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**Citation:** Garcez, A., Besold, T. R., Raedt, L., Foldiak, P., Hitzler, P., Icard, T., Kuhnberger, K-U., Lamb, L. C., Miikkulainen, R. & Silver, D. L. (2015). Neural-Symbolic Learning and Reasoning: Contributions and Challenges. Paper presented at the 2015 AAAI Spring Symposium Series, 23-03-2015 - 25-03-2015, Stanford University, USA.

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# Neural-Symbolic Learning and Reasoning: Contributions and Challenges

Artur d'Avila Garcez<sup>1</sup>, Tarek R. Besold<sup>2</sup>, Luc de Raedt<sup>3</sup>, Peter Földiák<sup>4</sup>, Pascal Hitzler<sup>5</sup>, Thomas Icard<sup>6</sup>,  
Kai-Uwe Kühnberger<sup>2</sup>, Luis C. Lamb<sup>7</sup>, Risto Miikkulainen<sup>8</sup>, Daniel L. Silver<sup>9</sup>

<sup>1</sup>City University London, UK; <sup>2</sup>Universität Osnabrück, Germany; <sup>3</sup>KU Leuven, Belgium; <sup>4</sup>Univ. of St. Andrews, UK; <sup>5</sup>Wright State University, OH; <sup>6</sup>Stanford University, CA; <sup>7</sup>UFRGS, Brazil; <sup>8</sup>UT Austin, TX; <sup>9</sup>Acadia University, Canada

## Abstract

The goal of neural-symbolic computation is to integrate robust connectionist learning and sound symbolic reasoning. With the recent advances in connectionist learning, in particular deep neural networks, forms of representation learning have emerged. However, such representations have not become useful for reasoning. Results from neural-symbolic computation have shown to offer powerful alternatives for knowledge representation, learning and reasoning in neural computation. This paper recalls the main contributions and discusses key challenges for neural-symbolic integration which have been identified at a recent Dagstuhl seminar.

## 1. Introduction

In order to respond to one of the main challenges of Artificial Intelligence (AI), that is, the effective integration of learning and reasoning (Valiant 2008), both symbolic inference and statistical learning need to be combined in an effective way. However, over the last three decades, statistical learning and symbolic reasoning have been developed largely by distinct research communities in AI (but see below for exceptions). More recently, developments in deep learning have been connected strongly with and have contributed novel insights into representational issues. So far these representations have been *low level*, and have not been integrated with the high-level symbolic representations used in knowledge representation. It is exactly in this area that neural-symbolic learning and reasoning has been relevant for over two decades, having addressed many relevant representational issues, e.g. the binding problem (Feldman, 2013; Sun, 1994). Neural-Symbolic Learning and Reasoning seeks to integrate principles from neural-networks learning and logical reasoning. It is an interdisciplinary field involving components of knowledge representation, neuroscience, machine learning and cognitive science. This note briefly overviews some of the achievements in neural-symbolic computation and outlines some

key challenges and opportunities. These challenges have been identified at a recent Dagstuhl seminar on Neural-Symbolic Learning and Reasoning, in Wadern, Germany (September 2014), which marked the tenth anniversary of the workshop series on Neural-Symbolic Learning and Reasoning, which started at IJCAI 2005 in Edinburgh. For details about the seminar presentations, please visit: <http://www.dagstuhl.de/14381>. For more information about the workshop series, please visit [www.neural-symbolic.org](http://www.neural-symbolic.org). Another area that is relevant to Valiant's challenge is that of statistical relational learning and probabilistic logic learning (Getoor et al., 2007; De Raedt et al., 2007), which aim at integrating probabilistic graphical models rather than connectionist methods with logical and relational reasoning.

The integration of the symbolic and connectionist paradigms of AI has been pursued by a relatively small research community over the last two decades and has yielded several significant results. Over the last decade, neural-symbolic systems have been shown capable of overcoming the so-called *propositional fixation* of neural networks, as McCarthy (1988) put it in response to Smolensky (1988); see also (Hinton, 1990). Neural networks were shown capable of representing modal and temporal logics (d'Avila Garcez and Lamb, 2006) and fragments of first-order logic (Bader, Hitzler, and Hölldobler, 2008; d'Avila Garcez, Lamb, and Gabbay, 2009). Further, neural-symbolic systems have been applied to a number of problems in the areas of bioinformatics, control engineering, software verification and adaptation, visual intelligence, ontology learning, and computer games (Borges, d'Avila Garcez, and Lamb, 2011; de Penning et al., 2011; Hitzler, Bader, and d'Avila Garcez, 2005). Most of the work on knowledge representation and learning in neural networks has focused on variable-free logic fragments. However, one should note that several approaches have dealt with alternative formalizations of variable binding, and the representation of relations (Bader, Hitzler, and Hölldobler, 2008; d'Avila Garcez, Lamb, and Gabbay, 2009; Pinkas, Lima, and Cohen, 2012; Franca, d'Avila Garcez and Zaverucha, 2014).

In deep learning (Hinton, Osindero, and Teh, 2006), the generalization of abstract representations from raw data may be a fundamental objective, but how it happens is not fully understood (Tran and d'Avila Garcez, 2013). Deep architectures seek to manage complex issues of representation abstraction, modularity, and the trade-off between distributed and localist representations. Several techniques developed under the umbrella of neural-symbolic computation can be useful towards this goal. For instance, *fibring neural networks* offer the expression of levels of symbolic abstraction. Connectionist modal logics are modular by construction (d'Avila Garcez, Lamb, and Gabbay, 2007).

In what follows, challenges and opportunities for neural-symbolic integration are outlined, as a summary of the discussions held at the Dagstuhl seminar. In a nutshell: (i) the mechanisms for structure learning remain to be fully understood, whether they consist of hypothesis search at the concept level, including (probabilistic) Inductive Logic Programming (ILP) and statistical AI approaches, or iterative adaptation processes such as Hebbian learning and contrastive divergence; (ii) the learning of generalizations of symbolic rules is a crucial process and not well understood - the adoption of neural networks that can offer degrees of modularity, such as deep networks, and the neural-symbolic methods for knowledge insertion and extraction from neural networks may help shed light into this question; (iii) effective knowledge extraction from large-scale networks remains a challenge - computational complexity issues and the provision of compact, expressive descriptions continue to be a barrier for explanation, lifelong learning and transfer learning. Items (i)-(iii) above open up a number of research opportunities, to be discussed next.

## 2. State-of-The-Art Results and Challenges

**Representation:** Most of the work on neural-symbolic learning and reasoning has focused on propositional logics. Early approaches were based essentially on the connectionist representation of propositional logic, a line of research which has since been substantially extended to other finitary logics (d'Avila Garcez, Lamb, and Gabbay 2009).

Some primary proposals for overcoming the propositional fixation of neural networks include the representation of variable-free predicate logic within neural networks using category-theoretic Topoi (Gust, Kühnberger, and Geibel, 2007), the use of encodings of predicate Horn logic programs with function symbols as vectors of real numbers mediated by the Cantor set (Bader, Hitzler, and Hölldobler, 2008), and learning first-order logic rules within neural networks by using an encoding of logical terms also as vectors (Guillame-Bert, Broda, and d'Avila Garcez, 2010). These systems have been shown to work in limited proof-of-concept settings or small examples, and attempts to achieve useful performance in practice have so far not been successful.

In order to make further progress, it may be necessary to consider logics of intermediate expressiveness, such as (a) description logics (DL), in particular logics in the Horn DL family (Krötzsch, Rudolph, and Hitzler, 2013), (b) propositionalization methods, as used by ILP (Blockeel et. al, 2011; França, Zaverucha, and d'Avila Garcez, 2014) and answer-set programming (Lifschitz, 2002), and (iii) modal logics (d'Avila Garcez, Lamb, and Gabbay, 2007), known to be more expressive than propositional logic and decidable. In particular, recent results regarding the integration of DL and rules (Krisnadhi, Maier, and Hitzler, 2011, Krötzsch et al., 2011) indicate the feasibility of representing DL within a neural-symbolic system (Hitzler, Bader, and d'Avila Garcez, 2005). The variable binding problem, though, and the question of how neural networks should reason with variables remain central to the question of adequate representation (d'Avila Garcez, Broda, and Gabbay 2002; Feldman, 2006; Pinkas, Lima, and Cohen, 2012).

Along with the efforts towards the representation of expressive logics within neural networks there has been work on the extraction of logical expressions such as logic programs or decision trees from trained neural networks (Craven and Shavlik, 1996; d'Avila Garcez, Broda, and Gabbay 2001, Lehmann, Bader, and Hitzler, 2010; Tran and d'Avila Garcez, 2013), including the use of such extracted knowledge to seed learning in other tasks. Meanwhile, there has been some suggestive recent work showing that neural networks can learn entire sequences of actions, thus amounting to "mental simulation" of some concrete, temporally extended activity. There is also a very well developed logical theory of action, for instance related to the basic propositional logic of programs PDL (Harel, Kozen, and Tiuryn, 2001), capturing what holds true after various combinations of actions. A natural place to extend the aforementioned work would be to explore extraction from a trained network exhibiting this kind of simulation behavior. As argued by Feldman (2006), if the brain is not a network of neurons that represent things, but a network of neurons that do things, action models should be playing a central role.

As regards knowledge representation in the brain, one of the key challenges is to understand how neural activations, which are widely distributed and sub-symbolic, give rise to behavior that is symbolic, such as language and logical reasoning. Recent advances in fMRI and MEG analysis make it possible to develop and test such theories. For instance, formal concept analysis (Ganter and Wille, 1999; Endres and Foldiak, 2009) leads to characterization of semantic structures in the brain, and conceptual attribute representations (Binder and Desai 2011) make it possible to model how semantics concepts map to brain areas. A major challenge for the future is to understand how such semantics are constructed and affected by context, such as a sequence of words in a sentence.

**Consolidation:** Learning to Reason (L2R) is a framework that makes learning an integral part of the reasoning process (Khardon and Roth, 1997). L2R studies the process of learning a knowledge base (KB) from examples of the truth-table of a logical expression, and reasoning with that knowledge base by querying it with similar examples. Learning is done specifically for the purpose of reasoning. L2R has close connections to the *neuroidal* model of Valiant (2000) which examines computationally tractable learning and reasoning given PAC constraints. These constraints limit the agent’s environment via a probability distribution over the input space. Despite interesting early findings (Valiant, 2008; Juba, 2013), there is much work to be done to make this a practical approach. A major question is how a L2R agent can develop a complete KB over time when examples of the logical expressions arrive with values for only part of the input space.

This suggests that a Lifelong Machine Learning (LML) approach is needed that can consolidate the knowledge of individual examples over many learning episodes (Silver, 2013a; Fowler, 2011). The consolidation of learned knowledge facilitates the effective retention and transfer of knowledge e.g. rapid and beneficial inductive bias (Silver, 2013b). This is a challenge for neural-symbolic integration because of the computational complexity of knowledge extraction and the need for compact representations that can enable efficient reasoning about what has been learned. Deep networks, however, by seeking to represent knowledge in a modular way, together with the representation adopted by connectionist modal logics, which are intrinsically modular (d’Avila Garcez, Lamb, and Gabbay, 2007), may offer a *sweet spot* in the complexity-expressiveness landscape (Vardi, 1996). Modularity of deep networks seems suitable for relational knowledge extraction, which can reduce the complexity of extraction further (Franca, d’Avila Garcez, and Zaverucha, 2015).

**Transfer:** Knowledge transfer between, at first site, unrelated domains is a crucial cornerstone of human learning. In this process, the use of analogy is considered essential (Gentner, Holyoak, and Kokinov, 2001). Whilst most of the prominent computational models of analogy are logic-based (Falkenhainer, Forbus, and Gentner 1989; Schmidt, Krumnack, Gust, Kühnberger, 2014), recent developments in structure learning in a neural-symbolic paradigm may open the way for an application of analogy at a sub-symbolic level. The expected gain is enormous: instead of having to retrain a network model on a new domain, insights already obtained could be transferred meaningfully between different networks. Yet, two fundamental questions remain: How can the knowledge-level notion of analogical transfer be implemented in connectionist architectures? How can possible analogies between different domains be discovered sub-symbolically in the first place?

Some work on heterogeneous transfer learning has been directed at these questions (Yang et al, 2009).

**Application:** Neural-symbolic integration has been applied to training and assessment in simulators, normative reasoning, rule learning, integration of run-time verification and adaptation, action learning and description in videos (Borges, d’Avila Garcez, Lamb, 2011; de Penning et al., 2011). Future application areas that seem promising include the analysis of complex networks, social robotics and health informatics, and multimodal learning and reasoning, such as e.g. combining video and audio tagged with ontological metadata. Overall, neural-symbolic integration seems suitable to applications where large amounts of heterogeneous data exist and knowledge descriptions are required. This is the case in robot navigation and communication, medical imaging diagnosis, genomics, hardware/software specification, earth observation, multimodal data fusion for information retrieval, big data understanding and, ultimately, language understanding. Several features illustrate the advantages of neural-symbolic computation: explanation capacity, no a-priori assumptions, and its comprehensive cognitive models integrating symbolic and statistical learning with sound logical reasoning. Ultimately, however, in each of the above application areas, measurable criteria of success should include accuracy and efficiency measures, as well as knowledge readability.

### 3. Conclusions

Neural-symbolic computation reaches out to two communities and seeks to achieve the fusion of competing views when such fusion can be beneficial. In doing so, it sparks new ideas and promotes cooperation. Further, neural-symbolic computation brings together an integrated methodological perspective, as it draws from both neuroscience and cognitive systems. Methodologically, it bridges gaps, as new ideas can emerge through *changes of representation*. In summary, neural-symbolic computation is a promising approach, both from a methodological and computational perspective to answer positively to the need for effective knowledge representation, reasoning and learning. Both its representational generality (the ability to represent, learn and reason about several symbolic systems) and its learning robustness can open interesting opportunities leading to adequate forms of knowledge representation, be they purely symbolic, or hybrid combinations involving probabilistic or numerical approaches implemented through neural networks.

**Acknowledgements:** We are grateful to Jerry Feldman for useful discussions which took place during Dagstuhl Seminar 14381 and have helped shape the challenges presented in this paper.

## References

- S. Bader, P. Hitzler, and S. Hölldobler, 2008. Connectionist model generation: A first-order approach. *Neurocomputing*, 71:2420-2432, 2008.
- J. R. Binder, R. Desai, 2011. The neurobiology of semantic memory. *Trends in Cognitive Sciences*, 15(11), 527-536.
- H. Blockeel, K.M. Borgwardt, L. De Raedt, P. Domingos, K. Kersting, X. Yan, 2011: Guest editorial to the special issue on inductive logic programming, mining and learning in graphs and statistical relational learning. *Machine Learning* 83(2):133-135.
- R.V. Borges, A. d'Avila Garcez and L.C. Lamb, 2011. Learning and Representing Temporal Knowledge in Recurrent Networks. *IEEE Transactions on Neural Networks*, 22(12):2409-2421.
- M. Craven and J. Shavlik, 1996. Extracting Tree-Structured Representations of Trained Networks. *Advances in Neural Information Processing Systems*, pp. 24-30, MIT Press.
- A. d'Avila Garcez, K. Broda, and D.M. Gabbay, 2001. Symbolic knowledge extraction from trained neural networks: A sound approach. *Artificial Intelligence*, 125:155- 207.
- A. d'Avila Garcez, K. Broda, and D.M. Gabbay, 2002. *Neural-Symbolic Learning Systems: Foundations and Applications*. Springer, Berlin.
- A. d'Avila Garcez, L. Lamb, 2006. A Connectionist Computational Model for Epistemic and Temporal Reasoning. *Neural Computation*, 18(7):1711-1738.
- A. d'Avila Garcez, L. C. Lamb and D. M. Gabbay, 2007. Connectionist modal logic: Representing modalities in neural networks. *Theoretical Computer Science* 371(1-2):34-53.
- A. d'Avila Garcez, L. C. Lamb, and D. M. Gabbay. 2009. *Neural-Symbolic Cognitive Reasoning*. Springer.
- L. de Penning, A.S. d'Avila Garcez, L.C. Lamb and J.J. Meyer, 2011. A Neural-Symbolic Cognitive Agent for Online Learning and Reasoning. In Proc. IJCAI-11, Barcelona, Spain.
- De Raedt, Luc, et al., 2008. *Probabilistic Inductive Logic Programming-Theory and Applications*, vol. 4911, LNCS, Springer.
- D. M. Endres and P. Foldiak, 2009. An application of formal concept analysis to semantic neural decoding. *Ann. Math. Artif. Intell.* 57:233-248.
- B. Falkenhainer, K. Forbus, and D. Gentner, 1989. The Structure Mapping Engine: Algorithm and Examples. *Artif. Intell.* 41:1-63.
- J. A. Feldman, 2006. *From Molecule to Metaphor: A Neural Theory of Language*, Bradford Books, Cambridge, MA: MIT Press.
- J. A. Feldman, 2013: The Neural Binding Problem(s). *Cogn. Neurodyn.*, 7:1-11.
- B. Fowler and D. Silver, 2011. Consolidation using Context-Sensitive Multiple Task Learning. 24th Conf. Canadian AI Assoc. Springer, LNAI 6657.
- M.V. França, G. Zaverucha and A.S. d'Avila Garcez, 2014. Fast relational learning using bottom clause propositionalization with artificial neural networks. *Machine Learning* 94(1):81-104.
- M.V. França, A.S. d'Avila Garcez and G. Zaverucha, 2015. Neural Relational Learning through Semi-Propositionalization of Bottom Clauses, AAAI Spring Symposium on Knowledge Representation and Reasoning: Integrating Symbolic and Neural Approaches, Stanford University, CA, March 2015.
- Ganter B., Wille R., 1999. *Formal Concept Analysis: Mathematical Foundations*. Springer, New York.
- D. Gentner, K. Holyoak and B. Kokinov, eds. 2001. *The Analogical Mind: Perspectives from Cognitive Science*. MIT Press.
- Getoor, Lise, and Ben Taskar, eds, 2007. *Introduction to statistical relational learning*. MIT press.
- M. Guillaume-Bert, K. Broda, and A. d'Avila Garcez, 2010. First-order logic learning in artificial neural networks. *IJCNN* 2010.
- H. Gust, K.-U. Kühnberger, and P. Geibel, 2007. Learning models of predicate logical theories with neural networks based on topos theory. In B. Hammer and P. Hitzler, editors, *Perspectives of Neural-Symbolic Integration*, pages 233-264. Springer.
- D. Harel, D. Kozen, J. Tiuryn, 2001: *Dynamic logic*. *SIGACT News* 32(1): 66-69.
- G. E. Hinton, S. Osindero, Y. Teh, 2006: A fast learning algorithm for deep belief nets. *Neural Computation* 18, pp 1527-1554.
- G. E. Hinton, 1990: Preface to the special issue on connectionist symbol processing. *Artificial Intelligence* 46, 1-4.
- P. Hitzler, S. Bader, and A. d'Avila Garcez, 2005. Ontology learning as a use case for neural-symbolic integration. In Proc. NeSy'05 at IJCAI-05.
- B. Juba, 2013. Implicit learning of common sense for reasoning. Proc. of IJCAI-13.
- R. Khardon and D. Roth, 1997. Learning to Reason, *Journal of the ACM*, vol. 44, 5, pp. 697-725.
- A. Krisnadhi, F. Maier, and P. Hitzler, 2011. *OWL and Rules. Reasoning Web: Semantic Technologies for the Web of Data*. 7th Summer School Galway, vol. 6848. LNCS. Springer.
- M. Krötzsch, F. Maier, A. A. Krisnadhi, and P. Hitzler, 2011. A better uncle for OWL: Nominal schemas for integrating rules and ontologies. Proc. WWW-2011, pages 645-654. ACM.
- M. Krötzsch, S. Rudolph, and P. Hitzler, 2013. Complexity of Horn description logics. *ACM Trans. Comput. Logic*, 14(1).
- J. Lehmann, S. Bader, and P. Hitzler, 2010. Extracting reduced logic programs from artificial neural networks. *Appl. Intell.* 32(3):249-266.
- V. Lifschitz, 2002. Answer set programming and plan generation. *Artif. Intell.* 138(1-2): 39-54.
- J. McCarthy, 1988. Epistemological problems for connectionism. *Behavioral and Brain Sciences*, 11:44.
- G. Pinkas, P.V. Lima, S. Cohen, 2012. A Dynamic Binding Mechanism for Retrieving and Unifying Complex Predicate-Logic Knowledge. *ICANN* (1) 2012: 482-490.
- M. Schmidt, U. Krumnack, H. Gust, and K.-U. Kühnberger, 2014. Heuristic-Driven Theory Projection: An overview. *Computational Approaches to Analogical Reasoning: Current Trends*. Springer. 163-194.
- D. Silver, 2013a. The Consolidation of Task Knowledge for Lifelong Machine Learning. Proc. AAAI Spring Symposium on Lifelong Machine Learning, Stanford University, pp. 46-48.
- D. Silver, 2013b. On Common Ground: Neural-Symbolic Integration and Lifelong Machine Learning. 9th Workshop on Neural-Symbolic Learning and Reasoning, NeSy'13 at IJCAI-13.
- P. Smolensky, 1988: On the proper treatment of connectionism. *Behavioral and Brain Sciences* 11 (1):1-23.
- R. Sun, 1994: *Integrating Rules and Connectionism for Robust Commonsense Reasoning*. John Wiley and Sons, New York.
- S. Tran and A. d'Avila Garcez, 2013. Knowledge Extraction from Deep Belief Networks for Images. Proc. IJCAI Workshop on Neural-Symbolic Learning and Reasoning, NeSy'13, Beijing.
- L.G. Valiant, 2000. A neuroidal architecture for cognitive computation. *J. ACM* 47(5): 854-882.
- L.G. Valiant, 2008. Knowledge Infusion: In Pursuit of Robustness in Artificial Intelligence. In *FSTTCS* (pp. 415-422).
- M. Y. Vardi, 1996: Why is Modal Logic So Robustly Decidable? *Descriptive Complexity and Finite Models*, 149-184.
- Yang, Q.; Chen, Y.; Xue, G.-R.; Dai, W.; and Yu, Y. 2009. Heterogeneous transfer learning for image clustering via the social web. In 47th Annual Meeting of the ACL.