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Dirichlet Branes on Orientifolds

N. Quiroz* †, B. Stefański, jr.‡

Max-Planck-Institut für Gravitationsphysik, Albert-Einstein Institut
Am Mühlenberg 1, D-14476 Golm, Germany

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Abstract

We consider the classification of BPS and non-BPS D-branes in orientifold models. In particular we construct all stable BPS and non-BPS D-branes in the Gimon-Polchinski (GP) and Dabholkar-Park-Blum-Zaffaroni (DPBZ) orientifolds and determine their stability regions in moduli space as well as decay products. We find several kinds of integrally and torsion charged non-BPS D-branes. Certain of these are found to have projective representations of the orientifold $\times$ GSO group on the Chan-Paton factors. It is found that the GP orientifold is not described by equivariant orthogonal K-theory as may have been at first expected. Instead a twisted version of this K-theory is expected to be relevant.

*E-mail address: quiroz@aei.mpg.de

†On leave from Departamento De Física, Centro de Investigación y Estudios Avanzados del IPN (CINVESTAV-IPN), México D.F., México

‡E-mail address: bogdan@aei.mpg.de

1 Introduction and Summary

The classification of D-branes is an important aspect of improving our understanding of string theory. It gives a better handle on what vacua are allowed, as well as on how these can decay into one another via tachyon condensation. For each model a suitable K-theory should exist and constructing branes from physical principles allows to test the K-theory hypothesis more extensively. Furthermore, minimally charged, stable non-BPS D-branes should have descriptions in dual theories and finding these is an important step in testing dualities beyond the BPS level. Finally, D-branes provide a way of introducing non-abelian gauge fields as well as a way of breaking supersymmetry, giving them phenomenological relevance. In this paper we investigate D-branes in orientifold theories. We find a surprisingly rich spectrum of BPS and non-BPS D-branes and discuss decays and transitions between them. We find large classes of torsion charged D-branes. It is precisely for torsion charges where K-theory and rational cohomology may differ allowing for non-trivial tests of K-theory.

Since 1995 Dirichlet branes [1] have played a central role in string theory. Initially D-branes were constructed as BPS objects with corresponding supergravity solutions. These were instrumental in testing dualities at the BPS level (for a review see [2]). Later it was realised that, in the first-quantised theory, stable non-BPS D-branes too could be constructed as coherent states in the closed string theory [3, 4, 5, 6, 7]. Such non-BPS D-branes have provided several non-BPS tests of dualities [8, 9, 10]. The description of D-branes by boundary states in a conformal field theory [11, 12, 13, 14] has been particularly useful when a geometric picture is not apparent [15].

Non-BPS D-branes often decay into BPS-anti-BPS pairs of D-branes when certain closed string marginal deformations are turned on. These brane descent relations have been shown to be marginal deformations in the open string conformal field theory [3] and as such conserve mass and charge in the process. The decays in the first quantised theory suggested that D-branes can annihilate with anti-D-branes into the closed string vacuum [3, 16] via tachyon condensation. This led to the description of D-branes in terms of K-theory [5, 17] where such annihilations are a central feature. A classification of D-branes via K-theory guarantees Dirac quantisation of charges. It may also provide a description of torsion charged D-branes where it is not always obvious how to characterise the charges (and decay products) of various (unstable) brane configurations. For torsion charges K-theory in general differs from cohomology. As such torsion D-branes offer a confirmation of the K-theory perspective. In the orbifolds of oriented theories studied so far no torsion branes were found. On the other hand the simplest orientifold - Type I theory - has four such branes the D-1,0,7- and 8-branes.

In this paper we classify D-branes in the $\Omega \times \mathcal{I}_4$ orientifold of Type IIB theory first considered by [18]. More recently these models were extensively investigated in the D-brane language by Gimon and Polchinski [19] (see also [20]) and Dabholkar, Park and Blum, Zaffaroni [21, 22]. Both models can be regarded either as orbifolds of Type I or as Ω projections of the $\mathcal{I}_4$ orbifold of Type IIB. Projecting by Ω does not introduce any new closed string states while there are $\mathcal{I}_4$-

twisted closed string sectors to which the D-brane may couple. Since we will construct D-branes as boundary states from various closed string sectors it is particularly useful to think of the GP orientifold as the Ω-projected K3 orbifold of Type IIB on $\mathbf{R}^6 \times T^4$. D-branes in the orientifold are Ω-invariant configurations of D-branes in the orbifold. Let us briefly review the kinds of D-branes present in the orbifold and discuss the action of Ω on them. In the orbifold [7] it is convenient to label Dp-branes as (r, s)-branes where s, $r+1$ are the number of directions along which the brane extends on which $\mathcal{I}_4$ does or does not act, respectively, with $r + s = p$. There are two elementary types of integrally charged[1] D-branes in this IIB orbifold: for s even there are BPS fractional branes while for s odd there are non-BPS truncated branes; in both cases r is odd. The fractional branes are charged under both untwisted and twisted R-R charges coming from anti-symmetric forms

$$C^{(r+1,s)}\,, \qquad C_t^{(r+1)}\,, \tag{1.1}$$

while the truncated branes are charged only under the twisted R-R forms. Here $C^{(r+1,s)}$ are the usual Type IIB R-R $r+s+1$-forms with r, s indices in the non-compact and internal directions, respectively; $C_t^{(r+1)}$ are the twisted R-R $r+1$-forms which have indices in the non-compact directions only. An (r, s)-brane couples to 2^s such twisted sectors. The BPS branes are stable - they do not develop open string tachyons on their world-volume. On the other hand the non-BPS branes have open string winding or momentum tachyons for certain values of the compactification radii and decay into BPS-anti-BPS pairs of fractional branes. Note also that two fractional branes with the same untwisted R-R charge and opposite twisted R-R charges can come together and move of the fixed points to form a bulk BPS brane charged only under the untwisted R-R sector.

Ω keeps only certain of the above R-R forms. In particular in the untwisted sector

$$C^{(r+1,s)}\,, \qquad \text{with } r+s = 1, 5, 9 \tag{1.2}$$

survive. In the twisted sector two choices are allowed; either we keep a hypermultiplet at each fixed point as in [19] *i.e.*

$$C^{(r+1)}\,, \qquad \text{with } r = -1, 3 \tag{1.3}$$

survive or we keep a tensor multiplet [21, 22] in which case

$$C^{(r+1)}\,, \qquad \text{with } r = 1, 5\,, \tag{1.4}$$

are Ω invariant. For definiteness we concentrate on the GP orientifold throughout this paper - in section 7 we discuss for completeness the DPBZ orientifold.

D-branes are stable due to the charges that they carry, and since only Ω-even configurations of branes from the orbifold are allowed in the orientifold, the spectrum of stable branes changes. Since truncated branes are charged only under twisted R-R fields the Ω projection will either keep

[1] There are no stable torsion branes in the K3 orbifold.

or remove them.[2] In particular from (1.3) we see that the truncated (r, s)-branes with $r = -1, 3$ are present in the orientifold while those with $r = 1, 5$ are removed.[3]

Fractional branes carry twisted and untwisted R-R charges and Ω can be odd or even on these separately. As a result there are four possible destinies for the fractional branes of the orbifold in the orientifold:

- $r = -1, 3$, $s = 0, 4$: the bulk R-R charge is not Ω invariant - the corresponding branes are only charged under twisted R-R charges, are non-BPS and very similar to the usual truncated branes;

- $r = -1, 3$, $s = 2$: these fractional branes are Ω invariant;

- $r = 1, 5$, $s = 0, 4$: the twisted charge is projected out - the branes are BPS but do not carry any twisted charges and can be thought of as a pair of fractional branes in the orbifold with opposite twisted charges. As we will see later such a pair of branes is nonetheless not allowed to move off the fixed points and we will refer to them as *stuck*. Note that for $r = 5$ and $s = 0, 4$ these are precisely the tadpole canceling 5- and 9-branes of the GP model;

- $r = 1, 5$, $s = 2$: both twisted and untwisted R-R charges are projected out by Ω - there are no integrally charged branes.

The lack of integral charges does not, in general, mean that a brane is unstable. There may be some torsion charge stabilising the brane as in the case of the D0- or D7-brane of Type I. The former is stable in the tadpole cancelled theory, while the latter is unstable due to the tachyonic open strings that stretch between it and the tadpole canceling D9-branes. The D7-brane decays into a gauge configuration on the D9-brane world-volume [23], demonstrating that even though the brane is unstable the corresponding charge is present in the theory - after all K-theory only predicts the presence of a charge rather than determine what kind of object carries it.

In the GP orientifold we encounter a large number of torsion charged branes. For example, the simplest Ω invariant pair of fractional $(5, 2)$-branes in the Type II orbifold, couples to the twisted and untwisted NS-NS sectors, has opposite twisted and untwisted R-R charges and decays into the vacuum in the orbifold. Hence above we were lead to conclude that there are no integrally charged $(5, 2)$-branes in the orientifold. In fact in the orientifold the configuration does carry a torsion charge and as a result is stable (in other words there are no tachyonic open strings that have both end-points on the (5,2)-brane configuration). In the non-compact theory the charge is $\mathbf{Z}_2 \oplus \mathbf{Z}_2$. As in the D7-brane of Type I the open strings that stretch between the tadpole canceling D9-branes and the (5,2)-brane have tachyons in their spectrum. We expect that as a result the

[2] Strictly speaking a brane may still be stable even though it carries no integral charge - there may be some torsion charge which stabilises it. For now we concentrate on integrally charged branes and discuss torsion charges below.

[3] s is odd since these are truncated branes in the orbifold [7].

(5,2)-brane does decay. However, this semi-stability of the (5,2)-brane indicates the presence of a torsion charge in the GP orientifold. In the DPBZ orientifold similar torsion branes are in fact stable in the tadpole cancelled theory. We also find a number of $\mathbf{Z}_2$ charged D-branes. These have $r = 5$ and $s = 1, 3$. We will show that there are decay channels between these indicating that they carry the same torsion charge.

Type II orbifolds are classified by equivariant complex K-theory. In all of the cases considered K-groups computed did indeed agree with the D-brane spectrum. Type I D-branes on the other hand are classified by orthogonal K-theory, which is also in agreement with the D-branes present in the theory.[4] One might expect that orbifolds of Type I should be classified by orthogonal equivariant K-theory. From the string theory perspective though it is clear that there is a potential subtlety in defining the action of Ω on the closed string twisted sectors. As we have already seen in the $\mathcal{I}_4 \times \Omega$ orientifolds there are two ways of defining the action of Ω on the $\mathcal{I}_4$-twisted sectors, which leaves either a tensor- or a hypermultiplet in the orientifold. This is very similar to the discrete torsion [24] one encounters in $\mathbf{Z}_2 \times \mathbf{Z}_2$ Calabi-Yau orbifolds [25]. In the Calabi-Yau models there are two theories with $(h^{11}, h^{21}) = (3, 51)$ and $(51, 3)$ which arise due to the presence of projective representations of the orbifold group $\mathbf{Z}_2 \times \mathbf{Z}_2$. The models are described by the equivariant complex K-theory and its twisted version. In the orientifold models considered in this paper, the orientifold group is also $\mathbf{Z}_2 \times \mathbf{Z}_2$, and indeed we have two possible models as in the Calabi-Yau orbifolds. As a result the equivariant orthogonal K-theory can only describe one of these. We show that it in fact gives the DPBZ model. The second model should be described by a suitably twisted version of this K-group, which to the best of our knowledge has not been studied in the mathematical literature.

The paper is organised as follows. In section 2 we discuss the construction of GSO, orbifold and Ω invariant boundary states in each of the closed string sectors; from these we construct boundary states corresponding to BPS D-branes in the orientifold: fractional, stuck and bulk. In sections 3 and 4 boundary states corresponding to integrally and torsion charged non-BPS D-branes are constructed. The stability conditions and decay channels of the branes are also analysed. In section 5 the open string perspective is discussed. In particular open strings that end on many of the branes constructed in the closed string channel have non-trivial Chan-Paton factors; the representation of the orientifold group as well as the GSO projection on these is discussed. For example $(-1)^F$ and Ω are found to form a projective representation on the Chan-Paton factors of the D7-brane of Type I. We show how this phase is compensated for by an opposite one on the world-sheet fields to give a genuine representation on the full open string Hilbert space. Section 6 contains a discussion of the relevant K-theories for the GP and DPBZ models. In particular it is argued that the equivariant orthogonal K-theory $KO_{\mathbf{Z}_2}$ is relevant to DPBZ rather than GP, where a twisted version of $KO_{\mathbf{Z}_2}$ has to be defined. In Section 7 branes on DPBZ are discussed and section 8 contains some conclusions and open problems. We have

[4]The D7 and D8-branes are somewhat subtle, and we refer the reader to [23] for discussions of these.

included several appendices which contain computational details to which we refer to throughout the paper.

In [26] certain aspects of D-branes in the GP model have been studied. In particular orientifold invariant boundary states in each of the closed string sectors were constructed. Further, the truncated branes were identified, though the stability of these branes differs from our results. We have also found a different, and much bigger, set of torsion charged branes to [26].

2 BPS D-branes

D-branes interact with closed string states, and as such can be described as coherent states in the closed string Fock space [11, 12, 13]. Since these represent a boundary in the world-sheet at which closed strings are absorbed or emitted, these coherent states are called boundary states. In each of these closed string sectors one constructs a GSO invariant coherent state. In orbifolds and orientifolds these boundary states also have to be invariant under the action of the symmetry group being modded out. Hence only GSO- and orbifold/orientifold- invariant closed string states couple to D-branes. These invariances place restrictions on the allowed boundary states. D-branes are also hypersurfaces on which open strings end; only certain combinations of boundary states give rise to a consistent open string spectrum. In this section we discuss the construction of orientifold invariant boundary states, and combine boundary states from various sectors to construct BPS branes.

2.1 Orientifold invariant boundary states

In each closed string sector one constructs two boundary states, corresponding to the two spin structure choices for fermions on the boundary. GSO invariance ensures that at most one linear combination survives. Orbifold and Ω invariance places further restrictions on the allowed r and s values. GSO and $\mathcal{I}_4$ invariance have been discussed in detail before [7], and in [26] Ω invariance has been investigated. In appendix B we review Ω invariance and summarise the results here. In the twisted sectors Ω can act in one of two ways, giving the GP or DPBZ orientifolds; here we state the results for the GP orientifold. Boundary states in each closed string sector are Ω, I_4 and GSO invariant for

- $|B(r,s)\rangle_{\text{NS-NS}}$ all (r,s),
- $|B(r,s)\rangle_{\text{R-R}}$ $r+s=1,5,9$,
- $|B(r,s)\rangle_{\text{NS-NS},\text{T}}$ $s=2$,
- $|B(r,s)\rangle_{\text{R-R},\text{T}}$ $r=-1,3$.

A D-brane is a consistent linear combination of boundary states from various closed string sectors. The consistency conditions determine the normalisation of boundary states as well as the allowed linear combinations from different sectors. In an oriented theory these consistency conditions come from the requirement that the cylinder diagram reproduce the annulus partition function of an open string ending on the D-brane with a projection operator inserted into the trace. For example a D-brane coupling to the untwisted and twisted R-R sectors would not lead to a consistent open string partition function, as the operator inserted in the corresponding open strnig trace would be proportional to $(-1)^F + \mathcal{I}_4(-1)^F$ which is not a projection operator; a brane that couples to both these sectors needs to couple to the untwisted and twisted NS-NS sectors as well.

In theories with Ω projections there are orientifold planes represented by crosscaps (for a construction of these coherent states see Appendix A); for example in the GP orientifold there are O9- and O5-planes. A coherent state corresponding to a consistent D-brane now has to reproduce the annulus and Möbius strip contributions to the one-loop partition function for an open, unoriented string ending on the D-brane. The annulus comes from the cylinder diagram corresponding to the exchange of a closed string between two boundary states, while the Möbius strip comes from the exchange of a closed string state between a crosscap and a boundary state. In practice the square of the normalisation of the D-brane and O-plane is obtained from the annulus and Klein bottle diagrams, while the relative sign follows from the Möbius strip. In [7] the normalisations of various boundary states were worked out rather explicitly. The computations are easily generalised to the case studied presently as summarised in Appendix C.

2.2 BPS D-branes in the GP orientifold

There are two kinds of elementary BPS D-branes in the GP orientifold - the fractional and stuck branes. Both are located at the transverse fixed points and can only have discrete Wilson lines in the compact directions along which they extend. They differ in that the fractional branes couples to both twisted and untwisted closed string sectors while the stuck branes couple only to the untwisted sectors. In this subsection we discuss both kinds of branes and comment on how bulk BPS branes come into the picture.

Fractional branes are charged under both twisted and untwisted R-R charges and so, as mentioned at the end of the previous subsection, in order that the open string partition function have a consistent projection operator inserted, these branes couple to all closed string sectors. In the GP orientifold there are only two such branes: the $(r,s) = (-1,2)$ and $(3,2)$, described by the boundary states

$$\begin{aligned} |D(r,s)\rangle \;\; &= \;\; \mathcal{N}_{(r,s),U}\left(|B(r,s)\rangle_{\text{NS-NS}} \; + \epsilon_1 \, |B(r,s)\rangle_{\text{R-R}}\right) \\ &\quad + \epsilon_2 \, \mathcal{N}_{(r,s),T} \sum_{\alpha=1}^{2^s} e^{i\theta_\alpha} \left(|B(r,s)\rangle_{\text{NS-NS}\,,\text{T}_\alpha} + \epsilon_1 \, |B(r,s)\rangle_{\text{R-R}\,,\text{T}_\alpha}\right) , \end{aligned} \tag{2.1}$$

where $\epsilon_i = \pm 1$, α labels the different fixed points between which the brane stretches, and $\theta_\alpha = 0, \pi$ is the Wilson line that is associated to the difference of the fixed point α and the origin. $\mathcal{N}_{(r,s),U}$ and $\mathcal{N}_{(r,s),T}$ are the normalisations of the untwisted and twisted sectors, determined most easily by requiring that the closed string cylinder diagram reproduce the one-loop open string partition function

$$\int_0^\infty dl \langle D(r,s)|e^{-l\pi H_c}|D(r,s)\rangle = \int \frac{dt}{2t} \mathrm{Tr}_{\text{NS-R}}\left(\frac{1}{2}\frac{1+(-1)^F}{2}\frac{1+\mathcal{I}_4}{2}e^{-2\pi t H_o}\right), \tag{2.2}$$

where $t = 1/2l$, $H_{c,o}$ are the closed and open string Hamiltonians and the extra factor of $1/2$ comes from the Ω projection.[5] As a result the normalistions naively would differ from [7] by an extra factor of $1/\sqrt{2}$, as is indeed the case for the $(-1,2)$-brane (see Table 32). The $(3,2)$-brane though is a 5-brane and for it the smallest representation of Ω on Chan-Paton factors is the 2×2 anti-symmetric matrix

$$\gamma_\Omega = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}. \tag{2.3}$$

This follows [19] from the fact that for D5-branes on world-sheet fields $\Omega^2 = -1$ which is compensated for by the same relation on the Chan-Paton factors with the above matrix; on the full open string Hilbert space Ω squares to one. Similar phases on the sub-sectors of the open string Hilbert space are quite common and we discuss them in section 5. Here we note that since the (3,2)-brane has 2×2 Chan-Paton factors its normalisation is actually a $\sqrt{2}$ bigger than that of the $\mathcal{I}_4$-orbifold $(3,2)$-brane; this ensures that the Dirac quantisation condition is minimally satisfied. The situation is rather reminiscent of the D1-D5 discussion in Type I. Further, it is straightforward to check that in the open string world-volume theories of these fractional branes there are no massless transverse scalars in the orbifolded directions and as a result the branes cannot move off the fixed-points.

A second class of BPS branes (which also cannot move off the fixed points) does not couple to twisted sectors.[6] We refer to these branes as stuck branes. In the orbifold they are a pair of fractional branes with the same untwisted R-R charge, but opposite twisted R-R charges and are mapped to one another by Ω, in other words they form a bulk brane with a single transverse scalar in each of the compact directions; this scalar is removed by the Ω projection giving in the orientifold a BPS brane that couples only to the untwisted sectors, yet is stuck at the fixed points. From section 2.1 it is immediate that such branes exist for $r = 1, 5$ and $s = 0, 4$ and are described by the boundary state

$$|D(r,s)\rangle = \mathcal{N}_{(r,s),U}\left(|B(r,s)\rangle_{\text{NS-NS}} + \epsilon\,|B(r,s)\rangle_{\text{R-R}}\right), \tag{2.4}$$

with $\epsilon = \pm 1$ and the normalisation constant $\mathcal{N}_{(r,s),U}$ given in Appendix C.[7] In particular note

[5] The full open string partition function has three projections - GSO, orbifold and Ω; the cylinder diagram is proportional to 1 and the Möbius strip to Ω.

[6] This can be contrasted with orbifold theories where branes stuck at fixed points do couple to twisted sectors.

[7] For branes with $r + s = 5$ the Chan-Paton factors are 2×2 as in the case of the $(3,2)$-brane.

that the GP orientifold has sixteen stuck $(5,0)$- and $(5,4)$-branes which cancel the untwisted 6- and 10-form R-R tadpoles. Further, neither the O-planes nor the tadpole canceling stuck branes couple to the twisted R-R sectors. This is consistent with the lack of a twisted sector six-form tadpole in the GP model [19, 20].

Finally, we note that single bulk branes are simply pairs of fractional or stuck branes which do not couple to twisted closed string sectors, and carry untwisted R-R charges. It can be easily checked that such objects have a single transverse scalar in each of the compact directions which allows them to move off the fixed points.

3 Integrally charged non-BPS $\hat{\text{D}}$-branes

Perturbatively stable D-branes have no open string tachyons on their world-volumes and carry an integral or torsion charge classified by a suitable K-theory. Integrally charged D-branes couple to suitable anti-symmetric R-R forms with which the charge is associated; in other words one may find an element of a rational cohomology from which the D-brane K-theory class was lifted. In the previous section we discussed integrally charged BPS branes in the GP model.[8] In this section we construct the remaining stable integrally charged branes in the GP model. As in the $\mathcal{I}_4$ orbifold [3, 4, 7] these are truncated non-BPS branes which couple to the untwisted NS-NS and twisted R-R sectors

$$|\hat{\text{D}}(r,s)\rangle = \mathcal{N}_{(r,s),U}\,|B(r,s)\rangle_{\text{NS-NS}} + \epsilon\,\mathcal{N}_{(r,s),T}\sum_{\alpha=1}^{2^s} e^{i\theta_\alpha}\,|B(r,s)\rangle_{\text{R-R},\text{T}_\alpha}\,, \tag{3.1}$$

and are only stable for certain values of the radii.[9] Above $\mathcal{N}$ are the normalisations of the boundary states, $\epsilon = \pm 1$ and $\theta_\alpha = 0, \pi$ are the Wilson lines associated to fixed point α. The open strings that end on such branes have the projection

$$\frac{1+\Omega}{2}\frac{1+(-1)^F\mathcal{I}_4}{2}\,. \tag{3.2}$$

The $r = -1, 3$ and $s = 1, 3$ ($r = 1, 5$ and $s = 1, 3$) truncated branes of the $\mathcal{I}_4$ orbifold are (not) Ω invariant and hence are (not) present in the GP orientifold. The second type of truncated branes are an Ω invariant pair of fractional $r = -1, 3$, $s = 0, 4$ branes from the $\mathcal{I}_4$ orbifold. For these values of r and s (see section (2.1)) The R-R untwisted and NS-NS twisted boundary states are Ω odd, so such pairs of fractional branes couple to the untwisted NS-NS and twisted R-R sectors only, justifying their name.

[8] BPS D-branes have to be integrally charged since they are massive.

[9] They are non-BPS as supersymmetry would require the presence of untwisted R-R (twisted NS-NS) sector boundary state to combine with the untwisted NS-NS (twisted R-R) one. Further the one-loop partition function for open strings ending on the brane does not generically vanish.

Since both kinds of truncated branes are non-BPS their stability is not guaranteed. Indeed, while $(1+(-1)^F\mathcal{I}_4)/2$ removes the ground-state tachyon from the spectrum of the open string ending on a truncated brane, it keeps winding and momenta states of the form

$$|\frac{n}{R_{||}}\rangle - |-\frac{n}{R_{||}}\rangle\,, \qquad |wR_\perp\rangle - |-wR_\perp\rangle\,, \tag{3.3}$$

where $n, w \in \mathbf{Z}$ and $R_{||}, R_\perp$ are radii of the compact directions parallel, transverse to the D-brane, respectively. The lightest such states are non-tachyonic for

$$R_{||} \leq \sqrt{2}\,, \qquad R_\perp \geq \frac{1}{\sqrt{2}}\,, \tag{3.4}$$

and outside of these regions the truncated branes of the orbifold decay into BPS-anti-BPS pairs of fractional branes [3, 4, 7]. In the GP orientifold the open strings that end on truncated branes are projected by $(1+(-1)^F\mathcal{I}_4)/2$ as well as $(1+\Omega)/2$ and it is immediate that this second projection removes either the momentum or the winding states of (3.3) depending on the action of Ω on the open-string ground state.[10] A good way to determine which states are removed is by computing the one-loop open string partition function in the closed string channel; since the computation is somewhat tedious we defer it to appendix C and summarize the results below. Minimally charged $r=-1$, $s=0,1$ truncated branes are stable for all values of $R_{||}$ and

$$R_\perp \geq \frac{1}{\sqrt{2}}\,, \tag{3.5}$$

while the $r=-1$, $s=3,4$ branes are stable for all values of $R_\perp$ and[11]

$$R_{||} \leq \sqrt{2}\,. \tag{3.6}$$

Non-minimally charged $r=-1$ branes have non-trivial CP factors and there will remain at least one winding and one momentum mode of the form (3.3). The results of Appendix C confirm that these branes have the same stability as in the orbifold (3.4).[12] As we will see below $r=3$ truncated branes have non-trivial Chan-Paton factors and so have the same stability conditions as the orbifold (3.4).

We now turn to the discussion of the decay channels of a minimally charged $r=-1$ brane. As was mentioned above the $s=3,4$ branes are T-dual to the $s=1,0$-branes and so we discuss

[10] In the above discussion we have taken the Chan-Paton factors to be trivial.

[11] Under four T-dualities in the internal directions an $(-1,s)$-brane becomes a $(-1,4-s)$-brane and the stability conditions are T-duality invariant with a winding mode becomes a momentum mode.

[12] Since truncated branes have non-vanishing forces between them the stability of a truncated brane with non-minimal R-R charge is somewhat complicated. By the stability of the non-minimally charged objects we simply mean the lack of open string tachyon in a hypothetical bound state.

only the latter. In the discussion of decays throughout this paper we will compare the mass and charge (if any) of the decaying brane with that of the proposed decay products at the critical radius. If the two agree we shall take it to mean that the decay is via a marginal deformation in the conformal field theory corresponding to turning on a vev for a suitable tachyonic mode which becomes massless at the critical radius [3].

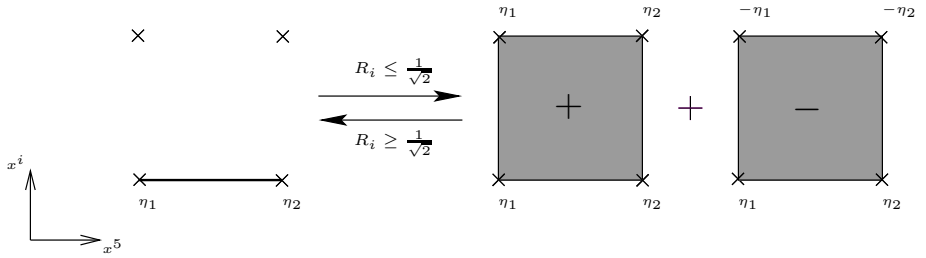

Figure 1: The decay of a truncated (-1,1)-brane into a D$\bar{\text{D}}$-pair of (-1,2)-branes. $\eta_i = \pm 1$ are the signs of the twisted R-R charges at the fixed points indicated by the crosses. The $\hat{\text{D}}(-1,1)$-brane has $\epsilon = \eta_1$ and $\theta_5 = \pi\eta_2/\eta_1$, the D$(-1,2)$-brane has $\epsilon_1 = +1$, $\epsilon_2 = \eta_1$, $\theta_5 = \pi(\eta_2 - \eta_1)/2$, $\theta_6 = 0$ and the $\bar{\text{D}}(-1,2)$-brane has $\epsilon_1 = -1$, $\epsilon_2 = \eta_1$, $\theta_5 = \pi(\eta_2 - \eta_1)/2$, $\theta_6 = \pi$ (*c.f.* equations (3.1), (2.1)).

Consider first a $\hat{\text{D}}(-1,1)$-brane stretching along x^5, say. This is stable for all values of R_5 and for $R_i \geq 1/\sqrt{2}$ with $i = 6, 7, 8$. For $R_i \leq 1/\sqrt{2}$ the brane decays into a fractional $(-1,2)$ brane-anti-brane pair stretching along x^5 and x^i in the same way as in the orbifold as shown in Figure 1.[13] From Table 32 the normalisations of the truncated brane's boundary states are

$$\mathcal{N}^2_{U,(-1,1)} = \frac{1}{64}\frac{R_5}{R_i R_j R_k}\,, \qquad \mathcal{N}^2_{T,(-1,1)} = \frac{8}{64}\,, \tag{3.7}$$

while each of the fractional branes is normalised as

$$\mathcal{N}^2_{U,(-1,2)} = \frac{1}{128}\frac{R_5 R_i}{R_j R_k}\,, \qquad \mathcal{N}^2_{T,(-1,2)} = \frac{4}{128}\,. \tag{3.8}$$

Since the untwisted (twisted) sector normalisation $\mathcal{N}_U$ ($\mathcal{N}_T$) is proportional to the mass (twisted R-R charge) of a brane one can easily verify that at the critical radius $R_i = 1/\sqrt{2}$ the truncated brane has the same mass and charges as the pair of fractional branes.

Similarly, a $\hat{\text{D}}(-1,0)$-brane is unstable for $R_i \leq 1/\sqrt{2}$; it decays into a pair of $\hat{\text{D}}(-1,1)$-branes which stretch along x^i as shown in Figure 2. The two $\hat{\text{D}}(-1,1)$-branes have the same twisted R-R

[13]The fractional branes have opposite bulk R-R charge (as a result the object is non-BPS) as well as two of the four twisted R-R charges (this corresponds to turning on a Wilson line on one of the two branes). The other two twisted R-R charges are the same on each of the fractional branes.

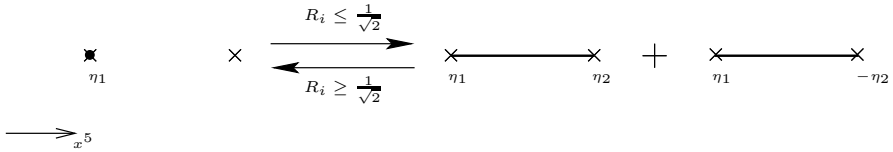

Figure 2: The decay of a truncated (-1,0)-brane into a pair of $\hat{\mathrm{D}}$(-1,1)-branes. $\eta_i = \pm 1$ are the signs of the twisted R-R charges at the fixed points indicated by the crosses. The $\hat{\mathrm{D}}$-branes have $\epsilon = \eta$, with opposite Wilson lines θ_5 on the $\hat{\mathrm{D}}(-1,1)$-branes.

charge at the fixed point at which the original $\hat{\mathrm{D}}(-1,0)$-brane was located and opposite twisted R-R charges at the other fixed point. Note that unlike a single $\hat{\mathrm{D}}(-1,1)$-brane, due to non-trivial Chan-Paton factors a pair of $\hat{\mathrm{D}}(-1,1)$-branes does indeed develop a tachyon from open-string momentum modes. Due to the relative Wilson line open strings with an endpoint on each of the two $\hat{\mathrm{D}}(-1,1)$-branes have half-integral momentum states which become tachyonic for

$$R_i \geq \frac{1}{\sqrt{2}} \tag{3.9}$$

and the configuration decays back into a $\hat{\mathrm{D}}(-1,0)$-brane.

Turning to the $r = 3$ truncated branes we note that a $\hat{\mathrm{D}}(3,1)$-brane decays into a pair of fractional $(3,2)$-branes of opposite bulk charges (with one of the fractional branes having a Wilson line so as to conserve twisted R-R charge in the decay) for $R_\perp \leq 1/\sqrt{2}$. However, the $(3,2)$-brane has 2×2 Chan-Paton factors since it is a 5-brane [19]. This makes it twice as massive and carry twice the twisted R-R charge.[14] In order to conserve energy and charge in this decay the $(3,1)$-brane has also got to have 2×2 Chan-Paton factors and as such its stability is given in equation (3.4). We hope to present a more detailed analysis of this and some of the other decay channels of the $r = 3$ truncated branes in the future.

4 Torsion charged D-branes

Torsion charged branes do not couple to R-R sectors. They have previously been encountered in Type I-like theories [3, 5, 6, 9], shown to carry $\mathbf{Z}_2$ charges (since a pair of them decays into the vacuum) and couple only to the untwisted NS-NS sector. In the first part of this section we find a new class of torsion branes coupling to both twisted and untwisted NS-NS sectors. In the decompactified theory these are $\mathbf{Z}_2 \oplus \mathbf{Z}_2$ charged. In the second part of this section we find $\mathbf{Z}_2$-charged branes in the GP model which couple only to the untwisted NS-NS sector much like the torsion branes of Type I. All the torsion branes in the GP orientifold give rise to open string

[14]This guarantees that the fractional $(-1,2)$ and $(3,2)$-branes satisfy the Dirac quantisation condition.

tachyons between them and the tadpole canceling $(5,0)$- or $(5,4)$-branes. Nonetheless this shows that, while the branes are unstable, the charges they carry are present in the theory. As we will see in section 7 in the DPBZ model, similar torsion branes do not have such tachyons and so are genuinely stable.

4.1 Torsion branes with twisted NS-NS couplings

Consider an $(r,2)$-brane of the form

$$|D(r,2)\rangle = \mathcal{N}_{(r,2),U}\,|B(r,2)\rangle_{\text{NS-NS}} + \epsilon\,\mathcal{N}_{(r,2),T}\sum_{\alpha=1}^{4} e^{i\theta_\alpha}\,|B(r,2)\rangle_{\text{NS-NS},\mathrm{T}_\alpha}\,, \tag{4.1}$$

where $\epsilon = \pm 1$, α labels the different fixed points between which the brane stretches, and $\theta_\alpha = 0, \pi$ is the Wilson line that is associated to the difference of the fixed point α and the origin.[15] We keep r and the normalisations $\mathcal{N}$ unspecified.[16] The one-loop partition function for an open string ending on such a brane is computed in Appendix D; one finds that for $r = 4, 5$ and suitable normalisations the ground-state tachyon is projected out stabilising the $(4,2)$ and $(5,2)$-branes. The $(4,2)$-brane is a D6-brane of Type I fixed under $\mathcal{I}_4$. Such D-branes have been shown to be inconsistent in Type I [5, 6] and so are inconsistent in the present orientifold. Expanding further in winding and momenta one finds that for

$$\frac{1}{R_{||}^2} + R_\perp^2 \geq \frac{1}{2} \tag{4.2}$$

the $(5,2)$-branes are stable. In particular in the non-compact orbifold the branes are always stable. Similar stability conditions have been previously encountered for integrally charged branes in [27, 28].

Since D-branes are described by K-theory, a complete set of branes transverse to a particular sub-manifold of spacetime has to form a group. In the non-compact theory we have found two branes with $\epsilon = \pm 1$ to which we will refer to as $g_\pm$.[17] In the next subsection we show that an $\epsilon = +1$ and an $\epsilon = -1$ brane join to form another stable brane, call it h, transverse to the same sub-manifold

$$g_+ + g_- = h\,, \tag{4.3}$$

[15] In particular $\theta_1 = 0$, $\theta_2 = \theta_5$, $\theta_3 = \theta_6$ and $\theta_4 = \theta_5 + \theta_6$ where θ_5, θ_6 are the Wilson lines on the brane in the x^5, x^6 directions.

[16] We take $s = 2$ as this is the only value of s for which the twisted NS-NS boundary state is Ω, GSO, and $\mathcal{I}_4$ invariant (see Appendix B for details).

[17] In the non-compact theory there is only one NS-NS twisted sector and the D-brane boundary state is

$$|D_{\mathbf{Z}_2\oplus\mathbf{Z}_2}(r,2)\rangle = \mathcal{N}_{(r,2),U}\,|B(r,2)\rangle_{\text{NS-NS}} + \epsilon\,\mathcal{N}_{(r,2),T}\,|B(r,2)\rangle_{\text{NS-NS},\mathrm{T}}\,.$$

and further that this brane is $\mathbf{Z}_2$ charged

$$h + h = 1\,, \tag{4.4}$$

where 1 is the vacuum. K-theory tells us that $\{1, g_+, g_-, h\}$ form a group. Since each of the branes is different from the vacuum this can be $\mathbf{Z}_2 \oplus \mathbf{Z}_2$ or $\mathbf{Z}_4$.[18] $\mathbf{Z}_4$ is impossible: for if that were the group equation (4.4) implies that $h = x^2$, where x is the generator of $\mathbf{Z}_4$, and (4.3) gives $g_+ = g_- = x$ (or $g_+ = g_- = x^3$). There should then be another brane corresponding to x^3 (or x), and since the complete list of branes is $\{1, g_+, g_-, h\}$ this is impossible. Hence the torsion branes in the decompactified theory are charged under $\mathbf{Z}_2 \oplus \mathbf{Z}_2$. As a result we have further learnt that the unstable configurations $g_+ + g_+$ and $g_- + g_-$ both decay into the vacuum.

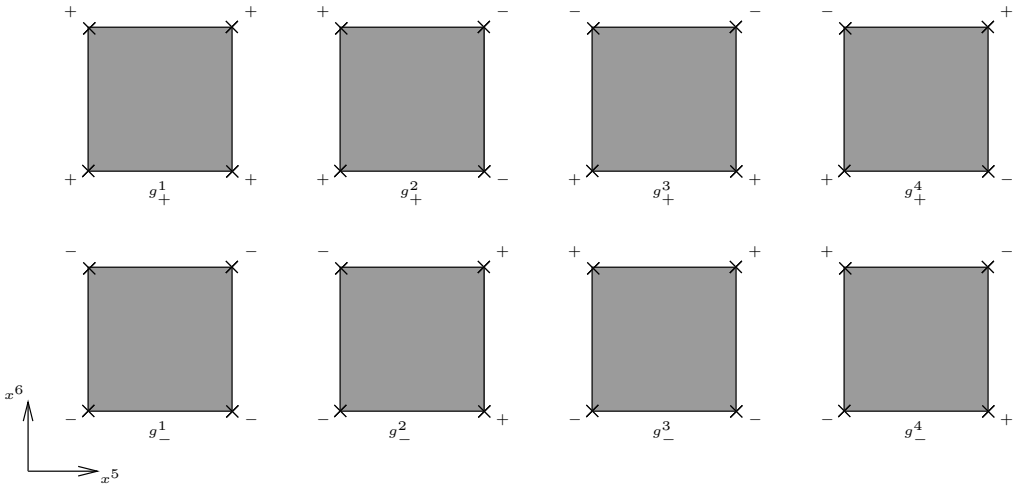

Figure 3: The eight $(r, 2)$-branes $(g_\pm^1, \ldots, g_\pm^4)$ coupling to both the twisted and untwisted NS-NS sectors with no constant gauge field in the internal directions The configurations correspond to the choices $\epsilon = \pm 1$ and $\theta_5, \theta_6 = 0, \pi$ in the boundary state (4.1). In the compactified space the branes are taken to stretch along x^5 and x^6 and the figure shows only these directions.

In the compact theory there are eight branes with twisted NS-NS couplings transverse to the same spacetime sub-manifold corresponding to the choices associated with ϵ and the two Wilson lines θ_i. Further by turning on a constant gauge field $F_{ij} = \pm 1$ in the two internal directions along which the brane stretches, we obtain eight more stable configurations. This is quite analogous to [29] (see also [30] for related orientifolds) where it was used to describe a certain decay product of a tadpole canceling brane-anti-brane pair into a truncated brane with a constant magnetic

[18] At first one might consider $\mathbf{Z}_2$ but equation (4.3) would imply that one of g_+, g_- would equal 1, the vacuum.

flux.[19] We refer to these sixteen branes as $g^i_\pm$ as shown in Figures 4 and 5. In the next subsection we show that as in the decompactified case there is a brane h such that

$$g^i_+ + g^i_- = h\,, \tag{4.5}$$

for all i and h is again order two (4.4). It would be interesting to determine the overall group structure in the compact case.

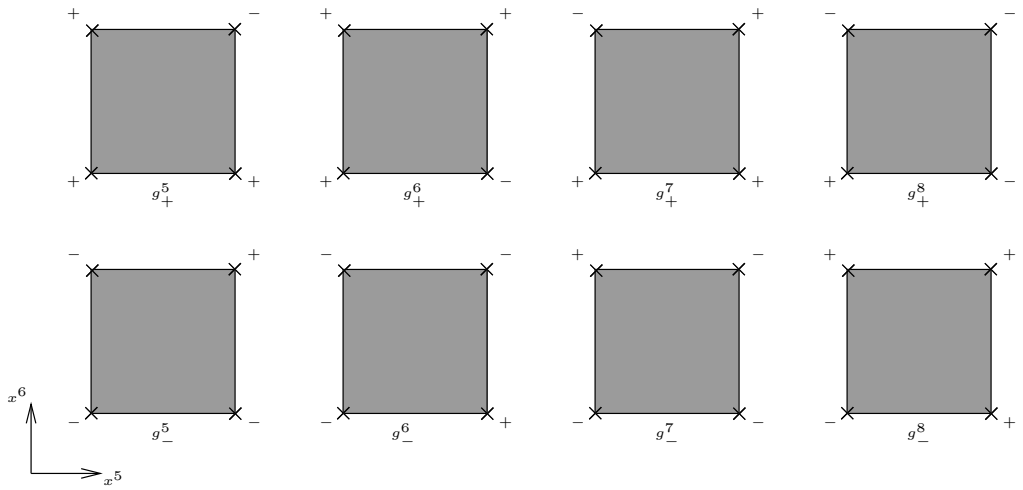

Figure 4: The eight $(5,2)$-branes $(g^5_\pm,\ldots,g^8_\pm)$ coupling to both the twisted and untwisted NS-NS sectors with a constant magnetic flux $F_{56}=\pm1$. The configurations correspond to the choices $\epsilon=\pm1$ and $\theta_5,\theta_6=0,\pi$ in the boundary state (4.1). Each is stable for a particular F_{56}. In the compactified space the branes are taken to stretch along x^5 and x^6 and the figure shows only these directions.

Before turning to the second class of torsion branes we mention a particularly useful way to think about the $(5,2)$-brane. This brane is an Ω-invariant configuration of two fractional $(5,2)$-branes in the $\mathcal{I}_4$ orbifold with opposite bulk and twisted R-R charges. Such a configuration condenses into the vacuum in the orbifold; in the orientifold however there is a torsion charge that this pair carries which stabilises it.

4.2 Torsion branes with no twisted NS-NS couplings

In this section we consider a D-brane coupling only to the untwisted NS-NS sector

$$|D_{\mathbf{Z}_2}(r,s)\rangle = |B(r,s)\rangle_{\text{NS-NS}}\ , \tag{4.6}$$

[19]We are grateful to M.R. Gaberdiel for bringing this construction to our attention and for suggesting its application here.

with arbitrary r, s, and normalisation. From the one-loop partition function computed in Appendix D one finds that for $r = 4, 5$ and $s = 1, 2, 3$ the ground-state tachyon is removed. We will show that the $(r, 1)$-, $(r, 2)$- and $(r, 3)$-branes decay into one another. Since the $(4, 2)$-brane is inconsistent (see previous subsection) we expect the $(4, 1)$- and $(4, 3)$-branes to be also inconsistent. The normalisations (listed in Appendix D) for the $s = 2$ branes indeed make them bound states of two branes from the previous subsection as described in equations (4.3) and (4.5).

The $(5, s)$-branes with no twisted NS-NS couplings are stable for

$$R_i \leq \sqrt{2}\,, i = 5, \ldots, 4 + s\,, \qquad R_j \geq \frac{1}{\sqrt{2}}\,, j = 5 + s, \ldots, 8\,, \tag{4.7}$$

where we have taken the brane to lie along $x^5, \ldots, x^{4+s}$ and be transverse to $x^{5+s}, \ldots, x^8$.

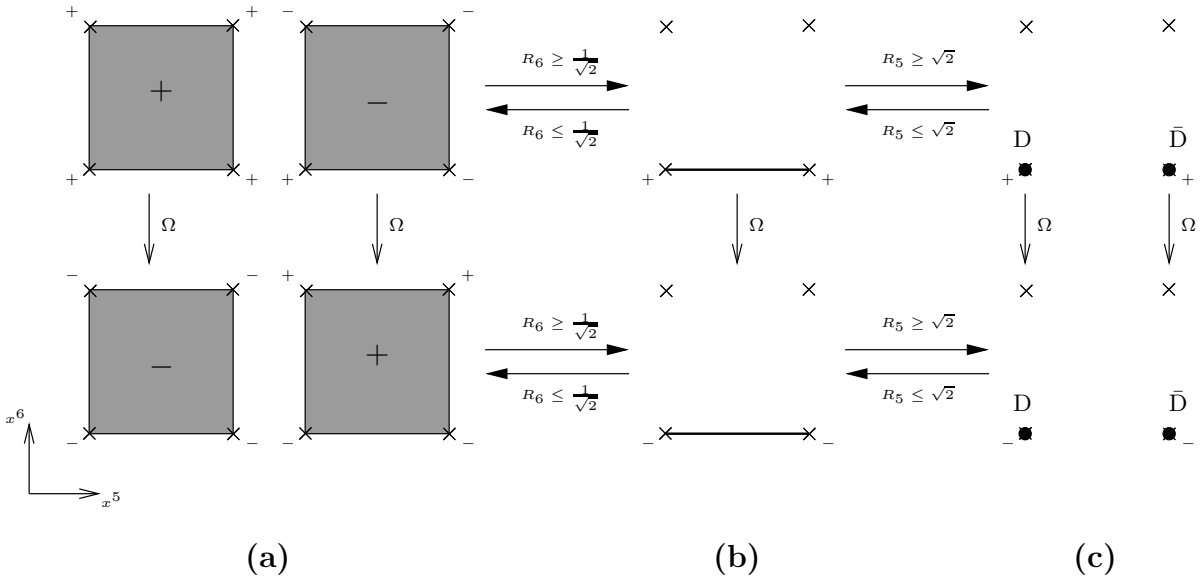

Figure 5: The decay channels of $\mathbf{Z}_2$-charged branes are most easily seen as an Ω invariant process in the $\mathcal{I}_4$ orbifold. The first line in the figure shows the standard decent of an $s = 2$ D$\bar{\text{D}}$pair (a), via an $s = 1$ $\hat{\text{D}}$-brane (b), into an $s = 0$ D$\bar{\text{D}}$pair. The second line is the Ω-image of this decay. Together, the diagrams show the decays between $\mathbf{Z}_2$-charged $s = 2, 1, 0$-branes in the GP orientifold. **(a)** A $\mathbf{Z}_2$-charged (5,2)-brane (called h in the text) is an Ω invariant configuration of four fractional $(5, 2)$-branes in the orbifold. The twisted R-R charges of each of the fractional branes are shown as $\pm$ next to the fixed points denoted by crosses. The untwisted R-R charge is ± 1 and shown in the middle of each brane. **(b)** A $\mathbf{Z}_2$ charged (5,1)-brane is an Ω invariant configuration of two truncated $(5, 1)$-branes in the orbifold. The twisted R-R charges of each of the $\hat{\text{D}}$-branes are shown as $\pm$ next to the fixed points denoted by crosses. **(c)** In the orbifold cover a stuck (5,0)-(anti-)brane is an Ω-invariant pair of fractional $(5, 0)$-(anti-)branes with opposite twisted R-R charges. Here we show a stuck brane at one fixed point with a stuck anti-brane at the other.

In order to analyse the decay products of these torsion branes a useful way to think about them is as Ω invariant bound states of fractional or truncated branes in the $\mathcal{I}_4$ orbifold. Consider a (5,1)-brane stretching in the x^5 direction for example. In the orbifold this is an Ω invariant pair of $\hat{\mathrm{D}}(5,1)$-branes with opposite twisted R-R charges at both fixed points, as can be seen in Figure 5 (b). Each of the $(5,1)$-branes is unstable for $R_5 \geq \sqrt{2}$ and $R_i \leq 1/\sqrt{2}$ $(i = 6,7,8)$. For $R_5 \geq \sqrt{2}$ each of the truncated branes decay into a brane-anti-brane pair of fractional $(5,0)$-branes at the two fixed points as is shown in figure 5 (c). The decay products are Ω invariant as well and form a stuck $(5,0)$-brane at one fixed point and a stuck $(5,0)$-anti-brane at the other. At the fixed planes in the GP orientifold there will typically be n_1, n_2 tadpole-canceling (5,0)-branes, which will combine with the brane-anti-brane pair from the decay of the $(5,1)$-brane to give a BPS tadpole canceling configuration of n_1+1, n_2-1 (5,0)-branes. This process looks very much like the brane transfer of [31]. Each of the truncated (5,1)-branes also decays for $R_i \leq 1/\sqrt{2}$ into a $\mathrm{D}\bar{\mathrm{D}}(5,2)$-pair as shown in Figure 5 (a). Again the decay configuration is Ω invariant and forms a $\mathbf{Z}_2$ torsion charged $(5,2)$-brane in the orientifold. By T-duality a similar argument can be made for the $s=3$ brane decaying into an $s=2$ brane as well as into a brane-anti-brane pair of stuck $(5,4)$-branes. Thus by considering the branes in the orbifolded cover we have related their torsion charges to one another.

5 Open strings and Chan-Paton factors

Many D-branes we have encountered throughout this paper can be thought of as bound states of branes from the $\mathcal{I}_4$ orbifold which are Ω invariant. These branes have non-trivial Chan-Paton factors on which the orientifold group as well as the GSO projection have to form, up to phases, a representation. Any such phases must be compensated for by opposite phases on the world-sheet fields. This was already encountered in [19, 32]. In particular in [19] it was argued that $\Omega^2 = -1$ on world-sheet fields for open strings stretching between the D9- and D5-branes. To cancel this phase the representation of Ω on Chan-Paton factors of the D5-brane is anti-symmetric giving an Sp instead of an SO gauge group on the D5-branes. In this section we discuss several cases where such phases enter the representation of the orientifold group and GSO projection. We first consider the $\mathbf{Z}_2$ D7-brane of Type I which is an Ω invariant pair of Type IIB D7-branes of opposite R-R charge. We show that Ω and $(-1)^F$ form a projective representation of $\mathbf{Z}_2 \times \mathbf{Z}_2$ on the Chan-Paton factors, and discuss how this phase is compensated for. In the second part of this section we show how this discussion generalises to certain branes in the GP model.

5.1 The D7-brane of Type I and projective representations of $\Omega \times (-1)^F$

The D7-brane of Type I is an Ω invariant bound state of D7-branes with opposite bulk R-R charge.[20] The Chan-Paton factors form a 2×2 matrix

$$\lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix} , \tag{5.1}$$

where a, d is the open string with both endpoints on the brane, anti-brane, respectively and b, c correspond to the open strings that stretch between the brane and the anti-brane. The GSO projection on a and d is $(1+(-1)^F)/2$, while on b and c it is $(1-(-1)^F)/2$. This can be compactly expressed as

$$\gamma_{(-1)^F} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{5.2}$$

and

$$(-1)^F : (\lambda) \to \gamma_{(-1)^F} \lambda \gamma^{-1}_{(-1)^F} = \begin{pmatrix} a & -b \\ -c & d \end{pmatrix} . \tag{5.3}$$

We want to find the representation of Ω on the Chan-Paton factors which together with $\gamma_{(-1)^F}$ should, up to $U(1)$ phases form a $\mathbf{Z}_2 \times \mathbf{Z}_2$ representation. These $U(1)$ phases will have to be cancelled by opposite phases on the world-sheet fields such that on the full open string Hilbert space we get a proper representation.[21] Thus

$$\gamma_{(-1)^F} \gamma_\Omega = e^{i\theta} \gamma_\Omega \gamma_{(-1)^F} , \tag{5.4}$$

for some θ. As a result γ_Ω has to be either

$$\begin{pmatrix} * & 0 \\ 0 & * \end{pmatrix} , \tag{5.5}$$

or

$$\begin{pmatrix} 0 & * \\ * & 0 \end{pmatrix} . \tag{5.6}$$

The one-loop partition function for an open string ending on the D7-brane was computed in [6] where one finds that the full Möbius strip amplitude with and without $(-1)^F$ inserted come with the same sign. On the world-sheet fields the two traces come with opposite signs due to the action of $(-1)^F$ on the vacuum. The Chan-Paton traces must compensate

$$\mathrm{tr}(\gamma_\Omega^T \gamma_\Omega^{-1}) = -\mathrm{tr}(\gamma^T_{\Omega(-1)^F} \gamma^{-1}_{\Omega(-1)^F}) , \tag{5.7}$$

[20] A similar analysis applies to the D-instanton of Type I.

[21] This is needed to show that $(1+\Omega)/2$ and $(1+(-1)^F)/2$ are genuine projection operators.

and so γ_Ω is off-diagonal (5.6).[22]

In order to see that this forms a projective representation rather than a proper representation we consider an open string stretching between the D7-brane and a brane on which Ω and $(-1)^F$ form a proper representation on the Chan-Paton sector. For simplicity we consider a single D-string which has trivial Chan-Paton factors. For a D7-D1 (D1-D7) string the Chan-Paton factors form a two dimensional column (row) vector. Ω exchanges a D7-D1 string with a D1-D7 string and vice-versa. It is then straightforward to check that for a D7-D1 string

$$\lambda^T \gamma_\Omega^{-1} \gamma_{(-1)^F}^{-1} = -(\gamma_{(-1)^F}\lambda)^T \gamma_\Omega^{-1}\,, \tag{5.8}$$

where λ is the Chan-Paton matrix for the D7-D1 string; a similar result holds for the D1-D7 string. In other words on Chan-Paton factors of open strings stretching between the D7- and D1- branes we have

$$\Omega(-1)^F = -(-1)^F\Omega\,. \tag{5.9}$$

On the full open string Hilbert space Ω and $(-1)^F$ commute and form a proper representation. At first it would appear that Ω and $(-1)^F$ should anti-commute on the world-sheet fields; in other words that Ω should be fermionic! On the other hand Ω maps the 17 sector to the 71 sector and so $(-1)^F$ on the left-hand side of equation (5.9) acts on a different sector to $(-1)^F$ on the right-hand side. By having a relative minus sign between the two sectors' $(-1)^F$'s, Ω and $(-1)^F$ will commute on the full Hilbert space. Since the 17 and 71 strings have $4k+2$ zero modes the action of $(-1)^F$ on them is only well defined up to a sign and so $(-1)^F$ can indeed be defined with an extra minus sign in the 17 sector relative to the 71 sector.

5.2 Chan-Paton factors and projective representations in the GP orientifold

The results of the last subsection easily generalise to the branes we have been constructing in the GP orientifold. In particular for the (5,2)-branes with twisted NS-NS couplings $\Omega \times (-1)^F$ also form a projective representation on Chan-Paton factors, while I_4 acts trivially. Similarly, since the truncated $(-1,0)$- and $(-1,4)$-branes are Ω invariant fractional brane-anti-brane pairs they too have non-trivial Chan-Paton factors. It is easy to see that on these both $(-1)^F$ and $\mathcal{I}_4$ act as

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{5.10}$$

From the Möbius strip diagrams in Appendix C, as in the previous subsection, one finds that Ω has an off-diagonal form. As above this gives a projective representation on the CP factors.

[22] This off-diagonal form of Ω is consistent with the geometric picture [5] where it is argued that Ω interchanges the a, d open strings and leaves invariant (up to phase) the b, c ones.

The phases encountered here have to be cancelled by opposite phases on the world-sheet fields if the orientifold $\times$ GSO group is to form a proper representation on the full open string Hilbert space. As in the previous subsection the only way that this can be done for a bosonic Ω is by having $(-1)^F$ and $\mathcal{I}_4$ acting on pp' strings with an extra relative minus sign as compared with the $p'p$ strings. Here p is the brane with a projective representation and p' is a brane with a proper representation of the orientifold $\times$ GSO group. In all cases there are $4k+2$ fermionic zero modes and so the action of $(-1)^F$ and $\mathcal{I}_4$ is again only well defined up to an overall sign.

6 Some comments on K-theory

D-branes can be viewed as bundles over sub-manifolds of spacetime. Physically one is only interested in isomorphism classes of these and, since brane-anti-brane annihilation is allowed, the classes of objects we are interested are best described by K-groups. Depending on the exact nature of the allowed bundles one may study many different types of K-groups: in Type II theories complex K-groups, K^*, in Type I orthogonal K-groups, KO^* and in orbifolds equivariant K-groups K^*_G [5] (see also [34]). It would be natural to conjecture that Type I orbifolds would be described by equivariant, orthogonal K-groups, KO^*_G. Yet we have seen that if the orientifold group $\Omega \times G$ admits projective representations (as can be seen by computing $H^2(\mathbf{Z}_2 \times G, U(1))$) there is a choice in picking the action of Ω on certain twisted sectors. In the case studied explicitly here there are two inequivalent theories with either a tensor or hypermultiplet in each twisted sector. As a result the D-brane spectrum is very different in the two theories, and so one expects two different K-groups. One of these will have to be $KO_{\mathbf{Z}_2}$ while the other will be a twisted version, $KO^{[c]}_{\mathbf{Z}_2}$ of this group in which the non-trivial two-cocycle c of $H^2(\mathbf{Z}_2 \times \mathbf{Z}_2, U(1)) = \mathbf{Z}_2$ is used to twist. Such a group, to the best of our knowledge, has unfortunately not been considered in the mathematical literature.

It is immediate that

$$KO_{\mathbf{Z}_2}(\mathrm{pt}) = \mathbf{Z} \oplus \mathbf{Z} \tag{6.1}$$

and we may conclude that there should be two charges associated with the $(5,4)$-brane. This happens in the DPBZ model where twisted R-R six-form potentials exist and the $(5,4)$-branes in the non-compact theory carry an untwisted as well as a twisted R-R charge giving $\mathbf{Z} \oplus \mathbf{Z}$. In the GP model on the other hand the $(5,4)$-branes are only charged under the untwisted R-R charge. Clearly then it is the DPBZ model which should be described by $KO_{\mathbf{Z}_2}$ while the GP model should be described by $KO^{[c]}_{\mathbf{Z}_2}$ for which we expect

$$KO^{[c]}_{\mathbf{Z}_2} = \mathbf{Z}\,. \tag{6.2}$$

The presence of twisted K-theories is related to the presence of a NS-NS flux [5]. Such a flux in orientifold theories changes the charge of an orientifold plane [33]. In our case the difference between GP and DPBZ comes precisely from the two types of O5-planes present in the theories.

The relationship between $KO_{\mathbf{Z}_2}$ and $KO^{[c]}_{\mathbf{Z}_2}$ is expected to be analogous to the relationship between $K_{\mathbf{Z}_2\times\mathbf{Z}_2}$ and $K^{[c]}_{\mathbf{Z}_2\times\mathbf{Z}_2}$. These last two K-theories describe D-branes in Type II on the $\mathbf{Z}_2\times\mathbf{Z}_2$ orbifold [23] with the twisted theory corresponding to the orbifold with discrete torsion. In $K^{[c]}_{\mathbf{Z}_2\times\mathbf{Z}_2}$ the two generators of $\mathbf{Z}_2\times\mathbf{Z}_2$ anticommute on the fibers, since they form a projective representation of the orbifold group. For $KO^{[c]}_{\mathbf{Z}_2}$ it is similarily expected that complex conjugation and the generator of $\mathbf{Z}_2$ should anticommute.

In the above we have restricted ourselves to a discussion of the orientifold group $\Omega\times\mathcal{I}_4$. It is however immediate that for any orientifold group $\Omega\times G$, where G has order 2 elements, the action of Ω on the respective twisted sectors will be ambigous.[24] As a result there should be twisted KO-theories corresponding to the various Ω actions on the twisted sectors. For example the for the $\Omega\times\mathbf{Z}_3$ orientifold Ω has a unique action on the twisted sectors and so we expect that $KO_{\mathbf{Z}_3}$ should describe D-branes in this background. However, for the $\Omega\times\mathbf{Z}_2\times\mathbf{Z}_2$ orientifold, as well as $KO_{\mathbf{Z}_2\times\mathbf{Z}_2}$ we expect three twisted K-theories $KO^{c_i}_{\mathbf{Z}_2\times\mathbf{Z}_2}$, $i=1,2,3$ corresponding to the four different choices of Ω actions on the twisted sectors.

7 Branes on the DPBZ orientifold

In this section we extend our analysis of the GP model to the DPBZ model. Since the computations are rather similar to the ones discussed above for the GP model we limit ourselves here to a summary of the stable branes in the DPBZ model. The BPS fractional branes are the $(1,0)$- and $(1,4)$-branes as well as the tadpole canceling $(5,0)$- and $(5,4)$-branes.[25] The BPS stuck branes are the $(-1,2)$ and $(3,2)$-branes.

The integrally charged non-BPS truncated branes exist for $r=1,5$ and $s=1,2,3$. There are again two kinds of torsion charged non-BPS branes. The branes that couple to twisted NS-NS sectors have $r=-1,0$ and $s=0$ or $r=3,4$ and $s=4$. However, unlike the GP model, these branes *do not* develop open string tachyons on their world-volumes for any values of the compactification radii (*c.f.* equation (4.2)). The open strings stretching between the $r=3,4$ and $s=4$ torsion branes and the tadpole canceling $(5,4)$-branes have tachyonic ground-states as in the GP model. On the other hand the $(-1,0)$- and $(0,0)$-branes (*i.e.* the D-particle and D-instanton) are stable in the presence of the tadpole canceling branes, with the D-particle carrying a spinor charge of one of the $SO(8)$'s. As in the GP model there are also $\mathbf{Z}_2$-charged branes coupling only to the untwisted NS-NS sector. They exist for $r=-1,0$ and $s=0,1$ as well as for $r=3,4$ and $s=3,4$. Most of these have the usual stability conditions (4.7). Only

[23] For details of various aspects of these orbifolds see [24, 25, 5, 27, 32].

[24] Similar results should hold for orientifolds in which Ω is always combined with some geometric involution, where the description in terms of KR rather than KO and its twisted analogues is more natural.

[25] In order to cancel the R-R twisted sector tadpole, the $(5,0)$- and $(5,4)$-branes have opposite twisted R-R charges.

the $(0,1)$- is stable for $R_i \leq \sqrt{2}$ $(i = 5, \ldots, 4+s)$ and the $(4,3)$-brane is stable for $R_8 \geq 1/\sqrt{2}$ with the branes stretching along $x^5, \ldots, x^{4+s}$ in the internal torus.

8 Conclusions and Outlook

In this paper we have constructed all stable BPS and non-BPS D-branes in the GP and DPBZ orientifolds. As well as the expected BPS branes, these include both integral and torsion charged non-BPS D-branes. We have found that integrally charged non-BPS D-branes come in two forms: either as truncated D-branes similar to the ones encountered in orbifolds [7], or as Ω invariant BPS-anti-BPS pairs of branes. We have also found a rich spectrum of stable torsion charged D-branes. In particular we have found a new class of torsion branes which couple to twisted and untwisted NS-NS sectors. The orientifold theories also have a large number of torsion branes coupling only to the untwisted NS-NS sector. These resemble the Type I-like theory torsion branes [31].

We have identified the stability conditions and decay products of the non-BPS D-branes. Due to the Ω projection the truncated branes have stability regions which are quite different from the orbifold stability regions. The Ω projection also has a significant effect on the torsion branes' stability regions. In the GP model, the stability of the torsion branes coupling to the twisted NS-NS sector resembles those encountered in [27, 28], while in the DPBZ model these branes are in fact always stable.

Many of the branes in orientifolds have non-trivial Chan-Paton factors and we have found that the Orientifold $\times$ GSO group can form projective representations on these. In fact even for the $\mathbf{Z}_2$ D7-brane and D-instanton of Type I $\Omega \times (-1)^F$, form a projective representation on the Chan-Paton factors.

We have discussed the role of K-theory in Type I orbifolds and have found that the equivariant orthogonal K-group KO_G does *not* always give the D-brane spectrum. In particular, if G has order two elements new twisted KO-groups describe the D-brane spectrum. This twisting should be similar to the twisted K-groups used in studying orbifolds of Type II theories with discrete torsion.

It would be interesting to understand better these twisted K-theories, as well as have a better understanding of the charges associated to branes coupling to twisted NS-NS sectors. Further the decay products of the GP-model $(5,2)$-brane with twisted NS-NS couplings have not been identified. Since the brane is really a pair of branes in the cover it may be that the analysis of [28] is applicable. We hope to return to these issues in the future.

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A Coherent states in orientifolds

In this appendix we construct coherent states corresponding to D-branes and O-planes in the GP orientifold. The coherent states have to be invariant under the closed-string GSO projection as well as under Ω and $\mathcal{I}_4$. Ω does not introduce any closed string twisted sectors, hence up to normalisation, the boundary states corresponding to D-branes constructed in [7] need simply be projected by $\frac{1}{2}(1+\Omega)$. This is discussed in Appendix B. The normalisation of the boundary states does change. Firstly, there will be an extra factor of $1/2$ for $\mathcal{N}^2$ coming from the orientation projection on the open strings partition function. Secondly, $\mathcal{N}^2$ will acquire an extra factor of $(\mathrm{Tr}(\gamma(1))^2$, coming from the Chan-Paton degrees of freedom. The boundary states' normalisation coefficients are given in Appendices C and D.

The crosscaps corresponding to O-planes can be constructed by perusal of the Klein-bottle amplitudes $\mathcal{K}$ in the tree channel. In the loop channel

$$\mathcal{K} = \int_0^\infty \frac{\mathrm{d}t}{2t} \mathrm{Tr}^{\mathrm{U+T}}_{\text{NS-NS ,R-R}} \left(\frac{\Omega}{2} \frac{1+(-1)^F}{2} \frac{1+\mathcal{I}_4}{2} e^{-2\pi t H_c} \right) , \tag{A.1}$$

where the trace is over all untwisted and twisted bosonic sectors, H_c is the closed string Hamiltonian, and the GSO projection is only taken in the left moving sector as the states have to be left-right symmetric. Evaluating we get

$$\begin{aligned}
\mathcal{K} &= \frac{V_6}{(2\pi)^6} \frac{1}{8} \int \frac{\mathrm{d}t}{2t} t^{-3} \frac{f_3^8(\tilde{q}^2) - f_4^8(\tilde{q}^2) - f_2^8(\tilde{q}^2)}{f_1^8(\tilde{q}^2)} \prod_{i=5}^{8} \left(\sum_{n_i \in \mathbf{Z}} e^{-\pi t (n_i/R_i)^2} + \sum_{w_i \in \mathbf{Z}} e^{-\pi t (w_i R_i)^2} \right) \\
&\quad + 16 \frac{V_6}{(2\pi)^6} \frac{1}{4} \int \frac{\mathrm{d}t}{2t} t^{-3} \frac{f_3^4(\tilde{q}^2) f_2^4(\tilde{q}^2) - f_2^4(\tilde{q}^2) f_3^4(\tilde{q}^2)}{f_1^4(\tilde{q}^2) f_4^4(\tilde{q}^2)} \\
&= \frac{1}{2} \frac{V_6}{(2\pi)^6} 2^4 \int \mathrm{d}l \frac{f_3^8(q) - f_2^8(q) - f_4^8(q)}{f_1^8(q)} \prod_{i=5}^{8} \left(R_i \sum_{w_i \in 2\mathbf{Z}} e^{-\pi t (w_i R_i)^2} + \frac{1}{R_i} \sum_{n_i \in 2\mathbf{Z}} e^{-\pi t (n_i/R_i)^2} \right) \\
&\quad + 32 \frac{V_6}{(2\pi)^6} \int \mathrm{d}l \frac{f_3^4(q) f_4^4(q) - f_4^4(q) f_3^4(q)}{f_1^4(q) f_2^4(q)} ,
\end{aligned} \tag{A.2}$$

where $\tilde{q} = e^{-\pi t}$, the f_i are defined in [19], $l = 1/(4t)$ is the tree channel modular parameter [19], $q = e^{-2\pi l}$, V_6 is the (infinite) volume of the six non-compact space-time directions, and R_i are the radii of T^4. The two parts of the first integral correspond to 1 and $\mathcal{I}_4$ insertions in the untwisted sector trace, respectively. The second integral comes from the trace over the sixteen twisted sector states. In the tree channel the two terms of the first integral correspond to the exchange between two O9- or two O5-planes, respectively, while the second integral corresponds to the O9-O5 interaction.

In non-compact space-time one defines in each (bosonic) sector of the theory the crosscap state

$$|C(r,s),k,\eta\rangle = \exp\left(\sum_{l>0}^{\infty}\left[\frac{(-1)^l}{l}\alpha^{\mu}_{-l}S_{\mu\nu}\tilde{\alpha}^{\nu}_{-l}\right] + i\eta\sum_{m>0}^{\infty}(-1)^m\left[\psi^{\mu}_{-m}S_{\mu\nu}\tilde{\psi}^{\nu}_{-m}\right]\right)|C(r,s),k,\eta\rangle^{(0)}\,, \tag{A.3}$$

where, depending on the sector, l and m are integer or half-integer, $\eta = \pm 1$, and k denotes the momentum of the ground state. The matrix S is diagonal and has (in Euclidean space-time) entries equal to $-1\,, 1$ for Neumann and Dirichlet boundary conditions, respectively. The zero-mode part of the crosscap state is the same as that of a boundary state, and we refer the reader to Appendix B and [7] for a discussion of these.

O-planes are localised in transverse space, so we Fourier transform the above crosscap state

$$|C(r,s),y,\eta\rangle = \int\left(\prod_{\mu\ \text{transverse}} dk^{\mu}e^{ik^{\mu}y_{\mu}}\right)|C(r,s),k,\eta\rangle\,, \tag{A.4}$$

y denotes the location of the O-plane. Compactifying some directions on circles modifies the zero mode part of the crosscap state. In particular, the momentum integrals become sums over $2n_i/R_i$ ($n_i \in \mathbf{N}$), and in compact Neumann directions the ground state becomes a sum over windings $2w_iR_i$ ($w_i \in \mathbf{N}$). The momenta and windings are even so as to match with the momentum and winding sums in the tree channel of equation (A.2).

As in the case of D-branes, closed string GSO invariance combines the $\eta = +, -$ crosscaps in each sector to give one state per closed string sector

$$|C(r,s)\rangle_{\text{NS-NS}} = \frac{1}{2}\Big(|C(r,s),+\rangle_{\text{NS-NS}} - |C(r,s),-\rangle_{\text{NS-NS}}\Big)\,, \tag{A.5}$$

$$|C(r,s)\rangle_{\text{R-R}} = \frac{4i}{2}\Big(|C(r,s),+\rangle_{\text{R-R}} + |C(r,s),-\rangle_{\text{R-R}}\Big)\,. \tag{A.6}$$

From the tree channel expression in equation (A.2) one can read off the form as well as normalisations of the GP O9- and O5-planes

$$\begin{aligned} |O(5,4)\rangle &= \mathcal{N}_9\left(|C(5,4)\rangle_{\text{NS-NS}} + |C(5,4)\rangle_{\text{R-R}}\right) \\ |O(5,0),\mathbf{y}\rangle &= \mathcal{N}_5\left(|C(5,0),\mathbf{y}\rangle_{\text{NS-NS}} + |C(5,0),\mathbf{y}\rangle_{\text{R-R}}\right)\,, \end{aligned} \tag{A.7}$$

where $\mathbf{y} = (y_5, \ldots, y_8)$ with $y_i = 0, \pi R_i$ fixing the location of the O5-plane to one of the sixteen fixed points of $T^4/\mathbf{Z}_2$. The normalisation constants are negative and given by[26]

$$\mathcal{N}^2_{\rm C9} = \frac{V_6}{(2\pi)^6} 8 \prod_{i=5}^{8} R_i \,, \qquad \mathcal{N}^2_{\rm C5} = \frac{V_6}{(2\pi)^6} \frac{1}{32} \prod_{i=5}^{8} \frac{1}{R_i} \,. \tag{A.8}$$

Note that, due to the sum over *even* momenta, each of the 16^2 O5-plane diagrams is the same. With the above definitions one may easily check that

$$\mathcal{K} = \int_0^\infty {\rm d}l \left(\langle C(5,4)| + \sum_{i=1}^{16} \langle C(5,0), \mathbf{y}_i| \right) e^{-lH_c} \left(|C(5,4)\rangle + \sum_{i=1}^{16} |C(5,0), \mathbf{y}_i\rangle \right) , \tag{A.9}$$

with $\mathbf{y}_i$ labeling the position of the sixteen fixed points.

B Ω invariance

In this appendix we study the Ω invariance of boundary and crosscap states. Defining the action of Ω on closed string states as in [19]

$$\Omega \alpha_r \Omega^{-1} = \tilde{\alpha}_r \,, \qquad \Omega \psi_r \Omega^{-1} = \tilde{\psi}_r \,, \qquad \Omega \tilde{\psi}_r \Omega^{-1} = \psi_r \,, \tag{B.1}$$

it is easy to see that the non-zero mode part of the boundary and crosscap states is Ω invariant. As in the case of orbifolds [7] constraints on r and s come from the analysis of the fermionic zero-modes. The various boundary states have to couple consistently to closed string states. For example in Type IIB the R-R sector boundary state has to couple to even R-R potentials. This in effect fixes the zero-mode part of the boundary state and is consistent with GSO-invariance. Let us see how this happens for Ω. In the untwisted R-R sector the Ω projection is represented on the zero-modes by[27]

$$\Omega_{\text{R-R}} = \kappa_{\text{R-R}} \prod_{i=1}^{8} \frac{1 - 2\psi_0^i \tilde{\psi}_0^i}{\sqrt{2}} \,. \tag{B.2}$$

This satisfies the relation $\Omega^2 = (-1)^{F+\tilde{F}}$, with $(-1)^F$ and $(-1)^{\tilde{F}}$ defined as in [4] for $\kappa^2_{\text{R-R}} = 1$. From this one obtains

$$\Omega |B(r,s), \pm\rangle^{(0)}_{\text{R-R}} = \kappa_{\text{R-R}}\, i^{11-p} |B(r,s), \pm\rangle^{(0)}_{\text{R-R}} \,. \tag{B.3}$$

Since we expect D(r,s)-branes with $p = r + s = 1, 5, 9$ to survive[28] we pick $\kappa_{\rm RR} = -1$. The crosscap states' zero-mode part is the same as that of a boundary state, so the above holds for it

[26] The fact that they are negative is to be expected; after all the O9 and O5-planes have negative tension and R-R charge. The sign of the normalisation constants follows from the Möbius diagram.

[27] For simplicity we work in the light-cone gauge here.

[28] This must be the case as the R-R potentials present in Type I are $C^{(2)}$ and $C^{(10)}$.

as well. In the twisted R-R sector Ω is defined as above with $i = 1, \ldots, 4$ and a different constant $\kappa_{\text{R-R,T}}$ (which also squares to one). When acting on the boundary state

$$\Omega|B(r,s),\pm\rangle^{(0)}_{\text{R-R,T}} = \kappa_{\text{R-R,T}}\, i^{5-r}|B(r,s),\pm\rangle^{(0)}_{\text{R-R,T}} \,. \tag{B.4}$$

In the GP orientifold the R-R,T 6-form and 2-form are removed by Ω. This fixes $\kappa_{\text{R-R,T}} = -1$. Finally, in the twisted NS-NS sector in the definition of Ω $i = 5, \ldots, 8$, and we have

$$\Omega|B(r,s),\pm\rangle^{(0)}_{\text{NS-NS,T}} = \kappa_{\text{NS-NS,T}}\, i^{6-s}|B(r,s),\pm\rangle^{(0)}_{\text{NS-NS,T}} \,. \tag{B.5}$$

Comparing to the massless spectrum fixes $\kappa_{\text{NS-NS,T}} = 1$. We have fixed κ in the above such that the boundary states couple to closed string states which are present in the theory. As a result we expect that the action of Ω defined in this way corresponds to the one used in [19]. In summary the following boundary states are Ω invariant

$$\begin{array}{ll} |B(r,s)\rangle_{\text{NS-NS}} & \text{for all } (r,s)\,, \\ |B(r,s)\rangle_{\text{R-R}} & r+s = 1,5,9\,, \\ |B(r,s)\rangle_{\text{NS-NS ,T}} & s = 2\,, \\ |B(r,s)\rangle_{\text{R-R ,T}} & r = -1,3\,, \end{array} \tag{B.6}$$

in agreement with [26].

C Normalisations and partition functions of integrally charged branes

In this appendix we compute the one-loop partition function for open strings that end on truncated branes so as to verify that they are tachyon free for certain ranges of the compactification radii. By comparing with the open string one-loop partition functions we also obtain the normalisation of these boundary states corresponding to the branes. The partition functions are computed in the tree channel and modular transformed to the loop channel *i.e.* the open string partition function. The relevant diagrams have the topologies of the annulus and Möbius strip

$$\mathcal{A} = \int_0^\infty \mathrm{d}l \langle \hat{\mathrm{D}}(r,s)|e^{-lH_c}|\hat{\mathrm{D}}(r,s)\rangle \,, \tag{C.1}$$

$$\mathcal{M}_9 = \int_0^\infty \mathrm{d}l \langle C(5,4)|e^{-lH_c}|\hat{\mathrm{D}}(r,s)\rangle + \langle \hat{\mathrm{D}}(r,s)|e^{-lH_c}|C(5,4)\rangle \,, \tag{C.2}$$

$$\mathcal{M}_5 = \int_0^\infty \mathrm{d}l \sum_{i=1}^{16} \left(\langle C(5,0),\mathbf{y}_i|e^{lH_c}|\hat{\mathrm{D}}(r,s)\rangle + \langle \hat{\mathrm{D}}(r,s)|e^{-lH_c}|C(5,0),\mathbf{y}_i\rangle \right) \,, \tag{C.3}$$

with $|\hat{\mathrm{D}}(r,s)\rangle$ defined in equation (3.1). Evaluating

$$\begin{aligned}
\mathcal{A} &= \frac{1}{2}\mathcal{N}_{(r,s),U}^2\int_0^\infty \mathrm{d}l\, l^{(r-5)/2}\frac{f_3^8(q)f_4^8(q)}{f_1^8(q)}\prod_{i=5}^{4+s}\sum_{w_i\in\mathbf{Z}} e^{-\pi l(w_iR_i)^2}\prod_{j=5+s}^{8}\sum_{n_j\in\mathbf{Z}} e^{-\pi l(n_j/R_j)^2} \\
&\quad - 2^{s-1}\mathcal{N}_{(r,s),T}^2\int_0^\infty \mathrm{d}l\, l^{(r-5)/2}\frac{f_2^4(q)f_3^4(q)}{f_1^4(q)f_4^4(q)}\,, \qquad \text{(C.4)}\\
\mathcal{M}_9 &= \mathcal{N}_{(r,s),U}\mathcal{N}_{\mathrm{C9}}\int_0^\infty \mathrm{d}l\frac{f_3^{r+s-1}(iq)f_4^{9-r-s}(iq)-f_4^{r+s-1}(iq)f_3^{9-r-s}(iq)}{f_1^{r+s-1}(iq)f_2^{9-r-s}(iq)2^{(r+s-9)/2}}\prod_{i=5}^{4+s}\sum_{w_i\in 2\mathbf{Z}} e^{-\pi l(w_iR_i)^2}\,, \qquad \text{(C.5)}\\
\mathcal{M}_5 &= 16\mathcal{N}_{(r,s),U}\mathcal{N}_{\mathrm{C5}}\int_0^\infty \mathrm{d}l\frac{f_3^{3+r-s}(iq)f_4^{5-r+s}(iq)-f_4^{3+r-s}(iq)f_3^{5-r+s}(iq)}{f_1^{3+r-s}(iq)f_2^{5-r+s}(iq)2^{(r-s-5)/2}}\prod_{j=5+s}^{8}\sum_{n_j\in 2\mathbf{Z}} e^{-\pi l(n_j/R_j)^2}\,, \qquad \text{(C.6)}
\end{aligned}$$

modular transforming to the open channel these become[29]

$$\begin{aligned}
\mathcal{A} &= \mathcal{N}_{(r,s),U}^2 2^4\frac{\prod_{j=5+s}^8 R_j}{\prod_{i=5}^{4+s}R_i}\int_0^\infty\frac{\mathrm{d}t}{2t}(2t)^{-(r+1)/2} \\
&\qquad\qquad \times\ \frac{f_3^8(\tilde{q})-f_2^8(\tilde{q})}{f_1^8(\tilde{q})}\prod_{i=5}^{4+s}\sum_{n_i\in\mathbf{Z}} e^{-2\pi t(n_i/R_i)^2}\prod_{j=5+s}^{8}\sum_{w_j\in\mathbf{Z}} e^{-2\pi t(w_jR_j)^2} \\
&\quad - \mathcal{N}_{(r,s),T}^2 2^{s+2}\int_0^\infty\frac{\mathrm{d}t}{2t}(2t)^{-(r+1)/2}\frac{f_4^4(\tilde{q})f_3^4(\tilde{q})}{f_1^4(\tilde{q})f_2^4(\tilde{q})}\,, \qquad \text{(C.7)}\\
\mathcal{M}_9 &= \frac{\mathcal{N}_{(r,s),U}\mathcal{N}_{\mathrm{C9}}}{2\prod_{i=5}^{4+s}R_i}\int_0^\infty\frac{\mathrm{d}t}{2t}(2t)^{-(r+1)/2}\prod_{i=5}^{4+s}\sum_{n_i\in\mathbf{Z}} e^{-2\pi t(n_i/R_i)^2} \\
&\quad\times\ \frac{e^{i\frac{\pi}{4}(r+s-5)}f_4^{r+s-1}(i\tilde{q})f_3^{9-r-s}(i\tilde{q})-e^{-i\frac{\pi}{4}(r+s-5)}f_3^{r+s-1}(i\tilde{q})f_4^{9-r-s}(i\tilde{q})}{f_1^{r+s-1}(i\tilde{q})f_2^{9-r-s}(i\tilde{q})2^{(r+s-9)/2}}\,, \qquad \text{(C.8)}\\
\mathcal{M}_5 &= 8\mathcal{N}_{(r,s),U}\mathcal{N}_{\mathrm{C5}}\prod_{j=5+s}^{8}R_j\int_0^\infty\frac{\mathrm{d}t}{2t}(2t)^{-(r+1)/2}\prod_{j=5+s}^{8}\sum_{w_j\in\mathbf{Z}} e^{-2\pi t(w_jR_j)^2} \\
&\quad\times\ \frac{e^{i\frac{\pi}{4}(r-s-1)}f_4^{3+r-s}(i\tilde{q})f_3^{5-r+s}(i\tilde{q})-e^{-i\frac{\pi}{4}(r-s-1)}f_3^{3+r-s}(i\tilde{q})f_4^{5-r+s}(i\tilde{q})}{f_1^{3+r-s}(i\tilde{q})f_2^{5-r+s}(i\tilde{q})2^{(r-s-5)/2}}\,. \qquad \text{(C.9)}
\end{aligned}$$

With the normalisations in Table 32 as well as the normalisations of crosscaps in Appendix A it is easy to see that the open string groundstate tachyon cancels for $s=0\,,4$ and all r and for

[29]The annulus and Möbius strip modular transformations are respectively $t=1/(2l)$ and $t=1/(8l)$.

$r = -1, 3$ and all s. As discussed in section 3 $\hat{\text{D}}$-branes exist only for $r = -1, 3$ and $s \neq 2$. Note that the tachyon ground-state cancellation is independent of the overall normalisation of the $\hat{\text{D}}$-brane boundary states confirming that the $\hat{\text{D}}$-branes are indeed integrally charged.

The normalisation of the boundary states can be obtained most easily by comparing with the one-loop partition functions computed in the loop channel. Particular attention needs to be paid to the Chan-Paton factors which will affect the normalisations - for example $\mathcal{N}_U$ will be proportional to the trace of the identity operator, tr(1), on the Chan-Paton states. We summarise the normalisations of truncated as well as fractional and stuck branes in Table 32.

(r, s)	(-1,0), (1,0), (-1,4), (3,2), (1,4), (5,0), (5,4)	(-1,1), (-1,3)	(3,0), (3,4)	(3,1), (3,3)	(-1,2)
n^2	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{128}$

Table 1: Normalisations of boundary states for minimally, integrally charged (r, s)-branes. n is related to the normalisations via $\mathcal{N}^2_{U,(r,s)} = \frac{V_{r+1}}{(2\pi)^{r+1}} n^2 \frac{\prod_{i=1}^{s} R_{j_i}}{\prod_{i=s+1}^{4} R_{k_i}}$ and $\mathcal{N}^2_{T,(r,s)} = \frac{V_{r+1}}{(2\pi)^{r+1}} 2^{4-s} n^2$.

Finally, expanding to suitable order in windings and momenta the stability conditions for $\hat{\text{D}}$-branes are found to match those given in Section 3.

D Normalisations and partition functions of torsion charged branes

In this section we compute the one loop partition functions for the torsion-charged branes which couple to the twisted NS-NS sectors[30]

$$\mathcal{A} = \int_0^\infty \mathrm{d}l \langle D(r,2)|e^{-lH_c}|D(r,2)\rangle \,, \tag{D.1}$$

$$\mathcal{M}_9 = \int_0^\infty \mathrm{d}l \langle C(5,4)|e^{lH_c}|D(r,2)\rangle + \langle D(r,2)|e^{-lH_c}|C(5,4)\rangle \,, \tag{D.2}$$

$$\mathcal{M}_5 = \int_0^\infty \mathrm{d}l \sum_{i=1}^{16} \langle C(5,0), \mathbf{y}_i|e^{-lH_c}|D(r,2)\rangle + \langle D(r,2)|e^{-lH_c}|C(5,0), \mathbf{y}_i\rangle \,, \tag{D.3}$$

and modular transform to the loop-channel, from which we may read of the values of r and the normalisation of the boundary state for which the ground-state tachyon cancels. Evaluating the

[30]For definiteness we consider the brane to be stretching along x^5 and x^6.

amplitudes[31]

$$\begin{aligned}
\mathcal{A} &= \frac{1}{2}\mathcal{N}_U^2\int_0^\infty \mathrm{d}l l^{(r-5)/2}\frac{f_3^8(q)-f_4^8(q)}{f_1^8(q)}\prod_{i=5,6}\sum_{w_i\in\mathbf{Z}} e^{-\pi l(w_iR_i)^2}\prod_{j=7,8}\sum_{n_j\in\mathbf{Z}} e^{-\pi l(n_j/R_j)^2} \\
&\quad + 2\mathcal{N}_T^2\int_0^\infty \mathrm{d}l l^{(r-5)/2}\frac{f_3^4(q)f_2^4(q)}{f_1^4(q)f_4^4(q)}\,, \qquad \text{(D.4)}\\
\mathcal{M}_9 &= \mathcal{N}_U\mathcal{N}_{\mathrm{C9}}\int_0^\infty \mathrm{d}l\frac{f_3^{r+1}(iq)f_4^{7-r}(iq)-f_4^{r+1}(iq)f_3^{7-r}(iq)}{f_1^{r+1}(iq)f_2^{7-r}(iq)2^{(r-7)/2}}\prod_{i=5,6}\sum_{w_i\in 2\mathbf{Z}} e^{-\pi l(w_iR_i)^2}\,, \qquad \text{(D.5)}\\
\mathcal{M}_5 &= 16\mathcal{N}_U\mathcal{N}_{\mathrm{C5}}\int_0^\infty \mathrm{d}l\frac{f_3^{r+1}(iq)f_4^{7-r}(iq)-f_4^{r+1}(iq)f_3^{7-r}(iq)}{f_1^{r+1}(iq)f_2^{7-r}(iq)2^{(r-7)/2}}\prod_{j=7,8}\sum_{n_j\in 2\mathbf{Z}} e^{-\pi l(n_j/R_j)^2}\,, \qquad \text{(D.6)}
\end{aligned}$$

and performing the modular transformation

$$\begin{aligned}
\mathcal{A} &= \frac{2^4\mathcal{N}_U^2R_7R_8}{R_5R_6}\int_0^\infty\frac{\mathrm{d}t}{2t}(2t)^{-(r+1)/2}\frac{f_3^8(\tilde{q})-f_2^8(\tilde{q})}{f_1^8(\tilde{q})}\prod_{i=5,6}\sum_{n_i\in\mathbf{Z}} e^{-2\pi t(n_i/R_i)^2}\prod_{j=7,8}\sum_{w_j\in\mathbf{Z}} e^{-2\pi t(w_jR_j)^2} \\
&\quad + 2^4\mathcal{N}_T^2\int_0^\infty\frac{\mathrm{d}t}{2t}(2t)^{-(r+1)/2}\frac{f_3^4(\tilde{q})f_4^4(\tilde{q})}{f_1^4(\tilde{q})f_2^4(\tilde{q})}\,, \qquad \text{(D.7)}\\
\mathcal{M}_9 &= \frac{\mathcal{N}_U\mathcal{N}_{\mathrm{C9}}}{2R_5R_6}\int_0^\infty\frac{\mathrm{d}t}{2t}(2t)^{-(r+1)/2} \\
&\quad\times\frac{e^{i\frac{\pi}{4}(r-3)}f_4^{r+1}(i\tilde{q})f_3^{7-r}(i\tilde{q})-e^{-i\frac{\pi}{4}(r-3)}f_3^{r+1}(i\tilde{q})f_4^{7-r}(i\tilde{q})}{f_1^{r+1}(i\tilde{q})f_2^{7-r}(i\tilde{q})2^{(r-7)/2}}\prod_{i=5,6}\sum_{n_i\in\mathbf{Z}} e^{-2\pi t(n_i/R_i)^2}\,, \qquad \text{(D.8)}\\
\mathcal{M}_5 &= 8\mathcal{N}_U\mathcal{N}_{\mathrm{C5}}R_7R_8\int_0^\infty\frac{\mathrm{d}t}{2t}(2t)^{-(r+1)/2} \\
&\quad\times\frac{e^{i\frac{\pi}{4}(r-3)}f_4^{r+1}(i\tilde{q})f_3^{7-r}(i\tilde{q})-e^{-i\frac{\pi}{4}(r-3)}f_3^{r+1}(i\tilde{q})f_4^{7-r}(i\tilde{q})}{f_1^{r+1}(i\tilde{q})f_2^{7-r}(i\tilde{q})2^{(r-7)/2}}\prod_{j=7,8}\sum_{w_j\in\mathbf{Z}} e^{-2\pi t(w_jR_j)^2}\,. \qquad \text{(D.9)}
\end{aligned}$$

With

$$\mathcal{N}_U^2=\frac{V_{r+1}}{(2\pi)^{r+1}}n^2\frac{R_5R_6}{R_7R_8}\,,\qquad \mathcal{N}_T^2=\frac{V_{r+1}}{(2\pi)^{r+1}}4n^2\,, \qquad \text{(D.10)}$$

for some number n the condition for the ground-state tachyon to cancel is

$$32n^2=4\sqrt{2}n\sin(\frac{\pi(r-3)}{4})\,. \qquad \text{(D.11)}$$

Since r is an integer between -1 and 5 one may easily verify that there are only two solutions (for positive $\mathcal{N}_U$)

$$r=4\,,\quad n^2=\frac{1}{64}\qquad\text{or}\qquad r=5\,,\quad n^2=\frac{1}{32}\,. \qquad \text{(D.12)}$$

[31] See equation (4.1)) for a definition of $|D(r,2)\rangle$.

Expanding to suitable order in momentum and winding one may confirm that the stability of these branes is as given in equation (4.2).

A similar computation has been done for D-branes that couple only to the untwisted NS-NS sector, and one finds that for $r = 4, 5$ and $s = 1, 2, 3$ the ground-state tachyon cancels for normalisations listed in Table 34 and the stability of these torsion branes is as given in 4.

(r, s)	(4,1), (4,3)	(4,2), (5,1), (5,3)	(5,2)
n^2	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{8}$

Table 2: Normalisations of boundary states for $\mathbf{Z}_2$ torsion charged D-branes. n is related to the normalisations via $\mathcal{N}^2_{U,(r,s)} = \frac{V_{r+1}}{(2\pi)^{r+1}} n^2 \frac{\prod_{i=1}^{s} R_{j_i}}{\prod_{i=s+1}^{4} R_{k_i}}$.

Note in particular that the $\mathbf{Z}_2$, $s = 2$ torsion branes have the same normalisation as a boundary state corresponding to a bound states of two torsion branes from equation (4.1) with opposite twisted NS-NS couplings.

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