Models of Corporate and Bank Default and Credit Migration

by

Paraskevi Dimou

Thesis

Submitted to City University for the degree of Doctor of Philosophy in Finance

CITY Cass Business School
City of London

December 2007

Table of Contents

CHAPTER 3: Structural models and the prediction of default probabilities

CHAPTER 4: Distance to Default as a predictor of bank credit ratings

List of Tables

CHAPTER 3: Structural models and the prediction of default probabilities

CHAPTER 4: Distance to Default as a predictor of bank credit ratings

CHAPTER 5: Are equity prices better predictors of rating changes than debt prices? A comparison of the Merton/KMV and BARRA models

List of Figures

CHAPTER 3: Structural models and the prediction of default probabilities

CHAPTER 4: Distance to Default as a predictor of bank credit ratings

CHAPTER 5: Are equity prices better predictors of rating changes than debt prices? A comparison of the Merton/KMV and BARRA models

Acknowledgements

I would like to thank my supervisor Dr Alistair Milne for his guidance during the development of this thesis.

I would like to thank my colleagues at the Joint Research Centre of the European Commission for all their comments and suggestions.

Special thanks to the participants of the following conferences and workshops, were parts of this thesis have been presented, for their useful comments and suggestions: International Conference on Computational Finance and its Applications 2004, took place in Bologna, Italy; Workshop at Bank of Finland 2005, held in Helsinki, Finland; International Conference on Computing in Economics and Finance 2006, held in Limassol, Cyprus; International Workshop on Computational and Financial Econometrics 2007, took place in Geneva, Switzerland;

I would like to acknowledge the support of Dr Olivier Renault, for providing part of the data used in this thesis.

Moreover, I would like to thank my examiners, Professor Esa Jokivuolle from Bank of Finland and Professor Roy Batchelor from Cass Business School, for their invaluable comments and suggestions regarding the thesis.

I am grateful to Giorgos for his love, patience and support during all these years.

I would like to dedicate this thesis to the three most important people in my life: to my mother and to my beloved sister for their unconditional love and support and especially to my father, who always believes in me.

Declaration

I grant powers of discretion to the University Librarian to allow this thesis to be copied in whole or in part without further reference to me. This permission covers only single copies made for study purposes, subject to normal conditions of acknowledgement.

Abstract

This thesis presents three studies on credit risk modelling. The first study compares the real default probabilities produced by three main structural models of default, Merton model, Longstaff and Schwartz model and Leland and Toft model, to the observed real default probabilities reported by Moody's for the BBB, BB and B rated bonds. We find that none of the models can accurately predict the default probabilities in all these cases. Merton as well as Leland and Toft models underpredict default probabilities. Longstaff and Schwartz model although it produces in some cases Expected Default Frequencies (EDFs) that are close to the observed ones, it tends to overestimate the default probabilities of riskier bonds as well as the default probabilities of bonds with the same rating but higher equity volatility. We also find that structural models tend to underestimate the default probabilities in early years.

The second study examines whether information from equity markets, as summarized in the distance to default measure derived from a Merton-Moody's KMV (MKMV) model, provides useful additional information over accounting variables for predicting changes in bank credit ratings. Using a dataset of 98 equity listed banks from 1997 to 2004, we find that distance to default measure has additional explanatory power for modeling current ratings, or predicting credit rating changes over a 6-month or 12-month horizon, but only for the smaller sized banks. We find no evidence that changes in distance-to-default have additional explanatory power for predicting rating categories, regardless of the size of the bank.

The third study compares two proprietary models, Moody's KMV (MKMV) and BARRA models that use information from the equity and debt market respectively for the estimation of market implied ratings that can be updated continuously. We compare the empirical performance of these models in terms of their ability to predict in a timely fashion changes in credit quality by employing a sample of 4594 bonds issued by 447 firms from US for a period of 3 years. We find that neither model provides a close mapping to observed ratings. Both however are useful for prediction of credit transitions.

CHAPTER 1: General introduction

Abstract of Chapter 1

This chapter presents an overview of the three main studies on credit risk modelling analysed in this dissertation. We present the objectives of the research and review how the three research studies contained in this thesis contribute to the existing literature.

1.1. Research objectives and contribution of research

The research reported in this thesis investigates the use of structural credit risk models for the prediction of default and credit rating transitions. It aims to examine both how different specifications of structural credit risk models affect default predictions and the empirical performance of the most widely used structural credit risk model, that of Moody's KMV, in relation to other models using accounting and bond market data.

The thesis contains a literature survey (Chapter 2) and three research studies. The objective of the first research study (Chapter 3) is to examine the differences in real default probabilities produced by different structural models. Three main structural models; Merton model, Longstaff and Schwartz model and Leland and Toft model, are compared. The main difference between these models is the different assumptions on the determination of default barrier and interest rates that they make. In Merton model, which is the basic structural model, default barrier is determined exogenously and risk free interest rates are constant. Longstaff and Schwartz model extends Merton model by assuming stochastic interest rates, while Leland and Toft model assumes that the default barrier is determined endogenously.

This chapter contributes to the current literature in several ways. During the last years, there has been considerable work on the comparison of different structural models in terms of the term structure of credit spreads they generate (or equivalently the term structure of risk-neutral default probabilities), i.e. the comparison of the pricing predictions of structural credit risk models. In contrast, aside from Leland (2004) there appears to have been no academic research on the comparison of the term structure of real default probabilities or Expected Default Frequencies (EDFs) produced by different structural models. Examining the ability of different structural models to generate accurate real default probabilities is an important contribution to current literature. Previous studies on structural models obtain mixed results on the ability of structural models to produce spreads that are in line with those observed in the market. The main consensus is that the structural models underpredict credit spreads. According to the literature reviewed in this thesis (see Chapter 2) this could be interpreted as suggesting either that structural models are inadequate for measuring credit risk or, as suggested by recent studies, that credit risk is only one of the components that explain yield spreads. In our study, we investigate in a theoretical setting whether structural models are adequate for the prediction of actual default probabilities and examine whether the models themselves are sound but the credit spreads are determined by other factors such as tax or liquidity.

The objective of the second study is to empirically determine whether information from equity markets, as summarized in the distance to default measure derived from Merton and similar to the one proposed by Moody's KMV Credit Monitor, provides useful additional information over accounting variables for the modelling and prediction of bank ratings and rating transitions in a sample of developed country banks. The use of distance to default (DD) measure for modelling and predicting credit ratings may have an advantage over the commonly used historical accounting variables: in contrast to the latter, that are released only annually or at best quarterly, the DD, which summarizes information from equity markets, can be continuously updated. For this reason, the distance-to-default measure is widely used by central banks, including European Central Bank (ECB), as an indicator of bank financial stability.

This study (Chapter 4) is an addition to an active recent literature on the assessment of bank credit quality. Although the incremental value of a distance to default measure over accounting variables in the prediction of default and credit ratings has already been examined for corporates, no study has investigated the usefulness of a distance to default measure for the prediction of changes in the credit quality of banks. Our study extends work by Gropp et al (2006) who examined the ability of different market indicators, including a distance to default measure based on equity prices, to discriminate between banks in two categories: financial fragile or not. Answering the question of whether distance to default is a useful indicator of changes in banks' credit quality is of utmost importance, since the distance to default measure is widely used by central banks, including European Central Bank (ECB), as an indicator of bank financial stability.

The increased importance of credit ratings in financial markets, both for valuation and for prudential regulation, has triggered the development of various models to predict rating changes. Two main proprietary models have been developed Moody's KMV (MKMV) and BARRA models that use information from the equity and debt market respectively. Both models yield market implied ratings that can be updated continuously. This paper compares the empirical performance of these models in terms of their ability to predict in a timely fashion changes in credit quality

In our third research study (Chapter 5), we empirically compare the predictive ability for credit rating changes of two leading proprietary models currently used by many financial institutions, Moody's KMV and BARRA models. Since the two models use information from the equity and bond market respectively, we are also comparing whether credit spreads or equity prices are better predictors of rating changes, subject to the restriction that these predictions are based on these particular proprietary models. This question is particularly important for practitioners. Both markets have their own advantages and disadvantages; the equity market may be characterized by bubbles and irrational behavior of investors, and the bond market suffers from low liquidity. It is therefore very useful to identify whether debt or equity market summarizes better publicly available information on firms' credit quality.

The contributions of this third research study are therefore both academic and practical. Previous academic studies have compared the empirical performance of structural and accounting models and also the performance of different theoretical structural and reduced form models. To our knowledge this chapter is the first research study to compare market-implied ratings derived from equity prices with those derived from bond prices. This chapter also seeks to make a practical contribution, helping practitioners assess the ability of two popular models to timely predict changes in the credit quality of firms.

CHAPTER 2: Literature on credit risk modelling

Abstract of Chapter 2

This chapter provides an overview of the current literature on credit risk modelling, focusing on research in two areas of the credit risk; pricing of debt instruments and prediction of default. There are two main groups of models developed for the pricing of individual debt instruments, which are described in this chapter; the firm-value-based models of credit risk or structural models and the so called reduced-form models. These pricing models and in particular structural models have also been used for the prediction of default together with credit scoring models.

2.1. Introduction

During the last decades, there has been a great interest both from researchers and academics for the development of credit risk modeling. Although credit risk has always been a major concern for banks and financial intermediaries, the recent increased focus on the development of new methodologies can be attributed to a number of factors. First of all, the recent growth of credit markets, both in size and complexity, by the introduction of OTC (Over the Counter) instruments led to a considerable interest in the development of credit risk models for the accurate pricing of defaultable instruments. Moreover, the introduction of the new Basel Accord supports the use of both external and internal ratings for determining regulatory capital requirements, hence creating a considerable interest in the development of models that predict credit ratings and rating transitions.

Therefore, both the need for better credit risk management and for a better understanding of the new financial instruments led to the development of two distinct areas of research; the pricing of defaultable instruments and the modelling of default risk and ratings transitions. In this chapter, we provide an overview of the available literature on credit risk models developed for both these purposes.

There are two main types of models that have been developed and used for the pricing of defaultable instruments; the structural models and the reduced form models. The structural models treat equity as an option to buy the company's assets and use an option pricing formula to derive the likelihood of default. In these models default occurs when the value of firm's assets hits an endogenously or exogenously specified threshold. The most widely known proprietary structural model is Moody's KMV Credit Monitor. The main advantage of these models is that they can use the latest market prices to provide an updated Expected Default Probability for individual companies. Their main disadvantage is that they become cumbersome to use for assets that have unusual capital structures or unusual payoffs.

To overcome this limitation, the reduced form models have been developed. In reduced form models, default is not linked to the firm asset value falling below a prespecified barrier-level as in structural models. They view the credit event as a perfectly unpredictable event and assume that the price of defaultable instruments follows a stochastic process. The price of the defaultable instrument is then derived by calibrating to market data. Reduced-form models are generally used in areas such as bond and derivative pricing rather than producing likelihood of default measures.

Both these models, and especially structural models, have been developed to complement more traditional econometric models of default prediction. These econometric models are simpler in nature using either information from firms' financial statements or macroeconomic variables to predict default. In such models, the default prediction involves the estimation of a regression to identify the variables that are most informative in the prediction of default. Recently, the econometric models have evolved from the calculation of default rates to the calculation of rating category transition probabilities. The main drawback of the econometric models is that they are not forward looking, since they model expectations of default based on past defaults not market prices.

This chapter is organized as follows. Section 2.2, describes the literature on the pricing of defaultable instruments. Section 2.3, presents the literature on the use of structural models, proprietary models and econometric models for the prediction of default and transition probabilities. Section 2.4, offers an overview of the empirical studies that focus on the comparison of different credit risk models.

2.2. Pricing of defaultable instruments

There are two main types of models developed for the pricing of individual defaultable instruments; the structural models and the reduced form models. Uhrig-Homburg (2002) offers a detailed description of the literature on structural and reduced form models. In this section, we build on his study and we add additional studies that are related to the pricing of defaultable instruments.

2.2.1. Structural models

Structural or firm value based models of credit risk describe the default as the explicit outcome of the deterioration of the value of the firm. Corporate securities are seen as contingent claims (options) on the value of the issuing firm. There are a number of structural models that have been developed for the pricing of debt instruments as well as for the assessment of default probabilities.

2.2.1.1. Merton model

Merton (1974) pioneered the structural credit risk approach since he was the first to use the option pricing theory (OPT) developed by Black and Scholes (1973) in the valuation of default risk spreads of fixed income instruments. His approach, which gave birth to a variety of models, is called structural because the default is triggered by the capital structure of the firm when the value of the firm's assets falls below its liabilities.

At this point, it is essential to describe the assumptions made by Merton's model since they play a crucial role in the understanding of the derivation of the price of a risky defaultable bond.

Assumption 1: Markets are frictionless, which means that there are no transaction costs, taxes or short-sales restrictions as well as bid-ask spreads. In addition, assets are perfectly divisible and are traded continuously.

Assumption 2: There is a riskless interest rate that is known and whose term structure is flat and constant.

Assumption 3: The value of the assets of the firm, denoted by V_a, is assumed to follow the following diffusion process:

$$dV_a = \mu V_a dt + \sigma_a dz \qquad [2.1]$$

where:

μ is the expected return of the firm's assets, σ_a is the volatility of firm's assets and z is a standard Wiener process.

Assumption 4: The value of the firm is equal to the equity and a zero-coupon non-callable debt contract, implying that the value of the firm is identical to the value of the assets.

Assumption 5: Managers act to maximise shareholders wealth.

Assumption 6: The debt contract is fixed with the initial hypothesis that the firm is not already at default.

Assumption 7: Default can only happen at maturity T, if the value of the assets is lower than the value of debt. Hence, the lower reorganisation boundary of the firm is determined exogenously.

Assumption 8: The absolute priority rule cannot be violated.

Taking into account the above assumptions, equity and debt payoffs can be represented as follows:

$$E_t = \max(V_t - P, 0) \qquad [2.2]$$

and

$$B_t = P - \max(P - V_t, 0) \qquad [2.3]$$

Hence, according to Merton's model, equity can be seen as a call option on the value of the firm for two reasons. First, equityholders have limited downside risk due to the limited liability rule. Also, they have a claim on the value of the assets if and only if at the maturity of debt the value of firm's assets is higher than the principal value of debt, P. On the other hand, bondholder's payoff can be represented by a long position in the face value of debt, which is its principal value P, minus a put option on the value of the firm with strike price P.

Since both the equity and bond payoffs can be seen as options to the value of the firm, Black and Scholes option pricing formulas can be used to obtain the values of equity and risky bond respectively. Therefore, the value of equity is:

$$V_e = V_a * N(d_1) - Pe^{-r(T-t)} * N(d_2) \qquad [2.4]$$

where V_a is the value of assets, V_e is the value of equity, T is the bond's time to maturity, P is the debt's principal value and

$$d_1 = \frac{\ln\left(\frac{V_a}{P}\right) + \left(r + \frac{\sigma_a^{\ 2}}{2}\right)(T-t)}{\sigma_a\sqrt{(T-t)}} \qquad [2.5]$$

$$d_2 = d_1 - \sigma_a\sqrt{(T-t)}\,. \qquad [2.6]$$

Furthermore, the price of a risky bond is the same as a riskless bond minus a put option on the value of the firm with strike price P and it is given by the following formula:

$$B = Pe^{-r(T-t)} + VN(-d_1) - Pe^{-r(T-t)}N(-d_2). \qquad [2.7]$$

The main implication of Merton's model is that it not only enables us to derive a term structure of credit spreads but also to investigate the impact of changes in leverage, volatility as well as maturity of debt on the credit spread. Although, the humped shape term structure is consistent with actual data, Merton's model suffers from important limitations. The simplistic capital structure, the absence of bankruptcy costs, the unique zero coupon bond issue as well as the fact that for very short maturities the model produces zero spreads (at the market, spreads are never equal to zero) are some of its unrealistic assumptions and results.

2.2.1.2. Extensions of Merton model

The various shortcomings of the Merton's model as well as its inability to estimate credit spreads that are in line with historical observations gave birth to a number of theoretical models, which are extensions of the original model, relaxing some of its assumptions and managing to overcome some of its limitations.

One of the main limitations of Merton's framework is that the risky bond is assumed to be a pure discount bond, despite the fact that in most cases firms issue coupon-paying bonds. Although for the risk-free bonds an accurate pricing of pure discount bonds would be sufficient to obtain the prices of coupon bonds, since they can be seen as a portfolio of zero-coupon bonds, this is not the case for risky coupon bonds. The reason is that if a firm defaults on one coupon payment then it automatically defaults on all subsequent payments.

Although, Merton (1976) tried to solve the coupon problem by demonstrating that the perpetual risky continuous coupon bond problem is similar to the valuation of a European option on a stock that pays dividends at a constant rate, his two assumptions of perpetual maturity and continuous coupon payments were still unrealistic.

To overcome this limitation, Geske (1977) introduced a model for the valuation of risky coupon debt with finite maturity by using the compound option (option on an option) approach. He suggested that if a firm issues a bond, where there are T years to maturity and there are n-1 individual coupon payments due before the principal plus interest at maturity, then the values of equity and bond at time t_{n-1}, just after the final individual coupon payment , are as follows:

$$E_{t_{n-1}} = \max(V_T - M, 0) \qquad [2.8]$$

where V_T is the value of the firm at the maturity and M is the sum of the principal and the interest that are due at the maturity of the bond.

$$B_{t_{n-1}} = V_{t_{n-1}} - E_{t_{n-1}}. \qquad [2.9]$$

On the other hand, according to Geske's analysis, just before the final coupon payment at $t_{(n-1)-}$ the values of equity and bond are as follows:

$$E_{t_{(n-1)-}} = \max\left(E_{t_{n-1}} - C, 0\right) \qquad [2.10]$$

since shareholders will not default at the coupon payment if and only if the value of the equity after the coupon payment is greater than the coupon payment.

Moreover, since $E_{t_{n-1}}$ is itself an option, for all dates before the final coupon payment, equity can be valued as a compound option.

$$B_{t_{(n-1)-}} = \min\left(V_{t_{n-1}}, C_{t_{n-1}} + B_{t_{n-1}}\right) \qquad [2.11]$$

bond has compound option characteristics as well.

Equations 2.10 and 2.11 for equity and the bond take the same form at all earlier coupon dates. Hence, Geske obtained an expression of the value of the risky bond by recursively solving the above expressions at each coupon date.

Despite the fact that Geske (1977) managed to overcome one of the most important limitations of Merton's model by deriving an analytic solution for the price of a risky coupon bond in discrete time with finite maturity, it cannot be applied to bonds of very long maturities, as the calculations are cumbersome.

Another important extension of Merton's original model is the Black and Cox (1976) valuation model. In this valuation framework, Black and Cox (1976) challenged some of the main assumptions made in the Merton model. As described above, in the latter default can only occur at the debt's maturity, while Black and Cox (1976) allow for the possibility of early bankruptcy. Additionally, one of the main limitations of Merton's model for coupon bonds with infinite maturity is that the firm has the right to repay the coupon payments by selling its assets. The implication of this is that the value of assets can fall to zero prior to default, resulting in zero recovery rates which is unrealistic. Black and Cox (1976) attempted to solve this problem, by making two separate assumptions. First of all, they assume that default occurs when the value of the firm's assets hit a lower threshold point. This threshold can be determined either exogenously by a covenant or endogenously, allowing in this case the equity investors to optimally choose the time to default. Secondly, the sale of assets for the payment of debt is forbidden and they assume that debt as well as dividends can only be paid by issuing new equity. This last assumption is more realistic, since in practise most corporate bonds have safety covenants that do not allow asset sales.

Specifically, they produce a valuation formula for bonds with safety covenants as well as they examine the effect of subordination arrangements and asset sales restrictions on the value of the bond. They conclude that the existence of subordinated claims give senior bonds a higher value compared to the value that they would have if they were a fraction of a homogeneous bond issue. Last but not least, they show that it is important in the valuation of bonds to consider how the stockholders are allowed to raise the money to make the payments to the bondholders.

In a later work, Kim, Ramaswamy and Sundaresan (1993), Longstaff and Schwartz (1995) as well as Brys and Varenne (1997) manage to find closed form solutions for coupon paying debt by extending the Black and Cox (1976) model in two ways. First of all, they developed a valuation framework for risky corporate fixed income securities that takes into account both default and interest rate risk by assuming stochastic interest rates. Moreover, following evidence from previous work, made by Franks and Torous (1994), Eberhart, Moore and Roenfeldt (1990), LoPucki and Whitford (1990), Weiss (1990), Betker (1995) and others, they assumed that the strict absolute priority rules can be violated in case of default. On the other hand, like Black and Cox (1976), they assume that default occurs when the value of the firm's assets reaches a constant or deterministic point as well as they allow default to occur prior to maturity. Despite all these similarities, it is essential to highlight the differences between these models.

Kim Ramaswamy and Sundaresan (1993) studied the valuation of risky non callable, subordinated and callable bonds. Although they developed a framework involving the valuation of a single debt issue, they managed to value subordinated debt by slightly modifying the idea developed in the Black and Cox (1976) paper, where the junior debt can be seen as a portfolio of suitably specified senior debt. Moreover, they allow for stochastic interest rates.

The additional contribution of Longstaff and Schwartz (1995) lays on the fact that they derive simple closed-form solutions for the valuation of both fixed and floating rate debt. Additionally, they found that there is a negative correlation between the asset values and the interest rates, which is in line with empirical evidence, resulting in a reduction of estimated yield spreads. Nevertheless, as in Kim, Ramaswamy and Sundaresan they found that the effect of stochastic interest rates on yield spreads is quite small.

On the other hand, the main objective of Brys and Varenne (1997) paper is to create a valuation model that would correct some limitations of previous models that incorporated stochastic interest rates into their analysis. They claim that their valuation model ensures that the payment received by bondholders in case of default is not greater than the value of the firm at that time. Moreover, they make sure that upon maturity the assets will always be of sufficient value to match the face value of the debt, which was not the case in the Longstaff and Schwartz (1995) model.

Another set of models that extended Merton (1974) and Black and Cox (1976) valuation models are models developed by Leland (1994) and Leland and Toft (1996). These two models extend Black and Cox (1976) valuation model by assuming that taxes and bankruptcy costs are not zero, since they include into their analysis the effect that they have on debt value. This implies that the value of the firm is no longer identical to the value of the firm's assets. In their analysis, the value of the firm is equal to the value of the firm's assets plus the value of tax deduction minus the value of bankruptcy costs.

Specifically, Leland (1994) introduced a model that examines the corporate debt values and optimal capital structure in a unified framework as well as it produces closed-form solutions relating their values to the firm risk, bankruptcy costs, taxes, bond covenants and other parameters. They assume that debt has infinite maturity and that the firm pays a non negative coupon continuously that is financed entirely by issuing new equity. The fact that in their model debt has infinite maturity implies that the return of the principal has no value, which allows them to assume time independence of cash flows of debt. This last assumption enables them to derive closed form solutions for risky corporate debt given capital structure. Last but not least, they consider an environment where the lower reorganisation boundary of the firm is determined endogenously, allowing equityholders optimally to choose the time to default. They argue that at each moment the equityholders have the choice of either make the coupon payment to bondholders or default their payment, leading the firm to bankruptcy. Since equityholders will try to maximise their value they will only meet the coupon payments if and only if the value of the firm's assets exceeds the default boundary.

Although, Leland (1994) made a substantial contribution by deriving closed form solutions for equity and debt values in the case of endogenous bankruptcy, their assumption regarding infinite life debt is restrictive since in practice firms choose the maturity as well as the amount of debt.

In their article, Leland and Toft (1996) extended Leland's model and they examined also the effect of debt maturity on bond values, credit spreads and the optimal amount of debt. To achieve this, they assume that debt is continuously rolled over, implying that the total outstanding debt principal as well as the average debt maturity will remain constant at any time as long as the firm remains solvent. The main contribution of this paper is that using the assumption that the debt is continuously rolled over enabled them not only to keep the assumption of time-independence but also to solve to some extent the problem of infinite maturity debt which was present in Leland (1994) analysis.

On the other hand, recent studies by Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) go a bit further, since in their models not only is the default barrier determined endogenously but also they take into account the strategic bargaining between shareholders and bondholders. Despite their similarity in the sense that they both incorporate into their analysis strategic debt service, they have some differences that are highlighted below.

Anderson and Sundaresan (1996) developed a framework in which equity- and debtholders interact strategically. They make several assumptions which are different from previous models. First of all, they assume that the firm undertakes a project that generates cash flows and that the value of firm is assumed to be equal to the present value of all future cash flows. Moreover, the interest payments are funded by the cash flows of that project and not by asset sales or equity issues, which was the case in previous models. Furthermore, they make the assumption that the firm cannot issue additional debt once the project starts. Last but not least, the main contribution of this paper and its main difference compared to the original Merton model is that they allow for renegotiations between equity and debtholders. Hence, their model gives the equityholders the ability not to pay the full amount of coupon or principal payments, even when the cash flows received enables them to fully meet their obligations. They argue that since liquidation is costly due to direct and indirect bankruptcy costs, equityholders can use that to their advantage. Hence, if the value of the payment that the bondholders receive is less than what they would get in case that the firm goes bankrupt, they would accept the payment regardless of the fact that it is less than the initial contracted amount that they should get. Anderson and Sundaresan (1996) found that the incorporation of strategic debt service into the analysis produces significantly higher default premia even at small liquidation costs.

One of the main disadvantages of Anderson and Sundaresan (1996) model is that although it enables us to find the equilibrium value of debt it does not provide closed form solutions which will improve the speed of the debt value calculation. Trying to overcome this limitation, in a later article, Anderson, Sundaresan and Tychon (1996) extended the Anderson and Sundaresan (1996) model by building its continuous time equivalent. In fact, they obtained analytical closed form solution for the value of a perpetual bond that pays coupon continuously.

Another model that incorporates strategic negotiations between equity and bondholders is the model introduced by Mella-Barral and Perraudin (1997). Unlike the discrete time finite debt maturity model by Anderson and Sundaresan (1996) they used a continuous time contingent claims asset pricing model with perpetual debt. They assume that equityholders can meet their obligations by issuing new equity. This gives them the possibility to choose when the firm will go bankrupt. If there was no renegotiation, the point of default would be the same as in Leland (1994) and Leland and Toft (1996). However, in this model it is assumed that if the value of firm's assets is less than the current value of debt, the bondholders will not be willing to unwind the firm and the equityholders are able to extract a surplus by offering debtholders a payment that is less than the promised amount.

Until this point, all of the structural models described above assume that default never comes as a surprise. One of the assumptions made by the Merton model, and followed by most of its subsequent models is that the value of the firm's assets follows a diffusion process. The implication of this assumption is that default can only occur gradually and cannot be caused by a sudden loss due to an unpredictable event. Zhou (2001) challenged this idea by assuming that the asset value follows both a diffusion and jump process. This allowed him to take into account the possibility of sudden defaults that are caused by unforeseen external shocks.

All the above models, except of Brys and Varenne (1997) model, assume that the capital structure of the firm remains unchanged until the maturity of debt. Furthermore, as mentioned before, with the exception of some cases most of the models assume that the value of assets follows a diffusion process. These two assumptions imply that as the value of assets increases over time, leverage ratios will be expected to decline over time, which contradicts recent empirical findings that leverage ratios are stationary or mean-reverting. Specifically, conducting empirical research at a firm level, Opler and Titman (1997) concluded that target leverage ratios exist within an industry. Moreover, Fisher, Heinkel and Zechner (1989) as well as Goldstein, Ju and Leland (2001) found that a firm can maximize its value if its leverage ratio is within a certain band. Given these findings, Collin-Dufresne and Goldstein (2001) introduced a dynamic restructuring model that extends the original Merton model in two ways. First of all, they allow the firm to change its debt, and hence its capital structure, as the value of assets changes. This implies that the default boundary is no longer assumed to be constant, since it is adjusted to reflect stationary leverage ratios. Additionally, the model accounts for stochastic interest rates. They found that their model generates higher credit spreads for low leverage firms as well as in this case they are less sensitive to firm value changes, which is more in line with those observed in practice compared to other structural models.

Another dynamic restructuring model was developed by Goldstein, Ju and Leland (1999). Although they do not assume stochastic interest rates as in Collin-Dufresne and Goldstein (2001), they assume that the firm is permitted to increase its debt levels. Moreover, unlike all the other structural models described above, they use Earnings before Interest and Tax (EBIT) as an alternative to the asset value variable. They concluded that by allowing the firm to increase its debt, the model produces yield spreads as well as optimal debt levels that are consistent with the empirical findings.

Additionally, Ericsson and Reneby (2001) developed another model where the firm is allowed to increase its debt over time. Although the increase in total debt is the result of many small debt issues, it is assumed that the growth in debt can be approximated by a continuous increase. In their model, the default barrier is determined by equityholders as in Leland and Toft (1996) model. The main difference is that in Ericsson and Reneby (2001) model the default barrier grows exponentially with time along with the total nominal amount.

2.2.1.3. Testing of structural models

All the models described above aim to derive closed form solutions for the price of debt and equity as well as to calculate the yield spreads. Although, as mentioned before, there are some authors who compared the spreads generated by their models with observed credit spreads, there have been other studies that intend to compare a variety of these models in terms of how well they predict yield spreads.

Merton model was tested empirically by Jones et al. (1984), implemented Merton's model on US corporate bond yields. They found that Merton's model produces credit spreads significantly lower than the observed credit spreads.

Wei and Guo (1997) compared Merton's model with Longstaff and Schwartz (1995) model. Using a limited sample of Eurodollar futures, they concluded that Merton's model outperforms Longstaff and Schwartz's model in terms of the credit spreads they produce. Lyden and Saraniti (2000) test the same two models and they conclude that both models overprice bonds.

Anderson and Sundaresan (2000) compared the perpetual coupon bond models of Merton (1974), Leland (1994) and Anderson, Sundaresan and Tychon (1996). Using aggregate yield data for the US corporate bond market, they estimate the spreads generated by these models for AAA, AA and BBB bonds. They found that the models of endogenous bankruptcy produced by Leland (1994) and Anderson, Sundaresan and Tychon (1996) generate spreads that are more in line with historical observations.

More recently, Eom, Helwege and Huang (2002) test the performance of five structural models: Merton (1974), Geske (1977), Longstaff and Schwartz (1995), Leland and Toft (1996) and Collin-Dufresne and Goldstein (2001). They found that structural models do not systematically underpredict credit spreads. They conclude that Merton (1974) model predicts low spreads. Nevertheless, they find that the newer structural models tend to overpredict spreads for the debt of firms with high volatility or high leverage. Except of Leland and Toft (1996) model, these models tend to underpredict spreads of safer bonds.

Therefore, despite previous studies (Jones, Mason and Rosenfeld; 1984), which showed that the original Merton model produces yield spreads that are below those that are historically observed, the results by Anderson and Sundaresan (2000) and Eom, Helwege and Huang (2002) produce mixed results on the performance of structural models of credit risk, since they show that in some cases the models with endogenous default boundaries can produce yield spreads that are in line with historical credit spreads. Similarly, other empirical studies, which were described above (Longstaff and Schwartz; 1995, Anderson and Sundaresan; 1996, Leland; 1994, Leland and Toft; 1996, Zhou; 2001), show that their models can generate credit spreads that are higher compared to those generated by the original Merton model.

Despite the fact that these studies produce encouraging results regarding the future of structural models of credit risk, they do not conclude, at least for bonds with low maturities, that the credit spreads generated by their models are exactly the same as the historical observations. Until recently, many researchers argued that this is a strong evidence that the models are wrong. However, the yield spreads in all of the above mentioned studies where calculated using models that account only for the credit risk of bonds. Hence, these studies do not take into account the fact that credit risk is only one of the factors that explain the yield spread between risky and non-risky bonds. Therefore, using credit risk models to explain the yield spread will result by definition in lower yield spreads than the historical observations.

Recent studies by Delianedis and Geske (2001), Ericsson and Renault (2000) and Ericsson and Reneby (2002), Yu(2003) support the idea that the bond yield spreads are determined, apart from credit risk, by a number of non credit risk factors such as liquidity and tax.

Recently, Huang and Huang (2003) adopted that idea and attempted to calculate the fraction of yield spreads that is due to credit risk. For investment grade and junk bonds they calibrated the models of Longstaff and Schwartz (1995), a model that incorporates elements by Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997) and Anderson, Sundaresan and Tychon (1996), as well as the stationary leverage model of Colin-Dufresne and Goldstein (2001) by standardizing the default probabilities, recovery rates and equity risk premiums for different leverage ratios for different credit ratings according to historical default loss experience. With empirically reasonable parameter choices, they showed that credit risk accounts for only a small fraction of the observed yield curves for investment grade bonds. Moreover, they showed that for this category of bonds the fraction explained by credit risk is even smaller for short maturity bonds. What is more, they found that, in the case of junk bonds, credit risk accounts for much higher fraction of the observed yield spreads. Last but not least, they showed that different structural models, which in theory can predict different credit spreads, in practice produce fairly similar spreads. This result however is in contrast to the Eom, Helwege and Huang (2002) findings, since they concluded that different structural models produce relatively different credit spreads.

One conclusion suggested by this review of the literature is that testing of structural models is likely to be difficult, since they can under- or over-predict credit spreads depending on the particular modelling assumptions and on whether other determinants of credit spreads are introduced. This in turn suggests that it is important to look not just at credit spreads but also at other predictions of these models such as default probabilities or consistency with rating changes, issues which are explored in the present thesis.

2.2.2. Reduced form models

This approach assumes that the firm's default time is unpredictable and is driven by an exogenous variable. The main disadvantage of reduced form models as opposed to the structural models is that they lack of clear economic rationale for defining the nature of default process. Nevertheless, they are able to model complex financial instruments due to their mathematical tractability.

In particular, in reduced form models, default probability occurs according to an exogenous hazard rate, or intensity rate, process that represents the frequency of defaults that can occur in a specific time interval. In this setting, default follows a jump process.

There is a large literature on reduced form models that includes Jarrow and Turnbull (1995), Jarrow, Lando, and Turnbull (1997), Das and Tufano (1996), Madan and Unal (1998), Lando (1998) and Duffie and Singleton (1999). The main difference between these models is the specification of the hazard rate, with models going from the most simple hazard rate to hazard rate that is linked to the to the risk-free rate or the recovery rate at default.[1]

2.2.2.1. The application of the hazard rate process

In the reduced form models, the time t price of a defaultable zero coupon bond with maturity t can be represented as follows:

[1] The analysis provided in this Section is based on the Uhrig-Homburg (2002) literature survey. For more detailed analysis of the reduced form models refer to Uhrig-Homburg (2002) and Cossin and Pirotte (2000).

$$u(t,T)=p(t,T)(\varphi+(1-\varphi)Q(\tau\succ T)) \qquad [2.12]$$

where $p(t,T)$ is the price of an otherwise identical default-free zero coupon bond and $(\varphi+(1-\varphi)Q(\tau\succ T))$ is the expected payment at maturity, with φ being the recovery rate and $(Q(\tau\succ T))$ being the risk neutral survival probability.

To price the risky coupon bond, the default time τ needs to be specified. Since, reduced form models assume that default is a surprise event, to model the default time τ, an adequate "jump" process needs to be specified.

Default time comes after the first jump $N(t)=1_{\{\tau\le t\}}$. N(t) can be represented as follows:

$$N(t)=M(t)+\int_0^t \lambda(s)ds, \qquad [2.13]$$

where M(t) is a martingale under the risk-neutral measure Q and $\lambda(t)$ is a non negative predictable stochastic process called intensity.

As mentioned before, reduced form models differ in their assumptions concerning the default intensity.

Jarrow and Turnbull (1995) model use the simplest specification, since they assume that the default intensity λ is constant. In this case, the default time τ can be interpreted as the first jump of a Poison process with parameter λ. In their model, they use default-free and defaultable term structures to obtain unique risk neutral and martingale default probabilities. One main disadvantage of Jarrow and Turnbull (1995) model is that they do not allow the credit quality of the bonds to change up to the default event. This limitation led to the development of new models.

Jarrow, Lando and Turnbull (1997) provide an extension of the Jarrow and Turnbull (1995) model, since they consider different credit classes and allow for both worsening and improving credit quality. The transition from one rating class to another is described by a stationary Markov chain. The main advantage of this model rests in its great flexibility to calculate the parameters to observable data. Nevertheless, due to the assumption of deterministic default intensity for a given rating together with the assumption of constant recovery rates, under this model changes in the credit spreads are only due to changes in ratings, even if evidence suggests that there is highly volatility in credit spreads of given rating classes.

One way to overcome this problem is to assume stochastic recovery rates as Das and Tufano (1996) and Madal and Unal (1998). The main advantage of these models is that credit spreads can change over time even if the rating remains constant. Nevertheless, these models are more complex to implement.

One of the most crucial assumptions of these reduced form models is the independence between the interest rate risk and timing risk of default. Although, due to this assumption, a more mathematically tractable model is obtained, the correlation between the interest rate risk and default is important. Lando (1998) relaxes the assumption of independence between default time and evolution of default free interest rate and presents a general modeling framework in which the default time is modeled through a Cox process. A second reduced form model that deals with correlations is the model by Duffie and Singleton (1999). They assume that the intensity parameter λ depends on the level of interest rates across time.

2.2.2.2. Testing of reduced form models

Some of the academic literature deals with the testing the performance of reduced form models to explain prices and the term structure of credit spreads.

Monkkonen (1998) compares six variations of reduced form models using the basic model of Jarrow and Turnbull as the benchmark model. The alternative models allow the default probability to depend on the risk free interest rate or they assume a stochastic recovery rate or both. He finds that results are similar only for short maturity bonds. Moreover, he shows that for investment grade bonds results remain almost the same across the different models, regardless of the assumptions for the relationship between default time, recovery rate and risk free interest rate.

Duffee (1999) provides an empirical study of reduced form models. He finds that the parameters based on Duffie and Singleton (1997) model change dramatically when they are calibrated to firms with different credit ratings.

2.3. Prediction of default and transition probabilities

Up to this point, in the above review of previous theoretical and empirical studies the focus has been on the use of reduced form and structural models for the calculation of prices for defaultable bonds and the prediction of yield spreads. Nevertheless, the accurate prediction of default and transition probabilities has become an increasingly important issue to regulators, investors and financial institutions. While the core measure of credit quality remains the ratings provided by rating agencies, like Moody's and Standard & Poors, agency ratings are adjusted only slowly in response to changes in firms' financial situation and business performance. Ratings have also proved quite inadequate for prediction of the collapse of several large companies, like Enron and Parmalat. These acknowledged weaknesses underpin the widespread interest in models that aim to timely predict changes in agency credit ratings. This Section provides an overview of the academic and industry models for the prediction of default and transition probabilities.

Delianedis and Geske (1999) attempted to compute risk neutral probabilities of default using the diffusion models of Merton and Geske. They argued that risk neutral probabilities of default might be more accurate than the actual probabilities of default due to the fact that they do not require an estimate of the firm's drift. The results were quite encouraging since they found that rating migrations or defaults can be detected months in advance.

One main disadvantage of this approach is that the risk neutral probabilities cannot be directly compared with historical default probabilities recorded by Moody's, since the latter represent real probabilities of default. Therefore, it is difficult to test the accuracy of these estimated risk neutral default probabilities.

Taking a different approach, Leland (2004) estimates the real default probabilities produced by different structural models. He compares the exogenous default boundary model of Longstaff and Schwartz (1995) with the endogenous default boundary model of Leland and Toft (1996) in terms of the Expected Default Frequencies (EDFs) they produce. The aim of the study is to compare the two models and ultimately to determine the difference in the Expected Default Frequency (EDF) if an endogenous default boundary is introduced. He showed that the endogenous default boundary model produces real default probabilities which are in line with the historical default probabilities, at least for long maturity bonds. For short maturity bonds, however, the model underestimates the historically observed default probabilities.

2.3.2. Proprietary models

Apart from Leland's (2004) work, who applied different theoretical structural models to calculate Expected Default Frequencies (EDFs), the increased interest in the prediction of default probabilities lead to the development of popular proprietary models, such as the Moody's KMV (MKMV) and BARRA models.

2.3.2.1. KMV model

This model has been developed by KMV Corporation, which is now part of Moody's, and its main purpose is to produce a firm specific Expected Default Frequency (EDF^{TM}) based on the structural approach. It is argued, that the main advantage of this model is that it produces EDF^{TM} that is a forward looking measure of actual default probability, since apart from balance sheet data they use the firm's equity price in order to predict firm's default probability. According to KMV, the calculation of EDF^{TM} can be summarized into the following three steps:

STEP 1: Estimation of the asset value and volatility using an option price base approach.

KMV model is based on Merton's (1974) structural model, where default is triggered by the capital structure of the firm. As an extension of Merton's model, KMV assumes a more realistic capital structure which is constituted of long term debt (infinite maturity), short-term debt (instantaneous maturity), convertible preferred shares and common stock. In this case, default occurs when the value of the firm's assets falls below an ad hoc default trigger level K, which is defined as follows:

$$K = \text{Short-term debt} + \tfrac{1}{2}\,\text{Long-term debt} \qquad [2.14]$$

Following the Merton model, KMV methodology estimates the value and volatility of assets from the market value and volatility of equity using an option pricing base approach (equity is viewed as a call option on the firm's assets with strike price K). To estimate the value and the volatility of assets two approaches can be followed.

According to the first approach, the following two equations (2.15 and 2.16) can be solved simultaneously and using an iterative technique the value and volatility of assets can be calculated:

$$V_e = V_a N(d_1) - Ke^{-rT} N(d_2) \qquad [2.15]$$

$$\sigma_e = N(d_1)\frac{V_a}{V_e}\sigma_a \qquad [2.16]$$

where:

V_a is the value of firm's assets, V_e is the value of firm's equity, r is the risk-free rate, σ_a is the volatility of firm's assets, T is the time to maturity and

$$d_1 = \frac{\log\left(\frac{V_a}{K}\right) + \left(r + \frac{\sigma_a^2}{2}\right)T}{\sigma_a\sqrt{T}} \qquad [2.17]$$

$$d_2 = d_1 - \sigma_a\sqrt{T} \qquad [2.18]$$

Nevertheless, due to the fact that the second equation holds instantaneously MKMV proposes another procedure to extract the value and volatility of a firm's asset. Using a complex iterative procedure, MKMV uses only equation 2.15 for the estimation of the unknown parameters.

STEP 2: Calculation of Distance to Default (DD)

At this stage, taking as inputs the value and the volatility of assets, the distance to default measure can be calculated. The distance to default is the number of standard deviations that the firm's asset value is away from default.

$$DD = \left[\frac{\ln\left(\frac{V_a}{K}\right) + \left(\mu - \frac{\sigma_a{}^2}{2}\right)T}{\sigma_a\sqrt{T}}\right] \qquad [2.19]$$

where μ is the expected return of the firm's assets.

STEP 3: Calculation of expected default frequency (EDF)

Instead of calculating default probabilities using N(-DD), as in the Merton case, KMV takes an alternative approach in order to correct for any biases that might result from the use of Merton model. Based on historical default and bankruptcy frequencies, using a huge database of more than 250,000 firms and 4,700 incidents of default, they calibrate the EDFs to match historical default data. This is done by estimating the proportion of firms with a given DD that actually defaulted in the past and use that proportion as the corresponding EDF™.

Although KMV Corporation claims that this model can actually predict default months in advance as well as that it can be applied with the same success in other countries apart from US, there is no independent study until now that tested the validity of that statement. This is due to the fact that MKMV model calculates EDF™ using a huge historical default database, which is not available for public use.

2.3.2.2. BARRA model

An alternative to the MKMV model is the use of debt market information in the estimation of credit risk, which may be more informative since bond spreads directly reflect the compensation required by investors for the risk due to credit rating changes and default. Although both structural and reduced form models can be used to obtain estimates of default probabilities from bond market data we present one of the most widely used, the proprietary model of BARRA that derives market implied ratings using the information from bond spreads.

This model is based on the assumption that on average the agency ratings are informative. It develops a mapping between observed bond spreads and ratings. From this mapping those issuers with implied ratings that differ markedly from their actual agency rating can be identified. This then leads to predictions of rating changes. The main advantage of this model, as in MKMV model, is that the ratings can be derived on a continuous basis and are taking advantage of the information regarding the credit quality of the firm that is already reflected in bond prices.

The BARRA model uses option-adjusted spreads for the derivation of market implied rating. Moreover, it uses issuers rather than individual bond issues. The spread of an issuer is computed as the average of the spreads of the issuer's outstanding bonds. Then, using the average spread per issuer, a distribution of the average issuer spreads over different rating categories is constructed.

BARRA shows that the resulting distribution of issuer average spreads exhibits large overlaps between individual ratings sub-distributions, observing a wide range of different ratings for the same average spread. The BARRA model does not try to explain these differences, rather it tries to use them to develop an implied classification and hence predict future ratings changes. The main estimation challenge in computing the implied ratings is to determine a sequence of spreads $b_{AAA/AA}$, $b_{AA/A}$, $b_{A/BBB}$ etc. that correspond to the boundaries between rating classes. Thus for example any issuer with an average spread s_j that lies between $b_{AAA/AA}$ and $b_{AA/A}$ will have an implied rating of AA; any issuer with an average spread $b_{AA/A}$ and $b_{A/BBB}$ will have an implied rating of A; etc.

More formally we can write these thresholds for the implied ratings as the vector b which can be represented as follows:

$$\begin{aligned} b &= \left(b_0^+, b_1^+, b_2^+, b_3^+, b_4^+, b_5^+\right) \\ &= \left(b_1^-, b_2^-, b_3^-, b_4^-, b_5^-, b_6^-\right) \\ &= \left(b_{AAA/AA}, b_{AA/A}, b_{A/BBB}, b_{BBB/BB}, b_{BB/B}, b_{B/CCC}\right) \end{aligned} \qquad [2.20]$$

To determine the vector b, the model minimises a penalty function that measures the gap between the observed spread s_j and the rating class boundaries, for any issuer j whose implied classification is different from its actual agency classification. The penalty function can be represented as below:

$$P(b) = \sum_j \left[w_j * \left(s_j - b_{i(j)}^+\right)^+ + w_j \left(b_{i(j)}^- - s_j\right)^+ \right] \qquad [2.21]$$

where:

$i(j)$: agency rating index of issuer j

s_j : spread of issuer j

b_l^- : lower threshold for implied rating index l

b_l^+ : upper threshold for implied rating index l

N : total number of issuers in the universe

N_l : number of issuers with rating l

$w_j = \dfrac{N}{N_{i(j)}}$: weight which is chosen to equalize the contribution of each rating bucket to the total penalty function.

The "plus" signs at the end of each parenthesis mean that the term is taken into account if and only if it is positive, i.e. only if the observed credit spread is respectively above or below the range of spreads consistent with the agency rating *i(j)* of issuer *j*.

The values of vector b that minimize the penalty function are the implied classification thresholds.

The main drawbacks of BARRA model is that it assumes a flat credit spread yield curve. However, they address the issue and show that even if one splits the sample for similar maturity bonds or for bonds in different industries the results obtained do not change substantially.[2]

2.3.2. Econometric models

Econometric models, or credit scoring models, are traditional models used for the prediction of default. They are quantitative models that rely on mathematical and statistical techniques.[3]

Fisher (1936) introduced the concept of discriminant analysis and Durant (1941) used discriminant analysis to separate good and bad consumer loans.

---

[2] A full description of BARRA model can be found on the paper "Market implied ratings", by Breger, Goldberg and Cheyette published in BARRA website: (www.barra.com/support/library/credit/market_implied_ratings.pdf).
The paper has also been published at Risk Magazine, July 2003.

[3] This Section provides an overview of the econometric techniques developed for the prediction of default. For a more detailed review of the credit scoring models refer to De Servigny and Renault (2004).

Beaver (1967) created bankruptcy prediction models and Altman (1968) introduced multiple discriminant credit scoring analysis. He developed a credit scoring system, called Z-score. The model uses a list of accounting ratios and multiple discriminant analysis to distinguish the accounting ratios that are more useful in the prediction of default. In principle, discriminant analysis estimates a function which can assign an observation to the correct population. Historical accounting and economic data are used to derive the discriminant function that will discriminate firms by placing them in one of the two populations.

Martin (1977), Ohlson (1980) and Wiginton (1980) were the firsts to introduce logit analysis for bankruptcy prediction.

Currently, the most widespread credit scoring techniques are: the logit/probit model, and the multiple discriminant analysis models. However, since Martin (1977) demonstrated that the discriminant analysis is just a special case of logit analysis, most of the current studies use the multinomial logit model for the prediction of default.

2.4. Empirical comparison of different models

The increased interest in credit risk modelling necessitates the empirical comparison different models on their performance in predicting bond and default probabilities. Sections 2.1, 2.2. and 2.3. provide an overview of the current literature on the testing of structural and reduced form models respectively. Although these empirical studies provide guidance on the performance of theoretical structural and reduced form models separately, they do not answer the question on which type of model is better for the derivation of corporate bond yields or the prediction of default probabilities. In this section we offer a description of the current studies on the comparison of different models.

Hillegeist et al (2004) compare the Altman's Z-Score and Ohlson's O-Score to the market based Merton/Black and Scholes model in terms of their ability to accurately predict corporate bankruptcy. They conclude that the market based structural default measure has more information relative to the traditional statistical measures for the prediction of corporate bankruptcy.

Du and Suo (2003) investigate the empirical performance of credit rating prediction based on Merton's (1974) structural credit risk model. They conclude that the distance to default measure, calculated by Merton's model, is not sufficient for the accurate prediction of credit ratings. They also find that a simple reduced form model outperforms Merton's model.

Bharath and Shumway (2004) study compares a simple Merton model with the hazard model of Shumway (2001). They conclude that the hazard rate model performs slightly better than the Merton model for the prediction of defaults.

Arora, Bohn and Zhu (2005) empirically compare two structural models; the basic Merton model and Vasicek-Kealhofer model with the Hull and White (2000) reduced form model based on their ability to discriminate defaulters from non defaulters. They find that Vasicek-Kealhofer and Hull and White models outperform the simple Merton model.

CHAPTER 3: Structural models and the prediction of default probabilities

Abstract of Chapter 3

In this chapter, three main structural models of default, Merton model, Longstaff and Schwartz model and Leland and Toft model, are compared in terms of the expected default frequencies (EDFs) they produce. We compare the EDFs produced by these models to the observed actual default probabilities reported by Moody's for the BBB, BB and B rated bonds. We find that none of the models can accurately predict the default probabilities in all cases. Merton as well as Leland and Toft models underpredict default probabilities in all cases. Longstaff and Schwartz model although it produces in some cases EDFs that are close to the observed ones, it suffers from important limitations. The model tends to overestimate the default probabilities of riskier bonds as well as the default probabilities of bonds with the same rating but higher equity volatility. Consistent with previous studies, it is found that structural models tend to underestimate the default probabilities in early years.

3.1. Introduction

In this chapter, we examine the differences in expected default frequencies (EDFs) produced by different structural models. Three main structural models; Merton model, Longstaff and Schwartz model and Leland and Toft model, are compared.

As described in Chapter 2, during the last years, there has been considerable work on the comparison of different structural models in terms of the term structure of credit spreads they generate (or equivalently the term structure of risk-neutral default probabilities) i.e. the comparison of the pricing predictions of structural credit risk models. In contrast, aside from Leland (2004) there appears to be have been no academic research on the comparison of the term structure of real default probabilities or Expected Default Frequencies (EDFs) produced by different structural models.[4]

There are two reasons why this question is of interest. First of all the prediction of default is of at least as much interest as the use of structural models for pricing. Default probabilities and associated rating transition probabilities are central to the calculation of credit value at risk, as in CreditMetrics or KMV Credit Monitor framework. With the new Basel Accord, banks will be able to use internal models to assign default probabilities on individual borrowers and hence to internally assess their economic capital. Therefore, there is a great need for the empirical testing and validation of the existing credit risk models in order to assess their ability to adequately help banks model default and credit rating changes.

Secondly previous studies of structural models of credit risk pricing obtain mixed results on the ability of structural models to produce spreads that are in line with those observed in the market.[5] The main consensus is that the structural models

[4] The term real default probability is used to indicate that we estimate real-world default probabilities as opposed to risk-neutral default probabilities. Real world default probabilities are calculated from historical data while the risk neutral default probabilities are backed out from bond prices. We focus on real world default probabilities since they are comparable to the actual default probabilities reported by Moody's. For the rest of the paper the terms default probability, real default probability and expected default frequencies are used interchangeably.

[5] Anderson and Sundaresan (2000) compared the perpetual coupon bond models of Merton (1974), Leland (1994) and Anderson, Sundaresan and Tychon (1996) and they found that the last two models that define the default boundary endogenously are generating spreads that are more in line with historical observations compared to the original Merton model. Lyden and Saraniti

underpredict credit spreads.[6] This could be interpreted as suggesting either that structural models are inadequate for measuring credit risk or, as suggested by recent studies, or that credit risk is only one of the components that explain yield spreads.[7] If structural models provide to be adequate for the prediction of actual default probabilities, then this gives greater credence to the view that the models themselves are sound but that credit spreads are determined by other factors such as tax or liquidity.

Two other types of models – apart from the structural credit risk models – have been proposed in the literature for the prediction of default and migration probabilities. The first approach is the so called 'accounting based' approach, pioneered by Altman (1968) in his use of discriminant analysis. There are other types of accounting based models include logit and neural network models. In these cases only past accounting data of the firm is used in order to calculate the default probability of a firm.

The more recent groups of credit risk measurement models are the reduced-form models and the firm-value-based models or structural models. The former set of models calculates the risk-neutral probability of default using information from the bond prices.[8] In the latter set of models default is triggered by changes in asset values and hence captured in the capital structure of the firm. Merton (1974) pioneered the structural credit risk models. He used the option pricing theory developed by Black and Scholes (1973) in the valuation of default risk spreads of fixed income instruments. Many of the models now used by the practitioners, such as KMV model and CreditMetrics, are variants of the original Merton model. Each approach has advantages and disadvantages. Reduced form models can more easily account for the observed credit spreads, especially at

---

(2000) implement Merton and Longstaff and Schwartz model. They find that both models underpredict credit spreads. Eom, Helwege and Huang (2003) compare Merton, Geske, Longstaff and Schwartz, Leland and Toft as well as Collin-Dufresne and Goldstein. They conclude that Merton and Geske model underpredict credit spreads. Contrary to previous research, they conclude that the rest of the models considerably overpredict credit spreads.

[6] See Elton, Gruber, Agrawal and Mann (2001), Huang and Huang (2000), Collin-Dufresne, Goldstein and Martin (2001), Ericsson and Renault (2001), Perraudin and Taylor (2002), Houweling, Mentink and Vorst (2002), and Yu (2003).

[7] Elton, Gruber, Agrawal and Mann (2001), Huang and Huang (2000), Collin-Dufresne, Goldstein and Martin (2001), Ericsson and Renault (2001), Perraudin and Taylor (2002), Houweling, Mentink and Vorst (2002), Yu (2003).

[8] Duffie and Singleton (1997), Duffee (1999), Das and Tufano (1996), Jarrow, Lando, Turnball (1997), Madan and Unal (1998), Duffie and Lando (2001) are some of the reduced-form models.

short-maturities, but calibration from risk-neutral to actual default probabilities remains a challenge. Structural approaches produce direct estimates of default, but as explored in this paper the standard structural models have difficulty explaining some features of default.

In this chapter, we focus on the comparison of the default predictions of different structural models and we build on Leland's (2004) work.[9] In his study, he compares the EDFs produced by Longstaff and Schwartz (LS) and Leland and Toft (LT) models for A, BBB and B rated bonds. He allows the models to differ only in terms of the default barriers they assume, by choosing common inputs (including asset volatility) across models. He finds that in all rating categories the endogenous default boundary model of Leland and Toft produces real default probabilities which are in line with the historical default probabilities at least for long maturity bonds. For LS model, he concludes that when its exogenous default boundary is chosen so as it matches the recovery rate of the endogenous default boundary of LT model, the model can also accurately predict the observed long-term default probabilities. Nevertheless, Leland (2004) shows that both Leland and Toft and Longstaff and Schwartz models tend to underestimate the short-term default probabilities.

Although, Leland's (2004) study provides useful insights on the impact of the different default boundaries (exogenous or endogenous) on the predicted EDFs, it does not answer what is arguably the more important question, of how well these different models perform as predictors of default, given available observable data on equity values and equity volatility. The main contributions of this study relative to Leland's are as follows. First, we examine how well the LT and LS models predict actual default frequencies when asset volatility differs across the models (whereas Leland (2004) imposes the same asset volatility in all models). For each model, we compute the unobserved asset values and volatility based on observable leverage, equity values and volatility, using an iterative method to obtain the implied parameters. Leland (2004) instead estimates the asset volatilities per rating class by finding the asset volatilities that best match

---

[9] Since this study has been completed, a paper by Tarashev (2005) has evaluated the empirical performance of six structural credit risk models by comparing the probabilities of default they deliver to ex post default rates. He concludes that the probabilities of default implied by some of the models match well the ex post default rates.

the actual default probabilities (apparently at a 10 year horizon). He, then, uses these estimated levels of asset volatility per rating class as common inputs to both LT and LS models for the estimation of default probabilities. His estimated asset volatilities for BBB, A and B rated bonds match the estimated asset volatilities of Schaefer and Strebulaev (2004).[10]

Despite the fact that the asset volatilities used by Leland (2004) are empirically reasonable, as explained they have been calibrated on the LT model to match empirical default over a 10 year horizon, a fair comparison with other models (e.g. LS) would require that the asset volatility for other models are calibrated separately in a similar fashion, something which Leland (2004) does not do. We allow the models to differ in terms of the asset volatility they produce (since asset volatility is unobservable) and calibrate to the observed equity volatilities for each rating class (see below Section 3.5 for a quantitative comparison). The choice of asset value and volatility is particularly critical for the LT model, since the default barrier is endogenous and depends on asset volatility.

A second contribution of our study compared to Leland (2004) is that we examine the sensitivity of the models' predictions to different levels of equity volatility within the same credit rating. This is of interest since equity volatility varies considerably over time. It is therefore worthwhile assessing the sensitivity of the models to changes of the equity volatility as a further key element that affects default probabilities.

The analysis then answers two main questions:

1. How different and accurate are the Expected Default Frequencies (EDFs) produced by different models assuming data input (equity volatility and market value leverage appropriate for BBB, BB and B rated bonds), as we move from the original Merton model to more sophisticated models that incorporate more realistic economic considerations and define the value of assets, volatility of assets and default barrier in a different way?

2. How do different assumptions about equity volatilities within each rating class affect the level and accuracy of EDFs produced by different

[10] Schaefer and Strebulaev (2004) use an empirical sample and estimate asset volatilities for different rating classes.

models? Here we focus on the LS model, since it turns out that the EDFs of the LT model are not very sensitive to equity volatilities.

The analysis of the paper is arranged as follows. The following section describes the models that are used in the comparison (with further technical details provided in Appendix B). Section 3 describes the formulas used for the estimation of the real default probabilities as well as the iterative technique used to estimate the two unobservable variables, the value and the volatility of assets. The estimation of these variables is vital for the prediction of default probabilities. Section 4 provides the parameter values that will be used for the estimation of the real default probabilities for the three models, including the mapping from the different rating categories. Section 5 shows the results and compares the term structure of EDFs produced by the models with the observed real default probabilities reported by Moody's for the BBB, BB and B rated bonds. Moreover, it presents the change in the term structure of EDFs produced by Longstaff and Schwartz model using different assumed equity volatilities to all rating categories of bonds. Finally, Section 6 summarises and concludes, discussing limitations of the analysis as well as routes for further research.

3.2. Theoretical framework

Throughout the chapter, except in Merton case which assumes a zero-coupon bond issue so that default can only occur at bond maturity, the firm is assumed to issue coupon paying finite maturity bonds. Moreover, we will assume that one firm has one risky bond. This implies that the firm has only one class of risky debt. Although this is an unrealistic assumption, it helps us to focus on the questions mentioned above and not on the issues arising from the seniority of bonds. The assumption of the discrete time does not allow us to use in the comparison some other continuous time models such as Anderson, Sundaresan, Tychon or Mella-Barral Perraudin. Hence, we restrict the comparison to three models: Merton model, Longstaff and Schwartz (LS) model and Leland and Toft (LT) model.

The original Merton model assumes that default can occur only at the date of maturity if and only if the value of assets is lower than the face value of debt. Merton (1974) model for coupon bonds would be more appropriate for the direct comparison with the LS and LT models but it cannot be used due to its assumption of infinite maturity debt. Although, we cannot eliminate the fact that the original Merton model assumes that default can only occur at maturity, we can use a modified Merton model in the calculation of default probabilities in order to allow for the estimation of real default probabilities.

As in the original Merton case, in this model, we assume that equity is a call option on the value of assets with strike price the default point. In addition, the default point is determined exogenously and is assumed to be the principal value of debt, P.

The firm's equity value can be represented as follows:

$$V_e = V_a N(d_1) - Pe^{-rT} N(d_2), \qquad [3.1]$$

where

$$d_1 = \frac{\log\left(\frac{V_a}{P}\right) + \left(r + \frac{\sigma_a^2}{2}\right)T}{\sigma_a \sqrt{T}} \qquad [3.2]$$

and

$$d_2 = d_1 - \sigma_a \sqrt{T}, \qquad [3.3]$$

where V_a is the value of assets, P is the total value of debt, T is the time to maturity and σ_a is the asset volatility.

The point where this modified Merton model is differentiated with respect to the original model is the way the Expected Default Frequency is calculated. As explained later in the chapter, instead of calculating the probability of default as $N(-d_2)$, the real default probability for this model is calculated using the Distance to Default (DD) measure that includes the actual drift μ (equals the asset risk premium plus the risk-free interest rate) instead of the risk-free interest rate. This allows us to derive real default probability instead of a risk neutral default probability estimate.

Longstaff and Schwartz model extends the original Merton model in two ways. First of all, they derive a closed form solution for the valuation of risky coupon bond with finite maturity. Moreover, they relax the assumption of constant interest rate and they assume that interest rates are stochastic. In this model, default occurs when the value of assets hits a lower threshold point, which is a fraction of the total value of debt. Moreover, the default barrier is determined exogenously and is equal to a fraction of the face value of debt. In our analysis, the fraction will be assumed to be 0.60 as in Huang and Huang (2003).[11]

In Leland and Toft model although the interest rate is assumed to be constant, the default barrier is determined endogenously. Default occurs when the asset value falls below that barrier. This model is an extension of Black and Cox (1976) and Leland (1994) models, which assume perpetual debt, but this model assumes that debt is continuously rolled over. This assumption implies that the average debt maturity remains constant, which allows us to compare this model with the other models of finite maturity debt.

Moreover, as described in the literature review in this model the decision not to pay the contracted amount and hence to default is made by managers, who try to maximize equity value. The decision to default or not default is at the equityholder's hands since they can meet coupon payments by additional equity contributions even if asset values are low. Hence, the default barrier is determined endogenously and it is the point that maximizes both the value of the equity and the value of the firm subject to the limited liability of equity. Appendix B offers a detailed description of Longstaff & Schwartz as well as Leland and Toft models.

3.3. Calculation of real default probabilities

The objective of this chapter is to calculate the real default probabilities produced by the three structural models. For the modified Merton model, the real default probability is given using the following equation.

---

[11] In their paper, Longstaff and Schwartz assume that that the default boundary is the face value of debt. Nevertheless, it is argued that this assumption is not reasonable since many firms continue to operate even after the asset value falls below the face value of debt. We follow Huang and Huang (2003) and assume that the Longstaff and Schwartz's default boundary is a fraction of the face value of debt, which is assumed to be 0.60. This is an empirically reasonable assumption, given empirical findings of recovery rates and bankruptcy costs for senior bonds (see for example Duffie and Singleton (2003) Figure 6.1).

$$EDF = \Pr\left(V_a \prec P / V_0\right) = \Pr\left(\ln\left(V_a\right) \prec \ln\left(P\right) / V_0\right) =$$

$$\Pr\left[\ln\left(V_0\right) + \left(\mu - \delta - \frac{\sigma_a^{\ 2}}{2}\right)t + \sigma_a \sqrt{t} e_t \prec \ln\left(P\right)\right] =$$

$$\Pr\left[\frac{\ln\left(V_0\right) + \left(\mu - \delta - \frac{\sigma_a^{\ 2}}{2}\right)t}{\sigma_a \sqrt{t}}\right] \prec -e_t =$$

$$N\left[-\frac{\ln\left(\frac{V_0}{P}\right) + \left(\mu - \delta - \frac{\sigma_a^{\ 2}}{2}\right)t}{\sigma_a \sqrt{t}}\right] = N\left(-DD\right) \qquad [3.4]$$

where V_0 is current value of assets, and δ is the payout rate.

It is worthwhile to mention that this model differs from the original Merton model in the way the Expected Default Frequency is calculated. Instead of calculating the probability of default as $N(-d_2)$, the real default probability for this model is calculated using the Distance to Default (DD) measure that includes the actual drift μ (equals the asset risk premium plus the risk-free interest rate) instead of the risk-free interest rate. This is crucial, since failure to include the actual drift μ, will result in a risk-neutral probability of default.

For the Longstaff and Schwartz as well as the Leland and Toft model the default probability can be determined as follows[12]:

$$\begin{aligned} EDF &= N\left(\left(-b - \left(\mu - \delta - 0.5 * \sigma_a^{\ 2}\right) * t\right) / \sigma_a \sqrt{t}\right) + \\ &e^{-2b\left(\mu - \delta - 0.5\sigma_a^{\ 2}\right)/\sigma^2} N\left(\left(-b + \left(\mu - \delta - 0.5 * \sigma_a^{\ 2}\right) * t\right) / \sigma_a \sqrt{t}\right) \end{aligned} \qquad [3.5]$$

where $b = \ln\left(\frac{V_a}{V_b}\right)$, and V_b will denote the default boundary level.

The reason why we cannot use the above equation for the Merton model is because the above equation denotes the cumulative default probability and as it is

[12] Refer to Leland (2004).

explained before Merton model does not give the possibility of default before the maturity day.

It is clear that in order to determine the EDFs, except of the parameter values given in the previous section we need to calculate three more parameters: the default boundary, the value of assets, the volatility of assets.

3.3.1. Calculation of default barrier

For Merton and LS models the determination of the default barrier value is straightforward as described in Section 2. For the LT model the default barrier is determined endogenously and will be calculated together with the value and the volatility of assets. This is summarized in the following Table 1.[13]

Table 1: Default barriers for Merton, Longstaff and Schwartz and Leland and Toft models

Merton-KMV	Face value of debt, P
Longstaff and Schwartz	Fraction of face value of debt: 0.6*P
Leland and Toft	$V_{LT}^{*} = \dfrac{(C/r)(A/(rT)-B)-AP/(rT)-\tau C x/r}{1+\alpha x-(1-\alpha)B}$

3.3.2. Calculation of value and volatility of assets

As it is described above, the term structure of EDFs produced by the three models for three categories of firms, BBB, BB and B rated will be calculated. Additionally, within each rating category we investigate two cases of equity volatility. This means that there are six cases: BBB firms with 25% equity volatility, BBB firms with 30% equity volatility, BB rated firms with 35% equity volatility, BB firms with 40% equity volatility, B rated firms with 45% equity volatility and B rated bonds with 50% equity volatility.

Due to the fact that Merton model does not allow for an early bankruptcy, since default can only happen at maturity, the value and the volatility of assets will be estimated in each case 10 times, for this model. Hence, each point in the graphs, for the Merton model, will represent the probability that a zero-coupon bond with maturity t = 1 to t = 10 will default. This is not the case for the LS and LT

[13] Refer to Appendix B for a detailed description of Leland and Toft model.

models, where the value and volatility of assets is estimated once for each case assuming a ten year debt maturity. This reason together with the fact that we are using different formulas for the calculation of Merton's and LS-LT's default probabilities, explain why EDFs produced by Merton model are not directly comparable to the EDFs produced by LS and LT models.

The estimation of the value and the volatility of assets can be done by solving the following equations simultaneously:

1st equation: The function for the value of equity. This is calculated using equation 3.1 for Merton model and for LT and LS models as follows:

$$V_e = V_a - B, \qquad [3.6]$$

where B is the value of a risky bond and is defined differently between the two structural models. The computations for the risky bonds and equity values are presented analytically in Appendix B for LS and LT models.

2nd equation: From Ito's lemma, we can extract a formula that connects volatility of equity to the volatility of assets. Hence, the second equation used is:

$$\sigma_e = \sigma_a * \frac{V_a}{V_e} * \frac{\partial V_e}{\partial V_a} \qquad [3.7]$$

It is clear that in the above formula the partial derivative of the value of equity to the value of assets will be different across models and will be calculated from the first equation. For the special case of Merton model, since the value of equity can be represented by a call option, the partial derivative of the value of equity to the value of assets is equal to the $N(d_1)$, where d_1 is given by equation 3.2.

3rd equation: The default barrier equation for LT model. As described above this is due to the fact that in LT model the default barrier is determined endogenously.

In the case of Merton and LS models we have two equations and two unknowns and in LT model we solve three equations with 2 unknowns. In all cases, having as inputs all the parameter values described in the following section, the value and the volatility of assets are calculated using an iterative technique in Matlab. In particular, Newton – Raphson iterations are used in order to derive these values. The results from the iterative technique are provided at the end of the paper in Appendix A, while the tables and figures of the estimated probabilities of default are given in Section 5 together with their interpretation.

3.4. Parameter and input values

At this point it is useful to determine the values of the parameters and assumed input values needed for the calculation real default probabilities. Our aim is to select empirically reasonable parameter values, where possible consistent with those typical of those observed for companies within each rating category. Since, the term structure of EDFs produced by the models will be compared with the default probabilities provided by Moody's for bonds over the period of 1970-1997, the chosen parameter values should represent those that applied over that period. In most of the parameter choices, we follow Huang and Huang (2003) and Leland (2004) papers.[14] For several of these parameters a range of values is observed within each rating class. Our goal has been to select representative values for three rating classes (BBB, BB, B) and also to explore the sensitivity to changes in these values, especially to changes in equity volatility.

Consider first those parameters and inputs that are common to all three models:

1. *Value of equity, V_e:* The value of equity for all cases is taken to be 100. This is simply a normalization, although lower rated firms tend to be smaller than higher rated firms, the default predictions of the various models depend only leverage and volatility measured as a proportion of the market value of equity.
2. *Riskless interest rate, r:* this is assumed to be 8%, which is the historical average of Treasury bills for the period of 1973-1998.
3. *Asset risk premium, λ:* This is assumed to be 4%, as in Leland (2004). Alternatively it would be possible to derive the asset risk premium is derived from the equity risk premium, which can in turn be expected to differ according to rating category (lower rated firms might for example have greater correlation with market). We could have assumed different

[14] Data for the observed real default probabilities has been collected from Moody's Investor Services: "Historical default rates of corporate bond issuers, 1920-1997", Special Comment, 1998.

asset risk premiums for the BBB, BB and B rated bonds basing these on an equity pricing model.

4. *Volatility of equity,* σ_e*:* There is no standard information on the equity volatilities of an average firm in different credit ratings. In order to be able to choose reasonable parameters for the equity volatility, daily equity prices of 20 BB rated US bonds and 16 B rated US bonds that are currently traded have been collected from Bloomberg. All these bonds are non perpetual non convertible and non callable bonds, with fixed coupon. In the majority of the firms, daily prices for the last seven years have been collected. Using these daily prices, the annualized volatility is calculated using the standard deviation approach and the Exponential Moving Average approach. The advantage of the latter approach is that it places geometrically declining weights on past observations, thus assigning importance to recent observations. By doing this, it produces smoother time series than the moving average approach. The following formula has been used:

$$\boxed{\sigma_{t-1}^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)\varepsilon_{t-1}^2 \text{, where } \lambda = 0.97}$$

   The results of using the two different methods for the calculation of the annualized volatility are very similar, as shown on Table 1 and Table 2 in Appendix A. The estimates in Tables 1 and 2 of Appendix A are based on a relatively short time series, not necessarily representative of the period 1970-1998 for which we have average default information. We want to allow for some variation relative to the values reported in these tables.

   Hence, for BBB bonds we investigate 25% and 30% equity volatility, for BB bonds we investigate 35% and 40% equity volatility and in the case of B rated bonds we investigate 45% and 50% equity volatility.

5. *Expected return on assets,* μ*:* 12%, which is the sum of risk-free rate and the asset risk premium.

6. *Payout rate,* δ*:* Is assumed to be 6% as in Huang and Huang (2003) paper.

7. *Face value of debt, P:* In our analysis, we use the average leverage ratios reported by Huang and Huang (2003) for BBB, BB and B rated, which are 43.3%, 53.53% and 65.70% respectively. This leverage ratio is the ratio of the market value of debt divided by the market value of assets. It is useful to highlight the fact that the most adequate measure of the face value of debt would be the average book value of liabilities for each rating category, or alternatively a ratio of the face value of debt divided by the market value of assets. Although this is a limitation in our analysis, data on the average book value of liabilities is not publicly available and these parameter values are reasonable to assume. Hence, the face value of debt is assumed to be 43.3, 53.53 and 65.70 in the case of BBB, BB and B rated bonds.

8. *Debt maturity, T:* We will work with bonds with 10 years to maturity (with the exception of the Merton where bond maturity will vary from 1 to 10 years, since EDF at time t in this case is the default probability of a zero-coupon bond maturing at time t)

Other parameter values are required for only one or two models.

9. *Coupon, C:* The coupon is calculated for each rating category as a 10% of the principal value. This variable is used in Leland and Toft model.

10. *Corporate tax rate,* τ *:* 15%, as in Leland (2004). This variable is used in Leland and Toft model.

11. *Fraction of default costs,* α: 30% as in Leland. This variable is needed for the Longstaff and Schwartz and the Leland and Toft models.

12. *Stochastic interest rate parameters, a, β,* η^2, ρ*:* These parameters are taken to be equal to 0.06, 1, 0.001 and -0.25 respectively, as in Longstaff and Schwartz paper. These variables are used only for the Longstaff and Schwartz model.

3.5. Results

Table 2 and Figure 1 present results for the case of BBB bonds with equity volatility of 25%. All three models underestimate the observed EDFs. At the shortest time horizons, of one to three years, the models predict effectively zero default, whereas actual default rates are in the range 0.12-0.75%. At medium term time horizons of four to seven years, default rates continue to be very low in all three models.

The Leland and Toft model continues to predict very low default rates, even at much longer time horizons. The default predictions from the Merton and Longstaff and Schwartz models are very similar, rising to over 1% at an eight year horizon and to nearly 2% at ten year; however these long-horizon default rates are still less than half those observed in the Moody's data.

Table 3 and Figure 2 present the equivalent results, also for BBB bonds, with the higher 35% equity volatility. Comparing with Tables 2 and 3 it is apparent that the default predictions of the Longstaff and Schwart (LS) model increase a lot in response to an increase in equity volatility, the Merton model default predictions also increase but by somewhat less than LS, whereas the default predictions of the Leland and Toft model change very little. There are in other words striking differences in the sensitivity to observed equity volatility.

All three models continue to substantially underestimate the observed EDFs, at shorter and medium term time horizons. Over one to three years, the three models predict very low default rates, still effectively zero for the Merton and LT models, and even the predictions of LS are well below the actual default rates are in the range 0.12-0.75%. At medium time horizons of four to seven years the predictions of the LS model are in line with the data while both the Merton model and especially LT underpredict. At the longest time horizons Longstaff and Schwartz (LS) model overestimate the EDFs, the Merton model is in line with the data and Leland and Toft (LT) model still underpredicts the Moody's real default probabilities.

Table 2: Real default probabilities for BBB bonds with equity volatility 25%

BBB rated bonds, 25% equity volatility				
Year	Merton	Longstaff and Schwartz	Leland and Toft	Observed data (Moody's)
1	0.00%	0.00%	0.00%	0.12%
2	0.00%	0.00%	0.00%	0.39%
3	0.01%	0.02%	0.00%	0.75%
4	0.05%	0.08%	0.00%	1.26%
5	0.22%	0.23%	0.01%	1.70%
6	0.40%	0.46%	0.03%	2.19%
7	0.62%	0.77%	0.06%	2.74%
8	1.16%	1.13%	0.11%	3.29%
9	1.49%	1.53%	0.16%	3.91%
10	1.81%	1.95%	0.22%	4.53%

Figure 1: Comparison of the term structure of EDFs produced by the different structural models with the term structure of real observed default probabilities reported by Moody's: case of BBB bonds with 25% equity volatility

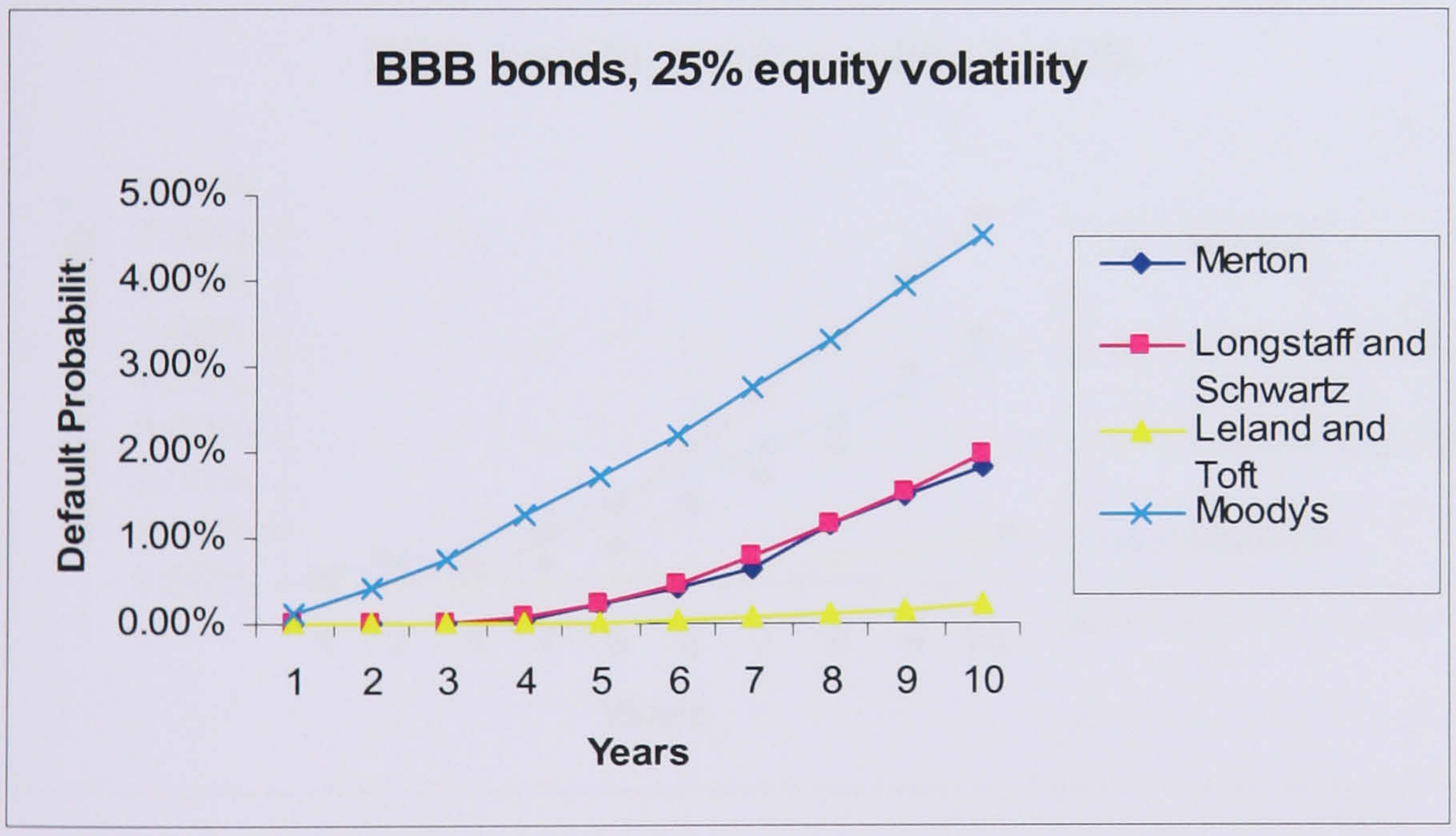
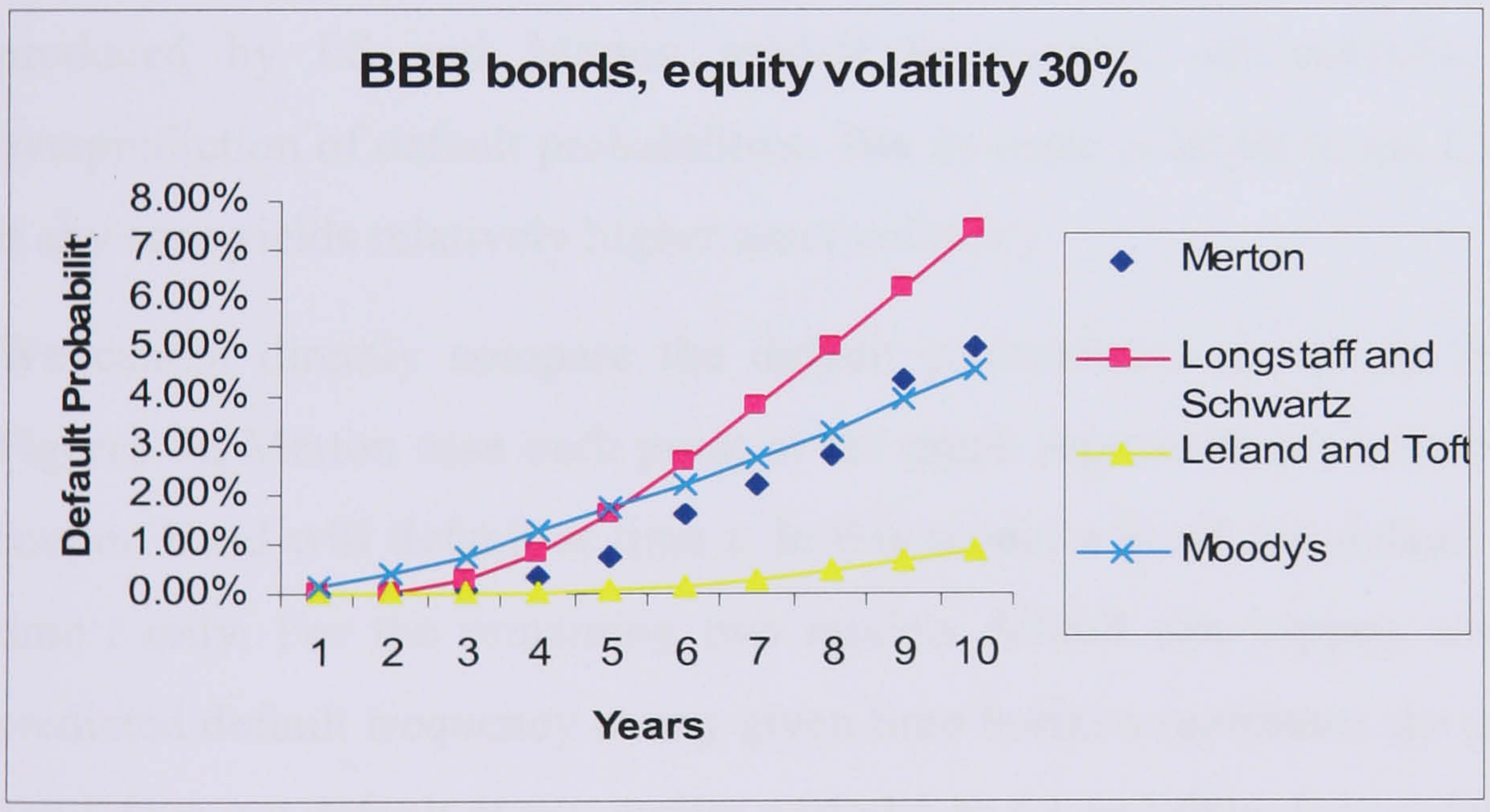


Table 3: Real default probabilities for BBB bonds with equity volatility 30%

BBB rated bonds, 30% equity volatility				
Year	Merton	Longstaff and Schwartz	Leland and Toft	Observed data (Moody's)
1	0.00%	0.00%	0.00%	0.12%
2	0.00%	0.02%	0.00%	0.39%
3	0.06%	0.24%	0.00%	0.75%
4	0.35%	0.78%	0.02%	1.26%
5	0.72%	1.62%	0.07%	1.70%
6	1.58%	2.65%	0.16%	2.19%
7	2.20%	3.80%	0.29%	2.74%
8	2.83%	5.00%	0.46%	3.29%
9	4.32%	6.21%	0.65%	3.91%
10	5.02%	7.41%	0.86%	4.53%

Figure 2: Comparison of the term structure of EDFs produced by by the different structural models with the term structure of real observed default probabilities reported by Moody's: case of BBB bonds with 30% equity volatility



Tables 4-5 and Figures 3-4 present results for bonds with BB characteristics. For the BB rated bonds when the volatility of a firm is 35%, LS model produces a term structure of default that is quite closely in line with the observed one (Table 4 and Figure 3). Only in the first years does it underpredict the default probabilities. Increasing the equity volatility to 40% the LS models then overpredicts EDFs for horizons of three years and over (Table 5 and Figure 4). The Merton model provides an accurate prediction only for the ten year horizon, at shorter horizons it underpredicts. Finally the LT model substantially underpredicts probabilities of default at all time horizons.

The reason why LT model produces such low probability of defaults (PDs) is that the endogenous default boundary is well below the face value of liabilities. The default rates predicted by the LT model do not alter greatly as we increase the equity volatility. This is due to the fact that the asset volatility increase generated by the model is offset by the decrease in the default barrier. The default barrier, chosen optimally by the firm, is reduced in order to provide greater protection against the volatility of assets. Hence, the net effect on the predicted default probability is quite small. In contrast, an increase in the volatility causes the term structure of EDFs produced by LS and Merton models to increase substantially, leading to an overprediction of default probabilities. The increase is larger in the LS model because it any case yields relatively higher asset volatility.

We cannot directly compare the default probabilities shown in these Tables and Figures. In Merton case each point at the graph represents the probability that a zero coupon bond will default at time t. In this model a bond can default at time *t* and at time *t* only. For the remaining two models default can happen earlier than *t*. The predicted default frequency at any given time horizon represents the probability that a bond does *not* default at any earlier period 1 to *t*-1 and then does default at period *t*. If the Merton model were instead applied to one-period bonds, with companies that avoid default rolling their bond issue over to each successive period, it would produce lower predicted default frequencies at longer time horizons than shown in these tables. This is because, in this situation of bond rollover, bonds could not default in more than one period. In effect there is an element of double-counting in the results reported here for the Merton model.

Table 4: Real default probabilities for BB bonds with equity volatility 35%

BB rated bonds, 35% equity volatility				
Year	Merton	Longstaff and Schwartz	Leland and Toft	Observed data (Moody's)
1	0.00%	0.00%	0.00%	1.34%
2	0.08%	0.93%	0.00%	3.71%
3	0.71%	3.31%	0.00%	6.21%
4	1.68%	6.42%	0.14%	8.77%
5	3.37%	9.68%	0.36%	11.44%
6	5.59%	12.84%	0.67%	13.72%
7	7.04%	15.80%	1.04%	15.53%
8	8.25%	18.54%	1.47%	17.44%
9	10.89%	21.07%	1.92%	19.19%
10	13.50%	23.38%	2.39%	20.88%

Figure 3: Comparison of the term structure of EDFs produced by the different structural models with the term structure of real observed default probabilities reported by Moody's: case of BB bonds with 35% equity volatility

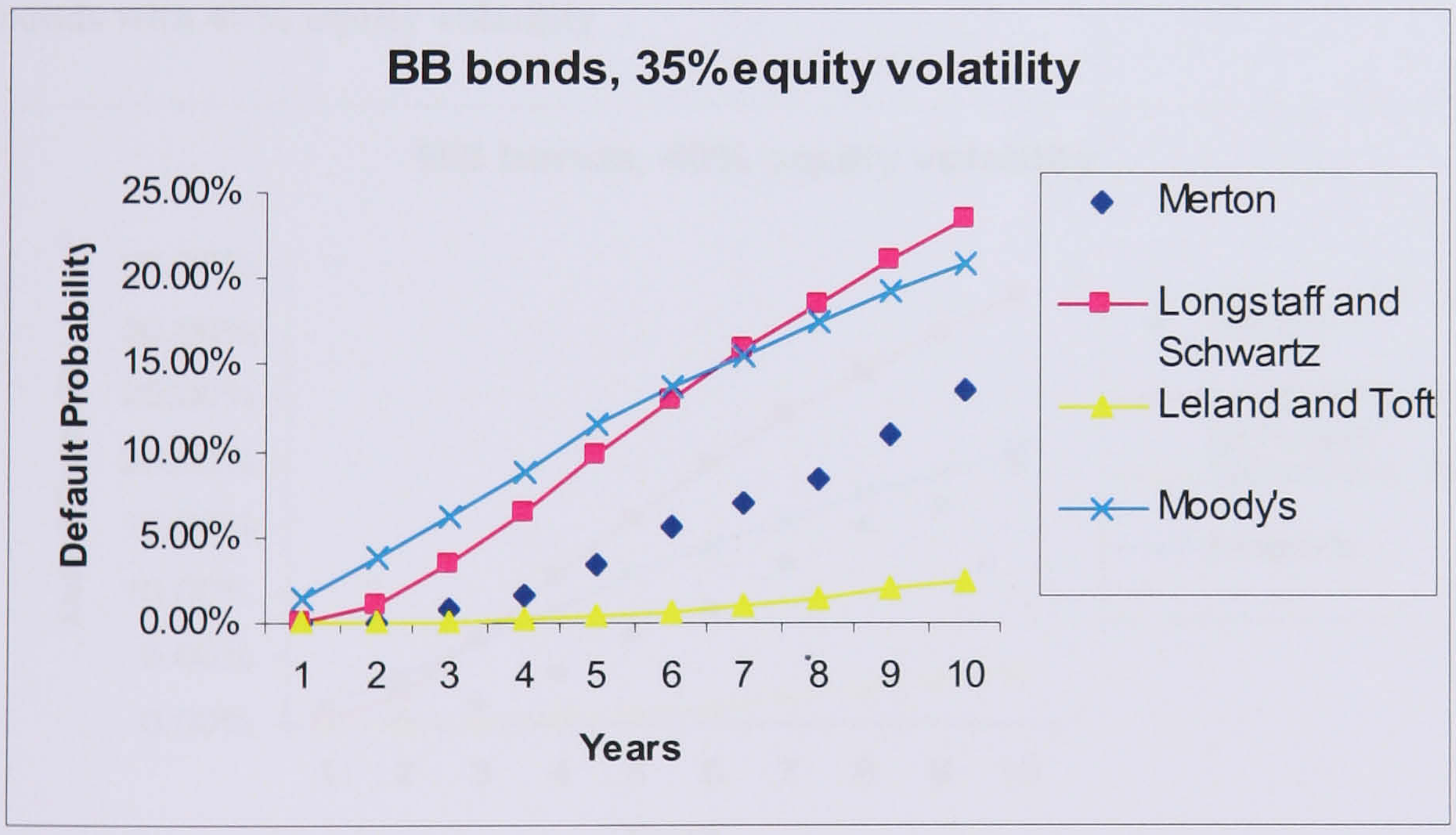
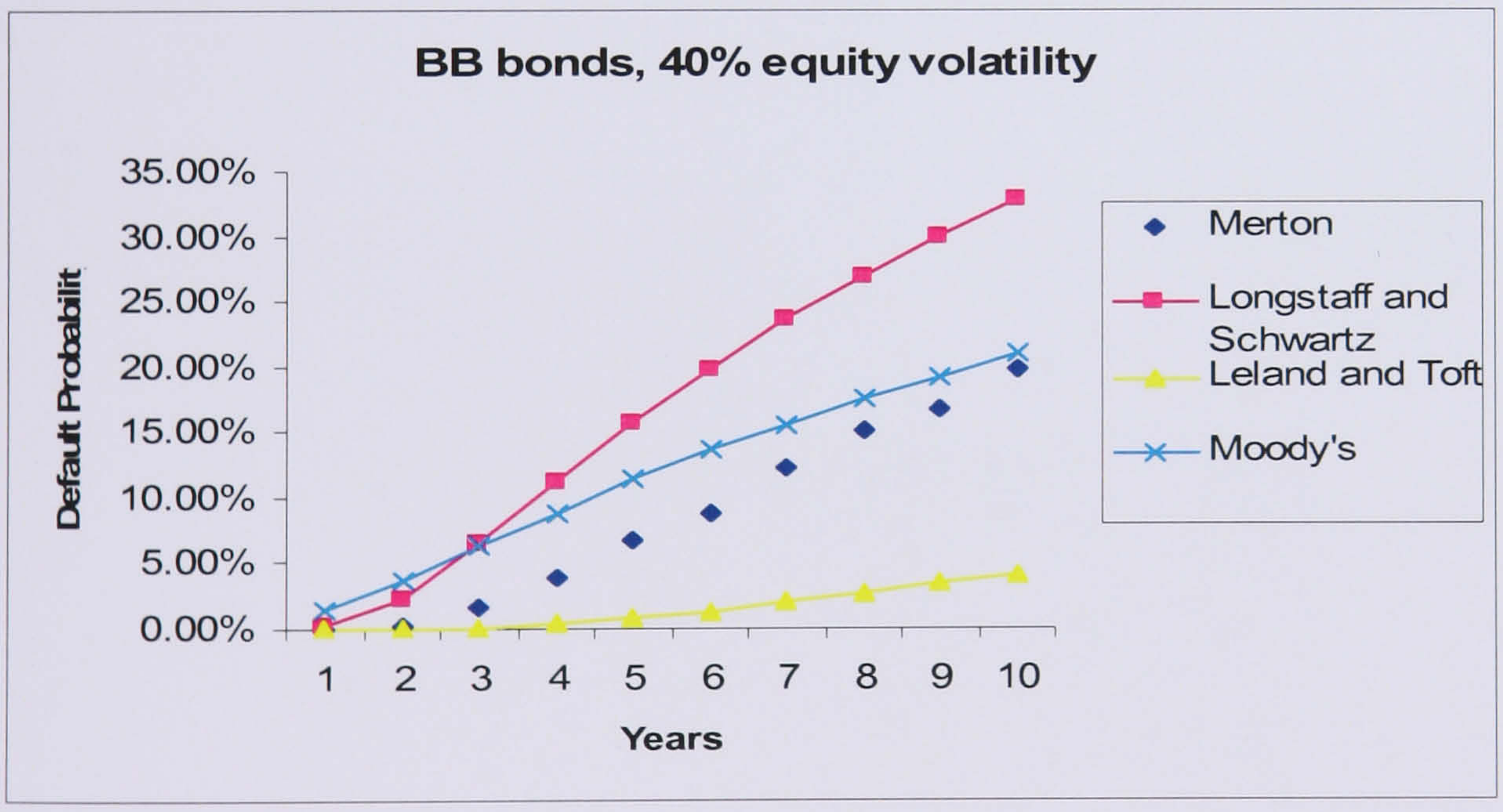


Table 5: Real default probabilities for BB bonds with equity volatility 40%

BB rated bonds, 40% equity volatility				
Year	Merton	Longstaff and Schwartz	Leland and Toft	Observed data (Moody's)
1	0.01%	0.12%	0.00%	1.34%
2	0.30%	2.31%	0.01%	3.71%
3	1.62%	6.50%	0.09%	6.21%
4	3.84%	11.18%	0.33%	8.77%
5	6.80%	15.69%	0.75%	11.44%
6	8.84%	19.83%	1.30%	13.72%
7	12.20%	23.56%	1.95%	15.53%
8	15.10%	26.91%	2.64%	17.44%
9	16.70%	29.94%	3.36%	19.19%
10	19.75%	32.67%	4.08%	20.88%

Figure 4: Comparison of the term structure of EDFs produced by the different structural models with the term structure of real observed default probabilities reported by Moody's: case of BB bonds with 40% equity volatility



Our final set of results are shown in Tables 6-7 and Figures 5-6, for bonds with the features of B rated bonds. The LS model now overpredicts EDFs from year three onwards, for the case of both low equity volatility (45%) and high equity volatility (50%). The Merton and LT models, on the other hand, underpredict default at all time horizons. Once again the LT model predicts exceptionally low default rates. The Merton model however comes fairly close to actual default probability at the longest time horizons of eight to ten years.

Table 6: Real default probabilities for B bonds with equity volatility 45%

B rated bonds, 45% equity volatility				
Year	Merton-KMV	Longstaff and Schwartz	Leland and Toft	Observed data (Moody's)
1	0.05%	2.04%	0.00%	6.78%
2	1.40%	10.84%	0.03%	13.19%
3	4.57%	19.84%	0.24%	19.13%
4	10.60%	27.37%	0.74%	24.11%
5	12.54%	33.54%	1.46%	28.59%
6	16.65%	38.65%	2.32%	32.56%
7	20.68%	42.94%	3.25%	35.91%
8	24.36%	46.60%	4.20%	38.62%
9	27.94%	49.76%	5.15%	41.02%
10	31.41%	52.52%	6.07%	43.85%

Figure 5: Comparison of the term structure of EDFs produced by the by the different structural models with the term structure of real observed default probabilities reported by Moody's: case of B bonds with 45% equity volatility

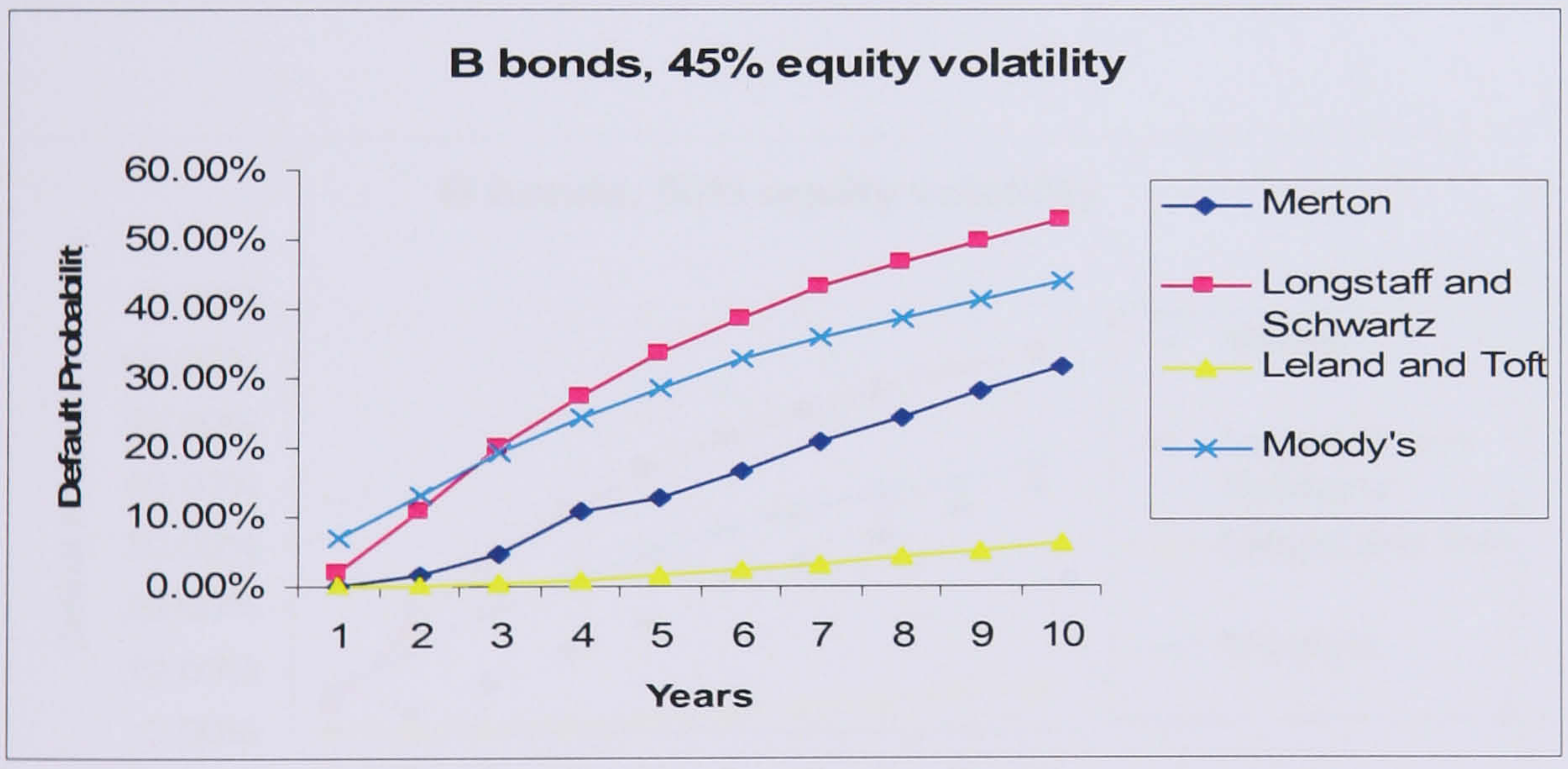
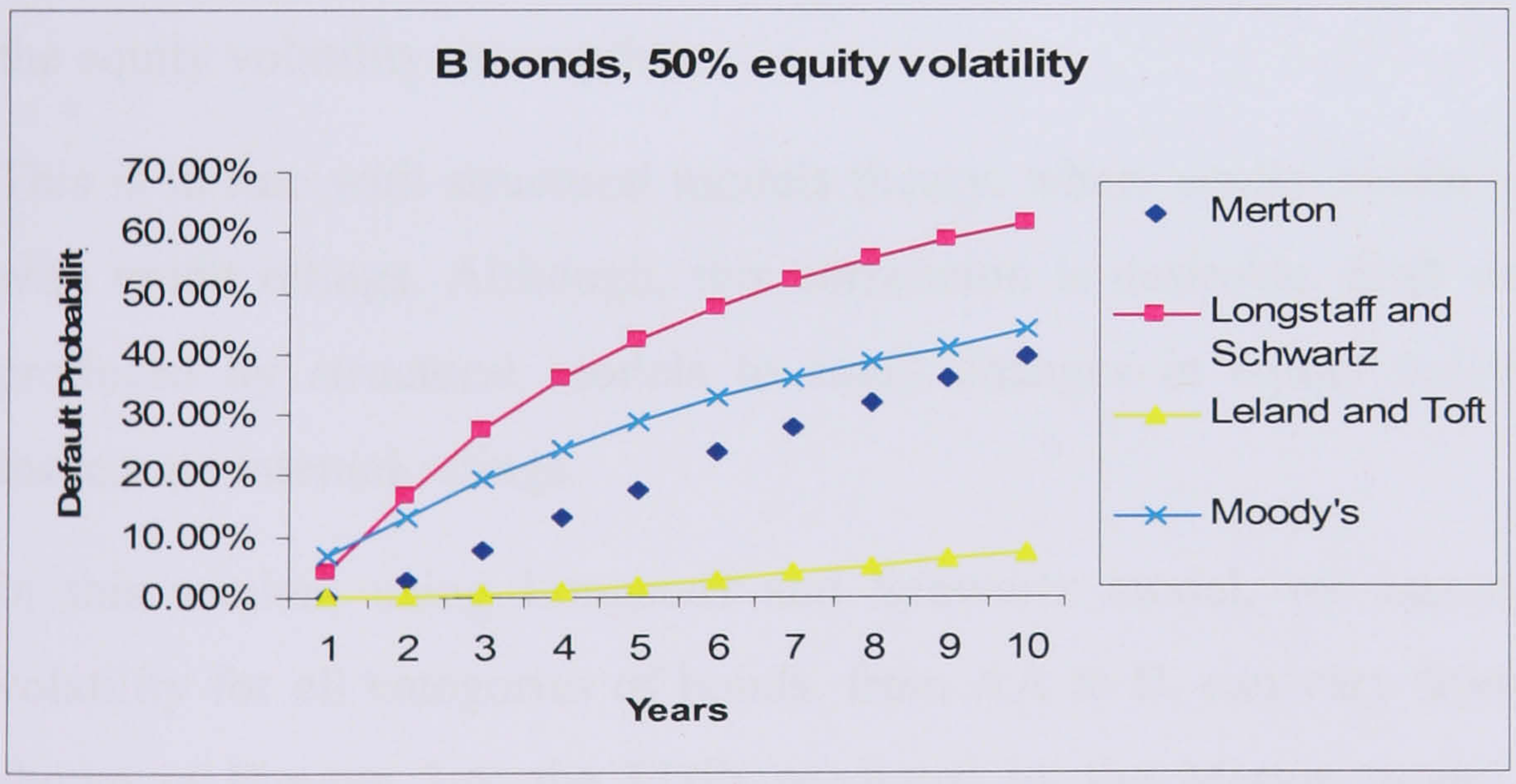


Table 7: Real default probabilities for B bonds with equity volatility 50%

B rated bonds, 50% equity volatility				
Year	Merton	Longstaff and Schwartz	Leland and Toft	Observed data (Moody's)
1	0.17%	4.20%	0.00%	6.78%
2	2.56%	16.58%	0.04%	13.19%
3	7.72%	27.35%	0.33%	19.13%
4	12.85%	35.74%	0.95%	24.11%
5	17.68%	42.34%	1.82%	28.59%
6	23.62%	47.66%	2.85%	32.56%
7	28.03%	52.04%	3.94%	35.91%
8	31.97%	55.72%	5.05%	38.62%
9	35.95%	58.87%	6.14%	41.02%
10	39.52%	61.58%	7.20%	43.85%

Figure 6: Comparison of the term structure of EDFs produced by the by the different structural models with the term structure of real observed default probabilities reported by Moody's: case of B bonds with 50% equity volatility



Comparing our results to the findings of Leland (2004), we arrive at a different conclusion concerning the ability of the LT and LS models to accurately predict default probabilities. As mentioned in the previous sections, the main difference between our study and Leland's work is that we use different approaches in estimating asset volatilities. While, Leland (2004) estimates asset volatilities per rating class by estimating the asset volatility that best matches the actual default probabilities and refers to the study of Schaefer and Strebulaev (2004) to confirm his findings, we estimate asset volatilities per rating class from observable data on equity volatility and leverage. Hence, we allow the asset volatility to differ across the models. This difference in the estimation is one of the causes of the difference between our findings and Leland's. Leland (2004) assumes that the asset volatility of a BBB bond is 23% while the asset volatility of a B rated bond is 32%. Using our estimation technique we find that for BBB bonds the estimated asset volatility by the LS model is on average 25% while for LT model is 19%, i.e. for both models we obtain asset volatilities that are fairly close to that assumed by Leland (2004). But we obtain much more different asset volatilities for B rated bonds, where our estimated asset volatility for LS model is around 45% while for LT model it is around 24%.

3.5.1. Term structure of EDFs under different observed equity volatilities

In the first part of this section, we showed that from the three structural models, Longstaff and Schwartz is the one to produce EDFs that are more in line with the observed ones. Nevertheless, except of Leland and Toft model, that produces quite stable EDFs using the different equity volatility assumptions, the other models and especially Longstaff and Schwartz model produce quite volatile EDFs, depending on the equity volatility assumption.

This is in line with structural models theory, where equity volatility should correlate with credit ratings. Although, this correlation is desirable, high sensitivity of EDFs produced by structural models to small changes in equity volatility may lead to inaccurate internal ratings.

In this section, using Longstaff and Schwartz model, we assume that the equity volatility for all categories of bonds, from AA to B, can vary from 30% to 50% (as shown in Figures 3-6, the EDFs predicted by the Merton model exhibit a similar sensitivity to equity volatility and Longstaff and Schwartz, but those of the LT model are almost unaffected). Figure 7, presents the resulting term structure of EDFs for AA, A and BBB rated bonds and demonstrates that the generated default probability is greatly affected by different equity volatilities of rated bonds especially in the long term. Similar is the result from Figure 8, where the term structure of EDFs for BBB, BB and B rated bonds is presented.

Figure 7: Term structure of EDFs of AA, A and BBB bonds using Longstaff and Schwartz model

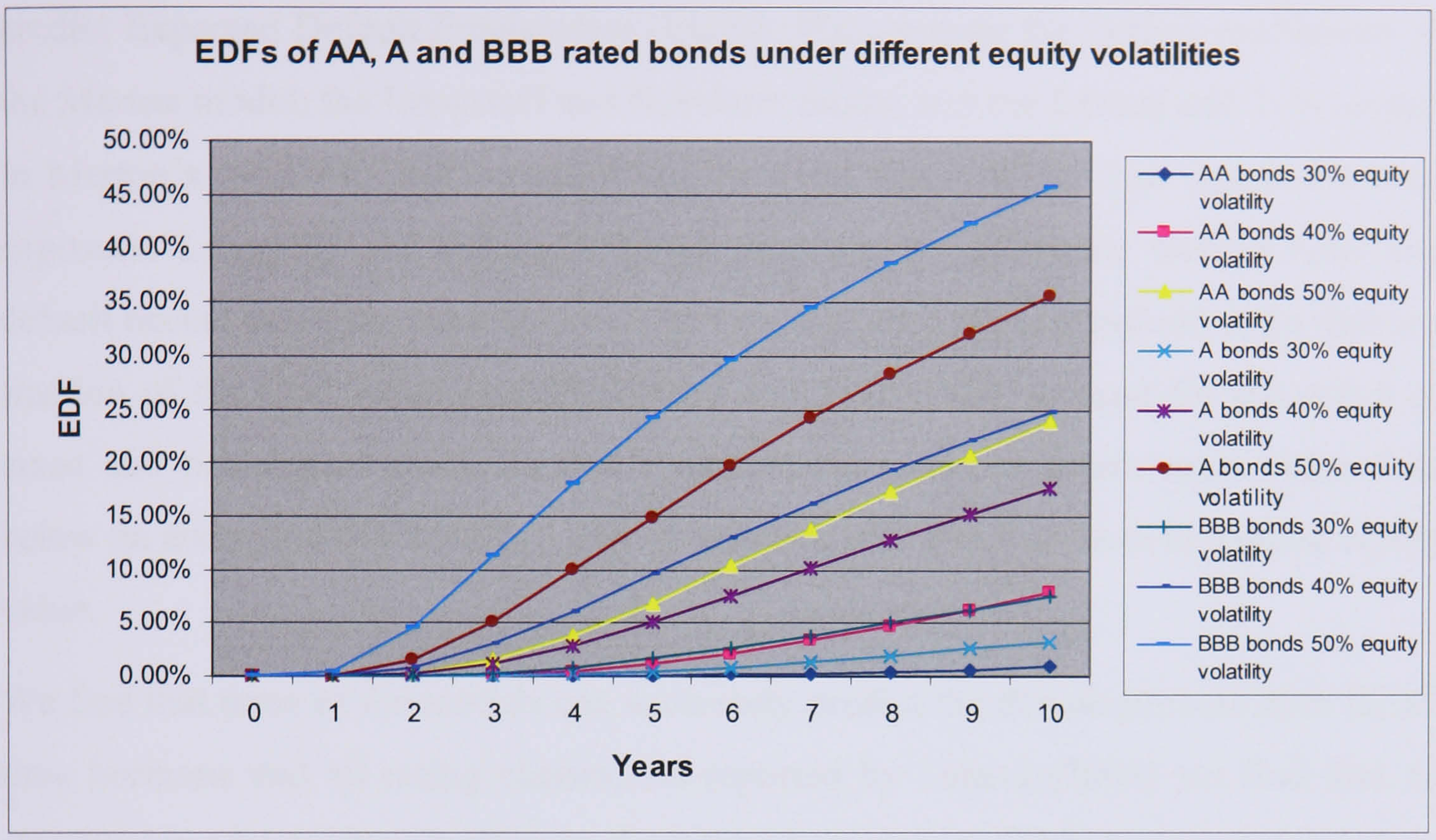
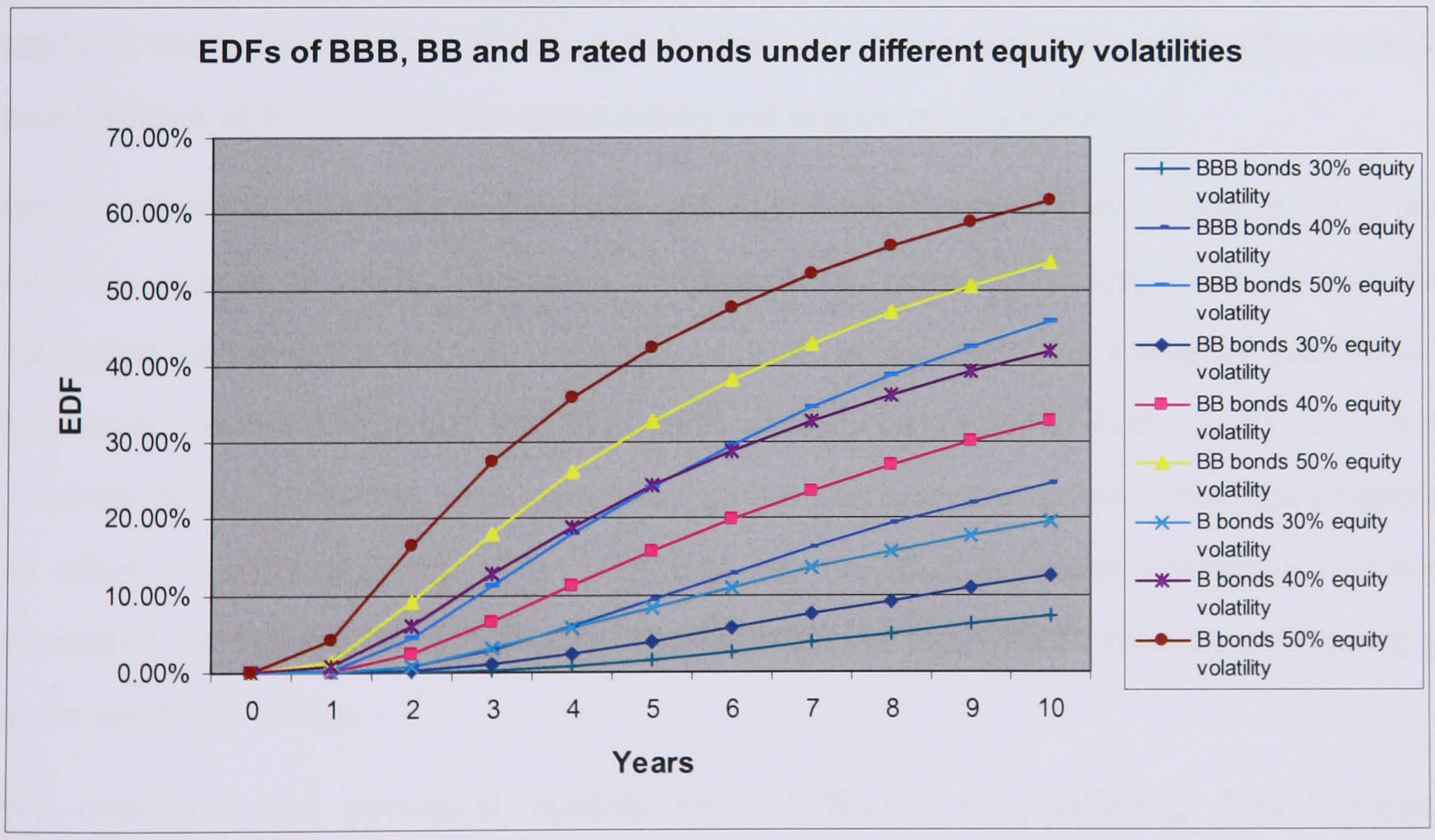


Figure 8: Term structure of EDFs of BBB, BB and B bonds using Longstaff and Schwartz model



3.6. Conclusions and further research

This paper has examined the ability of three structural models of corporate bonds to predict Expected Default Frequencies (EDFs). We compare the default predictions of the Merton model, the Longstaff and Schwartz model and the Leland and Toft model. In Merton's model default occurs when the asset value of the firm equals the total exposure. Longstaff and Schwartz model accounts for stochastic interest rates and default occurs when the asset value of the firm falls to a lower threshold point that is a fraction of the total value of debt. Leland and Toft model account for the effect of taxes and bankruptcy costs. In this model, default occurs where asset value falls below an endogenously specified barrier which is calculated so as to maximize equity value.

We find that none of the models can accurately predict the default probabilities for all time horizons and all rating classes. As reported by Leland (2004) we find that no structural model predicts well over short time horizons, possibly because of jumps in asset values over shorter time horizons. Longstaff and Schwartz model produces default predictions in line with observed data for a time horizons of over three years, but only for the case of BB bonds with 35% equity volatility. The model however, tends to overestimate the default probabilities of riskier bonds as well as the default probabilities of bonds with the same rating but higher equity volatility.

On the other hand, Merton and Leland and Toft model tend to always underpredict the EDFs in almost all cases. The main problem with Leland and Toft model is that the estimated endogenous default boundary is well below the face value of liabilities.. Moreover, unlike Longstaff and Schwartz model, Leland and Toft model does not generate highly different term structures as we increase the equity volatility keeping all other variables constant. This is due to the fact that the asset volatility increase generated by the model is offset by the decrease in the default barrier, producing a quite small net effect.

We conclude that structural models have difficulty in predicting default rates, especially at short time horizons, similar to the observed ones. This finding is in line with the earlier literature showing that structural models are not adequate for explaining observed credit spreads. While observed credit spreads might be explained by special features such as taxes and liquidity, these alternative explanations cannot explain the poor performance of structural models in predicting real default probabilities.

A further finding from this study is that structural models, especially Longstaff and Schwartz model, are highly sensitive to changes in the equity volatility. This is an important result since it suggests that banks should be cautious on the use of structural models for the calculation of their capital. The high sensitivity of the models to changes in the equity volatility will result on a high volatile assessment of the regulatory capital. This also suggests that predicted default rates will vary substantially over the business cycle, since in economic downturns both equity values and equity volatility can rise; however we do not explore this aspect of the models further since none of the models predict well over very short time horizons.

This study could be extended in several ways. First we agree with Leland (2004) that allowing for jumps in the asset values, would be useful in order to improve the performance of structural models in the prediction of real default probabilities over shorter time horizons. Another possible extension of our study would be the inclusion of a model of stochastic asset volatility e.g. as in GARCH specification, into a basic structural model. Also, since these are non-linear models, it would be appropriate to run simulations based on a joint distribution of parameter values rather than sample averages since this could increase average predicted EDFs. An attractive feature of the Leland and Toft model is that its default predictions are less sensitive with respect to observed equity volatilities, but the default rule appears to be much too tightly specified. Some generalization of their model with an endogenous default barrier that depends also on market conditions such as liquidity and hence generates higher EDFs may merit further study..

Appendix A

Appendix: Table 1: Average equity volatility of BB rated bonds collected from Bloomberg, using standard deviation and exponential moving average approaches.

BB rated bonds		
Firms	Standard deviation	Exponential Moving Average
FMC Corporation	31%	31%
Fresenius Med	44%	44%
Corning Inc	56%	54%
Hasbro	39%	41%
Starwood hotels	40%	38%
Hercules	35%	36%
JLG Industries	54%	54%
Millipore Corporation	37%	38%
NVR Inc	38%	38%
PEP boys	47%	47%
Potlatch Corporation	27%	27%
Pope and Talbot	39%	39%
R J Reynolds	46%	42%
Ryland Group	29%	31%
Smithfield foods	41%	42%
Saks Inc	48%	48%
Solectron Corporation	56%	56%
Service Corp Int	56%	57%
Tenet Healthcare	42%	37%
Avista Corporation	36%	38%
Average	41%	40%

Appendix: Table 2: Average equity volatility of B rated bonds collected from Bloomberg, using standard deviation and exponential moving average approaches.

B rated bonds		
Firms	Standard deviation	Exponential Moving Average
Amkor Tech Inc	86%	92%
Building material corporation	54%	54%
Continental Airlines	55%	50%
Great Atlantic and Pacific	41%	40%
IMC Global Inc	40%	40%
Interface	62%	60%
Northwest Airlines	55%	54%
Owens-Ill	49%	50%
Pioneer Standard	53%	53%
Rite Aid Corporation	59%	60%
Shopko Stores	47%	45%
Solutia	47%	54%
Williams Cos	53%	45%
Xerox Corp	51%	51%
CMS Energy	28%	26%
El Paso	41%	36%
Average (excluding Amkor)	51%	50%
Average (including Amkor)	54%	53%
Average total	51%	51%
Average total (excl Amkor)	49%	48%

Appendix: Table 3: Iteration results of Merton model in case of BBB bonds

Value and volatility of assets for Merton model: case BBB bonds with 25% equity volatility			Value and volatility of assets for Merton model: case BBB bonds with 30% equity volatility		
Years	Value of assets	Volatility of assets	Years	Value of assets	Volatility of assets
1	134	18%	1	140	21%
2	137	18%	2	137	22%
3	134	19%	3	134	22%
4	132	19%	4	131	23%
5	129	19%	5	129	23%
6	127	20%	6	127	24%
7	125	20%	7	125	24%
8	123	20%	8	123	24%
9	121	21%	9	121	25%
10	119	21%	10	119	25%

Appendix: Table 4: Iteration results of Merton model in case of BB bonds

Value and volatility of assets for Merton model: case BB bonds with 35% equity volatility			Value and volatility of assets for Merton model: case BB bonds with 40% equity volatility		
Years	Value of assets	Volatility of assets	Years	Value of assets	Volatility of assets
1	149	23%	1	149	27%
2	146	24%	2	146	27%
3	142	25%	3	142	28%
4	138	25%	4	139	29%
5	136	26%	5	135	30%
6	133	27%	6	132	30%
7	130	27%	7	128	31%
8	128	27%	8	127	32%
9	125	28%	9	125	32%
10	123	29%	10	123	33%

Appendix: Table 5: Iteration results of Merton model in case of B bonds

Value and volatility of assets for Merton model: case B bonds with 45% equity volatility			Value and volatility of assets for Merton model: case B bonds with 50% equity volatility		
Years	Value of assets	Volatility of assets	Years	Value of assets	Volatility of assets
1	161	28%	1	161	31%
2	156	29%	2	156	32%
3	151	30%	3	151	34%
4	147	31%	4	146	35%
5	143	32%	5	142	36%
6	139	33%	6	138	38%
7	135	34%	7	134	39%
8	132	35%	8	131	40%
9	129	36%	9	127	41%
10	126	37%	10	124	42%

Appendix: Table 6: Iteration results for the value and volatility of assets for LS and LT models in case of BBB bonds

	BBB rated bonds, 25% equity volatility			BBB rated bonds, 30% volatility		
	Value of assets	Volatility of assets	Default barrier	Value of assets	Volatility of assets	Default barrier
Longstaff and Schwartz	107	23%		107	28%	
Leland and Toft	139	18%	36	140	21%	34

Appendix: Table 7: Iteration results for the value and volatility of assets for LS and LT models in case of BB bonds

	BB rated bonds, 35% equity volatility			BB rated bonds, 40% volatility		
	Value of assets	Volatility of assets	Default barrier	Value of assets	Volatility of assets	Default barrier
Longstaff and Schwartz	107	33%		107	37%	
Leland and Toft	150	22%	42	150	24%	41

Appendix: Table 8: Iteration results for the value and volatility of assets for LS and LT models in case of B bonds

	B rated bonds, 45% equity volatility			B rated bonds, 50% volatility		
	Value of assets	Volatility of assets	Default barrier	Value of assets	Volatility of assets	Default barrier
Longstaff and Schwartz	107	42%		107	47%	
Leland and Toft	162	24%	50	162	25%	49

Appendix B

Longstaff and Schwartz model

According to Longstaff and Schwartz model, the price of a risky discount bond is:

$$B(X,r,T)=D(r,T)-\alpha * D(r,T) * Q(X,r,T),$$

where α is the recovery rate, X is the ratio of the value of assets to the default barrier, r is the risk-free rate, T is the maturity date and D(r,T) is the value of a riskless discount bond and can be represented by the following expression:

$$D(r,T)=\exp(A(T)-B(T) * r),$$

where

$$A(T)=\left(\frac{\eta^2}{2*\beta^2}-\frac{a}{\beta}\right)*T+\left(\frac{\eta^2}{\beta^3}-\frac{a}{\beta^2}\right)*(\exp(-\beta*T)-1)-\left(\frac{\eta^2}{4*\beta^3}\right)*(\exp(-2*\beta*T)-1)$$

and

$$B(T)=\frac{1-\exp(-\beta*T)}{\beta}.$$

where a, β, η^2, and ρ are stochastic interest rate parameters

In order to derive the solution of a risky discount bond, Q should also be defined. Hence,

$$Q(X,r,T,n)=\sum_{i=1}^{n} q_i,$$

where

$$q_1=N(a_1),$$

$$q_i=N(a_i)-\sum_{j=1}^{i-1}\left(q_j * N(b_{ij})\right),$$ where i=2, 3, ……., n

$$a_i=\frac{-\ln(X)-M(iT/n,T)}{\sqrt{S(iT/n)-S(jT/n)}}.$$

$$b_{ij} = \frac{M(jT/n,T) - M(iT/n,T)}{\sqrt{S(iT/n) - S(j*T/n)}}$$

and where,

$$M(t,T) = \left(\frac{a-\rho\sigma\eta}{\beta} - \frac{\eta^2}{\beta^2} - \frac{\sigma^2}{2}\right)t + \left(\frac{\rho\sigma\eta}{\beta^2} + \frac{\eta^2}{2*\beta^3}\right)\exp(-\beta T)(\exp(\beta t)-1)$$

$$+\left(\frac{r}{\beta} - \frac{a}{\beta^2} + \frac{\eta^2}{\beta^3}\right)(1-\exp(-\beta t)) - \left(\frac{\eta^2}{2*\beta^3}\right)\exp(-\beta T)(1-\exp(-\beta t))$$

and

$$S(t) = \left(\frac{\rho\sigma\eta}{\beta} + \frac{\eta^2}{\beta^2} + \sigma^2\right)t + \left(\frac{\rho\sigma\eta}{\beta^2} + \frac{2\eta^2}{\beta^3}\right)(1-\exp(-\beta t)) + \left(\frac{\eta^2}{2\beta^3}\right)(1-\exp(-2\beta t))$$

The value of a risky coupon bond will be the sum of discount bonds.

Leland and Toft

In Leland and Toft model, the valuation of a risky coupon bond as well as the formula for the endogenously determined default barrier is determined as follows:

The value of a risky coupon bond is:

$$D = \frac{C}{r} + \left(P - \frac{C}{r}\right)\left(\frac{1-e^{-rT}}{rT} - I(T)\right) + \left((1-\alpha)V_b - \frac{C}{r}\right)J(T)$$

Where α is the fraction of the assets that will be lost in case of bankruptcy, C is the coupon, r is the risk free rate, V_b is the default boundary, T is the maturity date

and

$$I(T) = \frac{1}{rT}\left(G(T) - e^{-rT}F(T)\right)$$

and

$$J(T) = \frac{1}{z\sigma\sqrt{T}}\left(-\left(\frac{V}{V_b}\right)^{-a+z} N(q_1(T))*q_1(T) + \left(\frac{V}{V_b}\right)^{-a-z} N(q_2(T))*q_2(T)\right)$$

where,

$$F(t) = N(h_1(t)) + \left(\frac{V}{V_b}\right)^{-2a} N(h_2(t))$$

$$G(t) = \left(\frac{V}{V_b}\right)^{-a+z} N(q_1(t)) + \left(\frac{V}{V_b}\right)^{-a-z} N(q_2(t))$$

where,

$$q_1(t) = \frac{(-b - z\sigma^2 t)}{\sigma\sqrt{t}},\ q_2(t) = \frac{(-b + z\sigma^2 t)}{\sigma\sqrt{t}};$$

$$h_1(t) = \frac{(-b - a\sigma^2 t)}{\sigma\sqrt{t}},\ h_2(t) = \frac{(-b + a\sigma^2 t)}{\sigma\sqrt{t}}$$

$$a = \frac{(r - \delta - (\sigma^2/2))}{\sigma^2},\ b = \ln\left(\frac{V_a}{V_b}\right),\ z = \frac{\left[(a\sigma^2)^2 + 2r\sigma^2\right]^{0.5}}{\sigma^2}$$

Therefore, in this model the value of debt equals the discounted expected value of the coupon flow plus the expected discounted value of the repayment of the principal plus the expected discounted value of the fraction of the assets which will go to debt with maturity t, if bankruptcy occurs.

The default barrier is determined endogenously and it is the point that maximizes both the value of the equity and the value of the firm subject to the limited liability of equity. Hence it is defined as:

$$V_b = \frac{(C/r)(A/(rT) - B) - AP/(rT) - \tau C x/r}{1 + \alpha x - (1-\alpha)B}$$

where,

$$A = 2ae^{-rT} N(a\sigma\sqrt{T}) - 2zN(z\sigma\sqrt{N}) - \frac{2}{\sigma\sqrt{T}} n(z\sigma\sqrt{T}) + \frac{2e^{-rT}}{\sigma\sqrt{T}} n(a\sigma\sqrt{T}) + (z - a)$$

and

$$B = -\left(2z + \frac{2}{z\sigma^2 T}\right) N(z\sigma\sqrt{T}) - \frac{2}{\sigma\sqrt{T}} n(z\sigma\sqrt{T}) + (z - a) + \frac{1}{z\sigma^2 T}$$

Last but not least the value of equity can be determined as follows:

$V_e = U - D$,

where

$U = V_a + \frac{\tau C}{r}\left[1 - \left(\frac{V_a}{V_b}\right)^{-x}\right] - \alpha V_b \left(\frac{V_a}{V_b}\right)^{-x}$ which is the total value of the firm. It is apparent from the equation that equals the asset value plus the value of tax benefits less the value of bankruptcy costs.

Note that $x = a + z$.

CHAPTER 4: Distance to Default as a predictor of bank credit ratings

Abstract of Chapter 4

This chapter examines whether information from equity markets, as summarized in the Distance-to-Default measure derived from Merton-MoodysKMV (MKMV), provides useful additional information for predicting changes in bank credit ratings. We use the BankScope banking accounting database together with Bloomberg to build a dataset comprising 98 equity listed banks, from 8 English-speaking and Scandinavian countries, with annual accounting, daily ratings and equity price data from 1997 to 2004. We divide bank ratings into four broad credit rating classes. We then build an ordered probit model of the current credit rating class, incorporating both accounting ratios and a Merton-type measure of distance to default. We find that distance to default has additional explanatory power for modeling current ratings, or predicting credit rating changes over a 6-month or 12-month horizon, but only for the smaller sized banks. We find no evidence that changes in distance-to-default have additional explanatory power for predicting rating categories, regardless of the size of the bank.

4.1. Introduction

This work addresses the usefulness of 'distance to default' as a measure of bank credit quality. The objective is to determine whether a 'Merton' style measure of distance to default, similar to the one proposed by the widely used Moody's KMV Credit Monitor, can improve the modeling and prediction of bank ratings and rating transitions in a sample of developed country banks. Bank supervisors are very interested in the use of such market based indicators as early warnings of bank fragility, especially in situations where accounting variables are available only after a long delay or are thought not provide a complete picture of the financial strength of an institution. But many questions remain unresolved. In particular are such indicators best regarded as convenient summaries of information on the soundness of a financial institution, information which could be obtained from the study of bank accounting statements or do they contain independent information not contained in accounting statements?

This analysis reported here extends work by Gropp et al (2006) who examined the ability of different market indicators, including a distance to default (DD) measure based on equity prices, to discriminate between banks in two categories: financial fragile or not. They assume that a bank is financial fragile if there has been a downgrade of its FitchIBCA individual rating to category C or below, which indicates a severe concern. They conclude that distance to default is a useful predictor of bank fragility either when used on its own, or when combined with an accounting based model.

This work examines instead whether the same indicator (DD) provides incremental value over accounting variables when the goal is modeling and prediction of external bank credit ratings and rating changes rather than bank fragility. Our findings suggest that DD contains only limited information that is not already available in company accounts.

The reason for focusing on banks' rating rather than on bank fragility can be summarized in the following. First, in the developed countries bank default is a rare event (in fact we have no defaults in our sample at all). Second, the new Basel Accord promotes the use of both external and internal ratings for determining regulatory capital requirements, hence creating a considerable interest in both corporate and bank credit ratings and rating transitions. While ratings are provided by rating agencies, like Moody's and Standard & Poors, their recent failure to timely signal the collapse of several large organizations, including Enron and Parmalat, has raised interest in the modeling and prediction of rating changes.

Moreover, the use of distance-to-default for modeling and predicting credit ratings may have an advantage over the commonly used historical accounting variables: in contrast to the latter, that are released only annually or at best quarterly, the DD, which summarizes information from equity markets, can be continuously updated. For this reason, the distance-to-default measure is widely used by central banks, including European Central Bank (ECB), as an indicator of bank financial stability.

We use the BankScope data base, together with equity market data, to construct a data sample of 98 equity listed banks from US, Canada, UK, Ireland and Scandinavian countries for a period of 6 years, from 1997 to 2004. We divide our observations into four broad credit rating classes, with approximately equal numbers of observations in each class, running from the weakest class 1 (B-BBB), through 2 (A1), 3 (A2-A3), to the strongest class 4 (AA1-AAA).[15]

First, as a preliminary test of whether DD is able to predict the "healthiness" of a bank: we split our bank's population into two sub-samples (banks above BBB – i.e. our rating classes 2-4; and below or equal to BBB our rating class 1) and examine whether there is a significant difference in the magnitude of DD for banks that migrate from classes 2-4 into class 1. This was done in order to confirm some of the findings of Gropp et al., verifying that in our dataset also DD measure has some power as a predictor of bank fragility.[16]

Then, we test the added value of DD in predicting credit rating and rating changes between our four classes, relative to a model based solely on accounting variables. To do so we develop an ordered probit model of bank credit ratings, which combines a number of accounting variables, taken from individual banks' financial statements, with a "distance-to-default" measure calculated using the structural Merton model. We eliminate a number of accounting variables and examine whether, in our preferred model, distance to default provides useful additional information about credit ratings, either relative to current or to "stale" (i.e. six month old) accounting data. Our results

[15] The classification is fully described at Section 4.
[16] Note that in Gropp et al. financial fragility is determined by a downgrade of the bank's FitchIBCA individual rating to C or below.

show that the additional value of DD with respect to normal accounting info is limited. However, if we eliminate from the sample the bigger and healthier banks and concentrate on the smaller ones, the added value provided by the DD with respect to standard accounting variables is statistically significant. We check the model validity of the resulting ordered probit model for the prediction of current rating by conducting out-of-sample specification tests.

Finally, we repeat our tests by focusing on the change in DD rather than the DD itself: the same model and data-set was used to examine whether changes in distance-to-default are useful as predictors of ratings changes. However, our results showed that for both small and big banks the added value provided by the change in DD with respect to accounting information is negligible.

The chapter is arranged as follows. Section 2 discusses the previous literature and the choice of the distance-to-default as an indicator of banks' ratings. Section 3 presents our econometric specification. Section 4 discusses the data employed in this paper and provides a variety of descriptive statistics and charts. Section 5 presents the estimation results. Section 6 concludes.

4.2. Previous literature

During the last decades, the assessment of banks' credit quality has become increasingly important not only to banks but also to regulators and investors. The importance of the accurate and timely prediction of bank credit quality generated a vast literature of papers that focus on modeling bank default.

There have been numerous studies that tried to identify the causes of bank failure using econometric models[17]. Most of these studies deal with bank insolvency so the dependent variable of the econometric model is binary. Much of the methodology is borrowed from the literature on corporate bankruptcy, where firm is either solvent or not. In case that the outcome is binary the two econometric methods commonly used are discriminant or logit/probit analysis. Early work on corporate bankruptcy made use mainly of discriminant analysis. However, since Martin (1977) demonstrated that

[17] Apart from the quantitative studies, there have been some qualitative studies that tried to define the determinants of bank failure. Poor management of assets, managerial problems, and fraud are some of the identified ones.

discriminant analysis is just a special case of logit analysis, most of the studies use the multinomial logit model.

In this literature it is possible to distinguish three main approaches, according to the different types of variables used to model bank fragility; the microeconomic approach, the macroeconomic and a hybrid approach[18]. The microeconomic approach focuses on the use of bank specific accounting data for the prediction of bank default.

One variation of the microeconomic approach examines the usefulness of US supervisor CAMEL scores for predicting bank default. CAMEL is the well known scorecard system employed by US bank supervisors to monitor the financial soundness of a bank. CAMEL ratings are constructed by supervisors using information from banks' financial statements as well as subjective valuations. CAMEL is a composite of five separate performance scores, covering Capital adequacy (C), Asset quality (A), Management or Administration (M), Earnings (E) and Liquidity (L). Most of these studies are concerned with the examination of whether CAMEL ratings have useful information on the ability of the banks to stay solvent and have reported mixed results on the link between CAMEL ratios and bank defaults.

Barker and Holdsworth (1993) showed that CAMEL ratings are useful in predicting bank failures, while Cole and Gunther (1998) found that although CAMEL ratings are useful in the prediction of bank defaults, as time passes their predictive power decays. On the other hand, Hirtle and Lopez (1999) reported these supervisory ratings are merely useful for monitoring the bank condition, while Gilbert, Meyer and Vaughan (1999, 2002) in a comparison of on site and off site examinations found that the CAMEL ratings that are used to the off site examination have higher predictive power than the on site examinations.

Another variation of the microeconomic approach focuses on the relationship between accounting ratios of the kind entered into CAMEL score and bank default. Lane et al. (1986), Berger, King and O'Brien (1991), Gibert (1993), Hempel, Simonson and Coleman (1994) as well as Gunther and Moore (2000) suggest as components of CAMEL the ratios of Equity/Total assets, Loan loss reserves/Non-performing loans,

[18] References regarding this distinction can be found in Shen & Hsieh (2004). Shen & Hsieh (2004) and Heffernan (2005) provide an excellent review of literature related to prediction of bank failure.

Non-interest expense/Total assets and ROA that best determine the capital adequacy, asset quality, management and earnings.

Other studies, examine a wider set of firm specific variables to examine the determinants bank failure. Measures of profitability, liquidity, capital adequacy, loan quality and loan growth rates were found to be significant in several studies. Among the measures of capital adequacy, Capital/Assets appears to be significant in most studies, while among the measures of loan quality, Reserve for possible loan losses/ Total loans, appears to be the most significant. Martin (1977), found three ratios to be significant for the prediction of bank default; the Net Income/Total Assets, Commercial Loans/Total Loans, Capital/Risky assets. Espahbodi (1991), finds that Loan revenue/Total income, Interest income/Total operational income, Interest paid on deposits/Total operating income are important predictors of band failure. Hwang et al. (1997) find that equity, profitability, liquidity and past due loans increase are important for any prediction of bank distress.

A similar microeconomic approach is used by Moody's in its proprietary accounting based model, RiskCalc, for privately held banks[19]. RiskCalc uses 7 categories of ratios to assess the credit quality of the banks; profitability, leverage, growth, efficiency, loan portfolio composition and concentration, liquidity and asset quality.

The macroeconomic approach uses macroeconomic variables for the prediction of bank fragility. This approach is based on the observation that changes in macroeconomic variables precede banking crises. This research has focused on the role of macroeconomic variables in triggering banking crises in emerging markets. Gavin and Hausman (1996) as well as Sachs, Tornell and Velasco (1996) found that lending booms preceded banking crises in Latin America. This result was reinforced by the work done by Kaminsky and Reinhart (1999). Calvo (1996) found that the ratio of broad money to foreign reserves is useful in explaining the Mexican 1994 crisis. Additionally, Honohan (1997), found that a higher loan to deposit ratio, a higher foreign borrowing-to-deposit rate and a higher growth rate in credit are correlated with banking crisis, using a sample of 24 countries. Additionally, Demirgüç-Kund and Detragiache (1998) showed that banking crises tend to erupt when the macroeconomic environment is weak and particularly when growth is low and there is

[19] Moody's KMV has developed a RiskCalc model for private companies as well as for private held banks. Due to the context of the paper, we only refer to the one concerned with privately held banks.

high inflation. Rossi (1999) suggested that liberalization decreases the likelihood of banking crisis. Eichengreen and Arteta (2000) found that the rapid domestic credit growth, the large ratio of M2 and foreign reserves and the deposit rate control are the robust causes of banking crises.

All the above studies focused on the determination of either the microeconomic or the macroeconomic variables that cause banking crises. This approach, although it gave a good insight on the usefulness of the different variables in the prediction of bank distress, has a main disadvantage. Increasing evidence suggests that banking failure is the result of the changes in both microeconomic and macroeconomic factors.

Banking failure may be the result of the changes in both microeconomic and macroeconomic factors. This problem has been addressed by some researchers, combining both micro and macro data (a hybrid approach) into their analysis (Gonzalez-Hermosillo (1999), Berger, Kyle and Scalise (2000), Heffernan (2003), Shen and Hsieh (2004)). These studies point to the general conclusion that both microeconomic and macroeconomic factors matter in bank default.

Apart from the econometric studies, there is an interest in establishing whether continuously updated measures of bank credit quality derived from equity or debt markets can be used to improve forecasts of bank default or a rating change. Some studies in the banking sector focused on the equity returns or the level of the equity prices prior to the banking crisis. Curry et al (2001) conclude that stock prices tend to decrease two years before a CAMEL downgrade. Berger et al (2000) examine the relationship between supervisory information and a number of market indicators including abnormal stock returns and other equity indicators. They find that supervisory assessments and equity indicators are not able to predict each other.

The study most similar to our own is the one by Gropp et al. (2006). They analyzed the ability of the distance to default measure, as given by Merton-MKMV, and subordinated bond spreads to signal bank fragility. Using a sample of EU banks they find that both distance to default and bond spreads are useful indicators of banks' fragility.

In this study, the choice of the distance-to-default, derived using option-pricing theory from the equity market data, was motivated by Gropp et al. (2006). They argue that the distance-to-default measure, (taken with a minus sign i.e. the negative of distance-to-default) is a market indicator with two good properties: it is *complete*, which means that is capable of capturing the major elements affecting default probability, and it is *unbiased*, which means that it is aligned with supervisors' interests.[20] Thus this indicator is preferred over biased direct equity price-based measures. Our aim is to follow the study by Gropp et al. (2006) and to try to assess whether the DD can also be useful to modeling and prediction of bank credit rating and rating changes, by providing additional information with respect to the bank accounting variables. A separate issue, which we do not address, is whether other information can usefully supplement banking accounting variables in predicting rating changes e.g. macroeconomic conditions. We note that the rating agencies themselves claim that these ratings are "through the cycle" i.e. they should not be affected by macroeconomic conditions but that there is evidence that macroeconomic conditions are a significant determinants of bank default probabilities (refer to Chapter 2, De Servigny and Renault).

4.3. Econometric model

To investigate the predictive power of the DD indicator, we use an econometric model combining a number of different accounting ratios, taken from each bank's financial statements, with a distant-to-default measure calculated using Merton's structural model. The dependent variable is both discrete and associated with a number (four in our case) of ordered outcomes. Hence, an appropriate estimation technique is the ordered probit or logit model.[21]

The ordered probit or logit model is specified as follows. Suppose that there are N banks (i=1,....N) and that banks' credit rating quality can be represented by the value of an unobserved continuous variable D_i, with higher values of D_i representing higher credit rating. This credit rating quality, D_i, is a linear function of K factors

[20] According to Gropp et al (2006) an indicator of bank fragility is called complete if it reflects three major determinants of default risk: market value of assets, leverage and volatility of assets. Moreover, an indicator of bank fragility is unbiased if it is decreasing in the earnings expectations and increasing in the leverage and asset risk.

[21] Ordered probit or logit models are more efficient for the modelling of multiple rating categories than the multinomial probit or logit models, commonly used for modelling multiple discrete outcomes. Multinomial probit or logit should be used only when there is no inherent ordering of the different choices. For a more detailed description of ordered probit/logit models refer to Greene (2000, Ch. 19).

whose values for individual i are X_{ik}, where k = 1,..., K. Hence, the credit rating quality can be represented using a latent regression:

$$D_i = \sum_{k=1}^{K} \beta_k X_{ik} + \varepsilon_i = Z_i + \varepsilon_i , \qquad [4.1]$$

where β_k is the coefficient associated with the k_{th} variable, ε_i is the randomly distributed error and

$$Z_i = \sum_{k=1}^{K} \beta_k X_{ik} \qquad [4.2]$$

As in every regression, D_i is unobservable. What we observe is an integer ordinal variable Y_i that can be associated with the different credit rating categories, such that $Y_i = 1$ if the bank has low credit quality, etc and belongs to $\{1,2,3,4\}$. The categorisation of each bank in terms of credit rating depends on the values of D_i in conjunction with the threshold values δ_1, δ_2, δ_3 such that

$$Y_i = 1......if.....D_i \le \delta_1$$

$$Y_2 = 2.....if.....\delta_1 \le D_i \le \delta_2 \qquad [4.3]$$

......

$$Y_i = 4.....if.....D_i \ge \delta_3$$

The threshold values δ_i are unknown parameters and have to be estimated along with β_k. We assume that all threshold values are above zero. A bank's classification in terms of its credit class depends on whether or not its credit score D_i crosses a threshold. Hence, the probabilities of Y_i taking the values 1 to 4 are given by:

$$\begin{aligned} &\Pr(Y_i = 1) = \Pr(Z_i + \varepsilon_i \le \delta_1) = \Pr(\varepsilon_i \le \delta_1 - Z_i) \\ &\Pr(Y_i = 2) = \Pr(\delta_1 \le Z_i + \varepsilon_i \le \delta_2) = \Pr(\delta_1 - Z_i \le \varepsilon_i \le \delta_2 - Z_i) \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\Pr(Y_i = 4) = \Pr(Z_i + \varepsilon_i \ge \delta_3) = \Pr(\varepsilon_i \ge \delta_3 - Z_i) \end{aligned} \qquad [4.4]$$

where $\delta_1 \prec \delta_2 \prec \delta_3$.

To fully specify the model, it is necessary to select a probability distribution for the error term, ε_i. We assume that ε_i is normally distributed across observations. This assumption yields the ordered probit model.[22] Following this assumption, the probabilities of Y_i taking the values 1 to 4 can be expressed as follows:

$$\begin{aligned}
&\Pr(Y_i = 1) = \Phi(\delta_1 - Z_i) \\
&\Pr(Y_i = 2) = \Phi(\delta_2 - Z_i) - \Phi(\delta_1 - Z_i) \\
&\cdots\cdots\cdots\cdots\cdots\cdots \\
&\Pr(Y_i = 4) = 1 - \Phi(\delta_3 - Z_i)
\end{aligned} \qquad [4.5]$$

where $\Phi(x) = \Pr(\varepsilon_i \prec x)$ is the cumulative standard normal probability distribution of the error terms. As mentioned before, our aim is to estimate the threshold values δ_i along with β_k. The nonlinearity of the ordered probit model implies that the use of algebraic expressions for the parameters' estimation is not possible. Instead, to estimate the required parameters the maximum likelihood estimation is used. The maximum likelihood estimation consists of specifying the probability of obtaining each observation as a function of the unknown parameters. We form the likelihood of observing the sample as the joint probability of obtaining all observations and can be expressed as follows:

$$L = [\Pr(Y_i = 1)]^{N_1} [\Pr(Y_i = 2)]^{N_2} [\Pr(Y_i = 3)]^{N_3} [\Pr(Y_i = 4)]^{N_4} \qquad [4.6]$$

where N_1 are the banks with credit rank 1, N_2 with credit rank 2 and so on.

The parameter values are estimated using an iterative procedure and are those that maximize the likelihood function. The ordered probit model provides maximum likelihood estimates of both the parameter weightings (the parameter vector β) and the ordered sequence of thresholds between the different ratings (δ1, δ2, δ3, …, etc.).

Using the estimated $\hat{\beta}_k$ allows us then to estimate a value of $\hat{Z}_i$ for each bank. These estimated values together with the estimated thresholds, provide, for each bank, the probability of being in each credit rank.

[22] The model can also be estimated using the logistic distribution for the error term. This assumption would yield the ordered logit model. In practice, the difference on the estimated values of the parameters from the use of the ordered probit or ordered logit model is trivial (refer to Greene (2000), Borooah (2001)).

4.4. Variables and data description

In order to perform our analysis, different accounting variables as well as the distance-to-default measure has been used. This section offers a detailed description of accounting and distance-to-default measures.

4.4.1 Accounting-based variables

All our accounting data is taken from the Bureau Van-Dyck "Bankscope" database. This database includes a large number of different accounting variables that can be employed for the modelling of bank credit quality. Our choice of accounting variables follows the specifications used in a number of previous studies, including Lane et al (1986), Berger, King and O'Brien (1991), Gilbert (1993), Hempel, Simonson and Coleman (1994) and Gunther and Moore (2000).

These choices can be discussed within the overall "CAMEL" framework. Even though we do not compute a CAMEL score for each bank (our preferred specification incorporates several accounting ratios, not a single summary statistic), we wished to ensure that we include variables in our specification that correspond to all five dimensions of the CAMEL classification, "C"= Capital adequacy, "A" = asset quality", "M"= management quality, "E"= earnings quality, and "L"= liquidity. The variables we use to measure these five dimensions are as follows:

In previous studies the most widely employed measure of capital adequacy is the ratio of *Equity/Total assets*. To this ratio we add three additional measures, the ratio of *Equity/Net loans* and the Basel weighted *Total Capital ratio* and *Tier 1 ratio*. For all these ratios, we expect that the higher the value, the higher will be the capital of the bank hence the lower will be the default probability of the individual bank.

The ratio of *Loan loss reserves/Non performing loans* has been employed by Berger, King and O'Brien (1991), Gilbert (1993) and Gunther and Moore (2000) to measure the quality of assets. There is uncertainty around the impact of this ratio to the credit quality of the bank. On the one hand it can be argued that a high ratio can show low credit risk since it means that the bank has sufficient capital to write off bad debt and on the other hand that it might increase the credit risk since it is an indication that in the future there will be an increase in bad loans. We prefer to use *Impaired loans/Loan loss reserves* due to the fact that data on non-performing loans was incomplete in Bankscope. We also include *Impaired loans/Gross loans* as an additional measure of asset quality.

Non interest expenses/Total assets is the most widely used accountancy ratio used as a proxy for the Management component of CAMEL. The sign of this ratio will be expected to be negative, since the lower this ratio is the lower the probability of default and hence the higher the anticipated rating.

ROAA (Return on Average Assets), which is equal to *Net income/Average assets*, can be used as a proxy for earnings. The higher the ratio is the higher is the credit rating since the profitability of the bank increases. We also include two other indicators commonly used by practitioners to measure earnings performance, ROAE (Return on Average Equity) and *Net Interest Margin*.

We include two liquidity ratios that offer an indication of the liquidity on the asset and liability side respectively. These are the ratios of *Liquid assets/Total assets* and *Short-term borrowing/Total assets*.

Finally we include into the analysis a measure of bank size, *ln(Total Assets)*. Larger banks are expected to be at lower risk of default because of the benefits of diversification across several markets and sources of revenue.

4.4.2 Distance to Default

The main purpose of this chapter is to introduce an equity based measure, the distance to default, into an accounting-based model of bank credit ratings and rating transitions. This measure is calculated using Merton's structural model of default. This model has already been discussed in Chapter 2 of the thesis, but for reasons of continuity we restate the main model here.

The original Merton model assumes that default occurs only at the date of maturity and if and only if the value of assets is lower than the face value of debt. The main advantage of this model is that equity markets can be used so as to estimate two unobserved variables: the market value and the volatility of the bank's assets. Such unobserved values may help explain for example why some firms with the same accounting ratios default and others stay solvent. Advocates of structural models claim that asset volatility plays a key role at the determination of default since a firm with the same leverage as another and higher asset volatility has higher probability of default.

As in the original Merton case, the model applied in this chapter assumes that equity is a call option on the value of assets with strike price the default point. In addition, the default point is determined exogenously and is assumed to be the principal value of debt, P. [23]

Hence, the value of equity is represented as follows:

$$V_e = V_a N(d_1) - Pe^{-rT} N(d_2) \ , \qquad [4.7]$$

where

$$d_1 = \frac{\log\left(\frac{V_a}{P}\right) + \left(r + \frac{\sigma_a^2}{2}\right)T}{\sigma_a \sqrt{T}} \qquad [4.8]$$

and

$$d_2 = d_1 - \sigma_a \sqrt{T} \ , \qquad [4.9]$$

where V_a is the value of assets, P is the total value of debt, T is the day to maturity and σ_a is the asset volatility.

It is clear that the value and the volatility of the assets are unknown. These parameters can be estimated by solving the following two equations simultaneously:

1st equation: $V_e = V_a N(d_1) - Pe^{-rT} N(d_2)$ [4.10]

2nd equation: From Ito's lemma, we can extract a formula that connects volatility of equity to the volatility of assets. Hence, the second equation used is:

$$\sigma_e = \sigma_a * \frac{V_a}{V_e} * \frac{\partial V_e}{\partial V_a} \qquad [4.11]$$

After value and the volatility of assets is estimated the distance-to-default measure (DD) measure, which denotes how many standard deviations is the market value of assets away from the default barrier, can be derived.

The formula used for the calculation of DD measure is the following:

[23] The actual measurement of variable P using banking data is described in Section 4.3.

$$DD = \left[\frac{\ln\left(\frac{V_a}{P}\right) + \left(r - \frac{\sigma_a^2}{2}\right)T}{\sigma_a \sqrt{T}} \right] \qquad [4.12]$$

4.4.3 Data description and descriptive statistics

For the purposes of this study, data from 98 bank institutions from US, UK, Canada, Ireland and Scandinavian countries has been collected from the BankScope database and from Bloomberg. Table 1 provides a description of the different types of banks included in our sample together with their respective frequency. For a bank to be included in our sample, the following criteria should be satisfied: to be rated by at least one major rating agency in the years from 1997 to 2004, to have available accounting information for all or some of the years between 1997 to 2004 and to be publicly traded within the same period. The choice of the countries included in our sample has been made based on the relative uniformity of their banking sector.

The credit ratings of the banks have been collected from 1997 to 2004 from Bloomberg. We note one limitation of this approach. Morgan (2002) argues that bank credit ratings are more noisy signals of creditworthiness than corporate ratings, due to the greater bank opacity. We use the credit ratings provided by the rating agencies as the only available measure of bank credit quality. We restrict credit rating changes to those announced by three major credit rating agencies: Moody's, S&P and Fitch. In each case, a credit rating change is determined when there was a change in the credit rating of the senior unsecured debt of the bank, which approximates the issuer rating. Mainly, Moody's has been used for the reporting of the credit rating changes, while information from S&P and Fitch has been employed when Moody's did not provide the required rating. Credit ratings are grouped into 4 categories. The choice of the rank categories has been made so as we have a relative uniform number of observations in each category. Table 2 shows the distribution of the ratings among the different categories. Tables 3 and 4 show the number of downgrades and upgrades per rating class per year. In total, there are 23 upgrades and 11 downgrades in our sample. Year 2001 is the year with the highest number of upgrades and downgrades.

All accounting ratios have been calculated from the Bankscope database for the period of 1997 to 2004. We removed a few outliers that may affect the regression results, for all accounting variables as well as distance to default measure so as the standard deviation and kurtosis of the distribution of each individual variable to be closer to a normal distribution. Descriptive statistics of the variables before and after the adjustment for outliers are presented on Tables 5 and 6 respectively[24].

Regarding the distance-to-default measure, as mentioned before, there are five variables that are necessary for the estimation of the unobserved value and volatility of assets: banks' face value of debt, market capitalization or "value of equity", equity volatility, time horizon and risk free interest rate.

Total liabilities that represent banks' face values of debt are collected from financial statements available in Bankscope. Although data is available from 1996 onwards, our sample is from 1998 to 2004 since most of the banks started having their shares publicly traded only after 1997. The length of the sample includes the recession period starting in 2001 when bank ratings and also equity prices and hence distance to default declined.

Monthly market capitalization data have also been collected when available from Bankscope database for the same period of time. In cases that the data have not been available in Bankscope, data on the number of outstanding shares and equity prices have been collected from Bloomberg. These two variables have then been used for the calculation of market capitalization.

In order to calculate the equity volatility, daily equity price data have also been collected from Bloomberg for the period between 1997 and 2004. The reason for collecting one additional year of equity data is due to the fact that we use a 12-month window in equity volatility calculations using standard deviation. Table 7 shows the evolution over time of the mean and standard deviation of the equity volatility for the banks in our sample. The mean value of equity volatility is increasing from 28.93% in 1998 to 40.55% in 2001 indicating the reaction of the equity market during the recession. In 2004, it is returning to 23.24%.

As a proxy for the risk-interest rate, we use 3-month Treasury bills or 3-month interbank rate. The data for this has been derived from the national banks of the

[24] Note that by comparing the results of the regression before and after the adjustment of the outliers we concluded that the reported trends are fully robust to the treatment of outliers.

respective countries in our sample. The time horizon T is set to 1 year, which is used in the literature as benchmark.

To calculate the Distance-to-Default measure, the value and the volatility of assets is first estimated and an iterative process is applied using equations [4.10] and [4.11]. Figure 1, shows the evolution of Distance-to-Default over time for different groups of banks. The sharp decrease in the Distance-to-Default in 2001 mainly reflects the increase in the equity volatility during that period (refer to Table 7). Additionally, commercial banks exhibit more stable Distance-to-Default measures compared to the bank holding institutions and mortgage banks. One possible explanation is that most of commercial banks are outside US and non-US banks behave differently than those in US.

4.5. Estimation of credit ratings

This chapter examines whether information from equity markets, as summarized in the distance-to-default measure derived from Merton-MKMV, has incremental value over the accounting variables for modeling and predicting bank credit ratings. We consider a number of different hypotheses about the predictive value of distance to default, differing in terms of the timing of accounting data, the availability of distance to default and the measurement of the credit rating. We first examine whether distance to default provides useful additional information about current credit ratings, either relative to current or to "stale" (i.e. six month old) accounting data. The reason that we test for the usefulness of distance to default six months after the release of the accounting statements is that the information in the accounting data should decay as time passes. Then, we test whether distance to default has incremental value over accounting variables for the prediction of future credit ratings and rating changes. Additionally, we test whether the change in DD rather than the DD itself is a useful predictor of future ratings and rating changes. Table 8, summarises all these hypotheses.[25]

[25] Note that we tested a number of related hypotheses that are not reported in this paper, since the results were similar to the ones reported in the paper. Among others, we tested whether DD alone has any explanatory power in predicting future ratings. We found that even when used alone, DD has no explanatory power.

Before presenting the econometric results from testing our different hypotheses (refer to Table 8), we conduct a preliminary test of whether DD is able to predict the "healthiness" of a bank: we split our bank's population into two sub-samples (banks above BBB – i.e. our rating classes 2-4; and below or equal to BBB our rating class 1) and examine whether there is a significant difference in the magnitude of DD for banks that migrate from classes 2-4 into class 1. This was done in order to confirm the findings of Gropp et al., and to verify that also in our dataset DD measure has the power to predict bank's fragility.[26]

Table 9, presents the results of a simple mean comparison test that we conduct to assess whether DD was able to detect weaker banks, i.e. banks with a tendency to downgrading equal or below rating class 1. We conclude that in our sample, under the assumption of unequal variances[27], the difference between the two means is statistically different at the 5% and 10% significance level both for 12 and 24 months prior to downgrade. This result indicates that also in our sample distance to default is a potentially useful indicator of bank fragility.

Then we investigate whether DD measure has incremental value over accounting variables for the prediction of credit ratings. In other words, we test whether an "extended" model that includes DD together with accounting variables has better predictive power compared to a "simple" accounting variables only model. To test this assumption we use a number of different hypotheses that differ in terms of the timing of accounting data, distance to default and credit rating (refer to Table 8). First, we test whether at the release of accounting statements DD measure has incremental value over accounting data for the prediction of current rating. Our second hypothesis is that distance to default has predictive power relative to stale accounting data (six months old). The reason that we test for the usefulness of distance to default six months after the release of the accounting statements is that in theory the information in the accounting data should decay as time passes. Apart from these two hypotheses that focus on the usefulness of DD in predicting the current rating, we examine three

[26] Note that Groop et al (2006) measures financial distress as the event of downgrading of FitchIBCA individual rating to category C or below.

[27] We tested on our sample the hypothesis that the variances of the two samples are equal. We found that we cannot reject the hypothesis that the variances are equal. Nevertheless, due to the low number of observations of the downgraded banks we believe that the probability of rejecting the hypothesis was low. We hence make the assumption of unequal variances in order to decrease the Type II error when testing for equal means.

more hypotheses that focus on the incremental value of DD at predicting future rating. In our third hypothesis we investigate whether one year lagged DD measure has incremental value at the prediction of bank credit ratings. Finally, for our last two hypotheses we focus on the change in DD rather than the DD itself and its ability in terms of predicting future rating. The main difference between the last two hypotheses is the timing of the accounting data. Our fourth hypothesis uses current accounting data at the release of the financial statements while the last hypothesis uses stale accounting data.

Before testing all these hypotheses, it was necessary to reduce the number of accounting variables used in the credit ratings model. There were two criteria for this reduction. The first criterion was to include variables used in previous studies of bank credit rating and credit quality. Since our objective is to examine the additional contribution of distance to default in an accounting model of credit ratings, we wanted to choose accounting variables that were representative of previous studies that built accounting models of bank credit quality or bank failure. Our literature review has revealed a very large number of different accounting variables used in this way. Given the high correlation among certain of the variables, we then reduce the number of accounting variables, that correspond to all five dimensions of the CAMEL classification (Capital adequacy, Asset quality, Management, Earnings quality and Liquidity), so that we work with a more parsimonious model that still retains most of the information in the accounting variables regarding the modeling of ratings.

Table 10 reports the correlation among the accounting variables. The table shows there are high correlations amongst certain groups of variables, suggesting that not all need to be included in the model. As expected, variables belonging to the same classification group are most highly correlated. In particular, ROAA and ROAE are highly correlated as well as Total Capital ratio and Tier 1 ratio. To decide which variables to keep, we first ran a regression including all variables using the current credit rating as a dependent variable and all the years from 1998 to 2004. Then looking at their individual statistical significance as well as considering their economic interpretation and the correlation among them we created our preferred model. This includes the following accounting variables: Tier 1 ratio, Net loans/Total assets, Impaired Loans/Gross Loans, ROAE, Net Interest Margin, Non-Interest Expense/average Assets, Short-term Borrowing/Total assets.

For the prediction of current rating (Hypotheses 1 and 2), we split the sample so as we can test for the robustness in our results by performing an out-of-sample test. We use years from 1998 to 2003 for the in sample parameter estimation and year 2004 is used for the out of sample test. Due to the fact that we only have seven years of sample data, we only perform out of sample tests to test the ability of distance to default to predict the current rating. For the prediction of future rating (Hypotheses 3 to 5), we use the full sample for an in-sample estimation.

Table 11, presents the in sample estimated coefficients and the z-values of the parameters for the different regressions that correspond to the different tested hypotheses.[28] The results of regressions indicate that all variable coefficients, except of the Tier 1 ratio and interest margin, have the expected sign. The negative sign of the Tier 1 ratio is in contrast with the standard theory, that the higher the capital a bank holds the higher the credit rating. It seems that there is an endogeneity issue; the rating is driving the ratio rather than the other way around. This finding is important since it shows that banks that are not near default may not hold as much capital relative to their risk adjusted assets as the ones that they are at the lower credit rating categories.

For the first three hypotheses (Tale 11 columns 1-3), where we investigate whether DD has incremental value for the prediction of current or future rating, we note that there are a number of statistically significant accounting variables. Our results show that amongst these accounting variables size is the most significant variable for the prediction of current or future rating. It is worthwhile to mention that the ratio of Non-interest expenses/Average assets, while it is statistically significant for the prediction of current rating, becomes less significant for the prediction of future credit ratings.

In contrast to many of the accounting variables, the distance to default measure is statistically insignificant (i.e. we cannot reject the null hypothesis that the regression coefficient of distance to default is not statistically different from zero when the other accounting variables are in the model). This conclusion is supported both by likelihood ratio tests for comparing the model with and without DD and by the z-statistic for the DD variable. To test the incremental value of including the distance to

[28] The z-statistic is a test statistic for the null hypothesis that an individual predictor's regression coefficient is zero, given that the rest of the predictors are in the model.

default into the different regressions, we use the likelihood ratio test. Using the likelihood ratio test we examine whether the difference between the log likelihood produced by the model that combines the accounting variables and the distance to default (extended model) and the log likelihood produced by the model with accounting only variables (simple model) is statistically significant. As shown on Table 11, the log likelihood ratio is not significant for 1%, 5% and 10% confidence level on all regressions, indicating that the distance to default adds little additional value for the prediction of credit ratings. This result is in line with the reported z-values for distance to defaults which are also statistically insignificant.

Our second hypothesis (Table 11 column 2) is that distance to default has predictive power relative to stale accounting data, reported six months previously. This hypothesis corresponding more closely to the practical situation facing supervisors where accounting information will only be available some months after market data. Here we find that while the log likelihood of the regression, unsurprisingly, has fallen (from -294.2. to -301.8) the current distance to default remains again statistically insignificant.

Our third hypothesis (Table 11 column 3) is that distance to default has incremental value over accounting variables for the prediction of future credit rating. This hypothesis aims to investigate whether DD measure is useful at the prediction of future credit rating. As in previous cases, we conclude that although most of the accounting variables are statistically significant the DD measure is statistically insignificant.

Additionally, when testing for the last two hypotheses (Table 11, columns 4-5), where we focus on whether the change in DD indicator is important for the prediction of future credit ratings. We note that, as in the case of the first three hypotheses, there are a number of statistically significant accounting variables. Our results show that amongst these accounting variables size is again the most significant variable. We conclude that as in the case of DD measure, the change in distance to default indicator is once more not statistically significant, even at the 10% confidence level for the prediction of future rating.

It is also worth examining whether DD affects the predictive ability of the model. Since logistic regression does not have an equivalent of R-squared one should be cautious on the test statistics used. We do this in two ways, first reporting different in sample prediction (goodness of fit) measures and then by reporting the mean probabilities of estimating a rating category using our different models. For in-sample goodness of fit measures we report the Prob>chi2 and McFadden's pseudo R-squared. The Prob>chi2 (or p-value) is the probability of getting a likelihood ratio test statistic as extreme as the one observed under the null hypothesis where we assume that all of the regression coefficients of the model are equal to zero. In all regressions the small p-values lead us to conclude that at least one of the coefficients in each of our models is different from zero. We also report the pseudo R-squared. Since, in the logistic regression the pseudo R-squared statistic does not mean what the R-squared means in OLS (the proportion of variance for the response variable explained by the predictors) we prefer not to draw any conclusions. As an additional test of the predictive power of our different models Tables 12 to 16 present the mean probabilities of estimating a rating category using our different models. In all cases, using our proposed model one has the highest probability at predicting the correct rating. We compare these results with the accounting only model results and we find that the inclusion of distance to default, as suggested by the likelihood ratio test, does not substantially improve the prediction power of the model.

Next, we split our sample to "big" and "small" banks to investigate whether distance to default has a statistically significant role in the prediction of the current rating of either smaller or larger equity-listed banks. The split between "big" and "small" banks have been made so as we leave roughly the same observations in each category. Tables 17 and 18 present the in sample estimation results for the small and big banks respectively. As Table 17 shows, distance to default becomes significant at 5% confidence level when used for the prediction of current rating using both current and "stale" accounting data. This result is confirmed by the likelihood ratio test, which indicates that the difference between the "extended" and "simple" model is statistically significant at 5% confidence level. For the prediction of future ratings using the change in the distance to default, Tables 17 and 18 (columns 4-5) show that the split between small and big banks did not affect the statistical significance of distance to default. Tables 19 and 20 confirm that for small banks using the "extended model" for the prediction of current rating using current and stale accounting data, the probability of predicting the correct rating is substantially increased compared to the "simple" model. Tables 21 and 22 show that for big banks this is not the case. The DD becomes statistically insignificant and its inclusion does not add to the predictive power of the model.

To test the validity of our in sample estimation we conduct out of sample tests using year 2004 data – which was not used for estimation – to examine the hypotheses 1 and 2 only. Tables 23 to 27 present the results of the out of sample tests for the prediction of current ratings. The out of sample estimation confirms our in sample estimation results. The inclusion of distance to default does not increase the predictive power of the model. However, when we use only the small banks the addition of the distance to default appears to be useful.

Finally we note that there is a problem of bias in the standard errors. This is possible for several reasons. We do not include year dummies (this is appropriate since our focus is on prediction). There may be further correlation of errors due to country or regional effects not included in the specifications. As a result the standard errors of our estimated variables are likely to be upwards biased, further weakening any inferences that can be drawn about the usefulness of DD for prediction.

Our finding that DD is not a useful indicator for the prediction of bank credit ratings (except for small banks) stands in sharp contrast to the conclusion of Gropp et al (2006) that DD is a useful indicator for the prediction of bank fragility. This difference can be attributed to several differences between our study and that of Gropp et al (2006). First, we use a different data set of mostly US banks, whereas their data is for European banks. Second, we use a dataset of 98 banks consisting of annual observations from 1998 to 2004, while Gropp et al (2006) uses a dataset of 84 banks consisting of monthly observations from 1991 to 2001. Moreover, Gropp et al (2006) examine whether DD provides incremental value over accounting variables when the goal is prediction of bank fragility as measured by a division of bank ratings into two categories (fragile and not fragile). Our study generalizes by examining the predictive power of DD for the modeling and prediction of a set of four ordered external bank credit ratings and of changes between these ratings. For comparison with Gropp et al (2006) we also conduct a mean comparison test of distance to default DD, but restricted to two ratings classes (our class 1 which is the most fragile against classes 2, 3, and 4 which are not fragile) finding a statistically significant difference, i.e. the DD measure used on its own seems to be useful as a predictor of bank fragility.

4.6. Conclusion

We have investigated a number of different ordered probit models of bank credit ratings, combining distance to default with bank accounting information. Despite an extensive specification search and the examination of several alternative hypotheses about the predictive ability of distance to default, we find that distance to default, when used for the modeling and the prediction of credit ratings for banks has relatively little incremental value over the accounting variables. We find that much the most important variable both for the prediction and the modeling of ratings is the size, as measured by the log of total assets. We find several other accounting variables are also statistically highly significant. We find distance to default is statistically significant only for small banks, for modeling current ratings, or predicting credit rating changes over a 6-month or 12-month horizon. We find no evidence that changes in distance-to-default have additional explanatory power for predicting rating category, either for small or large banks.

Our work complements that of Gropp et. al. (2006), who find that distance to default is a useful predictor of bank fragility when used on its own, as well as in the context of an accounting model. Our results indicate that distance to default, while it may be a useful summary statistic for bank supervisors and others monitoring financial sector stability, certainly does not supplant more traditional approaches to credit analysis when used for the prediction of credit quality of banks. As stated, we find that it is only for smaller banks that distance to default can help with the prediction of credit quality, relative to accounting variables. This could be either because accounting information is of lower quality or because the credit standing of small banks is driven by other factors (such as industry or country developments) that are not captured by accounting statements but are reflected in equity prices. For both these reasons, and because if (as is likely) supervisors are able to devote relatively less resources to a fully accounting based credit analysis for smaller institutions distance to default appears to be most useful for monitoring smaller banks. For larger banks, at least within the countries covered by our data, distance to default appears to be, at best, one of a range of indicators helpful to supervisors.

This study might be extended by the inclusion of lagged rating changes into the regressions. Bangia et al (2002) show that rating changes are serially correlated to certain extent. Hence, it would be useful to investigate whether the inclusion of lagged rating changes into the regression might even further reduce the predictive power of DD.

Tables and figures

Table 1: Type of banks included in the sample and their frequency

Bank type	Frequency
Bank holding and holding Company	71
Commercial Bank	19
Non Banking Credit Institutions	2
Real Estate/Mortgage bank	4
Savings bank	2

This table presents the number of banks split in different bank types. Data on the bank types has been collected from Bankscope.

Table 2: Categories of ranks in credit ratings

Category	Rating	Number of observations of a specific rating category	Number of observations
1	B	7	180
	BB	34	
	BBB	139	
2	A1	129	129
3	A2	86	184
	A3	98	
4	AA1	90	165
	AA2	53	
	AA3	8	
	AAA	14	

This table presents the number of observations in each rating class. Data on credit ratings has been collected from Bloomberg.

Table 3: Number of downgrades per rating class per year

	1999	2000	2001	2002	2003	2004	Total
AAA	0	0	0	0	0	0	0
AA	0	0	0	0	0	0	0
A	0	1	1	2	1	0	5
BBB	0	0	2	1	0	2	5
BB	0	0	0	0	0	0	0
B	0	0	0	1	0	0	1
Total	0	1	3	4	1	2	11

This table presents the number of downgrades per year in each rating class. Data on credit ratings has been collected from Bloomberg.

Table 4: Number of upgrades per rating class per year

	1999	2000	2001	2002	2003	2004	Total
AAA	0	0	0	0	0	0	0
AA	1	1	5	1	2	1	11
A	0	2	4	1	1	0	8
BBB	0	1	0	0	0	1	2
BB	0	0	1	0	0	0	1
B	0	0	1	0	0	0	1
Total	1	4	11	2	3	2	23

This table presents the number of upgrades per year in each rating class. Data on credit ratings has been collected from Bloomberg.

Table 5: Descriptive statistics of independent variables before accounting for outliers

Variable	N	p25	p50	p75	Mean	Sd	Skewness	Kurtosis
Equity/Total assets	722	5.84	7.69	9.42	10.06	23.68	12.03	165.29
Total Capital Ratio	654	11.10	12.10	13.40	12.62	3.42	6.92	107.07
Tier 1 Ratio	653	7.90	9.00	10.80	9.66	3.27	8.96	152.46
Equity/Net Loans	720	10.02	12.77	15.67	15.05	12.51	5.83	49.78
Impaired Loans/Loan Loss Reserves	645	0.00	0.01	0.01	0.01	0.06	10.98	127.56
Impaired Loans/Gross Loans	645	0.46	0.74	1.40	1.16	1.41	5.94	56.29
ROAA	722	0.80	1.17	1.47	1.19	0.75	-0.20	20.48
ROAE	722	12.58	15.55	19.06	15.50	7.80	-1.62	22.98
Interest Margin	722	2.41	3.60	4.26	3.47	1.44	1.63	11.50
Non-Interest Expenses/Average Assets	725	2.63	3.45	4.22	4.03	3.18	3.35	17.90
Liquid Assets/Total Assets	784	2.87	5.96	12.12	9.03	9.55	2.25	10.21
Short-term Borrowing/Total Assets	784	71.82	79.22	84.62	70.48	24.95	-1.96	5.64
ln(Total Assets)	717	23.36	24.38	25.92	24.68	1.76	0.70	3.31
Distance	605	2.79	3.36	4.10	3.43	0.84	0.15	2.65
Distance 6 months	596	2.80	3.42	4.08	3.46	0.87	1.02	10.92

This table presents the standard statistics of the different variables. N refers to the number of observations, while p25, p50 and p75 refer to the 25th, 50th and 75th quantile. The mean, standard deviation (denoted by sd) and kurtosis are also reported.

Table 6: Descriptive statistics of independent variables after accounting for outliers

Variable	N	p25	p50	p75	Mean	Sd	Skewness	Kurtosis
Equity/Total assets	713	5.83	7.67	9.34	7.75	2.57	0.76	4.97
Total Capital Ratio	646	2.42	2.49	2.60	2.52	0.16	1.48	7.62
Tier 1 Ratio	652	7.90	9.00	10.80	9.57	2.35	1.26	5.02
Equity/Net Loans	720	2.30	2.55	2.75	2.56	0.49	0.97	6.90
Impaired Loans/Loan Loss Reserves	637	-5.74	-5.26	-4.73	-5.21	0.75	0.34	3.01
Impaired Loans/Gross Loans	645	-0.79	-0.31	0.34	-0.23	0.82	0.28	3.39
ROAA	709	0.81	1.17	1.45	1.19	0.55	0.92	6.04
ROAE	708	12.67	15.61	19.04	15.78	5.48	0.21	5.08
Interest Margin	710	2.39	3.56	4.24	3.36	1.16	0.25	2.33
Non-Interest Expenses/Average Assets	684	2.57	3.34	4.06	3.42	1.55	0.95	5.16
Liquid Assets/Total Assets	724	1.24	1.88	2.57	1.85	0.99	0.38	3.03
Short-term Borrowing/Total Assets	668	75.65	80.64	85.57	80.05	7.41	0.85	4.05
ln(Total Assets)	692	23.34	24.32	25.73	24.51	1.52	0.27	2.13
Distance	599	2.78	3.35	4.09	3.42	0.81	0.03	2.16
Distance 6 months	595	2.80	3.41	4.07	3.45	0.81	0.01	2.30

This table presents the standard statistics of the different variables after accounting for the outliers. N refers to the number of observations, while p25, p50 and p75 refer to the 25^{th}, 50^{th} and 75^{th} quantile. The mean, standard deviation (denoted by sd) and kurtosis are also reported.

Table 7: Mean and standard deviation of the equity volatility from 1998 to 2004.

	1998	1999	2000	2001	2002	2003	2004
Mean	28.93%	37.97%	35.99%	40.55%	31.69%	33.76%	23.24%
Standard Deviation	5.84%	9.08%	7.92%	8.45%	13.02%	11.65%	6.52%

Table: 8, Hypotheses and Summary of Specifications

Note: all dates are given relative to most recent accounting year Jan – Dec XXXX

	Hypothesis	Format and date of dependent variable ()	Format and date of Distance to Default	Other explanatory variables
H1	**"Current distance to default helps explain current rating, relative to current accounting data"**	**R=One of four rating bands: as of March XXXX+1**	**DD: March XXXX+1**	**Various accounting ratios**
H2	**"Current distance to default helps explain current rating, relative to stale accounting data"**	**R=One of four rating bands: as of Sept XXXX+1**	**DD: Sept XXXX+1**	**Various accounting ratios**
H3	**"Current distance to default helps explain future rating, relative to current accounting data"**	**R=one of the four rating bands: as of March XXXX+2**	**DD: March XXXX+1**	**Various accounting ratios**
H4	**"Recent change in distance to default helps predict future rating, when accounting data is current"**	**R= September XXXX+1**	**Δ DD (DD March XXXX+1 *minus* DD Sept XXXX)**	**Various accounting ratios**
H5	**"Recent change in distance to default helps predict future rating, when accounting data is stale"**	**R= March XXXX+2**	**Δ DD (DD Sept XXXX+1 *minus* DD March XXXX+1)**	**Various accounting ratios**

Table 9: Ability of distance to default to predict downgrades

	Status	Nobs	Mean (DD)	t-test	p-value
12 months	0	71	3.059	3.938**	0.015
	1	3	2.239		
24 months	0	71	3.086	2.873**	0.032
	1	3	2.290		

This table presents the results of the mean comparison test. Status 1 represents the financial fragile banks while 0 represents the financially healthy banks. We present the number of observations (denoted by Nobs), the mean, t-test and p-value. Note that ** indicate statistical significance at 5% confidence level.

Table 10: Correlation matrix among the variables

Variable	Equity/ TA	Total Capital Ratio	Tier 1 Ratio	Equity/ Net Loans	Imp. Loans/ LLR	Imp. Loans/ Gross Loans	ROAA	ROAE	Int. Margin	Non-Interest Exp/A A	Liquid Assets/ TA	ST Borrow ing/TA	Ln (TA)	DD
Equity/ TA	1													
Total Capital Ratio	0.392	1												
Tier 1 Ratio	0.453	0.797	1											
Equity/ Net Loans	0.599	0.431	0.515	1										
Imp. Loans/ LLR	-0.402	-0.112	-0.227	-0.319	1									
Imp. Loans/ Gross Loans	-0.221	-0.100	-0.233	-0.063	0.825	1								
ROAA	0.617	0.095	0.156	0.395	-0.361	-0.249	1							
ROAE	-0.070	-0.170	-0.147	-0.060	-0.017	-0.052	0.656	1						
Int. Margin	0.612	0.130	0.213	0.250	-0.476	-0.279	0.496	0.044	1					
Non Interest Exp/A A	0.553	0.064	0.114	0.467	-0.314	-0.083	0.440	-0.003	0.539	1				
Liquid Assets/ TA	-0.002	0.119	0.019	0.277	-0.025	0.063	0.021	0.070	-0.076	0.070	1			
ST Borrow ing/TA	-0.051	0.156	0.262	-0.053	-0.210	-0.281	-0.072	-0.098	0.162	-0.036	-0.245	1		
ln(TA)	-0.339	-0.305	-0.454	-0.063	0.302	0.426	-0.124	0.146	-0.339	-0.064	0.117	-0.510	1	
DD	0.013	0.066	0.097	-0.053	-0.218	-0.192	0.046	0.010	0.108	-0.015	-0.175	0.077	-0.064	1

Table 11: Regression results using ordered probit model, total data

	H1	H2	H3	H4	H5
Tier 1 Ratio	-0.165 *(-3.70)*	-0.173 *(-3.93)*	-0.183 *(-4.06)*	-0.148 *(-3.50)*	-0.182 *(-4.09)*
Equity/Net Loans	1.002 *(4.01)*	1.237 *(4.70)*	1.193 *(4.49)*	1.388 *(5.41)*	1.183 *(4.49)*
Impaired Loans/Gross Loans	-0.384 *(-4.17)*	-0.509 *(-5.41)*	-0.513 *(-5.40)*	-0.504 *(-5.28)*	-0.516 *(-5.46)*
ROAE	0.073 *(5.52)*	0.079 *(5.94)*	0.079 *(5.92)*	0.088 *(6.54)*	0.081 *(5.96)*
Interest Margin	-0.373 *(-4.63)*	-0.382 *(-4.73)*	-0.382 *(-4.67)*	-0.344 *(-4.35)*	-0.377 *(-4.62)*
Non-Interest Expenses/Average Assets	0.140 *(2.17)*	0.162 *(2.51)*	0.126 *(1.96)*	0.070 *(1.09)*	0.129 *(2.00)*
Short-term Borrowing/Total Assets	0.042 *(3.39)*	0.056 *(4.39)*	0.055 *(4.27)*	0.059 *(4.69)*	0.056 *(4.34)*
ln(Total Assets)	1.111 *(13.83)*	1.151 *(14.02)*	1.156 *(14.02)*	1.218 *(14.31)*	1.162 *(14.04)*
Distance-to-Default	0.044 *(0.49)*	0.046 *(0.53)*	0.013 *(0.15)*	0.034 *(0.33)*	0.111 *(0.93)*
Number of obs	375	383	379	386	379
Log Likelihood	-294.181	-301.777	-291.723	-298.244	-291.303
Log Likelihood (accounting only model)	-294.300	-301.915	-291.734	-298.297	-291.733
LR chi2 (extended vs accounting model)	0.238	0.276	0.022	0.106	0.86
d.f.	1	1	1	1	1
Prob>chi2 (extended vs accounting model)	0.626	0.599	0.882	0.745	0.354
Prob>chi2 (extended vs null model)	0.000	0.000	0.000	0.000	0.000
Pseudo R2	0.427	0.425	0.436	0.431	0.437

H1: Current distance-to-default helps explain current rating relative to current accounting data
H2: Current distance-to-default helps explain current rating relative to stale accounting data
H3: Current distance-to-default helps explain future rating relative to current accounting data
H4: Recent change in distance-to-default helps predict future rating, when accounting data is current
H5: Recent change in distance-to-default helps predict future rating, when accounting data is stale

Table 12: Predict current rating in sample, total data

	Using DD for current rating				Accounting only model			
Ratings	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)
1	**0.661**	0.218	0.104	0.017	**0.655**	0.220	0.103	0.022
2	0.293	**0.375**	0.284	0.049	0.334	**0.368**	0.257	0.041
3	0.054	0.202	**0.477**	0.267	0.058	0.207	**0.475**	0.260
4	0.008	0.042	0.289	**0.661**	0.008	0.042	0.288	**0.662**

This table reports the in sample prediction of the mean probabilities of estimating a rating category using the model to test Hypothesis 1. Comparison of accounting only model and "extended" model that includes DD together with the accounting variables.

Table 13: Predict current 6 month rating in sample, total data

	Using DD for 6 month current rating				Accounting only model			
Ratings	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)
1	**0.644**	0.216	0.115	0.025	**0.649**	0.224	0.107	0.020
2	0.282	**0.380**	0.289	0.049	0.319	**0.371**	0.267	0.042
3	0.051	0.198	**0.456**	0.295	0.053	0.198	**0.456**	0.293
4	0.008	0.042	0.279	**0.671**	0.008	0.042	0.278	**0.671**

This table reports the in sample prediction of the mean probabilities of estimating a rating category using the model to test Hypothesis 2. Comparison of accounting only model and "extended" model that includes DD together with the accounting variables.

Table 14: Future rating prediction: using DD, total data

	Using DD				Accounting only model			
Ratings	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)
1	**0.675**	0.213	0.097	0.015	**0.693**	0.208	0.087	0.012
2	0.283	**0.379**	0.287	0.051	0.329	**0.365**	0.259	0.047
3	0.049	0.197	**0.467**	0.287	0.055	0.204	**0.469**	0.271
4	0.009	0.045	0.276	**0.671**	0.009	0.047	0.285	**0.660**

This table reports the in sample prediction of the mean probabilities of estimating a rating category using the model to test Hypothesis 3. Comparison of accounting only model and "extended" model that includes DD together with the accounting variables.

Table 15: Predict future rating using the difference of the DD rating 1999=distance1999-distance6m 1998, total data

	Using DD				Accounting only model			
Ratings	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)
1	**0.633**	0.228	0.122	0.017	**0.632**	0.231	0.117	0.020
2	0.282	**0.357**	0.308	0.053	0.319	**0.356**	0.282	0.043
3	0.056	0.194	**0.454**	0.297	0.054	0.189	**0.458**	0.299
4	0.007	0.034	0.244	**0.715**	0.007	0.034	0.251	**0.708**

This table reports the in sample prediction of the mean probabilities of estimating a rating category using the model to test Hypothesis 4. Comparison of accounting only model and "extended" model that includes DD together with the accounting variables.

Table 16: Predict future rating using the difference of the DD 6 months after release rating 1999=distance6m1998-distance 1998, total data

	Using DD				Accounting only model			
Ratings	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)
1	**0.674**	0.213	0.099	0.015	**0.693**	0.208	0.087	0.012
2	0.285	**0.379**	0.286	0.050	0.329	**0.365**	0.259	0.047
3	0.049	0.196	**0.468**	0.287	0.055	0.204	**0.469**	0.271
4	0.008	0.044	0.276	**0.672**	0.009	0.047	0.285	**0.659**

This table reports the in sample prediction of the mean probabilities of estimating a rating category using the model to test Hypothesis 5. Comparison of accounting only model and "extended" model that includes DD together with the accounting variables.

Table 17: Regression results using ordered probit model, small banks only

	H1	H2	H3	H4	H5
Tier 1 Ratio	-0.059 *(-1.12)*	-0.062 *(-1.18)*	-0.111 *(-2.00)*	-0.040 *(-0.80)*	-0.103 *(-1.86)*
Equity/Net Loans	0.131 *(0.40)*	0.305 *(0.91)*	0.571 *(1.65)*	0.535 *(1.64)*	0.541 *(1.57)*
Impaired Loans/Gross Loans	-0.507 *(-3.77)*	-0.664 *(-4.81)*	-0.672 *(-4.86)*	-0.569 *(-4.38)*	-0.675 *(-4.89)*
ROAE	0.052 *(2.56)*	0.066 *(3.25)*	0.071 *(3.47)*	0.073 *(3.76)*	0.074 *(3.64)*
Interest Margin	-0.402 *(-3.84)*	-0.369 *(-3.58)*	-0.401 *(-3.74)*	-0.283 *(-2.93)*	-0.387 *(-3.60)*
Non-Interest Expenses/Average Assets	0.184 *(2.44)*	0.209 *(2.74)*	0.137 *(1.80)*	0.106 *(1.43)*	0.132 *(1.74)*
Short-term Borrowing/Total Assets	0.011 *(0.65)*	0.037 *(2.10)*	0.038 *(2.17)*	0.028 *(1.61)*	0.036 *(2.08)*
ln(Total Assets)	1.57 *(9.55)*	1.608 *(9.77)*	1.150 *(9.21)*	1.474 *(9.49)*	1.485 *(9.21)*
Distance-to-Default	0.265 *(2.21)*	0.270 *(2.31)*	0.181 *(1.49)*	-0.041 *(-0.31)*	0.068 *(0.44)*
Number of obs	213	219	210	217	210
Log Likelihood	-154.852	-159.576	-154.202	-176.005	-155.216
Log Likelihood (accounting only model)	-157.328	-162.283	-155.31	-176.055	-155.314
LR chi2 (extended vs accounting model)	4.952	5.414	2.216	0.100	0.196
d.f.	1	1	1	1	1
Prob>chi2 (extended vs accounting model)	0.026	0.020	0.137	0.752	0.658
Prob>chi2 (extended vs null model)	0.000	0.000	0.000	0.000	0.000
Pseudo R2	0.401	0.408	0.401	0.350	0.397

H1: Current distance-to-default helps explain current rating relative to current accounting data
H2: Current distance-to-default helps explain current rating relative to stale accounting data
H3: Current distance-to-default helps explain future rating relative to current accounting data
H4: Recent change in distance-to-default helps predict future rating, when accounting data is current
H5: Recent change in distance-to-default helps predict future rating, when accounting data is stale
Note: Small banks; Banks that their ln(Total Assets) is less than 25

Table 18: Regression results using ordered probit model, big banks only

	H1	H2	H3	H4	H5
Tier 1 Ratio	-0.339 *(-2.60)*	-0.381 *(-2.81)*	-0.223 *(-1.98)*	-0.316 *(-2.77)*	-0.235 *(-2.22)*
Equity/Net Loans	2.117 *(4.27)*	2.422 *(4.70)*	1.847 *(3.83)*	2.414 *(4.75)*	1.834 *(3.90)*
Impaired Loans/Gross Loans	-0.314 *(-1.81)*	-0.405 *(-2.22)*	-0.365 *(-2.11)*	-0.633 *(-3.51)*	-0.356 *(-2.21)*
ROAE	0.107 *(4.83)*	0.119 *(5.15)*	0.109 *(4.81)*	0.116 *(5.27)*	0.112 *(5.03)*
Interest Margin	-0.795 *(-3.79)*	-0.934 *(-4.29)*	-0.809 *(-3.79)*	-0.819 *(-3.96)*	-0.798 *(-3.81)*
Non-Interest Expenses/Average Assets	0.397 *(2.12)*	0.467 *(2.44)*	0.456 *(2.45)*	0.275 *(1.52)*	0.463 *(2.54)*
Short-term Borrowing/Total Assets	0.084 *(3.89)*	0.078 *(3.66)*	0.068 *(3.24)*	0.084 *(3.92)*	0.070 *(3.36)*
ln(Total Assets)	1.220 *(6.25)*	1.099 *(5.70)*	0.976 *(5.52)*	1.027 *(5.43)*	0.987 *(5.56)*
Distance-to-Default	-0.091 *(-0.54)*	-0.011 *(-0.06)*	-0.138 *(-0.08)*	0.162 *(0.78)*	0.183 *(0.88)*
Number of obs	162	164	169	169	169
Log Likelihood	-108.748	-110.327	-115.542	-102.849	-115.154
Log Likelihood (accounting only model)	-108.892	-110.329	-115.545	-103.158	-115.545
LR chi2 (extended vs accounting model)	0.288	0.004	0.006	0.618	0.782
d.f.	1	1	1	1	1
Prob>chi2 (extended vs accounting model)	0.592	0.950	0.938	0.432	0.377
Prob>chi2 (extended vs null model)	0.000	0.000	0.000	0.000	0.000
Pseudo R2	0.303	0.306	0.274	0.321	0.276

H1: Current distance-to-default helps explain current rating relative to current accounting data
H2: Current distance-to-default helps explain current rating relative to stale accounting data
H3: Current distance-to-default helps explain future rating relative to current accounting data
H4: Recent change in distance-to-default helps predict future rating, when accounting data is current
H5: Recent change in distance-to-default helps predict future rating, when accounting data is stale
Note: Big banks; Banks that their ln(Total Assets) is more than 25

Table 19: Predict current rating in sample, small banks

	Using DD for current rating				**Accounting only model**			
Ratings	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)
1	**0.671**	0.219	0.075	0.034	**0.657**	0.226	0.079	0.038
2	0.236	**0.411**	0.278	0.075	0.295	**0.401**	0.245	0.597
3	0.027	0.161	**0.529**	0.283	0.027	0.169	**0.524**	0.278
4	0.004	0.026	0.202	**0.767**	0.006	0.026	0.195	**0.773**

This table reports the in sample prediction of the mean probabilities of estimating a rating category using the model to test Hypothesis 1. Comparison of accounting only model and "extended" model that includes DD together with the accounting variables using the subsample of "small" banks.

Table 20: Predict current 6 month rating in sample, small banks

	Using DD for 6 month current rating				**Accounting only model**			
Ratings	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)
1	**0.653**	0.212	0.093	0.042	**0.646**	0.231	0.089	0.033
2	0.224	**0.420**	0.292	0.064	0.273	**0.405**	0.269	0.052
3	0.0236	0.159	**0.533**	0.284	0.024	0.159	**0.531**	0.286
4	0.005	0.026	0.222	**0.746**	0.006	0.026	0.214	**0.754**

This table reports the in sample prediction of the mean probabilities of estimating a rating category using the model to test Hypothesis 2. Comparison of accounting only model and "extended" model that includes DD together with the accounting variables using the subsample of "small" banks.

Table 21: Predict current rating in sample, big banks

	Using DD for current rating				**Accounting only model**			
Ratings	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)
1	**0.842**	0.082	0.069	0.006	**0.847**	0.0739	0.069	0.009
2	0.571	**0.194**	0.201	0.033	0.614	**0.178**	0.178	0.030
3	0.175	0.151	**0.373**	0.301	0.192	0.149	**0.365**	0.294
4	0.029	0.033	0.225	**0.713**	0.028	0.033	0.225	**0.713**

This table reports the in sample prediction of the mean probabilities of estimating a rating category using the model to test Hypothesis 1. Comparison of accounting only model and "extended" model that includes DD together with the accounting variables using the subsample of "big" banks.

Table 22: Predict current 6 month rating in sample, big banks

	Using DD for 6 month current rating				**Accounting only model**			
Ratings	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)
1	**0.80**	0.101	0.084	0.011	**0.821**	0.094	0.074	0.010
2	0.516	**0.222**	0.221	0.042	0.556	**0.204**	0.198	0.042
3	0.158	0.152	**0.340**	0.349	0.163	0.150	**0.339**	0.348
4	0.020	0.035	0.214	**0.730**	0.020	0.035	0.214	**0.730**

This table reports the in sample prediction of the mean probabilities of estimating a rating category using the model to test Hypothesis 2. Comparison of accounting only model and "extended" model that includes DD together with the accounting variables using the subsample of "big" banks.

Table 23: Prediction of current rating out of sample, total data

	Using DD for current prediction				Accounting only model			
Ratings	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)
1	**0.478**	0.332	0.179	0.012	**0.4925**	0.328	0.169	0.010
2	0.296	**0.369**	0.305	0.030	0.310	**0.368**	0.294	0.028
3	0.042	0.202	**0.481**	0.275	0.046	0.211	**0.480**	0.263
4	0.003	0.033	0.282	**0.683**	0.003	0.035	0.289	**0.672**

This table reports the out of sample prediction of the mean probabilities of estimating a rating category using the model to test Hypothesis 1. Comparison of accounting only model and "extended" model that includes DD together with the accounting variables.

Table 24: Prediction of current rating out of sample, small banks

	Using DD for current prediction				Accounting only model			
Ratings	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)
1	**0.370**	0.391	0.225	0.013	**0.445**	0.380	0.167	0.008
2	0.208	**0.381**	0.332	0.078	0.278	**0.374**	0.280	0.067
3	0.008	0.118	**0.505**	0.368	0.016	0.165	**0.509**	0.309
4	0.000	0.011	0.166	**0.823**	0.001	0.018	0.204	**0.776**

This table reports the out of sample prediction of the mean probabilities of estimating a rating category using the model to test Hypothesis 1. Comparison of accounting only model and "extended" model that includes DD together with the accounting variables using subsample of "small" banks.

Table 25: Prediction of current rating out of sample, big banks

	Using DD for current prediction				Accounting only model			
Ratings	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)
1	**0.758**	0.137	0.099	0.006	**0.739**	0.142	0.111	0.008
2	0.633	**0.194**	0.164	0.009	0.616	**0.199**	0.174	0.011
3	0.185	0.183	**0.372**	0.260	0.169	0.176	**0.376**	0.277
4	0.017	0.046	0.284	**0.653**	0.014	0.276	0.276	**0.668**

This table reports the out of sample prediction of the mean probabilities of estimating a rating category using the model to test Hypothesis 1. Comparison of accounting only model and "extended" model that includes DD together with the accounting variables using subsample of "big" banks.

Table 26: Predict current 6 month rating out of sample

	Using DD for 6month current prediction				Accounting only model			
Ratings	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)
1	**0.481**	0.324	0.181	0.013	**0.490**	0.328	0.171	0.011
2	0.318	**0.366**	0.282	0.034	0.293	**0.368**	0.305	0.034
3	0.036	0.193	**0.477**	0.294	0.043	0.204	**0.466**	0.287
4	0.003	0.033	0.253	**0.712**	0.003	0.034	0.271	**0.693**

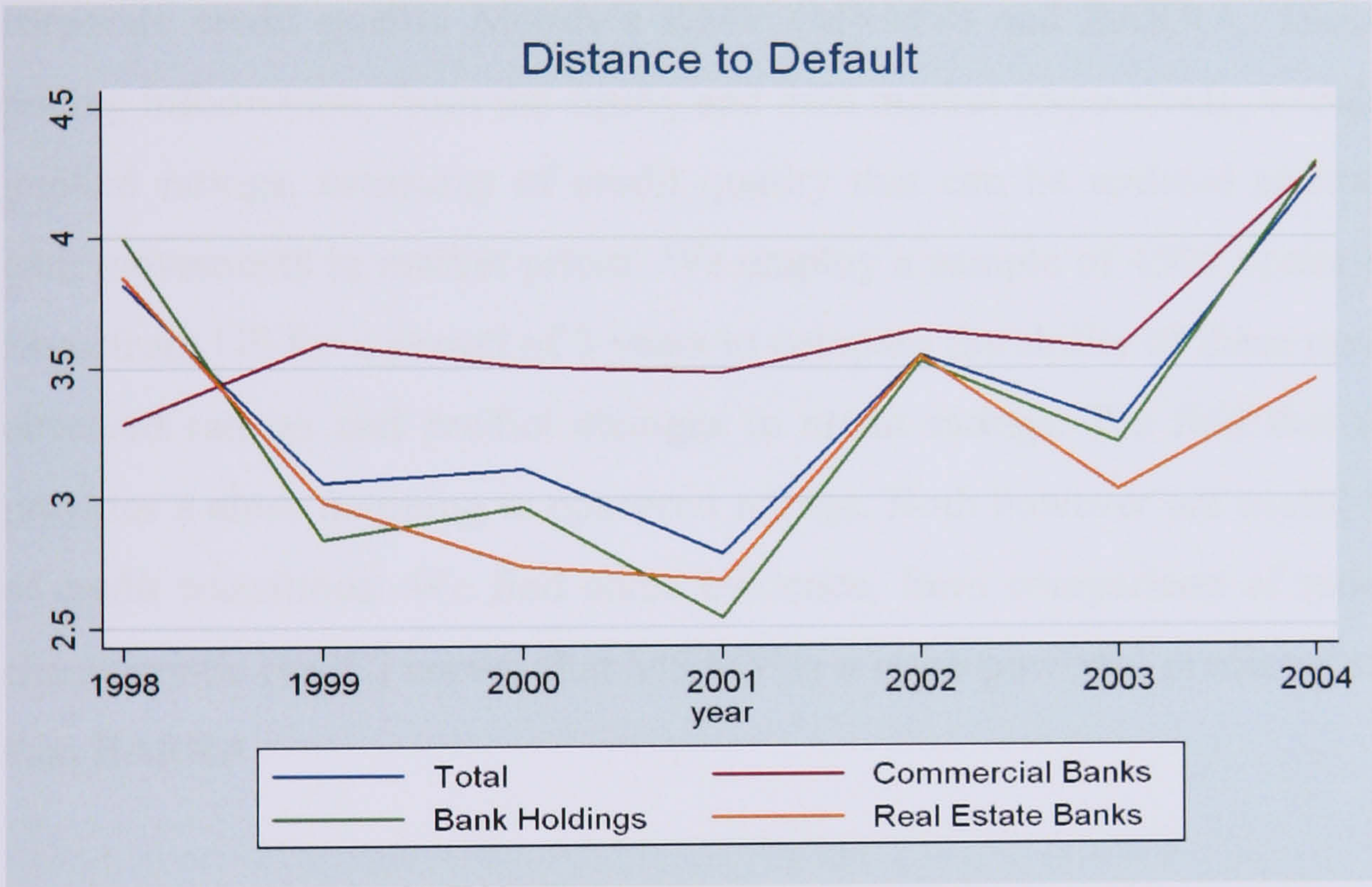
This table reports the out of sample prediction of the mean probabilities of estimating a rating category using the model to test Hypothesis 2. Comparison of accounting only model and "extended" model that includes DD together with the accounting variables.

Table 27: Predict current 6 month rating , out of sample, small banks

	Using DD for 6month current prediction				Accounting only model			
Ratings	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)	Mean (p1)	Mean (p2)	Mean (p3)	Mean (p4)
1	**0.387**	0.384	0.221	0.008	**0.444**	0.382	0.169	0.004
2	0.225	**0.394**	0.313	0.067	0.256	**0.373**	0.323	0.047
3	0.006	0.110	**0.533**	0.351	0.015	0.161	**0.523**	0.301
4	0.001	0.014	0.188	**0.797**	0.001	0.018	0.213	**0.768**

This table reports the out of sample prediction of the mean probabilities of estimating a rating category using the model to test Hypothesis 2. Comparison of accounting only model and "extended" model that includes DD together with the accounting variables using subsample of "small" banks.

Figure 1: The evolution of the mean of Distance-to-Default from 1998 to 2004 for the entire sample and for the bank holding banks, commercial banks and mortgage banks.



CHAPTER 5: Are equity prices better predictors of rating changes than debt prices? A comparison of the Merton/KMV and BARRA models

Abstract of Chapter 5

This study compares the performance of two widely used proprietary models of corporate credit quality Moody's KMV (MKMV) and BARRA. These models use pricing information, from the equity and debt market respectively, to calculate market implied ratings, measures of credit quality that can be updated continuously along with movements in market prices. We employ a sample of 4594 bonds issued by 447 firms from US for a period of 3 years to compare the ability of these models to mimic observed ratings and predict changes in credit ratings. We find that neither model provides a close mapping to observed ratings. Both however are useful for prediction of credit transitions. We find some evidence, from comparison of receiver operator characteristic (ROC) curves that MKMV is a more powerful predictor of downgrades than BARRA.

5.1. Introduction

The last decades have seen rapid growth of the credit markets, increased interest in credit as an investment class, and development of risk-sensitive procedures for assessing prudential capital requirements. As a result measures of credit quality have become increasingly important to regulators, investors and financial institutions. While the core measure of credit quality remains the ratings provided by rating agencies, like Moody's and Standard & Poors, these agency ratings are adjusted only slowly in response to changes in firms' financial situation and business performance. Ratings have also proved quite inadequate for prediction of the collapse of several large companies, like Enron and Parmalat. These acknowledged weaknesses underpin the widespread interest in models that aim to timely predict changes in agency credit ratings.

The objective of this work is to compare two widely used practitioner models, used to extract implied credit ratings from market data. These models are Moody's KMV (which obtains distance to default and by implication credit ratings from equity price data) and BARRA (which obtains ratings predictions directly from bond credit spreads). These models are estimated on a data set combining equity price and option-adjusted bond spreads, for 447 US firms, from January 2000 to December 2002.

Since these two models use information from the equity and bond market respectively, we are also comparing whether credit spreads or equity prices are better predictors of rating changes, subject to the restriction that these predictions are based on these particular proprietary models. Both markets have their own advantages and disadvantages. On one hand the equity market may be more affected by bubbles and irrational behavior of investors, which weakens the relationship between equity prices and credit performance. On the other hand the bond market suffers from low liquidity and hence (although the recent rapid growth of credit default swap markets since 2002 are making credit markets much more liquid than in the past.)

These are several other approaches to modeling ratings transitions and defaults, which will not be addressed in this study. Three main broad categories of default and ratings prediction are found in the literature; accounting models, structural models and reduced-form models

The accounting approach identifies accounting variables that can be used for the prediction of corporate default and rating changes. This approach includes the qualitative judgment by experienced credit professionals, which remains a central part of rating agency methodologies and a large variety of statistical models of rating changes and default based on accounting data.

Although, the accounting approach is informative it has many drawbacks. Financial statements are published at best quarterly, and for smaller firms only annually, and are backward looking. Accounting models of transition and default are also relatively difficult to operate and maintain. Concern about the stability of estimated relationships requires the regular updating of data and re-estimation as a check on model reliability. These models also do not offer any natural procedure for capturing default correlations and hence are relatively difficult to use for modeling distributions of portfolio credit returns.

These limitations explain the practitioner interest in using information from either equity or debt market to produce "market implied" ratings, and hence yield predictions of transition and default . The academic literature has proposed a large number of such models, adopting both structural and reduced form approaches to modeling credit risk. A number of proprietary models used by banks, have now been developed, broadly based on one of these two approaches, in order to predict ratings and default.

The two models compared here are the most widely used proprietary models of this kind. These are the Moody's KMV credit monitor (MKMV) and the BARRA model[29]. MKMV model is a structural model based on the Merton (1974) model of corporate default that views equity as a call option on the underlying assets of the firm. It uses the volatility and the value of equity extracted from the equity market to predict the distance to default (DD) and the expected default frequency (EDF). The most widely used practitioner model using debt prices for prediction of default and

[29] BARRA is now merged with Morgan Stanley Capital International Inc and operates under the name MSCI BARRA.

rating transition is that of BARRA. This can be viewed as a reduced form type model relating option-adjusted spreads to market implied ratings.[30]

There are a number of previous studies examining the predictive performance of different models of credit risk.[31] This literature includes comparisons of accounting models with structural models and of different structural and reduced-form specifications. For instance, Hillegeist et al (2004) find that structural models contain more information than accounting models (Altman's Z-Score or Ohlson's O-Score) for the prediction of firm default.

Other studies suggest that structural models are too restrictive and reduced form specifications can improve predictions of default or rating transition. Du and Suo (2003) find that distance to default measure is not sufficient for the prediction of credit quality, when used alone. They conclude that, when distance to default is used together with the firm market value, has higher power for predicting credit ratings than when distance to default is used on its own.

There do not appear to have been any previous studies which directly compare the predictive performance of implied ratings drawn from equity and from bond markets for corporates[32]. The relationship between market prices and credit ratings is however addressed in a another related literature, investigating the impact of credit ratings and credit rating changes on stock returns and on bond spreads (rather than as in this study the effectiveness of bond or equity prices as predictors or credit rating changes). Holthausen and Leftwich (1986), Hand et al (1992) and Dichev and Piotroski (2001) observe negative abnormal returns after a review for a downgrade but there is no significant abnormal equity return reaction following upgrades. Goh and Ederington (1999) conclude that the negative reaction to downgrades is stronger for downgrades to and amongst speculative grade (BB or lower) firms compared to downgrades within the investment grade category.

---

[30] A full description of BARRA model can be found on the paper "Market implied ratings", by Breger, Goldberg and Cheyette published in BARRA website:
(www.barra.com/support/library/credit/market_implied_ratings.pdf).
The paper has also been published at Risk Magazine, July 2003.

[31] Refer to Chapter 2 for a more detailed description of studies on the comparison of different credit risk models.

[32] Gropp et al (2006) examine the ability of both equity and bond market indicators (distance to default and bond spreads respectively) to signal bank fragility. Their study though focuses on banks while our study focuses on corporates.

Examining bond price reactions to rating changes, Katz (1974) and Grier and Katz (1976) conclude that in the industrial bond market there was some anticipation before decreases but not increases. Hand et al. (1992) find strong negative effect of downgrades on bond returns during the period just before and after the announcement. Wansley et al. (1992) find a linkage between significant abnormal returns and changes in credit ratings. Nevertheless, both Hite and Warga (1997) and Dynkin et al. (2002) find that bond prices reaction is stronger for downgrades to and amongst speculative grade (BB or lower) compared to downgrades within the investment grade category. Hull, Predescu and White (2004) examined the relationship between credit default swaps changes and credit rating changes. They find evidence that the credit default swap market anticipates downgrades, while the impact of upgrades is lower.

Other studies examined the relationship between credit ratings and credit spreads. West (1973) and Liu and Thakor (1984) show that on average bond spreads and credit ratings are negatively correlated, with a higher spread associated with higher default risk and hence a lower credit rating. Nevertheless, this relationship is not close. Taylor and Perraudin (2001) showed that on individual spreads even after accounting for liquidity and tax effects, are highly variable within credit rating classes.

This chapter is organized as follows. Section 2 provides a description of Moody's KMV equity price based model – including the estimation of "market" implied ratings using a Merton/MKMV model – and the BARRA credit spread model. Section 3 describes the data set and provides summary statistics. In Section 4, we present the empirical results, while in Section 5 we conclude.

5.2. Description of the two models

This section describes the two proprietary models compared in this work, Moody's KMV and BARRA, and explains how implied credit ratings and hence predictions of rating changes can be obtained from these two models. Both these models have already been discussed in Chapter 2 of the thesis, but for reasons of continuity we restate the main models here.

Both MKMV and BARRA models can be used to derive market implied credit ratings using market data for a single day. Thus, in contrast to accounting based models, they can be updated on a continuous basis and have the potential to provide timely information on changes in credit quality.

5.2.1. Moody's KMV model

Moody's KMV (MKMV) model is based on the structural credit risk Merton model. Merton (1974) pioneered the structural credit risk models since he was the first to use the option pricing theory (OPT) developed by Black and Scholes (1973) in the valuation of default risk spreads of fixed income instruments.[33]

As in the original Merton model, MKMV assumes that the firm's market value of assets follows a log-normal stochastic process. MKMV uses the same option-pricing formula for the value of equity as the Merton model. The value of equity V_e is seen as a call option on the value of assets V_a with strike price the default barrier P and is expressed using Black and Scholes option pricing formula:

$$V_e = V_a N(d_1) - Pe^{-rT} N(d_2), \qquad [5.1]$$

where

$$d_1 = \frac{\log\left(\frac{V_a}{P}\right) + \left(r + \frac{\sigma_a^2}{2}\right)T}{\sigma_a \sqrt{T}} \qquad [5.2]$$

and

$$d_2 = d_1 - \sigma_a \sqrt{T} = \frac{\log\left(\frac{V_a}{P}\right) + \left(r - \frac{\sigma_a^2}{2}\right)T}{\sigma_a \sqrt{T}}, \qquad [5.3]$$

and V_α is the market value of assets, σ_a is the volatility of assets, P is the default barrier, r is the risk-free rate and N(.) is the cumulative standard normal function.

Moody's KMV model uses an ad hoc formula for the calculation of the default barrier. While in Merton's case the default barrier is equal to the firm's book value of total liabilities in MKMV model is determined as the sum of short-term liabilities and half of the long-term debt:

[33] Merton's model has been thoroughly discussed in Chapters 2 and 3 of the thesis.

$$P = ShortTerm\_Liabilities + \frac{1}{2} LongTerm\_Liabilities \qquad [5.4]$$

The reason for this adjustment is that, in practice, many firms do not default when their market value of assets reaches their total liabilities, since the long-term nature of some of their liabilities allows them to operate and avoid default, at least in the short-term.

As in any structural model, the firm's default probability depends mainly on three variables; the asset value, the volatility of the assets and the default boundary. The default boundary can be extracted from the firm's financial statements, while the value and the volatility of the assets are unknown and have to be estimated.[34]

Using the estimated values of the value and the volatility of assets, the distance-to-default (DD) measure is estimated. The distance-to-default denotes how many standard deviations is the market value of assets away from the default barrier. Hence, the higher a firm's distance to default measure, the lower is the firm's probability of default.

The formula used for the calculation of DD measure is the following:

$$DD = \left[\frac{\ln\left(\frac{V_a}{P}\right) + \left(\mu - \frac{\sigma_a^2}{2}\right)T}{\sigma_a \sqrt{T}} \right] \qquad [5.5]$$

where μ denotes the expected return on the firm's assets and it is used to derive the default probability under the true probability measure.[35] Note that the formula for DD is the same as that previously given for the risk-neutral default probability d_2 except that the risk-free rate in the previous formula is replaced by the expected return on the firm's assets.

The main difference between Merton and MKMV models is on the final calculation of Expected Default Frequencies (EDFs). In Merton case it is calculated as N(-DD). This approach is based on the assumption that the firm's asset returns are normally distributed, which for the calculation of default probabilities is not so appropriate. The

[34] Techniques on how the value and volatility of assets are estimated will be analyzed in Section 5.2.2.
[35] Note that for the calculation of DD, the risk-free rate r can be used instead of μ. In this case, the default probability is computed under the risk neutral measure.

most important reason is that the default point is random. Since ex ante it is not possible to examine the behavior of the liabilities, MKMV tries to "correct" the bias in the EDFs caused by the use of the normal distribution, by mapping the distance to default measure to an EDF measure. Using a database of over 250,000 company-years observations with over 4,700 incidents of default, the model extracts a frequency table which relates the probability of default to different levels of distance to default. For instance, if the calculated distance to default for a firm is 5, then the corresponding EDF for 1 year horizon is calculated as the ratio of the firms with DD 5 that defaulted over 1 year time period to the total number of firms with DD 5.[36]

A principal attraction of the Merton/MKMV model is that information from the equity markets is used so as to predict the credit quality changes. Efficient market theory suggests that investors are rational and all publicly available information should be reflected on the equity prices. Hence, the information on the ability of a firm to repay its debts is reflected on the fluctuation and the level of its equity price. Assuming that the structural model is correctly specified, then equity market data can then be used so as to compute two unobserved variables: the market value of assets and the volatility of the bank's assets. In this way, equity data can be used for the calculation of default probabilities in a more timely fashion compared to the traditional accounting models, since distance-to-default and hence EDFs can be continuously updated.

5.2.2. Estimation of market implied ratings from equity data (Merton/MKMV model)

The purpose of MKMV model is to produce an EDF for each firm which can be continuously updated. Each firm's EDF can subsequently be mapped to a rating. This subsection describes the calculation used in this study for computing distance to default and implied ratings from the Merton/MKMV model.

For the calculation of firm's DD, two unknown variables need to be estimated: the value and the volatility of assets. In this paper, we follow the work by Vassalou and Xing (2004) and Du & Suo (2003) for the estimation of market value and volatility of a firm's asset who use a similar approach as MKMV.

[36] This adjustment is described in Peter M. Crosbie (2000) "Modelling Default Risk" chapter 9 of Satjayit Das (ed) Credit Derivatives and Credit Linked Notes, 2nd ed., John Wiley and Sons, pp 369-410.

We use an iterative procedure to extract the value and volatility of a firm's asset. We use one year estimation window of stock prices to estimate the value and volatility of a firm's assets. Using the value of the firm's equity volatility as an initial guess of the value of asset volatility, we solve equation 5.1 and a set of asset values is determined. Then for the second iteration we set the asset volatility equal to the annualized volatility of the set of asset returns derived from the first iteration. This iteration generates a new set of asset values and so on. The procedure continues until it converges. Using the derived value, volatility and expected return on assets, MKMV produces the distance-to-default measure as described above.

Alternatively, the value and the volatility of assets can be determined by solving the following equation together with equation 5.1. From Ito's lemma, we can extract a formula that connects volatility of equity to the volatility of assets:

$$\sigma_e = \sigma_a * \frac{V_a}{V_e} * \frac{\partial V_e}{\partial V_a} \qquad [5.6]$$

where σ_e is the equity volatility, calculated as the standard deviation of firm's returns and $\frac{\partial V_e}{\partial V_a} = N(d_1)$. In this case, the equations 5.1 and 5.6 are solved simultaneously to obtain the estimation of value and volatility of assets.[37]

While our methodology of computing the value and volatility of assets is the same as followed by MKMV, for the calculation of distance to default measure, we use the risk free rate instead of μ, which is the expected return on firm's assets. Hence, the distance to default measure is calculated as follows:

$$DD = \left[\frac{\ln\left(\frac{V_a}{P}\right) + \left(r - \frac{\sigma_a^2}{2}\right)T}{\sigma_a\sqrt{T}} \right] \qquad [5.7]$$

The main advantage of this approach, as pointed out by Vassalou and Xing (2005), is that we avoid induced estimation errors arising from the need to estimate μ.[38]

[37] Since for the estimation of the MKMV ratings it is the magnitude that counts instead of the absolute value of DD, we do not expect the final results to change much by estimating the value and the volatility of assets using equations 5.1 and 5.6.

As pointed out before, MKMV maps the distance to default measure to historical default data to determine the corresponding default probability credit ratings. We use instead an econometric model to related distance to default to the observed credit rates in our sample. In this case the dependent variable – the credit rating -- is both discrete and associated with a number (in our case the rating categories) of ordered outcomes. Hence, an appropriate estimation technique is the ordered probit model.[39]

Then for N firms (i=1,....N) the ordered probit model can be represented as follows:

$$Y_t = a + DD_{it} + \varepsilon_i \qquad [5.8]$$

where Y_i is an integer ordinal variable that can be associated with the different credit rating categories, such that $Y_i = 1$ if the bank has low credit quality, etc and belongs to $\{1,2,3,4,5,6\}$.

Estimating the ordered probit model, the probability that a particular rating will be assigned to a particular issuer can be derived. We take as the model implied rating to be the one with the highest assigned probability given by the model.

5.2.3. BARRA model

The use of the equity market data for the quantification of credit risk has its pitfalls, since equity markets are highly volatile. Moreover, behavioral finance offers several reasons for investor overreaction and helps explain the bubbles observed in the equity market. This generates some doubts as to whether equity market is the best available method for the prediction of credit rating changes.

An alternative is the use of debt market information in the estimation of credit risk, which may be more informative since bond spreads directly reflect the compensation required by investors for the risk due to credit rating changes and default. There are a number of theoretical models – both structural and reduced form – that can be used to obtain estimates of default probabilities from bond market data. We focus on one of the most widely used, the proprietary model of BARRA that derives market implied rating using the information from bond spreads.

---

[38] Vassalou and Xing (2005) show that their results remain relatively the same by using μ measure instead of the risk free rate.

[39] Ordered probit or logit models are more efficient for the modelling of multiple rating categories than the multinomial probit or logit models, commonly used for modelling multiple discrete outcomes. Multinomial probit or logit should be used only when there is no inherent ordering of the different choices. Chapter 4, Section 4.3 provides a detailed description of the ordered probit model.

This model is based on the assumption that on average the agency ratings are informative. It develops a mapping between observed bond spreads and ratings. From this mapping it can then identify those issuers with implied ratings that differ markedly from their actual agency rating. This then leads to predictions of rating changes. The main advantage of this model, as in MKMV model, is that the ratings can be derived on a continuous basis and are taking advantage of the information regarding the credit quality of the firm that is already reflected in bond prices.

The BARRA model uses option-adjusted spreads for the derivation of market implied rating.[40] Moreover, it uses issuers rather than individual bond issues. The spread of an issuer is computed as the average of the spreads of the issuer's outstanding bonds. Then, using the average spread per issuer, a distribution of the average issuer spreads over different rating categories is constructed.

As our data confirms, the resulting distribution of issuer average spreads exhibits large overlaps between individual ratings sub-distributions, observing a wide range of different ratings for the same average spread. The BARRA model does not try to explain these differences, rather it tries to use them to develop an implied classification and hence predict future ratings changes. The main estimation challenge in computing the implied ratings is to determine a sequence of spreads $b_{AAA/AA}$, $b_{AA/A}$, $b_{A/BBB}$ etc. that correspond to the boundaries between rating classes. Thus for example any issuer with an average spread s_j that lies between $b_{AAA/AA}$ and $b_{AA/A}$ will have an implied rating of AA; any issuer with an average spread $b_{AA/A}$ and $b_{A/BBB}$ will have an implied rating of A; etc.

More formally we can write these thresholds for the implied ratings as the vector b which can be represented as follows:

$$
\begin{aligned}
b &= \left(b_0^+, b_1^+, b_2^+, b_3^+, b_4^+, b_5^+\right) \\
&= \left(b_1^-, b_2^-, b_3^-, b_4^-, b_5^-, b_6^-\right) \\
&= \left(b_{AAA/AA}, b_{AA/A}, b_{A/BBB}, b_{BBB/BB}, b_{BB/B}, b_{B/CCC}\right)
\end{aligned}
$$

[40] Option-adjusted spreads (OAS) is a method of making spreads from different bonds more comparable. Hence, it is useful for comparing bonds with different characteristics on a more equal basis. More precisely, OAS is a measure of a bond's extra return over the return of a comparable Treasury bond net of the cost of any embedded options (refer to Cavallo and Valenzuela (2007)). We use the OAS analysis from Bloomberg.

To determine the vector b, the model minimises a penalty function that measures the gap between the observed spread s_j and the rating class boundaries, for any issuer *j* whose implied classification is different from its actual agency classification. The penalty function can be represented as below:

$$P(b) = \sum_j \left[w_j * \left(s_j - b^{+}_{i(j)}\right)^{+} + w_j \left(b^{-}_{i(j)} - s_j\right)^{+} \right]$$

where:

$i(j)$: agency rating index of issuer j

s_j : spread of issuer j

b_l^- : lower threshold for implied rating index l

b_l^+ : upper threshold for implied rating index l

N : total number of issuers in the universe

N_l : number of issuers with rating l

$w_j = \dfrac{N}{N_{i(j)}}$: weight which is chosen to equalize the contribution of each rating bucket to the total penalty function.

The "plus" signs at the end of each parenthesis mean that the term is taken into account if and only if it is positive, i.e. only if the observed credit spread is respectively above or below the range of spreads consistent with the agency rating *i(j)* of issuer *j*.

The values of vector b that minimize the penalty function are the implied classification thresholds.

The main drawback of BARRA model is that it assumes a flat credit spread yield curve. However, they address the issue and show that even if one splits the sample for similar maturity bonds or for bonds in the same industry the results do not change. In this chapter we estimate the market implied ratings as suggested by BARRA.

5.3. Data and summary statistics

For the purpose of this chapter, we have collected data for a sample of 4594 bonds issued by 447 firms from US for a period of 3 years, from January 2000 to December 2002. We have used daily option adjusted spreads from the 4594 bonds.

Credit ratings of each firm or issuer have been collected for the period between January 2000 and December 2002 from Bloomberg. We restrict credit rating changes to those announced by three major credit rating agencies: Moody's, S&P and Fitch. We use the issuer rating or senior unsecured debt rating, which is used frequently as a proxy of the issuer rating. Mainly, Moody's has been used for the reporting of the credit ratings and credit rating changes, while information from S&P and Fitch has been employed when Moody's did not provide the required rating. Credit ratings are grouped into 6 broad categories: AAA, AA, A, BBB, BB, B or less. In our sample we have 151 credit rating downgrades and 42 rating upgrades. Table 1, presents the frequency of each rating category in our sample while Table 2 shows the number of upgrades or downgrades to each rating category. We observe that rating categories A and BBB have the highest number of observations.

Figures 1, 2, and 3 show the relationship between the credit rating and average issuer spreads. For all years (2000-2003) we observe that on average the higher the spreads the lower the credit rating. There is however a considerable overlap between different credit ratings, it is common to observe both high spreads are found for low ratings and low spreads for high ratings. Table 3, presents the descriptive statistics for the credit spreads. The results of the percentiles confirm that on average the higher the spread the lower the rating but the relationship is not close because of these overlaps.

Regarding the MKMV model, for the calculation of distance-to-default measure, as mentioned before, there are five variables that are necessary for the estimation of the unobserved value and volatility of assets: the default barrier, market capitalization or "value of equity", equity volatility, time horizon and risk free interest rate.

We calculate the default barrier using COMPUSTAT database to extract information from firms' financial statements. We use the "Debt in one Year" and "Long Term Debt" COMPUSTAT data items as the firms' short-term and long-term liabilities respectively.

For the calculation of value of equity, data on the number of outstanding shares and have been collected from COMPUSTAT database for the same period of time. Daily equity price data have also been collected from Bloomberg for the period between 1999 and 2003. The reason for collecting one additional year of equity data is due to the fact that we use a 12-month window in equity volatility calculations using the standard deviation of equity returns. Note that we could use a GARCH type model for the calculation of equity volatility or an exponentially weighted average. Nevertheless, we use a 12 month window since this is the most widespread approach in the literature and our analysis in Chapter 3 suggested that this approach and the exponentially weighted average yield similar results.

A number of the 447 bond-issuing firms are not found in the COMPUSTAT database; while even for those that are included, in some cases equity price data is not available as far back as January 1999. For this reason the size of the sample for which we can compute MKMV predicted ratings is a good deal smaller than that for which we can compute BARRA predicted ratings. This means that for the estimation of Merton/MKMV model the number of firms used is reduced to 306 due to non availability of either equity or financial data and the number of daily observations falls from 381,188 to 273,416.

Following Vassalou and Xing (2004) and Du and Suo (2003) we use the monthly one year constant maturity Treasury bill rates as a proxy for the risk-interest rate. The data for this variable has been derived from the US Federal Reserve Bank.

The time horizon T is set to 1 year, which is used in the literature as benchmark.

To calculate the Distance-to-Default measure, the value and the volatility of assets are estimated using the iterative process described in Section 5.2.2.

Table 4 reports the descriptive statistics of the distance to default per rating category. As expected the average distance to default falls as the rating quality decreases. Nevertheless once again the relationship is far from close. The correlation between credit rating and distance to default is -0.4344.[41] As Figure 4 shows, just as with credit spreads, there is considerable overlap of distance to default in different ratings

[41] Note that the minus sign suggests that the higher the rating the higher the distance to default. The minus sign is due to the fact that in our analysis a higher rating is represented by a lower number, i.e. 1 is rating AAA while 6 denoted rating B or below.

categories. There is also some indication in Figure 4 of a non-linear relationship between the credit rating and the distance to default.

5.4. Empirical results

This section provides estimation results and compares BARRA and Merton/MKMV models in terms of their ability to predict ratings and rating changes.

As mentioned before, for the BARRA model the prediction of model implied ratings comes directly from the estimation of the model. For the prediction of model implied ratings using Merton/MKMV model we must use the further ordered probit model, specified in Section 5.2.2.

Table 5 presents the results from the estimation of ordered probit model, where distance to default measure is the only independent variable. The results confirm the relation between distance to default and credit ratings; the higher the distance to default measure the higher the credit rating.[42] We find that the distance to default measure is statistically highly significantly related to the credit rating with a very high value of the Z-statistic (195.84) and a p-value effectively equal to zero. As far the model's explanatory power, since logistic regression does not have an equivalent of R-squared, the test statistics used should be treated carefully. We report the Prob>chi2 and McFadden's pseudo R-squared. The Prob>chi2 (or p-value) is the probability of getting a likelihood ratio test statistic as extreme as the one observed under the null hypothesis where we assume that all of the regression coefficients of the model are equal to zero. Although we reject the null hypothesis (Prob>chi2 is equal to zero), the low reported value of McFadden's pseudo R-squared may indicate that the predictive ability of the model is low and that other variables, including possibly bond spreads, could help with the prediction of credit transitions.

As explained in Section 2, using the ordered probit model we compute the predicted probability of each firm falling in each rating category. The model implied rating from Merton/MKMV is assigned to be the rating with the highest predicted probability.

Tables 6 and 7 report respectively the BARRA and Merton/MKMV implied ratings compared to the Moody's rating. In both cases the relationship between the market

[42] Note that in this case only, a higher rating is represented by a higher number, i.e. 6 is rating AAA while 1 denoted rating B or below.

implied and actual Moody's rating is a weak one. Both models tend to allocate ratings towards the middle ratings banks, allocating many more companies to ratings categories 3 and 4 (A and BBB) than the actual Moody's ratings, and far less companies to ratings categories 1 and 2 (AAA and AA) or 5 and 6 (BB or B and below).

This tendency to re-allocate to central ratings is especially pronounced for Moody's KMV. For this reason the Merton-MKMV model produces an even weaker relationship with the actual rating than does BARRA. As indicated in Table 7, the Merton-KMV model hardly ever allocates an implied rating in either categories 1-2 (AAA and AA), even though these account for more than 5 per cent of the daily firm-observations. In fact, in Tables 6a and 7a it is shown that BARRA model allocates half of the observations to A and BBB ratings while Merton/Moody's KMV allocates around 99% of the observations to these rating categories.

For this reason the BARRA implied ratings seam to be somewhat more in line with Moody's ratings compared to the Merton/MKMV implied ratings. This is confirmed by Spearman's correlation measure between the BARRA and Moody's ratings is 0.767, while the corresponding correlation between Merton/MKMV and Moody's ratings is 0.361. The reason for the higher Spearman correlation between BARRA and Moody's ratings can be due to the fundamental differences between BARRA and Merton/MKMV model. BARRA model is calibrated to fit ratings thus it is an empirically originated model, conceptually different from the MKMV approach that is based on the structural model approach.

This "centralization" of credit ratings reflects the fact (documented in Figures 1-4) that the market data (credit spreads or distances to default) overlap considerably between different credit ratings. As a result both the direct estimation of market implied ratings boundaries in BARRA and in the ordered probit estimation from MKMV, the errors in rating classification are minimized by allocating a relatively large proportion of companies to the central ratings categories. The consequence of this tendency to "centralization" is that both models, but especially the KMV model, tend to predict movement towards the central ratings bands 3 and 4 (A and BBB) There are relatively few downgrade predictions from banks 3 and 4 into 5 and 6 or upgrades from 3 and 4 into 1 and 2.

We next test whether the models successfully predict Moody's rating 1 month, 3 months and 6 months prior the reported Moody's rating. These results are presented for BARRA in Tables 8, 9 and 10. These tables present the BARRA predicted credit ratings 1 month, 3 month and 6 months prior to the reported Moody's ratings using the entire sample. We find that, using the estimated BARRA boundaries, the model correctly predicts a large proportion of both downgrades (81 out of 138) and upgrades (19 out of 40) at a one month horizon. These proportions remain fairly high even at longer horizons, with correct predictions of 69 out of 138 downgrades at the three month horizon and 55 out of 134 at the six month horizons. Upgrade prediction actually improves slightly to 20 out of 40 at both three and six month horizons.[43]

However in all cases there are a very high number of false predictions of a rating change, in excess of 99.9% for both upgrades and downgrades at all time horizons. This indicates that the BARRA model cannot be used with the estimated boundaries as a practical forecasting tool, but that the sensitivity levels must be sharply reduced so that it makes fewer upgrade and downgrade predictions.

Similar results for predictive ability are obtained for the MKMV model downgrade predictions (Tables 11 – 13) which correctly predicts 42 out of 106 downgrades at one month horizon, 37 out of 106 at the three month horizon and 35 out of 102 at the six month horizon. There are very small numbers of observed upgrades, but the majority of these are predicted by the model. However as with the BARRA model, there are very high levels, now in excess of 99.95%, of false predictions of upgrade and of downgrade at all time horizons.

As a next step we compare the predictive power of BARRA and MKMV models using the same sample i.e. those observations for which both BARRA and MKMV implied ratings are available. The constraint we face is that for some of our firms we do not have equity pricing information, therefore to make this comparison we must restrict our BARRA results to the same subset of observations as for which we have observations on MKMV implied ratings. These are reported in Tables 14, 15 and 16, once again reporting the predictive performance of BARRA implied ratings 1 month, 3 months and 6 months prior the reported Moody's ratings. BARRA achieves slightly better performance than MKMV, with a greater proportion of correctly predicted

[43] The number of downgrades and upgrades is reduced for the 6 month period compared to the 1 and 3 month prediction period due to the fact that a bigger prediction window is chosen.

downgrades (eg 64 instead of 42 out of 106 at one month) and slightly lower but still unacceptably high level of false predictions (e.g. 99.86% at one month compared with 99.93% for MKMV)

Comparing the two models, we conclude that while there is some evidence that the BARRA model performs better at the prediction of downgrades while Merton/MKMV model performs better in the case of upgrades, nevertheless the main problem for both models is the high number of cases where they give a false signal. Although, BARRA model correctly predicted the majority of cases when there was actually a downgrade (most successfully 1 month prior to the downgrade by Moody`s) well over 99% of the cases that the model predicted a downgrade proved to be a false alarm since there was not a change in the credit rating. The same stands for Merton/MKMV model.

One of the main drawbacks of the above comparison of BARRA and Merton/MKMV model is that while BARRA market implied ratings are estimated on a daily basis, for the Merton/MKMV data we used the full sample for the estimation of the ordered probit model. Hence, in the above estimations Merton/MKMV model uses more information than the BARRA model. To overcome this problem, we also estimate the Merton/MKMV implied ratings on a daily basis (i.e. by running the ordered probit model daily and estimating daily thresholds). In Tables 17, 18 and 19 we report the predictive performance of Merton/MKMV implied ratings 1 month, 3 months and 6 months prior the reported Moody's ratings, when the implied ratings are calculated on a daily basis. We show that the results stay almost the same and the conclusions on the comparison of the two models remain unchanged.

This problem of excessive false prediction of rating transition indicates that it is necessary to compare the models not at their estimated sensitivity levels, but by altering sensitivity levels to vary the number of predictions and therefore alter the relative frequency of Type I and Type II errors. This comparison can be made using ROC (Receiver Operating Characteristic) curves. The main advantage of using ROC curves for the evaluation of models is their ability to discriminate between two states, while altering the sensitivity of predictions. Thus we can control the level of false predictions of a credit change at a reasonable level and then compare the models.

The ROC curve is constructed as follows. For all possible cut-off values the true positive rate (the proportion of rating transitions that have been correctly predicted) and the false negative rate (the proportion of all observations where no transition takes place, but which were predicted to transit). The ROC curve thus represents the tradeoff between Type I errors (one minus the vertical position of each point on the curve) and Type II errors (the horizontal position of each point on the curve). As the sensitivity of the cut-off point is increased we move up and to the right on the ROC curve with fewer Type I errors but more Type II errors. In principle the higher the area under the curve the better is the model performance. A purely random model would yield a curve close to the 45 degree line and would have an area under the curve of 0.5. A very powerful model that predicts most states with few false predictions would yield a curve close to the left-hand vertical and upper-horizontal axes, and have an area under the curve close to 1. Satchell and Xia (2006) and De Servigny and Renault (Chapter 3) offer a more detailed study of the ROC curves and their application to credit rating model validation.

Figures 5 and 6 show the ROC curves for BARRA and MKMV models in case of a downgrade.[44] To be able to compare the results we use only the cases where data exists both for BARRA spreads and distance to default measure.[45] We find that both models perform better than a random model. The most notable finding is that MKMV performs slightly better that BARRA model for the estimation of downgrades at reasonable levels of sensitivity the lower left part of the two curves (keeping the number of false positives to less than 50%). At these sensitivity levels MKMV clearly generates a higher rate of successful predictions than BARRA. This contrasts with our conclusions when using the estimated ratings boundaries, but recall that these estimated boundaries are associated with very high levels of Type II errors, since they often predict rating transitions that do not take place. Both models are more useful when the sensitivity is reduced so that they yield fewer predictions of rating changes. The ROC comparison shows that in this situation MKMV clearly performs better at predicting downgrades in our sample than BARRA.

[44] For the estimation of ROC curve, the STATA econometric program has been used. STATA has predetermined commands for the derivation of a ROC curve as well as the calculation of areas below the ROC curve.

[45] Note that to construct the ROC curve for MKMV model we use –DD instead of DD. This is due to the fact that in order to construct the ROC curve we need to use an ordinal variable where higher values represent higher risk.

Only when sensitivity is increased to very high levels, the upper right portion of the curves when the false positive reaches in excess of 90% or more, then BARRA outperforms MKMV. This is the portion of the curves consistent with the results reported in Tables 11-16.

Figures 7 to 8 present the ROC curves for BARRA and MKMV models in case of upgrade. Here we find that while neither model performs well, the BARRA model performs better than MKMV model. In fact it is appears that the MKMV model does not perform better than a random model in the case of upgrades.

It is notable that using the ROC curve analysis we conclude that MKMV model is slightly better predictor of downgrades compared to BARRA model, while at the beginning it has been shown that BARRA model performs better in the prediction of downgrades. This is due to the fact that ROC curves are constructed for different cut off points. Hence, in the first part of the analysis we compare the two models' predictions based on the model produced cut off points while using the ROC curves a comparison of the models can be done for different cut off points.

5.5. Conclusion

In this chapter, we compared two of the most widely used models for obtaining market-implied ratings, the MKMV model based on equity prices and the BARRA model based on credit spreads. It has estimated these models on a sample of 4594 bonds issued by 447 firms from US for a period of 3 years, from January 2000 to December 2002. In the case of BARRA the model estimation procedure itself allocates these companies to different "market implied" ratings categories. In the case of MKMV we use ordered probit estimation to obtain these implied ratings from the model estimated distances to default.

We find that credit spreads and MKMV distance to default are not closely related to the actual agency ratings recorded by Moody's. There are considerable overlaps in the distribution of both market measures of credit quality between ratings classes. As a result using these market measures of credit quality to obtain market implied ratings, we find that a relatively large proportion of companies are allocated to central ratings categories 3 (A) and 4 (Baa).

This centralization also means that, when the ratings models are used to predict ratings changes, based on their estimated ratings boundaries, then they yield excessive false predictions of ratings changes. More than 99% of predicted rating transitions over one-month, three-month, or six-month horizons proving to be false i.e. no rating transition in fact takes place.

We report ROC curves as a method of comparing the predictive ability of these models for capturing ratings changes at lower levels of sensitivity. These indicate that both models have some power for predicting downgrades but that, controlling for sensitivity to maintain a reasonable level of false positives MKMV is more successful than BARRA in predicting ratings downgrades. Perhaps due to the relatively small number of upgrades in this data sample, neither model is particularly good at predicting upgrades. In fact MKMV does not better than a random model while BARRA does only a little bit better.

Our main conclusion is a negative one – neither bond nor equity market information seems to be particularly helpful in overcoming the problem of a large number of false alarms i.e. predictions of rating changes that do not in fact take place. The data set contains relatively few actual transitions. Clearly it would help if the study could be extended, for example to a larger data set covering more years and companies from other markets than the US. This is not easy to do, particularly because of the inconvenience of collecting the required option-adjusted bond spreads. Moreover, since we show that none of the models captures all the information relevant to assessing corporate credit quality, it would be useful to examine the predictive power of the models by estimating, for each rating class, individual sensitivity barriers for MKMV and BARRA models. The choice of the barriers would aim to balance Type I and Type II errors.

Furthermore, there is evidence that ratings are serially correlated (refer to Bangia et al (2002)). By adding lagged ratings into the ordered probit model may help us to investigate how this would affect the predictive ability of the models.

Other model specifications could be entertained. For example it would also be worthwhile to examine how successfully equity price data can be used for modeling and predicting ratings changes in more reduced form framework than MKMV or the two sources of data, from equity and bond markets, might be combined in alternative structural or reduced-form models.

Nonetheless, on the basis of our limited data, we can conclude that neither MKMV nor BARRA are especially useful tools for the modeling and prediction of credit upgrades and downgrades. Neither model is close to capturing all the information relevant to assessing corporate credit quality. They would appear to be best used as one of a range of tools of credit risk analysis.

Tables and figures

Table 1: Frequency of each rating category in the data

Rating class	Actual Rating	Frequency	Percentage	Cumulative
1	AAA	1,630	0.38	0.38
2	AA	23,682	5.56	5.95
3	A	134,383	31.57	37.51
4	BBB	159,554	37.48	75
5	BB	70,287	16.51	91.51
6	B or below	36,153	8.49	100
Total		425,689	100	

Note: The total number of 425,689 observations refers to the number of daily observations

Table 2: Number of upgrades and downgrades to each rating category

	Actual Rating	Number of downgrades	Number of upgrades
Rating			
1	AAA	0	0
2	AA	1	10
3	A	11	9
4	BBB	50	12
5	BB	57	11
6	B or below	32	0

This table presents the total number of upgrades and downgrades per rating class. Data on the credit ratings has been collected from Bloomberg.

Table 3: Descriptive statistics of credit spreads

		Descriptive statistics of spreads				
Rating	Actual rating	Number of observations	Mean	25th percentile	50th percentile (median)	75^{th} percentile
1	AAA	1,158	92.16	59.02	97.65	123.79
2	AA	16,014	118.53	97.76	118.75	142.64
3	A	86,487	164.02	131.37	157.57	189.55
4	BBB	98,514	259.86	188.07	225.93	305.61
5	BB	41,820	422.19	308.25	379.9	479.21
6	B or below	20,613	586.88	404.06	516.37	670.78

This table presents the descriptive statistics of credit spreads. It shows the number of observations per rating class as well as the mean value of spreads (expressed in basis points). We present the 25^{th}, 50^{th} and 75^{th} percentile as well.

Table 4: Average distance to default per rating category

		Descriptive statistics of distance to default				
Rating	Actual rating	Number of observations	Mean	25th percentile	50th percentile (median)	75th percentile
1	AAA	1,065	7.37	5.74	6.72	9.07
2	AA	7,573	6.29	4.64	5.82	7.49
3	A	51,344	4.86	3.59	4.81	6.23
4	BBB	71,665	3.96	2.88	3.93	5.05
5	BB	36,272	3.25	2.41	3.22	4.14
6	B or below	17,462	2.53	1.50	2.38	3.43

This table presents the descriptive statistics of distance to default. It shows the number of observations per rating class as well as the mean value of distance to default (a measure that denotes how many standard deviations the asset value of a firm is away from the default boundary). We present the 25th, 50th and 75th percentile as well.

Table 5: Estimates of ordered probit model for panel data, 2000-2003

Parameter	**Coefficient**	**z value**	**p value**
Distance to default	0.255	195.84	0
Number of obs	185381		
Log Likelihood	-244439.73		
LR chi2	39454.16		
d.f.	1		
Prob>chi2	0		
Pseudo R2	0.074		
Cut-off points			
Cut_1	-0.438		
Cut_2	0.407		
Cut_3	1.537		
Cut_4	2.945		
Cut_5	3.939		

Note: these estimates are for the ordered probit model using daily DD data from beginning of 2000 to the end of 2002. The plus sign on the coefficient shows that the higher the distance to default the higher the credit rating (note that although throughout this Chapter it is not the case, for the estimation of ordered probit model, higher ratings are represented by higher numbers, i.e. 6 in this case denotes the higher rating category).

Table 6: BARRA implied rating versus Moody's rating

	Moody's rating						
BARRA implied rating	1	2	3	4	5	6	Total
1	1,353	8,266	10,130	655	402	32	20,838
2	253	8,064	26,377	1,775	319	73	36,861
3	24	5,876	56,930	33,756	743	426	97,755
4	0	299	26,818	74,920	13,489	2,039	117,565
5	0	0	2,031	22,985	29,343	6,915	61,274
6	0	0	168	7,823	16,644	22,260	46,895
Total	1,630	22,505	122,454	141,914	60,940	31,745	381,188

This table summarizes the number of observations that occur for each possible combination of BARRA implied rating and Moody's credit rating.

Table 6a: BARRA implied rating versus Moody's rating (as a percentage of total observations)

	Moody's rating						
BARRA implied rating	1	2	3	4	5	6	Total
1	0.35%	2.17%	2.66%	0.17%	0.11%	0.01%	5.47%
2	0.07%	2.12%	6.92%	0.47%	0.08%	0.02%	9.67%
3	0.01%	1.54%	14.93%	8.86%	0.19%	0.11%	25.64%
4	0.00%	0.08%	7.04%	19.65%	3.54%	0.53%	30.84%
5	0.00%	0.00%	0.53%	6.03%	7.70%	1.81%	16.07%
6	0.00%	0.00%	0.04%	2.05%	4.37%	5.84%	12.30%
Total	0.43%	5.90%	32.12%	37.23%	15.99%	8.33%	100%

This table summarizes the number of observations, expressed as a percentage of total observations, that occur for each possible combination of BARRA implied rating and Moody's credit rating.

Table 7: Merton/MKMV implied rating versus Moody's rating

	Moody's rating						
Merton/MKMV implied rating	AAA	AA	A	BBB	BB	B or below	Total
AAA	0	0	0	0	0	0	0
AA	0	0	54	0	0	0	54
A	1,275	6,130	26,751	16,481	2,212	992	53,841
BBB	285	4,902	47,028	88,974	49,469	21,908	212,566
BB	0	0	8	457	1,248	1,839	3,552
B or below	0	0	1,585	668	478	672	3,403
Total	1,560	11,032	75,426	106,580	53,407	25,411	273,416

This table summarizes the number of observations that occur for each possible combination of Merton/MKMV implied rating and Moody's credit rating.

Table 7a: Merton/MKMV implied rating versus Moody's rating (as a percentage of total observations)

	Moody's rating						
Merton/MKMV implied rating	AAA	AA	A	BBB	BB	B or below	Total
AAA	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA	0.00%	0.00%	0.02%	0.00%	0.00%	0.00%	0.02%
A	0.47%	2.24%	9.78%	6.03%	0.81%	0.36%	19.69%
BBB	0.10%	1.79%	17.20%	32.54%	18.09%	8.01%	77.74%
BB	0.00%	0.00%	0.00%	0.17%	0.46%	0.67%	1.30%
B or below	0.00%	0.00%	0.58%	0.24%	0.17%	0.25%	1.24%
Total	0.57%	4.03%	27.59%	38.98%	19.53%	9.29%	100.00%

This table summarizes the number of observations, expressed as a percentage of total observations, that occur for each possible combination of Merton/MKMV implied rating and Moody's credit rating.

Table 8: BARRA implied rating prediction 30 days prior to Moody's rating

	Moody's rating			Total	Moody's rating		
BARRA rating	Downgrade	Same rating	Upgrade	Total	Downgrade	Same rating	Upgrade
Downgrade	81	79763	2	79846	0.10%	99.90%	0.00%
Same rating	44	187145	19	187208	0.02%	99.97%	0.01%
Upgrade	13	103557	19	103589	0.01%	99.97%	0.02%

This table summarizes the number of observations that occur for each possible combination of BARRA implied rating change (downgrade, same rating, and upgrade) and Moody's credit rating change. In this table we investigate how well BARRA model predicts rating changes 30 days prior to Moody's rating changes.

Table 9: BARRA implied rating prediction 90 days prior to Moody's rating

	Moody's rating			Total	Moody's rating		
BARRA rating	Downgrade	Same rating	Upgrade	Total	Downgrade	Same rating	Upgrade
Downgrade	69	73946	3	74018	0.09%	99.90%	0.00%
Same rating	52	175452	17	175521	0.03%	99.96%	0.01%
Upgrade	17	100462	20	100499	0.02%	99.96%	0.02%

This table summarizes the number of observations that occur for each possible combination of BARRA implied rating change (downgrade, same rating, and upgrade) and Moody's credit rating change. In this table we investigate how well BARRA model predicts rating changes 90 days prior to Moody's rating changes.

Table 10: BARRA implied rating prediction 180 days prior to Moody's rating

	Moody's rating			Total	Moody's rating		
BARRA rating	Downgrade	Same rating	Upgrade	Total	Downgrade	Same rating	Upgrade
Downgrade	55	66061	3	66119	0.08%	99.91%	0.00%
Same rating	52	156690	17	156759	0.03%	99.96%	0.01%
Upgrade	27	95627	15	95669	0.03%	99.96%	0.02%

This table summarizes the number of observations that occur for each possible combination of BARRA implied rating change (downgrade, same rating, and upgrade) and Moody's credit rating change. In this table we investigate how well BARRA model predicts rating changes 180 days prior to Moody's rating changes.

Table 11: Merton/MKMV implied rating prediction 30 days prior to Moody's rating

	Moody's rating			Total	Moody's rating		
MKMV rating	Downgrade	Same rating	Upgrade	Total	Downgrade	Same rating	Upgrade
Downgrade	42	56819	0	56861	0.07%	99.93%	0.00%
Same rating	48	104478	3	104529	0.05%	99.95%	0.00%
Upgrade	16	79755	15	79786	0.02%	99.96%	0.02%

This table summarizes the number of observations that occur for each possible combination of Merton/MKMV implied rating change (downgrade, same rating, and upgrade) and Moody's credit rating change. In this table we investigate how well Merton/MKMV model predicts rating changes 30 days prior to Moody's rating changes.

Table 12: Merton/MKMV implied rating prediction 90 days prior to Moody's rating

	Moody's rating			Total	Moody's rating		
MKMV rating	Downgrade	Same rating	Upgrade	Total	Downgrade	Same rating	Upgrade
Downgrade	37	53111	1	53149	0.07%	99.93%	0.00%
Same rating	52	97892	2	97946	0.05%	99.94%	0.00%
Upgrade	17	76282	15	76314	0.02%	99.96%	0.02%

This table summarizes the number of observations that occur for each possible combination of Merton/MKMV implied rating change (downgrade, same rating, and upgrade) and Moody's credit rating change. In this table we investigate how well Merton/MKMV model predicts rating changes 90 days prior to Moody's rating changes.

Table 13: Merton/MKMV implied rating prediction 180 days prior to Moody's rating

	Moody's rating			Total	Moody's rating		
MKMV rating	Downgrade	Same rating	Upgrade	Total	Downgrade	Same rating	Upgrade
Downgrade	35	47206	0	47241	0.07%	99.93%	0.00%
Same rating	47	87211	3	87261	0.05%	99.94%	0.00%
Upgrade	20	69758	14	69792	0.03%	99.95%	0.02%

This table summarizes the number of observations that occur for each possible combination of Merton/MKMV implied rating change (downgrade, same rating, and upgrade) and Moody's credit rating change. In this table we investigate how well Merton/MKMV model predicts rating changes 180 days prior to Moody's rating changes.

Table 14: BARRA implied rating prediction 30 days prior to Moody's rating, same observations as in MKMV are considered

	Moody's rating			Total	Moody's rating		
BARRA implied rating	Downgrade	Same rating	Upgrade	Total	Downgrade	Same rating	Upgrade
Downgrade	64	46625	1	46690	0.14%	99.86%	0.00%
Same rating	33	119475	7	119515	0.03%	99.97%	0.01%
Upgrade	9	74952	10	74971	0.01%	99.97%	0.01%

This table summarizes the number of observations that occur for each possible combination of BARRA implied rating change (downgrade, same rating, and upgrade) and Moody's credit rating change. In this table we investigate how well BARRA model predicts rating changes 30 days prior to Moody's rating changes. For these calculations, only those observations for which both BARRA and Merton/MKMV implied ratings are available are used.

Table 15: BARRA implied rating prediction 90 days prior to Moody's rating, same observations as in MKMV are considered

	Moody's rating			Total	Moody's rating		
BARRA implied rating	Downgrade	Same rating	Upgrade	Total	Downgrade	Same rating	Upgrade
Downgrade	54	42225	1	42280	0.13%	99.87%	0.00%
Same rating	41	112427	7	112475	0.04%	99.96%	0.01%
Upgrade	11	72633	10	72654	0.02%	99.97%	0.01%

This table summarizes the number of observations that occur for each possible combination of BARRA implied rating change (downgrade, same rating, and upgrade) and Moody's credit rating change. In this table we investigate how well BARRA model predicts rating changes 90 days prior to Moody's rating changes. For these calculations, only those observations for which both BARRA and Merton/MKMV implied ratings are available are used.

Table 16: BARRA implied rating prediction 180 days prior to Moody's rating, same observations as in MKMV are considered

	Moody's rating			Total	Moody's rating		
BARRA implied rating	Downgrade	Same rating	Upgrade	Total	Downgrade	Same rating	Upgrade
Downgrade	42	35737	2	35781	0.12%	99.88%	0.01%
Same rating	41	99677	5	99723	0.04%	99.95%	0.01%
Upgrade	19	68761	10	68790	0.03%	99.96%	0.01%

This table summarizes the number of observations that occur for each possible combination of BARRA implied rating change (downgrade, same rating, and upgrade) and Moody's credit rating change. In this table we investigate how well BARRA model predicts rating changes 180 days prior to Moody's rating changes. For these calculations, only those observations for which both BARRA and Merton MKMV implied ratings are available are used.

Table 17: Merton/MKMV implied rating prediction 30 days prior to Moody's rating, daily estimation

	Moody's rating			Total	Moody's rating		
MKMV rating	Downgrade	Same rating	Upgrade	Total	Downgrade	Same rating	Upgrade
Downgrade	45	56827	0	56872	0.08%	99.92%	0.00%
Same rating	49	108711	3	108763	0.05%	99.95%	0.00%
Upgrade	12	75344	15	75371	0.02%	99.96%	0.02%

This table summarizes the number of observations that occur for each possible combination of Merton/MKMV implied rating change (downgrade, same rating, and upgrade) and Moody's credit rating change. In this table we investigate how well Merton/MKMV model predicts rating changes 30 days prior to Moody's rating changes. For these calculations, the Merton/MKMV model has been estimated on a daily basis.

Table 18: Merton/MKMV implied rating prediction 90 days prior to Moody's rating, daily estimation

	Moody's rating			Total	Moody's rating		
MKMV rating	Downgrade	Same rating	Upgrade	Total	Downgrade	Same rating	Upgrade
Downgrade	43	52088	0	52131	0.08%	99.92%	0.00%
Same rating	48	102170	3	102221	0.05%	99.95%	0.00%
Upgrade	15	72821	15	72851	0.02%	99.96%	0.02%

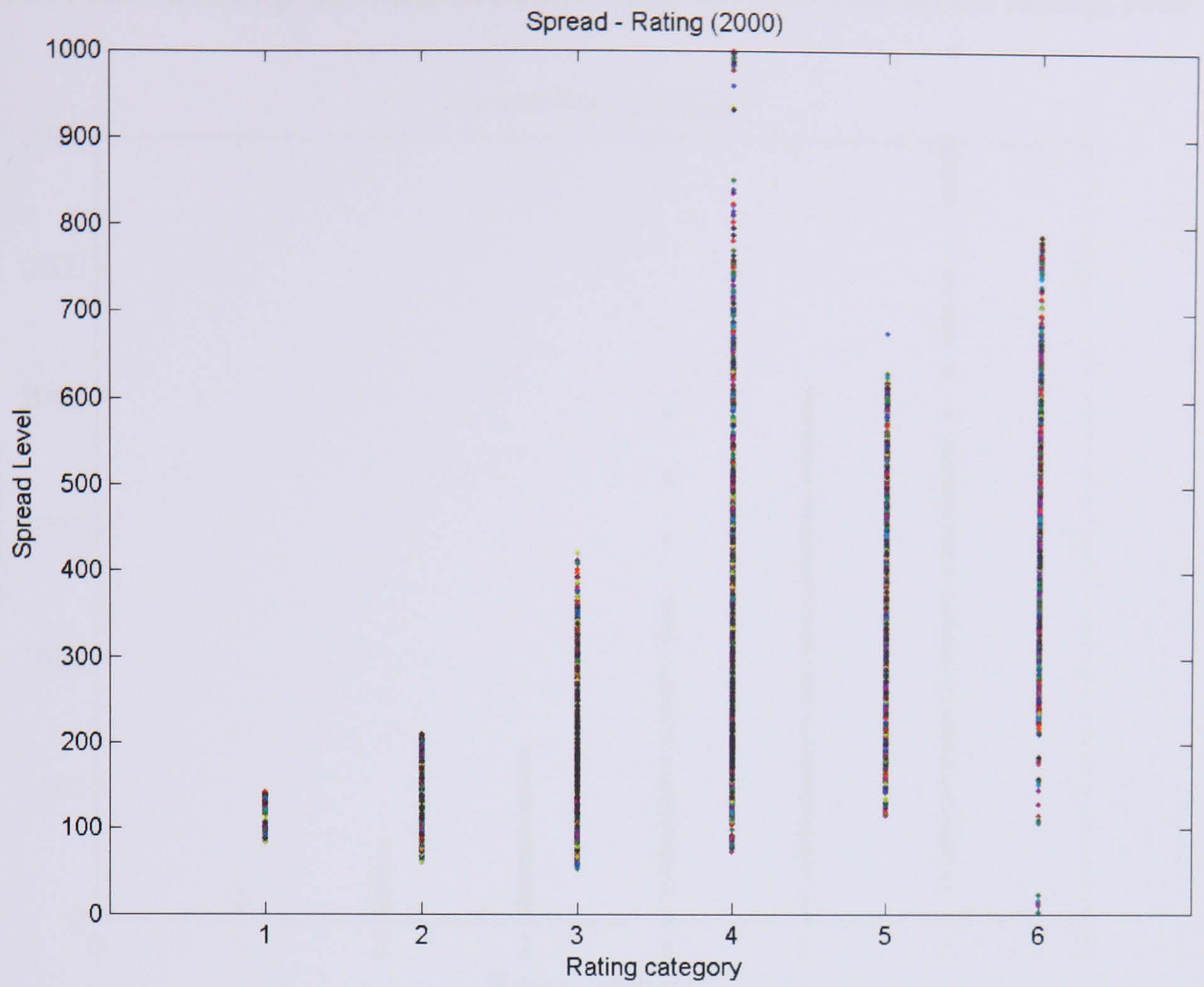
This table summarizes the number of observations that occur for each possible combination of Merton/MKMV implied rating change (downgrade, same rating, and upgrade) and Moody's credit rating change. In this table we investigate how well Merton/MKMV model predicts rating changes 90 days prior to Moody's rating changes. For these calculations, the Merton/MKMV model has been estimated on a daily basis.

Table 19: Merton/MKMV implied rating prediction 180 days prior to Moody's rating, daily estimation

	Moody's rating			Total	Moody's rating		
MKMV rating	Downgrade	Same rating	Upgrade	Total	Downgrade	Same rating	Upgrade
Downgrade	38	44467	0	44505	0.09%	99.91%	0.00%
Same rating	43	90998	3	91044	0.05%	99.95%	0.00%
Upgrade	21	68499	14	68534	0.03%	99.95%	0.02%

This table summarizes the number of observations that occur for each possible combination of Merton/MKMV implied rating change (downgrade, same rating, and upgrade) and Moody's credit rating change. In this table we investigate how well Merton/MKMV model predicts rating changes 180 days prior to Moody's rating changes. For these calculations, the Merton/MKMV model has been estimated on a daily basis.

Figure 1: Relationship between issuer average spreads and credit rating, year 2000



Note: The lower number indicates higher credit rating, where 1 is the AAA rating, 2 is the AA rating and so on.

Figure 2: Relationship between issuer average spreads and credit rating, year 2001

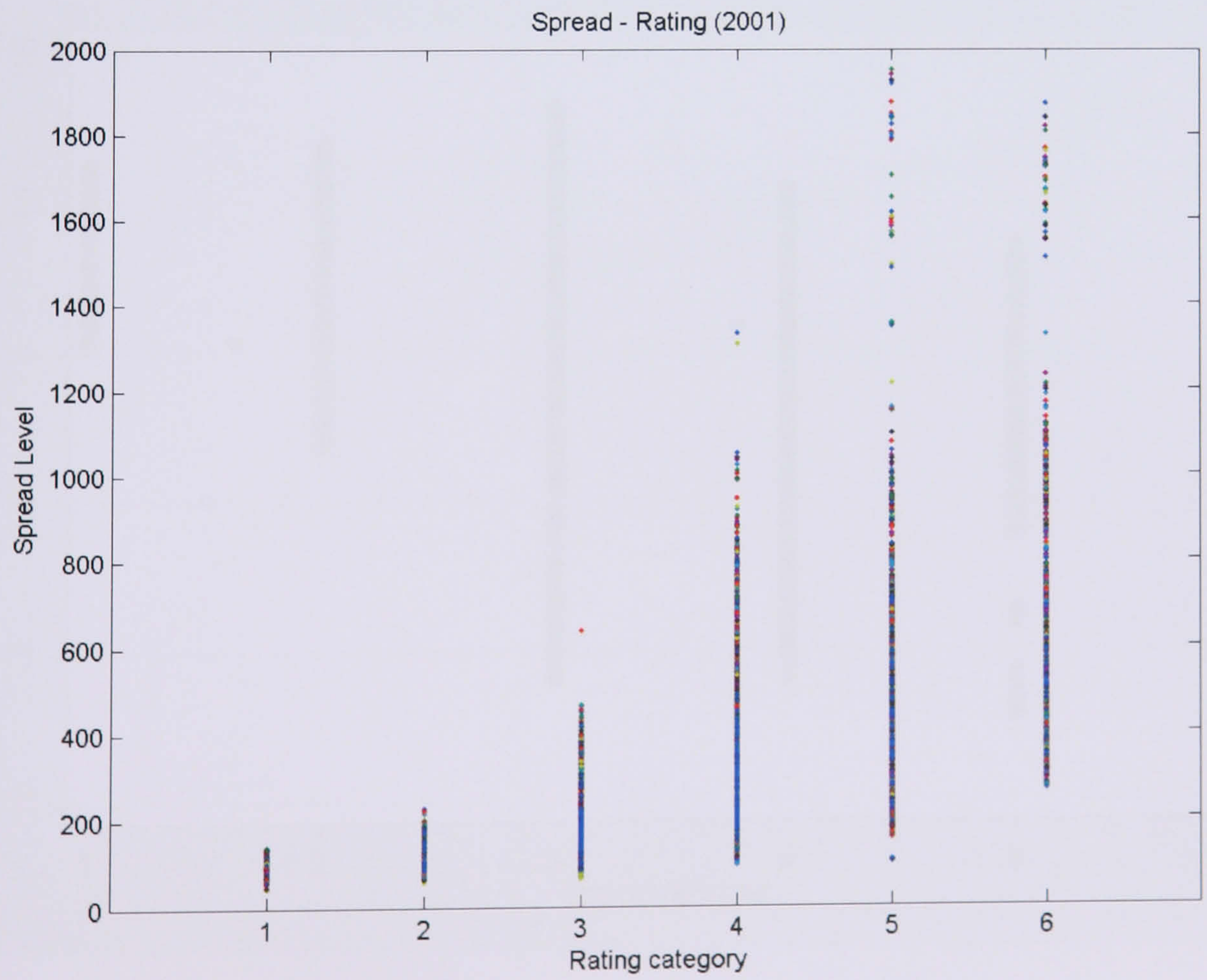


Figure 3: Relationship between issuer average spreads and credit rating, year 2002

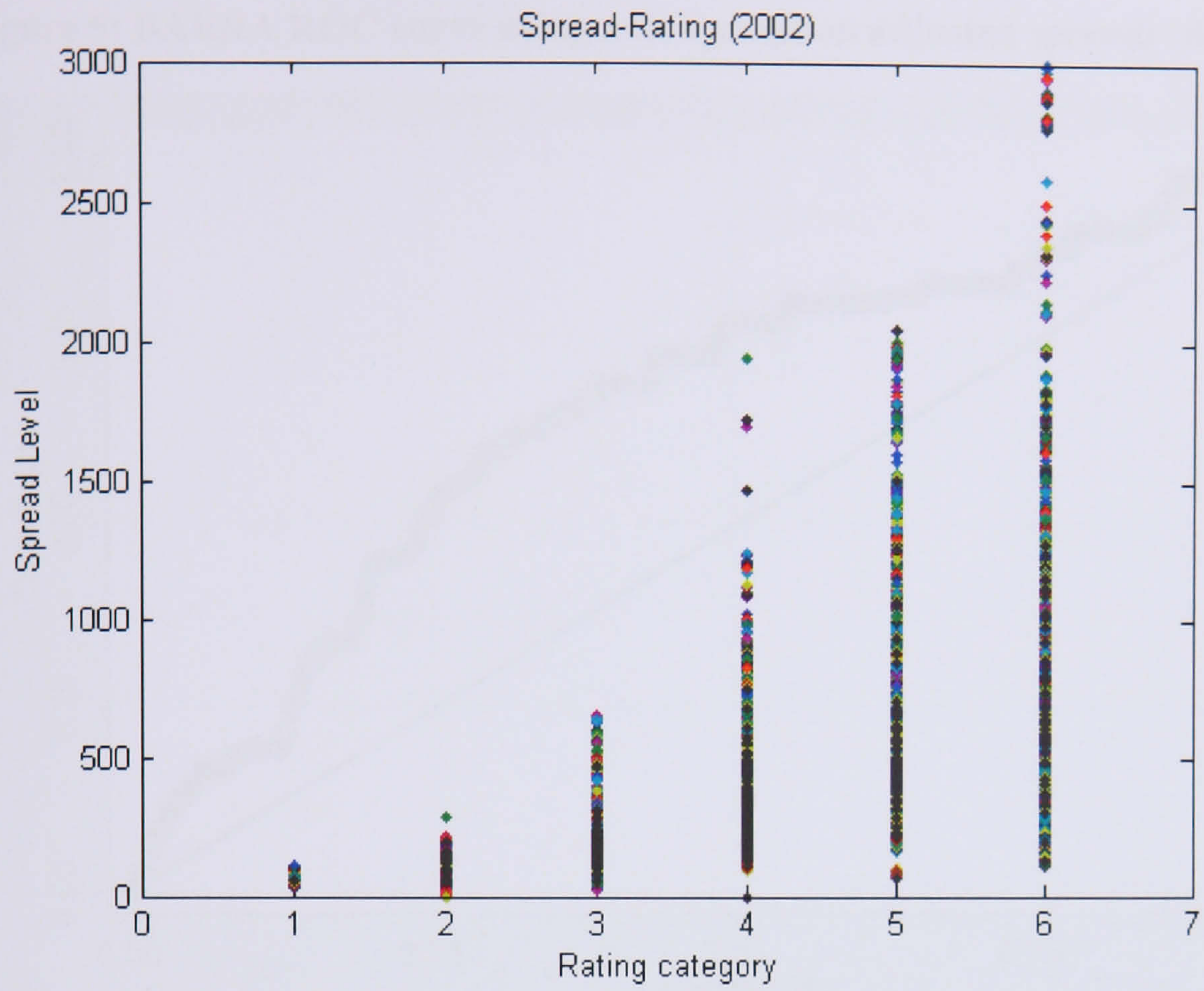


Figure 4: Relationship between issuer credit rating and distance to default

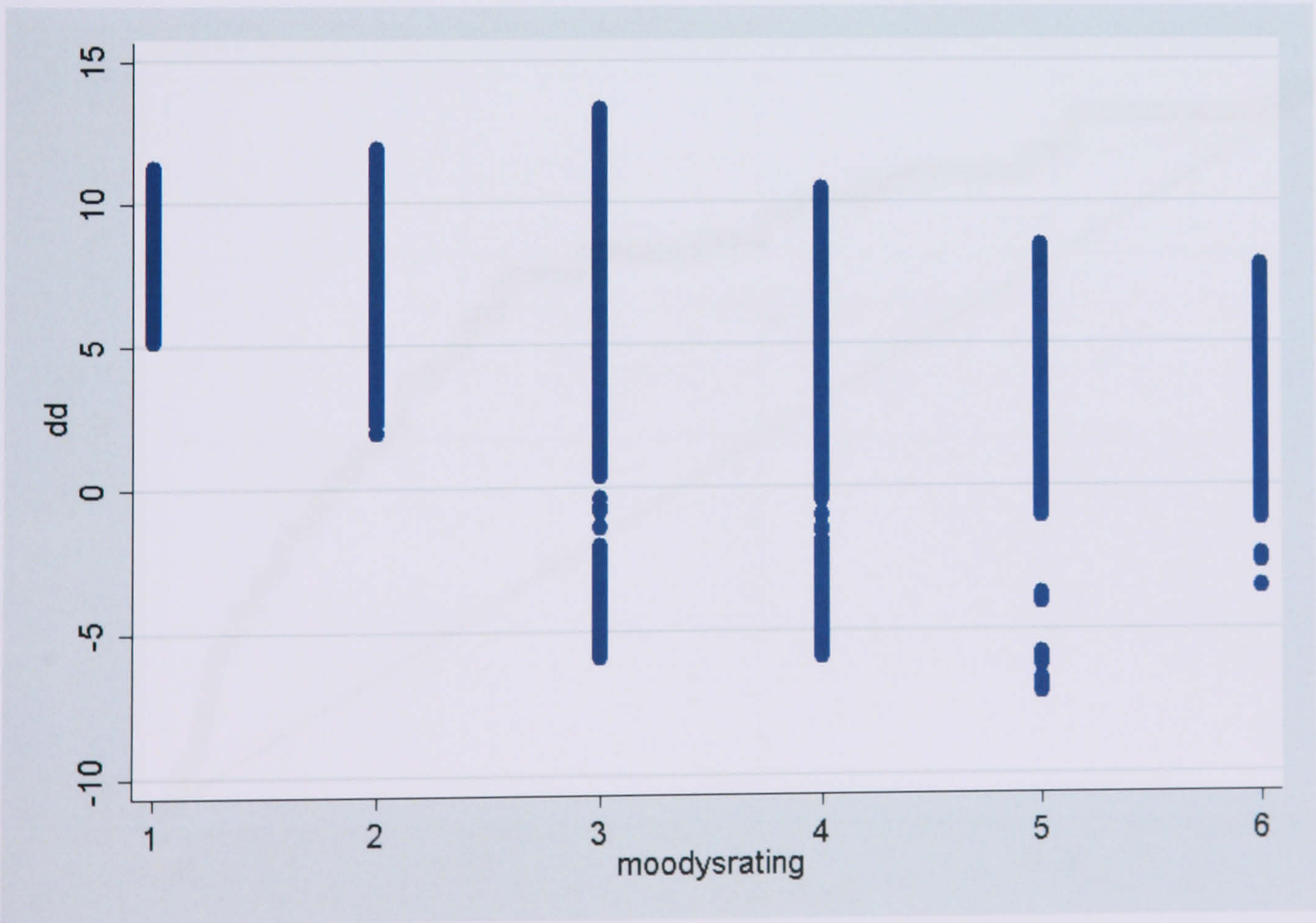


Figure 5: BARRA ROC curve using average option adjusted spreads (downgrade)

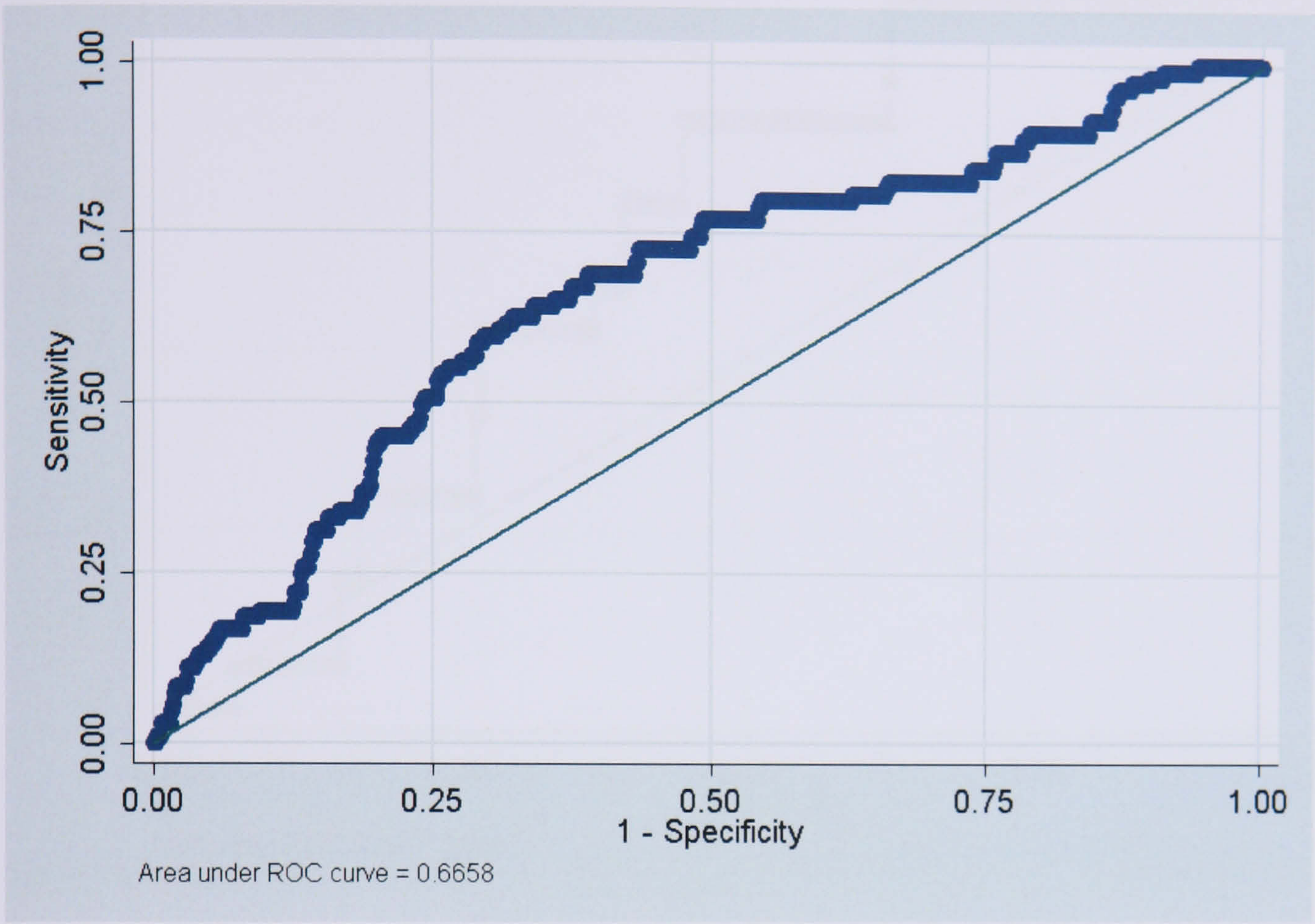


Figure 6: MKMV ROC curve using DD variable (downgrade)

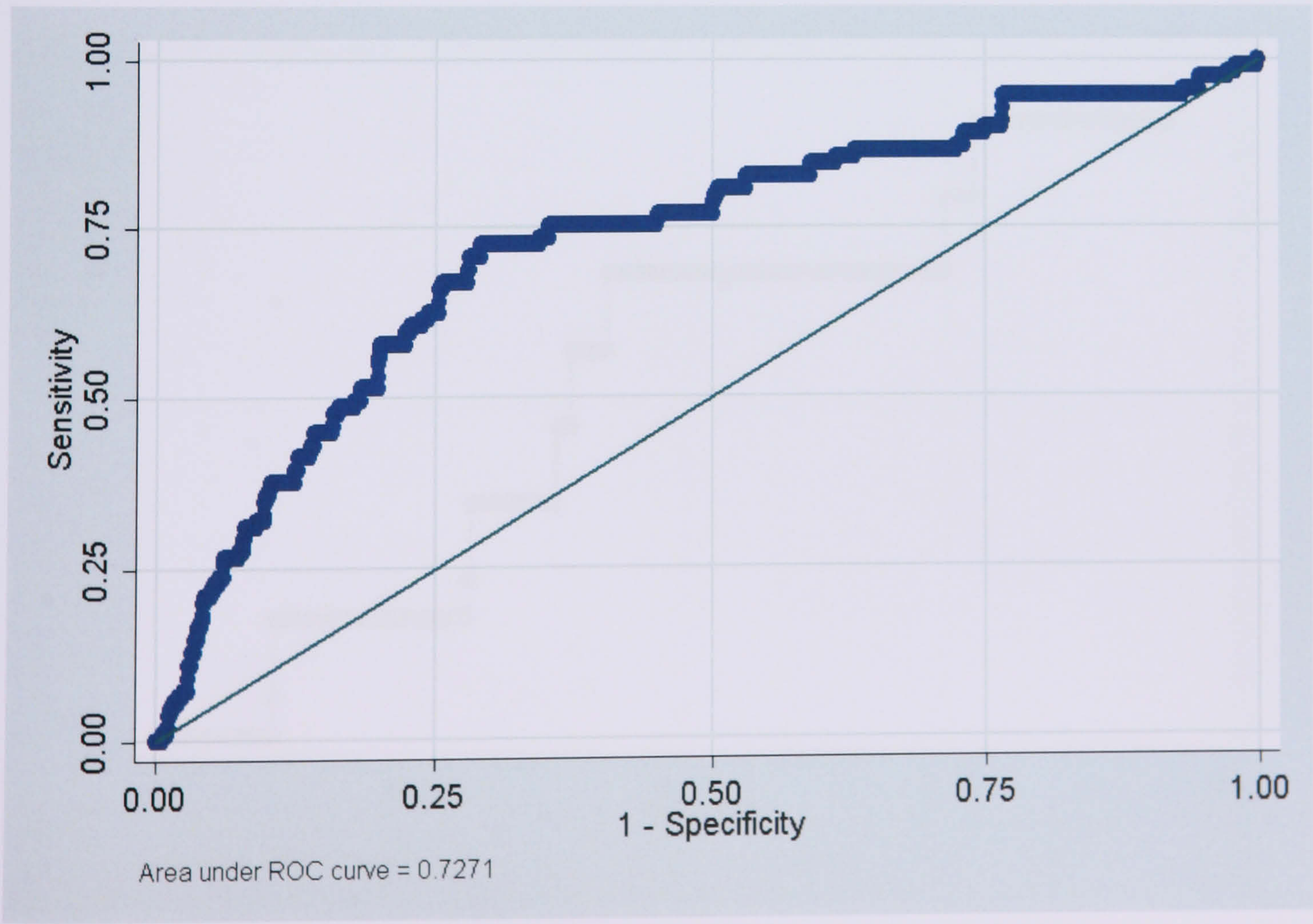
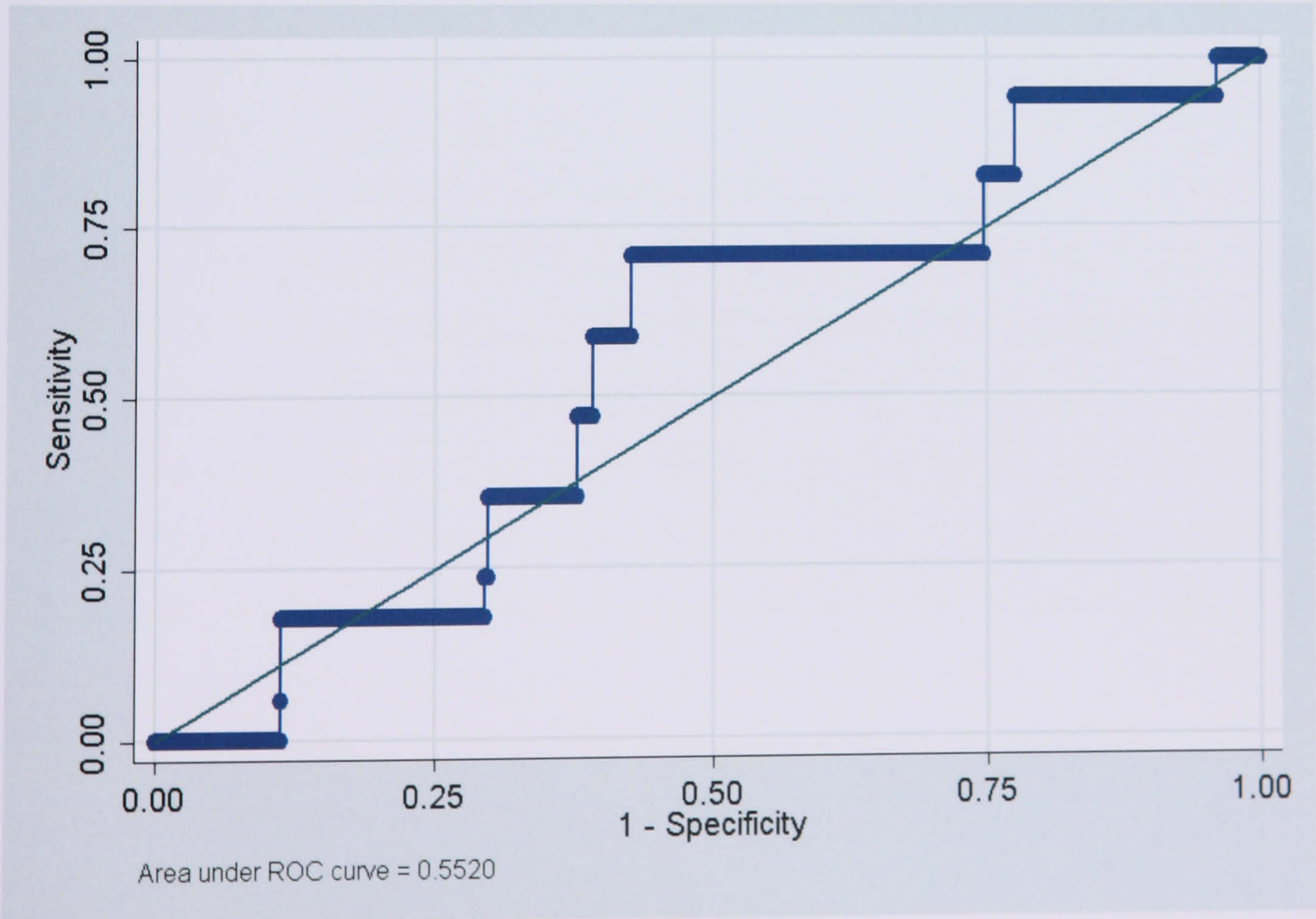


Figure 7: BARRA ROC curve using average option adjusted spreads (upgrade)

Figure 8: MKMV ROC curve using DD variable (upgrade)



CHAPTER 6: Conclusion

Abstract of Chapter 6

This chapter provides a review of the main findings of this thesis. We present again the three main studies and restate the contribution of each study to the current literature.

6.1. Main conclusions

As outlined in the general introduction the research reported in this thesis investigates the use of structural credit risk models for the prediction of default and credit rating transitions. It aims to examine both how different specifications of structural credit risk models affect default predictions and the empirical performance of the most widely used structural credit risk model, that of Moody's KMV, in relation to other models using accounting and bond market data.

The objective of the first research study is to examine the differences in real default probabilities produced by different structural models; Merton model, Longstaff and Schwartz model and Leland and Toft model. We find that none of the models can accurately predict the default probabilities in all cases. Longstaff and Schwartz model produces default predictions in line with observed data for time horizons of over three years in some cases but at the same time tends to overestimate the default probabilities of riskier bonds as well as the default probabilities of bonds with the same rating but higher equity volatility. On the other hand, Merton and Leland and Toft model tend to underpredict the default probabilities in almost all cases.

One of the main contributions of this study is the finding that structural models, especially Longstaff and Schwartz model, are sensitive to changes in the equity volatility. This is an important result since it suggests that banks should be cautious on the use of structural models for the calculation of their regulatory capital. The high sensitivity of the models to changes in the equity volatility will result on a high volatile assessment of the regulatory capital. Moreover, we conclude that structural models have difficulty, especially at short time horizons, in predicting default rates that are similar to the observed ones. This finding is in line with the earlier literature showing that structural models are not adequate for explaining observed credit spreads. While previous studies showed that observed credit spreads might be explained by special features such as taxes and liquidity, these alternative explanations cannot explain the poor performance of structural models in predicting real default probabilities.

Given these findings, for future research, the inclusion of another structural model, that assumes jumps in the asset values, would be useful in order to examine whether the inclusion of jumps in the asset values improves the performance of structural models in the prediction of expected default frequencies. Moreover, the inclusion of a structural model with stochastic volatility would enable us to investigate whether the inclusion of stochastic volatility can correct the high sensitivity of structural models to changes in the equity volatility.

The objective of the second study is to empirically determine whether information from equity markets, as summarized in the distance-to-default measure derived from Merton and similar to the one proposed by Moody's KMV Credit Monitor, provides useful additional information over accounting variables for the modelling and prediction of bank ratings and rating transitions.

Our study extends work by Gropp et al (2006) who have found that the distance to default measure based on equity price is a useful predictor of bank fragility when used on its own, as well as in the context of an accounting model. The main contribution of our study lies on the investigation of whether distance to default is a useful indicator of changes in banks' credit quality. Our results indicate that distance to default, while it may be a useful summary statistic for bank supervisors and others, monitoring financial sector stability, certainly does not supplant more traditional approaches to credit analysis when used for the prediction of credit quality of banks. We find that the most important variable both for the prediction and the modeling of ratings is the size, as measured by the log of total assets. We find that distance to default is statistically significant only for small banks, for modeling current ratings, or predicting credit rating changes over a 6-month or 12-month horizon and we find no evidence that changes in distance-to-default have additional explanatory power for predicting rating category, either for small or large banks. Hence, we conclude that for smaller institutions distance to default appears to be most useful for monitoring smaller banks, while for larger banks, at least within the countries covered by our data, distance to default appears to be, at best, one of a range of indicators helpful to supervisors.

For future research, our comparisons could be extended to a larger data set covering more years and companies from EU. Moreover, the investigation of whether the inclusion of lagged ratings or year dummies would affect the predictive power of distance to default would be beneficial.

In our third research study, we empirically compare the predictive ability for credit rating changes of two leading proprietary models currently used by many financial institutions, Moody's KMV and BARRA models. We find that credit spreads and MKMV distance to default are not closely related to the actual agency ratings recorded by Moody's since there are considerable overlaps in the distribution of both market measures of credit quality between ratings classes. As a result using these market measures of credit quality to obtain market implied ratings, we find that a relatively large proportion of companies are allocated to central ratings categories A and BBB. This centralization also means that, when the ratings models are used to predict ratings changes, based on their estimated ratings boundaries, they yield excessive false predictions of ratings changes. We report ROC curves as a method of comparing the predictive ability of these models for capturing ratings changes at lower levels of sensitivity. These indicate that both models have some power for predicting downgrades but that, controlling for sensitivity to maintain a reasonable level of false positives MKMV is more successful than BARRA in predicting ratings downgrades. Perhaps due to the relatively small number of upgrades in this data sample, neither model is particularly good at predicting upgrades. In fact MKMV does not better than a random model while BARRA does only a little bit better.

The contribution of this third research study is twofold. First, it demonstrates that market-implied ratings derived from equity prices are not more accurate compared to those derived from bond prices and vice versa. Second, we demonstrate that practitioners should be cautious when they use these rating models to predict ratings changes, since they yield excessive false predictions of ratings changes.

For future research, our comparisons could be extended to a larger data set covering more years and companies from other markets than the US. Moreover, since we show that none of the models captures all the information relevant to assessing corporate credit quality, it would be useful to examine the predictive power of the models by estimating, for each rating class, individual sensitivity barriers for MKMV and BARRA models. The choice of the barriers would aim to balance Type I and Type II errors. What is more, it is interesting to assess the predictive ability of a model that combines both distance to default and spreads and to investigate whether a "combined" model would work better than the individual models.

To summarize, from the three studies conducted in this thesis we can conclude that structural models, although they are useful tools of credit risk, they are far from close to capturing all the information relevant to assessing the credit quality of corporates or banks. Our findings suggest that practitioners and supervisors should use structural models as one of a range of tools of credit risk analysis.

References

Altman, E., 1968, "Financial ratios, discriminant analysis and the prediction of corporate bankruptcy", Journal of Finance, 23, 589-609.

Anderson, R., S. Sundaresan, 1996, "Design and valuation of debt contracts", Review of Financial Studies, 9, 37-68.

Anderson R., S. Sundaresan, 2000, "A comparative study of structural models of corporate bond yields: An explanatory investigation", Journal of Banking and Finance, 24, 255-269.

Anderson, R., S. Sundaresan and P. Tychon, 1996, "Strategic analysis of contingent claims", European Economic Review, 40, 871-881.

Andrade, G., S. Kaplan, 1998, "How costly is financial (not economic) distress? Evidence from highly levered transactions that became distressed", Journal of Finance, 53, 1443-1493.

Arora, N., Bohn, J.R., and F. Zhu, 2005, "Reduced form vs. structural models of credit risk: A case study of three models", Working Paper, Moody's KMV Company.

Bangia, A., F. Diebold, A. Kronimus, C. Schagen, and T. Schuermann, 2002, "Ratings migration and the business cycle, with application to credit portfolio stress testing", Journal of Banking and Finance, 26, 445-474.

Barker, D., and D. Holdsworth, 1993, "The causes of bank failures in the 1980s", Research Paper No.9325, Federal Reserve Bank of New York.

Beaver, W., 1967, "Financial ratios as predictors of failures", Empirical Research in Accounting: Selected Studies-1966, supplement to Journal of Accounting Research, 4, 71-111.

Berger, A. N., S. M. Davies, and M. J. Flannery, 2000, "Comparing market and supervisory assessments of bank performance: Who knows what when?", Journal of Money Credit and Banking, 32, 641-667.

Berger, A. N., K. K. King, and J. M. O'Brien, 1991, "The limitations if market value accounting and a more realistic alternative", Journal of Banking and Finance, 15, 753-783.

Berger, A. N., M. K. Kyle, and J. M. Scalise, 2000, "Did U.S. bank supervisors get tougher during credit crunch? Did they get easier during the banking boom? Did it matter to bank lending?", Research Paper, Board of governors of the Federal Reserve System.

Betker, B., 1995, "Management's incentives, equity bargaining power and deviations from absolute priority in Chapter 11 bankruptcies", Journal of Business, 68(2), 161-183.

Bharath, S., and T. Shumway, 2004, "Forecasting Default with the KMV-Merton model", Working Paper, University of Michigan.

Black, F., J. C. Cox, 1976, "Valuing corporate securities: Some effects of bond indenture provisions", Journal of Finance, 31, 351-367.

Black, F., M. Scholes, 1973, "The pricing of options and corporate liabilities", Journal of Political Economy, 81, 637-659.

Borooah, V. K., 2001, "Logit and probit: Ordered and Multinomial Models", Sage University Paper Series on Quantitative Applications in Social Sciences, 07-138.

Breger, L., Goldberg, L., and O. Cheyette, 2003, "Market implied ratings", Working paper, BARRA.

Brys, E., F. de Varenne, 1997, "Valuing risky fixed rate debt: An extension", Journal of Financial and Quantitative Analysis, 32, 239-248.

Calvo, G. A., 1996, "Capital flows and macroeconomic management: Tequila lessons", International Journal of Finance & Economics, 1, 207-224.

Cole, R. A., and J. W. Gunther, 1998, "Predicting bank failures: a comparison of on- and off-site monitoring systems", Journal of Financial Services Research, 13, 103-117.

Cavallo, E., and P. Valenzuela, 2007, "The determinants of corporate risk in emerging markets: an option-adjusted spread analysis", Working Paper, Inter-American Development Bank, US.

Collin-Dufresne, P., R. Goldstein, 2001, "Do credit spreads reflect stationary leverage ratios?", Journal of Finance, 56, 1929-1957.

Collin-Dufresne, Goldstein and Martin, 2001, "The determinants of credit spread changes", Journal of Finance, 56, 2177-2207.

Cossin, D., and H. Pirotte, 2000, "Advanced credit risk analysis: Financial approaches and mathematical models to assess price, and manage credit risk", John Wiley & Sons, Ltd.

Crosbie, P., J. Bohn, "Modelling Default Risk", 2002, KMV Corporation.

Curry, T.J., P. J. Elmer, and G.S. Fissel, 2003, "Using market information to help identify distressed institutions: A regulatory perspective", FDIC Banking Review, 15, 1-16.

Das, S.R., and P. Tufano, 1996, "Pricing credit sensitive spreads when interest rates, credit ratings and credit spreads are stochastic", Journal of financial Engineering, 5, 161-198.

De Servigny, A., and O. Renault, 2004, "Measuring and managing credit risk", McGraw-Hill.

Demirgüç-Kunt, A., and E. Detragiache, 1998, "The determinants of banking crises in developing and developed countries", IMF Staff Paper, 45, 81-109.

Delianedis, G., and R. Geske., 1999, "Credit risk and risk neutral default probabilities: Information about rating migrations and defaults", Working Paper, Anderson Graduate School of Management, UCLA.

Delianedis, G., and R. Geske., 2001, "The components of corporate credit spreads: default, recovery, tax, jumps, liquidity and market factors", Working Paper, Anderson Graduate School of Management, UCLA.

Du, Y., and W. Suo, 2003, "Assessing credit quality from equity markets: Can structural approach forecast credit ratings?", Working paper, Queens's University.

Duffee, G., 1999, "Estimating the price of default risk", Review of Financial Studies, 12, 197-226.

Duffie, D., K. Singleton, 1999, "Modelling the term structures of defaultable bonds", Review of Financial Studies, 12, 687-720.

Duffie, D., K. Singleton, 2003, "Credit Risk: Pricing, Measurement and Management", Princeton University Press.

Durand, D., 1941, "Risk elements in consumer instalment financing". Working Paper, NBER.

Dynkin, L., Hyman, J., and V. Konstantinovsky, 2002, "Sufficient diversification in credit portfolios", Journal of Portfolio Management, Fall, 89-114.

Eberhart, A., Moore, W., and R. Roenfeldt, 1990, "Security pricing and deviations from the absolute priority rule in bankruptcy proceedings", Journal of Finance, 45(5), 1457-1469.

Eichengreen, B. and C. Arteta, 2000, "Banking crises in emerging markets: Presumptions and evidence", University of California, Berkeley, August.

Elton, E., Gruber, M., Agrawal, D., and C. Mann, 2001, "Explaining the rate spread on corporate bonds", Journal of Finance, 56, 247-258.

Eom, Y., Helwege, J., and J. Huang, 2002, "Structural models of corporate bond pricing: An empirical analysis", Review of Financial Studies, 17, 499-544.

Ericsson, J., and O. Renault, 2001, "Liquidity and credit risk", working paper, Financial Market Group, London School of Economics.

Ericsson, J., and J. Reneby, 2001, "The valuation of corporate liabilities: Theory and tests", Working Paper, McGill University.

Ericsson, J., and J. Reneby, 2002, "Estimating structural bond pricing models", Working Paper, McGill University.

Espahbodi, P., 1991, "Identification of problem banks and binary choice models", Journal of Banking and Finance, 15, 53-71.

Fisher, E., Heinkel R., and J. Zechner, 1989, "Optimal dynamic capital structure choice: Theory and tests", Journal of Finance, 44, 19-40.

Fisher, L., 1936, "The use of multiple measurements in taxonomic problems", Annals of Eugenics, 7, 179-188.

Franks, J., W. Torous, 1994, "A comparison of financial recontracting in distressed exchanges and Chapter 11 reorganisations", Journal of Financial Economics, 35. 349-370.

Gavin, M., and R. Hausmann, 1996, "The roots of banking crises: The macroeconomic context", Working Paper, Inter-American Development Bank.

Geske, R., 1977, "The valuation of corporate liabilities as compound options", Journal of Financial and Quantitative Analysis, 541-552.

Gilbert, R. A., 1993, "Implications of annual examinations for the bank insurance fund", Federal Reserve Bank of St.Louis Review, January/February, 35-52.

Gilbert, R. A., Meyer A. P., and M. D. Vaughan, 1999, "The role of supervisory screens and econometric models in off-site surveillance", Federal Reserve Bank of St.Louis Review, November/December, 31-56.

Gilbert. R. A., Meyer, A. P., and M. D. Vaughan, 2002, "Could a Camels downgrade model improve off-site surveillance?", Federal Reserve Bank of St.Louis Review, January/February, 47-63.

Goh, J.C., and L.H. Ederington, 1999, "Cross sectional variation in stock market reaction to bond rating changes", Quarterly Review of Economics and finance, 39, 101-112.

Goldstein, R., Ju, N., and H. Leland, 2001, "An EBIT-based model of optimal capital structure", Journal of Business, 47, 483-512.

González-Hermosllo, B., 1999, "Determinants of ex-ante banking system distress: A macro-micro empirical exploration of some recent episodes", IMF Working Paper.

Greene, W. H., 2000, "Econometric Analysis", Prentice Hall International Inc, Fourth Edition.

Grier, P., and S. Katz, 1976, "The differential effects of bond rating changes among industrial and public utility bonds by maturity", Journal of Business, 49, 226-239.

Gropp, R., J. Vesala, and G. Vulpes, 2006, "Equity and bond market signals as leading indicators of bank fragility", Journal of Money, Credit and Banking, 38, 399-428.

Gunther, J. W., and R. R. Moore, 2000, "Financial Statements and Reality: Do troubled banks tell all?", Federal Reserve Bank of St.Louis Review, The Third Quarter, 30-35.

Hand, J.R.M., Holthausen R.W., and R.W. Leftwich, 1986, "The effect of bond rating agency announcements on bond and stock prices" Journal of Finance, 47, 733-752.

Heffernan, S. A., 2003, "The causes of bank failure", In: Mullineux A. W., Murinde, V. (Eds.), Handbook of International Banking. Edward Elgar: Cheltenham.

Heffernan, S. A., 2005, "Modern Banking", John Wiley & Sons, Ltd.

Hempel, B. J., Simonson, D. G., and A. B. Coleman, 1994, "Bank Management: Text and Cases", John Wiley & Sons, Inc, New York.

Hillegeist, S., Keating E., Cram D., and K. Lundstedt, 2004, "Assessing the Probability of Bankruptcy," Review of Accounting Studies, 9, 5-34.

Hirtle, B. J., and J. A. Lopez, 1999, "Supervisory information and the frequency of bank examinations", Federal Reserve Bank of New York Economic Policy Review, 5, 1-20.

Hite, G., and A. Warga, 1997, "The effect of bond-rating changes on bond price performance", Financial Analysts Journal, 35-51.

Holthausen, R.W., and R.W. Leftwich, 1992, "The effect of bond rating changes on common stock prices", Journal of Financial Economics, 17, 57-89.

Honohan, P., 1997, "Banking system failures in developing and transition economies: Diagnosis and Prediction", BIS Working Paper.

Houweling, Mentink and Vorst, 2002, "Is Liquidity Reflected in Bond Yields? Evidence from the Euro Corporate Bond Market", Working paper, Erasmus University.

Huang, J., M. Huang, 2003, "How much of the corporate-treasury yield spread is due to credit risk? Results from a new calibration approach", Working Paper, Stanford University.

Hwang, D. Y., C. F. Lee, and K. T. Liaw, 1997, "Forecasting bank failures and deposit insurance premium", International Review of Economics and Finance, 6, 317-334.

Hull, J. C. and A. White, 2000, "Valuing credit default swaps I: No counterparty default risk", Journal of Derivatives, 8, 29-40.

Hull, J. C., Predescu, M., and A. White, 2004, "The relationship between credit default swap spreads, bond yields and credit rating announcements", Journal of Banking and Finance, 28, 2789-2811.

Jarrow, R., S. Turnbull, 1995, "Pricing derivatives on financial securities subject to credit risk", Journal of Finance, 50, 53-86.

Jarrow, R., Lando, D. and S. Turnbull, 1997, "A Markov model for the term structure of credit risk spreads", Review of Financial Studies, 10, 481-523.

Jones, E., S. Mason and E. Rosenfeld, 1984, "Contingent claims analysis of corporate capital structures: An empirical analysis", Journal of Finance, 39, 611-625.

Kaminsky, G., and C. Reinhart, 1999, "The twin crises: The causes of banking and balance of payment problems", American Economic Review, 89, 473-500.

Katz, S., 1974, "The price and adjustment process of bonds to rating reclassifications: A test of bond market efficiency", Journal of Finance, 29, 551-559.

Kim, I., Ramaswamy, K., and S. Sundaresan, 1993, "Does default risk in coupons affect the valuation of corporate bonds? A contingent claims model", Financial Management, 117-131.

Lane, W. R., Looney S. W., and J. W. Wansley, 1986, "An application of Cox proportional hazard model to bank failure", Journal of Banking and Finance, 10, 511-531.

Lando, D., 1998, "On Cox processes and credit risky securities", Review of Derivatives Research, 2, 99-120.

Leland, H., 1994, "Corporate debt value, bond covenants and optimal capital structure", Journal of Finance, 49, 1213-1252.

Leland, H., 2004, "Predictions of default probabilities in structural models of debt", Journal of Investment Management, 2, 5-20.

Leland H., K. Toft, 1996, "Optimal capital structure, endogenous bankruptcy and the term structure of credit spreads", Journal of Finance, 51, 987-1019.

Liu, P., and A. Thakor, 1984, "Interest yields, credit ratings and economic characteristics of state bonds: An empirical analysis", Journal of Money, Credit and Banking, 16, 344-351.

Longstaff, F., E. Schwartz, 1995, "Valuing risky debt: A new approach", Journal of Finance, 50, 789-820.

LoPucki, Lynn M. and W. Whitford, 1990, "Bargaining over equity's share in bankruptcy reorganisation of large publicly held companies", University of Pennsylvania Law Review, 139(1), 125-196.

Lynden, S., and D. Saraniti, 2000, "An Empirical Examination of the Classical Theory of Corporate Security Valuation", Barclays Global Investors.

Madan, D., and H. Unal, 1998, "Pricing the risk of default", Review of Derivatives Research, 2, 121-160.

Martin, D., 1977, "Early warning of bank failures: A logit regression approach", Journal of Banking and Finance, 1, 249-276.

Mella-Barral, P., and W. Perraudin, 1997, "Strategic debt service", Journal of Finance, 52, 531-566.

Merton, R. C., 1974, "On the pricing of corporate debt: The risk structure of interest rates", Journal of Finance, 29, 449-470.

Moody's Investors Services, 1998, "Historical default rates of corporate bond issuers, 1920-1997", Special Comment.

Monkkonen, H., 1997, "Modeling default risk: Theory and empirical evidence", Doctoral dissertation, Queen's University, Canada.

Morgan, D., 2002, "Rating banks: risk and uncertainty in an opaque industry", The American Economic Review, 92, 874-888.

Ohlson, J., 1980, "Financial ratios and probabilistic prediction of bankruptcy", Journal of Accounting Research, 28, 109-131.

Opler, T., Saron, M., and S. Titman, 1997, "Designing capital structure to create shareholder value", Journal of Applied Corporate Finance, 7, 21-32.

Perraudin and Taylor, 2002, "Liquidity and Bond Market Spreads", Working paper, Birbeck College.

Rossi, M., 1999, "Financial fragility and economic performance in developing countries: Do capital controls, prudential regulation and supervision matter?", IMF Working Paper.

Sachs, J., Tornell, A., and A. Velasco, 1996, "Financial crises in emerging markets: The lessons from 1995", NBER Working Paper.

Satchel, S., and W. Xia, 2006, "Analytic models of the ROC curve: Applications to credit rating model validation", Working Paper, University of Technology, Sydney.

Schaefer, S., and I. Strebulaev, 2004, "Structural Models of Credit Risk are Useful: Evidence From Hedge Ratios of Corporate Bonds", Working Paper, London Business School.

Shen, C., and M. Hsieh, 2004, "Prediction of bank failures using combines micro and macro data", Working Paper, VanNung Institute of Technology.

Shumway, T., 2001, "Forecasting Bankruptcy More Accurately: A Simple Hazard Model", The Journal of Business, 74, 101-124.

Tarashev, N., 2005, "An empirical evaluation of structural credit risk models", Working Paper, Bank for International Settlements (BIS), No 179.

Taylor, A., and W. Perraudin, 2001, "Credit risk orderings implicit in ratings and bond market yields", Working Paper.

Vassalou, M., and Y. Xing, 2004, "Default risk in equity returns", Journal of Finance, 2, 831-868.

Vassalou, M., and Y. Xing, 2005, "Abnormal equity returns following downgrades", Working Paper, Columbia University.

Wasley, J.W., Glascock, J.L., and T.M. Clauretie, 1992, "Industrial bond pricing and information arrival: The case of bond rating changes", Journal of Business Finance and Accounting, 19, 733-750.

Wei, D., and D. Guo, 1997, "Pricing risky debt: An empirical comparison of the Longstaff and Schwartz and Merton models", The Journal of Fixed income, 8-28.

Weiss, L., 1990, "Bankruptcy Resolution: Direct costs and violation of priority claims", Journal of Financial Economics, 27, 285-314.

Wiginton, J., 1980, "A note on the comparison of logit and discriminant models of consumer credit behaviour", Journal of Financial and Quantitative Analysis, 15, 757-770.

Yu, F., 2003, "Accounting transparency and the term structure of credit spreads", Working paper, University of California, Irvin.

Zhou, C., 2001, "The term structure of credit spreads with Jump Risk", Journal of Banking and Finance, 25, 2015-2040.

Uhrig-Homburg, M., 2002, "Valuation of defaultable claims: A survey", Schmalenbach Business Review, 54, 24-57.

West, R., 1973, "Bond ratings, bond yields and financial regulation: Some findings", Journal of Law and Economics, 16, 159-168.