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# DYNAMICS OF THE IRISH GOVERNMENT SECURITIES MARKET 

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## Dissertation submitted in fulfilment of the requirement for the Degree of Doctor of Philosophy

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## Declaration

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#### Abstract

In this dissertation, the dynamics of the Irish government securities markets over the last eighteen years are analysed. The first chapter fits and models the Irish term structure of interest rates using a bootstrapping spline methodology. Problems such as a lack of sufficient data, particularly for the longer maturity dates, very significant outliers at the short end and the behaviour of bonds with embedded options are discussed and addressed. This is followed by estimates of the parameters for the stochastic process followed by the Irish term structure. The findings have important ramifications for the suitability of particular arbitrage free term structure models of the behaviour of bond values.

The microstructure underlying the term structure is examined together with the efficiency of the existing agency structure. Using the revenue and cost functions of the agency structure, the microstructure is analysed. The viability of market making is investigated. The moments of the primary dealers profit probabiity density function are identified and simulated on an annual basis. Primary dealing is developed and integrated with actuarial ruin theory to quantify market maker's capital requirements and the probability of the failure of a primary dealer system is estimated.

These findings are applied to different types of financial institutions in order to identify their investment freedom. A continuous trading model is used to develop a contingent immunisation analysis and apply it to portfolio management to model how these firms move from an immunised portfolio allocation.


## KEY TO MATHEMATICAL SYMBOLS

| $r(t)$ | - | vector of a set of spot rates |
| :---: | :---: | :---: |
| $d(t)$ | - | vector of a set of discount present value values |
| $P_{i}$ | - | price of a default free par bond |
| $C F_{1}(t)$ | - | expected cashflow at time t |
| T | - | length of time to maturity |
| $\alpha_{1}$ | - | coefficient applied to approximating function |
| $p(x)$ | - | polynomial of order $n$ or of degree less than $n$ |
| $B_{p}{ }^{k}(t)$ | - | a k-order B spline |
| $y(t)$ | - | gross redemption yield for bond maturing at time t |
| P | - | price vector |
| CF | - | cash flow matrix |
| V | - | the present value vector |
| $\mathrm{r}_{\text {t }}$ | - | spot rate of interest for maturity t |
| $f_{t}$ | - | forward rate of interest for maturity $\mathrm{t}-1$ to t |
| $u(x, t)-$ |  | temperature in a long thin uniform bar of material its temperature varies only |
|  |  | with distance $x$ along the bar and with time $t$ |
| $-\delta u / \delta x$ |  | temperature gradient |
| S | - | asset |
| $\mu$ | - | expected return on drift |
| $\sigma$ | - | volatility and represents the stochastic shock |
| $d z$ | - | infinitesimal change in a Wiener process |
| $d t$ | - | infinitesimal interval of time |
| $f(S)$ | - | function of the asset value is continuous |
| $P$ | - | Portfolio |

fraction of opposite position in other asset

| $d P \quad-$ | infinitesimal change in portfolio value |
| :---: | :---: |
| $\lambda d t$ | excess return earned over the spot risk free rate |
| $\alpha(\gamma-r)$ | instantaneous drift of the process brings the stochastic rate towards long term |
|  | average $\gamma$ with a power $\alpha$ which is proportional to the deviation of the |
|  | process from the mean |
| $p_{S}$ | Bid Price |
| $\lambda_{S}\left(p_{S}\right)-$ | Market Makers Demand For Sell Orders |
| $p_{\text {R }}$ | Offer Price |
| $\lambda_{B}\left(p_{B}\right)-$ | Market Makers Demand For Buy Orders |
| $I_{c}(0)-$ | Initial Cash Position |
| $p$ | Price |
| $r_{\text {Market Maker }}$ - | Required return on bond market making activities |
| $r_{\text {Risk free }}-$ | Required return on money market |
| $\beta_{\text {Market Making }}$ | Beta coefficient of bond market making activities |
| $r_{\text {Eiquity Market }}$ | Required return on equity market |
| $A_{t}$ | Present Value of all assets at time $t$ |
| $L_{t}$ | Present Value of all liabilities at time t |
| $\alpha_{t}$ | Proportion by which $A_{t}$ exceeds $L_{t}$ at time t |
| $t(x)$ | Duration of cash flow vector x |
| c | coupon payment on bond per unit time |
| y | yield or internal rate of return on the bond |
| $n$ | term to maturity of the bond |
| $P(c n ; y)$ - | price of the bond |
| $\checkmark \quad-$ | discount factor |


| $R_{t}$ | redemption value per unit nominal |
| :--- | :--- | :--- |
| $b_{t}$ | borrowing for the payment of the liability maturing at time $t$ |

Chapter 1
Introduction

### 1.1 Introduction

The objective of this dissertation is to investigate the Irish government bond market from several perspectives, namely: bond pricing, the evolution of the Irish term structure, the microstructure of the Irish sovereign debt market and an analysis of the Irish general insurance sector as a user of the market.

While Kearney (1985) modelled the gross redemption yield ${ }^{1}$ curve and the demand for money, there has been no work done on the measurement of Irish spot rates. Economists like Kearney (1985) have used the gross redemption yield curve in their econometric modelling, but did not question whether such yields are the most appropriate measure of Irish interest rates. Their approach is probably largely due to the difficulty in accessing the raw data to estimate the spot rates. Consequently, there is no body of research on the dynamics of Irish interest rates and the most efficient approach to the management of interest rate risks.

The cost of government borrowing in Ireland has been a focus of attention for the government, investors, other borrowers and academics for a very long time. Norton (1974) completed the first study on Irish government borrowing and his concern is the impact of varying debt service costs on future economic plans. If investor behaviour is correctly understood, the relationship between yields of different maturities can be investigated in relation to expectations ${ }^{2}$ about how interest rates change through time.

[^0]Early work ${ }^{3}$ by Fisher (1930) examined borrowing for different time periods, hence the name, 'term structure'. In Fisher's (1930) model, a two time-period horizon is assumed and there is an endowment of wealth in both time periods. Fisher (1930) modelled an agent who borrows or deposits against his present and future endowments in order to maximise utility over his time horizon. Fisher's (1930) model is extended to a series of time periods covering an individual's life with expectations about the level of income and consumption along with their respective variances. An individual seeks to maximise his utility and can expect to borrow or lend between different periods of his life. The term structure of yields acquires greater significance as governments and supranational organisations interact with investors (with international capital mobility and liquidity) who seek to redistribute or diversify their wealth on behaif of themselves or their principals.

### 1.2 Identification \& Quantification of Term Structure

In chapter two, the spot term structure of interest rates between 1980 and 1997 is estimated. Knowledge of spot rates is necessary in order to investigate the process underlying the term structure and to investigate the market's efficiency or bias in estimating future interest rates.

The development of the Irish government debt market over the past eighteen years is documented to serve as a database from which to estimate parameters. Considerable problems are faced due to lack of data. As mentioned above, there has been some work concerning gross redemption yields in the macro and monetary economic area ${ }^{4}$. The selection of term structure model is discussed along with sources of market data and sampling problems. Raw data from past dealing sheets has been collected for eighteen years.

[^1]In the Irish market the prices of zero coupon bonds are not observed, so their prices and corresponding term structure must be estimated from a set of coupon bonds issued by the government. The economic background to national debt management is described along with changing market conventions. A bootstrap methodology is used to generate the prices of notional zero coupon bonds from those of coupon-bearing bonds.

Mathematical functions known as B Splines with a number of knots are fitted to discount factors, spot rates and forward rates, and the optimal technique is chosen for the fitting of the bootstrapped spot rates. Significant outliers over the entire sample period are identified and excluded from the data set. The results of estimation are analysed and used to parameterise the discount function appropriate to bond pricing. Since only government securities denominated in their own currency are considered, default risk is not considered an issue. The Irish term structure has been estimated for the first time.

### 1.3 Modelling the Stochastic Process

With the spot rates estimated in chapter two, the process underlying the term structure is investigated and it is established that three factors describe the evolution of the term structure over the sample period. The principal components of the changes in spot rates are estimated and they demonstrate that the time series of Irish term structure is similar to that found in other studies. The results of the previous chapter are used to identify the parameters that drive the Irish term structure over a particular time horizon to generate a risk profile of the changes in value of government debt in chapter four.

[^2]
### 1.4 Microstructure of Irish Government Bond Market

The microstructure of the Irish government bond market, which generates the term structure, is investigated. The hypothesis to be tested is whether a competitive dealership market could be supported and would be preferable to the then existing agency microstructure. The Irish market is small relative to its European peers and constitutes around $1 \%$ of the debt of European Union sovereign country debt.

The issues concerning the authorities are: the different costs associated with different structures; immediacy; liquiditys and transparency. The relevant literature on microstructure is reviewed and the historical performance of the agency market is examined. An industry analysis for financial intermediation is developed to determine the capacity for transformation of the market from an agency-based order-driven system to a principal-based market-making system.

The approaches taken in the literature to financial intermediaries, trading limits and market making are discussed. The European Union Capital Adequacy Directive is investigated to see if it is adequate for the Irish market in terms of prescription of risk capital. This is followed by a simulation in a Monte-Carlo framework for a primary dealer to establish whether such a structure is viable using the term structure estimation methodology of chapter two and the analysis of the process in chapter three. A model of the daily profitability function appropriate to a primary dealer is fitted and examined. The likelihood of failure of a market maker over a trading year time horizon is simulated and the conclusion is that a competitive dealership market could be supported.

[^3]
### 1.5 Free Mismatch Reserves of Irish General Insurance Companies

The Irish general insurance market is analysed by using the estimates of the discount factors of the term structure from chapter two, to identify its immunising portfolio. To do this, a framework is developed in which managers attempt to maximise the value of the funds under management, subject to a minimum terminal value. The performance of the companies under such a strategy is compared with their actual achievements and those that would have occurred if their portfolios had been immunised. The performance is found to be highly varied and important implications for the insurance industry can be drawn. There is a greater investment risk taken relative to that required to immunise their liabilities and most of the excess investment return is used to subsidise underwriting losses. This is true for the majority of the insurers for all the data points in the time series.

A method of overcoming the problems of illiquidity and asset span in the set of possible Irish fixed income assets, is developed using interest rate swaps ${ }^{6}$. By using interest rate swaps to increase duration, long-term liabilities can be immunised. A reconciliation of duration is calculated to illustrate that immunisation had been achieved. A model portfolio with a simple liability is formulated and required immunisation by appropriate allocation of Irish bond assets.

The concept of mismatch reserve is developed over a one-year time horizon. The contingent claims analysis framework is used to value the mismatch reserve as an at the money relative performance option of two portfolios, the matching immunised portfolio and a mismatching portfolio. This is illustrated with an Irish general insurer's accounts and statutory returns.

[^4]The historical liability profile of the industry is reviewed and the actual investment performance by the industry as a result of mismatching is estimated under a given set of assumptions. The general insurer sector is investigated because it represents $10 \%$ of the bond market and its liabilities can only be matched with Irish assets. It is also very heavily dependent on investment performance

## Chapter 2

Term Structure of Irish Interest Rates: 1980-1997

### 2.1 Introduction

The term structure of interest rates represents the pricing relationship that exists at any point between default-free ${ }^{1}$ securities arrayed by maturity. The objectives of this chapter are; first, to identify discount factors from existing bonds trading in the secondary market using a bootstrapping methodology; second, to estimate the discount function; and third, from the discount function to estimate the spot ${ }^{2}$ curve along with the forward rates. The purpose is to present the first ever estimation of the Irish term structure.

This chapter is divided up as follows; section two reviews the background of the recent history of the Irish government securities market; section three examines the selection of a term structure model; section four discusses sources and inadequacies of market data; section five investigates the bootstrap estimation of the term structure and applies the estimation of the discount function to bond pricing; section six presents the summary and conclusions.

An estimation of the term structure is required to examine the evolution of Irish interest rates through time in chapter three. In that regard, apart from Exchequer Bills that have maturities less than 270 days and Exchequer Notes ${ }^{3}$ that have maturities less than 365 days, there have been no discount securities in the Irish market. It is proposed to introduce a strips market in 1999. The sample data cover the period 1980 to 1997 at six monthly intervals.

[^5]Daily Irish data on yield curves from 1980 to 1997 have been compiled from the dealing sheets of the four principal Irish brokers and the Irish Stock Exchange tickets that are reported to the Exchange. There remains a problem with paucity of data on an intra-day basis. Consequently, similar care is necessary in relation to tax effects that have been identified in the studies by Chambers, Carleton \& Waldman (1984) in the US or Steeley (1988) in the UK.

### 2.1.1 Overview

Fisher (1930) first examined the term structure in the context of deferred income or saving in a two period model. Term structure of interest rates is defined as the vector of spot rates $r(t)$ arrayed by maturity. The discount values $d(t)$ arrayed by maturity, corresponding to the present value or price of IR£1 to be received at time $t$ in the future is an alternative form of the term structure, being the inverse of the compounded spot rates.

Spot rates are used when valuing individual cash flows, in particular those of coupon bearing fixed income securities, (which are then seen as comprising a portfolio of individual zero coupon bonds). Term structure estimates are required to test the theories of the evolution of the term structure as in Ho and Lee (1986), and for normative uses such as the development of portfolio immunisation strategies as in Fisher and Weil (1971). Term structure estimates are used for direct valuation of cash flow streams and the pricing of fixed income securities (Houglet (1980)), pricing a bond as a series of individual stripped cash flows, and the valuation of futures contracts and contingent claims (Brennan and Schwartz (1977)).

Spot rates have been used to estimate a liquidity premium (McCulloch (1975a)); to assess the effect of taxation on bond yields (Schaefer (1981)); to assess consensus expectations of future interest rates, together with the analysis of the accuracy of such market implicit forecasts (Fama (1975)); and to arbitrage between bonds of different maturities.

[^6]However, because there are no pure discount Irish government securities (zero coupon bonds) for maturities exceeding one year, the term structure of spot rates cannot be immediately observed and, accordingly, an indirect approach must be followed. Thus implied discount factors may be extracted from interest rates payable on coupon bearing securities. As discussed later, such an extraction process is complicated by the pricing disturbances, such as the irregularity of the dates on which coupons are paid.

### 2.2 Recent History of Irish Government Securities Market

A market valuation of National Debt ${ }^{4}$ denominated in Irish pounds at six monthly intervals from Spring 1980 to Autumn 1997 inclusive has been constructed. In exhibit 2.1, the nominal amount of Irish debt outstanding from the Central Bank of Ireland's Annual Reports at certain dates is compared to the market's mark to market valuation. For most of the time period the market value lies below the nominal value, implying that the average yield is higher than the average coupon due to a rising yield environment.

[^7]

Exhibit 2.1 Irish National Debt Denominated in Domestic Treasuries
Source : ABN Amro Stockbrokers, Central Bank of Ireland \& Empirical data

The semi-annual increase in debt outstanding has been approximately linear, amounting to an extra $\operatorname{IR£374m}$ in nominal terms and $\operatorname{IR£444m}$ in market value terms. The secondary market value of the debt stood at a twenty percent discount to nominal value for the early eighties but fell to eight percent for the last decade. These changes are illustrated in exhibit 2.2 below;


Exhibit 2.2 Changes in Irish National Debt Denominated in Domestic Treasuries
Source : ABN Amro Stockbrokers \& Empirical data

Duration and maturity of the National Debt during the time period 1980 to 1997 is examined and shown in exhibit 2.3. While the maturity moves in a cyclical pattern between 5.25 years and 9.25 years, the duration is in a much narrower range. The financial crisis of the second half of 1986 can be clearly seen, when the government raised an extra IR£2.4bn after yields rose 600 basis points from the first half of 1986. This crisis compounded the country's financial problems because the funding involved issuing long dated maturity bonds reversing a policy of the previous decade which only issued short dated maturity bonds.


Exhibit 2.3 Characteristics of Irish Domestic Treasuries Market
Source : ABN Amro Stockbrokers \& Empirical data

The average duration of the debt over 1980 to 1997 is 4.02 years and, interestingly, it lay in a relatively narrow range of 3.28 years to 4.88 years throughout the study. The average maturity dropped from the start of the 1980's to April 1986 implying a decision by government authorities to fund in the shorter maturity sector in the expectation that interest rates would fall.

Throughout the period of 1980 to 1989 exchange controls were operative. These controls were substantially removed in 1989. Since investors, such as life offices and pension funds, match a proportion of their liabilities by investing in bonds with long maturities, it is probable, with the government's funding pattern, that a shortage of suitable bonds existed and rationing prevailed in the long maturity sector of the market. If a rationing premium existed, it could have been of such a magnitude that forward rates might have been negative though spot rates remained positive.

Historical changes in maturity are illustrated in exhibit 2.4. If the government funded longer than the average duration of the outstanding stock of national debt, duration would obviously increase.


Exhibit 2.4 Funding and Timing Impact on Duration Profile
Source: ABN Amro Stockbrokers \& Empirical data

The expansion in the size of the market can be explained by looking at the trend in Government borrowing over the past 15 years or so. Following the first oil crisis in the early 1970's, Ireland embarked on a policy of fiscal expansion in order to help offset the negative impact of spiralling oil prices on domestic output and demand.

The Exchequer Borrowing Requirement (EBR) increased substantially from 1975 onwards in both absolute terms and as a percentage of GNP, and, despite stringent attempts to bring it under control, the EBR remained in double digits as a percentage of GNP for most of the period up to 1987. Initially there was a large balance of payments deficit and a limited pool of domestic savings resulting in heavy reliance on foreign currency borrowing, but gradually this reliance on foreign borrowing diminished as the 1980 's progressed and the emphasis shifted towards Irish pound denominated debt. The increased emphasis on Irish pound funding was facilitated by ERM entry in 1989. The termination of the fixed exchange rate between Sterling and the Irish pound meant that the Irish pound then emerged for the first time effectively as a currency in its own right. However, it is only after 1987 that Irish pound denominated debt assumed a position of prominence, when foreign institutional investors began to invest in the market as a successful effort was made to reduce the Government's overall borrowing needs and new foreign borrowing was largely eschewed against a background of a much improved balance of payments position.

Gross government debt was estimated at just over IR£27bn at the end of 1991 and the government stock denominated in Irish pounds accounted for IR£13.8bn. The rest of the IR£13.2bn debt is accounted for by Exchequer bills, personal savings products, national instalment saving and foreign currency borrowing. In 1980 Irish pounds accounted for IR£2.9bn which rose to IR£9.5bn by 1986, jumped to IR£11.4bn in 1987 and is IR£18bn at the end of 1997.

### 2.3 Selection of Term Structure Model

Many attempts have been made, using a variety of methods to estimate the term structure of interest rates in other markets. The earliest approaches were simple gross redemption yield curves. In the following subsections, the different approaches are examined and consideration is given to the underlying assumptions, constraints, data requirements and overall simplicity or complexity.

### 2.3.1 Gross Redemption Yield to maturity

The first distinction which should be made is between attempts to construct gross redemption yield to maturity (GRY) curves, and those which endeavour to model the term structure of interest rates proper, (i.e. the array of spot rates, discounts and forward rates corresponding to their associated maturities). Among the early GRY estimates, Durand (1942) and Durand and Winn (1947) were prominent.

A number of criticisms may be directed at the concept of gross redemption yield to maturity, including the following; Malkiel (1966) and Buse (1968) state that only when the yield curve is flat can the GRY be so used as a surrogate measure, (i.e when all the spot rates are equal). Carleton and Cooper (1976) criticise the notion of GRY, stating that it is an "ambiguous concept", and that its "economic meaning was moot", in so far as the reinvestment of intermediate cash flows is expected to occur at this internal rate of return. However, this method was developed in the absence of present day computing power and has the redeeming feature of simplicity and ease of implementation.

The gross redemption yield curve is a complex mixture of discounts and coupons. Carleton and Cooper (1976) note that the averaging process implicit in calculation of a gross redemption yield destroys some important basic information, particularly with respect to coupon differences, the primary source of yield and price differentials.

Schaefer (1981) pointed out that, when the term structure of interest rates is upward sloping, the coupon effect comes into play, causing the GRY to underestimate spot rates of corresponding maturities. In particular when GRY are used as a proxy for spot rates, the errors introduced are related to the shape of the curve and typically are much greater when the term structure is steeply sloping.

Gross redemption yields are commonly used in less liquid markets (than say the US), especially in longer term. These failings are well known; GRY must be regarded as an inadequate estimate of the term structure of interest rates and this approach is rejected.

### 2.3.2 Discrete Estimation of the Term Structure

This procedure estimates the present value coefficients of each cash flow directly; in other words, the discounts associated with each flow represented by the following equation:

$$
\begin{equation*}
P=\sum_{t=1}^{T} C F(t) d(t) \tag{2.3.2.1}
\end{equation*}
$$

where t indexes time
$P_{i} \quad-\quad$ price of a default free par bond ${ }^{5}$,
$C F_{1}(t)$ - expected cashflow at time $t$,
$d(t) \quad-\quad$ present value coefficients,
T - length of time to maturity.
For analytic solution, there must exist an equal or greater number of bonds with linearly independent vectors of cash flows than there are payment dates ${ }^{6}$. Then, as prices and cash flows are known, the discount function can be derived and the spot rate curve estimated.

In many markets, this simple constraint is binding. Even for the US, Carleton and Cooper (1976) could only estimate a discrete version of the term structure for maturities up to 7 years.

They observed that the absence of cash flows at regular intervals (i.e. due to the nonexistence of securities at certain maturities) and the use of approximations could result in instabilities and implausible results.

[^8]Carleton and Cooper (1976) did, however, successfully employ this method, using for their estimation sample a selection of US Treasury Bonds. US Government Coupon Securities (i.e. notes and bonds) with rare exception, make regular semi-annual payments on only four days of each year, (i.e. February 15th, May 15th, August 15 th and November $15^{\text {th }}$ ), which facilitates this form of analysis.

Due to the relative paucity of issues and problems associated with consistent pricing in longer-term US securities, and in order to ensure the cash flow matrix had sufficient rank, the maximum maturity of the discount function estimation had to be severely restricted, i.e. to seven years. Carleton and Cooper (1976) demonstrated that their discount functions did exhibit the appropriate properties, i.e. discount factors that are non-negative and monotonically decreasing. However, Shea (1984) observed that Carleton and Cooper (1976) did not succeed in constraining their discount function to mature at par.

Vasicek and Fong (1982) address the question of transposition from discrete spot rates to forward rate curves. This curve may be saw-toothed in appearance and consequently unreliable. They add a requirement that the forward rate curves exist and are smooth.

McCulloch (1971), pointed out that, because of the multiplicity of payment dates, estimating a discrete discount function can encounter serious difficulties. Ireland, which has a multiplicity of payment dates, faces these difficulties.

Another major complication and reason for rejecting the discrete discount function is that, while the discount function meets at each point estimated, it is not continuous leading to an unstable forward curve.

As a result of the difficulties in estimation of discrete discount functions, Steeley (1988) developed linear approximation functions for the continuous discount or spot rate curves. Instead of attempting to use discrete discount factors which may be unavailable, an approximation for the discount function $d(t)$, in the following form is modelled:

$$
\begin{equation*}
d(t)=\sum_{l=1}^{L} \alpha_{l} f_{l}(t) \tag{2.3.2.2}
\end{equation*}
$$

where $\alpha, \quad$ coefficients are applied to the approximating functions.

### 2.3.3 Polynomial Approximations

The background and basis of polynomial approximations in numerical analysis lie with the Weierstrass Theorem, which holds that a continuously differentiable function can be approximated in some interval to within an arbitrary error by some polynomial defined over the same interval'.

Polynomials are used for approximation because they can be evaluated, differentiated and integrated easily using the basic arithmetic operations of addition, subtraction and multiplication. A polynomial of order $n$ a function in the form:

$$
\begin{equation*}
p(x)=a_{1}+a_{2} x+\ldots \ldots \ldots \ldots \ldots \ldots+a_{n} x^{n-1}=\sum_{j=1}^{n} a_{j} x^{j-1} \tag{2.3.3.1}
\end{equation*}
$$

Several criteria are available for choosing the 'goodness of fit of a polynomial'. Shea (1984) focuses on the 'least squares criterion'. In order to achieve good fit with the data, one could be tempted to use relatively high degree polynomials, endeavouring to reach the 'Weierstrass Ideal' of an approximation passing through all or very close to each observation points. Polynomials of at least order three must be used in order to ensure a smooth forward rate curve, which is twice continuously differentiable.

One of the great dangers associated with higher degree polynomials is that the approximation is likely to fluctuate wildly over its range when being fitted through limited data. The choice must be made between accuracy of fit and paucity of parameters - a trade-off of parsimony and error. Such polynomial approximations tend to weave around the exponential structure, leading to a set of unstable forward rates. This criticism is particularly apt with respect to the global nature of the fit. Vasicek and Fong (1982) criticised the use of polynomial approximations for the discount function, which may be considered principally an exponential decay.

It is difficult to fit both ends of the term structure simultaneously using polynomial approximation. When investigated by McCulloch (1971) using US data, this technique appeared to fit the long end best. Since the average weighted duration of the Irish national debt is 4.02 years over the entire sample time period, it was decided not to use this technique. Comparison later of polynomial splines and polynomial approximations also rejects the approximation on grounds of accuracy.

### 2.3.4 Polynomial Splines

Polynomial splines offer an alternative to general polynomial regression due to a concentration on local fit. In effect, sections of high degree polynomials can be closely approximated by several lower degree polynomials, thus eliminating the problems associated with higher order functions. Polynomial splines generate better solutions than polynomial approximations, since the latter need to oscillate widely in order to fit all or most of the points. This may be thought of as moving the entire function rather than merely spline sections of it. A knot to the next piece joins each piece of the approximation space, and it is customary to force these piecewise polynomiais to join smoothly at the knots. Thus, it is possible to approximate a continuous but complex shape.

Polynomial splines have uniform convergence properties and provide a high order of derivative continuity, with the added advantage of fixing some of the degrees of freedom and reducing the number of parameters to be estimated. In order to investigate whether a polynomial spline ${ }^{7}$ is superior to fitting a yield curve by a polynomial approximation, a spline with a single knot is fitted and compared with the polynomial approximation shown in 2.3.3.1. The cumulative deviance ${ }^{8}$ of annualised yields of a polynomial approximation for all the Irish yield 1980 to 1997 curve models is 0.1335141 which is greater than the worst fitting spline model at 0.1310494 . This means that spline functions are a better estimator of the effective GRY curve irrespective of the position of knots and as a result, the polynomial approximation approach to modelling the yield curve is rejected.

Hastie and Tibshirani (1990) note that piecewise cubic polynomials are a popular method, although many different configurations are possible. The spline output has two continuous derivatives that agree at the "knots", i.e., the spline has a continuous second derivative. Shea (1985) recommends the use of polynomial splines when the functional form of the term structure is unknown. This avoids some questions raised by assuming a functional form such as exponential decay.

Until the studies of McCulloch (1971) and Schaefer (1981), the problem of continuous yield curve approximation was rarely solved by techniques based in numerical analysis. Schaefer (1981) used a mathematical form known as Bernstein Polynomials. With Bernstein Polynomial functions, the term structure curve may be fitted to the entire range of available data or marurities, which compares favourably with polynomials which seem to accommodate longer maturities more than short

[^9]The primary advantage is that different parts of the term structure may be approximated without affecting other parts, and the discount factors are relatively easily constrained. Steeley (1988) pointed out that the speed of convergence of these Bernstein Polynomials is relatively slow as compared to spline functions.

McCulloch (1971) is among the first to pioneer the application of splines in the estimation of the term structure. He initially used a quadratic spline, i.e. a piecewise quadratic function to approximate the discount function. The main problem encountered with this function is that as it is only once continuously differentiable, discontinuous first derivatives or "knuckles" plagued the estimated forward rate curves.

As mentioned earlier, the main source of difficulty encountered when working with polynomial splines is the selection of the number and position of the knots, or breakpoints, with the most direct approach being referred to as cardinal splines, which require a single parameter, the number of interior knots. The positions can then be chosen, possibly uniformly over the range of the data. A slightly more adaptive version places the knots at appropriate quantiles of the predictor variable, whereas more complex schemes use data driven search criteria to select the number and position of the knots.

With respect to setting the within-sample knots, Steeley (1988) pointed to one, a priori guideline: when dividing the bonds between short ${ }^{9}$, medium and long bonds, it is important to remember that there exist market participants with different perceptions with respect to what can be regarded as short, e.g. 5 or 7 years, and such definitions of short can cause clustering around the short end, leaving the long end of the market poorly represented, and also that such clustering can warrant a subdivision within the short end.

[^10]The particular approach followed by Steeley (1988), referred to as the "general to specific" modelling approach was originally developed by Hendry (1979). The use of constraints is vital to the success of polynomial approximations, to avoid the estimated discount factors displaying undesirable properties.

Determining the number and location of polynomial pieces, along with the polynomial order that determines degree of continuity at the breakpoints, is the most difficult decision to be made. In addition, constraints can be altered, removed or added, as appropriate, if the model is yielding implausible results, such as negative forward rates ${ }^{10}$. There is one constraint that is invariant; that the discount function must contain the point $(0,1)$, i.e., par at maturity. The McCulloch (1971) cubic spline model yielded anomalous negative interest rate estimates. Schaefer (1981), using the Bernstein approach, counteracted the existence of negative forward rates by placing a negative slope constraint on the discount function.

Shea (1985) argued that, although preventing negative forward rates, this negative slope constraint is not helpful in obtaining stable forward rate structures, one of the primary objectives of term structure modelling. The primary motive behind polynomial approximation is to let the approximations over separate subintervals be to some degree independently determined. It is this dependence on local data that in turn argues against the use of negative slope constraints.

Experimentation with the order of the basis functions or variation of the number and position of the breakpoints, may also be used to iron out any offending part of the approximation, but Shea's (1985) opinion is that this approach can be self-defeating if it results in deterioration of measure, or quality, of fit. Also, lowering the order of the basis functions affects the higher order derivatives and may result in discontinuities.

[^11]The most widely favoured approach by econometricans like Johnston (1984) is to design additional constraints to reduce local dependence on data in the offending area. Experimentation with 'ad hoc' constraints can be easily accomplished with a restricted least squares approach. Spline bases facilitate the addition, deletion or alteration of constraints.

It is difficult to constrain estimation so that both the level and shape of a yield curve are satisfied simultaneously. With respect to this notion, Shea (1985) pointed out that the Vasicek and Fong (1982) exponential model is good at modelling the shape of the discount function when an exponential decay is indeed its true form. However, to be regarded as a generally reliable technique, it can need to be constrained in the levels or shapes of its associated yield curves.

Primarily Vasicek and Fong (1982) pioneered exponential splines. The logic behind their use is that discount functions are essentially exponential decays. Others, such as Shea (1985), doubt this to be their true form in a number of circumstances. Vasicek and Fong (1982) clearly state that the difference in curvature, which exists between the polynomial functions and discount functions, explains the previous findings of Shea (1984) and Rose and Schworm (1980) that term structures estimated using spline functions often generate forward rates that are unstable and fluctuate widely - frequently drifting to negative values.

Shea (1984), among other practitioners, disagreed with Vasicek and Fong (1982) pointing to Taylor series expansions. This series lies at the heart of the theory of local approximation to continuous functions. Shea (1984) felt that the entire logic behind the use of these complex functions is without adequate foundation and recommended instead the use of the ordinary polynomial splines methodology, which yields similar curves, without added complications.

[^12]Using exponential decays, Vasicek and Fong (1982) estimated that the discount function would be linear; thus complicated non-linear estimation procedures would be avoided. But, as Shea (1984) noted, if Vasicek and Fong (1982) are so committed to this belief, why use a polynomial spline (which has rarely been seen to be linear or near linear), rather than an ordinary regression line.

Vasicek and Fong (1982) made numerous claims about their model. Their term structure of interest rate approximation exhibits desirable asymptotic properties for long maturities. However, these asymptotic restrictions are of little use in defining an estimated discount function with the curvature of an exponential decay, and the asymptotic forward rate exhibits little influence over the shape or level of the forward rate curve within the estimation range. The asymptotic forward rate appears only to have relevance at maturities greater than 30 years for which there are no (relevant) bonds in issue.

The Vasicek and Fong (1982) model exhibited sufficient robustness to produce stable forward rates, and sufficient flexibility to fit a wide variety of shapes. Shea (1985) again admitted that it is difficult to fault the Vasicek and Fong (1982) model relative to any other spline model for its ability to smooth term structure data.

In an attempt to incorporate this exponential characteristic in a different manner to that proposed by Vasicek and Fong (1982), Carleton and Waldman (1984) have suggested that the spot rate curve rather than the discount function should be approximated, using an exponential function. The model is rejected for measuring the Irish term structure because non-linear estimation procedures are computationally more intensive than ordinary polynomial spline models, such as $B$ spline spot rates. These approach the same values of the forward rate curve as exponential splines and these are as stable as the exponential spline method, while computationally less onerous.

### 2.3.5 B Splines

In this case the discount factors are a discrete estimation from a bond vector where the $B$ splines model is again used, along the lines of cubic polynomial but with an appropriate choice of knots and constraints in order to generate a smooth forward curve. Spline approximation using the truncated power series, as portrayed by Chan, Karolyi, Longstaff and Saunders (1991c), is equivalent to approximations using other bases. Not all spline bases are equally capable of defining spline regressors useful for reliable estimation.

A number of basis functions generate a regressors matrix in which the columns are nearly collinear and are thus they are ill conditioned. Even if the error in the discount function $d(t)$ is small, the slope and level of the term structure of interest rates could still be in significant error. Thus, reliable spline approximation can depend crucially upon intelligent selection of the basis. Good corrective action in such circumstances would be to use a $B$ spline basis.

Since DeBoor's (1978) original work, $B$ splines have been recommended by Powell (1981) as a suitable alternative to the general polynomial splines counteracting the problems of collinearity. Deacon and Derry (1994) note that $B$ splines, which are identically zero over a large portion of the approximation space have good convergence properties and prevent the loss of accuracy due to cancellation.

The ease with which the $B$ spline can be constrained is another primary advantage associated with this function, although this can also be flexibly accomplished with a restricted piecewise polynomial structure.

However, some models do not exhibit such flexibility; e.g. with respect to the McCulloch (1971) model the particular cubic spline functions cannot simultaneously constrain the slope and the level of the yield curve.

Svensson (1993) estimates spot and forward rates using McCulloch's (1971) approach of fitting a discount function to a bond data set, but uses the Nelson and Siegel (1987) functional form instead of a spline. While he increased the flexibility of their model, he concluded that the original Nelson and Siegel (1987) model produced a satisfactory fit most of the time.

Steeley (1988), when using $B$ splines, adopted the following function:

$$
\begin{equation*}
B_{p}^{k}(t)=\sum_{l=p}^{p+k+1}\left[\prod_{h=p, h \neq 1}^{p+k+1} \frac{1}{\left(t_{h}-t_{l}\right)}\right]\left(t-t_{l}\right)_{+}^{k}-\infty<t<\infty \tag{2.3.6.1}
\end{equation*}
$$

which is known as a k -order $B$ spline, where the subscript " p " denotes that $\mathrm{B}_{\mathrm{p}}{ }^{k}(\mathrm{t})$ is only non zero if " t " is in the interval $\left[\mathrm{t}_{\mathrm{p}}, \mathrm{t}_{\mathrm{p}+k+1}\right]$.

Regression splines or piecewise polynomials, of which $B$ splines are a variant, are attractive because of their computational ease, when the "knots are given". In particular, standard linear model estimation is very convenient when using additive models. The main drawback of this approach is the difficulty associated with choosing the number and position of the knots. When a small number of knots are used, the smoother can show some disturbing nonlocal behaviour. With more knots, this global influence would be dampened, but frequently there are not many degrees of freedom to spare. Also, as discussed earlier, another problem with regression splines is that the smoothness of the estimate cannot be easily varied continuously as a function of single smoothing parameter. This is the primary advantage associated with other smoothers such as loess, kernel, running line or smooth spline.

The B Splines model that is chosen to measure the Irish term structure is constructed from bootstrapped discount factors and spot rates of the existing bonds' prices and the money market curve used for pricing government Exchequer Bills and Exchequer Notes.

There are a number of methods that can be employed to estimate spot rates;

- Bootstrapping,
- Exponential polynomials.
- Different spline methodologies,
- Various kernel smoothing techniques.

As discussed above, in the case of Ireland, where the term structure of interest rates cannot be directly observed (e.g. from zero coupon bonds), it can be indirectly estimated using a bootstrap approach to obtain the discount factors. Spline approximation can be used on the discount factors, spot rates or the forward rates with different assumptions about tax or liquidity effects. In the case of Ireland a two stage process was used. Firstly, the discount factors were bootstrapped using the exact date of each cash flow from the existing money market and bond set at each data point arrayed by maturity. Although not perfect, this approach provides an attempt to estimate the term structure of interest rates in a small bond market like Ireland with frequent 'gaps' in the maturity spectrum. The null hypothesis is that all Irish bonds are part of the data set held by institutional investors with a tax rate of zero and the identified outliers are placed in Appendix two. These outliers are excluded and the bootstrapped vector of present value coefficients ${ }^{11} d(t)$ is then estimated.

After the bootstrapping procedure has been estimated on the coupon bearing government bonds, the following data sets exist;

- Six-monthly data points at the end of each maturity bucket,
- Maturity of each bond's principal payment,
- Each individual bond's cash flows in a sequence of increasing maturity.

Since each sequential bond was used to iteratively to allow for different payment frequency to estımate the spline, a problem arises due the partial overlap of prior fitted spline functions. It is important to recall that any bond's final payment will have a significant influence when fitting any discount function to a bond's cash flows and the objective is to minimise the error with a bond's observed price.

Another problem that needs to be addressed was the problem of 'gaps' in the maturity spectrum (i.e. there is not always a suitable bond, or any bond maturing within a six-month maturity bucket). In order to fill the gaps, a decision on the appropriate trade-off between 'smoothness' (i.e. removing 'noise' from the data) and 'responsiveness' (i.e. flexibility to accommodate a genuine movement in the term structure) was required. This issue was overcome by interpolation from the previous known six-month maturity bucket.

The spline can be fitted to one of the data sets with constraints to ensure that the curve meets at each knot, is continuous at every point, contains the point $(0,1)$ (i.e. all discount bonds mature at par) and the first derivative is negative and approaches 0 in the asymptotic limit. (as time progresses to positive maturity.) The spline can use any of the following representations of the data set;

- Discount factors,
- Spot rates,
- Forward rates.

[^13]The spline was fitted to both the discount factors and the spot rates of the Irish data set and applied to the bond set at each data point to determine whether it was preferable to estimate the term structure via the discount factors or via the spot rates (see exhibit 2.8). Mastronikola (1991) developed a model for Bank of England's from estimating the term structure of spot rates by fitting a curve through redemption yields, derived directly from observed prices according to Deacon and Derry (1994). This invokes the assumption that the redemption yield curve is a realistic approximate to the par yield curve which may not be the case depending on market conditions. The less well this assumption matches the reality of the market place, then this approximation will perform poorly.

The spline requires knots in the data set and they may be specified as follows;

- Directly given to the estimation procedure
- Number of degrees of freedom specified

An important decision that has to be made when using any kind of spline function is the appropriate number and position of knot points. If the number is too low then the model will not fit the data closely when the term structure takes on difficult shapes, while if it is too high the estimated may be unduly influenced by unrepresentative outliers. When the specification is by degrees of freedom as in the Bank of England model, then the position of knot points will vary with the information content of the underlying data-set. This seems to have been little interest in the literature apart from Steeley (1991) in testing sophisticated techniques for specification of the optimal number and location of knot points. Knots are chosen (see Section 2.5) by using an iterative search method of the lrish data series employing one knot for each maturity point between one to seventeen years, then two and three knots and by investigation of the extra explanatory power of additional knots. Three sections are identified in a similar manner to the approach taken by Steeley (1991), as up to one year; up to five years; and over five years, with approximately equal quantities of bonds in each segment.

An issue arises with regard to the different possible treatments of the overlapping spline function:

- they may be given a weighting of zero and replaced with those from the currently fitted spline,
- they may be averaged at each data point by the numbers of bonds estimated to that data point,
- they may be weighted by some method such as the market value or turnover of the bond issue relative to the market values of all bonds fitted to that maturity point.

This approach explicitly constrained cashflows from different bonds due at the same time to be discounted at the same rate. and estimates a discount function from which the term structure can be derived.

While it would have been preferable to use bond turnover as a weight for price discovery concentration, the mixing of REPO activity in turnover measurement prior to 1995 ruled this out as a reasonable approach and the market value was used for weighting. This was caused by foreign European investors use to the funding market where the REPO was booked by the brokers as separate sale and repurchase and treated as three bond market transactions in the Stock Exchange and the Gilts Settlement Office of the Central Bank of Ireland. Then the investors sold the currency outright in the forward market for a maturity date coinciding with the 'REPO' and eliminating the currency exposure. This practice evolved at the brokers competed for market share and there was little surveillance by the authorities.

### 2.4 Data

Data is collected for all Irish government bonds in issue, half yearly over the perıod April 1980 to October 1997. It is not possible to obtain reliable data prior to 1980 other than the overnight rate, Exchequer Bill tender rate (provided by the Central Bank to the OECD database) and the "average" long government bond yield (provided by the Department of Finance to the IMF for their database).

The data is extracted from the published daily dealing sheets of ABN Amro, Davy, AIB Capıtal markets and NCB/Nat West stockbrokers, who coilectively held $90 \%$ market share of all bond dealings over the period April 1980 to October 1997. The information on the dealing sheets consisted of gross prices, coupon levels and term to maturity. From this information, the individual yields to maturity are estimated. The yields are shown on an Actual/365 bonds basis ${ }^{12}$, although the National Treasury Management Agency issued 30/360 annual basis bonds from 1990 to 1997 before conforming with European Actual/ Actual market practice.

Table 2.1 shows a list of all government bonds in issue at selected time periods throughout the fourteen-year sample. Since the Irish treasury bond market is relatively small in international terms, the collection of straight bonds arrayed by maturity is insufficient to employ the bootstrap procedure. As a result, straight bonds are supplemented with convertible, variable rate and dual redemption date bonds in order to increase the number of data points from which to model the structure.

[^14]With respect to dual ${ }^{13}$ redemption date bonds, where the market yield is very close to their coupon value ${ }^{14}$, these bonds are more likely to be identified as outliers, as can be seen in the next section. Where the market yield is greater than the corresponding coupon level, these bonds are positioned at their respective later redemption dates, since redemption is not likely to occur until then, and vice versa with respect to market yields lower than their coupon level. The yield curves and other market data are shown in Appendix 1. The last closing price is chosen (provided it is not marked as a small bargain of under $\operatorname{IR} £ 50,000$ ) from the dealing sheets and observations are confirmed by comparison with the previous and following days' trading levels.

This is done on a randomly chosen date. 18 April 1980, in the first six monthly interval in 1980 and repeated every six months until 1997. From Appendix 2, it is evident that short-term bonds dominate the sample, with a distinct lack of bonds at the longer end of the maturity range. There is a shortage of bonds with a maturity greater than 9 years in the sense that a debt obligation does not mature in every year. Over the sample period yields reached a high of $21 \%$ and a low of $5 \%$.

[^15]| Date | 18-A pril-80 | 18-October-97 |
| :---: | :---: | :---: |
| 1 | IR.FUNDING 9 1/2\% 1980 | 1 IR DEVELO. 11 1/2\% 1997/99 |
| 2 | IR.FINANCE 8 \% 1980 | 2 IR.FUNDING VAR\% 1998 |
| 3 | IR.NATION. $41 / 4 \%$ 1975/80 | 3 IR FUNDING VAR\% 2000 |
| 4 | IR.SAVING 5 \% 1971/81 | 4 IR.CAPITAL 9 3/4\% 1998 |
| 5 | IR.FUNDING 8 1/2\% 1981 | 5 IR.FINANCE 14 1/2\% 1998/00 |
| 6 | IREXCHEQR 10 \% 1981 | 6 IR.TREASU $61 / 4 \% 1999$ |
| 7 | IR.FINANCE 11 1/2\% 1981 | 7 IR.CAPITAL 7 1/2 \% 1999 |
| 8 | IREXCHEQR 11 1/2\% 1982 | 8 IR.CAPITAL 11 3/4\% 2000 |
| 9 | IR.FINANCE 10 1/2\% 1982 | 9 IR.DEVELO. 12 1/4\% 2000/03 |
| 10 | IR.CONVER $9 \% 1980 / 82$ | 10 IR.EXCHEQR 6 1/2\% 2000/05 |
| 11 | IR.FUNDING $113 / 4 \% 1983$ | 11 IR.TREASU 8 \% 2000 |
| 12 | IR FINANCE VAR\% 1983 | 12 IR.GOVER 9 \% 2001 |
| 13 | IR.FINANCE $12 \% 1984$ | 13 IR.CAPITAL 8 \% 2001 |
| 14 | IRNATION. 5 1/4\% 1979/84 | 14 IR.TREASU $61 / 2$ \% 2001 |
| 15 | IR.NATION. $14 \% 1985$ | 15 IR.DEVELO. 14 3/4\% 2002/04 |
| 16 | IR.EXCHEQR 6 \% 1980/85 | 16 IR.CAPITAL 9 1/4\% 2003 |
| 17 | IR.NATION $71 / 2 \% 1981 / 85$ | 17 IR.EXCHEQR $81 / 4 \% 2003$ |
| 18 | IR.NATION. $53 / 4 \% 1982 / 87$ | 18 IR.TREASU $61 / 4 \% 2004$ |
| 19 | IR.CONVER $81 / 2$ \% 1986/88 | 19 IR.CAPITAL 12 1/2\% 2005 |
| 20 | IR.NATION. $93 / 4 \%$ 1984/89 | 20 IR.TREASU 8 \% 2006 |
| 21 | IR.EXCHEQR 5 3/4\% 1984/89 | 21 IR.CAPITAL 9 \% 2006 |
| 22 | IR.NATION. 14 \% 1985/90 | 22 IR.CAPITAL 8 1/4\% 2008 |
| 23 | IR.EXCHEQR 6 \% 1985/90 | 23 IR.TREASU 6 \% 2008 |
| 24 | IR.NATION. 6 3/4\% 1986/91 | 24 IR.CAPITAL 8 1/2\% 2010 |
| 25 | IR.EXCHEQR 14 \% 1990/92 | 25 IR.CAPITAL 8 3/4\% 2012 |
| 26 | IR.NATION. 7 \% 1987/92 | 26 IR.TREASU $81 / 4 \% 2015$ |
| 27 | IR.DEVELO. $71 / 2 \%$ 1988/93 |  |
| 28 | IR.NATION. 9 1/4\% 1989/94 |  |
| 29 | IR.CONVER $12 \% 1995$ |  |
| 30 | IR.EXCHEQR 9 1/4\% 1991/96 |  |
| 31 | IR.NATION. $93 / 4 \% 1992 / 97$ |  |
| 32 | IR.NATION. 11 \% 1993/98 |  |
| 33 | IR DEVELO. 11 1/2\% 1997/99 |  |
| 34 | IR.FINANCE 14 1/2\% 1998/00 |  |
| 35 | IR.FINANCE $13 \% 1997 / 02$ |  |
| 36 | IR.DEVELO. 14 3/4\% 2002/04 |  |
| 37 | IR.EXCHEQR $61 / 2 \% 2000 / 05$ |  |

Table 2.1 Government Bonds in Issue 1980-1997
Source: ABN Amro Stockbrokers \& Empirical data

Schaefer (1981), when estimating the term structure in the UK, noted that the residual deviance/error term arising from high coupon bonds exhibits a negative slope with the coupon level. While the value of the debt increased from IR£2,856m to IR£18,382m between April 1980 and October 1997, the number of bonds declined from 37 to 26 . This occurred as a result of the declared consolidation strategy of the NTMA of designating a benchmark bond in each maturity spectrum and only issuing these bonds. There has been an increase in the number of singledated bullet bonds and a decline in bonds with small issue sizes with poor liquidity, embedded option feature such as dual dates or conversion options, and low coupon bonds carrying exemptions from capital taxes. The focus of the NTMA is on creating large liquid issues in which to concentrate the price discovery process.

### 2.5 Bootstrap Methodology

Before making any attempt to fit the term structure, the number and position of the knots or the degrees of freedom had to be selected to identify potential outliers in the yield curve data. The objective is to exclude them from the bootstrap methodology. As shown in exhibits 2.5 and 2.6 of the yield curve estimation deviance that includes outliers, knots are placed at a variety of positions within the maturity range. In exnibit 2.5 , the first fit had no knots, and the sum of the residuals is 0.1332591 . The knots are placed at yearly intervals from one to seventeen years and a knot at maturity one-year has a deviance of 0.1222294. A knot placed around year one is where the money market joins the bond market. The deviance of the gross redemption yield curves is summed for all observation points from 1980 to 1997. The deviance of one knot is an improvement of $8.27 \%$ in the size of the sum of residuals compared to having no knots (i.e. a polynomial).


Exhibit 2.5 Sum of Sample Deviance Residuals, 1980-97 for no knot and one-knot Source : Empirical data


Exhibit 2.6 Sum of Sample Deviance Residuals, 1980-97 for two knots
Source : Empirical data

In exhibit 2.6, two knots are used for 171 combinations from one to nineteen years. When exhibit 2.6 is examined, it can be seen that if the first knot is placed at a maturity of one year, this knot has the lowest cluster of deviance residuals. Then, as the knots are moved to greater maturities, the sum of the deviance residuals increased. Placing the second knot at a maturity of five years gives a deviance residual of 0.1175362 ; at a maturity of six years gives a deviance residual of 0.1176748 and finally at a maturity of seven years gives a deviance residual of 0.1178010 . The lowest of these is the combination of knots at one and five years with a deviance residual of 0.1175362 which is a $3.84 \%$ improvement over one knot and an $11.8 \%$ improvement over no knots. An example of this fitting procedure is shown in exhibit 2.7. A list of the different outliers is placed in Appendix 2 and the actual yield curves are shown in appendix 3. Outliers being found at the short end of the market would seem to be consistent with the findings Of Steeley (1988) for the UK gilt market.


Exhibit 2.7 Yield function fitted with knots at 1 and 5 years maturity
Source: Empirical data

All bonds with a maturity under one year are included in the money market data set and are used to bootstrap the first year. The money market data is taken as a proxy for government paper since no data is recorded for that period and Corrigan (1998) considers that this approach represents the cost for government paper given his experience with Variable bonds and Exchequer Bills. Each bond is arrayed by increasing maturity and their cash flow projected for every six-month period. Then the spot or forward rates are iteratively extracted corresponding to each coupon payment period starting with the first two period covered by the money market. Since the basis on which bonds had been issued changed three times over the sample period, care is needed when dealing with the new Treasury bonds issued by the NTMA. In certain periods several bonds matured giving more than one spot rate for that particular period. When this happens there are several possible approaches;

- Simple average of spot rates,
- Other weighting of spot rates,
- Use spot rate closest to the end of period.

The weighting may be in relation to the different market capitalisations of the bonds maturing for that period. The approach that is employed when several bonds mature is to use an average of spot rates for that period and to continue using that rate for bootstrapping. When no bonds matured in a period, the spot rate is found by interpolating on a linear basis between two nearest spot rates. When this process has been completed for the last bond, there should be a matrix of discount factors (i.e spot rates or forward rates) for each bond that relates it price to the time values of each of its cash flows.

## (2.5.1) <br> $[C F][V]=[P]$

where $P$ is the price vector, CF the cash flow matrix and $V$ the present value matrix. With this data set, a number of different methods may be used to estimate the term structure. It is assumed that the discount factors can be modelled using a mathematical function of a $B$ spline with two knots. The forward data presented a greater challenge for the fitting of any spline function than the spot rates or discount factors and is more suitable to some form of robust smoothing.

The $B$ spline is fitted to each bond in the data set after being fitted to the money market using the same approach of Fisher, Nychka and Zervos,(1995). This gave the lowest weighted deviation of bond prices compared to using any other data set or the forward rates. Where the data sets overlapped, the bootstrapped spot rates are replaced with the fitted spot rates and the $B$ spline is fitted through this data set. This process is repeated for each observation date of the time series using both discount and spot rate data sets.


Exhibit 2.8 Weighted Average of Bond Price Error by Capitalisation 1980-1997

## Source : Empirical data

The results for the spot rates is a cumulative bond price absolute error of 23.31 compared to a cumulative bond price absolute error of 25.18 for the discount factors as shown in exhibit 2.8. Each observation results is shown in appendix 3. Then using the estimated spot rate functions the time varying spot and forward rates from the period 1980 to 1997 are estimated and shown in exhibit 2.9 and 2.10

Spot Rates 1980 to 1997


Exhibit 2.9 Spot Surface 1980-1997
Source : Empirical data


Exhibit 2.10 Forward Surface 1980-1997

Source: Empirical data

In chapter three, the evolution of spot rates will be modelled from the discount functions and factor models will be developed to enable bond pricing. In this section, the estimated discount functions from previous sections will be applied to historical bond prices for October 1997 and the resulting bond price errors are shown in exhibit 2.11.

Outlier Bond Price Identification from Step-wise Spot Rate Data


Exhibit 2.11 October 1997 Bond Price Error
Source : Empirical data

The outliers are the high coupon $12 / 5 \%$ Capital bond 2005 and the $81 / 5 \%$ Capital 2010 with a small issue size. As can be observed in the exhibit 2.12, the bonds errors have an average of zero and are randomly scattered across the maturity spectrum.


Exhibit 2.12 October 1997 Comparison of Bond Prices
Source: Empirical data

When the actual market prices are compared to the price generated by the fitted discount function from the spot curve in exhibit 2.12, actual market prices can be observed to lie along a 45 -degree line containing the origin. This shows a good approximation to the relationship between the fitted spot curve and the observed gross redemption yield curve. The other years are shown in appendix 3 .

### 2.6 Summary and Conclusions

The Irish government treasury bond market from an historical perspective from 1980 to 1997 has been illustrated. The government has kept the duration close to 4 years over this period. The sample of avalable bonds is biased towards the shorter maturities. It is necessary to adjust some of the calculations of the gross redemption yield as data sources while having the same prices generates small differences in their yield calculations.

Different approaches taken to describe the term structure are explained. These include yield to maturity, discrete estimation of the term structure and polynomial approximations. When interpolating a number of points, a spline can be a much better solution than a polynomial interpolation, since the polynomial can oscillate wildly in order to hit all the points. The techniques of polynomial splines, $B$ splines and exponential splines are examined. After explaining why the $B$ splines are chosen, the methodology employed in the estimation of the observed term structure of interest rates is described.

Choosing two knots generates a basis matrix for a cubic spline. These enforce the constraint that the function be linear beyond the boundary knots, which are taken to be at the extremes of the data. An iterative search process, which identified the minimum residual deviance for different numbers of knots, is demonstrated. The model specifying five degrees of freedom with knots at a maturity of one and five years is chosen because it had the lowest residual and is superior to no knots and a knot place at a maturity of one year. A third knot is excluded since the additional explanatory power is marginal. There is a significant difference between the money market up to one-year maturity and the bond market beyond.

A bootstrapping methodology is employed to strip the curve and then a $B$ spline is fitted to the money market and bonds with a maturity greater than one year for both the discount factors and the spot rates. The results of the model using a two data sets are shown for the entire time series. Finally, the spot and forward surfaces are identified and estimated over the entire sample period.

The spot rate curve is applied to the sample data for October 1997 where the valuation of bonds by observed gross redemption rates and fitted spot rate curves are compared for the first time.

The spot rate curves produced in this chapter will be used to investigate the stochastic evolution of the term structure. The time series of the spot rates will be used to quantify the numbers of factors required to model the term structure stochastic process. In chapter four the spot rate curve determine the capital required by a Primary Dealer in government securities when the inventory has different positions in the maturity spectrum. Finally in chapter five, the discount function will be used to value general insurance liabilities. This will permit the calculation of their duration and consequently their matching index portfolio from market accepted indices.

Chapter 3
The Stochastic Process Underlying Observed Spot Rates

### 3.1 Introduction

### 3.1.1 Objectives

The objective of this chapter is to investigate the behaviour of the term structure of the Irish spot rates estimated in the last chapter and to use estimate the orthogonal factorswhich are associated with the changes in the term structure over time. This chapter is divided up as follows: section two reviews the background to stochastic processes; section three models the dynamics of the term structure; section four analyses the time series of spot rates; section five investigates the spread process and the orthogonality proposition for the Irish term structure and the summary and conclusions are in section six.

Over the past three decades two approaches to modelling the term structure have developed in the literature. The first approach is to start with a plausible stochastic process, or processes for the specified sources of uncertainty (i.e. stochastic factors) that drive the evolution of the spot rates through time. From these assumed processes, prices of pure discount bonds and bond yields are determined in the literature as functions of the specified state variables and risk adjusted parameters. These specifications allow a full span of spot rates to be determined, allowing in turn a value to be placed on ail bonds and any contingent claims of the stochastic term structure on a consistent basis. in modern financial institutions engaged in multicurrency asset/liability management, this is a very important prerequisite for efficient risk management of their capital and exposures.

The second approach involves modelling the term structure in a way which is consistent with the initially observed spot rate curve. This constrains the choice of stochastic process to the present level and shape of the term structure and ignores whether it is consistent with the timeseries of previous term structures.

When deciding on the choice of stochastic process a trade off must be made between a simple model that can only describe a subset of all possible paths of the term structure against a more complex model that can be difficult to identify and estimate. These latter models have frequently been referred to as whole yield curve models, and incorporate the models of Ho and Lee (1986), Hull and White (1990); and Heath, Jarrow and Morton (1992).

A term structure must perform two tasks for option valuation purposes. First, it must provide a stochastic process that can identify all possible future term structures. Second, it must be consistent with the term structure at any point in time. From the perspective of those wishing to develop Irish capital markets, this chapter can allow them to compare and contrast their present position with how it might be possible to proceed with the development of a derivatives market. In the last chapter, equation 2.4.8.6 denoted the relationship between spot rates and the discount function. This can be rewritten as:

$$
\begin{equation*}
r_{t}=\frac{1}{t} \ln v^{t} \quad \text { for time } t \tag{3.1.1}
\end{equation*}
$$

and the forward curve 2.4.8.7 can be rewritten as:

$$
\begin{equation*}
f_{t}=\frac{-}{t} \ln v^{\prime} \tag{3.1.2}
\end{equation*}
$$

While the approach taken by Cox, Ingersoll, and Ross (1985) is a single factor model, it can be viewed as a general equilibrium representation of the underlying economy in a stochastic framework.

Strickland (1993) holds that a single factor model is the equivalent of the arbitrage-free approach by invoking the "Fundamental Theorem of Asset Pricing" of Dybvig and Ross (1989). Vasicek's (1977) single factor model uses the arbitrage-free approach by assuming perfect correlation of spot rates. In the next chapter, the microstructure of term structure is examined together with how the marginal cost and revenue of price making maintain the arbitrage-free condition.

### 3.2 Stochastic Processes

In this section, the general background to stochastic processes in financial markets is reviewed. Stochastic processes use partial differential equations' to model the behaviour of a random variable through time. An equation is sought that is well behaved and consistent with the initial condition of the observed term structure and final boundary conditions of discount bonds maturing at par. Wilmott, Dewynne \& Howison (1996) discuss how partial differential equations modelling the term structure are developed from a model of the diffusion of heat flow in a continuous medium which are described by the equation:

$$
\begin{equation*}
\frac{u}{t}=\frac{{ }^{2} u}{x^{2}} \tag{3.2.1}
\end{equation*}
$$

In this model of the diffusion of heat in one space dimension where $u x, t$ denotes the temperature in a long thin uniform bar of material whose sides are perfectly insulated so that its temperature varies only with distance $x$ along the bar and with time $t$. Fourier's Law states that the heat flux is proportional to the temperature gradient $-u / x$ and this equation can be used to model the molecular diffusion of a substance through a substratum.

The theory of stochastic processes in financial markets is concerned with the investigation of the structure of families of random variables which denote the path of the value of an asset through time. According to Fama's (1963) efficient market hypothesis, prices in financial markets reflect past facts and the arrival of expected future information. This means that markets behave as a Markov process by being independent of the past and are only affected by the arrival of unexpected information.

[^16]A partial differential equation for the change in the value of an asset $S$ is;

$$
\begin{equation*}
d S=S d t+S d z \tag{3.2.1}
\end{equation*}
$$

where
is the expected return on dritt while
is called the volatility and represents the stochastic shock. Expressed in terms of a relative return on the asset, rather than in absolute terms, (3.2.1) gives the stochastic differential equation;

$$
\begin{equation*}
\frac{d S}{S}=d t+d z \tag{3.2.2}
\end{equation*}
$$

where $d z$ is an infinitesimal change in a Wiener ${ }^{2}$ process in an infinitesimal interval of time $d t$. This is a generalised mathematical model in continuous time of the behaviour of the return on any asset $S$. While (3.2.2) is not solvable for a particular market path because it follows a random walk, it allows the modelling all the possible paths of any market.

Ito's (1961) lemma is used to relate the infinitesimal change $d S$ in a function of a random asset price variable to the small change in the random variable itself. If the function of the asset value $f(S)$ is continuous and varies by an infinitesimal change $d S$ then $f(S)$ also changes by an infinitesimal amount. By using a Taylor series expansion about zero and ignoring the cubic and higher terms;

$$
\begin{equation*}
d f(S)=\frac{d f(S)}{d S} d S+\frac{1}{2} \frac{d^{2} f(S)}{d S^{2}} d S^{2}+\cdots \tag{3.2.3}
\end{equation*}
$$

From equation 3.2.1;

$$
\begin{equation*}
d S^{2}=S d t+S d z^{2} \tag{3.2.4}
\end{equation*}
$$

which expanded is;

$$
\begin{equation*}
d S^{2}={ }^{2} S^{2} d t^{2}+2 \quad S^{2} d t d z+{ }^{2} S^{2} d z^{2} \tag{3.2.5}
\end{equation*}
$$

According to Wilmott, Dewynne \& Howison (1993), the last term is the largest for small $d t$ and dominates the other two terms leaving;

[^17]\[

$$
\begin{equation*}
d S^{2}={ }^{2} S^{2} d z^{2}+\cdots \tag{3.2.6}
\end{equation*}
$$

\]

with $d z^{2} \rightarrow d t$, then 3.2 .6 becomes;

$$
\begin{equation*}
d S^{2} \rightarrow{ }^{2} S^{2} d t \tag{3.2.7}
\end{equation*}
$$

Substituting 3.2.7 into 3.2.3 and use 3.2.1, then;

$$
\begin{equation*}
d f(S)=\frac{d f(S)}{d S} d S+\frac{1}{2} \frac{d^{2} f(S)}{d S^{2}} d S^{2}+\cdots \tag{3.2.8}
\end{equation*}
$$

These equations are first exploited by Black \& Scholes (1973) with a view to pricing equity options. In their model, a portfolio of a long position in an option and a short position in a fraction of the underlying stock is equated to the risk free rate of return. If the portfolio is constantly rebalanced through time, then the portfolio would earn the risk free rate of return upon the maturity of the option. The option consists of two components, an intrinsic value ${ }^{3}$ and a time value ${ }^{4}$ and as the option approaches maturity, its value must approach the intrinsic value. Equities behave in a more independent manner than fixed income securities where there is a large positive correlation in adjoining maturities.

When the stochastic process of the spot rate curve appropriate to fixed income securities is being modelled, the spot rate curve or its equivalent discount function can be observed. The spot rate curve for April 1980 is illustrated in exhibit 2.10, and the equivalent discount function is shown in exhibit 3.1.

[^18]

Exhibit 3.1 April 1980-Irish Term Structure Discount Function

For a time interval $d t$, the arbitrage free drift of the spot rate curve is known. Assuming that unlimited short positions are possible, then an arbitrage free bar-bell portfolio can be formed with long positions around a particular maturity in which a short position is established (because its yieid is too low relative to the neighbouring spot rates of the long positions in the portfolio). The avoidance of arbitrage implies that a level of high correlation should exist between adjacent maturity points, and it can be possible to specify a simple model with few factors because all the different points on the discount function are related.

An explicit relationship is needed between these model factors and all other maturities of the spot rate curve which takes account of the high correlation and price risk ${ }^{1}$. This correlation is exploited by developing term structures that is driven by a few factors and all other maturities are derived by virtue of some strong relationship that exists between all other points and those driving factor maturities. The spot rate $r$ can be described as a lognormal walk in the stochastic differential equation:

$$
\begin{equation*}
d r=a(r, t) d t+b(r, t) d z \tag{3.2.9}
\end{equation*}
$$

The functions $a(r, t)$ and $b(r, t)$ describe the behaviour of the spot rate $r$. The different models of Merton (1973), Vasicek (1977), Dothan(1978) and Cox, Ingersoll, and Ross (1985) in section 3.3 have the common partial differential equation structure of 3.2 .9 but differ in their functional forms. When Black \& Scholes (1973) used 3.2.9, they had an underlying security which formed the offsetting hedge against the option in their portfolio. This underlying security does not exist in the case of spot rates so bonds of different maturities must be used for hedging.

Steeley (1989b) uses two bonds with different maturities $t$, and $t_{2}$, priced at $V$, and $V_{2}$ to form a portfolio. In the portfolio $P$, one unit of bond one is held against being short $\Delta$ units of bond two which gives:

$$
\begin{equation*}
P=V_{1}-\Delta V_{2} \tag{3.2.10}
\end{equation*}
$$

[^19]From 3.2.1, 3.2.3 and 3.2.8 Ito's (1961) lemma is used to relate the infinitesimal change $d P$ in portfolio value as a function of an infinitesimal interval of time $d t$ and an infinitesimal change in a Wiener process for the spot rate $d r$ :

$$
\begin{align*}
d P= & \frac{\partial V_{1}}{\delta t} d t+\frac{\partial V_{1}}{\delta r} d r+\frac{1}{2} b^{2} \frac{\partial^{2} V_{1}}{\delta r^{2}} d t  \tag{3.2.11}\\
& -\Delta\left(\frac{\partial V_{2}}{\delta t} d t+\frac{\partial V_{2}}{\delta r} d r+\frac{1}{2} b^{2} \frac{\partial^{2} V_{2}}{\delta r^{2}} d t\right)
\end{align*}
$$

If $\Delta$ is set equal to:

$$
\begin{equation*}
\Delta=\frac{\partial V_{1} / \delta r}{\partial V_{2} / \dot{\partial}} \tag{3.2.12}
\end{equation*}
$$

then 3.2.11 becomes;

$$
\begin{equation*}
d P=\left(\frac{\partial V_{1}}{\delta t}+\frac{1}{2} b^{2} \frac{\partial^{2} V_{1}}{\delta r^{2}}-\frac{\partial V_{1} / \delta r}{\partial V_{2} / \delta r}\left(\frac{\partial V_{2}}{\delta t}+\frac{1}{2} b^{2} \frac{\partial^{2} V_{2}}{\delta r^{2}}\right)\right) d t \tag{3.2.13}
\end{equation*}
$$

$$
\begin{equation*}
d P=r\left(V_{1}-\frac{\partial V_{1} / \delta r}{\partial V_{2} / \delta r} V_{2}\right) d t \tag{3.2.14}
\end{equation*}
$$

and the return on the portfolio is the risk free spot rate $r$;

$$
\begin{equation*}
d P=r P d t \tag{3.2.15}
\end{equation*}
$$

Equation 3.2.15 can be rewritten in terms of $V$, and $V_{:}$:

$$
\begin{equation*}
\left(\frac{V_{1}}{\delta t}+\frac{1}{2} b=\frac{\partial^{2} V_{1}}{\delta r^{2}}-r V_{1}\right) / \frac{\mathscr{D}}{\dot{\dot{d}}}=\left(\frac{\mathcal{N}_{2}}{\delta t}+\frac{1}{2} b^{2} \frac{\partial^{2} V_{i}}{\delta r^{2}}-r V_{=}\right) \frac{\partial N_{2}}{\delta t} \tag{3.2.16}
\end{equation*}
$$

While this is an equation with two unknowns $t$ and $t$, the equality holds iff both sides are independent of the maturity date giving;

$$
\begin{equation*}
c(r, t)=\left(\frac{\partial V}{\delta t}+\frac{1}{2} b^{2} \frac{\partial^{2} V}{\delta r^{2}}-r V\right) / \frac{\partial V}{\delta r} \tag{3.2.17}
\end{equation*}
$$

This can be expressed as;

$$
\begin{equation*}
c(r, t)=\lambda(r, t) b(r, t)-a(r, t) \tag{3.2.18}
\end{equation*}
$$

where $\lambda(r, t)$ is the market price of risk. This gives the partial differential fundamental zerocoupon pncing equation:

$$
\begin{equation*}
\frac{\partial V}{\delta t}+\frac{1}{2} b^{2} \frac{\partial^{2} V}{\delta r^{2}}+(a-\lambda b) \frac{\partial V}{\delta r}-r V=0 \tag{3.2.19}
\end{equation*}
$$

Equation 3.2.19 can be solved uniquely ${ }^{2}$ if the boundary condition is imposed that zerocoupon securities must mature at par. This becomes clearer in the case of a portfolio with a single bond maturing at $t$ and the infinitesimal change $d V$ in portfolio value as a function of an infinitesimal interval of time dt :

$$
\begin{equation*}
d V=\left(\frac{\partial V}{\delta t}+\frac{1}{2} b^{2} \frac{\partial^{2} V}{\delta r^{2}}\right) d t+b \frac{\partial V}{\delta r} d z \tag{3.2.20}
\end{equation*}
$$

From the partial differential fundamental zero-coupon pricing equation:

$$
\begin{equation*}
d V=\left(b \lambda \frac{\partial V}{\delta r}+r V\right) d t+b \frac{\partial V}{\delta r} d z \tag{3.2.21}
\end{equation*}
$$

which becomes:

$$
\begin{equation*}
d V-r V d t=b \frac{\partial V}{\delta r}(\lambda d t+d z) \tag{3.2.22}
\end{equation*}
$$

This expresses the return on the bond as an excess return earned over the spot risk free rate for taking an excess $\lambda d t$ of risk. The spot rate random walk in 3.2 .9 has coefficients that are more complex than those found in the equity market random walk. However, focusing on the price of pure discount securities (i.e. time equivalent inverse of spot rates), gives information about the behaviour of $r$.

[^20]There are different classes of solutions for $a(r, t)$ and $b(r, t)$ in 3.2.9. Two of these functional forms identified by Wilmott, Dewynne \& Howison (1996) are;

$$
\begin{equation*}
a(r, t)=\sqrt{\alpha(t) r-\beta(t)} \tag{3.2.24}
\end{equation*}
$$

and

$$
\begin{equation*}
b(r, t)=(-\gamma(t) r+\delta(t)+i(r, t) \sqrt{\alpha(t) r-\beta(t)}) \tag{3.2.25}
\end{equation*}
$$

The time dependent functions in 3.2.23 and 3.2.24 can be restricted to fit the data and still have a random walk for spot rates with the property of mean reversion such that the spot rates cannot become negative. The solution of the partial differential fundamental zero-coupon pricing equation 3.2.19 is;

$$
\begin{equation*}
V(r, t)=A(t) e^{-r \theta(t)} \tag{3.2.26}
\end{equation*}
$$

If the values of $\alpha, \beta, \gamma$ and $\delta$ are assumed to be constant then;

$$
\begin{gather*}
\frac{2}{\alpha} \log A=a \psi_{2} \log (a-B)+\left(\psi_{2}-\frac{1}{2} \beta\right) b \log ((B+b) / b)+\cdots  \tag{3.2.27}\\
\cdots+\frac{1}{2} B \beta-a \psi_{2} \log (a)
\end{gather*}
$$

and

$$
\begin{equation*}
B(t)=\frac{2\left(e^{v t}-1\right)}{(y+\psi,)\left(e^{v t}-1\right)+2 \psi,} \tag{3.2.28}
\end{equation*}
$$

where

$$
\begin{equation*}
b, a=\frac{ \pm \gamma+\sqrt{\gamma^{2}+2 a}}{a} \tag{3.2.29}
\end{equation*}
$$

and

$$
\begin{equation*}
\psi_{1}=\sqrt{\gamma^{2}+2 \alpha} \tag{3.2.30}
\end{equation*}
$$

and

$$
\begin{equation*}
\psi_{z}=\frac{\delta+a \beta / 2}{a+b} \tag{3.2.31}
\end{equation*}
$$

In the case of the term structure, the initial spot and forward curves and their associated volatilities can be observed from the market and a model would be expected to be consistent with these observations.

### 3.3 Modelling the Stochastic Process of the Irish Term Structure

In this section, the changes in the spot rates are examıned to see whether there are common factors in the Irish term structure. The parameters of the distribution of the Irish spot rates and their changes are identified to see whether they are normally distributed. The behaviour of the time series of spot rates and their changes have important implications for the specification of the stochastic process that seeks to describe their operation through time. The results can give some indication about arbitrage free term structures and the behaviour of bond prices for different hypotheses.

The main purpose of a factor analysis is to transform the correlated spot rate time series into factors that explain a substantial part of the common variance in the original data by a small number of common sources. It is important to understand the behaviour of each of the factors and their relative importance in explaining the comovement of the spot rate time series. The objective is to identify the first factor that reflected to the major movement of the whole term structure. Longstaff and Schwartz (1992) found it useful to simplify the analysis by considering a smaller number of linear combinations of the original spot rates.

Litterman and Scheinkman (1991) demonstrated that the stochastic process of the term structure can be represented by a small set of common factors that represent bond returns on US data. A considerable element of the variance of bond portfolios can be explained by three factors which represent shifts in level, steepness and curvature of the term structure. A variancecovariance matrix is used to identify the principal component for calculation of factor sensitivies and the correlation matrix is shown in exhibit 3.2.

The correlation matrix illustrates the fact that the comovement of spot rates decreases with increasing maturity. For example, the three month Irish spot rate changes and one year Irish spot rate changes are $93.6 \%$ correlated, the correlation between correlation between three month Irish spot rate changes and seventeen year Irish spot rate changes is $53.7 \%$ over the period 1980 to 1997.


Exhibit 3.2 Spot rate changes Correlation matrix 1980 to 1997
Source: Empirical

In exhibit 3.3, the Irish term structure of volatility is examined. Volatilities of spot rate changes decrease monotonically to a maturity of nine years; then increase monotonically to a maximum at a maturity of eighteen years before decreasing monotonically to a maturity of twenty years. This is an unexpected result that has not been borne out in empirical research in other markets


Exhibit 3.3 Volatility of Irish Spot rate changes 1980 to 1997
Source: Empirical

Furthermore, when we focus on the period when exchange controls are removed from 1989 to 1997 in exhibit 3.4, the volatility term structure falls strictly monotonically over the entire maturity range. This confirms that the Irish term structure behaved in an unusual manner between 1980 and 1989 and the findings in other European markets that long spot rate fluctuate less than short rates (Buhler and Zimmermann (1996)).


Exhibit 3.4 Volatility of Irish Spot rate changes 1989 to 1997
Source: Empirical

Since some of the factor models assume that volatilities and correlations are constant and stable, the standard deviation and correlations of the first differences are plotted for a moving five year period between 1985 to 1997 in exhibit 3.5 and 3.6 for the six month, 5 year, ten year and eighteen year spot rates


Exhibit 3.5 Volatilities of Irish Spot rate 1985 to 1997
Source : Empirical

During this period the spot rate is rather unstable and its behaviour can be broken into three very significantly different periods, firstly, 1980 to 1987, secondly, 1988 to 1991 and thirdly, 1991 to 1997. By using the standard two-sample t-test, the $t$-value of 2.207 ( $p$-value equal to $3.92 \%$ ), the first significant break in the time series of volatility is between the first period 1980 to 1987 and the second period 1988 to 1991 . The second significant break with a t-value of 5.1368 ( $p$-value equal to $0.02 \%$ ) is between the second period 1988 to 1991 and the third period 1991 to 1997. Volatility of the short rate is higher than the long rate, but both have fallen in recent years. While volatilities do not change in a parallel manner, there is clearly a strong relationship in adjoining spot rates.


Exhibit 3.6 Correlations of Irish Spot rates 1985 to 1997
Source: Empirical

In exhibit 3.6, the correlation is calculated for a moving five year period for the three month spot rate changes against the ten year spot rate changes, then for the three month spot rate changes against the eighteen year spot rate changes and finally for the ten year spot rate changes against the eighteen year spot rate changes. The results are similar to the volatility finding and the correlations are not very stable through time.

The first three components of the changes in the spot rates are shown in exhibit 3.7 and their importance is shown in table 3.1. It can be observed that the first factor is broadly similar for all maturities implying that it caused a paralled shift of the term structure. These results support the observation of Litterman and Scheinkman (1991) that the first factor represents a pure level shift or duration factor.

From exhibit 3.7, the short and long spot rates are inversely related to the second factor meaning that they move in opposite direction for a given factor shift. This second factor captures the inversion of the term structure and it can be described as a slope factor. The third factor captures curvature shifts in the term structure where the long and short spot rates move in a different direction from intermediate spot rates.

| Measure | Shift | Slope | Curvature |
| :---: | :---: | :---: | :---: |
| Standard deviatıon | 0.1433455 | 0.03991129 | 0.01848747 |
| Proportion of Variance | $90.76 \%$ | $7.04 \%$ | $1.51 \%$ |
| Cumulative Proportion | $90.76 \%$ | $97.80 \%$ | $99.31 \%$ |

Table 3.1 Irish Term Structure Factors Relative Importance 1980 to 1997
Source: Empirical

The degree to which each factor explains the variance of spot rate changes is of interest. The first factor explains $91 \%$ of the variance, and the first two factors together explain $98 \%$ of the variance. Overall, the factors explain $99 \%$ of the variations of the term strucutre of spot rates. The factor loadings are the coefficients of the principai component transformation and provide a summary of the influence of the original spot rate changes on the principal components. In terms of interpretation, a large coefficient in absolute terms corresponds to a high loading, while a coefficient near zero has a low loading. The loadings for the first factor are all of the same sign and a reasonable interpretation is that they represent a paralled shift or change in spot rates. The second component contrasts by being negative for the first seven years, then are positive from year 8 to year 18 and a reasonable interpretation is that it represents the slope in spot rates. Finally, the third factor is positive for the first two years, marginally negative between years 3 to 13 and positive from years 13 to 17 and a reasonable interpretation is that it represents the curvature in spot rates.


Exhibit 3.7 Irish Term Structure Factors 1980-1997

Source : Empirical

One of the issues in building a factor model is the number of factors to be considered. The purpose of factor analysis is to reduce the complexity of multivariate data by fransforming the spot rate data into the principal component space, and then choosing the first $n$ principal components that explains most of the variation in the onginal spot rates. There are different approaches to determing the number of factors required for term structure modelling:

- Cattell's criterion
- Include enough components to explain an arbitrary amount
- Kaisers criterion

Cattell's criterion is a screepiot where the eigenvalues $\lambda$, is plotted against $j$ and has the disadvantage that it can include too many components.


Exhibit 3.8 Cattell Plot of Eigenvalues for Irish Term Structure data 1980-1997
Source: Empirical

A screeplot plots the eigenvalues against their indices, and generally breaks visually into a steady downward slope and a gradual trailing away. The break from the steady downward slope indicates the break between the important principal components and the remaining components which make up the scree. The screeplot for the changes in spot rate by maturity is shown in exhibit 3.8 and only the first three components appear important, explaining $99 \%$ of the variance. If Kaiser's criterion for excluding eigenvalues is applied, all components except the first three are excluded. The $99 \%$ criterion suggests keeping the first three factors.

Exhibit 3.8 does not give a comprehensive view of both factors and the original data. The biplot in exhibit 3.9 allows the representation of both the original spot rate changes and the transformed observations on the principal components axes. By showing the transformed spot rate changes, the original data can be interpreted in terms of the principal components.


Exhibit 3.9 Biplot of Irish spot rate changes data 1980 to 1997
Source: Empirical

The biplot is interpreted with the x-axis representing the scores for the first principal component, the $y$-axis the scores for the second principal component. The original values are represented by arrows which graphically indicate the proportion of the original variance explained by the first two principal components. The direction of the arrows indicates the relative loadings on the first two principal components. For example, the variable spot rate changes of three months maturity has the largest loading in absolute value for both the first and second components. and the loading on the second component has a negative sign.

Thus, spot rate changes of three months maturity are represented by a long, downward sloping arrow. The variable spot rate changes of seven and three quarters years maturity has the smallest loadings on the two components with the second component having a value of zero. Thus, spot rate changes of seven and three quarters years maturity is represented by a short horizontal arrow. The variable spot rate changes of 17 years and nine months maturity has the largest loading in absolute value for both the first and second components, and the loading on the second component has a positive sign.

### 3.4 Examining the Irish Term Structure as a time series

In this section, univariate time series modeis are identified for short, long and spread rates of interest. The statistical properties of $r$, the six month short spot rate, $I$, twenty year long spot rate, and $s$, spread between long and short spot rate and their differences are described in table 3.2 with the appropriate statistical distribution for the spot rate time series.

|  | R | L | S | $\Delta \mathbf{R}$ | $\Delta \mathrm{L}$ | $\Delta \mathrm{S}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Minimum | $5.186 \%$ | $6.386 \%$ | $-4.367 \%$ | $-5.762 \%$ | $-4.320 \%$ | $-3.786 \%$ |
| Maxımum | $21.057 \%$ | $22.462 \%$ | $5.847 \%$ | $6.234 \%$ | $6.386 \%$ | $4.180 \%$ |
| Mean | $10.847 \%$ | $11.480 \%$ | $0.634 \%$ | $-0.050 \%$ | $-0.155 \%$ | $-0.105 \%$ |
| Std Deviation | $4.039 \%$ | $4.351 \%$ | $2.000 \%$ | $2.345 \%$ | $1.955 \%$ | $1.773 \%$ |
| Variance | $0.163 \%$ | $0.189 \%$ | $0.040 \%$ | $0.055 \%$ | $0.038 \%$ | $0.031 \%$ |
| Skewness | 0.595 | 1.167 | -0.032 | -0.058 | 1.211 | 0.148 |
| Kurtosis | 0.078 | 0.428 | 0.814 | 1.795 | 3.479 | 0.069 |

Table 3.2 Summary Distribution of Irish Spot Rate Parameter Factors 1980 to 1997
Source: Empirical

By using the Kolmogorov-Smirnov test for different statistical distributions, the statistical distribution identified to best fit the short spot rate time series from 1980 to 1997 is a Gamma distribution with the parameter estimates; $\alpha$ of 8.99 and $\beta$ of 0.0119 , with a Kolmogorov-Smirnov value of 0.0774 shown in table 3.10. The Gamma distribution is:

$$
\begin{equation*}
f(r)=\frac{\beta^{-\alpha}}{\Gamma(\alpha)} e^{-r / \beta} \Gamma^{\alpha-1} \quad(0 \leq r<\infty), 0<\alpha, 0<\beta \tag{3.4.1}
\end{equation*}
$$

In exhibit 3.2, the decline in the short spot rate from a high of over $20 \%$ in 1981 to a low of $5 \%$ in 1997 can be seen.


Exhibit 3.10 Irish Short Spot Rates - 1980 to 1997
Source: Empirical

Since the values of successive short spot rates tend to be close together, serial correlation is a problem. In exhibit 3.11, the lagged scatter plots consist of scatter plots of pairs of short rates $\left(r_{t}, r_{t+1}\right)$ of the time series separated by $j$ semi-annual units of time for $j=(1,2, \ldots 4)$


Exhibit 3.11 Irish Lagged Short Spot Rates - 1980 to 1997
Source : Empirical

An elliptical shape for up to 4 lags in the $45^{\circ}$ direction indicates positive correlation up to the fourth lag. The plot of the autocorrelation in exhibit 3.12 illustrates the estimation of the correlation between spot rate observations separated by a lag of $j$ semi-annual units of time.


Exhibit 3.12 Autocorrelation of Irish Short Spot Rates - 1980 to 1997
Source: Empirical

The horizontal dashed line is the $95 \%$ confidence interval for the autocorrelation estimate at each lag. The plot indicates autocorrelation for the first three semi-annual lags or one and a half years.

The distributions of the long spot rate, the spread between the long and short spot rates and the changes are in appendix four. When the kurtosis values for changes in the short, long and spread spot rates are examined in table 3.2 , they range from 3.54 to 5.41 indicating an element of leptokurtosis, particularly in the distribution of the long rates. However, the normal and Weibull distributions are the closest fitting distributions for changes in the short, long rates and changes in their spread. The other time series models are in appendix 4

### 3.5 Spread Process and the Orthogonality Proposition

In this section, the Ayres and Barry (1979) orthogonality proposition is examined in relation to the spread process. Brennan and Schwartz's observation of the short and long rates being permitted to influence each other is also examined. This orthogonality proposition originating from the Ayres and Barry (1979) study has been consistently observed and supported in a number of separate studies $\{$ Schaefer (1980); Nelson and Schaefer (1983).

Since a number of authors have found the long rate and short rate to be highly correlated, whereas the long rate and the spread between the long and the short rate, on many occasions is found to be orthogonal and thus more appropriate for use in a two factor model, spreads between the various rates are calculated and, along with the rates themselves are tested for correlation. The correlation table is thus used to aid the selection of possible factors.

|  | $\mathbf{r}$ | $\mathbf{l}$ | $\mathbf{s}$ | $\Delta \mathbf{r}$ | $\Delta \mathbf{l}$ | $\Delta \mathbf{s}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{r}$ | $100.00 \%$ |  |  |  |  |  |
| $\mathbf{l}$ | $89.90 \%$ | $100.00 \%$ |  |  |  |  |
| $\mathbf{s}$ | $-8.56 \%$ | $38.02 \%$ | $100.00 \%$ |  |  |  |
| $\Delta \mathbf{r}$ | $-30.40 \%$ | $-12.28 \%$ | $34.67 \%$ | $100.00 \%$ |  |  |
| $\Delta \boldsymbol{\Delta l}$ | $-30.98 \%$ | $30.09 \%$ | $-2.89 \%$ | $67.38 \%$ | $100.00 \%$ |  |
| $\Delta \boldsymbol{s}$ | $6.05 \%$ | $-16.93 \%$ | $-49.06 \%$ | $-57.97 \%$ | $21.14 \%$ | $100.00 \%$ |

Table 3.3 Orthogonality Tests of Factors 1980 to 1997 of Irish Term Structure
Source: Empirical

The correlation of the level of the short rate and the long rate is $90 \%$ and the correlation of their changes is $67 \%$ which is in keeping with the Brennan and Schwartz (1979) observation.

Their model is estimated in Appendix 4. The correlation is estimated between the levels of the long rate and the spread between the long and short rate at $38 \%$, indicating somewhat orthogonal behaviour. Despite the apparent significance of this value (Inverse F=0.33 for 35 degrees of freedom at the $5 \%$ level), the non-normality of both data sets can induce spurious correlation in a product moment correlation of more significant magnitude. At $\mathbf{- 1 7 \%}$ in table 3.3 , there is no relationship between the long rate and the change in the spread.

### 3.6 Summary and Conclusions

The nature of the stochastic processes that generated Irish yield curves is analysed in this chapter. One of the major problems is that the data have been confined to six monthly observations giving thirty-two sample points from which the spot rate curve factors describing the yield curve are identified. A number of different approaches to modelling the term structure of spot rates are then analysed. Each possible approach has advantages as well as disadvantages when compared on the basis of the tractability of the model solution, the number and ease of estimation of parameters, and the amount of market information used. While the hypothesis that changes in the short and long spot rates could not be rejected, they did display a high degree of leptokurtosis.

The dynamics of the Irish term structure are examined from 1980 to 1997 and three factors explained more than $99 \%$ of the term structure movement. The first factor implied a parallel shift of the term structure: the second factor implied a change in the slope of the term structure, and the final factor implies a change in the curvature of the term structure. It is interesting to note that the term structure of spot rate volatilities is not strictly monotonically decreasing until exchange controls are removed in 1989. This has strong implications for bond portfolio risk management which uses only duration measures for control purposes. However, the stability of correlation and volatilities across the term structure is a major concern between 1980 to 1989 from a risk management perspective, but with the advent EMU the correlation shouid increase between different European bond markets, reducing this risk.

Volatility is the most important model parameter and two factor models are better than single factor models. Two factor models of the term structure allow a more realistic representation of the yield curve than their single factor equivalents.

They can be characterised by their tractability and ease of use, but with the resulting disadvantages of unrealistic assumptions about the stochastic process for the short rate, and the limitation of possible shapes that the term structures can take. The changes in the spread between the short and long spot rates and the long spot rates are found to be orthogonal. This phenomenon has been observed in other markets and points to a two factor models involving these parameters. The parameters that are required for the Vasicek (1977), Cox, Ingersoll \& Ross (1985), Hull \& White (1990) are estimated in the final section. Futher research could be done if the spot rates are available on a weekly basis for the past ten years.

In the next chapter, the microstructure of the price discovery process of the term structure will be analysed using the findings of the second chapter to model the discount function and the findings of this chapter to build a Monte-Carlo simulation model of a dealer in government securities. Then in chapter five, the behaviour of a particular sector that invests in government securities will be analysed using the findings of the second chapter to quantify the duration of its liabilities. A matching immunised portfolio of goverment bonds using third party indices will be constructed and the findings of this chapter allow a Monte-Carlo simulation model to be built of a mismatch reserve for a non-matching portfolio.

## Chapter 4

Microstructure of the Irish Government Treasury Market

### 4.1 Introduction

The application of explicit trading rules to securities priced in line with processes outlined in chapters two and three is examined in this chapter. The hypothesis to be tested using simulation methods is whether a competitive dealership market could be feasible in terms of long-run bankruptcy risk, and preferable to the then existing agency microstructure. The efficient and effective operation of the price discovery process is an area of concern to both the borrower and investor.

The issues concerning the authorities are: the different costs associated with different structures; immediacy; liquidity ${ }^{1}$, ease of regulation and transparency. This chapter is divided up as follows: section two reviews the relevant literature on microstructure; section three reviews the microstructure of the agency bond market; section four investigates whether the European Union Capital Adequacy Directive is adequate for the Irish market; section five simulates a Monte-Carlo framework for a primary dealer and the summary and conciusions are in section six.

### 4.2 Literature Review

### 4.2.1 Definition of Microstructure

Garman (1976) defines market microstructure as the study of the process and outcomes of exchanging assets under explicit trading rules and of the resulting prices. Any price represents today's value for a future set of cash flows whose size and/or timing can be deterministic or stochastic.

For the Irish government, the risk-free ${ }^{2}$ cash flows are set down in the issue terms of its bonds of which $90 \%$ were fixed coupons with the remaining $10 \%$ being variable (due to the coupon setting mechanism or embedded options features). If there are any additional risks other than credit involved in the market microstructure, a higher return may be required to compensate for these risks. In this scenario, the government through the NTMA would have a greater cost of service on the National Debt, which could be lowered by changing the market microstructure.

There are three factors that can create a demand for the immediate execution of a trade. Firstly, as a result of analysis the asset/liability allocation of a portfolio can be re-aligned. Secondly, a response can be required to offset the risk of a new liability. Finally, a trader can be anticipating changes in the price of an asset not already fully discounted by other traders in the market. In all three cases there is a cost to the trader of delaying the execution of the particular trade. The most common cost in all three situations is the possible adverse price movement that could occur in the period prior to the execution of the trade. Price volatility can increase the demand for immediacy. In general, a bad price outcome is just as likely as a good one when prices are variable. The larger the underlying price variance per unit time, the greater the risk faced by the trader in the period before the trade is executed and the greater the demand for immediacy.

[^21]
### 4.2.2 Types of Microstructures

There are three broad categories of continuous or batch trading procedures. They are as follows. Firstly, a dealer, who has to continuously quote bid and ask prices at which he is willing to trade, dominates a dealer market. Secondly, the open auction is a continuous market clearing procedure under which traders continuously submit buy or sell orders to the market. Thirdly, the batch auction clearing house where traders submit orders to buy or sell specified quantities of the traded security either at the market price or subject to a limit price qualification. In batch trading, orders are allowed to accumulate over time, rather than being transacted immediately.

Continuous trading does not mean that transactions occur all the time, but that a transaction can occur whenever the orders of two traders cross. There are two distinctions made in continuous trading systems and matching systems. Firstly, in a dealer system, an intermediary "makes the market" by satisfying the customer's order from the intermediary's own account, while in a matching system, traders act as agents for the customer. Secondly, matching systems invariably have dealers (in the sense of professional traders who are usually willing to supply immediacy by trading to or from inventory), but public limit orders are given equal or preferred status.

There are a variety of stabilisation techniques to cope with excess demand or supply. The most common type of stabilisation is by the use of maximum price change limits, e.g., Chicago Board of Trade limit up and limit down on price movement within a trading session. Alternatives are to halt trading temporarily when an excessive price would otherwise occur, to indicate a price, or to accumulate orders for a time period and then resume trading with no price limit.

[^22]Market makers can have an affirmative obligation to stabilise security prices if transaction-to-transaction price changes exceed certain limits. The NTMA uses the stabilisation method of making purchases and sales in the market of it's own bonds.

The necessary climate for evolutionary innovation in electronic trading includes the availability of technology and economic gains from adoption but the failure of Irish Futures and Options Exchange (IFOX) in 1996 can mean waiting for this evolution until after the advent and development of the single EURO currency in European Monetary Union.

### 4.2.3 Liquidity and Bid/Ask Spread

One market variable long thought to be a factor in price adjustment is trading volume. McDermott (1993) maintains that volume is larger when prices move up than when they move down. The reporting of Irish volume has been made difficult because it contains a level of REPO activity to facilitate differences in one day Government Settlement Office ${ }^{3}$ (GSO) local settlement and one week Cedel or EUROCLEAR settlement. Empirical research has identified a strong link between volume and the absolute value of price changes.

Volume's role in the price adjustment process is to facilitate certainty. An important feature in this result is the common error in the information. If price and volume together revealed the true value of the risky asset, then higher volume need not necessarily accompany the absolute value of price changes: whatever volume arose would be sufficient to move prices to full information values.

Liquidity has long been recognised as an important determinant of market behaviour. While it is common today to ascribe only beneficial properties to liquidity, such a view has not always been held. Keynes (1936) said that:
"Of the maxims of orthodox finance, none, surely, is more anti-social than the fetish of liquidity, the doctrine that it is a positive virtue on the part of investment institutions to concentrate their resources on the holding of liquid securities ".

Ultimately, all assets are liquid over the time horizon of the assets' life, in that they can return all the cash flows on the designated payment dates, assuming no default risk. Liquidity is the possibility of facilitating the exchange of these future cash flows for one cash flow today by transacting with counterparty in the secondary market. The single cash flow today is the price of the asset and represents a discounted value of the sum of these future cash flows.

Liquid markets are generally viewed as those that allow trading with the least effect on price. In liquid markets it should be possible to trade, if not continuously then at least with some frequency, without unduly affecting prices. If prices move after trades, then these price revisions can provide a more accurate reflection of the costs of trading than do bid and ask prices. This view of liquidity involves a time series dimension quite distinct from the cross sectional properties normally associated with the earned spread. This is the focus of Grossman and Miller's (1988) analysis of liquidity. Their focus is on the role of liquidity as the price of immediacy, or essentially the notion that a trader willing to delay transacting commands a better price than one who demands immediate execution.

Grossman and Miller's (1988) view is the greater the number of speculators willing to provide immediacy, then the greater the liquidity of the market. Since the return to speculators increases due to increased price variance, markets with greater price volatility can have more speculators, but they in turn require a higher return to compensate them for the greater risk.

[^23]Liquidity can be enhanced in a market by improving the return to speculators (and thereby inducing more to enter) until the marginally revenue on a risk adjusted basis is equal to the marginal cost of maintaining a market presence.

If the number of traders and their risk capital affects liquidity, then the scale of trading can affect market performance. In particular, if prices are more transparent in a more liquid market, there should be a natural incentive for traders to converge on one market, rather than split their trades across markets.

Ho (1984) investigated the relationship between the bid-ask spread and market liquidity. In the case of the government debt market, the traders were involved in the price discovery process at two levels, firstly the individual bond values and secondly the underlying term structure described in chapter 2 which orders and put bounds on their prices in an arbitrage free framework. Ho (1984) believes ${ }^{4}$ that bond values and the underlying term structure were not independent of each other and the bid-ask spread represents the portion of the value of the transaction that pays for dealer services. He states that the return on capital of the dealer market determines the number of dealers, and hence the liquidity in the market.

If there is free entry and exit into and from the market, and there are no subsidies to dealers for their market making activities, there must be a direct relationship between the bidask spread and market liquidity. Ho (1984) showed this to be true when transaction volume is kept constant. That is, a tight spread can decrease dealer revenue, causing some of the dealers to leave the market leading to a loss of liquidity in the market. The number of market makers in a bond is central to the dealer market structure. Market makers could differ from each other with regard to their capitalisation and trading ponfolio size. However, the capital committed to a particular bond and the trading strategy of a market maker must meet a minimum level across all market makers to comply with the European Union Capital Adequacy Directive

[^24]Since there are few economies of scaie to trading, each dealer must reach his optimal capitalisation and pricing strategies for a particular bond. In Admati - Pfleiderer's (1988) model, uninformed liquidity traders were assumed to be of two types. There are non-discretionary liquidity traders who must transact a given amount at a specific time. The second group must also trade a given amount but they have some discretion with respect to the timing of their trades. Unlike Ho's model (1984), they recognise that by seeing the flow of orders from different types of counterparties, they could use information as input into their price making process.

### 4.2.4 Capital, Order Flow and Ruin Barrier

A seminal paper was written by Garman (1976) where he investigated the security market microstructure and argued that an exchange market could be characterised by a flow of orders to buy and sell. The question then arises: if buyers and sellers arrive at different points in time, to what time period do the supply and demand schedules refer? This flow of orders would arise as the solution to individual traders' underlying optimisation problems. As these orders go into the market, imbalances between the demand and supply of a certain good could temporally arise.

He examines two market clearing frameworks, a dealer structure and a double auction mechanism. The imbalance that would arise dictates an importance to the temporal microstructure and the requirement to carry an inventory. This is the essence of how the exchange between buyer and seller actually occurs at any point in time. In his model, Garman (1976) assumes that his position at any point in time is determined by the order arrival rates. If orders to buy and sell are not always balanced in the selected time period, how does the price change reflect the order flow?

Garman (1976) considers a single monopolistic market maker that sets prices, receives all orders and clears trades. The dealer's objective is to maximise expected profit per unit of time. Failure arises when the dealer runs out of inventory or cash.

Garman's (1976) assumptions were that the market maker is not allowed to borrow and the level of demand associated with these order processes is exogenous to the market maker. As the orders arrive, the dealers' cash position changes and it is this dynamic movement that is important for the dealer. Since he cannot augment his cash or bonds except through trading, the question is whether the market maker can avoid running his cash position to zero and thus failing. This is known as the Gambler's Ruin Problem. The gambler is assumed at the start to have some initial wealth $I_{c}(0)$ and wagers until he reaches a certain level, or loses all of his money.

As an embedded Markov chain the failure probability, provided that the value of the supply rate times the offer price exceeds the value of the demand rate times the bid price, can be expressed as:

$$
\begin{equation*}
p_{f}=\left[\left\{p_{S} \lambda_{S}\left(p_{S}\right)\right\} /\left\{p_{B} \lambda_{B}\left(p_{B}\right)\right\}\right]^{I_{c}(0) / p} \tag{4.1}
\end{equation*}
$$

where
$p_{S}-$ bid price,
$\lambda_{S}\left(p_{S}\right)$ - Market makers demand for sell orders,
$p_{B}-$ offer price,
$\lambda_{B}\left(p_{B}\right)$ - Market makers demand for buy orders,
$I_{c}(0)$ - initial cash position,
p-price.

Even with odds favouring winning the gambler faces a positive probability of ruin. If a significant fraction of the trader's capital is involved in a given transaction (an example being when a Government bond dealer buys a large inventory of bonds) then a given level of price variance can, during a period of delayed execution, be very costly as observed. This occurs when arbitraging or spreading on a large scale between two or more markets.

There can be little risk once all the components of the arbitrage are in place but much risk while one side is still open. Since the market maker is dealing in the efficiency of price discovery of the term structure, he can be exposed to three levels of risks. He can have an outright position in a bond, or one of two possible positions of where either he is long one bond and short another bond on an inter or intra maturity ${ }^{5}$ sector basis. In order to understand the risk caused by trading delays, the reason for trade must be understood. Bond holdings represent accumulated wealth and are often held for individuals by institutions such as trusts, insurance companies etc..

Daily trading volume for bonds can also be generated by trade between investors who want to move their portfolio of risky bonds into less risky bonds or from securities with mostly cash returns to those offering more price appreciation possibilities. Informational motives can also arise leading some to take a position in a bond because they feel they have some information not possessed by other traders.

The classic case of arbitrage occurs when an investor buys a security in one market where it is under priced and simultaneously sells the same security in another market where it is overpriced. Usually these trades involve narrow spreads, which means that profits depend on large volumes of activity. With the increased number of arbitrage participants and improved communications systems, markets have become efficient thus reducing the scope for such trades.

[^25]Stoll (1978) focuses on determining the costs the dealer faces in providing deater services or immediacy. These are: (1) the holding costs imposed by the sub-optimal portfolio position; (2) the order processing costs (e.g. fees, taxes etc.); (3) the cost of trading with informed traders. The first formal analysis of the dealer's problem was undertaken by Stoll (1978). The dealer must delineate the risks he faces and he must choose an optimal pricing strategy to maximise his utility. Stoll (1978) focuses on the portfolio risk that the dealer function entails. The dealer is assumed to be risk averse.

In this model, inventory matters largely because of the dealer's inability to hedge his inventory exposure. The model is simplistic, e.g., if the dealer were risk neutral or able to diversify then the cost of providing dealer services would fall and could fall to zero. It is not obvious how this theory would explain phenomena such as differences in spreads during the trading day in the same bond.

### 4.2.6 Information Signalling in Price Changes

Another issue in microstructure theory is the generation of new information and its reflection in trading volume and price. A distinguishing characteristic is the attempt to model trading out of equilibrium. The reason is that security market information arrives with great frequency; an attempt to delay trading would deny traders the speed of execution, which they demand. Assuming a Walrasian or other equilibrium price formation model then various observed phenomena such as market orders and bid-ask spreads cannot be explained. Glosten (1989) explores the revelation of inside information. He assumes a pure limit order book market and two classes of traders: the informed trader and the uninformed trader. If insider information is held monopolistically, an insider trader can offset his gains from trading against the probability that his trades can reveal the information. On the other hand, competition among insiders can drive up their collective rate of order arrival and the information can be revealed immediately.

If there exist other traders who are willing to provide immediacy, then a specific specialist need not be necessary in the market. For example, if traders can submit limit orders, then any market orders requiring immediate execution can be crossed with such orders, leaving no role for the specialist.

Cohen, Maier, Schwartz and Whitcomb (1981), propose a model which investigates the order strategies of traders who can choose between submitting a market order for immediate execution or a limit order which specifies a specific price for execution. There is no active specialist and they assume that the market ask (bid) price depends only on the last previous market ask, and hence is a Markov process.

If a trader submits a limit order between the current market bid and ask, what is the probability that the limit order can in fact execute over the next trading period? If it is one, then it can clearly be optimal for the trader to submit a limit order and hence reduce the price at which he trades. The authors however show that this is not the case. No matter how close the trader places his limit order to the current market price, the probability of the limit order executing is always less than one.

In Cohen, Maier, Schwartz and Whitcomb (1981) model, the limit orders held in the trading book determine the market spread. If the spread is wide then a trader has much to gain from submitting a limit order because, if it executes, the trader can have transacted at a much better price. There are two properties of this process. First, the "gravitational pull" of the market orders dictates that a non-zero spread is an equilibrium property of the market. Second. the size of the spread depends on the movement of traders between limits and markets, and this in turn partially depends on the execution probability of the limit order.

They noticed that there is a distinction in the market between market gains and trading gains. In market gains the notion is that when market prices go up in general, most investors gain; when they fall, most investors lose. In trading gains information costs can make an average investor lose money relative to the market return over time. This information loss arises because of the presence in the market of traders who have superior information of the market. These informed traders have the option not to trade, unlike the market maker who must always quote bid and ask prices. The market maker knows that when he is trading with an informed trader he usually loses. Therefore, in order to stay in business, he must be able to offset these loses by making gains from uninformed traders. These gains arise from the bid-ask spread that is adjusted to reflect their superior anticipation of order flow.

The first model to formalise this concept of information costs is by Copeland and Galai (1983). Their analysis develops a one period model of the market maker's pricing problem given that some traders have superior information. The model includes two approaches to viewing the bid-ask spread. One approach assumes a risk neutral dealer who sets bid and ask prices to maximise his expected profit. Another approach views the bid and ask prices as calls and put options provided by the dealer to the traders.

The most important result that emerges from this model is that even with risk neutral competitive dealers, a spread arises. The size of the spread differs with various market parameters, in particular the elasticities of traders demand functions. As long as there is a positive probability that some traders were informed, the spread is never zero. The Copeland and Galai (1983) model thus quantifies the concept introduced by Bagehot (1971) that information alone is sufficient to introduce market spreads.

If some traders have superior information, then the market maker loses on average to those traders. It is this insight that Glosten and Milgrom (1985) develop in their model of market maker's pricing decision that leads to three interesting conclusions. The first is that a spread arises that is independent of any exogenous transaction or inventory costs.

The spread arises because someone wishing to buy causes the market maker to revise his expectation of the assets value upward and his quotes move accordingly; the willingness of someone to sell causes the opposite revision.


#### Abstract

A second conclusion of the model is that transaction prices form a stochastic martingale. This means that a market observer following prices cannot do better in predicting the future price than by simply using the current price. This suggests a linkage between the price behaviour in the model and the concept of market efficiency


The final conclusion is that under some conditions the adverse selection induced by asymmetric information can cause the market to collapse or shut down. If there were too many informed traders, then the market maker can have to set the spread so large as to preclude any trading at all. But since information is reflected in prices through trades, this lack of trade results in a breakdown of the market system.

The advantage of this model is its ability to characterise the bid-ask spread. By demonstrating how market parameters such as the size of the market or the fraction of large to small trades affect quotes and spreads, the model shows how asymmetric information affects market behaviour.

Another aspect of the model is that it is possible to demonstrate that prices do indeed converge to full information values. However, this actual convergence takes place only in the limit. Hence, one limitation of this model is that it provides little insight into how long this adjustment process takes.

A third aspect of this model is the actual mechanics of the sequential trading process. In the model traders form a queue and trading takes place sequentiaily. How traders arrive at the queue is problematic. A final issue relates to the ability of the model to incorporate strategic behaviour. In the model traders and market makers were assumed to behave competitively.

For uninformed traders, the lack of any coherent trading motivation is clearly an area of major weakness in the model. The question of how a single informed trader can best exploit his informational advantage to maximise his profit needs to be considered. This strategic behaviour is analysed by Kyle (1985)

His model involves a framework in which a single risk neutral informed trader and a number of uninformed liquidity traders submit orders to a risk neutral market maker. The market maker aggregates the orders and clears all trades at a single price. Kyle's (1985) model therefore does not allow for a bid-ask spread. What his model does allow is the explicit characterisation of how an informed trader would choose to transact to maximise the value of private information.

It can be possible to calculate the effect on prices of trader's orders and hence permit investigation of the effect of multiple informed traders on market behaviour. This is the approach taken by Kyle (1984). There are two sources of information, one private and the other public. The public signal is observed by all market participants whereas the private signal is known only to the informed traders.

One aspect of the results found is that they are derived in an environment of risk neutrality. In the model, all traders and the market maker are assumed to be risk neutral. This assumption greatly simplifies trader's behaviour because only mean effects need be considered. In particular if informed traders are risk averse then the total scale of trading can affect each agents decision, leading to very different effects when the number of informed traders is allowed to vary.

Another important model is by Blume and Easley (1990). Using a game theoretic approach, they demonstrate that, regardless of the number of traders, if any trader has information which he alone possesses then there is no trading game or mechanism that can result in a rational expectations equilibrium for all standard economies. The difficulty is that if a trader can be an "information monopolist" then the prices predicted by the rational expectations models are unattainable.

A significant assumption is that the information is short lived. The public information arriving at the beginning of the next period dictates that private information is valuable for only one trading interval. Consequently, informed traders have no choice but to trade on their information in the period in which they receive it. Foster and Viswanathan (1990) consider an analysis of interday trading patterns. Their analysis involves a variant of the Kyle (1985) model in which trade occurs only once a day and information is "lumpy".

Given that information is "lumpy" it follows that uninformed traders might prefer to delay their trades and transact when the terms of trade are more favourable. Foster and Viswanathan (1990) assume that there were both discretionary and non-discretionary uninformed traders. Discretionary traders were allowed to delay their trades for at most one day. They were not permitted to split trades across trading days nor can they skip trading altogether if market prices seem unreasonable.

Grossman and Stiglitz (1980) noted that, if traders act competitively, their trades can result in prices impounding so much information that, in equilibrium, the price reveals all private information to uninformed traders. In this case, the issue of price adjustment is moot; prices instantly adjust to full information values and markets are full information efficient.

In actual markets such instantaneous adjustment is rarely observed. While uninformed traders recognise that prices are related to information, it can be difficult to isolate the pure information effects on security prices from the more transitory liquidity effects. While some trades can acquire information, it is not always obvious how that information relates to the ultimate value of the firm and hence not immediately apparent how unbiased is the information. These difficulties imply that simple models of price adjustment can yield little insight into the behaviour of actual asset markets. What underlies the difficulty in characterising the price adjustment process is that price movements depend on how market participants learn from the market information they obtained, and this in turn depends on other factors such as trader's risk preferences and endowments, the nature and extent of uncertainty and even the market structure itself.

In a noisy rational framework, prices are affected both by private information and by supply uncertainty. Information affects prices because some traders are assumed to receive a private signal of the asset's true value. The signal can be the truth or it too can contain noise that interferes with agents knowing with certainty the actual value. Supply uncertainty is incorporated to capture transitory effects on price that are not related to information.

This supply uncertainty can be introduced in a number of ways, but its role is always the same: with multiple sources of uncertainty, traders cannot immediately sort out the information effects on price from the supply effects on price.

Another possible explanation for a separate mechanism is the information problem inherent in large trades. If market participants interpret trade size as a signal of information, then a large seller can prefer some other trading approach than simply submitting a large order to the market maker. McDermott (1993) observes that block trades coming from the European continent do not have the same price sensitivity as local trades. A background review of the foreign government bond market microstructures in appendix five.

Which market structure better aids the price discovery function? In the quote driven market, dealers post prices before orders are submitted. Such a system is typified by NASDAQ, and is in effect a continuous dealer market. In an order driven market, orders are submitted and then trading prices determined.

### 4.3 Microeconomic Industry Structure of Irish Treasury Market

This section reviews the revenue and cost structure of the agency system, the requirement of the NTMA in a primary dealing system and the relative economic cost of price discovery in the Irish market relative to the German bond market.

As mentioned in chapter 2 , the Irish bond market is almost totally dominated by Government issues. In terms of market capitalisation, the government fixed interest market has grown from IR£4.15bn in 1980 to almost IR£16.2bn at the beginning of 1996. In terms of the actual structure of the bond market, there is a wide range of securities available to suit the various requirements of investors; bullet bonds, callable bonds, convertible bonds and variable rate bonds. Irish Government bonds are listed and dealt on the stock exchange in Dublin. Prices are formally fixed twice a day on the stock exchange floor, although these prices could fluctuate as market conditions dictate between fixings.

The NTMA is responsible since 1990 for issuing bonds to the market and it quotes bid prices for existing bonds, but it can also quote an offer price when it wishes to sell bonds. Bond sales were normally achieved through a tap system in the past, but the NTMA has instigated changes in bond issuing procedures. In the secondary agent market, a deal takes place without the involvement of the authorities.

This involves two investors dealing through one or two brokers. In this market, one broker can deal with another broker on behalf of a client. Most of the dealing in government securities takes place without the involvement of the NTMA in what is termed the 'secondary market' and market prices are effectively observed there.

The move from the downstairs floor to a upstairs trading room took place many years ago on the Irish stock exchange with off the floor put-throughs representing $95 \%$ of total volume. O'Connor (1993) believes that poor liquidity is generally cited as one of the major structural impediments to further convergence with other European bond markets, but the size of the market and the lack of liquidity in particular bonds is a more pressing problem. The costs of immediacy include the costs of providing brokerage services, costs of providing a central place where matching of customer orders can be effected, the costs of operating a clearing house that the different parties can trade in, the fixed costs to a market maker of maintaining a presence on the exchange floor, and the costs incurred by a market maker providing a customer with immediate execution of an order by trading directly with the customer or by trading a broker representing the customer.

The historical experience of the market is investigated in terms of revenue, costs and turnover on the historical agency basis in this section. From the perspective of a cash trader, the slope of the yield curve can influence cost of carry. Also of interest are the constituent sectors of the market and how they have changed through time. These are shown in exhibit 4.1 and show a slight decline in the shorter end of the market due to falling yields and funding at longer maturities of ten to twenty five years. It is necessary to calculate these to identify the returns distribution for all maturity bands.


Exhibit 4.1 Analysis of Irish Government Treasury Market Turnover
Source: Irish Stock Exchange

McDermott (1993) analysed daily volumes of dealings and market share for 1992 in exhibit 4.2 , to sample the average commission paid per transaction paid by his client base. This sample represents $30 \%$ of the market and can be extrapolated to other market participants. The average commission paid by each client per $£ 1 \mathrm{~m}$ traded is $£ 217.28$. The market shares are shown in exhibit 4.2 and are consistent with other known sources.


[^26]Source : McDermott (1993), Irish Stock Exchange \& IAIM

The Stock Exchange provided the history of turnover on a monthly basis of Irish treasury bonds over the past decade. A linear regression model is used on the data to forecast the turnover until 1995 (exhibit 4.3).


## Exhibit 4.3 Market Turnover

Source : Irish Stock Exchange

It is very difficult to estimate the turnover by foreign institutions other than the net inflows shown in the Central Banks report. However, foreign institutions are very significant because of their relatively large holdings and the consequent liquidity that they bring to an otherwise innately small market. Assuming the stockbrokers' earn a net price 10 p commission when dealing with non-residents and they represent $c .5 \%$ of the total turnover, non-residents represent revenue of $£ 2,350,984$ in 1993. The stock brokers as financial intermediaries for government bonds on an agency basis have total revenues as shown in table 4.1 below;

| Date | Actual (£m) | Fitted (£m) | Foreign (£m) | $\%$ <br> Foreign | Domestic <br> (£m) | £1,000.00 <br> Foreign <br> Revenue | £217.28 <br> Domestic <br> Revenue | Total <br> Revenue |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1985 | 19,326 | 22,137 | 856 | $443 \%$ | 18,470 | $£ 856,109$ | $£ 4,208,876$ | $£ 5,064,985$ |
| 1986 | 25,752 | 26,004 | 1,141 | $4,43 \%$ | 24,611 | $£ 1,140,770$ | $£ 5,608,350$ | $£ 6,749,120$ |
| 1987 | 33,526 | 29,871 | 1,485 | $4,43 \%$ | 32,041 | $£ 1,485,145$ | $£ 7,301,396$ | $£ 8,786,541$ |
| 1988 | 33,625 | 33,738 | 1,490 | $443 \%$ | 32,135 | $£ 1,489,531$ | $£ 7,322,956$ | $£ 8,812,487$ |
| 1989 | 41,016 | 37,604 | 1,817 | $4,43 \%$ | 39,199 | $£ 1,816,940$ | $£ 8,932,591$ | $£ 10,749,531$ |
| 1990 | 36,097 | 41,471 | 1,599 | $4,43 \%$ | 34,498 | $£ 1,599,031$ | $£ 7,861,288$ | $£ 9,460,319$ |
| 1991 | 43,215 | 45,338 | 1,869 | $432 \%$ | 41,346 | $£ 1,868,850$ | $£ 9,411,448$ | $£ 11,280,298$ |
| 1992 | 49,811 | 49,205 | 2,259 | $454 \%$ | 47,552 | $£ 2,259,000$ | $£ 10,848,092$ | $£ 13,107,092$ |
| 1993 |  | 53,072 | 2,351 | $4,43 \%$ | 50,721 | $£ 2,350,984$ | $£ 11,558,103$ | $£ 13,909,087$ |
| 1994 |  | 56,938 | 2,522 | $4,43 \%$ | 54,416 | $£ 2,522,277$ | $£ 12,400,230$ | $£ 14,922,507$ |
| 1995 |  | 60,805 | 2,694 | $443 \%$ | 58,112 | $£ 2,693,571$ | $£ 13,242,357$ | $£ 15,935,928$ |

Table 4.1 Estimated Total Revenue on an Agency Basis
Source: McDermott (1993) \& Empirical Work

The agency commission level scale is a function of the maturity with the cost increasing with maturity. The mix between shorts (i.e. less than five years to maturity) and longs (i.e. greater than five years to maturity) for 1992 is illustrated in exhibit 4.4. The Stock Exchange is unable to provide a breakdown of such data for earlier time periods.


Exhibit 4.4 Market Turnover Split by Maturity Sector
Source : McDermott (1993) \& Irish Stock Exchange

The sample is extended to estimate the stock brokers cost base. The cost structure is divided into short run costs (i.e. less than 90 days) and medium run (i.e. greater than 90 days). The long run costs are defined as greater than one year and these consisted of a risk-adjusted return to the equity capital base. The estimated costs are shown per dealer in table 4.2:

| Cost Category | Short run | Cost Category | Medium run |
| :---: | :---: | :---: | :---: |
| Office Services | $£ 1,200$ | Training | $£ 700$ |
| Lighting \& Heating | $£ 800$ | Rent (£53 per sq.m.) | $£ 5,000$ |
| Reuters | $£ 11,000$ | Insurance | $£ 500$ |
| Telephone | $£ 3,000$ | Bloomberg | $£ 1,000$ |
| Post | $£ 2,500$ | Datastream | $£ 3,000$ |
| Salary | $£ 45,000$ | Computer maintenance | $£ 500$ |
| PRSI | $£ 8,250$ | Business entertainment \& publications | $£ 2,000$ |
|  |  | Travel, accommodation \& Car expenses | $£ 5,000$ |
|  |  | Portion of operations cost | $£ 15,000$ |
|  | Computers deprecation | $£ 3,000$ |  |
|  |  | Car lease | $£ 5,000$ |
|  |  | Bonus | $£ 15,000$ |
|  |  | VHI | $£ 1,000$ |
|  |  | Pension | $£ 6,000$ |
|  |  | Total Medium Run cost | $£ 62,700$ |

Table 4.2 Estimated Operating Costs per Dealer per year
Source : McDermott (1993) \& Farrell (1993)

The term $\mathrm{PRSI}^{6}$ in table 4.3 refers to social insurance and the term VHI refers to voluntary health insurance. From tables 4.2 and 4.3 , the stockbrokers would be expected to have the following profit and loss accounts shown in table 4.4.

[^27]| Broker | Commission <br> Market Share | Revenue | Number of <br> Dealers | Short Run <br> Costs | Medium Run <br> Costs | Total Variable <br> Costs |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | $1.20 \%$ | $£ 157,285$ | 1 | $£ 71,750$ | $£ 62,700$ | $£ 134.450$ |
| B | $1.40 \%$ | $£ 183,499$ | 1 | $£ 71,750$ | $£ 62,700$ | $£ 134,450$ |
| C | $3.10 \%$ | $£ 406,320$ | 1 | $£ 71,750$ | $£ 62,700$ | $£ 134,450$ |
| A | $1.14 \%$ | $£ 149.421$ | 1 | $£ 71,750$ | $£ 62,700$ | $£ 134,450$ |
| D | $28.00 \%$ | $£ 3,669,986$ | 9 | $£ 645,750$ | $£ 564,300$ | $£ 1,210,050$ |
| E | $21.30 \%$ | $£ 2,791,811$ | 6 | $£ 430,500$ | $£ 376,200$ | $£ 806,700$ |
| F | $2.10 \%$ | $£ 275,249$ | 2 | $£ 143,500$ | $£ 125,400$ | $£ 268,900$ |
| G | $22.76 \%$ | $£ 2,982,555$ | 7 | $£ 502,250$ | $£ 438,900$ | $£ 941,150$ |
| I | $19.00 \%$ | $£ 2,490,347$ | 5 | $£ 358,750$ | $£ 313,500$ | $£ 672,250$ |
| J | N/A | N/A | 1 | $£ 71,750$ | $£ 62,700$ | $£ 134,450$ |
| Total | $100.00 \%$ | $£ 13,107,092$ | 34 | $£ 2,439,500$ | $£ 2,131,800$ | $£ 4,571,300$ |

Table 4.3 Estimated Costs of Agency Market Structure
Source : McDermott (1993) \& Empirical Work

In the long run, capital is employed to allow a broker to cover a quarter of his annual working capital requirement and the cost of default by a counterparty in a deal matched. The required return on capital employed is $14 \%$ derived from the capital asset pricing model using the money market, Murray's (1993) estimate of beta coefficients and the Riada total return on Irish equities from 1988 to 1993 . The difference is "excess return" which can represent excess reserve profits to the brokerage community.

| Broker | Profit Before <br> Tax | Tax @ 40\% | Profit After Tax | Capital <br> Employed | Return on <br> Capital | Excess <br> Return | Excess <br> Reserve Profits |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | $£ 22,835$ | $£ 9,134$ | $£ 13,701$ | $£ 33,613$ | $40.76 \%$ | $26.26 \%$ | $£ 8,827$ |
| B | $£ 49,049$ | $£ 19,620$ | $£ 29,429$ | $£ 33,613$ | $87.55 \%$ | $73.05 \%$ | $£ 24,556$ |
| C | $£ 271,870$ | $£ 108,748$ | $£ 163,122$ | $£ 33,613$ | $485.30 \%$ | $470.80 \%$ | $£ 158,248$ |
| A | $£ 14,971$ | $£ 5,988$ | $£ 8,983$ | $£ 33,613$ | $26.72 \%$ | $12.22 \%$ | $£ 4,109$ |
| D | $£ 2,459,936$ | $£ 983,974$ | $£ 1,475,962$ | $£ 302,513$ | $487.90 \%$ | $473.40 \%$ | $£ 1,432,097$ |
| E | $£ 1,985,111$ | $£ 794,044$ | $£ 1,191,067$ | $£ 201,675$ | $590.59 \%$ | $576.09 \%$ | $£ 1.161 .824$ |
| F | $£ 6,349$ | $£ 2,540$ | $£ 3,809$ | $£ 67,225$ | $5.67 \%$ | $0.00 \%$ | $£ 0$ |
| G | $£ 2,041,405$ | $£ 816,562$ | $£ 1,224,843$ | $£ 235,288$ | $520.57 \%$ | $506.07 \%$ | $£ 1.190 .726$ |
| I | $£ 1,818,097$ | $£ 727,239$ | $£ 1,090.858$ | $£ 168,063$ | $649.08 \%$ | $634.58 \%$ | $£ 1.066 .489$ |
| $J$ | $\mathrm{~N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ |
| Total | $£ 8,669,623$ | $£ 3,467,849$ | $£ 5,201,774$ | $£ 1,109.213$ |  |  | $£ 5,046,876$ |

Table 4.4 Profit \& Loss of Agency Market Structure
Source : McDermott (1993) \& Empirical Work

For the final analysis commission breakdown is represent in the following pie chart;


Exhibit 4.5 Commission Breakdown
Source : Empirical

McDermott (1993) observed that the market share division between foreigners and domestic institutions is probably too small at $5 \%$ versus $95 \%$ respectively, and found $25 \%$ of his trades were with foreigners. In addition, he pointed out that REPO's (Sell today and RePurchase of Government securities for a later date) distorted the overall turnover figures by $15 \%$ to $20 \%$.

Farrell (1993) has held the view that Irish term structure has been significantly influenced by the German term structure. On an empirical analysis, ten-year yields were found to have a correlation of between $65 \%$ to $90 \%$ from 1988 to 1992 on an annual basis. The Irish yield is assumed to consist of a European element which is proxied by the German Bund and a local element which is proxied by the spread in yield over the Bund then commission can be divided between the bund yield and the spread over the bunds. The correlation between Ireland and Germany is $75.35 \%$ between June 1988 and the start of 1994 .

In order to contrast the relative economic cost of price discovery in the Irish market relative to the German bond market, the cost of dealing in the ten year bond maturities of both markets is contrasted. A IR£1,000,000 Irish exposure equates to DM 2,495,000. A Bund 10-year contract is DM 250,000 so that 9.98 contracts is needed to achieve a $\operatorname{IR} £ 1,000,000$ exposure. It costs DM 18 to DM 20 to "round trip" (i.e. purchase and sale of contracts) which directly implies a total cost of DM 212 or IR£85. It would cost IR£360 to deal $£ 1,000,000$ of the Irish 10 -year bond, but under the 28 day rule where the trade would be closed out with the same broker within this time period, commission is only charged on one side of the transaction. In the brokers sample, it is found that $50 \%$ of deals have 'closing' so the cost would fall to IR£ 270 on average. The present yield on the 10 year Bund is $5.81 \%$ and the Irish 10 years is $6.29 \%$. This suggests that investors were paying $\operatorname{IR£185~for~the~} 48$ basis point spread. This means that $7.63 \%$ of the exposure is $68.52 \%$ of the cost and is illustrated in exhibit 4.6.


Exhibit 4.6 Components of Return by Cost of Exposure
Source: Empirical

The market was dominated by two firms up the end of the 1970's when there was a rapid increase in government borrowing and the break with Sterling in 1979 followed by membership of the ERM.

On the 18th March 1994, the Competition Authority ruled that the trading of Government gilts operates in an anti-competitive manner. According to Taylor (1994), "the Competition Authority criticises two key elements of the operation of the market for Government gilts ... the rule stipulating that all brokers charge investors the same fees for their dealing service is anticompetitive, because it stops price competition. Secondly, the current rule stipulating that stockbrokers should only act as agents for clients and must not buy or sell gilts on their own behalf also distorts competition".

Corrigan (1993) recognises that stockbrokers play a highly effective role in marketing and distributing bonds to a wide international base: in that respect their role was privileged in that the Agency offers and bids for bonds virtually exclusively through the Stock Exchange. The system of distribution works well as reflected in the exceptionally high non-resident institutional presence in the market.

However, liquidity on the Irish market is poor and is a cause of complaint to the NTMA. The "agency only" trading mechanism, which dates from 1799, gives rise to the following serious problems in the Irish market; firstly, lack of depth because of the absence of a natural pool of price makers means that the price discovery function is inefficient. With greater depth, the market would be relatively more stable.

Secondly, lack of 'immediacy' is a potential cost to the investor if the execution of a trade is delayed. In an agency only market, as exists in Ireland, where brokers match out buyers and sellers, immediacy is not usually provided by stockbrokers who operate to bring buyers and sellers together on the Stock Exchange. Under the existing system, there can be considerable delay in matching orders. If the market is more liquid then a higher degree of immediacy would be possible.

Thirdly, dealing costs: the bond market in Ireland was almost unique among OECD countries in operating until very recently on the basis of fixed minimum commissions with the exception being Greece. High dealing costs discourage active trading and consequently impair liquidity.

Fourthly, derivatives markets based on the Government bond market were seriously underdeveloped. This was largely due to the inefficiencies of price discovery in the underlying cash market and the lack of immediacy in that market.

The problems in the market are attributable to the fact that it does not have a natural pool of price makers. In seeking to promote a natural pool of price makers there are essentially two alternatives. The NTMA proposes a market making system consisting of primary dealers whom would be formally recognised as such by the NTMA. These dealers would commit themselves continuously to make two way prices in all market conditions.

The number of firms that might decide to become market makers was determined by the costs and benefits of being a market maker. As the number of market makers increases, the risk borne by each falls; but so does their expected return. On balance, the NTMA believes that the interests of the market generally would be served by 5-7 market makers on an on-going basis.

### 4.4 Risk Capital

In this section, the European Union Capital Adequacy directive is considered to determine whether it is sufficient for a small market like Ireland and a worked example is shown in appendix four. There are three types of capital requirements by a market maker:

Type 1 - Sector positioning (with intra sector hedging allowed).
Type 2 - Intra Sector positioning ( core sector non diversifable risk).
Type 3 - Inter Sector positioning ( hedging within a sector).

The first two types are combined to give the risk due to any potential change in the level and shape of the yield curve. They are distinguished in that if an equal but opposite nominal position was held in another sector of the yield curve, the type 1 risk capitals would be offsettable. The type 2 capital must always be carried irrespective of any other position in the portfolio. This core capital was required because of the different volatilities in each sector and the possibility of a non-parallel movement in the yield curve. In the case of type 3 , this is for when a position in a sector was hedged within the same sector.

A market maker needs capital because he cannot make a profit on every trade. On an inter day basis his trades can profit or lose, but these can be marked to market and settled the following day. On an overnight basis, the regulator calculates the marked to market value of his holdings, present capital and this should be within prescribed limits. If not there can be an immediate requirement of an equity share capital infusion and/or a reduction of the position. These results can be ready by the next trading session.

Any position can have two elements of capital backing it, debt and equity. In the case of debt capital, this can have a financing cost. The debt capital can be financed using a repurchase agreement (i.e. REPO) which was an agreement between a seller and buyer of government treasury's whereby the seller agrees to repurchase the government treasury's at an agreed price at some stated time in the future. The market maker borrows from an investor to finance his inventory using the securities as collateral. The development of a REPO and Reverse-REPO market for the market maker can directly influence the cost of funding and was undertaken by the NTMA which reached an agreement with the Revenue Commissioners on the tax treatment of REPOs, advanced a variation of the ISDA Master Legal agreement between all the principal market participants and offered a facility to Primary Dealers if the market agreed to change its microstructure.

If the market structure were changed, one area of interest is the amount of liquidity that capital invested in these market makers would generate. This was a difficult question to answer because the same amount of capital would allow a far smaller position to be held on an outright basis in the 2012's than if an equal (i.e. intra maturity) but opposite position was held in the 2010's.

In exhibit 4.7 the turnover for 1993 was $£ 36,465 \mathrm{~m}$ and was composed of shorts (i.e. 0 to 5 years) $£ 15,775 \mathrm{~m}$, mediums (i.e. 5 to 10 years) $£ 12,679 \mathrm{~m}$ and longs (i.e. $10+$ years) of $£ 8,011 \mathrm{~m}$. Overall, the Irish government bond market has turned over 2.52 times, 2.18 times in the shorts, 4.00 times in the mediums and 1.98 times in the tongs in 1994 and the seasonal variation in size and composition can be observed.


Exhibit 4.7 Average Daily Turnover
Source : McDermott (1993) \& Irish Stock Exchange

Each month is investigated to establish the capital required to facilitate 1993's turnover. Each month is assumed to have 30 days. Four market makers were hypothesised and $99 \%$ confidence is required that the market has adequate capacity. The shorts require capacity of $£ 76 \mathrm{~m}$ that needs $£ 2.1 \mathrm{~m}$ capital.

Then the mediums require capacity of $£ 72 \mathrm{~m}$ that needs $£ 2.7 \mathrm{~m}$ capital. Finally, the longs require capacity of $£ 50 \mathrm{~m}$ that needs $£ 3 \mathrm{~m}$ capital. The total for the market makers was $£ 7.8 \mathrm{~m}$. Market Makers would be expected to use $65 \%$ of their capital at any particular time. With market turnover growing at $15 \%$ compounded over the last eight years, then with an expected turnover in two year's time of $£ 48 \mathrm{bn}$ from 1995 , this implies that $£ 10.4 \mathrm{~m}$ would be needed for the market as a whole or $£ 2.6 \mathrm{~m}$ per individual Market Makers ${ }^{7}$.

In the long run, there would have to be an adequate return on capital employed. Stockbrokers until recently have been taxed as a partnership, which at high marginal tax rates would constitute a disincentive to retain the money in the business. Murray (1993) has estimated beta coefficients in table 4.5 for Irish financial companies.

| Stock | Raw | Method 1 | Method 2 | Method 3 | Methad 4 | Bayesian |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AlB | 0.840 | 0.770 | 0.780 | 1.150 | 1.150 | 0.810 |
| Bank of Ireland | 0.860 | 1.020 | 1.000 | 1.020 | 0.790 | 0.820 |
| Woodchester | -0.300 | 0.100 | 0.900 | 0.000 | 0.110 | 0.200 |
| Anglo-irish Bank | 0.500 | 0.200 | 0.600 | 0.800 | 0.500 | 0.500 |

Table 4.5 Betas of Irish Banks allowing for thinness of Equity Market
Source : Murray (1993)

Direct costs were estimated at $£ 4,878,400$ or 1.34 basis points per $£ 1 \mathrm{~m}$ traded. There were two sources of capital costs, borrowing from the money market at the overnight rate and the market makers own capital. The overnight rate is exhibited in 4.8 below;

[^28]

Exhibit 4.8 DIBOR Money Market as proxy for REPO Market
Source: Datastream

There were a number of approaches that can be taken to calculate the required return on the market makers capital. McDermott (1993) and O'Connor (1993) said that banks require a hurdle rate of return of $20 \%$ on the use of a bank's own capital. From Murray (1993), this implies that trading in government bonds is 16 times riskier than the rest of the banks activities over the past seven years of market returns. This hurdle rate of return should be provided by the return required by the shareholders using the Capital Asset Pricing Model (CAPM);

$$
\begin{equation*}
Y_{\text {Market Maker }}=r_{\text {Risk free }}+\beta_{\text {Market Making }}\left(r_{\text {EquityMarket }}-r_{\text {Risk free }}\right) \tag{4.5.3.1}
\end{equation*}
$$

### 4.5 Simulation Model of Primary Dealer

The motivation for this model of dealer profitability was the desire to answer a pragmatic question that was prevalent at the time of organisational revisions to the market structure in Ireland. This question was: Could the Irish market support a system of six primary dealers, rather than the previous agency arrangement, in the absence of an Irish futures market? In the modified Garman (1976) model of a primary dealer, the following assumptions are made: market makers would need to dedicate IR£30 million in total to the operation, or IR£5 million on average per market maker, each primary dealer is a price-setter who would be required to quote continuously in a range of bonds in a minimum size and within a maximum spread. and in all market conditions.

The purpose of the model is limited to an investigation of dealer profitability and the probability of ruin; it is unconcerned with the strategic behaviour of firms. The bid-offer spreads are in consequence fixed and the dealer firm is in effect a privileged price taker at bid or offer in a market where the mid price is an exogenous variable. The model does not consider any possible growth or decline in trading activity arising from strategic interactions; it does make the assumption that turnover within the Irish Government market would rise to the European Union average, an increase of $25 \%$ in the first year over that previously observed. (In retrospect this assumption proved conservative as an increase of $40 \%$ was observed - the causation of this increase is of course complex.)

In order to model dealer profitability, it was necessary to gather data on the level of market activity by number, timing, value, bond maturity band, price and volatility, as well as the proportion of trades closed out or covered within a calendar trading day. No record was available of dealer orders received but unexecuted. In the preliminary investigation of this data, it was evident that there was a particular form of large order, or block trade, which was sourced from continental Europe.

The practice in the market at the time was to work these orders on a best efforts basis and the primary dealer candidates indicated that this practice was likely to continue: however, in the model these were executed upon arrival and subsequently traded down by the securities dealer. These large or block trades were incorporated into the model by using a mixture of distributions, usual and large.

Notwithstanding this, the assumption within the model remains that dealer inventories are zero at close of business and that price is independent of inventory. This premise is supported by McDermott (1995), who observes that $85 \%$ of positions are closed within a day. The market practices at that time were in fact a mixture of agency and principal practices, a form of dual capacity. Dealers were observed to use foreign futures contracts, such as the LIFFE Bund, or Danish cash government bonds to hedge these positions based upon correlation estimation and assumptions.

The model seeks to identify the likelihood of ruin of a dealer and under the assumption that ruin is an independent process defines failure as the number of remaining primary dealers falling below four. Ruin is defined as the loss of all capital and retained profits within a trading year. NTMA regulations required a dealer to hold capital of $£ 5$ million and report their position quarterly.

There was a concern prevalent within the market that dealer strategies could include market domination. The specific rute to ensure active competition was to limit dealer capital invested to a maximum of $£ 8$ million. This constraint was replicated within the model by an upper bound of $£ 8$ million capital and retained profits above which funds were no longer available for position taking.

In order to describe this model mathematically, let;
ir $K_{t}$ - Total system capital at time $t$,
> sp-bid/ask price spread,
$>m-13$ bond buckets (corresponding to the EU Capital Adequacy Directive's maturity and coupon criteria),

- $x_{m}$ - minimum size for maturity bucket $m$,
r $d t$ - discrete time step,
خ $\quad \lambda$-Poisson arrival process - set at twenty minutes.
> $Q$ - empirical order size usual distribution,
$>J$ - empirical distribution of large order sizes,
$>\sigma_{m}$-standard deviation of intra-day bond prices within bucket $m$,
$>b_{m}$ - bond in bucket $m$,
$\Rightarrow I_{t}$ - inventory position at time t ,
$\geqslant \quad \mathbf{F}_{\mathrm{t}}$ - capital and retained profits at time t of firm i ,
- $r$-bond price correlation matrix,
; $n$ - number of dealers,

Ruin is therefore defined as:
(4.5.1) $\quad \mathbf{F}_{\mathbf{t}}=\mathbf{0}$
and total systemic capital as:
(4.5.2) $\quad \mathbf{K}_{\mathrm{t}} \equiv \sum_{\mathrm{i}=0}^{6}{ }_{\mathrm{i}} \mathbf{F}_{\mathrm{t}}$
where the maximum number of dealers is six. Market failure is defined as:

```
n<4
```

The objective of the model is to identify and quantify the probability of ruin of an individual dealer:
(4.5.2) $\quad \operatorname{PROB}\left(, F_{i} \leq 0\right)$

At the start of the trading year, there are six dealers each with an initial endowment of £5 million capital. Dealers are required to quote and deal at either bid or ask prices continuously and under all market conditions. Market practice was to quote and deal in round lots of $£ 2$ million in maturities of five years or less, and $£ 1$ million in longer maturities. The empirical analysis of actual trade data showed larger averages than these (see sections 4.3 and 4.4). The simulation used random numbers to generate a sample from the empirical distribution of trade sizes. The distribution was lower truncated at a $£ 2$ million order size. As the empirical distribution contained a proportion of large "outlier" trades, it was necessary to build a mixture of distributions to accommodate the usual order size distribution and the infrequent but large trades. The way in which this was achieved was by a randomly generated choice between the usual and large distributions, constrained to satisfy the historic distribution.

Time between trades was empirically observed to be 20 minutes, and accordingly the Poisson arrival process of orders within the model was calibrated to a 20 minute lambda. The average order size is taken from samples provided by McDermott (1995), the official publications of the Irish Stock Exchange and the internal records of the NTMA.

The buy/sell characteristic of a trade was determined by drawings from a Bernoulli distribution. The effect of unusual runs of successive sequences of purchases or sales, or market trends, was also investigated. This appeared to have little effect on profitability. In other words, in this model the effect of the spread earned by transaction execution, which was set at one half of the bid-offer spread, dominates the effects of market movements on open positions.

The market trending, bullish, bearish or stable, was limited to $15 \%$ of the trades executed by a dealer within the day. This end of day open position is consistent with the McDermott (1995) findings. Market share is the combination of order arrival and order (or trade) size processes. A base case of a $20 \%$ market share for a dealer was investigated. Sensitivity to market share was also investigated (see section 4.5.4).

Schematically the model can be represented as:


Exhibit 4.9 - Primary Dealer Simulation Model
Source: Empirical

The empirical data of market turnover was divided according to the following broad bands

```
>0 to 1 year
-1 to 5 years
> to 10 years
>10+ years
```

Obviously, within these bands there is a considerable variation of price sensitivity by security. The turnover within a broad band was further differentiated on the basis of market capitalisation. This differentiated turnover was then reallocated according to the risk sensitivity buckets as defined by the European Union's Capital Adequacy Directive. For the model simulation, the overall empirical distribution of transactions between these buckets was used to determine which security dealt.

The relationship between the security dealt, or bucket, and the price change was constrained by the correlation matrix between buckets and the probability distributions of price changes for each bucket. These distributions were assumed to be log-normally distributed. Opening values for all securities (buckets) were assumed to be $100 \%$. For maturity buckets 1 to 8, the minimum trading size was greater than or equal to $£ 2$ million and for all other maturities was greater than or equal to $£ 1$ million.

As in the Garman (1976) model all exchanges are made through one of the central market makers, which possesses a monopoly on all trading. No direct exchanges between buyers and sellers are permitted.

For

$$
\begin{gather*}
\text { For } m=1,8 \quad x_{m} \geq 2  \tag{4.5.3.}\\
m=9,13 \\
x_{m} \geq 1
\end{gather*}
$$

The dealing spread associated with a particular security or maturity bucket is


This dealing spread is the principal source of profit to a dealer. The profit or loss of a dealer is the sum of half the bid/offer spreads and the marked to market movement of open positions or inventory. This latter arises from the stochastic process governing term structure movements.

The model was constructed so that any transaction and price change would be reflected across the term structure. Thus any open positions in inventory would be revalued, or marked to market, during the trading day. Where a position was closed by a transaction the appropriate profit or loss was posted. At end of day the positions were marked to mid-market.

No transaction costs were included. Gross rather than net positions were used in the capital utilisation constraint. The dealer was allowed to leverage his capital up to the limits set down in the European Union Capital Adequacy Directive. In market practice, the NTMA operated a policy known as the "trailing market bid" which allowed dealers to close positions at end of day. The model had end of day mid market closure.

### 4.5.1 Identification of Primary Dealers Daily Profit Density Distribution

The main variables estimated are turnover, closing position, profit/loss, utilised capital, return on total capital (R.O.T.C.) and return on utilised capital (R.O.U.C.). Using a Latin Hypercube method, 10,000 Monte-Carlo simulations ${ }^{8}$ appeared to deliver convergence and stability of the parameters of the daily profit distribution.

| Minimum | $-£ 2,390,480$ |
| :---: | :---: |
| Maximum | $£ 1,567,592$ |
| Mean | $£ 34,481$ |
| Std Deviation | $£ 302,373$ |
| Skewness | -0.5563 |
| Kurtosis | 10.4983 |

Table 4.6 Moments of Profit Probability Density Function of the one day Monte-Carlo simulation

The mean daily turnover under this simulation was $£ 54.271$ million. The average daily profit is $£ 34,481$ and the risk or standard deviation is $£ 302,373$ that is a risk/reward ratio of 8.77 to 1. The utilised capital had a mean average of $£ 1.201$ million with a maximum of $£ 2.952$ million. It is interesting to note that the dealer was not bindingly constrained by Capital Adequacy even in the extremities of the simulation. The diagram in exhibit 4.9 illustrates the histogram of simulation results and a fitted normal distribution.

[^29]

Exhibit 4.10-Comparison of Input Distribution and Normal( $£ 34,500, £ 302,000)$
Source: Empirical

In order to estimate the ruin boundary, it is necessary to fit a continuous distribution to the histogram. The principle concern is accuracy of the tail estimation, the likely location of the ruin boundary.

The assumption of the model is that $20 \%$ of total exchanges are made through each market maker. The objective of the market maker is to earn his expected average profit per trade by earning the half the spread at the time of the trade and the other half of the spread when the opposite trade takes place or at the close of business. There can also be a trading profit or loss generated by the stochastic movement in security prices. The model assumed that the term structure changed in a continuous stochastic process with respect to time. There is no capital allowance made for offsetting hedges either within or across different maturity sectors.

If the first four moments (i.e. mean, standard deviation, skewness and kurtosis) of the profit function are fitted and compared to the equivalent moments of all possible profit probability density functions in table 4.6, the normal distribution with a mean of $£ 34,500$ and a standard deviation of $£ 302,000$ is the second closest fitting profit probability density function. The normal distribution is chosen over the logistic probability density function because of its over-malableness to fit any set of data. This is illustrated in exhibit 4.9.

The normal probability density function is chosen as the most appropriate profit probability density function by taking input data and converting to a density distribution. A first estimate of parameters is made using maximum-likelihood estimators from table 4.7. For example, the maximum-likelihood estimators of the normal function are $\mu$ equals mean and $\sigma$ equals standard deviation. Therefore, the mean and standard deviation of the input is used to form a first estimate of parameters.

The fit is optimised using the Levenberg-Marquardt method. This is an iterative nonlinear least-squares routine that minimises the chi-square goodness of fit statistic. The goodness-of-fit is measured for the optimised function and all functions are compared. (see table below) This optimisation requires an initial estimate of all parameters and it uses those generated by the maximum-likelihood estimators for each distribution. The values of the parameters are then varied in an attempt to minimise chi-square.

The Levenberg-Marquardt method does not find the absolute minimum for chi-square; rather, it finds a local minimum. The success of this method depends on the initial parameters used. The process of calculating maximum-likelihood estimators and optimising the chi-square value gives the best estimate for each distribution. Then each distribution function is ranked according to its chi-square value.

While the function with the lowest chi-square could have been chosen, two other measures of goodness-of-fit are calculated for the fitted distribution, the Kolmogorov-Smirnov statistic and the Anderson-Darling statistic. In certain cases, the best-fitting distributions selected by those tests can be different than those selected by the chi-square test because of the behaviour in the tails of the distribution an shown in table 4.7.

| Function | Chi-Square | Rank | K-s Test | Rank | A-D Test |
| :---: | :--- | :--- | :--- | :--- | :--- |
| Logistic (34500,166000) | 331.84 | 1 | 0.107 | 1 | 32.46 |
| Normal(34500,302000) | $7.96 \mathrm{E}+08$ | 12 | 0.127 | 3 | 48.40 |
| 3960000 Beta(24.26,15.3)-2390000 | $7.53 \mathrm{E}+16$ | 14 | 0.121 | 2 | 51.11 |
| PearsonVI(64.68,9.20e+3,3.45e+8)-2.39e+6 | $1.00 \mathrm{E}+34$ | 16 | 0.140 | 4 | 51.14 |
| ErrorFunction(0.00000234) | $3.46 \mathrm{E}+08$ | 11 | 0.170 | 9 | 60.08 |
| InverseGaussian(2.42e+6,1.41e+8)-2.39e+6 | $1.00 \mathrm{E}+34$ | 21 | 0.155 | 6 | 60.23 |
| Lognormal(2430000,323000)-2390000 | $1.00 \mathrm{E}+34$ | 20 | 0.153 | 5 | 60.71 |
| PearsonV(51.94,1.24e+8)-2.39e+6 | $1.00 \mathrm{E}+34$ | 17 | 0.167 | 8 | 71.46 |
| Lognormal2(14.71,0.14)-2390000 | $1.00 \mathrm{E}+34$ | 19 | 0.165 | 7 | 74.15 |
| ExtremeValue(-1.02e+5,2.36e+5) | $1.00 \mathrm{E}+34$ | 23 | 0.174 | 10 | 83.71 |
| Weibull(5.88,2510000)-2390000 | 846.69 | 2 | 0.246 | 11 | 138.30 |

Table 4.7 - Results of Fitting Different Function to Profit Data
Source: Empirical

The first step in interpreting the results is to consider the significance of the chi-square value; namely, how well the input data fit a certain distribution function. A lower chi-square value indicates a better fit. The quality of the results depends on the first estimate applied to the maximum-likelihood estimators because the Levenberg-Marquardt method does not find the absolute minimum for chi-square; rather, it finds a local minimum.

Since the profit density function is considered to be a continuous distribution, the fitted distribution is ranked by the Kolmogorov-Smirnov statistic and the Anderson-Darling statistic instead of by the chi-square value. As these tests (K-S \& A-D) compare the empirical distribution to the hypothesised distribution, they may be more powerful for some types of distribution. In exhibit 4.9 the comparison graph displays a good visual fit in areas that are important. This difference graph does display an acceptable magnitude of absolute error when visually inspected.

### 4.5.2 Annual Profit Distribution \& Ruin Barrier

Assuming that these probability distributions are independently distributed and that there are 252 trading days in a year, the simulation is run over a one year time span and these parameters are calculated on an annual basis. The probability of ruin of a primary dealers is estimated. With the initial capital of $£ 5 \mathrm{~m}$ and the daily profit probability density function, the sample paths evolution over one trading year is simulated. This allows the estimation of the probability of the primary dealer hitting the ruin barrier and exhausting all his capital. This is illustrated in 4.10.


Exhibit 4.11-Simulation of Primary Dealer over one Trading Year
Source : Empirical

The probability of the catastrophic ${ }^{9}$ event that primary dealing breaches the ruin barrier for a time horizon of one year is $0.28 \%$. On the other side a primary dealer could be worth $£ 30.446 \mathrm{~m}$ at the end of the year with a probability of $0.001 \%$. The original capital is exposed to a $4.67 \%$ risk of not being intact at year-end and there is a $12.29 \%$ risk that all costs including the return on capital cannot be covered. In table 4.9 the market structure as a whole and the probability that a number of primary dealers can not cover their costs is considered. There is a $2.5 \%$ risk that 3 primary dealers can fail to cover costs in a particular year. The NTMA said that it would consider the primary dealing system to have failed if the number of primary dealers fell below four.

[^30]It is assumed that the primary dealers would withdraw if they failed to cover other costs over a trading year, then the risk of the primary dealing system failing is $2.8 \%$ as shown in table 4.9.

| Primary | Loss Making |
| :---: | :---: |
| Dealer |  |
| 0 | $45.5 \%$ |
| 1 | $38.3 \%$ |
| 2 | $13.4 \%$ |
| 3 | $2.5 \%$ |
| 4 | $0.3 \%$ |
| 5 | $0.0 \%$ |
| 6 | $0.0 \%$ |

Table 4.9 Probability of different number of Primary Dealers Failing Source : Empirical

### 4.5.3. Risk/Reward Framework

This risk/reward ratio is useful for setting a probability framework of required success by the primary dealers. There is a $0.2 \%$ probability that he can make $£ 1.6 \mathrm{~m}$ or lose $£ 2.4 \mathrm{~m}$ on a single intra-daily basis. Apart from knowing the overall daily profit probability density function, he must be able to access the likelihood of making a profit with each individual client as shown in table 4.8 .

The objective of such market research is to see if they merit an increased or decreased allocation of relationship development resources. Some client's business objectives are so close to that of the primary dealer that they are quasi competitors.

The objective of such market research is to see if they merit an increased or decreased allocation of relationship development resources. Some client's business objectives are so close to that of the primary deaier that they are quasi competitors.

| $0 \%$ | $1000+$ to 1 | Not Possible |
| :--- | :--- | :--- |
| $10 \%$ | 9 to 1 | Highly Unlikely |
| $20 \%$ | 4 to 1 | Very Unlikely |
| $30 \%$ | 7 to 3 | Unlikely |
| $40 \%$ | 3 to 2 | Marginally Unlikely |
| $50 \%$ | 1 to 1 | Evenly Balanced |
| $60 \%$ | 2 to 3 | Marginally Probable |
| $70 \%$ | 4 to 7 | Probable |
| $80 \%$ | 1 to 4 | Very Probable |
| $90 \%$ | 1 to 9 | Highly Probable |
| $100 \%$ | 1 to $1000+$ | Certain |

Table 4.8 - Probability of Profitable Trade with Client
Source : Empirical
In terms of the overall daily profitability function, the probability of making a loss is $38.46 \%$

### 4.5.4 Profit Distribution Parameter Sensitivity

The model is simulated over a range of values of each parameter and the required adjustment in the spreads is identified in order to break even. Market share is simulated using $0.5 \%, 5 \%, 10 \%$ and $30 \%$. The optimal market share is seen to be just below $30 \%$. The primary dealer can find it very hard to make a profit if his market share drops below $5 \%$. Funding costs are simulated between $4 \%$ and $8 \%$.

There does not however seem to be a significant impact on the primary dealers profitability as the funding costs increase. Market trend does not seem to have a major impact on profitability. Longer trading hours can bring greater profitability to the primary dealers, but the level of capital risk involved can increase. Spread changes cannot effect the primary dealers until the spread drops below $25 \%$ of original levels.

Changes in market volume can increase the level of profitability of the primary dealers but the probability of a potential loss also increases. In return for these obligations of primary dealers, the dealers would obtain certain privileges. Competitive auctions can be open to bids from primary dealers, agency brokers and retail institutional investors with a non-competitive facility in relation to the retail market share of a Primary Dealer after each auction.

From time to time a primary dealer could have difficulty in obtaining bonds to enable him to cover a short position. The NTMA would, at its discretion, facilitate the dealer either by offering a reverse-REPO under which the NTMA would lend the dealer the bonds for a limited period.

The NTMA can also maintain continuous firm bids in IR£5 million size in each of the benchmark bonds designated by the NTMA for ongoing funding purposes. This bid can be confined to market makers only. Market makers can from time to time seek to improve the balance of their book by switching bonds. The NTMA would be prepared to facilitate such switches. They would have exclusive access to tap issuance and inter dealer broker.

### 4.6 Summary and Conclusions

The development of the literature in terms of bid/ask spread and inventory management has been discussed. The microstructure of the existing agency system has been investigated. The conclusion is that there are excess reserve profits earned above those required that have not been eroded by movements in the labour market or entry of new firms. Since Ireland has a small government bond market relative to its European peers with a small pool of investors, it has to import liquidity from foreign investors to facilitate adequate turnover.

The decision by the Competition Authority reflected their shared belief with the NTMA that the system lacked competivness in the context of European bond markets and European law. The capital requirement in a primary dealing structure would be $£ 25 \mathrm{~m}$ to have the capacity for turnover that would be required to compete with other European markets.

A market maker would have to capture a mean spread of 5 pence per $£ 100$ nominal to stay in business in the long run. The normal distribution with a mean of $£ 34,500$ and a standard deviation of $£ 302,000$ is the most appropriate distribution for modelling a Primary Dealer daily profit distribution. On a daily basis, the probability of making a loss is $38.46 \%$ and this means that a primary dealer is between marginally unlikely and unlikely to lose money in a particular day. The NTMA said that it would consider the primary dealing system to have failed if the number of primary dealers fell below four. By assuming that the primary dealers would withdraw if they failed to cover the costs over a trading year, then the risk of the primary dealing system failing is $2.8 \%$. The important profit distribution parameters are market share, earned spreads and volatility of the term structure. Having established the viability of a primary dealer market microstructure, the NTMA implemented the system and it survived for three years. At the end of the period two of the Primary Dealers exited and are replaced by two new entrants. In the next chapter the market will be examined from the perspective of an end user whose natural matching portfolio is to hold Irish government bonds, i.e., general insurance sector.

Chapter 5

The Impact of Contractual Liabilities on Investment Performance: The Case of Irish General Insurance Companies

### 5.1 Introduction

The objective of this chapter is to analyse the investment performance of the Irish general insurance market' whose matching portfolio is Irish government bonds. The general insurance sector is investigated and its interaction with the Irish bond market is examined because; (a) it represents $10 \%$ of the bond market and its liabilities can currently only be matched with Irish government bond assets, (b) it is very heavily dependent on investment performance, (c) there is a history of difficulties in this sector associated with spiralling underwriting losses. It is not possible for an insurer to achieve an immunised portfolio by increasing duration beyond that which is currently available from any fixed income bond.

To do this analysis, a framework is developed in which managers attempt to maximise the value of the funds under management, subject to a minimum terminal value. The minimum terminal value is determined as the sum of the products of their projected liabilities and the estimates of the term structure over an eighteen-year period. When the duration of the matching liability porffolio has been identified, the performance of the companies under such a strategy is compared with their actual achievements. The performance is found to be highly varied and important implications for the insurance industry of over reliance on investment performance to subsidise underwriting losses can be drawn.

This chapter is divided up as follows: section two demonstrates that interest rate swaps can increase duration to match long term liabilities; section three defines the concept of mismatch reserves; section four reviews the historical liability profile of the industry; section five investigates the investment performance by the industry from mismatching from the matching portfolio of Irish government bonds and the summary and conclusions are in section six.

The implied spot rate on such a portfolio is defined as the insolvency risk free rate of return ${ }^{2}$ which is different to the classical definition of risk free rate (i.e. the one period return of default risk free government paper). Even the prospect of being free of insolvency is only true for small changes in yields and continuous rebalancing of the portfolio. The second problem investigated is the approach taken to identifying the size of the mismatch reserve ${ }^{3}$ and the contribution from historical mismatching for the industry

### 5.2 Interest Rate Swaps \& Duration

In this section, an investigation of how an insurer can match his liabilities for long tailed insurance whose duration ${ }^{4}$ exceeds that of the assets with greatest maturity is undertaken. This is important because Redington (1952) demonstrated that an immunised portfolio would ensure solvency for a principal's liabilities when they are matched with the appropriate asset portfolio. The investment management risk free decision is to hold the duration matching portfolio unless the management believe that they can identify a superior portfolio in terms of incremental return or reduced risk.

As mentioned in chapter 2, there is a shortage of longer dated bonds in the Irish market. Certain lines of the insurance and assurance industry require these bonds because they have very long duration, e.g. liability insurance, re-insurance or pension liabilities.

In the Irish context, it is difficult but not impossible to achieve immunisation with the present structure of the government treasury market. Immunisation can be achieved using a combination of bank borrowings and interest rate swaps.

[^31]While this approach is not the most efficient in a perfect market with no transactions costs, no tax effects or no limitations on borrowing or lending, it is the only possible approach in a relatively illiquid and imperfect market such as the Irish government treasury market. Consider a fund whose sole liability is the payment of a known monetary amount at time $t$. Let;
$A_{t}$ is the present value of all assets at time $t$.
$L_{t}$ be the present value of all liabilities at time $t$,
$\alpha_{t}$ be the proportion by which $A_{t}$ exceeds $L_{t}$,
$\mathrm{t}(\mathrm{x})$ be the duration of cash flow vector x .
(5.2.1) $\quad \alpha_{t}=\left(\frac{A_{t}}{L_{t}}-1\right)$

Using Redington's (1952) immunisation, a portfolio of two or more assets would be constructed such that, for a given term structure at time $t$, then;
(5.2.2),$A_{t}=L_{1}$
(5.2.3) $\quad t\left({ }_{1} A_{t}\right)=t\left(L_{t}\right)$
(5.2.4) $\quad t^{2}\left({ }_{1} A_{t}\right)>t^{2}\left(L_{t}\right)$
where ${ }_{1} A_{t}$ is the present value at time zero of the specific immunisation assets with maturity t . There is an implicit assumption in the previous three equations that an investment return, regardless of timing of receipt, is capable of achieving an investment return equal to that obtained for investments of term $t$. Further impilied assumptions are that transaction' costs are zero, markets are frictionless and that the yield curve is flat for all maturities.

[^32]Redington (1952) has shown that, if the above assumptions and conditions are satisfied, the portfolio is immunised to the extent that small changes (i.e. a few basis points) in prevailing interest rates, spread uniformly along the yield curve, will produce small profits to the matching asset portfolio relative to the liability portfolio

However, the asset portfolio is exposed to the risk of changes in the yield curve shape from the assumed flat structure. If the yield curve changes to an upward or downward slope, then the reinvestment yields may not match those assumed in the valuation basis and a shortfall could occur.

To achieve an insolvency risk free portfolio, this reinvestment risk must be eliminated. In that regard there can be a shortage of assets with sufficient duration; moreover, in a small market the assets could be very illiquid. It is in this shortage of assets with sufficient duration scenario that the equivalent zero coupon model is developed, interest rate swaps and bank borrowing to achieve target duration in the asset portfolio. There is no secondary (or for that matter primary) market in zero-coupon securities in Ireland. An interest rate swap is a contract in which two counter-parties agree to exchange interest rate payments of differing character based on an underlying notional borrowing that is never exchanged. Liquidity is a very important consideration in small markets.

Interest rate swaps are traded on a spread in relation to government treasuries, and this spread is a function of the financial intermediary internal and external costs of processing the trade and monitoring the transaction, a risk premium for the credit risk process. This reflects the risk-adjusted return to this type of exposure and the balance sheet charge for using the scarce resource of capital.

In this context, the interest rate payment characters are short term DIBOR (Dublin Interbank Offered Rate) on the borrowing and a semi-annual ${ }^{5}$ fixed coupon on the bond. The following assumptions will be made;

1. A flat yield curve with no transaction costs, taxes or credit risks.
2. The asset portfolio consists of a coupon bond with a single redemption date at time $n$ and no embedded options.
3. A vector of borrowings and interest rate swaps for time periods 1 to $n-1$ exist for these periods.

Let:
c - coupon payment on bond per unit time,
$y$ - yield or internal rate of return on the bond,
$n$ - term to maturity of the bond,
$P(c, n ; y)$ - price of the bond,
v - present value factor,
$R$ - redemption value per unit nominal,
$b_{t}$ - borrowing for the payment of the liability maturing at time $t$.
$I_{t}$ - cost of borrowing for the period $(0, t)$,
$B_{0}$ - total borrowing at time 0 .

The present value factor is;
(5.2.4) $\quad v=\frac{1}{(1+y)}=(1+y)^{-1}$

Therefore, the price or value a bond that is a series of discounted cash flows is;
(5.2.5) $P(c, n ; y)=c v^{1}+c v^{2}+\ldots+(R+c) v^{n}$

[^33](5.2.6) $P(c, n ; y)=\sum_{t=1}^{t=n} c v^{t}+R v^{n}$

To eliminate the investment risk on the $(n-1)$ th coupon payment, a vector of borrowings is constructed which mature in time period $n-1$, such that the cash flows net to zero. The portfolio borrows at floating rates for $n-1$ periods and uses an interest rate swap such that it receives floating rates and pays fixed rates so all future cash flows are known with certainty. The net result is that a known cost of funding is generated.

## From assumption 1;

(5.2.7) $\quad i_{t}=y$

The required borrowing is;
(5.2.8) $\quad b_{n-1}=c\left(1+i_{n-1}\right)^{-1}=c(1+y)^{-1}$

The resultant cash flow in time period $\mathrm{n}-1$ is;
(5.2.9) $c+b_{n-1}+y b_{n-1}=0$

On a recursive basis moving to period $n-2$, and setting up a fresh borrowing after allowing for the cost of borrowing to be repaid in subsequent periods;
$(5.2 .10) b_{n-2}=\left(c+y b_{n-1}\right)(1+y)^{-1}$

For time period $n-3$;
$(5.2 .11) b_{n-3}=\left(c+y b_{n-1}+y b_{n-2}\right)(1+y)^{-1}$

## For time period 1

(5.2.12) $b_{1}=\left(c+y b_{n-1}+\ldots+y b_{2}\right)(1+y)^{-1}$

In general for time period t :
(5.2.13) $b_{t}=\left(c+y\left(b_{n-1}+\ldots+b_{n-t+1}\right)\right)(1+y)^{-1}$

This equation can be rewritten as;
(5.2.14) $b_{t}=\left(c+y \sum_{x=n-t+1}^{x=n-1} b_{x}\right)(1+y)^{-1}$
(5.2.15) $b_{1}=\left(c+y \sum_{x=2}^{x=n-1} b_{x}\right)(1+y)^{-1}$

By relaxing assumption 1 and interest rates are allowed to vary between periods;
(5.2.16) $b_{t}=\left(c+\sum_{x=n-t+1}^{x=n-1} i_{x} b_{x}\right)(1+y)^{-1}$

Then summing all the borrowings over time periods 1 to $n-1$;
(5.2.17) $B_{0}=\sum_{z=1}^{z=n-1} b_{z}$

Then;
(5.2.18) $(n-1) c+\sum_{z=1}^{==n-1} b_{z}+y \sum_{z=1}^{==n-1} \sum_{x=1}^{x=z} b_{z, x}=0$
(5.2.19) $(n-1) c+\sum_{z=1}^{:=i-1} b_{z}+\sum_{z=1}^{==n-i} \sum_{x=1}^{x=z} i_{z, x} b_{z, x}=0$

The practical result of equation (5.2.19) is to transform a coupon bond into a zero coupon bond. This process will have lengthened the duration and the duration of the initial coupon bond must be chosen so that its stripped duration matches that of the underlying liability.
(5.2.20) $\alpha_{2} A_{t}=A_{t}-{ }_{2} A_{t}$

The risk free asset portfolio has been established ${ }_{2} A_{t}$ (from a solvency perspective) and the mismatch reserve $\alpha A_{t}$, the excess of assets required to achieve the risk free of insolvency return. Specifically, with ${ }_{2} A_{t}-L_{t}=0$ and $\sigma^{2}=0$, the asset and liability portfolio will have identical distributions with regard to interest rate changes, hence the relative distribution will not exist.

If the endowment of assets is $A_{f}$;
(5.2.21) $A_{t}=(1+\alpha){ }_{2} A_{t}$ where $A_{t}>_{2} A_{t}$

Then the excess return earned on the asset portfolio is:
(5.2.22) $\left(\alpha_{2} A_{1}\right) \quad\left(r_{1} \sigma_{1}{ }^{1}\right)$

An example of a ten-year bond is set out below, with borrowings for years one to nine and the yield curve flat at a yield of $10 \%$. This is shown in table 5.1;

| Year | Cash Flow | Present Value | PV of CF | t by Cash Flow | PV of t by CF |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 10.00 | 0.9091 | 9.09 | 10.00 | 9.09 |
| 2 | 10.00 | 0.8264 | 8.26 | 20.00 | 16.53 |
| 3 | 10.00 | 0.7513 | 7.51 | 30.00 | 22.54 |
| 4 | 10.00 | 0.6830 | 6.83 | 40.00 | 27.32 |
| 5 | 10.00 | 0.6209 | 6.21 | 50.00 | 31.05 |
| 6 | 10.00 | 0.5645 | 5.64 | 60.00 | 33.87 |
| 7 | 10.00 | 0.5132 | 5.13 | 70.00 | 35.92 |
| 8 | 10.00 | 0.4665 | 4.67 | 80.00 | 37.32 |
| 9 | 10.00 | 0.4241 | 4.24 | 90.00 | 38.17 |
| 10 | 110.00 | 0.3855 | 42.41 | 1100.00 | 424.10 |
|  |  | Bond Price | 100.00 |  | 675.90 |
|  |  | Duration | 6.759 | years |  |

Table 5.1-10\% Bond with 10-Year Maturity
Source: Empirical

The duration is estımated to be 6.759 years. The sets of borrowings are determined by (5.2.13) \& (5.2.17). These are shown in the table 5.2 that also contains the net cash flows.

| Year | Borrowings | Bond | Net Cash Flow | In terms of 100 |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 57.59 | -100.00 | -42.40 | -38.55 |
| 1 | -4.24 | 10.00 | 0.00 | 0.00 |
| 2 | -4.67 | 10.00 | 0.00 | 0.00 |
| 3 | -5.13 | 10.00 | 0.00 | 0.00 |
| 4 | -5.64 | 10.00 | 0.00 | 0.00 |
| 5 | -6.21 | 10.00 | 0.00 | 0.00 |
| 6 | -6.83 | 10.00 | 0.00 | 0.00 |
| 7 | -7.51 | 10.00 | 0.00 | 0.00 |
| 8 | -8.26 | 10.00 | 0.00 | 0.00 |
| 9 | -9.09 | 10.00 | 0.00 | 0.00 |
| 10 |  | 110.00 | 110.00 | 100.00 |

## Table 5.2 Resultant Cash Flows with Borrowings

Source: Empirical

With the borrowings, the portfolio is now equivalent to a zero coupon bond. This has been rescaled in the final column so that it matures to the nominal $£ 100$. Since the longest duration is 6.76 years, and all the borrowings will have a shorter duration, how the duration becomes ten years is shown in the table 5.3;

| Asset | Portfolio Value | Portfolio Weight | Asset Duration | Portfolio Duration (years) |
| :---: | :---: | :---: | :---: | :---: |
| Bond | 100.00 | $235.8 \%$ | 6.76 | 15.94 |
| Borrowing Time 1 | -4.24 | $-10.0 \%$ | 1.00 | -0.10 |
| Borrowing Time 2 | -4.67 | $-11.0 \%$ | 1.91 | -0.21 |
| Borrowing Time 3 | -5.13 | $-12.1 \%$ | 2.74 | -0.33 |
| Borrowing Time 4 | -5.64 | $-13.3 \%$ | 3.49 | -0.46 |
| Borrowing Time 5 | -6.21 | $-14.6 \%$ | 4.17 | -0.61 |
| Borrowing Time 6 | -6.83 | $-16.1 \%$ | 4.79 | -0.77 |
| Borrowing Time 7 | -7.51 | $-17.7 \%$ | 5.36 | -0.95 |
| Borrowing Time 8 | -8.26 | $-19.5 \%$ | 5.87 | -1.14 |
| Borrowing Time 9 | -9.09 | $-21.4 \%$ | 6.33 | -1.36 |
| Total Value | 42.41 | $100.0 \%$ |  | 10.00 |

Table 5.3 Duration Reconciliation
Source: Empirical

Since the potfolio weight is only 42.41 rather than the original 100 that is spent in buying the bond, this will increase the duration to 10 years.

Settlement Date : 27-Mar-92

| Coupon Date | Time | GRY 8.70\% | Cash Flow | PV of Cash Flow | Time by PV of Cash Flow |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 30-Mar-92 | 0.008 | 0.9993 | 0.00 | 0.00 | 0.00 |
| 30-Sep-92 | 0.512 | 0.9573 | 4.38 | 4.19 | 2.14 |
| 30-Mar-93 | 1.008 | 0.9178 | 4.38 | 4.02 | 4.05 |
| 30-Sep-93 | 1.511 | 0.8792 | 4.38 | 3.85 | 5.81 |
| 30-Mar-94 | 2.007 | 0.8429 | 4.38 | 3.69 | 7.40 |
|  |  | . |  |  |  |
| 30-Sep-10 | 18.511 | 0.2067 | 4.38 | 0.90 | 16.74 |
| 30-Mar-11 | 19.006 | 0.1982 | 4.38 | 0.87 | 16.48 |
| 30-Sep-11 | 19.510 | 0.1899 | 4.38 | 0.83 | 16.21 |
| 30-Mar-12 | 20.008 | 0.1820 | 438 | 0.80 | 15.93 |
| 30-Sep-12 | 20.512 | 0.1743 | 104.38 | 18.20 | 373.22 |
|  |  |  | Dirty Price | 100.39 | 993.87 |
|  |  |  | Duration : | 9.90 | years |

Table 5.4 - Irish Bond before being stripped at $8.70 \%$ yields
Source : McDermott (1992) \& Empirical data

In table 5.4, the $83 / 4 \%$ Capital 2012 bond has a duration of 9.90 years. However, because the yield curve is not flat, the duration will not reconcile as in the previous example. To overcome this problem, the first ten years coupons are matched by borrowing and the internal rate of return of the portfolio is estimated at $8.44 \%$. The interest rate swap curve prevailing in the market on the 27 March 1992 is used. The duration is reestimated for all series of cash flows using $8.44 \%$.

This is shown in table 5.5. The duration of assets can be increased to match and immunise liabilities, for small changes in the yield curve. Such an asset with duration of 17.12 years did not exist, and has been synthetically created by the asset allocation in the portfolio.

| Asset | Portfolio Value | Portfolio Weight | Asset Duration | Portfolio Duration years |
| :---: | :---: | :---: | :---: | :---: |
| Bond | 102.96 | $230.5 \%$ | 10.02 | 23.10 |
| Borrowing Time 1 | -1.66 | $-3.7 \%$ | 0.51 | -0.02 |
| Borrowing Time 2 | -1.84 | $-4.1 \%$ | 0.98 | -0.04 |
| Borrowing Time 3 | -1.91 | $-4.3 \%$ | 1.44 | -0.06 |
| Borrowing Time 4 | -2.05 | $-4.6 \%$ | 1.87 | -0.09 |
| Borrowing Time 5 | -2.13 | $-4.8 \%$ | 2.29 | -0.11 |
| Borrowing Time 6 | -2.28 | $-5.1 \%$ | 2.68 | -0.14 |
| Borrowing Time 7 | -2.36 | $-5.3 \%$ | 3.07 | -0.16 |
| Borrowing Time 8 | -2.51 | $-5.6 \%$ | 3.43 | -0.19 |
| Borrowing Time 9 | -2.61 | $-5.8 \%$ | 3.78 | -0.22 |
| Borrowing Time 10 | -2.77 | $-6.2 \%$ | 4.11 | -0.26 |
| Borrowing Time 11 | -2.89 | $-6.5 \%$ | 4.44 | -0.29 |
| Borrowing Time 12 | -3.06 | $-6.9 \%$ | 4.74 | -0.32 |
| Borrowing Time 13 | -3.19 | $-7.1 \%$ | 5.04 | -0.36 |
| Borrowing Time 14 | -3.36 | $-7.5 \%$ | 5.34 | -0.40 |
| Borrowing Time 15 | -3.51 | $-7.9 \%$ | 5.60 | -0.44 |
| Borrowing Time 16 | -3.68 | $-8.2 \%$ | 5.87 | -0.48 |
| Borrowing Time 17 | -3.84 | $-8.6 \%$ | 6.12 | -0.53 |
| Borrowing Time 18 | -4.03 | $-9.0 \%$ | 6.37 | -0.57 |
| Borrowing Time 19 | -4.21 | $-9.4 \%$ | 6.61 | -0.62 |
| Borrowing Time 20 | -4.40 | $-9.9 \%$ | 6.83 | -0.67 |
| Total Value | 44.67 |  | Duration | 17.12 |
|  |  |  |  |  |

Table 5.5 - Duration Reconciliation of 8 3/4\% Capital 2012 Bond

Source : McDermott (1992) \& Empirical data

### 5.3 Derivation and Quantification of Mismatch Reserve

### 5.3.1 Theory

In the previous section, the asset porffolio free of any interest rate insolvency risk for small changes in yields is derived and borrowing is used to increase the portfolio duration. This section derives a mismatch reserve and quantifies for a major Irish insurer.


#### Abstract

In financial markets with intermediation by agents ${ }^{5}$ between principals ${ }^{7}$, the principals must provide their agents with expected time horizon for dissaving ${ }^{8}$ and the spread of the dissaving pattern. From these guidelines, the agent can communicate the expected return for this time horizon from the present implied internal rate of return on the principal's liability equivalent asset. He will also indicate an equivalent benchmark that reflects the target duration of dissaving from some class of market indices which the agent can replicate.


The agent receives his reward for Fama's (1968) standard role as a financial intermediary; collector and processor of financial data, retention of above market average expertise on market behaviour, economies of scale for transaction, execution and custodial services. Should the agent achieve an investment performance in excess of that initially expected, then there will be an excess of assets over liabilities. This excess solvency is defined as a mismatch reserve.

When such a successful mismatch occurs the principal has the choice to scale up the initial liability, maintain the same mismatch with a higher degree of solvency confidence, reduce contributions should future contributions be payable, or allow the agent more flexibility to maximise assets over liabilities while ensuring soivency with the same degree of confidence.

[^34]This is referred to as avoiding the insolvency ruin barrier that can be breached with $\varepsilon \%$ level of confidence. From this the agent's decision can be ranked on a hierarchical basis and the mismatch reserve allocated among the decisions which have been isolated to be independent events where possible. The allocation of mismatch per decision will then imply a set of limits per decision which the agent can take whilst ensuring solvency with ( $1-\varepsilon$ ) \% degree of confidence.

A mismatch reserve is the equivalent of holding an immunised portfolio and a relative performance option (Rainbow option) of an at the money call option on the return of the mismatched portfolio relative to the immunised portfolio for a specific time horizon. Alternatively, a mismatch reserve is the equivalent of holding a mismatched portfolio and a relative performance option (Rainbow option) of an at the money put option on the return of the mismatched portfolio relative to the immunised portfolio. When an excess of assets over liabilities exists, the principal decides to release this to the agent managing the asset portfolio in a timely manner. The principal can release the entire amount to be used in one time period, spread it as an annuity over the remaining life of the liability or release the return from the matched portfolio over infinite time periods.

A model of this approach is shown for an Irish general insurer in the next section. From this perspective, the yield curve is evaluated where the mismatch from the principals dissaving by the agent implies that the agent expects to earn an additional return. This additional premium can be evaluated using standard option pricing models or contingent claims analysis. At the macro level, the saving period by principals whereby they transfer their income from one time period to another has the open set of zero to infinity in terms of all individuals and organisations.

[^35]While the principal's time horizon can be unbounded, it is assumed truncated at the longest maturity of government debt. O'Connor's (1993) spectrum of the principal's liabilities time horizon is shown in table 5.6.

| Principal | Expected Time Horizon (years) |
| :---: | :---: |
| Property Insurer | 0.5 year |
| Bank | 2 years |
| Hire Purchase and Leasing | 3 years |
| Liability Insurer | 5 years |
| Building Society | 7 year |
| Life Insurance | 10 years |
| Pension Fund | 20 years |

Table 5.6 Principal Time Horizons
Source: O'Connor (1993) \& Empirical data

The mismatch reserve is the capital value of the asset profit in the fund. It is free to enhance liabilities by revising these upwards, or if they are still being funded, to reduce the contribution flow. If they are not used for either of these purposes, then the mismatch reserve can be used to assume greater risk than the immunised return. Since a 'notional' set of assets if not actual matched assets can be in a fund, departure from such matching can be viewed as borrowing at unspecified rates of interest, to create a leveraged position in the final asset allocation. If such borrowings from the matched asset position are to persist for the life of the portfolio, the cost of such borrowings is the internal rate of return required on the matched assets to meet the liability.

A one period model is investigated such that at the start of the period, the total value of assets, total value of mismatched assets and the mismatch reserve are known;

$$
\begin{equation*}
A_{t}=(1+\alpha)_{2} A_{t} \tag{5.3.1}
\end{equation*}
$$

The portfolio is allocated such that a proportion $\beta$ was not immunised. This departure is financed initially with regard to capital cost from the mismatch reserve and subsequently if required from the matched asset. Let ${ }_{3} A_{t}$ denote the mismatched assets. the risk free rate of return that would be earned on surplus assets is;
(5.3.2) $\quad r_{f} \alpha_{2} A_{f}$

The return actually earned on assets is;
(5.3.3) $\beta\left({ }_{3} A_{t+1}-(1+\alpha)_{2} A_{t}\right)+(1-\beta)(1+\alpha)\left({ }_{2} A_{t+1}-{ }_{2} A_{t}\right)$

Since the expected return on the risky portfolio must exceed the risk free portfolio;

$$
\begin{equation*}
E\left[\beta\left({ }_{3} A_{t+1}-(1+\alpha)_{2} A_{t}\right)+(1-\beta)(1+\alpha)\left({ }_{2} A_{t+1}-{ }_{2} A_{t}\right)\right]>E\left[r_{f}(1+\alpha)_{2} A_{t}\right] \tag{5.3.4}
\end{equation*}
$$

where $\beta>0$.

Because of the equality of starting assets ${ }_{3} A_{t}$ in (5.2.21), the value of asset allocation is equal to the initial portfolio at time $t$. If the condition that ${ }_{2} A_{t+1}=\left(1+r_{f}\right){ }_{2} A_{t}$ is imposed, i.e. the value of fully matched assets a time $t+1$ equals the value of these assets at time $t$ increased by the risk free internal rate of return.

Then, the first condition of risk assumption is;

$$
\begin{equation*}
E\left[\beta\left({ }_{3} A_{t+1}-(1+\alpha)_{2} A_{t}\right)+(1-\beta)(1+\alpha)\left({ }_{2} A_{t+1}-{ }_{2} A_{t}\right)\right]>E\left[r_{t}(1+\alpha)_{2} A_{t}\right] \tag{5.3.5}
\end{equation*}
$$

The expected return on the diversified portfolio must exceed the expected value of the fully matched portfolio return.
(5.3.6) $E(C)=C \quad$ so eliminating constraints implies.

$$
\begin{equation*}
E\left[\beta_{3} A_{t+1}-\beta\left(1+r_{f}\right)(1+\alpha)_{2} A_{t+1}\right]>0 \tag{5.3.7}
\end{equation*}
$$

$$
\begin{equation*}
E\left[\beta\left[{ }_{3} A_{t+1}-(1+\alpha)_{2} A_{t+1}\right]>0\right] \tag{5.3.8}
\end{equation*}
$$

The second condition of risk assumption, which limits jointly the proportion which can be diversified and the choice of asset for diversification, is that the probability of the asset portfolio value at time $t+1$ being less than the increased value of the required matching assets be of a low order. Let the threshold chosen for this probability be $\varepsilon$, so that;

$$
\begin{equation*}
P\left[\beta_{3} A_{t+1}+(1+\alpha)\left[(1-\beta)_{2} A_{t+1}-A_{2} A_{t}\right]-r_{f} \alpha_{2} A_{t}-\alpha_{2} A_{t}>0\right]=1-\varepsilon \tag{5.3.9}
\end{equation*}
$$

The limiting condition for this inequality can alternatively be expressed as;
(5.3.10) $\beta>(\beta+\alpha \beta-\alpha)_{2} A_{t+1} /_{3} A_{t+1}$
(5.3.11) $(1+\alpha-\alpha / \beta)_{2} A_{t+1} /_{3} A_{t+1}<1$ with probability $1-\varepsilon$

The extent to which risk can be assumed by the fund is a function of the insolvency ruin barrier $\varepsilon$, the size of mismatch reserve and subsequently of $\beta$, the proportion of mismatch and ${ }_{2} A_{t+1}$ divided by ${ }_{3} A_{t+1}$ the variability of the matched assets, relative to the chosen non-matched assets. If a high value for the mismatch reserve exists in the initial time period, this will increase the degree and type of allowable mismatch. Once the proportion of mismatch has been decided. the variability of assets allowable can be established. The variability of the asset allocation relative to the insolvency risk free rate of return on the matched asset is the product of a stochastic process.

In practice, the historical and implied returns and their respective volatilities will represent a guide to the expected behaviour of asset classes and different portfolios. However, the statutory authorities require different types of mismatch reserves for different financial intermediaries. The regulatory mismatch requirements for solvency maintenance are a three per cent rise in the yield curve and a twenty-five per cent decline in equities along with the second non-life directive of the European Union. Beyond the necessary solvency requirements, there is no specific mismatch requirement.

If the daily price histories for each of the asset portfolios under consideration re available, maximum likelihood estimates of probability density functions of daily price movements in each asset portfolio can be constructed.

Let:
$f(A)$ be the probability density function of daily price movement of the immunising asset portfolio,
$g_{A}\left({ }_{3} A\right)$ be the probability density function of daily price movement of the mismatched asset portfolio.

The expected return on the immunising asset is:
(5.3.12) $\int_{-\infty}^{\infty} A f(A) d A$
and that on the mismatched asset is;
(5.3.13) $\int_{-\infty}^{\infty} A g_{A}\left({ }_{3} A\right) d_{3} A$

In order to justify mismatching;
(5.3.14) $\int_{-\infty}^{\infty} A f_{A}(A) d A-\int_{-\infty}^{\infty} A g_{A}\left({ }_{3} A\right) d_{3} A<0$

Let ${ }_{1} A_{t}$ be the random value of the immunising asset portfolio at time $t$. Let ${ }_{3} A_{t}$ be the corresponding value of the mismatched asset portfolio. If $\alpha A_{+}$is the mismatch reserve, then;
(5.3.15) $f_{\alpha A}(\alpha A)=\int_{-\alpha}^{\alpha A} f_{A}(A+\alpha A) f_{3} A\left({ }_{3} A\right) d_{3} A$
is the probability of the daily distribution of the mismatch reserve. Since $\alpha A$ is the upper limit of the density function, a corresponding probability density mass is located at a value of zero. This mass cannot be more than $\varepsilon$, where $\varepsilon$ is the arbitrary insolvency probability.

A scaling factor $\beta$ is introduced, representing the proportion of the portfolio that can be mismatched while satisfying the above conditions. The scaling factor is identical to the factor $\beta$ used in departures from immunisation in the previous section.
(5.3.16) Probability $[\alpha A<0] \leq \varepsilon_{\beta}$ 'where $\varepsilon$ is scaled by proportion $\beta$.

With an assumption of the mismatch reserve being normally distributed, the standard normal coefficient can be used to deduce $\beta$. However, the assumption of normality cannot be appropriate. When the distribution of both portfolio valuations in isolation is considered as having a $\log$ normal distribution, their inter-relationship can be viewed from the perspective of Contingent Claims Analysis (C.C.A.). A notional call option on the outperformance of the mismatched portfolio is purchased, subject to a minimum payment of the mismatch reserve at expiry. As a development of the methodology of Stultz (1982);
(5.3.17) $\operatorname{Max}\left[{ }_{3} A_{t+1}-{ }_{2} A_{t+1}, \alpha A_{t}\right]$
for a one period option purchased at time t. $3^{A}(t+1)^{*}$ and $2^{A}(t+1)^{*}$ are jointly lognormal for a singie asset portfolio and compound lognormal otherwise and represent the portfolio valuations at expiration.

The option premium is;
(5.3.18) $C=e^{-r} E\left[\operatorname{Max}\left[{ }_{3} A_{(t+1)^{\circ}}-{ }_{2} A_{(t+1)^{*}}, \alpha A_{t} \mid\right]\right.$
or written as a double integral;
(5.3.19) $C-e^{-r} \alpha A_{t}=e^{-r} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \operatorname{Max}\left[{ }_{3} A e^{x}-{ }_{2} A_{t} e^{y}, 0\right] f(x, y) d x d y$
where $x=\log \left(\frac{{ }_{3} A_{(t+1)^{*}}}{{ }_{3} A_{t}}\right), y=\log \left(\frac{{ }_{2} A_{(t+1)^{*}}}{{ }_{2} A_{t}}\right)$ and $f(x, y)$ is the bivariate density function.

The expected value of the right hand side is the discount factor multiplied by the expected value of the mismatched portfolio given that the mismatched portfolio is more than the immunised portfolio at expiry.

This is given by;

where $f(x)=\left(\frac{1}{\sigma_{1} \sqrt{2 \pi t}}\right) e^{-\frac{1}{2} v_{1}^{2}} \quad \& v_{1}=\left(x-u_{1} t\right) / \sigma_{t} \sqrt{t}$
$f(y)=\left(\frac{1}{\sigma_{2} \sqrt{2 \pi} t}\right) e^{-\frac{1}{2} v_{2}^{2}} \quad \& v_{2}=\left(x-u_{2} t\right) / \sigma_{t} \sqrt{t}$,
$f(x \mid y)=\left(\frac{1}{\sqrt{2 \pi\left(1-\rho^{2}\right) \sigma_{1}^{2} t}}\right) e^{-\frac{1}{2} w_{1}} \quad \& w_{1}=\left[\left(x-u_{1} t\right)-\rho\left(\frac{\sigma_{1}}{\sigma_{2}}\right)\left(y-u_{2} t\right)\right]^{2} /\left(1-\rho^{2}\right) \sigma_{1}^{2} t$,

$$
f(y \mid x)=\left(\frac{1}{\sqrt{2 \pi\left(1-\rho^{2}\right) \sigma_{2}^{2} t}}\right) e^{-\frac{1}{2} w_{2}} \quad \& w_{2}=\left[\left(x-u_{2} t\right)-\rho\left(\frac{\sigma_{2}}{\sigma_{1}}\right)\left(y-u_{1} t\right)\right]^{2} \prime\left(1-\rho^{2}\right) \sigma_{2}^{2} t
$$

The formula utilises the normal density and conditional normal density functions. As a result, equation 5.3.20 becomes;
(5.3.21) $f(x, y)=\left(\frac{1}{\left(2 \pi \sigma_{1} \sigma_{2}\left(1-\rho^{2}\right) t\right)}\right) e^{-1 / 2 u}$
where $u=\frac{\left[\frac{\left(x-u_{1} t\right)^{2}}{\sigma_{1}^{2} t}-\frac{2 r\left(x-u_{1} t\right)\left(y-u_{2} t\right)}{\sigma_{1} \sigma_{2} t}+\frac{\left(y-u_{2} t\right)^{2}}{\sigma_{2}^{2} t}\right]}{\left(1-\rho^{2}\right)}$,
and $\quad u_{1}=\log \left(\frac{G}{d_{1}}\right)-\frac{1}{2} \sigma_{1}^{2}$ and $u_{2}=\log \left(\frac{G}{d_{2}}\right)-\frac{1}{2} \sigma_{2}^{2}$
where $d_{1}$ and $d_{2}$ are one plus the payoff rates of the two asset portfolios, $\sigma_{1}^{2}$ and $\sigma_{2}^{2}$ are the volatilities of the asset porffolios, $\rho$ is the correlation of asset portfolios, and $G$ is the natural logarithm of one plus the rate of return of the two underlying asset portfolios.

This implies;
(5.3.22) ${ }_{3} A_{t} e^{-r} \int_{\log \left(\frac{\alpha_{3} A_{4}}{A_{t}}\right)}\left[\int_{-\infty}^{\left.x-\log \left(\frac{\alpha_{3} A}{A_{t}}\right) f(y \mid x) d y\right] e^{-x} f(x) d x}\right.$
(5.3.23) $)_{3} A_{1} d_{2}^{-1}\left\{N\left[x_{2}\right]-N_{2}\left[-y_{2}, x_{2} ; r_{2}\right]\right\}$
in standard Black and Scholes (1973) terminology. $N(d)$ is the normal distribution, $N_{2}$ is the bivariate normal distribution.
(5.3.23) $x_{2}=\left[\log \left(\frac{\left({ }_{3} A_{t} d_{2}^{-1} / \alpha A_{t} e^{-r}\right)}{\sigma_{2}}\right)\right]+1 / 2 \sigma_{2}$
and
(5.3.24) $y_{2}=\left[\log \left(\left({ }_{3} A_{t} d_{2}^{-1} / A_{t} e^{-r}\right)\right)+\Sigma\right]+1 / 2 \Sigma$
where $\Sigma^{2}=\sigma_{1}^{2}+\sigma_{2}^{2}-2 \rho \sigma_{1} \sigma_{2}$ and $\rho_{2}=\left(\rho \sigma_{1}-\sigma_{2}\right) / \Sigma$ for a one period option.

On the left-hand side of the equation, the option premium amounts to $C-e^{-r} \alpha A_{t}$. The amount available for mismatching is limited to $\alpha A_{t}=C$. Thus the option premium allowing for perpetual option renewal is;
(5.2.25) $\alpha_{2} A_{t}-\frac{\alpha_{2} A_{t}}{1+r}=\frac{r \alpha_{2} A_{t}}{1+r}$

This is obviously a function of $\alpha A_{t}$, the size of the mismatch reserve. In order that the left hand side equal the right, i.e. that;

$$
(5.2 .26) \frac{r \alpha_{2} A_{t}}{1+r}=A_{t} d_{2}^{-1}\left\{N\left[x_{2}\right]-N\left[-y_{2}, x_{2}, r_{2}\right]\right\}
$$

and given that ail quantities have previously been defined for any specific mismatch, a scaling factor $b$ is required, under identical probability density function assumptions. This scaling factor is identical to the proportion of mismatch allowed referred to in the previous discussion.

The relationship between the variability of immunising assets as opposed to the allocated mismatched assets (i.e. ${ }_{2} A_{t+1} /{ }_{1} A_{t+1}$ ) is less obviously traceable in the above formulae, but would be reflected in the expression (1- 1 ), reflecting the difference between perfect and actual correlation between immunising and mismatched asset portfolios, and in the entries relating to $\sigma_{1}^{2}$ and $\sigma_{2}^{2}$, the variances of the mismatched and immunising assets.

Since ${ }_{2} A_{t+1} / 1 A_{t+1}$ appear initially in the context of a limiting condition for an inequality based on ruin probability $\varepsilon$, whereas $(1-r) \sigma_{1}^{2}$ and $\sigma_{2}^{2}$, appear in an option pricing formula, it is not possible to equate the random variable with its distribution. However, if the further assumption is made that ${ }_{2} A_{t+1}$ and ${ }_{1} A_{t+1}$ are jointly lognormally distributed then the ratio of these two assets will directly determine the payoff on the option in the contingent claims analysis section. It has been demonstrated that the size of the mismatch reserve dictates jointly the proportion and variability of mismatched assets. Further, it has been shown that the interest on the mismatch reserve can be regarded as a perpetuity of option premiums on the better performing of two portfolios of assets and that the proportionalities are identical under common assumptions whether departure from immunisation or contingent claims analysis is used.

### 5.3.2 Company Empirical Model

An example based on one of the major Irish general insurers is developed in this subsection from their returns to the Department of Enterprise and Employment ${ }^{9}$ over the sample time period of 1980 to 1997. The initial analysis is of the value and duration of their liabilities. These are Motor Vehicles. Fire and Property and Liability lines. Their liability claims ladder is estimated from their Form 8 's (i.e. projected time triangle of claim settlements) which is shown in table 5.7 and is sometimes referred to as the run off triangle or claims ladder. Claims ladders show the percentage being settled and paid in a particular year that is a given number of years after the year in which the business is written. The overall duration will be the duration profiles of each line of business and the amount written in a particular year and the portfolio mix of liability lines. The term structure from chapter 2 is used to estimate the present value and duration of the liabilities. It must be also borne in mind that the claims are estimated from the Department's Blue Book (i.e. Net of Re-insurance) while the F8's are Gross of Re-insurance, so that forecast errors are subject to changing claims and re-insurance arrangements.

Since re-insurance is also shown for outside the country (e.g. Lloyd's), the industry is investigated to see if the individual firm is significantly different from the industry expected duration. Their business breakdown is shown in table 5.7. The duration of the Credit \& Suretyship and Fire \& Property is estimated to be six months. Accident and Sickness and Marine and Transit is consolidated because they amounted to $0.1 \%$ of the liability portfolio. Over the sample period of 1980 to 1994 the company has managed to hold $10 \%$ of market share. With the present value and duration of the liabilities is estimated from the data in chapter two, and the estimated duration of the Riada Short Government treasury Index it is possible to identify the appropriate benchmark. This had to be weighted with short-term money (one month) and the benchmark is rebalanced on an annual basis when the duration of the liabilities are reestimated.

Motor

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Later |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Company | $45.7 \%$ | $35.7 \%$ | $8.7 \%$ | $5.4 \%$ | $2.0 \%$ | $0.9 \%$ | $0.9 \%$ | $0.7 \%$ |
| Industry | $50.9 \%$ | $33.6 \%$ | $8.7 \%$ | $3.7 \%$ | $1.6 \%$ | $0.8 \%$ | $0.3 \%$ | $0.4 \%$ |

Employer's Liability

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Later |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Company | $19.2 \%$ | $24.6 \%$ | $15.8 \%$ | $29.8 \%$ | $1.4 \%$ | $3.3 \%$ | $2.3 \%$ | $3.6 \%$ |
| Industry | $18.8 \%$ | $35.4 \%$ | $19.4 \%$ | $11.1 \%$ | $5.8 \%$ | $4.2 \%$ | $2.0 \%$ | $3.3 \%$ |

Public Liability

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Later |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Company | $26.4 \%$ | $41.7 \%$ | $14.2 \%$ | $9.6 \%$ | $3.9 \%$ | $2.1 \%$ | $0.0 \%$ | $2.1 \%$ |
| industry | $25.8 \%$ | $35.0 \%$ | $18.1 \%$ | $7.8 \%$ | $6.5 \%$ | $3.0 \%$ | $1.8 \%$ | $2.0 \%$ |

Total Liability

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Later |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Company | $22.4 \%$ | $32.4 \%$ | $15.1 \%$ | $20.6 \%$ | $2.6 \%$ | $2.8 \%$ | $1.2 \%$ | $2.9 \%$ |
| Industry | $22.1 \%$ | $35.2 \%$ | $18.8 \%$ | $9.6 \%$ | $6.1 \%$ | $3.6 \%$ | $1.9 \%$ | $2.7 \%$ |

Table 5.7 - Different Lines Run Off Triangles Percentage Settled for 1989
Source : Personal Communication (1993) - Insurance Corporation of Ireland

[^36]The split between employer's and public liability is $55 \%$ to $45 \%$. In the motor class the maturity profile is very similar to that of the industry and in the liability the only unusual year is that of year four where the liability jumped to $29.8 \%$. These cash flows can also be represented on a continuos time basis using either a gamma or loggamma function.

| Class | Share | Growth |
| :---: | :---: | :---: |
| Accident \& Sickness | $0.1 \%$ | $0.5 \%$ |
| Motor Vehicle | $39.2 \%$ | $8.0 \%$ |
| Fire \& Property | $44.7 \%$ | $12.5 \%$ |
| Marine \& Transit | $0.0 \%$ | $-2.1 \%$ |
| Liability | $12.2 \%$ | $12.0 \%$ |
| Credit \& Suretyship | $3.7 \%$ | $11.3 \%$ |

Table 5.8 Breakdown of Liability lines
Source : Cosgrove (1992) - Department of Industry \& Commerce

With Fire and Property, the standard actuarial approach is that the broker will have use of the money for the quarter, the claim will occur half way through the year and the settlement of the claim will take two months. Credit and Suretyship relate to short term Bills of Exchange, Specific Contract performance and have a normal life of six months. The liability duration, maturity and present value for 1994 is shown in table 5.9.

| Measure | Total | Motor | Fire | Liability | Other |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Maturity (years) | 1.32 | 1.42 | 1.00 | 2.26 | 1.00 |
| Present Value | $91 \%$ | $90 \%$ | $93 \%$ | $84 \%$ | $93 \%$ |
| Duration (years) | 1.24 | 1.17 | 0.91 | 1.72 | 0.91 |

Table 5.9 Present Value and Duration Profile
Source: Department of Industry \& Commerce \& Empirical data

[^37]The present value represents the discounted liability stream, although in practice insurers do not represent their liabilities as such to external bodies. This is an area that has led to much debate in the literature Daykin, Devitt, Khan \& McCaughan (1984) and Kahane (1979).

For immunisation, the liability portfolio is discounted by the appropriate discount factors from chapter two to obtain the present value and duration as set out in table 5.10. When this is done for 1989, the calculations are worked back to 1980 on a recursive basis. With the appropriate weights the required matched returns is estimated over the 1980-92 sample period on a gross basis since underwriting losses are offsetable against investment income.

| Asset | Duration | Weight | Weighed Duration |
| :---: | :---: | :---: | :---: |
| Three Month DIBOR | 0.25 | $50 \%$ | 0.13 |
| Riada Short Bond | 2.20 | $50 \%$ | 1.11 |
| Matched Portfolio |  | $100.00 \%$ | 1.24 |

Table 5.10 Matched Allocation for 1994
Source : ABN-Amro Riada Stockbrokers

After this it is assumed that the investment returns could be approximated using Hardy's formula and these are estimated by combining underwriting profits and losses and changes in shareholder's funds to impute the return. These returns required the assumption that there are no capital injections by their parent over the decade, the returns on equities are reduced by the $15 \%$ withholding credit on frish equity dividends and finally the accounting treatment of asset valuation is consistent throughout the period. In informal discussions with the company and the regulators they indicated that the approach taken is reasonable and consistent. The calculation of returns is shown in table 5.11.

| Date | Actual | Matched | Surplus/Deficit | Money | Short <br> Government <br> Treasuries |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1980 | $12.1 \%$ | $17.81 \%$ | $5.71 \%$ | $18.0 \%$ | $17.7 \%$ |
| 1981 | $14.7 \%$ | $10.33 \%$ | $-4.37 \%$ | $13.8 \%$ | $8.0 \%$ |
| 1982 | $4.6 \%$ | $18.62 \%$ | $14.02 \%$ | $18.8 \%$ | $18.6 \%$ |
| 1983 | $28.4 \%$ | $13.53 \%$ | $-14.87 \%$ | $15.5 \%$ | $11.0 \%$ |
| 1984 | $17.0 \%$ | $10.23 \%$ | $-6.77 \%$ | $12.1 \%$ | $7.4 \%$ |
| 1985 | $36.9 \%$ | $12.31 \%$ | $-24.59 \%$ | $15.0 \%$ | $11.3 \%$ |
| 1986 | $6.1 \%$ | $8.36 \%$ | $2.26 \%$ | $12.4 \%$ | $5.2 \%$ |
| 1987 | $15.7 \%$ | $11.63 \%$ | $-4.07 \%$ | $14.0 \%$ | $10.1 \%$ |
| 1988 | $14.2 \%$ | $7.84 \%$ | $-6.36 \%$ | $8.7 \%$ | $7.0 \%$ |
| 1989 | $8.69 \%$ | $5.75 \%$ | $-2.94 \%$ | $8.4 \%$ | $2.7 \%$ |
| 1990 | $4.99 \%$ | $8.27 \%$ | $3.28 \%$ | $12.1 \%$ | $5.1 \%$ |
| 1991 | $13.36 \%$ | $7.85 \%$ | $-5.51 \%$ | $11.4 \%$ | $5.8 \%$ |
| 1992 | $0.10 \%$ | $6.61 \%$ | $6.51 \%$ | $10.8 \%$ | $2.6 \%$ |
| 1993 | $6.78 \%$ | $13.73 \%$ | $6.95 \%$ | $17.0 \%$ | $9.7 \%$ |
| Average | $13.12 \%$ | $10.92 \%$ | $2.20 \%$ | $13.4 \%$ | $8.7 \%$ |
| Volatility | $9.78 \%$ | $3.95 \%$ | $9.83 \%$ | $3.2 \%$ | $4.8 \%$ |

Table 5.11 Matched versus Actual Returns
Source : Department of Industry \& Commerce \& Empirical data


Exhibit 5.1-Investment Performance over the time period 1980-1994
Source : Department of Industry \& Commerce \& Empirical data

As can be observed in exhibit 5.1, the matched investment performance is between the money and short government bond asset classes. The unmatched investment performance resulted in an increased return but with far greater risk. This is clearly seen by observing the surplus return and its risk (i.e. volatility). The next step is to choose a 'benchmark' portfolio that would either increase return and/or reduce risk relative to the matched portfolio.

Under the Irish regulators localisation rules, eighty percent of a general insurance companies assets must be held in the Irish market. Irish companies do not hold many foreign assets, though unlike life companies, they are not required to hold a mismatch reserve. Bounds are introduced into the portfolio in order that the theoretical mismatch reserve would be reduced.

These are shown in the table 5.12;

| Asset Class | Lower Bound | Benchmark | Upper Bound |
| :---: | :---: | :---: | :---: |
| Cash | $15 \%$ | $20 \%$ | $40 \%$ |
| Treasury's | $30 \%$ | $65 \%$ | $80 \%$ |
| Property | $0 \%$ | $5 \%$ | $10 \%$ |
| Equities | $5 \%$ | $10 \%$ | $20 \%$ |

Table 5.12 Bounded Benchmark Portfolio
Source : Empirical

From the constraints, portfolios are constructed to be low risk, matched, benchmark and high risk on a relative basis. Risk is defined in terms of absolute volatility that is estimated for each portfolio. These are shown in 5.13.

| Date | Matched | Low Risk | Benchmark | High Risk |
| :---: | :---: | :---: | :---: | :---: |
| Property | $0 \%$ | $0 \%$ | $5 \%$ | $5 \%$ |
| Money | $50 \%$ | $40 \%$ | $20 \%$ | $15 \%$ |
| Treasury's | $50 \%$ | $55 \%$ | $65 \%$ | $60 \%$ |
| Equities | $0 \%$ | $5 \%$ | $10 \%$ | $20 \%$ |
| Return | $9.64 \%$ | $10.35 \%$ | $6.45 \%$ | $11.44 \%$ |
| Volatility | $4.42 \%$ | $5.44 \%$ | $6.06 \%$ | $7.66 \%$ |

Table 5.13 Portfolio Weights and Performance Parameters 1980-1997
Source : Department of Industry \& Commerce \& Empirical data

| Time | Description | Low | Medium | High |
| :---: | :---: | :---: | :---: | :---: |
| 18 years | Return | $0.72 \%$ | $0.82 \%$ | $1.80 \%$ |
|  | Volatility | $2.21 \%$ | $3.73 \%$ | $6.30 \%$ |
| 9 years | Return | $0.32 \%$ | $0.45 \%$ | $1.43 \%$ |
|  | Volatility | $2.24 \%$ | $4.16 \%$ | $6.78 \%$ |
|  | Return | $0.95 \%$ | $2.14 \%$ | $4.01 \%$ |
|  | Volatility | $2.15 \%$ | $3.69 \%$ | $5.72 \%$ |

Table 5.14 Surplus over different time horizons 1980-1997
Source: Empirical

A simulation is run to identify the surplus/deficit returns for the benchmark and two boundary portfolios. These are conducted over three time horizons and they are graphed in exhibit
5.2.

Mismatch Excess Return


Exhibit 5.2 Historic Mismatch Excess Return/Risk Ratios
Source: Empirical

From these results and assuming a lognormal distribution for the portfolio returns, the required mismatch reserve is estimated. It is decided that two different levels of $\varepsilon \%$ would be used to illustrate the sensitivity of the insolvency ruin barrier. The formulae is set out in section 5.3 and the mismatch reserve matrix is set out in table 5.15 . For the lowest risk portfolio the annual mismatch requirement would be $2.91 \%$ and the main risk of this portfolio is for the government yield curve two rise in a non-paralleled manner whereby it becomes more positively shaped.

| Ruin | Low Risk | Benchmark | High Risk |
| :---: | :---: | :---: | :---: |
| $5 \%$ | $2.91 \%$ | $5.32 \%$ | $8.56 \%$ |
| $0.5 \%$ | $4.97 \%$ | $8.80 \%$ | $14.42 \%$ |

Table 5.15 Mismatch Reserve Matrix for Alternative Portfolios
Source: Empirical

### 5.3.3 Mismatch Reserve Estimation

The model included the matched portfolio, long sector of the Irish bond market, equally weighted bond and equity markets in Ireland, UK, US, Japan, Germany and France with unhedged foreign exchange exposure. Correlation and volatilities using monthly data from 1985 to 1995 are estimated and the mismatch reserve is quantified. As in the previous section, the mismatch reserve probability function is quantified and a ruin barrier of $5 \%$ and $0.5 \%$ chosen. The simulation results are shown in Appendix Six. They range from low risk reserves for the money market of $4.6 \%$ to $30.1 \%$ for a local market asset allocation of $50 \%$ in Irish bonds and $50 \%$ in Irish equities for a $0.5 \%$ ruin barrier. There are diversification opportunities from holding international equities, but equities are very risky relative to international bonds. This time period would have included two Irish pound currency crises, 1987 equity market crash, and 1994 bond bear market.

### 5.4 Industry Analysis

In this section, the sample set is identified, and analyse of each company within the industry undertaken to determine it immunising portfolio over the sample time period. As mentioned in the previous section, the companies are analysed using the 'Insurance Annual Report' over the time period 1980 to 1997. The following companies are included in the 'Insurance Annual Report 1980' (Blue Book) and have remained in the industry over the period of investigation. The list of companies below can be taken as the sample set:

1. Church and General
2. Hibernian
3. Insurance Corporation of Ireland
4. Irish National
5. Irish Public Bodies
. PMPA
. Combined
Assicurazioni Generali
Cornhill
6. Eagle Star
11.Ecclesiastical
7. General Accident
8. Guardian Royal Exchange
9. Methodist
10. Norwich Union Fire
11. Prudential
12. Royal Insurance
13. Sun Alliance and London
19.Zurich

From the above sample set, it is important to realise that PMPA and ICI are only there due to government intervention. PMPA Insurance Company collapsed in 1983, with an accumulated deficit of IR£ 203m, but is rescued by the government. In 1989 the PMPA name, underwriting book and assets is sold to Guardian Royal Exchange for IR£87m. PMPA changed its name to Primor which continues in operation to run off the claims liabilities. Insurance Corporation of Ireland is a former subsidiary of AIB group that coilapsed in 1985 with an accumulated deficit of IR£ 266 m . The coilapse is blamed mainly on the activities in the London reinsurance market. The Central Bank at the time warned the government that ICl's losses could have put the entire Irish insurance and banking system in jeopardy if the company is not rescued.

The government intervened and an administrator is appointed to ICl , who disposed of non-core and foreign-based parts of the group. Then after restoration of $I C I$ back to financial health, the name, underwriting book and assets of ICI is sold to Assurance Generales de France (AGF) in 1990 for $\operatorname{RR} 100 \mathrm{~m}$. The underwriting liabilities are retained and the company name changed to ICARCOM, which continues in operation to run off the claims liabilities. The companies listed are included in the 'Insurance Annual Report 1980' but left the market during the period of the investigation for some reason or other:

| 1. AFIA | - exit at the end of 1985 |
| :--- | :--- |
| 2. European Fed | - exit at the end of 1985 |
| 3. Phoenix | exit at the end of 1986 |
| 4. Insurance Company of North America - exit at the end of 1987 |  |
| 5. National Employers | - exit at the end of 1988 |
| 6. Shield | - exit at the end of 1990 |
| 7. American International | - exit at the end of 1990 |

Also during the period of investigation there are entrants into the industry, which are listed as follows

| 1.Celtic | - joined 1982 |
| :--- | :--- |
| 2. F.M. | - joined 1981 |
| 3.Construction Guarantee | - joined 1984 |
| 4.AMEV General | - joined 1984 |
| 5.Universal | - joined 1984 |
| 6.Ansvar | - joined 1984 |
| 7.Chubb | - joined 1985 |
| 8.CIGNA | -jined 1985 |
| 9.Lloyds | -joined 1985 |
| 10.De Montfort | - joined 1986 |
| 11.Financial Insurance | - joined 1988 |
| 12.Electra | - joined 1989 |
| 13.ICAROM | - joined 1990 |
| 14.Mutual Blood Stock | - joined 1990 |
| 15.NEM | - oined 1990 |
| 16.Primor | - oined 1990 |
| 17.Bankers Insurance | - joined 1990 |
| 18.Veterinary Defense Society | - joined 1990 |
| 19. Eagle Star Ireland | - joined 1991 |
| 2.AAfar Insurance Limited | - jined 1992 |
| 21.AG Europe | -joined 1992 |
| 22.XL Europe | - oined 1992 |
| 23.Colonia Versicherung | - joined 1992 |

representative of the population it only contains companies that are in the industry from 1980 through to 1997. The market share held by each company is identified and how it has changed over the period. From table 5.16, the top five largest general insurance companies in the industry based on their earned premium income are as follows: Guardian Royal PMPA (20.4\%), Hibernian $(13.4 \%)$, Royal \& Sun (13.1\%), Church \& General (8.7\%) and F.B.D. (8.2\%). The top five companies in this sample set account for $64 \% \%$ for the general insurance industry. Over the period of investigation, PMPA lost approximately $10 \%$ of its market share while ICl lost approximately $1.5 \%$. In 1980 PMPA heid the greatest market share (22.4\%) and Hibernian had $14.02 \%$ but after the collapse of PMPA in 1983 Hibernian took over as market leader until the Guardian Royal PMPA merger.

|  | 1980 | 1981 | 1982 | 1983 | 1984 | 1985 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Guardian Royal PMPA | $28.13 \%$ | $27.93 \%$ | $25.51 \%$ | $26.41 \%$ | $24.28 \%$ | $22.75 \%$ |
| Hibernian | $17.63 \%$ | $18.14 \%$ | $21.08 \%$ | $22.10 \%$ | $25.88 \%$ | $24.57 \%$ |
| Royal \& Sun Insurance | $6.32 \%$ | $6.19 \%$ | $6.18 \%$ | $5.45 \%$ | $5.58 \%$ | $6.01 \%$ |
| Church \& General | $3.38 \%$ | $4.17 \%$ | $3.75 \%$ | $3.75 \%$ | $4.39 \%$ | $5.00 \%$ |
| F.B.D | $1.90 \%$ | $1.95 \%$ | $2.36 \%$ | $2.72 \%$ | $3.70 \%$ | $4.62 \%$ |
| I.C.I | $9.13 \%$ | $9.27 \%$ | $9.74 \%$ | $11.70 \%$ | $7.44 \%$ | $9.53 \%$ |
| General Accident | $10.13 \%$ | $9.78 \%$ | $9.61 \%$ | $9.44 \%$ | $8.72 \%$ | $8.38 \%$ |
| Norwich Union Fire | $7.55 \%$ | $6.56 \%$ | $5.72 \%$ | $4.52 \%$ | $5.09 \%$ | $5.02 \%$ |
| Eagle Star | $0.08 \%$ | $0.07 \%$ | $0.06 \%$ | $0.05 \%$ | $0.05 \%$ | $0.05 \%$ |
| Irish National | $8.51 \%$ | $8.26 \%$ | $7.65 \%$ | $6.63 \%$ | $6.92 \%$ | $6.57 \%$ |
| Irish Public Bodies | $1.78 \%$ | $1.83 \%$ | $2.08 \%$ | $2.12 \%$ | $2.50 \%$ | $2.89 \%$ |
| Cornhill | $0.47 \%$ | $0.50 \%$ | $0.66 \%$ | $0.84 \%$ | $1.18 \%$ | $1.27 \%$ |
| Combined | $1.34 \%$ | $1.47 \%$ | $1.70 \%$ | $1.98 \%$ | $2.13 \%$ | $1.84 \%$ |
| Assicurazioni Generaii | $2.52 \%$ | $2.90 \%$ | $2.89 \%$ | $1.30 \%$ | $0.94 \%$ | $0.87 \%$ |
| Zurich | $1.13 \%$ | $1.01 \%$ | $1.01 \%$ | $0.98 \%$ | $1.18 \%$ | $0.63 \%$ |

Table 5.16 Market Share of Each Company 1980-1985
Source : Cosgrove (1997) - Department of Industry \& Commerce

|  | 1986 | 1987 | 1988 | 1989 | 1990 | 1991 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Guardian Royai PMPA | $19.80 \%$ | $18.31 \%$ | $17.32 \%$ | $9.15 \%$ | $18.59 \%$ | $18.02 \%$ |
| Hibernian | $23.77 \%$ | $23.33 \%$ | $22.38 \%$ | $24.68 \%$ | $23.26 \%$ | $19.54 \%$ |
| Royal \& Sun Insurance | $6.25 \%$ | $6.38 \%$ | $6.85 \%$ | $7.38 \%$ | $6.99 \%$ | $6.97 \%$ |
| Church \& General | $5.40 \%$ | $6.45 \%$ | $8.29 \%$ | $9.15 \%$ | $9.01 \%$ | $8.81 \%$ |
| F.B.D | $5.12 \%$ | $5.18 \%$ | $6.23 \%$ | $7.05 \%$ | $7.02 \%$ | $6.24 \%$ |
| I.C.I | $11.26 \%$ | $10.76 \%$ | $10.35 \%$ | $10.85 \%$ | $4.83 \%$ | $9.24 \%$ |
| General Accident | $8.46 \%$ | $8.15 \%$ | $8.39 \%$ | $9.01 \%$ | $9.03 \%$ | $7.50 \%$ |
| Norwich Union Fire | $5.03 \%$ | $5.11 \%$ | $5.27 \%$ | $5.88 \%$ | $6.09 \%$ | $5.36 \%$ |
| Eagle Star | $0.05 \%$ | $0.07 \%$ | $0.09 \%$ | $0.10 \%$ | $0.09 \%$ | $0.07 \%$ |
| Irish National | $6.47 \%$ | $7.25 \%$ | $5.79 \%$ | $6.66 \%$ | $5.43 \%$ | $9.24 \%$ |
| Irish Public Bodies | $3.92 \%$ | $4.48 \%$ | $4.49 \%$ | $4.98 \%$ | $4.55 \%$ | $3.96 \%$ |
| Cornhill | $1.37 \%$ | $1.35 \%$ | $1.30 \%$ | $1.39 \%$ | $1.53 \%$ | $1.60 \%$ |
| Combined | $1.59 \%$ | $1.53 \%$ | $1.55 \%$ | $1.82 \%$ | $1.76 \%$ | $1.46 \%$ |
| Assicurazioni Generali | $0.80 \%$ | $0.87 \%$ | $0.84 \%$ | $1.02 \%$ | $1.03 \%$ | $0.98 \%$ |
| Zurich | $0.73 \%$ | $0.79 \%$ | $0.87 \%$ | $0.87 \%$ | $0.79 \%$ | $1.00 \%$ |

Table 5.17 Market Share of Each Company 1986-1991
Source : Cosgrove (1997) - Department of Industry \& Commerce

|  | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Guardian Royal PMPA | $17.60 \%$ | $18.52 \%$ | $16.37 \%$ | $16.44 \%$ | $22.87 \%$ | $20.39 \%$ |
| Hibernian | $20.25 \%$ | $21.59 \%$ | $17.86 \%$ | $16.99 \%$ | $15.38 \%$ | $13.39 \%$ |
| Royal \& Sun Insurance | $7.52 \%$ | $0.47 \%$ | $5.68 \%$ | $5.76 \%$ | $4.95 \%$ | $13.05 \%$ |
| Church \& General | $9.95 \%$ | $9.70 \%$ | $8.32 \%$ | $8.50 \%$ | $8.32 \%$ | $8.67 \%$ |
| F.B.D | $7.10 \%$ | $8.22 \%$ | $7.47 \%$ | $8.00 \%$ | $8.23 \%$ | $8.25 \%$ |
| I.C.I | $9.28 \%$ | $10.90 \%$ | $9.87 \%$ | $10.11 \%$ | $8.62 \%$ | $7.92 \%$ |
| General Accident | $7.51 \%$ | $8.06 \%$ | $7.09 \%$ | $7.53 \%$ | $6.92 \%$ | $6.52 \%$ |
| Norwich Union Fire | $6.52 \%$ | $7.52 \%$ | $6.73 \%$ | $6.89 \%$ | $6.40 \%$ | $5.53 \%$ |
| Eagie Star | $0.08 \%$ | $0.07 \%$ | $7.70 \%$ | $6.96 \%$ | $6.66 \%$ | $5.52 \%$ |
| Irish National | $5.75 \%$ | $6.55 \%$ | $5.59 \%$ | $5.23 \%$ | $4.31 \%$ | $3.64 \%$ |
| Irish Public Bodies | $3.86 \%$ | $3.86 \%$ | $3.14 \%$ | $3.11 \%$ | $2.96 \%$ | $2.70 \%$ |
| Cornhill | $1.52 \%$ | $1.35 \%$ | $1.24 \%$ | $1.65 \%$ | $1.72 \%$ | $1.86 \%$ |
| Combined | $1.48 \%$ | $1.55 \%$ | $1.31 \%$ | $1.36 \%$ | $1.53 \%$ | $1.59 \%$ |
| Assicurazıoni Generali | $0.99 \%$ | $1.47 \%$ | $1.37 \%$ | $1.27 \%$ | $1.08 \%$ | $0.90 \%$ |
| Zurich | $0.60 \%$ | $0.18 \%$ | $0.25 \%$ | $0.20 \%$ | $0.05 \%$ | $0.06 \%$ |

Table 5.18 Market Share of Each Company 1992-1997
Source : Cosgrove (1997) - Department of Industry \& Commerce

| RANK | COMPANY | OVERALL | ANNUAL | CURRENT |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Guardian Royal PMPA | $-7.74 \%$ | $-0.43 \%$ | $20.39 \%$ |
| 2 | Hibernian | $-4.24 \%$ | $-0.24 \%$ | $13.39 \%$ |
| 3 | Royal \& Sun Insurance | $6.73 \%$ | $0.37 \%$ | $13.05 \%$ |
| 4 | Church \& General | $5.30 \%$ | $0.29 \%$ | $8.67 \%$ |
| 5 | F.B.D | $6.35 \%$ | $0.35 \%$ | $8.25 \%$ |
| 6 | I.C.I | $-1.22 \%$ | $-0.07 \%$ | $7.92 \%$ |
| 7 | General Accident | $-3.61 \%$ | $-0.20 \%$ | $6.52 \%$ |
| 8 | Norwich Union Fire | $-2.02 \%$ | $-0.11 \%$ | $5.53 \%$ |
| 9 | Eagle Star | $5.44 \%$ | $0.30 \%$ | $5.52 \%$ |
| 10 | Irish National | $-4.87 \%$ | $-0.27 \%$ | $3.64 \%$ |
| 11 | Irish Public Bodies | $0.92 \%$ | $0.05 \%$ | $2.70 \%$ |
| 12 | Cornhill | $1.39 \%$ | $0.08 \%$ | $1.86 \%$ |
| 13 | Combined | $0.25 \%$ | $0.01 \%$ | $1.59 \%$ |
| 14 | Assicurazioni Generali | $-1.62 \%$ | $-0.09 \%$ | $0.90 \%$ |
| 15 | Zurich | $-1.07 \%$ | $-0.06 \%$ | $0.06 \%$ |

Table 5.19 Market Share of Sample Set
Source : Cosgrove (1997) - Department of Industry \& Commerce

In table 5.19 , the focus is on the market share of the companies in the sample set rather than the total current industry. The changes in market share have not been significant with growth accounted for by mergers.

### 5.5 Mismatch Returns Performance

### 5.5.1 Comparing the Mismatch Return Within the Industry

In this section, the return of the immunised porffolio is estimated and the mismatched return for each company in each year of the sample period. For certain companies such as Cornhill, Eagle Star, Ecclesiastical, Methodist and Prudential, it is found to be difficult to estimate the actual return with a reasonable degree of confidence. It is possible to estimate the matched return for all companies. The average and standard deviation of actual returns for actual and matched returns are estimated.

It is interesting to observe how the matched returns of the industry over the eighteen years is between $12 \%$ and $20 \%$ and the risk of the portfolios is clustered between $4 \%$ and $7 \%$. The actual returns are considerably more scattered in relation to the matched returns. Some companies such as ICI, PMPA, Irish Public Bodies, Combined and Assicurazioni Generali would seem to have been better served if they had immunised their liability portfolios.

It is interesting to note the increased risk that they took to achieve greater returns by mismatching and investing in Irish equities and property. Church and General and Hibernian seem to have been very successful while others had considerable volatility in investment returns with a relatively modest increase in return.

Because of the different liability profiles of the companies in the industry, the incremental return for each company is estimated relative to its matching portfolio each year and the volatility or risk of such returns. In the case of ICI, Combined and Irish National, the finding suggests that they would have been better suited to matching (i.e. immunising) their portfolios.


Exhibit 5.3 Actual Outperformance relative to Matched Portfolio
Source: Empirical

In table 5.20 , the risk reward ratio is estimated by comparing the increase in percentage return for the risk taken to achieve the value added from mismatching. There is a marked contrast between Church \& General and Hibernian where the ratio exceeds one and on the downside Irish National that had a negative ratio in excess of minus one over the entire sample period.

| COMPANY | RISK | RETURN | RATIO |
| :---: | :---: | :---: | :---: |
| Church \& General | $20.48 \%$ | $5.46 \%$ | 3.75 |
| F.B.D | $13.55 \%$ | $5.97 \%$ | 2.27 |
| Hibernian | $19.12 \%$ | $6.06 \%$ | 3.16 |
| I.C.I | $11.00 \%$ | $6.61 \%$ | 1.66 |
| Irish National | $8.04 \%$ | $4.35 \%$ | 1.85 |
| Jrish Public Bodies | $12.10 \%$ | $6.55 \%$ | 1.85 |
| Guardian Royal PMPA | $17.46 \%$ | $11.53 \%$ | 1.51 |
| Combined | $\mathbf{4 . 0 5 \%}$ | $\mathbf{4 . 0 2 \%}$ | 1.01 |

Table 5.20 Risk/Reward Ratio of Actual Portfolios 1980-1997
Source: Empirical

In every year that is analysed, there has been a cross subsidisation of poor underwriting by good investment returns. A period has not occurred in which there is a downturn in the investment returns cycle and underwriting cycle to confirm research by Daykin and Bernstein (1985) which suggests that the cycles can be inversely related.

Between 1980 and 1997 the industry (excluding companies for whom underwriting results are unknown) has had premium income of $£ 7,128 \mathrm{~m}$ (in historical terms) on which it had underwriting losses of $£ 1,204 \mathrm{~m}$ but an investment income of $£ 1,539 \mathrm{~m}$ and capital gains of $£ 313 \mathrm{~m}$. Over the past five years the premium income is $£ 3,109 \mathrm{~m}$ with underwriting losses of $£ 331 \mathrm{~m}$ and investment income of $£ 661 \mathrm{~m}$ with capital gains of $£ 269 \mathrm{~m}$ which is a margin of safety cover of $54 \%$.

### 5.5.2 Impact of the value of Under \& Out-performance

It is assumed that firms achieved the matched return for those years in which it is not possible to identify the actual return.. This is performed over the periods 1980-97and in table 5.21.

|  | 1980 | 1997 | Return | Return | Deficit |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | INVESTMENTS | INVESTMENTS | MATCHED | ACTUAL | SURPLUS/ |
| Church \& General | $£ 8,064$ | $£ 331,059$ | $£ 38,507$ | $£ 322,995$ | $£ 292,552$ |
| Combined | $£ 2,525$ | $£ 17,429$ | $£ 12,057$ | $£ 14,904$ | $£ 5,372$ |
| F.B.D | $£ 5,583$ | $£ 297,535$ | $£ 26,660$ | $£ 291,952$ | $£ 270,875$ |
| General Accident | $£ 28,562$ | $£ 161,140$ | $£ 136,389$ | $£ 132,578$ | $£ 24,751$ |
| Hibernian | $£ 61,498$ | $£ 460,141$ | $£ 286,274$ | $£ 398,643$ | $£ 173,867$ |
| I.C.I | $£ 59,950$ | $£ 349,315$ | $£ 293,666$ | $£ 289,365$ | $£ 55,649$ |
| Irish National | $£ 29,560$ | $£ 113,838$ | $£ 141,155$ | $£ 84,278$ | $-£ 27,317$ |
| Irish Public Bodies | $£ 10,126$ | $£ 246,600$ | $£ 48,354$ | $£ 236,474$ | $£ 198,246$ |
| P.M.P.A | $£ 69,531$ | $£ 819,930$ | $£ 332,025$ | $£ 750,399$ | $£ 487,905$ |
| Zurich | $£ 4,071$ | $£ 24,354$ | $£ 19,440$ | $£ 20,283$ | $£ 4,914$ |

Table 5.21 Profit \& Loss Mismatching Contribution
Source : Department of Industry \& Commerce \& Empirical data

The capital is accumulated for the matched and actual returns with the deviance been translated into a profit or loss added value. Investments are assumed equal to the actual return and imputed the value of the matched return investments if such a policy had been pursued. In certain firms such as Cornhill or Prudential there seemed to be problems that are likely to be associated with capital injections in the overall period. It is also assumed that in the case of companies that did not split asset classes, that the current assets could be broken into cash and debtors by using the industry average. During the more recent five years the problem seemed to have reversed in the case of such firms and they could be withdrawing from the market with capital reductions. The investment operation has seemed to add considerable value in firms like Church \& General $+£ 293 m$, FBD $+£ 271$ m, Hibernian $+£ 174 m$.

In the case of General Accident, it underperformed by $£ 11.48 \mathrm{~m}, \mathrm{ICl}$ by $£ 34 \mathrm{~m}$, Irish National by $£ 7.09 \mathrm{~m}, \mathrm{PMPA}$ by $£ 24.46 \mathrm{~m}$, Royal Insurance by $£ 21.12 \mathrm{~m}$ and Sun Alliance by $£ 4 \mathrm{~m}$ over the past five years. There are other possible reasons to explain these variances such as the often mentioned reduction in capital, mergers \& reorganisations or being run down due underwriting losses. It has not been possible to eliminate these considerations from the analysis of investment performance.

### 5.5.3 Size of Investment Funds

Investment income has only covered underwriting losses by $2.25 \%$ annually over the entire sample period for Irish resident insurers and in the context of the volatility of investment returns it would be prudent to carry an explicit mismatch reserve. In table 5.22 , the asset breakdown of the industry is shown for those that are declared and using the ratios of premium incomes made an assumption about the size of the rest of the industry.

| Asset | Known | Industry (Investments) | Industry (Premium) |
| :---: | :---: | :---: | :---: |
| Bonds | $£ 1,722 \mathrm{~m}$ | $£ 2.389 \mathrm{~m}$ |  |
| Equities | $£ 533 \mathrm{~m}$ | $£ 739 \mathrm{~m}$ |  |
| Property | $£ 82 \mathrm{~m}$ | $£ 113 \mathrm{~m}$ |  |
| Cash | $£ 468 \mathrm{~m}$ | $£ 649 \mathrm{~m}$ |  |
| Total | $£ 2,805 \mathrm{~m}$ | $£ 3,891 \mathrm{~m}$ | $1,117 \mathrm{~m}$ |

Table 5.22 Investments of General Insurance Industry 1997
Source : Cosgrove (1997) - Department of Industry \& Commerce

In the context of the Irish government treasury market alone that is $£ 17 \mathrm{bn}$ the general insurance industry hoids about $10 \%$ of all issues by the Irish government. While it is known that investments must be at least $£ 2.8$ bn from the data in the Blue books, some investments are held under current assets the estimated bond holdings is revised up to be probably of the order of £3bn.

Over the past eighteen years the industry has increased its weighting in bonds by $13 \%$ and equities by $6 \%$, and reduced its exposure to property by $-10 \%$ and cash by $-8 \%$. It is not possible to tell from the Blue books whether these asset classes are in foreign currencies but exchange controls existed till 1990 along with a requirement for localisation of assets by the Department of Enterprise and Employment.

### 5.5.4 Returns on Shareholders Funds

This is difficult to estimate for the industry because of the defaults by ICI and PMPA and capital injections by the Irish government and changes in the Blue book returns in 1993. The sample set is reduced to Church \& General, Combined, FBD, Hibernian, ICI, Irish National, Irish Public Bodies, Guardian Royal PMPA and Zurich.

Shareholders return are defined as the Net Transfers to Reserves, Profit Retained and Dividends and shareholders funds as capital issued, reserves and the profit and loss account. The return of the matched return is subtracted to isolate the increased return over the asset matching risk for the shareholders by exposing themselves to the underwriting cycle. Two time periods are selected to gauge the overall performance and recent experiences. This is shown in table 5.23 .

Over the 1980-92 period the industry average shareholders funds had a return of $18.76 \%$ and the underwriting generated an additional return of $4.92 \%$ over the investment return. The recent five years show an improved return of $20.93 \%$. However, some companies such as Irish National, Irish Public Bodies, PMPA and Zurichs' shareholders would have seen better results by investing into the matching portfolio directly and not taking any exposure to the insurance industry.

| Company | 1980-92 | Shareholders | $1980-92$ | Insurance | $1988-92$ | Shareholders | 1988-92 | Insurance |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Return | Risk | Return | Risk | Return | Risk | Return | Risk |
| Church \& General | $18.89 \%$ | $13.54 \%$ | $5.02 \%$ | $15.88 \%$ | $22.68 \%$ | $10.80 \%$ | $12.14 \%$ | $10.82 \%$ |
| Combined | $27.88 \%$ | $9.14 \%$ | $14.33 \%$ | $8.54 \%$ | $28.82 \%$ | $11.50 \%$ | $18.08 \%$ | $10.81 \%$ |
| FBD | $35.00 \%$ | $33.06 \%$ | $21.12 \%$ | $34.70 \%$ | $66.17 \%$ | $26.73 \%$ | $55.61 \%$ | $26.11 \%$ |
| Hibernian | $24.88 \%$ | $12.98 \%$ | $10.98 \%$ | $12.12 \%$ | $19.52 \%$ | $17.82 \%$ | $8.93 \%$ | $17.87 \%$ |
| ICI | $24.16 \%$ | $30.21 \%$ | $10.35 \%$ | $30.47 \%$ | $29.93 \%$ | $36.99 \%$ | $11.44 \%$ | $37.17 \%$ |
| Irish National | $2.68 \%$ | $26.17 \%$ | $-11.17 \%$ | $27.24 \%$ | $10.23 \%$ | $9.72 \%$ | $-0.35 \%$ | $10.85 \%$ |
| Irish Public Bodies | $15.35 \%$ | $15.92 \%$ | $1.50 \%$ | $13.85 \%$ | $8.48 \%$ | $10.96 \%$ | $-2.00 \%$ | $9.67 \%$ |
| PMPA | $13.55 \%$ | $34.48 \%$ | $-0.37 \%$ | $34.39 \%$ | $10.26 \%$ | $28.05 \%$ | $-0.33 \%$ | $27.15 \%$ |
| Zurich | $6.45 \%$ | $30.88 \%$ | $-7.46 \%$ | $30.16 \%$ | $0.28 \%$ | $25.90 \%$ | $-10.30 \%$ | $24.39 \%$ |
| Industry | $18.76 \%$ | $22.93 \%$ | $4.92 \%$ | $23.04 \%$ | $20.93 \%$ | $19.83 \%$ | $10.36 \%$ | $19.43 \%$ |

Table 5.23 Shareholders Overall \& Underwriting Returns
Source : Department of Industry \& Commerce \& Empirical data

Earned premium income is divided by capital (i.e. shareholders funds) for each company by year since 1980. The industry average is $£ 2.33 \mathrm{~m}$ premium written per $£ 1 \mathrm{~m}$ capital with 1997 been one of the highest year on record. The most aggressive company seems to be FBD that writes $£ 3.304 \mathrm{~m}$ while Combined only writes $£ 1.628 \mathrm{~m}$. If rish National, Irish Public Bodies, PMPA and Zurich are to withdraw their capital of $£ 88 \mathrm{~m}$, this would reduce underwriting capacity by $£ 173 \mathrm{~m}$ to $£ 203 \mathrm{~m}$. Premium levels could be expected to rise by at least three per cent. The major firms would seem to be secure for the foreseeable future.

### 5.5.5 Value of Claims Paid

Claims settlement over the review period is approximated by adding the appropriate technical reserves and claims paid for each year. While earlier analysis indicates that claim inflation exceeded general inflation by c. $3 \%$ depending up on the class of business, it is conservatively assumed that the claims settlement would be the C.P.I over the period.

| Year | Original Claims Paid | Current Claims Paid |
| :---: | :---: | :---: |
| 1980 | £324,739 | £995,921 |
| 1981 | £411,704 | £1.159,439 |
| 1982 | £505,609 | £1,312,280 |
| 1983 | £600.513 | £1,426,034 |
| 1984 | £753,662 | $£ 1,643,449$ |
| 1985 | £827,672 | £1,657,332 |
| 1986 | £763,551 | £1,403,983 |
| 1987 | £838,861 | £1,416,400 |
| 1988 | £863,685 | £1,339,132 |
| 1989 | £927,648 | £1,320,758 |
| 1990 | £1,026,798 | £1,342,447 |
| 1991 | £1,212,775 | £1,456,010 |
| 1992 | £968,596 | £1,067,822 |
| 1993 | £1,153,298 | £1,253,771 |
| 1994 | £1,220,653 | £1,296,944 |
| 1995 | £1,055,314 | £1,093,537 |
| 1996 | $£ 1,129,814$ | £1,152,410 |
| 1997 | £1,230,749 | £1,230,749 |
| Total | $£ 15,815,641$ | £23,568,417 |

Table 5.24 Size of Gross Claims Settlement
Source : Department of Industry \& Commerce \& Empirical data

On an adjusted basis, the industry has paid $£ 15.82$ bn by 1997 and if when adjusted for inflation the value is about $£ 23.57$ b. In 1997 the national debt is $£ 28$ b. The run off for 1997 could be as high as c . $£ 4 \mathrm{bn}$ with investments at $£ 1.41$ bn and needing to be increased by $£ 653 \mathrm{~m}$.

While it can be criticised as an unreasonable comparison, the growth in this industry that redistributes risk is having a substantial negative impact on the Irish economy. Between 1980 and 1997, the share of claims settlement by domestic insurers has risen from $26 \%$ to $50 \%$.

### 5.6 Present Structure and Future of Insurance Market

Because of matching requirements, the general insurance investor's pool has continued to be a captive market even after exchange controls are removed. The regulators required localisation of assets and prescribed broad asset class headings, but in a regime of exchange controls international asset diversification is not an issue. This will cease to be the case when Ireland joins the Euro in 1999.

The population of all lrish insurance companies is identified and their individual importance in the market is identified. The nature of the liabilities is investigated and estimated the duration and liability of each individual class over the eighteen years. The top three classes are Motor, Liability and Property that accounted for $97 \%$ of all insurance underwritten. Over the entire sample period, the claims increase is compared to general inflation for each class and the claims rose in excess of inflation by c.3\% every year on average since 1980.

The only class of insurance that has fluctuated as a percentage of the overall business written has been Treaty that is very small at $2.82 \%$. The liability profile is estimated for each company since 1980 and the asset allocation of the matching portfolio identified. The risk/reward characteristics is investigated for each asset class over the sample period and the investment performance of each company and it's matching portfolio.

The performance of the matched is compared against the mismatched portfolio and the contribution of the mismatch return. The sample period is then subdivided into two and three periods to see if there is a difference in company's performance. Companies should know the portfolio that is expected to match their liabilities with its inherent risk versus reward characteristics. The annualised size of the mismatch reserve should be published along with duration of their liabilities in the Blue book.

An area of concern is the cross subsidisation of underwriting by investment performance and the risk of insolvency due to further increased mismatching. Against the background of the lifting of exchange controls and increasing pressure on management to increase returns, substantially greater investment risk may be taken than in the past. This raises the serious spectre of a third insurance company becoming insolvent.

This work could be developed if access is given to the form 8's for each company for each year over the past eighteen years for each class of business. Their individual experiences of claim inflation would be helpful along with knowledge of any material reinsurance's experiences. In relation to asset management, capital injections or disbursements, tax problems such as withholding tax on dividends or deposit interest retention tax and changes in asset allocation policies between Blue book publications.

### 5.7 Summary \& Conclusions

While a considerable amount of research has been dedicated to analysing management of either asset or llability portfolios, little work has been done on the interaction of assetliability management. It is this area that this chapter sought to address in relation to the Irish general insurance industry whose matching portfolio has always being determined by the Irish term structure. Some of the companies would have had a superior investment performance if they had matched their liabilities with Irish government bonds. There is also a substantial cross subsidisation between good investment returns and poor underwriting results. This means that the industry and its regulator that has prescribed mismatch reserving for the life assurance should consider mismatch reserving in the context of general insurance. Although general insurance companies do not discount liabilities, the duration and present value is estimated for liabilities and that of the matching portfolio consisting of the three month money market and the Riada Short bond index.

The mismatch reserve model uses the approach of contingent claims analysis (CCA) whereby the mismatch reserve is valued as an at the money call option on the relative outperformance of the mismatched portfolio against the matched portfolio for a particular time horizon. The time horizon should be the same as the review period by the asset/liability committee of the asset management performance. One company is chosen and analysed in detail. The benchmark asset allocation which is not a matched asset portfolio is chosen for a given mismatch reserve and limits are placed on the asset allocation consistent with risk/ruin theory, the size of the mismatch reserve and desire of the asset/liability committee to take greater investment risk than the matched portfolio. Information is confined to that of the Blue books and some general assumptions had to be made about different companies and the time period

The is very strong evidence to suggest that the insurance industry takes investment decisions which are much riskier than their liabilities require. In some cases the companies would have been better off by matching their assets to their liabilities. At a minimum, the regulator should consider introducing a requirement for companies that mismatch to carry additional reserves to reflect their increased investment risk in a similar fashion to the life insurance sector. In the event of a downturn in investment performance, there will be upward pressure on premium levels.

## Summary and Conclusions

### 6.1 Introduction

In this dissertation, the Irish term structure is identified and estimated. The stochastic process through which Irish interest rates evolve is estimated and different factor models are tested on Irish data. From this analysis the price discovery process of the term structure is investigated to see whether a primary dealership market microstructure is viable relative to the agency market microstructure. Finally, the behaviour of Irish general insurers is analysed in terms of their historical investment performance in Irish bond markets.

### 6.2 Summary

The dataset used comprised Irish government bond prices from 1980 to 1997. All data had to be sampled, collated and confirmed with the transactions passing through the Stock Exchange and the ante-sample and post-sample points.

Different approaches taken to term structure identification are examined and criticised including; yield to maturity, discrete estimation of the term structure, polynomial approximations and polynomial splines, $B$ splines and exponential splines. Since it is not possible to estimate the discount function directly in the Irish case, it is identified indirectly by bootstrapping the discount factors from the existing bonds. This fitted curve is constrained to generate a non-singular cash flow matrix.

After the term structure had been identified and estimated from 1980 to 1997 , the behaviour of the stochastic process governing Irish spot rates is investigated. The aim is to identify the risks facing investors in Irish bonds and the most plausible model of the stochastic process of the lrish term structure during this period. The background to stochastic processes and the different attempts to model the term structure stochastic process are reviewed. This is followed by modelling the dynamics of the term structure and the orthogonality proposition of the spread process for the Irish term structure.

The microstructure of the existing agency system of transacting is investigated. The hypothesis to be tested is whether a competitive dealership market could be supported and would be preferable to the existing agency microstructure. The findings favoured a competitive market due to: a) provision of immediacy,
b) price transparency,
c) limit on size of spreads,
d) capitalised primary dealer system recognised by NTMA,
e) NTMA allied commitment to develop REPO market,
f) minimum market depth leading to increased liquidity.

The investment performance of Irish general insurers is investigated. To do this a framework is developed in which managers attempt to maximise the value of the funds under management. subject to a minimum terminal value. The performance of the companies under such a strategy is compared with their actual achievements and those that would have occurred if their portfolios had been immunised. The performance is found to be highly varied, so important implications for the insurance industry can be drawn. These implications are that the framework developed in chapter five should be adopted and mismatching from the immunising portfolio should be tightly controlled.

### 6.3 Conclusions

There have been cycles in debt maturity and duration that have been in a maturity range of 5 years to 9 years and a duration range of 3.28 years to 4.88 years respectively. The average duration has been 3.94 years over this period. On examination of Irish government treasury market it is found that the authorities had funded at the shorter maturities when yields are high in the early 1980's. This resulted in observations at the sample points being clustered for short maturities.

The $B$ spline model specifying five degrees of freedom with knots at a maturity of one and five years is chosen to fit the lrish yield curve because it had the lowest residual deviance and is superior to both no knots and a knot placed at a maturity of one year. A third knot is excluded since the additional explanatory power is marginal. There is a significant difference between the money market up to one year maturity and the bond market beyond. When outtiers are identified and removed, the size of the residual deviance is reduced by up to $90 \%$.

The parameters of the fitted yield spline and the discount function are estimated empirically and tabulated since these results have not been achieved before in the case of Irish government treasury market. When the estimated term structure is used to value bonds from 1980 to 1997 and compared to the actual market prices the results are very good even with the originally identified outliers.

The first three factors explain more than $99 \%$ of the term structure movement. The first factor implies a parallel shift of the term structure, the second factor implied a change in the slope of the term structure and the final factor implies a change in the curvature of the term structure. Only three factors would be needed to explain the stochastic process of lrish interest rates. Heteroscedasticity is a problem and eliminates the use of single factor models to model the stochastic process. There is evidence of autocorrelation up to four lags in the case of the short and long rates. The hypothesis that changes of the short rate, long rate and spread are normally distributed could not be rejected. However, the kurtosis figure is greater than the three we would expect in the case of a normal distribution. The parameters of the stochastic process are estimated.

The conclusion is that there are excess reserve profits earned in the agency system above those required that have not been eroded by movements in the labour market or entry of new firms. The capital requirement in a primary dealing structure would be $£ 25 \mathrm{~m}$ to have the capacity for turnover that would be required to compete with other European markets. A market maker would have to capture a mean spread of 5 pence per $£ 100$ nominal to stay in business in the long run. The normal distribution with a mean of $£ 34,500$ and a standard deviation of $£ 302,000$ is the most appropriate distribution for modelling a Primary Dealer daily profit distribution. On a daily basis, the probability of making a loss is $38.46 \%$.

The NTMA stated that it would consider the primary dealing system to have failed if the number of primary dealers fell below four. By assuming that the primary dealers would withdraw if they failed to cover the costs over a trading year, then the risk of the primary dealing system failing is $2.8 \%$. The important profit distribution parameters are market share, earned spreads and volatility of the term structure.

While a considerable amount of research has been dedicated to analysing management of either asset or liability portfolios, little work has been done on the interaction of asset/liability management. The duration and present value is estimated for insurance liabilities and those of a matching asset portfolio consisting of the three month money market and the Riada Short bond index. The top three insurance classes are Motor, Liability and Property that accounted for $97 \%$ of all insurance underwritten.

The performance of the matched against the mismatched portfolio and the contribution of the mismatch return is examined with very mixed results. The sample period is then subdivided into two and three periods to see if there is a difference in company's performance. An area of concern is the cross subsidisation of underwriting by investment performance and the risks of insolvency due to further increased mismatching.

### 6.4 Areas for further research

The spot rates could be estimated monthly between 1980 and 1997 rather than on a semi-annual basis. Then a generalised additive model approach could be set up to investigate bond pricing errors in the discount function at each sample point. In chapter three the embedded option of outlier bond could be reverse engineered. The volatility of the term structure of the rest of Europe could be included in the analysis to identify cross currency influences. A Generalised Autocorrelation Conditional Heteroschedasticity (i.e. G-ARCH) model should be used in relation to the modelling of the time series of the volatility of the term structure.

In relation to the microstructure, the building of a database along the lines of the time stamped data series in the CRSP in the US would be very helpful. This would allow a more detailed study of the behaviour of the market on an intra-day basis and for event studies like funding decision by the authorities.

For mismatch reserving, if the data is available on a quarterly basis within the company, this would bring the sample series up to fifty two sample points rather than the thirteen that is used for modelling. Fifty two points are available when identifying the probability distribution of asset returns. A simple linear correlation process between asset classes is assumed, but it may be appropriate to investigate whether there is a need for a GARCH model that can handle changing variance and correlations.

Other methods than the chain ladder to estimate liability run offs should be used and compared to see if the results are superior. A time series model would be interesting to see the changes in frequency and severity of the Irish experience over the past thirteen years. A cross comparison of the individual components of a claim cost across the EC would be helpful to establish benchmarks of the most efficient approach to controlling this element of the cost.

## Appendix 1

Data for Term Structure Identification 1980-1997



|  | Trade $\quad 15 \mathrm{Apr}$-81 |  |  |  |  |  |  |  | Clean |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Sett: 21-Apr 81 | Coupon | Nomunal Issue | Market Price | $\begin{gathered} \text { Market } \\ \text { Yield } \end{gathered}$ | Volaulity | Duraton | Lite | Markel Value ( R Rem) | Market <br> Value <br> (IREm) | $\begin{aligned} & \text { Weight } \\ & \text { in } \\ & \text { Index } \end{aligned}$ | Weighted Volatility | Weighted Duration | Werghted Lite | $\begin{aligned} & \text { Ex-Div } \\ & \text { Date } \end{aligned}$ | Accrued Interest | Accrued Interest | $\begin{gathered} \text { First } \\ \text { Redempton } \end{gathered}$ Date | $\begin{gathered} \text { LasI } \\ \text { Redemption } \end{gathered}$ | Redempton |
|  | IR.FINANCE VAR\% 1983 | 14 16\% | 500 | 100.05 | 13654\% | 014 | 0140 | 0.11 | 5026 | 51.014 | 149\% |  |  |  |  |  |  |  |  |  |
|  | IR.EXCHEQR 10 \% 1981 | 1000\% | 1200 | 99.42 | 15594\% | 091 | 0114 | 011 | 119301 | 123934 | 363\% | 000 | 0.002 | 000 | 01 Mar 81 | 51 | 198 | ${ }^{01}-\operatorname{Sep} 83$ | 01 Sep.83 | 01 Sep 83 |
|  | IR FINANCE 11 12\% 1981 | 1150\% | 1600 | 99.05 | 14 415\% | 034 | 0367 | 036 | 158475 | 161045 | 472\% | 000 | 0004 | 000 | 01 Dec. 80 | 141 51 | 38 | 01. Jun 81 | ${ }^{01}$ Jun 81 | 01- Jun 81 |
|  | IR EXCHEOR 11 112\% 1982 | 1150\% | 800 | 9837 | 13934\% | 072 | 0770 | 078 | 78697 | 80687 | 236\% | 002 | 0018 | 002 | 01 Mar 81 | 51 | 161 | $01 . \operatorname{Sep} 81$ | 01 Sep 81 | 01 Sep 89 |
|  | IR FINANCE 10 12\% 1982 | 10 50\% | 1000 | 97.27 | 13810\% | 090 | 0957 | 098 | 97267 | 97440 | $266 \%$ | 003 | 0027 | 003 | 15 Apr 81 | 79 | 249 | 01 Feb 82 | 01 Feb 82 | 01 Feb 82 |
|  | [R NATION 9 14\% 1982 | 925\% | 980 | 86.68 | 23799\% | 102 | 1.141 | 1.19 | 84950 | 87680 | 257\% | 003 | 0029 | 003 | 15 Apr 81 | ${ }^{6}$ | 017 | 15 Apr 82 | 15-Apr 82 | 15 Apr 82 |
|  | IRCONVER 9\% 1980/82 | 900\% | 1510 | 94.35 | 13979\% | 125 | 1340 | 140 | 142469 | 143846 | 422\% | 005 | 0056 | 006 | 01-Jan 81 | 110 | 279 | 01-Jul 82 | 01-Jul 82 | 01 -Ju 82 |
|  | IR FUNDING 11 3/4\% 1983 | $1175 \%$ | 800 | 96.75 | 1402\% | 166 | 1780 | 194 | 71397 | 77938 | 228\% | O0s | 0056 | 006 | 15 Mar 81 | 37 | 091 | 15 Sep 8 | 15-Sep-82 | ${ }^{15} 5$ Sep. 82 |
|  | IR FUNDING 11 12\% 1983 | 1150\% | 98.0 | 9571 | 13960\% | 209 | 2240 | 253 | 93800 | 93492 | 274\% | 006 | 0041 | 004 | 31 Mar 81 | 1 |  | 31-Ma 83 | 31 Max 8 | ${ }^{31} \mathrm{Max} 83$ |
|  | IR FINANCE 12\% 1984 | 1200\% | 800 | 95.73 | 14331\% | 225 | 2413 | 278 | 76584 | 78661 | 231\% | 006 | 0061 | 007 | 01 May 81 | 10 | 031 | 01-Nov 83 | $01 . \operatorname{Hov}$ 日3 | 01 - Nov $\mathrm{Bl}^{\text {3 }}$ |
|  | IRCONVER 13\% 1984 | 1300\% | 980 | 97.91 | 14069\% | 249 | 2663 | 315 | 95952 | 98219 | 288\% | O09 | 0056 | 006 | $0^{01-F e b-81}$ | 79 | 260 | 01 Feb 84 | 01. Feb 84 | $01 .-\mathrm{Feb}$-84 |
|  | IR FINANCE $11314 \% 1984$ | 1175\% | 980 | 95.02 | 14.175\% | 260 | 2788 | 332 | 93115 | 95165 | 279\% | 007 | 0.7 | 009 | 15-Feb-81 | 65 | 231 | 15-Jun-84 | 15.Jun 84 | 15.Jun. 84 |
|  | IR NATION. 5 114\% 1979/84 | 5.25\% | 290 | 82.45 | 12.179\% | 300 | 3.185 | 3.57 | 23886 | 23.786 | 0.70\% | 002 | 0078 | 009 | 15 Feb 81 | 65 | 209 | 15-Aug-84 | 15-Aug.84 | 15 Aug.84 |
|  | IR NATION 14\% 1985 | 1400\% | 1308 | 94.47 | 16.719\% | 280 | 3.033 | 390 | 123563 | 125418 | $368 \%$ | 010 | 0022 | 002 | 15 May 81 | 24 | d | 15 Nov-79 | 15.Nov. 84 | 15.Nov 84 |
|  | IR EXCHEQR $12 \% 1985$ | 1200\% | 1308 | 95.04 | 14 173\% | 304 | 3251 | 407 | 124316 | 123285 | 361\% | 01 | 0117 | 014 | 15 Mar 81 | 37 | 142 | 15 Mar 85 | 15 Max 85 | 15 Mar 85 |
|  | IREXCHEOR 6 \% 1980/85 | 600\% | 715 | 81.60 | 12266\% | 364 | 3861 | 462 | 58385 | 60042 | 1.76\% | 006 | 017 | 015 | 15May 81 | 24 | 9 | 15-May-85 | 15 May 85 | 15 May 85 |
|  | IR NATION $71 / 12 \% 1981 / 85$ | 7.50\% | 591 | 84.33 | 12776\% | 384 | 4088 | 520 | 49838 | 51.173 | $150 \%$ | 006 | 0068 | 008 | 01-Dec 80 | 141 | 232 | 01- Dec-80 | 01 Dec 85 | 01 Dec 85 |
| $\omega$ | IR NATION 5 3/4\% 198287 | 5.75\% | 214 | 74.66 | 13107\% | 452 | 4821 | 649 | 15990 | 16010 | 047\% |  | 001 | 008 | 01.Jan-81 | 110 | 226 | 01.Jul 81 | $01 .-\mathrm{Ju} 86$ | 01-Ju1 86 |
|  | IR CONVER $81 / 2 \%$ 1986/98 | $850 \%$ | 602 | 80.91 | 14884\% | 422 | 4530 | 679 | 48741 | 49849 | $146 \%$ | 006 | 0.023 | 003 | 15 Apr 81 | 6 | 009 | 15-04182 | 15 Cat 87 | 15.04187 |
|  | IR NATION. 93/4\% 1984/89 | 975\% | 802 | 8457 | 15068\% | 452 | 4859 | 828 | 67818 | 69509 | 204\% | 006 | 0066 | 010 | 01 Feb 81 | 79 | 184 | 01-feb-86 | 01 Feb-88 | 01 Feb 88 |
|  | IR EXCHEOR 5 3/4\% 1984/89 | 575\% | 263 | 69.23 | 14256\% | 495 | 5298 | 854 | 18238 | 18196 | 053\% | 009 | 0099 | 017 | 01 Feb 81 | 79 | 211 | 01 Aug 84 | 01 Aug 89 | 01 Aug 89 |
|  | IR NATION 14\% 1985/90 | 1400\% | 1308 | 96.97 | 15287\% | 471 | 5067 | 890 | 126831 | 128686 | 377\% | 018 | 0028 | 005 | 01 May 81 | ) | 016 | 01- Nov 84 | 01 Nov 89 | 01 - Nov 89 |
|  | IREXCHEQR $6 \%$ 198599 | $600 \%$ | 598 | 68.14 | 15051\% | 488 | 5244 | 958 | 40748 | 40512 | 19\% | 006 | 0062 | 034 | 15-Mar al | 37 | 142 | 15 Mar 85 | 15 Mar 90 | 15 Max 90 |
|  | IR NATION 6 3/4\% $1986 / 91$ | 675\% | 636 | 70.83 | 15606\% | 471 | 5082 | 1045 | 45022 | 45257 | 133* | 006 | 0067 | 01 | 15 May 81 | 24 | -39 | 15-Now 85 | 15 Nov 90 | 15 Nov 90 |
|  | IREXCHEOR 14\% 1990,92 | 1400\% | 250 | 96.48 | 15636\% | 498 | 5369 | 1079 | 24120 | 24877 | $073 \%$ | 004 | 0039 | 014 | 01 Apr al | 20 | 037 | 01 ct 86 | 01 Oct 91 | 01.0191 |
|  | IR NATION 7\% 1987/92 | 700\% | 1062 | 72.77 | 15963\% | 478 | 5148 | 1116 | 17292 | 79877 | 234\% | 011 | 0120 | 026 | $15 \mathrm{eb} \mathrm{Cl}^{\text {a }}$ | 19 | 303 | 01 feb 90 | 01 Feb 92 | 01 feb 92 |
|  | IR DEVELO $711 \%$ 198893 | 150\% | 1768 | 7539 | 15538\% | 471 | 5074 | 1220 | 133271 | 137271 | 402\% | 019 | 0204 | 049 | 15 ec.al | 127 | 243 | $15 . \mathrm{Jun}$-87 | $15 . J u n 92$ | 15.Jun 92 |
|  | IR NATION. 9 1/4\% 1989/94 | 925\% | 36.5 | 92.88 | 11384\% | 649 | 6857 | 1320 | 33911 | 34928 | 102\% | 007 | 0070 | 0.14 | 01. Jan 81 | 110 | 226 | 01.Jul 88 | 01. Jut 93 | 01 Jul. 93 |
|  | RRCONVER 12\% 1995 | 1200\% | 316 | 91.99 | 1617\% | 486 | 5248 | 1441 | 29068 | 29452 | 086\% | 004 | 0045 | 012 | 0. 5 an 81 | 110 | 279 | 01. Jur 89 | 01- Jul 94 | 01. Jud 94 |
|  | (REXCHEQR 9 1/4\% 1991/96 | 9 25\% | 191.3 | 83.98 | 16500\% | 434 | 4699 | 1554 | 160627 | 160142 | 469\% | 020 | 0221 | 073 | ${ }^{15}$ Mar 81 | 37 | 122 | 15 Sep. 95 | 15 Sep. 95 | 15 Sep. 95 |
|  | IR NATION 9 9/4\% 199297 | 975\% | 2232 | 86.70 | 1633\% | 443 | 4791 | 1650 | 193509 | 193866 | $568 \%$ | 025 | 0272 | 094 | 15 Aor 81 | 10 | 025 | 01-Nov.91 | 01 Nov 96 | 01-Nov. 96 |
|  | IRNATION $11 \%$ 199396 | 1100\% | 2332 | 91.01 | 16379\% | 459 | 4966 | 1750 | 212186 | 212607 | $623 \%$ | 029 | 0309 | 109 |  | 6 | 016 | 15-0ct 92 | 15.0c197 | $15.0<197$ |
|  | IR DEVELO 11 1/2\% 199799 | 1150\% | 2100 | 92.64 | 1603\% | 456 | 4943 | 1858 | 194545 | 192958 | 565\% | 026 | 0280 | 105 |  | ${ }^{6}$ | 018 | 15.00193 | 150 Cc 98 | 15.08198 |
|  | IR FINANCE 14 1/2\% 1998/00 | 14 50\% | 250 | 9826 | 16193\% | 545 | 5888 | 1942 | 24566 | 24933 | 073\% | 004 | 0043 | 014 |  | 2 | 076 | 15-Nov. 97 | 15 Now 99 | 15 nov 99 |
|  | IR FINANCE 13\% 199702 | 1300\% | 2550 | 96.61 | 16 303\% | 503 | 5435 | 2096 | 246348 | 248164 | 727\% | 037 | 0395 | 152 |  | 37 | 147 | 15 Sep 98 | $15 \operatorname{Sep} 00$ | 15 Sep 00 |
|  | IR DEVELO 14 3/4\% 200204 | 1475\% | 250 | 99.16 | 15 889\% | 576 | 6215 | 2280 | 24791 | 25589 | 075\% | 004 | 0047 | 017 | 01 Apr 81 | 20 | 071 | 01 Acr 97 | 01 Apr 02 | 01 Apr 02 |
|  | IR EXCHEQR 6 112\% 2000/05 | 650\% | 1280 | 81.58 | 15 253\% | 378 | 4066 | 2420 | 104427 | 107047 | 314\% | 012 | 0129 |  | 01 feb 81 | 79 | 319 | 01 Feb-02 | 01 Feboct | 01 feb-04 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  | 076 | 27. Dec 80 | 115 | 205 | 27.Jun 00 | 27. Jun 05 | 27-Jun 05 |



|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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|  | Trade 16 Apr 82 |  |  |  |  |  |  |  | Clean |  |  |  |  |  |  |  |  |  |  |  |
|  | Sett $\quad 20$ Apr 82 |  |  |  |  |  |  |  | Market | Market | Weight |  |  |  |  |  |  |  |  |  |
|  |  |  | Nomunal | Market | Market |  |  |  | Value | Value |  | Werghed | Weighted | Werghled | Ex.Div | Accrued |  | $\begin{gathered} \text { Fust } \\ \text { Redempion } \end{gathered}$ | Last Redempton |  |
|  | Slock | Coupon |  |  | Yield | Volatily | Duraton | Lite | [REm] | ( R ¢ m ) | Index | Volatily | Duration | Lite | Date | $\underset{\text { Interest }}{ }$ | Accrued |  | Redempton Data | Redemplon Date |
|  | IR FINANCE VAR\% 1983 | 1952\% | 150.0 | 10010 | 18571\% | 013 | 0.137 | 0.11 | 150.143 | 154.151 | 368\% |  |  |  |  |  |  |  |  |  |
|  | IR.FINANCE VAR\% 1985 | 19.16\% | 210.0 | 10001 | 19080\% | 009 | 0099 | 0.15 | 210023 | 213989 | 5.11\% | 000 | 0005 | 000 | ${ }^{01} \cdot \mathrm{Mar}-82$ | 50 | 267 | ${ }^{01-S e p-83}$ | $01.5 e p .83$ | 01.5 Sep-83 |
|  | IR FINANCE VAR\% 1986 | 19.16\% | 2100 | 10006 | 18924\% | 003 | 0.030 | 028 | 210121 | 208909 | ${ }^{5} 4118 \%$ | 000 | 0005 | 001 | ${ }^{15} \mathrm{Mar}$-82 | 36 | 189 | ${ }^{15}$ Sepo.85 | 15-5ep 85 | 15-Sep-85 |
|  | IRCONVER 9\% 1980/82 | 900\% | 2410 | 96.30 | 19323\% | 037 | 0407 | 041 | 232084 | 234221 | 5898 | 000 | 0002 | 001 | 01-May-82 | 11 | 058 | 01-May-86 | ${ }^{01}$ May 86 | 01-May 86 |
|  | IR FUNOING 11 3/4\% 1983 | 11.75\% | 2100 | 9448 | 18979\% | 084 | 0.916 | 095 | 198408 | 199759 | $559 \%$ | 002 | 0023 | 002 | 15 -Ma 82 | 36 | 089 | 15.Sep 80 | 15 Sep-82 | $15 . \operatorname{Sep} 82$ |
|  | IR FUNDING 11 1/2\% 1983 | 1150\% | 1750 | 9172 | 18754\% | 131 | 1429 | 153 | 160510 | 1999004 | $47 \%$ | 004 | 0044 | 005 | 82 | 20 | 064 | $31-\mathrm{Ma-83}$ | $31 \mathrm{Ma-83}$ | ${ }^{31} \mathrm{Mar} \cdot 83$ |
|  | IR FINANCE $12 \% 1984$ | 1200\% | 2000 | 9136 | 18778\% | 149 | 1630 | 179 | 182729 | 1878 | , | 005 | 0055 | 006 | $01 . \mathrm{May} .8$ | 11 | 035 | 01-Nov 日3 | 01 Nov 83 | 01 Nov 83 |
|  | IR.CONVER 13\% 1984 | 1300\% | 1150 | 9175 | 18731\% | 174 | 1903 | 216 | 105517 | 11067 | 488 | 007 | 0073 | 08 | 01-Feb-82 | 78 | 256 | 01 Feos 84 | 01 Feb-34 | 01 Feb 84 |
|  | IR FINANCE 11 3/4\% 1984 | 1175\% | 1000 | 8930 | 1871\% | 186 | 2036 | 232 | 89297 | 35 | 264 | 005 | 050 | 06 | 15 Dec -81 | 126 | 448 | $15 . \mathrm{Jun}$-84 | 15-Jun- Ba | 15. Jun 84 |
|  | IR NATION. $51 / 4 \% \% 1979 / 84$ | 525\% | 29.0 | 7939 | 16236\% | 219 | 2366 | 258 | 23023 | 22919 | 2.18\% | 004 | 044 | 005 | 15 Feb 82 | 64 | 206 | 15-Aug 84 | 15-Aug-84 | 15 Aug. 84 |
|  | IRNATION 14\% 1985 | 1400\% | 1360 | 8599 | 22.745\% | 209 | 2325 | 290 | 116946 | 118823 | 055\% | 001 | 0013 | 001 | 15-May-82 | 25 | 036 | 15-Nov-79 | 15 Nov. 84 | 15 Nov-84 |
|  | IREXCHEQR 12 \% 1985 | 12.00\% | 1000 | 8799 | 18608\% | 231 | 2530 | 307 | 87987 | 87166 | 283\% | 006 | 0066 | 008 | ${ }^{15} \mathrm{Mar} \cdot 82$ | 36 | 138 | 15 Mar-85 | 15 Ma -85 | 15 Mar 85 |
|  | IREXCHEQR 6 \% 1980/85 | 600\% | 720 | 7620 | 16185\% | 287 | 3101 | 362 | 54862 | 56517 | 135\% | 005 | 0053 | 006 | ${ }^{15}$-Mar 8 | 25 | 082 | 15 May 85 | $15 . \mathrm{May}$-85 | 15 May 85 |
|  | IR NATION $7112 \%$ 1981/85 | 7.50\% | 640 | 77.59 | 16699\% | 310 | 3361 | 420 | 49655 | 51088 | 1 | $0{ }^{2}$ | 0042 | 005 | 01-Dec 81 | 140 | 230 | 01 Dec 80 | 01. Dec 85 | $01 . \mathrm{Dec}$.8s |
| $\checkmark$ | IRNATION 5 3/4\% 1982/87 | 575\% | 210 | 6896 | 16340\% | 382 | 4130 | 549 | 14482 | 14499 | 1.22\% | 004 | 0041 | 005 | 01. Jan-82 | 109 | 224 | 01-Ju1 81 | 01. Jul- 86 | 01.Jui 86 |
|  | IR CONVER 8 $1 / 2 \%$ 1986/88 | 850\% | 1050 | 7600 | 17552\% | 362 | 3941 | 579 | 79797 | 16499 81703 | 035\% | 001 | 0014 | 002 | 15-Apr 82 | 5 | 008 | 15.0ct-82 | 15.0 ct 87 | 150488 |
|  | IR NAJION. 9 3/4\% 1984/89 | 975\% | 1150 | 7935 | 17566\% | 392 | 4260 | 729 | 91250 | ${ }^{81763}$ | 229\% | 007 | 0077 | 011 | 01 - ec -82 | 78 | 182 | 01.Feb-86 | 01 feb-88 | 01. feb- 88 |
|  | IR EXCHEQR 53/4\% 1984/89 | 575\% | 260 | 6304 | 17485\% | 423 | 4596 | 754 | 16390 | 16345 | 0396 | 009 | 0095 | 016 | 01 Feb 82 | 78 | 08 | 01-Aug-84 | 01.Aug-89 | 01 Aug-8g |
|  | IR NATION. $14 \% 1985 / 90$ | 1400\% | 1460 | 9131 | 17990\% | 398 | 4333 | 791 | 133312 | 135326 | 3236 | 013 | 0018 | 003 | 01-Mar-82 | 11 | 17 | 01-Nov. 84 | 09. Hov 89 | $01 . \mathrm{Nov} 89$ |
|  | IREXCHEOR $6 \%$ 198590 | $600 \%$ | 600 | 6320 | 17655\% | 426 | 4637 | 858 | 37922 | 37676 | 090\% | 004 | 014 | 026 | 15-Mar-82 | 36 | 38 | 15 Mar 85 | 15 Max 90 | 15 Mar 90 |
|  | IR NATION. 6 34\% 1986/91 | 6.75\% | 690 | 6634 | 17927\% | 415 | 4522 | 945 | 45776 | 46018 | 110\% | 005 | 0050 | 008 | 15May 82 | 25 | 41 | 15 Nov. 85 | 15 Nov 90 | 15 Nov. 90 |
|  | IR EXCHEOR 14 \% 1990/92 | 1400\% | 850 | 9250 | 17 654\% | 431 | 4695 | 979 | 78624 | 81165 | 194\% | 008 | 0091 | 0 | 01-Apr-62 | 19 | 035 | 01-0ct-86 | 01-Cct 91 | $01.0 c 191$ |
|  | IR NATION. $7 \% 198792$ | 7.00\% | 126.0 | 68.56 | 17451\% | 423 | 4600 | 1016 | 86391 | 89433 | 2.13\% | 009 | 0098 | 019 | 01 Feb-82 | 78 | 299 | 01 Feb-90 | 01-Feb-92 | 01 Feb-S2 |
|  | IR DEVELO $71 / 2 \%$ 198883 | 750\% | 1910 | 7137 | 17.543\% | 415 | 4515 | 11.21 | 136320 | 140595 | 335\% | 014 | 0.151 | 022 | 15-0ec 81 | 126 | 241 | 15-Jun-87 | 15.Jun.92 | 15-Jun 92 |
|  | IR NATION 9 1/4\% 1989/94 | 9.25\% | 370 | 9803 | 9776\% | 699 | 7335 | 1221 | 36271 | 37292 | 089\% | 006 | 0065 | 011 | 01-Jar-62 | 109 | 224 | 01-Ju488 | 01-Ju1-93 | 01. Jut 93 |
|  | IR.CONVER 12\% 1995 | 1200\% | 2210 | 8974 | 18180\% | 415 | 4529 | 1341 | 196123 | 198737 | 474\% | 020 |  |  | 01-Jan 82 | 109 | 276 | 01.Ju1 89 | 01. Jux 901 | 01.Jul 94 |
|  | IR EXCHEQR 9 1/4\% 1991/96 | 9 25\% | 2210 | 8073 | $18554 \%$ | 370 | 4046 | 1455 | 178414 | 177798 | $424 \%$ |  | 0215 | 064 | 15-Mar 82 | 36 | 118 | 15-Sep-95 | 15-Sep. 95 | 15-Sep-95 |
|  | IR NATION 9 3/4\% 1992/97 | 975\% | 2430 | 8362 | 18452\% | 372 | 4065 | 1550 | 203194 | 203518 | $486 \%$ | 018 | 0172 | 062 | 01 May-82 | 11 | 028 | 01 - Nov 91 | 01 Nov 96 | 01 Nov. 96 |
|  | IR NATION 11\% 1993/98 | 1100\% | 2580 | 8836 | 18440\% | 384 | 4191 | 1650 | 227961 | 228349 | 545\% | 021 | 0197 | 075 | 15.Apr 82 | 5 | 013 | 15-0ct92 | 150C197 | 15.0 ct 97 |
|  | IR DEVELO 11 1/2\% 1997/99 | 11.50\% | 2750 | 9029 | 18732\% | 377 | 4122 | 1758 | 248302 | 246138 | 548\% | 022 | 0228 | 090 | 15-Apr 82 | 5 | 015 | 150 Ct 93 | 15.0 cl 98 | 15.0 ct 98 |
|  | IR FINANCE 14 1/2\% 1998/00 | 1450\% | 500 | 9643 | 18233\% | 449 | 4894 | 1842 | 48215 | 48930 | 5117\% | 022 | 0242 | 103 | 15-May 82 | 25 | 079 | 15-Nov. 97 | 15 Nov 99 | 15 Nov 99 |
|  | IR FINANCE $13 \% 199702$ | 1300\% | 2700 | 9490 | 18368\% | 411 | 4484 | 1996 | 256238 | 258064 | $616 \%$ | 025 | 0 27 | 022 | T5Ma 82 | 36 | 143 | 15-Sep-98 | 15 Sep 00 | ${ }^{15}$ Sepe 00 |
|  | IR DEVELO 14 3/4\% 200204 | 1475\% | 500 | 9791 | 17861\% | 469 | 5114 | 2180 | 48953 | 28 | 121\% | 025 | O27 | 123 | 01. AOP-82 | 19 | 06 | 01-Apr.97 | 01 Appr 02 | 01 Apt 02 |
|  | IR EXCHEQR $6112 \%$ 2000/05 | 650\% | 1330 | 7954 | 17.136\% | 309 | 3354 | 2320 | 105782 | 108481 | 259\% | 008 | 0087 | 026 | 01--eb-62 | 78 | 315 | 01 Feb 02 | 01 Feb-04 | 01 Feboas |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  | 060 | 27.0 ec 81 | 114 | 20 | 27.Jun 00 | 27.Jun-05 | 27. Jun 05 |


|  | Trade: $\quad 15-0 \mathrm{ct}-82$ |  |  |  |  |  |  |  | Clean | Dity | Stock |  |  |  |  |  |  |  |  |  |
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|  | Set: 1900 :82 | Coupon | Nominal Issue | Market Price | $\begin{gathered} \text { Market } \\ \text { Yield } \end{gathered}$ | Volabily | Duratoon | Life | Market Value (IREm) | Marke: <br> Value <br> (IREm) | Weight <br> in <br> Index | Weighted Volability | Weighted <br> Duration | Weighted Lite | $\begin{aligned} & \text { Ex-Div } \\ & \text { Date } \end{aligned}$ | Accrued Interest | Accrued Interest | $\begin{gathered} \text { First } \\ \text { Redemption } \\ \text { Date } \end{gathered}$ | $\begin{gathered} \text { Las! } \\ \text { Redempoton } \\ \text { Date } \end{gathered}$ | Redemption Date |
|  | IR FINANCE VAR\% 1985 | 1920\% | 1000 | 10000 | 19003\% | 025 | 0263 | 001 | 99998 | 105044 | 2.05\% |  |  |  |  |  |  |  |  |  |
|  | IR FINANCE VAR\% 1983 | 1765\% | 1350 | 10008 | 16877\% | 013 | 0131 | 012 | 135111 | 138242 | 270\% | 000 | 0005 | 000 | 15.Jul 88 | 96 | 505 | 15- $\tan 85$ | 15-Jan. 85 | 15.Jan. 85 |
|  | IR FINANCE VAR\% 1985 | 16.61\% | 2100 | 10001 | 16516\% | 009 | 0093 | 0.16 | 210026 | 213272 | 276\% | 000 | -0004 | 000 | ${ }^{01}$-Sep-82 | 48 | 23 | ${ }^{015} 5$ Sep. 83 | 01 Sep 83 | 01. Sep 83 |
|  | IR FINANCE VAR\% 1986 | 1837\% | 1600 | 10011 | 17916\% | 003 | 0036 | 028 | 160180 | 159134 | 311\% | 000 | 0001 | 001 | 15-Sep 82 | 34 | 15 | $15.5 e p .85$ | ${ }^{15} 5$ Sep 85 | 15.Sep 85 |
|  | IR FUNDING $11314 \% 1983$ | 1175\% | 2100 | 9938 | 13298\% | 042 | 0448 | 045 | 208706 | 209922 | $410 \%$ | 002 | 0018 | 002 | ${ }^{\text {01-Nov } 82}$ | 13 | 065 | ${ }^{01} \mathrm{May} 86$ | 01 May 86 | 01 May 86 |
|  | IR EXCHEQR $15 \%$ 1983 | 1500\% | 1400 | 10091 | 13529\% | 068 | 0725 | 074 | 141280 | 146799 | 287\% | 002 |  |  | ${ }^{\text {1 }}$ 1.-Jct 82 | 18 | 058 | 31 Mar 83 | ${ }^{31} \mathrm{Man} 83$ | ${ }^{31} \mathrm{Ma-83}$ |
|  | IR FUNDING 11 112\% 1983 | 1150\% | 2250 | 9839 | 13365\% | 094 | 1004 | 1.04 | 221388 | 220468 | 480\% | 002 004 | 0021 | 002 | ${ }^{15}$-Ju1-82 | 96 | 394 | $15 . \mathrm{Ju}$ 83 | $15 . \mathrm{Jut} 83$ | 15.Jut a3 |
|  | IR.FWNANCE $12 \% 1984$ | 1200\% | 2200 | 9833 | 13.615\% | 115 | 1228 | 129 | 216336 | 222046 | 433\% | 005 | 0043 | 004 | 01-Nov.82 | 13 | 041 | 01-Nov-83 | 01 Nov-83 | 01-Nov.83 |
|  | IRCONVER 13\% 1984 | 1300\% | 1150 | 9926 | 13590\% | 144 | 1541 | 1.66 | 114148 | 119305 | 233\% | 005 | 0053 | 006 | $01 . \mathrm{Aug} 82$ | 19 | 260 | 01-Feb-Ba | 01 Feb-84 | 01 Feb-84 |
|  | IR FINANCE 11 3/4\% 1984 | 11.75\% | 2200 | 9764 | 13469\% | 158 | 1687 | 182 | 214.809 | 219410 | 428\% | 007 | 0036 | 004 | 15-Jun-82 | 126 | 448 | $15 . J u n$ B4 | 15-Jun-84 | 15-Jun 84 |
|  | IR NATION. $514 \% \% 1979184$ | 525\% | 290 | 8926 | 11643\% | 186 | 1972 | 208 | 25885 | 25773 | 0.50\% | 007 | 0072 | 008 | 15 - Aug 82 | 65 | 209 | 15 Aug 84 | 15 Aug-84 | 15-Aug 84 |
|  | IR NATION 14\% 1985 | 1400\% | 1360 | 9988 | 14076\% | 198 | 2.123 | 241 | 135833 | 137605 | 269\% | 005 | 0010 | 001 | 15. Nov 82 | 27 | 039 | 15-Now 79 | $15 . \mathrm{Nov} 84$ | 15 Nov 84 |
|  | IR NATION 14\% 1985/90 | 14.00\% | 1460 | 10094 | 13425\% | 200 | 2130 | 241 | 147374 | 149.277 | 2.91\% |  | 0057 | 006 | $15 . \operatorname{Sep}$-82 | 34 | 130 | 15 Mar 8 s | $15 . \mathrm{Max} 85$ | 15 Maras |
|  | IR.EXCHECR $12 \% 1985$ | 12.00\% | 1000 | 9688 | 13780\% | 212 | 2267 | 257 | 96880 | 95.993 | 187\% | 004 | 0062 | 007 | 15 Sep 82 | 34 | 130 | 15Mar85 | 15-Mar 90 | 15 Mar 酐 |
|  | IR.FINANCE 12 174\% 1985 | 1225\% | 120.0 | 9742 | 13587\% | 241 | 2569 | 299 | 116899 | 118268 | 231\% | 006 | 0042 | 005 | 15. Nou-82 | 27 | 08 | 15 May-85 | 15 May 85 | 15-may-85 |
|  | IR EXCHEOR $6 \%$ 1980/85 | 600\% | 120 | 8537 | 12486\% | 264 | 2809 | 312 | 61468 | 63124 | 123\% | 003 | 0059 | 007 | 15-Sep 82 | 140 | 11 | 15.0ctis | 15.0 Ca 85 | 15.Oct 85 |
| $\infty$ | IR FUNDING 15 112\% 1986 | 1550\% | 1150 | 10350 | 13695\% | 259 | 2769 | 333 | 119030 | 119908 | 234\% | 006 | 0065 | 008 | 01.Jun-82 | 140 | 23 | 01. dec-80 | ${ }^{01}$ - Dece 85 | ${ }^{01.0 e c} 85$ |
|  | IR NATION 7 112\% 1981/85 | 7.50\% | 640 | 8618 | 13189\% | 297 | 3164 | 370 | 55157 | 56602 | 10\% | 003 | 0035 | 004 | ${ }^{01}$ 01-Ju182 | 18 | 076 | 15 Feb-86 | 15 Feb 86 | 15 Fed 86 |
|  | IR.EXCHEOR 12 12\% 1986 | 1250\% | 1200 | 9755 | 13590\% | 299 | 3.197 | 395 | 117055 | 118452 | 231\% | 007 | 0074 | 009 | 01-Ju182 | 110 | 226 | 01-Jul 81 | 01-Jut 86 | 01. Jud 86 |
|  | IR FUNDING 12 3/4\% 1986 | 1275\% | 1200 | 9799 | 13606\% | 322 | 3440 | 437 | 117590 | 119014 | 232\% | 007 | 0080 | 009 | ${ }^{15-5 e p-82}$ | 34 | 11 | 01.001.86 | 01.0486 | $01.0 c 886$ |
|  | IR FINANCE 16\% 1987 | 1600\% | 1200 | 10470 | 13891\% | 339 | 3625 | 474 | 125640 | 130686 | 255\% | 009 | 0092 | 012 | $15 \operatorname{sep} 82$ | 34 | 119 | 01 Mar 87 | 01 Max 87 | 01 Mar 87 |
|  | IR NATION 5 3i4\% 1992787 | 575\% | 210 | 7875 | 12691\% | 384 | 4088 | 499 | 16538 | 16551 | $032 \%$ | 001 | 0013 | 012 | ${ }^{15} 5 . \mathrm{Jul}-82$ | 96 | 421 | 15 -Jul 87 | $15 . \mathrm{Jum} 87$ | 15-Jut 87 |
|  | IR CONVER $81 / 12 \% 1986 / 88$ | $8.50 \%$ | 1050 | 8674 | 13094\% | 382 | 4070 | 529 | 91078 | 93008 | $182 \%$ | 007 | 0074 | 002 | $15-\mathrm{Cl}-82$ | ${ }^{4}$ | 006 | 15-0ct-82 | 15.0487 | 150 ct 87 |
|  | IR CONVER 15\% 1988 | 1500\% | 500 | 103.35 | 1362\% | 397 | 4237 | 587 | 51677 | 52375 | 102\% | 004 | 0043 | 0.10 | ${ }^{\text {1-RAug }} 8.82$ | 79 | 194 | 01.Feb-86 | 01 Feb-88 | $01 . \mathrm{eb}$-88 |
|  | IR NATION 9 14\%\% 1989/94 | 925\% | 370 | 10082 | 9018\% | 495 | 5169 | 670 | 37.303 | 38334 | $075 \%$ | 004 | 0 ¢ | 00 | 15-sep. 82 | 34 | 140 | 01 Sep. 88 | 01 Sep 88 | 01 Sep-88 |
|  | IR NAIION 9 3/4\% 1984/89 | 975\% | 1150 | 8936 | 13286\% | 439 | 4684 | 679 | 102763 | 105189 | 205\% | 004 | 0039 | 005 | 01. Jut 82 | 110 | 279 | 01-Jul 89 | 01-Jul 94 | 01-Jul 99 |
|  | IR EXCHEOR 5 3/4\% 1984/89 | 5.75\% | 260 | 7329 | 13303\% | 471 | 5021 | 704 | 19056 | 19002 | 0.37\% | 002 | 0096 | 014 | $0^{01-A v 9} 82$ | 79 | 211 | 01-Aug. 84 | 01-Aug 89 | 01-Aug-89 |
|  | IR EXCHEOR 14 \% 1990/92 | 1400\% | 850 | 10136 | 13473\% | 457 | 487 | 729 | 86160 | 88734 | 1.73\% | 008 | 0019 | 003 | 01-Nov-82 | 13 | 020 | 01-Nov. 84 | 01-Nov-89 | 01 . Nov-89 |
|  | IREXCHEQR $6 \% 198590$ | 60\%\% | 600 | 7290 | 13453\% | 497 | 5303 | 808 | 43741 | 43475 | 085\% | 004 | 0084 | 013 | ${ }^{01}$ - Aug 82 | 79 | 303 | 01.Feb-90 | 01 Feb 92 | 01 Feb-90 |
|  | IR NATION. 6 314\% 1986/91 | $675 \%$ | 69. | 7527 | 13727\% | 503 | 5379 | 896 | 51936 | 52165 | 102\% | 005 | 0045 | 001 | 15, ${ }^{\text {Nov. } 82}$ | 27 | 044 | 15-Nov-85 | 15 Nov 90 | 15-Nov.90 |
|  | IRNATION 7\% 1987/92 | 700\% | 1260 | 7720 | 13370\% | 522 | 5570 | 9.66 | 97271 | 100314 | $196 \%$ | 010 | 0055 | 009 | 01.04182 | 18 | 033 | 01-0ct 86 | 01.0 Cl .91 | $01.0 c+91$ |
|  | IR DEVELO $71 / 2 \%$ 198893 | $750 \%$ | 1910 | 7895 | 13615\% | 527 | 5625 | 10.71 | 150791 | 155105 | 303\% | 016 | Ors | 0 | 15.Jun 82 | 126 | 241 | 15-Jun-87 | 15 Jun 92 | 15.Jun 92 |
|  | IRCONVER $12 \% 1995$ | 1200\% | 960 | 9485 | 14179\% | 555 | 5947 | 1292 | 91.052 | 92125 | $180 \%$ | 010 | , | 032 | 01-Ju-82 | 110 | 226 | $01 . \mathrm{Ju}-88$ | 01 Jul-93 | 01. Jut 93 |
|  | IR EXCHEQR 9 14\% 199196 | 925\% | 2210 | 8602 | 1446\% | 520 | 5574 | 1405 | 190098 | 189.370 | 3.70\% | , | (10) | 02 | 15-Sep-82 | 34 | 112 | 15 Sep 95 | 15 Sep 95 | 15-Sep.95 |
|  | IR NATION 9 3/4\% 199297 | 9.75\% | 2430 | 明 51 | 14361\% | 531 | 5689 | 1500 | 215076 | 215335 | 4.20\% | , | 020 | 05 | 01-Nov 82 | 13 | 0 | 01 Nov91 | 01 Nov 96 | 01-Nov-96 |
|  | IR FINANCE 14 1/2\% 1998:00 | 1450\% | 500 | 10047 | 14212\% | 631 | 6757 | 15.92 | 50236 | 50.910 | $099 \%$ | . 22 | 029 | 063 | $15-\mathrm{ClH} 82$ | 4 | 01 | $15.0 \mathrm{Cl}-92$ | 15.0 ct 97 | 15.0ct 97 |
|  | IR NATION $11 \%$ 1993/98 | 1100\% | 258.0 | 9283 | 14369\% | 553 | 5922 | 1600 | 239506 | 239817 | 468\% | 80 | 0067 | 016 | ${ }^{15} 5$ Sep 82 | 34 | 13 | 15 Sep 98 | 15-Sep.00 | 15.5 ep 98 |
|  | IR DEVELO 11 1/2\% 199799 | 11.50\% | 2750 | 94.30 | 14.529\% | 557 | 5976 | 1708 | 259333 | 256.995 | 502\% | 028 | \% | 075 | $15 . \mathrm{Cct} 82$ | 4 | 012 | $15-0 c t 93$ | $15.0 c t 98$ | 150 ct 98 |
|  | IR. DEVELO 14 3/4\% 200204 | 1475\% | 500 | 10093 | 13.995\% | 684 | 7321 | 19.30 | 50467 | 52.062 | 102\% | 007 | 0300 | 086 | 15 NO O .82 | ${ }^{27}$ | 085 | 15-Nov. 97 | 15 Nov. 99 | 15-Nov 99 |
|  | IR FINANCE 13\% 199702 | 1300\% | 270.0 | 9817 | 14313\% | 614 | 6580 | 1946 | 265050 | 266780 | 521\% | 007 | 0074 | 020 | 01. Aug 82 | 79 | 319 | 01 Feb-02 | 01.Feb-04 | 01 - Feb-02 |
|  | IR. EXCHEQR 6 1/2\% 2000/05 | 650\% | 1330 | 7899 | 17.136\% | 3.13 | 3397 | 2270 | 105056 | 107.754 | 5210\% | 032 | 0343 | 101 | 01.0 ct 82 | 18 | 064 | 01 Apr-97 | 01-Apt-02 | 01-Apr-02 |
|  |  |  |  |  |  |  |  |  |  |  |  | 007 | 007 | 048 | 27. Jun- 82 | 114 | 203 | 27. Jun 00 | 27. Jun.05 | 27. Jun 05 |


|  | Trade $\quad 15$-Apr-83 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Sety: 19 Apr 83 |  |  |  |  |  |  |  | $\begin{aligned} & \text { Clean } \\ & \text { Marke! } \end{aligned}$ | Dirty Markel | Stock Weighı |  |  |  |  |  |  |  |  |  |
|  | Stock | Coupon | Nomina issue | Market Price | Market Yield | Volatily | Duraton | Lue | Value <br> (IREm) | Value <br> ( $\mathbf{R E m}$ ) | $\begin{aligned} & \text { weign } \\ & \text { in } \\ & \text { Index } \end{aligned}$ | Weighted Volatility | Weighted Duration | Weighted <br> Lhe | $\begin{aligned} & \text { Ex:Div } \\ & \text { Date } \end{aligned}$ | Accrued Interest | Accrued <br> Interest | $\begin{gathered} \text { Fiust } \\ \text { Redemptoon } \end{gathered}$ | Last Redemptoon | Redemption |
|  | IR FINANCE VAR\% 1985 | 1599\% | 1000 | 10000 | 15 422\% | 025 | 0257 | 001 | 99999 | 104.011 | 191\% | 000 |  |  |  |  |  |  |  |  |
|  | IR FINANCE VAR\% 1983 | 1531\% | 1650 | 9980 | 17198\% | 013 | 0134 | 012 | 164674 | 168063 | 309\% | 000 | 0005 | 000 | 15.Jan 83 | 94 | 401 | $15 \operatorname{Jan} 85$ | 15.Jan 85 | 15 Jan 85 |
|  | IR.FINANCE VAR\% 1985 | 1502\% | 2100 | 9998 | 15202\% | 009 | 0096 | 015 | 209948 | 212.971 | 301\% | 000 | 0004 | 000 | 01-Mar ${ }^{\text {d }}$ | 49 | 205 | $01 . \operatorname{Sep} 83$ | 01 Sep 83 | 01.5 Sep 83 |
|  | IR EXCHEOR 15 \% 1983 | 1500\% | 1400 | 10005 | 14779\% | 023 | 0243 | 024 | 140066 | 145471 | 267\% | 001 | 0006 | 001 | 15 Mar 83 | 35 | 144 | 15-Sep-85 | 15 Sep 85 | 15 Sep 85 |
|  | IR FINANCE VAR\% 1986 | 1731\% | 1600 | 10004 | 1715\% | 003 | 0033 | 028 | 160061 | 159151 | 292\% | 000 | 0006 | 001 | ${ }^{15}$-Jan. 83 | 94 | 386 | 15.Ju 83 | 15 Jut 83 | 15.Jul 83 |
|  | IR FUNDING 11 1/2\% 1983 | 1150\% | 2250 | 9868 | 14301\% | 050 | 0535 | 054 | 222028 | 221177 | $406 \%$ | 002 | 0022 | 001 | 01-May ${ }^{\text {83 }}$ | 12 | 057 | 01 May 86 | 01-May 86 | 01-May 86 |
|  | IR FINANCE 12\% 1984 | 1200\% | 2200 | 9852 | 14215\% | 072 | 0775 | 079 | 216736 | 222.301 | $408 \%$ | 003 | 0032 | 003 | 01-May ${ }^{\text {O }}$ S | 12 | 038 | 01 Nov 83 | 01 Nov 83 | 01 Nov 83 |
|  | IR CONVER 13\% 1984 | 1300\% | 2000 | 9924 | 13819\% | 104 | 1111 | 1.16 | 198470 | 207368 | $380 \%$ | 004 | 0042 | 003 | ${ }^{\text {01-Feb } 83}$ | 77 | 253 | 01 Fed 84 | 01 Feb 84 | 01 Feb 84 |
|  | IR FINANCE $11314 \%$ 1984 | 1175\% | 2200 | 9798 | 13662\% | 118 | 1262 | 133 | 215545 | 220004 | $404 \%$ | 004 | 0042 | 004 | 15.0 ec 82 | 125 | 44 | 15-Jun 84 | 15.Jun 84 | 15 Jun 84 |
|  | IR NATION. $5114 \%$ 197984 | 525\% | 290 | 9025 | 12654\% | 143 | 1523 | 158 | 26172 | 26064 | 048\% | 005 | 0051 | 005 | 83 | 63 | 203 | 15 Aug 84 | 15-Aug. 84 | 15-Aug $\mathrm{ad}_{4}$ |
|  | IR NATION 14\% 1985 | 1400\% | 1510 | 10229 | 12367\% | 165 | 1748 | 191 | 154457 | 156483 | 288\% | 001 | 0007 | 001 | 15 May 83 | 26 | 037 | 15-Now-79 | 15-Nov. 84 | 15 Nov. 84 |
|  | IR NATION 14\% 1995/90 | 1400\% | 1510 | 10229 | 12367\% | 165 | 1748 | 191 | 154457 | 156483 | 287\% | 005 | 0050 | 005 | 15 Mar 83 | 35 | 134 | 15 Max 85 | 15-Ma-85 | 15 Mar 85 |
|  | IR EXCHEOR 12\% 1985 | 1200\% | 2000 | 9828 | 13133\% | 17 | 1889 | 207 | 196566 | 194858 | 285\% | 005 | 0050 | 005 | ${ }_{15} \mathrm{Max}^{83}$ | 35 | 134 | 15-Mar 85 | 15 Mar 90 | 15 Mar 85 |
|  | IR FINANCE $1211.4 \% 1985$ | 1225\% | 1850 | 9846 | 13144\% | 208 | 2213 | 249 | 182147 | 182395 | 335\% | 006 | 0068 | 007 | 15 May 83 | 26 | 085 | ${ }^{15}$-May-85 | 15 May 85 | 15 May 85 |
|  | IREXCHEOR 6 \% 1980/85 | 600\% | 720 | 8668 | 12748\% | 227 | 2412 | 262 | 62411 | 64055 | 117\% | 003 | 0074 | 008 | 15 Apr 83 | 4 | 013 | 15.0 ct 85 | 15-0ct 85 | 15.0ct 85 |
|  | IR FUNDING 15 12\% 1986 | 1550\% | 1150 | 10396 | 13280\% | 228 | 2434 | 283 | 119550 | 122625 |  | 003 | 0028 | 003 | ${ }^{01}$ - Dec 82 | 139 | 228 | 01 Dec 80 | 01-Dec 85 | 01 Dec 85 |
|  | IR NATION $7112 \% 1981185$ | 750\% | 640 | 8837 | 12724\% | 265 | 2822 | 3.20 | 56556 | - 57976 | 225\% | 5 | 0055 | 006 | 15 Feb 83 | 63 | 267 | 15 Feb 86 | 15 Feb 86 | 15 Feb 86 |
|  | IR EXCHEOR 12 1/2\% 1986 | 1250\% | 1600 | 9871 | 13109\% | 272 | 2893 | 345 | 157939 | 158158 |  | 003 | 0030 | 003 | 01-dan-83 | 108 | 222 | 01. Jut 81 | 01 Jut 86 | 01-Jul-86 |
|  | IR FUNDING 12 3/4\% 1987 | 1275\% | 1600 | 9909 | 13156\% | 296 | 3157 | 387 | 158549 | 161285 | 296\% | 008 009 | 0084 | 010 | ${ }^{15} \cdot \mathrm{App}$ - 83 | 4 | 014 | 01 Cct 86 | $01.0 c t .86$ | 01.0486 |
| $\infty$ | IR FINANCE 16\% 1987 | 1600\% | 1050 | 10590 | 13283\% | 315 | 3364 | 424 | 11196 | 115520 | $296 \%$ | 009 | 0093 | 011 | 01. Ma 83 | 49 | 171 | 01 Mar 87 | 01 Max 8 | 01 Mar -87 |
| $\square$ | IR NATION 5 3/4\% 198288 | 575\% | 21.0 | 8021 | 1261\% | 356 | 3785 | 449 | 16844 | 16857 |  | 007 | 0071 | 009 | ${ }^{15}$ Jan 83 | 94 | 412 | 15-Jd 87 | 15-Ju4 87 | 15-Jur 87 |
|  | IR CONVER $81 / 2 \% 1986688$ | $850 \%$ | 1050 | 8765 | 12973\% | 358 | 3808 | 479 | 92030 | ${ }_{93912}$ | 172\% | 006 | 0012 | 001 | $15 . \mathrm{App}_{68} 83$ | 4 | 006 | 150482 | 15.0487 | 15.0487 |
|  | IR CONVER 15\% 1988 | 1500\% | 500 | 10617 | 12494\% | 383 | 4074 | 538 | 5308 | 54092 | 099\% | 000 | 0066 | 008 | 01 Feb 83 | 17 | 179 | 01-Feb-86 | 01 Feb 88 | 01 feb 88 |
|  | IR NATION 9 3 34\% 198489 | 975\% | 1250 | 91.72 | 12490\% | 429 | 4563 | 629 | 114653 | 117222 | $215 \%$ | -09 | 0040 | 005 | $01 . \mathrm{Mar} 83$ | 49 | 201 | $01.5 e p-88$ | 01 Sep 88 | 01 Sep 88 |
|  | IR.EXCHEOR 5 314\% 1984/89 | 575\% | 26.0 | 7652 | $12395 \%$ | 463 | 4916 | 6.54 | 19895 | 19846 |  | 002 | 0096 | 014 | 01 Feb 83 | 71 | 206 | 01-Aug 84 | 01-Aug 89 | 01.Aug 89 |
|  | IREXCHEOR 14\% 1990/92 | 1400\% | 950 | 10383 | 12.561\% | 450 | 4786 | 679 | 98638 | 101442 | 1 86\% | 002 008 | 0018 | 002 | 01-May ${ }^{83}$ | 12 | 019 | 01-Nov 84 | 01-Nov 89 | 01.-Nov.89 |
|  | IREXCHEOR 11 1/2\% 1990 | 1150\% | 730 | 9672 | 12619\% | 468 | 4974 | 733 | 70606 | 72123 | $132 \%$ | 008 | 0089 | 013 | 01. Feb-83 | 71 | 295 | 01-Feb-90 | 01 Feb 92 | 01-Feb-90 |
|  | IREXCHEOR $6 \%$ 1985/90 | 600\% | 650 | 7594 | 12524\% | 498 | 5292 | 758 | 49359 | 49081 | 090\% | 009 | 0066 | 010 | 12 Feb 83 | 66 | 208 | 15-Aug 90 | 15-Aug 90 | 15 Aug.90 |
|  | IR NATION. 6 3/4\% 1986/99 | 675\% | 690 | 7855 | 12598\% | 516 | 5486 | 846 | 54198 | 54428 | $100 \%$ | 004 | 0048 | 007 | 15 May 83 | ${ }^{26}$ | 043 | 15 Nov 85 | 15-Nov 90 | 15-Nov. 90 |
|  | IR NATION 7\% 1987/92 | 700\% | 1260 | 7916 | 12686\% | 530 | 5631 | 916 | 99747 | 102.765 | $188 \%$ | 010 | 0055 | 008 | 01-Apr-83 | 18 | 033 | 01-0¢186 | 01 Cct 91 | 01.0 ct 91 |
|  | IR DEVELO $71 / 2 \%$ 198893 | $750 \%$ | 1910 | 8083 | 12871\% | 542 | 5768 | 1021 | 154382 | 158618 | 291\% | 016 | -0168 | 0.17 | 15. Dec 82 | 125 | 240 | 15.Jun 87 | 15-Jun 92 | 15-Jun 92 |
|  | IR NATION 9 1/4\% 1989/94 | $925 \%$ | 230 | 8824 | 12936\% | 556 | 5921 | 11.21 | 20296 | 20.925 | 038\% | 002 | 0023 | 030 | 01- an - 83 | 108 | 222 | 01. Ju1-88 | 01 Jul 93 | 01 Ju193 |
|  | IRCONVER 12\% 1995 | 1200\% | 1060 | 9652 | 13371\% | 581 | 6196 | 1242 | 102310 | 103529 | $190 \%$ | 011 | 0023 | 004 | 01-ane 83 | 108 | 274 | 01-Ju 89 | 01.Ju94 | 01. Juel 94 |
|  | IR EXCHEOR 9 1/4\% 199196 | 925\% | 2260 | 8743 | 13609\% | 553 | 5910 | 1355 | 197585 | 196898 | 361\% | 020 | 0213 | 024 | 15 Mar 83 | 35 | 115 | 15 Sep. 95 | 15-Sep.95 | 15 Sep 95 |
|  | IR NATION 9 3/4\% 199297 | 975\% | 2430 | 8978 | 13559\% | 566 | 6044 | 1450 | 218154 | 210413 | 401\% | 023 | 0213 | 049 | ${ }^{01}$-May ${ }^{83}$ | 12 | 030 | 01 Nov-91 | 01 Nov 96 | 01 Nov 96 |
|  | IR FINANCE 14 1/2\% 1998/00 | 1450\% | 600 | 10203 | 13.378\% | 670 | 7150 | 1542 | 61217 | 62.051 | 1 14\% | 008 | 0242 | 058 | 15-Apr-83 | 4 | 011 | 15-0c192 | 150 CH 97 | 15.0197 |
|  | IR NATION. $11 \%$ 1993/r8 | 1100\% | 2630 | 9405 | 13567\% | 590 | 6306 | 1550 | 247363 | 247679 | $454 \%$ | 027 | 0286 | 018 |  | 35 | 139 | ${ }^{15} 5 \operatorname{Sep} 98$ | 15-Sep 00 | ${ }^{15} 5$ Sep 98 |
|  | IR DEVELO 11 112\% 199799 | 1150\% | 2750 | 9546 | 13696\% | 600 | 6.409 | 1659 | 262515 | 260264 | 477\% | 029 | 0306 | 079 | 15-Apr.83 | 4 | 012 | 1500193 | 15-0ct98 | 15-0ct 98 |
|  | IR DEVELO. 14 3/4\% 200204 | 14 75\% | 600 | 10221 | 13.164\% | 738 | 7866 | 1880 | 61328 | 63.193 | 1.16\% | 009 | 0091 | 022 | 15-May-83 | ${ }^{26}$ | 082 | 15-Nov-97 | 15-Nov.99 | 15 Nov 99 |
|  | IR FINANCE 13\% 199702 | 1300\% | 2700 | 9921 | 13509\% | 664 | 7.084 | 1896 | 267.857 | 269587 | 494\% | 03 |  | 022 | 01 - eb -83 | 77 | 311 | 01 Fet-02 | 01 Feb-04 | 01 Feb 02 |
|  | IR.EXCHEQR $6112 \%$ 2000/05 | $650 \%$ | 1330 | 8266 | 12467\% | 554 | 5885 | 2221 | 109933 | 112608 | 207\% | 011 | 0350 | 094 | 01-Apr-83 | 18 | 064 | 01-Apr-97 | 01-Apro2 | 01 Apr. 02 |
|  |  |  |  |  |  |  |  |  |  |  |  | 011 | 0122 | 046 | 27-Dec-82 | 113 | 201 | 27.Jun-00 | 27. Jun 05 | 27. Jun 05 |


|  | Trade $\quad 18.0 \mathrm{Cl}$ 83 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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|  | Sett 20 Oct 83 |  |  |  |  |  |  |  | Clean |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | Nomunal | Market | Market |  |  |  | Value | Value |  |  |  |  |  |  |  | Fust | Lası |  |
|  | Stock | Coupon | sue | Price | Yield | Volatily | Duraton | Lile | [(REm) | (1REm) | Index | Voralitity | Weighted Duration | Weighted Lide | $\begin{gathered} \text { Ex Div } \\ \text { Date } \end{gathered}$ | Accrued Interest | Accrued <br> Intresest | Redempton | Redempton | Redempton |
|  | IR FINANCE VAR\% 1985 | 1371\% | 1000 | 9999 | 13303\% | 026 | 0266 | 002 | 99994 | 103634 | 168\% | 000 |  |  |  |  |  |  |  |  |
|  | IR FUNDING 11 112\% 1983 | 1150\% | 2250 | 100.01 | 11.175\% | 003 | 0034 | 003 | 225023 | 224173 | 364\% | 000 | 0004 | 000 | 15.5 Ju 83 | 97 | 364 | ${ }^{15}$ Jan 85 | ${ }^{15-\operatorname{Jan} 85}$ | 15.Jan 85 |
|  | IR FINANCE VAR\% 1988 | 1395\% | 1700 | 10001 | 13846\% | 015 | 0153 | 010 | 170016 | 173652 | 282\% | 000 | 0001 | 000 | 01-Nov.83 | 12 | 038 | 01-Nov-83 | 01. Nov 83 | 01 Nov 83 |
|  | IR FINANCE VAR\% 1985 | 1362\% | 2100 | 10003 | 13408\% | 009 | 0096 | 015 | 210063 | 212805 | $346 \%$ | 000 | 0004 | 000 | 25-Aug 83 | 56 | 14 | 01.Jun 88 | 01-Jun 88 | 01 Jun 88 |
|  | IR FINANCE VAR\% 1986 | 14 18\% | 1600 | 10010 | 13792\% | 003 | 0033 | 028 | 160156 | 159411 | 259\% | 000 | 0003 | 001 | 15 Sep 83 | 35 | 131 | ${ }^{15}$ Sopp 85 | 15 Sep 85 | 15 Sep 85 |
|  | IR FINANCE $12 \% 1984$ | 1200\% | 2350 | 9976 | 12922\% | 021 | 0288 | 028 | 234439 | 240615 | 391\% | 001 | 0001 | 001 | O1 Nov 83 | 12 | 047 | 01 Mar 86 | 01 May 86 | 01 May 86 |
|  | IR.CONVER 13\% 1984 | 1300\% | 2400 | 10009 | 12836\% | 061 | 0649 | 065 | 240223 | 251072 | 408\% | 002 | 0011 | 001 | 01.4 .4983 | 80 | 263 | 01-Feb-84 | 01.Feb.84 | 01 Feb 88 |
|  | IR.FINANCE 11 3/4\% 1984 | 1175\% | 2500 | 9926 | 12806\% | 076 | 0807 | 082 | 248146 | 253454 | 412\% | 002 | 0026 | 003 | 15.Jun 83 | 127 | 452 | 15.Jun 84 | 15-Jun-84 | $15 . J$ Jun 84 |
|  | IR NATION. $51 / 4 \%$ 1979/84 | 525\% | 290 | 9333 | 12.358\% | 099 | 1054 | 1.07 | 27066 | 296957 | 4040\% | 003 | 0033 | 003 | 15.4 .89 .83 | 66 | 212 | 15 Aug 84 | 15.Aug 84 | 15 Aug 84 |
|  | IRNATION 14\% 1985 | 1400\% | 151.0 | 10193 | 12246\% | 125 | 1.326 | 140 | 153918 | 155944 | 253\% | 003 | 0005 | 000 | $15 \cdot \mathrm{Nov} 83$ | 26 | 037 | 15 Nov. 79 | 15 - Nov 84 | 15. Nov 84 |
|  | IRNATION 14\% 1985/90 | 1400\% | 1510 | 10193 | 12246\% | 1.25 | 1.326 | 140 | 153918 | 155944 | 2.53\% | 003 | 0034 | 004 | 15 Sep. 83 | 35 | 134 | 15 Mar -85 | 15 Mar 8 | ${ }^{15} \mathrm{Mar}$-85 |
|  | IR EXCHEQR 12\% 1985 | 1200\% | 2400 | 99.07 | 12.759\% | 139 | 1475 | 157 | 237769 | 235719 | 383\% | 005 | 0058 | 004 | 15 Sep -83 | 35 | 4 | 15-Mar 85 | 15 - | 15 Ma 85 |
|  | IR FINANCE 12 1/4\% 1985 | 1225\% | 2300 | 99.18 | 12806\% | 171 | 1821 | 199 | 228120 | 228506 | 371\% | 006 | 0056 | 006 | 15-Nov.83 | 26 | 085 | ${ }^{15}$-May.85 | 15 May 8 | 15-May 85 |
|  | IREXCHEQR6\% 1980/85 | 600\% | 720 | 8882 | 12694\% | 187 | 1992 | 212 | 63952 | 65620 | 1.07\% | 006 | 0068 | 007 | $15 .-01818$ | 5 | 017 | 15-Oct 85 | 15-OC1 85 | 15 Oct 85 |
|  | IR FUNDING 15 1/2\% 1986 | 1550\% | 1150 | 10412 | 12893\% | 194 | 2066 | 233 | 119734 | 122955 | 200\% | 002 |  | 002 | 01.Jun-83 | 141 | 232 | 01. Dec 80 | 01 Dec 8 | 01- Dec. 85 |
|  | IR EXCHEQR 10 3/4\% 1986 | 1075\% | 1250 | 9637 | 12827\% | 207 | 2197 | 245 | 120464 | 121163 | 197\% | 004 | 0041 | 005 | 15-A49.83 | 66 | 280 | ${ }^{15-F e b} .86$ | 15 Feb | 15 Feb 86 |
|  | IR NATION. $71 / 2 \%$ 1981/85 | 750\% | 640 | 8973 | 12.712\% | 229 | 2441 | 270 | 57425 | 58884 | 096\% | 002 | -0023 | 005 | 01.0c183 | 19 | 056 | 01 Apr 86 | 01 Apr 86 | 01 Apr-86 |
|  | IR EXCHEQR $121 / 12 \% 1986$ | 1250\% | 1600 | 9935 | 12 834\% | 240 | 2551 | 295 | 158967 | 159241 | 259\% | 006 | ${ }_{0}^{0} 0.065$ | 003 | 01. Jut 83 | 111 | 228 | 01. Ju-81 | 01. Jur 86 | $01 . \mathrm{Jul} 86$ |
|  | IR FUNDING 12 3/4\% 1987 | 1275\% | 1600 | 9965 | 12.919\% | 266 | 2834 | 3.36 | 159437 | 162174 | 263\% | 007 | 0075 | 008 | 15-0ct-83 | 5 | 017 | 01-0at 86 | $01.0 c t 8$ | $01.0 c 186$ |
|  | IR FINANCE 16\% 1987 | 1600\% | 1050 | 10619 | 13004\% | 287 | 3057 | 374 | 111499 | 115.961 | $188 \%$ | 005 | 0058 | 009 | 01.Sep-83 | 49 | 171 | 01 Mar 87 | 01 Mar | 09 Mar 87 |
|  | IR EXCHEQR $11 \% 1987$ | 1100\% | 1600 | 9541 | 12971\% | 299 | 3188 | 387 | 152662 | 155023 | 252\% | 008 | 0080 | 0010 | 15-Jul-83 | 97 | 425 | 15-Jul 87 | 15-Jul 87 | 15-Jul-87 |
| $\infty$ | IR NATION. 5 1/4\% 1982/日7 | 575\% | 21.0 | 8142 | 12705\% | 324 | 3447 | 399 | 17098 | 17115 | $028 \%$ | 001 | 0010 | $\bigcirc$ | 1-sep. 83 | 49 | 148 | 01 Sep 87 | 01.Sep-87 | 01. Sep 87 |
| $\infty$ | IR CONVER $81 / 2 \%$ 1986/88 | 850\% | 1050 | 8841 | 1295\% | 330 | 3513 | 429 | 92826 | 94781 | $154 \%$ | 005 | 0054 | 007 | $150 \times 183$ | 5 | 008 | 15.00 ct 82 | 1500487 | 15.0c1 87 |
|  | IR FUNDING 11 144\% 1988 | 1125\% | 800 | 9569 | 12967\% | 337 | 3588 | 453 | 76554 | 76258 | 124\% | 004 | 0044 |  |  | 80 | 186 | 01-Feb-86 | 01. Feb-88 | $01 . \mathrm{Feb} 88$ |
|  | IRCONVER 15\% 1988 | 1500\% | 500 | 10471 | 13.001\% | 52 | 3752 | 4.87 | 52354 | 53361 | 087\% | 003 | 0034 | 006 | 01-Nov 83 | 12 | 037 | 01 May 88 | ${ }^{01}$ May-88 | 01 May 88 |
|  | IR.NATION. 9 J/4\% 1984/89 | 975\% | 1350 | 9128 | 12.744\% | 404 | 4302 | 579 | 123226 | 126109 | 205\% | 008 | 0088 | 01 |  | 49 | 201 | 01. Sep. 88 | 01. Sep 88 | 01. Sep 88 |
|  | IR EXCHEQR 5 3/4\% 1984/89 | 575\% | 260 | 7601 | 12893\% | 435 | 4636 | 604 | 19763 | 19714 | 032\% | 001 | 0088 | 012 | ${ }^{01}$-Aug 83 | 80 | 214 | 01. Aug 88 | 01.Aug.89 | 01 Aug 89 |
|  | IREXCHEOR $14 \%$ 1990/92 | 1400\% | 1150 | 10280 | 12921\% | 423 | 4501 | 629 | 118222 | 121749 | 198\% | 008 |  | 002 | 01 - ${ }^{\text {d }}$ 818 83 | 12 | 019 | 01 -Nov 84 | 01. Nov 89 | 01 Nov 89 |
|  | IR EXCHEQR 11 112\% 1990 | 11.50\% | 1230 | 9536 | 13129\% | 441 | 4698 | 682 | 117297 | 119853 | 195\% | 00 | 0089 | 012 | 01.Aug. 83 | 80 | 307 | 01.Feb 90 | 01 Feb-92 | 01 Feb 90 |
|  | IREXCHEQR $6 \%$ 1985/90 | 6.00\% | 650 | 7513 | 13.024\% | 473 | 504 | 10 | 48 Q36 | 48559 | 079\% | 009 | 0091 | 013 | 15 Aug 83 | 66 | 208 | 15-Aug. 90 | 15 Aug 90 | 15-Aug 90 |
|  | IR NATION 6 3/4\% 1986/91 | 675\% | 690 | 7758 | 13060\% | 494 | 5258 | 79 | 53532 | 53.774 | 087\% | $0{ }^{0}$ | OO40 | 006 | 15. Nov 83 | 26 | 043 | 15-Nov 85 | 15-Nov. 90 | 15 Nov 90 |
|  | IRNATION 7\% 1987/92 | 700\% | 1260 | 7859 | 12963\% | 512 | 5450 | 866 | 99020 | 102087 | 166\% | 00 | 0046 | 007 | $01-\mathrm{Cct} 83$ | 19 | 035 | 01.0ct-86 | 01.0 ct 91 | $01-\mathrm{Cct} 91$ |
|  | IR FINANCE 11 1/2\% 1991/93 | 11.50\% | 250 | 9482 | 13 306\% | 515 | 5488 | 925 | 23706 | 24139 | 039\% | 008 | 0090 | 014 | $15-\mathrm{Jun} 83$ | 127 | 243 | 15.Jun 87 | 15.Jun 92 | 15 Jun 92 |
|  | IR DEVELO $71 / 2 \%$ 1988/93 | 750\% | 1910 | 8026 | 13095\% | 527 | 5619 | 970 | 153294 | 157647 | 256\% | 002 | 0022 | 004 | 26-Aug 83 | 55 | 113 | 15-Jan 91 | $15 . \operatorname{Jan} 93$ | 15 Jan 93 |
|  | IR NATION 9 1/4\% 1989/94 | 925\% | 630 | 8755 | 13167\% | 541 | 5765 | 1070 | 55158 | 56929 | 0.92\% | O | 0148 | 025 | 01-Ju4 83 | 111 | 228 | 01. Jui 88 | 01 Jut93 | 01 Juw 93 |
|  | IRCONVER 12\% 1995 | 12.00\% | 1410 | 9600 | 13561\% | 565 | 6037 | 1191 | 135358 | 136979 | 223\% | 005 | 0053 | 010 | 01.Jut 83 | 11 | 281 | 01. Jut 89 | 01. Ju-94 | 01. Jul 94 |
|  | IREXCHEQR 9 1/4\% 1991/96 | 9.25\% | 2460 | 8677 | 13813\% | 542 | 5.797 | 1304 | 213450 | 212703 | $346 \%$ | 013 | 0134 | 027 | 15-Sep-83 | 35 | 115 | 15-Sep. 95 | 15-Sep 95 | 15-Sep.95 |
|  | IR NATION 9 3/4\% 1992297 | 975\% | 2630 | 89.15 | 13756\% | 555 | 5927 | 14.00 | 234469 | 234820 | 382\% | O | 020 | 045 | 01-Nov. 83 | 12 | 030 | 01-Nov91 | 01 Nov. 96 | 01 Nov 96 |
|  | IR FINANCE 14 1/2\% 1998/00 | 14.50\% | 600 | 10173 | 13560\% | 652 | 6964 | 1492 | 61.041 | 61874 | 1.01\% | 021 | 0226 | 053 | 15-0ct-83 | 5 | 013 | 15-0ct-92 | 1500197 | $15.0 \mathrm{Cl}-97$ |
|  | IR NATION. 11 \% 1993/98 | 11.00\% | 2780 | 9353 | 13754\% | 578 | 6177 | 1500 | 260020 | 260439 | 423\% | 00 | 007 | 015 | $15-5$ Sep 83 | 35 | 139 | 15-Sep. 98 | 15 Sep 00 | 15-Sep-98 |
|  | IR DEVELO 11 1/2\% 1997/99 | 11.50\% | 2900 | 9492 | 13922\% | 585 | 6257 | 1608 | 275273 | 272899 | $443 \%$ | 024 | 0261 | 063 | $15-0 \mathrm{ct-83}$ | 5 | 015 | 15-0cl-93 | 15.0498 | 150 ct 98 |
|  | TR DEVELO 14 3/4\% 200204 | 1475\% | 700 | 10203 | 13330\% | 720 | 7680 | 1830 | 71423 | 73685 | 120\% | - | 027 | 071 | $15 . \mathrm{Nov-83}$ | 26 | 082 | 15 Nov. 97 | 15-Nov.99 | 15 Nov-99 |
|  | IR FINANCE 13\% 199702 | 1300\% | 2900 | 9892 | 13674\% | 649 | 6936 | 1846 | 286878 | 288839 | $469 \%$ | - | 0.92 | 022 | 01 -Aug 83 | 80 | 323 | 01-Feb-02 | 01 Feb 04 | 01 Feb-02 |
|  | IR DEVELO 12 1/4\% 2003 | 1225\% | 450 | 9860 | 13099\% | 674 | 7185 | 1967 | 44369 | 46361 | 075\% | 50 | 0325 | 087 | 010 Cl 83 | 19 | 068 | 01 Apr 97 | 01 Apre 02 | 01 Apr 02 |
|  | IR EXCHEOR 6 112\% 2000005 | 650\% | 1330 | 8175 | 12849\% | 532 | 5661 | 2170 | 108728 | 111450 | $181 \%$ | 010 | 0254 | 015 | 10. Jun-83 | 132 | 443 | $15 \cdot \mathrm{Jun} 03$ | 15 Jun 03 | 15 Jun 03 |
|  |  |  |  |  |  |  |  |  |  |  |  |  | 0103 | 039 | 27. Jun 83 | 115 | 205 | 27. Jun 00 | 27.Jun 05 | 27 Jun.05 |


|  | Trade: $\quad$ 17-Apr-84 |  |  |  |  |  |  |  | Clean | Dirty | Slock |  |  |  |  |  |  |  |  |  |
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|  | Sett 19-Apr-84 | Coupon | Nominal Issue | Market Price | $\begin{gathered} \text { Market } \\ \text { Yield } \end{gathered}$ | Volatility | Duration | Lile | Market <br> Value <br> (IREm) | Market Value <br> (IR£m) | $\begin{aligned} & \text { Weight } \\ & \text { in } \\ & \text { Index } \end{aligned}$ | Weighted Volatility | Weighted Duration | Weighted Life | $\begin{aligned} & \text { Ex Div } \\ & \text { Date } \end{aligned}$ | Accrued | Accrued Intrest | $\begin{gathered} \text { First } \\ \text { Redemption } \\ \text { Date } \end{gathered}$ | $\begin{gathered} \text { Last } \\ \text { Redempton } \\ \text { Date } \end{gathered}$ | Redempton |
|  | IR FINANCE VAR\% 1988 | 1256\% | 1700 | 100.02 | 12700\% | 0.37 | 0383 | -0.13 | 170.031 | 178213 | 282\% | 001 |  |  |  |  |  |  |  |  |
|  | IR.FINANCE VAR\% 1985 | 1254\% | 1000 | 10000 | 12688\% | 025 | 0260 | -0.01 | 100.002 | 103264 | 164\% |  |  | 000 | 01. Dec-83 | 140 | 481 | ${ }^{01-J u n} 88$ | 01-Jun-88 | 01.Jun-88 |
|  | IR FINANCE VAR\% 1985 | 1293\% | 2100 | 10000 | 12.930\% | 009 | 0096 | 0.15 | 210000 | 212602 | 164\% | 000 | 0004 | 00 | 15-Jan 84 | 95 | 326 | 15-Jan-85 | 15.Jan-85 | 15-Jan. 85 |
|  | IR CONVER 13\% 1984 | 1300\% | 2400 | 10021 | $11534 \%$ | 015 | 0160 | 016 | 240508 | 251271 | 398\% | 000 | 0003 | 001 | 15-Ma.84 | 35 | 124 | 15-Sep-85 | 15 Sep 85 | 15-Sep Bs |
|  | IR FINANCE VAR\% 1986 | 1273\% | 1600 | 10005 | 12525\% | 003 | 0033 | 028 | 160082 | 159.413 | 252\% |  |  | 001 | $15 . \operatorname{Dec} 83$ | 126 | 448 | 15-Jun 84 | 15.Jun 84 | $15 . \mathrm{Jun} 84$ |
|  | IR FINANCE 113/4\% 1984 | 11.75\% | 2700 | 9988 | 12171\% | 031 | 0327 | 032 | 269666 | 275.225 | 252\% | 000 | 0001 | 01 | 01 May 84 | -12 | 042 | 01-May-86 | 01 May 86 | 01-May-86 |
|  | IR NATION $5114 \%$ 1979/84 | 5.25\% | 290 | 96.16 | 12.580\% | 054 | 0574 | 0.58 | -27886 | 275.225 27778 | 436\% | 001 | 0014 | 001 | 15.Feb-84 | 64 | 206 | 15-Aug.84 | 15-Aug. 84 | 15 Aug 84 |
|  | IR FUNDING 11 1/2\% 1985 | 11.50\% | 155.0 | 9879 | 13210\% | 076 | 0.812 | 0.83 | 153.130 | 15625 | 0.44\% | 000 | 0.003 | 000 | 15 May-84 | 26 | 037 | 15 Nov. 79 | 15- Nov -84 | 15 Nov- 84 |
|  | IRNATION. 14 \% 1985 | 14.00\% | 1510 | 101.32 | 12.257\% | 083 | 0881 | 0.90 | 152.990 | 155016 | 24\% | 002 | 0020 | 002 | 15-Feb-84 | 64 | 202 | 15 Feb 85 | $15 . \mathrm{Feb} 85$ | 15-Feb-85 |
|  | IR NATION 14\% 1985/90 | 1400\% | 1510 | 101.32 | 12.257\% | 083 | 0.881 | 0.90 | 152.990 | 155016 | 2.46\% | 002 | 0022 | 002 | $15 . \mathrm{Mar} 84$ | 35 | 1.34 | 15-Ma 85 | $15 \mathrm{Mar85}$ | 15 Mar - 85 |
|  | IREXCHEQR 12\% 1985 | 1200\% | 2900 | 9900 | 13.133\% | 097 | 1035 | 107 | 287093 | 284615 | 2.46\% | 002 | 0022 | 002 | 84 | 35 | 1.34 | 15-Ma 85 | $15 . \mathrm{Mar} 90$ | 15 Mar 85 |
|  | IR FINANCE 12 1/4\% 1985 | 1225\% | 2800 | 9889 | 13201\% | 132 | 1404 | 149 | 276.895 | 272270 | 4 | 004 | 0047 | 5 | 84 | 26 | 085 | 15 May 85 | 15 May 85 | 15 May 85 |
|  | IREXCHEQR 6 \% 1980/85 | 600\% | 102.0 | 9193 | 12.001\% | 147 | 1555 | 1.62 | 93.769 | 115 | 4 $52 \%$ | 006 | 0062 | 07 | 15-Apr B4 | 4 | 013 | 15-0ct 85 | 15.0c185 | $15-\mathrm{Cc} 185$ |
|  | IR FUNDING 15 1/2\% 1986 | 15.50\% | 1150 | 10281 | 13373\% | 157 | 1671 | 1.83 | 118.231 | 121355 | 1526 | 002 | 0024 | 02 | 01-Dec-83 | 140 | 230 | 01-Dec-80 | 01. Dec-85 | 01. Dec. 85 |
|  | IREXCHEQR 10 $314 \% 1986$ | 1075\% | 1800 | 9640 | 13.202\% | 169 | 1798 | 195 | 173524 | 174477 | 1.92\% | 003 | 0032 | 04 | 15-Feb-84 | 64 | 272 | 15 Feb 86 | $15 . \mathrm{Feb}$ B | 15 Feb -86 |
|  | IR NATION. $711 / 2 \%$ 1981/85 | 750\% | 1190 | 9146 | 12548\% | 192 | 2040 | 2.20 | 108843 | 111506 | 27\% | 005 | 0050 | 05 | 01.Apr 84 | 18 | 053 | 01. Apr 86 | 01-Apr 86 | 01 Apr 86 |
|  | IREXCHEOR 12 1/2\% 1986 | 1250\% | 1800 | 9879 | 13.211\% | 204 | 2.179 | 2.4 | 177.826 | 178935 | 17\% | 003 | 0036 | 004 | 01.Jan. 84 | 109 | 224 | 01. Ju1 81 | 01. Jul 86 | 01-Jul 86 |
|  | IR FUNDING 12 3/4\% 1987 | 1275\% | 1800 | 9903 | 13.267\% | 233 | 2480 | 2.87 | 178255 | 181334 | 287\% | 006 | 0062 | 007 | 01.Ap. 84 | 18 | 062 | 01-0ct.86 | $01 . \mathrm{Crag}$ | 01.0486 |
|  | IR FINANCE 16\% 1987 | 1600\% | 1050 | 10495 | 13402\% | 254 | 2713 | 324 | 110.201 | 114570 | 181\% | 007 | 0071 | 0.08 | 01-Mar 84 | 49 | 171 | 01 Max 87 | $01 . \mathrm{Mar} 8$ | 01 Max 87 |
|  | IREXCHEQR 11 \% 1987 | 1100\% | 450 | 9512 | 13285\% | 268 | 2853 | 337 | 12803 | 43467 | 181\% | 005 | 0049 | 0.06 | 15-Jan: 84 | 95 | 416 | 15-Jul 87 | 15-Jul 87 | 15.Jul 87 |
|  | IR.NATION 5 $514 \%$ 1982/87 | 575\% | 21.0 | 8370 | 12384\% | 292 | 3098 | 349 | 17576 | 17.590 | 069\% | 002 | 0020 | 002 | 01.Mar-84 | 19 | 148 | 01-Sep-87 | 01-Sep 87 | 01.5 ep-87 |
| $\infty$ | IR.CONVER \& $112 \% 1986 / 88$ | 8 $50 \%$ | 1600 | 8917 | 12.968\% | 300 | 3196 | 3.79 | 142676 | 145581 | 228\% | 001 | 0.009 | 001 | 15-Apr 84 | 1 | 006 | 15 Oct 82 | 15.0487 | 15-0c187 |
|  | IR.FUNDING 11 1/4\% 1988 | 11.25\% | 100.0 | 95.18 | 13.301\% | 307 | 3277 | 4.04 | 95.176 | 185581 98007 | 230\% | 0.07 | 0074 | 0.09 | 01.-Fb. 84 | 78 | 182 | 01-Feb-86 | 01 Feb 88 | 01 Feb-88 |
|  | \|R.CONVER 15\% 1988 | 1500\% | 500 | 103.47 | 13.450\% | 322 | 3436 | 4.37 | 51.7 | 52741 | 0 04.4 | 005 | 0049 | 006 | 01-May 84 | 12 | 037 | 01-May-88 | 01. May 88 | 01 May-88 |
|  | IR.NATION. $93 / 4 \%$ 1984/89 | 975\% | 1800 | 90.90 | 13013\% | 379 | 4032 | 5.29 | 163612 | 167360 | 265\% | 003 | 0029 | 004 | 01 Max-84 | 49 | 201 | 01 Sep 88 | $01.5 e p .88$ | $01.5 e p .88$ |
|  | IR.EXCHECR $53 / 4 \% 1984 / 89$ | 575\% | 260 | 7899 | 12.144\% | 4.18 | 4432 | 554 | 20.538 | 20489 | 265\% | 001 | 0107 | 0.14 | $01 . \mathrm{Feb} .84$ | 78 | 208 | 01-Aus.84 | 01. Aug 89 | 01-Aug-89 |
|  | IREXCHEQR 14\% 199092 | 1400\% | 1250 | 10169 | 13329\% | 3.95 | 4215 | 579 | 127.115 | 130852 | 207\% | 001 | 0014 | 002 | 01. May -84 | -12 | 019 | 01 Nov-84 | 01.Nov-89 | 01 Nov- 89 |
|  | IR.EXCHEQR 11 1/2\% 1990 | 1150\% | 1430 | 9465 | 13.429\% | 4.17 | 4452 | 633 | 135352 | 138234 | 219\% | 009 | 0097 | 12 | 01 -reb 84 | 78 | 299 | $01 . \mathrm{Feb} 90$ | 01-Feb-92 | 01-Feb-90 |
|  | IR EXCHEQR $6 \% 1985 / 90$ | 600\% | 1000 | 7101 | 12591\% | 460 | 4887 | 658 | 77008 | 76581 | 121\% | 006 | 0059 | 014 | 15-eb 84 | 64 | 202 | 15.Aug 90 | 15.Aug 90 | 15-Aus 90 |
|  | IR.NATION 63 3\% 1986/91 | 675\% | 840 | 79.78 | 12768\% | 483 | 5.142 | 7.45 | 66.177 | 66.456 | 105\% | 005 | 0054 | 008 | 15-May ${ }^{\text {a }}$ | ${ }^{26}$ | 043 | 15-Nov-85 | 15- Nov 90 | 15 Nov. 90 |
|  | IR NATION 7\% 198792 | 7.00\% | 1410 | 7965 | 12.675\% | 505 | 5367 | 8.16 | 112.300 | 115.705 | 1.83\% | 0.09 | 054 | 0015 | 01-Apr-84 | 18 | 033 | 01 Oct-86 | 01-0ct.91 | 01.0ct-91 |
|  | IR.FINANCE 11 1/2\% 199193 | 11.50\% | 350 | 9448 | 13428\% | 498 | 5319 | 875 | 33.068 | 34115 | 054\% | 003 | 0098 | 015 | 15-Dec-83 | ${ }^{126}$ | 241 | 15-Jun-87 | 15-Jun-92 | 15-Jun-92 |
|  | IR DEVELO $71 / 12 \%$ 1988/93 | 750\% | 2260 | 80.26 | 13.123\% | 517 | 5513 | 921 | 181.389 | 186447 | 295\% | 015 | 0029 | 005 | 15-an- 84 | 95 | 299 | 15-Jan-91 | 15-Jar-93 | 15 Jan-93 |
|  | IR.NATION $9114 \% 1989 / 94$ | 925\% | 730 | 8650 | 13548\% | 522 | 5.570 | 10.21 | 63146 | 65162 | 103\% | 005 | O6s | 027 | 1) and | 109 | 224 | 01-Jut 88 | 01 Ju 93 | $01 . \mathrm{Jut} 93$ |
|  | IR.CONVER $12 \% 1995$ | 1200\% | 151.0 | 9496 | 13.973\% | 543 | 5806 | 11.41 | 14.388 | 145124 | 230\% | 012 | 005 | 011 | 01. an 84 | 109 | 276 | 01-Jur 89 | 01 July 94 | $01 . \mathrm{Jug} 94$ |
|  | IR.EXCHEQR 9 1/4\% 1991/96 | 925\% | 2460 | 8569 | 14.217\% | 524 | 5612 | 12.55 | 210.000 | 210053 | \% | 012 | O3 | 026 | 15 mar 84 | 35 | 115 | 15-Sep 95 | 15-Sep.95 | 15 Sep 95 |
|  | IR NATION 9 934\% 199297 | 975\% | 2630 | 8812 | 14.153\% | 5.35 | 5733 | 1350 | 231.766 | 232047 | , | 0 | (8) | 042 | 01 -May-89 | . 12 | 030 | 01 Nov. 91 | 01 - Nov 96 | 01. Now-96 |
|  | IR FINANCE 14 1/2\% 1998/00 | 14.50\% | 800 | 10095 | 13986\% | 623 | 6.663 | 14.42 | 80.761 | 81.873 | 130\% | 02 | 021 | 050 | 15-Apr-84 | 4 | 0.11 | 15-0ct-92 | 15-0ct-97 | 1500097 |
|  | IR NATION. 11 \% 1993/98 | 1100\% | 278.0 | 9249 | 14.209\% | 554 | 5930 | 14.50 | 257.136 | 257.471 | $408 \%$ | 000 | 0086 | 019 | 15 Mar -84 | 35 | 139 | 15 Sep. 98 | $15-\operatorname{sep} 00$ | 15 Sep- 98 |
|  | IR.DEVELO 11 1/2\% 1997/99 | 11.50\% | 2900 | 9395 | 14394\% | 5.59 | 5989 | 1558 | 272.469 | 270095 | 4288 | 23 | 0242 | 059 | 15-Apr-84 | 4 | 0.12 | 15-Oct-93 | 15-0ct-98 | 15.0ct.98 |
|  | IR DEVELO 14 3/4\% 200204 | 14.75\% | 800 | 10147 | 13.727\% | 6.87 | 7.345 | 1780 | 81.177 | 83697 | $133 \%$ | 0.24 | 0256 | 0.67 | ${ }^{15}$-May-84 | 26 | . 082 | 15-Nov-97 | 15-Now 99 | 15-Now. 99 |
|  | IR FINANCE 13\% 199702 | 13.00\% | 2900 | 9827 | 14079\% | 621 | 6645 | 17.96 | 284987 | 286845 | 4.54\% | 0.9 | 0.097 | 024 | $01 . \mathrm{Feb}$-84 | 78 | 315 | 01 -Feb-02 | 01. Fet-04 | 01 Feb-02 |
|  | IR DEVELO. 12 144\%2003 | 1225\% | 650 | 9795 | 13492\% | 645 | 6881 | 19.17 | 63.666 | 66.413 | 105\% | 08 | 0302 | 0.82 | 01 Apr 84 | 18 | 064 | 01. Apr 97 | 01 Aproz | 01 Apr 02 |
|  | IR EXCHEQR 6 1/2\% 2000/05 | 6.50\% | 1330 | 8112 | 13026\% | 524 | 5585 | 21.20 | 107895 | 110.593 | 175\% | 009 | 0072 | 02 | 15.00 c .83 | 126 | 423 | 15-Jun-03 | $15 . \mathrm{Jun} 03$ | 15. Jun 03 |
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|  |  |  | Coupon | Nominal Issue | Market Price | Market Yield |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| IR CAPITAL | 8\% | 1993 | 800\% | 3680 | 10003 | 6810\% | 003 | 0031 | 003 | 368127 | 367240 | 2 46\% | 000 | 000 | 000 | 01-Nov-93 | 11 |  |  |  |  |
| IR FINANCE | VAR\% | 1994 | 678\% | 6610 | 10000 | 6780\% | 011 | 0112 | 011 | 661000 | 667135 | 47\% | 000 | 001 | 001 | 01-Nov-93 | -11 | -024 | 01-Nov-93 | 01-Nov-93 | 01 Nov-93 |
| IR FUNDING | VAR\% | 1996 | $686 \%$ | 3170 | 10000 | 6850\% | 019 | 0192 | 019 | 317006 | 318256 | 213\% | 000 | 000 | 000 | 30-Sep-93 | 50 | 093 | 01-Jun.94 | 01. Jun 94 | 01-Jun.94 |
| IR FUNDING | VAR\% | 1995 | 678\% | 5038 | 10000 | 6780\% | 023 | 0236 | 024 | 503800 | 504268 | $338 \%$ | 001 | 001 | 001 | 16-0ct-93 | 5 | 009 | -sep-so | Saps | 30-Sep-96 |
| IR CAPITAL | 7\% | 1994 | 700\% | 6070 | 100.29 | 6220\% | 039 | 0399 | 040 | 608774 | 612962 | 411\% | 002 | 002 | 002 | 15-Scep-93 | ${ }^{5}$ | 009 | 16-Jun-95 | 16-Jun-95 | 16-Jun 95 |
| IR EXCHEOR | $13 \%$ | 1994 | 1300\% | 193 | 10582 | $6200 \%$ | 093 | 0961 | 098 | 20424 | 20465 | 014\% | 000 | 000 | 000 | 15-Oct-93 | 6 | 021 | 15-Oc1.94 | 15-Oat 94 | 5-Mar.94 |
| IR CONV | 1/2\% | 1995 | $950 \%$ | 3704 | 104.53 | 6030\% | 142 | 1462 | 152 | 387186 | 386319 | $259 \%$ | 004 | 004 | 004 | 30-0c1-93 | 9 | 023 | 30-A0r-95 | 30-A0t-95 | 30.A0ar-95 |
| IR CAPITAL | 121/4\% | 1995 | 1225\% | 7 | 108.22 | 6090\% | 148 | 1530 | 161 | 44045 | 45983 | 031\% | 00 | 000 | 000 | 01-Jun-93 | 142 | 476 | 01-Jun-95 | 01. Jun-95 | 01. |
| IR CONVER | 12\% | 1995 | 1200\% | 1490 | 109.09 | 6090\% | 173 | 1785 | 190 | 162551 | 164314 | 10\% | 002 | 002 | 002 | 15-Sep 93 | 36 | 118 | 15-Sep-95 | 15-Sep 95 | 15-Sep-95 |
| IR CAPITAL | 9\% | 1996 | $900 \%$ | 10580 | 10595 | 6260\% | 247 | 2549 | 278 | 1120979 | 1142617 | 766\% | 01 | 020 | 021 | 30-Jul-93 | 83 | 205 | 30.Jul-96 | 30-Jul-96 | 30-Jul. 96 |
| IR EXCHEQR | 8 $1 / 2 \%$ | 1996 | $850 \%$ | 987 | 10502 | 6310\% | 261 | 2696 | 295 | 103657 | 104139 | 070\% | 002 | 002 | 002 | 30-Sep-9 | 21 | 049 | 30-Sep-96 | 30-Sep-96 | 30-Sep-96 |
| IR FINANCE | $13 \%$ | 1997/02 | 1300\% | 1400 | 11556 | $6450 \%$ | 296 | 3052 | 345 | 161789 | 162786 | 109\% | 003 | 003 | 004 | $01-\mathrm{Oct} 93$ | 20 | 071 | 01-Apr-97 | 01 Apr-02 | 01-A0r-97 |
| $\cdots \mathrm{OR}$ Capital | $73 / 4 \%$ | 1997 | 775\% | 3380 | 10376 | $6400 \%$ | 324 | 3345 | 373 | 350696 | 357724 | 2 40\% | 008 | 008 | 009 | 15-Jul-93 | 98 | 208 |  |  |  |
| O ir exchear | 8 $314 \%$ | 1997 | 875\% | 13010 | 10651 | $6380 \%$ | 325 | 3351 | 377 | 1385649 | 1412452 | 947\% | 031 | 032 | 036 | 27-Jul-93 | 86 | 206 | 15-Jul. 97 | 15-Jul-97 | 15-Jul-97 |
| $\boldsymbol{\chi}_{\text {IR D Develo }}$ | 11 1/2\% | $1997 / 99$ | 1150\% | 1450 | 11355 | 6530\% | 343 | 3545 | 407 | 164649 | 171908 | 1.15\% | 004 | 004 | 005 | 15-May-93 | 159 | 501 | 15-Nov-97 |  |  |
| IR CAPITAL | 9 3/4\% | 1998 | 975\% | 9812 | 11039 | 6380\% | 386 | 3980 | 461 | 1083150 | 1120343 | 751\% | 029 | 030 | 035 | 01-Jun-93 | 142 | 379 | 01-Jun 98 | 01-Jun.98 | 15-Nou-97 |
| IR FINANCE | 14 1/2\% | 1998/00 | 14 50\% | 732 | 12239 | 6600\% | 404 | 4174 | 490 | 89593 | 90639 | 061\% | 002 | 003 | 003 | 15-Sep-93 | 36 | 143 | 15-Sep-98 |  | 01-Jun-98 |
| if treasue | 1/4\% | 1999 | 625\% | 7730 | 9919 | 6470\% | 454 | 4686 | 545 | 766776 | 793627 | 532\% | 024 | 025 | 029 | 01-Apr. 93 | 203 | 347 | 15-Sep-sa | 15-Sep-00 | 15-Sep-98 |
| IR CAPITAL | 71/2\% | 1999 | 750\% | 2270 | 104.28 | 6 $360 \%$ | 470 | 846 | 573 | 236706 | 241274 | 162\% | 008 | 008 | 009 | 15-Jul 93 | 980 | , | 11-Apr-99 | 01-Apr.99 | 01-Apr-99 |
| ir capital | 113/4\% | 2000 | 1175\% | 1309 | 11836 | 6500\% | 51 | 5317 | 649 | 154932 | 155185 | 104\% | 005 | 006 | 07 |  |  | 20 | 15-Jul-99 | 15-Jul-99 | 99 |
| ir develo | 121/4\% | 2000/03 | 12 25\% | 1035 | 11900 | 6740\% | 524 | 5414 | 665 | 123166 | 127609 | 086\% | 004 | 005 | 0 | 15-0c-93 | 6 | 019 | 15-Apr-00 | 15-Apr 00 | 15-Apr-00 |
| IR GOVER | $9 \%$ | 2001 | 900\% | 11334 | 109.68 | 6650\% | 592 | 6112 | 774 | 1243074 | 1270443 | $852 \%$ | 050 | 052 | 066 | -15 | 128 | 429 | $15 . \mathrm{Jun}-00$ | 15-Jun-03 | 15. Jun 00 |
| IR CAPItal | $8 \%$ | 2001 | $800 \%$ | 500 | 10563 | 6700\% | 607 | 6270 | 799 | 52815 | 52881 | 035\% | 002 | 002 | 003 | 15-0t-93 | 6 | 241 | 15-Jul-01 | 15.Jul 01 | 15 -Jul-01 |
| IR DEVELO | 143/4\% | $2002 / 04$ | 14 75\% | 714 | 126.94 | 6750\% | 647 | 6692 | 829 | 90635 | 92970 | 062\% | 004 | 004 | 005 | 01-Aug-93 | 81 | 327 | 5-0a-ol | 15-Oa-01 | 15-Oct-01 |
| IR CAPItal | 9 1/4\% | 2003 | 9 $25 \%$ | 13320 | 111.18 | 6700\% | 707 | 7319 | 973 | 1480893 | 1515309 | 1016\% | 072 | 074 | 099 | 11-Ju1-93 | 102 | 58 | 1-feb-02 | 01 Feb-04 | 01 -Fab-02 |
| IR EXCHEQR | 8 $1 / 4 \%$ | 2003 | $825 \%$ | 530 | 10694 | 6750\% | 720 | 7439 | 1003 | 56676 | 56568 | 0 38\% | 003 | 003 | 004 | 30-0ct-93 | . 9 | 20 | 30-0 | 1.Julos | 11-Ju-03 |
| IR TREASU 6 | 1/4\% | 2004 | 625\% | 2380 | 97.47 | 6740\% | 769 | 7950 | 1100 | 231971 | 232093 | 156\% | 012 | 012 | 017 | $18.0 \mathrm{Ct}-93$ | 3 | 005 | 30-Oc1-03 | 30-0a-03 | 30.0ad-03 |
| IR EXCHEQR | 612\% | 2000/05 | 650\% | 2360 | 98.73 | 6740\% | 800 | 8273 | 1169 | 233007 | 237878 | 159\% | 013 | 013 | 019 | 27-Jun-93 | 116 | 206 | 27.Jun-00 | 127 | 18.0a-04 |
| IR CAPItal | 121/2\% | 2005 | 1250\% | 414 | 121.20 | 6850\% | 876 | 9062 | 1216 | 50175 | 51988 | 035\% | 003 | 003 | 004 | 15.Jun-93 | 128 | 438 | 15.Dec-05 | 27.Jun-05 | 27-Jun-0 |
| IR CAPItal | $9 \%$ | 2006 | 900\% | 7712 | 109.82 | 6850\% | 863 | 8926 | 1287 | 846942 | 856443 | 574\% | 050 | 051 | 074 | 01-Spp-93 | 50 | 123 | , | Dec-os | 15-Dec-as |
| IR CAPItal | 81/4\% | 2008 | 825\% | 3196 | 10600 | 6980\% | 929 | 9616 | 1478 | 338777 | 344769 | 231\% | 021 | 022 | 034 | 30-Jul 93 | 93 | 187 | 1-Sep-o6 | -1-sep-06 | 1-Sep.06 |
| ir Capital | 8 112\% | 2010 | $850 \%$ | 3235 | 106.81 | 6990\% | 1017 | 10529 | 1696 | 345545 | 347050 | 233\% | 024 | 024 | 039 | 01-Oct-93 | O | 047 | -1-0. | -u-ob | --Jula |
| IR CAPITAL | 8 3/4\% | 2012 | 875\% | 8250 | 10727 | 7030\% | 1092 | 11308 | 1896 | 884950 | 889100 | $596 \%$ | 065 | 067 | 113 | 30-Sap-93 | 21 | 050 | 30-Sep-1 | - | 1-0c-10 |










Appendix 2
Outliers for Term Structure Identification 1980-1997

## A.2.1 April 1980

The bonds excluded are: Finance Variable\% 1983, National $41 / 4 \% 1975 / 80$. National 5 1/4\% 1979/84. Exchequer 6\% 1980/85, National 7\% 1987/92 and National $91 / 4 \%$ 1989/94.

## A.2.2 October 1980

The bonds excluded are: Finance Variable\% 1983, Nation. 4 1/4\% 1975/80, National 5 1/4\% 1979/84, Exchequer 6\% 1980/85, National 7\% 1987/92, National 9 1/4\% 1989/94, National 9 1/4\% 1982, National 9 1/4\% 1981, National 4 1/4\% 1975/80, National 14\% 1985/90, National 11\% 1993/98, National $5 \%$ 1971/81, Funding $81 / 2 \%$ 1981, Finance Variable\% 1983, Finance 14 1/2\% 1998/00, Exchequer 5 3/4\% 1984/89 and Exchequer 14\% 1990/92.

## A.2.3 April 1981

The bonds excluded are: the short maturity 10\% Exchequer 1981 bond, Finance 14 1/2\% 1998/00, Finance 11 3/4\% 1984, Finance 11 1/2\% 1981, Exchequer 6 \% 1980/85, Exchequer 5 3/4\% 1984/89, Exchequer 14\% 1990/92, a high coupon 14\% National 1985 bond and 9 1/4\% National 1982 and the 5 1/4\% National 1979/84 which had an embedded Conversion feature.

## A.2.4 October 1981

The bonds excluded are: high coupon $141 / 2 \%$ National 1988/00 bond, the $91 / 4 \%$ National 1989/94 bond and National 5 1/4\% 1979/84, Finance Variable\% 1986, Finance Variable\% 1983, Finance 10 1/2\% 1982, Exchequer 6 \% 1980/85, Exchequer 5 3/4\% 1984/89, Exchequer 11 1/2\% 1982, Conversion 8 1/2 \% 1986/88 and Conversion 13\% 1984

## A.2.5 April 1982

The bonds excluded are: National 9 1/4\% 1989/94, National 5 1/4\% 1979/84, National 14 \% 1985, Finance Variable\% 1986, Finance Variable\% 1985, Finance Variable\% 1983, Exchequer 6 \% 1980/85, Development 11 1/2\% 1997/99 and Conversion 9 \% 1980/82.

## A.2.6 October 1982

The bonds excluded are: National 9 3/4\% 1992/97, National 9 1/4\% 1989/94, National 5 3/4\% 1982/87, National 5 1/4\% 1979/84, National 11 \% 1993/98, Funding 11 3/4\% 1983, Finance Variable\% 1986, Finance Variable\% 1985, Finance Variable\% 1985, Finance Variable\% 1983, Exchequer 6 \% 1980/85 and Development 14 3/4\% 2002/04.
A.2.7 April 1983

The bonds excluded are: National 9 3/4\% 1992/97. National 5 3/4\% 1982/87, Nationai 11 \% 1993/98, Finance Variable\% 1986, Finance Variable\% 1985, Finance Variable\% 1985, Finance Variable\% 1983, Exchequer 15 \% 1983 And Development 14 3/4\% 2002/04.

## A.2.8 October 1983

The bonds excluded are: Funding 11 1/2\% 1983, Finance Variable\% 1988, Finance Variable\% 1986, Finance Variable\% 1985, Finance Variable\% 1985, Finance 12 \% 1984, Finance 11 1/2\% 1991/93, Exchequer 5 3/4\% 1984/89,Development 14 3/4\% 2002/04 And Development 12 1/4\% 2003.

## A.2.9 April 1984

The bonds excluded are: National 5 3/4\% 1982/87, Finance Variable\% 1988, Finance Variable\% 1986, Finance Variable\% 1985, Finance Variable\% 1985, Finance 11 3/4\% 1984, Exchequer 6 \% 1985/90. Exchequer 6 \% 1980/85, Exchequer 5 3/4\% 1984/89, Development 14 3/4\% 2002/04, Development 12 1/4\% 2003 And Conversion 13 \% 1984.

## A.2.10 October 1984

The bonds excluded are: National 5 3/4\% 1982/87, National 5 1/4\% 1979/84, National 14 \% 1985/90, Funding 11 1/2\% 1985, Finance Variable\% 1985, Finance Variable\% 1985, Finance Variable\% 1986, Finance Variable\% 1988, Exchequer 6 \% 1985/90, Exchequer 5 3/4\% 1984/89, Development 2 1/2\% 1989, Development 12 1/4\% 2003 And Development 11 1/2\% 1997/99.

## A.2.11 April 1985

The bonds excluded are: National 14 \% 1985/90, Finance Variable\% 1985, Finance Variable\% 1986, Finance Variable\% 1989, Finance Variable\% 1988, Finance 14 1/2\% 1998/00, Finance 12 1/4\% 1985, Exchequer 6 \% 1985/90, Exchequer 12 \% 1985, Development 2 1/2\% 1989 And Capital 11 3/4\% 2000.

## A.2.12 October 1985

The bonds excluded are: National 9 3/4\% 1992/97, National 14 \% 1985/90, National 11 \% 1993/98, Funding 15 1/2\% 1986, Funding 10 \% 1986, Finance Variable\% 1986, Finance Variable\% 1990, Finance Variable\% 1989, Finance Variable\% 1988, Exchequer 6 \% 1980/85, Exchequer 10 3/4\% 1986, Development 2 1/2\% 1989, Capital 9 1/2\% 1986 And Capital 11 3/4\% 2000.

## A.2.13 April 1986

The bonds excluded are: National 9 3/4\% 1984/89, National $71 / 2 \% 1981 / 85$, National 11\% 1993/98, Finance Variable\% 1990, Finance Variable\% 1989, Finance Variable\% 1988, Exchequer 6 1/2\% 2000/05, Exchequer 6\% 1985/90, Exchequer 12 1/2\% 1986, Development 2 1/2\% 1989 And Development 14 3/4\% 2002/04

## A.2.14 October 1986

The bonds excluded are: Funding 12 3/4\% 1987, Finance Variable\% 1990, Finance Variable\% 1989, Finance Variable\% 1988, Finance 14 1/2\% 1998/00, Finance 13\% 1997/02, Exchequer 13\% 1994, Development 2 1/2\% 1989, Development 14 3/4\% 2002/04, Development 12 ¼ 2000/03, Development 11 ½\% 1997/99, Capital $71 / 4 \% 1988$ and Capital 12 1/2\% 2005.

## A.2.15 April 1987

The bonds excluded are: National 9 3/4\% 1992/97, National 5 3/4\% 1982/87, National 14\% 1985/90, Finance Variable\% 1990, Finance Variable\% 1989, Finance Variable\% 1988, Finance 16\% 1987, Exchequer 9\% 1987, Exchequer 11\% 1987 and Capital 14\% 1987.

## A.2.16 October 1987

The bonds excluded are: Funding 11 1/4\% 1988, Finance Variable\% 1990, Finance Variable\% 1989, Finance Variable\% 1988, Finance 11 1/2\% 1991/93, Development 12 1/4\% 2000/03, Conversion $81 / 2 \%$ 1986/88, Capital 7 3/4\% 1997, Capital $71 / 2 \% 1999$, Capital 8 1/2\% 1991, Capital 11 3/4\% 2000, Capital 11\% 1988 and Capital 8\% 2001.

## A.2.17 April 1988

The bonds excluded are: National 9 3/4\% 1984/89, National 9 3/4\% 1992/97, Funding 11 1/4\% 1988, Finance Variable\% 1993, Finance Variable\% 1990, Finance Variable\% 1989, Finance Variable\% 1988, Finance 13\% 1997/02, Finance 11 1/2\% 1991/93, Exchequer 6 1/2\% 2000/05, Exchequer 5 3/4\% 1984/89, Development 2 1/2\% 1989, Conversion 15\% 1988. Capital 8\% 1993, Capital 7 1/4\% 1988. Capital 13\% 1990, Capital 12 1/2\% 2005, Capital 11\% 1988, Capital 9 1/4\% 2003 and Capital 9\% 2006.

## A.2.18 October 1988

The bonds excluded are: National 9 3/4\% 1992/97, National 11\% 1993/98, Finance Variable\% 1993, Finance Variable\% 1992, Finance Variable\% 1990, Finance Variable\% 1989, Finance 9\% 1989 and Development 2 1/2\% 1989.

## A.2.19 April 1989

The bonds excluded are: National 9 3/4\% 1992/97, National 9 1/4\% 1989/94, National 11\% 1993/98, Finance Variable\% 1993, Finance Variable\% 1992, Finance Variable\% 1991. Finance Variable\% 1990, Finance Variable\% 1989, Finance 13\% 1997/02, Exchequer 5 3/4\% 1984/89, Development 2 1/2\% 1989, Capital 11 3/4\% 2000, Capital 10\% 1989 and Capital 8\% 2001.

## A.2.20 October 1989

The bond excluded are: National 9 3/4\% 1992/97, National 11\% 1993/98, Finance Variable\% 1993, Finance Variable\% 1992, Finance Variable\% 1991, Finance Variable\% 1990, Exchequer 8 1/4\% 2003, Exchequer 5 3/4\% 1984/89, Exchequer 14\% 1990/92, Capital 7\% 1990, Capital 11 3/4\% 2000. Capital 9\% 2006 and Capital 8\% 2001.

## A.2.21 April 1990

The bonds excluded are: National 11\% 1993/98, Finance Variable\% 1994, Finance Variable\% 1993, Finance Variable\% 1992, Finance Variable\% 1991, Finance Variable\% 1990, Exchequer 9 1/4\% 1991/96, Exchequer 6\% 1985/90, Exchequer 11 1/2\% 1990, Development 14 3/4\% 2002/04, Capital 9 3/4\% 1998, Capital 13\% 1990 and Capital 11 3/4\% 2000.

## A.2.22 October 1990

The bonds excluded are: National 11\% 1993/98, Funding Variable\% 1995, Finance Variable\% 1994, Finance Variable\% 1993, Finance Variable\% 1992, Finance Variable\% 1991, Finance 11 1/2\% 1991/93. Exchequer 6\% 1985/90, Development 14 3/4\% 2002/04, Capital 8\% 1991 and Capital 7 1/2\% 1991.

## A.2.23 April 1991

The bonds excluded are: National 6 3/4\% 1986/91, Funding Variable\% 1995, Finance Variable\% 1994, Finance Variable\% 1993, Finance Variable\% 1992, Finance Variable\% 1991. Finance 12 1/2\% 1991, Capital 8 1/2\% 1991 and Capital 8 1/2\% 2010.

## A.2.24 October 1991

The bonds excluded are: Funding Variable\% 1995, Finance Variable\% 1994, Finance Variable\% 1993, Finance Variable\% 1992, Finance Variable\% 1991 and Capital 8\% 2001.
A.2.25 April 1992

The bonds excluded are: National 9 3/4\% 1992/97, National 7\% 1987/92, Funding Variable\% 1995, Finance Variable\% 1994, Finance Variable\% 1993, Exchequer $71 / 4 \%$ 1992, Capital 8 3/4\% 1992, Capital 11 3/4\% 2000 and Capital 8\% 2001.

## A.2.26 October 1992

The bonds excluded are: National 9 3/4\% 1992/97, National 11\% 1993/98, Funding Variable\% 1996, Funding Variable\% 1995, Finance Variable\% 1994, Finance Variable\% 1993, Finance 13\% 1997/02, Exchequer 13\% 1994, Capital 8 1/2\% 1992 and Capital $121 / 2 \%$ 2005.

## A.2.27 April 1993

The bonds excluded are: National 9 3/4\% 1992/97, National 9 1/4\% 1989/94, National $11 \%$ 1993/98, Government 9\% 2001, Funding Variable\% 1996, Funding Variable\% 1995, Finance Variable\% 1994, Exchequer 9 1/4\% 1991/96 and Development 7 1/2\% 1988/93.

## A.2.28 October 1993

The bond excluded are: Treasury 6 1/4\% 2004, Funding Variable\% 1996, Funding Variable\% 1995, Finance Variable\% 1994, Capital 8\% 1993, Capital 7\% 1994 and Capital 8 3/4\% 2012.
A.2.29 April 1994

The bonds excluded are: Funding Variable\% 1996, Funding Variable\% 1995, Finance Variable\% 1994, Exchequer 13\% 1994 and Capital 7 3/4\% 1997.

## A.2.30 October 1994

The bonds excluded are: Funding Variable\% 1998, Funding Variable\% 1996, Funding Variable\% 1995 and Capital 73/4\% 1997.

## A.2.31 April 1995

The bonds excluded are: Funding Variable\% 1998, Funding Variable\% 1996. Conversion $12 \%$ 1995, Conversion $91 / 2 \%$ 1995, Capital $121 / 4 \% 1995$, Capital 12 1/2\% 2005 and Capital 8\% 2001.

## A.2.32 October 1995

The bonds excluded are: Funding Variable\% 2000, Funding Variable\% 1998, Funding Variable\% 1996 and Capital 9\% 2006.
A.2.33 April 1996

The bonds excluded are: Funding Variable\% 2000, Funding Variable\% 1998, Funding Variable\% 1996. Finance 14 1/2\% 1998/00, Exchequer 8 1/2\% 1996, Capital 9\% 1996, Capital $81 / 4 \% 2008$ and Capital $81 / 2 \% 2010$.

## A.2.34 October 1996

The bonds excluded are: Funding Variable\% 2000, Funding Variable\% 1998, Finance 14 1/2\% 1998/00. Finance 13\% 1997/02, Capital 9\% 2006 and Capital 8\% 2001.

## A.2.35 April 1997

The bonds excluded are: Funding Variable\% 2000, Funding Variable\% 1998, Exchequer 8 3/4\% 1997, Exchequer 6 1/2\% 2000/05, Capital 8 1/2\% 2010, Capital 8\% 2001and Capital 7 3/4\% 1997.

## A.2.36 October 1997

The bonds excluded are: Funding Variable\% 2000, Funding Variable\% 1998, Exchequer 6 1/2\% 2000/05, Development 12 1/4\% 2000/03, Development 11 1/2\% 1997/99, Capital 8 1/4\% 2008 and Capital 8\% 2001.

## Appendix 3

Results of Term Structure Identification 1980-1997
Comparison of Actual v. Implied Bond Prices

Initial Yield Function fitted to All Yields


As explained in the methodology, the null hypothesis is that all bonds are part of the data set and outliers are bonds with highly significant yields who failed to be have acceptable yields within the $99 \%$ confidence interval for the appropriate degrees of freedom. Outliers distort the yield curve and are generally eliminated after being identified in the bootstrap procedure. The majority of outliers are generally composed of dual rate, low coupon, convertible or variable rate bonds. Dual rate bonds with coupons very close to market yields distort the curve since the embedded option is at the money. Since the holder is short the redemption option to the issuer, the prices is depressed by the present value of the option and biases the yield upwards as a consequence. Extremely low coupon bonds, as a result of their tax effect ${ }^{1}$, tend to distort the rates with the residual errors being negatively sloped, i.e. the error term is expected to increase as the coupon level falls.

[^38]To remove the tax effect, only high coupon bonds might have been included in the sample but due to the size and lack of liquidity of the Irish market during much of the period covered by the data, this approach would not have been viable. Variable rate bonds and convertible bonds, due to the money market index and the option effect respectively, tend to cause the bonds to trade above the yield curve.

Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Discount Data



Outlier Bond Price Identification from Step-wise Spot Rate Data


Initial Yield Function fitted to All Yields


Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Discount Data


Fitted Price v. Actual Price for Step-wise Spot Rate Data


Initial Yield Function fitted to All Yields


Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Spot Rate Data


Outier Bond Price Identification from Step-wise Discount Data


Fitted Price v. Actual Price for Step-wise Spot Rate Data


Outlier Bond Price Identification from Step-wise Spot Rate Data


Initial Yield Function fitted to All Yields


Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Discount Data


Fitted Price v. Actual Price for Step-wise Spot Rate Data


Outlier Bond Price Identification from Step-wise Spot Rate Data


Initial Yield Function fitted to All Yields


Fitted Price v. Actual Price for Step-wise Discount Data



Fitted Price v. Actual Price for Step-wise Spot Rate Data


Outlier Bond Price Identification from Step-wise Spot Rate Data


Initial Yield Function fitted to All Yields


Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Discount Data


Fitted Price v. Actual Price for Step-wise Spot Rate Data


Outlier Bond Price Identification from Step-wise Spot Rate Data


Initial Yield Function fitted to All Yields


Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Discount Data


Fitted Price v. Actual Price for Step-wise Spot Rate Data


Outier Bond Price Identification from Step-wise Spot Rate Data


Initial Yield Function fitted to All Yields


Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Discount Data


Fitted Price v. Actual Price for Step-wise Spot Rate Data


Outlier Bond Price Identification from Step-wise Spot Rate Data



Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Discount Data


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Outlier Bond Price Identification from Step-wise Spot Rate Data


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Outlier Bond Price Identification from Step-wise Spot Rate Data


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Outlier Bond Price Identification from Step-wise Spot Rate Data


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Initial Yield Function fitted to All Yields


Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Discount Data


Fitted Price v. Actual Price for Step-wise Spot Rate Data


Outlier Bond Price Identification from Step-wise Spot Rate Data


## Initial Yield Function fitted to All Yields



Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Discount Data


## Outlier Bond Price Identification from Step-wise Discount Data




Outlier Bond Price Identification from Step-wise Spot Rate Data




Outlier Bond Price Identification from Step-wise Discount Data


## Fitted Price v. Actual Price for Step-wise Spot Rate Data



Outlier Bond Price Identification from Step-wise Spot Rate Data


Initial Yield Function fitted to All Yields



Outtier Bond Price Identification from Step-wise Discount Data


Fited Price v. Actual Price for Step-wise Spot Rate Data


Outler Bond Price Identification from Step-wise Spot Rate Data


Initial Yield Function fitted to All Yields


Fitted Price v. Actual Price for Step-wise Discount Data



Fitted Price v. Actual Price for Step-wise Spot Rate Data


Outtier Bond Price Identification from Step-wise Spot Rate Data



Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Discount Data



Initial Yield Function fitted to All Yields


Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Discount Data


Fitted Price v. Actual Price for Step-wise Spot Rate Data


Outlier Bond Price Identification from Step-wise Spot Rate Data



Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Discount Data


Fitted Price v. Actual Price for Step-wise Spot Rate Data


Outlier Bond Price Identification from Step-wise Spot Rate Data



Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Discount Data



Outlier Bond Price Identification from Step-wise Spot Rate Data


Initial Yield Function fitted to All Yields


## Fitted Price v. Actual Price for Step-wise Discount Data



Outlier Band Price Identification from Step-wise Discount Data


Fitted Price v. Actual Price for Step-wise Spot Rate Data


Initial Yield Function fitted to All Yields


Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Discount Data


Fitted Price v. Actual Price for Step-wise Spot Rate Data


Outlier Bond Price Identification from Step-wise Spot Rate Data



Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Discount Data


Fitted Price v. Actual Price for Step-wise Spot Rate Data


Outlier Bond Price Identification from Step-wise Spot Rate Data


Initial Yield Function fitted to All Yields


Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Discount Data


Fitted Price v. Actual Price for Step-wise Spot Rate Data


Outlier Bond Price Identification from Step-wise Spot Rate Data


Initial Yield Function fitted to All Yields



Outlier Bond Price Identification from Step-wise Discount Data


Fitted Price v. Actual Price for Step-wise Spot Rate Data


Outlier Bond Price Identification from Step-wise Spot Rate Data


Initial Yield Function fitted to All Yields



Outlier Bond Price Identification from Step-wise Discount Data


## Fitted Price v. Actual Price for Step-wise Spot Rate Data



Outlier Bond Price Identification from Step-wise Spot Rate Data


Initial Yield Function fitted to All Yields


Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Discount Data


Fitted Price v. Actual Price for Step-wise Spot Rate Data


Outlier Bond Price Identification from Step-wise Spot Rate Data



Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Discount Data


Fitted Price v. Actual Price for Step-wise Spot Rate Data


Outlier Bond Price Identification from Step-wise Spot Rate Data



Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Discount Data


Fitted Price v. Actual Price for Step-wise Spot Rate Data


Outlier Bond Price Identification from Step-wise Spot Rate Data


Initial Yield Function fitted to All Yields


Fitted Price v. Actual Price for Step-wise Discount Data



Fitted Price v. Actual Price for Step-wise Spot Rate Data



Fitted Price v. Actual Price for Step-wise Discount Data


Outlier Bond Price Identification from Step-wise Discount Data


Fitted Price v. Actual Price for Step-wise Spot Rate Data


Outlier Bond Price Identification from Step-wise Spot Rate Data


## Appendix 4

Parameters of Stochastic Models for Bond Prices

## A.4.1 Distribution of Spot Rates \& their Changes

From exhibit A.4.1, a similar decline in the long spot rate from a high of over $20 \%$ in 1981 to a low of just under $8 \%$ in 1997 can be seen.


Exhibit A.4.1 Long Irish Spot Rates - 1980 to 1997
Source: Empirical

Then the statistical distribution which best fits the long spot rate time series from 1980 to 1997 is an Erlang distribution with the parameter estimates being $\xi$ of 8.99 and $\beta$ of 0.0119 shown in A.4.2;
(A.4.2) $\quad f(I)=\frac{\beta^{-\xi} l^{\xi-1} e^{-1 / \beta}}{\Gamma(\xi)}$

For the statistical distribution which best fits the spread spot rate time series from 1980 to 1997, by using Kolmogorov-Smirnov test the normal distribution is the closest fitting the data with the parameter estimates being $\mu$ of 0.00656 and $\sigma$ of 0.0169 shown in A.4.3;

$$
\begin{equation*}
f(s)=\frac{1}{\sqrt{2 \pi \sigma^{2}}} e^{-\frac{(s-\mu)^{2}}{2 \sigma^{2}}} \tag{A.4.3}
\end{equation*}
$$

From exhibit A.4.2, the term structure is normally shaped for $69 \%$ of the sample period and inverted for $31 \%$ of the time.


Exhibit A.4.2 Spread between Long and Short Irish Spot Rates - 1980 to 1997
Source: Empirical

The statistical distribution which best fits the changes in short spot rates time series from 1980 to 1997 is a normal distribution with the parameter estimates being $\mu$ of -0.00373 and $\sigma$ of 0.0119 shown in A.4.3;
(A.4.3) $\quad f(\Delta r)=\frac{1}{\sqrt{2 \pi \sigma^{2}}} e^{-\frac{(\Delta r-\mu)^{2}}{2 \sigma^{2}}}$

In exhibit A.4.3, the autocorrelation of the changes in short spot rates time series is investigated and the hypothesis of autocorrelation is rejected.

## Series: diff.spot.short.cts



Exhibit A.4.3 Autocorrelation of Short Irish Spot Rates - 1980 to 1997

Then the statistical distribution which best fits the change in long spot rates time series from 1980 to 1997 is a Log-normal distribution with the parameter estimates being $\mu$ of 2.99 and $\sigma$ of 0.32 and shifted by -0.0538 shown in A.4.4;
(A.4.4)

$$
f(\Delta I)=\frac{1}{\Delta I \sqrt{2 \pi \sigma^{2}}} e^{-\frac{(\ln (\Delta I)-\mu)^{2}}{2 \sigma^{2}}}
$$

For the statistical distribution which best fits the change in spread spot rates time series from 1980 to 1997, the Weibull distribution is identified to be the closest to the empirical data with the parameter estimates being $\mu$ of 4.29 and $\sigma$ of 0.0603 and shifted by -0.0533 shown in A.4.5;

$$
\left(\text { A.4.5) } \quad f(\Delta s)=\alpha \beta^{-\alpha}(\Delta s)^{\alpha-1} e^{-\left(\frac{\Delta s}{\beta}\right)^{\alpha}}\right.
$$

The estimation of the reversion of the short rate towards its six year moving mean is shown in table A.4.1.

| Date | Short rate | d(Short rate) | mean | reversion |
| ---: | ---: | ---: | ---: | ---: |
| $\mathbf{0 4 8 8 7}$ | $13.36 \%$ | $-1.40 \%$ | $13.84 \%$ | 0.4272 |
| $\mathbf{1 0 8 7}$ | $11.97 \%$ | $-1.14 \%$ | $13.66 \%$ | 0.2841 |
| $\mathbf{0 4 8 8}$ | $10.83 \%$ | $-2.74 \%$ | $1.44 \%$ | 0.1887 |
| $\mathbf{1 0 8 8}$ | $8.09 \%$ | $-0.19 \%$ | $9.77 \%$ | 0.2199 |
| $\mathbf{0 4 8 9}$ | $7.90 \%$ | $0.62 \%$ | $10.56 \%$ | 0.3890 |
| $\mathbf{1 0 8 9}$ | $8.52 \%$ | $2.01 \%$ | $10.71 \%$ | 0.2819 |
| $\mathbf{0 4 9 0}$ | $10.53 \%$ | $1.65 \%$ | $10.95 \%$ | 0.4004 |
| $\mathbf{1 0 9 0}$ | $12.19 \%$ | $-1.75 \%$ | $10.80 \%$ | 0.3556 |
| $\mathbf{0 4 9 1}$ | $10.43 \%$ | $-0.39 \%$ | $10.57 \%$ | 0.3809 |
| $\mathbf{1 0 9 1}$ | $10.04 \%$ | $0.12 \%$ | $10.40 \%$ | 0.4369 |
| $\mathbf{0 4 9 2}$ | $10.16 \%$ | $0.62 \%$ | $10.33 \%$ | 0.5448 |
| $\mathbf{1 0 9 2}$ | $10.77 \%$ | $3.36 \%$ | $11.19 \%$ | 0.3782 |
| $\mathbf{0 4 9 3}$ | $14.14 \%$ | $-5.76 \%$ | $10.43 \%$ | 0.7286 |
| $\mathbf{1 0 9 3}$ | $8.37 \%$ | $-1.67 \%$ | $9.79 \%$ | 0.6508 |
| $\mathbf{0 4 9 4}$ | $6.71 \%$ | $-0.52 \%$ | $9.29 \%$ | 0.5466 |
| $\mathbf{1 0 9 4}$ | $6.19 \%$ | $-0.35 \%$ | $8.83 \%$ | 0.4421 |
| $\mathbf{0 4 9 5}$ | $5.84 \%$ | $0.95 \%$ | $9.04 \%$ | 0.3838 |
| $\mathbf{1 0 9 5}$ | $6.79 \%$ | $-0.89 \%$ | $8.76 \%$ | 0.3366 |
| $\mathbf{0 4 9 6}$ | $5.90 \%$ | $-0.72 \%$ | $8.16 \%$ | 0.2698 |
| $\mathbf{1 0 9 6}$ | $5.19 \%$ | $0.63 \%$ | $7.50 \%$ | 0.2576 |
| $\mathbf{0 4 9 7}$ | $5.82 \%$ | $\mathbf{0 . 0 0 \%}$ | $6.93 \%$ | 0.2953 |
| $\mathbf{1 0 9 7}$ | $5.82 \%$ | $\mathbf{0 . 4 2 \%}$ | $6.97 \%$ | 0.2884 |

Table A.4.1 Discrete time equivalent of Short rate mean reversion
Source: Empirical

The estimation of the reversion of the long rate towards its six year moving mean is shown in table A.4.2.

| Date | Long rate | d(Long rate) | mean | reversion |
| ---: | ---: | ---: | ---: | ---: |
| $\mathbf{0 4 8 7}$ | $14.08 \%$ | $-2.88 \%$ | $12.83 \%$ | 0.1565 |
| $\mathbf{1 0 8 7}$ | $11.20 \%$ | $0.15 \%$ | $12.70 \%$ | 0.1580 |
| $\mathbf{0 4 8 8}$ | $\mathbf{1 1 . 3 5 \%}$ | $-2.21 \%$ | $10.13 \%$ | 0.1692 |
| $\mathbf{1 0 8 8}$ | $9.14 \%$ | $-1.01 \%$ | $10.27 \%$ | 0.2464 |
| $\mathbf{0 4 8 9}$ | $8.13 \%$ | $0.60 \%$ | $\mathbf{1 0 . 8 1 \%}$ | 0.3327 |
| $\mathbf{1 0 8 9}$ | $8.74 \%$ | $-0.25 \%$ | $9.93 \%$ | 0.2197 |
| $\mathbf{0 4 9 0}$ | $8.49 \%$ | $1.28 \%$ | $10.47 \%$ | 0.2600 |
| $\mathbf{1 0 9 0}$ | $9.77 \%$ | $0.51 \%$ | $10.45 \%$ | 0.3176 |
| $\mathbf{0 4 9 1}$ | $10.28 \%$ | $-1.36 \%$ | $10.02 \%$ | 0.3942 |
| $\mathbf{1 0 9 1}$ | $8.92 \%$ | $0.13 \%$ | $9.92 \%$ | 0.4635 |
| $\mathbf{0 4 9 2}$ | $\mathbf{9 . 0 4 \%}$ | $-0.61 \%$ | $9.67 \%$ | 0.5731 |
| $\mathbf{1 0 9 2}$ | $8.43 \%$ | $1.34 \%$ | $9.71 \%$ | 0.6761 |
| $\mathbf{0 4 9 3}$ | $\mathbf{9 . 7 7 \%}$ | $-1.58 \%$ | $9.61 \%$ | 0.6134 |
| $\mathbf{1 0 9 3}$ | $8.19 \%$ | $-0.43 \%$ | $8.88 \%$ | 0.5619 |
| $\mathbf{0 4 9 4}$ | $7.75 \%$ | $-0.02 \%$ | $8.75 \%$ | 0.5168 |
| $\mathbf{1 0 9 4}$ | $7.74 \%$ | $0.87 \%$ | $8.73 \%$ | 0.7801 |
| $\mathbf{0 4 9 5}$ | $8.61 \%$ | $0.00 \%$ | $8.73 \%$ | 0.7227 |
| $\mathbf{1 0 9 5}$ | $8.61 \%$ | $-0.20 \%$ | $8.78 \%$ | 0.6795 |
| $\mathbf{0 4 9 6}$ | $8.42 \%$ | $-0.42 \%$ | $8.68 \%$ | 0.6323 |
| $\mathbf{1 0 9 6}$ | $8.00 \%$ | $-0.11 \%$ | $8.64 \%$ | 0.5815 |
| $\mathbf{0 4 9 7}$ | $7.89 \%$ | $-0.78 \%$ | $8.24 \%$ | 0.4411 |
| $\mathbf{1 0 9 7}$ | $7.11 \%$ | $-0.73 \%$ | $7.83 \%$ | 0.4236 |

Table A.4.2 Discrete time equivalent of Long rate mean reversion

[^39]
## Appendix 5

## Microstructure Background

In this appendix the structure and instruments of the main bond markets that compete with the Irish government bond market for international portfolio asset allocations are examined. The foreign markets where the Irish government raised funds are identified.

## A.5.1 Germany

With an outstanding volume of more than Dm 2200bn at the end of 1991, the German bond market is one of Europe's largest. The government bond market, in particular, is given a strong boost following the economic and monetary union of the two German states in July 1990, and the subsequent unification in October 1990. In the wake of unification, new issuers came to the German capital market. These included Staatsbank, the German Unity Fund, Deutsche Reichsbahn and Treuhandanstalt.

The market comprises a wide range of investment instruments, with domestic bearer bonds playing the most important role and accounting for approximately 62 per cent of bonds outstanding. Ireland has been a regular issuer on the German capital market for over twenty years. In 1991, the National Treasury Management Agency (NTMA) tapped the "Schuldschein" market three times, raising funds totaling DM250 million. In 1992, extensive relationships continued to be developed in Germany, particularly with mortgage banks. This meant that the NTMA could tap the Schuldschein market both at short notice and for large volumes, including a single transaction of DM500 million. After the currency crisis had passed, the first public bond issue by the NTMA is in the Deutsche Mark market. The issue is for DM300 million, subsequently increased to DM500 million, with a maturity of 10 years.

In March 1993, the NTMA arranged a public bond issue of DM1,500 million which is Ireland's largest ever single issue in a foreign currency. Although not the largest issue in Deutsche Marks in 1993, it reflected the trend towards larger issues which has become evident in the major capital markets for reasons of liquidity and investor impact.

## A.5.2 France

The French Government Bond market has undergone a huge transformation since the 1980's, particularly since 1986, when the Treasury pursued an attractive issuing policy for the international investor. Three milestones can be identified over the last decade: In 1982, the tax on bond transactions in the stock exchange is abolished. In 1986, the French Treasury began a regular competitive bidding auction plan for Treasury bills and bonds, with the first Treasury swap auction being done in 1987. A futures market opened in 1986 and met with swift success.

Treasury auction procedures operate through open competitive bidding. In November 1986, the decision to set up a group of primary dealers in government securities (SVT) is made and in February 1987 the Treasury officially chose 13 institutions. They are required to ;

1. Ensure all auctions run smoothly by assessing global market demand,
2. Maintain the liquidity of the government securities market,
3. Inform Treasury of market developments regularly.

The Treasury issues three categories of standardised debt; OATs which are Treasury bonds issued in order for the government to raise long term funds, BTAN's are Treasury bonds issued by the government to raise medium term debt with a maturity date of between 2 and 5 tears and BTFs are notes with a maturity of up to one year and are used by the government for short term financing.

## A.5.3 United Kingdom

The UK government uses the gilt market as its major source of borrowing, as well as tapping retail investors via National Savings bonds. Gilts account for 60 per cent of the national debt. The first reform of the market occurred after the Big Bang in 1986, which primarily opened up the equity market but also led to some important improvements in the gilt market.

UK government securities are issued by the Treasury via the Bank of England. There are five issuing methods used:

1. Straight to the bank,
2. To the National Debt Commission,
3. To tender when the Bank invites bids for the stock,
4. By auction,
5. By Tap.

The government issues six types of bonds; Conventional gilts, Index-Linked Stocks, Optional Redemption gilts, Convertible gilts, Undated issues and Floating Rate Notes. Liquidity in the gilt market is managed by a system of gilt edged market makers otherwise known as GEMMs. GEMMs act as primary dealers in the market, and quote firm bid and offer prices at all times. They are able to trade stock positions acquired through market making, as they have access to Inter-dealer brokers.

In 1993, Ireland raised one bilateral loan of STG£73 million is contracted at a margin of 19 basis points below inter bank rates and with a maturity of 9 years. In May of the same year a four year EMTN (Euro Medium Term Note) of STG£10 million is issued at inter bank rates less 30 basis points. They have just introduced a REPO ${ }^{1}$ market and plan to develop a strips market next year.

[^40]
## A.5.4 Background to Capital Adequacy Requirement

The following variables are analysed to identify the market makers capital ;

- Source and type of price and/or yield data
- Time period from which data is drawn
- Exposed to risk time period
- Number of maturity bands
- Identification of loss distribution
- Bonds in which obligated to make a price
- $\quad$ Normal market liquidity, size and turnover
- $\quad$ Source and cost of debt funding
- Level of ruin barriers
- Number of Monte Carlo or Latin Hypercube simulations
- Treatment of non symmetrical loss distribution
- Risk Type I Capital Requirements
- Estimation of inter maturity hedging \& relationships
- List of acceptable instruments for hedging purposes
- Risk Type II Capital Requirements
- Model for yield curve estimation
- Combination of sector \& off the run exposures
- Risk Type III Capital Requirements
- Overall Master Model Approach for all Capital requirements
- Total cost of capital requirements
- Any special requirements for auction bids (i.e. taps, tranche, first or second price)


## A.5.9.2 Riada Actuaries Indices and Bank of England GEMMS

The second source of data is Riada's REDS system which stretches from 1979 to date and the fixed income indices are calculated on a simple weighted capital basis, as per the Financial Times - Institute of Actuaries formulae and used primary and secondary market prices. Data covering the period of the 1st January 1990 to the 5 th July 1993 inclusive is used. In assembling the data, the following assumptions are made; period of risk exposure is three days, funding cost is overnight DIBOR (i.e. Dublin Interbank Offered Rate), normal market liquidity, indices calculated on FT-Actuaries basis by Riada's and all returns are gross. As a direct result of the currency crisis, the short end of the yield curve is very volatile and consequently the proposed capital requirements in this area are quite high relative to the Bank of England model.

This three day time horizon is predominately influenced by the required reaction time as a regulator to a problem that may arise in a market makers position. The returns are calculated from a long or short position and the first four moments of the data are calculated in order that the loss distribution may be identified. Since the focus is short term movements, over a short time scale, i.e. market makers being subject to daily fluctuations in yields and remaining solvent over the year, a key decision is the most appropriate probability loss distribution. A market maker faces the very same potential loss profile as an insurance underwriter. In that regard, it must be appreciated that, if a probability distribution is not symmetrical, a different capital requirement would be needed for the same ruin barrier depending on whether the position is long or short.

In such a case, the larger of the two capital requirements which would be that for a short position is chosen since the distribution is negatively skewed. In the case of maturities, seven bands are used; one month, three months, one year, low coupon for five years, other five years, ten years and ten years plus. The risk can be calculated from the bottom up or the top down.

The standard deviation in the one month can be as volatile as the five to ten year of the yield curve. This is a direct result of the currency crisis. Over a three day holding the mean return is positive in the greater then one year area though is only about a penny. The Kurtosis is very high which means than the returns are very tightly distributed around the mean. A normal loss distribution would have the value of three. Finally, all the data displays negative skewness which implies that the mean is greater than the median. From our perspective, the implication is that the market will jump up rather than down and the risk is that the market maker is short rather than long.

Taking an assumption of normality and basing the price changes on the period of 1990 to date, the required capital requirements for Overall Open Position are set out in table A.5.1 along with the comparable Bank of England figures. Then with the appropriate ruin barrier of less than $0.000001 \%, \mathrm{viz}$, there is less than a million to one chance of a market maker becoming insolvent as a result of losses sustained over a one year period. The capital percentage is that of the nominal exposure, e.g. to hold $£ 1 \mathrm{~m}$ of the $83 / 4 \%$ Capital 2012 would require $£ 51,000$.

| Maturity Band | Capital | Odds of using Capital | B.O.E Capital |
| :---: | :---: | :---: | :---: |
| One Month | $2.30 \%$ | $9,758,612,410,337$ to 1 | $0.00 \%$ |
| Three Month | $0.62 \%$ | $8,830,587,504,648$ to 1 | $1.00 \%$ |
| One Year | $0.37 \%$ | $5,976,907,269,238$ to 1 | $1.50 \%$ |
| Low Coupon 1 to 5 Years | $1.68 \%$ | $7,531,103,055,804$ to 1 | $5.00 \%$ |
| 1 to 5 Years | $2.07 \%$ | $9,144,364,725,625$ to 1 | $6.00 \%$ |
| 5 to 10 Years | $3.55 \%$ | $9,908,910,071,222$ to 1 | $7.00 \%$ |
| $10+$ Years | $5.10 \%$ | $4,291,185,924,126$ to 1 | $8.00 \%$ |

Table A.5.1 Risk Weights for Overall Open Position
Source: Empirical

It is envisaged that market makers would carry offsetting (or hedging) positions. This would reduce their capital requirements. Using inter maturity bands (with different volatilities) would leave them with an exposure to a non-paralleled move in the yield curve. Since all exposures would be in a nominal sense (i.e. equivalent to Bank of England approach), there would be a net volatility exposure. These problems are tackled using three approaches;

- Correlation Matrix of sectors
- Comparisons of Durations
- Regression Hedges Ratios and their Correlations

The highest correlation is $89 \%$ between $5-10$ year area and the 10 year plus. The lowest is between the one month and the 10 year plus with a value of $17 \%$. There are two conclusions to this analysis. Firstly, there are no negative correlations that would present the opportunity for substantially reduced capital requirements in a Markovitz Mean-Variance framework. Secondly, there seems to be a difference in behaviour between the "money" end of the yield curve and the rest of the yield curve. The latter would seem to be substantially correlated. There would be some grounds for the belief that the shorter end of the maturity spectrum displays greater yield volatility than the longer end. The last element of the analysis is to investigate how stable the maturity band's durations have been over the past three years. This is illustrated in exhibit A.5.1.

Duration (yrs)


Exhibit A.5.1 Durations of Different Maturity Bands
Source: Empirical

The data from exhibit A.5.1 is shown in table A.5.2;

| Band | 0 to 5 | 5 to 10 | 10 to 15 | 15 to 25 |
| :---: | :---: | :---: | :---: | :---: |
| Mean | 1.53 | 5.25 | 7.64 | 9.28 |
| Std.Dev. | 0.25 | 0.15 | 0.18 | 0.29 |
| Minimum | 1.21 | 5.00 | 7.38 | 8.89 |
| Maximum | 1.82 | 5.42 | 7.86 | 9.64 |

Table A.5.2 Analysis of Duration for different maturity bands
Source: Empirical

From this it can be observed that the duration over the entire period did not change by more than three months in any six month period. The linear regressions are graphed in exhibit A.5.2. While the bond market shows some opportunities for cross hedging, the money market is very unstable.


Exhibit A.5.2 Regression Inter-Maturity Hedges
Source: Empirical

A portfolio that is long the asset with greater maturity and short the asset with close maturity is constructed. The net capital requirements are set out in table A.5.3. For example, a five to ten year exposure short position and a ten year long position would require $1.50 \%$ in a risk II scenario. However, they would have required $8.66 \%$ if short and long positions could not be offsetable, and there is a substantial reduction in capital requirement of $7.16 \%$.

| Hedge Exposure | 1 mth | 3 mths | 1 yr | Low $1 / 5 \mathrm{yr}$ | 1 to 5 Yrs | 5 to 10 Yrs |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| One Month |  |  |  |  |  |  |
| Three Month | $1.14 \%$ |  |  |  |  |  |
| One Year | $1.24 \%$ | $0.20 \%$ |  |  |  |  |
| Low Coupon 5 Years | $1.41 \%$ | $0.94 \%$ | $0.93 \%$ |  |  |  |
| 1 to 5 Years | $1.57 \%$ | $1.17 \%$ | $1.16 \%$ | $0.66 \%$ |  |  |
| 5 to 10 Years | $2.21 \%$ | $2.03 \%$ | $2.02 \%$ | $1.42 \%$ | $1.38 \%$ | $2.39 \%$ |
| $10+$ Years | $3.09 \%$ | $2.98 \%$ | $2.98 \%$ | $2.44 \%$ | $1.50 \%$ |  |

Table A.5.3 Net Capital for Exposures allowing offsetting Positions
Source: Empirical

The capital requirement for the reversed position of being long the asset with greater maturity being hedged by shorting the asset with the shorter maturity is almost symmetric. The Bank of England Requirements are set out in table A.5.4.

| Hedge Exposure | 1 mth | 3 mths | 1 yr | Low 1 / 5 yr | 1 to 5 Yrs | 5 to 10 Yrs |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| One Month |  |  |  |  |  |  |
| Three Month | 1.00\% |  |  |  |  |  |
| One Year | 1.50\% | 1.50\% |  |  |  |  |
| Low Coupon 5 Years | 5.00\% | 5.00\% | 4.50\% |  |  |  |
| 1 to 5 Years | 6.00\% | 6.00\% | 5.50\% | 5.00\% |  |  |
| 5 to 10 Years | 7.00\% | 7.00\% | 6.50\% | 6.00\% | 7.00\% |  |
| 10+ Years | 8.00\% | 8.00\% | 7.50\% | 7.00\% | 8.50\% | 5.00\% |

Table A.5.4 Bank of England Net Capital for Exposures allowing offsetting Positions
Source: Bank of England Regulations for GEMMS

The final area is the area of inter maturity band risk which is a limited stand alone risk. An example would be if a market maker wished to hedge a short position in the $83 / 4 \%$ Capital 2012 by going long $81 / 2 \%$ Capital 2010. This is all within the maturity band of the $10+$ area of the curve. There are three sources of price risk. Firstly, there are changes in the shape of the yield curve over the maturity band; secondly, movement of a particular stock closer to or further from its notional position on the benchmark yield curve and thirdly a small difference in the stock price volatilities. The main reasons for this risk are small issue size or tightly held stock by a foreigner, special estate duty privileges, dual dated or other option type features such as conversion rights. As the yield curve changes level and slope the option component of the bond with increase in value as it's option component moves from out of the money, through at the money to into the money and this will impact on the bond value. Finally, there tax effects such as small coupons for high rate tax payers high coupons for low rate tax payers. From chapter two, a cubic spline measured the type three price risk for each sector. An example is shown in exhibit A.5.3 below;


Exhibit A.5.3 April 1990 B Spline fitted to Yield Curve
Source: Empirical

These risks are shown in table A.5.5 by maturity band under the categories of "sector". The shorter maturity bands - overnight to one year - are treated in a similar manner to the 1 to 5 years sector. For the purposes of risk capital estimation these discrete risks have been treated as being 100\% correlated to err on the conservative side.

Then, using the standard deviation over the period of 1990 to date and a nomal distribution with a ruin barrier of 50,000 to 1, the intra maturity band capital requirement is calculated as follows;

| Sector | Risk Type III Capital |
| :---: | :---: |
| Up to 5 years | $1.078 \%$ |
| 5 to 10 years | $0.970 \%$ |
| $10+$ years | $0.917 \%$ |

Table A.5.5 Intra Maturity Band Capital
Source: Empinical

As indicated above, table A.5.3 sets out the net capital required for offsetting positions. This is the essential minimum capital required. However, the actual capital requirement may be greater than table A.5.5 indicates viz., using the Bank of England method. This method of calculating the capital required has also been computed by splitting the capital requirements derived in table A.5.6 into two elements. The first element is the amount available for hedging an offsetting position in a different maturity band and the second element is the core capital requirement. In computing the capital required for the long and short positions in a portfolio, the core capital for all stocks is added while the amount available for hedging is offset for equal nominal amounts of long and short positions. Table A.5.6 sets out the split between the amounts available for hedging and for core capital.

| Maturity Band | Risk Weights Hedging | Risk Weights Core | Risk Weights Total |
| :---: | :---: | :---: | :---: |
| Up to a month | $0.72 \%$ | $1.59 \%$ | $2.31 \%$ |
| 1 to 3 months | $0.40 \%$ | $0.23 \%$ | $0.63 \%$ |
| 3 to 12 months | $0.37 \%$ | $0.00 \%$ | $0.37 \%$ |
| Low Coupon 1 to 5 years | $1.55 \%$ | $0.14 \%$ | $1.69 \%$ |
| 1 to 5 years maturity | $2.07 \%$ | $0.00 \%$ | $2.07 \%$ |
| 5 to 10 years maturity | $3.55 \%$ | $0.00 \%$ | $3.55 \%$ |
| Over 10 years maturity | $4.18 \%$ | $0.92 \%$ | $5.10 \%$ |

Table A.5.6 Master Table for Market Makers Risk Exposures
Source: Empirical

The capital requirement for a portfolio can be derived from table A.5.6 as follows. For example, the capital required for a long position in the over ten year maturity band and a short position in the five to ten year maturity band would be $(4.18 \%+0.92 \%+0.00 \%)-3.55 \%$. This nets out at $1.55 \%$ which exceeds the minimum net capital for exposures under offsetting positions set out in table A.5.3 of $1.50 \%$. The capital for this two asset portfolio will be set at $1.55 \%$. Since this illustration does not involve intra maturity band risk, it follows that table A.5.6 capital is not required in this case.

The Council Directive 93/6/EEC of 15 March 1993 deals with the subject of the capital adequacy of investments firms and credit institutions. The main objective of Council Directive 93/22/EEC of 10 May 1993 on investment services in the securities field is to allow investment firms authorised by the competent authorities of their home Member States to establish branches and provide services freely in other Member States.

## A.5.2.4 EU Capital Adequacy Directive example for Primary Dealer

It is calculated as follows. Each nominal position is market to market by been multiplied by the closing clean price and accrued interest. The portfolio is organised into maturity bands and then zones for offset purposes. A worked example follows and this is the capital used in the next section.

The capital required by the market makers for a notional portfolio is shown in table A.5.7. This is done by a primary dealer netting his purchases and sales at the end of a trading day in a particular bond. Then, this net amount along with the start of the day's opening inventory is combined to arrive at the closing inventory position. They will adjust their settlement in the REPO and Reverse-REPO markets so that they can settle the following day. Profit or loss realised and recognised when marked to market, will be added to the capital dedicated to the business.

| Stock | Long Nominal Pasition | Shoft <br> Nominał <br> Position | Net <br> Nominal Position | Anaturif <br> y | 20no | Price | tong Nominal Position | Short Nominal Position |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Exch. NOTE 1M | £1.000,000 | E0 | £1,000,000 | 0.08 |  | 1 100.00\% | £1.000.000 | £0 |
| EXCHEQR 8.3/4\% 1997 | £5,000,000 | E0 | £5,000,000 | 1.74 |  | 2 100.00\% | £5,000.000 | £0 |
| CAPITAL 9.3/4\% 1998 | £10,000.000 | £0 | £10,000,000 | 2.58 |  | 2 100.00\% | £10,000,000 | £0 |
| TREASURY 6.1/4\% 1999 | £0 | $£ 20,000,000$ | -£20,000,000 | 3.42 |  | 2 100.00\% | £0 | £20,000,000 |
| TREASURY 8\% 2000 | £0 | £4,000,000 | - £4,000,000 | 4.97 |  | $3100.00 \%$ | £ | £4,000,000 |
| GOVER 9\% 2001 | £11,000,000 | £0 | £11,000,000 | 5.71 |  | 3 100.00\% | £11,000,000 | £0 |
| TREASURY 6.1/4\% 2004 | £0 | £22.000.000 | -£22,000,000 | 8.97 |  | 3 100.00\% | EO | £22,000,000 |
| TREASURY 8\% 2006 | £10,000,000 | £0 | £10,000,000 | 10.80 |  | 3 100.00\% | £10.000,000 | £0 |
| CAPITAL 8.3/4\% 2012 | £4.000,000 | £0 | £4,000,000 | 16.93 |  | 3 100.00\% | £4,000.000 | £0 |
| TREASURY 8.1/4\% 2015 | £0 | £7,000.000 | -£7,000,000 | 19.81 |  | 3 100.00\% | EO | £7,000.000 |
| Total Position | £41,000,000 | £53,000,000 | $-£ 12,000,000$ |  |  |  | £41,000,000 | £53,000,000 |

Table A.5.7 Primary Dealer Notional Portfolio
Source: Empirical
Exchequer Notes are included along with Variable Bonds which are treated as short term paper with their 'notional' maturity been the next reset of coupon. The assumed prices of all bonds is $100 \%$ to get agreement on the calculation. After initial netting with a long position of $£ 41 \mathrm{~m}$ and short position of $£ 53 \mathrm{~m}$, the net position is short $£ 12 \mathrm{~m}$. Individual bond positions are grouped into maturity bands and weighted according to the EU-CAD in table A.5.8.

| Maturity | Long Position | Short Posiation | Woighted Long Position | Weighted Sthort Position | \#hatched <br> Weightited <br> Posĭtion | Unmatched Waighted Position | Urmatched Welghted Long Positian | Unmatchod Weighted Shart Position |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.08 | £1,000,000 | £0 | £0 | £0 | £0 | £0 | £0 | EO |
| 0.25 | £ | £0 | £0 | £0 | £0 | £0 | £0 | £0 |
| 0.50 | £O | £0 | £0 | £O | EO | £0 | £0 | £ |
| 1.00 | £O | £0 | £0 | £O | EO | £0 | £0 | £ |
| 2.00 | £5,000,000 | £0 | £62,500 | £0 | £O | £62,500 | £62,500 | £ |
| 3.00 | £10.000,000 | £0 | £175,000 | £0 | EO | £175,000 | £175,000 | £0 |
| 4.00 | £0 | -£20,000,000 | £0 | - £450,000 | £0 | £450,000 | £0 | £450,000 |
| 5.00 | £0 | -£4,000,000 | £0 | -E110.000 | £0 | £110.000 | £ 0 | £110,000 |
| 7.00 | £11,000,000 | £0 | £357,500 | £0 | EO | £357,500 | £357.500 | £0 |
| 10.00 | £0 | $-£ 22,000,000$ | £ | -£825.000 | £0 | £825,000 | £O | £825,000 |
| 15.00 | £10,000,000 | £0 | £450,000 | £ | £0 | £450,000 | £450,000 | £0 |
| 20.00 | £4,000,000 | -£7,000,000 | £210,000 | -£367,500 | £210,000 | £157,500 | £0 | £157,500 |
| >20.00 | £0 | EO | £0 | £ | £0 | £0 | £0 | £0 |
|  | £44,000,000 | $-£ 53,000,000$ | £1,255,000 | -£1,752,500 | £210,000 | £2,587,500 | £1,045,000 | £1,542,500 |

Table A.5.8 Primary Dealer Netted Portfolio by Maturity
Source: Empirical

This gave a weighted long position of $£ 1,255,000$ and a weighted short position of $£ 1,752,500$. Then the long and short positions are matched within maturity band for $£ 210,000$ in the 15 to 20 years. This committed $£ 210,00$ of capital. All other positions are unmatched weighted positions to a value of $£ 2,587,500$ which are separated into $£ 1,045,000$ long and $£ 1,542,500$ short. After this the maturity bands are gathered into three zones of up to one year, one year to four years and four years plus. There are no positions in zone $1, £ 237,550$ long and $£ 450,000$ short in zone 2 , and $£ 807,500$ long and $£ 1,092,500$ short in zone three in table A.5.9.

| zone | Unmatehed long Posifion | Unmatched Short Fosfition | Batched Position | Unmatched Position | After <br> Zones <br> Long Posiltion | Attar <br> Zanes <br> Shert <br> Position | Matched Zones 1-2 | After <br> Zones 3 -2 <br> Long Position | After <br> Zones 1.2 <br> Short <br> position |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | £0 | £0 | £0 | £0 | £0 | £0 | £ | £0 | £O |
| 2 | £237.500 | -£450,000 | £237,500 | $£ 212.500$ | EO | -£212,500 | £0 | £0 | - £212,500 |
| 3 | £807,500 | -£1,092,500 | £807,500 | £285,000 | £O | -£285,000 | £0 | L0 | -£285.000 |
| Total | £1,045,000 | -£1,542,500 | £1,045,000 | £497,500 | £0 | -£497,500 | £ 0 | £0 | - $£ 497,500$ |

Table A.5.9 Primary Dealer Portfolio Positions by Zone
Source: Empirical

Consequently, zone 2 had $£ 237,500$ of matched positions and $£ 807,500$ of zone 3 matched positions. This committed $£ 71,250$ and $£ 242,250$ of capital respectively. The unmatched zone positions are $£ 212,500$ of zone 2 matched positions and $£ 285,000$ of zone 3 . From table A.5.10, there is no matching positions between zone 1 and zone 2 , between zone 2 and zone 3 or between zone 1 and zone 3 .

| Zone | Matched <br> Zones 2-3 | Attor <br> Zones 2-3 <br> Long <br> Position | After <br> Zames 2-3 <br> Short <br> Position | Matiched <br> Zones 1-3 | After <br> Zones 1-3 <br> Long <br> Position | After <br> Zonab t-3 <br> Short <br> Position | Residual |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | £0 | £0 | £0 | £0 | £ | £ | £0 |
| 2 | £0 | £0 | -£212,500 | £0 | £0 | -£212.500 | £212,500 |
| 3 | E0 | £0 | -£285,000 | £0 | EO | -£285.000 | £285,000 |
| Total | £0 | £0 | -£497,500 | £0 | £0 | -£497,500 | £497,500 |

Table A.5.10 Primary Dealer Portfolio Unmatched Positions by Zone

Source: Empirical

After this zone matching there is a residual position of $£ 497,500$. This committed $£ 497,500$ of capital. The total capital that a primary dealer would have to commit is $£ 832,000$ in table A.5.11.

| Matched Weighted Positions in all Bands | $£ 21,000$ |
| :---: | ---: |
| Matched Weighted Positions in Zone 1 | $£ 0$ |
| Matched Weighted Positions in Zone 2 | $£ 71,250$ |
| Matched Weighted Positions in Zone 3 | $£ 242,250$ |
| Matched Weighted Positions in Zone 1/Zone 2 | $£ 0$ |
| Matched Weighted Positions in Zone 2/Zone 3 | $£ 0$ |
| Matched Weighted Positions in Zone 1/Zone 3 | $£ 0$ |
| Residual Unmatched Weighted Positions | $£ 497.500$ |
| Required Capital | $£ 83,000$ |

Table A.5.11 Primary Dealer Portfolio EU-CAD Capital Requirement

Source: Empirical

Any individual bond position will initially attract the normal capital weighting. When a bond is matched within a maturity band by an equal and opposite position, this is given a $90 \%$ reduction of required capital. However, if it cannot be matched within a maturity band but within a zone band, the capital weight of the shorter maturity position will be given a $60 \%$ reduction in zone 1 and $70 \%$ reduction in zone 2 and 3.

The unmatched capital weight of the longer maturity bond within the zone band will attract $0 \%$ on the part greater than the capital weight of the shorter maturity position. This is followed by the matching on an inter zone basis rather than an intra basis, starting with zone 1 v. zone 2 , then zone 2 v . zone 3 and finally zone 1 v . zone 3 . You will notice that each zone in compared on a sequential basis starting with the shortest maturity and progressing to the longest. For matching between zone 1 v . zone 2 , the capital weight of the shorter maturity zone will be given a $60 \%$ reduction in zone 1 and the weight of the longer maturity bond in zone 2 will attract $0 \%$ reduction on the part greater than the capital weight of the shorter maturity position in zone 1. Whenthere is matching between zone 2 v . zone 3 , the capital weight of the shorter maturity zone will be given a $60 \%$ reduction in zone 2 and the weight of the longer maturity bond in zone 3 will attract $0 \%$ reduction on the part greater than the capital weight of the shorter maturity position in zone 2.

Finally, with matching between zone 1 v . zone 3 , you have to load the capital weight of zone 1 bond weight by $50 \%$ and the longer maturity bond in zone 3 will still attract $100 \%$ weighting. This penalises traders who borrow in the money market and run positions beyond four years as they would be better off not matching the positions. All residual positions are unmatched by elimination and hence attract a capital weighting of $100 \%$.

Appendix 6
Mismatch Reserve

In this appendix the approach is to use the data on a monthly basis from 1985 to 1997 and divide it into two sub-periods. The three month DIBOR represented the Irish money market, Riada total return indices represented short irish government bonds, long Irish government bonds, all Irish government bonds, Salomon World Government Indices on a local basis are used for UK,US, Japan, France and Germany. In the case of equity markets total return indices; UK is FT-All Share, US is S\&P Composite, Japan is Topix, France is CAC and Germany is Commerzbank. The monthly returns and risks are shown in table A.6.1.

| Monthly | Return 1985-89 | Risk 1985-89 | Return 1990-97 | Risk 1990-97 |
| :---: | :---: | :---: | :---: | :---: |
| Immunised | $0.65 \%$ | $0.58 \%$ | $0.44 \%$ | $0.37 \%$ |
| Irish Money | $0.89 \%$ | $0.19 \%$ | $0.72 \%$ | $0.30 \%$ |
| Irish Short | $0.58 \%$ | $0.72 \%$ | $0.37 \%$ | $0.46 \%$ |
| Irish Total | $0.59 \%$ | $0.75 \%$ | $0.39 \%$ | $0.49 \%$ |
| Irish Long | $0.74 \%$ | $1.43 \%$ | $0.39 \%$ | $0.74 \%$ |
| Irish Equity | $2.58 \%$ | $7.31 \%$ | $0.99 \%$ | $5.20 \%$ |
| Japan Bond | $0.79 \%$ | $2.48 \%$ | $0.79 \%$ | $4.15 \%$ |
| U.K. Bond | $0.70 \%$ | $2.91 \%$ | $0.97 \%$ | $2.57 \%$ |
| German Bond | $1.71 \%$ | $7.17 \%$ | $0.92 \%$ | $4.84 \%$ |
| US Bond | $0.36 \%$ | $4.06 \%$ | $0.68 \%$ | $3.42 \%$ |
| French Bond | $0.99 \%$ | $1.75 \%$ | $0.89 \%$ | $2.13 \%$ |
| Japan Equity | $2.46 \%$ | $5.96 \%$ | $-0.26 \%$ | $7.81 \%$ |
| U.K. Equity | $1.57 \%$ | $6.84 \%$ | $1.15 \%$ | $4.47 \%$ |
| German Equity | $1.71 \%$ | $7.17 \%$ | $0.92 \%$ | $4.84 \%$ |
| US Equity | $1.18 \%$ | $6.98 \%$ | $1.25 \%$ | $4.51 \%$ |
| French Equity | $2.33 \%$ | $7.26 \%$ | $0.83 \%$ | $5.01 \%$ |

Table A.6.1 Return and Risk of Total Return Indices in Irish Pounds
Source : Empirical

In table A.6.2 the correlation between the principal asset classes held by Irish general insurers is shown. There is a high correlation with the other bond classes and the immunised portfolio and with the UK bond market.

|  | Immunised Irish Money Irish Short |  | Irish <br> Total | Irish Long | Irish <br> Equity | Japan Bond U.K. Bond |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Immunised | $100.0 \%$ | $23.6 \%$ | $99.8 \%$ | $99.8 \%$ | $38.6 \%$ | $1.6 \%$ | $10.4 \%$ | $59.5 \%$ |
| Irish Money | $23.6 \%$ | $100.0 \%$ | $16.3 \%$ | $16.9 \%$ | $2.2 \%$ | $19.1 \%$ | $-2.6 \%$ | $-2.4 \%$ |
| Irish Short | $99.8 \%$ | $16.9 \%$ | $100.0 \%$ | $99.0 \%$ | $39.0 \%$ | $0.3 \%$ | $10.8 \%$ | $60.5 \%$ |
| Irish Total | $99.8 \%$ | $16.3 \%$ | $99.0 \%$ | $100.0 \%$ | $39.0 \%$ | $0.3 \%$ | $10.8 \%$ | $60.5 \%$ |
| Irish Long | $38.6 \%$ | $2.2 \%$ | $39.0 \%$ | $39.0 \%$ | $100.0 \%$ | $14.0 \%$ | $10.2 \%$ | $59.1 \%$ |
| Irish Equity | $1.6 \%$ | $19.1 \%$ | $0.3 \%$ | $0.3 \%$ | $14.0 \%$ | $100.0 \%$ | $-1.4 \%$ | $5.5 \%$ |
| Japan Bond | $10.4 \%$ | $-2.6 \%$ | $10.8 \%$ | $10.8 \%$ | $10.2 \%$ | $-1.4 \%$ | $100.0 \%$ | $30.8 \%$ |
| U.K. Bond | $59.5 \%$ | $-2.4 \%$ | $60.5 \%$ | $60.5 \%$ | $59.1 \%$ | $5.5 \%$ | $30.8 \%$ | $100.0 \%$ |
| German Bond | $3.4 \%$ | $5.8 \%$ | $3.1 \%$ | $3.1 \%$ | $-4.2 \%$ | $26.3 \%$ | $3.2 \%$ | $-11.5 \%$ |
| US Bond | $-0.6 \%$ | $-8.7 \%$ | $0.0 \%$ | $0.0 \%$ | $-32.0 \%$ | $7.7 \%$ | $17.8 \%$ | $1.3 \%$ |
| French Bond | $17.8 \%$ | $18.5 \%$ | $16.7 \%$ | $16.7 \%$ | $4.1 \%$ | $-2.7 \%$ | $52.4 \%$ | $17.4 \%$ |
| Japan Equity | $11.3 \%$ | $4.6 \%$ | $11.1 \%$ | $11.1 \%$ | $17.4 \%$ | $18.5 \%$ | $51.7 \%$ | $31.4 \%$ |
| U.K. Equity | $8.6 \%$ | $2.6 \%$ | $8.6 \%$ | $8.6 \%$ | $23.6 \%$ | $57.2 \%$ | $14.1 \%$ | $36.0 \%$ |
| German Equity | $3.4 \%$ | $5.8 \%$ | $3.1 \%$ | $3.1 \%$ | $-4.2 \%$ | $26.3 \%$ | $3.2 \%$ | $-11.5 \%$ |
| US Equity | $3.4 \%$ | $3.2 \%$ | $3.2 \%$ | $3.2 \%$ | $-13.0 \%$ | $49.0 \%$ | $9.7 \%$ | $2.8 \%$ |
| French Equity | $14.6 \%$ | $16.8 \%$ | $13.6 \%$ | $13.6 \%$ | $-0.5 \%$ | $37.4 \%$ | $16.6 \%$ | $7.7 \%$ |

Table A.6.2 Correlation of Total Return Indices in Irish Pounds 1985 to 1989

## Source: Empirical

In table A.6.3 the correlation between potential foreign asset classes held by Irish general insurers is shown. There is a high correlation between European bond classes and the US and UK equity market.

|  | German <br> Bond | US Bond | French <br> Bond | Japan <br> Equity | U.K. <br> Equity | German <br> Equity | US Equity | French <br> Equity |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Immunised | $3.4 \%$ | $-0.6 \%$ | $17.8 \%$ | $11.3 \%$ | $8.6 \%$ | $3.4 \%$ | $3.4 \%$ | $14.6 \%$ |
| Irish Money | $5.8 \%$ | $-8.7 \%$ | $18.5 \%$ | $4.6 \%$ | $2.6 \%$ | $5.8 \%$ | $3.2 \%$ | $16.8 \%$ |
| Irish Short | $3.1 \%$ | $0.0 \%$ | $16.7 \%$ | $11.1 \%$ | $8.6 \%$ | $3.1 \%$ | $3.2 \%$ | $13.6 \%$ |
| Irish Total | $3.1 \%$ | $0.0 \%$ | $16.7 \%$ | $11.1 \%$ | $8.6 \%$ | $3.1 \%$ | $3.2 \%$ | $13.6 \%$ |
| Irish Long | $-4.2 \%$ | $-32.0 \%$ | $4.1 \%$ | $17.4 \%$ | $23.6 \%$ | $-4.2 \%$ | $-13.0 \%$ | $-0.5 \%$ |
| Irish Equity | $26.3 \%$ | $7.7 \%$ | $-2.7 \%$ | $18.5 \%$ | $57.2 \%$ | $26.3 \%$ | $49.0 \%$ | $37.4 \%$ |
| Japan Bond | $3.2 \%$ | $17.8 \%$ | $52.4 \%$ | $51.7 \%$ | $14.1 \%$ | $3.2 \%$ | $9.7 \%$ | $16.6 \%$ |
| U.K. Bond | $-11.5 \%$ | $1.3 \%$ | $17.4 \%$ | $31.4 \%$ | $36.0 \%$ | $-11.5 \%$ | $2.8 \%$ | $7.7 \%$ |
| German Bond | $100.0 \%$ | $8.0 \%$ | $10.2 \%$ | $27.9 \%$ | $40.0 \%$ | $100.0 \%$ | $46.8 \%$ | $67.5 \%$ |
| US Bond | $8.0 \%$ | $100.0 \%$ | $27.9 \%$ | $7.3 \%$ | $18.8 \%$ | $8.0 \%$ | $62.9 \%$ | $11.7 \%$ |
| French Bond | $10.2 \%$ | $27.9 \%$ | $100.0 \%$ | $19.9 \%$ | $15.9 \%$ | $10.2 \%$ | $15.2 \%$ | $13.1 \%$ |
| Japan Equity | $27.9 \%$ | $7.3 \%$ | $19.9 \%$ | $100.0 \%$ | $40.5 \%$ | $27.9 \%$ | $38.7 \%$ | $42.5 \%$ |
| U.K. Equity | $40.0 \%$ | $18.8 \%$ | $15.9 \%$ | $40.5 \%$ | $100.0 \%$ | $40.0 \%$ | $67.0 \%$ | $50.4 \%$ |
| German | $100.0 \%$ | $8.0 \%$ | $10.2 \%$ | $27.9 \%$ | $40.0 \%$ | $100.0 \%$ | $46.8 \%$ | $67.5 \%$ |
| Equity |  |  |  |  |  |  |  |  |
| US Equity | $46.8 \%$ | $62.9 \%$ | $15.2 \%$ | $38.7 \%$ | $67.0 \%$ | $46.8 \%$ | $100.0 \%$ | $55.6 \%$ |
| French Equity | $67.5 \%$ | $11.7 \%$ | $13.1 \%$ | $42.5 \%$ | $50.4 \%$ | $67.5 \%$ | $55.6 \%$ | $100.0 \%$ |

Table A.6.3 Correlation of Total Return Indices in Irish Pounds 1985 to 1989
Source : Empirical

Over the second period of 1990 to 1997, the principal changes in correlation is an increase of $40 \%$ between the immunised porfolio and the French bond market and a decrease in correlation of $29 \%$ with the UK bond market. The correlation of the long end of the Irish bond market with the UK bond market fell from $60.5 \%$ to $32 \%$. The correlation of the long end of the Irish bond market with the US bond market fell from $-32 \%$ to $24.2 \%$.

The monthly mismatch reserves for individual classes and for a local asset class equally weighted in Irish bond and equities, then equally weighted in all bond markets and equity markets is shown in table A.6.4. The equity market require a mismatch reserve of twice the bond markets reflecting the balance between the greater return potential and the higher risk of holding these portfolios. The ruin barrier is set at a $5 \%$ and $0.5 \%$ level. It is interesting to note the risk attaching to the German bond market after the unification with East Germany and the merits of holding a diversified portfolio.

| Market | $5 \%$ (20 to 1) | $0.5 \%$ (200 to $\mathbf{1}$ ) |
| :---: | :---: | :---: |
| Irish Money | $0.76 \%$ | $1.34 \%$ |
| Irish Total | $1.56 \%$ | $2.39 \%$ |
| Irish Short | $1.62 \%$ | $2.33 \%$ |
| Irish Long | $2.45 \%$ | $4.05 \%$ |
| Bond Markets | $2.52 \%$ | $4.31 \%$ |
| French Bond | $2.76 \%$ | $4.39 \%$ |
| Equity Markets | $3.29 \%$ | $5.95 \%$ |
| Japan Bond | $4.12 \%$ | $6.34 \%$ |
| U.K. Bond | $4.91 \%$ | $7.65 \%$ |
| Local Market | $5.31 \%$ | $8.69 \%$ |
| US Bond | $6.96 \%$ | $10.91 \%$ |
| Japan Equity | $7.95 \%$ | $13.71 \%$ |
| Irish Equity | $10.25 \%$ | $17.03 \%$ |
| French Equity | $10.29 \%$ | $16.92 \%$ |
| U.K. Equity | $10.41 \%$ | $16.26 \%$ |
| German Bond | $10.56 \%$ | $17.22 \%$ |
| German Equity | $10.89 \%$ | $17.76 \%$ |
| US Equity | $11.13 \%$ | $17.48 \%$ |

[^41]Source : Empirical

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[^0]:    ${ }^{1}$ The gross redemption yieid of a bond is that discounting rate which when applied to all future cashflows of the bond will generate a present value equal to that of the current price of the bond in the market.
    ${ }^{2}$ There are at least three schools of thought on the term structure; namely: Preferred Habitat, Liquidity Preference and Rational Expectations.

[^1]:    ${ }^{3}$ Charging interest stretches back into ancient times. It is forbidden to members of the early Christian Church and left to the Judaic races to provide the service. Theologians such as St.Thomas Aquinas has spoken about the social justice of money lending and some Islamic countries have usury laws on their statute books.

[^2]:    ${ }^{4}$ Kearney (1985) examined the short and long gross redemption yields rather than the spot rates.

[^3]:    ${ }^{5}$ Liquidity is defined as the limit of the size of a transaction upon which the market price can be dealt.

[^4]:    ${ }^{6}$ The interest rate swaps would still have a residual credit risk, but this risk is minimised with the NTMA being the principal originator of $97 \%$ of all fixed rate bonds and the government's historical implicit guarantee of the local banking market.

[^5]:    ${ }^{1}$ This only holds for governments for debt denominated in their own currency which, as a last resort can be printed to repay the debt. A default free bond is defined as one for which at any point in time in the future, the probability of the occurrence of the cash flow is $100 \%$. Even this assumption is contingent on the political stability of the sovereign nation; since the world has seen defaults by Russia and China this century. At a more subtle level, there have been partial defaults in real terms over the past three decades by many nations with the inflation of the early 1970s and early 1980s.
    ${ }^{2}$ In financial markets, this is referred to as the zero coupon curve or pure discount or spot rates and is used when there is only one future cash flow being discounted to its current price.
    ${ }^{3}$ The National Treasury Management Agency (NTMA) which manages Irish government debt on a daily basis for the Minster for Finance is replacing the Exchequer bills which were only issued on a Wednesday for $1,3,6,9$ months via a

[^6]:    tender auction with notes which can be issued at any time for any maturity up to a year and in which they maintain a two way price. A strips program similar to that in the US is under consideration.

[^7]:    ${ }^{4}$ Debt which is denominated in foreign currencies or raised in the money market which is not an obligation of central government and had a first floating charge over receipts into the Central Fund of tax receipts has been excluded.

[^8]:    ${ }^{5}$ Sometimes, these are called dirty prices and they consist of the market principal prices and the accrued interest since the last dividend payment date.
    ${ }^{6}$ Otherwise the solution could have large standard errors or be indeterminate.

[^9]:    ${ }^{7}$ The spline technique is a piecewise polynomials with their explicit local fit that meet in a continuous fashion at breakpoints called knots.

[^10]:    ${ }^{8}$ Deviance is the residual sum of squares.

[^11]:    ${ }^{9}$ Short bonds have a remaining maturity of less than five years, medium bonds have a remaining maturity of more than five years and less than ten years and long bonds have a remaining maturity of more than ten years.

[^12]:    ${ }^{10}$ As already mentioned, special circumstances did exist in Ireland that can have lead to rationing of longer maturity bonds which would have allowed forward rates to be negative during the rationing period

[^13]:    ${ }^{11}$ Otherwise known as a discount function or zero curve.

[^14]:    ${ }^{12}$ This is calculated according to the standard set down by the Bank of England when Ireland had a fixed parity exchange rate against Sterling.

[^15]:    ${ }^{13}$ The issuer has the right to call the bonds after a certain exercise date from the issue of the bonds and this American style option cail exists up to three months before the maturity of the bonds. A notice period of three months exists if the issuer wishes to exercise this option.
    ${ }^{14}$ They are at the money in option pricing theory terminology and a highly valued embedded option, which distorts their yield.

[^16]:    ${ }^{1}$ A partial differential equation is an equation that relates in a non-trival manner two or more derivatives of that unknown function with respect to independent variables and its order is the of the order of the highest derivative. The degree of a partial differential equation which can be written as a polynomial in the derivatives is the degree of the highest ordered derivative which then occurs.

[^17]:    ${ }^{2}$ A Wiener process is a Markov process with a mean of zero and a variance of dt .

[^18]:    ${ }^{3}$ The intrinsic is the value the option would have if it is exercised immediately.
    ${ }^{4}$ The time value is the excess of the option over the intrinsic value.

[^19]:    ${ }^{1}$ In terms of any discount function. price risk is strictly increasing with maturity, and the rate of increase in price risk is strictly decreasing with maturity.

[^20]:    ${ }^{2}$ While the first concern is that a solution does exist. the second concern is that there is exactly only one solution to the differential equation that has the required properties.

[^21]:    ${ }^{1}$ Liquidity is defined as the limit on the size of a transaction upon which the market price can be dealt

[^22]:    ${ }^{2}$ The credit risk is assumed to be zero because bonds have the first fixed charge on tax receipts which flow into the Central Fund and do not require additional legislation in the government's budget Finance Act because the operation of the Central Fund is set out in the Constitution.

[^23]:    ${ }^{3}$ The legislation was passed in 1997 to rename the CBISS, which is an acronym for Central Bank of Ireland Settlement System.

[^24]:    4 Personai communication.

[^25]:    5 The risk capital requirement on an inter maturity basis will be greater than on an intra maturity basis.

[^26]:    Exhibit 4.2 Agency Broker Market Share of Commission

[^27]:    ${ }^{6}$ Pay Related Social Insurance.

[^28]:    7 Market Makers are called Primary Dealers in Ireland.

[^29]:    8 The software package utilised was @Risk which is an add in for Micrsoft Excel and Bestfit developed by Palisade corporation.

[^30]:    9 Catastrophe theory is the branch of actuarial ruin mathematics, which is the topological description of systems that display abrupt discontinuous change with very small probabilities. These probabilities a lie in the upper and lower tails of distributions and can lie more that three standard deviations from the mean.

[^31]:    'The raw date that made the empirical research in this chapter possible is provided by Martin Cosgrove, Principal Officer and Jimmy Joyce, Government Actury, Insurance Regulation, Department of Industry and Commerce and David O'Connor, AGF Insurance Corporation of Ireland along with discussions.
    ${ }^{2}$ Insolvency risk free rate of return refers to the return expected to be generated by an asset portfolio that guarantees the institutional solvency at the end of a particular time horizon.
    ${ }^{3}$ Mismatch reserve is defined as the excess of assets over those required to match the present value of liabilities or the Value at Risk required for holding a non-matching portfolio.

[^32]:    ${ }^{4}$ Duration is defined as the weighted average life of the class of business or portolio

[^33]:    ${ }^{5}$ NTMA policy is to move all bond coupons to an annual basis.

[^34]:    ${ }^{6}$ Agent is a person who is empowered to act for or represent another in a financial transaction.
    ${ }^{7}$ Principal is a person that has capital and empowers another to act as his representative in a financial transaction.

[^35]:    ${ }^{8}$ To reduce accumulated money from previous time periods.

[^36]:    ${ }^{9}$ Formerly. Department of Industry and Commerce which is the Irish equivalent of the UK Department of Trade and

[^37]:    Industry. According to the government actuary, Joyce (1998), the liabilities are stable over the time period.

[^38]:    ${ }^{1}$ Because of the low coupon, there is a yield short fall, which is made up by an increase in the bond price as it approached maturity. This increase in bond price is taken at a lower capital gains rate or not at all depending on the investor or tax time period and this distorted the bond market as investors sought to convert income tax liabilities into capital gain liabilities.

[^39]:    Source: Empirical

[^40]:    1 Repos are best understood as short term collateralised loans. Treasury securities are the collateral. A repo transaction occurs when an investor lends money to a dealer and takes securities. A reverse repo is a transaction in which an investor lends securities to a dealer and borrows money.

[^41]:    Table A.6.4 Mismatch Reserves of Excess of Assets over Liabilities 1985 to 1989

