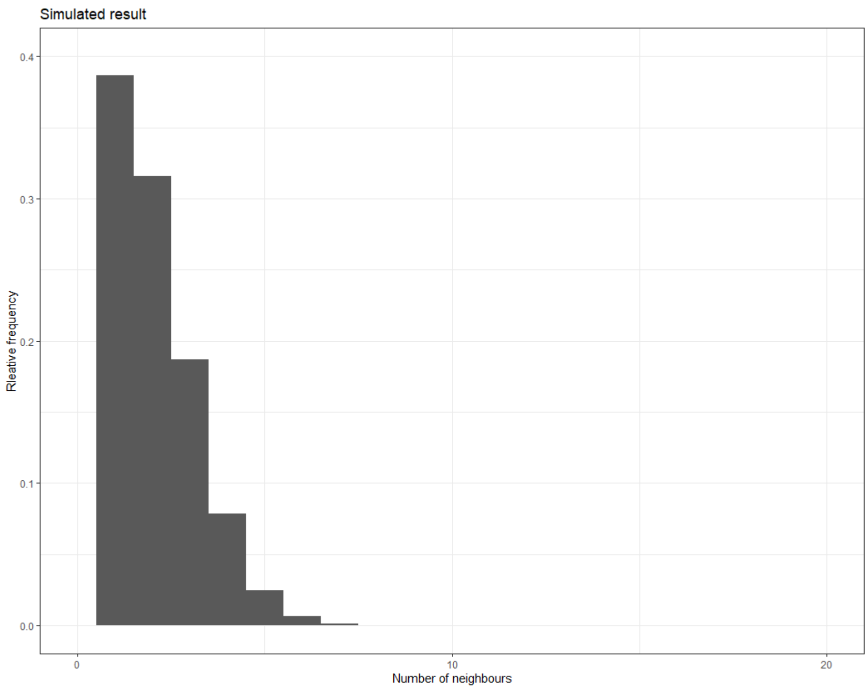
1.8 Conclusion

We build a model in which agents observe the deaths of other agents and update their subjective survival beliefs. The agent's simulation period is 110 years, so all agents can observe other agents' deaths. We ran 4,800, 1,600, and 4,900 simulations with three different grids of parameter combinations in the parameter estimation stage, where we use actual survey data on survival expectations. Our model was also validated against the data using a two-dimensional version of the Kolmogorov-Smirnov test. Simulations of our model demonstrate that observations of neighbours' death ages could be one of the primary sources of an individual's subjective survival expectation. Furthermore, two aspects of survival expectation are

[3]https://snap.stanford.edu/index.html



(a) Distribution of the number of network neighbours of each agent



(b) Number of death observed

Figure 1.10: Number of network neighbours.

reproduced in our model: the leading diagonal boundary and reporting behaviour such as rounding of ages to multiples of 5 years.

According to one of the simulations, which reproduces the SCF data, the network size required to reproduce realistic survival expectations appears smaller than the size of the real social network. This could suggest that people are selective about samples they observe to update important information, such as survival expectations.

We believe such selective agents' behaviour plays a vital role in reproducing two key characteristics of survival expectations of the SCF data. First, the leading diagonal boundary is driven by behaviour which removes observed death ages that they surpass to live. The leading diagonal boundary does not appear when agents update survival expectations based on a larger observation. A larger sample will lead to survival expectations close to the actuarial mortality rate. Second, horizontal lines observed in survival expectation of the SCF data result from the high-degree agents with specific ages instead of rounding behaviours. This happens when many agents are only affected by a high-degree agent.

We present an ABM model where agents update survival expectations based on their network structure. The model can be further developed to calculate the optimism of agents, and the optimism can be used to simulate annuity market circumstances. Next chapter, we use the model we presented to simulate an annuity market with a newly introduced index which measures the degree of optimism in relation to the subjective survival belief.

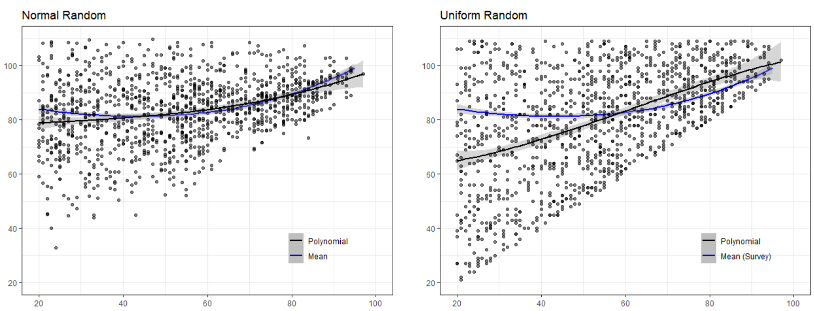
Appendix

We show that survival expectation cannot be reproduced when agents randomly select survival expectation. Two assumptions of randomness are normal random and uniform random. Firstly, it was assumed that agents choose survival expectations based on a normal distribution with the mean of objective life expectancy and a variance of five times the life expectancy. Second, it was assumed that agents pick survival expectations between their age and age 110. Figure 1.11 shows both normal random

Table 1.6: Uniform distribution

Statistic	Uniform	Normal
Res.Df	2,905	2,905
RSS	767,545	464,106
Df	4	4
Sum of Sq	46,344	1,061
F	43.851	1.6603
Pr(>F)	0.000	0.156

and uniform random models have polynomial fitting lines, which are quite deviated from the smoothed mean survival expectation curve of the SCF data. Therefore, it can be concluded that survey respondents did not choose survival expectations randomly.



(a) Normal random (b) Uniform random

Figure 1.11: Test of randomness

1.9 References

Ahern, K. R., Duchin, R. & Shumway, T. (2013), Peer effects in economic attitudes, *in* 'SSRN', Vol. 2085009.

Bissonnette, L., Hurd, M. D. & Michaud, P.-C. (2017), 'Individual survival curves comparing subjective and observed mortality risks', *Health Economics* **26**(12), e285–e303.

Bizjak, J., Lemmon, M. & Whitby, R. (2009), 'Option backdating and board interlocks', *The Review of Financial Studies* **22**(11), 4821–4847.

Bonabeau, E. (2002), 'Agent-based modeling: Methods and techniques for simulating human systems', *Proceedings of the National Academy of Sciences* **99**(suppl 3), 7280–7287.

Booth, H. & Tickle, L. (2008), 'Mortality modelling and forecasting: A review of methods', *Annals of actuarial science* **3**(1-2), 3–43.

Chung, F. & Lu, L. (2002), 'The average distances in random graphs with given expected degrees', *Proceedings of the National Academy of Sciences* **99**(25), 15879–15882.

Duflo, E. & Saez, E. (2002), 'Participation and investment decisions in a retirement plan: The influence of colleagues' choices', *Journal of public Economics* **85**(1), 121–148.

Elder, T. E. (2013), 'The predictive validity of subjective mortality expectations: Evidence from the health and retirement study', *Demography* **50**(2), 569–589.

Erdős, P. & Rényi, A. (1960), 'On the evolution of random graphs', *Publ. Math. Inst. Hung. Acad. Sci* **5**(1), 17–60.

Fasano, G. & Franceschini, A. (1987), 'A multidimensional version of the kolmogorov–smirnov test', *Monthly Notices of the Royal Astronomical Society* **225**(1), 155–170.

Gan, L., Hurd, M. D. & McFadden, D. L. (2005), Individual subjective survival curves, *in* 'Analyses in the Economics of Aging', University of Chicago Press, pp. 377–412.

Grevenbrock, N., Groneck, M., Ludwig, A. & Zimper, A. (2021), 'Cognition, optimism, and the formation of age-dependent survival beliefs', *International Economic Review* **62**(2), 887–918.

Hadwiger, H. (1940), 'Eine analytische reproduktionsfunktion für biologische gesamtheiten', *Scandinavian Actuarial Journal* **1940**(3-4), 101–113.

Hamermesh, D. S. (1985), 'Expectations, life expectancy, and economic behavior', *The Quarterly Journal of Economics* **100**(2), 389–408.

Heimer, R. Z., Myrseth, K. O. R. & Schoenle, R. S. (2019), 'Yolo: Mortality beliefs and household finance puzzles', *The Journal of Finance* **74**(6), 2957–2996.

Hong, H., Kubik, J. D. & Stein, J. C. (2004), 'Social interaction and stock-market participation', *The Journal of Finance* **59**(1), 137–163.

Hurd, M. D. & McGarry, K. (1995), 'Evaluation of the subjective probabilities of survival in the health and retirement study', *Journal of Human resources* pp. S268–S292.

Hurd, M. D. & McGarry, K. (2002), 'The predictive validity of subjective probabilities of survival', *The Economic Journal* **112**(482), 966–985.

Kastenbaum, R. (2000), *The psychology of death*, Springer Publishing Company.

Lee, R. D. & Carter, L. R. (1992), 'Modeling and forecasting us mortality', *Journal of the American statistical association* **87**(419), 659–671.

Lerner, J. & Malmendier, U. (2013), 'With a little help from my (random) friends: Success and failure in post-business school entrepreneurship', *The Review of Financial Studies* **26**(10), 2411–2452.

Lichtenstein, S., Slovic, P., Fischhoff, B., Layman, M. & Combs, B. (1978), 'Judged frequency of lethal events.', *Journal of experimental psychology: Human learning and memory* **4**(6), 551.

Ludwig, A. & Zimper, A. (2013), 'A parsimonious model of subjective life expectancy', *Theory and Decision* **75**(4), 519–541.

Ness-Cohn, E. & Braun, R. (2021), 'Fasano-franceschini test: an implementation of a 2-dimensional kolmogorov-smirnov test in r', *arXiv:2106.10539* .

O'Dea, C., Sturrock, D. et al. (2018), Subjective expectations of survival and economic behaviour, Technical report, Institute for Fiscal Studies.

O'Dea, C. & Sturrock, D. (2021), 'Survival pessimism and the demand for annuities', *The Review of Economics and Statistics* pp. 1–53.

Peracchi, F. & Perotti, V. (2011), Subjective survival probabilities and life tables: Evidence from europe, Technical report, Einaudi Institute for Economics and Finance (EIEF).

Peristera, P. & Kostaki, A. (2007), 'Modeling fertility in modern populations', *Demographic Research* **16**, 141–194.

Puri, M. & Robinson, D. T. (2007), 'Optimism and economic choice', *Journal of Financial Economics* **86**(1), 71–99.

Sacerdote, B. (2001), 'Peer effects with random assignment: Results for dartmouth roommates', *The Quarterly Journal of Economics* **116**(2), 681–704.

Salm, M. (2010), 'Subjective mortality expectations and consumption and saving behaviours among the elderly', *Canadian Journal of Economics/Revue canadienne d'économique* **43**(3), 1040–1057.

Seshadhri, C., Kolda, T. G. & Pinar, A. (2012), 'Community structure and scale-free collections of erdős-rényi graphs', *Physical Review E* **85**(5), 056109.

Simon, H. A. (1955), 'A behavioral model of rational choice', *The Quarterly Journal of Economics* **69**(1), 99–118.

Slovic, P. (1987), 'Perception of risk', *Science* **236**(4799), 280–285.

Smith, V. K., Taylor, D. H. & Sloan, F. A. (2001), 'Longevity expectations and death: Can people predict their own demise?', *American Economic Review* **91**(4), 1126–1134.

Smith, V. K., Taylor, D. H., Sloan, F. A., Johnson, F. R. & Desvousges, W. H. (2001), 'Do smokers respond to health shocks?', *The Review of Economics and Statistics* **83**(4), 675–687.
URL: *http://www.jstor.org/stable/3211762*

Sunstein, C. R. (2002), *Risk and reason: Safety, law, and the environment*, Cambridge University Press.

Tversky, A. & Kahneman, D. (1992), 'Advances in prospect theory: Cumulative representation of uncertainty', *Journal of Risk and uncertainty* **5**(4), 297–323.

Chapter 2

Subjective survival beliefs and annuity demand

2.1 Abstract

Subjective survival beliefs, which represent the degree of optimism that individuals have about when they will die, influence people's decision-making process. Puri & Robinson (2007) find that a measure of optimism positively correlates to numerous decisions, including labour, retirement, marriage, saving and investment. We use data from the Survey of Consumer Finance to test whether subjective survival beliefs affect an annuity purchase. An index which measures the degree of optimism/pessimism in survival expectations based on survival expectation and objective life expectancy is introduced here. The index is calculated in a similar way to the measure of optimism used by Puri & Robinson (2007). We use self-reported survival expectations from the Survey of Consumer Finance and compare them with the implied measure from actuarial life tables to calculate the subjective survival belief index. We find that the index significantly affects annuity purchase decisions when tested. Furthermore, we use an agent-based model simulation to reproduce the survival optimism

index in a simulated population, and we find that this induces low annuity purchase behaviour.

2.2 Introduction

Grevenbrock et al. (2021) assert that underestimation or overestimation of one's lifespan can affect economic decisions. Subjective survival beliefs affect an individual's economic decisions, such as investing in the stock market (Puri & Robinson 2007) to life-cycle saving and investing (Gan et al. 2015, Heimer et al. 2019).

Therefore, throughout the paper, we discuss the relationship between subjective survival beliefs and economic decisions. Furthermore, we introduce an index to measure the degree of over/underestimation of survival expectation to test whether optimism/pessimism affects financial decision-making. In particular, we test the relationship between subjective survival beliefs and the purchase of risk protection products such as annuities and life insurance.

The agent-based model (ABM) that we introduced in the previous chapter reproduces subjective survival beliefs, which are the basis of the index. Therefore, the index can be reproduced by the model. Furthermore, we use data from the Survey of Consumer Finance (SCF) with both males and females for the simulation since an environment where only female agents exist may not be realistic.

2.3 Literature review

2.3.1 Annuity demand

The demand among consumers for risk protection products has not always been high despite the utility of such products. Scholars have documented the mismatch between high theoretical utility values and the low demand for longevity risk protection products. Yaari (1965) demonstrates that full annuitisation is optimal under certain circumstances. He asserted, with fair-priced annuities and no bequest motive, that retirees should annuitise all their wealth upon retirement. However, voluntary purchases of annuities from insurance companies have remained low. In 2007 in the U.S., only $6.5 billion worth of annuity was purchased, while over $300 billion was transferred to Individual Retirement Accounts (Benartzi et al. 2011). Moreover, Inkmann et al. (2010) show that only 5.9% of the UK households hold voluntary annuity with 2002 data. Such suboptimal annuity purchase behaviour is referred to as the annuity puzzle.

There are a handful of explanations for the annuity puzzle. First, Mitchell et al. (1999) and Finkelstein & Poterba (2002) explain the lower-than-suboptimal annuity purchase derived from the lack of actuarially fair annuities. Second, Friedman & Warshawsky (1990) suggest strong bequest motive can be a reason for the suboptimal purchase of an annuity. Third, habit formation can be one explanation as consumers' consumption habits can prevent the purchase of annuity (Davidoff et al. 2005). Fourth, Bernheim (1991), Brown (2001) and Dushi & Webb (2004) claim that the existence of social security plans and private DB plans can reduce the demand for an annuity. Finally, Milevsky & Young (2002) argue that buying annuity limits household stock market participation; hence, issues regarding flexibility arise.

Scholars have proposed that behavioural biases can be another reason for the annuity puzzle. Cannon & Tonks (2008) presents evidence of framing effects in the annuity market and bounded rationality due to a lack of financial knowledge. Gottlieb (2012) uses prospect theory, and Holmer (2003) and Chen et al. (2019) use cumulative prospect theory, which is a variant of prospect theory, to explain the causes of the low demand for an annuity purchase. Chen et al. (2022) find the implication of the hyperbolic discount model for the annuitisation decision. Finally, Nenkov et al. (2016) suggests a novel interpretation of mortality salience. Mortality salience refers to the exhibition of psychological biases when people encounter fear of death. Experiments of Nenkov et al. (2016) uncover evidence that death-related thought affects financial decisions in later life and that consideration of the annuity product itself conjures thoughts of death.

2.3.2 Subjective survival belief and annuity demand

Subjective survival belief is a plausible feature that captures the behavioural biases listed above: bounded rationality, the cumulative prospect theory Tversky & Kahneman (1992) and mortality salience. Subjective survival belief often involves bounded rationality. People estimate subjective survival beliefs with limited information and make decisions with imperfect estimation. The cumulative prospect theory Tversky & Kahneman (1992) explains that people tend to overweight unlikely events while underweight average events. People exhibit similar patterns when they estimate subjective survival beliefs. Finally, mortality salience affects subjective survival belief as mortality salience leads to an overestimation of low probability at an older age.

Subjective survival belief affects saving and investing decisions to varying degrees. Puri & Robinson (2007) show that subjective survival belief can predict necessary economic behaviour, including stock market participation. Moreover, retirement decisions can be affected by subjective survival beliefs. For example, Van Solinge & Henkens (2009) show that subjective life expectations affect workers' retirement age. Hurd et al. (2004) find that people with low subjective survival beliefs are more likely to retire earlier and claim social security benefits earlier. Gan et al. (2015) illustrates that subjective life beliefs instead of actuarial life tables fit better when estimating a life-cycle model with a bequest. The study of Bateman et al. (2018) find evidence of subjective lifespan affecting annuity decision. Heimer et al. (2019), similarly, show that using subjective mortality can better explain under-saving for retirement and the slow decumulation of wealth towards the end of life when solving the life-cycle model. Further, O'Dea & Sturrock (2021) claim that under annuitisation, behaviour can be explained by subjective survival beliefs.

As described earlier, some researchers try to explain the suboptimal annuity purchasing behaviour using subjective survival belief. However, the method they employ to estimate subjective survival beliefs has some disadvantages. Heimer et al. (2019), for example, ask survey respondents about their 1, 2, 5, and 10-year survival beliefs and use the resulting four subjective survival probabilities to estimate the subjective survival curve. O'Dea & Sturrock (2021) use a similar approach, but they use responses from the English Longitudinal Study of Ageing (ELSA) and the ELSA respondents report two subjective survival probabilities. We contend that this method is not intuitive enough when surveying people about their mortality. In other words, we argue that people are likely to think about how many more years they will survive instead of what survival probability they have. Therefore, we analyse data from the U.S. Survey of Consumer Finance[1] (SCF) where subjective survival expectations are recorded as an age. So life insurance and annuity purchase behaviour can be studied in relation to subjective survival beliefs.

2.4 Data

The 2010 SCF data collects demographic and economic information from respondents. Survey questions include survival expectations, measured

[1]https://www.federalreserve.gov/econres/scfindex.htm

in terms of expected age at death and ownership of annuities other than state or workplace pension or term-life insurance policies. Therefore, the relationship between subjective survival belief and insurance-holdings can be tested. The characteristics of the survey respondents in which we are interested are reported in Table 2.1. Only 6.6% of survey respondents are annuity-holders, while over half of the respondents hold term-life insurance.

Table 2.1: Characteristics of the 6,482 respondents to the SCF survey.

Statistic	Mean	St. Dev.	Min	Max
Survival expectations	83.398	11.971	30	150
Proportion of annuity-holders	0.066	0.248	0	1
Proportion of term life lders	0.498	0.500	0	1
Objective life expectancy at age x	30.750	13.264	2.810	63.740
Subjective life expectancy at age x	32.708	17.790	0	126

2.5 Model description

In this section, we provide an index to measure subjective survival belief or survival optimism. The index that we introduce is a modified version of the measure used by Puri & Robinson (2007). After this, we adopt the relationship between the index and insurance purchase behaviour to create an ABM model which simulates the annuity market for the maximum period possible, 35 years, with the current data.

2.5.1 Optimism index λ_i

Puri & Robinson (2007) use life expectancy miscalibration as a measure of optimism. They use self-reported survival expectation to calculate subjective remaining lifespan and compare it to objective remaining lifespan from actuarial tables. The measure of optimism, in their notation, equals

$$\text{Optimism}_i = E_r(l|x) - E_a(l|x), \tag{2.1}$$

where $E_r(l|x)$ is individual i's subjective expectations of remaining lifespan l given condition x, a vector of personal characteristics. $E_a(l|x)$ is the expectation of l taken from an actuarial table given condition x.

We introduce an index λ_i to measure optimism or degree of under/overestimation implicit in subjective survival belief for an individual i. λ_i is similar to the optimism measure of Puri & Robinson (2007), but we formulate the degree of optimism in terms of multiplication. The new measure in our notation is

$$\lambda_i = \frac{e_{x_i}}{e_x}, \tag{2.2}$$

where e_x is the standard actuarial notation of the curtate expectation of life at age x and e_{x_i} is the remaining lifespan for aged x individual i, which is survival expectation minus current age as Puri & Robinson (2007). We refer to e_{x_i} as subjective life expectancy from this paper. Although λ_i changes over time as e_x changes over time, we do not index λ_i with time as our simulation results will be mostly based on a single point of time. The measure of Puri & Robinson (2007) represents optimism/pessimism when the measure is positive/negative. On the contrary, our index indicates optimistic survival beliefs when $\lambda_i \geq 1$, while pessimism when $\lambda_i \leq 1$. λ_i cannot be smaller than 0 as the survival expectation from the SCF data is at least the age of the respondents.

Figure 2.1 shows the distribution of λ_i for each cohort. The distributions for younger cohorts are left-skewed. This is because right tails become longer as we consider older cohorts. The oldest cohort, aged 89 to 99, has an almost flat line spread from 0 to 3. The longer right tails for the older cohorts confirm the hypothesis of optimism at older ages. In other words, older cohorts overestimate the probability of surviving.

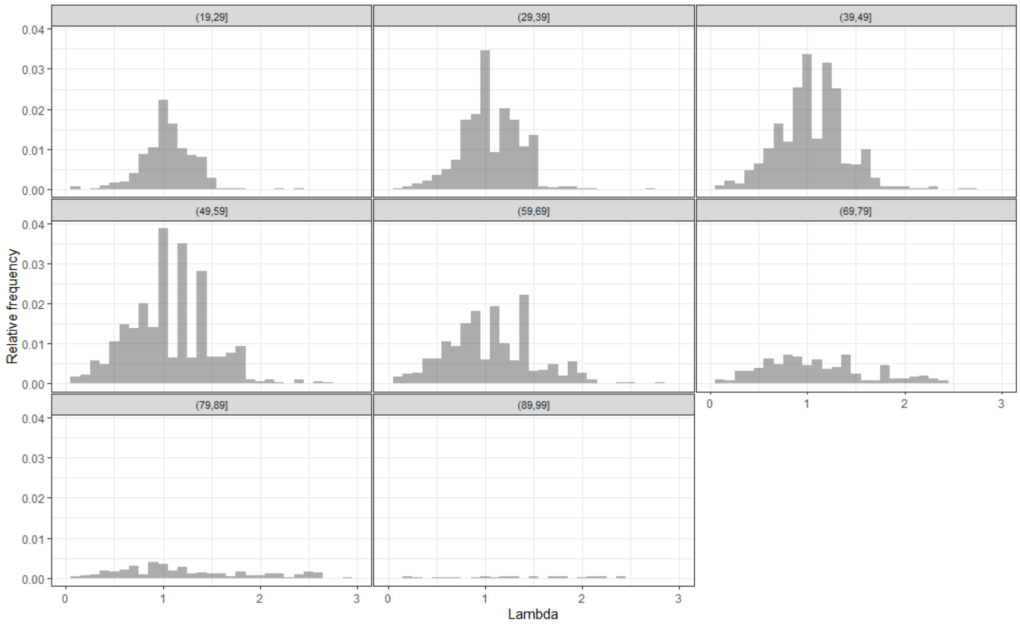


Figure 2.1: Histograms of calculated λ_i by cohort. (Source: The SCF, 2010)

2.5.2 Annuity market simulation

In the previous chapter, we presented an ABM where agents observe the death of neighbours within a network and establish survival expectations based on this information. We first use the model from the previous chapter to form survival expectations. We then employ the simulated survival expectations to simulate an annuity market. In the previous chapter, we used only female agents to simulate life expectations. However, in this chapter, male agents are included to simulate an annuity market. The ratio of male to female agents in the simulation is one-to-one. The simulation begins with the same number of male and female agents. Female agents give birth to male and female babies with equal chances. Therefore, the fertility rate is doubled from the previous paper. Other than this, we use the same model and the parameter values in the last chapter.

ABM again is employed to simulate an annuity market. All agents in the model purchase annuity each year with some probabilities. We run several simulations to determine what value of $P(A)$, where A is the event of purchasing an annuity, can reproduce realistic annuity market features.

The probabilities of individuals purchasing annuities are based on the optimism index as evidence of subjective survival beliefs affecting annuity purchases are reviewed in the previous section. Furthermore, we find evidence that gender plays a significant role in purchasing annuities, thus, we employ gender in our annuity market simulation.

2.6 Model calibration and validation

In this section, we show the distribution of the index by age using the SCF data in 2010 to determine if people become optimistic about survival as they age and whether the index affects the purchase of two different risk protection products: annuity and life insurance. Moreover, we introduce probability as a function of the optimism index and gender which we then use to simulate an annuity market.

2.6.1 Optimism index and insurance holdings

The histograms of λ_i for all agents, annuity-holders only, and term life insurance holders only are drawn as in Figure 2.2. The figure visually depicts the relationship between annuity and term life insurance purchases prior to statistical tests. The shape of the histogram of all agents and the histogram of term life holders is almost indistinguishable. However, the shape of the histogram for annuity-holders only, in the middle panel of Figure 2.2, is flatter and more right-skewed than in the left and right panels. The histograms suggest that annuity-holders generally have a greater value of λ_i than other survey respondents.

We carry out another visual check using Figure 2.3. The box-and-whisker plot on the left-hand side shows an average λ_i of annuity-holders, represented by 1 on the horizontal axis, which is slightly higher than individuals without any annuity, represented by 0 on the horizontal axis. Also, the

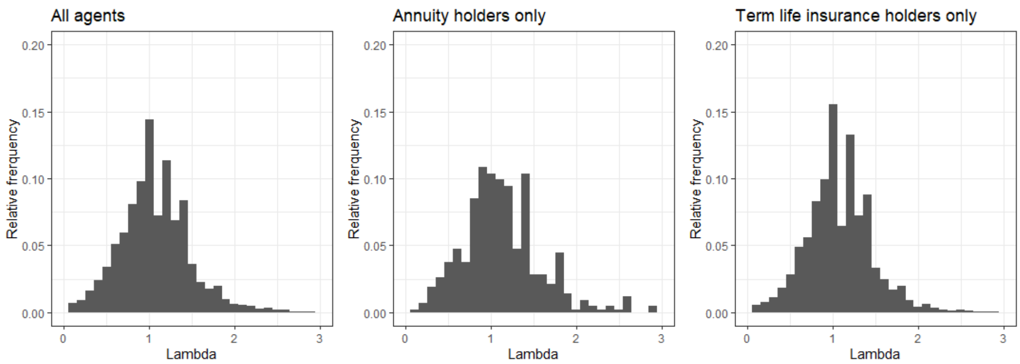


Figure 2.2: Histograms of calculated λ_i for all agents, annuity-holders only, and term life lders only. (Source: The SCF, 2010)

third and fourth quartiles of optimism index λ_i for the annuity-holders are slightly higher than their counterparts without annuity. On the contrary, it is difficult to conclude that the average λ_i for the holders of term-life insurance, represented by 1 on the horizontal axis of the right panel of Figure 2.3, is higher than the average λ_i for their counterparts without term life insurance. Term life insurance-holders with an optimism index close to 0 are more prevalent compared to annuity-holders with the same optimism index close to 0.

Visual checks on the box-and-whisker plots do not provide clear evidence to support that λ_i plays a significant role in either an annuity or a life insurance purchase. Therefore, this leads us to perform a logistic regression.

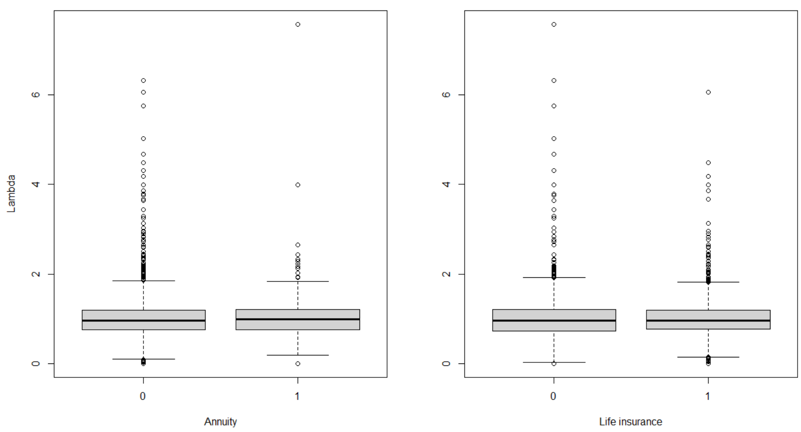


Figure 2.3: Box-and-whisker plots of annuity and life insurance-holder by calculated λ_i. (Source: The SCF, 2010)

Table 2.2 presents the result for a logistic regression of the purchase of annuities of term life insurance on the optimism index λ_i and gender (0= female, and 1 = male). The result suggests gender and λ_i are significant factors of annuity purchase. On the other hand, λ_i does not have a significant effect on a life insurance purchase. The logistic regression coefficient of λ_i is 0.223 with a standard error of 0.086, and the coefficient of gender is -0.281 with a standard error of 0.129 when the dependent variable is possession of annuity. We interpret the result as a 1 unit increase in λ_i will increase the chance of annuity purchase by 1.25, ($e^{0.223}$), times. Gender predicts both term life insurance and annuity purchase. A female has 24.5% lower chance to purchase an annuity and 45% lower chance to buy term life insurance than a male.

Subjective survival belief plays a significant role in an annuity purchase but is not a significant factor in a term life insurance purchase. Annuity-holders anticipate longevity and purchase annuities to protect against this. On the other hand, term life insurance-holders do not expect a shorter lifespan but instead may be preparing for an accidental death.

Table 2.2: Logistic regressions of annuity purchase (third column) or term-life insurance purchase (fourth column) on the optimism index and gender (0 = female, 1 = male). (Source: The SCF, 2010)

		Dependent variable:	*Dependent variable:*
		Annuity	Term-life
λ_i	coefficient	0.223^{***}	-0.017
	std.err	(0.086)	(0.051)
gender	coefficient	-0.281^{**}	-0.594^{***}
	std.err	(0.129)	(0.061)
Constant	coefficient	-2.843^{***}	0.147
	std.err	(0.115)	(0.064)
Observations		6,482	6,482
Log Likelihood		$-1{,}567.957$	$-4{,}443.625$
Akaike Inf. Crit.		3,141.915	8,893.251
Note:			*p<0.1; **p<0.05; ***p<0.01

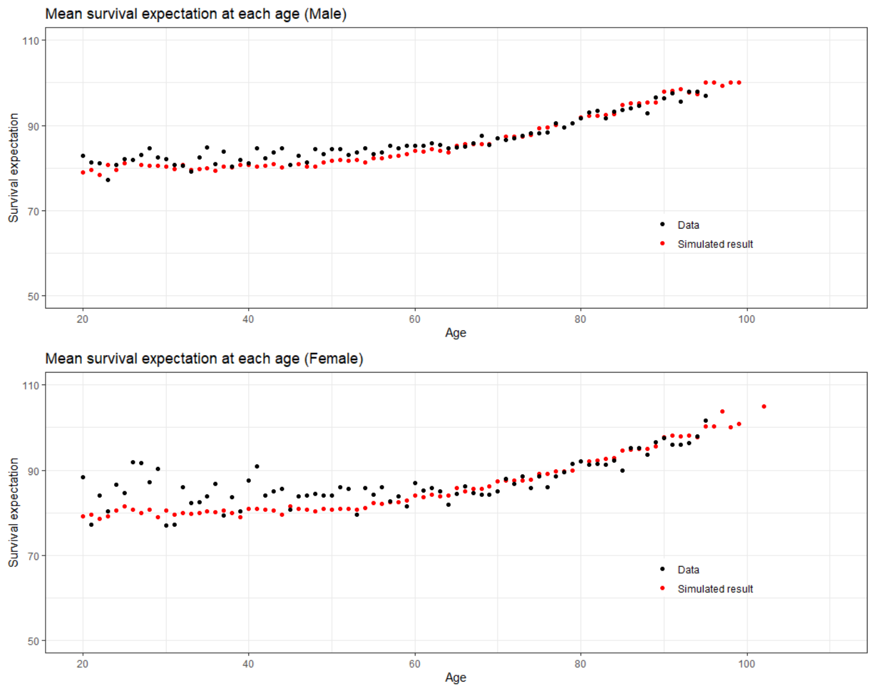
2.6.2 Parameters for survival expectations simulation

We run simulations with 2,000 agents. A combination of parameter values of $\delta = 0.12, \pi = 0.14, \kappa = 5$ which derived the highest mean squared deviation from the previous chapter. The mean squared deviation is slightly different, as males are included. Although we postulate that males and females have different weighting when selecting samples, we use the same parameter values as the estimation of gender-specific weights requires more sophisticated assumptions in family constructions and network creation.

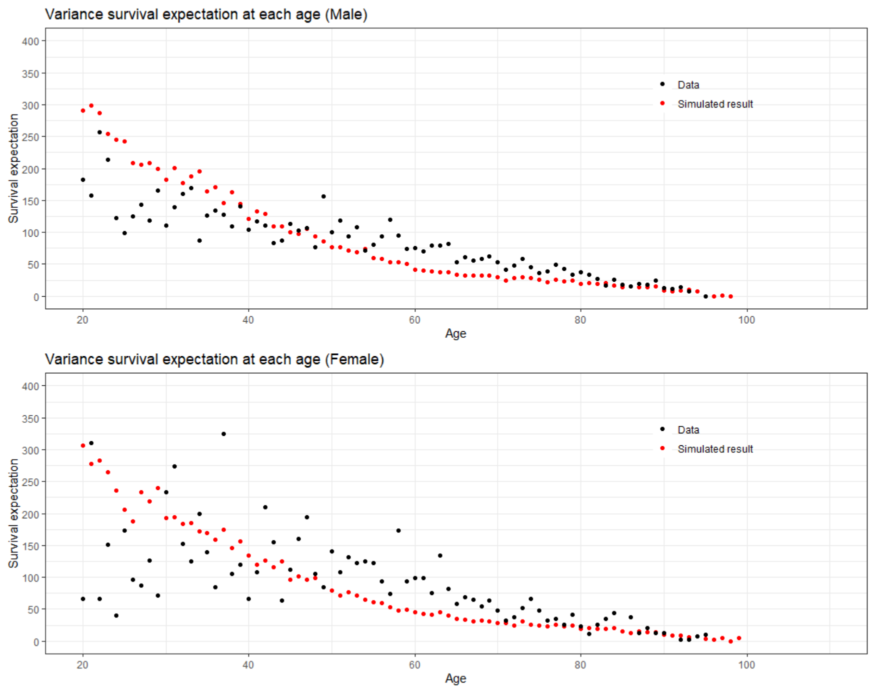
Figure 2.4 shows means (top two panels) and variances (bottom two panels) of the survival expectation at each age for males (first and third) and females (second and fourth). The survey data shows that the mean of female survival expectation is larger than that of male survival expectation, and the variance of female survival expectation is slightly larger than that of male survival expectation. This could imply that males and females sample differently the observations of ages at death of their network neighbours, or that they sample differently individuals of opposite gender to them. O'Dea et al. (2018) show father's age of death has a bigger impact on the subjective survival belief of a male, while mother's age of death has a more significant impact on the subjective survival belief of a female. Including such behaviour and identifying more factors affecting subjective survival belief are too complicated at this stage. Therefore, we leave further model refinements as future work and use the current model.

2.6.3 Probability for annuity market simulation

We assume the probability $P(A|i)$ of individual i purchasing an annuity is a function of gender and optimism index λ_i. Table 2.3 shows the logistic



(a) Mean of survival expectation(male& female)



(b) Variance of survival expectation(male& female)

Figure 2.4: Mean and variance of survival expectation for both male and female

regression result for the relationship between annuity possession and predictors in the odds ratio(OR). The OR measures the strength of the association between an outcome and an exposure.

Table 2.3: Results of logistic regression for annuity purchase. (Source: the SCF 2010)

		crude OR (95%CI)	adj. OR (95%CI)	P-value (Wald's test)	P-value (LR-test)
λ_i		1.27 (1.08,1.51)	1.25 (1.06,1.48)	0.01	0.014
gender		0.73 (0.57,0.94)	0.75 (0.59,0.97)	0.03	0.026
Observations	6,482				
Log Likelihood	$-1{,}567.957$				
Akaike Inf. Crit.	3,141.9148				

For example, the OR for λ_i is 1.25 and the OR for gender is 0.75. An increase in λ_i by one unit will increase the odds of having an annuity by 1.25 times, and a decrease in one unit of gender, male to female, will decrease the odds of having an annuity by 0.75 times. Using the OR, we estimate the probability of purchasing an annuity $P(A)$ and the probability of purchasing an annuity given characteristics i $P(A|i)$ as

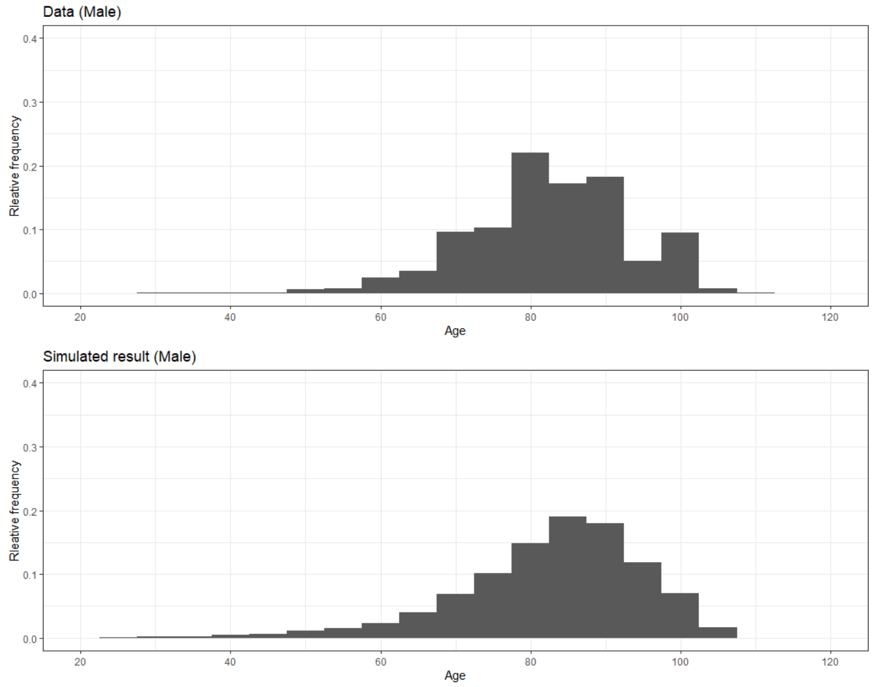
$$P(A|\mathrm{i}) = 0.75^{g} \cdot 1.25^{\lambda_i} \cdot \frac{P(A)}{1 - P(A)} \tag{2.3}$$

where $\frac{P(A)}{1-P(A)}$ is the odds of purchasing an annuity across the whole population.

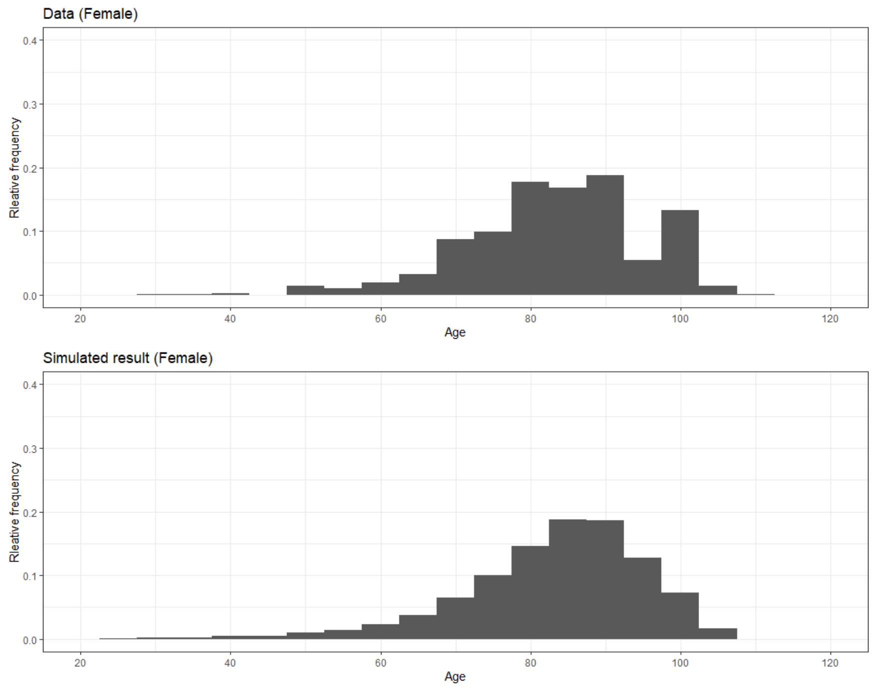
2.7 Results

Figure 2.5a and Figure 2.5b illustrate the histograms of survival expectations for all agents regardless of age from the 100 simulations and the data. The figures show that the simulated mean and variance of survival expectation and the figures of simulated survival expectation are close to the data. Therefore, we use the parameter values for an ABM of the annuity market simulation.

Figure 2.6 shows histograms of λ_i from the simulation, and the figure is comparable to Figure 2.1. The simulated figures exhibit similar quality to



(a) Histogram of male survival expectation from data (above) and simulation (below)



(b) Histogram of female survival expectation from data (above) and simulation (below)

Figure 2.5: Histograms of survival expectation for both male and female

the figures of the data. Longer right-tail and flatter shapes at older cohorts are common patterns observed from the simulation and the data.

We test the model outcome with different probabilities, $P(A)$ and $P(A|\mathrm{i})$

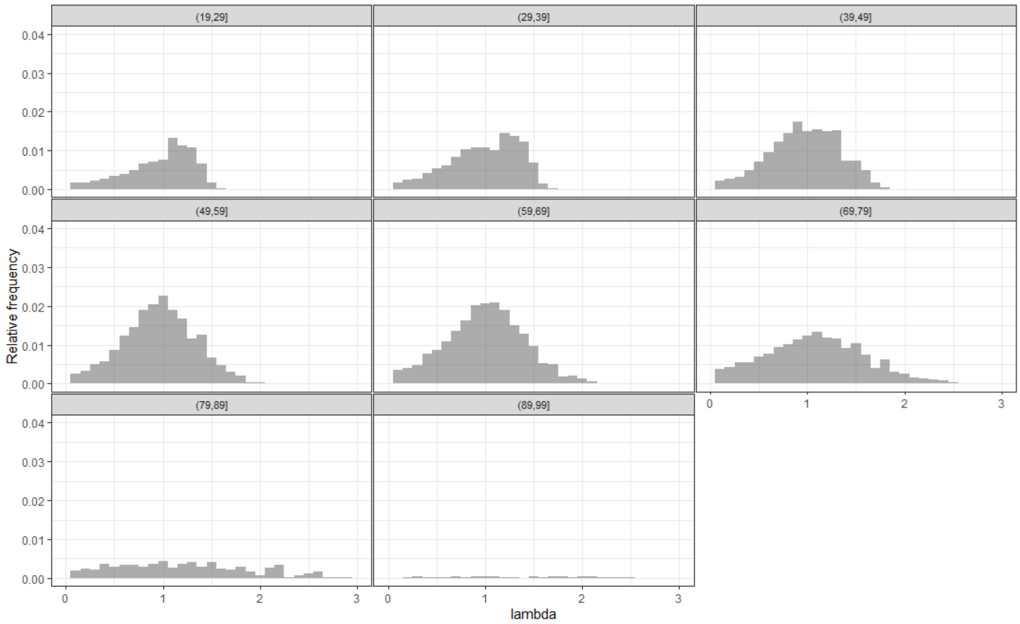


Figure 2.6: Histograms of λ_i by cohorts from the 100 simulations

by using the ABM developed in the previous chapter. We simulate an annuity market when agents are assumed to purchase an annuity with the probability, $P(A|\mathrm{i})$, each year. We use different values of $P(A)$ for every 100 simulations to create agents with annuities and calculate the proportion of annuity-holders each year. For annuity market simulation, we use the period from 1985 to 2020 as the minimum period of simulation required before the annuity market is 85 years. The simulation starts from 1900 for agents to learn survival expectations from the death of network neighbours. When the learning period is too short, there are not enough old agents to observe death from. For example, the oldest agent in 1965 is 85 years old, according to the settings of the simulation.

The proportion of annuity-holders with different probabilities is illustrated in Figure 2.7. We speculate that the increasing trend will slow down after a few years as annuity-holders start to die. Therefore, we expect the annuity-holders' proportion to stop growing after 2010. We reproduce the proportion of annuity-holders of the data, 6.6%, when the probability of an individual purchasing annuity each year, $P(A)$, is between 0.0075 and

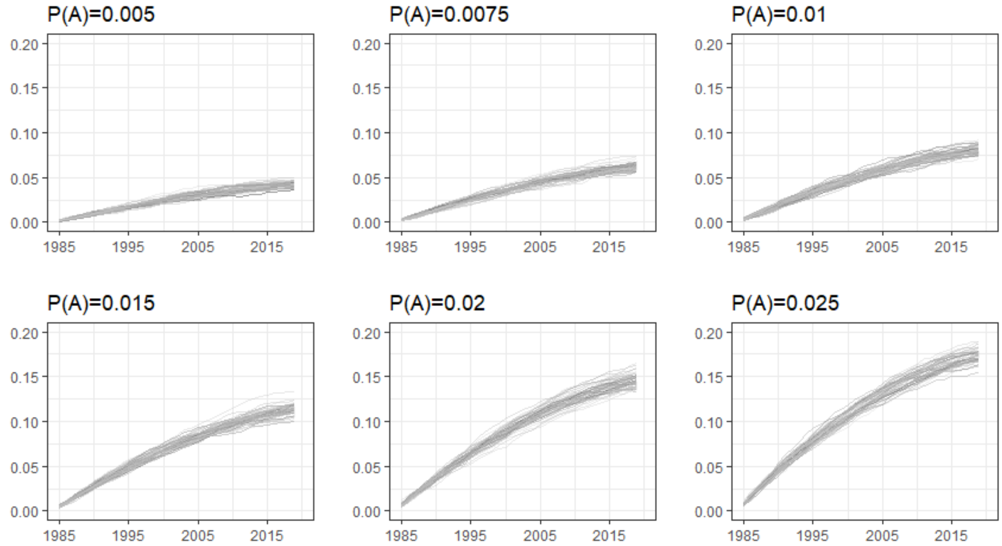


Figure 2.7: 100 simulations with $P(A)$ = 0.005, 0.0075, 0.01, 0.015, 0.02, 0.025 from 1985 to 2020

0.01.

Finally, Figure 2.8 illustrates the optimism indices for annuity-holders are higher than all agents. Furthermore, the simulated age structure of annuity-holders (bottom right panel) has a similar range similar modal age as the actual annuity-holders in 2010 (top right panel) and for both simulated agents and survey respondents. The age structure could have been even closer if simulated agents were allowed to purchase annuities a little bit earlier. Such results from the figure support individuals' probability of purchasing an annuity with odds ratio, $P(A|\mathrm{i})$, have some effects on the proportion of annuity-holders even on the small probability of 0.01. The histograms present that agents with annuities are slightly optimistic in both simulation and survey data.

2.8 Conclusion

In this chapter, we present an index which measures the degree to which an individual miscalibrates his survival expectation. The index λ_i measures the degree to which individual i overestimates subjective survival beliefs,

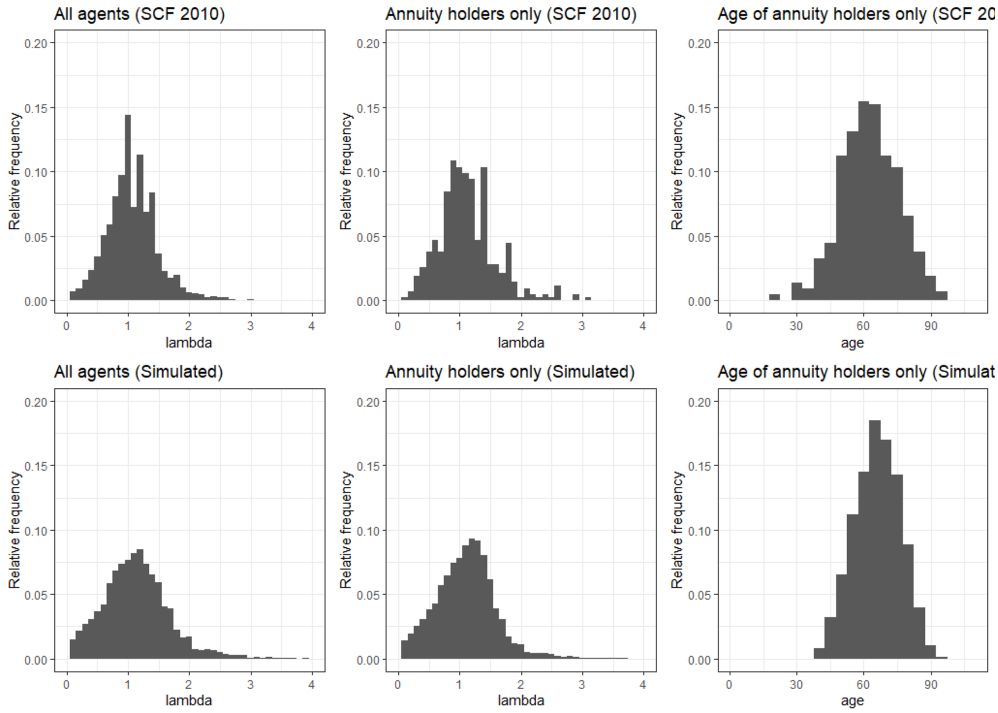


Figure 2.8: Comparison of histograms of data and simulation

i.e. the extent to which he is optimistic about survival. We show that λ_i has a significant effect on decisions to purchase an annuity. Also, the distribution of the optimism index for simulated agents is derived from the ABM simulation and this has some common characteristics to the distribution of the optimism index for the survey respondents.

Finally, simulations of the annuity market with male and female agents based on information regarding λ_i are conducted. The simulation results suggest that individuals are unlikely to buy an annuity each year. We reproduce the proportion of annuity-holders when individuals purchase annuities with the probability of less than one per cent each year. We have not discovered underlying components of probability $P(A)$. In future studies, we can further develop the probability, including extra variables such as interest rate, annuity rate and utility.

2.9 References

Bateman, H., Eckert, C., Iskhakov, F., Louviere, J., Satchell, S. & Thorp, S. (2018), 'Individual capability and effort in retirement benefit choice', *Journal of Risk and Insurance* **85**(2), 483–512.

Benartzi, S., Previtero, A. & Thaler, R. H. (2011), 'Annuitization puzzles', *Journal of Economic Perspectives* **25**(4), 143–64.

Bernheim, B. D. (1991), 'How strong are bequest motives? evidence based on estimates of the demand for life insurance and annuities', *Journal of political Economy* **99**(5), 899–927.

Brown, J. R. (2001), 'Private pensions, mortality risk, and the decision to annuitize', *Journal of Public Economics* **82**(1), 29–62.

Cannon, E. S. & Tonks, I. (2008), *Annuity markets*, Oxford University Press.

Chen, A., Haberman, S. & Thomas, S. (2019), 'Cumulative prospect theory and deferred annuities', *Review of Behavioral Finance* **11**(3), 277–293.

Chen, A., Haberman, S. & Thomas, S. (2022), 'Adaptive retirement planning, sustainable withdrawals, and deferred annuities', *The Journal of Retirement* .

Davidoff, T., Brown, J. R. & Diamond, P. A. (2005), 'Annuities and individual welfare', *American Economic Review* **95**(5), 1573–1590.

Dushi, I. & Webb, A. (2004), 'Household annuitization decisions: Simulations and empirical analyses', *Journal of Pension Economics & Finance* **3**(2), 109–143.

Finkelstein, A. & Poterba, J. (2002), 'Selection effects in the united kingdom individual annuities market', *The Economic Journal* **112**(476), 28–50.

Friedman, B. M. & Warshawsky, M. J. (1990), 'The cost of annuities: Implications for saving behavior and bequests', *The Quarterly Journal of Economics* **105**(1), 135–154.

Gan, L., Gong, G., Hurd, M. & McFadden, D. (2015), 'Subjective mortality risk and bequests', *Journal of Econometrics* **188**(2), 514–525.

Gottlieb, D. (2012), 'Prospect theory, life insurance, and annuities'.

Grevenbrock, N., Groneck, M., Ludwig, A. & Zimper, A. (2021), 'Cognition, optimism, and the formation of age-dependent survival beliefs', *International Economic Review* **62**(2), 887–918.

Heimer, R. Z., Myrseth, K. O. R. & Schoenle, R. S. (2019), 'Yolo: Mortality beliefs and household finance puzzles', *The Journal of Finance* **74**(6), 2957–2996.

Holmer, M. R. (2003), 'Simulation analysis of the decision to annuitize pension balances', *Policy Simulation Group* .

Hurd, M. D., Smith, J. P. & Zissimopoulos, J. M. (2004), 'The effects of subjective survival on retirement and social security claiming', *Journal of Applied Econometrics* **19**(6), 761–775.

Inkmann, J., Lopes, P. & Michaelides, A. (2010), 'How deep is the annuity market participation puzzle?', *The Review of Financial Studies* **24**(1), 279–319.

Milevsky, M. A. & Young, V. R. (2002), *Optimal asset allocation and the real option to delay annuitization: it's not now-or-never*, Citeseer.

Mitchell, O. S., Poterba, J. M., Warshawsky, M. J. & Brown, J. R. (1999), 'New evidence on the money's worth of individual annuities', *American economic review* **89**(5), 1299–1318.

Nenkov, G. Y. et al. (2016), 'Solving the annuity puzzle: The role of mortality salience in retirement savings decumulation decisions', *Journal of Consumer Psychology* **26**(3), 417–425.

O'Dea, C., Sturrock, D. et al. (2018), Subjective expectations of survival and economic behaviour, Technical report, Institute for Fiscal Studies.

O'Dea, C. & Sturrock, D. (2021), 'Survival pessimism and the demand for annuities', *The Review of Economics and Statistics* pp. 1–53.

Puri, M. & Robinson, D. T. (2007), 'Optimism and economic choice', *Journal of financial economics* **86**(1), 71–99.

Tversky, A. & Kahneman, D. (1992), 'Advances in prospect theory: Cumulative representation of uncertainty', *Journal of Risk and uncertainty* **5**(4), 297–323.

Van Solinge, H. & Henkens, K. (2009), 'Living longer, working longer? the impact of subjective life expectancy on retirement intentions and behaviour', *European journal of public health* **20**(1), 47–51.

Yaari, M. E. (1965), 'Uncertain lifetime, life insurance, and the theory of the consumer', *Review of Economic Studies* **32**(2), 137–150.

Chapter 3

Subjective survival beliefs and life-cycle model

3.1 Abstract

The notion of bounded rationality is increasingly employed within the framework of conventional economic theory. For example, Heimer et al. (2019) analyse household finance puzzles using a life-cycle model with subjective survival probability.

However, the estimation of the subjective survival probability approach used by Heimer et al. (2019) is questionable as the shape of the subjective survival curve deviates greatly from the objective survival curve using a life table. Furthermore, the probability estimation of individuals has problems such as focal point responses. Therefore, in this chapter, we use the optimism index introduced in the previous chapter to build subjective survival probabilities. In other words, we create survival probabilities based on survival expectations (ages at which agents expect to die). Our subjective survival probabilities are compared with two other measures derived using the hazard scaling (Gan et al. 2005) and the ordinary least squares regression (Heimer et al. 2019). Furthermore, a life-cycle model

of saving and consumption based on subjective survival measures is compared with the life-cycle model with objective survival probabilities. We show that the subjective survival probabilities that we develop do not lead to empirically verified behaviour such as the annuity puzzle or the household finance puzzle. We then introduce the decision state model, where agents make a transition to a mentally capable state where they contemplate annuity purchases (FC state). The simulations suggest a limitation on annuity purchase age can influence the annuity purchase. An increase in the age limit to buy an annuity from 65 to 85, increases the number of annuity holders by 30%.

3.2 Introduction

Financial models of consumption and savings often focus on expected return and risk aversion. However, laypeople cannot accurately estimate the probabilities that underlie financial and insurance markets. Simon (1955) has coined the term "bounded rationality". Bounded rationality suggests that people are rational but bounded by limited information. Similarly, people have limited information when they estimate probabilities of events and thus, they form subjective probabilities which deviate from the objective probabilities. A classic illustration of this is that individuals have subjective survival beliefs which differ from the objective survival probabilities derived from statistical mortality data. This is shown in a number of surveys: The English Longitudinal Study of Ageing (ELSA), the Health and Retirement Study (HRS), the Survey of Consumer Finance (SCF) and The Survey of Health, Ageing and Retirement in Europe (SHARE).

Life expectancy is one of the important factors that affect individual financial decisions. Individuals often fail to estimate their lifespan precisely. One of the early studies that reported the relationship between life expectancy miscalibration and the financial decisions of individuals is by Puri & Robinson (2007). They find that overly optimistic individuals tend to make imprudent financial decisions. Furthermore, O'Dea & Sturrock (2020) argue that the subjective survival perception, leading to underestimation of life expectancy at younger ages and overestimation of life expectancy at older ages, can be a reason for the sub-optimal purchase of an annuity. Finally, Heimer et al. (2019) argue that pessimism about mortality at younger ages causes younger people to spend more which results in sub-optimal savings for retirement.

Among the studies on subjective survival probabilities, Gan et al. (2015) and Heimer et al. (2019) use subjective survival probabilities to build life-cycle models. Their life-cycle models with subjective survival probabilities create lower-than-optimal annuity purchases. However, the method to find survival probabilities from their model is not robust. In particular, Heimer et al. (2019) use ordinary least squares regression using two to four data points to fit entire survival curve. Therefore, we propose a new method for building subjective survival probabilities based on the subjective survival belief index λ_i suggested in the previous chapter.

Finally, we show that subjective survival probability does not make a substantial difference in annuity purchases using a life-cycle model. Thus, we propose a life-cycle model where individuals form a view on annuities, which changes over time, and they make a decision to purchase an annuity at different points in time. The decision state model by Bateman et al. (2020) uses the consumer funnel theory to model the annuity purchasing process. Thus, consumers transition between states until they reach a state where they are willing to purchase an annuity. Using the decision state model, we assume agents initially disregard annuities in their financial decision-making but over time they become capable of considering the purchase of an annuity at retirement.

3.3 Literature review

In this section, first, we review two models of the life-cycle model where they employ subjective survival probability. Gan et al. (2005) and Heimer et al. (2019) describe life-cycle models with subjective survival probability. The methods that they use to estimate subjective survival probability are different from each other, so we review the methods they use. The second part of the literature review gives a brief introduction to the decision state model by Bateman et al. (2020). The theory is employed to back up a life-cycle model where agents change their asset mix throughout their lives.

3.3.1 Life-cycle model

A life-cycle model is often used when economists consider the intertemporal allocation of time, effort and money. The framework has been developed since the 1950s and it extends to include choices such as consumption, saving, education, human capital, marriage, fertility, labour supply and retirement decisions (Browning & Crossley 2001). An originator of the life-cycle hypothesis, Modigliani was awarded the Nobel Economic Sciences Prize in 1985. Since his first work in 1954 (Modigliani & Brumberg 1954), numerous studies related to the theory with different settings were published.

The life-cycle model of retirement savings we use in this chapter is based on the model set-up of Campbell et al. (2001). This includes time-separable Constant Relative Risk Aversion (CRRA) utility of investors, no bequest utility and borrowing constraints and the detailed settings are given in the Appendix. The model is built in FORTRAN based on the code provided by Fehr & Kindermann (2018). The code is modified to allow for different asset mixes at different times of life.

3.3.2 Life-cycle model with subjective survival probability

Gan et al. (2005) propose some optimism indices to account for subjective survival beliefs and they build a life-cycle model using one of the optimism indices: hazard scaling. Gan et al. (2015) investigate if subjective survival beliefs affect consumption and bequests motives. They find that employing subjective survival probabilities produces much better out-of-sample wealth level predictions than using objective life-table probabilities. Thus, they demonstrate that people's consumption and saving decisions are consistent with their subjective beliefs on survival. Finally, the authors find that bequest motives are not strong enough to consider most of the bequests to be accidental.

Gan et al. (2005) introduce optimism indices to measure the deviation of respondents' subjective belief from the underlying life table. Gan et al. (2005) define subjective survival probability to age $a+t$ of individual i aged a as:

$$s_{ia}(t) = \exp\big(-\int_0^t \mu_{ia}(a+r)dr\big). \tag{3.1}$$

According to Gan et al. (2005), $\mu_{ia}(a+t)$ is the subjective hazard rate at age $a+t$ for individual i, while life table hazard rate is written as $\mu_{0a}(a+t)$. Therefore, the subjective hazard rate can be written in terms of the objective hazard function and an optimism index γ_i for individual i:

$$\mu_{ia}(a+t) = \gamma_i \mu_{0a}(a+t). \tag{3.2}$$

The index value indicates that a person has a pessimistic perception compared to the life table when $\gamma_i > 1$. On the other hand, an individual is said to have an optimistic view of their survival when $\gamma_i < 1$.

Figure 3.1 shows the subjective survival curve using the optimism index provided by Gan et al. (2015) and the objective survival curve using 1995

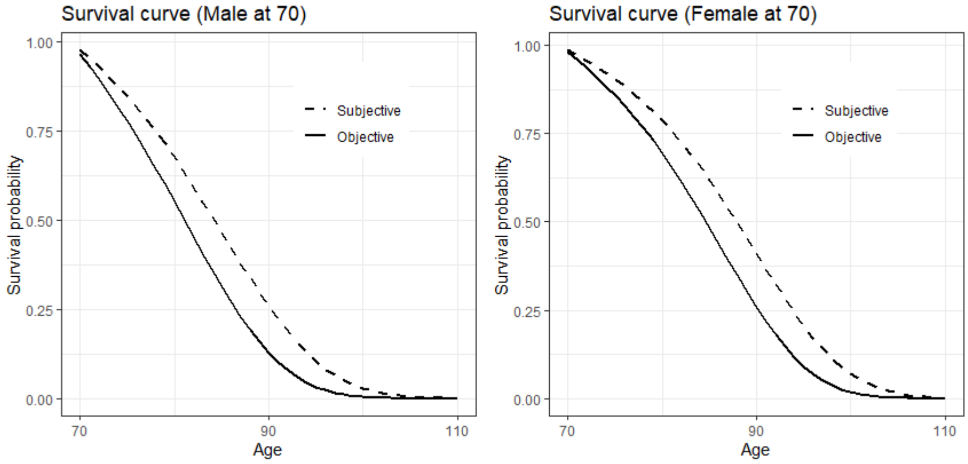


Figure 3.1: Reproduced subjective survival curves from Gan et al., 2015 (dotted) and objective survival curves from 1995 life table (solid).

mortality rate[1]. The left panel of Figure 3.1 shows the survival curve of males at age 70, whereas the right panel shows the corresponding curve for females of the same age. We find that agents, on average, overestimate life expectancy at older ages when subjective survival probability estimation by Gan et al. (2015) is used. A limitation of this model is that the optimism index is estimated by subjective survival probability to one target age. Also, the reliability of the model can be threatened when there is a problem with focal responses, which is a tendency of individuals to give probabilities as 0.0 or 1.0. Finally, the estimation of γ_i involves as they require calibration of hazard scaling parameters for all individuals for each age and use the average value. This can be burdensome when an optimism index for each individual is required.

Heimer et al. (2019) analyse the effects of subjective survival beliefs on life-cycle decisions. In their model, younger generations underestimate their survival view, while older cohorts overestimate their survival probabilities. As a result, Heimer et al. (2019) find those younger people under-save by 30 per cent relative to the optimal saving and retirees draw down their assets 34 per cent slower than the optimal choice when subjective survival

[1]U.S Male and female data for 1995 is collected from https://www.mortality.org/

beliefs are employed. Furthermore, they use consumption and net worth data from the Consumer Expenditure Survey (CEX)[2] to compare the result of their life-cycle model. The authors assert that subjective survival beliefs calibration yields a 78 per cent improvement in the model's fitting relative to the objective beliefs calibration.

To construct the survival probabilities used in the life-cycle model, Heimer et al. (2019) conduct a survey to collect financial and demographic data of individuals. They elicited beliefs from individuals about their 1, 2, 5, and 10 year survival probabilities as well as their financial status. Ordinary least squares regression was used to find the subjective survival probabilities based on the answers of elicited survival beliefs. The authors assert that mortality belief distortions provide a disincentive to save for the younger ones and to withdraw at a slower pace for retirees. One of their findings implies that bequest has as much impact as subjective beliefs. However, it is possible that the bequest motive is itself influenced by subjective survival beliefs as Gan et al. (2015) suggest.

Heimer et al. (2019) estimate the one-year survival probability using ordinary least squares regression. In the notation of Heimer et al. (2019), $s_{t+1|t}$ is the subjective survival probability that individuals attach to surviving to age $t+1$, conditional on having reached current age t, and it is regressed on age t according to

$$s_{t+1|t} = \psi_0 + \psi_1 t + \psi_2 t^2. \tag{3.3}$$

The parameters consisting of the subjective survival probability are estimated by the authors as $\psi_0 = 0.933, \psi_1 = 0.00149, \psi_2 = -0.000032$. We use these parameter values to find subjective survival curves.

Figure 3.2 shows the recreated subjective survival curve using the survival probability estimated by Heimer et al. (2019) and 2010 survival curve from

[2]https://www.bls.gov/cex/

the United States Social Security Administration (SSA) life table[3].

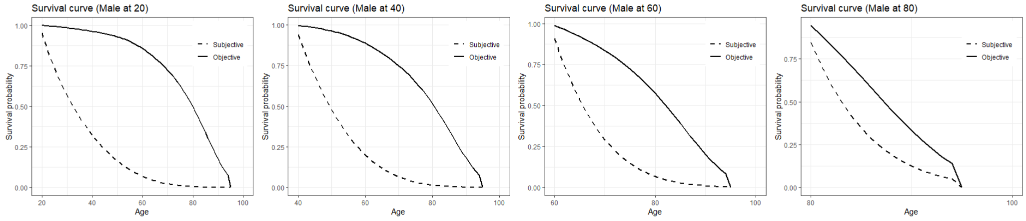


Figure 3.2: Reproduced subjective survival curves from Heimer et al., 2019 (dotted) and 2010 survival curves from the SSA life table (solid).

A problem of the method to build subjective survival probabilities by Heimer et al. (2019) can be easily detected from Figure 3.2. The subjective survival curves (dotted) present no resemblance to the objective survival curves. This suggests that the subjective survival probabilities by Heimer et al. (2019) are unrealistic. Although it is possible to have a subjective survival curve which deviates significantly from the objective survival curve, the ordinary least squares regression method based on a few points of probability is still questionable as the probability estimation of individuals has some problems stated in Gan's model.

3.3.3 Decision state model

Bateman et al. (2020) present the decision state model which is derived from the hierarchy of effects model in advertising. The hierarchy of effects model studies how consumers change their attitude toward a product when a marketing campaign takes place. Barry & Howard (1990) give a summary of popular hierarchy models. The models show stages of attitude towards a marketed good through which consumers proceed. For example, the AIDA model introduced by Lewis in 1900 distinguishes the stages into Attention, Interest, Desire and Action. Bateman et al. (2020) introduce the decision state model (DSM) with four stages: Pre-Aware, Aware but not interested, Interested but not capable and Capable.

[3]Data available from https://www.ssa.gov/oact/STATS/table4c6.html

The authors use surveys to illustrate that people are at different stages of life insurance purchase. Their analysis finds that financial literacy and financial experience are significantly associated with being in higher decision states, i.e. close to, or at the 'Capable' state. Furthermore, a higher future time perspective, which measures a person's perception of their future as being time-limited, also associates with higher decision states.

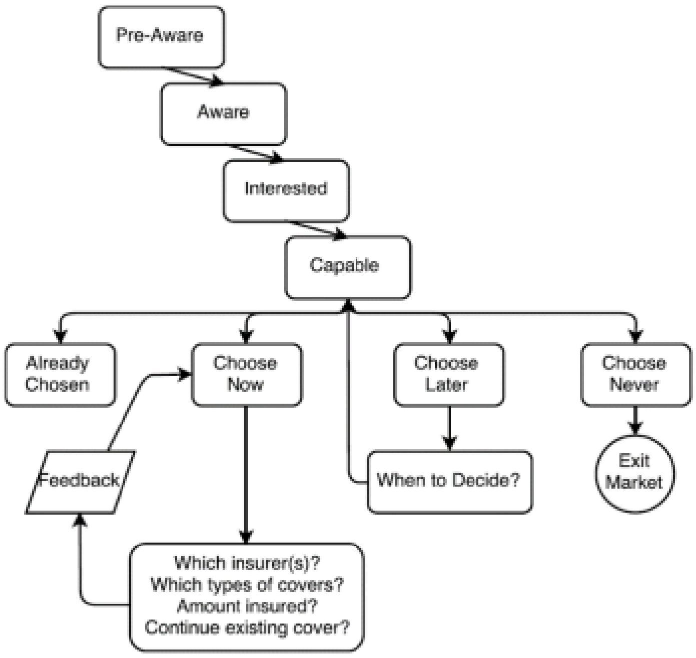


Figure 3.3: Flowchart of decision states model. (Source:Bateman et al. (2020))

3.4 Model description

In this section, we propose a method to calculate subjective survival probabilities based on the subjective survival belief index λ_i introduced in the previous chapter. Thereafter, we introduce a new model where individuals who consistently update survival beliefs change the timing of an annuity purchase accordingly.

3.4.1 Subjective survival probability with ω_i

We introduced an optimism index λ_i in the previous chapter. λ_i is the ratio between subjective life expectancy and objective life expectancy. In terms of actuarial notation, the index is

$$\lambda_i = \frac{e_{x_i}}{e_x}, \tag{3.4}$$

where e_{x_i} is subjective life expectancy for individual i aged x and e_x is actuarial life expectancy at age x. We would like to create a new index, ω_i, to measure the degree to which individuals over/underestimate subjective survival probabilities each year instead of the degree to which individuals over/underestimate life expectancy. Both Gan et al. (2015) and Heimer et al. (2019) find subjective survival probabilities based on probabilities that individuals conjectured. Survey respondents often give focal responses for probabilities such as 0 or 1. Furthermore, estimating survival probabilities for whole life using a few points can be inaccurate. Therefore we propose an approach starting from survival expectations, in ages, to calculate survival probabilities.

From equation 3.4, the subjective life expectancy in terms of the objective survival probability ${}_tp_x$ of an individual aged x for at least t years can be expressed as

$$\begin{aligned} e_{x_i} &= \lambda_i \sum_{t=1}^{\tau} {}_tp_x \\ &= \lambda_i(p_x + {}_2p_x + \cdots + {}_\tau p_x) \\ &= \lambda_i(p_x + p_x \cdot p_{x+1} + \cdots + p_x \cdots p_{x+\tau-1} \cdot p_{x+\tau}), \end{aligned} \tag{3.5}$$

since objective life expectancy is

$$e_x = \sum_{t=1}^{\tau} {}_tp_x \tag{3.6}$$

where τ is the remaining years until the terminal age, which is 110 in this case. Thus, $\tau = 110 - x$ in this case. Although λ_i can be used to find the degree which individuals under/overestimate ${}_tp_x$, We prefer to find an index which can estimate one-year subjective survival probability for all x's in p_x instead of ${}_tp_x$ as one-year subjective survival probability is easier to use and more useful.

We introduce another index ω_i for each individual i which satisfies the following condition

$$e_{x_i} = \omega_i \cdot p_x + \omega_i^2 p_x \cdot p_{x+1} + \cdots + \omega_i^{\tau+1} \cdot p_x \cdots p_{x+\tau-1} \cdot p_{x+\tau}. \tag{3.7}$$

Subjective one year survival probability for an individual i at age x, p_{x_i} can be seen as a multiplication of ω_i and objective one-year survival probability at age x, p_x. Furthermore, ω_i needs to meet another condition such that survival probabilities do not exceed 1. Therefore,

$$p_{x_i} = \min[1, \omega_i \cdot p_x], \quad \forall x. \tag{3.8}$$

The new measure is comparable to the optimism index with hazard scaling by Gan et al. (2005) as they both are a multiplication of a parameter and a survival probability. The subjective survival probability by Gan et al. (2005) can be re-written below

$$\begin{aligned} {}_tp_{x_i} &= \exp\big(-\int_0^t \gamma_i \cdot \mu_{0x}(x+r)dr\big) \\ &= ({}_tp_x)^{\gamma_i} \\ &= g_{it} \cdot {}_tp_x, \end{aligned} \tag{3.9}$$

where $g_{it} = ({}_tp_x)^{(\gamma_i - 1)}$, then life expectancy of i at age x corresponding survival probability is

$$e_{x_i} = \sum_{t=1}^{\tau} g_{it} \cdot {}_tp_x. \tag{3.10}$$

We prefer our method over hazard scaling of Gan et al. (2005) because equation 3.7 is simpler as it involves no exponential function. Furthermore, working with deterministic survival probabilities instead of hazard rates is easier to implement.

To calculate the ω_i for each individual i, we start multiplying all possible values of ω_i and p_x for all x. Assuming $\mathbf{w}$ is a vector of possible values of ω_i from 0.25 to 2 with 0.01 increases and $\mathbf{p}$ as a vector of survival probabilities for ages between 1 and 110, we find

$$\mathbf{W} = \mathbf{w} \otimes \mathbf{p} \tag{3.11}$$

for $\mathbf{p} = [p_1, p_2, \cdots, p_{109}, p_{110}]$ and $\mathbf{w} = [0.25, 0.26, \cdots, 1.99, 2]$. A matrix $\mathbf{W}$ is the outer product of the vector $\mathbf{w}$ and $\mathbf{p}$. We replace all elements of matrix $\mathbf{W}$ which are greater than 1 by 0 to eliminate these values as per the constraint in equation 3.8. Using equation 3.5, products of the elements of each row create a vector of weighted survival probabilities from age 1 to 110, ${}_{109}p_1$. Products of each row except the first column give vectors of weighted survival probabilities from age 2 to 110, ${}_{108}p_2$. By removing different columns in matrix $\mathbf{W}$, we can create a complete set of weighted values of ${}_tp_x$ for all possible values of ts and xs. We can then compute the complete set of weighted survival expectations using equation 3.7. Finally, we interpolate linearly between the two values from of survival expectation closest to the subjective survival expectation e_{x_i} from survey data about individual i, to find ω_i for individual i.

3.4.2 Life-cycle model with different asset mix

According to Bateman et al. (2020), people move through different stages when purchasing life insurance. Similarly, we assume people make a transition in their decision state when purchasing an annuity. Unaware consumers become aware of the annuity product, and they will be interested in the product before they become capable of purchasing an annuity. For example, when an agent learns and gains information on an annuity product the agent will make a transition state from unaware to interested or capable.

The transition of the state can be affected by many factors which cannot be easily justified. One could argue that there are more steps in the decision-making process before they become capable and purchase an annuity. For instance, private information can affect the perceived value and cause less or more demand for a financial product. As Wuppermann (2017) documents the demand for insurance may be dependent on private information. Information cascade can also cause decisions in stock market participations (Bikhchandani et al. 1998). Influence of the network or level of education can influence the decision process. However, these arguments are difficult to evaluate objectively, especially as data is sparse. Moreover, which information triggers people to move into what stage is impossible to distinguish. Thus, instead of justifying the factors which cause the transition, we consider how consumption, savings and investment change when the transition occurs.

In our model, we assume that individuals begin with a glide path which is the asset allocation mix of risk-free and risky assets until their demise. The individuals begin with a partially capable (PC) state in which they follow the optimal savings and withdrawal path with a mixture of risk-free and risky assets. An annuity is not available until they become fully capable (FC) state. Agents who become FC state of purchasing an annuity make a pre-commitment to purchase an annuity at 65. Agents make a transition from PC state to FC state based on a stochastic process in our model. Once agents can purchase an annuity, a new asset allocation will be made with three assets: risky and risk-free assets and annuity. Figure 3.4 illustrates flow chart of changes in asset mix.

Dynamic programming is employed to find optimal savings and withdrawal paths. The first path is risk-free and risky assets. It was assumed that all the agents, in the beginning, followed optimal savings and consumption paths solved by dynamic programming. Once they are triggered

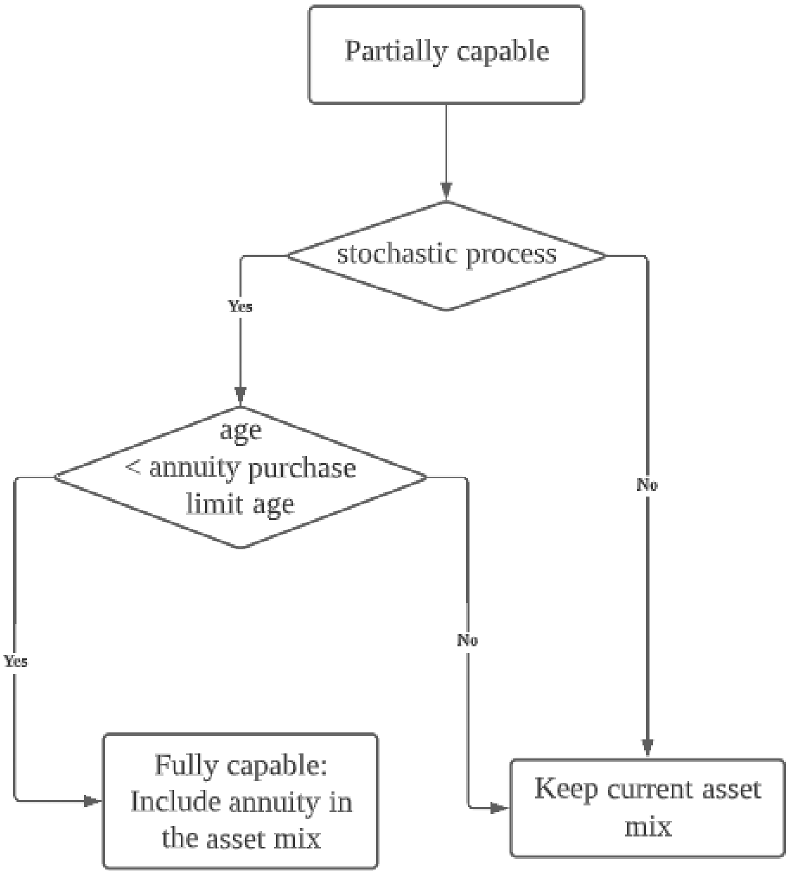


Figure 3.4: Flow chart of the state change

to purchase an annuity at a certain age, they follow new optimal savings and withdrawal paths where the annuity is included as an option. Therefore, firstly, we solve the optimal path without the annuity and calculate the saved assets at each age. Secondly, new optimal paths with the annuity starting from each age with the accumulated asset are calculated. Although it is ideal for keeping the original asset mix and then making the change within the dynamic programming, it does not apply to the current settings. So we divide the dynamic programming into two steps and carry out the calculation separately. In both problems, a household with time separable, expected, discounted utility maximises the utility function. Detailed settings of the life-cycle model are given in an appendix.

3.5 Model calibration and validation

3.5.1 Subjective survival probability with ω_i

Figure 3.5 illustrates calculated subjective survival curves (dotted) versus objective survival curves (solid)[4] of male (above) and female (below) agents at age 20, 40, 60, and 80. The coloured block indicates the subjective life expectancy of individuals. The darkest blue dotted lines show the subjective survival curves of agents when their survival expectation is 70 and the lightest blue dotted lines show the subjective survival curves of individuals with a survival expectation of 85. Figure 3.6 shows subjective life expectancy at each age decreases linearly as survival expectation is fixed, while objective life expectancy increases as people get older.

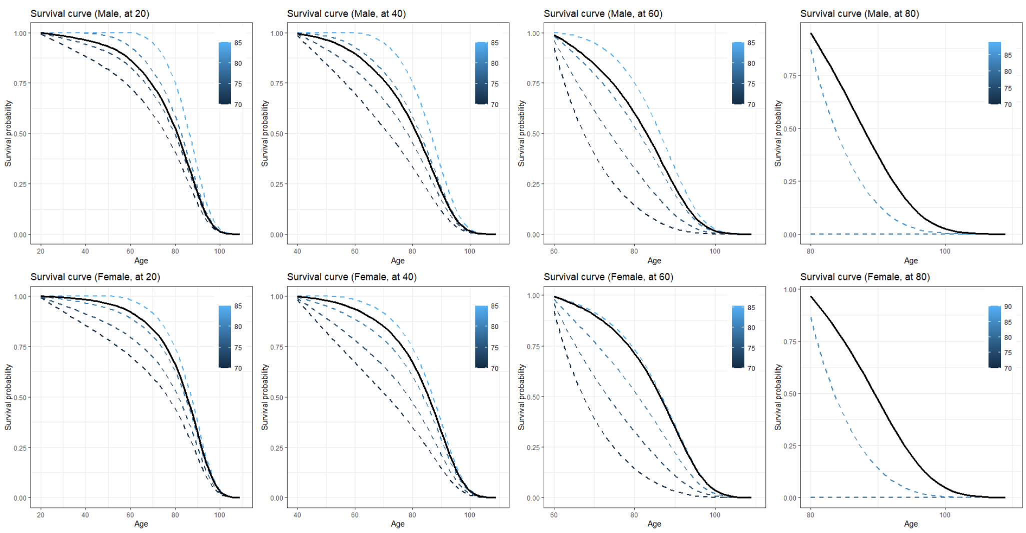


Figure 3.5: Subjective survival curves (dotted) with parameter ω_i corresponding to the survival expectations from age 70 to 85 and objective survival curves

Our subjective survival curves in Figure 3.5 and the subjective survival curves of Gan et al. (2005) in Figure 3.1 are close to objective survival curves, unlike the subjective survival curves of Heimer et al. (2019) in Figure 3.2. However, the survival curves of Gan et al. (2005) are harder to plot as they require calibration of hazard scaling parameters for all

[4] U.S Male and female data for 2019 is collected from https://www.mortality.org/

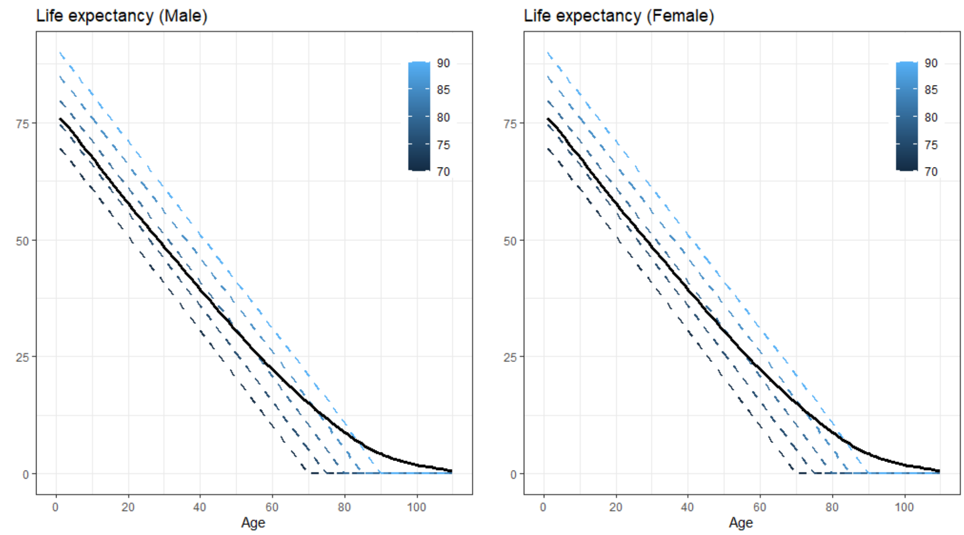


Figure 3.6: Life expectancy curves with survival expectation (dotted) and objective life expectancy curves (solid).

individuals for each age and use the average value, while our method only requires survival expectation and age of individuals.

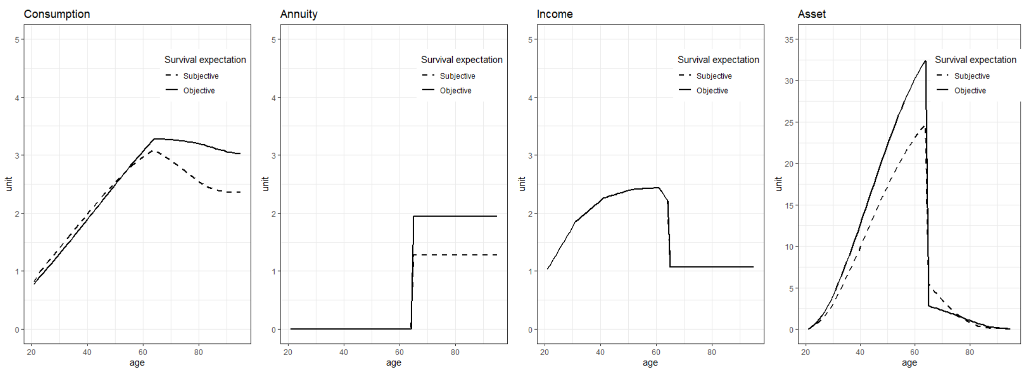
One of the features of our index is life expectancy. Our model is based on survival expectations given at one point in time. Therefore, subjective life expectancy at each age decreases every year. For example, when an agent expects to survive until the age of 90, his subjective life expectancy at age 20 is 70 years, his subjective life expectancy at age 21 is 69 years and so on. In other words, whichever age agents are in, they have consistent survival expectations as the age they think they will die is fixed. However, this is not true in many cases, as well as in our simulation. Since our agents learn and update survival expectations annually.

3.6 Results

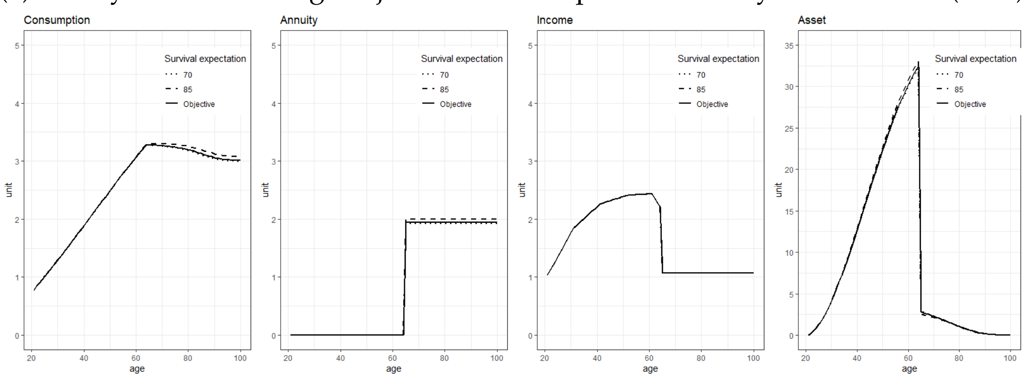
3.6.1 Life-cycle model simulation

Life-cycle model with different subjective survival probabilities estimation

We produce optimal consumption and savings paths using the subjective survival belief index λ_i and subjective survival beliefs by Heimer et al. (2019). In both problems, we assume an annuity is available for retirees at 65. Figure 3.7a illustrates that individuals with subjective survival beliefs spend more than what objective individuals would spend at younger ages, as Heimer et al. (2019) assert. As a result, individuals with subjective survival beliefs end up with lesser wealth to purchase an annuity at retirement, lesser annuity payment stream after retirement, and lesser consumption after retirement compare to the optimal saving path. Figure



(a) Life-cycle model using subjective survival probabilities by Heimer et al. (2019)



(b) Life-cycle model using subjective survival probabilities with ω_i

Figure 3.7: Life-cycle models with different subjective survival belief models

3.7 illustrates mean consumption, annuity, income and wealth using subjective survival probability estimated by Heimer et al. (2019) (top panels) and newly introduced optimism index ω_i (bottom panels). Figure 3.7a shows that pessimism in survival beliefs can reduce consumption over the lifetime and annuity purchases below the optimal level. However, the subjective survival probabilities assumption used in this life-cycle model by Heimer et al. (2019) seem too pessimistic, as shown in Figure 3.2. Therefore, we solve a life-cycle problem with subjective survival probabilities based on the optimism index ω_i. Figure 3.7b demonstrates that subjective survival has little impact on decisions on savings and consumption. Therefore, we assert that the subjective survival beliefs shown by the consumers of the SCF data are not strong enough to create a sub-optimal choice which differs greatly from the optimal choice according to objective mortality rates. On the other hand, we argue that the timing of an annuity purchase could reduce utility.

Life-cycle model with different timing of the transition in state

We assume agents can make a transition from PC state to FC state at any time,i.e include annuity in the asset mix at any time. Therefore, we solve the life-cycle model for the different ages at which agents make the transition to FC state. Although the transition to FC state can be made before the age of 65, an annuity purchase is allowed only at 65. Figure 3.8 shows consumption and income from the annuity regarding the age at which agents include annuities in their asset mix. The top left panel of the Figure 3.8 illustrates that the overall consumption level is minimum and the least smooth when the agent stays in FC state until death. Also, it is optimal to make a transition from PC to FC state at the age of 20 where the consumption is smoothest. Nevertheless, the consumption path is close to each other, where agents can purchase annuities.

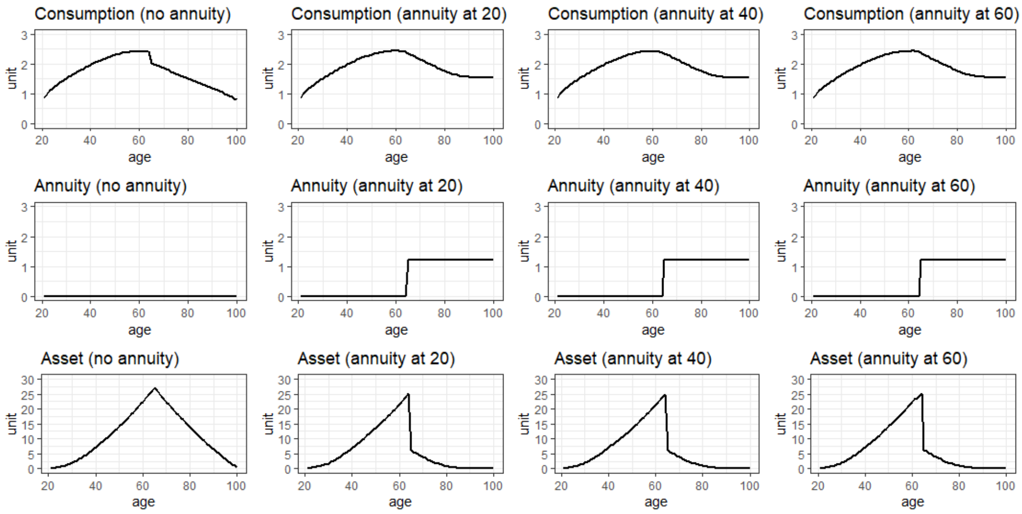


Figure 3.8: Changes in consumption annuity and wealth by annuity decision age

The total expected discounted utility is calculated for now and shown in Figure 3.9. Figure 3.9 shows the total expected discounted utility is decreasing as the timing to decide to purchase an annuity is delayed. 5 per cent of total utility is decreased when agents make a transition from PC to FC state at the age of 60 compared to the case where agents make a transition from PC to FC state at 20. However, the decrease in total utility gets drastic when agents miss out on the chance to purchase an annuity. This may be due to uneven consumption over the lifetime and a decrease in consumption after retirement.

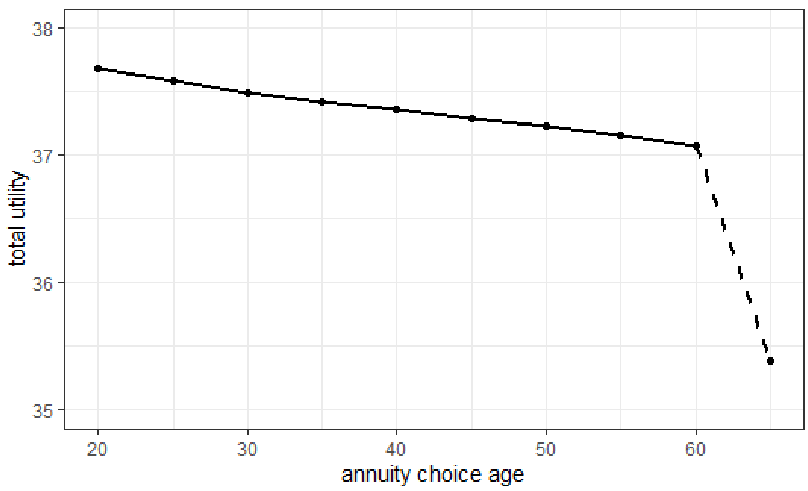


Figure 3.9: Changes in total expected discounted utility by changes in the timing of annuity decision.

Life-cycle model with different timing of the transition in the state (increase in annuity purchase age limit)

Now we investigate the cases when the agents are allowed to purchase annuities after retirement age. We let the agents to purchase annuities between ages 65 and 85 inclusive. Since the cohort becomes optimistic about its survival after retirement, there is a higher probability for the cohort to make a transition from PC to FC state after retirement. In the previous chapter, we showed the relationship between the optimism index and annuity purchases. Under this assumption, agents can make a transition from PC to FC state between 20 and 85. An annuity is purchased at age 65 if the transition is made before the age. Otherwise, annuity purchases will be at an age when the transition to FC is made.

Figure 3.10, 3.11 and 3.12 illustrate 3-dimensional plots of mean consumption, wealth and annuity paths with different annuity decision ages. The figures can be seen as the combinations of slices of graphs which in Figure 3.8 with all ages. z-axis, vertical axis, shows a level of consumption, wealth and annuity. The purple line is when the agent begins with an annuity decision at 20. The yellow line indicates when the individual misses the chance to purchase an annuity after 85.

Figure 3.11 shows wealth accumulation and decumulation throughout the lifetime concerning the timing of the annuity decisions. Again, the yellow line at the end shows the asset level when no annuity is purchased. The agent has to save up and withdraw a certain amount after retirement. Therefore, the agent has to save up a little more than when he considers purchasing an annuity. A drop in the wealth level shows the lump sum payment of an annuity at the time of purchase.

Figure 3.10 shows that consumption is smoothest when the agent considers purchasing an annuity from the beginning at age 20. The yellow line on

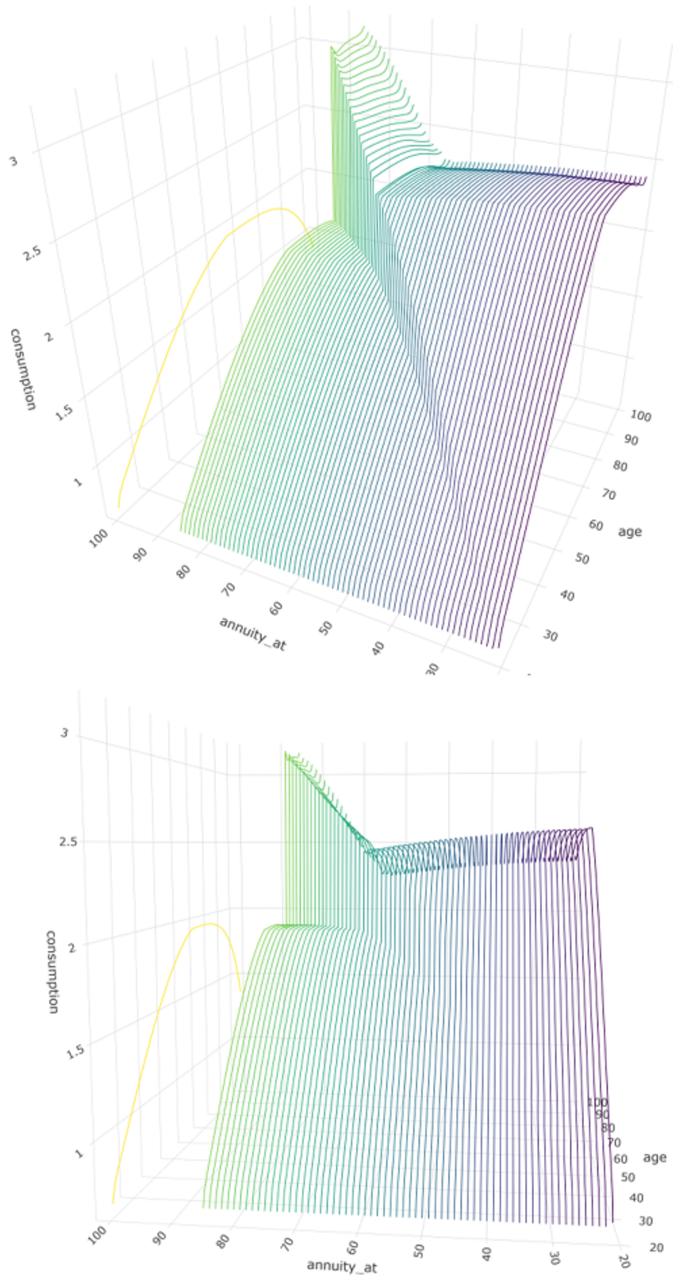


Figure 3.10: Mean consumption paths at each age of annuity choice

the left-hand side, when no annuity is purchased until age 100, has lower consumption than the cases with an annuity purchase. The diagonal cliff in the middle shows that a consumer has to consume less or in line with the yellow line until he decides to purchase an annuity. Once the decision is made, the consumer can spend more as the annuity provide a stream of income which is higher than the withdrawal of the wealth.

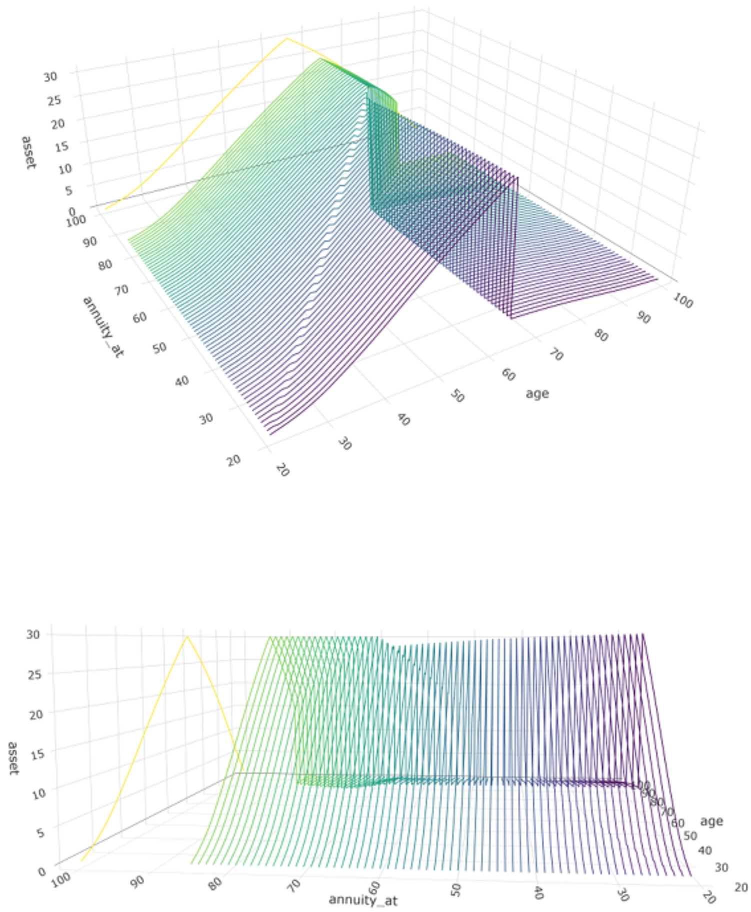


Figure 3.11: Mean wealth paths at each age of annuity choice

Finally, the annuity payment after retirement is shown in Figure 3.12. The purple line on the right-hand side is when the consumer has an annuity as a default from the beginning. The annuity payment is highest when the annuity decision is made from the beginning since individuals have lesser savings as they delay making an annuity purchase decision. Although annuity payment is higher when the decision is made after retirement, the total annuity payment has to be lower as they missed out on some payments. The diagonal line in the middle illustrates the delay as annuities are bought at ages after retirement.

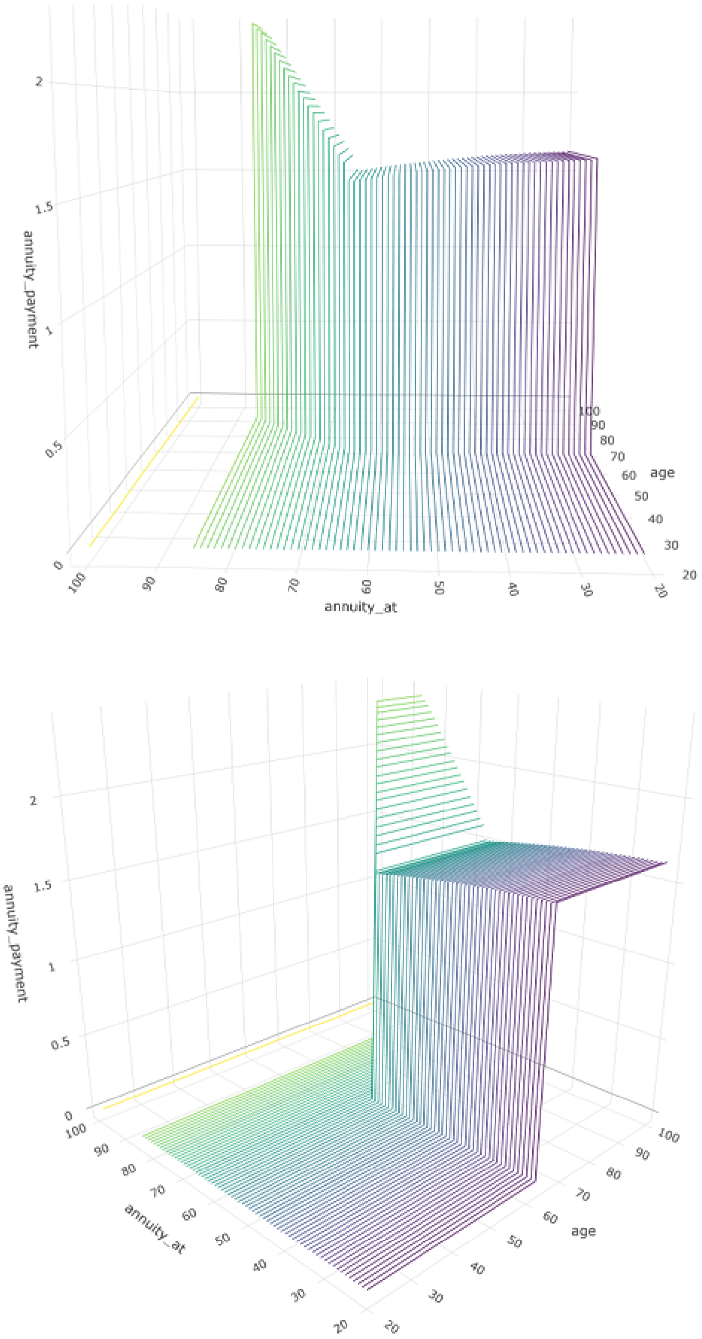


Figure 3.12: Mean annuity paths at each age of annuity choice

3.6.2 Agent-based simulation

We run the ABM simulations using the different assumptions on the age at which agents can purchase an annuity. Agents aged 40 to 65 are only allowed to purchase an annuity in our previous chapter. The simulation suggests the current level of annuitisation when agents purchase an annuity with the probability of one per cent each year. We change the assumptions on the age restrictions and simulate them to see how they affect the overall annuitisation. We test two different scenarios, annuity purchase ages between 40 and 75 and annuity purchase ages between 40 and 85. The 3.13 illustrated around 2.5 per cent increase in the proportion of annuity holders when the age limit was increased from 65 to 85. In other words, an increase in the annuity purchase age limit increases the number of annuity holders by 30%.

Figure 3.13: Changes in annuity purchase age limit

3.7 Conclusion

In this chapter, we introduced a method to build subjective survival probabilities based on the survival expectations of individuals. The method was compared with the existing methods by Gan et al. (2005) and Heimer et al. (2019) and exhibited a few strong points. The new method may be easier to calculate, more intuitive and more realistic. We use the subjective survival probabilities to find optimal life-cycle consumption and savings problems and find that the subjective survival probability itself may not be a cause of sub-optimal consumption and savings behaviour as Heimer et al. (2019) explain. We postulate that the agents can become interested in purchasing an annuity at any time through lifetime based on the decision state model by Bateman et al. (2020). We calculate a life-cycle model of savings and consumption when agents can prepare to purchase an annuity between 20 and 85. We find that the agents prepared to purchase an annuity sooner can enjoy higher consumption and utility. We run the agent-based model simulation to confirm that increase in the annuity purchase age limit can increase the proportion of annuity holders. More individuals benefit from higher utility coming from the annuity.

Appendix

3.7.1 Life-cycle model settings

All the notations used in this appendix are explicit and have no connections to the notations used in the main text. Moreover, I am suppressing the time index for all notations for simplicity. Utility function of consumption c is given as

$$u(c) = \frac{c^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} \tag{3.12}$$

, where risk aversion factor γ equals 0.1 in this example. The price of an annuity p_a is given as

$$p_a = (1+\xi) \sum_{j=j_r}^{J} \frac{\prod_{i=j_r}^{j} \psi_i}{(1+r_f)^{j-j_r-1}} \tag{3.13}$$

, where ξ denotes the loading factor and $\xi = 0$ as we assume actuarially fair annuity. ψ_j is the objective age-specific probability for an agent to survive from age $j-1$ to j. While, $\tilde{\psi}_j$ is one year subjective survival probability from $j-1$ to j. The unconditional probability of surviving to age j from the retirement age j_r is given by $\prod_{i=j_r}^{j}$. The retirement age j_r is 65.

We write the optimisation problem at each age j except for $j_r - 1$

$$\begin{aligned} V(j, X, a_r) = &\max_{c,a^+,\omega^+} u(c) + \beta\tilde{\psi}_{j+1} E[\exp(\epsilon^+)^{1-\frac{1}{\gamma}} V(j+1, X^+, a_r)] \\ &\text{s.t } X = c + a^+ \text{ , } a^+ \geq 0 \text{ , } 0 \leq \omega^+ \leq 1 \\ &X^+ = R_p(\omega^+, \vartheta^+) \frac{a^+}{exp(\epsilon^+)} + wh^+ + pen^+ + \frac{a_r}{p_a} \end{aligned} \tag{3.14}$$

a_r is a retirement asset, creating an annuity income stream after the retirement of $\frac{a_r}{p_a}$. β denotes the time discount factor and $\beta = 0.96$ in this model. X is cash on hand, a is the total amount of wealth, and ω is the proportion of stock holdings s in the portfolio, i.e., $\omega = \frac{s}{a}$. The risk-free rate r_f is 2 per cent. Risky stock follows a normal distribution with mean 4 and variance of $\vartheta = 0.024649$. w is the wage rate and labour productivity h_j is

$$h_j = e_j \exp(\eta_j + \zeta_j) \tag{3.15}$$

and productivity shock η is

$$\eta_j = \eta_{j+1} + \epsilon_j. \tag{3.16}$$

Earning process, e_j starts as 1 at age 20 and doubles after 30 years and decline afterwards. ζ follows a normal distribution with a mean of 0 and variance of 0.0738. ϵ follows a normal distribution with mean 0 and variance 0.0106. *pen* is the pension payment

$$pen = \begin{cases} \kappa w e_{j_r-1} \exp(\eta_{j_r-1}), & \text{for } j \geq j_r \\ 0, & \text{otherwise} \end{cases} \tag{3.17}$$

κ is 0.5 in this model.

At age $j_r - 1$, we have a different optimisation problem, just before retirement. The household has to decide how to split total wealth a^+ into liquid assets $a_l^+ = (1-\omega^+)a^+$ and retirement assets $a_r^+ = \omega_r^+ a^+$

$$\begin{aligned} V(j_r-1, X, 0) &= \max_{c,a^+,\omega^+,\omega_r^+} u(c) + \beta\tilde{\psi}_{j_r}, E[V(j_r, X^+, \omega_r^+ a^+] \\ \text{s.t}X &= c + a^+ \text{ , } a^+ \geq 0 \text{ , } 0 \leq \omega^+, \omega_r^+ \leq \\ X^+ &= R_p(\omega^+, \vartheta^+)(1-\omega_r^+)\frac{a^+}{exp(\epsilon^+)} + pen^+ + \frac{\omega_r^+ a^+}{p_a} \end{aligned} \tag{3.18}$$

3.8 References

Barry, T. E. & Howard, D. J. (1990), 'A review and critique of the hierarchy of effects in advertising', *International Journal of advertising* **9**(2), 121–135.

Bateman, H., Gerrans, P., Thorp, S. & Zeng, Y. (2020), *Life Insurance: Decision States, Financial Literacy, and the role of personal values*, Netspar, Network for Studies on Pensions, Aging and Retirement.

Bikhchandani, S., Hirshleifer, D. & Welch, I. (1998), 'Learning from the behavior of others: Conformity, fads, and informational cascades', *Journal of economic perspectives* **12**(3), 151–170.

Browning, M. & Crossley, T. F. (2001), 'The life-cycle model of consumption and saving', *Journal of Economic Perspectives* **15**(3), 3–22.

Campbell, J. Y., Cocco, J. F., Gomes, F. J. & Maenhout, P. J. (2001), Investing retirement wealth: A life-cycle model, *in* 'Risk aspects of investment-based Social Security reform', University of Chicago Press, pp. 439–482.

Fehr, H. & Kindermann, F. (2018), *Introduction to computational economics using Fortran*, Oxford University Press.

Gan, L., Gong, G., Hurd, M. & McFadden, D. (2015), 'Subjective mortality risk and bequests', *Journal of Econometrics* **188**(2), 514–525.

Gan, L., Hurd, M. D. & McFadden, D. L. (2005), Individual subjective survival curves, *in* 'Analyses in the Economics of Aging', University of Chicago Press, pp. 377–412.

Heimer, R. Z., Myrseth, K. O. R. & Schoenle, R. S. (2019), 'Yolo: Mortality beliefs and household finance puzzles', *The Journal of Finance* **74**(6), 2957–2996.

Modigliani, F. & Brumberg, R. (1954), 'Utility analysis and the consumption function: An interpretation of cross-section data', *Franco Modigliani* **1**(1), 388–436.

O'Dea, C. & Sturrock, D. (2020), Survival pessimism and the demand for annuities, Technical report, National Bureau of Economic Research.

Puri, M. & Robinson, D. T. (2007), 'Optimism and economic choice', *Journal of financial economics* **86**(1), 71–99.

Simon, H. A. (1955), 'A behavioral model of rational choice', *The quarterly journal of economics* **69**(1), 99–118.

Wuppermann, A. C. (2017), 'Private information in life insurance, annuity, and health insurance markets', *The Scandinavian Journal of Economics* **119**(4), 855–881.

Chapter 4

Community structured network with node-specific property

4.1 Abstract

Current studies of network focus on the topological property of node and edge, connection to other nodes, location and so on. This paper presents a new approach to constructing community structures. We analyse the relationship between age and friendship to create a model where the age gap, the age difference between two agents on a network, determines the community structures of the network. As a result, we construct a network with degrees of freedom and clustering coefficients which are close to those of a canonical small-sized network such as in villages. By creating a model where community structure is determined by the characteristics of individuals rather than by exogenous topological properties, we can provide a basis for network models where links between individuals relate to the characteristics of these individuals.

4.2 Introduction

Features that occur in networks which represent real system, as opposed to trivial topological features, are studied in network theory. Most research focuses on topological features such as degree distribution, clustering, community structure, and hierarchical structure. Therefore, existing studies of network models analyse such topological features within networks from real systems and seek to reproduce them. However, in reality, a network is often highly related to the properties that each node presents. People with similar socio-economic status often get closer together: socio-economic status includes education level, age, level of wealth, nationality and cultural and religious background. Newman (2006) claims that properties at the community level can be different from those at the level of the entire network. Therefore, analyses ignoring community structure may miss certain key network features. For example, some social network data such as Uganda village[1] data contains different villages with unique degrees of connections as each group has distinctive levels of sociability.

The data from Buijs & Stulp (2022) suggests that the age gap between friends is an important factor in friendship networks. The data contains Dutch women who are aged between 18 to 41 years, and shows that the age of 90% of their friends is within 10 years of the age of the Dutch women. In other words, a social network can include community groups based on the age gap. In this study, we use data from Buijs & Stulp (2022) to illustrate that friendship is highly dependent on the age of individuals and to construct a model where the age gap determines the community structure of the network.

We find the best-fitting distribution of the age difference between two agents in a network. The probability of having a friend with a particular

[1] https://networks.skewed.de/net/ugandan_ village

age gap can be calculated from the distribution. Using this probability distribution, we construct communities to which selected individuals are likely to have. Instead of using the desired degree of nodes, the probability of each node being connected within the community is used to create the network. Network model with node characteristics other than the topological property is barely studied. Therefore, such a model can be a useful tool for social network analysis.

4.3 Literature review

4.3.1 Network models

Network theory provides an important tool for analysing and describing complex systems in many areas of study. The social, physical, biological, information, and computer sciences broadly use network theory. Earlier studies of networks mainly focused on graphical representations of nodes and edges. The nodes and edges of the graphs show how entities or agents of a system are interconnected via links. Conventional network theory successfully illustrates real network properties such as heavy-tailed degree distribution, the small-world property or modular structures by using vertices and a single, static, unweighed single edge (Kivelä et al. 2014).

According to Kivelä et al. (2014), it has become necessary to investigate more complicated and more realistic connections among agents as network theory evolves and studies on complex systems advance. In other words, edges are considered to have heterogeneous features. For instance, the link may only be directed from one node to other nodes, thus, considered to be directed. The edges could have different strengths, or weights, of connectivity. Bipartite networks have edges which exist between different sets of nodes. Sometimes links are only active for a certain period of time. Also, Kivelä et al. (2014) state that edges are often considered to represent the relationships between nodes or actions that the agents take in a social network. Therefore, sociologists study social systems by analysing social networks using the same set of individuals with different types of ties. For example, as early as 1939, Roethlisberger & Dickson (1939) draw sociograms to depict relations between 14 individuals via 6 different types of social interactions.

In an effort to understand role relations in social groups, social psychologists and sociologists have included different types of nodes or hierarchical structures in social network analysis. In the sociology literature, networks in which each edge is categorised by its type are called 'multiplex networks' (Verbrugge 1979). On the other hand, networks in which nodes have different role relations are said to have 'multilevel networks' (Kivelä et al. 2014). One of the tools used to investigate the role relations of social networks is blockmodeling. Brusco et al. (2013) suggest that blockmodeling has become a useful tool for discerning the fundamental structure of any social network, even though blockmodeling was a tool designed to analyse the role relations in the social network. Blockmodeling methods group agents, or nodes, into clusters based on the distribution of their ties in their social group. In other words, blockmodeling partitions the nodes and edges of a network to analyse the basic structural properties of a network.

There are several properties that many real networks present. First, a scale-free network is one of the properties of the real networks (Barabási & Albert 1999). Networks typically have many vertices with a small number of edges and a small number of nodes with a high degree. Degree in network studies refers to the number of edges attached to a node, and degree distribution in a network often follows a power-law distribution with a heavy tail. The second property is the small world where the average distance between vertices in a network is short (Watts & Strogatz

1998). In both cases, clustering is important. Network clustering is shown when two friends, or neighbours, of an individual have a much higher probability of knowing one another than when two people are chosen randomly from the population. This effect is quantified by the clustering coefficient. A clustering coefficient of 1 means when a graph is completely connected. The final property is the community structure. Community structure is detected when a graph has nodes that are joined tightly within their group while groups have relatively loose connections to each other (Girvan & Newman 2002).

4.3.2 Random graph models

The Erdős-Rényi (ER) model (Erdős & Rényi 1959, 1960) is one of the seminal models in the study of networks. Large graphs arise in the studies of internet computing and share some aspects with the ER model in a natural way, despite the crucial differences such as high-degree vertices in real graphs Chung et al. (2006). One simple assumption of the ER model is that each pair of vertices is determined to have an edge independently with some fixed probability. The simplicity of the ER model enables the extraction of the essential behaviour of various graph properties. However, the limitation of the model arises from its very simplicity, which implies the same expected degree at every vertex.

The Chung-Lu (CL) model is one of the most widespread random graph models (Fasino et al. 2021). Chung & Lu (2002) originally propose a random graph model with weighted degrees, which is simple yet flexible. The CL model is characterised by a parameter vector $\mathbf{w} = (w_1, \ldots, w_n)^T$, which is a vector of non-negative real numbers. where there are n nodes in the network and w_i is the weight placed on node i.

$$p_{ij} = \frac{w_i w_j}{\sum_{k=1}^{n} w_k} \quad \text{for } i \neq j \tag{4.1}$$

and $p_{ii} = 0$ as the model does not allow a self-loop. Therefore, the expected degree for each node i is

$$\sum_{j=1}^{n} p_{ij} = \frac{w_i}{\sum_{k=1}^{n} w_k} \sum_{j=1}^{n} w_j = w_i. \tag{4.2}$$

Seshadhri et al. (2012) define a community to be a module with a large internal clustering coefficient where a module can be considered as a minimum substructure within a graph. A graph has a community structure when it can be broken up into communities, and community can be detected when a graph is broken up into modules. Seshadhri et al. (2012) propose that the simplest community is just a dense ER graph. The ER graph is considered dense when it has nodes connected with a constant probability, and it is said to be sparse when the probability is a fraction of the total number of vertices. The ER model, in general, is not a good model to represent interaction networks, as they tend to violate the tail behaviour of the degree distribution; i.e. nodes in the ER model have symmetric degree distribution. However, Seshadhri et al. (2012) claim that the ER model can be an important building block for the communities.

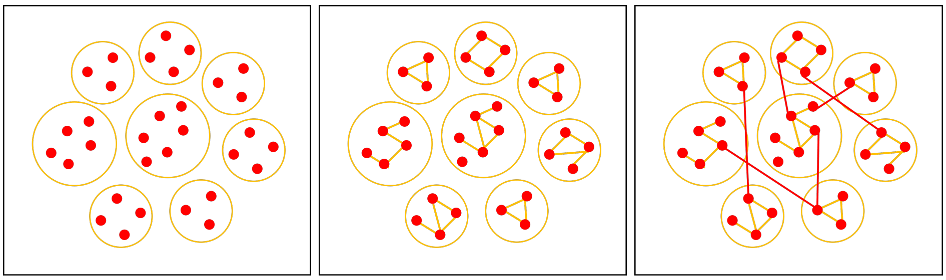
Based on the idea that a network consists of relatively small ER communities, Seshadhri et al. (2012) propose Block Two-Level Erdős-Rényi (BTER) model. The BTER model comprises nodes with short connections within communities and long-range connections from nodes to nodes in different communities. Short-connected communities tend to have high clustering coefficients, while long-range connections are sparse and lead to heavy-tailed degree distribution. The BTER model construction is a three-step process. In the first step, each node with a degree higher than two is assigned to a community. The nodes are all partitioned into these communities, resulting in a structure shown in Figure 4.1a.

The model assumes the desired degree distribution $\{d_i\}$ is given to nodes to match a specific degree distribution from data. Nodes will have the desired degree distribution comparable to a chosen real network. As a result, node i will have the desired degree d_i. Nodes with the same desired degree will be partitioned into the same community. G_k denotes the kth community and k_i denotes the community assignment for node i.

The next step is to create links within each community, as shown in Figure 4.1b. Seshadhri et al. (2012) argue that degree of nodes and the clustering coefficients are highly related. Small-sized communities tend to have a higher clustering coefficient, hence are more complete. The authors suggest edge probability within community k as Equation 4.3.

$$\rho_k = \rho \left[1 - \eta \left(\frac{\log(\bar{d}_k + 1)}{\log(d_{max} + 1)} \right)^2 \right]. \tag{4.3}$$

$\bar{d}_k = min\{d_i | i \in G_k\}$ is the minimum degree of a node within community k, and d_{max} is the maximum degree of any node in the entire graph. ρ and η are parameters that can be calibrated to fit a real-world network. Seshadhri et al. (2012) select the parameters by manual experimentation.



(a) Distribution of nodes into communities (b) Local links within each community (c) Global links across communities

Figure 4.1: BTER model construction

The final phase of the model construction is to make the excess degree connections across communities in Figure 4.1c. Nodes within a community form edges with nodes in other communities based on desired degrees. The excess degree e_i is $d_i - d_i'$. Node i should have d_i incident edges based on input degree distribution and d_i' edges from the first step of making connections based on the ER model. In other words, $d_i' = \rho_{k_i}(n(G_{k_i}) - 1)$.

CL model, which is a weighted ER model, is used to form the edges that connect communities.

$$e_i = \begin{cases} 1, & \text{if } d_i = 1, \\ d_i - \rho_{k_i}(n(G_{k_i}) - 1), & \text{otherwise,} \end{cases} \tag{4.4}$$

where $n(G_{k_i})$ is the size of community k_i. The probability of selecting nodes to from other communities to form an edge with node i is $e_i / \sum_j e_j$.

Seshadhri et al. (2012) argue that a dense ER graph is a must-have community feature. Also, they show that dense ER graphs, which have a small number of nodes, connected with a weighted ER model generate heavy-tailed degree distribution using real-world network data. However, the model requires the desired degree distribution. Furthermore, the model requires extra rules when implemented because some features are not extant in the mathematical model. For instance, when nodes with a high desired degree are partitioned, they have to be with the closest community, as an exact matching community may not exist. In the next section, we suggest a model where community structures are determined by the characteristics of nodes and edges are created by the given probabilities instead of desired degree nodes.

4.4 Data

4.4.1 Dutch women friendship data

Buijs & Stulp (2022) survey 706 Dutch women aged between 18 and 41. The respondents are asked for details of 25 acquaintances, where details include the age of the acquaintances and whether they consider them as a friend. The mean number of friends is 10.4, with a standard deviation of 5.3. The most interesting feature of the data is the age gap between friends. We use simple imputation to fill out the missing values of the age gap. The questionnaire asks for the exact age of friends whose age is between 19 and 50. Ages over 50 are grouped as 50+, and ages under 18 are grouped as 18-. Two values of 18- were imputed with the value 17, as it is natural to assume that adults are not likely to have adolescent friends. The 50+ group is more complicated to impute. l_x, the number of persons surviving to exact age x, from the life table [2] is used to calculate the probability of having friends at each age.

$$P(\text{friend at age } x \text{ over } 50) = \frac{l_x}{\sum_{k \in (51,110)} l_k} \tag{4.5}$$

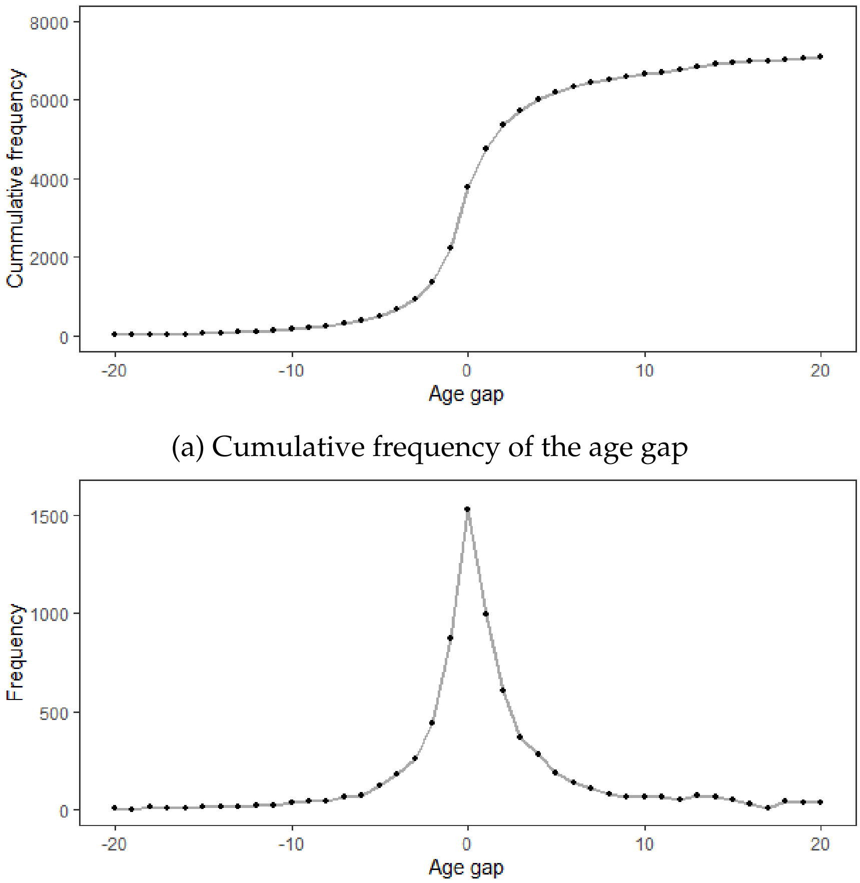
where 110 is the terminal age in the life table.

Simple imputation is used as the size of the 50+ group is not large; only 471 friends out of 7353 friends are classified in the 50+ group.

4.4.2 Ugandan Villages data

The data from Buijs & Stulp (2022) does not include any information regarding the network topology part from degree distribution. Therefore, a similar type of real network is required to compare with the generated networks. Chami et al. (2017) collect data from 3,491 households in 17 villages bordering Lake Victoria in Mayuge District, Uganda. Friendship and health advice networks within their own villages are measured by the survey. Nodes are households, and edges represent either a close friendship or a trusted health advisor connection, obtained via a name generator questionnaire. The data is selected as the friendship network is based on offline relationships. Also, the graphs are undirected and unipartite in line with the model that we will propose next. The Uganda network does not exhibit properties of a scale-free network as most online data does. Broido & Clauset (2019) argue that social networks are either not scale-free or weakly scale-free, whereas technology-based networks are strongly scale-free.

[2] 2017 Dutch women https://www.mortality.org/



(a) Cumulative frequency of the age gap

(b) Frequency of the age gap

Figure 4.2: Distribution plots of age gap between Dutch women friends. (Source: Buijs & Stulp (2022))

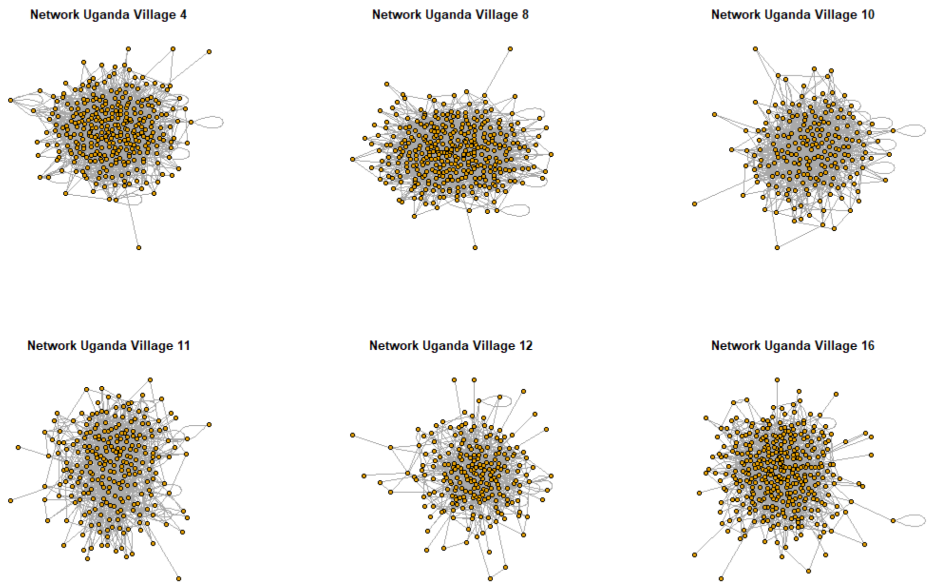


Figure 4.3: Ugandan village network graphs. (Source: Chami et al. (2017))

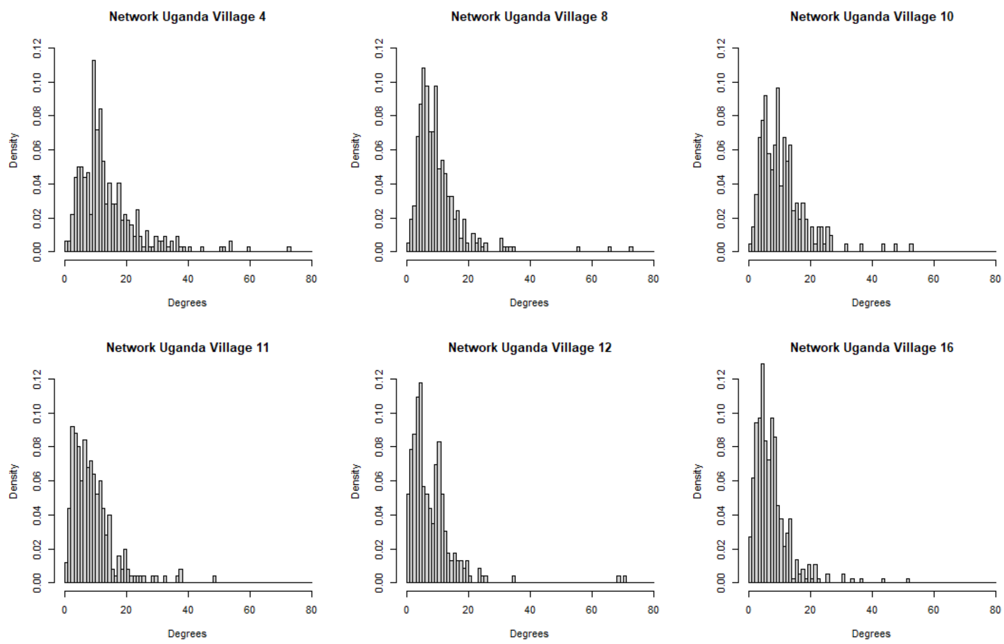


Figure 4.4: Ugandan village degree distribution. (Source: Chami et al. (2017))

We present network graphs in Figure 4.3 and corresponding degree distributions in Figure 4.4 for the six largest villages from the data. Table 4.1 presents statistics of the real networks of the Ugandan villages and the Dutch women friendship data. The mean degree and the standard deviation of degree for Dutch women data are somewhat lower than Ugandan villages data. This is expected as the number of friends in Dutch women is capped.

	Nodes	Edges	Mean degree	Sd degree	Diameter	Mean distance	Louvain		Transivity
							Groups	Modularity	
Dutch women	706	NA	10.4	5.3	NA	NA	NA	NA	NA
Village 4	320	2302	14.39	9.98	5	2.53	9	0.31	0.089
Village 8	369	1902	10.31	7.59	5	2.80	10	0.34	0.056
Village 10	207	1180	11.40	7.62	5	2.54	8	0.3	0.10
Village 11	250	1183	9.46	6.85	5	2.79	9	0.4	0.093
Village 12	229	962	8.40	7.96	6	2.88	9	0.35	0.083
Village 16	372	1475	7.93	6.06	7	3.13	13	0.38	0.050

Table 4.1: Statistics of real friendship networks. (Source: Buijs & Stulp (2022) and Chami et al. (2017))

Diameter, mean distance, Louvain measure of modularity and transitivity measure are calculated and shown in Table 4.1. The calculated measures are compared with the measure of simulated networks in a later section. The measures provide a view of how the networks are structured. The diameter of a graph is the length of the largest shortest path (largest geodesic). In other words, the diameter of a graph is the maximum number of vertices that must be traversed when moving from one vertex to another. Similarly, distances refer to the length of all the shortest paths from the vertices in the network. The mean distance is the average length of all the shortest paths from the vertices. The Louvain measure gives the number of groups or communities, as well as modularity, where modularity pertains to the strength of the division of a network into modules. For example, networks with high modularity imply dense connections within modules, along with sparse connections between nodes in different modules. The method to calculate the Louvain measure is derived from Blondel et al. (2008). Finally, transitivity is the probability of a vertex being connected to adjacent vertices. Transitivity is also called the clustering coefficient. The transitivity measure that we present in Table 4.1 is the global transitivity which is the ratio of the count of triangles and connected triples in the graph. We also calculate the local transitivity, a ratio of the count of triangles connected to the vertex and the triples centred on the vertex, of all the vertices of the six Ugandan villages depicted in Figure 4.3. The local transitivity for the vertices plotted against their degree is shown in 4.5. All of the measures are calculated using an R package [3].

4.5 Model description

In this section, we present a model where community structure is determined by characteristics that are owned by nodes. In the BTER model, a community is determined by the desired degrees of nodes. We consider a model in which the community is constructed by property other than degrees of nodes. In this way, we can test how blocks or communities within a network are constructed. Furthermore, we can use the model to

[3]https://igraph.org/r/

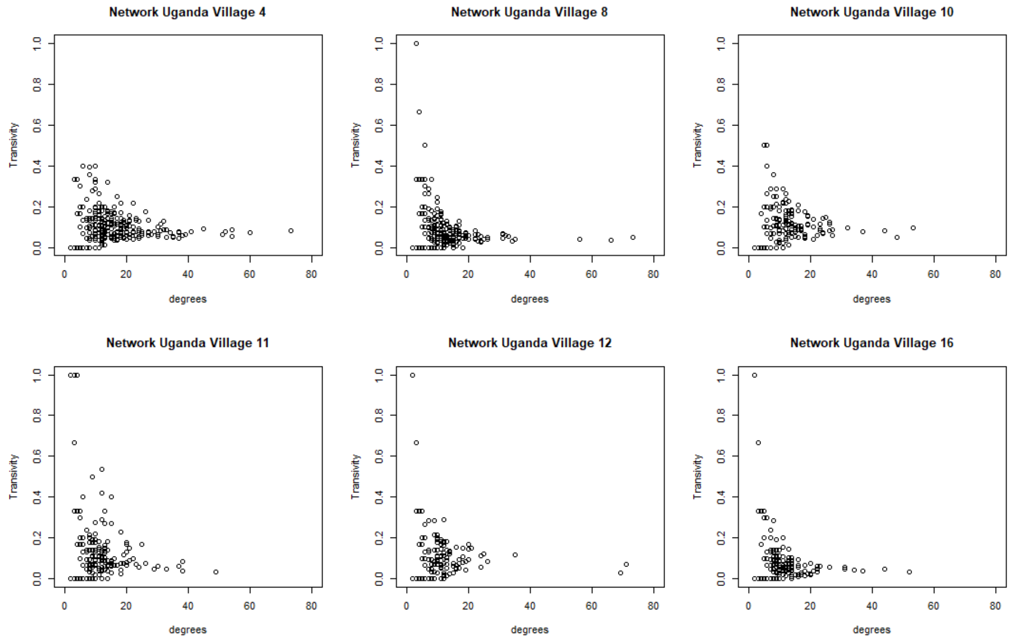


Figure 4.5: Plot of local clustering coefficient against degree for each node in the six Ugandan village networks depicted in Figure 4.3.

create realistic networks for agent-based model simulation. We can postulate a case where the network model affects the decisions of individuals and network creation is affected by some characteristics or behaviour of individuals. Such complexity may lead to emergent phenomena which cannot be anticipated.

The first step of model construction is to create a community, as in the BTER model. However, the topological features of nodes are not pre-determined; only the total number of nodes and the age of agents at each node are determined in our model. We use the t-distribution with the estimated parameter values in Table 4.2 to construct communities in our model. Cumulative distribution of a non-central t-distribution with df v and mean μ is given as $F_{v,\mu}(x)$. The probability of connection between agents or nodes i and j, aged x_i and x_j respectively with age gap of $s_{i,j} = x_i - x_j$, is

$$\psi(s_{i,j}) = F_{v,\mu}(s_{i,j} + 1) - F_{v,\mu}(s_{i,j}). \tag{4.6}$$

Although the probabilities can be written in matrix form, it is easier to ex-

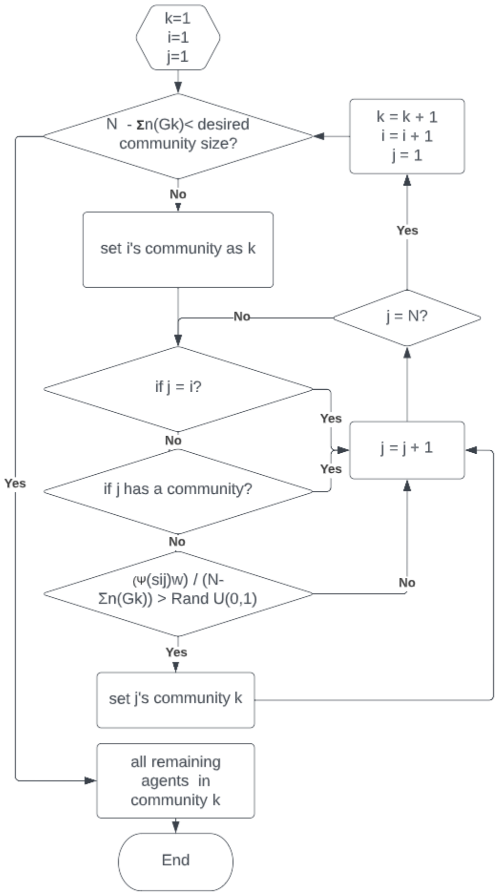


Figure 4.6: Community matching algorithm

plain community construction using the flow chart in Figure 4.6, especially as community construction is based on an agent-based approach. The flow chart is constructed similarly to the illustration of Grimm et al. (2006). To construct a community, when the total number of nodes is N and $n(G_k)$ is the size of community k, agent i invites another agent j who does not yet belong to any community to join his community k_i with probability

$$P(j \in G_{k_i} | j \notin \cup_k G_k) = \min\left(\frac{w\psi(s_{i,j})}{N - \sum n(G_k)}, 1\right) \tag{4.7}$$

According to Equation 4.7, agents whose age is closer are more likely to form a community together, as the probability is monotonically increasing with $\psi(s_{i,j})$. The term $\frac{w}{N-\sum n(G_k)}$ is required to normalise the number of nodes from which agents choose. The number of agents set without a community will keep decreasing as agents are assigned to communities. The normalising factor can make the probability higher when the number of agents who are not assigned to a community decreases so that the increasing probability can lead to a consistent size of communities. w weights the number of starting nodes and $N - \sum n(G_k)$ is the number of nodes left when kth community is created. w is one of the crucial parameters as it determines the distribution of the size of communities. The minimum function is required as the probability can exceed one when the number of remaining individuals without community gets smaller.

Once communities are constructed with a given distribution of ages, nodes will make connections to other nodes from the same community. The edge probability within community k is the same as the original BTER model. However, as desired degree distribution is not pre-determined, the minimum degree within the community $\bar{d}_k + 1$ is replaced with the size of the community of k to $n(G_k)$ and the maximum desired degree from the entire graph $\log(d_{max})$ is changed to the size of the largest community within the entire graph $n(G_{max})$.

$$\rho_k = \rho\left[1 - \eta\left(\frac{\log(n(G_k))}{\log(n(G_{max}))}\right)^2\right]. \tag{4.8}$$

The final step of the model is to connect excess degrees outside of the communities. Similarly, as nodes information is unavailable, the excess degree is unnecessary as in Equation 4.4. Therefore, we suggest a probability of an individual i from the community k_i connecting to an agent j from the other community k_j as the CL model

$$\phi_{i,j} = \min\left(\frac{t_i t_j}{n(G_{k_i}) + n(G_{k_j})}, 1\right) \tag{4.9}$$

where t_i and t_j are numbers of connected edges from the first step for agent i and j.

4.6 Model calibration and validation

4.6.1 Analysis of Dutch women friendship data

According to section 5.4.1, the age gap plays a crucial role in forming a friendship. Therefore we use age gap as a node characteristic and create communities based on the age gap. The age gap between friends in Dutchwomen data is analysed using a distribution fitting. Theoretically, the age gap does not follow any continuous distribution and can hardly be fitted to continuous distributions because of the asymmetry and discrete values. Nevertheless, we carry out parametric estimation using R packages [4]. The *t*-distribution is selected for fitting as the frequency and cumulative frequency plots in Figure 4.2 shares some characteristics with the *t*-distribution. We find that the best fitting parameters of as in Table 4.2. The graphical validation of parameter values is presented in Figure 4.7. Theoretical density and cumulative distribution on the left panels are illustrated as a red line, and the red lines are close to the empirical density and cumulative distribution. The q-q plot on the top right panel has a gradual slope for quantiles less than 0 as the age gap is limited when having younger friends.

	estimate (Std. Error)
v: degrees of freedom	1.1385 (0.0272)
μ: mean	0.3215 (0.0297)
σ standard deviation	1.8583 (0.0393)

Table 4.2: Parameter values for t distribution fitting

[4]https://cran.r-project.org/web/packages/metRology/index.html and https://cran.r-project.org/web/packages/fitdistrplus/index.html are used

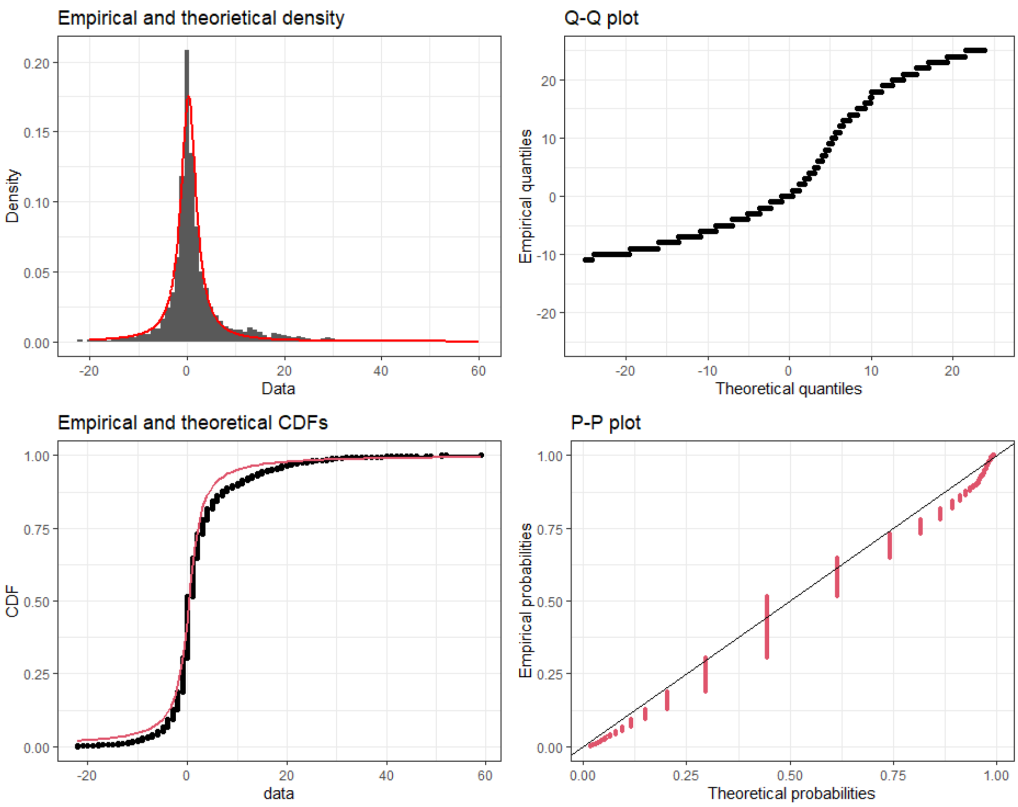
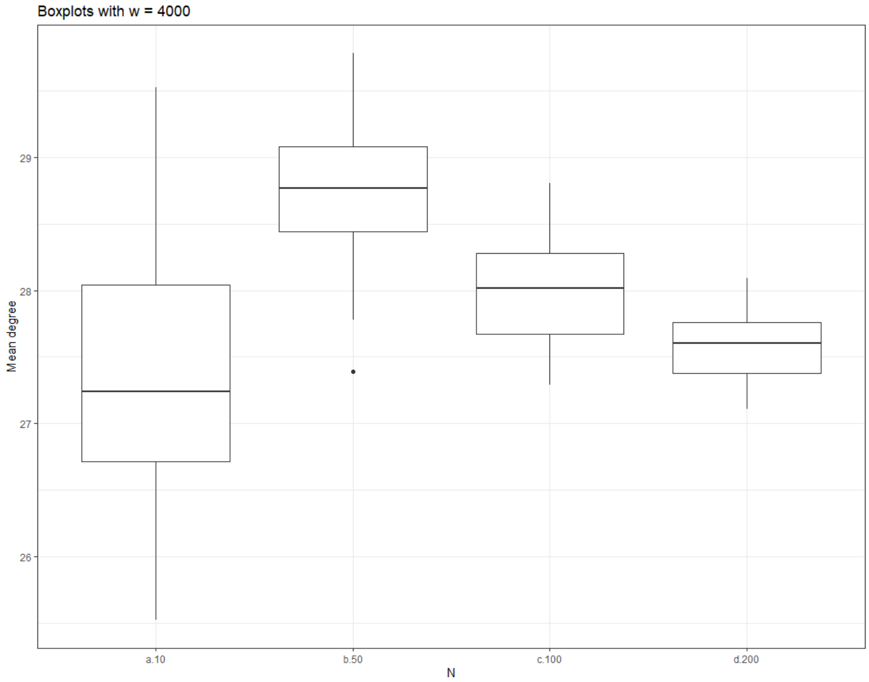


Figure 4.7: Fitting of the t-distribution to the Dutch women friendship age gap data.

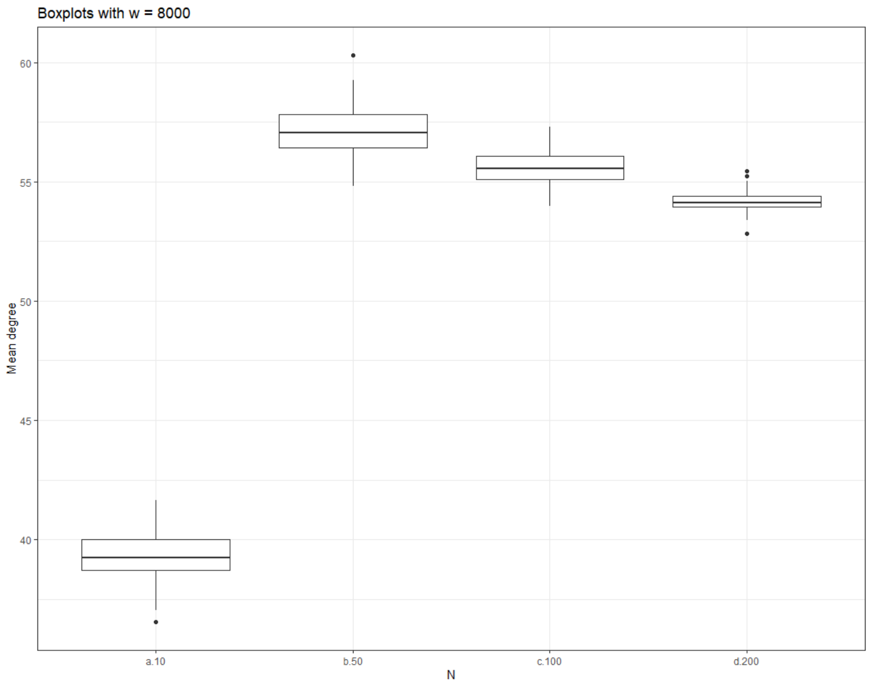
4.6.2 Test of community sizes

Our model has four key parameters, of which two parameters, w and N, are for community structure. The other two parameters, ρ and η, are for edge probabilities. 50 times of simulation for each w and N is performed to analyse effects of w on community size. The model we build consists of some individuals aged between 20 and 100. The number of individuals at each age is fixed. Hence the number of total agents is multiple of 80. $N = 8000$ when 100 agents are at each age. Therefore we test values of w and N as multiples of 80.

$w = 4000$ gives mean community size about 28 and $w = 8000$ gives mean community size around 51. The total number N has an effect on mean community size as a bigger population would give a smaller standard deviation in sizes. Figure 4.8 compares mean sizes of communities with different populations and w; network models were simulated 50 times each.



(a) Boxplots of mean degree, w = 4000



(b) Boxplots of mean degree, w = 8000

Figure 4.8: Boxplots of mean degree with N = 800, 4000, 8000, 16000

One simulation with 8000 nodes is comparable to 10 simulations with 800 nodes. However, the result of multiple simulations with a smaller sample is not quite the same as one simulation with a larger sample. Agents with smaller samples have smaller neighbours which whom they can make a group. For example, 100 individuals are at one age when $N = 8000$ compare to 10 individuals at one age when $N = 800$. We expect weight, w, should have a greater impact when the size of the total node gets smaller. However, a smaller mean degree on the left-hand side of Figure 4.8 implies that when the size of the total node is as small as 800, groups cannot be formed with the total capacity.

We test combinations of parameter values ρ and η from 0 to 1, increasing 0.1 each step and w from 1000 to 8000, increasing 1000 each step. We find a combination of parameter values of $\rho = 0.45$, $\eta = 0.2$, and $w = 2000$ found to produce a mean degree of 14.51. We use these parameter values for simulation.

4.7 Results

In this section, we examine the most commonly studied properties of a realistic network: degree distribution, the average distance between vertices and clustering coefficients of the generated network. Two cases of different types of simulations are performed. Firstly one large number of total nodes N, instead of multiple small N, is simulated. Secondly, 6 simulations with small N are carried out. The first scenario is to see the smooth distribution as if simulated multiple times. We first set N as 8000 instead of having 10 simulations with 800 nodes. We know that one large-sized sample simulation and multiple small-sized sample simulations are not equivalent, However, having a simulation with $N = 8000$ will show smoother distribution than 10 simulations with $N = 800$. Moreover, the small-sized sample simulation is hard to produce power-law distribution.

Figure 4.9 shows the histogram of degrees from the data (top panel) and the simulated model (bottom panel). The number of friends in the friendship data is capped at 25 by the survey design. We can speculate that the tails

	Nodes	Edges	Mean degree	Sd degree	Diameter	Mean distance	Louvain Groups	Louvain Modularity	Transivity
Dutch women	706	NA	10.4	5.3	NA	NA	NA	NA	NA
Village 4	320	2302	14.39	9.98	5	2.53	9	0.31	0.089
Village 8	369	1902	10.31	7.59	5	2.80	10	0.34	0.056
Village 10	207	1180	11.40	7.62	5	2.54	8	0.3	0.10
Village 11	250	1183	9.46	6.85	5	2.79	9	0.4	0.093
Village 12	229	962	8.40	7.96	6	2.88	9	0.35	0.083
Village 16	372	1475	7.93	6.06	7	3.13	13	0.38	0.050
Simulation 0	8000	58056	14.51	8.77	6	3.60	51	0.39	0.044
Simulation 1	320	2022	12.64	7.77	4	2.52	14	0.32	0.081
Simulation 2	320	1967	12.29	6.62	5	2.58	11	0.34	0.088
Simulation 3	320	1841	11.51	7.17	5	2.60	10	0.35	0.076
Simulation 4	320	1901	11.88	7.41	5	2.58	13	0.34	0.081
Simulation 5	320	1786	11.16	6.51	5	2.65	12	0.36	0.086
Simulation 6	320	2036	12.73	8.01	5	2.54	12	0.33	0.092

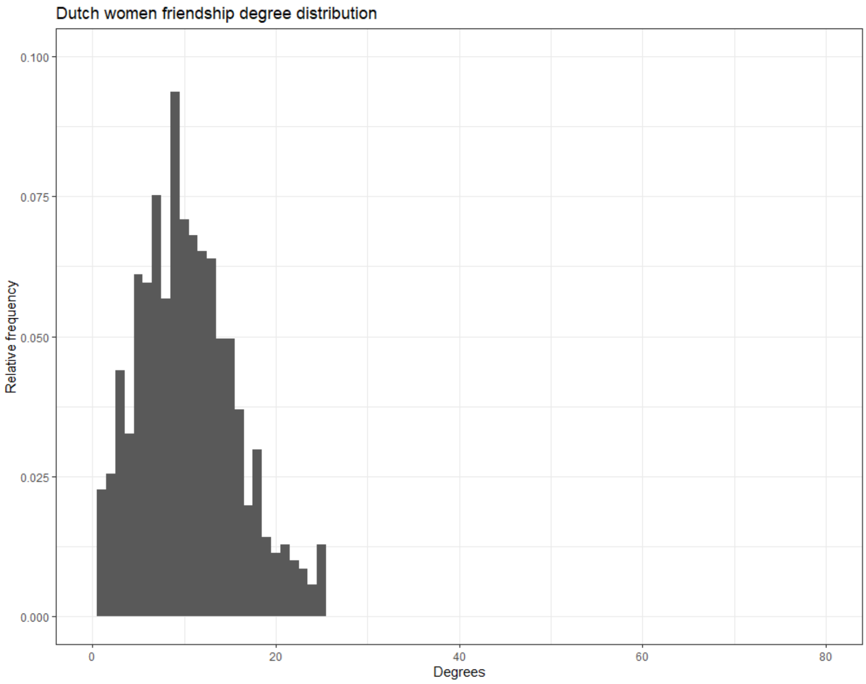
Table 4.3: Statistics of simulated networks.

would have been longer, but how far the tail spread cannot be conjectured precisely. Except for the long tail, the shape of the two graphs is very similar. A smaller number of total nodes, N, in simulation could create a less smooth shape like in real data.

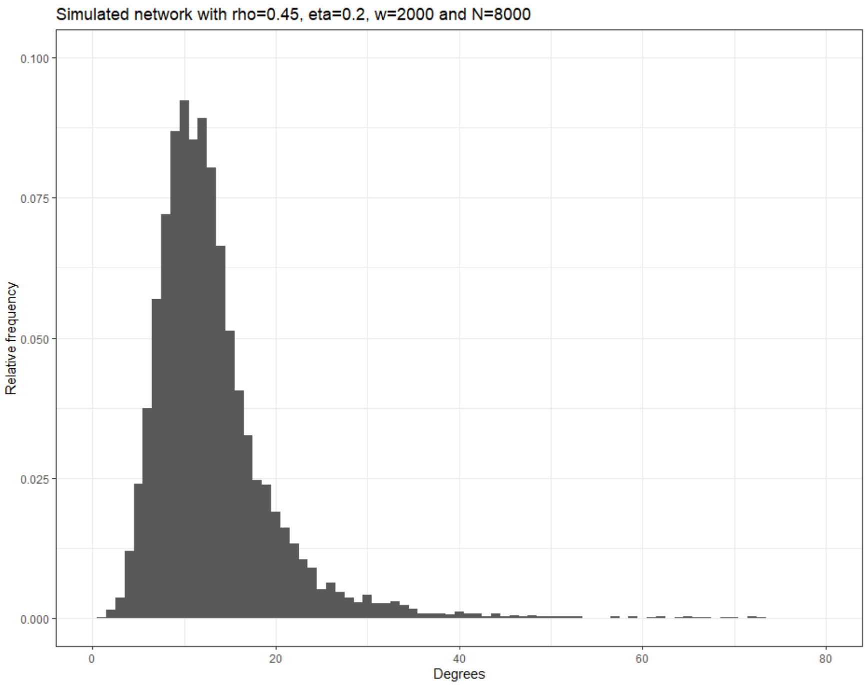
Figure 4.10 shows the distribution of ages of agents within a community. We can see that agents with similar ages form a majority of a community.

To examine the second case where the number of small networks is created, we generate 6 graphs with parameter values $N = 320, \rho = 0.45, \eta = 0.2, w = 2000$. By creating 6 small networks we can also compare the model with the network graphs in Figure 4.3, A small size of N is selected to compare with some Ugandan village data in 4.3. We can also test how the model works in an environment where N is small. First, plotted graphs in Figure 4.3 and Figure 4.11 of both simulated and actual networks are similar as they have few clusters in the centre of mass and peripheral nodes have much less dense connections to others. Clustered nodes are noticeable in both Figures in that groups of nodes are separated and have some gap between them.

Second, degree distributions are illustrated in Figure 4.4 and 4.12. We can observe some common features, including right-tailed shape and peak at degree 10 with a close frequency level. Finally, the local clustering



(a) Dutch women friendship data



(b) Simulated network

Figure 4.9: Histogram of degrees for the data and the simulated result

coefficient, or local transitivity, is compared with the degrees of nodes. Figure 4.5 and 4.13 suggest that generated networks have a similar level of clustering coefficient and corresponding degree.

Table 4.3 shows measures of simulated networks comparable to measures

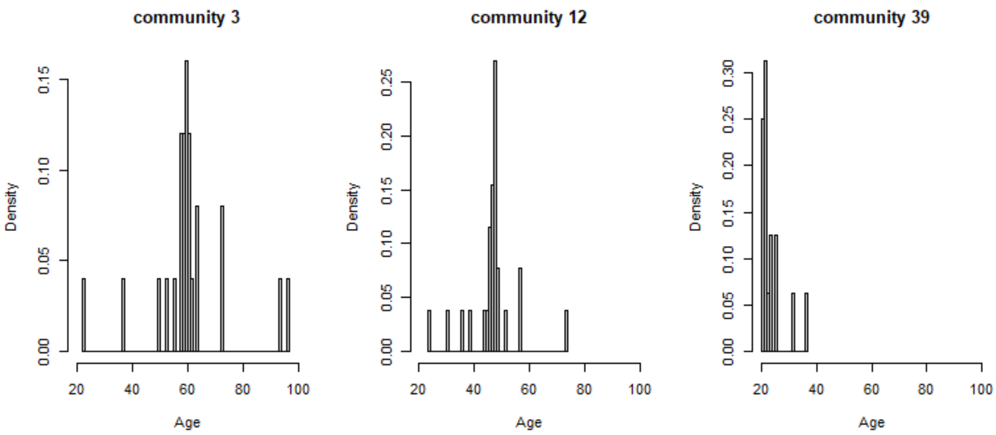


Figure 4.10: Distribution of agents' age within a community

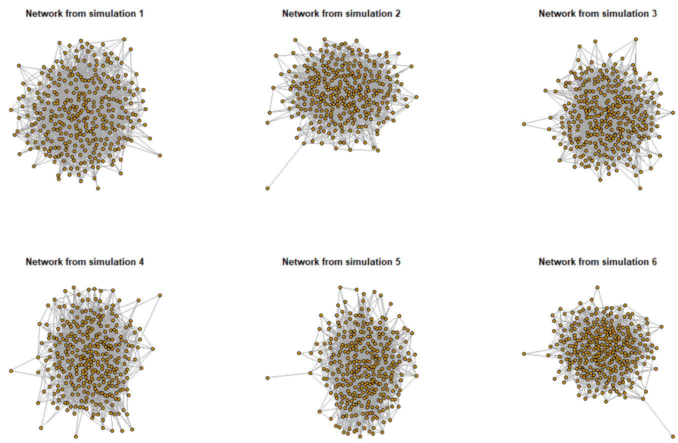


Figure 4.11: Network graphs of the simulations

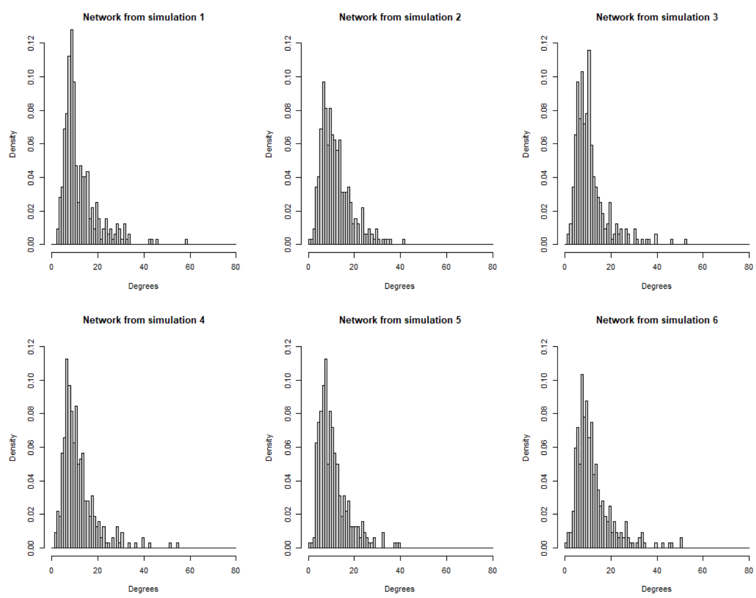


Figure 4.12: Degree distribution of the simulations

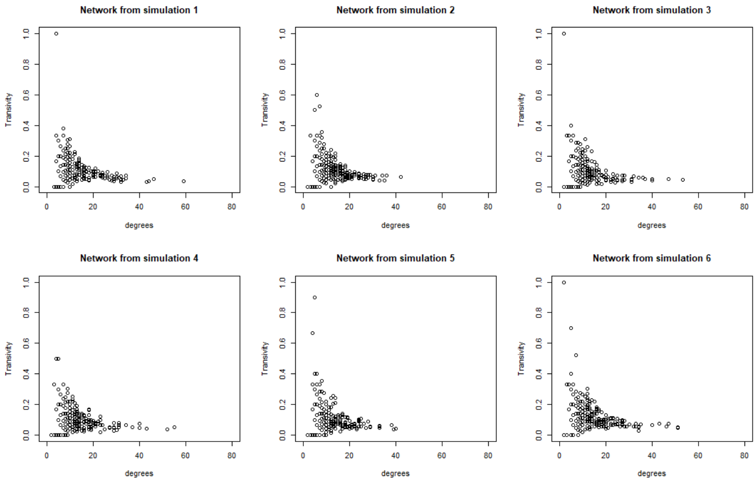


Figure 4.13: Local clustering coefficient for degrees of nodes from simulations

from Table 4.1. All of the measures of simulated networks are within a range of the measures of networks from the data. The mean degree of the networks from the data ranges between 7.93 and 14.39, while the mean degree of the simulated networks is between 11.16 and 12.73. The standard deviation of real networks is from 6.06 to 9.98, and the standard deviation of simulated networks is from 6.51 to 8.01. Other measures of the simulated networks are also very close to the measure of the real networks. The dispersion of the real network seems slightly larger than the simulated network.

4.8 Conclusion

In this paper, we have presented a model where community structure is determined by the characteristics of nodes, and all nodes are linked with the ER model at two different levels; links within communities determine the level of transitivity, while links outside the communities generate longer tails. Therefore, the simulated graphs satisfy two contradicting characteristics. Community structure, particularly, is determined by a pattern observed in data of Buijs & Stulp (2022). The authors find that people befriend others who are close in age to their own. We, therefore, allow agents to cluster in communities according to age. The simulated networks have measures and shapes close to real networks. In our future study, a network model with the age gap will be used for an agent-based model simulation.

Appendix

Network measures such as transitivity, modularity and distance and diameter are calculated using iGraph (Csardi et al. 2006). A detailed equation is given in Appendix.

4.8.1 Transitivity

The probability of two adjacent vertices of a vertex being linked is defined as transitivity. The clustering coefficient is another term used to refer to transitivity. There are a number of ways to calculate transitivity. In this paper, we use the definition by Barrat et al. (2004) as

$$C_i^w = \frac{1}{s_i(k_i - 1)} \sum_{j,h} \frac{w_{ij} + w_{ih}}{2} a_{ij} a_{ih} a_{jh}. \tag{4.10}$$

s_i is the strength of vertex i, a_{ij} are elements of the adjacency matrix A, k_i is the vertex degree, w_{ij} are the weights. Strength of vertex is calculated by summing up the edge weights of the adjacent edges for each vertex.

4.8.2 Modularity

When a graph is broken up into modules, modularity determines how modular each subgraph is. In other words, the modularity of a graph measures how vertices are separated in types from each other. It is defined as

$$Q = \frac{1}{2n} \sum_{i,j} (A_{ij} - \gamma \frac{k_i k_j}{2m} \delta(c_i, c_j)). \tag{4.11}$$

m is the number of edges, a_{ij} is the element of the A adjacency matrix in row i and column j, k_i is the degree of i, k_j is the degree of j, c_i is the type (or component) of i, c_j is the type (or component) of j, the sum is over every pair (i, j) of vertices, and $\delta(x, y)$ is 1 if $x = y$ and 0 other wise (Newman 2006).

4.9 References

Barabási, A.-L. & Albert, R. (1999), 'Emergence of scaling in random networks', *Science* **286**(5439), 509–512.

Barrat, A., Barthelemy, M., Pastor-Satorras, R. & Vespignani, A. (2004), 'The architecture of complex weighted networks', *Proceedings of the national academy of sciences* **101**(11), 3747–3752.

Blondel, V. D., Guillaume, J.-L., Lambiotte, R. & Lefebvre, E. (2008), 'Fast unfolding of communities in large networks', *Journal of statistical mechanics: theory and experiment* **2008**(10), P10008.

Broido, A. D. & Clauset, A. (2019), 'Scale-free networks are rare', *Nature* **10**(1), 1–10.

Brusco, M., Doreian, P., Steinley, D. & Satornino, C. B. (2013), 'Multiobjective blockmodeling for social network analysis', *Psychometrika* **78**(3), 498–525.

Buijs, V. L. & Stulp, G. (2022), 'Friends, family, and family friends: Predicting friendships of dutch women', *Social Networks* **70**, 25–35.

Chami, G. F., Ahnert, S. E., Kabatereine, N. B. & Tukahebwa, E. M. (2017), 'Social network fragmentation and community health', *Proceedings of the National Academy of Sciences* **114**(36), E7425–E7431.

Chung, F., Fan, R., Chung, F. R., Graham, F. C., Lu, L. & Chung, K. F. (2006), *Complex graphs and networks*, number 107, American Mathematical Society.

Chung, F. & Lu, L. (2002), 'The average distances in random graphs with given expected degrees', *Proceedings of the National Academy of Sciences* **99**(25), 15879–15882.

Csardi, G., Nepusz, T. et al. (2006), 'The igraph software package for complex network research', *InterJournal, complex systems* **1695**(5), 1–9.

Erdős, P. & Rényi, A. (1959), 'On random graphs. i', *Publicationes Mathematicae* pp. 290–297.

Erdős, P. & Rényi, A. (1960), 'On the evolution of random graphs', *Publ. Math. Inst. Hung. Acad. Sci* **5**(1), 17–60.

Fasino, D., Tonetto, A. & Tudisco, F. (2021), 'Generating large scale-free networks with the chung–lu random graph model', *Networks* **78**(2), 174–187.

Girvan, M. & Newman, M. E. (2002), 'Community structure in social and biological networks', *Proceedings of the National Academy of Sciences* **99**(12), 7821–7826.

Grimm, V., Berger, U., Bastiansen, F., Eliassen, S., Ginot, V., Giske, J., Goss-Custard, J., Grand, T., Heinz, S. K., Huse, G. et al. (2006), 'A standard protocol for describing individual-based and agent-based models', *Ecological modelling* **198**(1-2), 115–126.

Kivelä, M., Arenas, A., Barthelemy, M., Gleeson, J. P., Moreno, Y. & Porter, M. A. (2014), 'Multilayer networks', *Journal of Complex Networks* **2**(3), 203–271.

Newman, M. E. (2006), 'Finding community structure in networks using the eigenvectors of matrices', *Physical Review E* **74**(3), 036104.

Roethlisberger, F. & Dickson, W. (1939), 'Management and the worker.'.

Seshadhri, C., Kolda, T. G. & Pinar, A. (2012), 'Community structure and scale-free collections of erdős-rényi graphs', *Physical Review E* **85**(5), 056109.

Verbrugge, L. M. (1979), 'Multiplexity in adult friendships', *Social Forces* **57**(4), 1286–1309.

Watts, D. J. & Strogatz, S. H. (1998), 'Collective dynamics of 'small-world'networks', *Nature* **393**(6684), 440–442.

Chapter 5

Impact of social interactions in retirement asset decision

5.1 Abstract

Since 1956 and until 2015, most individuals in the UK with a defined-contribution (DC) pension were under compulsion to buy an annuity, or some restriction to purchase an annuity at retirement (Cannon et al. 2016). Since 2015, almost 85% of the people who accessed their pension contributions chose non-annuity decumulation, including withdrawal and drawdown. The result is surprising as a survey which took place in May 2014 before the reform, shows that around 57% of the respondents preferred a regular income, while 19% to 33% of the respondents answered that they would take non-annuity payments. We contend that social learning can explain this discordance. Mimicking behaviour may arise when people make a decision to save for retirement. We build an agent-based model where agents decide on retirement savings based on neighbours' decisions. Our agent-based model simulations show that as little as 5% of individuals may have a preferene for annuities. It confirms previous simulation results of low probability of buying an annuity.

5.2 Introduction

The UK, since 1956 and until 2011 effectively, required the compulsory purchase of annuities upon retirement (Cannon et al. 2016). There are pros and cons to compulsory annuitisation. Annuities, traditionally, have been considered to have high utility. Yaari (1965) suggest full annuitisation is optimal for most of the people when they have no bequest motive. Compulsion in annuitisation also prevents moral hazard and cost of myopia (Cannon et al. 2016). However, there is an argument against the obligation as richer individuals with higher lifespans will likely benefit more from the compulsory annuity than individuals with shorter lifespans.

In 2011, the UK Government announced that the necessity to annuitise was removed, subject to a minimum income requirement (MIR) of a minimum total pension income of £20,000. In 2015, the UK government announced making even more flexible use of pension savings. After the reform, the retirement market completely changed. Until 2015, 90% of pension contributions purchased annuities[1]. In 2019 only 10% of pension pots were converted into annuities. The trend reversed completely within 4 years. The trend, however, is hard to explain fully. Especially when the survey by the National Association of Pension Funds (NAPF) before the reform and after the announcement in May 2014 illustrates only 19% to 33% of the individuals intend to convert part or full pension savings into non-annuity decumulation while 57% of the respondents answered that they prefer regular income [2].

We postulate that individuals may make a decision without thoughtful consideration, given the contrasting results before and after the reform becomes effective. The Financial Conduct Authority (2018) find that 32%

[1] https://www.fca.org.uk/data/retirement-income-market-data-2020-21

[2] https://www.plsa.co.uk/portals/0/Documents/0387_Workplace_pension_survey_May_2014_3DOCUMENT.pdf

of individuals accessed to withdraw from their pension savings without advice after the reform, while only 5% of individuals withdrew without advice before the reform. We often find cases where friends and family influence more than experts on our decisions. Savings and retirement decisions can be the same. For example, Park & Banerjee (2020) finds that coworkers can influence individuals' annuitisation decisions. We assume individuals are influenced to withdraw by their peers and construct a model where decisions are influenced by their peers.

We present an agent-based model to reproduce the current phenomenon when the individuals are divided into two groups: isolated and imitating groups. isolated individuals make decisions based on their evaluation while mimicking group mimic the behaviour of their peers within their network. According to the simulation, annuity demand for the isolated group is as low as 10% to 15%, while the demand for a lump sum is twice as high as the annuity demand. Furthermore, once one asset starts to dominate another, it is extremely difficult to retrieve the share.

5.3 Literature review

5.3.1 Compulsory purchase annuity market and the reforms

Most individuals in the UK with Defined Contribution (DC) pensions had no choice but to purchase annuities until 2011. Individuals who had accumulated DC contributions were required to convert their savings into an annuity. This is a so-called compulsory purchase annuity (CPA) market in place since 1956. On the other hand, individuals could also convert any savings into a life annuity in a voluntary annuity market (purchased-life annuity, PLA). These two markets were differentiated as they were taxed differently until the CPA market was completely removed in 2015 after the reform (Cannon et al. 2016).

Such a compulsory annuity market in the UK was unusual compared to other international markets. International researchers like Poterba (2001) and Yermo (2001) claim that this policy has benefits which outweigh its costs. Arguments for compulsory annuitisation include the prevention of moral hazard and protection against myopia; CPA prevents adverse selection, and tax advantage given during the accumulation phase encourages long-term savings plans. On the other hand, a compulsory annuity could redistribute wealth away from the poor to the rich as the poor tend to have shorter life expectation than the rich, thereby receiving annuity payments for a shorter period.

An annuity is often considered to be providing protection against uncertain lifespans and offers welfare benefits. Yaari (1965) suggest it is optimal to purchase an annuity for all individuals without bequest motive. The UK policymakers let individuals make their own decisions on pension savings, despite the correlation between compulsion and high annuity purchases. Bateman & Piggott (2010) argue that the removal of incentives to annuitise in Australia over 2006-2007 led to the collapse of the Australian annuity market.

Before the policy changes in 2011 and 2015, there were exceptions to the compulsory annuitisation requirement. For example, tax-free lump sums up to 25% of pension, deferral of annuitisation up to 75 years old, exemptions for the small pots and trivial commutation for sufficiently small pension savings. Furthermore, income drawdown has been allowed as an alternative since 1994. Until the first reform, pensioners are allowed to withdraw a proportion of their savings up to a limit each year. The principal change in the 2011 reform was the removal of the compulsory annuitisation requirements for individuals who could prove that they would have an annual income of £20,000 or more in their retirement. The presence of the minimum income requirement (MIR) left most individuals without any option but annuitisation. Since income drawdown and MIR were considered suitable for wealthy retirees, the Financial Services Authority recommended income drawdown only for individuals with relatively large pension pots (Cannon et al. 2016).

The second reform in 2015 completely removed the annuitisation requirement. The reform, first of all, allowed all individuals to access their pension savings as a cash withdrawal from the age of 55, regardless of their income restriction. However, marginal tax rates on cash withdrawal remained a strong disincentive to prevent conversion at once. Also, the Pension Wise service was started to provide free pension guidance for consumers to make better choices (The Financial Conduct Authority 2017).

The Financial Conduct Authority (2018) identifies some issues after the reform. First of all, consumers are losing choices because of annuity providers leaving the market and limited product innovation. Consumer irrational behaviours are causing another problem. A majority of customers take the path of least resistance and buying drawdown without advice from experts. We analyse if peer influences may contribute to such irrational behaviour in the next section.

5.3.2 Peer influences on decision making

Influences of others' behaviour on one's decision-making are broadly illustrated in various areas of study. Moreover, people's reasoning is based on objective outer data, which incorporates others' behaviour to certain degrees. Benartzi & Thaler (2007) claim that family members and friends who are not necessarily experts may act as advisors. Sorensen (2006), Hvide & Östberg (2015), Lu & Tang (2019), Park & Banerjee (2020) suggest social influence on decision making process. Sorensen (2006) use data from the University of California to examine social learning in health plan choice. Hvide & Östberg (2015) study influence of coworkers on investment choices. Lu & Tang (2019) investigate the effects of social interactions on individual asset allocation decisions using 401(k) enrolment data. Park & Banerjee (2020) study the coworker influence on an individual's annuitisation decision. Friends and colleagues with similar levels of benefit, intimacy, and modal retirement age at work are heavily influenced by each other's annuity decisions.

Duflo & Saez (2002) investigate peer effects on savings decisions. The authors consider the role of peer effects on participation in a Tax Deferred Account (TDA) and on decisions related to the TDA plan. They ask employees of a large university whether their decisions to enrol in the TDA plan to choose a specific vendor are influenced by the decisions of their colleagues in the same department. An interesting result is that there are significant differences in the TDA plan participation rates of the staff from different libraries, even though salaries and benefits are similar across the libraries. Duflo & Saez (2002) argue that this result may suggest peer effect, although these could be other causal factors.

Duflo & Saez (2003) conduct an experiment to show how the role of information and social interactions can affect employees' decisions to enrol in a TDA retirement plan. A large university holds a fair every year to promote enrolment in TDA and invites all of its employees. The authors select a group and several employees from the group to send an invitation letter with $20 reward for attending the fair. As a result, treated individuals (with the reward) are more than five times more likely to attend the fair than the controlled group (without the reward). More interestingly, untreated individuals within the treated group, the group with treated individuals, are three times more likely to attend the fair compared to the controlled group without individuals with the reward. The experiment of Duflo & Saez (2003) shows a spill-over social effect on their colleagues within departments. Also, people who attended the fair have a higher probability of joining the TDA 5 to 11 months after the fair. Therefore, the social influence may increase registration as it brings more peers to the fair.

Vermeer et al. (2014) study a survey which includes self-assessment and vignette questions, which shows that individual preferences are affected by preferences and actual retirement behaviour of the social environment, which include relationships, institutions, culture, and physical structures. They find the timing of retirement depends on the retirement age of relatives, friends, colleagues and acquaintances. A majority of respondents postpone retirement when their peers delay retirement. The result of Vermeer et al. (2014) suggests that a one-year delay of retirement age within a social environment leads to an increase of three months in the individual's own retirement age.

5.3.3 Coordination game forming a social norm: age of retirement

When agents respond to other agents, collective behaviour is difficult to predict. A system with many interacting agents can be described on two different levels: the microscopic and the macroscopic level. At the microscopic level, individual agents make decisions, and at the macroscopic level, collective behaviour is observed. It is the interactions of agents that connect the two different levels. Although the rational choice theory is a powerful tool for analysing agents' decision-making, it has certain limitations. Agents often exhibit bounded rationality as people cannot process information fully. For example, externalities arise when agents' choices produce unexpected side effects on the social activities of other agents who have no direct connection with them.

In this section, we introduce a decision-making process described by Namatame & Chen (2016). For a binary choice problem, whether to buy a product or not, each agent has his own idiosyncratic preferences regarding the product. Namatame & Chen (2016) assume individual idiosyncratic preference is linked to an agent's willingness to pay. Idiosyncratic preferences lead agents to buy the product given a preference level and the agent's willingness to pay. Furthermore, individuals are subject to social influence from other agents when they consider the choices of others. Let x_i be the binary decision variable of agent i. $x_i = 1$ means agent i buys the product, whereas $x_i = 0$ means that agent i does not purchase the good. The utility function, $y_i(t)$ at time t, can be expressed in terms of idiosyncratic preferences h_i, price q and a social influence component as below

$$y_i(t) = h_i - q + \frac{\omega}{k_i} \sum_{j \in N_i} a_{ij} x_j(t). \tag{5.1}$$

Namatame & Chen (2016) give the social influence term which is composed of a positive constant ω, degree k_i of agent i, set of neighbours N_i, and a binary value a_{ij} showing the connectivity between agents i and j. The binary value a_{ij} takes 1 if agents i and j are connected, and it takes value 0 if agents i and j have no connection. An agent's choices will be determined only by the willingness to pay and the price only if the agent does not care about others' choices: i.e. $\omega = 0$. However, with some social influences, the binary decision variable x_i becomes

$$x_i(t) = \begin{cases} 1 \text{ if } y_i(t) \geq 0 \\ 0 \text{ otherwise.} \end{cases} \tag{5.2}$$

We can rewrite Equation 5.2 as below by using Equation 5.1 and introducing $p_i = \sum_{j \in N_i} a_{ij} x_j / k_i$ and $\phi_i = \frac{(h_i - q)}{\omega}$

$$x_i(t) = \begin{cases} 1 \text{ if } p_i > \phi_i \\ \\ 0 \text{ otherwise,} \end{cases} \tag{5.3}$$

where $0 \leq \phi_i \leq 1$. We can consider p as the fraction of neighbours of agent i who decide to purchase an annuity. The choice rule depends on p_i and ϕ_i. In this case, an individual benefits from the higher utility with the increased number of neighbours with conforming choices. More neighbours with the same choice mean higher p_i and the level of utility grows as p_i increases.

The binary choice problems with social influence can be modelled using a game theoretic framework. Table 5.1 shows the payoffs of the 2 X 2 coordination game. Two participants' choices in a game appear in the first row and column of the table. A participant gains ϕ_i or $1 - \phi_i$ depending upon whether his choice matches with his opponent's choice. Agents' payoff increases with the number of neighbours with the same choices as the game is played between all neighbours.

	A	B
A	$1 - \phi_i, 1 - \phi_j$	$0, 0$
B	$0, 0$	ϕ_i, ϕ_j

Table 5.1: Payoffs to each agent with a binary choice

Epstein (2006) use the same decision-making process and present an agent-based model to illustrate the timing of retirement converging to an optimal level despite low levels of individual rationality. In 1961, the US government reduced the minimum age at which workers can claim social security from 65 to 62. In spite of the big impact of the policy on individuals, the transition of modal age from 65 to 62 took almost 30 years (Epstein 2006). The model of Axtell and Epstein reproduces the shift in retirement age norm using an agent-based approach where agents without rationality such as expected utility maximisation based on income and consumption are employed. The agents in the model are divided into three groups: the majority group of imitators mimicking a member of their social network and two minority groups of random and rational agents. The model has agents with ages from 20 to 100. Each agent is heterogeneous and has his own social network. Randomly connected agents play a coordination game on the retirement age. The payoff for the players is proportionate to the number of players with conforming behaviours within their networks. Simulation of Epstein (2006) find the transition of modal age from 65 to 62 was accomplished within 35 years when rational agents consist of 1 to 4 per cent of the population.

5.3.4 Adaptive choices with reinforcement

Bendor et al. (2009) construct a binary choice model using social influence. When there is a binary choice A or B, the probability of choosing A at time $t+1$ is given as the weighted average of personal preference and social influence as below

$$P(\text{agent choose } A \text{ at } t+1) = \hat{\alpha} \cdot \hat{p} + (1-\hat{\alpha}) \cdot \frac{NA_t}{(NA_t + NB_t)} \tag{5.4}$$

where NA_t denotes the number of people who choose A at time t and NB_t the number of people who choose A at time t. Similarly the probability of choosing B at time $t+1$ is

$$P(\text{agent choose } B \text{ at } t+1) = \hat{\alpha} \cdot (1-\hat{p}) + (1-\hat{\alpha}) \cdot \frac{NB_t}{(NA_t + NB_t)}. \tag{5.5}$$

$\hat{p}$ reflects existing preferences or merits of A over B usually $p \in (0,1)$. Isolated agents without social influences make decisions based on $\hat{p}$, while social influence is determined by the proportion of people with each choice. The model assumes that an agent's choice is a weighted average of the two components.

The model of Namatame & Chen (2016) consists of adaptive agents who are continually influencing choices of each other. The adaptive choices of agents can be incorporated with both individual preference and social influence. A logit function is employed for an individual choice. Assuming a collection of N agents face binary choice A or B in a sequence. Let U_A and U_B represent the utilities of choosing A and B, then the probability of choosing A is

$$\mu = \frac{1}{1 + \exp(-(U_A - U_B)/\lambda)}. \tag{5.6}$$

Now let the utility of choosing A at time t be a_t, and B as $b_t = 1 - a_t$. The probability of choosing A at time t becomes

$$\mu_t = \frac{1}{1 + \exp(-(2a_t - 1)/\lambda)} \tag{5.7}$$

where $\lambda \in (0, 1]$ is the parameter governing the level of rationality of agents. Heterogeneity in preference is captured by the parameter a_t, which is the preference level of choosing A over B. Namatame & Chen (2016) assume a_t to be normally distributed with a mean of 0.5. When λ gets closer to 1, the probability becomes closer to 0.5 for all levels of utility and λ gets closer to 0 the probability becomes sensitive to changes in utility level and forms a more step-wise function. The probability of choosing A is exhibited in Figure 5.1.

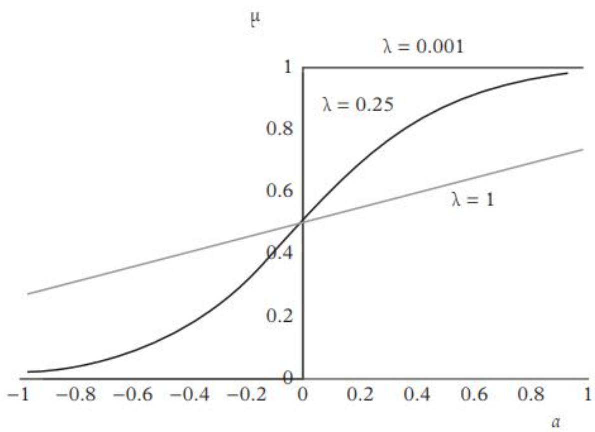


Figure 5.1: Probability of choosing A at time t, μ_t on Y-axis and a_t on X-axis

The model of Namatame & Chen (2016) assumes individuals prefer repeated choice and repetition leads to an increase in payoff which gives rise to a form of reinforcement learning. The utility of choosing A or B after one-period increases or decreases as below

$$a_{t+1} = \begin{cases} a_t + \delta & \text{agent choose } A \text{ at } t \\ a_t - \delta & \text{agent choose } B \text{ at } t \end{cases} \tag{5.8}$$

Using the probability μ_{t+1}, Namatame & Chen (2016) denote probability of agents choose A at $t+1$ as

$$P(\text{agent choose } A \text{ at } t\text{+1}) = \tilde{\alpha}\mu_{t+1} + (1-\tilde{\alpha})S_t \tag{5.9}$$

where S_t is a term for a social influence. Therefore, the probability of choosing from a binary choice is a weighted average of social influence and individual preference. Namatame & Chen (2016) assume the impact of social influence is linearly increasing with the number of neighbours with the same choices. Therefore the social influence factor, S_t is given by the ratio $\frac{A_t}{(A_t+B_t)}$ as in the model of Bendor et al. (2009).

The weight between individual preference and social influence are the most important factors which determine the decision-making process in the models of Bendor et al. (2009), Namatame & Chen (2016). We examine how the balance between social influence and individual preference affects the decision-making process and the outcome. Firstly, a pure social influence is a decision factor when $\alpha = 0$. In this case, the collective choice is unpredictable as there is no reinforcement of individual preference. When the choice depends purely on social influence, the outcome at $t+1$ is purely dependent on the predominance of existing choices before t.

The second case is where agents exhibit both individual preference reinforcement and social influence. When social influence is stronger, the choices of most agents converge on a particular alternative and a large-scale cascade occurs. When individual preference reinforcement is stronger, the collective outcome is determined by the initial preference distribution. In particular, if agents' preferences split into two groups: one initially favouring A over B and one initially favouring B over A. There is a tendency for two distinct groups of agents to form along these lines. In the final case where there is no social influence and only individual preference reinforcement exist, the collection of heterogeneous agents with diverse preference will evolve into two extremes.

Namatame & Chen (2016) also show that the network topology may impact the adaptive choice model. For example, in a complete network, each agent connects to all other agents. In other words, everyone knows each other. Therefore, in this case, the choice of A will be certain when α is 0. Also, when α is lower than 0.5, thus, social influence is stronger than individual preference reinforcement, and most cases will converge to probability 0 or 1. On the contrary, networks with core-periphery structures show that they do not exhibit probability convergence to 0 or 1 even with the strongest social influence. The exemplified processes suggest that degree distribution and clustering affect convergence.

5.4 Data

The pension reform in 2011 affected a limited amount of people, thus, the result of the change was limited. However, the change in the 2015 reform influenced many individuals with greater flexibility and changed the retirement market completely. After the second reform, the demand for drawdown has become much more popular than annuities as twice as many pots chose drawdown over annuities (The Financial Conduct Authority 2018). Annuity used to take up 90% of the market share before the reform.

The effect of the first policy change in 2011 is insignificant compared to the second change in 2015. Cannon et al. (2016) quantify total premiums of annuities increased by £307 million and total premiums of drawdown decreased by £213 million after the first reform, whereas premiums of annuities decreased by £1.8 billion and total premiums of drawdown increased by £651 million after the second reform. Similarly, The Financial Conduct Authority (2017) illustrates the impact of the second reform in 2015. According to the report, accessing pots early and taking lump sum has become the new norm. 72% of pots were accessed by consumers under 65 and they take lump sums. 53% of pots accessed were fully withdrawn and twice as many pots chose drawdown over annuities. After 3 years, the FCA retirement income market data[3] shows that annuity purchase is kept decreasing since 2015 down to 10%.

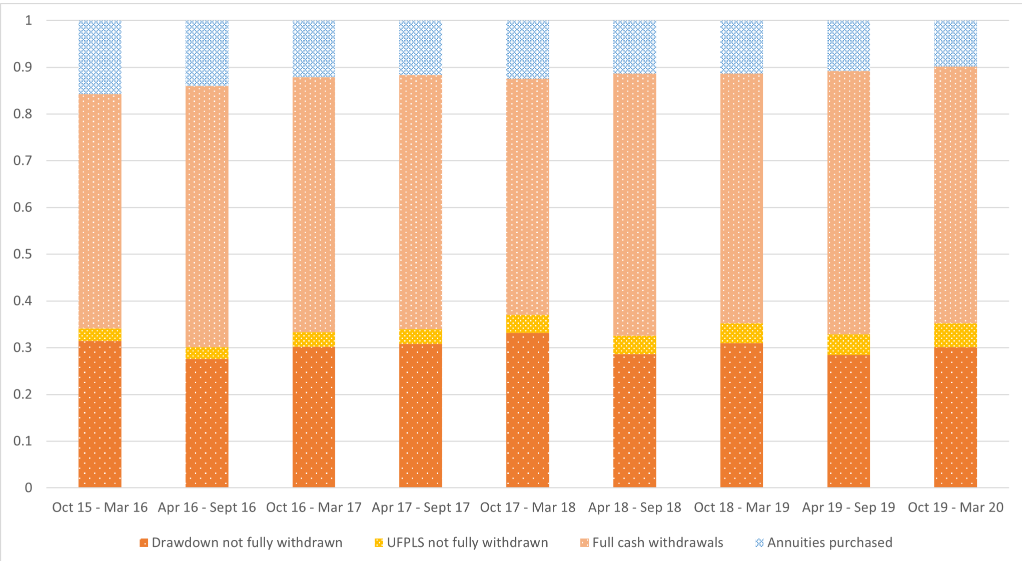


Figure 5.2: Proportion of savings account for each option. Source: The FCA Retirement income market data

A complete reversal in the trend of annuity purchases is interesting. The survey from the The National Association of Pension Funds (2014) tried to monitor the public view on the pension reform announced in the Budget

[3]https://www.fca.org.uk/data/retirement-income-market-data-2020-21

2014. The survey was conducted in April 2014, which is just after the Budget 2014, with 1009 respondents who were in employment. According to the survey, 24% of the respondents said that they would convert all pension savings into lump sum as they have other income, 19% of the people would take lump sum regardless of other income and 33% would take a lump sum when needed. In other words, only 19% to 33% of people reported that they would take some form of lump sum just after the pension freedom was announced. The prevailing choices of withdrawal after 2015 are not consistent with the The National Association of Pension Funds (2014) survey as well as the theoretical frameworks of the high utility of annuity suggested by Yaari (1965).

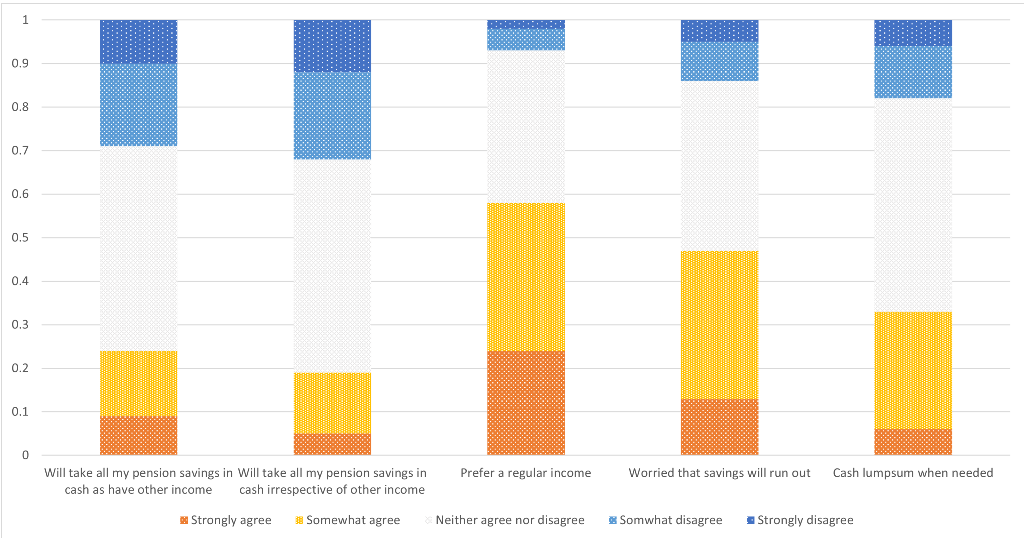


Figure 5.3: NAPF Survey on accessing pension savings at retirement

5.5 Model description

In this section, we present an agent-based model where agents show adaptive behaviour and make a decision about whether to buy an annuity or take a lump sum based on both individual preference and social influence. In this model, we build a society where the population consists of cohorts ranging from age 20 to 100, and each cohort contains C agents. At the end of each time period, agents get older and agents aged between 55 and

64 make a choice between an annuity and a lump sum. The model only focuses on agents who have an established network at time 0. Therefore, agents do not make any more connections to other agents but can lose connections because of death.

In this model, we assume agents have social influence and individual preference is given and stable as in Bendor et al. (2009). The probability of choosing an annuity $P(an_{i,t+1})$, the probability of choosing a lump sum $P(lm_{i,t+1})$ and the probability of making no choice $P(n_{i,t+1})$ are given as

$$P(an_{i,t+1}) = \alpha \cdot \rho + (1-\alpha) \cdot \frac{An_{i,t}}{(An_{i,t} + Lm_{i,t} + No_{i,t})}, \tag{5.10}$$

$$P(lm_{i,t+1}) = \alpha \cdot (1-\rho) + (1-\alpha) \cdot \frac{Lm_{t}}{(An_{i,t} + Lm_{i,t} + No_{i,t})}, \tag{5.11}$$

$$P(n_{i,t+1}) = 1 - P(an_{i,t+1}) - P(lm_{i,t+1}). \tag{5.12}$$

where total number of connected nodes of agent i with annuity is $An_{i,t}$, total number of neighbour of agent i with lump sum is $Lm_{i,t}$ and total number of neighbour of agent i with no choice as $No_{i,t}$. ρ works same as $\hat{p}$ in Bendor et al. (2009). This exhibits pre-existing preferences for annuity products over a lump sum. In other words, when there is no social influence, agents will make decisions based on ρ.

The transition matrix for individual i at time t, $W(i,t)$ can be written as

$$W(i,t) = \begin{pmatrix} P(n_{i,t+1}) & P(an_{i,t+1}) & P(lm_{i,t+1}) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{5.13}$$

where the first row represents the transition from no decision state to other states. The first element, $W(i,t)_{1,1}$ denotes the transition from no decision to no decision. The second element, $W(i,t)_{1,2}$, shows the transition from no decision to annuity and the third element, $W(i,t)_{1,3}$, represents the transition from no decision to lump sum. The second row and third row represent the transition from annuity decision and lump sum decision to other states, respectively. The last two rows show that the decision is irreversible and the agent cannot make any further decision once a decision is made.

5.5.1 Agent types

The model has two pillars of decision-making: individual preference and social influence. Individual preferences are determined by ρ. Agents choose an annuity or a lump sum based on the probability ρ. On the other hand, agents with $\alpha = 0$ make a decision purely based on social influence. We refer to agents with $\alpha = 0$ as imitator agents and agents with $\alpha = 1$ as isolated agents. We assume that the model has only two types of agents: imitator agents and isolated agents.

Imitator agents make a choice using a simple coordination game within their social networks. In other words, the timing and outcome of the decision are based on their unique social network and members of their network. We assume that imitator agents seek conformity within the social network as Epstein (2006) and Bendor et al. (2009) also assume. Therefore, imitator agents gain utility by coordinating their behaviour with the other members within their social network. However, unlike the binary options in the models of Epstein (2006), and Bendor et al. (2009), agents have three options in this case. As Table 5.2 shows, both agents i and j gain utility when their decision matches with each others' decision. Unlike Table 5.1, the payoff for making any decisions is equal, thus, we use payoff 1 for simplicity.

	no decision	annuity	lump sum
no decision	1, 1	0, 0	0, 0
annuity	0, 0	1, 1	0, 0
lump sum	0, 0	0, 0	1, 1

Table 5.2: Payoff of coordination game between agent i and j for retirement asset choice when retirement is not imminent

5.5.2 Payoff of coordination game

Let, $N_{i,t}$ be a set of nodes connected to agent i and $x_{i,t}$ be the state of agent i at time t, then utility of agent i at time t with state $x_{i,t}$, $U(x_{i,t})$ is

$$U(x_{i,t}) = \sum_{j \in N_{i,t}} u(x_{i,t}, x_{j,t}) \tag{5.14}$$

$u(x_{i,t}, x_{i,t})$ is the utility of i's interaction with j at time t, and the payoff function can be described as 3 X 3 game illustrated in Table 5.2. Table 5.2 highlights that agents gain utility as they act like their peers. In other words, the utility can be derived from not making any decisions when their peers are not making any. The settings above, however, are unreasonable when retirement is imminent. Making no decision about the retirement asset at retirement derives no utility in general, which is shown in Table 5.3. Therefore, we reduce the choice to the binary option one year before the retirement age of 65. Imitator agents aged 64 make a decision based on probabilities derived from $\frac{An_{i,t}}{An_{i,t}+Lm_{i,t}}$ and $\frac{Lm_{i,t}}{An_{i,t}+Lm_{i,t}}$.

	no decision	annuity	lump sum
no decision	0, 0	0, 0	0, 0
annuity	0, 0	1, 1	0, 0
lump sum	0, 0	0, 0	1, 1

Table 5.3: Payoff of coordination game between agent i and j for retirement asset choice when retirement is imminent for both i and j

5.5.3 Network

The social network of an agent comprises a number and a list of other agents connected to the agent. As discussed in the previous section, a social network is one of the factors influencing the decision process. We use the agent-based model presented in Chapter 5, where the community is constructed before, and the connection is made with Erdos-renyi model Erdős & Rényi (1960) in two steps Seshadhri et al. (2012) with the same parameter values.

5.6 Model calibration and validation

For simulation, we use T agents in total from the beginning, with ΠT isolated agents and $(1 - \Pi)T$ imitator agents. We set T equal to 800 for computational efficiency. We do not use realistic mortality assumption, and every agent dies at age 100. Agents lose connection with dead agents, and no new connection is made for simplicity. Parameter values to create networks are the same as in Chapter 5. Agents start to make a choice from age 55, and the choice to purchase an annuity or to receive a lump sum is irreversible.

There are two main parameters which we can change to test the model. The first parameter is Π, and the parameter determines the proportion of isolated agents and imitator agents. Π affects the probability of agents without any choice. Since isolated agents will always make a decision based on their preferences. The second parameter ρ affects the proportion of isolated agents who choose annuity and lump sum. We test Π and ρ so that the result of the The National Association of Pension Funds (2014) survey can be replicated.

Isolated agents in the model choose annuity ρ when a random number is smaller than ρ, and agents choose a lump sum if the uniform random real number between 0 and 1 is greater than ρ. Imitator agents make a choice based on a probability derived from $An_{i,t}$, $Lm_{i,t}$, $No_{i,t}$ and $N_{i,t}$. Imitator agents purchase an annuity when $\frac{An_{i,t}}{N_{i,t}}$ is less than a random number from

$Uniform(0,1)$. They choose lump sum payment when $\frac{An_{i,t}}{N_{i,t}}$ is greater than the random number and $\frac{An_{i,t}+Lm_{i,t}}{N_{i,t}}$ is less than the random number. Imitator agents make no decision when the random number is greater than $\frac{An_{i,t}+Lm_{i,t}}{N_{i,t}}$.

The range of parameter value ρ starts from 0.05. Simulation results from Chapter 3 suggest that individuals may have the probability to purchase an annuity yearly as low as 1%. We increase ρ up to 0.5 from 0.05 as we can conjecture the ρ greater than 0.5 would result in the mirroring result. Similarly, we test parameter Π from 0.05 to 0.5. We do not need to test larger Π values as increased isolated agents converge the result close to individual preference ρ.

5.7 Results

The result of 50 simulations with each parameter value is exhibited in Figure 5.4, Figure 5.5, Figure 5.6 and Figure 5.7. As expected, an increase in isolated agents causes the proportions of annuity holders and lump sum holders to converge towards ρ and $1-\rho$ respectively. First, lump sum dominance, over 90% of lump sum holders, can be observed when ρ is 0.05 or 0.2. The result is expected, as we find the probability of 0.05 can recreate the current annuity market situation. The second case is when 95% of agents are imitator agents, and the probability of choosing an annuity is equal to the probability of choosing a lump sum. The second case is shown in the right-hand side panel of Figure. There are some cases when a lump sum dominates and the annuity with a low chance. However, the second case is not probable as the pace of dominance is not rapid as the current situation.

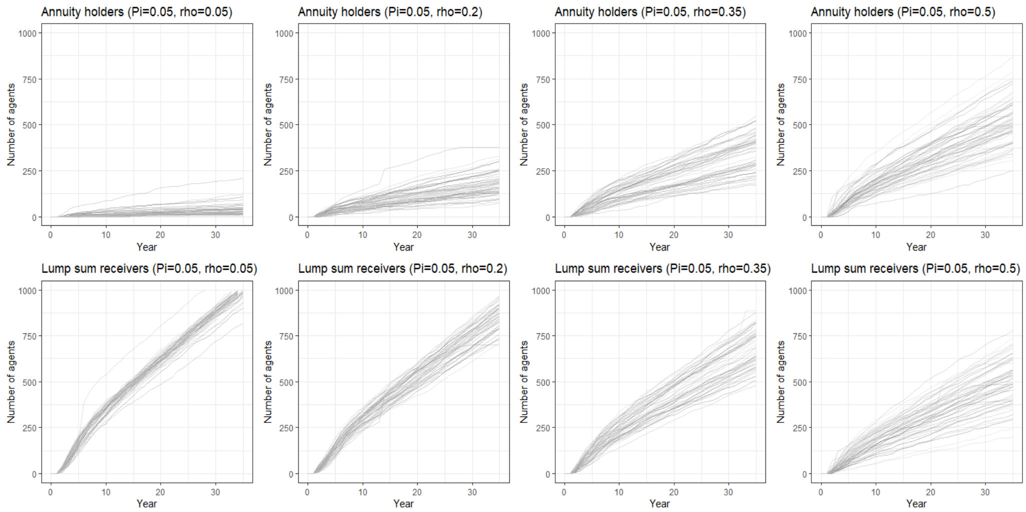


Figure 5.4: Number of annuity holders and lump sum receivers from 50 simulations with $\Pi = 0.05$ and $\rho = (0.05, 0.2, 0.35, 0.5)$

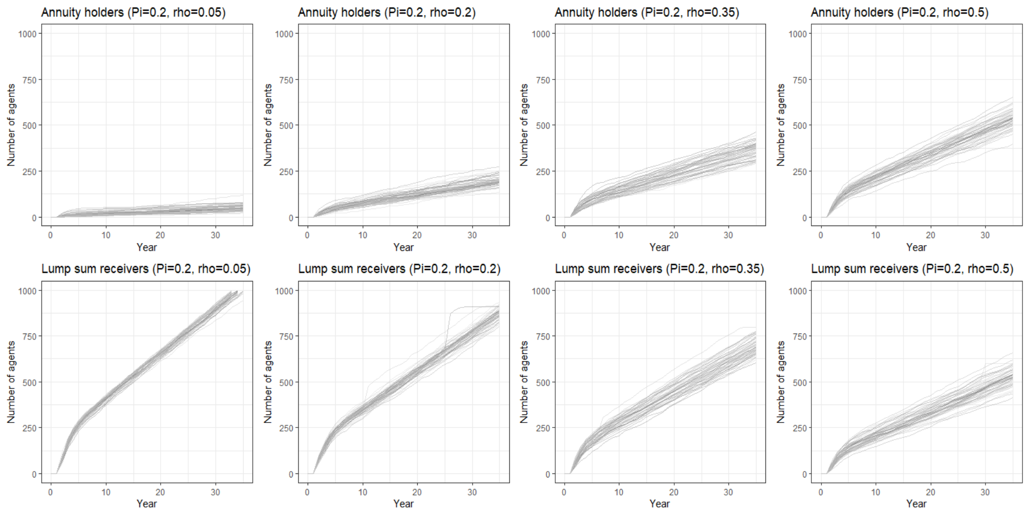


Figure 5.5: Number of annuity holders and lump sum receivers from 50 simulations with $\Pi = 0.2$ and $\rho = (0.05, 0.2, 0.35, 0.5)$

5.8 Conclusion

We build a model based on the adaptive choices model by Bendor et al. (2009), Namatame & Chen (2016) and the coordination game model by Epstein (2006), Namatame & Chen (2016). The model we build is an agent-based model, and it reproduces the inverted trend in the annuity market. The survey of The National Association of Pension Funds (2014) before the pension reform in 2015 and after the announcement in 2014 shows that only one-third of the respondents would receive a lump sum when required. However, within 4 years of reform, 90% of pension pots were

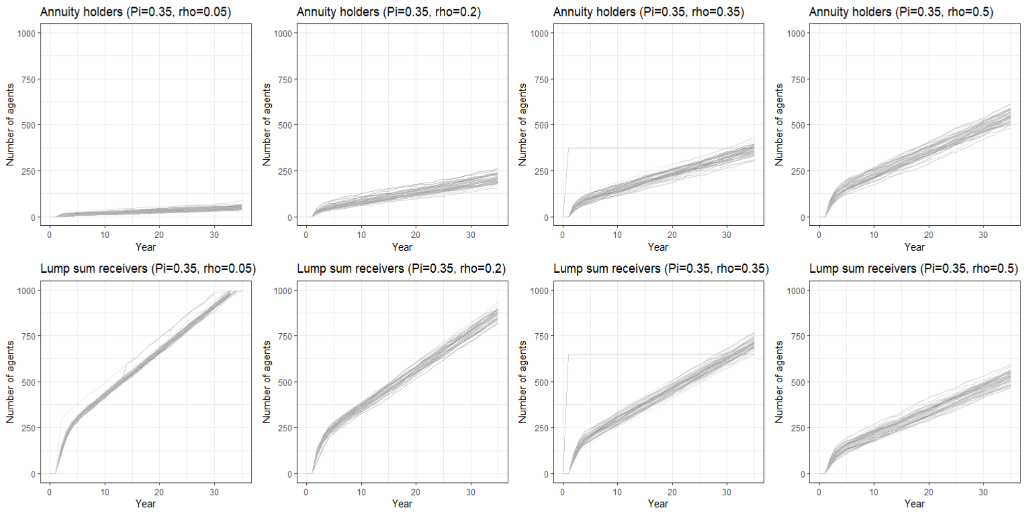


Figure 5.6: Number of annuity holders and lump sum receivers from 50 simulations with $\Pi = 0.35$ and $\rho = (0.05, 0.2, 0.35, 0.5)$

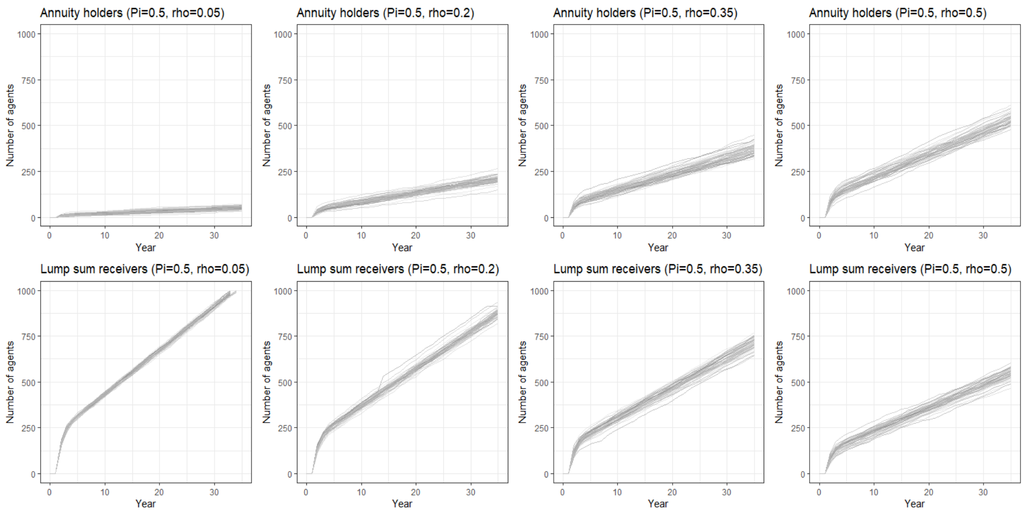


Figure 5.7: Number of annuity holders and lump sum receivers from 50 simulations with $\Pi = 0.5$ and $\rho = (0.05, 0.2, 0.35, 0.5)$

withdrawn in some form of a lump sum. To recreate the phenomenon, we produce an agent-based model where a realistic network model is adopted. The model has two types of agents: isolated agents and imitator agents. The simulation result suggests that lump sum receivers can dominate by chance when imitator agents take 95% of the population and the probability of choosing annuity and lump sum are half. However, this case is inappropriate as it takes over 10 years for one asset to dominate. Finally, the model confirms low individuals' preference for annuity purchases, as shown in Chapter 3. Very low individuals' preference, as low as five per cent, for an annuity can be a suitable topic for further studies.

5.9 References

Bateman, H. & Piggott, J. (2010), 'Too much risk to insure? the australian (non-) market for annuities'.

Benartzi, S. & Thaler, R. (2007), 'Heuristics and biases in retirement savings behavior', *Journal of Economic Perspectives* **21**(3), 81–104.

Bendor, J., Huberman, B. A. & Wu, F. (2009), 'Management fads, pedagogies, and other soft technologies', *Journal of Economic Behavior & Organization* **72**(1), 290–304.

Cannon, E., Tonks, I. & Yuille, R. (2016), 'The effect of the reforms to compulsion on annuity demand', *National Institute Economic Review* **237**, R47–R54.

Duflo, E. & Saez, E. (2002), 'Participation and investment decisions in a retirement plan: The influence of colleagues' choices', *Journal of Public Economics* **85**(1), 121–148.

Duflo, E. & Saez, E. (2003), 'The role of information and social interactions in retirement plan decisions: Evidence from a randomized experiment', *The Quarterly Journal of Economics* **118**(3), 815–842.

Epstein, J. M. (2006), 'Generative social science: Studies in agent-based computational modeling', **146**, 146–174.

Erdős, P. & Rényi, A. (1960), 'On the evolution of random graphs', *Publ. Math. Inst. Hung. Acad. Sci* **5**(1), 17–60.

Hvide, H. K. & Östberg, P. (2015), 'Social interaction at work', *Journal of Financial Economics* **117**(3), 628–652.

Lu, T. J. & Tang, N. (2019), 'Social interactions in asset allocation decisions: Evidence from 401 (k) pension plan investors', *Journal of Economic Behavior & Organization* **159**, 1–14.

Namatame, A. & Chen, S.-H. (2016), *Agent-based modeling and network dynamics*, Oxford University Press.

Park, Y. & Banerjee, S. (2020), 'Coworker influence on annuitization decisions: Evidence from defined benefit plans', *Journal of Economic Behavior & Organization* **178**, 582–606.

Poterba, J. M. (2001), 'Annuity markets and retirement security', *Fiscal Studies* **22**(3), 249–270.

Seshadhri, C., Kolda, T. G. & Pinar, A. (2012), 'Community structure and scale-free collections of erdős-rényi graphs', *Physical Review E* **85**(5), 056109.

Sorensen, A. T. (2006), 'Social learning and health plan choice', *The Rand Journal of Economics* **37**(4), 929–945.

The Financial Conduct Authority (2017), 'Retirement outcomes review', **Interim report**.

The Financial Conduct Authority (2018), 'Retirement outcomes review', **Final report**.

The National Association of Pension Funds (2014), 'Workplace pension survey', **May**.

Vermeer, N., Van Rooij, M. & Van Vuuren, D. J. (2014), 'Social interactions and the retirement age'.

Yaari, M. E. (1965), 'Uncertain lifetime, life insurance, and the theory of the consumer', *The Review of Economic Studies* **32**(2), 137–150.

Yermo, J. (2001), Private annuities in oecd countries, *in* 'OECD 2000 Private Pensions Conference', p. 299.

Conclusion

An agent-based model has a bottom-up approach. It explains a phenomenon when it is reproduced because of the adoption of particular behavioural rules. The model and the employed learning processes in this paper might be too simple and lack numerical accuracy. However, it is also true that laypeople could not accurately calculate mortality probability and allocate assets optimally. Throughout the paper, we have introduced two different agent-based models, which stretch the model to five different applications. The first agent-based model successfully reproduces the subjective survival expectation of the Survey of Consumer Finance data. The result of annuity market simulation with the first model implies that individuals have 1% probability of choosing an annuity each year. The second agent-based model is constructed based on the age of each node. The constructed network is realistic and comparable to some real networks of Uganda villages. Annuity market simulation with the second model also leads to a conclusion of low individual preference for annuity. Assumptions of our models are simple, as we did not evaluate individual preferences using personal characteristics. For further research, we can incorporate the evaluation of preferences based on the heterogeneity of agents. So we can find a way to influence individual preferences.