Designing the edges of holes (with bias flow) to maximise acoustic damping

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Abstract

Circular holes with a mean bias flow passing through them can amplify or damp acoustic energy and this property is relevant for many industrial applications. In this work, we propose a methodology to design the edges of such holes so that the acoustic damping is maximised. The approach relies on a Bayesian optimisation framework and is illustrated in a short circular hole with a mean laminar bias flow. The acoustic response of the perforation is characterised numerically using a two-step approach where, first, a steady mean flow is computed as the solution of the incompressible Navier-Stokes equations. Second, small-amplitude acoustic perturbations are superimposed on this mean flow and their dynamics are obtained as the solution of the linearised compressible Navier-Stokes equations. Both the upstream and downstream edges of the hole are modified with 45°chamfers. The sizes of these two chamfers are the control parameters optimised to maximise the acoustic absorption coefficient. The results of this letter show that the careful design of the edges can dramatically increase the acoustic energy that holes can damp: the hole investigated here goes from one generating 55% more acoustic energy than incident upon it at a given frequency to one that damps 46% of this acoustic energy.

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1. Introduction

Circular holes with a mean bias flow passing through them can amplify or damp acoustic energy. This property is relevant for many industrial applications and for many of them, e.g. acoustic liners in gas turbines [1] or fuel injectors in rocket engines [2], it is desirable to maximise the acoustic energy damped by the individual holes.

The acoustic response of short holes, where the mean flow separates at the upstream edge and remains detached within the length of the hole, was found to be dramatically sensitive to small modifications of the edges of the hole [3, 4] for turbulent bias flows. This sensitivity was confirmed experimentally for laminar

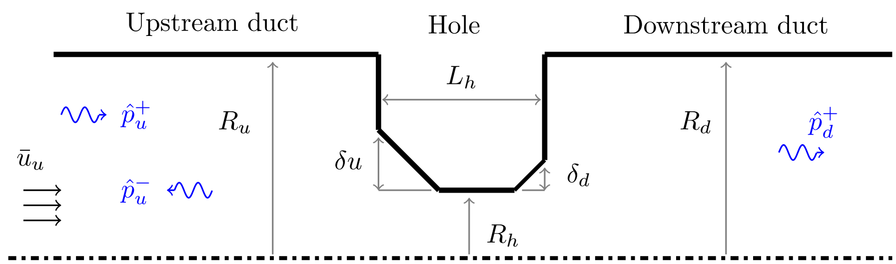

Figure 1: Schematic of the configuration. The dash-dotted line denotes the axis of revolution. The chamfers are defined by 45° angles.

flows [5]. The effect of the hole edge geometry on acoustic damping was also explored by several authors [6–10].

In this letter, we propose an efficient methodology to leverage this sensitivity and design the edges of such holes so that the damped acoustic energy is maximised. To illustrate the technique, a short circular hole with a laminar bias flow (Sec. 2) is considered. The optimisation approach, based on a Bayesian optimisation framework [11], and the optimal geometries are presented in Sec. 3.

2. Problem formulation

We consider a circular hole of radius, R_h, perforated on a flat plate of thickness, L_h, that separates two concentric, circular ducts of radii, R_u and R_d, as sketched in figure 1. The edges of the hole are modified with chamfers of 45° angles with sizes denoted δ_u and δ_d for the upstream and downstream edges, respectively. A subsonic, uniform flow oriented in the axial direction is imposed on the left-hand side of the upstream duct. This produces a bias flow through the hole with bulk velocity $\bar{u}_h$. A small-amplitude, incoming acoustic plane wave coming from the upstream side and propagating in the downstream direction, $\hat{p}_u^+$, is superimposed on this mean flow. The hole scatters this wave and reflected, $\hat{p}_u^-$, and transmitted, $\hat{p}_d^+$, acoustic waves are generated. For frequencies below the cut-off frequency of the ducts, as considered here, these acoustic waves are plane waves far from the area expansion.

The flow variables can be naturally decomposed into a steady mean, denoted by $\bar{(\cdot)}$, and a perturbation component, denoted by $\tilde{(\cdot)}$. Numerically, a two-step approach is used here, where the mean flow is computed first and, subsequently, the dynamics are linearised around this mean flow to obtain the governing equations for the perturbations. The local Mach number is assumed small across the domain which allows the following simplifications: (i) the mean flow is assumed incompressible and (ii) an isentropicity relation between density and pressure [4] is assumed in the linearised compressible Navier-Stokes equations (LNSE). The incompressible non-linear Navier-Stokes equations are discretised using finite elements and the non-linear system is solved using the

Newton method. The LNSE are recast in the frequency domain and discretised and solved again with the finite element method. The same mesh is used for the mean and perturbation parts. A detailed description of the numerical approach is given by [5].

The boundary conditions for the mean flow are as follows. At the inlet and outlet, we impose a Poiseuille velocity profile and an outflow condition, respectively. At the walls, a non-slip condition, i.e. $\mathbf{u} = 0$, is imposed. For the acoustics, non-reflecting boundary conditions are imposed at the inlet and outlet of the domain. Moreover, a time-periodic incoming acoustic wave is superimposed at the inlet. The amplitude of this wave is defined in the frequency domain as $\tilde{p}(t) = \hat{p}_u^+ \exp(\mathrm{i}\omega t)$, where ω is the angular frequency, t is time, i is the imaginary unit and $\hat{p}_u^+$ is the Fourier coefficient. All the walls of and around the hole and the walls of the upstream and downstream ducts are treated with a non-slip and a slip boundary condition, respectively.

The problem is defined by the following geometrical parameters: $R_u/R_h = R_d/R_h = 5.0$ and $L_h/R_h = 1.0$. The Reynolds number is set to $Re = (\bar{u}_h R_h)/\nu = 694$ and the Mach number to $M_u = \bar{u}_h/c = 0.046$, with ν the kinematic viscosity and c the speed of sound. The adiabatic heat index is $\gamma = 1.4$, corresponding to air.

To analyse the energy balance of the perforation, we use the absorption coefficient [12], Δ, defined as

$$\Delta = \frac{|W_u^+| - \left(|W_u^-| + |W_d^+|\right)}{|W_u^+|}, \tag{1}$$

where $W_j^{\pm}$ (for j = u, d) is the time-averaged surface-averaged acoustic energy flux associated with the plane waves. The acoustic absorption coefficient compares the average amount of acoustic energy entering and leaving the domain of the hole. If there is no outgoing acoustic energy on average then $\Delta = 1$ and the hole can be interpreted as fully absorbing the incoming acoustic energy. When the amount of acoustic energy entering and leaving the system is the same, then $\Delta = 0$ and the hole is providing neither acoustic energy damping nor generation. Finally, acoustic energy is generated in the domain for $\Delta < 0$, which corresponds to whistling.

3. Optimisation results

In the following, we seek to determine the sizes of the chamfers that maximise the acoustic absorption coefficient at a given frequency. To achieve this, we use Bayesian optimisation [11], a global optimisation approach. Formally, the optimisation problem is written as:

$$arg \max_{\delta_u, \delta_d} \Delta(\delta_u, \delta_d), \tag{2}$$

subject to $\delta_u \in [0, L_h]$, $\delta_d \in [0, L_h]$, and $(\delta_u + \delta_d) \leq L_h$. The optimisation algorithm is implemented in the open-source library `scikit-optimize` (see

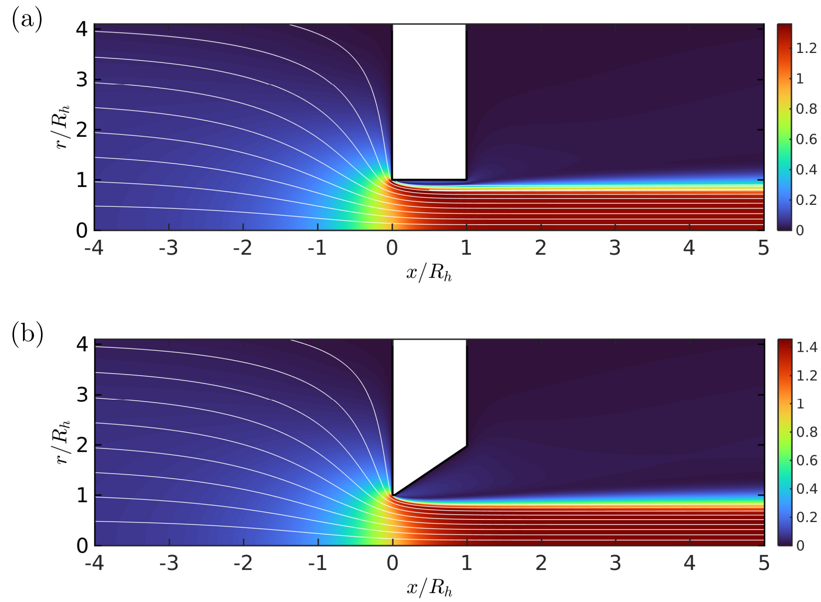

Figure 2: Normalised mean-flow velocity magnitude, $|\bar{\mathbf{u}}|/\bar{u}_h$, obtained for (a) a straight and (b) an optimally-chamfered hole.

https://scikit-optimize.github.io/stable/) and is based on Bayesian optimisation using Gaussian processes [11]. In Bayesian optimisation, the objective function (here the absorption coefficient) needs to be evaluated for given inputs (the sizes of the chamfers). This process is known as sampling. Our numerical approach requires to perform the following four steps for each sampling point: first, a mesh is generated for a given pair of inputs (δ_u, δ_d); second, a mean flow is computed; third, the acoustic field is obtained by solving the LNSE and; fourth, the value of the absorption coefficient is determined. The optimisation algorithm works as follows. First, the algorithm is initialised by sampling the objective function a prescribed number of times. Then, a Gaussian process which describes the objective function is obtained by fitting it to the initial samples. Based on this Gaussian process, an acquisition function is defined and the next sample point is determined by minimising it. The next sample is computed, the Gaussian process is updated and the acquisition function is minimised again to determine the next sampling point. This process continues until a prescribed number of iterations is reached. The parameters used for the optimisation are the following: the Gaussian process estimator is a Matern kernel, the algorithm is called 300 times and is initialised with 20 points which are uniformly distributed random numbers, the acquisition function is set to `"gp_hedge"`, and the acquisition optimiser is set to `"lbfgs"`. In our implementation, we assume that the objective function is exact (i.e. zero variance) for each sampling point.

First, we consider the baseline configuration that we seek to optimise: a straight hole. Figure 2(a) shows the mean flow obtained for this configuration.

The flow cannot follow the sharp turn at the upstream rim of the hole and separates, creating a low-speed re-circulation zone adjacent to the wall. For short holes, such as the one considered in this letter, the flow remains separated within the hole's length. A strong shear layer separates this detached region from a jet that develops in the central part of the hole. Figure 3 shows the absorption coefficient. At low frequencies, the hole damps around a 50% of the incoming acoustic energy, $\Delta \approx 0.5$. For increasing frequencies ($St > 0.7$), the absorption coefficient sharply drops and for $1.15 \leq St \leq 2.05$ acoustic energy is generated. The global minimum of the absorption coefficient is obtained in this region at $St = 1.4$ and corresponds to $\Delta = -0.55$. A local maximum is obtained at $St = 3.6$, and a second whistling region extends between $4.2 \leq St \leq 5.94$. These results have been compared with an incompressible approach [13] and the agreement is excellent. They have also been compared with experimental data and the trends and overall values of the experiments are correctly captured [5]. Moreover, the linearised approach used here was shown to accurately reproduce the acoustic response of experiments with turbulent bias flows when combined with turbulent computations of the mean flow [4, 14].

We now optimise for the frequency of minimum acoustic absorption, i.e. $St = 1.4$. The optimal sizes of the chamfers are $\delta_u^*/R_h = 0.0044$ and $\delta_d^*/R_h = 0.9696$, yielding $\Delta^* = 0.46$. The effect of optimisation is to alter the acoustics to go from a hole which generates 55% more acoustic energy than incident upon it to one that damps 46% of this acoustic energy. Figure 3 shows that the acoustic damping increases for the majority of frequencies investigated, except at $3.0 \leq St \leq 4.1$. The two regions of acoustic energy amplification are completely suppressed: in the first and second regions, absorption coefficients of around $\Delta \approx 0.4 - 0.5$ and $\Delta \approx 0.1$. are achieved, respectively. Note that for the results of the optimised configuration, there is strong shear instability [15] at $1.8 \leq St \leq 2.8$ which manifests as small wiggles in the absorption coefficient. This instability could be removed using the complex mapping proposed by [15], without affecting the results and conclusions of this letter.

Physically, the differences between the two cases are explained as follows. For the sharp-edged hole, the incoming acoustic wave impinges the hole and, due to this, vorticity is shed from the leading edge [3], i.e. acoustic energy is transformed into kinetic energy. This vorticity convects along the hole and produces sound. The sound generated, however, remains weak in the majority of the hole, but becomes stronger close to the downstream edge, where the acoustic field is more receptive to acoustic sources due to the geometrical sharp edge. When the sound generated close to the downstream edge surpasses the acoustic energy damped at the leading edge, a net generation of acoustic energy occurs (whistling). Having a large chamfer at the downstream edge and a small chamfer at the leading edge (as in the optimal configuration), reduces the intensity of sound generated close to the downstream edge (the geometrical singularity is moved away from the source), and does not modify the transfer of acoustic to kinetic energy happening at the leading edge. In practical terms, the optimal configuration behaves as a very thin hole [15].

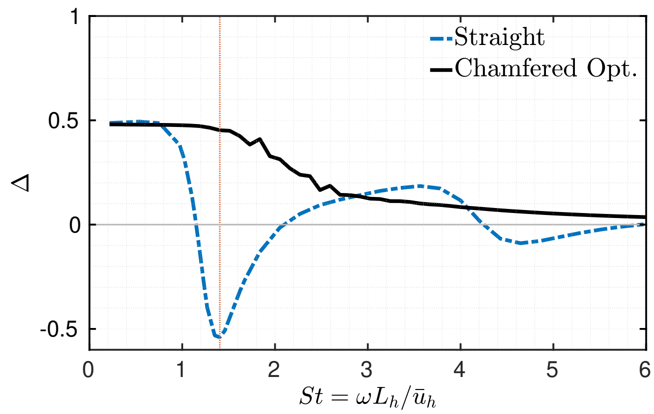

Figure 3: Absorption coefficient for the straight (blue dash-dotted line) and optimally-chamfered holes (black solid line). The red dotted line indicates the frequency considered for the optimisation.

4. Conclusions

In this letter, we demonstrate for the first time that the acoustic damping of short holes with bias flow can be dramatically increased by carefully designing their edges. The design methodology requires: (i) characterising the acoustic response of the hole, for which we used a numerical two-step approach, and (ii) the optimisation of the objective function – here the absorption coefficient – for which we employed Bayesian optimisation. Using this method, the acoustic absorption coefficient of a laminar hole at a given frequency can be altered to go from exhibiting strong whistling to one which strongly damps acoustic energy.

The present methodology can be readily combined with any computational or experimental methodology to characterise the acoustic response of the hole. For instance, the mean flow could be obtained efficiently using the Reynolds-Averaged Navier-Stokes (RANS) equations [4] to optimise holes with turbulent bias flow. An exciting alternative is multi-fidelity optimisation [16], where several numerical approaches with different levels of fidelity are combined together in the optimisation process.

In addition, different control parameters and objectives functions could be defined for the optimisation. For instance, we could define the angles of the chamfers as additional control parameters or assume rounded edges and optimise their curvature. As for the objective functions, one possibility could be to maximise the acoustic absorption while keeping the mean pressure losses to a minimum. Another possibility could be to combine the numerical model of the hole with acoustic networks and/or acoustic analogies to optimise objective functions defined for complete acoustic systems, such as the growth rate of thermoacoustic instabilities in combustors, the noise radiated to the far field, or the acoustic absorption of acoustic liners [17]. All those directions are currently under active research by the authors.

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