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# AN EMPIRICAL EXAMINATION OF BILATERAL SEABORNE TRADE FLOWS IN THE WORLD ECONOMY 

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Ph.D.

## A thesis submitted for the degree of Ph.D. <br> in Applied Economics.

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#### Abstract

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## DECLARATION

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#### Abstract

The aim of this thesis is to construct a disaggregated econometric model of the pattern of bilateral seaborne trade flows. Commodities are classified into 5 categories according to the type of ship used in their transportation. Exports and imports are classified into 30 regions, according to the major sea-lanes used by ships.

An understanding of the determinants of trade flows at this level of disaggregation is important for shipowners. The use of disaggregated data also helps in the estimation of the price elasticities of traded goods, an issue of more general interest to exporters and policy makers.

Our theoretical model borrows the ideas of multistage budgeting from consumer demand theory. The total imports of each importing region are allocated amongst their trade partners, depending on relative prices and trends in tastes. Our econometric implementation of the model uses the very general Constant Ratio of Elasticities of Substitution Homogeneous (CRESH) functional form. This encompasses the CES, LES, Cobb-Douglas and Leontief forms, more commonly used in trade models.

Empirical implementation of the model has resulted in elasticity estimates which are much higher than those estimated in earlier trade models. This indicates a high degree of competition in international markets. The pattern of these elasticities suggest that importing regions establish a few trade partners internationally for the main bulk of their imports, while the proportion of their imports allocated to the remaining trade partners, is highly sensitive to relative prices.


## ABBREVIATIONS:

| AE | Adaptive Expectations |
| :---: | :---: |
| ALH | Average Length of Haul |
| BLUE | Best Linear Unbiased Estimators |
| BTN | Brussels Tarif Nomenclature |
| CCCN | Cust.oms Co-operation Council Nomenclature |
| CES | Cunstabt Elasticity of Substitution |
| cif | Cost-Insurance-Freight |
| CMSE | Constant Market Share Elasticities |
| CPC | Central Product Classification |
| CRESH | Constant Ratio of Elasticity of Substitution Homogeneous |
| CRS | Constant Returns to Scale |
| CV | Coefficient of Variation |
| DGP | Data Generating Process |
| DR | Democratic Republic |
| EEC | European Economic Community |
| EFTA | European Free Trade Association |
| fob | Free-On-Board |
| FOC | First Order Conditions |
| FR | Federal Rapublic |
| FRB | Federal Reserve Board |
| GDP | Gross Domestic Product |
| GDP-CON | Gross Domestic Product in Constant prices |
| GLS | Generalized Least Squares |
| GNP | Gross National Product |
| HS | Harmonized System |
| IFS | International Financial Statistics |
| IMF | International Monetary Fund |
| ISIC | International Standard Industrial Classification |
| LES | Linear Expenditure System |
| MCV | Modified Coefficient of Variation |
| ML | Maximum Likelihood |
| MLE | Maximum Likelihood Estimator |
| MTC | Maritime Transport Commodity (Classification) |
| MTCAC | Maritime Transport Coastal Area Classification |
| OECD | Organization for Economic Co-operation and Development |
| OLS | Ordinary Least Squares |
| PA | Partial Adjustment |
| RRSS | Restricted Residual Sum of Squares |
| ROW | Rest Of the World |
| SGM | Statistical Generating Mechanism |
| SITC | Standard International Trade Classification |
| SOC | Second Order Conditions |
| s.t. | subject to |
| SURE | Seemingly Unrelated Regression Equations |
| UK | United Kingdom |
| UN | United Nations |
| URSS | Unrestricted Residual Sum of Squares |
| USA | United States of America |
| USSR | United Soviet Socialist Republics |
| w.r.t. | with respect to |

## SYMBOLS:

| $e_{i 1}$ | Own import price elasticity of demand |
| :---: | :---: |
| $e_{i 1}^{\text {xp }}$ | Estimated own export price elasticity of demand |
| $e_{i h}$ | Cross import price elasticity of demand |
| $e_{\text {ih }}^{\text {xp }}$ | Estimated cross export price elasticity of demand |
| $e^{\mathrm{wp}}$ | Estimated world export price elasticity of demand |
| $\varepsilon_{\text {it }}$ | White noise error regression term |
| $m_{i j}^{k}=m_{i j}=m_{i}$ | Volumes of bilateral imports of commodity $k$ by country $j$ from partner $i$ |
| $m_{j}^{k}=m_{j}=m$ | Volume of total imports of commodity $k$ by country $j$ |
| $\mathrm{m}^{\mathrm{k}}$ | Volume of total world imports of commodity k |
| M, | Value of imports of $j$ |
| MP, | Total import prices of j |
| $\mu_{j}^{k}=\mu$ | Index of total imports of country $j$ for commodity $k$ |
| $p_{i j}=p_{i}$ | Bilateral prices of imports of commodity $k$ by country j from partner $\mathfrak{i}$ |
| $p_{j}^{k}=P_{j}$ | Total import prices of commodity $k$ by country $j$ |
| $\mathrm{P}_{\mathrm{W}}$ | World price index |
| t | Time trend |
| $\sigma_{i 1}$ | Partial Allen own price elasticity of substitution |
| $\sigma_{\text {ih }}$ | Partial Allen cross price elasticity of substitution |
| $T^{\text {k }}$ | World trade volume of commodity $k$ |
| T | Volume of aggregate world trade |
| $w_{i j}=w_{i}$ | Bilateral shares of imports of commodity $k$ by country $j$ from partner i |
| $w_{i j}^{0}=w_{i}^{o}$ | Bilateral shares of imports of commodity $k$ by country $j$ from partner $i$ at the base year |
| $W_{1 j}$ | Nominal trade share |
| $W^{\text {k }}$ | Total world prices of commodity $k$ |
| $x_{1}{ }^{\text {k }}$ | Volume of total exports of commodity $k$ from country $i$ |
| $x^{k}$ | Volume of total world exports of commodity $k$ |
| $X P_{i}$ | Total export prices of country $\mathfrak{i}$ |

## MATHEMATICAL SYMBOLS:

| $d x$ | Derivative of $x$ |
| :---: | :---: |
| $\partial x$ | Partial derivative of $x$ |
| $X_{t-1}$ | Lag of variable $X$ |
| $\Delta X_{t}=X_{t}-X_{t-1}$ | Change in $X$ between periods $t-1$ and $t$ |
| $E X_{t}$ | Mathematical expectation of $x$ |
| exp $=e$ | $=2.718$ |
| $\lim x$ | Limit of $x$ |
| $\ln x$ | Natural logarithm of $x$ |
| $\Pi_{1}$ | Product over $\mathfrak{i}$ |
| $\Sigma_{i}$ | Sum over i |
| $\mathbf{x}^{\prime}$ | Transpose of vector $\mathbf{x}$ |
| $\Sigma^{+}$ | Generalized inverse of matrix $\Sigma$ |
| $\otimes$ | Kroneker product |
| $\forall$ | For all |
| $\therefore$ | Therefore |
| 三 | Identical |
| \# | Different to |
| $\infty$ | Inf inity |
| $\|x\|$ | Determinant or absolute value of $x$, where appropriate |
| > | Greater than |
| $<$ | Less than |
| $\geq$ | Greater or equal to |
| $\leq$ | Less or equal to |

## CHAPTER

INTRODUCTION

## 1.0). Introduction.

This chapter is an introduction to the thesis. In its three main sections it considers respectively, (i) the central question which we attempt to answer, (ii) the proposed outline of the answer and (iii) the structure of the thesis.
1.1). The question.

The division of labour was seen by Adam Smith(1776) as the basis of specialization in the production of commodities. However, the extent to which the division of labour takes place was seen to be limited by the size of the market. The only way to extend markets beyond the national boundaries is by international trade.

The significance of international trade in the world economy has grown enormously since the 2nd World War. World GDP has increased in real terms by 377\% between 1950 and 1986 (UN (1990)). World imports have increased in real terms by $919 \%$ over the same period (IMF (1990)).

At the time of Adam Smith, the easiest and safest way of trading amongst nations of the world was by ships. This continues to be the case today. The vast majority of goods that are traded internationally are transported by ships.

Over the years, well defined sea-lanes have developed, linking the ports of major trading regions. Specialized types of ships have also been developed, to handle different types of cargo. UN statistics on
seaborne trade identify 30 trading regions, and 5 cargo types (bulk dry, bulk liquid, refrigerated goods, general cargo dry and other dry cargo). Details are given in Chapter 5, Appendices 5.1 and 5.2.

Our objective in this thesis is to understand the factors determining the demand for these different ship tupes on each shipping lane. In particular, we are interested in how sensitive demand is with respect to the prices of the goods being transported, and the competitiveness of the exporting and importing regions. This information is an important input in the decisions of shipowners and exporters in general.

## 1.2). An outline of the answer.

The demand for seaborne transport is a derived demand. A particular type of ship, over a sea-lane, at each point in time, is demanded because there is a demand for the commodity the ship can carry. If one is to explain the demand for seaborne transport in an import 'market' (where an import market is described by the importing region and the 'commodity' that is demanded by the region), then the problem is equivalent to explaining the demand for trade in that market.

In this thesis, we take as given the general level of demand in an import market at each point in time, and we explain the way this demand is allocated over trade partners. We assume that the changing patterns of allocation are driven by the relative prices offered by competing exporters in each import market, and by a time trend which describes the changing taste of importers for different exporters.

We derive our models from neoclassical optimization principles. For each country and each commodity, the structure of preferences of a representative economic agent over imports from other countries is represented by a function of the imported quantities, in the same way as consumers' preferences are represented by a utility function.

The economic properties of the system depend of course on the particular objective function chosen. We choose a function, the Constant Ratio of Elasticity of Substitution Homogeneous (CRESH) function, which is general enough to allow for a distinct bilateral price effect for each exporter, and which holds trade partners as competitors in import markets.

The possibility that trade between two regions may be influenced by the export prices of regions other than the exporter and importer is also allowed in our model. That is, apart from own price elasticities, cross price elasticities of seaborne trade demand are computed. However, the number of estimated coefficients increase only linearly with the number of trade partners, allowing us to parameterize an apparently 'large' system, using a relatively short set of data, for the years 1969-1986.

## 1.3). Organization.

The thesis is in 8 chapters.

In chapter two, since there are no similar studies in seaborne trade linkage models, we survey existing studies on international import allocation models.

In chapter three, we extend the theory of budget allocation of the consumer to seaborne import allocation of the regional importer, thus providing the theoretical rationale of seaborne import allocation models. We use a particular objective (or aggregator) function, which allows for relatively 'general' substitution patterns amongst trade partners and at the same time leaves the system feasible for estimation. This system encompasses most existing models of international trade as special cases.

In chapter four, we examine the econometric specification of the derived seaborne import allocation system. We turn the theoretical model of chapter three into an empirical form by linearizing it,
introducing dynamics and taking into account the data constraints. A methodology for selecting among alternative models is proposed, and the theoretical consequences of the empirical formulation are examined.

In chapter five, we describe the data used in the estimation of our econometric models. Apart from the dependent variables on bilateral seaborne trade flows, we have to construct data on relative prices. Methodological and practical problems are examined in the process, and the correspondence of the data to the theoretical variables is discussed.

In chapter six, we show that our system falls into the class of Seemingly Unrelated Regression Equations (SURE), with a functional adding up constraint, which translates into a constraint on parameters across equations. Estimation methods for linear and non-linear systems of this type are proposed.

In chapter seven, we focus on the estimated systems and the results of the empirical exercise. We derive sets of own and cross export and world price elasticities of demand for each trade partner in each import market. Trends and dynamics are also discussed.

Finally, in chapter eight, we present a brief summary and conclusions of the thesis, and reflect on the problems faced. We also suggest possible drawbacks and improvements that may be made on this research front.

## LITERATURE SURVEY

## 2.0). Introduction.

In this chapter, we survey linkage models of the world economy, with emphasis on trade linkage. Each model is evaluated in terms of our objective, of explaining bilateral seaborne trade flows in the international economy. Ideally, we like to construct a world economy system, that can explain trade among 30 partners and 5 commodity groups. The model should have a strong theoretical structure, both economically and econometrically, and be feasible to turn into an empirical form.

This chapter is in three main sections. In the first, we look at conceptual aspects of trade linkage models in terms of an import-export matrix, and we set up the mathematical notation which is used throughout the thesis. A list of collective surveys of such models is presented and a summary of the major linkage models of the 1980's is provided in appendix 2.1. In the second, we concentrate on trade linkage systems and examine the pros and cons of different approaches. Based on this discussion, we select the class of Estimation Consistent Bilateral Import Allocation Models as the class of models which can be most fruitfully used in satisfying our objective. In the third section, we examine more closely alternative functional forms of the above class and select one of these as being the most interesting in terms of modeling our seaborne trade flows.

## 2.1). International Trade Linkage Systems.

2.1.1). World Trade Flows in an Import-Export Matrix.

An import-export matrix is usually employed, of Taplin(1967),

Magee(1975) etc, to illustrate world trade flows. Table 2.1 shows such a matrix. Rows of the table record export volumes of a commodity group from countries towards their trade partners. Columns of the table record import volumes of countries for that commodity coming from all their trade partners.

## Table 2.1.

World Import-Export Matrix for commodity $k$.

| Exporting country | Importing country |  |  |  | Total Exports |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | n |  |
| 1 | $m_{11}^{k}$ | $\mathrm{m}_{12}^{\mathrm{k}}$ | $m_{13}^{k}$ | $\mathrm{m}_{1 \mathrm{n}}^{\mathrm{k}}$ | $x_{1}$ |
| 2 | $\mathrm{m}_{21}^{\mathrm{k}}$ | $\mathrm{m}_{22}{ }^{\text {k }}$ | $\mathrm{m}_{23}^{\mathrm{k}}$ | $m_{2 n}^{k}$ | $x_{2}^{k}$ |
| - | . $\cdot$ | . . | . . | -•• | - |
|  |  | $\ldots$ |  |  | . |
| $n$ | $m_{n 1}^{k}$ | $m_{n 2}^{k}$ | $m_{n 3}^{k}$ | $m_{n n}^{k}$ | $x_{n}^{k}$ |
| Total Imports | $m_{1}^{k}$ | $m_{2}^{k}$ | $m_{3}^{k}$ | $m_{n}^{k}$ | $T^{k}$ |

We use the following notation:
$\mathfrak{i}=$ exporting country or region.
$j=$ importing country or region.
$k=$ commodity group.
$m_{i j}^{k}=$ bilateral volumes of imports of commodity $k$ by country $j$ from partner $i$, or equivalently bilateral volume of exports of commodity $k$ from country $i$ to partner $j$.
$x_{i}^{k}=$ total exports of commodity $k$ from country $i$ to all its trade partners.
$m_{j}^{k}=$ total imports of commodity $k$ by country $j$ from all its trade partners.
$x^{k}=$ total world exports of commodity $k$.
$m^{k}=$ total world imports of commodity $k$.
$T^{\mathbf{k}}=$ world trade of commodity $k$.
$T=$ aggregate world trade.
where 'totals' refer to the sum over trade partners, while aggregates refer to the sum over 'commodities' (or 'goods').

The following identities follow by definition. The sum of bilateral imports and exports add up to their totals per category of goods.

$$
\begin{align*}
& \sum_{j} m_{1 j}^{k}=x_{i}^{k}  \tag{2.1}\\
& \sum_{i} m_{i j}^{k}=m_{j}^{k} \tag{2.2}
\end{align*}
$$

The sum of these total exports and imports should be equal to world trade per category of goods.

$$
\begin{equation*}
x^{k}=\sum_{i} x_{i}^{k}=\sum_{j} m_{j}^{k}=m^{k}=T^{k} \tag{2.3}
\end{equation*}
$$

Finally, the aggregate over all commodity groups of the above totals should be equal to aggregate total world trade.

$$
\begin{equation*}
\sum_{k} x^{k}=\Sigma_{k} m^{k}=T \tag{2.4}
\end{equation*}
$$

A world trade matrix for commodity group $k$ at a particular period of time, a year say, is shown in table 2.1. Similar matrices can be constructed at different points in time to show volumes of world trade for the same commodity over a number of years. To increase the dimension of the problem, a series of world trade flow matrices can be constructed for each commodity group to cover the whole world, over a period of time. Thus, a series of three dimensional import-export matrices, one for each year, would represent such a situation. The horizontal axes of these matrices would record importing and exporting countries respectively, while the vertical axis would record commodity groups.

Often, people are interested in values of trade flows, in which case the entries in table 2.1 are in terms of values instead of volumes. Conventionally, capital letters are used to show the same
table in value terms.

If we divide $m_{i j}^{k}$ by $m_{j}^{k}$ we obtain tables similar to 2.1 in shares form, where the entries in these new tables are $w_{i j}^{k}\left(\equiv m_{i j}^{k} / m_{j}^{k}\right)$ with the property $\Sigma_{i} w_{i j}^{k}=1$.

The challenge then is to build a model that can explain the bilateral trade flows inside the table (or the corresponding values, or the bilateral shares in either volume or value terms) in such a way so that identities (2.1)-(2.3) hold. Then the above identities and (2.4) may be used to calculate consistently the totals on the margins of the tables, and aggregate world trade.

There are several approaches to solving this problem. In our attempt to give an overview of the evolution of these approaches we present a survey of surveys of world trade models. This is followed by a list of the most important trade linkage models in operation over the past decade. This helps us build a picture of the complexity and size of the currently operating linkage models, and plan for the form of our own model.

### 2.1.2). Surveys of World Trade Models.

Building an Import Export matrix similar to that of table 2.1, and attempting to explain its elements, narrows down the set of international trade models under consideration to the group of linkage models. That is, to models of more than two trade partners. We shall refer to these simply as trade models.

Early attempts to survey models of this type can be found in Cheng(1959). He provides an exhaustive survey of empirical estimates of international trade elasticities and propensities in the 1930's and 1940's, estimated by single equation methods. Comments on work prior to this can be found in Brown(1951). Prais(1962) surveys econometric work in international trade up to 1962, and discusses the estimation techniques used. He proposes, in line with

Orcutt(1950), that simultaneous equation methods should be used to model bilateral trade flows.

Taplin(1967) surveys a variety of approaches to the study of world trade. He compares these approaches, and discusses their pros and cons. This leads him to set the outlines for the desirable properties, a world trade model should satisfy. In broad terms, a disaggregated model that can distinguish between a large number of trade partners, and a number of commodity groups.

Leamer and Stern(1970) provide the most extensive survey of econometric work in international trade, part of which is devoted to the subset of world trade models, in chapters 5 and 6 . They note that relatively small trade models, with not well understood properties, dominated the scene prior to the 1970's. However, with developments in computing and accumulated knowledge, larger and better understood models were built in the 1970's and 1980's.

Magee (1975) presents a survey of international trade models up to 1975. A section deals with world trade models, distinguishing between those using cross section and those using time series data. He chooses to discuss the latter as the most interesting, since they describe changes in the patterns of trade over time.

Stern's et al(1976) impressive work, covers the period 1960-1975, including 130 items of research. Three bibliographical indices are used to classify existing models by: (i). country or region, (ii). commodity or commodity group and (iii). author. Their bibliography contains most of the known work.

Amano et al(1980) survey and compare alternative approaches to estimating world trade models. Such approaches include the estimation of bilateral trade flows directly in volume or value terms, in shares form, or indirectly through total import or export equations.

Courbis(1981) in a round table discussion with 'eminent world
trade model builders', presents a collection of the major existing multinational, econometric, and general equilibrium trade models.

Hickman(1983b) provides a cross section of global international economic models. He, conveniently, classifies them according to their nrincipal features (i.e. by type of model, endogenous variables, exogenous variables, purpose of the model), and the type of the endogenous linkage mechanisms between countries/regions in each model. That exercise shows that trade flows are included in every model, with other linkage mechanisms, often, supplementing these in the completion of the system. A summary discussion is presented at the beginning, with the authors of each paper expanding on each project in the individual chapters of the volume.

Heliwell and Padmore(1984) cover the most important linkage macroeconometric models, of two or more countries, that were in operation in the 1970's and early 1980's. They classify them according to the type of linkage and country coverage. Studies on partial, general equilibrium, and single economy trade models are, therefore, overlooked.

Italianer(1986) in chapter 1, offers a good literature survey of trade models, which he selects out of the broader set of linkage models.

Bryant et al(1988) provide a collection of the world's most advanced multicountry econometric models up to 1988. Simulation results from these models are reported and compared, in an effort to enhance the empirical understanding of cross-border macroeconomic interactions. The reliability of the results are evaluated and controversial analytical issues are discussed.

### 2.1.3). Major Currently Operating World Linkage Models.

Models in the above studies have often been classified according to whether they are:

Macroeconometric, in which case a set of elaborate econometric models are linked together, for example LINK (Sawyer(1979)), etc, General Equilibrium, which model the whole world as one country, for example Artus(1986), Parkin and Zis(1976), Piganiol(1979), Beenstock and Dicks(1983), etc,

Input-Output models, which are closely associated to a Leontief type input-output analysis, for example INFORUM(Nytus and Almon(1983)), etc,

Single economy models, which emphasize the bilateral linkages between one country and its trade partners in a world framework, for example Batchelor and Bowe(1974), National Ports Council(1975), etc, Hybrid models contain elements from more than one type of approach, cf COMLINK (Adams and Marquez(1983)), etc.

The most important world linkage models in operation in the 1980's are listed in appendix 2.1. A number of these models were built before the 1980's and continue to be in operation today. Many of these are refined and elaborated further to provide forecasts and simulations for international organizations. A reference on each model is provided, which, when followed, leads to a literature guide and description of the model in question. We also refer to the number of trade partners distinguished, and to the level of disaggregation of trading commodity groups in each case.

## 2.2). Towards an Estimation Consistent Import Allocation Model.

In this section, we analyse the progress in the state of the art towards the development of bilateral world trade models, which can accomodate a large number of trade partners and a number of disaggregated commodity groups. We evaluate alternative models in terms of size, that is, according to whether they are disaggregated enough in terms of our study, and in terms of the sensible economic properties that we expect out of such models. Elements of the desirable ecomomic properties of large scale world trade linkage

models can be found in Taplin(1967), Rhomberg(1970, 1973), Waelbroeck(1973), Barten and d'Alcantara(1977), Courbis(1981), Italianer(1986), Bryant et al(1988).

Table 2.2 summarizes the current section in a tree diagram. It can be interpreted as a flow diagram of the progress on building large scale trade linkage models of the world economy. Along its main column, it shows the development of international trade models towards the possible estimation of a large scale, bilateral, trade flow system. Each entry on that column is leading to two/three possible developments, shown by the entries one level down. The entries, on the main column, as we move down, correspond to a step towards our target. Entries on the side(s), at each level, show alternative approaches to that of the main entry on the same level. We discuss the reason we have chosen to follow the particular approach at each level, at the section listed on the far right hand column of the table, and as we leave behind the alternative routes, we give brief references on work done on each approach. It will be evident from the discussion that follows the reason for prefering a particular approach over the alternatives, in our effort to construct our world seaborne trade model.

### 2.2.1). Traditional Theories vs Trade Linkage Models.

Starting with the group of International Trade Models out of the set of economic models we choose, for our study, to estimate trade among many countries. Thus, the Traditional theories of international trade, such as those of Ricardo(1817), Heckscher(1919), Ohlin(1933), etc, are not discussed here. These theories consider trade between two countries, for two traded commodities at a single point in time. They are static. They fail to account for the possibility of a single economy importing the same good from more than one trade partner. They also fail to explain the situation of the same good being imported and exported by the same country. These theoretical considerations preclude them from being
directly applicable for our task. However, they provide important ideas of what variables can explain trade internationally. A survey of these theories can be found in Jones and Kenen(1984).

We confine ourselves to the set of trade linkage models, which can account for the possibility of multiple trade partners, more than one commodities and dynamic elements. The beginning of interest in Trade Linkage Models can be associated with the increasing interdependence of national economies due to international trade, which took-off especially in the post 2 nd world war period.

It is worth mentioning here the early efforts to compile Import-Export Matrices, without any analysis of mathematical equations and econometric estimation, by the League of Nations' (1942) 'Network of World Trade'. Import Export matrices were filled for the years 1928, 1935 and 1938, dividing the world in 17 regions. Later, Beckerman(1956) filled Import-Export matrices for 1938-1953. Woolley(1965) provided transaction matrices on payments for trade services and capital flows for 75 countries, for 1950 to 1954. Today, the compilation of such Import-Export matrices is standard work carried out by international organizations such as the UN, IMF, OECD etc.

The empirical research accelerated in the interwar period, with single equation methods used to evaluate bilateral elasticities and propensities, see Brown(1951) and later work surveyed in Cheng (1959). The need soon became apparent to study the possible interlinkages between countries in the world economy simultaneously, both for economic and econometric reasons. International trade had been taking an increasingly important part in countries' economic life, with countries being involved in trade for many commodities, with a number of partners. This made important the explicit modelling of world trade linkage in the economic models. On the econometric front, Orcutt(1950) identified the possible biases in estimation when single equation methods are used. These amount to the simultaneity bias problems when supply factors are ignored. Estimated demand elasticities by single equation methods would then
be a mixture of both demand and supply elasticities, rendering inferences invalid. The solution is to use simultaneous equation techniqes.

### 2.2.2). Structural vs Transmission Models.

Trade linkage models were originally estimated by using cross section data, thus, effectively explaining the structure of world trade at a point in time, a year say. These type of models are called Structural Models, and are reviewed in Taplin(1967), Leamer and Stern(1970) and Italianer(1986). Examples of these are the models by Savage and Deutsch(1960), Tinbergen(1962), Poytonen(1963), Pulliainen(1963), Linneman(1966), Waelbroeck(1967), Olsen(1971), Aitken(1973), Signora(1981), De Vos and Bikker(1982).

Prices and other variables are fixed at a point in time, and they play no role in determining patterns of world trade in the structural models. However, we also like our system to explain the way these patterns are changing over time due to more complicated inter-relationships between prices, incomes, imports, and other variables that come together to change the structure of world trade in the longer run. Such a model would establish the relationships between these variables, and trace the transmission of changes of a variable in a country into the rest of the world. We call these Transmission Models, and the type of data used to build them are time series or pooled data.

### 2.2.3). Bilateral vs Global Linkage Models.

Transmission models can be distinguished between Global linkage and Bilateral linkage models. Global Linkage Models explain total trade flows. That is, total imports of each country are explained, no matter the origin (assuming import equations are estimated). Trade might be disaggregated, but import/export shares are not explained in the system; they are only used to calculate other
variables and in that respect being considered exogenous. When these shares are explained, we have Bilateral Trade Linkage. In terms of the Import-Export matrix, of table 2.1, a global model explains the totals, $m_{j}^{k,} s$ or $x_{i}^{k \prime} s$, on the margins of the matrix, while a bilateral model explains the $m_{i j}^{k}$ 's (or the shares), inside the matrix.

Often, global linkage models are used because people are interested in explaining total import/export equations. In this case the transmission mechanism can be captured by a variable such as world trade. It is also possible that people might resort to global models because it is not possible to find bilateral data, or even if these are obtained, it might be too difficult to handle them. Global models that have been built due to the former reason are those of Neisser and Modigliani(1953), Polak(1953), Beckerman(1956), Mennes(1967), Carrin et al(1980), Waelbroeck and Ginsburgh(1981), METEOR(Kooynan, 1982). Global models built due to data problems are the IMF model(Artus and McGuirk, 1981), INTERLINK (Richardson, 1988), SIMLINK (Hicks(1976a, 1976b)), MARCO II(Guillaume, 1981), Tsukuba-FAIS(Shishido, 1983).

It is suggested that global linkage is second best in comparison to bilateral linkage. This is because the latter permit a more detailed representation of the interdependency of countries through world trade. A similar argument to that made by Orcutt(1950), on aggregation over commodities, can be made here relating to aggregation over trade partners. When total imports are analysed, there is no attention paid to the differing elasticities of the component economic partners, which results in aggregation errors. If all imports from trade partners of a country vary equally over the period of observation, or if there is no correlation between the amount of variation and their elasticities, there is no bias. However, elasticity estimates based on aggregate index numbers are influenced by exporting countries with high or low elasticities of substitution. The answer to this aggregation bias problem, is to disaggregate trade. That is, to use a bilateral trade linkage model. More crudely, economic influences between closely inter-related
economies (eg EEC countries) are too important to be transmitted through global variables, such as world trade, in a global model.

The importance of Orcutt's critisism was recognised early, and crude attempts were made to include elements of bilateral linkage in the estimation of global models. This was achieved by using constant bilateral shares, or utilizing shares from previous years. Examples of this are found in the models by Tyszinski(1951), Fleming and Tsiang(1956-57), Waelbroeck(1962), Tims and Meyer-zu-Schlochtern(1962), and Kuznets(1964), as they are conveniently summarized in Taplin(1967).

### 2.2.4). Total vs Bilateral Import Allocation Models.

The approach taken in the estimation of bilateral trade linkage models, is critical for their feasible estimation with respect to their size. There are two possible approaches. That of Total Import Allocation, in which case total income is allocated over domestic inputs and trade partners, and that of Bilateral Import Allocation Models, in which case total income is allocated between a domestic aggregate and over trade partners.

The size of a total allocation model can grow very large, when compared to its counterpart bilateral import allocation model. In an import allocation system, the number of parameters that determine the allocation of income between domestic factors and imports, is not directly entering the system of the final bilateral import demand equations. This reduces considerably the number of parameters entering the model, allowing us to include more trade partners. Therefore, when the number of trade partners increases, and as more commodity groups are distinguished, it pays to use an import allocation model. Size is the critical factor in explaining the relative decline in the use of total allocation in comparison to import allocation models.

Early efforts to estimate bilateral trade linkage systems have
concentrated on total import allocation models, in theoretical studies, such as those of Metzler(1950) and Armington(1969a, 1969b, 1973), and later in the empirical models of the IMF (Polak and Rhomberg(1962), Rhomberg and Boissonneault(1964), Rhomberg and Fortucci(1964) and Rhomberg(1968)), Resnick(1968), Morishima and Murata(1972), IMF-MERM (Artus and McGuirk(1981), MCM(Edison et al(1987)) and Viaene(1983).

But how could bilateral import allocation models be implemented? The answer to this is given by Taplin(1967), proposing a two stage budgeting procedure. This approach is widely used today in the estimation of systems of demand equations, of Baker et al(1989). At the first stage, the allocation of income over domestic factors and total imports is determined. At the second stage, total imports of each good are allocated over trade partners. This requires bilateral imports, determined at the second stage, to add up to total imports, determined at the first stage. In effect, this implies that identities (2.1)-(2.3) hold. It is important to realise, that the assumption of separability of preferences between domestic inputs and imports is underlying the two stage budgeting procedure, of Barten and d'Alcantara(1977). Separability of preferences refers to the situation where we may partition domestic inputs and imports into separate groups, so that preferences within each group can be described independently of the preferences of the other groups. We elaborate on the latter in chapter 3.

High disaggregation of goods in international trade allows one to distinguish influences that are specific to individual commodity groups, which otherwise disappear in an aggregate study. Also, economic policies can be argued to influence different sectors of the economy in different ways. These sectors can be studied more effectively when disaggregated data are analyzed. On the estimation side, Orcutt(1950), and later Magee(1975), and others, assert that elasticities and estimation results are biased when aggregate data are used; the aggregation bias problem. Along the econometric arguments for using disaggregated data in preference over aggregate figures, one might be interested in the sectoral analysis of world
2.2.5). Inconsistent vs Consistent Bilateral Import Allocation Models.


#### Abstract

Bilateral trade import allocation models can be segregated according to whether the adding up identities (2.1)-(2.3) hold. If they do, the models are termed Consistent, while if they fail, we are dealing with Inconsistent Models. In effect, the adding up conditions require bilateral imports per category of goods, determined at the second stage of the two stage budgeting procedure, to add up to total imports for each good, determined at the first stage. There is no question of inconsistency for total allocation models, since total imports are determined as the sum of bilateral imports over all trade partners. The only requirement is that the world trade balance is zero. The same requirement is also sufficient for global models, in this case because no bilateral variables appear.


It is important that the adding up constraints are satisfied, if the results of the two stage budgeting procedure are to be mutually compatible. The satisfaction of the adding up constraints also ensures, that the effects of the international transmission process are not distorted due to inconsistencies in our data. The possible inconsistency that could arise due to differences in c.i.f, f.o.b. measurement of data does not exist in our model, since volumes of seaborne trade will be examined, although, a problem ever present in international trade value data.

Examples of inconsistent linkage models in the literature are: EUROLINK (Ranuzzi, 1981), IMF World trade model(Ripley, 1981), INTERLINK (Richardson, 1988), LINK (Filatov et al, 1983), MARCOII(Guillaume, Modigliani(1953) 1981) Marwah(1976) Mennes (1967), Neisser and Tsukuba-FAIS(Shishido, 1983) etc.
2.2.6). Simulation vs Estimation Consistent Import Allocation Models.

Two possibilities are further distinguished within the class of consistent bilateral import allocation models, according to the way the adding up constraints are imposed. Estimation consistent and simulation consistent import allocation models.

The adding up conditions may be imposed in Simulation Consistent Import Allocation Models, in a number of ways, at the simulation stage. It is common practice in a number of studies, of Amano et al(1980), to consider the Rest of the World(ROW) imports as a residual in the system of import equations. Any differences that might arise in the allocation of total imports (between the total and the sum of the bilateral flows, excluding the ROW), are assigned to this residual, thus solving the problem of consistency. This is a convenient, but a somewhat arbitrary decision to treat one of the trade partners in an asymmetric way.

Another way to impose simulation consistency is to allocate the residuals that might arise between the sum of bilateral and total imports, in an arbitrary way, over given elements. This allocation could be achieved in a multiplicative or additive manner. For example, Samuelson and Kurihara(1980) apply the RAS method to modify shares in the trade matrix. The RAS method, developed by Stone and Brown(1964), consists of multiplying iteratively all rows and columns of an initial matrix by varying factors, until they both add up to the corresponding totals. For each row, the multiplicative factor is defined as the given row total divided by the row total after the previous multiplication. And similarly for the columns of the matrix. Bacharach(1965) shows that, starting with an initial (trade) matrix, this is a convergent procedure with a unique solution, subject to a given normalization.

The class of Simulation Consistent Import Allocation Models may be considered second best in comparison to Estimation Consistent Import Allocation Models. This is because consistency is imposed in the latter at the estimation stage with no concept of arbitrariness
involved. Consistency for this class of models is a direct consequence of the classical optimisation problem and not a result of some arbitrary method as discussed above. The adding up constraint is taken directly into account in the optimization problem of the importer, by recognizing that the system of bilateral import demand equations should be estimated subject to the linear constraint, which requires bilateral imports to add up to total imports. The question here is whether the functional form of the objective function, permits the explicit inclusion of the constraints at the estimation stage (this is taken up more formally in the next chapter). Resort to simulation consistency then, can be argued, to take place, when the functional form of the optimization problem does not permit the explicit satisfaction of the adding up constraints at the estimation stage.

There are about 15 estimation consistent import allocation models in the literature, which we list later under two different subsets of the above class of models.

### 2.2.7). Simple vs More General Functional Form Import Allocation Models.

The main stream of estimation consistent import allocation models, is allowing for a single price effect only between trade partners in an importing market. We call these, 'Simple Functional Form Import Allocation Models'. Thus, systems such as those based on a CES (Constant Elasticity of Substitution), a Cobb-Douglas, or a Stone-Geary (Linear Expenditure System) objective function belong to this category. By definition, the single price effect, only allows for a constant elasticity of substitution in an import market. This elasticity of substitution is at times constrained to take specific values, such as zero or one. Since only one price parameter need to be estimated, these systems are attractive for empirical purposes.

Alternatively, one may use functional forms which allow both for estimation consistency and more than one relative price effect. That is, 'more general' substitution patterns between trade partners are
allowed in the same import market. Such models may be derived from functions which are either: (1) second order approximations to a 'flexible' functional form objective function, or (2) from a 'specific' objective function. The problem with (1) is that the number of parameters increase quadratically with the number of trade partners. This would be a problem for an empirical model of our dimensions (30 partners, 5 commodities), and the fact that our time series is short. Also, when weak-separability (the assumption needed for import allocation) is imposed on flexible functional forms, most of them may no longer be considered as second order approximations of weakly separable objective functions. So, there remains (2). Here, for $n$ partners, there are less than $3 n$ parameters to be estimated (Hanoch (1975, p396)).

Thus, models based on 'More General Functional Forms' (in particular, on 'specific' objective functions) are almost as easy to estimate as their 'Simple Functional Form' counterparts. At the same time, the former allow for more general substitution patterns between trade partners as compared to the latter. At the outset then, systems based on a 'More General Functional Form' objective function are more attractive for adoption for our purposes. We expand more on the a priori choice of the functional form in the next chapter.

In the next section, we focus on the alternative functional forms that have been used in international trade to derive estimation consistent bilateral trade import allocation models. We select one of these to apply to our seaborne trade model.

## 2.3). Main Estimation Consistent Systems.

The question of the functional form of the objective function has been a long standing question in the literature. It may be argued that the choice of the functional form is, ultimately, a matter of empirical evidence. A number of empirical studies have been devoted to the subject. Gana et al(1979), Amano et al(1980), Italianer(1982)
and Sarma(1983) are examples of studies, which compare different approaches of estimating bilateral trade flows with consistency requirements.

The different approaches that have been compared in the above studies are:
-The 'naive' approaches of Constant Quantity Shares and Constant Value Shares, which are essentialy based on a Cobb-Douglas function.
-The Klein and Van Peeterssen(1973) approach, based on the foundations of the Linear Expenditure System(LES) in consumer demand theory, as developed by Stone(1954) and reviewed in Deaton and Muellbauer(1980). A similar approach which is developed later by Samuelson and Kurihara(1980) is used in estimating total export equations. A variant of the above is the approach of Johnson(1978), who uses an extended LES function to estimate bilateral trade flow equations.
-The approach of Hickman and Lau(1973) based on a CES function. The approach of Constant Market Share Elasticities(CMSE), Samuelson(1973), which is another CES system in trade market shares form.
-Finally, the approach of COMET (Italianer, 1982), which examines each bilateral import market, deriving a set of bilateral import reduced form equations.

Since we have selected the class of estimation consistent import allocation models in the previous section, we focus more carefully on the approaches that satisfy these criteria and we leave out the inconsistent systems. As a matter of reference we present the estimation consistent systems based on 'simple' functional forms. Thus, we start by presenting some of these estimation consistent 'simple' functional form systems, and then we turn into estimation consistent systems which allow for more 'general' substitution patterns between trade partners.
2.3.1). The 'Naive' Constant Quantity/Value Share models.

The constant quantity/value share models are tested in Gana et al(1979), Amano et al(1980) and Sarma(1983).

We start with the approach of the Constant Quantity Share. The trade share of some good $k$ (the index is omitted) is defined as,

$$
\begin{equation*}
w_{i j}=m_{i j} / m_{j} \tag{2.5}
\end{equation*}
$$

The basic assumption is that the trade share matrix in the base period is fixed. That is,

$$
\begin{equation*}
w_{i j}=w_{i j}^{0} \tag{2.6}
\end{equation*}
$$

where the superscript ' $o$ ' denotes the base period.

When the $m_{j}$ are determined by some regional model (or given exogenously in the system) the $m_{i j}$ can be determined by:

$$
\begin{equation*}
m_{i j}=w_{i j}^{0} m_{j} \tag{2.7}
\end{equation*}
$$

and the $x_{i}$ are determined by the following identity.

$$
\begin{equation*}
x_{i} \equiv \Sigma_{j} m_{i j} \equiv \Sigma_{i} w_{i j}^{0} m_{j} \tag{2.8}
\end{equation*}
$$

Also, since $\sum_{i} w_{i j}^{0}=1$, total world trade (for commodity $k$ ) can be consistently determined through,

$$
\begin{equation*}
\sum_{i} x_{i} \equiv \sum_{i} \sum_{j} w_{i j}^{0} m_{j} \equiv \sum_{j} m_{j}=T \tag{2.9}
\end{equation*}
$$

Thus, identities (2.1)-(2.4) are automatically satisfied rendering the system estimation consistent. The disadvantage of this model is that the $w_{i j}$ 's are considered constant, and this would lead to errors when this assumption is violated.

On the price side, by definition,

$$
\begin{equation*}
P_{j} m_{j}=\sum_{i} p_{i j} m_{i j} \tag{2.10}
\end{equation*}
$$

Dividing through by $m_{j}$ yields the import price of the $j$ th region as an expression of the exogenously given export prices $p_{1 j}$ :

$$
\begin{equation*}
P_{j}=\sum_{i} w_{i j}^{0} p_{i j} \tag{2.11}
\end{equation*}
$$

Similarly, denoting the world price index by $P_{W}$, since by definition,

$$
\begin{equation*}
P_{W} T=\Sigma_{i j} p_{i j} m_{i j} \tag{2.12}
\end{equation*}
$$

dividing through by $T$ yields the world price index:

$$
\begin{equation*}
P_{W}=\Sigma_{i} v_{i j}^{0} p_{i j} \tag{2.13}
\end{equation*}
$$

where $v_{i j} \equiv m_{i j} / T$, which is the share of the ith region in the world trade of some commodity $k$ (index omitted).

The nominal trade share of the ith region in the $j$ th import market is determined by:

$$
\begin{equation*}
W_{i j}=\left(p_{i j} m_{i j} / P_{j} m_{j}\right)=W_{i j}^{0}\left(p_{i j} / P_{j}\right) \tag{2.14}
\end{equation*}
$$

where $\sum_{i} W_{i j}=1$ is satisfied.

The Constant Value Share approach assumes that the nominal trade share is fixed at its base year period value:

$$
\begin{equation*}
W_{i j}=W_{i j}^{0} \tag{2.15}
\end{equation*}
$$

Assuming that $M_{j}$ and $P_{i j}$ are exogenous, real bilateral import flows are determined by:

$$
\begin{equation*}
m_{i j}=W_{i j}^{0} M_{j} / P_{i j} \tag{2.16}
\end{equation*}
$$

and real total imports of $j$ are determined through the identity:

$$
\begin{equation*}
m_{j} \equiv \sum_{i} m_{i j} \tag{2.17}
\end{equation*}
$$

Since $\quad W_{i j}=\left(m_{i j} / m_{j}\right)=\left(W_{i j}^{0} M_{j} / p_{i j}\right) /\left[\Sigma_{i}\left(W_{i j}^{0} M_{j} / p_{i j}\right)\right]$
the condition $\Sigma_{i} W_{i j}=1$ is satisfied. As a result, $P_{j}$ and other world trade variables can be determined consistently, as in the Constant Quantity method. The same drawbacks as with the latter method are also true here.

### 2.3.2). The Linear Expenditure System(LES).

The Linear Expenditure System was first used empirically in consumption theory with the work of Stone(1954). Examples of estimation consistent LES models in bilateral import allocation models are the Interdependence model of Hieronymi(1983) and the model of Johnson(1978). These are extentions of the LES from the theory of the consumer to linkage models.

Let us assume that the importer optimizes a Stone-Geary objective function,

$$
\begin{equation*}
\Pi_{i}\left(m_{i j}-\gamma_{i j}\right)^{\beta}{ }_{i j} \quad \text { subject to } \quad \sum_{i} p_{i j} m_{i j}=M_{j} \tag{2.19}
\end{equation*}
$$

where $\beta_{i j}$ and $\gamma_{i j}$ are the parameters of interest with $\Sigma_{i} \beta_{i j}=1$, and $M_{j}$ is the total import expenditure of $j$ on some good $k$ (index omitted).

Then, the typical LES estimating equation is:

$$
\begin{equation*}
p_{i j} m_{i j}=\gamma_{i j} p_{i j}+\beta_{i j}\left(\Sigma_{i j} p_{i j} m_{i j}-\Sigma_{i j} \gamma_{i j} p_{i j}\right) \tag{2.20}
\end{equation*}
$$

Estimating the above as a system, subject to $\sum_{i} \beta_{i j}=1$, maintains adding up. Thus, (2.20) is an estimation consistent import allocation model.

Johnson(1978) extends the above to incorporate the effects of a US dock strike. Also, dynamics are introduced by assuming a price expectations mechanism. However, when the latter is applied to (2.20), short-run elasticities are necessarily larger than their long-run counterparts (see Johnson(1978,p.73)). To overcome the problem, it is assumed that $\gamma_{i j}=\gamma_{i} m_{i j}^{0}$ in (2.20), and assuming dynamics the final estimating equation is:

$$
\begin{align*}
p_{i j} m_{i j} & =\tilde{\gamma}_{i} m_{i j}^{0} p_{i j}+\beta_{i j}\left[\Sigma_{i j} p_{i j} m_{i j}-\left(\Sigma_{i} \tilde{\gamma}_{i} m_{i j}^{0} p_{i j}\right)_{-1}\right]  \tag{2.21}\\
& +\left(1-\lambda_{j}\right)\left\{\left(p_{i j} m_{i j}\right)-1-\left(m_{i j}^{0} p_{i j}\right){ }_{-1}-\beta_{i j}\left[\left(\Sigma_{i j} p_{i j} m_{i j}\right)_{-1}-\left(\sum_{i m_{i j}}^{0} p_{i j}\right){ }_{-1}\right]\right\}
\end{align*}
$$

where $\tilde{\gamma}_{i}=\lambda_{j}\left(\gamma_{i}-1\right)+1$. An iterative procedure is used to estimate the above, where, for estimation is assumed that $\lambda_{j}=\lambda$. Simple OLS on the pooled (2.21) satisfies the constraint on the $\beta_{i j}$ 's, preserving adding up.

### 2.3.3). Constant Elasticity of Substitution(CES) Models.

Within the class of estimation consistent import allocation systems based on 'Simple Functional Forms', there is a particular class of models which has dominated the scene. The class of Constant Elasticity of Substitution(CES) models. It is the most general of the Simple Functional Form models, since, as we see in the next chapter, it contains the Cobb-Douglas and the LES as special cases.

The application of the CES function to import allocation models starts with the pioneering work of Hickman and Lau(1973). They use a CES objective function to derive a system of import demand equations. The objective function, or import quantity index (total imports of commodity $k$ by country $j$, denoted by $\mu_{j}^{k}$, which we simplify to $\mu$ ) as is known in terms of import allocation models, takes the form:

$$
\begin{equation*}
\mu=\left[\sum_{\left.i \delta_{i j} m_{i j}^{-\rho_{j}}\right]}^{-\left(1 / \rho_{j}\right)} \quad \text { where } \rho_{j}>-1\right. \tag{2.22}
\end{equation*}
$$

$\delta_{i} \in R$, and $\sigma_{j}$ is the Allen partial elasticity of substitution in the jth market, measuring the response of the ratio of imports from countries 1 and 2, say, to a change in the ratio of import prices from 1 and 2, holding all other import prices constant.

The importer's problem is then to optimize the objective function (2.22)/(2.23) subject to the budget constraint $M_{j}=\sum_{i=1}^{n} m_{i j} p_{i j}$, where $M_{j}$ is the value of imports corresponding to $\mu$.

The cost minimizing quantities of reaching a given level of imports (represented by the CES function), are then given by the following bilateral import demand equations:

$$
\begin{equation*}
m_{i j}=\delta_{i j}^{\sigma_{j}} \mu\left(p_{i j} / P_{j}\right)^{-\sigma_{j}} \tag{2.24}
\end{equation*}
$$

where $P_{j}$ is the 'composite' CES price index,

$$
\begin{equation*}
P_{j}=\left[\sum_{i} \delta_{i j}^{\sigma_{j}} p_{i j}^{1-\sigma_{j}}\right]^{1 /\left(1-\sigma_{j}\right)} \tag{2.25}
\end{equation*}
$$

with the additive price aggregation property that

$$
\begin{equation*}
P_{j} \mu=\sum_{i} p_{i j} m_{i j}=M_{j} \tag{2.26}
\end{equation*}
$$

Thus, adding up is ensured by the definition of the 'composite' price index $P_{j}$ at the estimation stage, rendering the system estimation consistent. It can be seen that the relative price effect -the Allen elasticity of substitution- is the same for all bilateral import equations. This is an obvious advantage for estimation, although a disadvantage in terms of the economic possibilities of substitution allowed.

The popularity of this class of models then amounts to their theoretical consistency, being derived from classical optimization
techniques, satisfying the adding-up conditions, and due to the induced convenience of being easily turned into empirical models. They can incorporate a large number of trade partners and commodity groups, thus an apparent solution to the requirements of large disaggregated world linkage models.

Hickman and Lau(1973) estimate an empirical model based on the CES function involving 27 trade partners. They extend (2.24) to include a time trend, which takes account of non-price factors. The system is then linearized by a first order Taylor expansion around some base year values, thus eliminating the unobservable CES indices $\mu$ and $P_{j}$. Price expectations are also incorporated in the form of adaptive expectations, rendering the system dynamic. Their typical estimating equation is of the form:

$$
\begin{align*}
\left(m_{i j}-w_{i j}^{0} m_{j}\right)= & -\sigma_{j} m_{i j}^{0} \beta_{i j}-\sigma_{j}\left(1-\lambda_{j}\right) m_{i j}^{0}\left(p_{i j}-\sum_{i j} w_{i j}^{0} p_{i j}\right)  \tag{2.27}\\
& -\sigma_{j} m_{i j}^{0} \gamma_{i j} t+\lambda j\left(m_{i j}-w_{i j}^{0} m_{j}\right)
\end{align*}
$$

Adding up is still preserved by estimating (2.27) as a system, and constraining the sum of the coefficients of the time trend and the constants to sum to zero. So the system remains estimation consistent.

Other CES models in the literature, or variants of it are: Hickman(1973), DESMOS III(Dramais(1981)), the Interfutures project (Halttunen and Warner(1979a), (1979b)), the EPA world economic model(Amano et al(1982)), Geraci and Prewo(1982), GLOBUS(Kirkpatric(1983)), Samuelson(1973) and the INTERLINK model(Richardson(1988)).
2.3.4). The Model of Resnick and Truman(1975).

Resnick and Truman(1975) use a multistage approach to estimate bilateral trade flows for non-food imports by 10 Western European countries. The model is specified in four stages for each country.

In the first stage, total real imports of each country are described by:

$$
\begin{equation*}
\ln M_{T}=a_{0}+a_{1} \ln P_{T}+a_{2} \ln Y+a_{3} \ln Q_{D} \tag{2.28}
\end{equation*}
$$

where $P_{T}=\sum_{i} W_{i} P_{i}=$ Relative price of imports for the world, where

$$
P_{i}=P_{i}^{x}(1+T) / P_{d} \text {, with }
$$

$P_{i}^{x}=\$$ export price index for the ith country, $P_{d}=G N P$ deflator of the importing country in \$'s, $(1+T)=$ Tariff index applied to the source of import,
$Y=$ Real GNP at base year prices, $Q_{D}=$ Pressure of demand variable measured as the difference between actual demand and trend demand.

Given the total import demand from the 1st stage, imports are allocated between Europe (EUR) and the Rest Of the World (ROW), according to the equations:

$$
\begin{align*}
& \left(M_{E U R} / M_{T}\right)=d_{10}+d_{11} P_{E U R}+d_{12} P_{R O W}  \tag{2.29}\\
& \left(M_{\text {ROW }} / M_{T}\right)=d_{20}+d_{21} P_{E U R}+d_{22} P_{R O W} \tag{2.30}
\end{align*}
$$

focusing more on the the European countries, $M_{E U R}$ are allocated between the EEC and the EFTA by:

$$
\begin{align*}
& \left(M_{E E C} / M_{E U R}\right)=b_{10}+b_{11} P_{E E C}+b_{12} P_{E F T A}  \tag{2.31}\\
& \left(M_{E F T A} / M_{E U R}\right)=b_{20}+b_{21} P_{E E C}+b_{22} P_{E F T A} \tag{2.32}
\end{align*}
$$

At the final stage, the total imports of the EEC and EFTA are allocated amongst their member countries through the equations:

$$
\begin{align*}
& \left(M_{i} / M_{E E C}\right)=c_{i 0}+\sum_{j=2}^{5} c_{i j} P_{j} ; i=2, \ldots, 5  \tag{2.33}\\
& \left(M_{i} / M_{E F T A}\right)=c_{i 0}+\sum_{j=6}^{10} c_{i j} P_{j} ; i=6, \ldots, 10 \tag{2.34}
\end{align*}
$$

where $P_{\text {EUR }}, P_{\text {ROW }}, P_{\text {EEC }}, P_{\text {EFTA }}$, are the relative price indices from Europe, the ROW, the EEC and the EFTA, respectively, defined in a similar way to $P_{T}$ above.

The equations are estimated by OLS with the following restrictions imposed on the parameters. (i) the sum of the constants is constrained to equal 1 and (ii) the sum of coefficients on the individual price variables is constrained to be 0 . Own and cross price elasticities are computed from the model.

As a critique, we should say that this multi-stage import allocation produces estimation consistent results. However, the presence of the GNP deflator in all the relative price indices violates the separability conditions (implicit in multi stage import allocation, see chapter 3), since a change in the GNP price index might also influence the allocation of income between domestic goods and imports.

### 2.3.5). The Model of Snella(1979).

Snella(1979) assumes that the preferences of the importer can be described by some kind of utility function. He thus, starts from an indirect utility function and by using Roy's theorem (Roy(1942)) he arrives at the import demand equations (see chapter 3, for these duality properties). This, index of preferences has the form:

$$
\begin{equation*}
\psi_{j}=\psi_{j}\left(p, M_{j}^{-1}\right)=\phi_{j}\left(M_{j} / A_{j}\right)\left(A_{j} / B_{j}\right), \quad j=1, \ldots, n \tag{2.35}
\end{equation*}
$$

where $\quad \mathrm{p}=$ vector of prices in the jth market,
$\phi_{j}=$ a continuous, but arbitrary function, $A_{j}$ and $B_{j}=$ two functions of the vector of prices $p$, homogeneous of degree one and normalized to 1.

Using Roy's theorem, the import demand functions are:
$m=\left(\partial A_{j} / \partial \mathbf{p}\right) C_{j}\left(\tilde{M}_{j}\right)+\left(\partial B_{j} / \partial \mathbf{p}\right)\left(A_{j} / B_{j}\right)\left[\tilde{M}_{j}-C_{j}\left(\tilde{M}_{j}\right)\right]$
where $m=$ vector of imports in the $j$ th market,
$\tilde{M}_{j}=M_{j} / A_{j}$, that is, total imports of $j$ in real prices,
$C_{j}=$ a function defined by $\phi_{j}\left(\tilde{M}_{j}\right)=\left\{\partial\left[\phi_{j}\left(\tilde{M}_{j}\right)\right] / \partial \tilde{M}_{j}\right\}\left[\tilde{M}_{j}-C_{j}\left(\tilde{M}_{j}\right)\right]$

Under the hypothesis $\tilde{M} \geq C(\tilde{M}) \geq 0$, the function $C$; may be interpreted as a committed amount of imports, in real terms, while $\left[\tilde{M}_{j}-C_{j}\left(\tilde{M}_{j}\right)\right]$ represents the real excess amount of imports. $C_{j}$ may be nonlinear and hence the import demand functions may be nonlinear.

In order to turn the family of systems defined in (2.36) into an empirical form, the functional form of the price indices $A, B$, and the function $C_{j}$ must be specified. The following linear and geometric specifications are proposed for $A_{j}$ and $B_{j}$, respectively:

$$
\begin{equation*}
A_{j}=\Sigma_{i} a_{i} p_{i}, \quad B_{j}=\Pi_{i} p_{i}^{b_{i}}, \quad \text { where } \quad \sum_{i} a_{i}=\Sigma_{i} b_{i}=1 \tag{2.37}
\end{equation*}
$$

Four alternative specifications are proposed for $C_{j}$ :

$$
\begin{gather*}
C_{j}\left(\tilde{M}_{t}\right)=\gamma  \tag{2.38}\\
C_{j}\left(\tilde{M}_{t}\right)=(1+\gamma)^{t} C_{0}  \tag{2.39}\\
C_{j}\left(\tilde{M}_{t}\right) / \tilde{M}_{t}=S_{j}\left(\tilde{M}_{t}\right)=\exp \left\{-\gamma / \tilde{M}_{t}\right)  \tag{2.40}\\
C_{j}\left(\tilde{M}_{t}\right) / \tilde{M}_{t}=S_{j}\left(\tilde{M}_{t}\right)=1-\exp \left\{-\tilde{M}_{t} / \gamma\right\} \tag{2.41}
\end{gather*}
$$

Thus, for example, using (2.37) and (2.38) in (2.36) we may arrive at the LES (see equation (2.20)). Other specifications of the above would give different models. In fact, Snella(1979), using 11 trade regions and 6 commodity groups, finds that the best performing version of $C_{j}$ is that of (2.41).

The important properties of the family of models specified by (2.36) is that they are derived from the optimizing behaviour of the importers, and the conditions for separability (used implicitly here) are satisfied. The latter due to the indirect utility function
being of the Gorman polar form (see chapter).
2.3.6). The INFORUM model (Nyhus(1978)).

The INterindustry FORecasting of the University of Maryland (INFORUM) system is determining bilateral import shares through the following equation.

$$
\begin{equation*}
w_{i j}=w_{i j}^{0} p_{i j}^{\beta_{i j}} \tag{2.42}
\end{equation*}
$$

where $p_{i j}$ here is defined as the ratio of the effective price of some good in region $i$, $p e_{i j}$, relative to the world price as seen by the importing region $j, P_{j}$. Thus, $p_{i j}=p e_{i j} / P_{j}$, and (2.42) becomes:

$$
\begin{equation*}
w_{i j}=w_{i j}^{0}\left(p e_{i j} / P_{j}\right)^{\beta} \tag{2.43}
\end{equation*}
$$

Next, define the 'composite' price index $P_{j}$ implicitly by:

$$
\begin{equation*}
\Sigma_{i} w_{i j}^{o}\left(p e_{i j} / P_{j}\right)^{\beta}=1 \tag{2.44}
\end{equation*}
$$

This index is linearly homogeneous in prices, since if all domestic prices are doubled, then a doubling of the world price leaves the price ratio unchanged. Also, the ratio of the shares of any two regions will change if the price of a third region is changed, provided neither region's share is zero, and both do not have identical $\beta_{i j}$ 's.

Adding up is satisfied as a result of the implicit definition of the price index $P_{j}$. This can be easily seen by summing both sides of (2.43) yielding,

$$
\begin{equation*}
\Sigma_{i} w_{i j}=\Sigma_{i} w_{i j}^{0}\left(p e_{i j} / P_{j}\right)^{\beta} \tag{2.45}
\end{equation*}
$$

Using the definition of $P_{j},(2.44)$, in the above, the right hand side becomes one, and the model can be seen to be estimation
consistent.
(2.44) may also be expressed in terms of bilateral imports (not shares) as:

$$
\begin{equation*}
m_{i j}=w_{i j}^{0} m_{j}\left(p e_{i j} / P_{j}\right)^{\beta} \tag{2.46}
\end{equation*}
$$

The properties of (2.43) are still preserved.

Nyhus(1978) adds a time trend in the above, to take account of non-price factors. Also, the effective price term pe for a given commodity, is defined as a weighted average of present and past domestic prices. Since the estimating equation is highly nonlinear, an iterative estimation technique is used to arrive at estimation consistent results.

The advantages of this model over the Simple Functional Form models, is that it allows for a differing price effect between trade partners in an importing market. At the same time the number of parameters only increase linearly with the number of trade partners, and thus the model remains feasible for estimation. Also, another interesting feature, is the imposition of estimation consistency through the implicit definition of the 'composite' price index $P_{j}$. The model is not derived from classical optimization principles. An empirical model is actually estimated distinguishing between 10 trade partners and 119 commodities.

### 2.3.7). The CRESH system (Hanoch(1971), Italianer(1986)).

The Constant Ratio of Elasticities of Substitution Homothetic(CRESH) function was first introduced by Hanoch(1971) in production theory. The function is homothetic (or homogeneous), and the derived elasticities of substitution vary between different factors (trade partners), but are in fixed ratios. Italianer(1986) applies this function to an import allocation model, distinguishing between 7 trade partners and 5 commodity groups. The derived system
is similar to that of Nyhus(1978), except that the estimating equations are derived from neoclassical optimization principles.

Assume that the importer optimizes the following, implicitly defined, objective function:

$$
\begin{equation*}
F\left(\mu, m_{1 j}, \ldots, m_{n j}\right)=\sum_{i} \delta_{i j}\left(m_{i j} / \mu\right)^{-\rho_{i j}}-1 \equiv 0 \tag{2.47}
\end{equation*}
$$

where $\delta_{i}, \rho_{i} \in R$. Then, the derived import demand equations are of the form:

$$
\begin{equation*}
m_{i j}=\mu\left[\left|\delta_{i j} \rho_{i j}\right|\left(p_{i j} / P_{j}\right)^{-1}\right]^{a_{i j}}, \quad a_{i j}=1 /\left(1+\rho_{i j}\right) \tag{2.48}
\end{equation*}
$$

Finally, the 'composite' price index $P_{j}$ is defined implicitly by:

$$
\begin{equation*}
G\left(P_{j}, P_{1 j}, \ldots, P_{n j}\right)=\sum_{i} \delta_{i j}\left[\left|\delta_{i j} \rho_{i j}\right|\left(p_{i j} / P_{j}\right)^{-1}\right]^{-a_{i j} \rho_{i j}}-1 \equiv 0 \tag{2.49}
\end{equation*}
$$

This definition of $P$, ensures that adding-up is satisfied, yielding an estimation consistent import allocation model. The substitution possibilities allowed by this function are quite general, since there is a distinct price effect for each trade partner. At the same time the number of estimated parameters rise linearly with the number of trade partners, and the model remains feasible for estimation.

Following the tradition of Hickman and Lau(1973), Johnson(1978), Nyhus(1978) etc, Italianer(1986) includes a time trend in order to take account of non-price factors in the allocation of imports. He then linearizes (2.48), and incorporates price expectations, rendering the system dynamic. The final estimating equation has the form:

$$
\begin{align*}
\Delta \ln \left(m_{i j} / \tilde{m}_{j}^{0}\right)= & \lambda j \ln \left(m_{i j} / \tilde{m}_{j}^{0}\right)_{-1}+b_{i j}\left(1-\lambda_{j}\right)-a_{i j}\left(1-\lambda_{j}\right)\left[\Delta \ln p_{i j}-\sum_{h} w_{h j}^{0} \Delta \ln p_{h j}\right]  \tag{2.50}\\
& +\left(1-\lambda_{j}\right) \Sigma_{h} w_{h j}^{0} a_{h j}\left[\Delta \ln p_{h j}-\Sigma_{i} w_{l j}^{0} \Delta \ln p_{l j}\right]
\end{align*}
$$

where

$$
\begin{equation*}
\Delta \ln \left(\tilde{m}_{j}^{0}\right)=\sum_{i} w_{i j}^{0} \Delta \ln m_{i j} \tag{2.51}
\end{equation*}
$$

A property of $(2.50)$ is that $\Sigma_{i} w_{i j}^{0} \Delta \ln \left(m_{i j} / \tilde{m}_{j}^{0}\right)=0$. When the above is estimated as a system and the constraint $\sum_{i} w_{i j}^{0} b_{i j}=0$ is imposed on the constants, adding up is satisfied, and the system remains estimation consistent. Regarding the substitution possibilities allowed in the system, trade partners stand as competitors in import markets.

The last approach seems to be the most fruitful in terms of our objectives in this study. In the rest of the thesis we use the CRESH function to derive more formally, from neoclassical optimization principles, an import allocation model for seaborne trade flows in the international economy.

The aim of this chapter has been to survey the literature of world trade linkage models, and to select the system with the most desirable properties, which we can then apply into seaborne trade. The survey is related to our aim of constructing a relatively large seaborne trade model of the world economy, of 30 trade partners and 5 commodity groups. We started the chapter by explaining the concept of world trade flows in the context of an import-export matrix. The various methodological directions which have been followed in the past to build linkage models have been examined and compared, subsequently.

In conclusion, it is argued that a theoretically consistent and empirically feasible world linkage model is desirable. The class of Estimation Consistent Bilateral Import Allocation Models based on General Functional Forms is the best choice, a priori. In particular, the CRESH objective function of Hanoch(1971), may be used to derive a set of bilateral seaborne import demand equations from neoclassical optimization principles. The system is estimation consistent, allows for more than one price effect amongst trade partners in the same import market, the number of estimating parameters increases linearly with the number of equations (trade partners), and trade partners are held as competitors in the same import market. Thus, the CRESH system, while allowing for a sound theoretical structure, it is possible to estimate a large scale seaborne bilateral trade flow model based on it.

## Appendix 2.1.

Major Linkage Models of the 1980's.

LINK (USA-International) - Sawyer(1979), Filatov et al(1983) - 28 countries and 4 regions, 4 commodity groups. The LINK project is continuously elaborated to include more national models.

INTERLINK (OECD model) - Richardson(1988) - 23 countries and 8 regions, 4 categories of goods.

Fair(1987)- 63 countries and rest of the world (64 partners), aggregate trade. The largest existing model, in terms of trade partners.

COMET IV(EEC, etc) - d'Alcantara and Italianer(1982) - 13 countries with 5 commodity groups, and the rest of the world divided in 5 regions with aggregate trade.

DESMOS(EEC, etc) - Dramais(1981)- 10 countries and 3 regions, aggregate trade.

METEOR(EEC, etc) - Kooyman(1982)- 9 countries and 5 regions, aggregate trade.

EPA(Japan -World model) - Kaneko and Yasuhara(1986) - 9 countries and 6 regions, aggregate trade.

EUROLINK - Ranuzzi(1981) - 4 countries and Rest of the World, disaggregated trade.

MCM(USA - F.R.B.) -Edison et al(1987) - 5 countries and rest of the world, aggregate trade.

IMF World Trade Model -Ripley(1981) -14 countries and 4 regions, 4 categoties of goods.

IMF Multilateral Exchange Rate Model(MERM) -Artus and McGuirk(1981), Theoretical model, 20 countries, 6 commodity groups.

MARCO II(Brussels) -(Guillaume(1981)) -7 partners, 5 commodity groups.

WEP(UK-Treasury) -Horton(1984)- 9 countries and 5 regions, aggregate trade.

GEM(UK-National Institute Global Econometric Model) -Wren-Lewis(1987) - An extension of the WEP model with new equations describing the exchange rates.

LIVERPOOL model -Canzoneri and Minford(1988), Minford et al (1986)-9 countries and 3 regions, aggregate trade. A rational expectations model.

Tsukuba-FAIS - Shishido(1983) - Linked trade models for 15 trade partners.

FUGI(Japan) -Kaya et al(1983)- 28 trade partners.

DYNAMICO(UN) -Costa(1983), 10 trade partners, 10 sectors.

MOISE- (Lafay and Brender(1981)) -8 countries and 12 regions, 12 commodity groups.

EXPLOR-MULTITRADE -(Sallin-Kornberg and Fontela(1981)) -10 countries and 5 regions, 31 commodity groups.

## THEORY

## 3.0). Introduction.

In this chapter we examine more formally the theoretical basis of import allocation models, and we develop a model which can allow for differing elasticities of substitution amongst trade partners.

This chapter is in three main sections. In the first, we focus at the experience from consumer demand theory in estimating systems of consumer demand equations. Theoretical and practical problems in estimating such systems are examined. In the second, we extend the results to the case of systems of seaborne import demand equations. An aggregator function is defined, which plays the role of the utility or production function according to whether the importer is a producer or a consumer. The principles of multistage budgeting and the related problems of consistency of aggregation, and adding-up are discussed. In the third section, we propose the use of a specific form of the aggregator function, the Constant Returns to Scale(CRS) version of the Constant Ratio of Allen Elasticities of Substitution Homogeneous Homothetic(CRESH) function. This kind of function permits relatively 'general' substitution patterns between trade partners, while at the same time it leaves the derived model feasible to estimate.

We derive fully the properties of the import allocation model based on the CRESH function. One of the nice properties of this model is that it can encompass most of the import allocation models of the literature as special cases. Thus, systems such as the CES, the Cobb-Douglas, the LES and the Leontief can be obtained from CRESH, by placing restrictions on the parameters of the latter. At the same time we show that CRESH is flexible enough to incorporate factors other than relative prices, as the determinants of the

## 3.1). The Experience from the Theory of the Consumer.

The theory of consumer choice was developed in order to describe how a fixed total expenditure is allocated over a number of goods. The problem of the consumer consists of maximizing utility $u(x)$, subject to a budget constraint $p^{\prime} x \leq y$, where $x$ is a vector of quantities of consumption goods $x=\left(x_{1}, \ldots, x_{n}\right), p$ is the vector of the associated prices $p=\left(p_{1}, \ldots, p_{n}\right)$, and $y$ is the consumers' total expenditure (or income, provided there is no saving). Solving the primal problem (as defined above) results in a set of Marshallian demand equations $x_{i}^{*}=g_{i}(p, y), i=1, \ldots, n$. If these optimal quantities are substituted into the direct utility function, we obtain a function which shows the maximum utility attainable given $\mathbf{p}$ and $y$. This new function $\psi(p, y)$ is known as the indirect utility function. The Marshallian demand functions are related to the indirect utility functions through Roy's(1942) identity;
$g_{i}(p, y)=-\left[\partial \psi(p, y) / \partial p_{i}\right] /[\partial \psi(p, y) / \partial y]$.

The problem of the consumer can be stated in an alternative form, that of minimizing $p^{\prime} x$ subject to $u(x) \geq \bar{u}$. In words, the consumer wants to find the minimum expenditure necessary to reach the level of utility $\bar{u}$. The solution to the dual problem yields the Hicksian demand functions $h_{i}(p, \bar{u})$. When these are substituted in the minimized function $p^{\prime} \mathbf{x}\left(=\sum_{i} p_{i} h_{i}(p, \bar{u})\right)$, they yield the cost (or expenditure) function $y=e(p, \bar{u})$. This function shows the minimum expenditure necessary to reach a fixed level of utility $\bar{u}=\psi(p, y)$. The Hicksian demand equations are related to the expenditure function through Shephard's Lemma; $h_{i}(p, \bar{u})=\left[\partial \mathbf{e}(p, \bar{u}) / \partial p_{i}\right]$.

These two consumer problems exhibit duality in the sense that the Marshallian and the Hicksian demand functions are equivalent at the optimum. The Marshallian demands at income $y$ are the same as the Hicksian demands at utility $\psi(p, y)$. That is, $g_{i}(p, y) \equiv h_{i}(p, \psi(p, y))$. Similarly, the Hicksian demands at utility $\bar{u}$ are the same as the

Marshallian demands at expenditure e(p, $\bar{u})$; that is, $h_{i}(p, \bar{u}) \equiv g_{i}(p, e(p, \bar{u}))$. As a result, the expenditure function can be derived by inverting the indirect utility function $\psi, \psi(\mathbf{p}, \mathrm{e}(\mathbf{p}, \bar{u})) \equiv \bar{u}$, which states that the maximum utility from expenditure $e(p, \bar{u})$ is $\bar{u}$. Similarly, the indirect utility function can be derived by inverting the expenditure function $e, \quad e(p, \psi(p, y)) \equiv y$, which states that the minimum expenditure necessary to reach $\psi(p, y)$ is $y$.

Important properties of the Marshallian and Hicksian demand equations are the following:
1). Adding-up: The total value of both Hicksian and Marshallian demands is equal to expenditure. That is, $\sum_{i} p_{i} h_{i}(p, \bar{u})=\sum_{i} p_{i} g_{i}(p, y)=y$
2). Homogeneity: The Hicksian demands are homogeneous of degree zero in prices, while the Marshallian demands are homogeneous of degree zero in both total expenditure and prices. That is, for any scalar $\lambda>0, \quad h_{i}(\lambda p, \bar{u})=h_{i}(p, \bar{u})=g_{i}(\lambda p, \lambda y)=g_{i}(p, y)$.
3). Symmetry: The cross-price derivatives of the Hicksian demands are symmetric. That is, for all $i \neq j, \quad\left[\partial h_{i}(p, u) / \partial p_{j}\right]=\left[\partial h_{j}(p, u) / \partial p_{i}\right]$.
4). Negativity: The ( $n \times n$ ) matrix of substitution terms, with elements $\left[\partial h_{i}(p, u) / \partial p_{j}\right]$, is negative semidefinite. Negativity is a direct consequence of the concavity of the cost function in prices $\mathbf{p}$, the latter being due to the fact that costs are minimized, or equivalently, due to utility being maximized.

Empirically, early efforts to model consumer demand equations focus on explaining the demand for each good, as a function of total expenditure and the prices of all other commodities in the system. For a large multigood system, lack of degrees of freedom is a problem in estimation. Stone(1954), solves the problem by transforming the Marhallian demands of the primal problem into Hicksian compensated demands, and applying the homogeneity property to a logarithmic form of the system. This enables him to reduce the set of the cross price effects entering each equation to the subset relevant to the good in question. Thus, the application of the theoretical properties of Marshallian demands enables the conservation of degrees of freedom, and the feasible estimation of such systems of equations. This approach does not, however,
necessarily preserve the symmetry properties of the demand equations.

Alternatively, all the theoretical restrictions, of adding-up, homogeneity and symmetry can be imposed from the outset. This reduces the $\left(n^{2}+n\right)$ estimated parameters of a system of Marshallian demand equations to $\left[\left(n^{2}+n\right) / 2\right]-1$. An example of this is Stone's(1954) Linear Expenditure System (LES). In this case the number of independent parameters is further reduced to $2 n-1$, due to the selection of a particular functional form. A similar gain in degrees of freedom can be obtained with the choice of other functional forms, notably the Constant Elasticity of Substitution(CES) function.

However, the considerable savings in degrees of freedom in empirical systems is achieved by decomposing the consumer's consumption decision into smaller problems, which result in the same choice as the original problem. In general, two sets of conditions must be observed in order to guarantee the existence of commodity aggregates, which let us decompose the problem of the consumer in two ways:
1). Hicksian separability, (Hicks (1936)). This is based on the composite commodity theorem. It asserts that if a group of prices move in parallel, then the corresponding group of commodities can be treated as a single good. This would enable us to aggregate such commodities into a single group, distinct from all other commodities in the economy. It can be shown (Varian 1984, p 147) that the new preferences defined over the decomposed sets of commodities leads to the same choices as the original ones. In practice, in considering demands of a particular commodity we take the composite commodity to be all goods in the economy except the one we are interested in. However, in open competitive economies, when we are modelling choices over long time periods, relative prices are unlikely to be stable.
2). Functional separability. This is based on the concept of
multi-stage budgeting, introduced by Strotz(1957) and Gorman(1959). There are two ideas behind this: i) separability of preferences, and ii) consistency of aggregation over goods.
(i). Separability of preferences refers to the possible partitioning of commodities into groups, so that preferences within groups can be described independently of the quantities on other groups. This implies that closely related goods can be grouped together. (ii). Consistency of aggregation requires that the commodities purchased by optimizing the group objective functions, should be the same as those purchased if the consumer allocated his consumer budget directly to individual commodities.

If such conditions are satisfied, a number of subutility functions can be optimized, each of which involves only quantities related to the group in question. This increases considerably the degrees of freedom available. However, it is important to be aware that separability of preferences imposes restrictions on behaviour that limit the possible substitution effects between goods in different groups. To be able to derive such a model mathematically, the utility function must have a special functional form, whence the term functional separability. These possible functional forms are discussed below, in the context of import allocation models.

## 3.2). Seaborne International Import Allocation Models.

The experience of estimating multigood systems of demand equations in consumption theory leads us to select multistage budgeting procedures, as the most fruitful way of turning demand theory, consistently, into empirical models of demand. The lesson from the theory of the consumer is that multistage budgeting can allow for a compromise between the requirements of theory and empirical work.

In international trade linkage models, a fixed total expenditure on imports is allocated over a number of trade partners. In this section, we extend the ideas of multistage budgeting from the theory
of the consumer to derive import allocation models of seaborne trade. The results discussed so far in the context of the consumer theory apply, mutatis mutandis, to this problem.

### 3.2.1). The Regional-National Aggregator Function.

Before we examine the multistage budgeting procedures in the context of import allocation models, it is necessary to look more carefully at the microfoundation problem of the individual importer.

Imports may be regarded either as inputs in the production process, or as consumption goods, which satisfy directly aggregate demand. In the former case, imports are inputs in the production function, in the cost minimizing problem of the producer-importer. In the latter case, imports are consumption goods in the utility maximizing problem of the consumer-importer. Fortunately, it is not necessary to make this distinction in order to derive bilateral import demand equations.

An objective function can be introduced, called the aggregator function after Diewert(1976, 1982), which takes the form of either a production or a utility function when needed. The aggregator function can be taken to represent the preferences of each geographical region (the national economy, in the case of one country) with respect to its expenditures on domestic goods and imported goods. The aggregator function for region (country) $j$ takes the form (The index $j$ of the importing region is omitted for notational convenience):

$$
\begin{equation*}
y=f\left(q_{1}, \ldots, q_{g}, m_{1}^{1}, \ldots, m_{n}^{1}, \ldots, m_{1}^{g}, \ldots, m_{n}^{g}\right) \tag{3.1}
\end{equation*}
$$

where $q_{1}, \ldots, q_{g}$ are goods produced domestically, while the $m_{i}^{k}$ are imports of goods $k=1, \ldots, g$, from trade partners $\mathfrak{i = 1}, \ldots, n$, with $\mathfrak{i} \neq j$.

Two assumptions are used in writing the aggregator function as in (3.1). The Armington(1969a) assumption, that no matter how detailed
the category of goods may be, the 'same' goods supplied by different countries are imperfect substitutes, and therefore deserve a separate place in the aggregator function. The second assumption, is that employed by Barten(1971), that domestically produced goods are imperfect substitutes for the same $k$ ind of imported goods, so that domestic goods also deserve a separate place in the aggregator function.

It is assumed that the aggregator function, $y=f($.$) obeys the$ regularity properties usually imposed on utility and production functions. That is, $f($.$) is twice continuously differentiable, it is$ strictly quasi-concave and has decreasing positive marginal derivatives.

Optimization of this aggregator function leads to demand functions for the domestically produced goods, $q_{1}, \ldots, q_{g}$, and the bilateral imports, $m_{1}^{1}, \ldots, m_{n}^{1}, \ldots, m_{1}^{g}, \ldots, m_{n}^{g}$. In each of these demand functions, the demand for each good is explained in terms of the total outlay of the region, the prices of all the goods entering the aggregator function, and possibly other variables. Empirical implementation of such a system would be impossible for a world model of the dimensions we set as 'ideal' in chapter 2; that is, 5 commodities and 30 regions. There would not be enough degrees of freedom to estimate such a model.

### 3.2.2). Separability of Preferences and Multistage Budgeting.

We assume that commodities can be aggregated into groups, so that preferences within each group can be described independently of the quantities in other groups. Preferences are, thus, assumed to be separable with respect to different groups of goods. For example, if dry bulk goods produced and imported in a region is taken as one group of goods, the decision maker (consumer or producer), is assumed to be able to rank individual dry bulk goods in a well defined utility ordering, which is independent of his decision on other domestic or imported goods.

This implies that we can define subaggregator functions for each group. The values of these subaggregator functions then combine to give the regional aggregator function. Assuming separability of preferences between different types of goods, and consistent aggregation of the subaggregator functions into the regional aggregator function, (3.1) can be written as:

$$
\begin{equation*}
y=v\left[f^{1}\left(q_{1}, m_{1}^{1}, \ldots, m_{n}^{1}\right), \ldots, f^{g}\left(q_{g}, m_{1}^{g}, \ldots, m_{n}^{g}\right)\right] \tag{3.2}
\end{equation*}
$$

where $v($.$) is an increasing function in all its arguments, and$ $f^{k}(),. k=1, \ldots, g$, are the subaggregator functions. Optimizing one subaggregator function, $f^{k}(),. k=1, \ldots, g$, is a separate or independent action from optimizing any other one. Optimization of $f^{1}($.$) , say, generates demand functions for the domestic good q_{1}$ and imports $m_{1}^{1}, \ldots, m_{n}^{1}$, in terms of total expenditure on good 1 , the domestic price of the good, all bilateral import prices and possibly other variables.

So far, a two-stage budgeting procedure has been defined: At the first or higher stage, total expenditure is allocated to the broad groups of goods, 1,...,g. At the second, or lower stage, group expenditures are allocated over domestic goods and imports.

A third disaggregation of the aggregator function can further increase the degrees of freedom. It amounts to defining sub-subaggregator functions, which distinguish in each subaggregator function the good produced domestically from the whole set of the same imported good. That is, it is assumed, Barten(1971), that preferences are separable with respect to whether each good is produced domestically or abroad. The decision to produce or consume the domestic good $k, k=1, \ldots, g$, is independent from the decision to produce or consume the 'same' imported good $k$ (as well as being independent from quantities on other groups).

This enables us to write (3.2) further as:
$y=v=u\left[f_{D}^{1}\left(q_{1}\right), \ldots, f_{D}^{g}\left(q_{g}\right), f_{I}^{1}\left(m_{1}^{1}, \ldots, m_{n}^{1}\right), \ldots, f_{I}^{g}\left(m_{1}^{g}, \ldots, m_{n}^{g}\right)\right]$
where $f_{I}^{k}($.$) describe preferences over imports from each of the$ $i=1, \ldots, n$, trade partners for good $k$, while $f_{D}^{k}($.$) describe$ preferences over the domestic product $k$. $u($.$) is increasing in all$ its arguments. At the final stage of the multi-stage budgeting procedure, import expenditure for each imported good is allocated amongst trade partners. This is in line with the Armington(1969a) assumption that the same goods imported from different trade partners are imperfect substitutes.

Bilateral import demand equations may now be derived, by optimizing independently each of the $g$ sub-subaggregator functions, in the regional aggregator function. Each of these bilateral import demand equations, for good $k$, is explained in terms of total import outlay for the particular good, the bilateral prices for that good coming from trade partners $i=1, \ldots, n$, and possibly other relevant variables. The increase in the degrees of freedom is significant. In terms of our ideal model, for say, 5 commodity groups and 30 regions, we would have had to include $5+(30 \times 5)=155$ prices in the system. Use of multistage budgeting, reduces the included price variables in the system to 30 bilateral import prices related to a particular commodity $k$, thus, saving 125 degrees of freedom.

A utility (aggregator) tree describing this three stage budgeting procedure is shown on Table 3.1. The importing region-country allocates total expenditure in three stages. At the first or highest stage, expenditure of region $j$, say, is allocated to the broad groups of goods, $1, \ldots, g$. Thus, $g$ subaggregator functions, one for each group, are determined at this stage. At the second or middle stage, the expenditure for each good is allocated between domestic goods and imports. An import quantity index (or sub-subaggregator function) for each good is defined at this stage, and this forms the basis for the existence of import allocation models. At the third or lowest stage, total import expenditure for each good is allocated between trade partners, leading to bilateral import demand equations from each trade partner of $j$, for each $k$ ind of good $k$.

Table 3.1.
The Utility (Aggregator) Tree.


Three stage-budgeting involves both consistent aggregation (to construct the broad groups at the different stages) and separability of decision making (for each of the subgroup problems). Separability of preferences is both necessary and sufficient for the third stage of the procedure. The existence of the sub-subaggregator function at the second stage, is then sufficient for the existence of an import allocation model. But separability of preferences, and two - or three - stage budgeting, are not equivalent. Neither implies the other.

It is important to be aware of the limitations which separability imposes on the resultant bilateral import demand equations. These limitations refer to the possible substitution effects between goods in different groups. Apart from income effects, a change in the imported price of dry bulk (say) from partner $i$, for example, will affect the demand for liquid bulk goods in the same way as a change in the price of dry bulk from any other partner $\mathrm{j} \neq \mathrm{i}$.

In practice, in estimating import allocation models, only the last two stages of the three-stage budgeting procedure are utilized in order to derive the bilateral import demand equations. It is assumed that, preferences are separable with respect to imports and domestic goods, and aggregation over imports is consistent. These two assumptions are necessary and sufficient for the existence of import allocation models.

### 3.2.3). Consistency of Aggregation.

Consistent aggregation (over imports) occurs when bilateral imports derived by the optimization of the sub-subaggregator functions, lead to the same solution as that derived by the optimization of the subaggregator functions themselves. This requires us to be able to define a single, aggregate, import quantity and price index for each sub-subaggregator function, which is then used to allocate import expenditures for each good amongst
bilateral trade partners. It is not generally true that, given separability of preferences, aggregation will be consistent. It requires restrictions to be placed on the functional form of the sub-subaggregator functions. It is worth mentioning here that since the demand functions can be derived from the indirect utility or expenditure functions, we may, alternatively, impose the restrictions on one of these functions. An example of the former is the model of Snella(1979), see section (2.3.5).

Gorman(1959) derived the required conditions for consistent aggregation in consumption theory. If we think of the subaggregator functions as general utility functions, the sub-subaggregator functions as group utility or subutility functions, and bilateral imports as the individual commodities, we can use the consumption theory terminology to describe the Gorman conditions for consistent aggregation.

To understand the Gorman conditions, some further terminology is required. Assume that $\mu_{k}=\theta_{k}\left(f_{k}\right)$ are group quantity indices, where $\mu_{k}$ is the composite quantity index of imports for good $k$. Goldman and Uzawa(1964), show that:
if the utility function $f($.$) is weakly separable, it can be written$ in the form $u\left(f_{1}^{-1}, \ldots, f_{g}^{-1}\right)$, where $f_{k}^{-1}$ is the subutility function, $k=1, \ldots, g$.
Also, if $f$ is strongly or additively separable it can be written in the form $u\left(f_{1}^{-1}+\ldots+f_{g}^{-1}\right)$.
The utility function $f$ is defined as weakly homothetic separable if it can be written as, $u\left(f_{1}^{-1}, \ldots, f_{q}^{-1}\right)$ (i.e., if it is weakly separable), and if $f_{1}^{-1}, \ldots, f_{g}^{-1}$, are each homothetic.
The function $u\left(f_{1}, \ldots, f_{g}\right)$ in turn, is homothetic if it is a monotonically increasing transformation $u\left[g\left(f_{1}, \ldots, f_{g}\right)\right]$ of a function $g\left(f_{1}, \ldots, f_{g}\right)$, which is homogeneous of degree one. $f=u\left(f_{1}, \ldots, f_{g}\right)$ is therefore homothetic if and only if the expenditure function can be written as $e(u, p)=\theta(u) b(p)$.

The first condition for consistent aggregation derived by Gorman(1959), is that each group expenditure function can be written
as a homothetic homogeneous expenditure function: $e_{k}\left(f_{k}, p_{k}\right)=\theta_{k}\left(f_{k}\right) b_{k}\left(p_{k}\right)$. If we think of $\mu_{k}=\theta_{k}\left(f_{k}\right)$ as group quantity indices, and $b_{k}\left(p_{k}\right)$ as the corresponding price indices, then if preferences are weakly separable, the utility maximization problem,

$$
\max u=u\left(f_{1}, \ldots, f_{g}\right) \quad \text { subject to } \quad \Sigma_{k} e_{k}\left(f_{k}, p_{k}\right)=y_{k} \text {, }
$$

becomes:

$$
\max u=f\left[\theta_{1}^{-1}\left(\mu_{1}\right), \ldots, \theta_{g}^{-1}\left(\mu_{g}\right)\right] \text { subject to } \sum_{k} \mu_{k} b_{k}\left(p_{k}\right)=y_{k}
$$

Thus, weak separability with homotheticity (in other words, weak homothetic separability) of the group utility functions implies consistent aggregation. This form of cost function imposes stringent conditions on behaviour. It implies that the composition of the budget is independent of total expenditure or of utility. This would produce linear Engel curves through the origin, implying that all expenditure elasticities are unity.

An alternative solution proposed by Gorman(1959) is that the group indirect utility function should take, what is now called, the Gorman generalized polar form:

$$
\psi_{k}\left(y_{k}, p_{k}\right)=F_{k}\left[y_{k} / b_{k}\left(p_{k}\right)\right]+a_{k}\left(p_{k}\right)
$$

for some monotone increasing function $F_{k}($.$) , and some explicitly$ additive utility function:

$$
u=\theta^{-1}\left(\mu_{1}\right)+\ldots+\theta^{-1}\left(\mu_{g}\right)
$$

A utility function of the above form implies that preferences are strongly or additively separable. The consumer's optimization problem is then reduced to:

$$
\max u=\sum_{k} F_{k}\left(\mu_{k}\right)+\sum_{k} a_{k}\left(p_{k}\right) \quad \text { subject to } \quad \sum_{k} \mu_{k} b_{k}\left(p_{k}\right)=y_{k}
$$

where $\mu_{k}=y_{k} / b_{k}\left(p_{k}\right)$ is the group quantity index, and $b_{k}\left(p_{k}\right)$ is the
corresponding price index. Note that the additive structure of the utility function results in the $a_{g}$ being irrelevant to the maximization problem. However, their presence makes the Gorman polar form of the indirect utility function less restrictive, in the sense that it allows nonlinear relationships between within group expenditures and group outlay.

Nonetheless, additivity of the utility function is also a severe restriction. The problem lies in the fact that, due to additivity, any group in the utility function can be a combination of any other groups. This effectively prevents the existence of any special relationships between any pairs of groups. In particular, only substitutes are permitted, but not complements, and there is an approximate proportionality between expenditure and price elasticities. It can be argued that, both proportionality and absence of complementarity (and inferiority) are not too severe restrictions for broad groups of goods. This argument is quite relevant in the context of import allocation models, where imports of the same kind of good from different trade partners are likely to be substitutes in world import markets.

A third way of imposing consistent aggregation is by assuming weak separability and quasi-homotheticity of the expenditure function, where quasi-homotheticity implies for the expenditure function: $e_{k}\left(f_{k}, p_{k}\right)=\theta_{k}\left(f_{k}\right) b_{k}\left(p_{k}\right)+a_{k}\left(p_{k}\right), \quad(k=1, \ldots g)$. It can be shown (Deaton and Muellbauer 1980, p 132-133) that total group expenditure can then be written in terms of total expenditure and two sets of price indices constructed from the functions $a_{g}$ and $b_{g}$. In practice, if prices move together, the two price indices may be proxied by one. In this system, Engel curves are again linear, but are not constrained to go through the origin. Income erasticities only tend to unity as total expenditure increases. Barten and Turnovsky(1966), Barten(1970) and Theil(1975), working with differential demand systems, have used this idea, together with local approximations, to estimate systems of consumer demand equations. The conditions required are less stringent than in the other two cases.

To summarize, given weak separability, for consistent aggregation of the import allocation model, one of the following conditions should be true: i) the expenditure function should be homothetic homogeneous, $i$ i) the indirect utility function should be of the Gorman polar form, and the utility function should be additive (the latter implying strong separability of preferences), or iii) the expenditure function should be quasi-homothetic.

### 3.2.4). The Adding-up Conditions.

From equation (3.3), the sub-subaggregator functions $f_{I}^{k}($.$) , or$ partial aggregator functions $\mu_{k}($.$) , or import quantity indices, as$ they are more widely known, form the basis of the import allocation model. We simplify our notation to write:

$$
f_{I}^{k}\left(m_{1}^{k}, \ldots, m_{n}^{k}\right) \equiv \mu_{k}\left(m_{1}^{k}, \ldots, m_{n}^{k}\right) \equiv \mu
$$

When we defined the regional aggregator function we asserted that for the purpose of deriving import allocation models, it does not matter whether it is viewed as a utility or a production function, in the consumption or production theory, respectively. The same is true for the regional sub-subaggregator functions. We call these functions from now on partial aggregator functions or partial import quantity indices, or simply objective functions.

If the partial aggregator functions for some good $k$ play the role of the utility function in the problem of the consumer-importer, then his problem is to:

$$
\max \mu(.) \quad \text { subject to } \quad M_{j}=\sum_{i=1}^{n} m_{i} p_{i} \quad i=1, \ldots, n \text {, }
$$

where $p_{i}$ are the bilateral seaborne import prices corresponding to the bilateral seaborne import quantities $m_{i}$, and $M_{j}$ is the import bill of some commodity $k$, in current prices. The bilateral import demand equations, can be derived as functions of total expenditure on imports (of country $j$ on good $k$ ) and all bilateral import prices
for the particular good. That is:

$$
m_{i}=g_{i}\left(M_{j}, p_{1}, \ldots, p_{n}\right) ; \quad i=1, \ldots, n .
$$

If the aggregator function plays the role of the production function in the problem of the producer-importer, then his problem is to minimize the cost of the import bill $M_{j}$ as follows:

$$
\min M_{j}=\sum_{i} m_{i} p_{i} \quad \text { s.t. } \quad \mu=\mu(.)
$$

where $\mu=\mu\left(m_{1}, \ldots, m_{n}\right)$, is the import quantity index for some good $k$, which describes the technical relationship between $m_{1}, \ldots, m_{n}$ and $\mu$. Solution of the above problem gives factor demand equations for the bilateral imports-inputs in the form:

$$
m_{i}=g_{i}\left(m_{j}, p_{1}, \ldots, p_{n}\right) ; \quad i=1, \ldots, n .
$$

where $m_{j}$ is the total volume of imports of good $k$ (index omitted) by $j$.

In any case, we would expect that identity (2.2), of the previous chapter, should hold by definition of the data. That is, bilateral imports (for some good $k$ ) should add up to total imports in volume terms:

$$
\sum_{i} m_{i}=m_{j}
$$

and also bilateral import expenditures should add up in value terms:

$$
M_{j}=\sum_{i} m_{i} p_{i}
$$

The last condition is automatically satisfied, since it constitutes the binding budget constraint in the utility maximizing problem, while it is the cost function in the cost minimizing problem of the producer. Also the first condition is satisfied for the import demand equations derived from the cost minimizing problem
of the producer-importer. The total volume of imports by country $j$ for good $k$, expressed by $m$, enter directly each of the bilateral import demand equations.

This is along the line of the adding-up property we expect systems of demand equations to satisfy. Satisfaction of the adding-up constraint in this way makes the model estimation consistent. Adding-up is imposed by virtue of the optimization problem of the importer. The rest of the identities of world trade as defined in equations (2.1), (2.3), (2.4) are then automatically satisfied as a result.

## 3.3). A Specific Import Allocation Model.

Having provided the theoretical framework of import allocation models, we need to determine a specific functional form of the aggregator function, which allows for separability of preferences and satisfies tr.e consistent aggregation requirements. The choice of such a function is critical for the substitution possibilities allowed in the estimated model.

### 3.3.1). The Choice of the Aggregator Function.

As we saw in the previous chapter, Hickman and Lau(1973) introduced a CES partial aggregator function and by classical optimization techniques derived an estimation consistent import allocation model for some good $k$, say. The CES partial aggregator function, which describes the preferences of the importer $j$ for some good $k$, takes the form (the indices $j$ and $k$ are omitted):

$$
\mu=\left[\sum_{i} \delta_{i} m_{i}(\sigma-1) / \sigma\right] \sigma /(\sigma-1) \quad \text { where } \quad \sigma=(1 / 1+\rho), \quad \rho>-1
$$

This function satisfies both the assumptions of weak separability and homotheticity, which are needed in order to derive consistent import allocation models (see Hickman and Lau(1973, p350)).

Optimization of the CES function leads to the following import demand equations.

$$
m_{i}=\mu \delta_{i}^{\sigma}\left(p_{i} / P\right)^{-\sigma}
$$

where $P$ is the 'composite' CES price index,

$$
P=\left[\Sigma_{i} \delta_{i}^{\sigma_{p}}(1-\sigma)\right]^{1 /(1-\sigma)}
$$

with the additive price aggregation property that

$$
P \cdot \mu=\sum_{i} p_{i} m_{i}=M
$$

However, as we can see from the derived demand equations the CES function allows for only one relative price effect (a single elasticity of substitution), which is what makes it empirically attractive to estimate, but at the same time restrictive in the substitution possibilities it allows in the system.

The CES import quantity index is not the only function that satisfies the weak separability and homotheticity assumptions, necessary for import allocation models, contrary to the assertion of Hickman and Lau(1973, p 350).

One set of functions which satisfies these conditions is the 'flexible functional form' family of Diewert(1973), which may be considered as second order approximations to a general aggregator function. The number of parameters in these models increase quadratically with the number of trade partners. This make it unsuitable for empirical import allocation models, especially when the length of the available time series data is short and the number of trade partners large. Examples of such models is the translog model of Christensen et al(1973), models based on the quadratic function, Diewert's(1971, 72, 73, 74) Generalized, Linear and Leontief models, and other similar models referenced in Blackorby et al(1978, ch. 8).

An alternative to the flexible functional form approach is to derive the import allocâtion model from a 'specific aggregator function'. If the aggregator function contains $n$ factors, the derived demand equations will have less than $3 n$ parameters, of Hanoch (1975, p 396). This makes the model feasible to estimate. Furthermore, the $n$ factor CES function and the functions derived as special cases of it are special cases of the specific aggregator set of functions (Arrow(1960-1961), Arrow et al(1961), Uzawa(1962)). There are two classes of models based on a specific aggregator function, defined by Hanoch(1975).

First, 'explicit additive models' derived from an explicitly (or strongly or additively separable) additive aggregator function. Hanoch(1975, p 396), suggests that this form of the aggregator function is very restrictive, because of the existence of multiplicative or additive relationships between each substitution effect and its related income effect. To this class, belong functions such as the CES, the Cobb Douglas, or the Leontief type function (Bergson(1936), Berndt and Cristensen(1973)), with elasticities of substitution constant and equal for all trade partners, and unit value income elasticities.

Second, 'implicit additive models' derived from a corresponding type of direct or indirect aggregator function. These allow for non-constancy and non-homotheticity of the Allen elasticities of substitution (the latter are defined formally later in the chapter). Imposing the restrictions of Constant Ratio of Elasticities of Substitution and linear Homogeneity, results in the CRS version of the CRESH model, of Hanoch(1971). The assumption of linear homogeneity, as discussed earlier is restrictive, although, perhaps not so much in the context of import allocation models. Still, it is convenient in making the model estimation consistent; that is, for consistency of aggregation, and as a result for adding-up to hold.

The CRESH function retains the advantages of being theoretically consistent and estimation feasible, while it allows for more than one relative price effects. The CRESH function has been used by

Italianer(1986) to derive an empirical import allocation model. This function contains the CES as a special case, and in consequence the special cases of the latter are special cases of CRESH - that is, the Cobb-Douglas, the Leontief and the linear functions. CRESH in turn belongs to a wider class of functions which have a constant ratio of elasticities of substitution, but are not necessarily homogeneous or homothetic cf Hanoch(1971). We work with a special form of the CRESH function which makes it homogeneous of degree one - the CRS (Constant Returns to Scale) form.

It may be noted here that, other functional forms of the aggregator function satisfy the theoretical conditions of import allocation models, but may not be practical for estimation, because the number of parameters increase more than proportionately with the number of trade partners.

### 3.3.2). The CRESH Import Allocation Model.

Earlier in the chapter, in dealing with multistage budgeting, we noted that at the second stage of the three stage budgeting procedure, an import quantity index or partial aggregator function $\mu$ for each category of goods is determined. This is then allocated at the third stage amongst trade partners, thus deriving a set of bilateral import demand equations. We use the CRESH function for this partial aggregator function. The properties of the derived import allocation model depend on the properties of the CRESH function.

Given a set of exogenously determined bilateral import prices, $p_{1}, \ldots, p_{n}$, for the good $k$, say, the problem of the importer is to minimize import costs for product $k$ subject to a technical relationship between $\mu$ and $m_{1}$, where $m_{i}$, $i=1, \ldots, n$, are the bilateral import quantities of good $k$.

Let $\mu$ be defined implicitly by the following CRESH function of Hanoch(1971), where implicit additivity refers to the fact that the
aggregator function is defined by an identity of the following form:

$$
\begin{equation*}
F\left(\mu, m_{1}, \ldots, m_{n}\right)=\sum_{i} \delta_{i}\left(m_{i} / h(\mu)\right)^{-\rho_{i}}-1 \equiv 0 \tag{3.4}
\end{equation*}
$$

with $m_{1}>0,0 \leq \mu \leq \bar{\mu} \leq \infty$, where $h(\mu)$ is continously differentiable with $h(0)=0, h(\bar{\mu})=\infty$ and $h^{\prime}(\mu)>0$, where $\bar{\mu}$ denotes the maximum of the import quantity index. Also $F(0, \ldots, 0)=0$. Following Arrow et al (1961, p230), we refer to $\rho_{i}$ as the substitution parameters (since they determine the substitution possibilities between the $m_{1} ' s$ for a given $h(\mu)$ ), and to $\delta_{i}$ as the distribution parameters (since $\delta_{i}$ determine the distribution of $h(\mu)$ for a given set of $\left.\rho_{i}^{\prime} s\right)$.

Hadley(1964, p47) shows that, using the implicit function theorem, (3.4) gives a unique, continuously differentiable import quantity index $\mu=f\left(m_{1}, \ldots, m_{n}\right)$ if and only if $\max _{i} \delta_{i}>0$ and $\delta_{i} \rho_{i}$ are of the same sign for all $i$, or if $\rho_{1}=0$ for some $i$. We assume that the parameters satisfy these restrictions.

The function $\mu=f\left(m_{1}, \ldots, m_{n}\right)$ is homothetic, satisfying $h\left[f\left(t m_{1}, \ldots, t m_{n}\right)\right]=\operatorname{th}\left[f\left(m_{1}, \ldots, m_{n}\right)\right]$. That is, $h(\mu)$ is linearly homogeneous with respect to $m_{1}, \ldots, m_{n}$, which is necessary for consistent aggregation (since the expenditure or cost function of a homogeneous function of degree one is written as $e(\mu, p)=h(\mu) b(p)$, which is one way of satisfying the assumption of consistent aggregation, see section (3.2.3)). If we choose $h(\mu)=\mu^{1 / r}$ then $f\left(m_{1}, \ldots, m_{n}\right)$ is homogeneous of degree $r$; that is, $f\left(t m_{1}, \ldots, t m_{n}\right)=t^{r} f\left(m_{1}, \ldots, m_{n}\right)$. Here we choose $h(\mu)=\mu$, in which case $f\left(m_{1}, \ldots, m_{n}\right)$ is homogeneous of degree one, which is the CRS (Constant Returns to Scale) version of (3.4) - the CRESH function giving:

$$
\begin{equation*}
F\left(\mu, m_{1}, \ldots, m_{n}\right)=\Sigma_{i} \delta_{i}\left(m_{i} / \mu\right)^{-\rho_{i}}-1 \equiv 0 \tag{3.5}
\end{equation*}
$$

In many empirical studies so far (cf Hickman and Lau(1973), Nyhus(1978), Italianer(1986)), the introduction of a trend term seems to be indispensable in the explanation of import demand. This takes account of systematic non-price factors that affect imports,
such as changes in tastes or technology, for instance. This time trend $t$, can be inserted directly in (3.5), where $t=0$ in the base year. The trend coefficient $\gamma_{i}$ are interpreted then as shifts in the import quantity index due to non-price factors, such as changes in tastes, technology etc.

$$
\begin{equation*}
F\left(\mu, m_{1}, \ldots, m_{n}, t\right)=\sum_{i} \delta_{i} e^{\gamma_{i} t}\left(m_{i} / \mu\right)^{-\rho_{i}}-1 \equiv 0 \tag{3.6}
\end{equation*}
$$

The regularity conditions of (3.4) are assumed to hold for (3.5) and (3.6) also.

Having defined the aggregator function, we can express mathematically the problem of the importer as min $\sum_{i} p_{i} m_{i}$ subject to (3.6). This is equivalent to minimizing the following Lagrangean w.r.t. $m_{1}, \ldots, m_{n}, \lambda$, where $\lambda$ is a linear homogeneous function of prices $p_{1}, \ldots, p_{n}$ :

$$
\begin{equation*}
L=\Sigma_{i} p_{i} m_{i}+\lambda\left[1-\Sigma_{i} \delta_{i} e^{\gamma_{i}^{t}}\left(m_{i} / \mu\right)^{-\rho_{i}}\right] \tag{3.7}
\end{equation*}
$$

The first and second order conditions for minimization of (3.7) are equivalent to the conditions for quasi-concavity of $f\left(m_{1}, \ldots, m_{n}\right)$. The First Order Conditions (F.O.C.) for minimizing (3.7) are:

$$
\begin{gather*}
\partial L / \partial m_{i}=p_{i}+\lambda \delta_{i} \rho_{i} e^{\gamma_{i} t}\left(m_{i} / \mu\right)^{-\rho_{i}} m_{i}^{-1}=0 ; i=1, \ldots, n .  \tag{3.8}\\
\partial L / \partial \lambda=\sum_{i} \delta_{i} e^{\gamma_{i} t}\left(m_{i} / \mu\right)^{-\rho_{i}}-1 \equiv 0 \tag{3.9}
\end{gather*}
$$

For a solution of the above system $\delta_{i} \rho_{i}$ should have the same sign for all $i$, and this should be the sign of $\lambda$, because $\lambda \delta_{i} \rho_{i}>0$, for all i. This system of equations gives bilateral import demand equations $m_{i}, i=1, \ldots n$, and an equation for $\lambda$, in terms of $\mu, p_{i}$ and $t$. Because $\mu$ is a linearly homogeneous homothetic import quantity index, the elasticity of $m_{i}$ with respect to $\mu$ at the optimum is 1 . This implies:

$$
\begin{equation*}
|\lambda|=\mu P\left(p_{1}, \ldots, p_{n}, t\right) \tag{3.10}
\end{equation*}
$$

where $P($.$) is a price index which is a real positive function of the$ bilateral prices $p_{i}$ and time $t$.

Hence, solving (3.8) for $m_{i}$ and eliminating $\lambda$ by using (3.10) we get the system of bilateral import demand equations at a given $\mu$. These equations are shown below and are homogeneous of degree 0 in prices.

$$
\begin{equation*}
m_{1}=\mu\left[\left|\delta_{1} \rho_{1}\right| e^{\gamma_{1} t}\left(p_{1} / P\right)^{-1}\right]^{a_{1}}, \quad a_{1}=1 /\left(1+\rho_{1}\right) \tag{3.11}
\end{equation*}
$$

where the price index $p>0$ is defined implicitly, by substituting (3.11) in the budget constraint, equation (3.9), resulting in:
$G\left(P, p_{1}, \ldots, p_{n}, t\right)=\sum_{i} \delta_{i}\left[\left|\delta_{i} \rho_{1}\right| e^{\left(-\gamma_{i} / \rho_{i}\right) t}\left(p_{1} / P\right)^{-1}\right]^{-a_{i} \rho_{1}}-1 \equiv 0$
Thus, we have defined a specific import allocation model through the CRS version of the CRESH function. Because of the way we defined the CRESH 'composite' price index $P$ in (3.12), the system has the property of consistent aggregation and adding up. That is $\mu$ and $P$ are defined so that the additive price aggregation property P. $\mu=\sum_{i} m_{1} p_{i}=M$ holds.

Since $\quad \delta_{1}>0$, also $\delta_{i} e^{\gamma_{1} t}>0$, and $C=\Sigma_{i} \delta_{i} e^{\gamma_{1} t}>0$. Since minimizing $\sum_{i} p_{i} m_{i}$ subject to $\mu$ or $\mu / C$ leads to exactly the same solution we may divide the aggregator function $\mu$ by $\sum_{i} \delta_{i} e^{\gamma_{i} t}$. This is equivalent to a normalization of these parameters so that $\sum_{i} \delta_{i} e^{\gamma_{i}^{t}}=1$.

### 3.3.3). Properties of the model.

In order to derive the properties of the model we work here for computational ease with a general aggregator function of the form $\mu=f\left(m_{1}, \ldots, m_{n}\right)$. We then extent our general results to the CRESH
model. Let us assume that $\mu=f\left(m_{1}, \ldots, m_{n}\right)$ is twice continuously differentiable. The cost minimizing importer wants to minimize the import cost subject to the relation described by the aggregator function. He thus has to solve a problem similar to (3.7). Mathematically, his aim is to:

$$
\begin{equation*}
\min L\left(m_{i}, \lambda\right)=\sum_{i} p_{i} m_{i}+\lambda\left[\mu-f\left(m_{1}, \ldots, m_{n}\right)\right] \tag{3.13}
\end{equation*}
$$

Let $f_{i}=\partial f / \partial m_{i}$. The FOC are:

$$
\begin{align*}
& \partial L / \partial m_{i}=p_{i}-\lambda f_{i}=0, \quad \mathfrak{i}=1, \ldots, n  \tag{3.14}\\
& \partial L / \partial \lambda=\mu-f\left(m_{1}, \ldots, m_{1}\right)=0 \tag{3.15}
\end{align*}
$$

These constitute a set of ( $n+1$ ) equations to solve for the $(n+1)$ unknowns $m_{1}, \ldots, m_{n}, \lambda$ in terms of the $(n+1)$ parameters $p_{1}, \ldots, p_{n}, \bar{\mu}$. When solved they provide import demand equations in terms of the specified total imports $\bar{\mu}$ and bilateral prices. These equations have the form:

$$
\begin{equation*}
m_{i}=f_{i}\left(p_{1}, \ldots, p_{n}, \bar{\mu}\right) \quad i=1, \ldots, n \tag{3.16}
\end{equation*}
$$

(3.14) may be rewriten in two different ways:
a)

$$
\begin{equation*}
\left(f_{i} / f_{j}\right)=\left(p_{i} / p_{j}\right) \tag{3.17}
\end{equation*}
$$

which gives us the optimal input combination as the point where the marginal rate of technical substitution between any pair of imports, equals the corresponding import price ratio.
b)

$$
\begin{equation*}
\left(f_{1} / p_{1}\right)=\ldots=\left(f_{n} / p_{n}\right)=(1 / \lambda) \tag{3.18}
\end{equation*}
$$

which indicates that the optimal input combination is at the point where the marginal productivity of the last $\$$ spent on imports $(1 / \lambda)$, is the same everywhere.
3.3.3.1). Comparative Statics.

In order to describe the effects of changes of total import requirements $(\mu)$ or of changes of import prices $(P)$ on the optimal import mix of an importing region, we derive the following results

Taking the total differentials of the FOC (and using $p_{1}=\lambda f_{i}$ to eliminate $p_{i}$ ) yields:

$$
\begin{align*}
& \sum_{i} f_{i} d m_{i}=d \mu  \tag{3.19}\\
&-f_{i} d \ln \lambda+d f_{i}=(1 / \lambda) d p_{i},  \tag{3.20}\\
& i=1, \ldots, n
\end{align*}
$$

which we may write in matrix notation as:

$$
\left[\begin{array}{cccc}
0 & f_{1} & \ldots & f_{n}  \tag{3.21}\\
f_{1} f_{11} & \ldots & f_{1 n} \\
f_{n} f_{n 1} & \ldots & f_{n n}
\end{array}\right]\left[\begin{array}{c}
-d \ln \lambda \\
d m_{1} \\
d m_{n}
\end{array}\right]=(1 / \lambda)\left[\begin{array}{c}
\lambda d \mu \\
d p_{1} \\
d p_{n}
\end{array}\right]
$$

Equivalently: $\quad F \underline{m}=\lambda^{*} \underline{q}$
or

$$
\underline{m}=\lambda^{*} F^{-1} \underline{q}=K \underline{q}
$$

where the definitions of $F, \underline{m}, \lambda^{*}, \underline{q}$ and $K$ are obvious from (3.2.1).

### 3.3.3.2). Elasticities.

The effect of changes of total imports on the imports of $j$ from $i$, ceteris paribus, can be measured by the total import elasticity (income elasticity in consumption, output elasticity in production) defined by:

$$
\begin{equation*}
e_{i \mu}=\left(\partial m_{i} / m_{i}\right) /(\partial \mu / \mu) \equiv \partial \ln m_{i} / \partial \ln \mu \tag{3.22}
\end{equation*}
$$

It measures the percentage change in imports from country $\mathfrak{i}$ for a percentage change in the total import requirements $\mu$ in the

For the matrix equation the elasticities take the form (for $\mathrm{i}=1, \ldots, \mathrm{n}$ ):

$$
\begin{equation*}
e_{i \mu}=K_{i 0}\left(\mu / m_{i}\right)=\left(\lambda^{*} F^{-1}\right)_{i 0}\left(\mu / m_{i}\right)=\left[(1 / \lambda)\left(F_{10} /|F|\right)\right]\left(\mu / m_{i}\right) \tag{3.23}
\end{equation*}
$$

where $F_{10}$ is the (i0) cofactor, and $|F|$ is the determinant of the bordered Hessian F. Similarly $K_{10}$ is the (10) cofactor of K.

The effect of changes of import prices on import demands in an importing market $j$ may also be derived. Thus, using the own and cross price elasticities of bilateral imports we can measure, respectively, the percentage change in the import quantity of partner $\mathfrak{i}$ for a percentage change in the own import price or the import price of another partner, ceteris paribus. Mathematically, these price elasticities are defined as:

$$
\begin{align*}
& e_{i i}=\left(\partial m_{i} / m_{i}\right) /\left(\partial p_{i} / p_{i}\right) \equiv \partial \ln m_{i} / \partial \ln p_{i}  \tag{3.24}\\
& e_{i h}=\left(\partial m_{i} / m_{i}\right) /\left(\partial p_{h} / p_{h}\right) \equiv \partial \ln m_{i} / \partial \ln p_{h}
\end{align*}
$$

For the matrix equation they are (for $i, h=1, \ldots, n$ ):

$$
\begin{equation*}
e_{i h}=K_{i h}\left(p_{h} / m_{i}\right)=\left(\lambda^{*} F^{-1}\right)_{i h}\left(p_{h} / m_{i}\right)=\left[(1 / \lambda)\left(F_{i h} /|F|\right)\right]\left(p_{h} m_{i}\right) \tag{3.25}
\end{equation*}
$$

From (3.18) we know that $\left(f_{h} / p_{h}\right)=(1 / \lambda)$, which, when subsituted in (3.25) yields:

$$
\begin{equation*}
e_{i h}=\left[\left(F_{i h} /|F|\right)\right]\left(f_{h} / m_{i}\right) \tag{3.26}
\end{equation*}
$$

Now, if we divide the elasticity by the optimal value share (the proportion of total imports spent on imports from region $h$ ), that is by $w_{h}=\left(p_{h} m_{h}\right) /\left(\sum_{h} p_{h} m_{h}\right)$, which at the optimum (using (3.17)) is equivalent to $w_{h}=\left(f_{h} m_{h}\right) /\left(\sum_{h} f_{h} m_{h}\right)$, we obtain:

$$
\begin{equation*}
e_{i h} / w_{h}=\left[\left(\sum_{h} f_{h} m_{h}\right) /\left(m_{i} m_{h}\right)\right]\left[\left(F_{i h} /|F|\right)\right] \equiv \sigma_{i h} \tag{3.27}
\end{equation*}
$$

This is the partial elasticity of substitution between two trade partners (factor inputs) $i$ and $h$ as against all other imports (factors) as defined by Allen(1938, p 503), in production theory.

For a linearly homogeneous function $\mu=f\left(m_{1}, \ldots, m_{n}\right)$, such as our CRESH furiction, we have from Euler's theorem, $\mu=f_{1} m_{1}+\ldots+f_{n} m_{n}$ $=\Sigma_{h} f_{h} m_{h}$. As a result (3.27) reduces to:

$$
\begin{equation*}
\sigma_{i h} \equiv\left(\mu / m_{i} m_{h}\right)\left(F_{i h} /|F|\right) \tag{3.28}
\end{equation*}
$$

It is common practice, in production (consumption) theory, to use the elasticity of substitution as a measure of the degree of substitutability between two factors (two consumer goods in consumption), for a change in their relative prices, ceteris paribus. It may be interpreted as describing the percentage change in the relative quantities of two importing regions for a percentage change in their relative prices.

An important deficiency of the elasticities of import demands $e_{i h}$ as defined in (3.24)-(3.27) is that they are not symmetric. The Allen elasticities of import substitution, as normalized import demand elasticities (since the former are obtained by dividing the latter by the optimal value share, as shown in (3.27)), solve this problem. Thus $\sigma_{i h}$ are symmetric with respect to the two imports $m_{i}$ and $m_{h}$, and positive.

There are two limiting cases in the concept. When $m_{i}$ and $m_{h}$ are perfect substitutes $\sigma_{i h}$ tend to infinity, while when $m_{i}$ and $m_{h}$ can not be substituted $\sigma_{i h}=0$.

We also showed that $\sigma_{i h}$ can be obtained from $e_{i h}$ by:

$$
\begin{equation*}
\sigma_{i h}=\left(1 / w_{h}\right) e_{i h} \quad i, h=1, \ldots, n \tag{3.29}
\end{equation*}
$$

which is very convenient mathematically since we can obtain estimates of $\sigma_{i h}$ from $e_{i h}$ and the optimal share $w_{h}$.

From the properties of the determinants, we know that if each element of the 1st row of $F$ is multiplied by the cofactor of the corresponding element of another row, then the sum of the products is zero. That is,

$$
\begin{equation*}
f_{1} F_{h 1}+f_{2} F_{h 2}+\ldots+f_{n h n} F_{h}=0 ; \quad k=1, \ldots, n \tag{3.30}
\end{equation*}
$$

Multiplying both sides of $(3.26)$ by $w_{h}$ yields:

$$
\begin{equation*}
w_{h} \sigma_{i h}=\left(f_{h i h} F_{i h}\right) / m_{i}|F| \text { and hence } \sum_{h} w_{h} \sigma_{i h}=0 \tag{3.31}
\end{equation*}
$$

From the SOC, $|F|$ should be positive definite, that is the elements of $F$ should alternate in sign. Thus, $F_{i h}$ and $|F|$, say, should be opposite in sign. But $\sigma_{i h}$ is proportional to the ratio $F_{i i}$ to $|F|$ and so $\sigma_{i i}<0, i=1, \ldots, n$. As a result,

$$
\begin{equation*}
\Sigma_{i \neq h} W_{h} \sigma_{h 1}>0, \quad i=1, \ldots, n \tag{3.32}
\end{equation*}
$$

This implies that the elasticities can be positive or negative, but the weighted positive elasticities must outweight the negative ones.

### 3.3.3.3). Introducing Market Conditions.

So far we have looked at the cost minimizing decision of the producer importer without considering the total import price $P$ he is faced with (the price at which he sells his product in production theory). The total demand for imports is determined as a function of the total import price index. The importer, under perfect competition, equates $P$ to $A C$ and $M C$ (where $A C$ and $M C$ are equal for a linear production function) in order to determine his optimum. Assuming that the elasticity of demand for total imports is $\eta=-(P / \mu)(d \mu / d P)$, then equilibrium is described by the following equations:

$$
\begin{equation*}
f\left(m_{1}, \ldots, m_{n}\right)=\phi(P) \tag{3.33}
\end{equation*}
$$

$$
\begin{equation*}
\left(p_{1} / f_{1}\right)=\ldots=\left(p_{n} / f_{n}\right)=p \tag{3.34}
\end{equation*}
$$

where $\phi(P)$ is the demand function for total imports. Using (3.33) and (3.34) it can be shown that the own and cross price elasticities of demand are (see for example Allen(1938, p503-509)):

$$
\begin{equation*}
e_{i h}=w_{h}\left(\sigma_{i h}+\eta\right) \quad i, h=1, \ldots, n \tag{3.35}
\end{equation*}
$$

Hence, if the price of imports from partner $\mathfrak{i}$ rises then the demand from this or any other partner is affected in two ways:
a) The overall cost of imports is higher and total imports are dearer. For decreasing demand ( $\eta<0$ ) less are imported and there is a proportional fall in imports from all sources (this is the output effect in production, or the income effect in consumption) as shown by the negative factor $w_{h} \eta$, in (3.35).
b) Demand of imports from partner $\mathfrak{i}$ is reduced relative to the other regions since it is relatively more expensive to import from the relatively more expensive partner. Hence, demand from $h$ is reduced, as shown by $w_{h} \sigma_{h h}$ in (3.35), where $\sigma_{h h}<0$. The effect on the demand for the other regions' imports depends on the sign of the $\sigma_{i h}$ : (i) if $\sigma_{i h}>0$ then demand from $h$ increases and $i$ and $h$ are substitutes, (ii) if $\sigma_{i h}<0$ then demand from $h$ decreases and $i$ and $h$ are complements. Thus, the sign of $\sigma_{i h}$ indicates substitutability or complementarity. In general, substitutes must be more than complements, as we saw from (3.32).

### 3.3.3.4). Extension of Results to CRESH.

For the CRESH model, the values of the total, own and cross price elasticities of import demand may be obtained from (3.11) by applying (3.22) and (3.24). We already know that since the CRESH function is linearly homogeneous its total import elasticity is 1. The own and cross import price elasticities are:

$$
\begin{equation*}
e_{i i}=\left(w_{i} a_{i}^{2} / a\right)-a_{i}, \quad i=1, \ldots, n \tag{3.36}
\end{equation*}
$$

and

$$
\begin{equation*}
e_{i h}=a_{i} w_{h} a_{h} / a, \quad i \neq h=1, \ldots, n \tag{3.37}
\end{equation*}
$$

where $a=\Sigma_{j} w_{j} a_{j}$ is $a$ weighted average of the $a_{i}^{\prime} s$. The proof of (3.36) and (3.37) is relegated to appendix 3.1.

The own and cross partial Allen elasticities of substitution for the CRESH import allocation model can also be derived by using (3.36) and (3.37) in (3.29). Thus,

$$
\begin{equation*}
\sigma_{i i}=\left(a_{i}^{2} / a\right)-\left(a_{i} / w_{i}\right) \tag{3.38}
\end{equation*}
$$

and

$$
\begin{equation*}
\sigma_{i h}=\left(a_{i} a_{h} / a\right) \tag{3.39}
\end{equation*}
$$

### 3.3.3.5). Second Order Conditions(SOC).

In order to verify that the solution of the Lagrangean problem defined in (3.13) gives a minimum, we need to check the SOC. The SOC require that the bordered Hessian $F$ defined in (3.21)/(3.21') is negative semi definite, or equivalently that the matrix with element $i, h$, equal to $\partial m_{i} / \partial p_{h}$, is negative semi definite. But from the definition of $\sigma_{i h}$ in (3.27) we observe that we can equivalently write the SOC in terms of the $(n \times n)$ matrix $\Sigma=\left[\sigma_{i h}\right]$. Thus [ $\left.\sigma_{i n}\right]$ must be negative semi definite for a minimum, which in turn requires the principal minors of $\left[\sigma_{i h}\right]$ to alternate in sign (see for example Allen (1938, p 505)). Thus,

$$
(-1)^{m}\left|\Sigma_{m}\right| \geq 0, \quad m=1, \ldots, n-1
$$

and

$$
\left|\Sigma_{n}\right|=0
$$

where $\left|\Sigma_{m}\right|$ is any principal minor determinant of $\Sigma$ of order $m$

For the CRESH model, using (3.38) and (3.39) in (3.40) yields:

$$
\begin{equation*}
(-1)^{m}\left|\sum_{m}\right|=\left(\prod_{i=1}^{m} a_{i} / w_{i}\right)\left(1-\sum_{i=1}^{m}\left(w_{i} a_{i} / a\right)\right) \tag{3.41}
\end{equation*}
$$

Thus, (3.40) becomes:

$$
\begin{equation*}
\left(\Pi_{i=1}^{m} a_{i}\right)\left(1-\sum_{i=1}^{m}\left(w_{i} a_{i} / a\right)\right) \geq 0 \tag{3.42}
\end{equation*}
$$

This condition excludes the possibility of more than one $a_{1}<0$, (Hanoch(1971, p700), Italianer(1986, p175-178)).
When one $a_{1}<0$, let us say $a_{1}$, while all others are positive, then $m_{1}$ is a substitute for all the other imports, which form a set of complements.

When all $a_{1}>0$, (3.42) always holds, in which case all bilateral imports are substitutes, since $\sigma_{i h}>0, \forall i \neq h$.

### 3.3.4). Models Nested in CRESH.

As noted earlier, a nice feature of the CRESH aggregator function, presented in (3.6), is that a number of well known models may be obtained as nested functions by placing restrictions on the substitution parameters $\rho_{i}$ (or equivalently on the $a_{i}{ }^{\prime} s$ ). This amounts to placing restrictions on the partial Allen elasticities of substitution, as we can see from (3.38) and (3.39). As a consequence, the bilateral import demand equations of (3.11) and the own and cross price elasticities of import demands of (3.36) and (3.37) alter appropriately. We derive the results below for each nested model.
1). The CES aggregator function may be derived by letting $\rho_{i}=\rho$ (ie $a_{i}=a$ ) in (3.6). Thus (3.6) becomes:

$$
\begin{equation*}
\mu=\left[\sum_{i} \delta_{i} e^{\gamma_{i}^{t}} m_{i}-\rho\right]^{-1 / \rho} \tag{3.43}
\end{equation*}
$$

or equivalently if we let $\rho=(1-\sigma) / \sigma$ (since $a_{i}=1 /\left(1+\rho_{i}\right)$ ),

$$
\begin{equation*}
\mu=\left[\sum_{i} \delta_{i} e^{\gamma_{i}^{t}} m_{i}^{(\sigma-1) / \sigma_{j} \sigma /(\sigma-1)} \quad \sigma=1 /(1+\rho)\right. \tag{3.44}
\end{equation*}
$$

Similarly, the derived bilateral import demand equations (3.11)
become:

$$
\begin{align*}
& m_{i}=\mu\left[\left|\delta_{i} \rho\right| e^{\gamma_{i} t}\left(p_{i} / P\right)^{-1}\right]^{\sigma}  \tag{3.45}\\
& m_{i}=\mu\left[\left|\delta_{i} \rho\right| e^{\gamma_{i} t}\left(p_{i} / P\right)^{-1}\right]^{1 /(1+\rho)} \tag{3.46}
\end{align*}
$$

The own and cross partial Allen elasticities of substitution are obtained from (3.38) and (3.39):

$$
\begin{equation*}
\sigma_{i i}=\sigma\left(1-\left(1 / w_{i}\right)\right) \quad \text { and } \quad \sigma_{i h}=\sigma \tag{3.47}
\end{equation*}
$$

The own and cross price elasticities of import demands are also derived with the use of (3.36) and (3.37):

$$
\begin{equation*}
e_{i i}=\sigma\left(w_{i}-1\right) \quad \text { and } \quad e_{i h}=\sigma w_{h} \tag{3.48}
\end{equation*}
$$

The above special case of the CRESH model is the CES model of Hickman and Lau (1973), outlined in section (2.3.3). It is more restrictive than CRESH since the substitution parameter $\rho$ is constant over trade partners.
2). The fixed-input Leontief model may be derived as the limit of the CRESH model as $\rho_{i} \rightarrow \infty$ (ie $a_{i} \rightarrow 0$ ), or equivalently as the limit of the CES as $\rho \rightarrow \infty$ (ie $\sigma \rightarrow 0)$. This is the limiting case mentioned earlier, when we considered the properties of the elasticities of substitution, where there is no substitutability between imports.

The aggregator function is obtained by considering the limit of (3.43) as $\rho \rightarrow \infty$ :

$$
\begin{aligned}
\lim _{\rho \rightarrow \infty} \mu= & \lim _{\rho \rightarrow \infty}\left\{1 /\left[\sum_{i} \delta_{i} e^{\gamma_{i} t} / m_{i}^{\rho}\right]^{1 / \rho_{1}}\right. \\
= & \lim _{\rho \rightarrow \infty}\left\{1 /\left[\left(\delta_{1} e^{\gamma_{1} t} / m_{1}^{\rho}\right)+\left(\delta_{2} e^{\gamma_{2} t} / m_{2}^{\rho}\right)+\ldots+\left(\delta_{n} e^{\gamma_{n} t} / m_{n}^{\rho}\right)\right]^{1 / \rho}\right. \\
= & \lim _{\rho \rightarrow \infty}\left\{m_{1} /\left[\delta_{1} e^{\gamma_{1} t}+\delta_{2} e^{\gamma_{2} t}\left(m_{1} / m_{2}\right)^{\rho}+\ldots+\delta_{n} e^{\gamma_{n} t}\left(m_{1} / m_{n}\right)^{\rho}\right]^{\left.1 / \rho_{0}\right\}}\right.
\end{aligned}
$$

When $m_{1}<m_{i}$ for $i \neq 1$, then $\lim \left(m_{1} / m_{i}\right)^{\rho}=0$, and since also $\lim _{\rho \rightarrow \infty}\left(\delta_{i} e^{\gamma, t}\right)^{1 / \rho}=1, \quad$ then $\quad \lim _{\rho \rightarrow \infty} \mu$ for $m_{1}<m_{2}, \ldots, m_{n}, \quad$ is $m_{1}$.

$$
\begin{aligned}
& \text { When } m_{2}<m_{1}, m_{3}, \ldots, m_{n} \\
& \lim _{\rho \rightarrow \infty} \mu=\lim _{\rho \rightarrow \infty}\left\{m_{2} /\left[\delta_{1} e^{\gamma_{1} t}\left(m_{2} / m_{1}\right)^{\rho}+\delta_{2} e^{\gamma_{2} t}+\ldots+\delta_{n} e^{\gamma_{n} t}\left(m_{2} / m_{n}\right)^{\rho}\right]^{1 / \rho}\right\}=m_{2}
\end{aligned}
$$

and so on, for the rest of imports. So, the aggregator function for the n importing regions is:

$$
\begin{equation*}
\mu=\min \left\{m_{1}, \ldots, m_{n}\right\} \tag{3.49}
\end{equation*}
$$

The bilateral import demand equations (3.45)/(3.46) become:

$$
\begin{equation*}
m_{i}=\mu \tag{3.50}
\end{equation*}
$$

indicating that imports from partner $\mathfrak{i}$ are not related to prices, but depend solely on the total import requirements $\mu$ of the importing region.

The partial elasticities of substitution are zero, reflecting the non-substitutability between imports from different regions. The own and cross import elasticities are also zero; the demand for imports from any region does not depend on prices.
3). The Cobb-Douglas model can be derived from CRESH when $a_{i}=1\left(\rho_{i}=0\right)$, or $\sigma=1(\rho=0)$ for the CES model. The limit of (3.43) as $\rho \rightarrow 0$ is indeterminate. Taking logs of both sides of (3.43) yields:
$\ln \mu=-(1 / \rho) \ln \left[\sum_{i} \delta_{i} e^{\gamma_{i} t_{i}} m_{i}^{-\rho}\right]=-(1 / \rho) \ln \left[\sum_{i} \delta_{i} e^{\gamma_{i}} e^{-\rho \ln m_{i}}\right]$

By L'Hopital's rule we have the following results:

$$
\lim _{\rho \rightarrow 0} \ln \mu=\lim _{\rho \rightarrow 0}[f(\rho) / g(\rho)]=\lim _{\rho \rightarrow 0}\left[f(\rho)^{\prime} / g(\rho)^{\prime}\right]
$$

where

$$
g(\rho)=\rho \quad \text { and }
$$

$$
f(\rho)=-\ln \left[\sum_{i} \delta_{i} e^{\gamma_{i} t} e^{-\rho \ln m_{i}}\right]
$$

Thus, $g(\rho)^{\prime}=1$, and $f(\rho)^{\prime}=\left[\sum_{i} \delta_{i} e^{\gamma_{i} t} \operatorname{lnm_{i}} e^{-\rho \ln m_{i}}\right] /\left[\sum_{i} \delta_{i} e^{\gamma_{i} t} e^{-\rho \ln m_{i}}\right]$
$\therefore \quad \lim _{\rho \rightarrow 0} \ln \mu=\lim _{\rho \rightarrow 0}\left\{\left[\sum_{i} \delta_{i} e^{\gamma_{i} t}\left(\ln m_{i}\right) m_{i}^{-\rho}\right] /\left[\sum_{i} \delta_{i} e^{\gamma_{i}} m_{i}^{-\rho}\right]\right\}$
$=\left(\sum_{i} \delta_{i} e^{\gamma_{i} t} \ln m_{i}\right) /\left(\Sigma_{i} \delta_{i} e^{\gamma_{i} t}\right)=\ln \left(\prod_{i} m_{i} \delta_{i} e^{\gamma_{i}}\right) /\left(\Sigma_{i} \delta_{i} e^{\gamma_{i}}\right)$
Using the normalization $\Sigma_{i} \delta_{i} e^{\gamma_{i}^{t}}=1$, we have:

$$
\lim _{\rho \rightarrow 0} \ln \mu=\ln \left(\prod_{i} m_{i} \delta_{i} e^{\gamma_{i} t}\right)
$$

Hence,

$$
\begin{equation*}
\mu=\prod_{i} m_{i} \delta_{i} e^{\gamma_{i} t} \tag{3.51}
\end{equation*}
$$

The partial Allen elasticities of substitution are $\sigma=1$, and the import elasticities of demand are:

$$
\begin{equation*}
e_{i 1}=\left(w_{i}-1\right) \quad \text { and } \quad e_{i h}=w_{h} \tag{3.52}
\end{equation*}
$$

4). The Linear Expenditure System (LES) is derived when
$a_{i} \rightarrow \infty\left(\rho_{1} \rightarrow-1\right)$ for the CRESH or when $\sigma \rightarrow \infty$ for the CES model. This is the other limiting case of the elasticities of substitution. Here, there is perfect substitutability between the imports of two regions.

The LES aggregator function is derived by letting $\rho=-1$ in (3.43).

$$
\begin{equation*}
\mu=\sum_{i} \delta_{i} e^{\gamma_{i} t} m_{i} \tag{3.53}
\end{equation*}
$$

Both the partial elasticities of substitution and the import demand elasticities tend to infinity, reflecting the perfect substitutability between the imports of different trade partners. If the price of imports from a partner is lower than the prices of
other partners, then all the imports in the importing region would come from the partner with the lowest price. The result is a corner solution.
3.3.5). Extensions of the simple CRESH model.

The CRESH aggregator function (3.6) used to derive our import allocation model is describing the 'technical' relationship between total imports of a region and its trade partners, assuming a producer importer. For a consumer importer, the aggregator function describes the relationship between total imports and the preferences of the importer. In either case, the simplicity or complexity of these relationships depends on the factors included in the aggregator function. Thus, imports and a time trend are included in (3.6). These factors are reflected later in the derived demand equations of the optimizing importer, as can be seen from the time trend, total imports and relative prices that appear in (3.11).

In this section we extend the CRESH aggregator function to incorporate factors, other than relative prices and time trends, which might be important in the optimizing behaviour of the importer. Such factors could reflect demand or supply elements, and they are often, in the international markets, as important as relative prices in the allocation of imports amongst trade partners.

In international trade, information is an important element in the decision to import from a particular region rather than some other one. An exporter might be better known in an import market if he sells a variety of products, thus establishing a name in the market. This is achieved easier by countries with large potential capacities, relative to other regions. Also, one of the important factors in international trade is the marketing of a product. Economies of scale are very important in marketing internationally, and it is expected that such economies show more in countries with higher capacity. Thus, the production capacity of an exporting region relative to that of its competitors is, expected to affect
positively its bilateral exports in the jth import market.

Another factor that reflects the competitive position of exporters in an import market, is their ability to fulfil the import requirements of the importer, relative to their competitors. Thus, if production is at full employment in a certain region delivery times might increase, sales promotions might fall and also the ability to meet export requirements might fall because of domestic demand pressure. The competitive position of such an exporter with high degree of capacity utilization (or equivalently, with high demand pressure) would deteriorate relative to its competitors in a certain importing market.

We may incorporate such non-price factors in the CRESH model by introducing extra terms in (3.6). A relative capacity (potential output) and/or a relative capacity utilization (demand pressure) factor may thus be included in (3.6). Let $\psi_{i}$ denote the index of exporting capacity of region $i$, and $\Psi$ the corresponding CRESH 'composite' capacity index of all trade partners. Similarly, let $\phi_{i}$ be an index of the pressure of demand for region $i$, while $\Phi$ is the corresponding 'composite' CRESH index over all trade partners. Then (3.6), the CRESH aggregator function becomes:
$F(\mu, \underline{m}, t, \Psi, \Psi, \Phi, \phi)=\sum_{i} \delta_{i} e^{\gamma_{i} t}\left(m_{i} / \mu\right)^{-\rho_{i}}\left(\psi_{i} / \Psi\right)^{+\zeta_{i}}\left(\phi_{i} / \Phi\right)^{-\theta_{i}}-1 \equiv 0$
where $m, \psi$ and $\phi$ are ( $1 \times n$ ) vectors. The same regularity conditions as with (3.6) are assumed to hold here too.

We eliminate for simplicity the pressure of denand variable $\phi$, in order to consider the solution of the problem of the optimizing importer. The results are then easily extended to functions with more variables. (3.54) becomes:

$$
\begin{equation*}
F(\mu, \underline{m}, t, \Psi, \Psi)=\sum_{i} \delta_{i} e^{\gamma_{i}^{t}}\left(m_{i} / \mu\right)^{-\rho_{i}}\left(\psi_{i} / \Psi\right)^{+\zeta_{i}}-1 \equiv 0 \tag{3.56}
\end{equation*}
$$

The cost minimizing importer wants to min $\sum_{i} m_{i} p_{i}$ s.t. (3.56). This amounts to minimizing the following Lagrangean:

$$
\begin{equation*}
\min L=\sum_{i} p_{i} m_{i}+\lambda\left[1-\sum_{i} \delta_{i} e^{\gamma_{i} t}\left(m_{i} / \mu\right)^{-p_{i}}\left(\psi_{i} / \Psi\right)^{+\zeta_{i}}\right] \tag{3.57}
\end{equation*}
$$

The F.O.C. for minimizing (3.57) are:

$$
\begin{gather*}
\partial L / \partial m_{i}=p_{i}+\lambda \delta_{i} \rho_{i} e^{\gamma_{i} t}\left(m_{i} / \mu\right)^{-\rho_{i}} m_{i}^{-1}\left(\psi_{i} / \Psi\right)^{+\zeta_{i}}=0 ; \quad 1=1, \ldots, n  \tag{3.58}\\
\partial L / \partial \lambda=\sum_{i} \delta_{i} e^{\gamma, t}\left(m_{i} / \mu\right)^{-\rho_{i}}\left(\psi_{i} / \Psi\right)^{+\zeta_{i}}-1 \equiv 0 \tag{3.59}
\end{gather*}
$$

Since the elasticity of $m_{1}$ with respect to $\mu$ is 1 , we can write:

$$
\begin{equation*}
|\lambda|=\mu P\left(p_{1}, \ldots, p_{n}, \psi_{1}, \ldots, \psi_{n}, t\right) \tag{3.60}
\end{equation*}
$$

where $P($.$) is a price index which is a real positive function of the$ bilateral prices $P_{i}$, capacity outputs $\psi_{i}$, and time $t$.

Solving (3.58) for $m_{i}$ and eliminating $\lambda$ by using (3.60) we get the system of bilateral import demand equations at a given $\mu$.

$$
\begin{equation*}
m_{i}=\mu\left[\left|\delta_{i} \rho_{i}\right| e^{\gamma_{i} t}\left(p_{i} / P\right)^{-1}\right]^{a_{i}}\left(\psi_{i} / \Psi\right)^{+\zeta_{i}}, \quad a_{i}=1 /\left(1+\rho_{i}\right) \tag{3.61}
\end{equation*}
$$

where the CRESH price and capacity-output indices ( $P, \Psi$ ) are defined implicitly, by substituting (3.61) in the budget constraint, equation (3.59), resulting in:
$G\left(P, P_{i}, \Psi, \psi_{i}, t\right)=$
$=\sum_{i} \delta_{i}\left[\left|\delta_{i} \rho_{i}\right| e^{\left(-\gamma_{i} / \rho_{i}\right) t}\left(p_{i} / P\right)^{-1}\right]^{-a_{i} \rho_{i}}\left(\psi_{i} / \Psi\right)^{+\zeta_{i}\left(1-\rho_{i}\right)}-1 \equiv 0$

Because of the way we defined the CRESH 'composite' price and capacity-output indices $P$ and $\Psi$ in (3.62), the system still has the property of consistent aggregation and adding up. That is, $\mu, P$ and $\Psi$ are defined so that the additive price aggregation property P. $\mu=$ $\sum_{i} m_{i} p_{i}=M$ holds.

It is a simple matter to extend the above framework, to
incorporate more relevant factors (such as the relative pressure-of-demand of a region) in the decision function of the optimizing importer. Incorporating the relative pressure of demand in the aggregator function, and solving the problem of the optimizing importer yields the following bilateral import demand equations:
$m_{1}=\mu\left[\left|\delta_{1} \rho_{1}\right| e^{\gamma_{1} t}\left(p_{1} / P\right)^{-1}\right]^{a_{i}}\left(\psi_{i} / \Psi\right)^{+\zeta_{1}}\left(\phi_{1} / \Phi\right)^{-\theta}, \quad a_{1}=1 /\left(1+\rho_{i}\right)$
where the CRESH 'composite' price, capacity-output and pressure-of-demand indices ( $P, \Psi$ and $\Phi$ ) which ensure adding up, are defined implicitly in the following:
$G\left(P, P_{i}, \Psi, \psi_{i}, \phi_{i}, \Phi, t\right)=$
$=\sum_{i} \delta_{i}\left[\left|\delta_{i} \rho_{i}\right| e^{\left(-\gamma_{i} / \rho_{i}\right) t}\left(\rho_{i} / P\right)^{-1}\right]^{-a_{i} \rho_{i}}\left[\left(\psi_{i} / \Psi\right)^{+\zeta_{i}}\left(\phi_{1} / \Phi\right)^{-\theta_{i}}\right]^{\left(1-\rho_{1}\right)}-1 \equiv 0$

The pressure of demand variable for some exporting region $\mathfrak{i}$ is measured by the ratio of actual output to capacity output. Let us denote this ratio for region $i$ by $q_{i} / \psi_{i}$, and the corresponding CRESH ratio in some importing region $j$ by $Q / \Psi$, where $Q$ is again defined implicitly in a similar way to $\Psi$. Let us further take the last two terms of (3.63), which become now:

$$
\begin{equation*}
\left(\psi_{i} / \Psi\right)^{+\zeta_{i}}\left[\left(q_{i} / \psi_{i}\right) /(Q / \Psi)\right]^{-\theta_{i}} \tag{3.65}
\end{equation*}
$$

In the long run, we have full employment and $q_{1}=\psi_{1}$. The last term in the above becomes unity, and we go back to model (3.61).

Since the estimates of $\zeta_{i}$ and $-\theta_{i}$ are close to unity, (3.65) reduces to $q_{i} / Q$, and the import demand equations become:

$$
\begin{equation*}
m_{i}=\mu\left[\left|\delta_{i} \rho_{i}\right| e^{\gamma_{i} t}\left(p_{i} / P\right)^{-1}\right]^{a_{i}}\left(q_{i} / Q\right), \quad a_{i}=1 /\left(1+\rho_{i}\right) \tag{3.66}
\end{equation*}
$$

which is the gravity model.

In the short run capacity is fairly constant. That is, $\left(\psi_{i} / \Psi\right)=K$. In this case the import demand function becomes:

$$
\begin{equation*}
m_{i}=\mu\left[\left|\delta_{1} \rho_{1}\right| e^{\gamma_{i} t}\left(p_{1} / P\right)^{-1}\right]^{a_{1}} K\left(q_{1} / Q\right)^{-1}, \tag{3.67}
\end{equation*}
$$

Thus, the model behaves initially opposite to that of a gravity model.

The aim of this chapter has been to develop an estimation consistent import allocation model, which can be turned into an empirical model. To achieve this, we have applied to the theory of import allocation models the ideas of multistage budgeting. Multistage budgeting allowed us to break the problem of the consumer-producer importer into smaller, more manageable sub-problems. In order to apply such a problem 'consistently' we had to select an aggregator function that satisfied the criteria of consistent aggregation and adding up.

Our final import allocation model is based on the CRESH function. We have adopted this function since it satisfies the conditions necessary for the existence of a consistent import allocation model, and is 'general enough' in terms of the substitution possibilities it allows between different trade partners. The CRESH function leads to a model with 'normal' looking bilateral import demand equations, for each category of goods, derived by using neoclassical optimization techniques. Economically, the important thing is the differing relative price effect between trade partners in the model. Empirically, the important thing is the high level of disaggregation we may achieve with such a specification, which is our goal in terms of our world seaborne trade model. Furthermore, The CRESH model encompasses most of the other import allocation models used in the literature, as nested models which appear when restrictions are placed on the substitution parameters between the trade partners.

## Appendix 3.1.

The CRESH Import Elasticities of Demand.
The import elasticities of demand for the CRESH model are easiest derived by taking logs of (3.11), yielding:

$$
\begin{equation*}
\ln m_{i}=\ln \mu+a_{i} \ln \left(\left|\delta_{i} \rho_{i}\right|\right)+a_{i} \gamma_{i} t-a_{i} \ln \left(p_{i} / P\right) \tag{1}
\end{equation*}
$$

From the definition of the own and cross import elasticities of demand of (3.24) we have:

$$
\begin{align*}
e_{i 1} \equiv \partial \ln m_{i} / \partial \ln p_{i} & =-a_{i}\left(P / p_{i}\right)\left[\partial\left(p_{i} / P\right) / \partial p_{i}\right]  \tag{i}\\
& =-a_{i}\left(P / p_{i}\right)\left\{\left[P-p_{i}\left(\partial P / \partial p_{i}\right)\right] / P^{2}\right\}
\end{align*}
$$

At the base year values $(t=0)$, prices are 1 . The above then reduces to:

$$
\begin{equation*}
\left.e_{i i}\right|_{t=0}=-a_{i}\left[1-\left.\left(\partial P / \partial p_{i}\right)\right|_{t=0}\right] \tag{2}
\end{equation*}
$$

From equation (32), Appendix 4.1 we have:

$$
\begin{equation*}
\left.\left(\partial P / \partial p_{i}\right)\right|_{t=0}=\left(w_{i}^{0} a_{i}\right) /\left(\sum_{h} w_{h}^{0} a_{h}\right) \tag{3}
\end{equation*}
$$

which, when used in (2) yields:

$$
\begin{equation*}
\left.e_{i 1}\right|_{t=0}=\left(w_{i}^{0} a_{i}^{2} / a\right)-a_{i} \tag{4}
\end{equation*}
$$

The latter is (3.36) at the base year values.
(ii) Similarly, $e_{i h} \equiv \partial \ln m_{i} / \partial \ln p_{h}=-a_{i}\left(P / p_{i}\right)\left[\partial\left(p_{i} / P\right) / \partial p_{h}\right]$

$$
\begin{aligned}
& =a_{i}\left(P / p_{i}\right)\left(p_{i} / P^{2}\right)\left(\partial P / \partial p_{h}\right) \\
\therefore \quad & \left.\quad e_{i h}\right|_{t=0}=\left.a_{i}\left(\partial P / \partial p_{h}\right)\right|_{t=0}
\end{aligned}
$$

using $\left.\quad\left(\partial P / \partial p_{h}\right)\right|_{t=0}=\left(w_{h}^{0} a_{h}\right) /\left(\Sigma w_{j} a_{j}\right)$ we get (3.37) at the base year values:

$$
\begin{equation*}
\left.\therefore \quad e_{i h}\right|_{t=0}=a_{i} a_{h} w_{h}^{0} / a \tag{5}
\end{equation*}
$$

## ECONOMETRIC SPECIFICATION

## 4.0). Introduction.

In the previous chapter we derived a set of nonlinear, but theoretically consistent, bilateral import demand equations in (3.11). The implicit CRESH indices $\mu$ (the aggregator function) and $P$ (the composite price index) have to be defined empirically in order to be able to estimate the equations. In this chapter we use first order Taylor approximations to find analytical solutions to this problem. At the same time the nonlinearities in the system are reduced considerably, a result that helps in the computational solution of the system. The non-availability of data on bilateral seaborne import prices force us to reformulate the model.

This chapter is in six main sections. In the first, we consider the Taylor linearization of equations (3.11). A similar approximation is also derived for equations in logarithms and logarithmic first differences. In the second, we introduce dynamics and we discuss a sequence of nested models, created by restrictions on the dynamic mechanisms. In the third, we further linearize the system in order to able to identify all the structural parameters, and also arrive at a more easily estimable system. In the fourth section, we consider restrictions placed on the model by the non-availability of bilateral import prices for each commodity. Proxies are introduced and a framework for testing the restricted models and the functional form is suggested. In the fifth section, we consider some estimation problems which arise due to the introduction of these proxies. The concept of the bilateral import price elasticities is also altered, and the new price elasticities arising are derived here. In the sixth section, we examine a number of nested models (CES, Cobb-Douglas, Linear Expenditure System, Leontief fixed factors models) by placing restricions on the
parameters of the system. This reflects restrictions on the initial preference/technology structure of the optimizing trade partners.

## 4.1). Linearizing the Import Allocation Model.

The bilateral import demand equations resulting from the classical optimization problem of the importer were derived in the previous chapter in equations (3.11), repeated here for convenience.

$$
\begin{equation*}
m_{i}=\mu\left[\left|\delta_{i} \rho_{i}\right| e^{\gamma_{i} t}\left(p_{i} / P\right)^{-1}\right]^{a_{i}}, \quad a_{i}=1 /\left(1+\rho_{i}\right) \tag{4.1}
\end{equation*}
$$

These equations are nonlinear and the CRESH indices $\mu$ (the aggregator -utility or production- function) and $P$ (the composite price index) are unobservable. These problems must be overcome in order to be able to turn the system into an empirical form.

We may eliminate $\mu$ by summing both sides of (4.1) over the exporting regions $\mathfrak{i}$ to get:

$$
\Sigma_{i} m_{i}=\mu \Sigma_{i}\left[\left|\delta_{i} \rho_{i}\right| e^{\gamma_{i} t}\left(p_{i} / P\right)^{-1}\right]^{a} \quad \text { where } \quad \sum_{i} m_{i} \equiv m
$$

Solving this equation for $\mu$ and substituting back into (4.1) eliminates the unobservable $\mu$, giving:

$$
\begin{aligned}
& m_{i}=m\left[\left|\delta_{i} \rho_{i}\right| e^{\gamma_{i}^{t}}\left(p_{i} / P\right)^{-1}\right]^{a}\left\{\sum_{k}\left[\left|\delta_{k} \rho_{k}\right| e^{\gamma_{k}^{t}}\left(p_{k} / P\right)^{-1}\right]^{a_{k}}\right\}^{-1} \quad \text { (4.2) } \\
& \text { It may be easily verified that adding up is satisfied by summing } \\
& \text { up both sides of }(4.2) \text {. }
\end{aligned}
$$

If we can further make the assumption that the combined price index $P$ is a fixed weighted average of the bilateral import prices $P_{1}$, where the weights are, say, the base year import shares $W_{i}^{0}=\left(m_{i}^{0} / m^{0}\right)$, then we can write $P=\sum_{i} W_{i}^{0} p_{i}$ as the empirical counterpart of $P$ in (4.2).

All the variables in (4.2) are now defined empirically but we are faced with the estimation of a highly nonlinear system of equations. Estimating such a system may be achieved by a number of methods, cfr Maddala(1977, p 171-174). The Taylor approximation around some set of values seems to be the most fruitful answer empirically. This involves linearizing the set of equations around some initial parameter values chosen in an iterative procedure. For practical purposes, when dealing with large models, convergence might be computationally expensive and often not possible. As a result, it is desirable to derive an analytical solution to the linearization problem, reduce the system to a more linear version, and let further linearizations if necessary be achieved computationally.

Following Italianer(1986), we propose three different approximations to (4.1), which eliminate the unobservable $\mu$, solve the problem of defining the price index $P$ (in a non ad-hoc manner), and at the same time linearize the import allocation model. These approximations involve variables in levels, logarithms and logarithmic first differences.

### 4.1.1). Variables in Levels.

When the unobserved import quantity index $\mu$ is eliminated we obtain equation (4.2), with the composite price index $P$ still to be defined for empirical purposes. A first order Taylor expansion of (4.2) around the base year values, say, $m=m^{0}, t=0$ and $p_{i}=1, \forall i$ gives. (The proof of this is relegated to appendix 4.1).

$$
\begin{equation*}
m_{i}=w_{i}^{0} m+m_{i}^{0} c_{i} t-m_{i}^{0} a_{i}\left[p_{i}-\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) p_{h}\right], \quad \forall i=1, \ldots, n, \tag{4.3}
\end{equation*}
$$

where

$$
\begin{equation*}
\left.c_{i}=a_{i}\left[\gamma_{i}+\Sigma_{h}\left(w_{h}^{0} a_{h} / a^{0}\right)\left(\gamma_{h} / \rho_{h}\right)\right]-\Sigma_{h} w_{h}^{0}\left(\gamma_{h} / \rho_{h}\right)\right], \quad \forall i=1, \ldots, n, \tag{4.4}
\end{equation*}
$$

and $a^{0}$ is a weighted average of the $a_{i}{ }^{\prime} s$

$$
\begin{equation*}
a^{0}=\sum_{h} w_{h}^{0} a_{h} \tag{4.5}
\end{equation*}
$$

and $w_{i}^{0}=\left(m_{i}^{0} / m^{0}\right)$ are the base year import shares with the property that $\sum_{i} W_{i}^{0}=1$.

From (4.4) we can see that $\Sigma_{i} w_{i}^{0} c_{1}=0$, and as a consequence $\Sigma_{i} m_{1}^{0} c_{1}=0$. Summing both sides of (4.3) over $\mathfrak{i}$ gives,

$$
\Sigma_{i} m_{i}=\Sigma_{i} w_{i}^{0} m+\sum_{i} m_{i}^{0} c_{i} t-\Sigma_{i} m_{i}^{0} a_{i} p_{i}+\sum_{i} m_{i}^{0} a_{i}\left(1 / \Sigma w_{j}^{0} a_{j}\right)\left(1 / m^{0}\right) \Sigma_{h} m_{h}^{0} a_{h} p_{h}
$$

which, with the use of $\Sigma_{i} m_{i}^{0} c_{1}=0$, results in $\Sigma_{i} m_{i}=m$. That is, adding up is automatically satisfied. Thus, attaching an error term to (4.3) and estimating the system of equations will guarantee adding up by construction. On the other hand, defining $P$ arbitrarily by $P=\sum_{i} W_{i}^{O} P_{i}$, as suggested earlier, fails to meet adding up.

### 4.1.2). Variables in Logarithms.

Transforming the model to log-linear form by taking logarithms of both sides of (4.2) violates adding-up. The alternative is to proceed from (4.1) by taking logs of both sides, giving:

$$
\begin{equation*}
\ln m_{i}=\ln \mu+a_{i}\left[\ln \left|\delta_{i} \rho_{i}\right|+\gamma_{i} t-\ln \left(p_{i} / P\right)\right] ; i=1, \ldots, n \tag{4.6}
\end{equation*}
$$

where the corresponding CRESH price index $\ln P$, which guarantees consistency of aggregation, is now defined implicitly (through the first order conditions derived in chapter 3) by:

$$
\begin{equation*}
\Sigma_{i} \delta_{i} \exp \left\{a_{i}\left[-\rho_{i}\left[\ln \left|\delta_{i} \rho_{i}\right|+\gamma_{i} t+\rho_{i} \ln \left(p_{i} / P\right)\right]\right\}-1 \equiv 0\right. \tag{4.7}
\end{equation*}
$$

Now assume that the import quantity index $\bar{m}$ is defined as a weighted geometric average of the bilateral imports with weights the base year values.

$$
\begin{equation*}
\bar{m}=\prod_{i} m_{i}^{w_{i}^{0}} \quad \text { that is } \quad \ln \bar{m}=\sum_{i} w_{i}^{0} \ln m_{i} \tag{4.8}
\end{equation*}
$$

Multiplying both sides of (4.6) by $w_{i}^{0}$, summing over $\mathfrak{i}$ and solving for $\ln \mu$ gives,

$$
\ln \mu=\ln \bar{m}-\Sigma_{i}\left\{w_{i}^{0} a_{i}\left[\ln \left|\delta_{i} \rho_{i}\right|+\gamma_{i} t-\ln \left(p_{i} / P\right)\right]\right\}
$$

This may be substituted into (4.6) to eliminate $\ln \mu$, resulting in:

$$
\begin{equation*}
\ln m_{i}=\ln \bar{m}+\ln \eta_{i}+\bar{c}_{i} t-a_{i} \ln \left(p_{i} / P\right)+\sum_{h} w_{h}^{0} a_{h} \ln \left(p_{h} / P\right) \tag{4.9}
\end{equation*}
$$

where

$$
\begin{gathered}
\bar{c}_{i}=a_{i} \gamma_{i}-\sum_{h} w_{h}^{0} a_{h} \gamma_{h} \\
\ln \eta_{i}=a_{i} \ln \left|\delta_{i} \rho_{i}\right|-\Sigma_{h} w_{h}^{0} a_{h} \ln \left|\delta_{i} \rho_{i}\right|
\end{gathered}
$$

Equation (4.9) in the base year, when $t=0, m_{i}=m_{i}^{0}, \bar{m}=\bar{m}$ and $p_{i}=1$, becomes:

$$
\ln m_{i}^{0}=\ln \bar{m}^{-0}+\ln \eta_{i}
$$

Defining the share $\bar{w}_{1}=m_{i} / \bar{m}$, we observe that $\eta_{i}=\bar{w}_{i}^{0}$.
Hence (4.9) reduces to:

$$
\begin{equation*}
\ln m_{i}=\ln \left(\bar{m} \bar{w}_{i}^{0}\right)+\bar{c}_{i} t-a_{i} \ln \left(p_{i} / P\right)+\sum_{h} w_{h}^{0} a_{h} \ln \left(p_{h} / P\right) \tag{4.10}
\end{equation*}
$$

as a result $\ln \mu$ does not appear in (4.10) and $\ln P$ has still to be defined for empirical purposes. A first order Taylor expansion of (4.10) around the base year values, $t=0, \quad \ln \bar{m}=\ln \bar{m}^{-0}$ and $\ln p_{i}=0$ yields:

$$
\begin{equation*}
\ln m_{i}=\ln \left(\bar{w}_{i}^{0} \bar{m}\right)+c_{i} t-a_{i}\left[\ln p_{i}-\Sigma_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) \ln p_{h}\right] \tag{4.11}
\end{equation*}
$$

where the trend parameters $c_{i}$ are defined in (4.4) and satisfy the condition $\sum_{i} W_{i}^{0} C_{i}=0$.

Multiplying both sides of $(4.11)$ by $w_{i}^{0}$ and summing over $\mathfrak{i}$ yield:

$$
\sum_{i} w_{i}^{0} \ln m_{i}=\sum_{i} w_{i}^{0} \ln \left(w_{i}^{-0}\right)+\sum_{i} w_{i}^{0} c_{i} t-\sum_{i} w_{i}^{0} a_{i} \ln p_{i}+\sum_{i} w_{i}^{0} a_{i}\left(1 / a^{0}\right) \sum_{h} w_{h}^{0} a_{h} \ln p_{h}
$$

Since $\Sigma_{i} w_{i}^{0} \ln \left(\bar{w}_{i}^{-0} m\right)=\Sigma_{i} w_{i}^{0} \ln m_{i}$ and $\Sigma_{i} w_{i}^{0} c_{i}=0$, the adding up restrictions are satisfied. That is, $\quad \Sigma_{i} W_{i}^{0} \ln m_{i}=\ln \bar{m}$.
4.1.3). Variables in Logarithmic First Differences.

An alternative form of the system may be obtained by taking the first differences of the variables in (4.11), yielding:
$\Delta \ln m_{i t}=\Delta \ln \left(\bar{w}_{i}^{0} \bar{m}_{t}\right)+c_{i} \Delta t-a_{i}\left[\Delta \ln p_{i t}-\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) \Delta \ln p_{h t}\right]$
where the operator $\Delta$ for some variable $x$, say, is $\Delta x_{t}=x_{t}-x_{t-1}$ and $\Delta t=1$. Adding $u p$ in the above equation is satisfied provided both sides of the equation are multiplied by $w_{1}^{0}$ and summed over $i$, and the first term on the right hand side (total imports) is $\sum_{i} W_{i}^{0} \Delta \ln m_{i t}$. The latter is the first difference form of the logarithm of the total imports variable, defined in (4.8) as a weighted geometric average of the bilateral imports $m_{i}$. Thus, rewriting (4.12) using $\Delta \ln \tilde{m}_{t} \equiv \Sigma_{i} W_{i}^{0} \Delta \ln m_{i t}$ instead of $\Delta \ln \left(\bar{w}_{i}^{0} \bar{m}_{t}\right)$ yields:

$$
\begin{equation*}
\Delta \ln m_{i t}=\Delta \ln \tilde{m}_{t}+c_{i}-a_{i}\left[\Delta \ln p_{i t}-\Sigma_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) \Delta \ln p_{h t}\right] \tag{4.13}
\end{equation*}
$$

We can see below that when (4.13) is multiplied through by $w_{1}^{0}$ and summed over $i$, it satisfies adding up:
$\sum_{i} W_{i}^{0} \Delta \ln m_{i t}=\sum_{i} w_{i}^{0} \Delta \ln \tilde{m}_{t}+\sum_{i} W_{i}^{0} c_{i}-\sum_{i} W_{i}^{0} a_{i} \Delta \ln p_{i t}+\sum_{i} W_{i}^{0} a_{i}\left(1 / a^{0}\right) \sum_{h} W_{h}^{0} a_{h} \Delta \ln p_{h t}$
where $\sum_{i} w_{i}^{0} c_{i}=0$, is used.

Alternatively, we may arrive at (4.13), with a little algebra, from the first order conditions (equations (3.8) and (3.9), derived in chapter 3 of the optimization problem of the importer. (4.13) resembles the well known Rotterdam model of consumer demand, Barten (1967).

The static equations defined by (4.3) and (4.11) and the dynamic equation defined by (4.13) can be written in terms of differences in growth rates, between bilateral imports from partner $\mathfrak{i}$ and total imports, with respect to the base year. Since the coefficients of the total trade variables is 1 (a property of the underlying CRESH aggregator function), we obtain respectively:

$$
\begin{align*}
& \left(m_{i}-w_{i}^{0} m\right)_{t} / m_{i}^{0}=c_{i} t-a_{i}\left[p_{i t}-\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) p_{h t}\right] \\
& \ln \left(m_{i} / w_{i}^{0-}\right)_{t}=c_{i} t-a_{i}\left[\ln p_{i t}-\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) \ln p_{h t}\right] \\
& \Delta \ln \left(m_{i} / \tilde{m}_{t}=c_{i}-a_{i}\left[\Delta \ln p_{i t}-\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) \Delta \ln p_{h t}\right]\right.
\end{align*}
$$

We may write the above specifications as the single equation with a stochastic term $\varepsilon_{i t}$ :

$$
\begin{equation*}
y_{i t}=c_{i} T-a_{i}\left[L P_{i t}-\Sigma_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) L P_{h t}\right]+\varepsilon_{i t} \tag{4.14}
\end{equation*}
$$

where the new variables $y_{i t}$, $T$, and $L P$ it are defined appropriately. It is easily checked that adding up holds in (4.14) in the sense of $\Sigma_{i} w_{i}^{0} y_{i t}=0$. Estimates of the 'structural' parameters $a_{i}$ and $c_{i}$ may be obtained directly from the coefficients of the estimated equations. Thus, price elasticities of demand and elasticities of substitution may be estimated through the $a_{i}$ 's (using equations (3.36)-(3.39)).

The $\gamma_{i}$ parameters represent trend shifts in imports for country $i$. Overall, trend shifts away from one partner are allocated to the rest of the trade partners. On average, over all trade partners, trends should cancel each other out. Thus, $\Sigma_{i} w_{i}^{0} \gamma_{i}=0$. As will be seen later, the adding up requirement enables only $(n-1)$ of the $c_{i}^{\prime} s$ to be estimated since one of the equations has to be deleted for estimation. As a result, given estimates for the $c_{i}$ and the $a_{i}$ parameters, the $n \quad \gamma_{i}^{\prime} s$ may be estimated from the ( $n-1$ ) equations (4.4), and $\Sigma_{i} w_{i}^{0} \gamma_{i}=0$ being the last equation needed.
4.2.1). Simple One Period Lags and Two Possible Variants.

The specification of the models represented by (4.14) are overly simple. However, dynamics and time lags between changes in variables that determine trade flows and the flows themselves are often important in international trade. Lags may be important because of imperfect information in the market, especially when importing from markets which are geographically and culturally distant. Production and delivery lags are important for certain categories of products. Random, one-off, events (such as the oil crisis) also have a role to play. We may thus extend the static model to include simple one period lags. Assuming common lag coefficients, $\lambda, \xi$, between trade partners write (4.14) as:

$$
\begin{align*}
y_{i t} & =c_{1} T-a_{1}\left[L P_{i t}-\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) L P_{h t}\right]+\lambda y_{i t-1}  \tag{4.15}\\
& +\xi\left\{c_{i}(T-1)-a_{i}\left[L P_{i t-1}-\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) L P_{h t-1}\right]\right\}+\varepsilon_{i t}
\end{align*}
$$

By placing restrictions on the above simple dynamic structure we may obtain a number of economically meaningful models:
a). When $(1+\xi)\left(c_{1}-a_{1}\right)=1$ we have the error correction model:

$$
\begin{align*}
\Delta y_{i t} & =\left\{c_{i} \Delta T-a_{i}\left[\Delta L P_{i t}-\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) \Delta L P_{h t}\right]\right\}  \tag{4.16}\\
& +(1-\lambda)\left\{\left[(T-1)-\left(L P_{i t-1}-\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) L P_{h t-1}\right)\right]-y_{i t-1}\right\}+\varepsilon_{i t}
\end{align*}
$$

This equation indicates that $y_{i t}$ is obtained from $y_{1 t-1}$ by adding a random shock $\varepsilon_{i t}$, a portion of the change in $\left\{T-\left[L P_{i}-\Sigma_{h}\left(w_{h} a_{h} / a^{0}\right) L P_{h}\right]\right\}$, and a portion of the deviation from the equilibrium value of the previous period, $\left(\left\{(T-1)-\left[L P_{i t-1}-\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) L P_{h t-1}\right]\right\}=y_{i t-1}\right)$. Despite its nice economic interpretation, adding up does not hold. We can see that by multiplying through by the base year import share and summing over i:

$$
\begin{aligned}
\sum_{i} w_{i}^{0} \Delta y_{i t}= & \sum_{i} w_{i}^{0}\left\{c_{i} \Delta T-a_{i}\left[\Delta L P_{i t}-\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) \Delta L P_{h t}\right]\right\} \\
& +(1-\lambda)\left[\sum_{i} w_{i}^{0}(T-1)-\sum_{i} w_{i}^{0} L P_{i t-1}+\sum_{i} w_{i}^{0}\left(1 / a^{0}\right) \sum_{h} w_{h}^{0} a_{h} L P_{h t-1}-\sum_{i} w_{i}^{0} y_{i t-1}\right] \\
& =(1-\lambda)\left[\sum_{i} w_{i}^{0}(T-1)-\sum_{i} w_{i}^{0} L P_{i t-1}+\sum_{i} w_{i}^{0}\left(1 / a^{0}\right) \sum_{h} w_{h}^{0} a_{h} L P_{h t-1}\right] \\
& \neq 0 .
\end{aligned}
$$

b). When $\xi=0$ in (4.14) we have the partial adjustment model:

$$
\begin{equation*}
y_{i t}=c_{i}^{T}-a_{i}\left[L P_{i t}-\Sigma_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) L P_{h t}\right]+\lambda y_{i t-1} \tag{4.17}
\end{equation*}
$$

Adding up holds here since $\sum_{i} w_{i}^{0} y_{i t}=0$. We further discuss the economic interpretation of this model below when we combine it with a model of adaptive expectations. In that way we form a general dynamic structure against which we can test more restrictive, parsimonious models.

### 4.2.2). A 'Mixed' System of Adaptive Expectations and Partial Adjustment.

Dynamics may, alternatively, be introduced in (4.14) by assuming some expectations formation mechanism for the price variables entering the equations. The introduction of expectations is justified since the importer may not have perfect information about prices. He might thus have to speculate on future delivery prices on the basis of past price information. In particular, we assume Adaptive Expectations first proposed by Cagan(1956). It is asserted that expectations of prices are generated by revising the previous period's expectations, by a fraction of the error in expectations in the previous period (the difference between the actual and expected prices of the previous period). Mathematically:

$$
\begin{equation*}
L P_{t}^{e}=L P_{t-1}^{e}+\lambda\left(L P_{t-1}-L P_{t-1}^{e}\right) ; 0 \leq \lambda \leq 1 \tag{4.18}
\end{equation*}
$$

Essentially this amounts to forming expected current prices as a
function of weighted past prices, where the weights are declining geometrically as we go back in time:

$$
\begin{equation*}
L P_{t}^{e}=(1-\lambda) \sum_{h=0}^{\infty} \lambda^{h} L P_{t-h} \tag{4.19}
\end{equation*}
$$

If production and delivery lags are also important $w$ can let the dependent variable $y_{i t}^{*}$ represent the desired rather than the actual value of $y_{i t}$. Assuming then that there are some costs of adjustment which involve adjusting $y_{i}$ by part of the discrepancy between $y_{i t-1}$ and $y_{i t}^{*}$, we may write the partial adjustment mechanism as:

$$
\begin{equation*}
y_{i t}=(1-\theta) y_{i t-1}+\theta y_{i t}^{*} \quad 0 \leq \theta \leq 1 \tag{4.20}
\end{equation*}
$$

or equivalently

$$
y_{i t}=y_{i t-1}+\theta\left(y_{i t}^{*}-y_{i t-1}\right)
$$

Thus, if both adjustment costs and price expectations are important we may rewrite (4.14) as:

$$
\begin{equation*}
y_{i t}^{*}=c_{i}^{T}-a_{i}\left[L P_{i t}^{e}-\Sigma_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) L P_{h t}^{e}\right]+\varepsilon_{i t} \tag{4.21}
\end{equation*}
$$

Incorporating (4.19) and (4.20) in (4.21) yields a 'mixed' model of Adaptive Expectations (AE) and Partial Adjustment (PA):

$$
y_{i t}=(1-\theta) y_{i t-1}+\theta c_{i} T-\theta a_{i}\left[(1-\lambda) \sum_{h=0}^{\infty} \lambda^{h} L P_{i t-h}-\sum_{1}\left(w_{1}^{0} a_{1} / a^{0}\right)(1-\lambda) \sum_{h=0}^{\infty} \lambda^{h} L P_{i t-h}\right]+\theta \varepsilon_{i t}
$$

The infinite price lags may be truncated for' estimation purposes by the Koyck transformation (Koyck(1954)). This amounts to subtracting a multiple $\lambda$ of the above equation from itself, yielding:

$$
\begin{align*}
y_{i t}= & (1-\theta+\lambda) y_{i t-1}-\lambda(1-\theta) y_{i t-2}+\theta(1-\lambda) c_{i}^{T}+\lambda \theta c_{i}  \tag{4.22}\\
& -\theta(1-\lambda) a_{i}\left[L P_{i t}-\Sigma_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) L P_{h t}\right]+\theta\left(\varepsilon_{i t}-\lambda \varepsilon_{i t-1}\right)
\end{align*}
$$

Writing the system in general form for estimation purposes yields:
$y_{i t}=A y_{i t-1}+B y_{i t-2}+C_{i} T+D_{i}+E_{i}\left[L P_{i t}-\Sigma_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) L P_{h t}\right]+\theta\left(\varepsilon_{i t}-\lambda \varepsilon_{i t-1}\right)$

Note that in (4.22)/(4.23):
a). Adding up is satisfied since $\sum_{i} w_{i}^{0} y_{i t}=0$.
b). for the model in logarithmic differences there is no trend term, while the constant is $\theta(1-\lambda) c_{1}$. The latter picks up any trends in this model.
c). the error term has now become first order moving average $M A(1)$, as a result of applying the Koyck transformation. Thus, if the original stochastic term is white noise an MA(1) would be introduced, while if serial correlation is originally present it could be removed as a result. This is a matter of empirical testing.
d). Only ratios of the parameters $a_{i}$ and $c_{i}$ may be calculated. They can not be identified individually.
e). the structural parameters of the expectations' mechanisms, $\theta$ and $\lambda$, cannot be identified. We can see that by solving for $\theta$ and $\lambda$ from: $A=1-\theta+\lambda$ and $B=-\lambda(1-\theta)$, resulting in the quadratic equation $\lambda^{2}-A \lambda-B=0$ with roots:

$$
\lambda_{1}=(1 / 2)\left[A+\sqrt{ }\left(A^{2}+4 B\right)\right] \quad \lambda_{2}=(1 / 2)\left[A-\sqrt{ }\left(A^{2}+4 B\right)\right]
$$

and the corresponding solutions for $\theta$ are:

$$
\left(1-\theta_{1}\right)=(1 / 2)\left[A-\sqrt{ }\left(A^{2}+4 B\right)\right] \quad\left(1-\theta_{2}\right)=(1 / 2)\left[A+\sqrt{ }\left(A^{2}+4 B\right)\right]
$$

Thus, because of the way $\theta$ and $\lambda$ enter symmetrically in the system we cannot determine which value is $\theta$ and which is $\lambda$.

Despite not being able to identify the structural parameters from (4.23), the system may be estimated using appropriate econometric techniques. If forecasting is the primary aim of estimation, identification of the structural parameters may not be essential, as long as the model can predict the data 'well'.

The adaptive expectations, partial adjustment and static systems
may be obtained by restrictions on the coefficients of the 'mixed' model (4.22)/(4.23).

### 4.2.3). Adaptive Expectations(AE) Systems:

When $\theta=1$, we have the Adaptive Expectations model:

$$
\begin{align*}
y_{i t} & =\lambda y_{i t-1}+(1-\lambda) c_{i} T+\lambda c_{i}  \tag{4.24}\\
& -(1-\lambda) a_{i}\left[L P_{i t}-\Sigma_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) L P_{h t}\right]+\left(\varepsilon_{i t}-\lambda \varepsilon_{i t-1}\right)
\end{align*}
$$

This may be tested against the 'mixed' model by a $t$ test on $B$ in equation (4.23). The estimating model is now:
$y_{i t}=A^{\prime} y_{i t-1}+C_{i}^{\prime} T+D_{i}^{\prime}+E_{i}^{\prime}\left[L P_{i t}-\Sigma_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) L P_{h t}\right]+\left(\varepsilon_{i t}-\lambda \varepsilon_{i t-1}\right)$

The structural parameters may be identified in this model from the estimated parameters of $\left(4.24^{\prime}\right)$ as: $\lambda=A^{\prime}, a_{i}=E_{i}^{\prime} /\left(A^{\prime}-1\right), c_{i}=D_{i}^{\prime} / A^{\prime}$.

The adaptive expectations model of (4.24) with the variables substituted in is shown below:

$$
\begin{aligned}
& \left(m_{i}-w_{i}^{0} m_{t} / m_{i}^{0}=\lambda\left(m_{i}-w_{i}^{0} m\right)_{t-1} / m_{i}^{0}+c_{i}(1-\lambda) t+c_{i} \lambda-a_{i}(1-\lambda)\left[p_{i t}-\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) p_{h t}\right]\right. \\
& \ln \left(m_{i} / w_{i}^{-0}\right)_{t}=\lambda \ln \left(m_{i} / w_{i}^{-0-}\right)_{t-1}+c_{i}(1-\lambda) t+c_{i} \lambda-a_{i}(1-\lambda)\left[\ln p_{i t}-\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) \ln p_{h t}\right] \\
& \Delta \ln \left(m_{i} / \tilde{m}\right)_{t}=\lambda \Delta \ln \left(m_{i} / \tilde{m}\right)_{t-1}+c_{i}(1-\lambda)-a_{i}(1-\lambda)\left[\Delta \ln p_{i t}-\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) \Delta \ln p_{h t}\right]
\end{aligned}
$$

### 4.2.4). Partial Adjustment(PA) Systems:

when $\lambda=0$, we have the Partial Adjustment Model:

$$
\begin{equation*}
y_{i t}=(1-\theta) y_{i t-1}+\theta c_{i}^{T-\theta a_{i}\left[L P_{i t}-\Sigma_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) L P_{h t}\right]+\theta \varepsilon_{i t}, ~} \tag{4.25}
\end{equation*}
$$

The estimating model is in this case:

$$
y_{i t}=A^{\prime \prime} y_{i t-1}+C_{i}^{\prime \prime} T+E_{i}^{\prime \prime}\left[L P_{i t}-\Sigma_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) L P_{h t}\right]+\theta \varepsilon_{i t}
$$

The 'structural' parameters can be identified in this model through: $\theta=1-A^{\prime \prime}, \quad c_{i}=C_{i}^{\prime \prime} /\left(1-A^{\prime \prime}\right)$ and $a_{i}=E_{i}^{\prime \prime} /\left(1-A^{\prime \prime}\right)$. Here the error term has become white noise again.

The PA model may be tested against the 'mixed' model by an $F$ test on $B=D_{1}=0$ in equation (4.23). Also, the partial adjustment model may be tested against the more general $A E^{\prime} s$ model by a $t$ test on $D_{1}^{\prime}=0$.

Substituting the actual variables in (4.25) yield:

$$
\begin{aligned}
& \left(m_{i}-w_{i}^{0} m_{t} / m_{i}^{0}=(1-\theta)\left(m_{i}-w_{i}^{0} m\right)_{t-1} / m_{i}^{0}+\theta c_{i} t-\theta a_{i}\left[p_{i t}-\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) p_{h t}\right]\right. \\
& \ln \left(m_{i} / w_{i}^{-0}-\right)_{t}=(1-\theta) \ln \left(m_{i} / w_{i}^{0-} m\right)_{t-1}+\theta c_{i} t-\theta a_{i}\left[\ln p_{i t}-\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) \ln p_{h t}\right] \\
& \Delta \ln \left(m_{i} / \tilde{m}\right)_{t}=(1-\theta) \Delta \ln \left(m_{i} / \tilde{m}\right)_{t-1}+\theta c_{i}-\theta a_{i}\left[\Delta \ln p_{i t}-\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) \Delta \ln p_{h t}\right]
\end{aligned}
$$

### 4.2.5). Static Systems:

When $\lambda=0$ and $\theta=1$ we end up with the static models of (4.14).
This may be tested against the 'mixed' model with an $F$ test on $A=B=D_{i}=0$. Similarly, an $F$ test on $A^{\prime}=D_{i}^{\prime}=0$, will test the Static system against the $A E$ systems. $A$ test on $A^{\prime \prime}=0$ will test the Static versus the PA systems.

## 4.3). Linearizing further.

The equations of the 'mixed' model (4.22) and its restricted versions (equations (4.24), (4.25) and (4.14)) are still highly nonlinear in the 'structural' parameters $a_{i}$. Let us linearize the system further by taking a first order Taylor expansion around some value $a=\sum_{i} w_{i} a_{i}$, $a$ weighted average of the $a_{i}$ 's. The resulting equations are:

$$
\begin{align*}
y_{i t}= & (1-\theta+\lambda) y_{i t-1}-\lambda(1-\theta) y_{i t-2}+\theta(1-\lambda) b_{i} T+\lambda \theta b_{i} \\
& -\theta(1-\lambda) a_{i}\left[L P_{i t}-\sum_{h} w_{h}^{0} L P_{h t}\right]+\theta(1-\lambda) \sum_{i} w_{i}^{0} a_{i}\left[L P_{i t}-\sum_{h} w_{h}^{0} L P_{h t}\right]+\theta\left(\varepsilon_{i t}-\lambda \varepsilon_{i t-1}\right) \tag{4.26}
\end{align*}
$$

where

$$
\begin{equation*}
b_{i}=a_{i} \gamma_{i}-\sum_{h} w_{h}^{0} a_{h} \gamma_{h}+[a /(1-a)]\left(a_{i}-\sum_{h} w_{h}^{0} \gamma_{h}\right) \sum_{h} w_{h}^{0} \gamma_{h} \tag{4.27}
\end{equation*}
$$

Again, for the model in logarithmic differences, there is no trend term, while the constant is $\theta(1-\lambda) c_{i}$.

Using $\sum_{i} w_{i}^{0} \gamma_{i}=0$ (as in section 4.2), (4.27) reduces to:

$$
\begin{equation*}
b_{i}=a_{i} \gamma_{i}-\Sigma_{h} w_{h}^{0} a_{h} \gamma_{h} \tag{4.28}
\end{equation*}
$$

As with the $c_{i}^{\prime} s$ earlier on, it can be easily seen that the sum of $a$ weighted average of the $b_{i}$ 's is also zero:

$$
\begin{equation*}
\sum_{i} w_{i}^{0} b_{i}=0 \tag{4.29}
\end{equation*}
$$

As a result, adding $u p$ is automatically satisfied since $\sum_{i} w_{i}^{0} y_{i t}=0$. (4.26) shows that differences of growth rates of bilateral imports, of region $j$ from $i$ and total imports, with respect to the base year depend on the last two years' differences in growth rates, on time trend factors, on the relative import price from $\mathfrak{i}$ with respect to a weighted import price from all trade partners, and on a weighted factor of the latter relative import price where the weights are $w_{i}^{0} a_{i}$.

We may write the further-linearized 'mixed' model for estimation purposes as follows:

$$
\begin{align*}
y_{i t} & =A y_{i t-1}+B y_{i t-2}+C_{i} T+D_{i}+E_{i}\left[L P_{i t}-\sum_{h} W_{h}^{O} L P_{h t}\right] \\
& +F \sum_{i} W_{i}^{0} a_{i}\left[L P_{i t}-\sum_{h} W_{h}^{0} L P_{h t}\right]+\theta\left(\varepsilon_{i t}-\lambda \varepsilon_{i t-1}\right)
\end{align*}
$$

The structural parameters of the system are then identified by:

$$
\begin{equation*}
a_{i}=-\left(E_{i} / F\right), \quad b_{1}=\left(C_{i} / F\right), \tag{4.30}
\end{equation*}
$$

and there remain the $\gamma_{i}^{\prime} s, \lambda$ and $\theta$ to be determined. Again given estimates for the $b_{i}$ 's and $a_{i}$ 's the $\gamma_{i}$ 's may be estimated from $\sum_{1} w_{i}^{0} \gamma_{i}=0$ and the ( $n-1$ ) equations (4.28).

However, $\lambda$ and $\theta$ enter symetrically the equation and they still cannot be determined. Only one of them may be determined at the time, which amounts to assuming either adaptive expectations or partial adjustment only. Still, the system can be estimated in the usual way. If $\lambda$ and $\theta$ are not of direct interest to us, we observe that the rest of the structural parameters needed to calculate elasticities and trends can be well identified. Also, nothing impeeds us to use (4.26') for forecasting.

Note also that $-E_{i}=a_{1} \theta(1-\lambda)$ may be interpreted as the short run $a_{i}$ 's, and we may therefore calculate short run own and cross price elasticities of substitution.

Again, placing restrictions on the coefficients of the 'mixed' model enables us to arrive at either the adaptive expectations, the partial adjustment or the static system.
a). When $\theta=1$, we obtain the Adaptive Expectations model:

$$
\begin{align*}
y_{i t}= & \lambda y_{i t-1}+(1-\lambda) b_{i} T+\lambda b_{i}-(1-\lambda) a_{i}\left[L P_{i t}-\sum_{h} w_{h}^{0} L P_{h t}\right]  \tag{4.31}\\
& +(1-\lambda) \sum_{i} w_{i}^{0} a_{i}\left[L P_{i t}-\sum_{h} w_{h}^{0} L P_{h t}\right]+\left(\varepsilon_{i t}-\lambda \varepsilon_{i t-1}\right)
\end{align*}
$$

That is, $\mathrm{B}=0$ in (4.26').

For the model in logarithmic differences there is no trend term and the constant becomes $(1-\lambda) b_{i}$.
(4.31) in its estimating form becomes:

$$
\begin{align*}
y_{i t} & =A^{\prime} y_{i t-1}+C_{i}^{\prime} T+D_{i}^{\prime}+E_{i}^{\prime}\left[L P_{i t}-\sum_{h} W_{h}^{0} L P_{h t}\right] \\
& +F^{\prime}\left\{\sum_{i} w_{i}^{0} a_{i}\left[L P_{i t}-\sum_{h} W_{h}^{0} L P_{h t}\right]\right\}+\left(\varepsilon_{i t}-\lambda \varepsilon_{i t-1}\right)
\end{align*}
$$

where the structural parameters are identified through: $\lambda=A^{\prime}$, $a_{1}=-\left(E_{i}^{\prime} / F^{\prime}\right)$ and $b_{i}=\left(D_{1}^{\prime} / A^{\prime}\right)$.
b). When $\lambda=0$, we obtain the Partial Adjustment model:

$$
\begin{align*}
y_{i t}= & (1-\theta) y_{i t-1}+\theta b_{1} T-\theta a_{1}\left[L P_{i t}-\Sigma_{h} w_{h}^{0} L P_{h t}\right] \\
& +\theta \sum_{1} w_{i}^{0} a_{1}\left[L P_{i t}-\Sigma_{h} w_{h}^{0} L P_{h t}\right]+\theta \varepsilon_{i t} \tag{4.32}
\end{align*}
$$

That is, $B=D_{1}=0$ in (4.26'). For the model in logarithmic first differences $T=1$.
(4.32) in its estimating form is:

$$
\begin{align*}
y_{i t}= & A^{\prime \prime} y_{i t-1}+C_{i}^{\prime \prime} T+E_{i}^{\prime \prime}\left[L P_{i t}-\Sigma_{h} w_{h}^{0} L P_{h t}\right]  \tag{4.32'}\\
& +F^{\prime \prime}\left\{\Sigma_{i} w_{1}^{0} a_{i}\left[L P_{i t}-\Sigma_{h} w_{h}^{0} L P_{h t}\right]\right\}+\theta \varepsilon_{i t}
\end{align*}
$$

where the structural parameters are identified by: $\theta=F^{\prime \prime}$, $a_{i}=-\left(E_{i}^{\prime \prime} / F^{\prime \prime}\right)$ and $b_{i}=\left(C_{i}^{\prime \prime} / F^{\prime \prime}\right)$.
c). When both $\theta=1$ and $\lambda=0$, we have the Static model:

$$
\begin{equation*}
y_{i t}=b_{i} T-a_{i}\left[L P_{i t}-\sum_{h} w_{h}^{0} L P_{h t}\right]+\sum_{i} w_{i}^{0} a_{i}\left[L P_{i t}-\sum_{h} w_{h}^{0} L P_{h t}\right]+\varepsilon_{i t} \tag{4.33}
\end{equation*}
$$

That is, $B=D_{i}=A=0$ and $F=1$ in (4.26 $)$.

Finally, the models represented by (4.26)/(4.26') and its restricted versions are still nonlinear in the parameters, except when $\theta=1, \lambda=0$; that is, when there are no dynamics in the system. However, the nonlinearities are considerably reduced and that makes it a lot easier to estimate the model computationally. In addition, all the structural parameters, including $\gamma_{i}$, are identified (except
of course of the dynamic parameters $\lambda$ and $\theta$ in the 'mixed' model, as explained earlier on).

## 4.4). Restrictions Imposed by the Data.

In the systems developed above, the $L P_{i}^{\prime} s$ represent bilateral seaborne import prices of region $j$ from its trade partners $i$ $(i=1, \ldots, n)$ for good $k(k=1, \ldots, g)$ in either levels, logarithms or logarithmic first differences. As we discuss in the next chapter values of these bilateral price variables (for either seaborne trade or general trade) are not available. The alternative is to use proxies based on the available data, which purport to describe the actual required prices.

Let the bilateral import prices of $j$ from $i$ for good $k\left(L P_{i} \equiv L P_{i j}^{k}\right.$ ) be a weighted average of the total export prices of region $i$, the total import prices of region $j$ and the world prices of good $k$. That is, $L P_{i j}^{k}=f\left(X P_{i}, M P_{j}, W P_{k}\right)$. We may further assume that we can use a Cobb-Douglas function to represent the relationship.

Let the variables be in levels:

$$
\begin{equation*}
p_{i j}^{k} \equiv p_{i}=x p_{i}^{\alpha} m p_{j}^{\beta} w p_{k}^{(1-\alpha-\beta)} \tag{4.34}
\end{equation*}
$$

with variables in logarithms:

$$
\begin{equation*}
\ln p_{i j}^{k} \equiv \ln p_{i}=\alpha \ln \times p_{i}+\beta \operatorname{lnmp}+(1-\alpha-\beta) \ln w p_{k} \tag{4.35}
\end{equation*}
$$

with variables in logarithmic first differences:

$$
\begin{equation*}
\Delta \ln p_{i j}^{k} \equiv \Delta \ln p_{i}=\alpha \Delta \ln \times p_{i}+\beta \Delta \ln m p_{j}+(1-\alpha-\beta) \Delta \ln w p_{k} \tag{4.36}
\end{equation*}
$$

where $\alpha$ and $\beta$ are parameters to be determined at the estimation stage (the alternative is to impose a priori values on these parameters), and $0 \leq \alpha, \beta \leq 1$. We also assume that $\alpha$ and $\beta$ are common across trade partners.

### 4.4.1). 'Mixed' Systems.

Substituting (4.34)-(4.36) into equation (4.26') -the 'mixed' model- yields:
a). Variables in Levels:

$$
\begin{align*}
y_{i t} & =A y_{i t-1}+B y_{i t-2}+C_{i} T+D_{i}+E_{i}\left[\times p_{i}^{\alpha} m p_{j}^{\beta}{ }_{w p}^{(1-\alpha-\beta)}-\sum_{h} W_{h}^{0} \times p_{h}^{\alpha} m p_{j} \beta_{w p_{k}}^{(1-\alpha-\beta)}\right] \\
& +F\left\{\sum_{i} w_{i}^{0} a_{i}\left[\times p_{i}^{\alpha} m p_{j}^{\beta} w p_{k}^{(1-\alpha-\beta)}\right]-\sum_{h} w_{h}^{0}\left[\times p_{h}^{\alpha} m p_{j}^{\beta} w p_{k}^{(1-\alpha-\beta)}\right]\right\} \tag{4.37}
\end{align*}
$$

b). Variables in Logarithms:
$y_{i t}=A y_{i t-1}+B y_{i t-2}+C_{i} T+D_{i}$
$+E_{i}\left\{\left[\alpha \ln \times p_{i}+\beta \operatorname{lnmp} p_{j}+(1-\alpha-\beta) \ln w p_{k}\right]-\sum_{h} w_{h}^{0}\left[\alpha \ln \times p_{h}+\beta \ln m p_{j}+(1-\alpha-\beta) \ln w p_{k}\right]\right\}$
$+F\left\{\sum_{i} w_{i}^{0} a_{i}\left[\alpha \ln \times p_{i}+\beta \ln m p_{j}+(1-\alpha-\beta) \ln w p_{k}\right]-\sum_{h} w_{h}^{0}\left[\alpha \ln \times p_{h}+\beta \ln m p_{j}+(1-\alpha-\beta) \ln w p_{k}\right]\right\}$
c). Variables in Logarithmic First Differences:
$y_{i t}=A y_{i t-1}+B y_{i t-2}+D_{i}$
$+E_{i}\left\{\left[\alpha \Delta \ln \times p_{i}+\beta \Delta \ln m p_{j}+(1-\alpha-\beta) \Delta \ln w p_{k}\right]-\sum_{h} w_{h}^{0}\left[\alpha \Delta \ln \times p_{h}+\beta \Delta \ln m p_{j}+(1-\alpha-\beta) \Delta \ln w p_{k}\right]\right\}$
$+F\left\{\sum_{i} w_{i}^{0} a_{i}\left[\alpha \Delta \ln \times p_{i}+\beta \Delta \ln m p_{j}+(1-\alpha-\beta) \Delta \ln w p_{k}\right]-\sum_{h} W_{h}^{0}\left[\alpha \Delta \ln \times p_{h}+\beta \Delta \ln m p_{j}+(1-\alpha-\beta) \Delta \ln w p_{k}\right]\right\}$
where the definitions of $A, B, C_{i}, D_{i}, E_{i}$ and $F$ are the same as in (4.26'). As a result, all the properties of the system before the introduction of the proxies still hold. Thus, adding up for example is satisfied for all three equations in the sence of $\sum_{i} w_{i}^{0} y_{i t}=0$.

The question that arises here, is whether these new formulations
can be derived from the preference structures of the individual importer, which are now altered. That is, $x p_{1}, m p$, and $w p_{k}$ take the place of $p_{i j}^{k}\left(\equiv p_{i}\right)$ in the objective function of the importer. In fact, it is not difficult to derive (4.37)-(4.39) from this altered objective function. The proof is slightly more complicated but runs along the same lines as that of appendix 4.1 and the steps followed to derive (4.26). The formulation of this new problem of the importer is presented in appendix 4.2.

The advantage of defining bilateral import prices in this way is that the weights of its constituent parts are determined at the estimation stage (simultaneously with the other parameters), subject to the properties of the available data and the overall specification of the model. Furthermore, consistency is maintained in the system at the estimation stage.

However, having defined the bilateral import prices in this way the non-linearities in the model are increased once again. We do not attempt to further linearize the model analytically. Instead we let these linearizations be determined computationally during estimation.

We once more place restrictions on the above -'mixed'- systems of the final estimating equations, to present a clear picture of the final equations that may be chosen for estimation.
4.4.2). Adaptive Expectations Systems.

When $\theta=1$ in (4.26), we have the adaptive expectations model of (4.31)/(4.31'). Thus, when (4.34)-(4.36) are substituted in for the LP's we have:
a'). Variables in Levels:
$y_{i t}=A^{\prime} y_{i t-1}+C_{i}^{\prime} T+D_{i}^{\prime}+E_{i}^{\prime}\left[\times p_{i}^{\alpha} m p_{j}^{\beta}{ }_{w p_{k}}^{(1-\alpha-\beta)}-\sum_{h} w_{h}^{0} \times p_{h}^{\alpha}{ }_{m p}^{\beta}{ }_{j}{ }_{j p}{ }_{k}^{(1-\alpha-\beta)}\right]$
$+F^{\prime}\left\{\sum_{i} w_{i}^{0} a_{i}\left[\times p_{i}^{\alpha} m p_{j}^{\beta} w p_{k}^{(1-\alpha-\beta)}\right]-\sum_{h} w_{h}^{0}\left[\times p_{h}^{\alpha} m p_{j}^{\beta} w p_{k}^{(1-\alpha-\beta)}\right]\right\}$
$\left.b^{\prime}\right)$. Variables in Logarithms:
$y_{i t}=A^{\prime} y_{i t-1}+C_{i}^{\prime} T+D_{i}^{\prime}$
$+E_{i}^{\prime}\left\{\left[\alpha \ln \times p_{i}+\beta \ln m p_{j}+(1-\alpha-\beta) \ln w p_{k}\right]-\sum_{h} w_{h}^{0}\left[\alpha \ln \times p_{h}+\beta \ln m p_{j}+(1-\alpha-\beta) \ln w p_{k}\right]\right\}$
$+F^{\prime}\left\{\sum_{i} w_{i}^{0} a_{i}\left[\alpha \ln \times p_{i}+\beta \ln m p_{j}+(1-\alpha-\beta) \ln w p_{k}\right]-\sum_{h} w_{h}^{0}\left[\alpha \ln \times p_{h}+\beta \ln m p_{j}+(1-\alpha-\beta) \ln w p_{k}\right]\right\}$
c'). Variables in Logarithmic First Differences:
$y_{i t}=A^{\prime} y_{i t-1}+D_{i}^{\prime}$
$+E_{i}^{\prime}\left\{\left[\alpha \Delta \ln \times p_{i}+\beta \Delta \operatorname{lnm} p_{j}+(1-\alpha-\beta) \Delta \ln w p_{k}\right]-\sum_{h} w_{h}^{0}\left[\alpha \Delta \ln \times p_{h}+\beta \Delta \ln m p_{j}+(1-\alpha-\beta) \Delta \ln w p_{k}\right]\right\}$
$+F^{\prime}\left\{\sum_{i} w_{i} a_{i}\left[\alpha \Delta \ln \times p_{i}+\beta \Delta \operatorname{lnmp} p_{j}+(1-\alpha-\beta) \Delta \ln w p_{k}\right]-\sum_{h} w_{h}^{0}\left[\alpha \Delta \ln \times p_{h}+\beta \Delta \ln m p_{j}+(1-\alpha-\beta) \Delta \ln w p_{k}\right]\right\}$

The structural parameters of these models are determined in the same way, as those of equation (4.31'). It should also be noted that the following restrictions are placed across the estimating parameters of (4.40) and (4.41): $A^{\prime}+F^{\prime}=1$ and $C_{i}^{\prime} A^{\prime}-D_{i}^{\prime} F^{\prime}=0$. For model (4.42) only the linear restriction (ie $A^{\prime}+F^{\prime}=1$ ) is relevant. The error term in all the equations is the same as in (4.31).

### 4.4.3). Partial Adjustment Systems.

When $\lambda=0$ in (4.26), we have the partial adjustment model of (4.32)/(4.32'). Thus, when (4.34)-(4.36) are substituted in for the LP's we have:
a''). Variables in Levels:

$$
\begin{align*}
y_{i t} & =A^{\prime \prime} y_{i t-1}+C_{1}^{\prime} T+E_{1}^{\prime \prime}\left[x p_{1}^{\alpha} m p_{j}^{\beta} p_{k}^{(1-\alpha-\beta)}-\Sigma_{h} w_{h}^{0} \times p_{h}^{\alpha} m p_{j}^{\beta} w p_{k}^{(1-\alpha-\beta)}\right] \\
& +F^{\prime \prime}\left\{\Sigma_{1} w_{1}^{0} a_{1}\left[\times p_{1}^{\alpha} m p_{j}^{\beta} w p_{k}^{(1-\alpha-\beta)}\right]-\Sigma_{h} w_{h}^{0}\left[\times p_{h}^{\alpha} m p_{j}^{\beta} w p_{k}^{(1-\alpha-\beta)}\right]\right\} \tag{4.43}
\end{align*}
$$

b''). Variables in Logarithms:
$y_{i t}=A^{\prime \prime} y_{i t-1}+C_{i}^{\prime \prime}{ }^{\prime} T$
$+E_{i}^{\prime \prime}\left\{\left[\alpha \ln \times p_{1}+\beta \operatorname{lnmp} p_{j}+(1-\alpha-\beta) \ln w p_{k}\right]-\sum_{h} w_{h}^{0}\left[\alpha \ln \times p_{h}+\beta \operatorname{lnmp} p_{j}+(1-\alpha-\beta) \ln w p_{k}\right]\right\}$
$+F^{\prime}\left\{\sum_{i} w_{i}^{0} a_{i}\left[\alpha \ln \times p_{i}+\beta \ln m p_{j}+(1-\alpha-\beta) \ln w p_{k}\right]-\sum_{h} w_{h}^{0}\left[\alpha \ln \times p_{h}+\beta \ln m p_{j}+(1-\alpha-\beta) \ln w p_{k}\right]\right\}$
c''). Variables in Logarithmic First Differences:
$y_{i t}=A^{\prime \prime} y_{i t-1}+C_{1}$
$+E_{i}{ }^{\prime \prime}\left\{\left[\alpha \Delta \ln \times p_{i}+\beta \Delta \operatorname{lnmp}{ }_{j}+(1-\alpha-\beta) \Delta \ln w_{p_{k}}\right]-\sum_{h} w_{h}^{0}\left[\alpha \Delta \ln \times p_{h}+\beta \Delta \operatorname{lnmp} p_{j}+(1-\alpha-\beta) \Delta \ln w p_{k}\right]\right\}$
$+F^{\prime}\left\{\Sigma_{i} w_{1}^{0} a_{1}\left[\alpha \Delta \ln \times p_{i}+\beta \Delta \ln m p_{j}+(1-\alpha-\beta) \Delta \ln w p_{k}\right]-\Sigma_{h} w_{h}^{0}\left[\alpha \Delta \ln \times p_{h}+\beta \Delta \ln m p_{j}+(1-\alpha-\beta) \Delta \ln w p_{k}\right]\right\}$

The sructural parameters of these systems are identified in the same way as those of equation (4.32'). Furthermore, the linear restriction $A^{\prime \prime}+F^{\prime \prime}=1$ is relevant in the models.

### 4.4.4). Static Systems.

When $\lambda=0$ and $\theta=1$ in (4.26), we have the not so interesting static model of (4.33). When (4.34)-(4.36) are substituted in for the LP's we have:
a'"). Variables in Levels:

$$
y_{i t}=C_{i}^{\prime \prime \prime} T+E_{i}^{\prime \prime \prime}\left[\times p_{i}^{\alpha} m p_{j}^{\beta} p_{k}^{(1-\alpha-\beta)}-\Sigma_{h} w_{h}^{0} \times p_{h}^{\alpha} m p_{j}^{\beta} p_{k}^{(1-\alpha-\beta)}\right]
$$

$+\sum_{i} w_{i}^{0} a_{i}\left[\times p_{i}^{\alpha} m p_{j}^{\beta} w p_{k}^{(1-\alpha-\beta)}\right]-\sum_{h} w_{h}^{0}\left[\kappa p_{h}^{\alpha} m p_{j}^{\beta} w p_{k}^{(1-\alpha-\beta)}\right]$
b'"'). Variables in Logarithms:
$y_{\text {it }}=C_{i}^{\prime \prime \prime T}$
$+E_{i}^{\prime \prime} \quad\left\{\left[\alpha \ln \times p_{i}+\beta \operatorname{lnm} p_{j}+(1-\alpha-\beta) \ln w p_{k}\right]-\sum_{h} w_{h}^{0}\left[\alpha \ln \times p_{h}+\beta \operatorname{lnmp} p_{j}+(1-\alpha-\beta) \ln w p_{k}\right]\right\}$
$+\sum_{i} w_{i}^{0} a_{i}\left[\alpha \ln \times p_{i}+\beta \ln m p_{j}+(1-\alpha-\beta) \ln w p_{k}-\sum_{h} w_{h}^{0}\left[\alpha \ln \times p_{h}+\beta \ln m p_{j}+(1-\alpha-\beta) \ln w p_{k}\right]\right.$
c','). Variables in Logarithmic First Differences:
$y_{i t}=C_{i}^{\prime \prime}$
$+E_{i}^{\prime \prime \prime}\left\{\left[\alpha \Delta \ln \times p_{i}+\beta \Delta \ln m p_{j}+(1-\alpha-\beta) \Delta \ln w p_{k}\right]-\sum_{h} w_{h}^{0}\left[\alpha \Delta \ln \times p_{h}+\beta \Delta \ln m p_{j}+(1-\alpha-\beta) \Delta \ln w p_{k}\right]\right\}$
$+\sum_{i} w_{i}^{0} a_{i}\left[\alpha \Delta \ln \times p_{i}+\beta \Delta \ln m p_{j}+(1-\alpha-\beta) \Delta \ln w p_{k}\right]-\sum_{h} W_{h}^{0}\left[\alpha \Delta \ln \times p_{h}+\beta \Delta \ln m p_{j}+(1-\alpha-\beta) \Delta \ln w p_{k}\right]$
where $b_{i}=C_{i}^{\prime \prime \prime}$ and $a_{i}=E_{i}^{\prime \prime \prime}$.
4.5). Methodology, Problems and Properties of the Empirical Systems.

### 4.5.1). Methodology.

A number of possible estimating versions of the CRESH model have been defined in terms of Mixed systems, Adaptive Expectations(AE), Partial Adjustment(PA) and Static systems.

Ideally, concerning the dynamics of the CRESH model, we like to start from the Mixed model and by placing restrictions on its parameters arrive at some more parsimonious system of $A E, P A$ or Static.

Once a particular system has been chosen, by placing further restrictions on its parameters we could move from the 'statistical' form of the model to the corresponding 'econometric' version. Thus, we refer to the estimating version of the $A E$ system, say, equation (4.31'), as the statistical model, while by placing restrictions on its coefficients we arrive at what we call the econometric version of the $A E$ model, equation (4.31). These restrictions are, $A=\lambda$, $C_{i}=(1-\lambda) b_{i}, \quad D_{i}=\lambda b_{i}, \quad E_{i}=-(1-\lambda) a_{i}, F=(1-\lambda)$.

The functional form of the selected system should also be decided upon. Should the system be in levels, in logs, or in first differences? We discuss more of this in chapter 7.
Restrictions on the estimated coefficients may reflect
restrictions on the initial preferences of the optimizing agents.
Thus, the significance of the trend term, for example, may be
questioned by letting $\gamma_{i}=0$ in the above systems. Further restricted
versions of the above systems are considered later in the chapter.

### 4.5.2). Estimation Problems of the Empirical Systems.

Data restrictions (only 18 time series observations in a system of 30 equations) have forced us to compromise. The Mixed Model involves two lags of the dependent variable. Lack of degrees of freedom forbids identification of the individual coefficients of the statistical Mixed system during estimation. Imposing the theoretical restrictions enables estimation, but the coefficients $\theta$ and $\lambda$ cannot be identified individually (see section (4.2.2)).

As a result, we have to define as our most general model, a system with at most one lagged dependent variable as a regressor. This leaves us with the $A E$ as the most general model compared to PA and Static systems. When we attempt to estimate the statistical version of this system we run into difficulties once more, due to lack of degrees of freedom. The theoretical restrictions are then imposed and this enables estimation of the econometric system.

Another problem incured during estimation, is that the price indices of total exports, $x p_{i}$, and imports, $m p_{j}$, are highly collinear. As a result only $\alpha$ or $\beta$ can be identified at any time. In order to be able to distinguish the individual price effect of each export partner, we choose to drop the total import price index mp, setting $\beta=0$ throughout. Thus, the definition (4.34) of $p_{i j}^{k}$ in its Cobb-Douglas form is now.

$$
\begin{equation*}
p_{i j}^{k}=x p_{i}^{\alpha} w p_{k}^{(1-\alpha)} \tag{4.49}
\end{equation*}
$$

where $\ln \left(p_{i j}^{k}\right)$ and $\Delta \ln \left(p_{i j}^{k}\right)$ alter apropriately. Similarly, in all the estimating systems $\beta=0$, and the $m p$ term is dropped as a result.

Thus, for some import market, $j$, and some traded good, $k$, (the subscripts $j, k$, omitted through out), the AE systems become ( $\beta=0$ ):
$\left.a^{\prime}\right)$. Variables in Levels:

$$
\begin{align*}
y_{i t} & =\lambda y_{i t-1}+(1-\lambda) b_{i} t+\lambda b_{i}-(1-\lambda) a_{i}\left\{\left[\times p_{i}^{\alpha} w p_{k}^{(1-\alpha)}\right]-\sum_{h} w_{h}^{0}\left[\times p_{h}^{\alpha} w p_{k}^{(1-\alpha)}\right]\right\}  \tag{4.50}\\
& +(1-\lambda)\left\{\sum_{i} w_{i}^{0} a_{i}\left[\times p_{i}^{\alpha} w p_{k}^{(1-\alpha)}\right]-\sum_{h} w_{h}^{0}\left[\times p_{h}^{\alpha} w p_{k}^{(1-\alpha)}\right]\right\}
\end{align*}
$$

$\left.b^{\prime}\right)$. Variables in Logarithms:

$$
\begin{align*}
y_{i t}= & \lambda y_{i t-1}+(1-\lambda) b_{i} t+\lambda b_{i}  \tag{4.51}\\
& -(1-\lambda) a_{i}\left\{\left[\alpha \ln \times p_{i}+(1-\alpha) \ln w p_{k}\right]-\sum_{h} w_{h}^{0}\left[\alpha \ln \times p_{h}+(1-\alpha) \ln w p_{k}\right]\right\} \\
& +(1-\lambda)\left\{\sum_{i} w_{i}^{0} a_{i}\left[\alpha \ln \times p_{i}+(1-\alpha) \ln w p_{k}\right]-\sum_{h} w_{h}^{0}\left[\alpha \ln \times p_{h}+(1-\alpha) \ln w p_{k}\right]\right\}
\end{align*}
$$

c'). Variables in Logarithmic First Differences:

$$
\begin{equation*}
y_{i t}=\lambda y_{i t-1}+(1-\lambda) b_{i} \tag{4.52}
\end{equation*}
$$

$$
\begin{aligned}
& -(1-\lambda) a_{i}\left\{\left[\alpha \Delta \ln \times p_{i}+(1-\alpha) \Delta \ln w p_{k}\right]-\sum_{h} w_{h}^{0}\left[\alpha \Delta \ln \times p_{h}+(1-\alpha) \Delta \ln w p_{k}\right]\right\} \\
& +(1-\lambda)\left\{\sum_{i} w_{i}^{0} a_{i}\left[\alpha \Delta \ln \times p_{i}+(1-\alpha) \Delta \ln w p_{k}\right]-\sum_{h} w_{h}^{0}\left[\alpha \Delta \ln \times p_{h}+(1-\alpha) \Delta \ln w p_{k}\right]\right\}
\end{aligned}
$$

and similarly for the Mixed Model, the Partial Adjustment and the Static systems.

### 4.5.3). The Elasticities Under the New Formulation.

Having altered the system, slightly, to take account of the lack of data for bilateral prices and the empirical problems incured during estimation, we need to re-interpret the elasticities derived in section (3.3.3.4). In this new specification, the bilateral (own and cross) import price elasticities, in some import market $j$, are decomposed into an import elasticity (own and cross) with respect to a change in the total export price of the exporting region, and an import elasticity with respect to a change in the world price of the traded good.

Thus, the own import price elasticity, in market $j$ from $i$ for some good $k$, with respect to a change in the total export price of partner $\mathfrak{i}$, which we will call the Own Export Price Elasticity, is:

$$
\begin{equation*}
e_{i i}^{x p}=\alpha e_{i i}=\left\{\alpha\left[\left(w_{i} a_{i}^{2} / a\right)-a_{i}\right]\right\}, \quad i=1, \ldots, n \tag{4.53}
\end{equation*}
$$

The cross import price elasticity, in market $j$ from $\mathfrak{i}$ for some good $k$, with respect to a change in the total export price of partner $h, i \neq h=1, \ldots, n$, which we will call the Cross Export Price Elasticity, is:

$$
\begin{equation*}
e_{i h}^{x p}=\alpha e_{i h}=\left\{\alpha\left[\left(a_{i} w_{h} a_{h} / a\right)\right]\right\}, \quad i \neq h=1, \ldots, n \tag{4.54}
\end{equation*}
$$

The import price elasticity, in market $j$ from $i$ for some good $k$, with respect to a change in the total world price of good $k$, which we will call the World Export Price Elasticity, is:

$$
\begin{equation*}
e^{w p}=(1-\alpha) e_{i 1}=\left\{(1-\alpha)\left[\left(w_{i} a_{i}^{2} / a\right)-a_{i}\right]\right\}, \quad i=1, \ldots, n \tag{4.55}
\end{equation*}
$$

In (4.53)-(4.55) we observe that the newly defined elasticities are multiples of the elasticities derived in chapter 3 (before the introduction of the proxies for bilateral prices) with the multiplicative factors being $\alpha$ and (1- $\alpha$ ). The proof of (4.53)-(4.55) is relegated to appendix 4.3.

## 4.6). Restrictions on the CRESH function.

At the end of chapter 3, we derived a sequence of nested aggregator functions as special cases of the preferences described by the CRESH function. Similarly, a sequence of nested demand equations may be derived by placing restrictions on the CRESH models derived above. Thus, when $a_{i}=\sigma$ (a constant), we have the class of Constant Elasticity of Substitution(CES) models. When $\sigma$ takes the limiting values of $\sigma=0$ and $\sigma=\infty$, we obtain the Leontief and the Linear Expenditure Systems(LES), respectively. The Leontief technology characterizes systems where there is no substitutability between imports from different trade partners, while the LES imposes perfect substitutability between trade partners. When $\sigma=1$ we arrive at the Cobb-Douglas model of unitary elasticities. We present as examples of nested versions of CRESH the estimating systems of the CES and the Cobb-Douglas models.
4.6.1). Constant Elasticity of Substitution(CES) Systems.

Take the static models specified by (4.14). These were derived as first order Taylor approximations to the bilateral import demand equations of chapter 3 . We repeat the equation here for convenience.

$$
\begin{equation*}
y_{i t}=c_{i} T-a_{i}\left[L P_{i t}-\Sigma_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) L P_{h t}\right] ; \quad a_{i}=1 /\left(1+\rho_{i}\right) \tag{4.14}
\end{equation*}
$$

where $c_{i}$ were defined in (4.4), also repeated here.

$$
\begin{equation*}
\left.c_{i}=a_{i}\left[\gamma_{i}+\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right)\left(\gamma_{h} / \rho_{h}\right)\right]-\sum_{h} w_{h}^{0}\left(\gamma_{h} / \rho_{h}\right)\right] \quad \forall i=1, \ldots, n . \tag{4.4}
\end{equation*}
$$

When $a_{i}=\sigma$ (also, using $\Sigma_{i} w_{i}^{0}=1$ and $\sigma-1=\sigma \rho$ ) then (4.4) becomes:

$$
c_{i}=\sigma\left[\gamma_{i}+\left(\sigma / \Sigma_{j} w_{j}^{0} \sigma\right)(1 / \rho)\left(\Sigma_{h} w_{h}^{0} \gamma_{h}\right)\right]-(1 / \rho) \Sigma_{h} w_{h}^{0} \gamma_{h}=\sigma\left(\gamma_{i}+\Sigma_{h} w_{h}^{0} \gamma_{h}\right)
$$

To avoid confusion we write $c_{1}$ for the CES case as:

$$
\begin{equation*}
r_{i}=\sigma\left(\gamma_{i}-\sum_{h} w_{h}^{0} \gamma_{h}\right) \tag{4.56}
\end{equation*}
$$

Then leting $a_{i}=\sigma$ (and $c_{i}=r_{i}$ ) in (4.14) we have

$$
\begin{gather*}
y_{i t}=r_{i} T-\sigma\left[L P_{i t}-\left(1 / \sigma \sum_{j} w_{j}^{0}\right) \sigma \sum_{h} W_{h}^{0} L P_{h t}\right] \\
y_{i t}=r_{i}^{T}-\sigma\left[L P_{i t}-\sum_{h} W_{h}^{0} L P_{h t}\right] \tag{4.57}
\end{gather*}
$$

which is the CES equivalent of the static models described in (4.14). Hence, there is a single common price effect $\sigma$ between the trade partners in the same importing market $j$. (4.57) is equation (4.28) of Hickman and Lau(1973, p354) derived by optimizing a CES production function.

Similarly, the dynamic CES version of the 'mixed' model of equation (4.22) reduces to:

$$
\begin{align*}
y_{i t} & =(1-\theta+\lambda) y_{i t-1}-\lambda(1-\theta) y_{i t-2}+\theta(1-\lambda) r_{i}^{T}+\lambda \theta r_{i} \\
& -\theta(1-\lambda) \sigma\left[L P_{i t}-\sum_{h} W_{h}^{0} L P_{h t}\right]+\theta\left(\varepsilon_{i t}-\lambda \varepsilon_{i t-1}\right) \tag{4.58}
\end{align*}
$$

where the trend term and the constant for the model in logarithmic first differences become one term, equal to: $\theta(1-\lambda) r_{i}$.

As would be expected, when we consider the CES version of equation (4.26), that is, the further linearized ('mixed') model in the $a_{i}$ parameters (now $\sigma$ ), we get back to equation (4.58). That is, the same model, as if we hadn't linearized the system in the $\alpha_{i}$
parameters. Hence, our final estimating import demand equations would be (4.58) with the appropriate LP's substituted to take account of data restrictions, in line with the analysis of the previous sections. It should be noted here that none of $\lambda, \theta, r_{i}$ or $\sigma$ can be identified from (4.58).
4.6.2). Cobb-Douglas Systems.

The corresponding equations to (4.57) and (4.58) when $\sigma=1$ are:

$$
\begin{equation*}
y_{i t}=\left(\gamma_{i}-\sum_{h} w_{h}^{0} \gamma_{h}\right) T-\left[L P_{i t}-\sum_{h} w_{h}^{0} L P_{h t}\right] \tag{4.59}
\end{equation*}
$$

and

$$
\begin{align*}
y_{i t} & =(1-\theta+\lambda) y_{i t-1}-\lambda(1-\theta) y_{i t-2}+\theta(1-\lambda)\left(\gamma_{i}-\sum_{h} w_{h}^{0} \gamma_{h}\right) T \\
& +\lambda \theta\left(\gamma_{i}-\sum_{h} w_{h}^{0} \gamma_{h}\right)-\theta(1-\lambda)\left[L P_{i t}-\sum_{h} W_{h}^{0} L P_{h t}\right] \tag{4.60}
\end{align*}
$$

Thus, the Cobb-Douglas systems are nested in CRESH, since they are special cases of the CES.

The aim of this chapter has been to turn the system of the bilateral seaborne import demand equations derived in the previous chapter, to a system of equations which can be implemented empirically. In doing so we were careful not to lose the theoretical properties imposed by the standard theory of systems of demand equations. Thus, adding-up, for instance, is maintained throughout.

By linearizing the system of theoretical equations we were able to define empirically the composite price index $P$. At the same time, the cost of computing time in estimation is reduced, which for large systems of equations is considerable. It often makes the difference between converging to a solution or not. Dynamics and proxies of the unobservable bilateral import prices are introduced. The introduction of dynamics makes the system considerably richer. It also provides us with a framework of moving from some general model that can describe the data sufficiently well, to a more restricted theoretically consistent econometric model. The introduction of proxies for the missing bilateral prices introduces empirical and theoretical problems. The collineariry between the proxies and the lack of degrees of freedom problems are solved by imposing the theoretical restrictions at the outset. The elasticities of demand are further examined under the new formulation. Furthermore, as examples of the encompassing properties of the CRESH model, we derive the CES and the Cobb-Douglas import demand equations by placing restrictions on the parameters of CRESH.

Appendix 4.1.
First Order Taylor Linearizations of the Import Demand Equations.

We want to linearize equation (4.2), which is repeated here for convenience.

$$
\begin{equation*}
m_{i}=m\left[\left|\delta_{i} p_{i}\right| e^{\gamma_{1} t}\left(p_{i} / P\right)^{-1}\right]^{a_{i}}\left\{\Sigma_{h}\left[\left|\delta_{h} \rho_{h}\right| e^{\gamma_{h} t}\left(p_{h} / P\right)^{-1}\right]^{a_{h}-1}\right. \tag{1}
\end{equation*}
$$

where the composite price index $P$ is defined implicitly by:

$$
\begin{align*}
& G\left(P, p_{1}, \ldots, P_{n}, t\right)=\sum_{i} \delta_{i}\left[\left|\delta_{i} \rho_{1}\right| e^{\left(-\gamma_{1} / \rho_{i}\right) t}\left(p_{1} / P\right)^{-1}\right]^{-a_{1} \rho_{1}}-1=0  \tag{2}\\
& \text { and } a_{i}=1 /\left(1+\rho_{i}\right)
\end{align*}
$$

Using (2) we derive the following results at the base year values (when $t=0, m=m^{0}, p_{i}=1$ ), which will be useful later:

$$
\begin{align*}
& \partial G / \partial p_{1}=\delta_{1} a_{i} \rho_{1}\left[\left|\delta_{1} \rho_{1}\right|^{-\rho_{1} e^{\gamma_{1} t}}\left(p_{i} / P\right)^{\rho_{1}}\right]^{a_{1}} p_{1}^{-1}  \tag{3}\\
& \partial G /\left.\partial p_{i}\right|_{t=0}=\delta_{i} a_{i} \rho_{i}\left|\delta_{i} \rho_{i}\right|^{-\rho_{i} a_{i}} \\
& \partial G / \partial P=-\sum_{h} \delta_{h} a_{h} \rho_{h}\left[\left|\delta_{h} \rho_{h}\right|^{-\rho_{h} e^{\gamma_{h}}{ }^{t}}\left(\rho_{h} / P\right)^{\rho_{h}}\right]_{h}{ }^{h_{h}} P^{-1}  \tag{4}\\
& \partial G /\left.\partial P\right|_{t=0}=-\Sigma_{h} \delta_{h} a_{h} \rho_{h}\left|\delta_{h} \rho_{h}\right|^{-\rho_{h} a_{h}} \\
& \partial G / \partial t=\sum_{h} \delta_{h} a_{h} \gamma_{h}\left[\left|\delta_{h} \rho_{h}\right|^{-\rho_{h} e_{h} \gamma^{t}}\left(p_{h} / P\right)^{\rho_{h}}\right]_{h}  \tag{5}\\
& \partial G /\left.\partial t\right|_{t=0}=\sum_{h} \delta_{h} a_{h} \gamma_{h}\left|\delta_{h} \rho_{h}\right|^{-\rho_{h} a_{h}} \tag{5'}
\end{align*}
$$

By the implicit function theorem:

$$
\partial \mathrm{P} / \partial \mathrm{p}_{1}=-\left(\partial \mathrm{G} / \partial \mathrm{p}_{\mathrm{i}}\right) /(\partial \mathrm{G} / \partial \mathrm{P})
$$

```
\partialP/\partialt=-(\partialG/\partialt)/(\partialG/\partialP)
```

$\partial p_{i} / \partial t=-(\partial G / \partial t) /\left(\partial G / \partial p_{i}\right)$

At the base year values:

$$
\begin{align*}
& \partial P /\left.\partial p_{i}\right|_{t=0}=\left(\delta_{i} a_{i} \rho_{i}\left|\delta_{i} \rho_{i}\right|^{-\rho_{i} a_{i}}\right) /\left(\sum_{h} \delta_{h} a_{h} \rho_{h}\left|\delta_{h} \rho_{h}\right|^{-\rho_{h} a_{h}}\right)  \tag{6}\\
& \partial P /\left.\partial t\right|_{t=0}=\left(\sum_{h} \delta_{h} a_{h} \gamma_{h}\left|\delta_{h} \rho_{h}\right|^{-\rho_{h} a_{h}}\right) /\left(\sum_{h} \delta_{h} a_{h} \rho_{h}\left|\delta_{h} \rho_{h}\right|^{-\rho_{h} a_{h}}\right)  \tag{7}\\
& \partial p_{i} /\left.\partial t\right|_{t=0}=\left(\sum_{h} \delta_{h} a_{h} \gamma_{h}\left|\delta_{h} \rho_{h}\right|^{-\rho_{h} a_{h}}\right) /\left(\delta_{i} a_{i} \rho_{i}\left|\delta_{i} \rho_{i}\right|^{-\rho_{i} a_{i}}\right) \tag{8}
\end{align*}
$$

Let us write (1) for notational convenience as $m_{i}=f\left(m, t, p_{i}\right)=f(\underline{x})$. A first order Taylor approximation around the base year values takes the form:
$m_{i} \simeq f\left(\underline{x}^{0}\right)+\left[\partial f\left(\underline{x}^{0}\right) / \partial m\right]\left(m-m^{0}\right)+\left[\partial f\left(\underline{x}^{0}\right) / \partial t\right](t-0)+\sum_{h}\left[\partial f\left(\underline{x}^{0}\right) / \partial p_{h}\right]\left(p_{h}-1\right)$

Let us find each term in (9):
A). First, find $f\left(\underline{x}^{0}\right)\left(=m_{i}^{0}\right)$.

$$
\begin{equation*}
f\left(\underline{x}^{0}\right)=f\left(m^{0}, 0,1\right)=m^{0}\left(\left|\delta_{i} \rho_{i}\right|^{a}\right) /\left(\Sigma_{h}\left|\delta_{h} \rho_{h}\right|^{a}\right) \tag{10}
\end{equation*}
$$

which implies for the base year share

$$
\begin{equation*}
w_{i}^{0}=\left(\left|\delta_{i} \rho_{i}\right|^{a_{i}}\right) /\left(\Sigma_{h}\left|\delta_{h} \rho_{h}\right|^{a_{h}}\right) \tag{11}
\end{equation*}
$$

Hence, (10) becomes: $\quad m_{i}^{0}=m^{0} w_{i}^{0}$
B). $\partial f\left(\underline{x}^{0}\right) / \partial m$ is:

$$
\begin{equation*}
\partial f\left(\underline{x}^{0}\right) / \partial m=\left(\left|\delta_{i} \rho_{i}\right|^{a}\right) /\left(\Sigma_{h}\left|\delta_{h} \rho_{h}\right|^{a_{h}}\right)=w_{i}^{0} \tag{12}
\end{equation*}
$$

C). To find $\partial f\left(\underline{x}^{0}\right) / \partial t$ :

Let

$$
\begin{equation*}
K_{1}=\left|\delta_{1} p_{1}\right|^{a_{1}} p_{1}^{-a_{1}} \tag{13}
\end{equation*}
$$

which at the base year values is

$$
\begin{equation*}
\left.K_{i}\right|_{t=0}=\left|\delta_{i} \rho_{1}\right|^{a_{1}} \tag{14}
\end{equation*}
$$

and from (12) we have: $\left.\quad\left(K_{i} / \sum_{h} K_{h}\right)\right|_{t=0}=w_{1}$
With the use of (13), (1) can be simplified to:

$$
\begin{equation*}
m_{1}=\left(m K_{1} e^{a_{1} \gamma_{1} t_{P} a_{1}}\right)\left(\Sigma_{h} K_{h} e^{a_{h} \gamma_{h} t}{ }_{P} a_{h}\right)^{-1} \tag{1'}
\end{equation*}
$$

$\partial m_{1} / \partial t=\left(\sum_{h} K_{h} e^{a_{h} \gamma_{h} t} P_{h}\right)^{-1}\left\{\left[m K_{1}\left[a_{1} \gamma_{i} e^{a_{1} \gamma_{1} t} P^{a_{1}}+e^{a_{1} \gamma_{i} t} a_{i} P^{a_{i}-1}(\partial P / \partial t)\right]\right\}\right.$
$-\left(m K_{i} e^{a_{i} \gamma_{i} t_{P} a_{i}}\right)\left\{\Sigma_{h} K_{h}\left[a_{h} \gamma_{h} e^{a_{h} \gamma_{h} t_{P}} a_{h}+e^{a_{h} \gamma_{h} t} a_{h} P^{a_{h}-1}(\partial P / \partial t)\right]\right\}\left(\sum_{h} K_{h} e^{a_{h} \gamma_{h} t_{P} a_{h}}\right)^{-2}$

$$
\begin{align*}
\partial m_{i} /\left.\partial t\right|_{t=0}= & \left(\left.\sum_{h} K_{h}\right|_{t=0}\right)^{-1}\left\{\left.m K_{i}\right|_{t=0}\left[a_{1} \gamma_{1}+\left.a_{i}(\partial P / \partial t)\right|_{t=0}\right]\right\}  \tag{17}\\
& -\left(\left.m K_{i}\right|_{t=0}\right)\left\{\left.\sum_{h} K_{h}\right|_{t=0}\left[a_{h} \gamma_{h}+\left.a_{h}(\partial P / \partial t)\right|_{t=0}\right]\right\}\left(\left.\sum_{h} K_{h}\right|_{t=0}\right)^{-2}
\end{align*}
$$

which, with the use of (15), we can write as:

$$
\begin{equation*}
\partial m_{1} /\left.\partial t\right|_{t=0}=m w_{1}^{0}\left\{a_{i}\left[\gamma_{1}+\left.(\partial P / \partial t)\right|_{t=0}\right]\right\}-m w_{i}^{0}\left\{\sum_{h} w_{h}^{0} a_{h}\left[\gamma_{h}+\left.(\partial P / \partial t)\right|_{t=0}\right]\right\} \tag{18}
\end{equation*}
$$

From

$$
\begin{equation*}
a_{1}=1 /\left(1+\rho_{1}\right), \quad-\rho_{1} a_{1}=a_{1}-1 \tag{19}
\end{equation*}
$$

Dividing top and bottom of (7) by $\left(\Sigma_{h}\left|\delta_{h} \rho_{h}\right|^{-\rho_{h} a_{h}}\right.$ and using (19) and (10') we may write:

$$
\partial P /\left.\partial t\right|_{t=0}=\left(\sum_{h} w_{h}^{0} a_{h} \gamma_{h} / \rho_{h}\right) /\left(\sum_{h} w_{h}^{0} a_{h}\right)
$$

Use ( $7^{\prime}$ ) and $w_{i}^{0}=\left(m_{i}^{0} / m^{0}\right)$ in (18) to get:

$$
\begin{aligned}
\partial m_{i} /\left.\partial t\right|_{t=0}= & m_{i}^{0}\left\{a_{i}\left[\gamma_{i}+\left(\sum_{h} w_{h}^{0} a_{h} \gamma_{h} / \rho_{h}\right) /\left(\sum_{h} w_{h}^{0} a_{h}\right)\right]\right. \\
& \left.-\left[\sum_{h} w_{h}^{0} a_{h} \gamma_{h}+\sum_{h} w_{h}^{0} a_{h}\left(\sum_{h} w_{h}^{0} a_{h} \gamma_{h} / \rho_{h}\right) /\left(\sum_{h} w_{h}^{0} a_{h}\right)\right]\right\} \\
& =m_{i}^{0}\left\{a_{i}\left[\gamma_{i}+\left(\sum_{h} w_{h}^{0} a_{h} \gamma_{h} / \rho_{h}\right) /\left(\sum_{h} w_{h}^{0} a_{h}\right)\right]-\left\{\sum_{h} w_{h}^{0} a_{h} \gamma_{h}\left[1+\left(1 / \rho_{h}\right)\right]\right\}\right\}
\end{aligned}
$$

which, with the use of $\left[1+\left(1 / \rho_{h}\right)\right]=1 / a_{h} \rho_{h}$ becomes:

$$
\begin{equation*}
\partial m_{i} /\left.\partial t\right|_{t=0}=m_{i}^{0}\left\{a_{i}\left[\gamma_{i}+\left(\sum_{h} w_{h}^{0} a_{h} \gamma_{h} / \rho_{h}\right) /\left(\sum_{h} w_{h}^{0} a_{h}\right)\right]-\sum_{h} w_{h}^{0} \gamma_{h} / \rho_{h}\right\} \tag{20}
\end{equation*}
$$

Further, let $a^{0}=\sum_{i} w_{i}^{0} a_{i}$, then (16) can writen as:

$$
\begin{align*}
\partial m_{i} /\left.\partial t\right|_{t=0} & =m_{i}^{0}\left\{a_{i}\left[\gamma_{i}+\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right)\left(\gamma_{h} / \rho_{h}\right)\right]-\sum_{h} w_{h}^{0}\left(\gamma_{h} / \rho_{h}\right)\right\}  \tag{21}\\
& =m_{i}^{0} c_{i} \tag{22}
\end{align*}
$$

where, $\quad c_{i}=a_{i}\left[\gamma_{i}+\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right)\left(\gamma_{h} / \rho_{h}\right)\right]-\sum_{h} w_{h}^{0}\left(\gamma_{h} / \rho_{h}\right)$
D). Finally, we need to find $\sum_{h}\left[\partial f\left(\underline{x}^{0}\right) / \partial p_{h}\right]$

Let $\quad B_{i}=\left[\left|\delta_{i} \rho_{i}\right| e^{\gamma_{i} t}\right]_{i}$
which at the base year values becomes:

$$
\begin{equation*}
\left.B_{i}\right|_{t=0}=\left|\delta_{i} \rho_{i}\right|^{a_{i}} \tag{25}
\end{equation*}
$$

From (12) we have: $\left.\quad\left(B_{i} / \sum_{h} B_{h}\right)\right|_{t=0}=w_{i}^{0}$

Using (24), we may write (1) as:

$$
m_{i}=m B_{i}\left(p_{i} / P\right)^{-a} i\left[\sum_{h} B_{h}\left(p_{h} / P\right)^{-a} h\right]^{-1}
$$

Distinguish between 1) $i=h$ and 2) $i \neq h$ :
1). Let $i=h$, then:

$$
\begin{align*}
\partial m_{i} / \partial p_{1} & =m\left\{B_{1}\left[\sum_{h} B_{h}\left(p_{h} / P\right)^{-a_{h}}\right]^{-1}\left\{\partial\left[\left(p_{1} / P\right)^{-a}{ }_{1}\right] / \partial p_{1}\right\}\right. \\
& \left.\left.-B_{i}\left(p_{1} / P\right)^{-a} \sum_{h} B_{h}\left\{\partial\left[\left(p_{h} / P\right)^{-a_{h}}\right] / \partial p_{1}\right\}\left[\Sigma_{h} B_{h}\left(p_{h} / P\right)^{-a}\right]_{h}^{-2}\right\}\right\} \tag{27}
\end{align*}
$$

where

$$
\left.\begin{array}{l}
\partial\left[\left(p_{1} / P\right)^{-a_{1}}\right] / \partial p_{1}
\end{array}=-a_{1}\left(p_{1} / P\right)^{-a_{1}-1}\left[\partial\left(p_{1} / P\right) / \partial p_{1}\right]\right] .
$$

Next, find $\partial\left[\left(p_{h} / P\right)^{-a}{ }_{h}\right] / \partial p_{1}$ in the $\sum_{h} B_{h} \partial\left[\left(p_{h} / P\right)^{-a_{h}}\right] / \partial p_{1}$ term of (24):
a). For $i \neq h$ : $\quad=-a_{h}\left(p_{h} / P\right)^{-a_{h}}{ }^{-1}\left[\partial\left(p_{h} / P\right) / \partial p_{1}\right]$

$$
\begin{align*}
& =-a_{h}\left(p_{h} / P\right)^{-a_{h}-1}\left[-p_{h} p^{-2}\left(\partial P / \partial p_{i}\right)\right] \\
\therefore & \partial\left[\left(p_{h} / P\right)^{-a_{h}}\right] /\left.\partial p_{i}\right|_{t=0}=\left.a_{h}\left(\partial P / \partial p_{i}\right)\right|_{t=0} \tag{29}
\end{align*}
$$

b). For $\mathrm{i}=\mathrm{h}$, the expression is the same as in (28).

Combining (28) and (29) we find:
$\left\{\left.\Sigma_{h} B_{h}\left\{\partial\left[\left(p_{h} / P\right)^{-a_{h}}\right] / \partial p_{i}\right\}\right|_{t=0}=\left\{\left.\left.\Sigma_{h} B_{h}\right|_{t=0} a_{h}\left(\partial P / \partial p_{i}\right)\right|_{t=0}\right\}-\left.a_{i} B_{i}\right|_{t=0}\right.$

Using (28) and (30) in (27) we get:

$$
\begin{aligned}
& \partial m_{i} /\left.\partial p_{i}\right|_{t=0}=m\left\{\left.B_{i}\right|_{t=0}\left(\left.\sum_{h} B_{h}\right|_{t=0}\right)^{-1}\left\{-a_{i}\left[1-\left.\left(\partial P / \partial p_{i}\right)\right|_{t=0}\right]\right\}\right. \\
& \left.-\left.B_{i}\right|_{t=0}\left\{\left[\left.\left.\sum_{h} B_{h}\right|_{t=0} a_{h}\left(\partial P / \partial p_{i}\right)\right|_{t=0}\right]-\left.a_{i} B_{i}\right|_{t=0}\right\}\left[\left.\sum_{h} B_{h}\right|_{t=0}\right]^{-2}\right\}
\end{aligned}
$$

With the use of (26) we have:

$$
=m w_{1}^{0}\left\{\left\{-a_{i}\left[1-\left.\left(\partial P / \partial p_{1}\right)\right|_{t=0}\right]\right\}-\left\{\left[\left.\sum_{h} w_{h}^{0} a_{h}\left(\partial P / \partial p_{i}\right)\right|_{t=0}\right]-a_{i} w_{1}^{0}\right\}\right\}
$$

$\therefore \partial m_{i} /\left.\partial p_{i}\right|_{t=0}=m_{i}^{0}\left[-a_{i}+a_{i} w_{i}^{0}+\left.a_{i}\left(\partial P / \partial p_{i}\right)\right|_{t=0}-\left.\sum_{h} w_{h}^{0} a_{h}\left(\partial P / \partial p_{i}\right)\right|_{t=0}\right]$

Finally, (6), with the use of (19) and the fact that $\operatorname{sign}\left(\delta_{i} \rho_{i}\right)$ is the same for all $\mathfrak{i}$ (see section 3.3 .2 of chapter 3 ), becomes:

$$
\partial P /\left.\partial p_{i}\right|_{t=0}=\left[a_{i}\left(\delta_{i} \rho_{i}\right)^{a}\right] /\left[\Sigma_{h} a_{h}\left(\delta_{h} \rho_{h}\right)^{a}{ }^{h}\right]
$$

dividing top and bottom by $\Sigma_{j}\left(\delta_{j} \rho_{j}\right)^{\mathrm{a}}$ ) and using (11) we obtain:

$$
\begin{equation*}
\partial P /\left.\partial p_{i}\right|_{t=0}=\left(w_{i}^{0} a_{i}\right) /\left(\Sigma_{h} w_{h}^{0} a_{h}\right) \tag{32}
\end{equation*}
$$

which, when substituted in (31) gives:

$$
\begin{gather*}
\partial m_{i} /\left.\partial p_{i}\right|_{t=0}=m_{i}^{0}\left\{-a_{i}+a_{i} w_{i}^{0}+a_{i}\left[\left(w_{i}^{0} a_{i}\right) /\left(\sum_{h} w_{h}^{0} a_{h}\right)\right]-\sum_{h} w_{h}^{0} a_{h}\left[\left(w_{i}^{0} a_{i}\right) /\left(\sum_{h} w_{h}^{0} a_{h}\right)\right]\right\} \\
\therefore \quad \partial m_{i} / \partial p_{i} l_{t=0}=m_{i}^{0} a_{i}\left\{-1+\left[\left(w_{i}^{0} a_{i}\right) /\left(\sum_{h} w_{h}^{0} a_{h}\right)\right]\right\} \tag{33}
\end{gather*}
$$

2). Let $i \neq h$, then:

$$
\begin{align*}
\partial m_{i} / \partial p_{h} & =m\left\{B _ { i } \left[\Sigma _ { j } B _ { j } ( p _ { j } / P ) ^ { - a } h ^ { - 1 } \left\{\partial\left[\left(p_{i} / P\right)^{-a} i d / \partial p_{k}\right\}\right.\right.\right.  \tag{34}\\
& \left.\left.-B_{i}\left(p_{i} / P\right)^{-a_{i}} \Sigma_{j} B_{j}\left\{\partial\left[\left(p_{j} / P\right)^{-a} j\right] / \partial p_{h}\right\}\left[\sum_{j} B_{j}\left(p_{j} / P\right)^{-a}\right]^{-2}\right\}\right\}
\end{align*}
$$

where, from (29), by interchanging the $i$ and $k$ subscripts, we have the result:

$$
\begin{equation*}
\partial\left[\left(p_{1} / P\right)^{-a_{i}}\right] /\left.\partial p_{h}\right|_{t=0}=\left.a_{1}\left(\partial P / \partial p_{h}\right)\right|_{t=0} \tag{35}
\end{equation*}
$$

Next, find $\partial\left[\left(p_{j} / P\right)^{-a}{ }_{j}\right] / \partial p_{h}$ in $\sum_{j} B_{j}\left\{\partial\left[\left(p_{j} / P\right)^{-a} j\right] / \partial p_{h}\right\}$ of (34):
a). For $h \neq j$ we have a result similar to (29) by substituting the subscripts $h$ and $i$ with $j$ and $h$ respectively:

$$
\begin{equation*}
\therefore \quad \partial\left[\left(p_{j} / P\right)^{-a a_{j}}\right] /\left.\partial p_{h}\right|_{t=0}=\left.a_{j}\left(\partial P / \partial p_{h}\right)\right|_{t=0} \tag{36}
\end{equation*}
$$

b). For $h=j$, the expression is the same as in (35).

Combining (35) and (36) we find:
$\left\{\Sigma,\left.B_{j}\left\{\partial\left[\left(p_{j} / P\right)^{-a}{ }_{h}\right] / \partial p_{h}\right\}\right|_{t=0}=\left[\left.\left.\sum_{j} B_{j}\right|_{t=0} a_{j}\left(\partial P / \partial p_{h}\right)\right|_{t=0}\right]-\left.a_{h} B_{h}\right|_{t=0}\right.$
Using (35) and (37) in (34) we get:

$$
\begin{align*}
& \partial m_{i} /\left.\partial p_{k}\right|_{t=0}=m\left\{\left.B_{i}\right|_{t=0}\left(\left.\sum_{j} B_{j}\right|_{t=0}\right)^{-1}\left[\left.a_{i}\left(\partial P / \partial p_{h}\right)\right|_{t=0}\right]\right.  \tag{38}\\
& \left.\quad-\left.B_{i}\right|_{t=0}\left\{\left[\left.\left.\sum_{j} B_{j}\right|_{t=0} a_{j}\left(\partial P / \partial p_{h}\right)\right|_{t=0}\right]-\left.a_{h} B_{h}\right|_{t=0}\right\} \quad\left(\left.\Sigma \beta_{j}\right|_{t=0}\right)^{-2}\right\}
\end{align*}
$$

With the use of (26) we have:

$$
\begin{array}{r}
=m w_{i}^{0}\left\{\left.a_{i}\left(\partial P / \partial p_{h}\right)\right|_{t=0}-\left[\left.\sum_{j} w_{j}^{0} a_{j}\left(\partial P / \partial p_{h}\right)\right|_{t=0}-a_{h} w_{h}^{0}\right]\right\} \\
\therefore \quad \partial m_{i} /\left.\partial p_{h}\right|_{t=0}=m_{i}^{0}\left[a_{h} w_{h}^{0}+\left.a_{i}\left(\partial P / \partial p_{h}\right)\right|_{t=0}-\left.\sum_{j} w_{j}^{0} a_{j}\left(\partial P / \partial p_{h}\right)\right|_{t=0}\right] \tag{39}
\end{array}
$$

Using (32) in (39) we obtain:

$$
\begin{equation*}
\left.\therefore \quad \partial m_{i} /\left.\partial p_{h}\right|_{t=0}=m_{1}^{0} a_{i}\left[\left(w_{h}^{0} a_{h}\right) /\left(\Sigma w_{j}^{0} a_{j}\right)\right]\right\} \tag{40}
\end{equation*}
$$

Thus, using (10'), (12), (22), (33) and (40) in (9) we obtain:

$$
\begin{align*}
& m_{i}=m^{0} w_{1}^{0}+w_{1}^{0}\left(m-m^{0}\right)+m_{i}^{0} c_{i}(t-0)-m_{i}^{0} a_{i}\left(p_{i}-1\right)+m_{i}^{0} a_{i} \Sigma_{h}\left(w_{h}^{0} a_{h} / a^{0}\right)\left(p_{h}-1\right) \\
& \therefore m_{i}=w_{i}^{0} m+m_{i}^{0} c_{i} t-m_{i}^{0} a_{i}\left[p_{i}-\Sigma_{h}\left(w_{h}^{0} a_{h} / a^{0}\right) p_{h}\right] \quad \forall i=1, \ldots, n . \tag{41}
\end{align*}
$$

where

$$
\begin{equation*}
\left.c_{1}=a_{i}\left[\gamma_{i}+\sum_{h}\left(w_{h}^{0} a_{h} / a^{0}\right)\left(\gamma_{h} / \rho_{h}\right)\right]-\sum_{h} w_{h}^{0}\left(\gamma_{h} / \rho_{h}\right)\right] \quad \forall i=1, \ldots, n . \tag{42}
\end{equation*}
$$

and

$$
\begin{equation*}
a^{0}=\Sigma_{h} w_{h}^{0} a_{h} \tag{43}
\end{equation*}
$$

These are equations (4.3) - (4.5) of section (4.1.1).

## Appendix 4.2.

The Underlying Problem of the Importer Given the Data Restrictions.

By substituting $P_{i j}^{k}$ as defined in (4.49), repeated here for convenience,

$$
\begin{equation*}
p_{i j}^{k}\left(\equiv p_{i}\right)=x p_{i}^{\alpha} w p_{k}^{(1-\alpha)} \tag{1}
\end{equation*}
$$

for $P_{i}$, in the bilateral import demand equations, we, essentialy, redefine the classical optimization problem of the importer as:

$$
\begin{equation*}
\min \sum_{i}\left(x p_{i}^{\alpha} w p_{k}^{(1-\alpha)}\right) m_{i} \quad \text { s.t. } \quad \sum_{i} \delta_{i} e^{\gamma_{i} t}\left(m_{i} / \mu\right)^{-\rho_{i}}-1 \equiv 0 \tag{2}
\end{equation*}
$$

From the solution of this problem (following the same procedures as we did in chapter 3 - the proof is easy and therefore omitted) we obtain bilateral import demand equations of the form:

$$
\begin{equation*}
m_{i}=\mu\left[\left|\delta_{i} \rho_{i}\right| e^{\gamma_{i}}\left(\times p_{i}^{\alpha} w p_{k}^{(1-\alpha)} / P\right)^{-1}\right]_{i}^{a_{i}} \tag{3}
\end{equation*}
$$

where the composite CRESH price index which guarantees adding-up is defined implicitly by:
$G\left(P, x p_{1}, \ldots, x p_{n}, W p_{k}, t\right)=\sum_{i} \delta_{i}\left[\left|\delta_{i} \rho_{i}\right| e^{\left(-\gamma_{i} / \rho_{i}\right) t}\left(x p_{i}^{\alpha} w p_{k}^{(1-\alpha)} / P\right)^{-1}\right]^{-a_{i} \rho_{i}}-1 \equiv 0$

It is easy, but messy, to show (following the same steps as in appendix 4.1 (That is, by a 1st order Taylor approximation around the base year values of the variables), and then with a 2nd linearization around the value $a=\sum_{i} w_{i} a_{i}$ of the parameters) that the empirical estimating equations are those presented in (4.37)-(4.39), with $\beta=0$.

## Digression:

In order to find the elasticities of substitution (in appendix
4.3) we need to derive $\partial P / \partial \times p_{i}, \partial P / \partial \times p_{h}$ and $\partial P / \partial w p_{k}$ at the base year values. These are defined as:

$$
\begin{align*}
& \partial P / \partial \times p_{i}=-\left(\partial G / \partial \times p_{1}\right) /(\partial G / \partial P) \\
& \partial P / \partial \times p_{h}=-\left(\partial G / \partial \times p_{h}\right) /(\partial G / \partial P)  \tag{5}\\
& \partial P / \partial w p_{k}=-\left(\partial G / \partial w p_{k}\right) /(\partial G / \partial P)
\end{align*}
$$

$$
\partial G /\left.\partial w p_{i}\right|_{t=0}=(1-\alpha) \delta_{i} a_{i} \rho_{i}\left|\delta_{i} \rho_{i}\right|^{-\rho_{i} a_{i}}
$$

$$
\begin{equation*}
\partial G /\left.\partial P\right|_{t=0}=-\sum_{h} \delta_{h} a_{h} \rho_{h}\left|\delta_{h} \rho_{h}\right|^{-\rho_{h} a_{h}} \tag{8}
\end{equation*}
$$

Using, also the result of equation (32) of appendix 4.1, we have:

$$
\begin{align*}
& \partial P / \partial \times\left. p_{i}\right|_{t=0}=\alpha\left[\left(\delta_{i} a_{i} \rho_{i}\left|\delta_{i} \rho_{i}\right|^{-\rho_{i} a_{i}}\right) /\left(\sum_{h} \delta_{h} a_{h} \rho_{h}\left|\delta_{h} \rho_{h}\right|^{-\rho_{h} a_{h}}\right)\right] \\
& \therefore \quad \partial P / \partial \times\left. p_{1}\right|_{t=0}=\alpha\left[w_{1}^{0} a_{i} / \Sigma_{h} w_{h}^{0} a_{h}\right]=\alpha\left[w_{i}^{0} a_{i} / a\right]  \tag{10}\\
& \partial P / \partial \times\left.\rho_{h}\right|_{t=0}=\alpha\left[\left(\delta_{h} a_{h} \rho_{h}\left|\delta_{h} \rho_{h}\right|^{-\rho_{h} a_{h}}\right) /\left(\Sigma_{1} \delta_{1} a_{1} \rho_{1}\left|\delta_{1} \rho_{1}\right|^{-\rho_{1} a_{1}}\right)\right] \\
& \therefore \quad \partial P / \partial \times\left. p_{h}\right|_{t=0}=\alpha\left[w_{h}^{0} a_{h} / \Sigma_{1} w_{1}^{0} a_{1}\right]=\alpha\left[w_{h}^{0} a_{h} / a\right]  \tag{11}\\
& \partial P /\left.\partial w p_{k}\right|_{t=0}=\alpha\left[\left(\delta_{i} a_{i} \rho_{i}\left|\delta_{i} \rho_{i}\right|^{-\rho_{i} a_{i}}\right) /\left(\Sigma_{h} \delta_{h} a_{h} \rho_{h}\left|\delta_{h} \rho_{h}\right|^{-\rho_{h} a_{h}}\right)\right] \\
& \therefore \quad \partial P /\left.\partial w p_{k}\right|_{t=0}=(1-\alpha)\left[w_{i}^{0} a_{i} / \sum_{h} w_{h}^{0} a_{h}\right]=(1-\alpha)\left[w_{i}^{0} a_{i} / a\right]  \tag{12}\\
& \text { where } a=\sum_{i} w_{i}^{0} a_{i} \text {. }
\end{align*}
$$

## Appendix 4.3.

The New Elasticities of Substitution Given the Data Restrictions.

We want to derive the own and cross export price elasticities with respect to a change in the total export price of the trade partners, and the world price elasticity with respect to a change in the world price of the traded good $k$, in some import market $j$.

These are easiest derived by taking logs of (3) in appendix 4.2.

$$
\begin{equation*}
\left.\ln m_{i}=\ln \mu+a_{i} \ln \left|\delta_{i} \rho_{i}\right|+\gamma_{i} t-a_{i}\left[\alpha \ln \times p_{i}+(1-\alpha) \ln w p_{k}-\ln P\right)\right] \tag{1}
\end{equation*}
$$

a). Thus, the own and cross export price elasticities with respect to a change in the total export price of the trade partners, are defined mathematically as:

$$
\begin{equation*}
e_{i i}^{x p}=\partial \ln m_{i} / \partial \ln x p_{h}, \quad i=h=1, \ldots, n \tag{2}
\end{equation*}
$$

From (1) we have:

$$
\begin{aligned}
& \partial \ln m_{i} / \partial \ln \times p_{i}=-a_{i}\left[\alpha \times p_{i}^{-1}-p^{-1}\left(\partial P / \partial \times p_{i}\right)\right] \\
& \partial \ln m_{i} / \partial \ln \times\left. p_{i}\right|_{t=0}=-a_{i}\left[\alpha-\left.\left(\partial P / \partial \times p_{i}\right)\right|_{t=0}\right]
\end{aligned}
$$

which, with the use of (10) of appendix 4.2, yields:

$$
\begin{equation*}
e_{i i}^{x p}=\left\{\alpha\left[\left(w_{i} a_{i}^{2} / a\right)-a_{i}\right]\right\}=\alpha e_{i i}, \quad i=1, \ldots, n \tag{3}
\end{equation*}
$$

From the definition of the cross price elasticity:

$$
\begin{aligned}
& \partial \ln m_{i} / \partial \ln \times p_{h}=a_{i} P^{-1}\left(\partial P / \partial \times p_{h}\right) \\
& \partial \ln m_{i} / \partial \ln \times\left. p_{h}\right|_{t=0}=\left.a_{i}\left(\partial P / \partial \times p_{i}\right)\right|_{t=0}
\end{aligned}
$$

which, with the use of (11) of appendix 4.2, yields:

$$
\begin{equation*}
e_{i h}^{x p}=\left\{\alpha\left[\left(a_{i} w_{h} a_{h} / a\right)\right]\right\}=\alpha e_{i h}, \quad i \neq h=1, \ldots, n \tag{4}
\end{equation*}
$$

b). The world price elasticity, with respect to a change in the world price of the traded good $k$, is defined by:

$$
\begin{array}{r}
e^{w p}=\partial \ln m_{i} / \partial \ln w p_{k}, \quad i=1, \ldots, n  \tag{5}\\
\therefore \quad \partial \ln m_{i} / \partial \ln w p_{k}=-a_{i}\left[(1-\alpha) w p_{k}^{-1}-p^{-1}\left(\partial P / \partial w p_{k}\right)\right] \\
\partial \ln m_{i} /\left.\partial \ln w p_{k}\right|_{t=0}=-a_{i}\left[(1-\alpha)-\left.\left(\partial P / \partial w p_{k}\right)\right|_{t=0}\right]
\end{array}
$$

which, with the use of (12) of appendix 4.2, yields:

$$
\begin{equation*}
e^{w p}=\left\{(1-\alpha)\left[\left(w_{i} a_{i}^{2} / a\right)-a_{i}\right]\right\}=(1-\alpha) e_{i i}, \quad i=1, \ldots, n \tag{6}
\end{equation*}
$$

## 5.0). Introduction.

In chapter 3 we derived sets of bilateral seaborne import demand equations, in terms of equation (3.11). Each of these systems of equations describe the bilateral seaborne import-flows of a region of the world from all other regions, for a certain commodity group. Ideally, we would like to obtain data for the theoretical variables described in (3.11). That is, for each commodity group we need:

- Volumes of bilateral seaborne trade-flows,
- the corresponding bilateral trade prices,
- the total volume of trade of each region,
- other relevant variables specific to the import region/good,
- intermediate variables needed to construct the final variables.

A feature of empirical international trade models of this type is that data for bilateral import prices are not available. Total import prices are used instead. We face a similar problem in this thesis. This has forced us in chapter 4 to respecify our theoretical model in order to take account of the lack of data in the empirical model.

This chapter is in three main sections. In the first, we describe the international systems which are used to classify commodities and regions of the world into aggregates, and are relevant to the objectives of our study. In the second, the seaborne bilateral and total volumes of trade used are described, and their peculiarities are examined. A snap-shot of world trade patterns is presented in terms of 5 tables of bilateral seaborne trade shares for 1980. In the third section, we explain the construction of the final data of import prices used in the estimation of the model. The
appropriateness of a number of alternative series of index numbers and the inherent problems involved are discussed. Other relevant and intermediate variables used in the construction of the model, and the transformations of the original data into the final dataset are also discussed here.
5.1). The SITC, MTC and MTCAC Classifications of the UN.

Data for Commodities traded internationally such as motorcars, salt, tobacco etc, are aggregated into groups in order to reduce the information into smaller more manageable proportions, which are easier to understand, compile and analyze. A classification system is required to aggregate commodities.

The most commonly known classification system is the UN (United Nations) SITC (Standard International Trade Classification) system (UN, 1951). This is a multi-digit system introduced in 1950, which is used to classify internationally traded commodities. Total trade is divided into 10 (1-digit) 'sections' as follows:

0 - Food and live animals chiefly for food
1 - Beverage and tobacco
2 - Crude materials, inedible, except fuel
3 - Mineral fuels and related materials
4 - Animal and vegetable oils, fats and waxes
5 - Chemicals and related products
6 - Manufactured goods by materials
7 - Machinery and transport equipment
8 - Miscellaneous manufactured articles
9 - Commodities not included elsewhere

Each of these 'sections' is further divided into 'divisions', 2-digit commodity aggregates, providing a greater detail of disaggregation. For example, 'section' 1 is disaggregated into 2 'divisions' as follows:
'Divisions' are subdivided into 3-digit 'groups', 'groups' into 4-digit 'subgroups' and 'subgroups' into 5-digit 'items', at the most detailed level.

The need soon became apparent to take account of changes in the volume and patterns of international trade over time, and to make the SITC compatible with other internationally used commodity classification systems. Such systems are the BTN (Brussels Tariff Nomenclature, Customs Co-operation Council(1955)), the CCCN (Customs Co-operation Council Nomenclature, Customs Co-operation Council(1976)), the HS (Harmonized System, Customs Co-operation Council(1985)), the CPC (Central Product Classification, UN(1988)), and the ISIC (International Standard Industrial Classification, UN(1968)). Thus, the 'original' SITC system is reshaped and extended gradually, to more and more headings of disaggregated commodities: Thus, SITC Revised (UN, 1961) revise the 'original' SITC in order to make it compatible with the BTN classification. The latter is classifying commodities according to the nature of the material commodities are made from. This makes the data from this classification not readily available for economic analysis. SITC Revised provides a one to one correspondence with the BTN. The former containes information for 1312 commodity aggregates.
Beginning with data in 1976, SITC Revision 2 (UN, 1975) was introduced to accommodate for changes in the volume, composition and geographical patterns of trade. The BTN is also revised resulting in the CCCN, under which there is a one to one correspondence betweem the CCCN and the SITC Revision 2's headings. The number of commodity aggregates in the SITC Revision 2 are extended to 1924.
The need for more detail and greater harmonization of commodity systems produce a revision of CCCN to give the HS. Along these lines, the SITC Revision 2 is harmonized with the HS, the ISIC and the CPC (A classification intended for use for all kinds of statistics; not for any specific reason, such as international trade) classifications. The resultant SITC Revision 3 (UN, 1985) is
in use since 1988. In this classification, greater disaggregation expands the number of basic headings to 3118 (10 'sections', 67 'divisions', 261 'groups', 1033 'subgroups', 720 of the latter divided further into 2805 basic 'items', which gives us 3118 basic headings in total).

Another commodity classification system, inore relevant to shipping analysts, is the MTC (Maritime Transport Classification) system (see for e.g. UN, 1980, p23-27). This is a 5 -digit system, which is used to classify total seaborne-trade according to the characteristics of its ocean carriage. Thus, total seaborne-trade is divided into 5 (1-digit) commodity sections shown in table 5.1.

Table 5.1.
Commodity Sections of the MTC.

```
1 - Bulk, Dry
2 - Bulk, Liquid
3-Refrigerated Goods
4 - General cargo, Dry
5 - Other Dry Cargo
```

These sections are divided into 37 3-digit 'categories', which are further subdivided into 128 5-digit commodity 'items' at the highest level of disaggregation. The full MTC commodity classification system is listed in appendix 5.1. The appendix includes the commodity break-down of seaborne-trade in terms of the MTC classification codes, and the corresponding codes for SITC Revised and SITC Revision 2.

In a similar way to commodity classification, the whole world is divided into 30 coastal regions along the lines of compatibility with geographical and national constraints. The segregation of the world into 30 major regions is defined by the UN MTCAC (Maritime Transport Area Classification) listed in appendix 5.2 (see for e.g. UN 1980, p28-30). These coastlines determine the major sea-lanes
over which international seaborne trade is conducted.

The seaborne exports/imports of landlocked countries are credited to the coastline which they use for their seaborne trade transactions. Landlocked countries are thus, classified under the coastal region which they use to conduct their trade. Some of these countries use more than one coastlines for their seaborne trade. For example, Switzerland uses both the ports of the North Sea and Italy. In that case the country is classified under both coastal regions.

Countries which use widely separated seaports, within their own boundaries, for their seaborne trade transactions have separate coastlines. This is essential for the definition of a sea-lane to be meaningful. Examples of such countries are the USSR, USA, Colombia etc (see appendix 5.2). In such a situation, one country is classified under more than one coastlines.

Along the above arguments, the $U N$ guidelines of appendix 5.2, and from the answers to questionnaires, which we have received from the embassies in London of landlocked countries, we have classified the countries of the world in the 30 coastline regions defined in appendix 5.2. The outcome is shown in Table 5.2. Some countries in certain regions are in brackets. The reason for that is explained later on in the chapter.

Table 5. 2.
Classification of Countries According to the MTCAC.

01 - Canada, USA.
02 - Canada.
03 - USA.
04 - USA.
05 - USA.
06 - USA.
07 - Canada, USA.
08 - Belize, Costa Rica, El Salvador, Guatemalla, Honduras, Mexico, Nicaragua, Panama.

11 - Antigua and Barbuda, Bahamas, Barbados, Dominica, Dominican Republic, Grenada, Haiti, Jamaica, Netherlands Antilles(that is Aruba, Bonaire, Curacao), St Kitts and Nevis, St Lucia, St Vincent, Trinidad and Tobago, (Anguilla, Bermuda, Cuba, Guadeloupe, Martinique, Virgin Islands).
12 - Colombia, Guyana, Surriname, Venezuela, (French Guiana).
13 - Argentina, Bolivia, Brazil, Paraguay, Uruguay, (Falklands, St Georgia) .

14 - Colombia, Chile, Ecuador, Peru.
15 - Iceland, Ireland, UK, (Faeroe Is land).
16 - Belgium, Denmark, Finland, Germany F.R., Hungary, Luxembourg, Netherlands, Norway, Sweden, Switzerland.

17 - Hungary, Poland, (Czechoslovakia, German D.R., USSR).
18 - France, Portugal, Spain.
19 - Austria, France, Greece, Hungary, Italy, Malta, Spain, Switzerland, Yugoslavia, (Albania, Andora, Gibraltar, Monaco).

20 - Romania, (Bulgaria, USSR).
21 - Cyprus, Israel, Lebanon, Syria, Turkey.
22 - Algeria, Egypt, Libya, Morocco, Tunisia.
23 - Benin, Cameroon, Central African Republic, Congo, Chad, Ivory Coast, Gabon, Gambia, Ghana, Guinea Bissau, Liberia, Mauritania, Mali, Niger, Nigeria, Senegal, Sierra Leone, Togo, Zaire, Zambia, (Angola, Equatorial Guinea, Guinea, Namibia, Sao Tome and Principe, Upper Volta).

24 - Lesotho, South Africa, Swaziland.
25 - Botswana, Burundi, Kenya, Madagascar, Malawi, Mauritius, Rwanda, Seychelles, Somalia, Swaziland, Tanzania, Uganda, Zaire, Zambia, Zimbabwe, (Comoros, Mozambique, Zanzibar).

26 - Egypt, Ethiopia, Israel, Jordan, Saudi Arabia, Sudan, Yemen, (Djibouti, Democratic Yemen).

27 - Bahrain, Iran, Iraq, Kuwait, Oman, Qatar, Saudi Arabia, United Arab Emirates.

28 - Afganistan, Bangladesh, Burma, India, Maldives, Nepal, Pakistan, Sri Lanka, (Bhutan).

29 - Indonesia, Malaysia, Phillipines, Singapore, Thailand, (Brunei Darussalam, Democratic Kampuchea, East Timor).

30 - China, (Laos, Dem. People's Republic of Korea, USSR, Viet Nam).
31 - Hong Kong, Japan, Republic of Korea, (Macau).
32 - Australia, Fiji, New Zealand, Papua New Guinea, Solomon Islands, Western Samoa, Vanuatu, (Kiribati, Nauru, New Hebrides, Tonga, Tuvalu).

33 - Unspecified.

Data for Volumes (in metric tons), Ton-miles and ALH (Average Length of Haul, in nautical miles) of bilateral and total seaborne trade flows have been obtained from the UN Maritime Tansport Study (called 'International trade statistics yearbook' after 1983, see for example UN(1983)), for the years 1969 to 1986. The published data can be thought of as matrices of the form of table 2.1, in chapter 2. The importing-exporting regions of the table are the coastal areas specified in table 5.2, while the different commodities are the types of cargo listed in table 5.1. The published data have been compiled by collecting information of seaborne trade from coastline countries of the world (85 in 1982). The main features of this compilation, the limitations of the study and the accuracy of the results are described in the volumes of the Maritime Transport Study (see for example chapters II-IV).

The main limitations of the study are the same as those encountered in international trade statistics, plus those specific to seaborne trade statistics (Chapter III). For example import matrices for a country may be different from export matrices because of the different trade systems used in different countries (general, special etc). c.i.f., f.o.b. differences, common in trade values, are not a problem here since we are dealing with volumes of trade. Another limitation of the study is the 2-3 years time lag in publishing the data.

Dividing the world in 30 coastlines and distinguishing amongst 5 types of cargo, amounts to considering 30 bilateral seaborne trade flows for each coastline (the intra-seaborne trade of the importing region plus the 29 flows from the other regions), per type of cargo. That is, $(30 \times 5=) 150$ bilateral trade flows for each region. The bilateral trade of each region should add-up to the total trade of the region, for the specific type of cargo ('commodity'). This is identity (2.2) of chapter 2, and should hold for each year in the time period considered. For the whole world, we obtain data for bilateral trade volumes, over the 18 years.)

By definition, a number of identities should hold for these data. These are described in section 2.1, in terms of the import-export matrix of table 2.1. Thus, apart from identity (2.2), (2.3) and (2.4) should also hold for our data. (2.3) amounts to the sum of the total imports of all regions, per commodity, adding-up to total world imports (for that commodity), for every year. (2.4) amounts to the sum of the total world imports of each commodity adding-up to total world imports for all commodities.

In terms of estimating our import allocation models (with respect to the bilateral imports of one region for one commodity), it suffices for identity (2.2) to hold. That is, the bilateral imports of a region for a certain commodity should add-up to the total imports of the commodity by that region.

The nature of our study, estimation of highly disaggregated import allocation models of 5 goods and 30 trade zones in the international economy has consequences for our data. An importing region has a pre-specified total amount of imports, of a particular commodity, which it wishes to allocate amongst its 30 trade partners (including intra-trade). The number of alternative sources of imports for most goods is large (30), with no one region predominating trade internationally (for most goods, with the exception of Bulk Liquid). Thus, often, seaborne imports from trade partners at certain time periods are zero. Outliers and zero trade are a common feature in the data. This produces very volatile time series, increasing the convergence problems of the nonlinear estimating systems.

Often in import markets, there are no seaborne imports over the entire period from one or more regions. The consequences for estimation is that these regions are dropped from the system and we have less than 30 equations to estimate. This is quite common for Good 2 (not many regions produce Bulk Liquid goods) and Good 3. Also, there is no seaborne trade in the import regions $03,04,05$,

06 from 03, 05 and 06, for all goods (freight is transported by land). It is important to note here, that even though there are situations when there is trade between two trade partners for only one year, the equation is kept in the system. This is because, even one years' trade amongst two regions implies that the particular exporting region is in the information set of the relevant importer, albeit not offering relatively 'good' terms of trade for more than one year over the considered period. Statistically, keeping the equation in the system ensures adding-up (that is identity (2.2) holds), for every time period.

Finally, we may obtain a picture of the patterns of seaborne trade by looking at the base-year (1980) import shares for the 5 commodities, shown on tables 5.3-5.7. The columns of these seaborne import share tables are used as the base-year weights $\left(w_{i}^{0}\right)$ in the estimation of the individual models (see for example (4.50)-(4.52)). These shares are also used in the calculation of the elasticities (see (4.53)-(4.55)).

A number of patterns of trade in the international economy are revealed by observing the seaborne import share tables.

We observe no one region predominating trade for most goods. That is, there is no single exporter which has monopolistic power internationally. The exception is for good 2(Bulk Liquid), particularly regions 27 (Persian Gulf) and 29 (South-East Asia), and also regions 11 (Caribbean Area) and $12(\mathrm{~N}$. Coast of South America), which capture the largest shares in the international import markets.

For, good 1(Bulk Dry), region 05 (US Gulf) is in a strong position internationally

There is a high export penetration of region 08(Central America) in the neighboring 03, 04, 05 and 06 (USA) for good 3(Refrigerated goods) and other goods to a lesser extent.

The strong export position of the industrialized regions $16,19,15$ (Europe), and 31(Far East Asia), for good 5(Other Dry Cargo) in the world markets is to be noticed.

| Table 5.3. |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PARTNER I | 01 | 02 | 03 | 04 | 05 | 06 | 07 | 08 | 11 | 12 | 13 | 14 | 15 | MPPORTING REGION J |  |  | $18$ | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  | 16 | 17 | 18 |  |  |  |  |  |  |  |  |  |  | 29 | 30 | 31 | 32 |
| 01 | 77.365 | 29.699 | 0.3312 | 0.2559 | 0.0773 | 0.0466 | 2.2398 | 0.9037 | 4.3820 | 3.7538 | 1.5920 | 1.2792 | 1.4471 | 1.0481 | 1.5872 | 1.2669 | 1.5920 | 0.5068 | 0.0261 | 1.3517 | 0.9246 | 3.9336 | 0.0000 | 0.1749 | 0.5589 | 0.8607 | 0.4925 | 0.5032 | 0.1319 | 0.0078 |
| 02 | 16.584 | 0.0538 | 42.245 | 30.087 | 8.7502 | 0.0694 | 1.3880 | 1.2800 | 18.159 | 1.5053 | 7.0105 | 0.6557 | 12.615 | 5.8785 | 6.5102 | 3.9115 | 6.6440 | 3.7120 | 4.1995 | 4.5976 | 6.3878 | 0.4195 | 0.6770 | 2.2598 | 2.6016 | 1.7378 | 1.3517 | 2.3337 | 0.7573 | 0.5614 |
| 03 | 0.0147 | 3.2889 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0858 | 3.4985 | 1.4798 | 3.0678 | 16.492 | 7.9246 | 9.2417 | 6.1640 | 3.3837 | 14.040 | 12.842 | 6.2784 | 10.482 | 9.7834 | 0.2172 | 2.5670 | 4.0844 | 0.9394 | 0.5564 | 1.5753 | 0.7856 | 2.3402 | 5.6051 | 0.4256 |
| 04 | 0.0185 | 0.8477 | 0.0000 | 0.0000 | 0.0079 | 0.0000 | 1.0427 | 1.3978 | 5.4064 | 2.6795 | 0.8157 | 2.3739 | 0.2685 | 0.5864 | 0.7978 | 0.4884 | 0.7479 | 0.7206 | 1.4331 | 0.3717 | 0.2608 | 1.8273 | 1.2885 | 0.1648 | 0.0531 | 1.2716 | 0.14170 | 0.7975 | 0.2482 | 0.5330 |
| 05 | 0.3488 | 11.919 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 6.1037 | 50.837 | 20.445 | 43.628 | 23.377 | 45.125 | 3.3133 | 12.430 | 8.3934 | 6.9120 | 7.7293 | 14.851 | 19.130 | 8.5081 | 20.191 | 20.498 | 20.639 | 10.434 | 4.7163 | 11.971 | 4.1471 | 13.766 | 6.1326 | 5.8897 |
| 06 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.2952 | 5.2201 | 0.0010 | 0.2734 | 0.0589 | 3.7936 | 0.0372 | 0.2290 | 0.0699 | 0.0323 | 0.1714 | 0.0410 | 0.0169 | 0.3016 | 0.0061 | 4.1841 | 0.0020 | 0.3474 | 0.0907 | 1.2341 | 0.7200 | 2.9270 | 2.3106 | 2.4525 |
| 07 | 0.0849 | 0.0000 | 2.9519 | 3.2315 | 0.2358 | 5.0462 | 34.828 | 7.0780 | 1.3895 | 1.5764 | 7.4211 | 6.3834 | 2.0469 | 0.9217 | 1.6910 | 0.3879 | 1.5818 | 0.8961 | 0.4442 | 6.3786 | 3.9054 | 23.859 | 4.8869 | 0.6241 | 1.9909 | 15.489 | 8.1786 | 13.844 | 14.079 | 13.268 |
| 08 | 0.0677 | 0.9280 | 1.6266 | 0.8314 | 2.5157 | 20.898 | 12.875 | 0.0820 | 1.3619 | 0.5166 | 0.2968 | 0.4502 | 0.1155 | 0.1420 | 0.1413 | 0.1383 | 0.1256 | 0.0000 | 0.1217 | 0.0601 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.1117 | 0.0000 | 0.0000 | 1.0545 | 0.0577 |
| 11 | 0.3654 | 7.0724 | 7.6389 | 13.133 | 33.350 | 1.2490 | 1.1144 | 0.3620 | 3.7074 | 9.2372 | 0.1825 | 0.1479 | 1.9092 | 0.4313 | 6.1410 | 0.2341 | 0.2320 | 0.8657 | 1.0707 | 1.5779 | 2.8499 | 0.0000 | 0.0173 | 0.2750 | 1.4878 | 0.0361 | 0.2027 | 1.2215 | 0.0714 | 0.8622 |
| 12 | 0.2323 | 0.6893 | 12.868 | 1.7464 | 7.3468 | 0.2274 | 0.3052 | 2.3368 | 11.548 | 2.3658 | 0.7621 | 0.2093 | 1.9343 | 1.8146 | 0.7357 | 1.3639 | 2.3379 | 0.7503 | 0.0000 | 0.0156 | 0.1039 | 0.6350 | 0.1553 | 0.0001 | 0.0015 | 0.1165 | 0.0145 | 0.0000 | 0.0749 | 0.0694 |
| 13 | 0.3825 | 13.712 | 5.7488 | 9.8864 | 6.8326 | 2.3741 | 0.6178 | 3.2022 | 10.659 | 2.3231 | 18.119 | 9.6666 | 4.3621 | 11.351 | 27.160 | 10.504 | 10.514 | 6.0103 | 9.3882 | 2.3514 | 1.7735 | 5.4528 | 1.1759 | 0.9285 | 5.0883 | 2.6019 | 8.22623 | 3.6069 | 7.9943 | 0.0643 |
| 14 | 0.0000 | 0.2364 | 2.0733 | 2.7349 | 3.0002 | 0.5447 | 0.4955 | 1.2889 | 1.1905 | 2.1794 | 1.9394 | 2.4271 | 0.1861 | 0.4702 | 0.1245 | 0.1986 | 0.7316 | 1.3347 | 0.3230 | 0.1637 | 0.0507 | 0.0063 | 0.1635 | 0.4772 | 1.3720 | 0.0979 | 0.12820 | 0.3976 | 2.2553 | 0.0005 |
| 15 | 0.3405 | 1.4788 | 0.4442 | 0.0822 | 0.3792 | 0.6476 | 0.5305 | 0.1954 | 0.9694 | 0.6371 | 0.4213 | 0.1623 | 3.8383 | 6.1674 | 2.0061 | 6.1924 | 2.3138 | 0.5253 | 0.9604 | 1.2238 | 3.2528 | 4.4515 | 1.1205 | 0.6440 | 0.8512 | 1.4730 | 0.41970 | 0.0783 | 0.0182 | 0.4593 |
| 16 | 1.9687 | 3.2807 | 3.7337 | 11.811 | 3.3862 | 10.242 | 1.3395 | 4.6821 | 2.5082 | 4.7377 | 4.8705 | 1.9382 | 23.443 | 18.846 | 22.203 | 9.8309 | 5.2582 | 4.2198 | 9.5532 | 15.121 | 13.759 | 14.371 | 9.4680 | 12.896 | 15.457 | 11.983 | 4.00822 | 2.1684 | 0.1447 | 0.7888 |
| 17 | 0.0140 | 0.0972 | 0.2339 | 0.9001 | 0.9971 | 0.0000 | 0.0015 | 0.0027 | 7.7813 | 2.3366 | 6.3251 | 0.2487 | 5.0143 | 7.4287 | 3.5671 | 6.5707 | 3.4719 | 0.0000 | 0.5461 | 1.8924 | 0.2194 | 0.0000 | 0.1733 | 1.0447 | 0.7302 | 1.5959 | 0.5180 | 0.5437 | 0.1222 | 0.0047 |
| 18 | 0.4657 | 0.3296 | 0.5555 | 0.5813 | 1.0808 | 1.1607 | 0.2256 | 1.0055 | 2.5101 | 0.6426 | 0.6883 | 0.5094 | 8.0621 | 2.7498 | 4.4250 | 1.1008 | 1.4629 | 3.8440 | 2.9288 | 13.268 | 12.520 | 0.2789 | 5.9412 | 4.8646 | 6.0916 | 2.1814 | 0.04330 | 0.8350 | 0.0199 | 0.2886 |
| 19 | 0.1839 | 2.5397 | 0.6795 | 5.0365 | 1.4536 | 0.3283 | 0.3891 | 3.1912 | 0.9025 | 4.9074 | 1.1813 | 0.7112 | 2.2951 | 0.9380 | 0.0634 | 1.0100 | 3.8678 | 7.6731 | 19.034 | 15.387 | 9.3868 | 8.2153 | 7.8293 | 7.8866 | 9.1730 | 3.1141 | 0.8804 | 1.3617 | 0.0837 | 0.1925 |
| 20 | 0.0037 | 0.0053 | 0.0059 | 1.6373 | 0.3156 | 0.0000 | 0.0000 | 0.0000 | 1.1220 | 0.0003 | 0.1959 | 0.1912 | 0.1510 | 0.0950 | 0.0000 | 1.2926 | 5.0352 | 24.337 | 5.9385 | 4.6375 | 0.0797 | 0.0000 | 0.4494 | 3.1894 | 0.3970 | 4.4639 | 0.6770 | 0.0000 | 0.0065 | 0.0259 |
| 21 | 0.0000 | 0.0142 | 0.6045 | 4.4928 | 0.4384 | 0.1113 | 0.0015 | 0.0540 | 0.0033 | 0.0004 | 0.9330 | 0.0000 | 0.4716 | 0.3874 | 0.1738 | 0.8500 | 1.4129 | 1.9931 | 1.3914 | 1.0905 | 0.0646 | 0.0736 | 0.9614 | 0.4908 | 0.1357 | 0.0716 | 0.17990 | 0.0226 | 0.0565 | 0.0174 |
| 22 | 0.0080 | 0.1716 | 0.2868 | 0.0006 | 1.5565 | 0.0537 | 0.0000 | 6.0553 | 0.5337 | 0.7935 | 1.9921 | 0.0000 | 2.3810 | 2.0730 | 2.9317 | 5.5422 | 4.8838 | 3.3842 | 4.3356 | 0.6635 | 2.1090 | 0.0061 | 0.0000 | 0.2071 | 1.2537 | 3.5718 | 0.01060 | 0.8610 | 0.1728 | 0.2446 |
| 23 | 0.0771 | 18.330 | 7.2284 | 0.2834 | 8.5590 | 0.0019 | 0.0011 | 0.6362 | 0.0003 | 0.1773 | 0.2040 | 0.0056 | 3.3454 | 6.3753 | 5.2489 | 9.6985 | 10.869 | 2.8567 | 1.9834 | 0.6548 | 4.8360 | 0.1748 | 0.1120 | 0.0410 | 0.0091 | 0.5016 | $\begin{array}{lll} 6 & 0.39390 \\ 8 & 0.1592 & 0 \end{array}$ | 0.0847 | 0.5117 | 0.0233 |
| 24 | 0.4302 | 2.5234 | 1.1924 | 1.9822 | 4.6899 | 2.3838 | 0.5431 | 0.0359 | 0.0084 | 3.9085 | 0.2284 | 1.1575 | 1.9786 | 6.6124 | 0.0036 | 13.396 | 6.2077 | 0.0372 | 0.1421 | 0.0658 | 0.0187 | 0.0000 | 0.6082 | 0.1355 | 0.0400 | 0.5568 |  | 0.0000 | 4.5291 | 1.4966 |
| 25 | 0.0173 | 0.1428 | 0.9425 | 2.0395 | 1.2568 | 0.0001 | 0.0000 | 0.0000 | 0.0012 | 0.2677 | 0.0085 | 0.0350 | 1.4352 | 0.0702 | 0.0168 | 0.2142 | 0.2624 | 0.1073 | 0.0833 | 0.1419 | 0.2226 | 0.9635 | 11.526 | 1.6769 | 0.2413 | 0.4065 | $\begin{array}{lll} 8 & 0.1592 & 0 . \\ 5 & 2.2622 & 0 . \end{array}$ | 0.0394 | 0.0412 | 0.0246 |
| 26 | 0.0000 | 0.0000 | 0.1245 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0008 | 0.0008 | 0.0000 | 0.0000 | 0.0000 | 0.1183 | 0.0837 | 0.5884 | 0.0485 | 0.3097 | 1.1604 | 3.6481 | 0.0750 | 0.0008 | 0.0000 | 2.3800 | 4.5731 | 0.1525 | 5.5646 | $\begin{array}{lll} 6 & 1.1888 & 0 . \\ 8 & 0.3246 & 0 . \end{array}$ | $\begin{aligned} & 0.4454 \\ & 0.3897 \end{aligned}$ | 0.1361 | 0.0008 |
| 27 | 0.0000 | 0.0036 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0023 | 0.0000 | 0.0003 | 0.0000 | 0.0612 | 0.0108 | 0.0235 | 0.0022 | 0.0491 | 0.0000 | 0.0810 | 0.5227 | 0.0000 | 0.0127 | 0.5369 | 0.7228 | 4.36646 | 6.9918 |  |  | 0.0207 | 0.1089 |
| 28 | 0.0285 | 0.0109 | 0.1429 | 0.0520 | 0.5003 | 0.0731 | 0.0626 | 0.1932 | 0.9907 | 0.0013 | 0.2338 | 0.4329 | 0.3776 | 0.4312 | 1.4785 | 0.2684 | 1.0679 | 11.972 | 0.4665 | 0.1222 | 2.7862 | 0.0839 | 6.1946 | 1.9067 | 4.3749 | 1.5489 | $\begin{aligned} & 0.3246 \\ & 1.4664 \end{aligned}$ | 1.5554 | 4.9661 | 0.0965 |
| 29 | 0.0000 | 0.4412 | 2.1136 | 1.2256 | 0.6529 | 0.8830 | 0.7257 | 0.2681 | 0.4629 | 0.1089 | 0.5747 | 0.1494 | 0.4709 | 1.1574 | 0.0304 | 0.4534 | 0.6258 | 0.0170 | 0.2435 | 0.4580 | 6.0687 | 0.1448 | 7.4999 | 9.6571 | 7.0687 | 2.7148 | $\begin{array}{lll} 9 & 1.4664 & 1 . \\ 3 & 18.389 & 3 . \end{array}$ | 3.4922 | 9.3895 | 3.3441 |
| 30 | 0.0000 | 0.0000 | 0.1312 | 0.0232 | 0.8596 | 0.1100 | 0.1770 | 0.0189 | 0.0483 | 0.0225 | 0.0000 | 0.0077 | 0.3057 | 0.3156 | 0.4039 | 0.1408 | 0.3214 | 0.0000 | 0.0473 | 0.0694 | 0.1909 | 0.0000 | 0.9167 | 2.0357 | 0.5738 | 1.4651 | 13.7552 | 0.1491 | 3.4432 | 0.3907 |
| 31 | $0.6145$ | 0.2721 | 2.9180 | 4.2481 | 8.9973 | 38.150 | 9.7614 | 6.0995 | 2.3842 | 8.3438 | 3.4626 | 11.271 | 0.6523 | 0.2557 | 0.0917 | 0.2060 | 0.7972 | 1.1292 | 1.8525 | 1.7057 | 7.7988 | 7.5699 | 9.5596 | 22.406 | 17.6018 | 8.9950 | $\begin{array}{r} 0 \quad 26.3621 \\ 6 \quad 14.571 \quad 3 \\ \hline \end{array}$ | 15.226 | 2.9092 | 18.230 |
| 32 | 0.3825 | 1.9095 | 3.1785 | 3.6950 | 2.7590 | 15.397 | 23.850 | 0.0724 | 0.0376 | 0.0085 | 0.6103 | 2.7419 | 8.1222 | 4.5330 | 0.0061 | 3.7015 | 6.5309 | 0.7731 | 0.1255 | 7.4361 | 0.0127 | 0.2692 | 1.6335 | 8.9940 | 12.9635 | 5.6926 |  | $31.009$ | 32.711 | 50.068 |


| Table 5 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | MAT | TRIX | OF S | A | R | E IM | OR | S | R | F | R | R | ERA | E | GOO | S | 980). |  |  |  |  |  |  |  |
| PARTNER |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 01 | 02 | 03 | 04 | 05 | 06 | 07 | 08 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
| 01 | 71.119 | 0.0028 | 0.0050 | 0.0014 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2127 | 0.0000 | 0.0000 | 0.1723 | 0.1835 | 0.3897 | 0.0000 | 0.6363 | 0.0007 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0264 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0004 | 0.0000 |
| 02 | 0.0135 | 2.0521 | 1.4987 | 0.0034 | 0.0000 | 0.0000 | 0.4899 | 0.8288 | 17.704 | 20.017 | 4.9371 | 2.5905 | 1.2478 | 2.1333 | 0.1145 | 1.5369 | 1.0264 | 0.0216 | 0.3603 | 7.3915 | 6.7366 | 4.1748 | 0.1270 | 0.0000 | 0.0429 | 0.1762 | 0.1715 | 0.0000 | 0.8903 | 0.9897 |
| 03 | 0.1694 | 6.9328 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0114 | 4.1801 | 4.0589 | 0.3345 | 0.3446 | 0.6038 | 0.3595 | 2.0544 | 0.0327 | 1.1427 | 0.2705 | 0.3869 | 1.4176 | 0.7718 | 0.7068 | 1.9142 | 0.0130 | 0.1355 | 0.3414 | 0.0057 | 0.1799 | 0.0000 | 0.5947 | 0.1067 |
| 04 | 0.0000 | 0.0017 | 0.0000 | 0.0096 | 0.0000 | 0.0000 | 0.0000 | 20.610 | 18.482 | 25.125 | 0.3043 | 0.4538 | 0.0502 | 1.1138 | 0.0643 | 1.5795 | 0.6473 | 0.0000 | 0.0729 | 0.2131 | 0.4388 | 0.9438 | 0.0235 | 0.0710 | 0.1364 | 0.0000 | 0.1058 | 0.0000 | 1.5562 | 0.1969 |
| 05 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.8716 | 0.9489 | 2.0035 | 0.3997 | 1.2393 | 0.3085 | 1.4706 | 0.0000 | 1.5906 | 0.0887 | 0.5983 | 7.0818 | 6.8424 | 0.7416 | 7.2708 | 0.0013 | 3.4034 | 0.4537 | 0.0000 | 0.0066 | 0.0000 | 6272 | 0.0074 |
| 06 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.2738 | 2.2081 | 0.0056 | 2.6314 | 0.2675 | 1.7502 | 0.7588 | 2.3771 | 0.3960 | 1.3235 | 0.9214 | 0.0000 | 0.0412 | 0.0000 | 0.0000 | 2.0675 | 0.0392 | 0.5206 | 0.5829 | 0.1936 | 8.0855 | 0.0000 | 15.221 | 6.5491 |
| 07 | 1.5456 | 0.0000 | 0.0158 | 0.0000 | 0.0000 | 0.0174 | 18.325 | 0.0713 | 0.0001 | 4.8219 | 0.6608 | 4.9844 | 0.2960 | 0.7898 | 0.0000 | 0.7853 | 0.2195 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.3886 | 0.0000 | 2.7351 | 0.9319 | 0.0606 | 0.5491 | 0.9351 | 7.6555 | 1.4296 |
| 08 | 0.0000 | 0.7514 | 38.216 | 54.640 | 72.622 | 29.084 | 0.0418 | 6.6590 | 0.1990 | 0.0338 | 4.8898 | 0.0163 | 0.4788 | 16.785 | 0.0000 | 2.5915 | 13.775 | 0.0000 | 1.1911 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 3.3288 | 0.2358 | 0.0000 | 0.0013 | 0.0000 | 0.5362 | 0.0544 |
| 11 | 0.0000 | 1.3600 | 0.3679 | 7.8476 | 0.0153 | 0.0138 | 0.0000 | 6.1353 | 2.7095 | 0.1084 | 0.0000 | 0.0097 | 2.3817 | 0.1661 | 16.712 | 7.3179 | 0.8128 | 0.0000 | 0.0000 | 0.0000 | 0.0486 | 0.0290 | 0.0000 | 0.0088 | 0.0153 | 0.0000 | 0.0022 | 0.0000 | 0.0667 | 0.0082 |
| 12 | 0.0000 | 0.0000 | 3.3906 | 5.1281 | 17.151 | 0.0000 | 0.0000 | 0.0144 | 5.3677 | 1.8244 | 2.6888 | 0.0402 | 2.3552 | 2.0022 | 9.8610 | 6.6413 | 3.3984 | 1.9185 | 4.6361 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0280 | 0.0080 | 0.0000 | 0.0001 | 0.0000 | 0.1188 | 0.0000 |
| 13 | 0.0000 | 1.0837 | 1.5942 | 3.8074 | 0.0589 | 0.3301 | 0.3462 | 2.2560 | 0.3548 | 0.7392 | 53.422 | 15.889 | 0.6205 | 6.0491 | 13.462 | 2.5918 | 5.5385 | 0.3352 | 6.4556 | 4.4748 | 8.3636 | 2.3293 | 0.0000 | 6.8590 | 7.9187 | 0.0000 | 0.0306 | 0.0000 | 1.4513 | 0.0411 |
| 14 | 0.0000 | 0.0211 | 17.106 | 2.5598 | 6.6215 | 38.395 | 3.1042 | 3.7222 | 0.0907 | 3.8727 | 11.064 | 56.263 | 1.2450 | 5.1939 | 13.248 | 2.8000 | 11.620 | 0.0000 | 5.3964 | 0.0764 | 0.5393 | 2.8554 | 0.0013 | 7.4055 | 4.1124 | 0.3337 | 0.0017 | 1.0903 | 0.9742 | 11.708 |
| 15 | 0.0087 | 2.2559 | 4.8118 | 0.0674 | 0.0039 | 0.0192 | 0.2559 | 7.9680 | 3.8702 | 0.3768 | 0.2318 | 2.2910 | 7.7717 | 9.3693 | 14.280 | 17.982 | 4.5588 | 3.4469 | 8.2849 | 13.468 | 15.032 | 2.1363 | 0.5750 | 0.2184 | 0.6354 | 0.1820 | 0.4880 | 0.0000 | 0.4885 | 0.3736 |
| 16 | 1.8371 | 21.415 | 5.2203 | 1.0302 | 0.1504 | 0.7129 | 3.5209 | 16.941 | 11.180 | 25.508 | 18.693 | 8.9434 | 20.557 | 20.089 | 15.288 | 6.7020 | 7.3405 | 1.1199 | 27.579 | 28.800 | 23.920 | 7.0858 | 11.071 | 6.9696 | 15.615 | 38.388 | 4.3754 | 0.1483 | 2.1700 | 2.1444 |
| 17 | 0.0000 | 0.0000 | 0.3788 | 0.0000 | 0.0000 | 0.0000 | 0.0019 | 2.8231 | 13.390 | 0.0018 | 0.0000 | 0.1114 | 0.1228 | 0.3826 | 0.0000 | 1.5193 | 0.2006 | 0.0000 | 4.1282 | 1.1220 | 2.9042 | 0.0000 | 0.0000 | 0.1622 | 0.1094 | 0.3901 | 0.0679 | 0.0000 | 0.1172 | 0.0005 |
| 18 | 0.0087 | 4.1755 | 0.5178 | 0.0630 | 0.0048 | 0.1285 | 0.6583 | 2.9278 | 15.756 | 4.0282 | 0.9829 | 0.9610 | 26.502 | 2.6848 | 4.9019 | 0.3553 | 0.7781 | 14.911 | 1.2952 | 11.133 | 11.104 | 1.2479 | 21.425 | 10.256 | 5.7133 | 6.5168 | 0.7630 | 0.0000 | 0.1987 | 1.0412 |
| 18 | 3.4912 | 6.3864 | 0.7821 | 0.3470 | 0.0064 | 0.2797 | 0.3367 | 0.1180 | 0.3314 | 1.3229 | 0.3466 | 0.0701 | 6.5802 | 2.5284 | 5.6536 | 0.2432 | 2.7416 | 15.161 | 6.6384 | 16.906 | 7.3689 | 3.1304 | 12.417 | 9.4190 | 5.6884 | 2.2064 | 0.0717 | 0.0062 | 1.1121 | 0.8752 |
| 20 | 0.0000 | 0.0365 | 0.1036 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0171 | 0.0198 | 0.0000 | 0.1264 | 0.2621 | 14.806 | 0.0000 | 3.5633 | 7.3391 | 0.0000 | 20.617 | 3.0116 | 0.6752 | 0.0000 | 0.0008 | 0.0000 | 0.0051 | 0.3002 |
| 21 | 0.0000 | 1.3949 | 0.2429 | 0.0137 | 0.0152 | 0.0934 | 0.0000 | 0.0000 | 0.0138 | 0.0006 | 0.0007 | 0.0000 | 11.566 | 8.9625 | 3.0985 | 1.5784 | 12.302 | 9.8352 | 9.5578 | 2.8973 | 0.0380 | 0.0000 | 0.0799 | 9.8952 | 5.9594 | 0.1459 | 1.2291 | 0.0000 | 0.1161 | 0.0664 |
| 22 | 0.0271 | 3.3847 | 0.0277 | 0.0032 | 0.0000 | 0.0000 | 0.0000 | 0.0167 | 0.0009 | 0.0000 | 0.0000 | 0.0000 | 3.4125 | 5.6215 | 1.4584 | 12.402 | 10.401 | 37.217 | 11.989 | 0.0952 | 0.5420 | 0.0000 | 0.0000 | 7.4598 | 2.0770 | 0.0000 | 0.0687 | 0.0000 | 0.2102 | 0.0280 |
| 23 | 0.0000 | 1.8414 | 0.0665 | 6.9244 | 0.0101 | 0.0000 | 0.0000 | 2.6805 | 0.0000 | 0.0024 | 0.0000 | 0.9316 | 0.4593 | 0.5956 | 0.0000 | 5.7381 | 8.2717 | 0.0926 | 0.0016 | 0.1062 | 7.0665 | 0.2247 | 0.0013 | 0.0359 | 0.0047 | 0.0014 | 0.0000 | 0.0000 | 0.1763 | 0.2298 |
| 24 | 10.283 | 12.857 | 0.5451 | 2.5968 | 0.0172 | 0.2736 | 0.1874 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0211 | 4.9371 | 5.5977 | 0.0000 | 2.6739 | 3.6078 | 0.0000 | 0.0000 | 0.0000 | 0.5408 | 0.0000 | 8.8474 | 4.3429 | 1.8751 | 0.0939 | 0.0001 | 0.0000 | 0.3342 | 2.2050 |
| 25 | 0.0000 | 0.0205 | 0.0023 | 1.3938 | 0.0033 | 0.0000 | 0.0000 | 0.0000 | 0.0003 | 0.0024 | 0.0000 | 0.0000 | 0.3530 | 0.1317 | 0.7895 | 0.2182 | 2.7786 | 0.0001 | 0.0049 | 0.0026 | 0.0826 | 3.4609 | 3.2997 | 0.7780 | 1.0214 | 1.5909 | 0.0041 | 0.0000 | 0.0747 | 0.0000 |
| 26 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0097 | 0.0000 | 0.0002 | 0.0054 | 0.0005 | 0.0214 | 0.0000 | 0.0000 | 0.0753 | 0.0085 | 0.0412 | 0.0837 | 0.0426 | 0.5975 | 1.2248 | 0.5371 | 3.7091 | 0.4407 | 0.0236 | 0.0000 | 0.1350 | 0.0093 |
| 27 | 0.0000 | 0.2009 | 0.1299 | 0.7419 | 0.0000 | 0.0095 | 0.0000 | 0.0000 | 0.0013 | 0.0024 | 0.0010 | 0.0000 | 0.1085 | 0.0758 | 0.0993 | 0.0548 | 0.4649 | 0.0000 | 0.3428 | 0.0922 | 0.0009 | 0.0000 | 0.2004 | 0.2319 | 1.6277 | 12.380 | 1.8553 | 0.0102 | 0.1317 | 1.2566 |
| 28 | 0.0745 | 0.3574 | 0.3704 | 0.0384 | 0.0046 | 0.0440 | 0.1036 | 0.0000 | 0.0056 | 0.0629 | 0.0000 | 0.0005 | 0.1339 | 0.1524 | 0.0009 | 0.0143 | 0.2768 | 0.0026 | 0.3054 | 0.0654 | 0.1610 | 0.1057 | 1.5994 | 2.0800 | 20.958 | 31.315 | 4.3497 | 7.6629 | 1.1853 | 0.1505 |
| 29 | 10.317 | 0.5315 | 0.1876 | 0.7997 | 0.0000 | 2.6631 | 1.6163 | 0.6907 | 0.0000 | 0.0949 | 0.1057 | 0.0000 | 0.0448 | 0.3788 | 0.0000 | 17.872 | 2.7661 | 0.1386 | 0.1871 | 0.0360 | 0.0001 | 0.0000 | 0.5659 | 13.683 | 5.9894 | 1.4637 | 20.185 | 0.0170 | 26.396 | 2.7959 |
| 30 | 0.0000 | 0.0000 | 0.0502 | 0.0062 | 0.0000 | 0.0098 | 0.0627 | 0.0089 | 0.0060 | 0.0229 | 0.0000 | 0.0000 | 0.1425 | 0.3469 | 0.1625 | 0.3149 | 1.6897 | 0.0000 | 0.0000 | 0.0000 | 0.0033 | 0.0000 | 0.4807 | 0.4894 | 0.3246 | 0.0028 | 21.734 | 7.6817 | 4.5061 | 0.3835 |
| 31 | 0.0000 | 3.9785 | 1.9460 | 1.6820 | 0.0191 | 11.498 | 23.898 | 0.5771 | 1.1695 | 2.0724 | 0.6430 | 0.1712 | 0.0566 | 0.3739 | 0.0000 | 1.3212 | 1.9694 | 0.0000 | 0.0819 | 0.0549 | 5.6712 | 50.782 | 9.1592 | 0.2832 | 0.2914 | 1.5894 | 15.988 | 0.3803 | 19.144 | 18.771 |
| 32 | 1.1049 | 28.955 | 22.421 | 10.293 | 3.2946 | 16.427 | 45.763 | 16.689 | 4.1283 | 4.9894 | 0.0134 | 4.4794 | 6.9469 | 2.1602 | 0.3969 | 0.3437 | 1.1923 | 0.0000 | 2.9084 | 1.8032 | 0.6045 | 9.2274 | 8.2278 | 5.6484 | 12.943 | 2.5200 | 19.859 | 82.067 | 12.80 | 48.275 |



| Table 5.7. |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PARTNERI | 01 | 02 | 03 | 04 | 05 | 08 | 07 | 08 | 11 | 12 | 13 | 14 | IMPORTING REGION j |  |  |  | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  | 15 | 16 | 17 |  |  |  |  |  |  |  |  |  |  |  | 29 | 30 | 31 | 32 |
| 01 | 53.344 | 0.1327 | 0.0192 | 0.0019 | 0.0000 | 0.0001 | 0.0093 | 0.0000 | 0.7714 | 2.5939 | 0.1550 | 0.3568 | 0.6617 | 0.1582 | 0.6118 | 0.3357 | 0.3585 | 0.0438 | 0.0196 | 0.1514 | 0.0556 | 0.8050 | 0.0451 | 0.0300 | 0.0001 | 0.0772 | 0.0706 | 0.0135 | 0.0757 | 0.0161 |
| 02 | 2.4647 | 0.2303 | 0.7088 | 0.0222 | 0.0061 | 0.00130 | 0.0000 | 6.9962 | 5.8553 | 12.551 | 4.2898 | 2.9795 | 6.7207 | 6.7887 | 9.0431 | 4.5970 | 6.0596 | 0.0078 | 3.0622 | 0.8551 | 1.9251 | 1.4480 | 2.4494 | 0.3303 | 0.3689 | 4.3495 | 0.7776 | 5.2145 | 3.9391 | 2.0625 |
| 03 | 0.0205 | 0.1728 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0016 | 1.0184 | 1.8300 | 8.3730 | 6.2849 | 8.2585 | 1.6331 | 2.4790 | 0.3898 | 3.6892 | 5.9436 | 0.6232 | 5.3181 | 1.6742 | 3.1032 | 5.4028 | 2.5536 | 5.8671 | 7.9872 | 3.5645 | 1.0790 | 0.9051 | 2.7476 | 4.0013 |
| 04 | 0.0306 | 0.0130 | 0.0008 | 0.0052 | 0.0016 | 0.0014 | 0.0007 | 6.9768 | 20.878 | 21.979 | 2.5636 | 6.3810 | 0.9675 | 3.1028 | 0.0159 | 5.3946 | 4.6542 | 2.0410 | 2.0059 | 1.4750 | 0.2573 | 4.9026 | 0.7025 | 0.8305 | 0.4087 | 1.5374 | 0.3300 | 0.8498 | 1.1635 | 0.4907 |
| 05 | 0.0000 | 0.0350 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 21.109 | 5.2207 | 13.874 | 6.5430 | 9.2071 | 2.1402 | 3.8824 | 0.3724 | 1.5214 | 3.7939 | 6.2026 | 4.5346 | 2.8290 | 3.3496 | 2.9920 | 1.6784 | 2.7409 | 4.2329 | 2.5921 | 1.6172 | 2.7608 | 4.0094 | 1.7125 |
| 06 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0057 | 0.9016 | 0.0201 | 1.3029 | 0.4202 | 0.6845 | 0.2192 | 0.3855 | 0.0525 | 0.8257 | 1.4668 | 0.0000 | 0.0985 | 0.0001 | 0.0048 | 1.0212 | 0.0044 | 0.0858 | 0.1580 | 0.6930 | 4.2443 | 0.9046 | 15.600 | 6.9920 |
| 07 | 0.0121 | 0.0000 | 0.1708 | 0.0005 | 0.0000 | 0.34107 | 7.7874 | 2.7464 | 1.0981 | 5.9053 | 2.2485 | 3.1427 | 6.5240 | 6.6244 | 0.6131 | 3.3149 | 12.160 | 0.7199 | 0.0000 | 0.5723 | 0.0000 | 4.7872 | 0.0540 | 0.0251 | 0.3903 | 4.1360 | 2.8777 | 21.794 | 25.212 | 8.6322 |
| 08 | 0.0000 | 1.2138 | 0.0209 | 0.7006 | 1.5358 | 0.0428 | 0.1087 | 0.8140 | 0.2660 | 0.9309 | 2.5962 | 1.6279 | 0.1757 | 0.1779 | 0.0013 | 0.5597 | 0.4729 | 0.0002 | 0.0005 | 0.0012 | 0.0018 | 0.0000 | 0.0013 | 0.0006 | 0.0001 | 0.0030 | 0.1339 | 0.0040 | 0.2950 | 0.0002 |
| 11 | 0.0000 | 0.1196 | 0.0497 | 1.1177 | 0.2437 | 0.0070 | 0.0000 | 0.8694 | 3.6574 | 0.9946 | 0.0064 | 0.0235 | 0.0211 | 0.0520 | 0.0000 | 0.0158 | 0.0142 | 0.0000 | 0.0057 | 0.0135 | 0.0010 | 0.0000 | 0.0136 | 0.0268 | 0.0035 | 0.0018 | 0.0043 | 0.0012 | 0.0080 | 0.0093 |
| 12 | 0. 1098 | 0.0118 | 0.2621 | 1.5041 | 0.2839 | 0.0057 | 0.0077 | 1.1598 | 0.4478 | 0.1174 | 0.6172 | 1.0668 | 0.0949 | 0.4601 | 0.0023 | 0.1671 | 0.2106 | 0.0000 | 0.0001 | 0.0000 | 0.0005 | 0.0000 | 0.0008 | 0.0140 | 0.0510 | 2.1040 | 0.0002 | 0.0000 | 1.9717 | 0.0011 |
| 13 | 0. 2303 | 6.5360 | 2.4302 | 7.3720 | 4.1389 | 1.4833 | 0.4237 | 6.5655 | 1.2557 | 3.7090 | 7.6687 | 12.489 | 0.6434 | 3.5136 | 0.1119 | 1.0130 | 1.8202 | 0.0000 | 0.0542 | 1.0984 | 3.4126 | 1.8210 | 2.6694 | 0.6913 | 0.8619 | 1.0993 | 0.2838 | 1.0246 | 3.0331 | 0.3544 |
| 14 | 0.0000 | 0.8309 | 5.0095 | 2.5157 | 6.3304 | 0.1825 | 0.0200 | 3.8221 | 0.1066 | 4.2920 | 27.670 | 10.948 | 1.8445 | 2.3517 | 1.4769 | 4.3200 | 5.6748 | 0.0000 | 0.0026 | 0.0012 | 0.1385 | 0.0559 | 0.0025 | 0.0000 | 0.0000 | 0.0086 | 0.0385 | 5.9308 | 2.4886 | 0.0001 |
| 15 | 10.991 | 17.891 | 4.0247 | 3.1047 | 2.7742 | 1.0021 | 0.8347 | 2.8287 | 6.3478 | 1.6968 | 4.2970 | 1.1833 | 5.5059 | 10.989 | 8.5605 | 17.029 | 6.4173 | 4.2235 | 9.5601 | 8.0369 | 10.444 | 20.682 | 17.131 | 6.0554 | 6.5549 | 10.477 | 2.9139 | 0.4279 | 1.0810 | 6.4755 |
| 16 | 17.989 | 52.252 | 26.438 | 11.208 | 18.710 | 10.787 | 5.7351 | 14.359 | 8.1970 | 5.3026 | 10.343 | 7.1847 | 40.799 | 38.085 | 67.776 | 28.120 | 19.837 | 11.878 | 30.824 | 18.719 | 19.070 | 18.899 | 16.763 | 14.546 | 16.735 | 7.7484 | 6.7622 | 12.132 | 3.5756 | 8.4985 |
| 17 | 3.0517 | 2.9046 | 0.5342 | 0.2441 | 0.2832 | 0.0019 | 0.0012 | 0.1521 | 6.7032 | 1.4544 | 0.3087 | 0.3965 | 0.9308 | 1.9107 | 0.0000 | 1.0507 | 0.3043 | 0.0000 | 3.1794 | 1.1373 | 0.2363 | 0.0000 | 0.0097 | 0.3492 | 0.7551 | 1.3227 | 0.0759 | 1.6297 | 0.0366 | 0.0147 |
| 18 | 6.9147 | 4.3451 | 3.7725 | 2.6074 | 1.7812 | 0.5742 | 0.5040 | 2.8717 | 7.4598 | 2.6415 | 1.7944 | 2.6045 | 11.372 | 3.1371 | 4.3031 | 6.8396 | 2.4857 | 13.783 | 2.8738 | 4.9191 | 16.851 | 6.6528 | 8.4722 | 1.1881 | 1.3772 | 0.8601 | 0.7494 | 0.2765 | 0.2244 | 2.0237 |
| 19 | 2.0033 | 3.6 ${ }^{\circ}$ | 11.510 | 3.4613 | 2.7901 | 1.2809 | 0.4720 | 5.6353 | 6.2493 | 6.2340 | 9.0173 | 5.3909 | 5.5035 | 2.7181 | 1.3913 | 5.4539 | 8.0136 | 22.325 | 27.267 | 39.528 | 16.631 | 5.0579 | 20.671 | 15.225 | 13.176 | 7.1104 | 2.3964 | 2.9790 | 1.0161 | 1.8845 |
| 20 | 0.0000 | 0.1131 | 0.2775 | 0.1121 | 4.3583 | 0.0000 | 0.0000 | 0.0867 | 0.0340 | 0.1192 | 0.0179 | 0.0284 | 0.0485 | 0.0931 | 0.0000 | 2.3147 | 1.9304 | 29.582 | 2.3498 | 5.7239 | 0.0862 | 0.0000 | 0.4638 | 1.2438 | 0.0347 | 2.8341 | 0.0921 | 0.0000 | 0.3091 | 0.1009 |
| 21 | 0.0010 | 0.1727 | 0.2331 | 0.0487 | 0.1802 | 0.0685 | 0.3269 | 0.0051 | 0.0020 | 0.0082 | 0.0006 | 0.0113 | 0.1228 | 0.1236 | 0.0019 | 0.0163 | 0.3463 | 0.8472 | 0.1422 | 0.0952 | 0.0086 | 0.0058 | 0.4733 | 0.1606 | 0.2904 | 0.0422 | 0.1187 | 0.0188 | 0.0169 | 0.0144 |
| 22 | 0. 1285 | 0.0407 | 0.0990 | 0.0039 | 0.1980 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.4060 | 0.2490 | 0.0066 | 0.7074 | 1.9116 | 4.3896 | 0.0351 | 0.1518 | 0.0196 | 0.0046 | 0.0002 | 0.4557 | 0.1170 | 0.0025 | 0.0000 | 0.0000 | 0.0244 | 0.0000 |
| 23 | 0.0000 | 0.3301 | 0.1602 | 0.0015 | 7.5490 | 0.0000 | 0.0000 | 0.0000 | 0.0019 | 0.0000 | 2.0805 | 0.0002 | 0.0852 | 2.3625 | 0.0000 | 1.2230 | 1.4726 | 0.0000 | 0.0161 | 0.0672 | 0.9569 | 0.0234 | 0.0392 | 0.0068 | 0.0008 | 0.5845 | 0.3609 | 0.0000 | 0.8354 | 0.0002 |
| 24 | 1.7115 | 1.1762 | 1.5712 | 0.5049 | 10.350 | 0.0615 | 0.1377 | 0.0073 | 0.0000 | 0.3953 | 0.7761 | 1.8951 | 4.4181 | 1.0522 | 0.0000 | 0.8417 | 2.2676 | 0.0000 | 0.0014 | 0.0000 | 0.0023 | 0.0000 | 0.2494 | 0.0018 | 0.0004 | 0.0997 | 0.3398 | 0.0000 | 1.1423 | 0.1163 |
| 25 | 0.0711 | 0.0081 | 1.6957 | 0.2146 | 1.7194 | 0.0000 | 0.0000 | 0.0000 | 0.0140 | 0.0000 | 0.1534 | 0.0726 | 0.6392 | 0.6567 | 0.0001 | 3.2795 | 3.0534 | 0.0384 | 0.0647 | 0.2140 | 0.0184 | 0.0804 | 0.5451 | 5.7339 | 0.2782 | 3.3258 | 0.3666 | 0.0347 | 1.5835 | 0.0004 |
| 26 | 0.0000 | 0.0000 | 0.0594 | 0.0002 | 0.0444 | 0.0001 | 0.0000 | 0.0000 | 0.0038 | 0.0000 | 0.0000 | 0.0000 | 0.0849 | 0.0856 | 0.0932 | 0.0323 | 0.3247 | 0.4299 | 0.0396 | 0.6615 | 0.0142 | 0.0034 | 1.1856 | 3.2858 | 0.1553 | 0.0098 | 0.0000 | 0.0159 | 0.0140 | 0.0000 |
| 27 | 0.0000 | 0.0000 | 0.1382 | 0.0623 | 0.6713 | 0.0096 | 0.0080 | 0.0002 | 0.0165 | 0.0022 | 0.0522 | 0.0000 | 0.0489 | 0.1691 | 0.0001 | 0.0070 | 0.3250 | 0.0215 | 0.9730 | 0.1591 | 0.0000 | 0.0099 | 0.4967 | 0.9711 | 3.8958 | 2.1647 | 0.2518 | 0.0030 | 0.5206 | 0.0318 |
| 28 | 0.3757 | 0.1542 | 0.1038 | 0.0407 | 0.0462 | 0.0945 | 0.6533 | 0.0064 | 0.0557 | 0.0255 | 0.0329 | 0.0268 | 0.0251 | 0.0260 | 0.0907 | 0.0363 | 0.2622 | 0.9719 | 0.0684 | 0.6676 | 0.9147 | 0.0278 | 5.5401 | 0.5093 | 2.1421 | 1.8583 | 1.1289 | 0.4549 | 0.0668 | 0.0658 |
| 29 | 0.0000 | 0.1026 | 1.6268 | 0.0727 | 0.2349 | 1.8476 | 0.1521 | 0.0829 | 0.0079 | 0.0134 | 0.0463 | 0.0273 | 0.1567 | 0.4368 | 0.1718 | 0.0690 | 0.1505 | 0.1542 | 0.0823 | 0.0509 | 0.3652 | 0.0011 | 0.6846 | 0.4290 | 0.6240 | 2.8295 | 6.6521 | 1.3801 | 1.2539 | 1.5235 |
| 30 | 0.0000 | 0.0000 | 0.0578 | 0.0278 | 0.0020 | 0.0358 | 0.0685 | 0.0365 | 0.0349 | 0.0502 | 0.0077 | 0.1171 | 0.0168 | 0.2215 | 0.3364 | 0.0719 | 0.2796 | 0.0000 | 0.4263 | 0.3517 | 0.2971 | 0.0008 | 0.6732 | 1.0218 | 0.2551 | 0.8717 | 1.5777 | 0.0000 | 2.2249 | 0.0880 |
| 31 | 0.5413 | 6.2187 | 37.835 | 64.724 | 33.436 | 79.838 | 82.069 | 20.595 | 23.198 | 5.4137 | 9.8909 | 23.828 | 5.7352 | 7.1214 | 1.7312 | 6.7380 | 6.1677 | 1.3311 | 6.8083 | 8.3533 | 21.764 | 24.709 | 15.801 | 33.980 | 31.857 | 21.645 | 55.493 | 36.541 | 15.903 | 37.066 |
| 32 | 0. 0074 | 1.3864 | 1.1883 | 0.3212 | 2.3287 | 2.3315 | 0.6712 | 0.3523 | 0.2667 | 0.0169 | 0.1153 | 0.0654 | 2.4525 | 0.6040 | 2.8447 | 0.4132 | 1.8183 | 0.3837 | 0.1843 | 2.4898 | 0.0697 | 0.6032 | 0.6212 | 4.1904 | 7.2845 | 16.044 | 9.2612 | 4.7018 | 9.6254 | 17.821 |

We observe very strong ties (high shares) between the neighboring regions 29, 30, 31, 32(the Pacific rim countries). Also, a similar relationship exists between regions $15,16,17,18$ and 19 ; that is, the EFTA/EEC regions.

We notice the almost self-sufficient 01 in terms of seaborne trade (over $70 \%$ for all goods except for good 2 with a $50 \%$ intra-seaborne trade), due to the special type of ships needed in the Great Lakes. Of course, 01 is also supplied through the road and rail system.
Also the high intra seaborne trade (around 50\%) of region 32 for goods 1 and 3 is worth noting. These are Bulk Dry and Refrigerated goods which are produced 'locally' and need to be transported between areas of Oceania.

## 5.3). Bilateral Import Prices.

The specification of the theoretical equations of the model (equation (3.11) and its linerized form, equation (4.14)) requires us to obtain bilateral import prices, for each commodity group which correspond to the bilateral volumes of seaborne trade flows. To the best of our knowledge, these data are not published anywhere. To be able to construct these aggregate price indices (for each region/good) ourselves using raw data, we must have information on bilateral import prices for each individual good and country of the world. Information for this type of data is limited and partial (data are only available for certain countries, goods, years over the period considered).

This has forced us to change the specification of our model, to take account of these data restrictions. In altering the specification of the model, (in chapter 4) bilateral import prices of a region for a type of commodity are thought of as a function of the total import prices of the importing region, the total export prices of the exporting region and the world price of the type of commodity considered. According to this specification, we have to obtain the following data for our import allocation model:

1) Total import data for each of the 30 regions,
2) Total export data for each of the 30 regions and
3) World prices for each of the 5 types of goods considered.

### 5.3.1). Total Import-Export Price Aggregates for a Region.

The procedure to answering 1) and 2) is similar. It involves finding total import/export prices for each country in each region, and calculating a weighted average for every region.

We obtain these data (126 countries) from the World Bank publications, in machine readable form, after the kind permission of the Bath university computer centre to use their database. Additionally, some data from the IFS (International Financial Statistics) of the IMF (International Monetary Fund) are obtained for another 14 countries. This brings the list of countries for which we have data to 140. In table 5.2 the countries inside the brackets, in each region, are countries for which there are no data available. These countries are relatively unimportant (in terms of size and resources) in influencing the total trade of the region they belong to, with the exception of, probably, Cuba in region 08 and the Eastern block countries in regions 17, 20 and 30.

Total import(c.i.f.)/export(f.o.b.) price indices (1980=100), dollar exchange rates and GDP-CON's (Gross Domestic Products, in constant 1980 prices), in local currencies, are collected for each country. The published world bank total import/export price indices for industrial economies come from the UN 'Yearbook of International Trade Statistics' (e.g., UN(1989)), the 'Munthly Bulletin of Statistics' (eg UN(1990)) and the 'International Financial Statistics Yearbook' (e.g. IMF(1990)). The total import/export price indices for developing countries are world bank estimates. They are based on international prices for primary commodities and unit value indices for manufactures. In order to ensure consistency between data for a group of countries and those for individual countries, the published indices are aggregated by broad commodity groups for each country.

A number of alternatively weighted total import/export price indices may be constructed for a region, depending on the weighting scheme used. Constant or varying weights may be used, each reflecting certain underlying assumptions. The weights used must reflect the relative importance of the country in the region, according to some criterion. We use the dollar GDP-CON of each country in the region, as the criterion. We arrive at the \$GDP-CON, from the original data, by dividing the GDP-CON in local currency by the dollar exchange rate of the country in question, also obtained from the world bank publications.

In general, with price indices, the decision has to be made as to whether to use base year or current year weights. A price index number is meant to show changes in prices over time. This requires prices only to change over time. If both prices and the weights ('quantities') are allowed to vary, the change in the price index might be a combined effect of both variables changing.

Laspeyres price indices use base year weights in constructing the index. That is, they measure the change in prices of a given set of 'quantities', those of the base year. However, as tastes, technology and other factors in the economy change over time, the base year weights become outdated as we move away from the base year. Mathematically, the Laspeyres import (export) price index of a region is:

$$
\begin{equation*}
L_{p t}=\left[\left(\sum_{i=1}^{n} q_{i 0} p_{i t}\right) /\left(\sum_{i=1}^{n} q_{i 0} p_{i 0}\right)\right] \times 100 \tag{5.1}
\end{equation*}
$$

where $p_{\text {it }}$ are the input import (export) indices of each country $i=1, \ldots, n$ in the region at time $t$, and $q_{10}$ are the 1980 (\$GDP-CON based) weights of each country in the region.

Paasche price indices use current year weights in constructing the index. This remedies the critisism of the Laspeyres indices because the weights are revised in each time period, to take account of changes in tastes, technology, etc. Strictly speaking, these are not
price indices since they do not reflect changes in prices only ('quantities' are allowed to vary). Mathematically, the Paasche import (export) price index of a region is:

$$
\begin{equation*}
P_{p t}=\left[\left(\sum_{i=1}^{n} q_{i t} p_{i t}\right) /\left(\sum_{i=1}^{n} q_{i t} p_{i 0}\right)\right] \times 100 \tag{5.2}
\end{equation*}
$$

where $p_{i t}$ are defined as before, and $q_{i t}$ are the (\$GDP-CON) current period ( $t$ ) weights of each country in the region.

In general, there is likely to be a discrepancy between the Laspeyres and the Paasche price (and quantity) indices. The following results are due to Bortkiewicz(1923, 1924), also reproduced in Allen(1975).

Let $L_{\text {qt }}, P_{q t}$ denote the Laspeyres and Paasche quantity indices respectively, where the weights now are prices instead of 'quantities' (as they are in the price indices). From the mathematical definitions (leaving aside the factor of 100) we observe that,

$$
\begin{equation*}
L_{\mathrm{pt}} \times P_{\mathrm{qt}}=L_{\mathrm{qt}} \times P_{\mathrm{pt}}=\mathrm{V}_{\mathrm{t}} \tag{5.3}
\end{equation*}
$$

where $V_{t}$ is the change in value between year 0 and $t$. That is,

$$
\begin{equation*}
V_{t}=\left[\left(\sum_{i=1}^{n} q_{i t} p_{i t}\right) /\left(\sum_{i=1}^{n} q_{i 0} p_{i 0}\right)\right] \tag{5.4}
\end{equation*}
$$

(5.3) indicates that the good of the Laspeyres price index times the Paasche quantity index is equivalent to the Laspeyres quantity index times the Paasche price index. Therefore we can write:

$$
\begin{equation*}
\left(P_{\mathrm{pt}} / L_{\mathrm{pt}}\right)=\left(P_{\mathrm{qt}} / L_{\mathrm{qt}}\right) \tag{5.5}
\end{equation*}
$$

(5.5) allows one to state that whatever divergence there is between the price indices there is an equivalent one between the quantity indices. To qualify the statement, let us start by writing the Laspeyres price and quantity indices in their relatives form. For
convenience we omit the summation index $i$, and also assume that the $q$ 's represent quantities.

$$
\begin{equation*}
\left.\left.L_{\mathrm{pt}}=\left[\Sigma w_{0} \bar{p}_{\mathrm{p}}\right) /\left(\Sigma w_{0}\right)\right] \quad \text { and } \quad L_{\mathrm{qt}}=\left[\Sigma w_{0} \bar{q}_{\mathrm{q}}\right) /\left(\Sigma w_{0}\right)\right] \tag{5.6}
\end{equation*}
$$

where $w_{0}=p_{0} q_{0}$ are the iter by item base year weights.

The indices in (5.6) can be interpreted as the weighted means of relatives. The corresponding weighted variances are:
$\left.\sigma_{p}^{2}=\Sigma w_{0}\left(\bar{p}_{\mathrm{p}}-L_{p t}\right)^{2}\right] /\left(\Sigma w_{0}\right)$ and $\left.\sigma_{q}^{2}=\Sigma w_{0}\left(\frac{q_{t}}{q_{0}}-L_{q t}\right)^{2}\right] /\left(\Sigma w_{0}\right)$

The weighted covariance times $\Sigma w_{0}$ is:

$$
\begin{aligned}
& \sigma_{p q}=\Sigma w_{0}\left(\frac{p_{t}}{p_{0}}-L_{p t}\right)\left(\frac{q_{t}}{q_{0}}-L_{q t}\right) \\
& \quad=\Sigma w_{0} \bar{p}_{t} \bar{p}_{0} \bar{q}_{0}-L_{p t} \Sigma w_{0} \bar{q}_{0}-L_{q t} \Sigma w_{0} \bar{p}_{0}+L_{p t} L_{q t} \Sigma w_{0}
\end{aligned}
$$

Using (5.6) in the above yields,

$$
\begin{equation*}
\sigma_{p q}=\Sigma w_{0} \bar{p}_{p_{0}} \bar{q}_{t}-L_{p t} L_{q t} \Sigma w_{0} \tag{5.8}
\end{equation*}
$$

The weighted correlation coefficient between prices and quantities is:

$$
\begin{equation*}
r=\sigma_{p q} / \sigma_{p} \sigma_{q} \Sigma w_{0} \tag{5.9}
\end{equation*}
$$

Using (5.8), (5.9) becomes:

$$
\begin{equation*}
r=\left[1 /\left(\sigma_{p} \sigma_{q}\right)\right]\left\{\left[\left(\Sigma w_{0} \bar{p}_{\mathrm{p}}^{\mathrm{p}} \bar{q}_{\mathrm{t}}\right) / \Sigma w_{0}\right]-\left(L_{\mathrm{pt}} L_{q t}\right)\right\} \tag{5.10}
\end{equation*}
$$

Using $w_{0}=p_{0} q_{0}$ and (5.3) in $\left[\left(\Sigma w_{0} \bar{p}_{t} \bar{q}_{t}\right) / \Sigma w_{0}\right]$ of (5.10) we get:
$\left[\left(\Sigma w_{0} \frac{p_{t} q_{0}}{q_{0}}\right) / \Sigma w_{0}\right]=\left[\left(\Sigma q_{t} p_{t}\right) /\left(\Sigma q_{0} p_{0}\right)\right] \equiv v_{0 t}=P_{p t} L_{q t}$

Put (5.11) into (5.10) to get:

$$
\begin{aligned}
r & =\left[1 /\left(\sigma_{\mathrm{p}} \sigma_{\mathrm{q}}\right)\right]\left[\left(P_{\mathrm{pt}} L_{\mathrm{qt}}\right)-\left(L_{\mathrm{pt}} L_{\mathrm{qt}}\right)\right] \\
& =\left[1 /\left(\sigma_{\mathrm{p}} \sigma_{\mathrm{q}}\right)\right]\left[\left(L_{\mathrm{pt}} L_{\mathrm{qt}}\right)\right]\left[\left(P_{\mathrm{pt}} / L_{\mathrm{pt}}\right)-1\right]
\end{aligned}
$$

## Rearranging gives:

$$
\begin{equation*}
\left(P_{\mathrm{pt}} / L_{\mathrm{pt}}\right)=1+r\left(\sigma_{\mathrm{p}} / L_{\mathrm{pt}}\right)\left(\sigma_{\mathrm{q}} / L_{\mathrm{qt}}\right) \tag{5.12}
\end{equation*}
$$

Thus, the ratio of the Paasche to the Laspeyres price index is 1 plus a term, which consists of the good of the correlation coefficient between prices and quantities times two coefficients of variation, one for $L_{p t}$ the other for $L_{q t}$. The coefficients of variation are always positive. This leaves the correlation coefficient to determine whether the ratio of the Paasche to the Laspeyres price (or quantity) index is greater or less than one; that is, whether the Paasche or the Laspeyres index is the greater of the two.

The Paasche index is the greater if $r>0$; that is, when prices and quantities move in the same direction. This would be typical of a market dominated by suppliers so that an increase in prices leads to increased supplies and sales. Examples of such rare markets are those of exporters selling on a large international market, and farmers selling on a market comprising both home produced and imported food.
The Laspeyres index is the greater of the two if $r<0$; that is, when prices and quantities move in opposite direction. This would be typical of a demand dominated market where buyers buy less as prices rise and more as prices fall. Such markets are those for imports or consumer goods, and is the more usual situation.

The magnitude of the discrepancy between the Paasche and the Laspeyres indices depends on the strength of the correlation between
prices and quantities (as shown by $r$ ), and on the dispersion of the price and quantity relatives (as shown by the coefficients of variation of $L_{p t}$ and $L_{q t}$ ).

One way of reconciling the problem of the Laspeyres and Paasche price indices, is to use the Fischer or Perfect price index structures. These are geometric averages of Laspeyres and Paasche price indices. In this way, the average of a changing set of numbers is obtained, the new index spliting the discrepancy between the Laspeyres and Paasche indices. Mathematically they are defined by:

$$
\begin{equation*}
F_{\mathrm{t}}=\sqrt{\left(L_{\mathrm{t}} \times P_{\mathrm{t}}\right)} \tag{5.13}
\end{equation*}
$$

We use the above principles to find averages of the import/export price indices for the set of countries in each region. Both Laspeyres and Paasche indices are calculated, with the weights used being the \$GDP-CON of each country. These indices are then used to calculate the Fischer's price index used during estimation.
5.3.2). World Price Aggregates for the 5 Types of Products.

Our solution to part 3) above, to finding world prices for each of the 5 types of goods, consists of two alternatives:
a) Constructing world price indices for each type of good,
b) Constructing unit value indices for each type of good.

Ideally, we like price indices to reflect the prices at which 'commodities' are exchanged, as opposed to unit values. Value changes are shown in (5.3) to be split into a product of a Laspeyres quantity (price) times a Paasche price (quantity) index, and conversely. In the division of the value into quantity and price components by means of index numbers, the price index should relate to 'pure' price quotations, leaving the quantity index to represent changes both in the number of units (e.g. tonnes) and in the quality of items.

Unit values are values per unit of quantities for the most disaggregated commodities (homogeneous 'commodities') entering trade. However, it is impossible to find data for every single commodity entering trade (e.g. Ford cars, JVC stereos, etc). As a result, there is no way of ensuring that a change in an aggregate unit value of a 'commodity' reflects a change in its price. This is because even if prices are constant and quantities change (or if there is a shift from, say, cheaper to dearer goods) within a certain type of aggregate good, the unit value of the good may change. But this would reflect changes in quantities (composition of the aggregate), rather than prices.

Another potential problem with unit value indices (Kravis and Lipsey, 1971, Introduction) is that they might not show the price at which the commodity may be obtained today, but the price at which it was negotiated when the commodity was contracted. This would be a problem, when we explain current import flows in terms of past contracted prices.

The answer as to what to do is not straight forward. However, we suggest a number of rules which we follow:

When price indices are available then they must be the first choice. Price indices for an aggregate 'good' may be constructed as a weighted average of the prices of the constituent goods.

If there are no sufficient data of prices for our aggregate goods use unit values. If data are available base these unit values on as many fine and homogeneous subdivisions of items as possible. Unit values can be acceptable when items are homogeneous or because different brands and qualities move together over time.
a) Constructing world price indices for each type of good:

World export price indices for (i) 66 primary commodities, (ii) 6 non-ferrous base metals, and (iii) machinery and transport equipment are published by the UN. Indices for the first two groups are available at the UN Statistical papers, Series M, No 82, for 1950 to 1985 (UN, 1987), and are updated every year at the UN Monthly Bulletin of Statistics. Data for the last group are available at the

UN Monthly Bulletin of Statistics. A list of all the commodities is presented in appendix 5.3. The first column shows the aggregate commodity groups for which there is a price index. This has been using the indices of the commodity subgroups of the second column and/or the commodity classes, shown in the third column of the appendix.

The methods used in the construction of the above published world price indices are described on pages 2 to 16 of the UN Series $M$, No 82 publication (UN, 1987).

In sum: The indices are world export price indices for commodities entering international trade. They are Laspeyres price indices with 1980 $=100$, calculated in US $\$$. The weights used are designed to reflect patterns of world trade, and are adjusted on average every 5 years (e.g. beginning in 1977, the 1980 patterns of world trade are used). Often, an index represents the average price movement of an aggregate of goods larger than the aggregate on which the computation of the index is based (e.g. an index for beef, fresh is assumed to represent price changes of meat of bovine animals, fresh, chilled or frozen). This permits us to use the index for a wider class of goods than the headings indicate.

The aggregate world price indices, which we construct for our 5 types of goods (MTC 'sections'), are weighted Laspeyres price indices of the published UN indices, with 1980=100. Their construction involves:
i) Associating each of the headings of appendix 5.3 with the appropriate MTC heading it represents (The more detailed (than MTC) SITC has also been used as an aid, to classify precisely the headings of appendix 5.3 into MTC 'categories'/'items'). In some cases more than one MTC 'items' are represented oy a single heading of appendix 5.3. In other situations, more than one headings of appendix 5.3 are needed to represent a single item of MTC.
ii) Constructing weights, which are used in the calculation of the Laspeyres price indices. Since the base year in the original indices is 1980, we use the shares of each MTC 'categoty'/'item' in world seaborne quantities of the 'section' (type of good) that it belongs
to in 1980, as weights. That is, the world seaborne quantity of each 'item' in 1980, is divided by the total world seaborne quantity of the MTC 'section' it belongs to, to create the weight for that 'item'.
iii) Using these weights, together with the original price indices, to construct the aggregate price indices for each of the 5 'sections' of the MTC (our 5 types of goods). A note should be made here that, whenever the original price indices fell short of covering the 1969-1986 period, in order to avoid sudden jumps in the final price indices, we extrapolated those indices to cover our period of estimation. Thus, if the observations $x_{t}$ and $x_{t+1}$ were known, to obtain the value of the unknown $\hat{x}_{t+2}$ we used:

$$
\begin{equation*}
\hat{x}_{t+2}=\left(x_{t+1} / x_{t}\right) x_{t+1} \tag{5.14}
\end{equation*}
$$

A notable feature of the final price indices is that, by construction, the greater the coverage of 'items' within each 'section' the closer the final index is to 100 in 1980. This is because the original price indices used are 100 in 1980, and also because the weights in each aggregate index are designed to sum-up to one. A quick look at the time series of the constructed final index allows us to infer about the representativeness of the index for the type of 'good' it intends to describe. The time series of the price indices of the 5 type of 'goods' is shown in figures 5.1-5.5, and their representativeness of the type of 'good' they purport to describe is summarised in table 5.8.

Table 5.8.
Percentage Coverage of Price Indices of MTC 'sections'.

| Bulk, Dry | $79 \%$ |
| :--- | :--- |
| Bulk, Liquid | $84 \%$ |
| Refrigerated Goods | $100 \%$ |
| General cargo, Dry | $30 \%$ |
| Other Dry Cargo | $56 \%$ |

Figure 5.1.


Figure 5.2.
TIME


Value of indices for 1980 show \% coverage of commodities in each index

Figure 5.3.


Figure 5.4.
GENERAL DRY CARGO WORLD PRICE INDEX


Jalue of indices for 1980 show \% coverage of commodities in each index

Figure 5.5. OTHER DRY CARGO WORLD PRICE INDEX


Value of index for 1980 shows \% coverage of commodities

Along the lines of our rules above it would be reasonable (due to the absence of data) to construct unit value indices for General cargo Dry (since only $29 \%$ of the total is explained by the aggregate real price index constructed), possibly Other Dry Cargo (56\% coverage), and use these in our model.
b) Constructing unit value indices for each type of cargo:

We defined the unit value index of a 'commodity' as the ratio of its traded value to the traded volume. World unit value indices for our 5 types of 'goods' are the ratios of the values of each of the world seaborne traded 'goods' to the corresponding world seaborne traded volume of each 'good'. The seaborne traded volumes of each of the 5 types of 'goods' are available from the MTS, as described earlier in section 5.2. However, the values of seaborne traded commodities, to the best of our knowledge, are not available anywhere. Since internationally traded commodities are largely carried by sea, the best proxy available for the latter are total values of international trade, irrespective of the mode of transport.
Values of world trade by commodity (in thousand U.S.dollars),
according to the SITC classifications are available at the UN 'Yearbook of International Trade Statistics'. The correspondence between the MTC and the SITC Revised and SITC Revised 2, are listed in appendix 5.1. These classifications are used to classify and collect data for individual headings under the appropriate 'good' type. Finally, the values for each heading obtained, under each 'good' type, are added to give us the total value of world trade, for each of the 5 types of commodities. Note that, we do not have readily available data for volumes of individual MTC 'items' in order to calculate the unit values from relatively more homogeneous 'commodities', and then find an average for our 5 types of 'goods'.

There have been two minor difficulties in the above in terms of the compatibility and availability of data of the SITC Revised and SITC Revised 2 classifications. First, data are not available for
some years or at all for certain headings in the more detailed 4 or 5-digit levels (data exist for all 'groups', 3-digit headings). Second, data are only disaggregated up to the 3-digit level for 1969, and that creates a problem of a missing value in 1969 when classifying commodities of greater detail. The solution to both these problems is necessary in order to avoid jumps in the value series (due to missing observations) from one time period to the next.

Our solution to the former problem is to extrapolate back to the years for which data are not available, by using the percentage increase in the first two years for which data are available, as shown in equations (5.14). The value of the headings for which there exist no data at all are often derived by deduction (e.g. subtracting the values of the available 'subgroups' from the value of the 'group' comprising them). This is not possible for certain commodity headings, which however, are of minor importance in the total value of the type of good they belong to.

Our solution to the latter problem consists of finding the share of the higher than 3-digit disaggregated commodity in the 3-digit 'group', it belongs to, in 1970, and using that to calculate its value in 1969.

Having overcome these problems, we divide the total value of world trade (in thousands of US dollars) for each 'good' by the corresponding total seaborne trade quantity (in metric tons). The outcome is a unit value index for each of the 5 'goods' we are interested in. Multiplying the index by 1000 turns out conveniently the unit of measurement to $\$$ 's per ton. Thus, the $\$ /$ ton unit values for 1980 for our 5 goods are: 163 for Bulk Dry, 233 for Bulk Liquid, 1679 for Refrigerated Goods, 2563 for General Dry Cargo and 4594 for Other Dry Cargo.

The question is whether these unit values can be used instead of the price indices, in our model when the coverage of the price indices is low. We attempt a direct comparison by rescaling both the price indices and the unit values, for each good, into new series
with $1980=100$. This is achieved by multiplying each series with the appropriate factor that makes it 100 in 1980 (for example the unit value series for Bulk Dry has to be multiplied by 0.6134969 , and similarly for the other goods). Thus, the original series are shifted up or down without changing the underlying trends.

Both the unit value and price indices for each type of good are shown and contrasted in figures 5.6-5.10. We observe that the unit values and price indices move together for Bulk Dry, Bulk Liquid, Refrigerated Goods and quite close for Other Dry Cargo. The coverage for all these indices is over $56 \%$. However, the unit value index for General Cargo Dry moves differently from the corresponding world price index. The coverage of this price index in terms of the available individual commodity indices is only $30 \%$. Observing the close resemblance of the unit values to the price indices for the goods for which there is a high \% coverage, we suggest that unit values can be good substitutes of the price indices for General Dry Cargo and for Other Dry Cargo. As a final note, we should say here that all the final price indices used in the model are rescaled so that they take the value of 1 in the base year, 1980.

Figure 5.6.
Price Index vs Unit Value of Bulk Dry


Figure 5.7.
Price Index vs Unit Value of Bulk Liquid


Figure 5.8.


Figure 5.9.
Price Index vs Unit Value of General Dry Cargo


Figure 5. 10.



#### Abstract

The aim of this chapter has been to describe the final data used in the estimation of the model, and their construction from the published data. We found it useful to start the chapter by explaining the standard international classifications used, which are related to the data required by our model. Then the availability of the data was examined with respect to the theoretical variables. The major drawback of the available dataset is the lack of bilateral import prices, which are needed in order to explain the bilateral seaborne trade flows of the world economy. In consequence, the model had to be respecified in terms of the available published data.


The advantages and disadvantages of using various weighting schemes in the construction of price index numbers have been examined. The Fishers' price index is a compromise between the Laspeyres and the Paasche indices. On the question of whether to use price indices or unit values, unit value series are considered second best to real price quotations. These considerations have been taken into account when we constructed the final data set of the model. However, the constructed unit values seem to follow the same trends as the corresponding price indices. This allows us to think of unit values as good substitutes for 'goods' for which no price indices are available.

Commodity Classification for Maritime Transport Statistics (MTC).

| CODE | COMMODITY | SITC COVERAGE |  |
| :---: | :---: | :---: | :---: |
|  |  | REVISED | REVISED 2 |
| 0 | All Commodities |  |  |
| 1 | Bulk, Dry |  |  |
| 101 | Grains |  |  |
| 10101 | Wheat and meslin, unmilled | 041 |  |
| 10102 | Rice | 042 |  |
| 10103 | Cereals n.e.s., unmilled | 043-045 |  |
| 102 | Sugar |  |  |
| 10201 | Raw sugar, beet and cane | 061.1 |  |
| 10202 | Refined sugar etc. | 061.2 |  |
| 103 | Oil seeds, nuts \& Kernels |  |  |
| 10301 | Groundnuts, green | 221.1 | 222.1 |
| 10302 | Soya beans | 221.4 | 222.2 |
| 10303 | Oil seeds etc. n.e.s. | $\begin{aligned} & \text { 221.2-211.3; } \\ & 221.5-221.9 \end{aligned}$ | $\begin{aligned} & 222 \cdot 3-222 \\ & 223 \end{aligned}$ |
| 104 | Timber |  |  |
| 10401 | Pulpwood | 242.1 | 246.01; 246. |
| 10402 | Logs, conifer | 242.2 | 247.1 |
| 10403 | Logs, non-conifer | 242.3 | 247.2 |
| 10404 | Lumber, shaped | 243 | 248 |
| 10405 | Other wood, n.e.s. | 242.4;242.9 | 247.9 |
| 105 | Ores |  |  |
| 10501 | Iron ores | 281 |  |
| 10502 | Copper ores | 283.1 | 287.1 |
| 10503 | Bauxite | 283.3;513.65 | 287.3 |
| 10504 | Manganese ores | 283.7 | 287.7 |
| 10505 | Non-ferrous ores n.e.s. | $\begin{aligned} & 283.2 ; 283.9 \\ & 283.4-283.6 ; 283.9 \end{aligned}$ | $\begin{array}{r} 286 ; 287.2 \\ 287.4-287.6 ; 28 \end{array}$ |
| 106 | Metal scrap |  |  |
| 10601 | Iron and steel scrap | 284282 |  |
| 10602 | Non-ferrous metal scrap | 284 | 288; 686.3 |
| $\begin{aligned} & 107 \\ & 10701 \end{aligned}$ | Coal and coke etc. Coal | 321.4 | 322.1;322 |


| 10702 | Coke | 321.8 | 323.2 |
| :---: | :---: | :---: | :---: |
| 10703 | Other solid fuels n.e.s. | 241;321.5-321.7 | 245;246.0 |
|  |  |  | 322.3;322 |
|  |  |  | 323.1 |
| 108 | Fertilisers |  |  |
| 10801 | Natural phosphates | 271.3 |  |
| 10802 | Natural fertilisers n.e.s. | 271.1;271.2;271.4 |  |
| 10803 | Fertilisers, manufactured | 561 | 562 |
| 10y | Ferrous base metals |  |  |
| 10901 | Pig iron | 671.2 |  |
| 10902 | Other ferro-alloys n.e.s. | 671.1;671.3-671.5 | 671.3;671 |
| 10903 | Products of ferrous base metal | 672-679 |  |
| 110 | Animal feeding stuff |  |  |
| 11001 | Animal feeding stuff | 081 |  |
| 111 | Other bulk, dry |  |  |
| 11101 | Gypsum, plasters etc. | 273.2 |  |
| 11102 | Mineral sands | 273.3 |  |
| 11103 | Sulphur | 274.1 |  |
| 11104 | Iron pyrites, unroasted | 274.2 |  |
| 11105 | Salt | 276.3 | 278.3 |
| 11106 | Asbestos, crude | 276.4 | 278.4 |
| 11107 | Other crude minerals n.e.s. | 273.1;273.4; | 273.1;273. |
|  |  | 276 less | 278.2;278 |
|  |  | 276.3-276.4 | less 278.9 |
|  |  |  | 278.5-278 |
| 2 | Bulk, Liquid |  |  |
| 201 | Crude petroleum etc. |  |  |
| 20101 | Crude petroleum etc. | 331 | 333 |
| 202 | Energy petroleum products, liquid |  |  |
| 20201 | Gasolines | 332.1 | 334.1 |
| 20202 | Kerosene and jet fuels | 332.2 | 332.4 |
| 20203 | Distillate fuels | 332.3 | 334.3 |
| 20204 | Residual fuel oils | 332.4 | 334.4 |
| 203 | Fuel gases, liquefied |  |  |
| 20301 | Fuel gases, liquefied | 341 |  |
| 204 | Other bulk liquid |  |  |
| 20401 | Molasses | 061.5 |  |
| 20402 | Oils, distilled from mineral tars | 521.4 | $\begin{aligned} & 335.2 \text { less } \\ & 335.21 \end{aligned}$ |
| 3 | Refrigerated Foods |  |  |
| 301 | Refrigerated foods |  |  |
| 30101 | Meat fresh, chilled or frozen | 011 |  |
| 30102 | Milk and cream, fresh | 022.3 |  |
| 30103 | Butter and cheese | 023-024 |  |
| 30104 | Eggs, fresh | 025 |  |
| 30105 | Fish fresh, chilled or frozen | 031.1;031.3 | 034;036 |


| 30106 | Oranges, tangerines, clementines | 051.1 | 007.1 |
| :--- | :--- | :--- | ---: |
| 30107 | Bananas, fresh | 051.3 | 057.3 |
| 30108 | Potatoes, fresh |  |  |
| 30109 | Other fresh fruit, vegetables | $051.2 ; 051.4 ; 051.5$ | $054.4-054$ |
|  |  | $051.9 ; 054.4-054.6$ | $057.92-057$. |
|  |  |  | $057.2 ; 057$ |


334.5; 335
335.3; 335
335.21


| 415 | Other manufactures |  |  |
| :---: | :---: | :---: | :---: |
| 41501 | Leather, rubber manufactures | 61;62 |  |
| 41502 | Veneer sheets and plywood | 631.1;631.2 | 634.1;634. |
| 41503 | Wood \& cork manufactures n.e.s. | 63 less | 634.3-634 |
|  |  | 631.1 and631.2 | 635 |
| 41504 | Cement | 661.2 |  |
| 41505 | Non-metal, mineral manufactures | 661 ess 661.2 |  |
| 41506 | Misc. metallic products | 694-698 | 694-699 |
| 41507 | Miscellaneous manufactures | 8 | 751.82;759.1 |

## 5 Other Dry Cargo

| 501 <br> 50101 | Woodpulp and paper waste Woodpulp and paper waste | 251 |  |
| :---: | :---: | :---: | :---: |
| 502 | Crude minerals n.e.s. |  |  |
| 50201 | Crude minerals n.e.s. 275;285 |  | 277;278.9; |
| 503 | Non-ferrous base metals |  |  |
| 50301 | Copper | 682 |  |
| 50302 | Aluminium | 684 |  |
| 50303 | Tin | 687 |  |
| 50304 | $\begin{array}{ll}\text { Non-ferrous base metals n.e.s. } & 681 ; 685-686 ; \\ \\ 683 ; 688-689\end{array}$ |  | $\begin{gathered} 68 \text { less } 682,68 \\ 686.33 \text { \& } \end{gathered}$ |
| 504 | Manufactures of metal |  |  |
| 50401 | Finished structures and parts | 691 |  |
| 50402 | Metal containers, wire products 6 | 692-693 |  |
| 505 | Machinery and equipment |  |  |
| 50501 | Agricultural machinery 712 |  | 721-722 |
| 50502 | Metal working \& special machinery 715;718 |  | $\begin{array}{r} \text { 723less } 723.48 ; \\ 725-727 ; 728 \\ 728.3 ; 728 \\ 736.1-736 . \\ 737 \text { less } 737 . \end{array}$ |
| 50503 | Passenger motor cars etc. 732.1;732.6 |  | 781 |
| 50504 | Motorcycles and parts 732.9 |  | 785.9 |
| 50505 | Other road motor vehicles n.e.s732.2-732.5; 732.7-732.8 |  | 782-784 |
| 50506 | Railway \& non-motor vehicles 731-733 |  | $\begin{aligned} & 785.2-785 \\ & 786 ; 791 \end{aligned}$ |
| 50507 | Aircraft, boats and their parts734-735 |  | 792-793 |
| 506 | Miscellaneous |  |  |
| 50601 | Live animals | 001 |  |
| 50602 | Hides and skins, undressed | 211 |  |
| 50603 | Explosives and pyrotech products 571 |  | 572 |
| 50604 | Commodities n.e.s. | 9 |  |

00 World
01 Great Lakes
Canada Atlantic
US Norh Atlantic
US South Atlantic
US Gulf
US South Pacific
North Pacific of North America
Central America
Caribbean Area
N Coast South America
E Coast South America
W Coast South America
British Isles
Northern Europe
Centr. Planned Eur. Baltic Sea Atlantic Europe

Mediterranean Europe

Centrally Planned Europe Black Sea
21 Mediterranean Asia
22 Mediterranean Africa
23 Western Africa
24 Southern Africa
25 Eastern Africa
26 Red Sea Area

27

28
Persian Gulf Area

Southern Asia
South East Asia

Centrally Planned
North Pacific
Far East Asia
Oceania
Unspecified

Great Lakes and upper St. Laurence of North
America river ports, above Montreal
St. Laurence river ports. Montreal and below; Greenland, St Pierre and Miquelon From Maine to Virginia, inclusive From N.Carolina to Miami, Florida inclusive and Puerto Rico and US Virgin islands
From Key West, Florida to Texas, inclusive California and Hawaii
Washington, Oregon and Alaska of United States of America and Canadian West Coast From coasts of Mexico to that of Panama incl All the carribean islands and Bermuda, excluding Puerto Rico and US Virgin islands From Caribbean Colombia to French Guinea inc Coasts of Brazil, Uruguay and Argentina and the nearby islands
From Pacific Colombia to Chile inclusive United Kingdom; Ireland; Iceland \& Faeroe is 1 Belgium; Netherlands; Germany F R; Denmark;
Norway; Sweden and Finland
USSR; Poland and German Dem. Rep.
French Atlantic coast; Spanish north coast and Portugal
From Spanish south coast, including the Canary islands to that of Greece inclusive and Malta
Bulgaria, Romania and USSR
From coasts of Turkey (including northern) to that of Israel inclusive, and Cyprus From Egypt to Morocco inclusive From Western Sahara to Namibia, inclusive, and the nearby islands
Republic of South Africa
From Somalia to Mozambique, inclusive, and the nearby islands
Egypt; Sudan; Ethiopia; Djibouti; Israel; Jordan; Yemen; Dem Yemen and Saudi Arabian west coast
Iran, Islamic Republic of; Iraq; Kuwait;
Bahrain; Oman; Saudi Arabian east coast;
Qatar and United Arab Emirates
From Pakistan to Burma inclusive
Malaysia; Singapore; Thailand; Dem Kampuchea
Indonesia; East Timor; Phillipines and
Brunei Darussalam
Viet Nam; China; Dem. People's Republic of Korea and USSR
Hong Kong; Macau; Japan \& Republic of Korea Australia; New Zeland and islands of Oceania

Appendix 5.3. Source: UN(1987)
Commodities for which world prices are available. Primary commodities



Road Motor Vehicles

## 6.0). Introduction.

In chapter 4 we derived sets of empirical seaborne bilateral import demand equations for a region of the world and a certain type of product. Whether 'static' or 'dynamic' systems are chosen for estimation, there is an adding-up constraint of the sum of the left hand side variables multiplied by the base year shares to equal zero. This fact has implications for the econometric estimation of the system. We explore these implications here.

This chapter is in four main sections. In the first, we turn the system of equations into a stochastic form by attaching a random term which describes the 'non-systematic' behaviour of the importers. We introduce the underlying principles of such a stochastic framework in the context of single equation models. In the second, we generalize the results to systems of equations, and we identify our own model to fall under the class of Seemingly Unrelated Regression Equations (SURE). We examine the appropriate estimation techniques of linear and nonlinear models and their properties. In the third, we examine SURE systems with a singular covariance matrix of the error terms. This singularity is a common problem in import allocation models, under which there is an adding-up constraint requiring bilateral imports to add-up to total imports. It is suggested that the singularity may be removed by reformulating the problem as a restricted Generalized Least Squares problem, or assuming normality, as a Maximum Likelihood estimation problem of a system of $(n-1)$ equations. In the fourth section, we provide test procedures for testing hypotheses on the parameters of interest. Thus, test statistics for contemporaneous correlation and linear restrictions on the coefficients across equations are suggested. The latter provide a framework for testing the

## 6.1). The Stochastic Environment of Import Allocation Models.

Let us take the simplest form of the empirical import demand equations as derived in the 'static' model in levels, say, the static version of equation (4.50), which we repeat here for convenience.

$$
\begin{align*}
y_{i t} & =c_{i} T-a_{i}\left[x p_{i}^{\alpha} w p_{k}^{(1-\alpha)}-\sum_{h} w_{h}^{0} \times p_{h}^{\alpha} w p_{k}^{(1-\alpha)}\right] \\
& +\sum_{i} w_{i}^{0} a_{i}\left\{\left[x p_{i}^{\alpha} w p_{k}^{(1-\alpha)}\right]-\sum_{h} w_{h}^{0}\left[\times p_{h}^{\alpha} w p_{k}^{(1-\alpha)}\right]\right\}+\varepsilon_{i t} \tag{6.1}
\end{align*}
$$

This specification describes the bilateral seaborne import differences of growth rates, of bilateral imports from partner $i$ $(i=1, \ldots, n)$ and total imports, with respect to the base year, $y_{\text {it }}$, of region $j$, in terms of the 'systematic' components described by the time trend and relative prices. Furthermore, a 'random' or 'non-systematic' component $\varepsilon_{i t}$ is included to take account of the unsystematic behaviour of the importer. That is, of all factors other than time and prices that affect his import allocation decision. Thus, (6.1) consists of a set of $n$ equations, one for each trade partner (assuming there are intra seaborne trade movements).

In order to examine the appropriate econometric techniques of estimating such systems of equations, we first introduce the principles underlying the general stochastic framework of single equations. In terms of our model we examine the imports of $j$ from a single partner i. We then extend these principles to multiequation systems, considering all trade partners $i=1, \ldots, n$, in the importing region j .

Let us assume that observations on import growth rates $y$ from a certain region, are random outcomes of experiments (performed by society), with a probability distribution conditional on the (kx1) vector of fixed variables $\mathbf{x}$ and associated parameters $\theta$. In terms of
(6.1) $x$ consists of $T, \quad x p_{i}, \quad W p_{k}$, and $\theta$ consists of the coefficient parameters $c_{i}, a_{i}, \alpha, \beta$ and the variance of $y, \sigma^{2}$. That is, the random variable $y$ takes values from the conditional distribution $f(y / x ; \theta)$. Since $y$ is a random variable, its value, at any time, is subject to error. Mathematically,

$$
\begin{equation*}
y=f(y / x ; \theta)+\varepsilon \tag{6.2}
\end{equation*}
$$

where $\varepsilon$ is the random error term.

Assuming for simplicity that the Statistical Generating Mechanism (SGM) (the mathematical form of the function that purports to describe the true Data Generating Process (DGP)), that describes $y$ is a linear function of $\mathbf{x}$ (Linearity of the model (6.1) is true if there are no data constraints, as we saw in chapter 4. The static model then reduces to (4.33), we can write the linear version of (6.1) or equivalently, (6.2) as:

$$
\begin{equation*}
y=\beta^{\prime} x+\varepsilon \tag{6.3}
\end{equation*}
$$

where $\beta$ is a $(k \times 1)$ vector of parameters, $\beta^{\prime}=\left(\beta_{1}, \ldots, \beta_{k}\right)$. By construction, the non-systematic part $\varepsilon$ is orthogonal to the systematic component, $\beta^{\prime} x$, of $y$. The conditional distribution of $y$ is:

$$
\begin{equation*}
(y / x) \sim\left(\beta^{\prime} x, \sigma^{2}\right) \tag{6.4}
\end{equation*}
$$

As a consequence, the distribution of the error term, $\varepsilon$, is,

$$
\begin{equation*}
\varepsilon \sim\left(0, \sigma^{2}\right) \tag{6.5}
\end{equation*}
$$

The SGM together with the probabilty model (the assumed family of parametric distributions, where $y$ takes values from) define the statistical model. This provides the theoretical stochastic framework to study the economic phenomenon we are interested in.

Let $\mathbf{y}$ be a sample of $T$ observations independently and identically drawn from the above distribution. Then the conditional distribution of the ( $T_{x}$ ) random vector $\mathbf{y}$ is,

$$
\begin{equation*}
(y / X) \sim\left(\beta^{\prime} x_{t}, \sigma^{2} I_{T}\right) \tag{6.6}
\end{equation*}
$$

where the variance-covariance matrix of the distribution of $\mathbf{y}$, $\left(E\left[(y-X \beta)(y-X \beta)^{\prime}\right]=\sigma^{2} I_{T}\right)$, is due to the assumption of selecting an independent and identical sample from the population. Given the Statistical Model and the Sampling Model (the way the sample is drawn), the elements of the error term are independently and identically distributed with a distribution,

$$
\begin{equation*}
\underline{\varepsilon} \sim\left(0, \sigma^{2} I_{T}\right) \tag{6.7}
\end{equation*}
$$

Given the Statistical Model, the sample observations may be used to estimate the unknown parameter vector $\theta \equiv\left(\beta, \sigma^{2}\right)$. Minimizing the sum of squared deviations of the actual data, $\mathbf{y}$, from their average value (the fitted values), that is minimizing the objective function $S(\beta)=(y-X \beta)^{\prime}(y-X \beta)$, yields the OLS (Ordinary Least Squares) estimator $\mathbf{b}$ of $\beta$. The estimator $\mathbf{b}$ is $\mathbf{a}$ random variable itself, with a distribution:

$$
\begin{equation*}
\text { b } \sim\left(\left(x^{\prime} x\right)^{-1} x^{\prime} y, \sigma^{2}\left(x^{\prime} x\right)^{-1}\right) \tag{6.8}
\end{equation*}
$$

An estimator for $\sigma^{2}$ is:

$$
\begin{equation*}
\hat{\sigma}^{2}=\hat{e}^{\prime} \hat{e} / T-k=(y-X b)^{\prime}(y-X b) /(T-k) \tag{6.8'}
\end{equation*}
$$

The OLS estimators of $\theta \equiv\left(\beta, \sigma^{2}\right), \hat{\theta} \equiv\left(\mathbf{b}, \hat{\sigma}^{2}\right)$, are Best Linear Unbiased Estimators (BLUE). They are unbiased (on average they are equal to the true parameters) and are best in the sense that no other estimator of $\beta$ or $\sigma^{2}$ has smaller variance.

Furthermore, if we are prepared to make a specific assumption
about the distribution of $y$, such as that it is normal, we can write the conditional distribution of $\mathbf{y}$ as:

$$
\begin{equation*}
(y / X) \sim N\left(\beta^{\prime} x_{t}, \sigma^{2} I_{T}\right) \tag{6.9}
\end{equation*}
$$

and the OLS estimator $\mathbf{b}$ has a distribution,

$$
\begin{equation*}
b \sim N\left(\left(X^{\prime} X\right)^{-1} X^{\prime} y, \sigma^{2}\left(X^{\prime} X\right)^{-1}\right) \tag{6.10}
\end{equation*}
$$

Normality thus, provides us with a more specific probability model for estimation (of $\theta$ ) and inference.

The assumption of normality of the conditional distribution of $y$ allows us to write:

$$
\begin{equation*}
f(y / x)=\left[\left(2 \pi \sigma^{2}\right)^{(-1 / 2)}\right] \exp \left\{-\left(1 / 2 \sigma^{2}\right)(y-x \beta)^{2}\right\} \tag{6.11}
\end{equation*}
$$

The assumption of an independent and identical sample of $T$ observations implies that the distribution of the sample vector $\mathbf{y}$, or equivalently its Likelihood Function is:

$$
\begin{equation*}
L(\theta, y) \equiv \prod_{t=1}^{T} f(y / x)=\left[\left(2 \pi \sigma^{2}\right)^{(-T / 2)}\right] \exp \left\{-\left(1 / 2 \sigma^{2}\right)(y-X \beta)^{\prime}(y-X \beta)\right\} \tag{6.12}
\end{equation*}
$$

The Maximum Likelihood (ML) rule may be used to provide estimators for $\beta$ and $\sigma^{2}$. This rule ensures that the parameter vector $\theta$ chosen, maximizes the probability of randomly drawing the sample used. Mathematically, $\theta$ is chosen by maximizing function (6.12), or more easily the logarithm of the likelihood function. The latter takes the form:

$$
\begin{equation*}
\ln [L(\theta, y)]=-(T / 2) \ln 2 \pi-(T / 2) \ln \sigma^{2}-\left(1 / 2 \sigma^{2}\right)(y-X \beta)^{\prime}(y-X \beta) \tag{6.13}
\end{equation*}
$$

Maximizing $\ln [L(\theta, y)]$ with respect to $\beta$ is equivalent to maximizing $-\left(1 / 2 \sigma^{2}\right)(y-X \beta)^{\prime}(y-X \beta)$, (since $\sigma^{2}$ is a constant). Given the negative sign and the constancy of $\sigma^{2}$ the above problem is equivalent to minimizing $S(\beta)=(y-X \beta)^{\prime}(y-X \beta)$. This is the OLS rule for finding $\beta$. The corresponding MLE for $\sigma^{2}$ is:

$$
\begin{equation*}
\hat{\sigma}^{2}=(y-X b)^{\prime}(y-X b) / T \tag{6.14}
\end{equation*}
$$

If the latter estimator is corrected for the degrees of freedom by replacing the $T$ with ( $T-k$ ), we have shown that the MLE estimators are equivalent to the OLS estimators under normality.

### 6.1.2). Significance of the Assumptions and GLS Estimation.

(i). An implicit assumption used so far in the estimation of $b$, is that $\operatorname{Rank}\left(X^{\prime} X\right)=\operatorname{Rank}(X)=k$, where $T>k$. The Rank of the ( $T_{x k}$ ) matrix $X$ is defined as the largest in order of all square submatrices of $X$ that has a non-zero determinant. Since T>k, the order of the largest square submatrix of $X$ is ( $k x k$ ), and therefore the largest possible Rank of $X$ is $k$. In the latter case we say that the matrix $X$ has full Rank.

Now, the determinant of the submatrix ( $k x k$ ) is non-zero if its columns and rows are linearly independent. That is, if there exist a set of scalars $\lambda_{1}, \ldots, \lambda_{k}$ with $\lambda_{1} x_{1}+\ldots+\lambda_{k} x_{k}=0$, where $x_{i}$ are the columns of $X$, then $\mathbf{x}_{1}, \ldots, x_{k}$ are linearly independent if and only if $\lambda_{1}=\ldots=\lambda_{k}=0$. In words, no single column or row can be derived as a linear combination of the others. $\mathbf{x}_{1}, \ldots, \mathbf{x}_{\mathbf{k}}$ are linearly dependent if at least one $\lambda_{i} \neq 0$.

Thus, if the $k$ columns of the ( $T_{x k}$ ) matrix $X$ are linearly independent, the determinant of any square submatrix of order ( $k x k$ ) is non-zero, and as a result Rank $(X)=k$. Since $\left(X^{\prime} X\right)$ is ( $k x k$ ) and $X$ has full rank, then $\operatorname{Rank}\left(X^{\prime} X\right)=k$. That is, its determinant is non-zero and the square matrix $\left(X^{\prime} X\right)$ is invertible. $b=\left(X^{\prime} X\right)^{-1} X^{\prime} y$, and $V(b)=\sigma^{2}\left(X^{\prime} X\right)^{-1}$ can be computed. Of course, the assumption of $T>k$, guarantees that the number of parameters, $k$, can be computed from the available number of observations, $T$.
(ii). We have also assumed that the covariance matrix of the vector $y$, and therefore of the vector $\underline{\varepsilon}$, in the SGM, is of the fixed
scalar identity type. That is, $E\left[(y-X \beta)(y-X \beta)^{\prime}\right]=E\left[\underline{\varepsilon} \varepsilon^{\prime}\right]=\sigma^{2} I_{T}$. The assumption that the elements of the random vector $\underline{\varepsilon}$ are independent and identically distributed (that is, they are uncorrelated and have identical variance) may be changed, to involve more general structures. Let us assume that $E\left[\underline{\varepsilon} \underline{\varepsilon}^{\prime}\right]=\sigma^{2} \Omega$, where $\Omega$ is a known real positive definite symmetric matrix. Depending on the precise form of the matrix $\Omega$, the error term may be heteroskedastic (varying variance, $\sigma^{2}$, between observations), autocorrelated (the elements of the error term are correlated), a combination of both, or perhaps follow complicated VAR (Vector Auto-Regressive) processes.

Thus, assuming that the SGM is $y=X \beta+\underline{\varepsilon}$, where the random term $\underline{\varepsilon}$ now has a distribution $\underline{\varepsilon}_{\sim}\left(0, \sigma^{2} \Omega\right)$, OLS estimation of $\beta$ amounts to minimizing the following objective function $S(\beta)=(y-X \beta)^{\prime} \Omega^{-1}(y-X \beta)$ with respect to $\beta$. The result is an estimator with a distribution:

$$
\begin{equation*}
\mathbf{b} \sim\left(\left(X^{\prime} X\right)^{-1} X^{\prime} y, \sigma^{2}\left(X^{\prime} X\right)^{-1} X^{\prime} \Omega X\left(X^{\prime} X\right)^{-1}\right) \tag{6.15}
\end{equation*}
$$

This estimator is unbiased, but its variance is different from the OLS estimator (when the error term is assumed white noise). As a result the estimator will be inefficient. It can be shown, Judge et al(1988, appendix $A$, section $A 11$ ), that since $\Omega$ is a positive definite matrix, a matrix $P$ with the property $P \Omega P^{\prime}=I_{T}$ always exists. This matrix can be used to transform the model to:

$$
\begin{equation*}
P y=P X \beta+P \underline{\varepsilon} \quad \text { or } \quad \mathbf{y}^{*}=X^{*} \beta+\underline{\varepsilon}^{*} \tag{6.16}
\end{equation*}
$$

where $y^{*}=P y, X^{*}=P X$ and $\underline{\varepsilon}^{*}=P \underline{\varepsilon}$. The transformed error vector has a distribution $\underline{\varepsilon}_{\sim}^{*}\left(0, \quad \sigma^{2} I_{T}\right)$, since $E\left[\underline{\varepsilon}^{*} \underline{\varepsilon}^{* \prime}\right]=\sigma^{2} E\left[P \underline{\varepsilon} \underline{\varepsilon}^{\prime} P^{\prime}\right]=\sigma^{2} P \Omega P^{\prime}$ $=\sigma^{2} I_{T}$. OLS may now be applied to the transformed model. This estimator of $\beta$ is the GLS (Generalized Least Squares) or Aitken estimator, which has a distribution:

$$
\begin{equation*}
b^{*} \sim\left(\left(x^{* \prime} x^{*}\right)^{-1} x^{* \prime} y, \sigma^{2}\left(x^{* \prime} x^{*}\right)^{-1}\right) \tag{6.17}
\end{equation*}
$$

or, in terms of the original data,

$$
b^{*} \sim\left(\left(X^{\prime} \Omega^{-1} X\right)^{-1} X^{\prime} \Omega^{-1} y, \sigma^{2}\left(X^{\prime} \Omega^{-1} X\right)^{-1}\right)
$$

The GLS estimator is BLUE. Furthermore, if we are prepared to assume that the random variable $\mathbf{y}$ is normally distributed, the distribution of $\mathbf{b}^{*}$ is normal. That is,

$$
b^{*} \sim_{\sim} N\left(\left(X^{\prime} \Omega^{-1} X\right)^{-1} X^{\prime} \Omega^{-1} y, \sigma^{2}\left(X^{\prime} \Omega^{-1} X\right)^{-1}\right)
$$

Assuming normality, the latter estimator may be obtained by maximum likelihood methods: The distribution of $\mathbf{y}$ and hence its likelihood function under normality is:

$$
\begin{equation*}
L(\theta, y)=\left[\left(2 \pi \sigma^{2}\right)^{(-T / 2)}\right]|\Omega|^{(-1 / 2)} \exp \left\{-\left(1 / 2 \sigma^{2}\right)(y-X \beta)^{\prime} \Omega^{-1}(y-X \beta)\right\} \tag{6.18}
\end{equation*}
$$

Following the same arguments as with the OLS estimation (when the variance of $\varepsilon$ is $\sigma^{2} I_{T}$ ), maximizing the logarithm of the above function with respect to $\theta$ is equivalent to minimizing the objective function $S(\beta)=(y-x \beta)^{\prime} \Omega^{-1}(y-x \beta)$. Therefore, the MLE obtained are equivalent to the GLS estimators under normality.

The assumption of normality provides us with a more specific probability model for estimation and inference. As with the case of the OLS estimator, in order to ensure that $\mathbf{b}^{*}$ and $\mathrm{V}\left(\mathrm{b}^{*}\right)$ can be estimated, the condition of Rank(X)=k must be fulfilled. In addition, $\Omega$ has to be inverted to obtain the GLS estimators (see (6.17')). A prerequisite for the inversion of $\Omega$ is that $\operatorname{Rank}(\Omega)=T$. The latter requires that no column or row of $\Omega$ is a linear combination of the others.

## 6.2). Systems of Equations.

We can extend the single equation framework of the one trade partner to our complete import allocation model of (6.1), which is a system of $n$ equations, one for each trade partner. Since common coefficients appear across equations, the system must be 'pooled' together for estimation. Assuming a sample of $T$ observations for
each of the $n$ regions, then we can write the system in stacked form as follows:

$$
\left[\begin{array}{l}
y_{1}  \tag{6.19}\\
\vdots \\
y_{n}
\end{array}\right]=\left[\begin{array}{llll}
x_{1} & 0 & \ldots & 0 \\
0^{1} & x_{2} & :: & 0 \\
\vdots & : & : & : \\
0 & 0 & \cdots & \vdots \\
0 & \cdots & x_{n}
\end{array}\right]\left[\begin{array}{l}
\beta_{1} \\
\beta_{2}^{1} \\
\vdots \\
\beta_{n}
\end{array}\right]+\left[\begin{array}{l}
\varepsilon_{1} \\
\underline{\varepsilon}_{2} \\
\vdots \\
\varepsilon_{n}
\end{array}\right]
$$

where $y_{i}$ and $\varepsilon_{i}$ are ( $T_{x} 1$ ) vectors, $\beta_{i}$ are $(k \times 1)$ vectors and $X_{i}$ are (Txk) matrices, for each of the $i=1, \ldots, n$ equations in the system. Each equation explains the variation of the dependent variable $\mathbf{y}_{1}$, $i=1, \ldots, n$, over time, where the definition of the variables and parameters corresponds to that of equation (6.2). Assume, for simplicity, that for each equation, $\underline{\varepsilon}_{-1} \sim\left(0, \sigma_{1}^{2} I_{T}\right)$. That is, the 'classical' assumptions of the single equation model are true. The above system in matrix form becomes:

$$
\begin{equation*}
\mathbf{Y}=\mathrm{XB}+\mathrm{E}, \quad \text { with } \quad \mathrm{E}_{\sim}\left(0, \Sigma \otimes \mathrm{I}_{\mathrm{T}}\right) \tag{6.20}
\end{equation*}
$$

where $Y$ and $E$ are ( $n T_{x} 1$ ) vectors of dependent variables and error terms, respectively, $X$ is the ( $n T \times n k$ ) matrix of explanatory variables and $B$ is the ( $n k \times 1$ ) vector of coefficients. Also, $\Sigma$ is the variance-covariance matrix of the residuals between equations. It takes the form $\Sigma=\left[\sigma_{i j}\right]=E\left[\varepsilon_{-i} \varepsilon_{j}^{\prime}\right], \forall i, j=1, \ldots, n$, where $\left[\sigma_{i j}\right]$ denotes the ( $n \times n$ ) matrix with elements ranked by $\mathfrak{i}, j$. More specifically:

$$
\Sigma=\left(\begin{array}{cccc}
E\left[\underline{\varepsilon}_{1} \underline{\varepsilon}_{1}^{\prime}\right] & E\left[\underline{\varepsilon}_{1} \varepsilon_{2}^{\prime}\right] & \ldots & E\left[\underline{\varepsilon}_{1} \underline{\varepsilon}_{n}^{\prime}\right]  \tag{6.21}\\
E\left[\underline{\varepsilon}_{2} \underline{\varepsilon}_{1}^{\prime}\right] & E\left[\underline{\varepsilon}_{2} \underline{\varepsilon}_{2}^{\prime}\right] & \ldots & E\left[\underline{\varepsilon}_{2} \underline{\varepsilon}_{n}^{\prime}\right] \\
\vdots & \vdots & \vdots & \vdots \\
E\left[\underline{\varepsilon}_{n} \underline{\varepsilon}_{1}^{\prime}\right] & E\left[\underline{\varepsilon}_{n} \underline{\varepsilon}_{2}^{\prime}\right] & \ldots & E\left[\underline{\varepsilon}_{n} \underline{\varepsilon}_{n}^{\prime}\right]
\end{array}\right)=\left[\begin{array}{ccc}
\sigma_{11} & \sigma_{12} & \ldots \\
\sigma_{21} & \sigma_{22}^{12} & \cdots \\
\sigma_{2 n}^{1 n} \\
\vdots & \vdots & :: \\
\sigma_{n 1} & \sigma_{n 2} & \cdots \\
\sigma_{n n}
\end{array}\right]
$$

If there is no relationship between the individual equations (i.e. if there are no cross-equation correlations in the errors $\Sigma=\left[\sigma_{11}\right]=0$, $\forall i \neq j$ in (6.21)), the error term in (6.20) becomes $E_{\sim}\left(0, \sigma_{11}^{2} \otimes I_{T}\right)$. The OLS estimators may be obtained from the 'pooled' system (6.20) by minimizing the objective function $S(B)=(Y-X B)^{\prime}(Y-X B)$. The estimators are: $B=\left(X^{\prime} X\right)^{-1} X^{\prime} Y$, which are BLUE. These are equivalent to applying

OLS separately to each individual equation, and are the same as the MLE estimators, for the same reasons as with the single equation case. A prerequisite for the estimation of $B$ is that $\operatorname{Rank}\left(X^{\prime} X\right)=\operatorname{Rank}(X)=n k$, so that $\left(X^{\prime} X\right)$ is invertible.

### 6.2.1). Seemingly Unrelated Regression Equations (SURE).

Often, even though the equations may seem unrelated, if they are used to model a set of related economic functions (e.g. our set of import demand equations), there might be some relationship between the equations not immediately apparent. For example there might be some parameter restrictions across-equations, the variables between equations may be required to satisfy certain constraints, or there might be some common unmeasurable or omitted variable that is included in the error terms. In such cases, the error terms, $\varepsilon_{i}$, are correlated across individual equations.

We can see such a situation with respect to our import allocation models. Take for example the 'static' model of (6.1), a system of $n$ equations. We can write it in a more concrete form as in equation (6.20). More generally, to allow for the fact that the model in its estimating form (as well as the dynamic models) is nonlinear (see chapter 4) we write:

$$
\begin{equation*}
Y=F(X, \Theta)+E \tag{6.22}
\end{equation*}
$$

where the definitions of $Y, X$ and $E$ are the same as in (6.20), and $\Theta \equiv(B, \Sigma)$ are the parameters of interest.

Apart from the common coefficients, there is an adding up constraint across the individual equations in the system. As we showed in chapter 4, all our systems satisfy the following constraint.

$$
\begin{equation*}
w^{\prime} Y=w^{\prime} F(X, \Theta) \equiv 0 \tag{6.23}
\end{equation*}
$$

where $w^{\prime}=\left(w_{1}^{0}, \ldots, w_{n}^{0}\right)$, is the vector of base year shares. (6.23) is equivalent to:

$$
\begin{equation*}
W^{\prime}[Y-F(X, \Theta)] \equiv 0 \tag{6.24}
\end{equation*}
$$

As a result $W^{\prime} E \equiv 0$. In words, the fact that $W^{\prime} Y \equiv 0$ in the system makes the left hand side nonstochastic. Hence the right hand side is nonrandom and $w^{\prime} E \equiv 0$. The error terms are linearly dependent. Also, apart from the common coefficients across equations the constraint $\Sigma_{i} W_{i}^{0} b_{1}=0$ is placed across the time trend parameters of the estimating systems. Therefore, because of the adding-up constraint amongst the observable random variables, contemporaneous correlation is introduced in the error terms. In order to improve the efficiency of the estimates the equations must be estimated jointly to take account of these correlations.

The simplest assumption one can make is that the errors are correlated across equations at a given point in time (contemporaneous correlation), but errors are not correlated across-equations over time. When cross-equation correlations are allowed in the covariance, the system of equations is known as SURE (Seemingly Unrelated Regression Equations), Zellner(1962).

### 6.2.2). Estimation of General SURE Systems.

Just as with the single equation case (when the covariance matrix of the vector $\varepsilon$ is not of the fixed scalar identity type), OLS estimators in a SUR model are unbiased and consistent, but their efficiency can be improved by using the information contained in the contemporaneously correlated errors, and in the possible inter-relationships between the explanatory variables across equations. This contemporaneous correlation is reflected in the error term $E_{\sim}(0, \Omega)$, where $\Omega=\Sigma \otimes I_{T}$ (with $\Sigma \neq 0$, for some $i \neq j$ ) is some known real positive definite symmetric matrix of order ( $n T x n T$ ). Then, estimating the equations jointly by GLS yields the BLUE, Zellner(1962). The SURE estimators are obtained by minimizing the
objective function $\mathbf{S}(\mathrm{B})=(\mathbf{Y}-\mathrm{XB})^{\prime}\left(\boldsymbol{\Sigma}^{-1} \otimes \mathrm{I}_{\mathrm{T}}\right)(\mathbf{Y}-\mathrm{XB})$. They are distributed as:

$$
\begin{equation*}
B_{\sim}^{*}\left(\left(X^{\prime} \Omega^{-1} X\right)^{-1} X^{\prime} \Omega^{-1} Y,\left(X^{\prime} \Omega^{-1} X\right)^{-1}\right) \tag{6.25}
\end{equation*}
$$

or in terms of the original data:

$$
B_{\sim}^{*}\left(\left(X^{\prime}\left(\Sigma^{-1} \otimes I\right) X\right)^{-1} X^{\prime}\left(\Sigma^{-1} \otimes I\right) Y,\left(X^{\prime}\left(\Sigma^{-1} \otimes I\right) X\right)^{-1}\right)
$$

Analytically:
where $\sigma^{i j}$ is the $i, j$ th element of $\Sigma^{-1}$ in (6.25'). Also,

$$
V\left(B^{*}\right)=\left(X^{\prime} \Omega^{-1} X\right)^{-1}=\left(\begin{array}{llll}
\sigma^{11} X_{1}^{\prime} X_{1} & \sigma^{12} X_{1}^{\prime} X_{2} & \cdots & \sigma^{1 n} X_{1}^{\prime} X_{n}  \tag{6.26}\\
\sigma^{21} X_{2}^{\prime} X_{1} & \sigma^{22} X_{2}^{\prime} X_{2} & & \sigma^{2 n} X_{2}^{\prime} X_{n} \\
\vdots & & \vdots & \vdots \\
\sigma^{n 1} X_{n}^{\prime} X_{1} & \sigma^{n 2} X_{n}^{\prime} X_{2} & \cdots & \sigma^{n n} X_{n}^{\prime} X_{n}
\end{array}\right)^{-1}
$$

Assuming that $\mathbf{Y}$ follows a multivariate normal distribution, the SURE estimators (6.25) are distributed as:

$$
\begin{equation*}
B_{\sim}^{*} N\left(\left(X^{\prime}\left(\Sigma^{-1} \otimes I\right) X\right)^{-1} X^{\prime}\left(\Sigma^{-1} \otimes I\right) Y, \quad\left(X^{\prime}\left(\Sigma^{-1} \otimes I\right) X\right)^{-1}\right) \tag{6.27}
\end{equation*}
$$

The GLS estimators of (6.25) are equivalent to the MLE: Assuming normality of $Y$ we can write its distribution as:
$F(Y / X, \Theta)=\left[(2 \pi)^{(-n / 2)}\right]|\Omega|^{(-1 / 2)} \exp \left\{-(1 / 2)(Y-X B)^{\prime} \Omega^{-1}(Y-X B)\right\}$

Taking a sample of $T$ idependent and identically distributed random variables $Y$, the likelihood function of the sample is:
$L(Y / X, \Theta)=\left[(2 \pi)^{(-n T / 2)}\right]|\Omega|^{(-1 / 2)} \exp \left\{-(1 / 2)(Y-X B)^{\prime} \Omega^{-1}(Y-X B)\right\}$
or equivalently:
$L=\left[(2 \pi)^{(-n T / 2)}\right]\left|\Sigma^{-1}\right|^{(-T / 2)} \exp \left\{-(1 / 2)(Y-X B)^{\prime}\left(\Sigma^{-1} \otimes I\right)(Y-X B)\right\}$

Maximizing the logarithm of (6.29') with respect to B is equivalent to minimizing the objective function:

$$
\begin{equation*}
S(B)=(Y-X B)^{\prime}\left(\Sigma^{-1} \otimes I\right)(Y-X B) \tag{6.30}
\end{equation*}
$$

A result which is a direct extension of the single equation GLS estimation of the previous section. Thus, assuming normality, the ML and GLS estimators are equivalent.

The MLE of $\Sigma$ is an ( $n \times n$ ) matrix with elements:

$$
\begin{equation*}
\sigma_{i j}=\left(y_{i}-X_{i} \beta_{i}^{*}\right)^{\prime}\left(y_{j}-X_{j} \beta_{j}^{*}\right) / T \tag{6.31}
\end{equation*}
$$

An unbiased estimator of $\Sigma$ is obtained by replacing $T$ with ( $T-k$ ) in the denominator of the above, assuming that $k$ explanatory variables are used in each equation.

Normality of the multivarite distribution of $\mathbf{Y}$ allows one, apart from the ability of using $M L$ methods, to use a more specific stochastic framework to draw inferences about the estimated parameters.

Again, a prerequisite for the estimation of $B^{*}$ and $V\left(B^{*}\right)$ is that Rank (X) $=n k$, with $n T>n k$ (i.e. with $T>k$ ). In addition, $\Sigma$ must be invertible, that is, its columns must be linearly independent. Mathematically, $\operatorname{Rank}(\Sigma)=n$. In terms of $\Omega, \operatorname{Rank}(\Omega)=n T$.

### 6.2.3). SURE Estimation with Unknown Covariance Matrix.

In most cases $\Sigma$ is unknown and $B^{*}$ cannot be estimated. An estimate of $\Sigma$ must be formulated. Then $\hat{\mathbf{B}}^{*}$ may be obtained in two stages. In the first stage, run each individual equation in (6.22) by least
squares and obtain the residuals $e_{i}=\left(\mathbf{y}_{1}-X_{i} b_{i}\right)$. Use these to find an estimate $\hat{\Sigma}$ of $\Sigma$, where $\hat{\Sigma}=\left[\hat{\sigma}_{i j}\right]$ for all $i, j=1, \ldots n$, and $\hat{\sigma}_{i j}=e_{i}^{\prime} e_{j} / T$ are the estimated variances and covariances of $\Sigma$ in (6.21). In the second stage, use $\hat{\Sigma}$ to obtain $\hat{B}^{*}$ as in (6.25') with $\Sigma$ replaced by $\hat{\boldsymbol{\Sigma}}$. These are the SUR estimators proposed by Zellner(1962).

In the above two step procedure, $\hat{\sigma}_{i j}$ is biased in finite samples because of the $T$ in the denominator. An unbiased estimator of $\sigma_{i j}$, when the number of explanatory variables in each equation are the same, is $\hat{\sigma}_{i j}=e_{i}^{\prime} e_{j} /(T-k), \quad \forall i, j=1, \ldots, n$. When the number of explanatory variables across equations are different one may find the average number of coefficients per equation ( $K / n$ ) (where $\left.K=\Sigma_{i} k_{i}\right)$, and use $(T-(K / n))$ as a divisor to obtain an unbiased $\hat{\sigma}_{i j}$.

An alternative, iterative estimator of $B$ may be obtained as an extension of the above SUR estimator. Thus, given the first two steps above, in the third step use $\hat{B}^{*}$ obtained in the second step to find a new estimate of $\Sigma, \hat{\nu}$, through $\hat{\sigma}_{i j}=\left(y_{i}-X_{i} \hat{\beta}_{i}^{*}\right)^{\prime}\left(y_{j}-X_{j} \hat{\beta}_{j}^{*}\right) / T$. In the fourth step use $\hat{\sigma}_{i j}$ to find a new estimator for $B$. This procedure is continued until convergence. Under normality, the estimator obtained is consistent and asymptotically efficient, Gallant(1975). It can be shown, Magnus(1978), that when $\mathbf{E}$ (or equivalently $Y$ ) follows a multivariate normal distribution the latter estimator is the MLE. Under appropriate conditions both the SUR and the iterative SUR estimator have the same limiting distribution.

Asymptotically, the choice of the divisor in estimating $\hat{\boldsymbol{\sigma}}_{i j}$ does not influence the properties of $\hat{\mathbf{B}}^{*}$; it remains an unbiased and consistent estimator of $B$. The small sample properties have been investigated by a number of authors, with different conclusions. Their common ground is that $\hat{\mathbf{B}}^{*}$ is more efficient than the OLS estimator, provided the correlation between the disturbances amongst equations is not too low, cfr Judge et al (1985, p469-470).

Once data restrictions are taken into account in our 'static' seaborne import allocation models, or when we consider their 'dynamic' versions (when expectations are incorporated) the systems become nonlinear. The presence of non-linearities in the system requires numerical iterative methods to reach a solution. Such an iterative procedure has just been considered in the previous subsection of SUR estimation for an unknown covariance matrix. A number of other algorithms are suggested in the literature, see Madalla(1977, p171-174), Judge et al(1988, Ch.12), Pindyck and Rubinfeld(1981, p262-265). Thus, the methods of 'direct search', 'steepest descent', 'direct optimization', 'scoring', 'Gauss-Newton', 'Newton-Raphson' etc are possible alternatives. We concentrate here on the 'Gauss-Newton' algorithm which is the most efficient computationally, and is the one we use for estimation. It is based on Taylor approximations of the original nonlinear system.

Assuming a system of equations which has the form of (6.22), obtaining estimates of the parameters $\theta \equiv(B, \Sigma)$ involves minimizing the objective function:

$$
\begin{equation*}
S(B)=[Y-F(X, \Theta)]^{\prime}\left(\Sigma^{-1} \otimes I_{T}\right)[Y-F(X, \Theta)] \tag{6.32}
\end{equation*}
$$

We write the function $\mathbf{F}(\mathbf{X}, \Theta)$ for convenience as $\mathrm{F}(\mathrm{B})$. The FOC for minimization of the above are:

$$
\begin{align*}
\partial[S(B)] / \partial B & =-2\{\partial[F(B)] \cdot / \partial B\}\left(\Sigma^{-1} \otimes I_{T}\right)[Y-F(B)]=0  \tag{6.33}\\
& =Z(B)\left(\Sigma^{-1} \otimes I_{T}\right)[Y-F(B)]=0 \tag{6.33'}
\end{align*}
$$

This a system of $n k$ equations. They are evaluated at some initial estimates of $B$, which we provide, say $B_{0}$. Taking a first order Taylor approximation of the model, equation (6.22), around $B_{0}$, yields:

$$
\begin{equation*}
Y \simeq F_{0}\left(B_{0}\right)+\left\{\partial\left[F_{0}\left(B_{0}\right)\right] \cdot / \partial B\right\}\left[B-B_{0}\right]+E \tag{6.34}
\end{equation*}
$$

$\therefore \quad\left\{Y-F_{0}\left(B_{0}\right)-\left\{\partial\left[F_{0}\left(B_{0}\right)\right], / \partial B\right\} B_{0}\right\} \simeq\left\{\partial\left[F_{0}\left(B_{0}\right)\right]^{\prime} / \partial B\right\} B+E$

To simplify the notation write the above as:

$$
\begin{equation*}
\bar{Y}_{0}=Z_{0}\left(B_{0}\right) B+E \tag{6.35'}
\end{equation*}
$$

where the definitions of $\bar{Y}_{0}$ and $Z_{0}\left(B_{0}\right)$ are obvious by comparing (6.35) with (6.35').

Next use least squares in (6.35') to find a second estimate of B, $B_{1}$, say. That is, minimize the objective function:

$$
\begin{equation*}
S(B)=\left[Y-Z_{0}\left(B_{0}\right) B\right]^{\prime}\left(\Sigma^{-1} \otimes I\right)\left[Y-Z_{0}\left(B_{0}\right) B\right] \tag{6.36}
\end{equation*}
$$

The FOC of the above yield a second estimate of $B, B_{1}$ say. Thus,

$$
\begin{gather*}
B_{1}=\left\{\left[Z_{0}\left(B_{0}\right)\right]^{\prime}\left(\Sigma^{-1} \otimes I_{T}\right)\left[Z_{0}(B)\right]\right\}^{-1} Z_{0}\left(B_{0}\right)^{\prime}\left(\Sigma^{-1} \otimes I_{T}\right) \bar{Y}\left(B_{0}\right)  \tag{6.37}\\
=B_{0}+\left\{\left[Z_{0}\left(B_{0}\right)\right]^{\prime}\left(\Sigma^{-1} \otimes I_{T}\right)\left[Z_{0}(B)\right]\right\}^{-1} Z_{0}\left(B_{0}\right)^{\prime}\left(\Sigma^{-1} \otimes I_{T}\right)\left[Y-F\left(B_{0}\right)\right] \tag{6.37'}
\end{gather*}
$$

Continuing this process we get the nth estimate of $B$ :

$$
\begin{equation*}
B_{n}=B_{n-1}+ \tag{6.38}
\end{equation*}
$$

$+\left\{\left[Z_{n-1}\left(B_{n-1}\right)\right]^{\prime}\left(\Sigma^{-1} \otimes I_{T}\right)\left[Z_{n-1}\left(B_{n-1}\right)\right]\right\}^{-1} Z_{n-1}\left(B_{n-1}\right)^{\prime}\left(\Sigma^{-1} \otimes I_{T}\right)\left[Y-F\left(B_{n-1}\right)\right]$

When the process has converged the FOC are satisfied, $\left[Z_{n-1}\left(B_{n-1}\right)\right] '\left(\Sigma^{-1} \otimes I_{T}\right)\left[Y-F\left(B_{n-1}\right)\right]=0$, and $B_{n}=B_{n-1}$.

When $\Sigma$ is unknown a two step procedure similar to that we described for the linear SUR estimator may be followed. The least squares estimates of the first step are obtained by the nonlinear iterative procedure described above.

Assuming $Y$ is normally distributed, its maximum likelihood function resembles that of (6.29') with (Y-XB) replaced by
$(\mathrm{Y}-\mathrm{F}(\mathrm{X}, \otimes))$. The log of the likelihood function becomes:
$L L=(n T / 2) \ln 2 \pi-(T / 2) \ln |\Sigma|-(1 / 2)\left\{(Y-F(X, \Theta))^{\prime}\left(\Sigma^{-1} \otimes I\right)(Y-F(X, \Theta))\right\}$

Maximizing the above can be seen to be equivalent to minimizing the objective function (6.32). Thus, under normality, the nonlinear GLS and ML estimators are equivalent.

### 6.2.5). Properties of SURE Estimators.

There are two conditions under which both the linear and the nonlinear SUR estimators are identical to the OLS:
1). When there is no contemporaneous correlation, in which case the covariance matrix $\Sigma$ is diagonal and $\Sigma=\left[\sigma_{1]}\right]=0$, all $i \neq j$.
2). When the explanatory variables across equations are identical. That is, for all equations $X_{1}=\ldots=X_{N}=\bar{X}$ (say), so that the matrix $X$ of $(6.19) /(6.20)$ becomes $X=\left(I_{n} \otimes \bar{X}\right)$. Then $\left(X^{\prime}\left(\Sigma^{-1} \otimes I\right) X\right)$ of (6.25') becomes: $\quad\left(X^{\prime}\left(\Sigma^{-1} \otimes I\right) X\right)=\left(I \otimes \bar{X}^{\prime}\right)\left(\Sigma^{-1} \otimes I\right)(I \otimes \bar{X})=\Sigma^{-1} \otimes(\bar{X}, \bar{X})$. Using this in (6.25') to find $B^{*}$ we get: $B^{*}=\left(\Sigma^{-1} \otimes(\bar{X}, \bar{X})\right)^{-1}(I \otimes \bar{X})\left(\Sigma^{-1} \otimes I\right) Y$ $=\left(\Sigma \otimes(\overline{\mathrm{X}}, \overline{\mathrm{X}})^{-1}\right)(\mathrm{I} \otimes \overline{\mathrm{X}})\left(\Sigma^{-1} \otimes \mathrm{I}\right) \mathrm{Y}=\left[I \otimes\left(\overline{\mathrm{X}} \bar{X}^{\prime}\right)^{-1} \bar{X}^{\prime}\right] \mathrm{Y}$, which is nothing but the OLS estimator applied to each equation separately.

In either case there is no gain in efficiency by estimating the equations jointly, and it is 'best' if OLS is used.

Another point that is worth making is that, if equations are arranged in two or more groups such that the disturbances of each equation in each group are only correlated with the disturbances of equations in the same group (but are uncorrelated with disturbances of equations in other groups), then the matrix $\Sigma$ is block-diagonal. Then (6.25) can be applied separately to each group of equations.

So far, we have seen that, if there is any type of relationhip between the equations (such as a system with error related equations in some way), taking account of this information by estimating the equations jointly results in more efficient estimates. More generally, if there are time-invariant or individual-invariant
parameters, then 'pooling' the system together results in more efficient estimates (due to the increased degrees of freedom).

There are other ways of estimating 'pooled' data models apart from SURE. In SURE we assumed separate coefficient estimates (but fixed over time) for each exporting region and contemporaneous correlation in the error term. Other 'pooling' methods make alternative assumptions about the disturbance terms and about the way the coefficient vector changes over cross-section or time. Thus, the Dummy Variable (or Covariance) model assumes a white noise error term and an identical coefficient vector, except from the constant terms which are allowed to vary. The Error Components model is an extension of the Dummy Variable model, by assuming that the constant terms are random, effectively providing for more complex structures of the error term. Another alternative is the Swamy(1970) random coefficient model. It can be regarded as an extension of SURE by assuming that the individuals represent a random sample from some larger population (which is not true for our models, since we consider all the regions of the world). It is then appropriate to regard the different coefficient vectors as random drawings from some probability distribution.

## 6.3). Import Allocation Models as SURE with Singular Covariance Matrix.

Equation (6.22) expresses our import allocation models in a general form, which we repeat here for convenience:

$$
\begin{equation*}
Y=F(X, \Theta)+E, \quad E_{\sim}\left(0, \Sigma \otimes I_{T}\right) \quad \text { or } \quad E_{\sim}(0, \Omega) \tag{6.40}
\end{equation*}
$$

That is, it is assumed that for the error term:

$$
\begin{equation*}
E[E]=0 \quad \text { and } \quad E\left[E E^{\prime}\right]=\Omega \equiv \Sigma \otimes I_{T} \tag{6.41}
\end{equation*}
$$

The following functional form constraint applies to the system:

$$
\begin{equation*}
\mathbf{w}^{\prime}[\mathrm{Y}-\mathrm{F}(\mathrm{X}, \Theta)] \equiv 0 \tag{6.42}
\end{equation*}
$$

The constraint carries over to the vector of disturbances:

Hence: $\quad w^{\prime} E[E] \equiv 0$
and

$$
\begin{equation*}
w^{\prime} \Omega=w^{\prime}\left(\Sigma \otimes I_{T}\right)=w^{\prime} E\left[E E^{\prime}\right]=E\left[w^{\prime} E E^{\prime}\right] \equiv 0 \tag{6.44}
\end{equation*}
$$

That is, the rows and columns of $\Omega$ (or equivalently of $\Sigma$ ) are linearly dependent, with $\operatorname{Rank}(\Omega)<n T$ (equivalently, Rank $(\Sigma)<n)$. The sum of any row or column of the covariance matrix is zero, and the matrix is singular. This singularity of $\Omega(\Sigma)$ prevents us from finding the SUR estimators of (6.25)/(2.25').

Intuitively, because we are faced with an import allocation problem (with the total imports of a region for a commodity allocated amongst its trade partners), the equations are linearly dependent. It is sufficient to allocate imports to the $(n-1)$ regions, the last being a residual. The nth equation is completely redundant, since, by using the information contained in the ( $n-1$ ) equations, we can obtain the $n$th equation as a linear combination of the rest.

The natural solution to the singularity problem is to delete one of the equations in the system. Then $\sum_{i} w_{i}^{0} \varepsilon_{i}=0, i=1, \ldots, n-1$, is not satisfied anymore, the $(n-1)$ errors are independent, and the rank of the covariance of the errors of the $(n-1)$ equations is $(n-1) ; \Sigma$ is invertible. It makes no difference which equation is deleted, since the equations may be put in any order.

However, if for a general system, the linear dependence of the error terms implies linear constraints on the parameters, deleting equations leads to loss of information. Ignoring information results in inefficient estimates, unless the implied parameter restrictions are imposed or fulfilled automatically. This suggests approaching the singular covariance matrix problem, as a restricted least squares problem. In this way all the available information (sample and a priori) can be incorporated in the formulation of the problem.

In that direction, we provide a solution by using the generalized inverse of the singular covariance matrix instead of the ordinary inverse (when $\Sigma$ is invertible) of the restricted GLS formula.

In our import allocation problem, the functional form restrictions imply restrictions on the coefficient vector of $b_{i}, \quad \sum_{i} w_{i}^{0} b_{i}=0$, $i=1, \ldots, n$ (see Chapter 4). Let us take for convenience the linear SUR model, (6.20), and let there be non-sample information available in terms of exact linear restrictions on the parameters, in the form RB=r. Then, (6.20) can be estimated efficiently by formulating the problem as, minimising the objective function:

$$
S(B)=E^{\prime}\left(\Sigma^{-1} \otimes I_{T}\right) E \quad \text { subject to } \quad R B-r=0
$$

Assuming that the rank conditions are satisfied, the Restricted GLS estimator of $B$ is:

$$
\begin{equation*}
B_{R}^{*}=B^{*}+\left[X^{\prime}\left(\Sigma^{-1} \otimes I\right) X\right]^{-1} R^{\prime}\left\{R\left[X^{\prime}\left(\Sigma^{-1} \otimes I\right) X\right]^{-1} R^{\prime}\right\}^{-1}(R B-r) \tag{6.45}
\end{equation*}
$$

where $B^{*}$ is the Unrestricted GLS estimator as defined in (6.25'). Thus, the Restricted GLS estimator differs from the Unrestricted GLS estimator of (6.25'), by a linear function of the restrictions. When the latter are satisfied, that is, when $R B-r=0, B_{R}^{*}$ is equivalent to $B^{*}$.

Let us define the generalised inverse (or Moore-Penrose inverse) of the singular covariance matrix $\Sigma$ as the matrix $\boldsymbol{\Sigma}^{+}$, which satisfies the following four conditions:
i).
ii).
iii).
iv).

$$
\begin{aligned}
& \Sigma \Sigma^{+} \Sigma=\Sigma \\
& \Sigma^{+} \Sigma \Sigma^{+}=\Sigma^{+} \\
& \left(\Sigma \Sigma^{+}\right)^{\prime}=\Sigma \Sigma^{+} \\
& \left(\Sigma^{+} \Sigma\right)^{\prime}=\Sigma^{+} \Sigma
\end{aligned}
$$

It can be shown that for any matrix $\Sigma$, there exists a unique generalized inverse as defined above, Theil(1971, p267). It may be obtained using the orthogonal transformation procedure. This
involves diagonalizing $\Sigma$. The latter can be achieved by finding the positive eigenvalues of $\Sigma((n-1)$, since $\operatorname{Rank}(\Sigma)=n-1)$, and forming the $F$ matrix whose columns are the eigenvectors of $\Sigma$, corresponding to its positive eigenvalues. Similarly, take the remaining 0 eigenvalue of $\Sigma$ and form the zero vector $G$, which corresponds to the 0 eigenvalue. The augmented matrix $U=[F, G]$, whose columns are the characteristic vectors of $\Sigma$, is orthogonal and will diagonalize the covariance matrix $\Sigma$, with the characteristic roots of the latter on the diagonal. The columns of $U({\underset{-1}{1}}$, say) are the distinct characteristic vectors of $\Sigma$, and are orthogonal in the sense that their inner product $\underset{-1}{x_{1}^{\prime}}=0, i, j=1, \ldots n$. In matrix form, $U=[F, G]$ is orthogonal, since, $U^{\prime} U=U U^{\prime}=I_{n}$. That is,

$$
U^{\prime} U=\left[\begin{array}{l}
F^{\prime} F F^{\prime} G  \tag{6.46}\\
G^{\prime} F G^{\prime} G
\end{array}\right]=\left[\begin{array}{ll}
I_{n-1} & 0 \\
0 & I_{1}
\end{array}\right]=I_{n}
$$

Thus, the quadratic (or Jordan Canonical) form U' $\Sigma U$ diagonalizes $\Sigma$, with the characteristic roots of $\Sigma$ on the diagonal.

$$
\therefore \quad U^{\prime} \Sigma U=\Lambda=\left(\begin{array}{ll}
\Lambda_{n-1} & 0  \tag{6.47}\\
0 & 0
\end{array}\right)
$$

where $\Lambda_{n-1}$ is the diagonal matrix with the ( $n-1$ ) positive eigenvalues of $\Sigma$ as its elements.
(6.20) may now be transformed by premultiplying it by $U^{\prime}$, to get:

$$
\begin{equation*}
U^{\prime} Y=U^{\prime} X B+U^{\prime} E, \quad \text { where } \quad E_{\sim}\left(0, \Sigma \otimes I_{T}\right) \tag{6.48}
\end{equation*}
$$

Equivalently,

$$
\left[\begin{array}{l}
F^{\prime} Y \\
G^{\prime} Y
\end{array}\right]=\left[\begin{array}{l}
F^{\prime} X \\
G^{\prime} X
\end{array}\right] B+\left[\begin{array}{l}
F^{\prime} E \\
G^{\prime} E
\end{array}\right]
$$

From (6.47), the covariance of $G^{\prime} E, E\left[G^{\prime} E E^{\prime} G\right]=G^{\prime} \Sigma G=0$. That is, $G^{\prime} E$ is a random vector 0 , and (6.48') can be thought of as estimating,

$$
\begin{equation*}
F^{\prime} Y=F^{\prime} X B+F^{\prime} E \tag{6.49}
\end{equation*}
$$

subject to the restrictions:

$$
\begin{equation*}
G^{\prime} Y=G^{\prime} X B \tag{6.50}
\end{equation*}
$$

From (6.47), the covariance of $F^{\prime} E$ is, $E\left[F^{\prime} E E^{\prime} F\right]=F^{\prime} \Sigma F=\Lambda_{n-1}$. We define the generalized inverse of $\Sigma$, to be the new matrix $\Sigma^{+}=F \Lambda_{n-1}^{-1} F^{\prime}$. It can be checked that $\Sigma^{+}$satisfies conditions (i)-(iv). Hence, the generalized restricted least squares estimator takes the form (Theil(1971, p285)):

$$
\begin{equation*}
\mathrm{B}_{\mathrm{R}}^{*}=\mathrm{B}^{*}+\left[\mathrm{X}^{\prime}\left(\Sigma^{+} \otimes I\right) X\right]^{-1} X^{\prime} G\left\{G^{\prime} X\left[X^{\prime}\left(\Sigma^{+} \otimes I\right) X\right]^{-1} X^{\prime} G\right\}^{-1}\left(G^{\prime} Y-G^{\prime} X B\right) \tag{6.51}
\end{equation*}
$$

where $\mathrm{B}^{*}$ is the unrestricted generalized estimator of B ,

$$
\begin{equation*}
B^{*}=\left[X^{\prime}\left(\Sigma^{+} \otimes I\right) X\right]^{-1} X^{\prime}\left(\Sigma^{+} \otimes I\right) Y \tag{6.52}
\end{equation*}
$$

that is,

$$
\begin{equation*}
B^{*}=\left[X^{\prime}\left(F \Lambda_{n-1}^{-1} F^{\prime} \otimes I\right) X\right]^{-1} X^{\prime}\left(F \Lambda_{n-1}^{-1} F^{\prime} \otimes I\right) Y \tag{6.52'}
\end{equation*}
$$

assuming that $X^{\prime} F$ has full rank.

Also, the variance of $B_{R}^{*}$ is:
$V\left(B_{R}^{*}\right)=\left(X^{\prime} \Omega^{+} X\right)^{-1}-\left(X^{\prime} \Omega^{+} X\right)^{-1} X^{\prime} G\left[G^{\prime} X\left(X^{\prime} \Omega^{+} X\right)^{-1} X^{\prime} G\right]^{-1} G^{\prime} X\left(X^{\prime} \Omega^{+} X\right)^{-1}$

It can be seen that the Restricted GLS estimator (6.45) is equivalent to that of (6.51), where the $R B=r$ have become $G^{\prime} X B=G^{\prime} Y$, and the covariance matrix $\Sigma^{-1}$ is reduced to $\Sigma^{+}$.

When $G^{\prime} X=0$ the constraint (6.50) on the parameters is eliminated. Then $B_{R}^{*}=B^{*}$. If we denote the generalized inverse of $\Omega$ by $\Omega^{+}$, where $\Omega^{+}=\left(\Sigma^{+} \otimes I\right)=\left(F \Lambda_{n-1}^{-1} F^{\prime} \otimes I\right)$ and assuming that $G^{\prime} X=0$, the generalized estimator (6.51) and (6.52) may be written in compact form as:

$$
\begin{equation*}
B^{*} \sim\left(\left(X^{\prime} \Omega^{+} X\right)^{-1} X^{\prime} \Omega^{+} Y, \quad\left(X^{\prime} \Omega^{+} X\right)^{-1}\right) \tag{6.54}
\end{equation*}
$$

The above estimator is also valid if $G^{\prime} X \neq 0$ but the restrictions on the parameters are automatically fulfilled in $\mathbf{B}^{*}$.

In practice, deleting one of the equations makes the rest linearly independent (This automatically imposes the constraint $\sum_{i} w_{i}^{0} c_{1}=0$ in the import allocation model). The usual procedures can be applied in the subsystem of the $(n-1)$ remaining equations. Thus, we overcome the problem of estimating the generalized inverse $\boldsymbol{\Sigma}^{\boldsymbol{+}}$.

Assuming that the conditional distribution of $Y / X$ is normal, we can compute confidence intervals and test hypotheses on the parameters.

Furthermore, assuming normality of $Y / X$, we can formulate the problem in terms of maximizing a restricted likelihood function of $(n-1)$ equations. Thus, the likelihood function (6.29) (or in terms of the nonlinear model the log-likelihood function (6.39)) becomes:

$$
\begin{equation*}
L(Y / X, \Theta)=2 \pi^{[-(n-1) T / 2]}\left|\Omega^{+}\right| \exp \left[(-1 / 2) E^{\prime} \Omega^{+} E\right] \tag{6.55}
\end{equation*}
$$

where $\Omega^{+}$is the generalized inverse of $\Omega^{-1}$. That is, the $T(n-1)$ full rank matrix obtained by deleting one of the equations in the system.

Thus, given the functional form constraints (6.24)/(6.24'), the problem of the importer is to maximize (6.55) subject to:

$$
\begin{equation*}
\mathbf{w}^{\prime} \Omega \equiv 0 \quad \text { (equivalently, } \quad \mathbf{w}^{\prime} \Sigma \equiv 0 \text { ) } \tag{6.56}
\end{equation*}
$$

and $\quad a_{i} \geq 0 \quad$ (see Chapter 3)

This problem, based on the same arguments as before, can be seen to be equivalent to minimizing the objective function:

$$
S(B)=E^{\prime}\left(\Sigma^{-1} \otimes I_{T}\right) E \quad \text { subject to }(6.56) \text { and }(6.57)
$$

or equivalently,

```
min S(B)= E'( }\mp@subsup{\Sigma}{}{+}\otimes\mp@subsup{I}{T}{})E\mathrm{ subject to (6.56) and (6.57)
```

Therefore, the ML and the GLS estimators of B for the 'restricted' problem are equivalent; those of (6.52)/(6.52')/(6.54)

The ijth term of the MLE of $\Sigma$ is given by:

$$
\begin{equation*}
\sigma_{i j}=\left(y_{i}-x_{i} \beta_{i}\right)^{\prime}\left(y_{j}-x_{j} \beta_{j}\right) / T \tag{6.58}
\end{equation*}
$$

These principles apply to both linear and nonlinear models.

## 6.4). Tests of Hypotheses on the Statistical Parameters $\Theta=\left(B_{2} \Sigma\right)$.

When there is no type of cross equation correlations in the residuals, then applying OLS separately to each equation gives the BLUE. When the model is 'pooled' together (due, perhaps, to common coefficients across equations), then a diagonal matrix would also give the BLUE estimators, provided there are no cross equation correlations. We suggest tests for the nature of the covariance model here, which is useful in order to determine the 'best' method of estimation.

From the optimization problem of the importer, we observe that the constraint $a_{i} \geq 0$ should be satisfied. In practice, negative $a_{i}$ 's are set to zero. This involves restrictions on the parameters of the model, and should be tested. Also, in chapter 4 we developed a framework of nested models. At the theoretical level, the CRESH model was constrained by placing restrictions on the elasticities of substitution between trade partners. Thus, the CES, the LES, the Cobb-Douglas and the Leontief models were obtained by placing coefficient restrictions across the equations of the system. Similarly, a number of nested models were also suggested as restricted versions of a more general 'Mixed' model by assuming alternative dynamic mechanisms. Such restricted versions of the 'Mixed' system were the AE (Adaptive Expectations), the PA (Partial

Adjustment), the Static models, or systems where the time trend is droped. Effectively, moving from a 'general' to a more 'restricted' model implies coefficient restrictions on the more general model. These coefficient restrictions can be tested statistically and we suggest a number of test statistics for that purpose here.

### 6.4.1). Tests for Contemporaneous Correlation.

The simplest type of cross-equation correlation is contemporaneous correlation. Tests for contemporaneous correlation (that is, for a diagonal $\Sigma$ ) amounts to testing the following hypothesis.

$$
\begin{align*}
& H_{0}: \sigma_{i j}=0 \quad \forall \quad i \neq j, \quad i, j=1, \ldots, n  \tag{6.59}\\
& H_{A}: \text { At least one } \sigma_{i j} \neq 0, \forall i \neq j .
\end{align*}
$$

Assuming normality, Breusch and Pagan(1980) show that a Lagrange multiplier test may be used with a test statistic:

$$
\begin{equation*}
\lambda_{L M}=T \sum_{i=2}^{n} \sum_{j=1}^{i-1} r_{i j}^{2} \underset{\sim}{H_{0}} \underset{\sim}{\text { as }} \chi^{2}((n(n-1)) / 2) \tag{6.60}
\end{equation*}
$$

where $r_{i j}$ is the correlation coefficient $r_{i j}=\hat{\sigma}_{i j} /\left(\hat{\sigma}_{i i} \hat{\sigma}_{j j}\right)^{(1 / 2)}$, with $\sigma_{i j}=\left(y_{i}-x_{i} \beta_{i}\right) \prime\left(y_{j}-x_{j} \beta_{j}\right) / T$, and a rejection region specified by $\lambda_{L M}>\chi_{\alpha}^{2}((n(n-1)) / 2)$, where $\alpha$ is the prespecified significance level. Note, that $n(n-1) / 2$ is half the off diagonal elements in $\Sigma$.

Alternatively, the Maximum Likelihood ratio may be used to test $H_{0}$. The test statistic is:

$$
\begin{equation*}
\lambda_{L R}=T \ln \left[\left|\hat{\Sigma}^{*}\right| /|\hat{\Sigma}|\right] \underset{\hat{H}_{0}}{\text { as }} \chi^{2}((n(n-1)) / 2) \tag{6.61}
\end{equation*}
$$

where $\hat{\Sigma}^{*}$ is the constrained MLE of $\Sigma$ (that is, a diagonal matrix with $\hat{\sigma}_{i 1}$ on the diagonal), based on the least squares residuals. $\hat{\Sigma}$ is the unconstrained estimator (the matrix $\hat{\boldsymbol{\Sigma}}$ with at least one $\hat{\sigma}_{i j} \neq 0$, for $i \neq j$ ) of $\Sigma$. It may be adequate to approximate $\hat{\Sigma}$ with an
estimator based on the residuals of the SURE estimator. The rejection region remains the same as with the LM test.

When we estimate our complete system with one equation droped, then in all the above results $n$ shoud be replaced with ( $n-1$ ).

### 6.4.2). Tests of Restrictions Across Equations.

Tests for the validity of linear restrictions of the coefficients across equations, of the form $R B=r$, are considered here. The test involves a comparison of the residual sum of squares of the restricted and unrestricted models. If the increase in the residual sum of squares, as we move from the unrestricted to the restricted model, is not 'significant' we accept the restricted model, otherwise we reject it. To test the significance of the change in the residual sum of squares we need a test statistic whose distribution is known.

Judge et al(1988, p456-459) show that the appropriate test statistic, when the covariance matrix is of the scalar identity type, is:

$$
\begin{equation*}
F=\frac{\left[\left(Y-X \hat{B}_{R}^{*}\right)^{\prime}\left(Y-X \hat{B}_{R}^{*}\right)-\left(Y-X \hat{B}^{*}\right)^{\prime}\left(Y-X \hat{B}^{*}\right)\right] / J}{\left[\left(Y-X \hat{B}^{*}\right)^{\prime}\left(Y-X \hat{B}^{*}\right)\right] /(n T-K)} \sim_{H_{0}} F(J, n T-K) \tag{6.62}
\end{equation*}
$$

where $J$ are the number of restrictions and ( $n T-K$ ) are the number of degrees of freedom of the unrestricted model (where $K=\sum_{i} k_{i}$ ).

When there is contemporaneous correlation in the residuals (such as the SURE model) the test statistic becomes:

$$
\begin{equation*}
F=\frac{\left[\left(Y-X \hat{B}_{R}^{*}\right)^{\prime}\left(\Sigma^{*^{-1}} \otimes I\right)\left(Y-X \hat{B}_{R}^{*}\right)-\left(Y-X \hat{B}^{*}\right)^{\prime}\left(\Sigma^{*^{-1}} \otimes I\right)\left(Y-X \hat{B}^{*}\right)\right] / J}{\left[\left(Y-X \hat{B}^{*}\right)^{\prime}\left(\Sigma^{*^{-1}} \otimes I\right)\left(Y-X \hat{B}^{*}\right)\right] /(n T-K)} \sim_{H_{0}}^{\sim} F(J, n T-K) \tag{6.63}
\end{equation*}
$$

with the rejection region for both statistics specified by $F_{\alpha} F_{\alpha}(J, n T-K)$.

That is, the test statistic is of the general form:

$$
\begin{equation*}
F=[(R R S S-U R S S) / J] /[U R S S /(n T-K)] \underset{H_{0}}{{\underset{H}{0}}^{F}} \underset{(J, n T-K)}{ } \tag{6.64}
\end{equation*}
$$

where

```
RRSS=Restricted Residual Sum of Squares,
URSS=Unrestricted Residual Sum of Squares,
    J =Number of Restrictions,
nT-K=Number of Degrees of Freedom of the Unrestricted Model.
```

In general $\Sigma$ is unknown and has to be replaced by $\hat{\Sigma}$, and thus $\hat{\mathbf{B}}_{\mathrm{R}}^{*}$ and $\hat{\mathbf{B}}^{*}$ need to be replaced by $\hat{\hat{B}}_{\mathrm{R}}{ }^{*}$ and $\hat{\hat{B}}^{*}$, respectively. The above statistics have an approximate $F$ distribution. Note that, when $\Sigma$ is unknown, in order to avoid deriving a misbehaving $F$ statistic (that is, a negative $F$ statistic), $\hat{\Sigma}$ needs to be held constant between the unrestricted and restricted models.

Also, the denominator in (6.64) converges in probability to one, and we may instead use the following statistic to test our hypotheses (assuming contemporaneous correlation):

$$
\begin{equation*}
\left.g_{1}=\left[\left(Y-X_{B}^{*}\right)^{*}\right)^{*^{-1}}\left(Y-X \hat{B}_{R}^{*}\right)-\left(Y-X \hat{B}^{*}\right)^{\prime} \Omega^{-1}\left(Y-\hat{X B}^{*}\right)\right]_{H_{0}^{\sim}}^{\sim} \chi^{2}(J) \tag{6.65}
\end{equation*}
$$

We can write $g_{1}$ in terms of the restrictions in the model as:

$$
\begin{equation*}
g_{2}=\left[(r-R \hat{R})^{\prime}\left[R\left(X^{\prime} \Omega^{-1} X\right)^{-1} R^{\prime}\right]^{-1}(r-R \hat{B})\right] \underset{H_{0}}{\sim} \chi^{2}(J) \tag{6.66}
\end{equation*}
$$

In finite samples, dividing either $g_{1}$ or $g_{2}$, which we may call $g$, by J yields:

$$
\begin{equation*}
\lambda=(g / J) \underset{H_{0}}{\underset{\sim}{x}} \underset{(J, n T-K)}{ } \tag{6.67}
\end{equation*}
$$

which gives a more 'cautious' statistic, rejecting the Null in a
smaller number of cases.

Asymptotically, the following likelihood ratio test may be used:

$$
\begin{equation*}
L R=T \ln \left[\left|\tilde{\Sigma}^{*}\right| /|\tilde{\Sigma}|\right] \underset{\underset{\sim}{H_{0}}}{\text { as }} \chi^{2}(J) \tag{6.68}
\end{equation*}
$$

where $\tilde{\Sigma}$ and $\tilde{\Sigma}^{*}$ are the unrestricted and the restricted MLE of $\Sigma$, respectively.
The aim of this chapter has been to specify the statistical
environment of the import allocation models specified in chapter 4 .
These are systems of $n$ import demand equations with $k$ predetermined
variables and common coefficients between equations. As a result,
the system needs to be estimated jointly. Furthermore, possible
omitted variables (in conjunction with the common coefficients) and
functional form conmstraints, introduce cross-equation correlations
and this classifies the system as SURE.

[^0]
## RESULTS

## 7.0). Introduction.

In the previous chapters we have derived three alternative empirical forms of our CRESH seaborne import allocation model. In this chapter we discuss the empirical difficulties in estimating these models, and we let our data decide on the 'best' functional form model in each import market. Bilateral import elasticities of demand are calculated, and we use these to make inferences about the competitiveness of international markets.

This chapter is in five main sections. In the first, we present the alternative functional forms that may be used for estimation of our seaborne import allocation model. In the second, we put together all the points related to estimation, as developed in the previous chapters, to present a program logic for estimation. In the third section, we specify the criteria for selection of the 'best' functional form, and we present the empirical results of this exercise for the 5 goods. In the fourth, we present the coefficient estimates of the selected functional form models related to the calculation of the import elasticities of demand, and we discuss the elasticity estimates. We estimate bilateral import elasticities of demand, with respect to changes in the own export prices of the exporting regions, with respect to changes in prices of other trade partners, and with respect to changes in the world prices of goods. In the fifth section, we discuss results related to trends and dynamics of the estimated systems.

## 7.1). The Estimating Seaborne Import Allocation Systems.

In chapter 4, we argued that the most general set of systems,
which the length of our data allow us to estimate are those of Adaptive Expectations(AE), derived in equations (4.50)-(4.52). Three functional forms are distinguished, levels, logs and logarithmic first differences. Thus, for some import market, defined by the importing region $j$ and the traded good $k$ (the subscripts $j, k$, omitted throughout), we have the following estimating systems.
a). Variables in Levels:

$$
\begin{align*}
y_{i t} & =\lambda y_{i t-1}+(1-\lambda) b_{i} t+\lambda b_{i}-(1-\lambda) a_{i}\left\{\left[x p_{i}^{\alpha} w p_{k}^{(1-\alpha)}\right]-\sum_{h} w_{h}^{0}\left[x p_{h}^{\alpha} w p_{k}^{(1-\alpha)}\right]\right\} \\
& +(1-\lambda)\left\{\sum_{i} w_{i}^{0} a_{i}\left[x p_{i}^{\alpha} w p_{k}^{(1-\alpha)}\right]-\sum_{h} w_{h}^{0}\left[x p_{h}^{\alpha} w p_{k}^{(1-\alpha)}\right]\right\} \tag{7.1}
\end{align*}
$$

b). Variables in Logarithms:

$$
\begin{align*}
y_{i t} & =\lambda y_{i t-1}+(1-\lambda) b_{i} t+\lambda b_{i}  \tag{7.2}\\
& -(1-\lambda) a_{i}\left\{\left[\alpha \ln \times p_{i}+(1-\alpha) \ln w p_{k}\right]-\sum_{h} w_{h}^{0}\left[\alpha \ln \times p_{h}+(1-\alpha) \ln w p_{k}\right]\right\} \\
& +(1-\lambda)\left\{\sum_{i} w_{i}^{0} a_{i}\left[\alpha \ln \times p_{i}+(1-\alpha) \ln w p_{k}\right]-\sum_{h} w_{h}^{0}\left[\alpha \ln \times p_{h}+(1-\alpha) \ln w p_{k}\right]\right\}
\end{align*}
$$

c). Variables in Logarithmic First Differences:

$$
\begin{align*}
y_{i t} & =\lambda y_{i t-1}+(1-\lambda) b_{i}  \tag{7.3}\\
& -(1-\lambda) a_{i}\left\{\left[\alpha \Delta \ln \times p_{i}+(1-\alpha) \Delta \ln w p_{k}\right]-\sum_{h} w_{h}^{0}\left[\alpha \Delta \ln \times p_{h}+(1-\alpha) \Delta \ln w p_{k}\right]\right\} \\
& +(1-\lambda)\left\{\sum_{i} w_{i}^{0} a_{i}\left[\alpha \Delta \ln \times p_{i}+(1-\alpha) \Delta \ln w p_{k}\right]-\sum_{h} w_{h}^{0}\left[\alpha \Delta \ln \times p_{h}+(1-\alpha) \Delta \ln w p_{k}\right]\right\}
\end{align*}
$$

where $y_{\text {it }}$ in levels, logs and logarithmic first differences is defined, respectively, as:

$$
\begin{equation*}
\left(m_{i}-w_{i}^{0} m_{t}\right)_{i}, m_{i}^{0}, \quad \ln \left(m_{i} / \bar{w}_{i}^{0} \bar{m}_{t}, \quad \Delta \ln \left(m_{i} / \tilde{m}\right)_{t}\right. \tag{7.4}
\end{equation*}
$$

where, $w_{i}^{0}=m_{i}^{0} / m^{0}, \quad \ln \bar{m}_{t}=\sum_{i} w_{i}^{0} \ln m_{i t}, \quad W_{i}^{-0}=m_{i}^{0} / \bar{m}^{0} \quad$ and $\quad \Delta \ln \tilde{m}_{t}=\sum_{i} w_{i}^{0} \Delta \ln m_{i t}$.

## 7.2). The Estimating Program Logic.

Regarding estimation of the above, for each importing region and each commodity group, we are faced with sets of 30 equations to estimate. For 30 importing regions, over three functional forms, there are 90 systems to estimate. Over 5 goods, it involves 450 estimating systems. Economies of scale in computer programming for such a mass operation are vital. With this in mind, we have developed a computer program which is based on the following ideas.

The systems satisfy the functional adding-up constraint of $\Sigma_{i} W_{i}^{0} y_{i t}=0$. As a result, SURE methods should be employed for estimation (see chapter 6), and the restriction $\sum_{i} W_{i}^{0} b_{i}=0$ is placed on the parameters of each system (see chapter 4). From the definition of the bilateral price index through a Cobb-Douglas function we have the condition $0 \leq \alpha \leq 1$ (see chapter 4). Regarding dynamics, the condition should be satisfied by the definition of the A.E. mechanism (see chapter 4). Finally, the theoretical conditions for the existence of the CRESH composite price index, imply $a_{i} \geq 0$ (see chapter 3 ).

Thus, for some import market $j, k$, we estimate each of (7.1)-(7.3) by nonlinear Least Square SURE methods subject to the

| adding up condition: | $\sum_{i} w_{i} b_{i}=0$, |
| :--- | :--- |
| the condition involving the price proxies: | $0 \leq \alpha \leq 1$, |
| the condition involving dynamics: | $0 \leq \lambda \leq 1$, |
| and the second order conditions of: | $a_{i} \geq 0$. |

The following points are important for the estimation of each model:

- The adding-up condition is enforced by excluding one equation from the complete system. The $b_{i}$ for the excluded equation is obtained through the condition $\sum_{i} W_{i}^{0} b_{i}=0$, while the corresponding $a_{i}$ is estimated directly through the composite price index.
- Since the model is nonlinear in the parameters with contemporaneous correlation of the residuals across equations, we employ the Gauss iterative methods for SURE models outlined in chapter 6.
- Having fewer time series observations (18) than equations (30), we impose an arbitrary constant (across equations) diagonal starting covariance matrix of the form $\sigma^{2} \otimes I_{T}$.
- In practice, the crucial coefficient, in order to reach convergence in these large nonlinear systems, turns out to be $\alpha$. In our experience, when $\alpha$ is fixed, the systems converge. We start with $\alpha=0.5$ (thus giving an equal weight to $x p_{i}$ and $w p_{k}$ ), in order to obtain better estimates (than the starting values) of the other coefficient parameters $\left(a_{i}, b_{i}\right.$ and $\lambda$ ) in the system. At the same time, the initial covariance matrix is improved upon (in order to get better estimates of the standard errors of the estimated coefficients), by using $\sigma_{i i}^{2} \otimes I_{T}$ from the results of the first estimation. In fact, $\sigma_{i i}^{2} \otimes I \mathrm{I}$ is improved in succesive estimations by using its previous estimating value when we estimate the system again.

Taking into account the above points, we have written a computer program for each import region, the logic of which runs as follows:
1). We estimate the system once (allowing up to 20 iterations) with $\alpha=0.5$ and $\sigma^{2} \otimes I_{T}$ (and some initial values for $a_{i}, b_{i}$ and $\lambda$ ), in order to get estimates of the covariance matrix of the residuals and the coefficient parameters $a_{i}, b_{i}$ and $\lambda$.
2). We estimate the system a second time (up to 20 iterations) with $\alpha=0.5$, starting with the previous covariance matrix estimates $\sigma_{i 1}^{2} \otimes I \mathrm{I}$ (where $\sigma^{2}$ is now allowed to vary across equations) and
coefficient estimates of $a_{i}, b_{i}, \lambda$. This gives us a more realistic set of estimates of the parameters of interest (with $\alpha=0.5$ ), as compared to the 'arbitrary' values we have started with.
3). The third time (up to 60 iterations), we let $\alpha$ become a parameter, using the previous estimates of the converged model. This model gives us new estimates of all the coefficient parameters (including $\alpha$ ).
4). Next, we test whether $\alpha$ satisfies the condition $0 \leq \alpha \leq 1$. If $\alpha$ takes a value outside this range then it is set to a constant and equal to its closest boundary ( 0 or 1 ). If $\alpha$ is set equal to either 0 or 1 , the system converges in the next estimation. If $\alpha$ takes a value within the boundaries, then it remains a parameter in the next estimation with an initial value equal to the value from the previous SURE.
5). Similarly, we test the condition $0 \leq \lambda \leq 1$. If $\lambda$ takes a value outside its boundaries, it is set equal to one of them, as with $\alpha$ above. $\lambda$ has never become greater than 1. Note that, whether $\lambda$ is constant or not is not important for convergence.
6). Now, we distinguish two possibilities according to whether $0 \leq \alpha \leq 1$, or not.

6a). When $\alpha$ is set equal to one of its boundaries the model converges very quickly, usually in 2 to 3 iterations. We estimate the system twice to get better estimates of the parameters, corresponding to this new value of $\alpha$. Next, we impose, succesively, the second order conditions $a_{i} \geq 0$, by seting the negative $a_{i}$ 's to zero, until we are left with only positive ones, and the model has converged.

6b). When $\alpha$ stays within its boundaries then there are two possibilities. The model will have either converged or not.
i). If the model had not converged, then $\alpha$ is set to a constant and equal to its latest estimate. The system will now converge. It is now estimated twice (up to 60 iterations each time) with $\alpha$ as a constant and equal to its previous value, before the negative $a_{1}$ 's are set to 0 , in succesive estimations.
ii). If the model converges with $\alpha$ as a parameter, then we go through the same loop described in 6a). That is, the system is estimated twice (up to 60 iterations each time), ensuring that the
value of $\alpha$ stays within its boundaries in each estimation. Then the second order condition is imposed by starting to set the negative $a_{i}$ 's to 0 , until $a_{i} \geq 0, \forall i=1, \ldots, n$.

Finally, a word on the excluded equations from each system. These fall into two categories:
i) equations which are dropped from the system due to lack of data over the entire estimation period, which can be identified by the NA (Non Available) entry in the tables of the estimated $a_{i}$ coefficients, presented later in the chapter.
ii) the equation excluded from each complete system in order to avoid singularity (see chapter 6). Since there is no individual region that is distinguished for its size and no absent trade throughout the import markets, we may drop any region, preferably the same over a single good. A complication that may arise here is to exclude the equation of a region which has zero weight (zero base-year trade-share), and attempt to calculate the missing $b_{i}$ through $\sum_{i} w_{i}^{0} b_{i}=0$. We are careful to avoid such a situation. This excluded equation can be identified in the results by its present $b_{i}$ coefficient, but the missing corresponding $t$ statistic (which we do not calculate), denoted by NA.

## 7.3). Selection of the 'Best' Functional Form.

The first choice we have to make during estimation relates to the functional form of the selected system, at each import market. Does the system perform 'better' in levels, in logs or in first differences? The answer to this depends on the nature of the data, and is to be provided by empirical testing. For 5 'goods' and 30 importing regions, over three functional forms, it involves estimating 450 nonlinear systems of 29 equations (at times less, when equations are dropped due to the absence of bilateral trade over the entire period).

The question that naturally arises is the criterion of selection. The fit of the alternative functional forms might provide a
criterion for selection. Another criterion might be a combination of fit with the natural fulfilment of the second order conditions ( $a_{i} \geq 0$ ). The number of significant $a_{i}$ 's may provide extra help. Our a priori expectations of the elasticity estimates can also play a role in our decision. Note, that we refer to the a-priori expectations of the elasticities rather than those of the estimated coefficients $a_{i}$. The latter, are used together with the estimated $\alpha$ and the base year shares $W_{i}^{0}$, in order to calculate the elasticities (see (4.53)-(4.55) or (7.7)-(7.9)).

With respect to the criteria of $f i t$, and the combination of $f$ it and fulfilment of the second order conditions $a_{i} \geq 0$, we have three non-nested models to compare for each import market. The fitted values of the left hand side variables of the systems are not comensurate (see (7.4)), but we may transform them to get back to the original fitted bilateral imports, $\hat{m}_{i}$. Thus, for each functional form we calculate the CV (Coefficient of Variation), and the MCV (Modified Coefficient of Variation) defined by:

$$
\begin{align*}
& C V=100 \times\left\{\left[\sum_{i} \sum_{t}\left(m_{i t}-\hat{m}_{i t}\right)^{2}\right] / n T\right\}^{1 / 2} /\left(\sum_{i} \sum_{t} m_{i t} / n T\right)  \tag{7.5}\\
& M C V=100 \times\left[\left\{\left[\sum_{i} \Sigma_{t}\left(m_{i t}-\hat{m}_{i t}\right)^{2}\right] / n T\right\}^{1 / 2} /\left(\sum_{i} \sum_{t} m_{i t} / n T\right)\right] / A \tag{7.6}
\end{align*}
$$

where $\hat{m}_{i t}$ are the fitted values of the levels of bilateral imports, calculated from the fitted $\hat{y}_{i t}$ (see (7.4). $m_{i t}$ are the corresponding actual levels of bilateral seaborne imports. $A$, is the number of $a_{i}$ coefficients, which satisfy naturally the second order sufficiency condition (that is, without being set to zero during estimation). Thus, the functional form with the lowest value of these statistics is considered as 'best'. Note that, (7.5)-(7.6) are calculated over the $n$ equations of the complete system, including the fitted values of the equation which is excluded in order to avoid singularity of the system.

Complications may arise in the calculation of these statistics,
because of the zero seaborne trade for quite a large number of regions and time periods. As a result, it is possible at times, for the average explained variability to be large in relation to the mean, resulting in values of the CV exceeding 100. We interpret this, as the particular functional form behaving less well than its competitors in that import market.

By using the MCV, we combine both the fit of the model and the satisfaction of the second order conditions into a single number. A combination of these measures with the number of significant $a_{1}$ 's as a helping aid enables us to choose the 'best' functional form of our model in each import market. Such 'Tables of diagnostics of all functional forms' for each good are presented in appendix 7.1. Columns of the tables refer to import regions. For each import region we present the estimated $\alpha$ coefficient, the $C V$, the $M C V$ and the number of significant $a_{1}$ 's at the $5 \%$ and $1 \%$ levels of significance, for all functional forms. The bottom row of the tables refer to the selected functional form for each import market. Any further results we present (such as the calculation of elasticities etc) will be based on these functional forms.

Table 7.1 summarizes the results on these 'best' selected functional forms for all goods. We observe that for individual goods the functional form in levels comes first in all cases, with either the logarithmic form or the first difference form coming second, depending on the individual good. Over all goods, the levels form comes first by being selected 116 times (or 77\%), the logarithmic form comes second best by being selected 22 times (or $15 \%$ ), while the first difference form comes third with only 12 systems (or $8 \%$ ) being selected out of the total of 150 selected.

Table 7.1.
Summary Table of the Selected Functional Forms.

| FUNCTIONAL FORM | GOOD |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 1 | 2 | 3 | 4 | 5 | All Goods |
| LEVELS | 23 | 21 | 25 | 23 | 24 | 116 |
| LOGS | 5 | 4 | 2 | 6 | 5 | 22 |
| DIFFERENCES | 2 | 5 | 3 | 1 | 1 | 12 |

## 7.4). Estimation Results of the 'Best' Models.

7.4.1). The Estimated $\alpha$ and $a_{i}$ Coefficients.

In combination with $\alpha$ and the $w_{i}^{0}$, the $a_{i}$ coefficients are used in the calculation of elasticity estimates. In the tables of appendix 7.2 we present both the estimated $a_{i}$ and $\alpha$ coefficients and their $t$ statistics. Columns of the tables show the estimated values of these parameters for the related importing regions.

The estimated coefficient $\alpha$ (when it can be estimated) shows the relative importance of the total export prices of a region and the world price of the good, in the import allocation decision of the importer. When $\alpha$ takes a boundary solution, we can estimate either the export price elasticities $(\alpha=1)$ only, or the world price elasticity only ( $\alpha=0$ ), see (4.53)-(4.55) or equivalently (7.7)-(7.9) below. $\alpha$ turns out to be critical for convergence during estimation, and whether it has been estimated as a parameter or as a constant may be seen by the availability of its $t$ statistic in the tables that follow.

In the 150 selected systems $\alpha$ is estimated as a parameter 9 times only. It falls in the range 0 to 1,76 times, while on 74 occasions it is set equal to the boundary solutions 0 or 1 . In the
latter case, it is set to 0 five times and 69 times to 1.

### 7.4.2). The Estimated Elasticities of Import Demands.

By allowing the estimation of a distinct price effect for each trade partner $i \quad(i=1, \ldots 8,11, \ldots, 32)$, in an import market $j, k$ $(j=1, \ldots, 8,11, \ldots, 32, k=1, \ldots 5)$, we are able to evaluate how the imports of the importing region, $j$, from its 30 trade partners are affected with changes in the own prices of each of these regions, and also with respect to changes in other regions' prices. By virtue of the introduction of the world price of the good as a determinant of bilateral imports, we are also able to estimate the bilateral effect of changes in world prices of the good on the imports of $j$ from each of its trade partners.

In fact, the sensitivity of the above price effects is calculated in terms of elasticities, using (4.53)-(4.55), repeated here for convenience:

$$
\begin{array}{ll}
e_{i i}^{x p}=\alpha e_{i i}=\left\{\alpha\left[\left(w_{i} a_{i}^{2} / a\right)-a_{i}\right]\right\}, & i=1, \ldots, n \\
e_{i h}^{x p}=\alpha e_{i h}=\left\{\alpha\left[\left(a_{i} w_{h} a_{h} / a\right)\right]\right\}, & i \neq h=1, \ldots, n \\
e^{w p}=(1-\alpha) e_{i i}=\left\{(1-\alpha)\left[\left(w_{i} a_{i}^{2} / a\right)-a_{i}\right]\right\}, & i=1, \ldots, n \tag{7.9}
\end{array}
$$

When these elasticities are estimated from the adaptive expectations systems (7.1)-(7.3), they will be interpreted as long run elasticities. The short run equivalents may be obtained by multiplying the long run elasticities by (1- $\lambda$ ). Given the constraint $0 \leq \lambda \leq 1$, the short run elasticities will be smaller than their long run counterparts

For each import market $j, k$, we calculate long run export (own and cross) and world price elasticities of bilateral seaborne imports from each of its 30 trade partners. For an importing region $j$ then (for some good k), there are 930 estimated elasticities. These are
arranged into a table, where columns of the table refer to the import elasticities of $j$ from $i(i=1, \ldots, 8,11, \ldots, 32)$, with respect to changes in the own export price of the region $i$, changes in the export prices of other trade partners of $j$ and changes in the world price of the traded good $k$. The main diagonal of this table reports the own price elasticities, while the off diagonal elements are the cross price elasticities. The bottom row of the table reports the bilateral import elasticities of $j$ from $i$, with respect to changes in the world price of the good.

Over 30 regions (for some good k), there are 30 of these tables, reporting 27900 elasticities. Such tables for good 1 are shown in appendix 7.3. A number of points are worth making here. The NA entries indicate non availability of the elasticity estimates, due to the absence of the corresponding $a_{i}$ 's (bilateral trade is zero between these regions over the entire period of estimation). Entries of (****) in tables indicate that the bilateral elasticity corresponding to the cell is infinitely large (a very large number). Also, when the entire bottom row of these tables consists of zero's, the estimated $\alpha$ coefficient is 1 (see (7.9)) and the world price of the good has no role to play in the allocation decision of the importer. The allocation of imports between trade partners then, is determined purely by the total export prices offered in the market (see for example, the corresponding tables of importing regions 02 , $04,05,22$ etc). The opposite is true, when $\alpha=0$.
Having estimated bilateral seaborne trade import allocation models
for 30 regions and 5 goods, it involves calculating 139500
$(=27900 \times 5)$ elasticities. This is too much information to include for
the space permitted in this thesis (tables of elasticities for the
rest of the goods can be provided on request).

We do not attempt to go too much into the results, besides giving guidelines and an overview of the estimates. The following points emerge by reading the elasticity tables.

The bilateral own import price elasticities are negative, since
increases in the own export price of a region $\mathfrak{i}$ reduces the bilateral amount of imports of $j$ from that region. It is relatively cheaper now to import from the rest of the 29 competitors of $\mathfrak{i}$. The bilateral cross import price elasticities are positive, since the bilateral imports of $j$ from $i$ increase if the prices of the competitors of $\mathfrak{i}$ go up. It is now relatively cheaper for $j$ to import from $i$ than from the competitor of $i$, $h \neq i$. Similarly, the bilateral world import price elasticities are negative, since increases in the world price of the traded good reduce overall trade, and as a consequence the bilateral amount of imports of $j$ from its trade partners is reduced.

By looking at individual columns of the elasticity tables we observe that the value (in absolute terms) of the own price elasticity (corresponding to a trade partner, in an import market $j, k$ ) is larger than the values of the corresponding cross price elasticities. Changes in the own export price of $\mathfrak{i}$ to $j$ have a much larger impact on the exports of $i$ to $j$, than changes in the export prices of the other trading regions. The effect of the latter is 'indirect', and is therefore reflected in the smaller cross price elasticities.

A striking feature of our results is that for each import market (a combination of importing region and traded commodity), there is a wide variation in price elasticities across trading partners (exporting regions). Broadly, price elasticities tend to be relatively low, and import flows relatively stable, for exporters which are on average major suppliers. Price elasticities tend to be high, and trade flows highly variable, for exporters which are on average small suppliers.

For example, import region 15 (British Isles) for Good 1, elasticities from trade partners such as $03,05,06,16$ and 19 (with which 15 has great institutional ties), are much lower in absolute value than the corresponding elasticities from regions 28, 30 and 32 (which are quite far from region 15). The latter group of countries need thus, much larger changes in their prices in order to penetrate
the British Isles market. A similar story is observed in other import regions, such as in 18 (Atlantic Europe), with trade partners 25, 26, 27, 30 and 31 yielding sets of relatively higher elasticities, as compared to regions $02,03,05,12,17,18,20$ and 23, the latter being much closer economically to 18.

It may be argued here that importing regions have some well established sources of imports internationally where they import the bulk of their imports from, while exploiting the relative better terms of trade offered from time to time by other trade partners (suppliers). These latter trade partners would, perhaps, need to offer 'substantial' reductions in prices in order to capture shares in international import markets. This result is reinforced later when we discuss the elasticity results for the rest of the goods.

There is no space here to discuss all the results and conclusions that may be derived by examining every elasticity table of appendix 7.3. However, it is useful to summarize these results and the results of the other goods in terms of frequency distributions and the related histograms. Also, a number of statistics such as averages, and measures of variability of the elasticity estimates are presented in the tables/graphs that follow. These graphs and summary statistics help us compare the competitiveness of goods internationally.

The average values and variabilities of the elasticities are calculated from the frequency distributions, by assuming that the average value of the open-ended classes of the distributions is 20 (in absolute terms). In this way, outliers (high value elasticities) are averaged-out, since we are interested in comparing these statistics across individual goods. The results are tabulated below.


Table 7.3



We observe the average values of the own bilateral import price elasticities for individual goods to vary between -7.07 and -5.83 , indicating a high own elasticity of import demand in the international markets. Perhaps, this is because there are a large number of exporters (30) competing to capture increasingly larger shares in world import markets. Even small changes in export prices offered by some exporting region lead in reallocation of imports to alternative sources internationally.

On average, cross price elasticities are close to 1. This reinforces the result of the competitiveness in world import markets; other regions' changes in prices (apart from the exporter $\mathfrak{i}$ to $j$ ) are quite important in the allocation decision of the importer j.

The magnitude of the world price elasticities across import markets is again higher in absolute terms than the cross price elasticities. The magnitude of these elasticities run more in line with the own price elasticities, even though they are smaller in absolute value, ranging between -6.58 and -2.15 . The effect of changes in world prices of the traded good affect bilateral imports in a 'direct' way, in comparison to cross price elasticities. However, the effect of changing world prices on bilateral trade is relatively smaller, compared to the effect of changes in the own bilateral prices of the exporting region.

Another interesting measure relates to the variability of the calculated price elasticities for individual goods. Such measures are provided by the standard deviation (s) and the coefficient of variation $(s / \bar{x})$ of the data. The high variability of own, cross and world price elasticities across the international markets is related to the high disaggregation of the traded goods. We suspect that, had we estimated aggregate (over goods) trade models, we would be working with smoother bilateral trade data, returning smoother elasticity estimates. One would also expect that these aggregate elasticity estimates would be an average of the individual good
elasticities.

It is instructive to compare the results of our estimated elasticities with previous studies:

- Houthakker and Magee(1969), Adams et al(1969), and Taplin(1967) calculate own price elasticity estimates to be around -1 .
- Armington(1969b) working with 11 trade partners finds an average of own price elasticities of -1.9. His estimated cross price elasticities range from 0 to .29 with an outlier of 0.4.
- Branson(1972) calculates an average own price elasticity of -2.9.
- Samuelson(1973) using semi-annual data for 1960-1972, distinguishing between 19 (OECD) trade partners, calculates own elasticities which are typically less than 1 in absolute values.
- Hickman and Lau(1973) estimate elasticities from a CES model using Taplin's(1967) data for the period 1961-1969, they distinguish between 27 partners. In their 'best' model the long run elasticities range between 0 and -9 , with an average of -2.5 . The estimated short run elasticities are lower, ranging from 0 to -4.5 , with an average value of -1.5 .
- Resnick and Truman(1975) working with 11 partners (10 European countries and a ROW region), find average own price elasticities for 1958 to average -2.53 , and for $1968-1.9$. The estimated cross price elasticities are close to 0 , ranging from 0 to 0.389 with outliers (for ROW 2.037 etc).
- Nyhus(1978) working on the INFORUM model, distinguishes between 10 partners and 1 digit SITC commodities with an estimating period 1962-1972. The estimated elasticities range for each commodity between -0.65 and -2.6 , returning an average of -1.58 .
- Gana et al(1979) compare the elasticity estimates from 7 different models applied to a common set of IMF quarterly data for the period 1971-1977, distinguishing between 7 countries. The elasticity estimates from the estimated systems such as those of the LES, CES etc return elasticities which range from -0.3 to -1
- Samuelson and Kurihara(1980) working in the EPA model(Japan) distinguish between 15 trade partners. The reported elasticities of their prefered model average -0.65.
- Italianer(1986) estimates a model for 8 partners. His 1975 own
price elasticity estimates range for aggregate trade from -0.5 to -1.5, while for the 5 individual goods distinguished they range between 0 and -4 . The corresponding estimated cross price elasticities for aggregate data range from 0 to 0.7 , while for individual goods they range between 0 and 2.7.

Thus, from previous studies we observe that on average the own price elasticity estimates range approximately from -0.5 to -2.5 , while the cross price elasticities are below 0.5 and close to 0 . A much higher degree of competition across world markets is suggested by our own higher elasticity estimates.

These results may be due to the special features of our model. Compared to other studies, there is a high disaggregation of goods and trade partners, we estimate cross price elasticities (very few other studies do), and we do not have a 'large' region playing the role of the Rest of the World(ROW) in the system. With respect to the latter, there is no trade partner (a big monopolist) predominating trade 'artificially' in the international economy (as described by the model), and perhaps distorting the effects of competition in world markets.

## 7.5). Other Estimated Results.

Besides the price elasticities based on the $a_{i}$ coefficients, we have estimated the $b_{i}$ and $\lambda$ coefficients in each system. Estimates of these have been arranged in the tables of appendix 7.4, one table for each individual good. Columns of the tables refer to import markets. The first row presents the estimated $\lambda$ coefficient, while the rest of the rows refer to the estimated $b_{i}$ 's (multiplied by 100 in order to show percentages), related to individual exporting regions. A separate set of tables present the estimated $t$ statistics of the above estimated coefficients. We do not attempt to discuss the individual estimates in detail. Instead we give guidelines with examples on the way the coefficients should be interpreted, and let the reader explore the tables.

The $b_{i}$ coefficients are estimated subject to the restriction $\Sigma_{i} w_{1}^{0} b_{i}=0$. They reflect trend like changes in bilateral imports over the estimation period. Since we are dealing with import allocation models, switches in imports towards a trade partner over the time period (indicated by a positive $b_{i}$ coefficient), is at the expense of some other trade partner(s), and visa versa. It is a zero-sum 'game'. Thus, by comparing the $b_{i}$ 's of a single column we can infer about the percentage changes in the exports of each exporting region over the examined period in the relevant import market.

In general, the $b_{i}$ coefficients take values between 0 and 100 (in absolute terms), with some exceptions. Very large coefficients are denoted by ${ }^{* * * *}$, with a sign in front to indicate the direction of change. Small $b_{i}$ 's for an importing region (and a particular good), indicate that there have been no major changes in the allocation of its imports amongst its trade partners over the examined period. The opposite is true for large $b_{i}$ 's.

It would be instructive to give some examples on how to interpret the tables: Thus, we observe, by looking at the column of importing region 15 (British Isles) for good 1, that imports from the USA regions and also from the EEC areas have increased at the expense of the Commonwealth regions, such as those of Africa the Far East and Oceania. Similar trends are observed for the other goods, columns 15 on the other tables.

Individual rows of the tables reflect the changing strength of exporting regions in international markets. Thus, by looking for example at the 27th row (Persian Gulf) of the table for good 2 we observe almost every coefficient to be negative. This reflects the diminishing importance of Bulk liquid exports from the area during 1969-1986. Other regions have entered the world market for these products capturing increasingly larger shares of world markets at the expense of region 27. On the other hand, for the same good,
exporting regions 04,05 and 15 , say, have increased their export shares in world markets, as seen by the number of positive coefficients of the corresponding rows.
7.5.2). Estimated Dynamics and 'Diagnostics'.

The $\lambda$ coefficient is introduced by virtue of the dynamics (Adaptive Expectations) assumed in chapter 4. It is estimated in each import market subject to the restriction $0 \leq \lambda \leq 1$. In our experience $\lambda$ never exceeds 1, and it takes values below zero in very few systems (in 21 out of 150 ), in which case it is set to 0 . It is highly significant in almost all the estimated models. The average (over import regions) values of $\lambda$ for the individual goods are, for Good 1, .45, Good 2, . 28, Good 3, . 37, Good 4, .52, and for Good 5, . 43

One may think of $\lambda$ as reflecting short run adjustments between changes in prices and changes in imports. Short run price elasticities may be obtained by multiplying the long run elasticity estimates of appendix 7.3 by $(1-\lambda)$. Thus, judging from the above average values of $\lambda,(1-\lambda)$ takes values between .5 and 1. As a result the short run elasticities will be over $1 / 2$ of their long run counterparts, but still substantially lower compared to the latter.

Finally, the mean lag response of imports to changing prices, in each import market, is calculated as $\lambda /(1-\lambda)$, see appendix 7.5 . This can be interpreted as the long run adjustment coefficient. We observe that this varies between 0 and 1 to 2 years with some outliers reaching even the very extreme value of 5.65 years.

Besides the estimated coefficients and their $t$ statistics, one might be interested in indications of fit, the satisfaction of the underlying assumptions (such as lack of heteroskedasticity, autocorrelation, tests of mispessified dynamics etc), and tests of restrictions on the coefficient estimates of the selected systems.

Due to lack of space, we only include some of these tests here. These are listed in appendix 7.5 in separate tables for each individual good.

Thus, tests of restrictions on the $a_{1}$ coefficients in terms of the $F$ statistic suggested in (6.62)-(6.64) are calculated. The first row of the tables refers to the estimated value of the statistic, the second and the third row refer to the critical values of these statistics at the $5 \%$ and $1 \%$ levels of significance, while the fourth and the fifth row refer to the degrees of freedom. Thus, the fourth row refers to the number of restrictions (the number of $a_{1}$ 's set to zero, while the fifth row refers to the degrees of freedom of the unrestricted model. On average, the restrictions are accepted in $50 \%$ of the 150 estimated systems.

The Coefficient of Variation(CV) of each individual system is also presented. It gives an indication of fit of the whole system (adjusted coefficients of multiple determination for each individual equation in each system are estimated, but not presented here). The final row of these tables present the estimated mean lag coefficients $\lambda /(1-\lambda)$.

The aim of this chapter has been to focus on the empirical problems encountered during estimation of our seaborne import allocation models, and to discuss the results of this exercise. A computer program logic has been developed, which is used to estimate the systems. The 'best' performing functional form in each import market is determined by the data. Based on that, we derive bilateral seaborne own, cross and world import price elasticities of demand.

Because of the high disaggregation in goods and trade partners and the fact that we do not have a large monopolistic trade partner in the system, the values of the estimated elasticities support the argument that competition in world import markets is intense. The values of own and world price elasticities are several points above 1, in absolute terms, while the cross price elasticities take values as high as 1 for individual goods. These are relatively higher than the elasticities estimated in other similar studies in the literature.

## APPENDIX 7.1

5 TABLES OF DIAGNOSTICS OF ALL FUNCTIONAL FORMS

5 GOODS






## APPENDIX 7.2

```
5 TABLES OF ALPHA(j) AND a(ij) COEFFICIENTS
AND
    5 TABLES OF ESTIMATED t STATISTICS
FOR THE ALPHA(j) AND a(ij) COEFFICIENTS
```



|  |  |  | TABLE OF |  |  | ESTIMATED "Alpha(j)" |  |  |  |  | AND |  | "a(ij)" |  | COEFFICIENTS FOR GOOD 3 ( 30 IMPORTING REGIONS ). |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PARTNERI |  |  |  |  |  |  |  |  |  |  |  |  |  |  | IMPORTING PEGION ] |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 01 | 02 | 03 | 04 | 05 | 06 | 07 | 08 | 11 | 12 |  | 13 | 14 | 15 | 16 | 17 | 18 | . 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
| Alpha(0) | 0.0006 | 0.0000 | 0.0028 | 0.50 | 0.20570 | 0.2907 | 0 | 0 | 1 | 1 |  | 0.5 | 10 | 0.0259 | 1 | 0.0054 | 0.5 | 5 | 0 | 0.0016 | 0 | 1 | : 1 | - 1 | - 1 | - 1 | 0.0042 | - 1 | 1 | 1 | 1 |
| 01. | 40648. | $1 \mathrm{E}+12$ | 3E+07 | 0 | NA | 328.58 | $2 \mathrm{E}+11$ | 0 | 0 | 0 |  | . 762 | 2862.1 | 34967. | 0 | 0 | 1044.4 | 40 | NA | $2 \mathrm{E}+07$ | 0 | $6 \mathrm{E}+06$ |  | 20249. | NA | 30023. |  | 95204. | NA |  | 0 |
| 02 | 0 | 1E+08 | 0 | 0 | 91.2829 | 96.851 | 3E+11 | 1E+10 | 0 | 0 |  | 3.537 | 0.5530 | 228.31 | 2.8148 | 0 | 0 | 0 | 0 | 0 | $2 \mathrm{E}+12$ | 1.9845 | 1.5090 | 1758.8 | 0 | 0 | 00 | 0 | NA | 8.7277 | 121.88 |
| 03 | 3078.3 | $1 \mathrm{E}+07$ | NA | NA | NA | NA | $3 \mathrm{E}+11$ | 1E+10 | 0 | 0 |  | 0 | 0 | 228.31 | 1.9778 | 0 | 0 | 0 | 2E+08 | 0 | $2 \mathrm{E}+12$ | 2.1459 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 04 | 0 | 0 | NA 2 | 20.287 | NA | NA | 3E+11 | $1 \mathrm{E}+10$ | 03 | 3.0279 |  | 4.570 | 0.0855 |  | 2.8215 | 0 | 9.4756 | 6.7282 | 0 | 0 | $2 \mathrm{E}+12$ | 1.9405 | 0 | 8.2885 | 1.8168 | - 0 | 0 | 0 | 0 | 3.5625 | 9.5088 |
| 05 | NA | 0 | NA | NA | NA | NA | $3 \mathrm{E}+11$ | $1 \mathrm{E}+10$ | 0 |  |  | . 759 | 0 | 20.546 | 0.6388 | 0 | 8.9400 | 0 | $2 \mathrm{E}+08$ | 3435.2 | $2 \mathrm{E}+12$ | 1.6973 | 3.6156 | 9.0308 | 3.7780 | 0 | 0 | 0 | 0 | 3.2164 | 334.27 |
| 08 | 0 | 0 | NA | NA | NA | NA | $3 \mathrm{E}+11$ | $1 \mathrm{E}+10$ | 05 | 5.5543 |  | 2.320 | 3.6508 | 298.05 | 4.2080 | 0 | 13.602 | 8.0421 | 0 | 0 | $2 \mathrm{E}+12$ | NA | 2.9535 | 7.5985 | 4.3882 | 4.3778 | 1085.1 | 2.1199 | 0 | 0.4729 | 1.9590 |
| 07 | 0 | 0 | 17456. 1 | 101.43 | 01 | 147.39 | $3 \mathrm{E}+11$ | $1 \mathrm{E}+10$ | 2477.2 |  |  | 5.353 | 0 | 0 | 0 | 3E+08 | 20.703 | 32.371 | 0 | 4E+07 | $2 \mathrm{E}+12$ | 6285.9 | 36.863 | 40973. |  | 23.018 | 0 | 0 | 0 |  | 15.770 |
| 08 | 0 | 0 | 0 | 0 | 0 | 0 | $3 \mathrm{E}+11$ | $1 \mathrm{E}+10$ | 8.28198 | 835.74 |  | 0 | 6378.8 | 248.58 | 0 | 2E+09 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4E+06 | 0 | 0 | 13.730 | 0 |
| 11 | $\cdots \cdots$ | 0 | 01 | 1.9169 | 0 | 0 | 3E+11 | $1 \mathrm{E}+10$ | 3.99864 | 47.677 |  | 38.76 | 0 |  | 21.506 | 0 | 6.9782 | 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 42.515 | 0 |
| 12 | 0 | 0 | 0 | 0 | 0.7741 | 0 | 0 | 0 | 0 | 0 |  |  | 256.30 | 384.62 | 0 | 0 | 84.172 | 24.083 | $2 \mathrm{E}+08$ | 0 | 0 | 0 | 123.05 | NA | 0 | 0 | 0 | 0 | NA | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 | 28.555 | 36.638 | 3E+11 | 1E+10 | 9.10822 | 26.363 |  |  | 8.4949 | - |  | 2014.7 | 0 | $0 \quad 0$ | 0 | 0 | $2 \mathrm{E}+12$ | 1.2205 | 15.454 | 0 | 0 | 5.7126 | - | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 | 0 | 0 | $3 \mathrm{E}+11$ | $1 \mathrm{E}+10$ | 0 |  |  | 0.084 | 6.1923 | 0 | 0 | 0 | 35.611 | 10.914 | 0 | 0 | $2 \mathrm{E}+12$ | 0 |  | 211.31 | 7.3622 | 2.3851 | 6131.4 | 617.14 | 0 | 0 | 3.3081 |
| 15 | 0 | 0 | 2910.2 | 0 | 0 | 0 | 3E+11 | $1 \mathrm{E}+10$ | 01 | 15.583 |  | 0 | 0 | 0 | 0 | 0 | 13.602 | 2.1771 | 2E+08 | 0 | $2 \mathrm{E}+12$ | 0 | 0 | 0 | 0 | 0 | 0 | 8.9880 | 0 | 0 | 0 |
| 16 | 0 | 0 | 5772.27 | 7.5141 | 16.692 | 0 | $3 E+11$ | $1 \mathrm{E}+10$ | 6.8877 | 0 |  |  | 6.5879 | 477.03 | 9.6954 | 2508.2 | 0 | 1.3604 | $2 \mathrm{E}+08$ | 5359.6 | 2E+12 | 28.986 | 42.367 | 7.9454 | 31.269 | 4.9036 |  | 29.200 |  | 24.382 | 15.046 |
| 17 | 0 | 0 | 0 | 0 | NA | NA | 3E+11 | $1 \mathrm{E}+10$ | 01 | 18.486 |  | 0 | 0 |  | 0 | NA | 23.915 | 50 | NA | 7918.9 | $2 \mathrm{E}+12$ | 3.7580 | 30.205 | 0 | 0 | 0 | 3163.5 | 0 | 0 | 0 | 0 |
| 18 | 0 | 7E+08 | 3477.8 | 5.5683 | 0 | 13.262 | 3E+11 | $1 \mathrm{E}+10$ | 2.07448 | 8.7435 |  | 0 | 0 | 185.59 | 3.2711 | 3744.3 | 40.283 | 30 | $2 \mathrm{E}+08$ | 6274.4 | $2 \mathrm{E}+12$ | 0 | 0 | 0 | 10.908 |  | 291.11 | 0 | 0 | 0 | 0 |
| 19 | 0 | $2 \mathrm{E}+07$ | 8549.0 | 0 | 0 | 0 | 3E+11 | 9E+09 | 23.184 | 19.555 |  | 4.134 | 484.23 | 4371.7 | 22.741 | 1338.0 | 0 | $0 \quad 0$ | $2 \mathrm{E}+08$ | 3478.6 | $2 \mathrm{E}+12$ | 9.8811 | 2.8233 | 0 | 10.219 | 0 | 0 | 0 | 0 | 40.748 | 29.648 |
| 20 | 0 | 0 | 8873.2 | NA | NA | NA | NA | 7E+09 | $3 \mathrm{E}+07$ | 0 |  | 0 | 0 | - | - | NA | 98.677 | 7 | $2 \mathrm{E}+08$ | 0 | $2 \mathrm{E}+12$ | 0 | NA | 6.0094 | 0 | 0 | 0 | 0 | NA |  | 0.8605 |
| 21 | 0 | 0 | 0 | 90.051 | 90.494 | 51.775 | 0 | 0 | 0 | 9891.8 |  | 0 | 0 | 0 | 0 | 0 | 12.361 | 1 | 2E+08 | 313.61 | $2 \mathrm{E}+12$ | 5.5942 | 0 | 0 | 0 | 0 | 4308.7 | 15.293 | 0 | 0 | 0 |
| 22 | 0 | 2E+07 | 0 | 42.263 | 30 | 0 | 3E+11 | 1E+10 | 1878.6 |  |  | 63.42 | 23434. | , | 0 | 8357.4 |  | 04.1621 | 2E+08 | 0 | $2 \mathrm{E}+12$ | 26.145 | NA | 18694. | 0 | 0.1110 | 0 | 22.911 | 5598.0 | 10.698 | 0 |
| 23 | 4580.4 | 0 | 7505.3 | 2.3621 | 87.486 | 0 | 0 | $1 \mathrm{E}+10$ | 0 | 16.041 |  | 01.55 | 50 | 344.68 | 10.191 | 2E+08 | 3.0184 | 40 | 2E+08 | 41378 | $2 \mathrm{E}+12$ | 3.8178 |  | 87.805 | 0 | 0 | 0 | 857.88 | 18158. | 44.287 | 0 |
| 24 | 0 | 0 | 0 | 12.776 | 79.619 | 0 | 3E+11 | 3E+09 | - ${ }^{\text {anc. }}$ |  |  | 7.740 | 0 | 278.03 | . | NA | 0 | $0 \quad 0$ | NA | $9 \mathrm{E}+08$ | NA | 0 | NA | 0 | 0 | 2.3404 | 0 | 6'914. | NA | 8.1552 | 0 |
| 25 | NA | 1E+08 | $\cdots$ | 5.1388 | 275.35 | 118.04 | 2E+11 | NA | 384.43 |  |  | 7.601 | 5481.8 | 57.871 | 6.9652 |  | 7.6918 | 85.4174 | $2 \mathrm{E}+08$ | 36973. | $2 \mathrm{E}+12$ | 0 | 0 | 11.371 | 12.410 | 18.730 | 1859.4 | 0 |  | 2.1018 | 0 |
| 28 | NA | 2E+10 | 0 | , | $0 \quad 0$ | 33.137 | $2 E+11$ | , | 0 | NA |  | 0 | $0 \quad 0$ | - $\cdots$.... | 149.21 | NA | 463.68 | 88.8306 | $2 \mathrm{E}+08$ | 880.79 | $2 \mathrm{E}+12$ | 0 | 0 | 0 | 15.791 | 22.520 | 0 | 1585.7 | 0 | 0 | - |
| 27 | NA | 7E+06 | 3618.5 | 5 | $0 \quad 0$ | 2.7784 | - 0 |  | 2.4967 |  |  | 0 | 02052.4 |  |  | 567.36 |  | 03.6773 | 0 | 0 | $2 \mathrm{E}+12$ | 152.12 | ...... | 42.237 | 13.814 | 8.7388 | 3610.6 | 0 | 10.568 | 0 | 0 |
| 28 | ..... | 0 | 0 | 0 | $0 \quad 0$ | 55.047 | 3E+11 |  | 4.8449 |  |  | 14.58 | 88094.5 | 51701.1 | 16.120 | 22821. | 10.007 | 7128.55 | 0 | 0 | $2 \mathrm{E}+12$ | 0 |  | 12.135 | 0 | 17.093 | 0 |  | 17.203 | 0 | 51.749 |
| 29 | 0 | 0 | 192.64 |  | $0 \quad 0$ | 0 | - 3E+11 | $1 \mathbf{E}+10$ |  |  | 0 | 0 | $0 \quad 0$ | - 1068.3 | 171.29 | 1308.5 |  | 019.947 | $2 \mathrm{E}+08$ | 0 | 2E+12 | 66.244 | 8810.6 | 0 | 0 | 0 | 5353.0 | 4.4999 | 6383.6 | 0 | 0 |
| 30 | 0 | - $1 \mathrm{E}+12$ | 20 | 0 | $0 \quad 0$ | $0 \quad 0$ | - 3E+11 | $1 \mathrm{E}+10$ | 76.593 |  |  | 7.504 | 4 ...... | - 165.83 |  | 29860. |  | 014.707 | - 0 | $2 \mathrm{E}+07$ |  | 22.142 | 12872. | 0 | 0 | - 0 | ...... | 0 | 0 | 0 | 28.024 |
| 31 |  | 6E+06 | $8 \quad 0$ | 0 | $0 \quad 0$ | 00 | - 3E+11 | $1 \mathrm{E}+10$ | 13.167 |  | 0 |  | 025.833 | 1157.0 | 2.8653 | 2E+07 | 12.410 | 0 14.549 | 0 | 31674. | $2 \mathrm{E}+12$ | 0 |  | 6.5019 | 8.5729 | 0 | 6357.1 | 0 | 0 | 0 | 12.598 |
| 32 | ..... | - 0 | 0 | 0 | 0 0 | - 0 | 0 - $3 \mathrm{+}+11$ | $1 \mathrm{E}+10$ | 10.932 |  | 087 | 7.144 | 425.982 | 2641.13 | 39.5683 | 12298. | 8.9280 | - 42.727 | 0 | 0 | $2 \mathrm{E}+12$ | 0 | 19.204 | 4.5209 | 0 | 7.9849 | 3463.0 | 0 | 0 | 0 | 0 |





APPENDIX 7.3

30 TABLES OF ELASTICITY ESTIMATES FOR GOOD 1

30 IMPORTING REGIONS
TABLE OF EXPORT (OWN AND CROSS) AND WORLD PRICE ELASTICTTIES FOR 1980
WITH RESPECT TO A CHANGE IN THE PRICE OF PARTNER h, AND A CHANGE IN THE WORLD PRICE OF GOOD 1.

| h, WP(1) |  |  |  |  |  |  |  |  |  |  | BLAT | ERNL | ORN | NE IMP |  | REGIO | ON 01 | FROM P | ARTNER | 01-3 |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 01 | 02 | 03 | 04 | 05 | 06 | 07 | 08 | 11 | 12 | 13 | 14. | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
| 01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 03 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 04 | 0 | 0 | 0 | -10.21 | 0 | 0 | 0 | 0.0250 | 0.0111 | 0.0838 | 0.0433 | 0.1168 | 0 | 0.0489 | 0 | 0.0197 | 0.0279 | 0.1274 | 0.0948 | 0 | 0.0002 | 0 | 0.0256 | 0 | 0.0164 | 0 | 0.3726 | 0.5452 | 0.0580 | 0 |
| 05 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 08 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 07 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 08 | 0 | 0 | 0 | 0.0915 | 0 | 0 | 0 | -6.981 | 0.0279 | 0.2102 | 0.1085 | 0.2925 | 0 | 0.1227 | 0 | 0.0495 | 0.0701 | 0.3195 | 0.2379 | 0 | 0.0006 | 0 | 0.0642 | 0 | 0.0412 | 0 | 0.9347 | 1.3676 | 0.1455 | 0 |
| 11 | 0 | 0 | 0 | 0.2198 | 0 | 0 | 0 | 0.1506 | -3.061 | 0.5050 | 0.2809 | 0.7029 | 0 | 0.2948 | 0 | 0.1190 | 0.1886 | 0.7877 | 0.5717 | 0 | 0.0015 | 0 | 0.1543 | 0 | 0.0990 | 0 | 2.2458 | 3.2858 | 0.3497 | 0 |
| 12 | 0 | 0 | 0 | 1.0514 | 0 | 0 | 0 | 0.7207 | 0.3210 | -21.12 | 1.2478 | 3.3818 | 0 | 1.4099 | 0 | 0.5693 | 0.8065 | 3.6718 | 2.7344 | 0 | 0.0072 | 0 | 0.7379 | 0 | 0.4738 | 0 | 10.740 | 15.714 | 1.6728 | 0 |
| 13 | 0 | 0 | 0 | 0.8945 | 0 | 0 | 0 | 0.6131 | 0.2731 | 2.0548 | -11.10 | 2.8598 | 0 | 1.1994 | 0 | 0.4843 | 0.8881 | 3.1236 | 2.3262 | 0 | 0.0061 | 0 | 0.6277 | 0 | 0.4031 | 0 | 9.1370 | 13.368 | 1.4230 | 0 |
| 14 | 0 | 0 | 0 | 0.0000 | 0 | 0 | 0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | -32.78 | 0 | 0.0000 | 0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0 | 0.0000 | 0 | 0.0000 | 0 | 0.0000 | 0 | 0.0000 | 0.0000 | 0.0000 | 0 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18 | 0 | 0 | 0 | 5.2018 | 0 | 0 | 0 | 3.5653 | 1.5882 | 11.949 | 8.1732 | 16.631 | 0 | -6.768 | 0 | 2.8168 | 3.9900 | 18.185 | 13.527 | 0 | 0.0357 | 0 | 3.6508 | 0 | 2.3443 | 0 | 53.134 | 77.741 | 8.2755 | 0 |
| 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18 | 0 | 0 | 0 | 0.4969 | 0 | 0 | 0 | 0.3405 | 0.1517 | 1.1415 | 0.5897 | 1.5887 | 0 | 0.6863 | 0 | -5.279 | 0.3811 | 1.7352 | 1.2823 | 0 | 0.0034 | 0 | 0.3487 | 0 | 0.2239 | 0 | 5.0758 | 7.4285 | 0.7905 | 0 |
| 18 | 0 | 0 | 0 | 0.2780 | 0 | 0 | 0 | 0.1905 | 0.0849 | 0.6388 | 0.3300 | 0.8890 | 0 | 0.3728 | 0 | 0.1505 | -7.647 | 0.9710 | 0.7231 | 0 | 0.0019 | 0 | 0.1961 | 0 | 0.1253 | 0 | 2.8404 | 4.1559 | 0.4423 | 0 |
| 20 | 0 | 0 | 0 | 0.0258 | 0 | 0 | 0 | 0.0177 | 0.0078 | 0.0593 | 0.0306 | 0.0828 | 0 | 0.0346 | 0 | 0.0139 | 0.0198 | -35.69 | 0.0872 | 0 | 0.0001 | 0 | 0.0181 | 0 | 0.0116 | 0 | 0.2839 | 0.3862 | 0.0411 | 0 |
| 21 | 0 | 0 | 0 | 0.0000 | 0 | 0 | 0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0 | 0.0000 | 0 | 0.0000 | 0.0000 | 0.0000 | -28.65 | 0 | 0.0000 | 0 | 0.0000 | 0 | 0.0000 | 0 | 0.0000 | 0.0000 | 0.0000 | 0 |
| 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 23 | 0 | 0 | 0 | 0.0010 | 0 | 0 | 0 | 0.0007 | 0.0003 | 0.0024 | 0.0012 | 0.0033 | 0 | 0.0014 | 0 | 0.0005 | 0.0008 | 0.0036 | 0.0027 | 0 | -0.070 | 0 | 0.0007 | 0 | 0.0004 | 0 | 0.0108 | 0.0156 | 0.0016 | 0 |
| 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 25 | 0 | 0 | 0 | 0.0240 | 0 | 0 | 0 | 0.0164 | 0.0073 | 0.0551 | 0.0285 | 0.0788 | 0 | 0.0322 | 0 | 0.0130 | 0.0184 | 0.0839 | 0.0824 | 0 | 0.0001 | 0 | -7.175 | 0 | 0.0108 | 0 | 0.2454 | 0.3591 | 0.0382 | 0 |
| 28 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 27 | 0 | 0 | 0 | 0.0000 | 0 | 0 | 0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0 | 0.0000 | 0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0 | 2E-10 | 0 | 0.0000 | 0 | -4.818 | 0 | 0.0000 | 0.0000 | 0.0000 | 0 |
| 28 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 29 | 0 | 0 | 0 | 0.0000 | 0 | 0 | 0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0 | 0.0000 | 0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0 | 0.0000 | 0 | 0.0000 | 0 | 0.0000 | 0 | -104.6 | 0.0000 | 0.0000 | 0 |
| 30 | 0 | 0 |  | 0.0000 | 0 | 0 | 0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0 | 0.0000 | 0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0 | 0.0000 | 0 | 0.0000 | 0 | 0.0000 | 0 | 0.0000 | -153.1 | 0.0000 | 0 |
| 31 | 0 | 0 | 0 | 1.9263 | 0 | 0 | 0 | 1.3203 | 0.5881 | 4.4250 | 2.2860 | 6.1587 | 0 | 2.5830 | 0 | 1.0430 | 1.4775 | 6.7287 | 5.0095 | 0 | 0.0132 | 0 | 1.3519 | 0 | 0.8681 | 0 | 19.676 | 28.788 | $-13.23$ | 0 |
| 32 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| WP(1) | 0 | 0 | 0 | -10.21 | 0 | 0 |  | -6.961 | -3.061 | -21.12 | -11.10 | -32.78 |  | -6.768 | 0 | -5.279 | -7.647 | -35.69 | -28.65 | 0 | $-0.070$ | 0 | -7.175 | 0 | -4.618 | 0 | -104.6 | -153.1 | -13.23 | 0 |





TABLE OF EXPORT (OWN AND CROSS) AND WORLD PRICE ELASTICTTIES FOR 1980







## WITH RESPECT TO A CHANGE IN THE PRICE OF PARTNER h, AND A CHANGE IN THE WORLD PRICE OF GOOD

|  |  |  |  |  |  |  |  |  |  |  | BLAT |  | Or |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 01 | 02 | 03 | 04. | 05 | 06 | 07 | 08 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 26 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
| 01 | -68.15 | 0.5375 | 0.4079 | 0.8097 | 0.37430 | 0.5884 | 0 | 0.3074 | 0 | 0 | 0.3196 | 0 | 0.71420 | 0.5888 | 0 | 0 | 12.375 | 0 | 0 | 0.2874 | 0.9254 | 0 | 1.9013 | 2.0529 | 3.9909 | 0.8771 | 1.3476 | 1.9313 | 10.445 | 3.4315 |
| 02 | 3.0147 | -1.936 | 0.0507 | 0.1008 | 0.0485 | 0.0728 | 0 | 0.0382 | 0 | 0 | 0.0397 | 0 | 0.0887 | 0.0731 | 0 | 0 | 1.5382 | 0 | 0 | 0.0357 | 0.1150 | 0 | 0.2363 | 0.2551 | 0.4980 | 0.1090 | 0.1675 | 0.2400 | 1.2983 | 0.4285 |
| 03 | 2.3900 | 0.0531 | -1.480 | 0.0800 | 0.0370 | 0.0580 | 0 | 0.0304 | 0 | 0 | 0.0316 | 0 | 0.07080 | 0.0582 | 0 | 0 | 1.2240 | 0 | 0 | 0.0284 | 0.0915 | 0 | 0.1880 | 0.2030 | 0.3947 | 0.0867 | 0.1332 | 0.1910 | 1.0331 | 0.3394 |
| 04 | 0.4530 | 0.0100 | 0.0078 | -3.003 | 0.00690 | 0.0109 | 0 | 0.0057 | 0 | 0 | 0.0059 | 0 | 0.01330 | 0.0109 | 0 | 0 | 0.2311 | 0 | 0 | 0.0053 | 0.0172 | 0 | 0.0365 | 0.0383 | 0.0745 | 0.0163 | 0.0251 | 0.0360 | 0.1951 | 0.0840 |
| 06 | 4.4397 | 0.0983 | 0.0748 | 0.1482 | -1.328 | 0.1073 | 0 | 0.0582 | 0 | 0 | 0.0585 | 0 | 0.13070 | 0.1077 | 0 | 0 | 2.2853 | 0 | 0 | 0.0528 | 0.1694 | 0 | 0.3480 | 0.3757 | 0.7305 | 0.1605 | 0.2486 | 0.3535 | 1.9120 | 0.6281 |
| 06 | 0.1281 | 0.0028 | 0.0021 | 0.0042 | 0.0019 | -2.182 | 0 | 0.0018 | 0 | 0 | 0.0016 | 0 | 0.00370 | 0.0031 | 0 | 0 | 0.0853 | 0 | 0 | 0.0015 | 0.0048 | 0 | 0.0100 | 0.0108 | 0.0210 | 0.0046 | 0.0071 | 0.0102 | 0.0551 | 0.0181 |
| 07 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 08 | 0.0418 | 0.0009 | 0.0007 | 0.0013 | 0.00060 | 0.0010 | 0 | -1.146 | 0 | 0 | 0.0005 | 0 | 0.00120 | 0.0010 | 0 | 0 | 0.0212 | 0 | 0 | 0.0004 | 0.0015 | 0 | 0.0032 | 0.0035 | 0.0088 | 0.0015 | 0.0023 | 0.0033 | 0.0179 | 0.0058 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13 | 3.4618 | 0.0767 | 0.0582 | 0.1155 | 0.05340 | 0.0837 | 0 | 0.0438 | 0 | 0 | -1.145 | 0 | 0.10190 | 0.0840 | 0 | 0 | 1.7683 | 0 | 0 | 0.0410 | 0.1320 | 0 | 0.2713 | 0.2930 | 0.5698 | 0.1251 | 0.1923 | 0.2756 | 1.4908 | 0.4887 |
| 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 | 4.2024 | 0.0931 | 0.0708 | 0.1403 | 0.08480 | 0.1016 | 0 | 0.0532 | 0 | 0 | 0.0553 | 0 | -2.538 0 | 0.1020 | 0 | 0 | 2.1442 | 0 | 0 | 0.0497 | 0.1603 | 0 | 0.3294 | 0.3656 | 0.6914 | 0.1519 | 0.2335 | 0.3346 | 1.8098 | 0.5945 |
| 18 | 10.587 | 0.2346 | 0.1780 | 0.3534 | 0.1634 | 0.2559 | 0 | 0.1341 | 0 | 0 | 0.1395 | 0 | 0.3117 | -1.937 | 0 | 0 | 5.4022 | 0 | 0 | 0.1254 | 0.4039 | 0 | 0.8290 | 0.8981 | 1.7421 | 0.3829 | 0.5882 | 0.8430 | 4.5597 | 1.4979 |
| 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 19 | 11.075 | 0.2454 | 0.1862 | 0.3697 | 0.1709 | 0.2877 | 0 | 0.1403 | 0 | 0 | 0.1459 | 0 | 0.32610 | 0.2888 | 0 | 0 | -40.47 | 0 | 0 | 0.1312 | 0.4225 | 0 | 0.8881 | 0.9373 | 1.8223 | 0.4005 | 0.6153 | 0.8818 | 4.7895 | 1.5868 |
| 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22 | 0.5684 | 0.0125 | 0.0095 | 0.0189 | 0.0087 | 0.0137 | 0 | 0.0072 | 0 | 0 | 0.0074 | 0 | 0.0167 | 0.0138 | 0 | 0 | 0.2900 | 0 | 0 | -1.064 | 0.0216 | 0 | 0.0445 | 0.0481 | 0.0935 | 0.0205 | 0.0315 | 0.0452 | 0.2448 | 0.0804 |
| 23 | 5.6290 | 0.1247 | 0.0946 | 0.1879 | 0.0888 | 0.1360 | 0 | 0.0713 | 0 | 0 | 0.0741 | 0 | 0.1857 | 0.1366 | 0 | 0 | 2.8721 | 0 | 0 | 0.0687 | -3.234 | 0 | 0.4412 | 0.4784 | 0.9282 | 0.2035 | 0.3127 | 0.4482 | 2.4241 | 0.7963 |
| 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 25 | 0.1274 | 0.0028 | 0.0021 | 0.0042 | 0.0019 | 0.0030 | 0 | 0.0018 | 0 | 0 | 0.0018 | 0 | 0.0037 | 0.0030 | 0 | 0 | 0.0850 | 0 | 0 | 0.0015 | 0.0048 | 0 | -7.077 | 0.0107 | 0.0209 | 0.0046 | 0.0070 | 0.0101 | 0.0549 | 0.0180 |
| 28 | 0.1640 | 0.0036 | 0.0027 | 0.0054 | 0.0025 | 0.0039 | 0 | 0.0020 | 0 | 0 | 0.0021 |  | 0.0048 | 0.0039 | 0 | 0 | 0.0837 | 0 | 0 | 0.0019 | 0.0062 | 0 | 0.0128 | -7.638 | 0.0269 | 0.0059 | 0.0091 | 0.0130 | 0.0706 | 0.0232 |
| 27 | 0.0414 | 0.0009 | 0.0008 | 0.0013 | 0.0006 | 0.0010 | 0 | 0.0005 | 0 | 0 | 0.0005 | 0 | 0.0012 | 0.0010 | 0 | 0 | 0.0211 | 0 | 0 | 0.0004 | 0.0015 | 0 | 0.0032 | 0.0035 | -14.86 | 0.0014 | 0.0023 | 0.0032 | 0.0178 | 0.0058 |
| 28 | 0.3608 | 0.0079 | 0.0060 | 0.0120 | 0.0055 | 0.0087 |  | 0.0045 | 0 | 0 | 0.0047 | 0 | 0.0106 | 0.0087 | 0 | 0 | 0.1841 | 0 | 0 | 0.0042 | 0.0137 | 0 | 0.0282 | 0.0305 | 0.0593 | -3.256 | 0.0200 | 0.0287 | 0.1554 | 0.0510 |
| 29 | 1.4882 | 0.0329 | 0.0250 | 0.0496 | 0.0229 | 0.0359 | 0 | 0.0188 | 0 | 0 | 0.0196 | 0 | 0.0438 | 0.0381 | 0 | 0 | 0.7593 | 0 | 0 | 0.0178 | 0.0567 | 0 | 0.1188 | 0.1259 | 0.2448 | 0.0538 | -4.940 | 0.1185 | 0.6409 | 0.2105 |
| 30 | 0.5815 | 0.0128 | 0.0097 | 0.0194 | 0.0089 | 0.0140 |  | 0.0073 | 0 | 0 | 0.0076 | 0 | 0.0171 | 0.0141 | 0 | 0 | 0.2987 | 0 | 0 | 0.0088 | 0.0221 | 0 | 0.0465 | 0.0492 | 0.0958 | 0.0210 | 0.0323 | -7.152 | 0.2504 | 0.0822 |
| 31 | 2.5490 | 0.0564 | 0.0428 | 0.0851 | 0.0393 | 0.0818 |  | 0.0323 | 0 | 0 | 0.0335 | 0 | 0.0750 | 0.0618 | 0 | 0 | 1.3006 | 0 | 0 | 0.0302 | 0.0972 | 0 | 0.1898 | 0.2157 | 0.4194 | 0.0921 | 0.1416 | 0.2029 | -37.83 | 0.3608 |
| 32 | 14.840 | 0.3288 | 0.2496 | 0.4954 | 0.2290 | 0.3588 |  | 0.1880 | 0 | 0 | 0.1965 | 0 | 0.4369 | 0.3602 | 0 | 0 | 7.5721 | 0 | 0 | 0.1758 | 0.5862 | 0 | 1.1633 | 1.2580 | 2.4419 | 0.5367 | 0.8245 | 1.1817 | 6.3911 | $-10.69$ |
| WP(1) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |






TABLE OF EXPORT (OWN AND CROSS) AND WORLD PRICE ELASTICITIES FOR



| PRICE OF <br> h, WP(1) |  |  |  |  |  |  |  |  |  |  | LATE | ERNL SE | OPN | MPO | orts or | F REGIO | ON 27 | M P | ARTNER | 1-32. |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 01 | 02 | 03 | 04 | 05 | 06. | 07 | 08. | 11 | 12. | 13 | 14 | 15 | 16 | 17 | 18 | - 19 | 20 | 21 | 22 | 23 | 24 | 26 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
| 01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 - 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 03 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 04 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 06 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 08 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 07 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 08 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 - 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -450.0 | 0 | 0.0101 | 0 | 0 | 0.0031 | 0.0085 | 0.0075 | 0 | 0.0163 | 0 | 0 | 0.3558 | 0.0051 | 0.0513 | 0 | 0.0132 | 0 | 0 | 0 | 0.0380 |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8.8112 | 0 | -5.376 | 0 | 0 | 0.0328 | 0.0902 | 0.0793 | 0 | 0.1716 | 0 | 0 | 3.7384 | 0.0637 | 0.6308 | 0 | 0.1393 | 0 | 0 | 0 | 0.3998 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.4398 | 0 | 0.0175 | 0 |  | -1.678 | 0.0147 | 0.0129 | 0 | 0.0280 | 0 | 0 | 0.6109 | 0.0087 | 0.0881 | 0 | 0.0227 | 0 | 0 | 0 | 0.0853 |
| 18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 32.946 | 0 | 0.4006 | 0 | 0 | 0.1230 | -4.280 | 0.2986 | 0 | 0.6416 | 0 | 0 | 13.978 | 0.2009 | 2.0179 | 0 | 0.5210 | 0 | 0 | 0 | 1.4951 |
| 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 43.612 | 0 | 0.5303 | 0 | 0 | 0.1628 | 0.4486 | - -3.687 | 0 | 0.8494 | 0 | 0 | 18.504 | 0.2080 | 2.6712 | 0 | 0.6897 | 0 | 0 | 0 | 1.9791 |
| 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.3088 | 0 | 0.0169 | 0 | 0 | 0.0052 | 0.0143 | 0.0125 | 0 | -8.755 | 0 | 0 | 0.6026 | 0.0085 | 0.0856 | 0 | 0.0220 | 0 | 0 | 0 | 0.0633 |
| 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8.9889 | 0 | 0.1090 | 0 | 0 | 0.0334 | 0.0918 | 0.0807 | 0 | 0.1746 | 0 | 0 | -187.5 | 0.0547 | 0.5493 | 0 | 0.1418 | 0 | 0 | 0 | 0.4070 |
| 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.7774 | 0 | 0.0094 | 0 | 0 | 0.0029 | 0.0079 | 0.0069 | 0 | 0.0151 | 0 | 0 | 0.3298 | -2.748 | 0.0478 | 0 | 0.0122 | 0 | 0 | 0 | 0.0352 |
| 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4.9341 | 0 | 0.0599 | 0 | 0 | 0.0184 | 0.0505 | 0.0444 | 0 | 0.0981 | 0 | 0 | 2.0935 | 0.0301 | -27.31 | 0 | 0.0780 | 0 | 0 | 0 | 0.2239 |
| 27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 28 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 36.541 | 0 | 0.4443 | 0 | 0 | 0.1364 | 0.3742 | 0.3289 | 0 | 0.7118 | 0 | 0 | 15.504 | 0.2229 | 2.2381 | 0 | -6.553 | 0 | 0 | 0 | 1.6582 |
| 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 32 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 310.67 | 0 | 3.7777 | 0 | 0 | 1.1599 | 3.1816 | 82.7988 | 0 | 6.0507 | 0 | 0 | 131.81 | 1.8952 | 19.028 | 0 | 4.9134 | 0 | 0 | 0 | -6.365 |
| WP(1) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\cdots$ | 0 | -7815. | 0 |  | -2439. | -6222. | - -5330 | 0 | $\cdots$ | 0 | 0 | $\cdots$ | -3902. | $\cdots$ | 0 | -9627. | 0 | 0 | 0 | -9252. |

TABLE OF EXPORT (OWN AND CROSS) AND WORLD PRICE ELASTICTTIES FOR 1980



| $\begin{array}{\|l\|} \hline \text { PRICE OF } \\ h, \text { WP(1) } \end{array}$ |  |  |  |  |  |  |  |  |  |  | Late | ERNL SE | OP1 | NE IMPO | O | O REGIO | N 30 F | FROM PA | NEP | 2 01-32. |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 01 | 02 | 03 | 04 | 05 | 06 | 07 | 08 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18. | 18 | 20. | 21 | 22 | 23 | 24 | 26 | 28 | 27 | 28 | 29 | 30 | 31 | 32 |
| 01 | -263.9 | 0.0680 | 0.60640 | 0.71080 | 0.47830 | 0.75014 | 4.5251 | 4E+06 | 2.2221 | 0 | 0 | 2.0284 | 0 | 2.7707 | 0 | 2.3399 | 1.3078 | $1 \mathrm{E}+06$ | 0 | 2.7641 | 0 | $\cdots$ | 1.4902 | 0 | 0 | 0 | 0.4916 | 0 | 2.7413 | 0 |
| 02 | 0.3063 | $-0.550$ | 0.00510 | 0.0060 | 0.00400 | 0.0083 | 0.0385 | 33474. | 0.0189 | 0 | 0 | 0.0172 | 0 | 0.0235 | 0 | 0.0199 | 0.0111 | 11009. | 0 | 0.0235 | 0 | 2589.5 | 0.0128 | 0 | 0 | 0 | 0.0041 | 0 | 0.0233 | 0 |
| 03 | 2.82040 | 0.0051 | -5.009 0 | 0.05570 | 0.03730 | 0.0588 | 0.3547 | $\cdots$ | 0.1742 | 0 | 0 | 0.1588 | 0 | 0.2172 | 0 | 0.1834 | 0.1025 | ..... | 0 | 0.2167 | 0 | 23654. | 0.1168 | 0 | 0 | 0 | 0.0385 | 0 | 0.2149 | 0 |
| 04 | 1.12620 | 0.0020 | 0.0189 - | -5.902 0 | 0.01490 | 0.02340 | 0.1416 | - 0 | 0.0695 | 0 | 0 | 0.0634 | 0 | 0.0887 | 0 | 0.0732 | 0.0409 | 40471. | 0 | 0.0885 | 0 | 9445.4 | 0.0468 | 0 | 0 | 0 | 0.0153 | 0 | 0.0858 | 0 |
| OS | 13.0290 | 0.02390 | 0.21960 | $0.2573-$ | -3.798 0 | 0.27171 | 1.6390 | $1 \mathrm{E}+08$ | 0.8048 | 0 | 0 | 0.7340 | 0 | 1.0038 | 0 | 0.8475 | 0.4737 | $\cdots \cdots$ | 0 | 1.0012 | 0 | ...... | 0.5398 | 0 | 0 | 0 | 0.1780 | 0 | 0.9829 | 0 |
| 06 | 4.36300 | 0.0080 | 0.07350 | 0.08610 | 0.0577 | -6.1630 | 0.5488 | $\cdots \cdots$ | 0.2695 | 0 | 0 | 0.2457 | 0 | 0.3360 | 0 | 0.2838 | 0.1586 | $\cdots \cdots$ | 0 | 0.3352 | 0 | 38592. | 0.1807 | 0 | 0 | 0 | 0.0596 | 0 | 0.3325 | 0 |
| 07 | 124.49 | 0.2288 | 2.09892 | 2.45821 | 1.04842 | 2.5980 | -22.08 | $1 \mathrm{E}+07$ | 7.6001 | 0 | 0 | 7.0129 | 0 | 9.5888 | 0 | 8.0979 | 4.5263 | $4 \mathrm{E}+06$ | 0 | 9.5859 | 0 | $1 \mathrm{E}+06$ | 5.1574 | 0 | 0 | 0 | 1.7015 | 0 | 9.4873 | 0 |
| OB | b. 1881 | 0.0003 | 0.00310 | 0.00360 | 0.00240 | 0.00380 | 0.0234 | $\cdots$ | 0.0114 | 0 | 0 | 0.0104 | 0 | 0.0143 |  | 0.0121 | 0.0067 | 6688.6 | 0 | 0.0143 | 0 | 1581.0 | 0.0077 | 0 | 0 | 0 | 0.0025 | 0 | 0.0141 | 0 |
| 11 | 5. 3937 | 0.0099 | 0.09090 | 0.10850 | 0.07140 | 0.11240 | 0.6785 | $\cdots$ | -18.19 | 0 | 0 | 0.3038 | 0 | 0.4154 | 0 | 0.3508 | 0.1981 | $\cdots$ | 0 | 0.4144 | 0 | 45237. | 0.2234 | 0 | 0 | 0 | 0.0737 | 0 | 0.4110 | 0 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14 | 1. 0012 | 0.0029 | 0.0280 | 0.03160 | 0.02120 | 0.03330 | 0.2014 | $\cdots$ | 0.0989 | 0 | 0 | -18.80 | 0 | 0.1233 | 0 | 0.1041 | 0.0582 | 57541. | 0 | 0.1230 | 0 | 13429. | 0.0083 | 0 | 0 | 0 | 0.0218 | 0 | 0.1220 | 0 |
| 16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18 | 11.927 | 0.0219 | 0.20110 | 0.2356 | 0.15790 | 0.2487 | 1.5004 | 1E+08 | 0.7368 | 0 | 0 | 0.6719 | 0 | -22.18 | 0 | 0.7758 | 0.4338 | $\cdots$ | 0 | 0.9165 | 0 | $\cdots$ | 0.4041 | 0 | 0 | 0 | 0.1630 | 0 | 0.9089 | 0 |
| 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18 | 3.8825 | 0.0071 | 0.0654 | 0.0768 | 0.05140 | 0.08090 | 0.4884 | $\cdots$ | 0.2398 | 0 | 0 | 0.2187 |  | 0.2990 |  | -19.25 | 0.1411 | $\cdots \cdots$ | 0 | 0.2983 | 0 | 32602. | 0.1608 | 0 | 0 | 0 | 0.0530 | 0 | 0.2958 | 0 |
| 19 | 3.6380 | 0.0084 | 0.05080 | 0.06990 | 0.0488 | 0.0738 | 0.4451 | $\cdots$ | 0.2188 | 0 | 0 | 0.1993 |  | 0.2725 |  | 0.2302 | -10.77 | ..... | 0 | 0.2719 | 0 | 29881. | 0.1408 | 0 | 0 | 0 | 0.0483 | 0 | 0.2697 | 0 |
| 20 | 0.0812 | 0.0001 | 0.0010 | 0.00120 | 0.0008 | 0.0012 | 0.0077 | e688.6 | 0.0037 | 0 | 0 | 0.0034 | 0 | 0.0047 |  | 0.0039 | 0.0022 | $\cdots$ | 0 | 0.0047 | 0 | 513.42 | 0.0025 | 0 | 0 | 0 | 0.0008 | 0 | 0.0046 | 0 |
| 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22 | 4.7294 | 0.0088 | 0.0797 | 0.0934 | 0.0828 | 0.0988 | 0.6949 | $\cdots$ | 0.2921 | 0 | 0 | 0.2884 |  | 0.3642 |  | 0.3078 | 0.1719 | $\cdots \cdots$ | 0 | -22.68 | 0 | 30885. | 0.1960 | 0 | 0 | 0 | 0.0846 | 0 | 0.3804 | 0 |
| 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 24 | 0.0142 | 0.0000 | 0.0002 | 0.0002 | 0.0001 | 0.0002 | 0.0017 | 1581.0 | 0.0008 | 0 | 0 | 0.0008 | 0 | 0.0011 |  | 0.0009 | 0.0005 | 513.42 | 0 | 0.0010 | 0 | $\cdots$ | 0.0005 | 0 | 0 | 0 | 0.0001 | 0 | 0.0010 | 0 |
| 25 | 0.1160 | 0.0002 | 0.0019 | 0.0023 | 0.0015 | 0.0024 | 0.0147 | 12774. | 0.0072 | 0 | 0 | 0.0085 | 0 | 0.0090 | 0 | 0.0078 | 0.0042 | 4201.4 | 0 | 0.0089 | 0 | 980.55 | -12.42 | 0 | 0 | 0 | 0.0015 | 0 | 0.0089 | 0 |
| 28 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 28 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 29 | 3.4118 | 0.0082 | 0.0575 | 0.0673 | 0.0461 | 0.0711 | 0.4291 | -....* | 0.2107 | 0 | 0 | 0.1821 |  | 0.2627 |  | 0.2218 | 0.1240 | $\cdots$ | 0 | 0.2821 | 0 | 28614. 0 | 0.1413 | 0 | 0 | 0 | -4.052 | 0 | 0.2800 | 0 |
| 30 | 0 | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 31 | 32.944 | 0.1523 | 1.3984 | 1.6385 | 1.0983 | 1.7298 | 10.434 | 9E+08 | 5.1237 | 0 |  | 4.6725 |  | 6.3887 |  | 5.3064 | 3.0157 | $3 \mathrm{E}+08$ | 0 | 8.3735 | 0 | $\cdots$ | 3.4382 | 0 | 0 | 0 | 1.1338 | 0 | $-16.63$ | 0 |
| 32 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| WP(1) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

TABLE OF EXPORT (OWN AND CROSS) AND WORLD PRICE ELASTICTTIES FOR 1980

| PRICE OF <br> h, WP(1) |  |  |  |  |  |  |  |  |  |  | LATE | L SE | ORN | MPO |  | F REQIO | ON 31 | M P | NEI | 1-32 |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 01 | 02 | 03 | 04 | 05 | 06 | 07 | 08 | 11 | 12 | 13 | 14. | 15 | 16. | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 26 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
| 01 | -525.10 | 0.18031 | 1.8586 | 1.2888 | 0 | 2.95491 | 1.5649 | 0 | 2.1363 | 0 | 0 | 0 | 0 | 0 | 0 | 7.5837 | 4.3810 | 0 | 0 | 0 | 0.7940 | 0 | 0 | 0 | 0.9978 | 2.0077 | 0 | 0.4670 | 1.1007 | 0.4173 |
| 02 | 1.0930 | $-0.5740$ | 0.0078 | 0.0054 | 0 | 0.01240 | 0.0085 | 0 | 0.0090 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0319 | 0.0184 | 0 | 0 | 0 | 0.0033 | 0 | 0 | 0 | 0.0042 | 0.0084 | 0 | 0.0019 | 0.0046 | 0.0017 |
| 03 | 78.974 | 0.0579 | -5.054 | 0.3823 | 0 | 0.8985 | 0.4783 | 0 | 0.6503 | 0 | 0 | 0 | 0 | 0 | 0 | 2.3085 | 1.3336 | 0 | 0 | 0 | 0.2417 | 0 | 0 | 0 | 0.3037 | 0.6111 | 0 | 0.1421 | 0.3350 | 0.1270 |
| 04 | 2.42580 | 0.00170 | 0.0173 | -3.885 | 0 | 0.02780 | 0.0146 | 0 | 0.0189 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0709 | 0.0409 | 0 | 0 | 0 | 0.0074 | 0 | 0 | 0 | 0.0093 | 0.0187 | 0 | 0.0043 | 0.0102 | 0.0039 |
| 05 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 08 | 51.761 | 0.03790 | 0.37080 | 0.2571 | 0 | -8.346 | 0.3122 | 0 | 0.4262 | 0 | 0 | 0 | 0 | 0 | 0 | 1.5130 | 0.8740 | 0 | 0 | 0 | 0.1584 | 0 | 0 | 0 | 0.1990 | 0.4005 | 0 | 0.0931 | 0.2198 | 0.0832 |
| 07 | 167.020 | 0.1225 | 1.18850 | 0.8297 | 0 | 1.9024 | -3.725 | 0 | 1.3753 | 0 | 0 | 0 | 0 | 0 | 0 | 4.8824 | 2.8205 | 0 | 0 | 0 | 0.5112 | 0 | 0 | 0 | 0.6424 | 1.2928 | 0 | 0.3006 | 0.7086 | 0.2886 |
| 08 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11. | 1.1579 | 0.0008 | 0.0082 | 0.0057 | 0 | 0.01310 | 0.0069 | 0 | -6.451 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0338 | 0.0195 | 0 | 0 | 0 | 0.0035 | 0 | 0 | 0 | 0.0044 | 0.0089 | 0 | 0.0020 | 0.0049 | 0.0018 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18 | 1.1484 | 0.0008 | 0.0082 | 0.0057 | 0 | 0.0130 | 0.0069 | 0 | 0.0094 | 0 | 0 | 0 | 0 | 0 | 0 | -22.90 | 0.0193 | 0 | 0 | 0 | 0.0035 | 0 | 0 | 0 | 0.0044 | 0.0088 | 0 | 0.0020 | 0.0048 | 0.0018 |
| 18 | 2.7828 | 0.0020 | 0.0199 | 0.0138 | 0 | 0.0318 | 0.0187 | 0 | 0.0229 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0813 | -13.20 | 0 | 0 | 0 | 0.0085 | 0 | 0 | 0 | 0.0107 | 0.0215 | 0 | 0.0050 | 0.0118 | 0.0044 |
| 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $?$ | 0 | 0 | 0 |
| 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 23 | 3.0808 | 0.0022 | 0.0220 | 0.0153 | 0 | 0.0350 | 0.0185 | 0 | 0.0253 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0800 | 0.0520 | 0 | 0 | 0 | -2.391 | 0 | 0 | 0 | 0.0118 | 0.0238 | 0 | 0.0055 | 0.0130 | 0.0049 |
| 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 25 | 0 | $0 \quad 0$ | 0 | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 28 | 0 | $0 \quad 0$ | $0 \quad 0$ | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 27 | 0.1572 | 0.0001 | 0.0011 | 0.0007 | 0 | 0.0017 | 0.0009 | 0 | 0.0012 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0045 | 0.0028 | 0 | 0 | 0 | 0.0004 | 0 | 0 | 0 | -3.017 | 0.0012 | 0 | 0.0002 | 0.0008 | 0.0002 |
| 28 | 75.587 | 0.0554 | 0.5414 | 0.3755 |  | 0.8609 | 0.4569 | 0 | 0.6224 | 0 | 0 | 0 | 0 | 0 |  | 2.2095 | 1.2764 | 0 | 0 | 0 | 0.2313 | 0 | 0 | 0 | 0.2907 | $-5.488$ | 0 | 0.1360 | 0.3208 | 0.1215 |
| 20 | 0 | - 0 | $0 \quad 0$ | 0 | 0 | $0 \quad 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 30 | 12.191 | 0.0089 | 0.0873 | 0.0605 |  | 0.1388 | 0.0735 | 0 | 0.1003 | 0 | 0 | 0 | 0 | 0 |  | 0.3563 | 0.2058 | 0 | 0 | 0 | 0.0373 | 0 | 0 | 0 | 0.0468 | 0.0943 | 0 | $-1.390$ | 0.0517 | 0.0196 |
| 31 | 24.276 | 80.0178 | 80.1739 | 0.1205 |  | 0.2785 | 0.1464 | 0 | 0.1998 | 0 | 0 | 0 | 0 | 0 | 0 | 0.7096 | 0.4099 | 0 | 0 | 0 | 0.0742 | 0 | 0 | 0 | 0.0933 | 0.1878 | 0 | 0.0437 | -3.225 | 0.0390 |
| 32 | 103.48 | 0.0759 | 0.7413 | 0.5140 |  | 01.1788 | 0.6241 | 0 | 0.8521 | 0 | 0 | 0 | 0 | 0 |  | 3.0248 | 1.7474 | 0 | 0 | 0 | 0.3167 | 0 | 0 | 0 | 0.3980 | 0.8008 | 0 | 0.1862 | 0.4390 | -1.095 |
| WP(1) | -707.5 | 5-0.774 | 4-6.811 | -5.235 |  | $0-11.24$ | -6.019 | 0 | -8.692 | 0 | 0 | 0 | 0 | 0 |  | $-30.85$ | -17.78 | 0 | 0 | 0 | -3.222 | 0 | 0 | 0 | -4.065 | -7.393 | 0 | -1.873 | -4.346 | -1.476 |

TABLE OF EXPORT (OWN AND CROSS) AND WORLD PRICE ELASTICTTIES FOR 1980


## APPENDIX 7.4

```
5 TABLES OF LAMDA(j) AND b(ij) COEFFICIENTS
    AND
    5 TABLES OF ESTIMATED t STATISTICS
FOR THE LAMDA(j) AND b(ij) COEFFICIENTS
```

TABLE OF ESTIMATED "Lamda(j)" AND "b(ij)"x100 COEFFICIENTS FOR GOOD 1 (30 IMPORTING REGIONS).



TABLE OF ESTIMATED "t" STATISTICS FOR THE "Lamda(j)" AND "b(ij)" COEFFICIENTS FOR GOOD 1 (30 IMPORTING REGIONS).

| PARTNERI | IMPORITING REGION ; |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 01 | 02 | 03 | 04 | 05 | 06 | 07 | 08 | 11 | 12 | 13 | 14 | 15 | 18 | 17 | 18 | 19 | 20 | 21 | 22 | - 23 | 24 | 25 | 28 | 27 | 28 | - 29 | - 30 | - 31 | . 32 |
| Lamda() | 11.4381 | . 671 | NA 1 | 59 | 41 | NA | NA 8 | 8.6525 | 6886 | 14.645 | . 101 | NA | 19.656 | 25.259 | 10.270 | 15.625 | 19.597 | 14.103 | 10.836 | 12.663 | NA | 7.6540 | 8.5090 | 22.797 | 15.735 | 13.816 | 17.619 | 18.786 | 30. | 17.396 |
| 01 | NA | NA | NA | NA | NA | NA | NA | NA | NA | NA | NA | NA | NA | NA | NA | NA | - NA | A | A | NA | NA | NA | 30 | A | NA | NA | IA | A | A | A |
| 02 | 3160 | 0.6200 | 0.54881 | 1.94811 | 1.50650 | 0.3013 | 3.4245 | 1.6745 | 2809 | A11 | 0.4453 | 718 | 63 | 0.8112 | 488 | 1.0358 | 0.5488 | 3.6514 | -2.484 | -0.755 | 3. | -0.153 | NA | 1. | -0.131 | -0.286 | 0.2807 | 0.0106 | -0. | 1.0858 |
| 03 | -2.844 0 | 0.279 | NA | NA | NA | NA | 523 | -0.679 | 0.583 | -0.444 | 489 | -0.906 | 0.0503 | 1.0857 | 0.817 | 0.19 | 2.12 | 1.36 | 5.2031 | 3.2336 | -5.305 | . 1655 | 2.6340 | 58 | -0.56 | 1.61 | 85 | 0.3316 | 0.9418 | -0.165 |
| $04 . . . \mid$ | 1.07282 | 2.8678 | NA | -0.078 | 1.9888 | - | -2.294 0 | 0.6063 | 7.7283 | 0812 | 0.1378 | 2.4540 | 1.5432 | 0.8431 | 2.511 | 0.5901 | 0.4419 | 1.70 | 3.8411 | 1.3026 | 1.4855 | 1.4497 | 3.6933 | 1.4338 | -2.692 | 2.1092 | -0.878 | 1.1034 | 2.5634 | 968 |
| 05. | -0.779 1 | 1.9339 | NA | NA | NA | NA | -0.678 | 1.3694 | 7.9887 | 1.7299 | 0.0832 | 0.8131 | 1.0301 | 0.6008 | 0.2168 | 0.9458 | 0.3 | 0.7142 | -0.799 | 1.2268 | 8.7273 | 2. | 4.18 | 4.228 | 1.82 | -1.523 | -2.50 | 0.5134 | 0.331 | 1.9836 |
| 08 | -0.678 | 1.4333 | NA | NA | : NA | NA | 0.2123 | 2.7239 | -0.436 | -0.013 | 51 | 1.9150 | 0.0845 | -0.822 | -0.423 | 0.2 | 0.85 | 0.0425 | -0.892 | 1.13 | 2.2506 | 2.0 | 0.3975 | 0.37 | -0.16 | 0.56 | -0.4 | 1.3 | 3.31 | -0.145 |
| 07 | 0. 2311 - | -0.801 | 0.4447 | $-1.681 \quad 0$ | 0.8738 | -0.084 | -7.096 | 2.0817 | 0.9472 | 1.4958 | 6.7800 | -0.885 | 1.3360 | 1.0074 | 0.9010 | 0.2857 | 0.1287 | 0.7455 | 0.7708 | 2.1590 | 1.5159 | 1.3638 | 0.4240 | 0.1614 | -0.518 | 0.1641 | 0.7810 | 1.951 | 0.0171 | 1.6210 |
| 08 | 0. 17120 | 0.8 | -0.191 | 220 | -0.203 | -0.214 | 2.5107 | -3.607 | 1.915 | 0.3419 | -0.002 | -2.502 | -2.008 | 1.0540 | 1.0650 | -0.168 | $-0.452$ | -0.768 | -0.795 | 0.4571 | -1.306 | -1.586 | -0.129 | 0.467 | 0.0083 | 0.6904 | -0.278 | . 39 | -0.40 | -0.968 |
| 11 | 1.86490 | 0.3422 | 0.8182 | -2.083 | $-1.792$ | 0.3114 | -0.938 | 0.4285 | 192 | 1.4336 | 3.8115 | -3.112 | 0.1185 | 0.7758 | 0.6560 | -0.6 | -0.40 | 1.9701 | -2.888 | 0.4705 | 1.478 | -0.397 | -1.97 | 1.0383 | -0.918 | 1.51 | -1.855 | 0.516 | 0.3591 | 0.336 |
| 12 | 1.1394 - | $-1.523$ | $-1.369$ | 2.8325 | -3.752 | 0.1713 | -0.713 | 2.4163 | -6.304 | 2.3018 | 0.4999 | 1.8969 | -0.248 | 0.9340 | -0.631 | 3.453 | -2.216 | 0.7693 | 0.6255 | -0.107 | -0.967 | 1.4850 | 2.638 | 0.1451 | 0.3448 | 2.726 | 2.3300 | 1.6041 | 1.2690 | -0.185 |
| 13 | 4.342 | 4.5877 | 0.1913 | 0.8035 | 2.5450 | 0.5328 | 0.0322 | -0.757 | 8.9102 | -0.341 | -2.271 | -1.459 | 0.0000 | 2.0308 | 10 | 2.97 | -1.27 | -0.027 | -0.190 | 0.5585 | 0.412 | -0.382 | 0.097 | 0.447 | 1.7195 | 780 | 1.6395 | . 08 | . 60 | 1.0116 |
| 14 | -0.463 | 1.6833 | -1.018 | 0.3415 | -0.944 | -0.227 | -1.586 | -2.748 | 0.0005 | $-0.544$ | 0.4335 | 1.3587 | -0.319 | 0.9018 | -1.713 | -1.6 | 0.121 | 0.2421 | $-0.195$ | 0.681 | -2.528 | 0.1018 | -0.504 | -0.720 | 1.0880 | 0.2884 | 0.7946 | 0.617 | -1.30 | -0.318 |
| 15 | -2.481 | 1.5146 | 1.160 | -1.279 | -0.337 | 0.4308 | 1.0802 | -1.475 | $-0.857$ | -1.153 | -4.934 | -3.507 | 1.6044 | 0.1985 | 1.0707 | -0.299 | -0.577 | 1.3423 | -1.425 | -0.368 | -9.756 | -1.338 | -3.180 | 1.2563 | 0.0384 | 0.3579 | 1.2880 | . 87 | 9408 | -3.982 |
| 18 | 0.3997 | 1. | 0.4 | -0.694 | 0.7429 | 0.4751 | 1.1521 | -1.204 | -2.143 | $-1.287$ | -3.846 | -3.589 | 0.1508 | -0.700 | -0.240 | 1.6 | -2.390 | 2.5483 | -0.908 | 0.6115 | -2.898 | -1.075 | 1.7529 | 3.9846 | 1.2883 | 3.729 | 2.1040 | 0.753 | 97 | 878 |
| 17 | -0.892 | $-0.687$ | 0.6237 | 1.2882 | 0.0168 | -0.123 | -0.971 | -1.41 | 2.1370 | 1.4386 | 0.8934 | 0.2288 | -0.125 | -0.524 | -3.754 | -2.407 | -2.155 | A | -0.975 | -0.807 | -9.645 | NA | -4.898 | -0.182 | 721 | -1.165 | 0.1647 | 1.328 | -3.308 | -0.109 |
| 18 | -0.148 | 1.943 | 1.331 | 2.7109 | 1.2335 | 1.3489 | 5.8414 | 0.3612 | -0.313 | -0.448 | -0.877 | -4.658 | 1.2827 | 0.7979 | 0.5434 | -0.4 | 2.2198 | 0.0308 | -0.21 | -0.125 | -2.453 | -4.17 | -0.21 | 0.973 | -0.104 | 0.4592 | 0.8459 | 1.1902 | 1.574 | -0.780 |
| 19 | 0. 1338 | 1.79 | 0.0388 | 3.9505 | 3.7087 | 0.5782 | 2.0977 | 0.9358 | -0.188 | -0.011 | -0.817 | -0.479 | 1.5850 | -0.347 | -2.637 | 1.1674 | 3.3484 | 1.1712 | 0.4865 | -1.158 | -1.743 | 1.6145 | 2.0396 | 1.4109 | 0.7070 | 2.2729 | 0.9899 | 1.3772 | 1.733 | 1.2433 |
| 20 | 1.4694 | -0.441 | 0.1947 | -0.156 | 3.1183 | 0.4938 | 0.7110 | -1.191 | -3.959 | 1.2818 | -0.300 | -2.054 | 0.9108 | 0.6284 | 0.365 | -0.13 | -2.455 | -0. | -1.103 | -0.845 | -4.271 | NA | -0.810 | -1.038 | -1.194 | -0.568 | 0.11 | -0.507 | 0.810 | -2.771 |
| 21 | -0.218 | 0.881 | 0.7049 | 1.0656 | 1.1987 | 0.2920 | 2.5870 | -1.024 | -0.218 | 0.9280 | 1.3222 | $-0.957$ | 0.2843 | 2.7445 | -1.112 | 2.8725 | 0.3219 | 1.7329 | -2.282 | 0.6778 | -1.316 | 0.7036 | 2.8610 | 1.9424 | 0.2087 | 1.5227 | 0.3477 | 1.6231 | 0.6905 | 1.2271 |
| 22 | 0.2526 | 1.7418 | $\rightarrow .283$ | -1.157 | 0.4192 | -0.022 | 2.1478 | 0.1468 | 0.506 | 2.0102 | -0.981 | -2.642 | -2.159 | -0.780 | -1.014 | -0.679 | -2.786 | -0.7 | $-2.280$ | -0.688 | -2.117 | 2.760 | 0.2627 | 0.9821 | 0.4357 | 1.4344 | 3.2073 | 0.7982 | -0.145 | 0.0025 |
| 23 | 0. 6827 | 1.1528 | -0.430 | 0.4287 | 4.0733 | -0.064 | 2.1324 | 0.7190 | -1.155 | 0.8398 | -1.780 | -1.950 | $-1.773$ | -1.997 | 0.3238 | -3.831 | $-0.398$ | 0.333 | 0.3889 | -0.791 | 0.4438 | -4.980 | -0.880 | 0.1974 | -0.682 | 2.3128 | $-0.168$ | -2.760 | -4.640 | -0.148 |
| 24 | 2.6379 | 0.3367 | 0.3948 | 2.3428 | 3.6392 | 0.3488 | 3.1691 | -1.480 | -0.284 | 0.2241 | $-0.230$ | 3.1850 | 0.0752 | 2.0415 | 0.5627 | 1.0514 | 4.7880 | 0.6994 | 3.7681 | 1.8284 | -0.638 | NA | -2.508 | -0.053 | $-3.100$ | 1.8370 | 0.3795 | -0.778 | 2.5804 | -0.653 |
| 25 | -0.787 | -0.578 | -0.595 | -0.527 | -1.659 | -0.100 | -1.820 | -1.373 | -1.785 | -0.608 | $-1.123$ | 1.2817 | -0.110 | -1.873 | -0.471 | -3.748 | -1.675 | 1.6104 | -0.078 | 0.0935 | -2.456 | 1.3442 | 1.0148 | 0.1457 | -2.047 | 1.0873 | -0.785 | 0.4943 | . 258 | 1.5219 |
| 28 | 0.0817 | 0.0634 | -0.095 | 0.2233 | -0.087 | -0.896 | NA | -0.620 | 0.1229 | -0.130 | 0.1195 | 0.2505 | -0.328 | -0.910 | 2.9584 | -1.013 | 1.6818 | 4.8886 | 0.8571 | 0.9860 | $-0.978$ | 0.1784 | -1.145 | -1.357 | -1.368 | 3.2911 | 2.6516 | 0.3723 | 1.3514 | 2.3634 |
| 27 | -0.061 | 1.5830 | 0.1085 | 1.0777 | 3.0362 | 0.4909 | 0.8959 | -0.239 | 1.4541 | $-0.418$ | 1.4812 | NA | 0.2078 | 0.9034 | -2.524 | -0.859 | -0.140 | 0.8638 | 1.4570 | 0.7307 | $-0.681$ | -2.964 | -0.888 | -1.682 | 0.5256 | 0.3182 | -0.044 | -1.101 | 0.7538 | 0.6821 |
| 28 | -1.170 | -2.708 | -0.748 | -1.217 | -0.541 | 0.0983 | 31.6465 | 1.2500 | 2.2258 | -0.858 | 0.9487 | 2.3408 | -1.157 | -0.408 | -6.122 | 0.4205 | 0.0940 | -0.284 | -0.049 | -0.842 | 1.3805 | -0.881 | -0.843 | -2.081 | -0.662 | -3.775 | -2.695 | -1.277 | -1.095 | 0.0759 |
| 29 | -1.564 | -1.178 | -0.540 | 1.8085 | -1.451 | -0.529 | -2.704 | 0.8878 | 2.8731 | -0.280 | 1.4098 | 1.6469 | 0.3282 | 0.1054 | -0.578 | -0.932 | -0.850 | -0.774 | 2.1340 | 1.0271 | 4.2280 | -6.746 | 1.0103 | 0.2948 | 1.6584 | 2.4194 | 0.1080 | 1.0338 | -2.495 | . 7698 |
| 30 | -1.316 | 80.3114 | 1.0689 | -0.219 | 0.7901 | 10.7574 | 2.9546 | - -1.135 | 2.6894 | -0.578 | -3.680 | -1.122 | -0.116 | 0.6924 | 2.0145 | 0.5670 | 3.2728 | -0.648 | 1.9535 | -0.161 | -2.081 | -0.187 | -3.685 | -1.380 | 0.0528 | -0.825 | -0.709 | -2.358 | 0.7592 | $-0.833$ |
| 31 | -2.487 | -1.564 | -0.088 | 8-1.237 | 0.0182 | $2-1.079$ | -0.433 | -0.215 | -0.699 | -2.333 | -4.945 | -0.646 | -0.286 | -2.312 | -4.236 | -2.651 | -1.685 | 0.5012 | -3.499 | -1.283 | -1.624 | -3.339 | -2.545 | -0.845 | -0.276 | -0.178 | -1.989 | -1.046 | 0.3901 | -0.913 |
| 32 | 1.6632 | 2-1.919 | $9-0.088$ | 61.7511 | $1-1.514$ | - -0.215 | 5.9241 | $1-0.625$ | -1.650 | -1.915 | -0.839 | -3.889 | -1.796 | -1.086 | -1.023 | 0.2446 | -0.251 | 0.6250 | $-1.401$ | 2.5463 | $-2.410$ | -1.187 | $-6.71$ | -3.278 | -0.463 | 0338 | 0.697 | 211 | . 68 | -1.182 |


TABLE OF ESTIMATED "t" STATISTICS FOR THE "Lamda(j)" AND "b(ij)" COEFFICIENTS FOR GOOD 3 (30 IMPORTING REGIONS).

TABLE OF ESTIMATED "t" STATISTICS FOR THE "Lamda(j)" AND "b(ij)" COEFFICIENTS FOR GOOD 5 ( 30 IMPORTING REGIONS).

| artneri | IMPORTING REGION |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 01 | 02 | 03 | 04 | 05 | 08 | 07 | 08 | 11 | 12 | 13 | 14 | 15 | 18 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 28 |  | 28 | 29 | 30 | 31 |  |
| Lamda() | 321 | 15. | 14 | 14. | NA | 1 | 10.78 | 7.41989 | 2900 | 17.178 | . 089 | 55 | 7.698 | 48 | 4892 | 15.824 | 18.5 | 5.4313 | 10.2 | 1.8 | 15.4 | . 71 | 13.831 | 14. | 11. | 14.7 | NA | 12. | 32 | 7.7494 |
| 01 | NA | NA | NA | NA 0 | 0.5781 | NA | NA | -2.655 | NA | NA | NA | NA | NA | NA | NA | NA | NA | NA | NA | NA | NA | NA | NA | NA | NA | NA | NA | NA | NA | NA |
| 02 | 0.378 | -0.891 | - | -0.65 | NA | -5. | 0.1679 | NA | 3.3802 | 07 | 0.8624 | -0.05 | 1.4426 | 3404 | . 864 | . 35 | 0.6 | 0.97 | -0. | -0.166 | 0.8 | -3.736 | 1.3922 | 0.5 | 1. | -1.879 | -0.154 | 0.6166 | 0.2957 | 0.4955 |
| 03 | -0.217-1 | -1.204 | NA | NA | NA | NA | -0.528 | -1.756 | -1.337 | 5.109 | -2.530 | -3.341 | 0.2391 | -0.977 | -0.843 | -1.883 | 1.3879 | -0. | -2.1 | 1.41 | -1.948 | -2.10 | -0.1 | -1. | -0.2 | 1.14 | 0.0 | 1.63 | 3.5128 | 1.4799 |
|  | $1.0780-$ | -1.067 | 1. | 1.0 | 2.1684 | 3.0 | - | -0.514 | 3.7978 | 1.8802 | 2.13 | 3.4888 | 0.44393 | 7445 | -6.2 | A | 1.98 | -1 | 2.5282 | -0.07 | -1.091 | -0.2 | -2.3 | 0.6 | 0.9 | -0.6 | 0.7297 | 0.0362 | 4.0838 | 0.4121 |
| 05 | -1.355 - | -1.300 | NA | NA | NA | NA | -0.891 | 4.0341 | 0.78 | - | -0.877 | -1.717 | -1.357 | . 0163 | -2.801 | 0.0835 | 1.56 | -0.0 | 2.85 | -0.054 | -1.007 | 1.77 | 0.0621 | -0.8 | -0.1 | -0.4 | 0.04 | 0.30 | 0.7752 | -3.281 |
| 06 | 0.64530 | 0.5927 | NA | NA | NA | NA | 0.3653 | 0.32 | -0.908 | 1.2414 | -0.967 | -2.655 | -1.365 | -0.577 | 1.5415 | -0.015 | 1.18 | -0.412 | -1.32 | -0.6 | 0.9429 | -1.17 | -0.5 | -0.0 | -1.4 | 1.2 | -0, | 2.5798 | 0.9465 | 5.6567 |
| 07 | -0.776 - | -3.217 | -0.442 | 0.3996 | 1.0755 | -8.228 | -7.298 | -0. | 0 | 0 | -5.563 | -3.198 | -1.234 | -0.333 | 0.0325 | $-0.381$ | 0 | -0.519 | -0.749 | 0.9595 | -0.610 | 2. | 0.2794 | -1.716 | -1.208 | - | -0.156 | 0.8777 | -0.249 | 1.4919 |
| 08 | -0.045 | 0.5240 | 65- | -1.374 | 0.5805 | -1.889 | 76 | -1 | -1.661 | -2.760 | -0.113 | -1.182 | -0.483 | -0.546 | -0.467 | -1.000 | -1. | -1.639 | 0.5662 | 0.1 | -2.086 | -2.6 | 0.0 | 0.8 | -0.8 | -0.8 | 0.6543 | -0.657 | -0.326 | -0.873 |
| 11 | 0.2543 1 | 1.8059 | 0.1678 - | -1.200 - | -0.3 | -1.726 | -0.6 | 4.93 | 1.25 | -1.7 | 2.27 | -3.8 | 1.3 | 1.4641 | 0.42 | 0.93 | 1.8 | -0.884 | 0.9750 | -0. | 0.9265 | 0.0 | -0.23 | -1 | 1.1171 | 1.3712 | 0.7510 | 0.0907 | 2.7467 | 808 |
| 12 | 0.0867 | 1.5844 | 1.7243 | 1.9179 | 0.9809 | 0.7902 | 1.4725 | 3. | -0.612 | -1.353 | -0.580 | -0.121 | 0. | 0.9863 | 0.6 | 0.5082 | 0.3677 | -0.76 | 0.1 | -0.249 | 1.1095 | 0.46 | 0.6716 | 0.3287 | 0.5032 | -0.01 | -0.124 | 0.0685 | 0.4209 | -0.875 |
| 13 | 1.2357 | -1.903 | 5.10362 | 2.5179 | 4.0310 | 6.0168 | 1.7977 | 3. | -0.990 | 1.9206 | 0.8905 | 1.98 | 2.2795 | -0.600 | -2.23 | 0.7 | 0.0 | -1. | -0.4 | -0.281 | 0. | 1.4737 | 0.28 | -0. | 0.5927 | -0.123 | 1.3818 | 0.8165 | 0.5225 | 3.8711 |
| 14 | . 65 | 93 | 0.08521 | 1.2818 | -0.838 | -0.739 | -0.253 | -0.375 | 0.3343 | 5.9684 | 2.4066 | -0.083 | 1. | 0.6403 | 0.5399 | 880 | -0.126 | -1.22 | -0.255 | 0.1610 | -0.20 | 0.89 | -0.5 | -0.0 | 0.73 | 1.4260 | 0.5206 | 0.60 | . 0 | . 64 |
| 15 | . 68 | -1.043 | -2.057 - | -1.729 | -0.914 | -6.357 | -1.973 | -1.659 | -2.426 | -1.388 | 0.1878 | -6.248 | -0.452 | -1.750 | 0.6837 | 1.1977 | -0.478 | 1.6793 | -2.62 | 0.3304 | -2.580 | -1.236 | -1.84 | -1.7 | -0.976 | -1.61 | -2.00 | -1.05 | -3.18 | -5.600 |
| 16 | 3.4118 | 0.7692 | 0.64680 | 0.8916 | -5.340 | -4.424 | -4.408 | -1.347 | -2.773 | -2.701 | -2.218 | -0.400 | 1.2188 | -0.758 | 1.6564 | -1.629 | -2.843 | 0.9468 | 0.61 | 2.17 | 0.782 | 3.31 | 2.82 | 1.76 | 1.69 | 4.77 | -0.5 | 2.09 | -0.45 | 2.0 |
| 17 | -1.655 | -1.450 | -2.435-1.7 | -1.740 | $-3.698$ | -1.974 | -1.302 | -0.468 | 1.4789 | 0.1380 | -1.586 | 1.1196 | 0.1750 | 0.4486 | -3.068 | -1.229 | -1.163 | NA | 1.2919 | -0.36 | -1.03 | 0.9705 | -0.47 | 0.09 | 0.816 | -1.85 | -0.05 | 3.59 | -0.12 | -1. |
| 18 | 0.8728 | 8183 | 0.75 | 0.0227 | 5174 | -0.043 | 0.0887 | -3.053 | 0.8639 | -0.502 | -0.614 | 0.1910 | 1.29 | -0.118 | -0.790 | -1.063 | -0.005 | 3.7970 | -0.710 | -4.135 | -0.246 | 2.0 | 0.0950 | -0.245 | -1.316 | -0.07 | 0.57 | . 29 | 3818 | -3.146 |
| 18 | 5136 | -1.819 | -0.838 | 0.2291 | 1055 | -1.289 | -1.100 | 0.0712 | -1.547 | -1.702 | 0.3268 | 712 | 0. | -0.693 | $-0.910$ | 0.8710 | 1.8844 | 2.3743 | 0.3983 | 0.8314 | 0.0465 | -0.051 |  | 3.124 | . 1 | $-0.311$ | . 14 | . 74 |  |  |
| 20 | . 060 | , 333 | 0.8356 | 0.124 | 2.0820 | 2.2750 | NA | -0.788 | $-3.743$ | -0.409 | -0.892 | -1.420 | -2.132 | 1.2040 | NA | -2.391 | -1.087 | -3.718 | -1.515 | -1.248 | -5.287 | -0.189 | 0.10 | -0.841 | -1.405 | -2.46 | 0.0178 | -0.510 | -2.56 | -0.873 |
| 21 | 596 | 583 | 06 | 1.084 ¢ | 05 | 2.8082 | 0.8650 | 170 | . 717 | -0.487 | -1.750 | 1.0872 | -1.020 | 1.1757 | -2.524 | -1.020 | -0.140 | -2.137 | -0.894 | -0.399 | -0.585 | . 25 | 1.458 | 0.5 | 0.8 | -0.65 | 0.98 | 0.1401 | 1.2290 | 0.9698 |
| 22 | 0.6172 | 1.1057 | 0.1817 | 1.0 | 0.666 | 0.9158 | 0.90 | , 320 | 0.7535 | 0.59 | -0.149 | 0.4129 | -0.002 | 3.5084 | 1.1424 | -0.254 | 4.6347 | 1.882 | 2.95 | 0.2428 | 0.1850 | D. 14 | 2.06 | 0.80 | 0.70 | . 84 | 0.03 | -0.978 | 055 | - 53 |
| 23 | . 997 | 0.3108 | -0.000 | -0.687 | -0.18 | 2.911 | 0.82 | -1.723 | 0.5541 | -0.433 | -2.523 | 0.0166 | -2.944 | -0.845 | -3.362 | -2.348 | -1.601 | -0.780 | -0.593 | 1.0184 | -2.255 | 0.8062 | -1.000 | -0.908 | -1.742 | -2.890 | 0.0428 | -0.891 | -2.450 | -1.087 |
| 24 | 18 | 0. | 2. | 0.9931 | -0.32 | 2.1488 | 0.0107 | -0.573 | -0.052 | -2.046 | 2.2239 | 1.8869 | -0.773 | 0.1280 |  | 0.2494 | 0.3210 |  | 0.1992 | 0.2 | 0.2365 | NA | 0.3598 | 0.1013 | -2.592 | 0.1819 | 0.7746 | NA | 0.3785 | 0.5 |
| 25 | 0. 28 | -2.3 | -0. | -1 | 8.5352 | -0.911 | -1.897 | -0.645 | -0.090 | -0.613 | 0.7191 | -0.494 | 0.2328 | -0.068 | -0.607 | 3.18 | 1.83 | 0.3075 | 1.36 | 0.3 | -0.024 | 1.8294 | -3.113 | -1.582 | -0.919 | -3.02 | 0.5888 | -0.575 | -1.134 | 0.4 |
| 28 |  | A 0.6361 | 0.7574 | 1.0 | 1.0947 | 5. | 1.07 | -0.274 | -0.241 | -0.414 | 0 | 0.7318 | 1.5607 | 0.1339 | 3.5828 | -0.655 | 0.86 | 2. | 0.863 | 0.9 | -0.207 | 1.8 | 0.78 | -1.717 | 0.33 | 0.3 | 0.1878 | -0.19 | 0.3506 | 0.7459 |
| 27 | b. 1717 | 73.7135 |  | 4.0 | 23 | 5. | 4.9481 | 1 | 0.1590 | -0.528 | -0.339 | 2 | , | 1.5470 | -1.842 | 1.1815 | 3.103 | -2.152 | 1.25 | 2.18 | 0.4534 | -0.39 | 2.3781 | -1.259 | 1.76 | 1.978 | 0.9044 | -0.852 | . 34 | 0.6307 |
| 28 | -1.188 | 80.2272 | 0.1381 | -0.669 | - -2.439 | -0.690 | -1.884 | -1.082 | -0.623 | 1.3747 | -0.094 | 0.1488 | 8. 18 | 0.0658 | -0.100 | 0.9824 | -0.428 | -1.388 | -0.843 | -0.056 | -2.228 | 0.1300 | -0.258 | -3.22 | -1.9 | -1.239 | -0.180 | -1.156 | 0.52 | -1. |
| 29 | . 024 | 0.2409 | -2.523 | 3.1059 | $9-1.615$ | -3.488 | 0.79 | 3.1219 | 1.0114 | 0.7071 | 1.7828 | -0.611 | 11.5144 | 1.2350 | -1.485 | -1.229 | 0.4154 | -0.927 | 1.215 | 0.55 | 2.4801 | 1.128 | 0.8027 | -0.340 | 1.74 | 8. 178 | 0.4878 | 2.095 | 2.305 | 4.52 |
| 30 | 0.747 | -1.333 | -0.492 | $2-0.232$ | 21.2308 | -1.477 | -0.4 | 0.9698 | 0.9537 | -0.975 | -0.712 | 2.0243 | 30.2842 | -3.271 | 4.7817 | 1.3937 | -1.114 | NA | 0.9934 | 0.2517 | -1.083 | 1.7741 | -4.522 | -2.342 | 0.203 | 0.6331 | 0.8029 | -0.568 | -0.604 | 0.6189 |
| 31 | -1.17 | -1.189 | -0.957 | -1.797 | 7-1.092 | 3.4993 | 5.4228 | 0.7385 | 0.714 | 4.0053 | 1.8638 | 4.2637 | 11.9225 | 1.3043 | 1.7213 | 1.5419 | 0.1476 | -0.718 | 0.5499 | 0.8842 | 1.1625 | 1.2488 | 1.8518 | -2.036 | 0.803 | 2.0737 | -0.392 | -1.640 | -0.022 | 4.2829 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

APPENDIX 7.5

5 TABLES OF DIAGNOSTICS FOR THE SELECTED FUNCTIONAL FORMS

5 GOODS






## CHAPTER 8

## CONCLUSIONS

## 8.0). Introduction.

This chapter is a brief summary and conclusion of the thesis. In the first section, we review the contributions to seaborne trade theory by this work. In the second section, we summarize the empirical results from the estimation of the bilateral seaborne trade flow model. The third section suggests fruitful areas of research on this topic.

## 8.1). Theory.

In order to model the demand for specialized ships over the sea-lanes of the world, we developed a theory of allocation of total seaborne trade, for different cargo types over a number of sources trade partners. Based on this theory, we estimated econometric models of seaborne trade and quantified responses of demand to changes in prices and tastes.

Since there is no experience in model building of bilateral seaborne trade flows, and since the demand for sea transport is a derived demand which stems from the demand for commodities across borders, we borrowed the principles of allocation of a fixed total over a number of sources, from the theories of consumer behaviour and international trade. The work presented here is novel to seaborne trade.

A major problem in this 'new' approach to detailed seaborne trade, is that at this high level of disaggregation over goods and trade partners, there are not enough degrees of freedom to estimate the models. The problem is solved by the assumption of multi-stage
budgeting, which entails the assumptions of separability of preferences and consistent aggregation. Consistent aggregation and separability are enforced by selecting an objective function of a special form.

In particular, the selected CRESH function, by being linear homogeneous, satisfies the assumptions needed for multi-stage budgeting. This function has the rare feature of allowing the estimation of distinct price effects between trade partners, in world import markets. However, the number of estimated coefficients in the systems increase only linearly with the number of trade partners. This has permitted the empirical implementation of our models. Furtheremore, the CRESH function contains most of the known work in trade models as special cases.

We have extended and generalized the known mathematical properties of the CRESH function, in order to take account of factors other than relative prices and time trends, which may be important in the allocation decisions of importers. Factors, such as relative production capacities and capacity utilizations have been introduced, and systems of demand equations have been derived from neoclassical optimization principles.

In order to allow for a richer structure of importers' preferences, in terms of their intertemporal reactions to price changes and/or other variables that enter the system, we introduced dynamics into the model. Thus, a 'mixed' system of an adaptive expectations and partial adjustment framework has been developed. Restrictions on this, resulted in a series of nested empirical models of seaborne trade. Particular attention is paid into ensuring that these systems are derivable from neoclassical optimization principles, and that the theoretical properties of demand equations are still preserved.

The theoretical dynamic models were turned into empirical econometric models, in order to quantify bilateral seaborne trade flows. Systems of demand equations have been estimated for 150 import markets over the period 1969-1986.

The lack of availability of bilateral import price data has been a major constraint in our models. We used total export prices and world prices of individual goods as proxies for bilateral prices. We were careful in this process to ascertain that the respecified models were compatible with the classical optimization problem of importers. The final data used for estimation were constructed from data on individual units, countries and individual commodities. Problems and implications related to such procedures were examined in the process.

Regarding the estimation methods of these systems, they are identified to fall into the class of complete systems of demand equations, which satisfy functional form constraints. The constraint that bilateral imports should add-up to total imports in each import market, entails restrictions on the coefficient parameters and a singular covariance matrix of the residuals. It is recognized that SURE methods should be employed for estimation, and one of the equations should be deleted in order to permit estimation of the whole system.

Unique features of our estimating systems are, the high disaggregation over goods and trade partners, the absense of a large monopolistic region (a Rest Of the World (ROW) region) in the system, and the estimation of cross price elasticities. These properties have implications for our empirical results.

In trade theory, it is almost axiomatic that trade is so highly price sensitive that a 'law of one price' applies to internationally traded goods. However, empirical studies have consistently failed to support this theory. The own price elasticity estimates in previous
studies take average values which vary between -0.5 and -2.5 , while the averages of the cross price elasticities take values in the range 0 to +0.5. In our results, the own price elasticities take values, on average, in the range -5.83 to -7.05 . The range of values for the cross price elasticities is +0.75 to +1.06 . Also, the average values of the elasticities with respect to changes in world prices range from -2.15 to -6.58 . These elasticity estimates are much higher than any previous estimates. They suggest that competition in international markets is indeed much higher than previously indicated by empirical studies. Our results provide support for the law of one price in world markets.


#### Abstract

Another interesting result is the high degree of variability of individual elasticity estimates. This variability of the elasticities around their average values, as measured by the coefficients of variation for individual cargo groups, range for the own price elasticities between $120 \%$ and $145 \%$ (see chapter 7). For the cross price elasticities these values are $366 \%$ to $446 \%$. This result is also observed by the bimodal nature of the distributions of elasticities (chapter 7).


This is because individual importers seem to have some well established sources of imports where they import the bulk of their imports from, and import the rest from other regions according to the relative prices offered. Thus, in an import region, elasticities are low for the well established regions (relatively low competition), while elasticities are high (price sensitive) for the marginal regions. This reflects the stable pattern of demand for ships over certain sea-lanes, and the 'occasional' demand for transport from the marginal partners.

## 8.3). Recommendations for Future Research.

It would be naive of us to suggest that every possible facet of modeling bilateral seaborne trade flows has been covered, by the work of this thesis. Space and time limitations have imposed a
compromize. However, there are questions left open for further research, and we suggest some here.

Separability of preferences, used in multi-stage budgeting is a maintained hypothesis in the thesis. Such an assumption should be tested for our data. However, this entails finding domestic variables for our categories of goods, for each country (which are then aggregated to give regional aggregates), and test a total allocation model versus the import allocation, which we estimate. Such a task is almost impossible, but interesting to see whether this underlying assumption holds.

The theoretical background for the inclusion of other variables (besides time trends and relative prices), as determinants of bilateral imports has been developed. Thus, we have developed a theoretical structure which enables us to extend the CRESH function to include variables such as relative production capacities, relative capacity utilizations etc, that may influence the allocation decision of the importer. It would be interesting to find how the models perform when such variables are included for estimation.

Another interesting question is also the testing of CRESH versus more simple functional form models, such as the CES, the LES, the Cobb-Douglas and the Leontief systems. In theory, the simpler models are nested versions of CRESH. They are more restrictive since they do not allow for estimation of cross price elasticities. However, if one is not interested in the estimation of bilateral price effects, it might be interesting to test how well these models perform compared to CRESH, for a common set of data.

The high volatility in our data (and in consequence the high variability of the elasticity estimates) is ascribed to the high disaggregation over countries and goods involved. It would be of interest to estimate and compare the results of aggregated (over goods) models with the disaggregated ones. Would the elasticity estimates from the aggregate models be smoother, and provide some
kind of average of total world seaborne trade elasticities?

Finally, the CRESH bilateral seaborne trade model could be used, to derive the unavailable bilateral price indices, which correspond to the trade flows described.

## CHAPTER 8 ADDENDUM

In the introduction to the thesis, we set as our aim to examine and understand the factors that determine the structure and changing patterns of demand for different ship types on each shipping lane. Since demand for these ships is a derived demand for the commodities ships can carry, we are effectively interested in the structure and changing patterns of demand of world seaborne-trade. We find that relative bilateral prices offered by exporting regions, changing tastes over time, and dynamics of adjustment (in terms of adaptive expectations), are the driving forces in explaining the allocation of demand for each type of ship in international import markets.

The sheer scale of the study, and the high level of disaggregation used, means that we have been obliged to make some simplifying assumptions in developing the model.

In order to implement the problem empirically we had to assume separability of preferences. That is, in each region the demand for each type of commodity-cargo is independent from the demand for any other commodity. For example, the demand for dry cargo is independent from the demand of liquid cargo(tankers) and other goods. It is further assumed that, for each type of cargo, the demand over the part of the good produced domestically is independent(distinct) from the demand over the part of the good imported. Finally, it is assumed that the same types of cargo coming from different geographical regions are imperfect substitutes. We implement our models at this last, disaggregated, stage.

In the estimated model it is assumed that the supply function is horizontal, so no supply factors have been incorporated in the system. However, they can in principle be incorporated. A theoretical model which can allow for supply factors has been developed in the third chapter of the thesis. The mathematical properties of CRESH when relative capacity utilizations or relative export capacities are included in it, are fully explained there. Further research on this front could provide interesting empirical results.

Extensions of the model may also be envisaged in order to incorporate the potential substitutability between seaborne trade and other forms of transport. For example, further developments of the road or railway systems in certain regions may alter the seaborne import allocation decisions of importers.

Our analysis has been geared to explaining the past - to estimating elasticities of demand, trends and dynamics in the allocation decision of importers. It would be very desirable if the model could be used to forecast the future. However, two practical problems arise in such an exercise.

The first issue is that when our systems are estimated in levels form, the possibility of negative values for the forecasts of bilateral tonnages of seaborne trade is not ruled out. The problem does not arise in logs or logarithmic first differences, since bilateral volumes are constrained to take positive values only. A possible solution is to use alternative methods of estimation of the levels form, which allow for truncated dependent variables (see for example Maddala, G. S. (1983) 'Limited-dependent and qualitative variables in economics', Econometric Society Monographs 3).

The second, and more serious problem relates to the availability of data. It is unfortunate that in 1989 the UN stopped producing the detailed data on seaborne trade used here. For forecasting purposes, we therefore need to update our database. It may be possible to do this by obtaining UN data for overall trade, as opposed to seaborne trade, and classifying them into the same aggregate groups as in the Maritime transport study above. The SITC correspondence to the MTC classifications, developed in chapter 5 , could be used to guide this task.

Adams, F. G., Eguchi, H. and Meyer-Zu-Schlochtern, F. (1969), 'An econometric analysis of international trade', OECD, Paris.

Adams, F. G., and Marquez, J. (1983), 'The impact of petroleum and commodity prices in a model of the world economy' in Hickman, B., (eds.), 'Global international economic models', Amsterdam, N. Holland, 203-218.

Aitken, N. D. (1973), 'The Effect of the EEC and EFTA on European trade: a temporal cross-section analysis', American Economic Review, 63, 881-892.
d'Alcantara, G. and Italianer, A. (1982), 'European project for a multinational, macrosectoral model', Commission of the European Communities D G XII, Reference MS II, Brussels, (March).

Allen, R. G. D. (1938), 'Mathematical analysis for economists', St. Martin's Press, London. Reprinted by Macmillan, London.

Allen, R. G. D. (1975), 'Index numbers in theory and practice', Mac Millan Press Ltd.

Amano, A., Kurihara, E., and Samuelson, L. (1980), 'Trade linkage sub-model in the EPA world economic model', Economic Bulletin, 19, Economic Research Institute, Economic Planning Agency, Tokyo.

Amano, A., Maruyama, A., and Yoshitomi, M. (eds.) (1982), 'EPA world economic model', Vols. I and II, Economic Research, Institute, Economic Planning Agency, Tokyo.

Armington, P. S. (1969a) 'A theory of demand for products distinguished by place of production', IMF Staff papers, 6, 159-178.

Armington, P. S. (1969b), 'The geographic pattern of trade and the

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effects of price changes', IMF Staff papers, 16, 179-201.
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Armington, P. S. (1973), 'A note on the income-compensated price elasticities of demand used in the multilateral exchange rate model', Appendix to Artus J. R., and Rhomberg R. R., 'A multilateral exchange rate model', IMF Staff papers, 20, 608-611.

Arrow, k. J. (1960-61), 'Additive logarithimic demand functions and the Slutsky relations', Review of Economic Studies, 28, 176-181.

Arrow K. J., Chenery, H. B., Minhas, B., S., and Solow, R., M. (1961), 'Capital-labour substitution and economic efficiency', Review of Economics and Statistics, 63, 225-250.

Artus, J. R., and McGuirk, A. K. (1981), 'A revised version of the multilateral exchange rate model', IMF Staff Papers, 8, 275-309.

Artus, P. (1986), 'An aggregate model of the world economy', in Artus, P., and Guvenen, O., (eds.), 'International macroeconomic modelling for policy decisions', Martinus Nijhoff Publishers, AD Dordrecht: Netherlands, 105-127.

Artus, P., and Guvenen, O., (in collaboration with Cagey) (1986), 'International macroeconomic modelling for policy decisions', Martinus Nijhoff Publishers, Dordrecht: Netherlands.

Bacharach, M. (1965), 'Estimating non-negative matrices from marginal data', International Economic Review, 6, 294-310.

Baker, P., Blundell, R., Micklewright, J. (1989), 'Modelling household energy expenditures using micro-data', Economic Journal, 99, 720-738.

Barten, A. P. (1967), 'Evidence on the Slutsky conditions for demand equations', Review of Economics and Statistics, 49, 77-84.

Barten, A. P. (1970), 'Reflexions sur la construction d'un systeme empirique des fonctions de demande', Cahiers du Seminaire

Barten, A. P. (1971), 'An import allocation model for the Common Market', Cahiers Economiques de Bruxelles, 50, 153-164.

Barten, A. P., and d'Alcantara, G. (1977), 'Models of bilateral trade flows' in Albach, H., Helmstadter, E., and Henn, R., (eds.) Quantitative Wirtschaftsforschung. Wilhelm Krelle Zum Co. Geburtstag, (J.C.B. Mohr, Tubingen), 43-57.

Barten, A. P., and Turnovsky, S. (1966), 'Some aspects of the aggregation problem for composite demand equations', International Economic Review, 7, 231-259.

Batchelor, R. A., and Bowe, C. (1974), 'Forecasting UK international trade: a general equilibrium approach' Applied Economics, 6, 109-141.

Beckerman, W. (1956), 'The world trade multiplier and the stability of world trade, 1938 to 1953', Econometrica, 24, 239-252.

Beenstock, M., and Dicks, G. (1983) 'An aggregate monetary model of the world economy', European Economic Review, 21, 261-285.

Bergson, A. P. (1936), 'Real income, expenditure proportionality, and Frisch's 'new method' ', Review of Economic Studies, 4, 33-52.

Berndt, E. R., and Christensen, L. R. (1973), 'The internal structure of functional relationships: separability, substitution and aggregation', Review of Economic Studies, 40, 403-410.

Blackorby, C., Primont, D., and Russel, R. R. (1978), 'Duality, separability and functional structure: theory and economic applications', North-Holland Publishing Company, New York.

Bortkiewicz, L. Von (1923/1924), 'Zweck and Struktur einer preisindexzahl', Nordisk Statistik Tidskrift.

Branson, W. H. (1972), 'The trade effects of the 1971 currency realingments', Brookings Papers on Economic Activity, 1, 15-69.

Brown, A. J. (1951), 'The fundamental elasticities in international trade', in Wilson, T., and Andrews, P. (eds.), 'Oxford studies in the price mechanism', Oxford University Press.

Bryant, R. C., Henderson, D. W., Holtham, G., Hooper, P., and Symansky, S. A. (1988), 'Empirical macroeconomics for interdependent economics', The Brookings Institution, Washington, D. C.

Cagan, P. (1956) 'The monetary dynamics of hyperinflation' in Friedman, M. (ed.) 'Studies in the quantity theory of money', University of Chicago Press.

Canzoneri, M. B., and Miford, P. (1988), 'When international policy co-ordination matters: an empirical analysis', Applied Economics, (Sept). 20(9), 1137-1154.

Carrin, G. J., Gunning J. W., and Waelboeck, J. (1980), `A general equilibrium model for the world economy: some preliminary results', Paper presented at the 8 th IIASA Global Modelling Conference, (July) 22-25, Laxenburg, Austria.

Cheng, H. S. (1959), 'A collection of statistical estimates of elasticities and propensities in international trade', Staff Papers, 7, 107.

Christensen, L. R., Jorgenson D. W., and Lau, L. J. (1973), 'Transcedental logarithmic production frontiers', Review of Economics and Statistics, 55, 28-45.

Costa, A. M. (1983), 'DYNAMICO: A multilevel programming model of world trade and development', in Hickman, B. (eds.): 'Global international economic models', Amsterdam: N. Holland, 259-279.

Courbis, R. (eds.) (1981a), 'International trade and multicountry models, 'Models et macroeconomic appliquee' Serie "Travaux du
G.A.M.A. ", No. 3, Paris, Economica.

Courbis, R. (1981b), 'Introduction', in COURBIS, R. (ed.), 'International trade and multicountry models', Economica, Paris, 21-38.

Courbis, R. (1981c), 'La construction de modeles multinationau: problems methodoloques', in COUBIS, R. (ed.), 'International trade and multicountry models', Economica, Paris, 243-247.

Customs Co-operation Council (1955), 'Brussels tariff nomenclature of the customs co-operation council: nomenclature for the classification of goods in customs tariffs', Brussels.

Customs Co-operation Council (1976), 'Nomenclature for the classification of goods in customs tariffs', 5 th edition, Brussels.

Customs Co-operation Council (1985), 'The harmonized commodity description and coding system', Brussels.

Deaton, A. S., and Muellbauer, J. (1980), 'Economics and consumer behavior', Cambridge University Press.

Diewert, W. E. (1971), 'An application of the Shephard Duality Theorem: a generalized Leontief production function', Journal of Political Economy, 79, 481-507.

Diewert, W. E. (1972), 'International consumer theory and the demand for durables', Working Paper No 14, IMSSS, Stanford University, August.

Diewert, W. E. (1973), 'Funstional forms for profit and transformation functions', Journal of Economic Theory, 6, 284-316.

Diewert, W. E. (1974), 'Application of duality theory', in Intriligator, M. D., and Kendrick, D. A. (eds.), 'Frontiers of quantitative economics', Vol II, Amsterdam: N. Holland, 106-171.

Diewert, W. E. (1976), 'Exact and superlative index numbers', Journal of Econometrics, 4, 115-146.

Diewert, W. E. (1982) 'Duality approaches to microeconomic theory', in Arrow, K. J. and M. D. Intriligator (eds.) 'Handbook of mathematical economics', vol. II, North-Holland Publishing Company, Amsterdam, 535-599.

Dramais, A. (1981), 'The DESMOS model', in Courbis, R., (eds.), 'International trade and multicountry models', 221-234.

Edison, H. J., Marquez, J. R., and Tryon, R. W. (1987), 'The structure and properties of the Federal Reserve Board multicountry model', Economic Modelling, (Apr.) 4, No. 2.

Fair, R. C. (1987), 'Properties of a multicountry econometric model', Journal of Policy Modelling, 9(1), 83-123.

Filatov, V., Hickman, B. and Klein, L. (1983), 'Long-run simulations with the project LINK system, 1978-1985', in Hickman (eds.) 'Global international economic models', North-Holland, Amsterdam.

Fleming, J. M., and Tsiang, S. C. (1956/57), 'Changes in competitive strength and export shares of major industrial countries', IMF staff papers, 5, 218-248.

Gallant, A. R. (1975), 'Seemingly unrelated nonlinear regressions', Journal of Econometrics, 3, 35-50.

Gana, J. L., Hickman, B. G., Lau, K. J., Jacobson, L. K. (1979), 'Alternative approaches to linkage of national econometric models', in Sawyer, J. A. (ed.), 'Modelling the international transmission mechanism', North-Holland Publishing Company, Amsterdam, 9-43.

Geraci, V., and Prewo, W. (1982), 'An empirical demand and supply model of multilateral trade', Review of Economics and Statistics, 64, 432-441.

Goldman, S. M. and $H$. Uzawa (1964), 'A note on separability in demand analysis', Econometrica, 32, 387-398.

Gorman, W. M. (1959), 'Separable utility and aggregation', Econometrica, 27, 469-481.

Guillaume, Y. (1981), 'MARCO II un modele structure-conjoncture de l'economie mondiale', in Courbis, R. (ed.), 'International trade and multicountry models', Economica, Paris.

Hadley, G. (1964), 'Nonlinear and dynamic programming', Addison-Wesley, Reading Massachusetts.

Halttunen, H. and D. Warner (1979a), 'A model of trade and exchange rate projections', N.B.E.R. Working paper No. 389, National Bureau of Economic Research, Cambridge, Massachusetts.

Halttunen, H. and D. Warner (1979b), 'A model of trade and exchange rate projections: equations and parameters', N.B.E.R. Working paper No. 390, National Bureau of Economic Research, Cambridge, Massachusetts.

Hanoch, G. (1971), 'CRESH production functions', Econometrica, 39, 696-712.

Hanoch, G. (1975), 'Production and demand models with direct and indirect implicit additivity', Econometrica, 43, 395-419.

Heckscher, E. F. (1919), 'The effects of foreign trade on the distribution of income', Econnomisk Tidskrift, 21: 497-512, Reprinted in Ellis, H. S., and Metzler, L. A. (1949 eds.) 'Readings in the theory of international trade', Blakiston, London, 272-300.

Helliwell, J. F., and Padmore, T. (1984), 'Empirical studies of macroeconomic interdependence', in Kenen, P. (eds.), Handbook of International Economics, (Ch 21), North-Holland.

Hickman, B. G., (1973), 'A general linear model of world trade', Ch

3 in Ball, R. J. (eds.), 'The International linkage of national economic models', Amsterdam, N. Holland ,21-43.

Hickman, B. G., (1983a), 'A cross section of global international economic models', in Hickman B. G. 'Global international economic models', North-Holland, Amsterdam , 3-26.

Hickman, B. G., (eds.) (1983b), 'Global international economic models: selected papers from an IIASA conference' Contributions to Economic Analysis Series, No. 147, Amsterdam; New York and Oxford, North Holland.

Hickman, B. G., and Lau, L. J. (1973), 'Elasticities of substitution and export demand in a world trade model', European Economic Review, 4, 347-380.

Hicks, J. R. (1936), 'Value and capital', Oxford University Press.

Hicks, N. L. (1976a), 'A model of trade and growth for the developing world', European Economic Review, 239-255.

Hicks, N. L. (1976b) 'The SIMLINK model of trade and growth for the developing world', World Bank Staff Working Paper No. 220, (World Bank, Washington D C USA).

Hieronymi, 0. (1983), ' The balance-of-payments model in the interdependence system', paper presented at the 4 th International Workshop of the Applied Econometric Associationn, December 8-9, Brussels.

Horton, G. (1984), 'Modelling the world economy', Treasury Working Paper, No. 33.

Houthakker, H. S., and Magee, S. P. (1969), 'Income and price elasticities in world trade', Review of Economics and Statistics, (May) 51, 111-125.

International Monetary Fund (1990), 'International Financial

Statistics Yearbook', IMF, Washington D.C., USA.

Italianer, A. (1982), 'An evaluation of the bilateral trade flows in the COMET model', Discussion paper D/1982/2020/15, Catholic University of Louvain, Centre for Economic Studies, Louvain.

Italianer, A. (1986), 'Theory and practice of international trade linkage models', Advanced Studies in Theoretical and Applied Econometrics Series, Vol 9 Norwell, Mass., Dordrecht and Lancaster. Martinus, Nijhoff.

Italianer, A., and d'Alcantara, G. (1983), 'Modelling bilateral sectoral trade flows', paper presented at the 4 th International Workshop of the Applied Econometric Association, (Dec) 8-9, Brussels.

Johnson, K. N. (1978), 'Balance of payments equilibrium and equilibrating exchange rates in a world econometirc model', unpublished Ph.D. dissertation, University of Pennsylvannia, Philadelphia, USA.

Jones, R., and Kenen, P. (1984), `Handbook of international economics', Vol 1, Amsterdam, N. Holland.

Judge, G. G., Griffiths, W. E., Hill, R. C., Lutkepohl, H., and Lee, T. C. (1985), 'The theory and practice of econometrics', second edition, J. Wiley \& Sons.

Judge, G. G., Hill, R. C., Griffiths, W. E., Lutkepohl, H., and Lee, T. C. (1988), 'Introduction to the theory and practice of econometrics', second edition, J. Wiley \& Sons.

Kaneko, T., and Yasuhara, N. (1986), 'Exchange rate simulations with the EPA world economic model', European Economic Review, (Feb), 30(1), 237-259.

Kaya, Y., Onishi, A., and Suzuki, Y. (1983), 'Project FUGI and the future of ESCAP developing countries' in Hickman B. (eds.) 'Global
international economic models', Amsterdam, N. Holland, 237-257.

Kirkpatric, G. (1983), 'A multicountry OECD/Comecon simulation model', paper presented at the 4 th International workshop of the Applied Econometric Association (December 8-9), Brussels.

Klein, L. R. and Van Peeterssen, A. (1973), 'Forecasting world trade within project LINK', in Ball, R. J. (eds.) 'The international linkage of national economic models', Amsterdam, N. Holland.

Kooyman, J. (1981), 'The METEOR model', in Courbis, R. (eds.), 'International trade and multicountry models', Economica, Paris, 235-242.

Koyck, L. M. (1954), 'Distributed lags and investment analysis', North-Holland Publishing Company, Amsterdam.

Kravis, I. and Lipsey, R. (1971), 'Price competitiveness in world trade', National Bureau of Economic Research, New York.

Kuznets, S. (1964), 'Quantitative aspects of the economic growth of nations: IX. Level and structure of foreign trade: comparison for recent years', in Economic Development and Cultural Change, Vol. XIII, Part II.

Lafay, G., and Brender, A. (1981), 'Models and forecasting in a crisis period: the case of the MOISE Model', in Courbis, R. (eds.), 'International trade and multicountry models', Economica, Paris, 295-307.

League of Nations (1942), 'The network of world trade', Princeton, $N$. Jersey.

Leamer, E., and Stern, R. (1970), 'Quantitative international economics', Boston, Allyn \& Bacon.

Linneman, H. (1966), 'An econometric study of international trade flows', Amsterdam, N. Holland.

Maddala, G. S. (1977), Econometrics, McGraw Hill, Tokyo.

Magee, S. (1975), 'Prices incomes and foreign trade', in Kenen, P. (eds.), 'International trade and finance: frontiers for research', Cambridge University Press, Cambridge, 175-252.

Magnus, J. R. (1978), 'Maximum likelihood estimation of the GLS model with unknown parameters in the disturbance covariance matrix', Journal of Econometrics, 7, 281-312.

Marwah, K. (1976), 'A world model of international trade: forecasting market shares and trade flows', Empirical Economics, 1, 1-39.

Mennes, L. B. M. (1967), 'A world trade model for 1970', Weltwirtschaftliche Archiv, 99, 225-255.

Metzler, L. A. (1950), 'A multiple region theory of income and trade', Econometrica, 18, 329-354.

Minford, P., Agenor P. R., and Nowell, E. (1986), 'A New classical econometric model of the world economy', Economic Modelling, Vol. 3, No. 3, (July).

Morishima, M., and Murata, Y. (1972), 'An estimation of the international trade multiplier, 1954-1965' in Morishima, M., Murata, Y., Nosse, Y., and Saito, T. (eds.). 'The working of econometric models' Cambridge University Press, 303-329.

National Ports Council (1975), 'UK international trade' Economics Division, National Ports Council, London.

Neisser, H., and Modigliani, F. (1953), 'National incomes and international trade: A quantitative analysis', Urbana, Illinois.

Nytus, D. E. (1978), 'A detailed model of bilateral commodity trade and effects of the exchange rate changes', in Stone, R., and

Peterson W. (eds.), 'Econometric contributions to public policy' Macmillan, London, 130-150.

Nyhus, D. E., and Almon, C. (1983), 'Linked input-output models for France, the Federal Republic of Germany and Belgium' in Hickman, B. (eds.), 'Global international economic models' Amsterdam, $N$ Holland, 183-200.

Ohlin, B. (1933), 'Interregional and international trade', Harvard University Press, Cambridge, Massachusetts.

Olsen, E. (1971), 'International trade theory and regional income differences, United states 1880-1950', North-Holland Publishing Co., Amsterdam.

Orcutt, G. H. (1950), 'Measurement of Price elasticities in international trade', Review of Economics and Statistics, 32, 117-132.

Parkin, M., and Zis, G. (1976), 'Inflation in the world economy', Manchester University Press and University of Toronto Press.

Piganiol, B. (1979), ' Le modele mondeco', CEPRI, Universite de Paris IX - Dauphine.

Pindyck, R. S., and Rubinfeld, D. L. (1981), `Econometric models and economic forecasts', second edition, McGraw Hill.

Polak, J. J. (1953), 'An international economic system', University of Chicago Press, Chicargo.

Polak, J. J., and Rhomberg, R. R. (1962), `Economic instability in an international setting', American Economic Review, 52, 160-218.

Poyhonen, P. (1963), 'A tentative model for the volume of Trade between Countries', Weltwirtschaftliches Archiv, Band XC, 93-99.

Prais, S. J. (1962), 'Econometric research in international trade: A

Pulliainen, K. (1963), 'A world trade study: an econometric model of the pattern of the commodity flows in international trade in 1948-1960', Ekonomiska Samfundets Tidskrift, 2, 78-91.

Ranuzzi, P. (1981), 'The experience of the EEC Eurolink project in modelling bilateral trade linkage equations', Journal of Policy Modelling, 3, 153-173.

Resnick, S. (1968), 'An empirical study of economic policy in the Common Market', in Ando, A., Brown, E. C., and Friedlaender, A. F.(eds.), 'Studies in economic stabilization', The Brookings Institution, Washington, D C, 184-214.

Resnick, S. A., and Truman, E. M. (1975), 'An empirical examination of bilateral trade in Western Europe', in Balassa, B. (eds.), 'European economic integration', North-Holland Publishing Company, Amsterdam, 41-78.

Rhomberg, R. R. (1968), 'Transmission of business fluctuations from developed to developing countries.' International Monetary Fund Staff papers, 15, 1-29.

Rhomberg, R. R. (1970), 'Possible Approaches to a Model of World Trade and Payments', IMF Staff Papers, 17, 1-26.

Rhomber, R. R. (1973), 'Towards a general trade model', in Ball, R. J. (eds.), 'The international linkage of national economic models', N. Holland, 9-20.

Rhomber, R. R., and Boissonneault, L. (1964), 'Effects of income and price changes on the U.S. balance of payments', IMF Staff Papers, March 1974, Vol. XI, 59-124.

Rhomberg, R. R., and Fortucci, P. (1964), 'Projections of US current account balances for 1964 from a world trade model', IMF Staff Papers, 11, 414-433.

Ricardo, D. (1817), 'The principles of political economy and taxation', Dent \& Sons Ltd, London.

Richardson, P. (1988), 'The structure and simulation properties of OECD's INTERLINK model', OECD Economic Studies., Spring 1988 (10), 57-122.

Ripley, D. M. (1981), 'The IMF world trade model: A progress report', in Courbis, R. (eds.), 'International trade and multicountry models', Economica, Paris, 123-136.

Roy, R. (1942), 'De l'utilite, contribution a la theorie des choix', Paris, Hermann.

Sallin-Kornberg, E., and Fonntella. E. (1981), 'Scenarios building with the EXPLOR-MULTITRADE 85 model', in Courbis, R. (eds.), 'International trade and multicountry models', Economica, Paris, 309-325.

Samuelson, L. (1973): 'A new model of world trade', OECD Economic Outlook Occasional Studies, (Dec.) 1973, 3-22.

Samuelson, L., and Kurihara, E. (1980), 'OECD trade linkage methods applied to the EPA world economic model', Economic Bulletin, No. 18, Economic Research Institute, Economic Planning Agency, Tokyo.

Sarma, K. S. (1983), 'Alternative methods of trade linkage', Paper presented to the 4th International Workshop of the Applied Econometric Association, (Dec.) 8-9, Brussels.

Savage, I. R. and K. W. Deutsch (1960), 'A statistical model of the gross analysis of transaction flows', Econometrica, 28, 551-572.

Sawyer, J. A. (1979), 'Modelling the international transmission mechanism', N. Holland Publishing Company, Amsterdam.

Shishido, S. (1983), 'Long term forecast and policy implications:
simulations with a world econometric model', in Hickman, B. (1983) (eds.), 'Global international economic models'. Amsterdam, N. Holland.

Signora, A. (1981), 'Les lecons d'une experience d'application de modeles mondiaux au niveau sectoriel', in COURBIS, R. (eds.), 'Internationnal trade and multicountry models', Economica, paris, 163-170.

Smith, A. (1776), 'The Wealth of Nations', London, Penguin books.

Snella, J. J. (1979), 'An econometric model of international trade', Paper presented at the Seventh International Conference on Input-Output techniques, April 9-13, Innsbruck, Austria,

Stern, R., Francis, J., and Schumacher, B. (1976), 'Price elasticities in international trade: an annotated bibliography', Macmillan, London.

Stone, J. R. N. (1954), 'Linear expenditure systems and demand analysis': an application to the pattern of British demand.' Economic Journal, 64, 511-527.

Stone, J. R. N., and Brown, A. (1964), 'A computable model for economic growth', Chapman and Hall, London.

Strotz, R. H. (1957), 'The empirical implications of a utility tree', Econometrica, 25, 269-280.

Swamy, P. A. V. B. (1970), 'Efficient inference in a random coefficient regression model', Econometrica, 38, 311-323.

Taplin, G. B. (1967), 'Models of world trade', IMF Staff Papers, 14, (Nov.), 433-453.

Taplin, G. B. (1973), 'A Model of world trade', in Ball, R, J. (eds.), 'The international linkage of national economic models', Amsterdam, N. Holland, 177-223.

Theil, H. (1971), 'Principles of econometrics', Wiley \& Sons, New York.

Theil, H. (1975), 'Theory and measurement of consumer demand', 2 Vols., North-Holland Publishing Company, Amsterdam.

Tinbergen, J. (1962), 'Shaping the world economy: suggestions for an international economic policy', The Twentieth Century Fund, New York., 262-93.

Tims, W., and Meyer-zu-Schlochtern, F. J. M. (1962), 'Foreign demand and the development of Dutch exports', Cahiers Economiques de Bruxelles, 15, 389-395.

Tyszinski, H. (1951), 'World trade in manufactured commodities 1889-1950', Manchester School of Economic and Social Studies, 19, 272-304.

United Nations (1951), 'Standard International Trade Classificaton', Statistical Papers, series $M$, No. 10, second edition, United Nations, New York.

United Nations (1961), 'Standard International Trade Classification Revised', Statistical Papers series M, No. 34, United Nations, New York.

United Nations (1968), 'International Standard Industrial Classification of all economic activities', Statistical Papers series M, No. 4, United Nations, New York.

United Nations (1975), 'Standard International Trade Classification Revision $2^{\prime}$, Statistical Papers Series M, No. 34/Rev. 2, United Nations, New York.

United Nations (1980), 'Commodity trade (by sea) statistics: Results of the Maritime transport study for the years 1969-1972', Statistical Papers, Series D, No 2, United Nations, New York.

United Nations (1983), 'International seaborne trade statistics yearbook (Maritime Transport)' Statistical Papers, Series D, No 2, United Nations, New York.

United Nations (1986), 'Standard International Trade Classification Revision 3', Statistical Papers Series M, No. 34/Rev. 3, United Nations, New York.

United Nations (1987), 'Methods used in compiling the UN price indices for external trade', Statistical Papers Series M, No. 82, Vol I, United Nations, New York.

United Nations (1988), 'Final draft of the Central Product Classification (CPC)', Statistical Papers series M, No. 77, United Nations, New York.

United Nations (1989), 'Yearbook of international trade statistics', Statistical Papers, United Nations, New York.

United Nations (1990), 'Monthly bulletin of statistics', United Nations, New York.

Uzawa, H. (1962), `Production functions with constant elasticity of substitution', Review of Economic Studies, 29 (1962), 291-299.

Varian, H. R. (1984), 'Microeconomic analysis', second edition, W. W. Norton \& Company.

Viaene, J. M. (1983), 'Product availability, price discrimination and interdependent import flows', Discussion Paper 8317/G, Institute for Economic Research, Erasmus University, Rotterdam.

Vos, A. F. de, and J. A. Bikker (1982), `Interdependent multiplication model for allocation and aggregates: a generalization of gravity models', Research Paper No. 80, Interfaculty of Actuarial Sciences and Econometrics, Free University, Amsterdam.

Waelbroeck, J. (1962), 'La demande exterieure et l'evolution des exportations Belges', Cahiers Economiques de Bruxelles, 15, 397-412.

Waelbroeck, J (1967), 'On the structure of international trade interdependence', Cahiers Economiques de Bruxelles, 36, 495-511.

Waelbroeck, S. (1973), 'The methodology of linkage', in Ball, R. J. (eds.) 'The international linkage of national economic models', North Holland, Amsterdam, 45-61.

Waelbroeck, J., and Ginsburgh, J. (1981), 'Some calculations of the impact tariffs on world trade and welfare with a general equilibrium model the world economy', in Courbis, R. (eds.) 'International trade and multicountry models', Economica, Paris, 279-294.

Woolley, H. B. (1965), 'Measuring transactions between world areas', Studies in International Economic Relations, No 3, National Bureau of Economic Research, New York.

Wren-Lewis S, (1987), 'On the development and exploitation of a world econometric model', National Institute of Economic and Social Research, (Nov.).

Zellner, A. (1962), 'An efficient method of estimating Seemingly Unrelated Regressions and tests of aggregation bias', Journal of the American Statistical Association, 57, 348-368.


[^0]:    The functional form - data - constraint makes these cross equation correlations more specific. It also results in a singular covariance matrix of the error terms, rendering estimation impossible. However, by reformulating the original problem as a restricted GLS problem we are able to obtain the required parameters from a system of ( $n-1$ ) equations using the usual estimation procedures. Assuming normality, the estimators obtained are equivalent to the MLE. Tests of restrictions on the parameters of the system of $n$ equations are presented. These enable us to provide test procedures for a number of theoretical considerations put forward in the thesis.

