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Colour valued Scattering Matrices

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Abstract

We describe a general construction principle which allows to add colour values to a coupling constant dependent scattering matrix. As a concrete realization of this mechanism we provide a new type of S-matrix which generalizes the one of affine Toda field theory, being related to a pair of Lie algebras. A characteristic feature of this S-matrix is that in general it violates parity invariance. For particular choices of the two Lie algebras involved this scattering matrix coincides with the one related to the scaling models described by the minimal affine Toda S-matrices and for other choices with the one of the Homogeneous sine-Gordon models with vanishing resonance parameters. We carry out the thermodynamic Bethe ansatz and identify the corresponding ultraviolet effective central charges.

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1 Introduction

The bootstrap principle [1] has turned out to be a successful method to compute scattering matrices in 1+1-dimensions. Solving the set of bootstrap equations and giving a consistent explanation to the singularity structure in the complex rapidity plane, the scattering matrices are determined uniquely up to the so-called CDD-factors [2]. The latter factors are constituted in such a way that they solve the bootstrap equations but do not introduce additional poles in the physical sheet. Therefore they are neglected in most situations. However, they may also by utilized in order to include coupling constants into the scattering matrices, as for instance in [3]. We will show in the following that the CDD-factors can also be employed consistently to add colour values to the scattering matrices. In the context of the Homogeneous sine-Gordon models Fernández-Pousa and Miramontes [4] proposed a new type of S-matrix which violates parity invariance and describes the scattering of particles which carry two quantum numbers. The main quantum number governs the fusing structure while for certain values of the colour quantum numbers the particles interact solely via a CDD-factor, which could be trivial in some cases. We will provide a systematic construction principle for colour valued scattering matrices and give explicit realizations which include the ones of [4] as a particular case. These type of theories are related to two different Lie algebras $\mathbf{g}$ and $\tilde{\mathbf{g}}$, where the former relates to the main and the latter to the colour quantum number. We refer to these theories by $\mathbf{g}|\tilde{\mathbf{g}}$.

Our manuscript is organized as follows: In section 2 we describe the general construction principle which attaches colour values to an S-matrix and provide a concrete realization of this. In section 3 we carry out a TBA-analysis in order to identify the corresponding ultraviolet effective central charges. We provide an explicit example in section 4. In section 5 we state our conclusions.

2 Construction Principle

We recall that the two-particle S-matrix which describes the scattering between particles of type a and b as a function of the rapidity difference θ is often* of the general form $S_{ab}(\theta) = S_{ab}^{\min}(\theta) S_{ab}^{\mathrm{CDD}}(\theta, B)$. Here $S_{ab}^{\min}(\theta)$ is the so-called minimal S-matrix, related for instance to scaling theories of statistical models [7], which satisfies the unitarity, crossing and fusing bootstrap equations

$$S_{ab}(\theta) S_{ba}(-\theta) = 1, \qquad S_{\bar{a}b}(\theta) = S_{ba}(i\pi - \theta), \qquad \prod_{l=a,b,c} S_{dl}(\theta + \eta_l) = 1 \ , \tag{1}$$

with η_l being the fusing angles. The CDD-factor $S_{ab}^{\mathrm{CDD}}(\theta, B)$ depends on the effective coupling constant and is chosen in such a way that it also satisfies these equations

*This is not necessarily the case as for instance for affine Toda field theories related to non-simply laced Lie algebras, which was first worked out in [5]. See also [6] and references therein for a recent treatment of these type of theories.

without introducing additional poles in the physical sheet, i.e. $0 \leq \mathrm{Im}\,\theta \leq \pi$. We may now modify the usual expression to

$$\hat{S}_{ab}^{ij}(\theta) = S_{ab}^{\min}(\theta) S_{ab}^{\mathrm{CDD}}(\theta, B_{ij}) \;. \tag{2}$$

Here we have introduced an additional dependence of the effective coupling constant on the quantum numbers i and j, which we refer to as colours. It is clear by construction that $\hat{S}_{ab}^{ij}(\theta)$ also satisfies the crossing, unitarity and fusing bootstrap equations, but now each particle carries two quantum numbers (a, i), which may take their values in different ranges, for definiteness say $1 \leq a \leq \ell$ and $1 \leq i \leq \tilde{\ell}$. This means, now we have in total $\tilde{\ell} \times \ell$ different particle types. Alternatively, we can define an S-matrix which coincides with one or the other factor in (2) for certain colour values

$$S_{ab}^{ij}(\theta) = \begin{cases} S_{ab}^{\min}(\theta) = (S_{ab}^{CDD}(\theta, B_{ii} = 0))^{-1} & \text{for } i = j \\ S_{ab}^{CDD}(\theta, B_{ij}) & \text{for } i \neq j \end{cases} \;. \tag{3}$$

This means whenever $i = j$ we simply have $\tilde{\ell}$ copies of theories which interact via a minimal scattering matrix and for $i \neq j$ the particles interact purely via a CDD-factor. Clearly by construction also (3) satisfies the consistency equations (1). It should be noted here that (2) and (3) still describe scattering processes for which backscattering is absent. Hence, these type of colour values play a different role as those which occur for instance in S-matrices related to affine Toda field theories [9] with purely imaginary coupling constant, e.g. [10]. Despite the fact that the relative mass spectra related to (3) are degenerate, this is consistent when we encounter $\tilde{\ell}$ different overall mass scales or the particles have different charges.

We will now generalize the structure just outlined, which we already encountered in [8], and provide a concrete realization for $S_{ab}^{ij}(\theta)$, which is of affine Toda field theory type, involving a pair of simply laced Lie algebras. It is clear, however, from our previous comments that the forms (2) and (3) are of a more general nature. We associate the main quantum numbers a, b to the vertices of the Dynkin diagram of a simply laced Lie algebra $\mathbf{g}$ of rank ℓ and the colour quantum numbers i, j to the vertices of the Dynkin diagram of a simply laced Lie algebra $\tilde{\mathbf{g}}$ of rank $\tilde{\ell}$ and refer from now on to these theories as $\mathbf{g}|\tilde{\mathbf{g}}$[†].

We define now the general building blocks

$$[x, B]_{\theta,ij} = e^{\frac{i\pi x \varepsilon_{ij}}{h}} \left(\frac{\sinh \frac{1}{2}(\theta + i\pi(x - 1 + B)/h) \sinh \frac{1}{2}(\theta + i\pi(x + 1 - B)/h)}{\sinh \frac{1}{2}(\theta - i\pi(x - 1 + B)/h) \sinh \frac{1}{2}(\theta - i\pi(x + 1 - B)/h)} \right)^{\frac{1}{2}} \;. \tag{4}$$

with ε_{ij} being the anti-symmetric tensor, i.e. $\varepsilon_{ij} = -\varepsilon_{ji}$. It is this property of the ε's which is responsible for the parity breaking of the S-matrix. This block has the obvious properties

$$[x, B]_{\theta,ij} \, [x, B]_{-\theta,ji} = 1 \qquad \text{and} \qquad [h - x, B = 1]_{\theta,ij} = [x, B = 1]_{i\pi-\theta,ji} \;. \tag{5}$$

[†]This should of course not be understood as a coset.

We understand here in a slightly loose notation that in the second equality we first take the square root and thereafter perform the shifts in the arguments. Note further that the order of the colour values is relevant. From (4) we construct the $\mathbf{g}|\tilde{\mathbf{g}}$–scattering matrix

$$S_{ab}^{ij}(\theta) = \prod_{q=1}^{h} \left[2q - (c_a + c_b)/2), \tilde{I}_{ij} \right]_{\theta,ij}^{-\frac{\tilde{K}_{ij}}{2} \lambda_a \cdot \sigma^q \gamma_b} \quad . \tag{6}$$

This is of the form (3) apart from a phase factor and a square root taken when $i \neq j$. Here the λ_a's are fundamental weights, the γ_a's are simple roots times a colour value $c_a = \pm 1$, h is the Coxeter number and σ is the Coxeter element related to the Lie algebra $\mathbf{g}$. $\tilde{K}$ is the Cartan matrix and $\tilde{I} = 2 - \tilde{K}$ the incidence matrix of the $\tilde{\mathbf{g}}$ related Dynkin diagram. For more details on the notation and properties of the quantities involved see [11, 6]. For $i = j$ we recover with $\tilde{I}_{ii} = 0$ and $\tilde{K}_{ii} = 2$ the known form of the minimal scattering matrix of affine Toda field theory. Whenever i and j are not linked on the $\tilde{\mathbf{g}}$–Dynkin diagram S becomes 1, i.e. the particles interact freely. Instead when i and j are linked on the $\tilde{\mathbf{g}}$–Dynkin diagram, we have $\tilde{I}_{ij} = 1$ and $\tilde{K}_{ij} = -1$ such that the corresponding blocks are inverted. Comparing (4) with the conventional minimal blocks, we have introduced the parity breaking phase factor and also taken the square root to minimize the powers of the poles since in $[x, B = 1]_{\theta,ij}$ the two factors in the denominator and as well as in the numerator coincide. Hence for $i \neq j$ the expression (6) corresponds to the square root of the usual affine Toda field theory CDD-factor for $B = 1$. It is this operation of taking the square root which is the reason for the occurrence of the phase factor in (4), since only with its presence the consistency equations are satisfied.

There is no need to introduce the phase to satisfy the unitarity equation in (1), since the first property in (5) is satisfied with or without it. However, already in order to satisfy the crossing relation the introduction of the phase factor is crucial since the second property in (5), which is needed to establish it, only holds when it is included. Assuming the validity of the ADE-fusing rules one may verify by the usual shifting arguments, e.g. [11, 6], that the fusing bootstrap equations are satisfied. It is further clear that (6) is hermitian analytic [12].

For many applications, like the thermodynamic Bethe ansatz or form factors, it is most convenient to employ the scattering matrix in form of an integral representation instead of the blockform (4). In [13, 6] it was demonstrated how to derive one formulation from the other and by specifying the analysis in there to the present situation it follows immediately that we can express the scattering matrix (3) alternatively as

$$S_{ab}^{ij}(\theta) = e^{i\pi\varepsilon_{ij} K_{ab}^{-1}} \exp \int_{-\infty}^{\infty} \frac{dt}{t} \left(2\cosh\frac{\pi t}{h} - \tilde{I} \right)_{ij} \left(2\cosh\frac{\pi t}{h} - I \right)_{ab}^{-1} e^{-it\theta} \quad . \tag{7}$$

The pre-factor results from a similar computation as may be found in section 4.2.1. of [13].

We note that when we choose $\tilde{\mathbf{g}}$ to be A_1 the colour values become identical for all particles and the system reduces to the one described by $S_{ab}^{\min}(\theta)$. This is the only example for which (6), (7) does not violate the parity invariance. Choosing instead $\mathbf{g}$ to be A_n we recover the S-matrix of the Homogeneous sine-Gordon models for vanishing resonance parameter at level $(n+1)$ [4, 8].

Similar as in the case for which the universal scattering matrix (6) coincides with models already known, also all S-matrix elements which belong to the new theories are well-behaved meromorphic functions. At first sight the power 1/2 in the definition of the building block (4) seems to suggest the presence of square root branch cuts. For the $\mathbf{g}|A_1$–model the 1/2 is familiar for instance from [11] where it is kept as a power in relation (6). A detailed analysis which explains how the building blocks combine to meromorphic functions may be found in there. For the case $B = \tilde{I}_{ij} = 1$ the square root can be taken directly in (4) and the remaining power 1/2 in (6) is once again compensated by the same mechanism as in [11].

It is straightforward to include also resonance parameters into the scattering matrix (6), (7) which could in principle be colour value dependent and may also break the parity invariance [4, 8].

3 TBA Analysis for the $\mathbf{g}|\tilde{\mathbf{g}}$ S-matrix

According to the standard arguments of the thermodynamic Bethe ansatz [14] the TBA-equations for a system which interacts dynamically via the scattering matrix (7) and statistically via Fermi statistics read

$$rm_a^i \cosh\theta = \varepsilon_a^i(\theta) + \sum_{b=1}^{\ell}\sum_{j=1}^{\tilde{\ell}} \int_{-\infty}^{\infty} d\theta' \varphi_{ab}^{ij}(\theta-\theta') \ln\left(1+e^{-\varepsilon_b^j(\theta')}\right) . \tag{8}$$

Here r is the inverse temperature and m_a^i the mass of particle (a,i). The pseudoenergies are denoted as usual by $\varepsilon_a^i(\theta)$ and the kernels are obtained from (7)

$$\varphi_{ab}^{ij}(\theta) = -i\frac{d}{d\theta}\ln S_{ab}^{ij}(\theta) = \int_{-\infty}^{\infty} dt \left[\delta_{ab}\delta_{ij} - \left(2\cosh\frac{\pi t}{h} - \tilde{I}\right)_{ij}\left(2\cosh\frac{\pi t}{h} - I\right)_{ab}^{-1}\right] e^{-it\theta} . \tag{9}$$

One of the most direct informations the thermodynamic Bethe ansatz provides is the effective central charge $c_{\text{eff}} = c - 24h_0$ of the underlying ultraviolet conformal field theory, with c being the Virasoro central charge and h_0 the smallest conformal dimension of the theory. Then, provided that the solutions of the TBA-equation develop the usual "plateau behaviour"‡, e.g. [14], one may approximate $\varepsilon_a^i(\theta) = \varepsilon_a^i = const$ in a large region for θ when r is small. By standard TBA arguments [14] follows that the effective central charge is expressible as

$$c_{\text{eff}} = \frac{6}{\pi^2}\sum_{a=1}^{\ell}\sum_{i=1}^{\tilde{\ell}} \mathcal{L}\left(\frac{x_a^i}{1+x_a^i}\right) \tag{10}$$

‡This is not always the case as for instance in affine Toda field theories with generic effective coupling constant [13].

with $\mathcal{L}(x) = \sum_{n=1}^{\infty} x^n/n^2 + \ln x \ln(1-x)/2$ denoting Rogers dilogarithm [15] where the $x_a^i = \exp(-\varepsilon_a^i)$ are obtained as solutions from the constant TBA-equations in the form

$$x_a^i = \prod_{b=1}^{\ell} \prod_{j=1}^{\tilde{\ell}} (1 + x_b^j)^{N_{ab}^{ij}} \ . \tag{11}$$

The matrix N_{ab}^{ij} is defined via the asymptotic behaviour of the scattering matrix which for the case at hand may be read off directly from (9)

$$N_{ab}^{ij} = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\theta \ \varphi_{ab}^{ij}(\theta) = \ \delta_{ab}\delta_{ij} - K_{ab}^{-1} \tilde{K}_{ij} \ . \tag{12}$$

In regard to finding explicit solutions for the set of coupled equations (11), it turns out to be convenient to introduce new variables because they may be related to Weyl characters of the Lie algebra $\mathbf{g}$ or $\tilde{\mathbf{g}}$. Following [16, 17] we define

$$Q_a^i = \prod_{b=1}^{\ell} (1 + x_b^i)^{K_{ab}^{-1}} \qquad \Leftrightarrow \qquad x_a^i = \prod_{b=1}^{\ell} \left(Q_b^i\right)^{K_{ab}} - 1 \tag{13}$$

such that the constant TBA-equations (11) acquire the more symmetric form

$$\prod_{b=1}^{\ell} \left(Q_b^i\right)^{I_{ab}} + \prod_{j=1}^{\tilde{\ell}} \left(Q_a^j\right)^{\tilde{I}_{ij}} = \left(Q_a^i\right)^2 \ . \tag{14}$$

The effective central charge (10) is then expressible in various equivalent ways

$$c_{\mathrm{eff}}^{\mathbf{g}|\tilde{\mathbf{g}}} = \frac{6}{\pi^2} \sum_{a=1}^{\ell} \sum_{i=1}^{\tilde{\ell}} \mathcal{L}\left(1 - \prod_{b=1}^{\ell} \left(Q_b^i\right)^{-K_{ab}}\right) = \ell\tilde{\ell} - \frac{6}{\pi^2} \sum_{a=1}^{\ell} \sum_{i=1}^{\tilde{\ell}} \mathcal{L}\left(1 - \prod_{j=1}^{\tilde{\ell}} \left(Q_a^j\right)^{-\tilde{K}_{ij}}\right) \tag{15}$$

$$= \ell\tilde{\ell} - \frac{6}{\pi^2} \sum_{a=1}^{\ell} \sum_{i=1}^{\tilde{\ell}} \mathcal{L}\left(\prod_{b=1}^{\ell} \left(Q_b^i\right)^{-K_{ab}}\right) = \frac{6}{\pi^2} \sum_{a=1}^{\ell} \sum_{i=1}^{\tilde{\ell}} \mathcal{L}\left(\prod_{j=1}^{\tilde{\ell}} \left(Q_a^j\right)^{-\tilde{K}_{ij}}\right) \tag{16}$$

where we used the well-known identity $\mathcal{L}(x) + \mathcal{L}(1-x) = \pi^2/6$, see e.g. [15]. It is also clear that having solved the equations (14) for the case $\mathbf{g}|\tilde{\mathbf{g}}$ we have immediately a solution for the case $\tilde{\mathbf{g}}|\mathbf{g}$ simply by interchanging the role of the two algebras. Supposing now that $c_{\mathrm{eff}}^{\mathbf{g}|\tilde{\mathbf{g}}} = \mu \, c_{\mathrm{eff}}^{\tilde{\mathbf{g}}|\mathbf{g}}$ for some unknown constant μ, it follows directly from (15) and (16) that $c_{\mathrm{eff}}^{\mathbf{g}|\tilde{\mathbf{g}}} = \mu\ell\tilde{\ell}/(1+\mu)$. We conjecture now this constant to be $\mu = \tilde{h}/h$ such that

$$c_{\mathrm{eff}}^{\mathbf{g}|\tilde{\mathbf{g}}} = \frac{\ell\tilde{\ell}\,\tilde{h}}{h + \tilde{h}} \ . \tag{17}$$

As expected from the observations concerning the scattering matrix we recover several known cases when we fix some of the algebras. For instance we obtain $c_{\mathrm{eff}}^{\mathbf{g}|A_1} = 2\ell/(h+2)$ which is the well known formula for the effective central charge of the minimal affine Toda theories. Furthermore we recover the effective central charge for the Homogeneous sine-Gordon models $c_{\rm eff}^{A_n|\tilde{\mathbf{g}}} = n\tilde{\ell}\tilde{h}/(n+1+\tilde{h})$ [8]. It should be noted that this is independent of whether a resonance parameter is present or not despite the fact that the TBA-equation are not parity invariant in that case, see [8]. Numerically we also solved (14) explicitly for numerous examples with $\mathbf{g}\neq A_n$ and confirmed (17).

$\mathbf{g}\vert\tilde{\mathbf{g}}$	A_m	D_m	E_6	E_7	E_8
A_n	$\frac{nm\,(m+1)}{n+m+2}$	$\frac{nm\,(2m-2)}{n+2m-1}$	$\frac{72\,n}{n+13}$	$\frac{126\,n}{n+19}$	$\frac{240\,n}{n+31}$
D_n	$\frac{nm\,(m+1)}{2n+m-1}$	$\frac{nm\,(m-1)}{n+m-2}$	$\frac{36\,n}{n+5}$	$\frac{63\,n}{n+8}$	$\frac{120\,n}{n+14}$
E_6	$\frac{6\,m\,(m+1)}{m+13}$	$\frac{6\,m\,(m-1)}{m+5}$	18	$\frac{126}{5}$	$\frac{240}{7}$
E_7	$\frac{7\,m\,(m+1)}{m+19}$	$\frac{7\,m\,(m-1)}{m+8}$	$\frac{84}{5}$	$\frac{49}{2}$	35
E_8	$\frac{8\,m\,(m+1)}{m+31}$	$\frac{8\,m\,(m-1)}{m+14}$	$\frac{96}{7}$	21	32

Table1: Effective central charges $c_{\rm eff}^{\mathbf{g}|\tilde{\mathbf{g}}}$ of the $\mathbf{g}|\tilde{\mathbf{g}}$-theories.

4 An Explicit example: $D_4|D_4$

In order to illustrate the working of our general formulae it is instructive to evaluate them for a concrete model. We chose the $D_4|D_4$-model which is an example for a therory hitherto unknown. The model contains 16 different particles labeled by (a,i) with $1 \leq a,i \leq 4$. The Coxeter number is 6 for D_4. Naming the central particle in the D_4–Dynkin diagram by 2 the S-matrix elements according to (6) are computed to

$$
\begin{array}{ll}
S_{11}^{ii}(\theta) = S_{33}^{ii}(\theta) = S_{44}^{ii}(\theta) = [1,0]_{\theta,ii}^{2}[5,0]_{\theta,ii}^{2} & \text{for } i=1,2,3,4, \\
S_{12}^{ii}(\theta) = S_{23}^{ii}(\theta) = S_{24}^{ii}(\theta) = [2,0]_{\theta,ii}^{2}[4,0]_{\theta,ii}^{2} & \text{for } i=1,2,3,4, \\
S_{13}^{ii}(\theta) = S_{14}^{ii}(\theta) = S_{34}^{ii}(\theta) = [3,0]_{\theta,ii}^{2} & \text{for } i=1,2,3,4, \\
S_{22}^{ii}(\theta) = [1,0]_{\theta,ii}^{2}[3,0]_{\theta,ii}^{4}[5,0]_{\theta,ii}^{2} & \text{for } i=1,2,3,4, \\
S_{11}^{2j}(\theta) = S_{33}^{2j}(\theta) = S_{44}^{2j}(\theta) = [1,1]_{\theta,2j}^{-2}[5,1]_{\theta,2j}^{-2} & \text{for } j=1,3,4, \\
S_{12}^{2j}(\theta) = S_{23}^{2j}(\theta) = S_{24}^{2j}(\theta) = [2,1]_{\theta,2j}^{-2}[4,1]_{\theta,2j}^{-2} & \text{for } j=1,3,4, \\
S_{13}^{2j}(\theta) = S_{14}^{2j}(\theta) = S_{34}^{2j}(\theta) = [3,1]_{\theta,2j}^{-2} & \text{for } j=1,3,4, \\
S_{22}^{2j}(\theta) = [1,1]_{\theta,2j}^{2}[3,1]_{\theta,2j}^{4}[5,1]_{\theta,2j}^{2} & \text{for } j=1,3,4, \\
S_{ab}^{ij}(\theta) = 1 & \text{for } a,b=1,2,3,4;\ i\neq j;\ i,j\neq 2\,.
\end{array}
$$

The solutions to the constant TBA-equations (11) read

$$
\begin{aligned}
x_1^1 &= x_3^1 = x_4^1 = x_1^3 = x_1^4 = x_3^3 = x_3^4 = x_3^4 = x_4^4 = x_2^2 = 1 && (18)\\
x_2^1 &= x_2^3 = x_2^4 = 1/2 && (19)\\
x_1^2 &= x_3^2 = x_4^2 = 2 && (20)
\end{aligned}
$$

such that the effective central charge according to (10) is

$$
c_{\rm eff} = \frac{6}{\pi^2}\left(10\mathcal{L}\left(\frac{1}{2}\right) + 3\mathcal{L}\left(\frac{2}{3}\right) + 3\mathcal{L}\left(\frac{1}{3}\right)\right) = 8\ . \qquad (21)
$$

This result confirms the general formula (17).

5 Conclusions

We have shown that the proposed scattering matrices (6), (7) provide consistent solutions of the bootstrap equations (1). In comparison with (3) we have taken the square root of the CDD-factor which lead to the introduction of the non-trivial parity breaking phase factors. The main motivation for this was to recover the known scattering matrices which were mentioned at the end of section 2. It is clear though that when we view (2) as the usual affine Toda field theory scattering matrix related to simply laced algebras we can write down immediately colour valued S-matrices related to two different algebras. When we do not take the square root this is straightforward and also works for theories related to non-simply laced algebras. We leave a systematic investigation of these type of theories for future investigations.

A further open question is to identify the corresponding Lagrangian for the $\mathbf{g}|\tilde{\mathbf{g}}$-theories. The knowledge of the ultraviolet central charge (17) will certainly be useful in this search since it provides the renormalization fixed point. As we know from the Homogeneous sine-Gordon models the $A_n|\tilde{\mathbf{g}}$–theory may be viewed as perturbed $\tilde{\mathbf{G}}_{n+1}/U(1)^{\otimes\tilde{\ell}}$-coset WZNW theories. In analogy, we could view for instance the "dual" theory of this, i.e. the $\tilde{\mathbf{g}}|A_n$–theory, formally as perturbed $\tilde{\mathbf{G}}_1^{\otimes(n+1)}/\tilde{\mathbf{G}}_{n+1}$-coset WZNW theory. Besides the identification of the fixed point theory for the situation in which $\mathbf{g}\neq A_n$, it remains open to find the precise form of the perturbing operators. We do not expect that they will turn out to be irrelevant, since the colour giving CDD-factors are different in nature than the ones recently discussed in [18].

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References

[1] B. Schroer, T.T. Truong and P. Weisz, *Phys. Lett.* **B63** (1976) 422;
M. Karowski, H.J. Thun, T.T. Truong and P. Weisz, *Phys. Lett.* **B67** (1977) 321;
A.B. Zamolodchikov, *JETP Lett.* **25** (1977) 468.

[2] L. Castillejo, R.H. Dalitz and F.J. Dyson, *Phys.Rev.* **101** (1956) 453.

[3] A.E. Arinshtein, V.A. Fateev and A.B. Zamolodchikov, *Phys. Lett.* **B87** (1979) 389;
H.W. Braden, E. Corrigan, P.E. Dorey and R. Sasaki, *Nucl. Phys.* **B338** (1990) 689.

[4] C.R. Fernández-Pousa and J.L. Miramontes, *Phys. Lett.* **B472** (2000) 392.

[5] G.W. Delius, M.T. Grisaru and D. Zanon, *Nucl. Phys.* **B382** (1992) 365.

[6] A. Fring, C. Korff and B.J. Schulz, *Nucl. Phys.* **B567** (2000) 409.

[7] R. Köberle and J.A. Swieca, *Phys. Lett.* **86B** (1979) 209;
A.B. Zamolodchikov, *Int. J. Mod. Phys.* **A1** (1989) 4235.

[8] O.A. Castro-Alvaredo, A. Fring, C. Korff and J.L. Miramontes, *Thermodynamic Bethe Ansatz of the Homogeneous Sine-Gordon Models,* hep-th/9912196.

[9] A.V. Mikhailov, M.A. Olshanetsky and A.M. Perelomov, *Commun. Math. Phys.* **79** (1981) 473;
G. Wilson, *Ergod. Th. Dyn. Syst.* **1** (1981) 361;
D.I. Olive and N. Turok, *Nucl. Phys.* **B257** [FS14] (1985) 277.

[10] T. Nakatsu, *Nucl. Phys.* **B356** (1991) 499;
T.J. Hollowood, *Nucl. Phys.* **B384** (1992) 523;
G.M. Gandenberger, *Nucl. Phys.* **B449** (1995) 375;
G.M. Gandenberger and N.J. MacKay, *Nucl. Phys.* **B457** (1995) 240;
G.M. Gandenberger, N.J. MacKay and G. Watts, *Nucl. Phys.* **B465** (1996) 329.

[11] A. Fring and D.I. Olive, *Nucl. Phys.* **B379** (1992) 429.

[12] D.I. Olive, *Nuovo Cimento* **26** (1962) 73;
G. Takács and G. Watts, *Nucl. Phys.* **B547** (1999) 538;
J.L. Miramontes, *Phys. Lett.* **455B** (1999) 213.

[13] A. Fring, C. Korff and B.J. Schulz, *Nucl. Phys.* **B549** (1999) *579.*

[14] Al.B. Zamolodchikov, *Nucl. Phys.* **B342** (1990) 695
T.R. Klassen and E. Melzer, *Nucl. Phys.* **B338** (1990) 485; **B350** (1991) 635.

[15] L. Lewin, Dilogarithms and associated functions (Macdonald, London, 1958).

[16] A.N. Kirillov and N.Y. Reshetikhin, *J. Sov. Math.* **52** (1990) 3156.

[17] A. Kuniba, *Nucl. Phys.* **B389** (1993) 209;
A. Kuniba and T. Nakanishi, *Mod. Phys.* **7** (1992) 3487;
A. Kuniba, T. Nakanishi and J. Suzuki, *Mod. Phys.* **8** (1993) 1649; *Int. J. Mod. Phys.* **9** (1994) 5215; *Int. J. Mod. Phys.* **9** (1994) 5267.

[18] G. Mussardo and P. Simon, *Bosonic-type S-matrix, Vacuum Instability and CDD Ambiguities,* hep-th/9903072.