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The Optimality of Uniform Pricing in IPOs: An Optimal Auction Approach*

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Abstract

This paper uses an optimal auction approach to investigate the conditions under which uniform pricing in IPOs is optimal. We show that the optimality of a uniform price in IPOs depends crucially on whether the (optimal) allocation rule is restricted. These restrictions may stem from the retail investors' budget constraint and/or from the institutional investors' preferences. We show that the main determinant of the optimality of a uniform pricing rule is the existence and the *shape* of the retail investors' budget constraint. In contrast, institutional investors' preferences are shown to mainly affect the optimal allocation rule.

Keywords: Initial Public Offering, Price Discrimination, Rationing, Optimal Auction.

JEL Classification Numbers: D8, G2.

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1 Introduction

The literature on Initial Public Offerings (IPOs) has grown remarkably over the past few decades. Most of this literature, both theoretical (e.g. Rock, 1986, Allen and Faulhaber, 1989, and Benveniste and Spindt, 1989) and empirical (e.g. Beatty and Ritter, 1986, Ritter 1987, and Cornelli and Goldreich, 2001), has focused on explaining a number of apparent market anomalies surrounding IPOs such as underpricing, long-term underperformance, and hot issues markets.[1] Much less attention has been devoted to understand whether the current IPO format, which imposes uniform pricing, is actually efficient. Firms that go public are currently forbidden from using price discrimination; they can however quantity discriminate by choosing an allocation rule and potentially rationing some investors. Virtually all papers in the IPO literature, to be consistent with current practice, simply assume uniform pricing. The only paper addressing this issue (Benveniste and Wilhelm, 1990) challenges the efficiency of the current regulatory constraints, and suggests that firms could improve on their IPO performance (raise the IPO proceeds) if they were allowed to price discriminate across investors.

The objective of this paper is to investigate the conditions under which uniform pricing is optimal, where optimality is defined from the point of view of an issuer who wants to maximize the sale's expected proceeds. Our results show that the optimality of uniform pricing depends on whether the (optimal) allocation rule is subject to certain restrictions. If this is the case, then discriminatory pricing may be required to elicit information from the informed investor. Otherwise (i.e. if the issuer has enough control over the allocation rule), quantity discrimination alone suffices to achieve optimality. We consider two types of restrictions on the allocation rule. The first are direct and exogenous allocation constraints. The second type of restrictions endogenously result from assuming non-linear preferences of institutional investors. Our results indicate that only allocation constraints critically matter for the optimality of uniform pricing. We also show that the optimality of uniform pricing depends on the existence of an allocation constraint as well as on the shape of this constraint, since this will define the extent to which

[1] Ritter and Welch (2002) provide a concise and useful review of the IPO literature. For a fully comprehensive review of the theory and evidence on IPO activity see Jenkinson and Ljungqvist (2001).

the allocation rule is restricted. On the other hand, institutional investors' preferences only affect the optimal allocation rule. Specifically, we show that with risk-neutral preferences, the optimal allocation rule gives priority to retail investors, with institutional investors being residual claimants. The opposite holds when preferences are non-linear.

In our IPO a firm wants to place a fixed number of unseasoned shares. There are two potential groups of investors: n institutional investors and a continuum of retail investors. Each institutional investor receives a private signal about the value of the asset for sale, whereas both the issuer and the retail investors are uninformed. The seller designs the IPO mechanism in order to elicit private information from institutional investors and maximize the expected IPO proceeds. We apply an optimal auction approach to derive the optimal IPO, which consists in identifying the optimal allocation and pricing rule. We solve two different models. In the first, institutional investors are assumed to be risk neutral. This is in line with most of the IPO literature. In the second, institutional investors are assumed to have preferences that are concave in quantity but may place a higher valuation on the shares than do retail investors. The non-linear preferences case represents the main novelty of the paper. Each of these models is then solved under three different assumptions on the allocation constraint: (i) no allocation constraint; (ii) a quantity constraint, i.e. a maximum (minimum) amount of share is to be allocated to retail (institutional) investors; and (iii) a cash constraint, i.e. retail investors only have a maximum amount of cash that they can spend in the IPO.[2]

We show that the optimal IPO cannot be implemented with a uniform pricing rule under the second assumption (i.e. when retail investors are quantity constrained). In the other cases, we show that uniform pricing is optimal in equilibrium. Additionally, our results imply that underpricing always occurs when discriminatory pricing is needed to achieve optimality. Conversely, this need not be the case when uniform pricing is optimal. The explanation behind these results is that quantity constraints impose tighter restrictions on the seller's ability to use quantity discrimination as a tool to elicit information from institutional investors, which

[2]We do not report the results for the case of non-linear preferences and cash constraints since due to complexity of the technical problem, we cannot produce a closed form solution.

renders the complementary use of price discrimination necessary to achieve optimality. Another contribution of the paper is the full characterization of the optimal allocation rule in all of the different environments considered, with interesting implications for the use of rationing in equilibrium.

The only other paper in the IPO literature that has dealt with the issue of optimal pricing is Benveniste and Wilhelm (1990). However, our paper differs from theirs in many respects, the most important of which is the methodology used. We apply optimal auction theory, which enables us to characterize the optimal IPO mechanism in a very general environment with respect to both the informational structure and investors' characteristics. The paper is also closely related to Maksimovic and Pichler (2006), hereafter MP. Although the focus of their paper is not on optimal pricing rule, they show that the existence and the size of underpricing depends on the existence of allocation constraints. In this paper, we show that allocation constraints also affect the optimal pricing rule and we are able to say something about the link between pricing rules and underpricing. Finally, we extend their model on three dimensions i) we consider a continuous state space while MP is a discrete model; ii) we assume non-linear preferences of institutional investors, which implies that, in our model, allocation constraints may effectively arise endogenously whereas in MP they are exogenously imposed and informed investors are risk neutral; and iii) we also look at cash constraints, while they only consider quantity constraints.

Our work yields a number of empirical predictions.

- The most direct implication is that uniform pricing and discriminatory pricing will lead to the same IPO proceeds if the constraints on the allocation rule are not too tight. As discriminatory pricing is forbidden in most countries, a direct test of this prediction is only possible in the small number of countries where discriminatory pricing has been used in IPOs. For example, Jagannathan and Sherman (2006) document the use of discriminatory IPO auctions in Taiwan and Japan.

- We should observe greater underpricing in IPOs when there are tighter restrictions on the allocation rule, e.g. the requirement that a minimum quantity of shares be allocated to institutional investors.[3] This is consistent with the results of MP.

- Quantity constraints would result in larger underpricing than cash constraints. This raises the issue of understanding which of these two types of constraints is more relevant in practice, which is another interesting empirical research question.

The paper is organized as follows. In the next section, we set up the model and derive the sellers' maximization problem for generic institutional investor preferences with no constraints on the retail investors. The following sections then solve the model under different assumptions regarding institutional investors' preferences and allocation constraints. Section 3 analyzes the case of linear preferences with and without an allocation constraint. In Section 4 the same analysis is conducted in the case of non linear preferences. Finally, Section 5 investigates the impact of cash constraints on retail investors with linear preferences of institutional investors. The last section concludes with a discussion of the results. All proofs are in the Appendix.

2 The Model

A firm would like to sell Q shares in an IPO, with Q fixed and, without loss of generality, normalized to 1. An intermediary is in charge of marketing the new shares. He is assumed to act in the firm's best interest. Hereafter, we will simply refer to the *seller* to denote the firm-intermediary coalition. This is a standard way of modelling the role of an intermediary in the IPO literature. The seller wishes to maximize the proceeds from the sale. He faces both $n(> 2)$ large institutional investors who hold private information about the firm's market value, and a fringe of retail investors, who are uninformed.

Institutional investors have private information in that each agent i receives a signal s_i about the market value of the new shares. Signals are i.i.d. according to a uniform distribution

[3] This practice is supported by theoretical arguments (Stoughton and Zechner, 1998) in that the issuer wants to have a minimum institutional ownership because institutional investors tend to monitor the firm more closely. The empirical relevance is more difficult to prove, although it has been, for instance, observed in some OpenIPOs run by WR Hambrecht. (We thank the referee for providing us with this useful piece of information).

defined on $\Omega_i = [\underline{s}; \overline{s}]$ with $\underline{s} > 0$, so the cumulative distribution function is $F_i(s_i) = \dfrac{s_i - \underline{s}}{\Delta s}$, with $\Delta s = \overline{s} - \underline{s}$, and the density function $f_i(s_i) = \dfrac{1}{\Delta s}$. Let us also denote by $f(s)$ the joint density function so that $f(s) = f(s_1, ..., s_n) = \prod_i f_i(s_i)$, with $s = (s_1, ..., s_n) \in \Omega = \bigotimes_i \Omega_i = [\underline{s}, \overline{s}]^n$.[4] Each signal received by an institutional investor represents a piece of information about the market value of the new shares. We therefore assume that the value of the new shares, v, is a function of the vector of signals received by institutional investors. More precisely, we assume that $v(\cdot)$ is a function defined over Ω and is given by the average of all the signals, that is

$$v(s) = \frac{1}{n} \sum_i s_i. \tag{1}$$

The above function has two main properties: *a)* $\frac{\partial v(s)}{\partial s_i} > 0$, i.e. the asset value is increasing in each signal, and *b)* $\frac{\partial v(s)}{\partial s_i} = \frac{\partial v(s)}{\partial s_j}$ for any $i \neq j$, i.e. signals are equally weighted in the valuation function. This kind of informational structure is very common in auction theory (e.g. Bulow and Klemperer, 1996 and 2002) and is a straightforward generalization of the simple binomial informational structure adopted in other IPO papers (e.g. Benveniste and Wilhelm, 1990; Biais and Faugeron-Crouzet, 2002) to a continuous signal space.[5] It can furthermore be proved that our results hold for any generic valuation function $v(s) = \Psi(s)$ which satisfies properties *a)* and *b)* above (i.e. $\frac{\partial \Psi(s)}{\partial s_i} > 0$ and $\frac{\partial \Psi(s)}{\partial s_i} = \frac{\partial \Psi(s)}{\partial s_j}$ for any $i \neq j$). For the sake of tractability, we prefer a simple specification.

Given a vector of signals s, each institutional investor i's preferences are given by the following utility function:

$$u_i(p_i, q_i, v(s)) = z(q_i, v(s)) - q_i p_i \qquad \text{for all } i \in \{1, 2, ...n\}, \tag{2}$$

where q_i is the quantity assigned to investor i and p_i is the price per share he has to pay. We denote by $T_i = p_i q_i$ the total payment from investor i to the seller. The utility function

[4] Note that the uniform distribution of private signals satisfies the *increasing hazard rate assumption.* Indeed, we have that for the uniform distribution $\dfrac{\partial}{\partial s_i}\left[\dfrac{f_i(s_i)}{1 - F_i(s_i)}\right] = \dfrac{\partial}{\partial s_i}\left[\dfrac{1}{(\overline{s} - s_i)}\right] \geq 0$ for all i and all s_i. We show later in the paper that several of our results (in particular those in Section 3) do not change qualitatively if we consider a more general distribution of signals satisfying the increasing hazard rate assumption.

[5] These papers assume that the signals investors receive can be either good or bad and the stock market value is monotonic in the number of good signals. We make the same assumption, but we use a continuum of signals.

in Equation (2) is linear in the transfer T_i. We make the following assumptions about the institutional investors' valuation function z (the subscripts denote derivatives with respect to variables):

A1 $z_1 > 0$ and $z_2 > 0$;

A2 $z_{11} \leq 0$;

A3 $z(0, v) = 0$ for all v;

A4 $z_{12} > 0$ (*single-crossing condition*);

A5 $z_{122} \leq 0$ and $z_{112} \geq 0$;

A6 $z_{12}(0, v) \geq 1$.[6]

We assume a continuum of competitive, uninformed, and risk-neutral *retail* bidders. The total mass of these retail bidders is normalized to one. In the following sections, we will consider the possibility that these investors face an allocation constraint, with the constraint taking a number of different forms. Neither retail investors nor the seller hold any private information about the market value of the asset and only observe the density $f(\cdot)$ of signals.

In order to extract the information from the institutional investors, the seller designs a mechanism specifying the allocation and pricing rules for both institutional and retail investors. By using the *revelation principle,* we can focus on *direct mechanisms* in which the seller asks each institutional investor to announce his signal and then fixes quantities and prices as function of their announcements in such a way as to induce them to reveal their information truthfully (see Fudenberg and Tirole, 1991).

A mechanism is described by a pair of outcome functions (p, q) of the form $p : \Omega \to \mathbb{R}^{n+1}$ and $q : \Omega \to [0, 1]^{n+1}$ where $p = (p_1, ..., p_n, p_R)$ is a vector of prices and $q = (q_1, ..., q_n, q_R)$ is a vector of allocations. Thus if s is the vector of signals announced by institutional investors, each investor $i \in \{1, ..., n\}$ receives $q_i(s)$ shares and pays a price per share of $p_i(s)$, while retail

[6] Most of these assumptions are standard in the mechanism design literature. See Fudenberg and Tirole (1991) for a discussion.

investors receive $q_R(s)$ and pay a price per share of $p_R(s)$. We assume that all of the shares issued must be allocated to either institutional or retail investors. The seller's choice of the vector $\{q_i\}_{i=1,..n}$ implicitly determines the number of shares allocated to retail investors, q_R, which is given by:

$$q_R = 1 - \sum_i q_i. \tag{3}$$

The set of possible strategies for the institutional investor i with signal s_i is Ω_i. Faced with a mechanism (p, q), his expected utility if he misrepresents his signal by announcing $\widehat{s_i}$ to the seller rather than his true signal s_i is

$$\begin{aligned} U_i(\widehat{s_i}, s_i) &= E_{s_{-i}}\{u_i(p_i(\widehat{s_i}, s_{-i}), q_i(\widehat{s_i}, s_{-i}), v(s_i, s_{-i}))\} \\ &= \int_{\Omega_{-i}} [z(q_i(\widehat{s_i}, s_{-i}), v(s_i, s_{-i})) - q_i(\widehat{s_i}, s_{-i})p_i(\widehat{s_i}, s_{-i})] \, f_{-i}(s_{-i})ds_{-i} \,, \end{aligned} \tag{4}$$

where s_{-i} is the vector of all of the other institutional investors' signals, i.e. $s_{-i} = (s_1, .., s_{i-1}, s_{i+1}, ..., s_n)$ and $f_{-i}(s_{-i}) = \prod_{j \neq i} f_j(s_j)$.[7]

The optimal IPO mechanism for the seller is the solution to the following optimization program:

$$\max_{(p,q)} U_F = E_s \left[\sum_i T_i(s) + T_R(s) \right] = \int_\Omega \left(\sum_i T_i(s) + T_R(s) \right) f(s)ds \,, \tag{5}$$

subject to the following standard constraints:

- *Retail Investors' Participation Constraint* (RPC). This requires the expected payoff for retail investors to be greater than their reservation utility, which equals zero.

  $$E_s \left[q_R(s)(v(s) - p_R(s)) \right] = \int_\Omega q_R(s) \left[v(s) - p_R(s) \right] f(s)ds \geq 0. \tag{RPC}$$

  Thus the constraint is conditional on the initial distribution of the signals which implies that retail investors commit to buying the share without ever learning the reported signals. This in turn implies that they do not play strategically in the IPO game.

[7] Note that the expected utility of agent i depends on the mechanism offered by the seller (p, q), which is omitted in our notation for the sake of simplicity.

- *Institutional Investors' Participation Constraint* (IPC). The IPC ensures that each institutional investor is willing to participate in the offering conditional on his own signal. The expected utility of each institutional investor, conditional on his signal, should be greater than his expected utility when he does not participate in the IPO. The IPC is then written as follows:

$$\begin{aligned} U_i(s_i, s_i) &= E_{s_{-i}} \{z(q_i(s), v(s)) - q_i(s)p_i(s)\} \\ &= \int_{\Omega_{-i}} [z(q_i(s), v(s)) - p_i(s)q_i(s)] \, f_{-i}(s_{-i}) ds_{-i} \geq 0. \end{aligned} \tag{IPC}$$

  This must be satisfied for all i and all s_i.

- *Institutional Investors' Incentive Compatibility Constraint* (IIC). This constraint ensures that each institutional investor has no incentive to misrepresent his type - the signal he receives - to the firm. The IIC then requires that each agent i be better off by truthfully announcing his signal. Using Equation (4) this may be written as follows:

$$U_i(\widehat{s}_i, s_i) \leq U_i(s_i, s_i) \qquad \text{for all } s_i, \widehat{s}_i \text{ and } i, \tag{IIC}$$

  or, equivalently,

$$s_i \in \arg\max_{\widehat{s}_i} U_i(\widehat{s}_i, s_i) \qquad \text{for all } s_i, \widehat{s}_i \text{ and } i. \tag{IICa}$$

- *Full Allocation Constraint* (FAC).

$$\sum_i q_i(s) + q_R(s) = 1 \text{ for all } s\,, \tag{FAC}$$

  and the quantity non-negativity constraints:

$$q_i(s) \geq 0 \qquad \text{for all } i \text{ and } s. \tag{6}$$

Finally, we will also introduce the appropriate allocation constraint.

The above optimization program can be simplified by re-arranging the constraints. In the Appendix, we show that the seller's optimization program can be written as follows

$$
\begin{cases}
\max\limits_{\{q_i\}_1^n} \int_\Omega \left\{ v(s) + \sum_i \left[z(q_i(s), v(s)) - \dfrac{1}{n}(\overline{s} - s_i) z_2(q_i(s), v(s)) - v(s) q_i(s) \right] \right\} f(s) ds \\
s.t: \\
(i) \quad U_i(\underline{s}, \underline{s}) = 0 \qquad \text{for all } i \\
(ii) \quad \frac{1}{n} E_{s_{-i}} \left[z_{12}(q_i(s), v(s)) \dfrac{\partial q_i(s)}{\partial s_i} \right] \geq 0 \qquad \text{for all } i \text{ and all } s_i \\
(iii) \quad q_i(s) \geq 0 \qquad \text{for all } i \text{ and all } s \\
(iv) \quad \sum\limits_i q_i(s) \leq 1 \qquad \text{for all } s.
\end{cases}
\tag{$\mathcal{P}$}
$$

Note that this program depends only on quantities. Once optimal quantities have been determined, optimal prices can be obtained from the participation constraints of both the institutional and the retail investors. For all s_i and all i, prices for institutional investors satisfy

$$
\begin{aligned}
\int_{\Omega_{-i}} p_i(s) q_i(s) f_{-i}(s_{-i}) ds_{-i} &= \int_{\Omega_{-i}} z(q_i(s), v(s)) f_{-i}(s_{-i}) ds_{-i} - \\
&\quad \int_{\Omega_{-i}} \left\{ \frac{1}{n} \int_{\underline{s}}^{s_i} z_2(q_i(\widetilde{s}_i, s_{-i}), v(s)) d\widetilde{s}_i \right\} f_{-i}(s_{-i}) ds_{-i}.
\end{aligned}
\tag{7}
$$

Likewise, for retail investors, the optimal pricing rule must satisfy the (binding) participation constraint

$$
\int_\Omega p_R(s) q_R(s) f(s) ds = \int_\Omega v(s) q_R(s) f(s) ds. \tag{8}
$$

Equations (7) and (8) above will typically admit multiple solutions. We are interested in verifying whether there exists at least one equilibrium requiring a uniform price for all investors, i.e. a price function $p(s)$ such that $p(s) = p_i(s) = p_R(s)$ for all i and all s.

It is worth noticing that the seller's program in this setup is quite different from that in a standard auction design problem where an uninformed seller faces usually only informed bidders. The participation of a class of uninformed bidders in the auction makes the problem rather different and interesting in the sense that it mitigates the adverse-selection problem *vis-à-vis* the informed investors and, thus, lowers the seller's cost of extracting their private information.[8] What really matters, however, is not that retail investors are uninformed but

[8] In a very similar framework, Malakhov (2006) investigates the impact of retail investors' participation on the issuer's revenues. It is shown that the seller's revenues are increasing in the number of uninformed investors participating in the offering, since more uninformed investors lowers the outside option of informed investors and, consequently, as in our model, reduces the cost of gathering information.

rather that their information, if any, differs from that of the institutional investors and, more importantly, that they do not play strategically, i.e. it is impossible to elicit their information. This point has previously been made by Maksimovic and Pichler (2006).

3 Risk-neutral institutional investors

We start by analyzing the standard case in the IPO literature of risk-neutral institutional investors. We thus assume that the informed investors' valuation function is

$$z(q, v(s)) = v(s)q\,, \tag{9}$$

and the utility function of investor i is

$$u_i(p_i, q_i, v(s)) = [v(s) - p_i]\, q_i. \tag{10}$$

We also assume that the allocation rule is restricted. We model this restriction as a quantity constraint on retail investors, i.e. there exists a maximum quantity of shares $K < 1$ they can buy. In other words,

$$q_R(s) \leq K. \tag{11}$$

Notice that, in our model where the number of shares issued is fixed, this constraint is equivalent to requiring that a minimum quantity of shares be allocated to institutional investors. In other words we could just as well write it as $\sum_i q_i(s) \geq 1 - K$, which represents a quite common, though implicit, practice in IPOs. There may be several reasons why this is the case: due to the monitoring role played by institutional shareholders which potentially enhances the firm value (Mello and Parsons, 1998, Stoughton and Zechner, 1998), or because of the tight links with the underwriter who tends in turn to favor his institutional clientele over retail demand in IPOs (Aggarwal, 2003; Aggarwal et al., 2002).

In this case the seller's problem is the following:

$$
\max_{\{q_i\}_1^n} \int_{\Omega} \left\{ v(s) - \textstyle\sum_i \left[\frac{1}{n}(\overline{s} - s_i) q_i(s) \right] \right\} f(s) ds
$$

$$
\begin{aligned}
&s.t: \\
&(i) \quad U_i(\underline{s}, \underline{s}) = 0 && \text{for all } i \\
&(ii) \quad E_{s_{-i}} \left[\frac{\partial q_i(s)}{\partial s_i} \right] \geq 0 && \text{for all } i \text{ and all } s_i \\
&(iii) \quad q_i(s) \geq 0 && \text{for all } i \text{ and all } s \\
&(iv) \quad \sum_i q_i(s) \leq 1 && \text{for all } s \\
&(v) \quad q_R(s) \leq K && \text{for all } s.
\end{aligned}
\tag{$\mathcal{P}1$}
$$

In the next proposition we characterize the optimal IPO mechanism in terms of allocation and pricing rules:

PROPOSITION 1 *If institutional investors are risk neutral and retail investors can buy at most K shares, the optimal IPO is characterized as follows:*

1. *(**Allocation rule**) For all $s \in \Omega$, let $s_m = \max\{s_1, ..., s_n\}$. In equilibrium, the issuer allocates as many shares as possible to retail investors and the remaining shares to the institutional investor reporting s_m, that is $q_R(s) = K$ and $q_m(s) = 1 - K$.*[9]

2. *(**Pricing rule**) The optimal IPO requires discriminatory pricing such that $p_m(s) < p_R(s) = v(s)$.*

In the Appendix, the above Proposition is proved for a generic distribution of private signals satisfying the increasing hazard rate assumption.

To understand the logic behind the above result notice that the seller's objective function is decreasing in the term $\int_{\Omega} \sum_i \left[\frac{1}{n}(\overline{s} - s_i) q_i(s) \right]$ which represents the institutional investors' information rents and can be easily shown to be increasing in the quantity $q_i(s)$ and in the signal s_i. These properties together ensure that the seller optimally allocates as much as possible to retail investors, i.e. up to their quantity constraint, with any residual quantity going to the

[9]Note that as a result of the continuous distributions we consider in our model, the probability of having more than one agent announcing s_m is zero.

institutional investor with the highest reported signal who then receives a strictly positive information rent.[10] To see why he also pays a lower price than do retail investors, simply note that with linear preferences, after replacing the optimal quantity, the ICC becomes

$$[v(s) - p_m(s)](prob\ s_i > s_{-i})q_m(s) \geq [v(s) - p_i(\widehat{s}_i, s_{-i})](prob\ \widehat{s}_i > s_{-i})q_m(s)$$

given that $q_m(s) > 0$ and $v(s) = p_R(s)$, it must be that $p_R(s) > p_m(s)$.

In the absence of an allocation constraint, the result below follows directly from Proposition 1:

COROLLARY 1 *When institutional investors are risk neutral and retail investors are not constrained, the optimal offering is such that all the shares are sold to retail investors irrespective of the signals reported by the informed investors, at a marginal price of* $p_R = v(s)$.

This result is a generalization of MP to a continuous state space and implies that risk-neutral institutional investors receive no information rents because, in equilibrium, they are actually excluded from the offering. These results can be shown to hold for all distributions satisfying the increasing hazard rate condition.

With respect to this section's results, it is important to note that retail investors have priority in the allocation. This highlights the important role that uninformed investors play in the IPO process. Their presence mitigates the adverse-selection problem vis-a-vis the institutional investors, and consequently allows the seller to lower the cost of eliciting their private information by exploiting the competition between the two groups of investors. If retail investors have unlimited buying capacity, then the cost of information gathering is zero (Corollary 1).[11] This in turn implies that the existence of an allocation constraint is in fact detrimental to the efficiency of the selling mechanism.

In reality, however, we never observe an IPO in which all of the shares are placed with retail investors. In the light of this empirical regularity, our results then suggest that: a) the

[10] It should be clear that the assumption of an increasing hazard rate is crucial for this result because it ensures that information rents increase in signals. Different assumptions regarding the behaviour of the hazard rate would of course affect the optimal allocation rule.

[11] The role of retail investors has been already stressed by Benveniste et al. (1996), Biais and Faugeron-Crouzet (2002), Maksimovic and Pichler (2006) and Bennouri and Falconieri (2006).

most relevant empirical situation is that of maximum (minimum) quantity constraints on retail (institutional) investors; or b) institutional investors' preferences may not be linear. In the next section, we look at the impact of assuming that institutional investors have non-linear preferences on our results.

4 Non-Linear Preferences

The previous section suggests that the optimality of uniform pricing essentially depends on whether the seller can freely use the allocation rule to discriminate among investors. It is then natural to ask whether introducing risk aversion or some kind of concavity in institutional investors' preferences may also affect optimal pricing, to the extent that this reduces the seller's discretion in allocating shares. We therefore relax the risk-neutrality assumption in this section, and introduce concavity in the institutional investors' valuation function, which becomes:[12]

$$z(q_i, v(s)) = q_i(\alpha v(s) - \frac{\delta}{2} q_i). \tag{12}$$

When $\alpha \geq 1$, this new valuation function $z(\cdot, \cdot)$ is concave in quantity, satisfies Assumptions A1-A6, and produces a marginal valuation which is decreasing at an exogenously given rate, $\delta > 0$.[13] We consider this specification of the utility function because concavity indirectly restricts the discretion of the seller to allocate the shares and, as such, it may affect the optimality of uniform pricing. Further, concavity in quantity may be interpreted as aversion to *inventory risk,* i.e. the risk associated with portfolio composition.[14] We also assume $\alpha > 1$, which means that institutional investors value the shares more than do retail investors for small quantities, but with a decreasing marginal valuation. That is, at $q_i = 0$, their marginal valuation is αv whereas the marginal valuation of retail investors is v. In other words, institutional investors are very keen to participate in the IPO and obtain a positive quantity of shares, but their marginal utility decreases as the allocation increases, possibly due to inventory risk. If there

[12]Risk neutrality is the most common assumption in the IPO literature. Other papers that have, like us, assumed non-linear preferences are Stoughton and Zechner (1998) and Benveniste and Wilhelm (1997).

[13]Notice that the new valuation function does not describe standard risk-averse preferences, which are typically concave in wealth.

[14]This is well documented for instance in the market-microstructure or foreign-exchange market literatures (see for instance O'Hara, 1995, and Lyons, 2003).

is no cash constraint on retail investors, the optimal allocation rule with $\alpha = 1$ is the same as that under risk neutrality, i.e. the seller allocates the entire quantity to retail investors. In this case, institutional investors' preferences do not matter.

We first analyze, in the next section, the optimal mechanism in the absence of constraints on retail investors in order to isolate the impact of institutional investors' non-linear preferences. We subsequently add the allocation constraint.

4.1 NO CONSTRAINTS ON RETAIL INVESTORS

The seller's optimization program in this case can be written as:

$$\begin{array}{ll}
\max\limits_{\{q_i\}_1^n} \displaystyle\int_\Omega \left\{ v(s) + \textstyle\sum_i \left[q_i(s) \left((\alpha - 1)\, v(s) - \dfrac{\delta}{2} q_i(s) - \dfrac{\alpha}{n} (\overline{s} - s_i) \right) \right] \right\} f(s) ds & \\
s.t: & \\
(i) \quad U^i(\underline{s}, \underline{s}) = 0 & \\
(ii) \quad E_{s_{-i}} \left[\dfrac{\partial q_i(s)}{\partial s_i} \right] \geq 0 \qquad \text{for all } i \text{ and all } s & (\mathcal{P}2) \\
(iii) \quad q_i(s) \geq 0 \qquad \text{for all } i \text{ and all } s & \\
(iv) \quad \sum\limits_i q_i(s) \leq 1 \qquad \text{for all } s. &
\end{array}$$

Thus, *($\mathcal{P}2$)* is identical to *($\mathcal{P}1$)* except for the different utility function and the absence of condition *(v)*.

The next proposition describes the optimal selling mechanism.

PROPOSITION 2 *Assume that institutional investors have non-linear preferences described by Equation (12) and that there is no allocation constraint. The optimal IPO is characterized by the following allocation and pricing rules:*

1. ***(Allocation Rule)*** *For all i and all s, let $\widetilde{q}_i(\cdot)$ be such that*

$$\widetilde{q}_i(s) = \begin{cases} \dfrac{(\alpha - 1)\, nv(s) - \alpha\, (\overline{s} - s_i)}{n\delta} & \text{for all } s_i \geq s_i^\circ \\ 0 & \text{otherwise} \end{cases}$$

where $s_i^\circ = \frac{\alpha\overline{s} - (\alpha-1)v_{-i}}{2\alpha - 1}$, with $v_{-i} = \sum\limits_{j \neq i} s_j$ then

- *If* $\sum_i \widetilde{q}_i \leq 1$, *only the institutional investors reporting a signal above* s_i° *receive a positive quantity equal to* $\widetilde{q}_i(s)$. *Retail investors receive the remaining shares;*

- *If instead* $\sum_i \widetilde{q}_i > 1$ *(oversubscription), the institutional investors reporting a signal above* s_i° *receive a positive quantity* $\widehat{q}_i(s) = \widetilde{q}_i(s) - \mathcal{Q}$, *where* $\mathcal{Q}$ *is the amount of shares by which they are rationed.*[15] *Retail investors obtain no shares.*

2. ***(Pricing Rule)*** *The optimal IPO mechanism can be implemented by a uniform pricing rule with* $p \geq v(s)$. *If* $\sum_i \widetilde{q}_i \leq 1$, *then* $p = v(s)$. *If* $\sum_i \widetilde{q}_i > 1$, *then* $p > v(s)$.

Contrary to the case of risk neutrality, the institutional investors now have priority in the allocation rule. The result stems not from the non-linear preferences of the institutional investors, but rather from assuming that their marginal valuation of the asset at $q_i = 0$ is larger than the marginal valuation of the retail investors. Notice that the threshold s_i° is decreasing in the parameter α which implies that as the institutional investors' marginal valuation of the asset at $q_i = 0$ increases, a larger fraction of them receive a positive number of shares. Furthermore, the optimal quantity $\widetilde{q}_i(s)$ is decreasing in δ and increasing in the signal s_i reported by the institutional investor. In other words, the seller rewards better information about the stock value (i.e. higher signals) with a larger quantity of shares. In conclusion, the results of Proposition 2 crucially depend on the assumption of $\alpha > 1$. Indeed, as previously noted, with $\alpha = 1$, non-linear preferences would lead to the same results as in the case of risk neutrality.

The pricing rule is due to the fact that in this model where there is no allocation constraint, all the shares can always be sold to retail investors at a price $p = v(s)$, so that the optimal price need not be set below this minimum value.

The next section analyzes the case of non-linear preferences with the addition of an allocation constraint similar to the one considered in Section 3.

[15] Specifically, as we show in the Proof in the Appendix, $\mathcal{Q} = \frac{\beta(s)}{\delta}$ where $\beta(s)$ is the Kuhn-Tucker multiplier associated to the constraint (iv) of $(\mathcal{P}2)$.

4.2 QUANTITY CONSTRAINTS

We now introduce a quantity constraint as defined in Equation (11). The seller's optimization problem then becomes

$$
\begin{array}{ll}
\max\limits_{\{q_i\}_1^n} \int_\Omega \left\{ v(s) + \sum_i \left[q_i(s) \left((\alpha-1)\, v(s) - \frac{\delta}{2} q_i(s) - \frac{\alpha}{n} (\overline{s} - s_i) \right) \right] \right\} f(s) ds & \\
s.t: & \\
(i) \quad U^i(\underline{s}, \underline{s}) = 0 & \\
(ii) \quad E_{s_{-i}} \left[\frac{\partial q_i(s)}{\partial s_i} \right] \geq 0 \qquad \text{for all } i \text{ and all } s & \\
(iii) \quad q_i(s) \geq 0 \qquad \text{for all } i \text{ and all } s & \\
(iv) \quad \sum\limits_i q_i(s) \leq 1 \qquad \text{for all } s & \\
(v) \quad \sum\limits_i q_i(s) \geq 1 - K \qquad \text{for all } s. &
\end{array}
\tag{$\mathcal{P}3$}
$$

Problem *($\mathcal{P}3$)* is the same as *($\mathcal{P}2$)* with the addition of constraint *(v)*. The optimal mechanism is then described by the following proposition:

PROPOSITION 3 *When institutional investors have non-linear preferences and retail investors are quantity constrained, the optimal IPO is characterized by the following allocation and pricing rules:*

1. *(**Allocation Rule**) there exists a threshold value of the signals* $s_i^\circ = \frac{\alpha \overline{s} - (\alpha-1) v_{-i}}{2\alpha - 1}$*, with* $v_{-i} = \sum\limits_{j \neq i} s_j$*, such that,*
   - *If* $1 - K \leq \sum\limits_i \widetilde{q}_i(s) \leq 1$*, all the institutional investors reporting a signal above the threshold* s_i° *obtain a positive quantity* $\widetilde{q}_i(s) = \dfrac{(\alpha-1)\, nv(s) - \alpha\,(\overline{s} - s_i)}{n\delta}$*. The remaining shares are allocated to retail investors.*
   - *If* $\sum\limits_i \widetilde{q}_i(s) < 1 - K$*, then retail investors receive* K *shares; the remaining* $1 - K$ *shares are allocated to all the institutional investors reporting a signal* $s_i > s_i^K$ *with* $s_i^K < s_i^\circ$*.*

- *If* $\sum_i \widetilde{q}_i(s) > 1$ *(oversubscription), the institutional investors with signals above the threshold* s_i° *obtain a quantity* $\widehat{q}_i < \widetilde{q}_i$ *(rationing). The retail investors receive no shares..*

2. *(**Pricing Rule**) The optimal pricing rule is such that*

   - *If the allocation constraint is not binding, the optimal IPO can be implemented by uniform pricing;*
   - *Conversely, if the constraint is binding, then a discriminatory pricing rule is optimal, such that:* $p_i(v) = p_I < v(s)$ *for all* i *and* $p_R = v(s)$.

The allocation rule result is quite intuitive. When there is a restriction on the allocation rule, the seller may be forced to allocate to the institutional investors more shares than would otherwise be optimal. Specifically, in order to place all the shares, the issuer will have to allocate a positive amount of shares to some institutional investors reporting a signal below the threshold s_i°.

As for the pricing rule, in the presence of an allocation constraint, whether the optimal price is discriminatory or uniform depends on whether the constraint is binding or not in equilibrium, which in turn depends on the value of the parameters α and δ. For sufficiently high values of α and/or low values of δ the allocation constraint will not be binding, and thus, we are back to the case of no constraints where a uniform price is optimal. An alternative reading of this result is that if parameter K were to be determined endogenously, it would be optimally set so that the constraint never binds.

5 Cash-constrained retail investors

The results of the previous sections suggest that the form of the optimal pricing scheme depends on whether there is a *binding* allocation constraint, which is in turn affected by the specific characteristic of the institutional investors' preferences. We can also think about different types of constraints. Hence, in this section, we analyze the case of cash-constrained retail investors that can afford/are willing to invest at most K in the sale, in the context of linear preferences of institutional investors.

The new optimization program for the seller is similar to program $(\mathcal{P}1)$, except that the last constraint, *(v)*, is replaced by the cash constraint for retail investors.

$$\begin{array}{ll}
\max\limits_{\{q_i\}_1^n} \displaystyle\int_\Omega \left\{ v(s) - \textstyle\sum_i \left[\dfrac{1}{n}(\overline{s} - s_i) q_i(s) \right] \right\} f(s) ds & \\
s.t: & \\
(i) \quad U_i(\underline{s}, \underline{s}) = 0 \qquad \text{for all } i & \\
(ii) \quad E_{s_{-i}} \left[\dfrac{\partial q_i(s)}{\partial s_i} \right] \geq 0 \qquad \text{for all } i \text{ and all } s_i & \\
(iii) \quad q_i(s) \geq 0 \qquad \text{for all } i \text{ and all } s & (\mathcal{P}4) \\
(iv) \quad \sum\limits_i q_i(s) \leq 1 \qquad \text{for all } s & \\
(v) \quad \left(1 - \sum\limits_i q_i(s)\right) p_R(s_i, s_{-i}) \leq K \qquad \text{for all } s. &
\end{array}$$

We are aware that with a continuum of retail investors, it is less plausible to justify the existence of such a constraint at the aggregate level, even though the constraint holds individually. However, the purpose of our analysis is to investigate whether different kinds of allocation constraints affect differently the optimal pricing rule. In other words, we want to understand whether it is just the existence of such a constraint that matters or also its shape (the functional form). We expect this to be the case, as different forms of allocation constraints will determine the tightness of the allocation rule. For instance, contrary to the maximum quantity constraint previously considered, the above *cash* constraint depends on both the number of shares purchased and their price, thereby providing, by construction, more flexibility to the issuer when it comes to IPO design, as he can rely on both price and quantity to achieve optimality.

The optimal mechanism is described in the following proposition:

PROPOSITION 4 *With risk neutral institutional investors and cash constrained retail investors, the optimal IPO is characterized as follows:*

1. ***(Allocation rule)*** *The issuer satisfies retail investors up to their cash constraint. The remaining shares are allocated to the institutional investor reporting the highest signal*

$s_m = \max\{s_1, s_2, ..., s_n\}$.

2. (***Pricing rule***) *There exists at least one uniform price that implements the optimal IPO.*

The above results imply that, as with linear preferences, retail investors have priority in the allocation of shares. In this case, though, the allocation rule depends on the price. We thus need to solve for the optimal pricing rule and then step back to obtain explicitly the equilibrium quantities. This suggests that the issuer has enough leeway to choose an allocation rule that can be supported by an optimal uniform price.

More generally, the cash constraint restricts less the allocation rule than the quantity constraint. The flexibility gained by the issuer is enough to allow uniform pricing at the optimum. Again, we do not obtain a unique optimal price. We find that there may be more than one optimal uniform price, and that equilibria may exist with discriminatory pricing.

6 Discussion

This paper has investigated the conditions under which the uniform pricing rule in IPOs is optimal. The issuer can potentially use both quantity and price discrimination to elicit information from informed investors and achieve optimality (i.e. maximize the sale proceeds). Our findings show that as long as quantity discrimination is sufficiently unrestricted, the issuer does not also need price discrimination, so that the optimal IPO can be implemented with uniform pricing. The allocation rule may be restricted because of direct allocation constraints and/or because institutional investors have non-linear preferences. Allocation constraints in our model can be of two types, cash or quantity constraints. We show that the most important determinant of the optimal pricing rule is the existence of an allocation constraint and its *shape.* Conversely, institutional investors' preferences are shown to have a direct impact on the optimal allocation rule. Specifically, when non-linear preferences are considered, in equilibrium, contrary to the case of risk neutrality, institutional investors have priority in share allocation over retail investors who become residual claimants. Furthermore, the institutional investors' preferences also indirectly affect the optimal pricing rule to the extent that the specific values of parameters

α and δ determine whether or not the allocation constraint is binding in equilibrium.

The model delivers a number of empirical implications:

- The most direct implication is that uniform pricing and discriminatory pricing will lead to the same IPO proceeds if the allocation rule is unrestricted. Seeing as discriminatory pricing is forbidden in most countries, a direct test of our results will only be possible in the few countries that have allowed discriminatory pricing in IPOs. For example, both Taiwan and Japan have used discriminatory IPO auctions (i.e. pay-your-bid auctions, see Jagannathan and Sherman, 2006), that could allow a test of our results. However, there are still no examples of discriminatory pricing in bookbuilding.

- A second empirical implication predicts that we should observe greater underpricing in IPOs characterized by quantity constraints rather than cash constraints. This raises the issue of understanding which of these two types of constraints is more relevant in practice, which is another interesting empirical research question.[16]

- Related to the previous point, our results predict that underpricing should be more important in IPOs where the existence of a quantity constraint is associated with a low demand for the shares by the institutional investors.

Appendix

The seller's optimization program ($\mathcal{P}$)**.** From the RPC and the maximand we can see that the seller's profit is increasing in the retail investors' payments, therefore at the optimum the RPC binds. We can then rewrite the RPC as follows:

$$\int_{\Omega} p_R(s)q_R(s)f(s)ds = \int_{\Omega} v(s)q_R(s)f(s)ds = \int_{\Omega} v(s)\left(1 - \sum_i q_i(s)\right) f(s)ds\,, \tag{13}$$

where we have replaced $q_R(s)$ from the FAC. We now turn to the IIC. By applying the envelope theorem to the maximization problem in Equation (IICa) and then taking expectations over

[16] Derrien (2005) is, to our knowledge, the only paper to document that in some French IPOs a fraction of the shares on sale is explicitly reserved to retail investors.

s_i, we have:[17]

$$U_i(s_i, s_i) = U_i(\underline{s}, \underline{s}) + \int_{\underline{s}}^{s_i} \left\{ \frac{1}{n} \int_{\Omega_{-i}} z_2(q_i(\widetilde{s}_i, s_{-i}), v(s)) f_{-i}(s_{-i}) ds_{-i} \right\} d\widetilde{s}_i. \tag{14}$$

Since the seller's payoff is decreasing in the information rents paid to the informed investors, at the optimum he will set $U_i(\underline{s}, \underline{s}) = 0$, so that the informed investors with the lowest evaluations receive zero rents at the optimum. Inverting integrals, taking the expectation over s_i and applying Fubini's theorem[18] to Equation (14) yields

$$\int_{\Omega_i} U_i(s_i, s_i) f_i(s_i) ds_i = \frac{1}{n} \int_{\Omega_{-i}} \left\{ \int_{\Omega_i} \left[\int_{\underline{s}}^{s_i} z_2(q_i(\widetilde{s}_i, s_{-i}), v(s)) d\widetilde{s}_i \right] f_i(s_i) ds_i \right\} f_{-i}(s_{-i}) ds_{-i}. \tag{15}$$

Integration by parts of the integral of the term in brackets on the l.h.s. yields the following

$$\int_{\Omega_i} U_i(s_i, s_i) f_i(s_i) ds_i = \frac{1}{n} \int_{\Omega} (\overline{s} - s_i) z_2(q_i(s), v(s)) f(s) ds. \tag{16}$$

Finally, from the definition of the expected utility of institutional investors, we have for all i and all s_i

$$\int_{\Omega_{-i}} p_i(s) q_i(s) f_{-i}(s_{-i}) ds_{-i} = \int_{\Omega_{-i}} z(q_i(s), v(s)) f_{-i}(s_{-i}) ds_{-i} - U_i(s_i, s_i) \geq 0. \tag{17}$$

Taking expectations over s_i and using Equations (16) and (13) we obtain the seller's objective function as stated in the optimization program ($\mathcal{P}$). Constraint *(ii)* in ($\mathcal{P}$) is a monotonicity condition which is a sufficient condition for truth-telling to be optimal.[19] For prices paid by institutional investors, plugging Equation (14) into Equation (17) yields Equation (7). Prices for retail investors satisfy their (binding) participation constraint. ■

Proof of Proposition 1. For generality, we prove the result using a generic distribution $f(\cdot)$ satisfying the *increasing hazard rate assumption.* Writing Equation (16) with the general distribution function gives the following objective function for the seller

$$\max_{\{q_i\}_1^n} \int_{\Omega} \left\{ v(s) - \sum_i \left[\frac{1}{n} \left[\frac{1 - F_i(s_i)}{f_i(s_i)} \right] q_i(s) \right] \right\} f(s) ds. \tag{18}$$

[17] Notice that the equation below holds at the optimum, i.e. for $\widehat{s}_i = s_i$.

[18] Fubini's theorem states that we can invert integrals whenever the integrand is finite.

[19] The monotonicity condition is derived by using Assumptions 4 and 5. In the mechanism design literature, it is often referred to as an *implementability condition.*

This objective function is decreasing in the quantity allocated to institutional investors ($q_i(s)$). The cost of allocating a positive quantity to them, measured by

$$\int_{\Omega} \left\{ \sum_i \left[\frac{1}{n} \left(\frac{1 - F_i(s_i)}{f_i(s_i)} \right) q_i(s) \right] \right\} f(s) ds, \tag{19}$$

is decreasing in the signal s_i because of the increasing hazard rate assumption. Consequently, the allocation rule maximizing the seller's revenues consists in allocating as much as possible to retail investors (i.e. up to their budget constraint) and any residual quantity to the institutional investor(s) having (and reporting) the highest announced signal (i.e. the agent with the signal $s_m = \max\{s_1, s_2, ..., s_n\}$). Since signals are continuously distributed only one agent announces s_m. Denote by $q_m(s)$ the quantity allocated to this agent.

With risk neutral investors, the pricing conditions (7) and (8) become

$$\int_{\Omega_{-i}} p_i(s) q_i(s) f_{-i}(s_{-i}) ds_{-i} = \int_{\Omega_{-i}} \left\{ v(s) q_i(s) - \frac{1}{n} \left[\int_{\underline{s}}^{s_i} q_i(\widetilde{s}_i, s_{-i}) d\widetilde{s}_i \right] \right\} f_{-i}(s_{-i}) ds_{-i} \quad \text{for all } s_i \text{ and all } i \tag{20}$$

and

$$\int_{\Omega} p_R(s) f(s) ds = \int_{\Omega} v(s) f(s) ds. \tag{21}$$

We prove the result by contradiction. Assume the existence of a uniform price function, such that $p(s) = p_m(s) = p_R(s)$ for all s, that implements the optimal mechanism. Applying this to Equation (20), taking expectations over s_i and summing over n gives

$$\int_{\Omega} p(s) \left[\sum_i q_i(s) \right] f(s) ds = \int_{\Omega} \left\{ v(s) \left[\sum_i q_i(s) \right] - \left\{ \sum_i \frac{1}{n} \left[\int_{\underline{s}}^{s_i} q_i(\widetilde{s}_i, s_{-i}) d\widetilde{s}_i \right] \right\} \right\} f(s) ds. \tag{22}$$

Since $\sum_i q_i(s) = 1 - K$ and $\int_{\Omega} p(s) f(s) ds = \int_{\Omega} v(s) f(s) ds$, a necessary condition for the existence of a uniform price is

$$\int_{\Omega} \left\{ \sum_i \frac{1}{n} \left[\int_{\underline{s}}^{s_i} q_i(\widetilde{s}_i, s_{-i}) d\widetilde{s}_i \right] \right\} f(s) ds = 0.$$

However, $q_i(s)$ is always non-negative and strictly positive for one investor. So this last equation never holds which contradicts the initial assumption about the existence of an optimal uniform price function. ■

Proof of Proposition 2. For the first part of this proposition, we consider the relaxed problem (i.e. we drop the monotonicity constraint in program ($\mathcal{P}2$)) and check it ex post. As such, the objective function becomes an ordinary maximand with the constraints defined at each point and can be maximized pointwise on Ω. The number of shares, $q_i(s)$, the seller must assign to investor i in order to elicit his information is given by the following maximization problem, for each $s \in \Omega$,

$$\begin{aligned}
&\max_{\{q_i\}_{i=1,..n}} \sum_i \left[q_i(s) \left((\alpha - 1)\, v(s) - \frac{\delta}{2} q_i(s) - \frac{\alpha}{n}(\overline{s} - s_i) \right) \right] \\
&s.t: \\
&U_i(\underline{s}, \underline{s}) = 0 \qquad \text{for all } i \\
&q_i(s) \geq 0 \qquad \text{and} \qquad \sum_i q_i(s) \leq 1 \qquad \text{for all } i \text{ and } s.
\end{aligned}$$

The Kuhn-Tucker conditions for this problem are

$$\begin{cases}
(\alpha - 1)v(s) - \delta q_i - \dfrac{\alpha}{n}(\overline{s} - s_i) + \lambda_i(s) - \beta(s) = 0, & \text{for all } i \\
\lambda_i(s) q_i(s) = 0 \\
\beta(s)[1 - \sum_i q_i(s)] = 0.
\end{cases} \tag{23}$$

With λ_i and β being the Kuhn-Tucker multipliers associated to the feasibility constraint and the FAC, respectively. Now denote the seller's objective function by $H(q, v(s))$, that is

$$H(q, v(s)) = \sum_i \left[q_i(s) \left((\alpha - 1)\, v(s) - \frac{\delta}{2} q_i(s) - \frac{\alpha}{n}(\overline{s} - s_i) \right) \right], \tag{24}$$

with $q_i \in [0, 1]$ and $s \in \Omega = [\underline{s}, \overline{s}]^n$. This function is concave in q_i for all i, since $\dfrac{\partial^2 H}{\partial q_i^2} \leq 0$. Now let $s^{\circ}{}_i(v_{-i}) = \frac{\alpha \overline{s} - (\alpha-1) v_{-i}}{(2\alpha - 1)}$ for all i and v_{-i} and define the following sets for all $s \in \Omega$,

$$\begin{aligned}
N^-(s) &= \{i \in N \mid s_i \leq s^{\circ}{}_i(v_{-i})\}, \\
N^+(s) &= \{i \in N \mid s_i > s^{\circ}{}_i(v_{-i})\}.
\end{aligned}$$

We can easily show that for each $i \in N^-(s)$ it must hold that $q_i(s) = 0$. Then, for each $i \in N^+(s)$, define the quantity $\widetilde{q}_i$ by

$$(\alpha - 1)v(s) - \delta \widetilde{q}_i(s) - \frac{\alpha}{n}(\overline{s} - s_i) = 0. \tag{25}$$

For each $i \in N^+(s)$, $\widetilde{q}_i(s)$ is positive. If, $\sum_{i \in N^+(s)} \widetilde{q}_i(s) \leq 1$, then the quantity $\widetilde{q}_i(s)$ is the solution of our mechanism.[20] If however, $\sum_{i \in N^+(s)} \widetilde{q}_i(s) > 1$, which corresponds to *oversubscription* of the new shares, then the quantity $\widetilde{q}_i(s)$ cannot be optimal as it violates the FAC. The optimal quantities, which we denote by $\widehat{q}_i(s)$, are given by the solution to the following system of equations,

$$\begin{cases} \widehat{q}_i(s)[(\alpha - 1)v(s) - \delta \widehat{q}_i(s) - \dfrac{\alpha}{n}(\overline{s} - s_i) - \beta(s)] = 0 \\ \displaystyle\sum_{i \in N^+(s)} \widehat{q}_i(s) = 1; \qquad \beta(s) > 0, \end{cases} \tag{26}$$

which implies that $\widehat{q}_i(s)$ is either zero or is positive and solves the following equation

$$(\alpha - 1)v(s) - \delta \widehat{q}_i(s) - \frac{\alpha}{n}(\overline{s} - s_i) - \beta(s) = 0. \tag{27}$$

Clearly, in this case, all the shares are allocated to institutional investors. Retail investors receive nothing in equilibrium.

To prove the existence of an optimal uniform price function, we proceed in three steps. In the first step (Step 1), we derive a condition for the existence of a uniform pricing rule for institutional investors, i.e. $p_i(s) = p_I(s)$ for all i and s. Subsequently (Step 2), we show the existence of a unique uniform pricing rule; last, in Step 3, we show that the same pricing rule can be applied to retail investors, i.e. $p_I(s) = p_R(s)$.

Step 1: The linear transfer for institutional investors must satisfy Equation (7). Consider the following price function

$$p_i^0(s) = \frac{z(q_i(s), v(s)) - \frac{1}{n}\left[\int_{\underline{s}}^{s_i} z_2(q_i(\widetilde{s}_i, s_{-i}), v(\widetilde{s}_i, s_{-i}))d\widetilde{s}_i\right]}{q_i(s)}, \tag{28}$$

for each s_i and each $q_i(s)$ such that $q_i(s) \neq 0$.[21] Any price satisfying Equation (7) can also be written as $p_i(s) = p_i^0(s) + \Phi_i(s)$, where Φ_i satisfies the following equation

$$\int_{\Omega_{-i}} \Phi_i(s) q_i(s_i, s_{-i}) f_{-i}(s_{-i}) ds_{-i} = 0, \qquad \text{for all } i \text{ and } s_i. \tag{29}$$

[20] In this case, the equation defining $\widetilde{q}_i(s)$ is the FOC of our objective function H, since the Kuhn-Tucker multipliers, $\lambda_i(s)$ and $\beta(s)$ are both zero.

[21] Otherwise $p_i^0(s) = 0$.

The existence of a uniform price function for institutional investors implies that the marginal effects of changes in the private signal of different investors on prices are equal, that is $\frac{\partial p(s)}{\partial s_i} = \frac{\partial p(s)}{\partial s_j}$ for all i and j.[22] Applying this uniformity condition to an admissible pricing function gives

$$\frac{\partial p_i^0(s)}{\partial s_i} + \frac{\partial \Phi_i(s)}{\partial s_i} = \frac{\partial p_i^0(s)}{\partial s_j} + \frac{\partial \Phi_i(s)}{\partial s_j}. \tag{30}$$

Multiplying both sides by $q_i(s)$, writing $\frac{\partial p_i^0(s)}{\partial s_i}$ and $\frac{\partial p_i^0(s)}{\partial s_j}$, and considering from Proposition 2 the fact that $\frac{\partial q_i(s)}{\partial s_i} - \frac{\partial q_i(s)}{\partial s_j} = \frac{\alpha}{n\delta}$ for each s and each i, allows us to write Equation (30) as follows

$$\frac{\partial}{\partial s_i}\left[\Phi_i(s) - \frac{\alpha}{n}\frac{\int_{\underline{s}}^{s_i} q_i(\widetilde{s}_i, s_{-i})d\widetilde{s}_i}{q_i(s)}\right] = \frac{\partial}{\partial s_j}\left[\Phi_i(s) - \frac{\alpha}{n}\frac{\int_{\underline{s}}^{s_i} q_i(\widetilde{s}_i, s_{-i})d\widetilde{s}_i}{q_i(s)}\right] + \frac{\alpha}{2n}, \tag{31}$$

for each i and each j. This is a partial differential equation in $\Phi_i(s) - \frac{\alpha}{n}\frac{\int_{\underline{s}}^{s_i} q_i(\widetilde{s}_i, s_{-i})d\widetilde{s}_i}{q_i(s)}$ whose generic solution is given by

$$\Phi_i(s) = \varphi(s_1 + ... + s_n) + \frac{\alpha}{n}\frac{\int_{\underline{s}}^{s_i} q_i(\widetilde{s}_i, s_{-i})d\widetilde{s}_i}{q_i(s)} + \frac{\alpha}{2n}s_i, \quad \text{for all } i \tag{32}$$

where $\varphi(\cdot)$ is a twice-differentiable function defined on the set $[n\underline{s}, n\overline{s}] = n\Omega$. So, we have shown that the optimal mechanism may be implemented by a uniform price schedule if and only if

$$p(s) = p_i(s) = p_i^0(s) + \varphi(s_1 + ... + s_n) + \frac{\alpha}{n}\frac{\int_{\underline{s}}^{s_i} q_i(\widetilde{s}_i, s_{-i})d\widetilde{s}_i}{q_i(s)} + \frac{\alpha}{2n}s_i\,, \tag{33}$$

for all i, where φ is a twice differentiable function satisfying the following integral equation

$$\begin{aligned}&\int_{\Omega_{-i}} \varphi(s_1 + ... + s_n) q_i(s_i, s_{-i}) f_{-i}(s_{-i}) ds_{-i} = \\ &-\int_{\Omega_{-i}}\left[\frac{\alpha}{n}\frac{\int_{\underline{s}}^{s_i} q_i(\widetilde{s}_i, s_{-i})d\widetilde{s}_i}{q_i(s)} + \frac{\alpha}{2n}s_i\right] q_i(s_i, s_{-i}) f_{-i}(s_{-i}) ds_{-i} = g(s_i).\end{aligned} \tag{34}$$

[22] This is true in our model where all signals are equally informative. In this case, the uniform price, if it exists, must be equally sensitive to a change of the private signal of any investor. In other words, what matters is by how much the signal has changed and not whose signal it was.

Proving the existence of a uniform price schedule is equivalent to proving the existence of this function φ.

STEP 2: We first show that Equation (34) can be written as a Volterra integral equation of the first kind.[23] Then, by simply applying the general properties of this kind of integral equation we can prove the existence and the *uniqueness* of the function φ. To do so, we first need to transform Equation (34) into a simple integral equation, i.e. with the support defined on $\mathbb{R}$. Notice that, since $q_i(s_i, s_{-i}) = 0$ when $s_i \leq s_i^0(v_{-i})$, the support of the integral Equation (34) is equal to $\Omega^0_{-i} = \{(s_1, s_2, .., s_{i-1}, s_{i+1}, .., s_n) \mid \sum_{j\neq i} s_j = v_{-i} \geqslant v^0_{-i}(s_i)\}$ where $v^0_{-i}(s_i)$ is defined as the inverse of $s_i^0(v_{-i})$, i.e.

$$v^0_{-i}(s_i) = \frac{\alpha\overline{s} - (2\alpha - 1)s_i}{\alpha - 1}. \tag{35}$$

Also, from the definition of the optimal quantity, the l.h.s. of Equation (34) equals $\int_{\Omega^0_{-i}} \varphi(s_i + v_{-i})q_i(s_i, v_{-i})f_{-i}(s_{-i})ds_{-i}$. By applying the Generalized Change Variable Theorem (GVCT) we can set $v_{-i} = \gamma_i(s_{-i})$ for each i, which finally implies that $\gamma_i(\Omega^0_{-i}) = [v^0_{-i}(s_i); (n-1)\overline{s}] \in \mathbb{R}$.[24] One key implication of the GVCT is that there exists a measure λ defined over $[v^0_{-i}(s_i); (n-1)\overline{s}]$ such that

$$\int_{\Omega^0_{-i}} \varphi(s_i + v_{-i})q_i(s_i, v_{-i})f_{-i}(s_{-i})ds_{-i} = \int_{v^0_{-i}(s_i)}^{(n-1)\overline{s}} \varphi(s_i + v_{-i})q_i(s_i, v_{-i})\lambda(dv_{-i}). \tag{36}$$

Last, by applying the Radon-Nikodým theorem[25], we are also able to prove the existence of a density function ρ associated with the measure λ such that

$$\int_{\Omega^0_{-i}} \varphi(s_i + v_{-i})q_i(s_i, v_{-i})f_{-i}(s_{-i})ds_{-i} = \int_{v^0_{-i}(s_i)}^{(n-1)\overline{s}} \varphi(s_i + v_{-i})q_i(s_i, v_{-i})\rho(v_{-i})dv_{-i}. \tag{37}$$

From the above result, the integral Equation (34) reduces to

$$\int_{v^0_{-i}(s_i)}^{(n-1)\overline{s}} \varphi(s_i + v_{-i})q_i(s_i, v_{-i})\rho(v_{-i})dv_{-i} = g(s_i). \tag{38}$$

[23] A Volterra integral equation of the the first kind is defined in the following way:

$$\int_{y_0}^{\tau(x)} f(x, y)h(x, y)dy = g(x);$$

in other words, one of the integral limits must depend on the variable x.

[24] See Dunford and Schwartz (1988, 3rd Ed.), chapter 3, lemma 8, page 182.

[25] See, for example, Dunford and Schwartz (1988, 3rd Ed.), chapter 3, theorem 2, page 176.

This is a Volterra integral equation of the first kind which ensures that, as long as the function g is well behaved, a solution in φ always exists. This shows the existence of a unique uniform pricing rule for institutional investors. We denote such a pricing rule by $p_I(s)$ in the following.

Step 3: To complete the proof it remains to show that the uniform price for institutional investors also applies to retail investors. This is equivalent to showing that $p_I(s)$ satisfies the retail investors' participation constraint. This problem is relevant only in the cases for which both retail and institutional investors receive positive amounts of shares at the optimum. We start by defining the following sets:

- $\Omega^- = \{s \mid q_i = 0 \text{ for all } i\}$, i.e. all the quantity is distributed to retail investors;
- $\Omega \backslash \Omega^- = \{s \mid q_i(s) \neq 0 \quad \text{for at least one } i\}$.

Recall that the retail investors' participation constraint can be written as follows

$$\int_\Omega p_R(s) q_R(s) f(s) ds = \int_\Omega v(s) \left(1 - \sum_i q_i(s)\right) f(s) ds. \tag{39}$$

Proving that uniform pricing applies to all investors is then equivalent to demonstrating the existence of a price function $p_R(s)$ solving the above integral equation and such that:

$$p_R(s) = \begin{cases} p^-(s) & \text{for all } s \in \Omega^- \\ p_I(s) & \text{for all } s \in \Omega \backslash \Omega^-. \end{cases} \tag{40}$$

which requires that retail investors are charged different prices depending on whether they receive the whole quantity or not. The problem then boils down to proving the existence of a price $p^-(s)$ such that

$$\int_{\Omega^-} p^-(s) f(s) ds = \int_\Omega v(s) \left(1 - \sum_i q_i(s)\right) f(s) ds - \int_{\Omega \backslash \Omega^-} p_I(s) \left(1 - \sum_i q_i(s)\right) f(s) ds. \tag{41}$$

We then prove the following:

LEMMA 1 *A price function $p^-(s)$ as defined in Equation (41) exists if and only if it satisfies the following equation*

$$\int_{\Omega^-} p^-(s) f(s) ds = \int_\Omega \{v(s) + H(q, v(s))\} f(s) ds - \int_{\Omega \backslash \Omega^-} p_I(s) f(s) ds\,, \tag{42}$$

where the function $H(q, v(s))$ defines the seller's payoff at the optimum.

Proof of Lemma 1:

For the *only if* part, suppose there exists a price $p^-(s)$ which satisfies the participation constraint of retail investors. This implies that the seller's expected payoff at the optimum is given by

$$\int_{\Omega^-} p^-(s)f(s)ds + \int_{\Omega\setminus\Omega^-} p_I(s)f(s)ds = \int_{\Omega} \{v(s) + H(q, v(s))\} f(s)ds \tag{43}$$

due to the uniform pricing rule and the fact that all of the shares are always sold.

For the *If* part, taking the expectation of Equation (7) over s_i and summing over i gives

$$\int_{\Omega} \sum_i p_i(s)q_i(s)f(s)ds = \int_{\Omega} \sum_i \left\{ z(q_i(s), v(s)) - \frac{1}{n}\left[\left[\int_{\underline{s}}^{s_i} z_2(q_i(\widetilde{s}_i, s_{-i}), v(\widetilde{s}_i, s_{-i}))d\widetilde{s}_i\right]\right]\right\} f(s)ds. \tag{44}$$

Using the fact that all institutional investors pay the same price and adding $-\int_{\Omega} p_I(s)f(s)ds$ and $\int_{\Omega} v(s)\left(1 - \sum_i q_i(s)\right) f(s)ds$ to both sides yields the following

$$\begin{array}{c} \int_{\Omega} p(s)\left(1 - \sum_i q_i(s)\right) f(s)ds - \int_{\Omega} v(s)\left(1 - \sum_i q_i(s)\right) f(s)ds = \\ \int_{\Omega} p(s)f(s)ds - \int_{\Omega} \{v(s) + H(q, v(s))\} f(s)ds. \end{array} \tag{45}$$

If the uniform price exists then the r.h.s. of the above equation is equal to zero which immediately implies that the retail investors' participation constraint, on the l.h.s. of the equation, holds. This ends the proof of Lemma 1 ■

The right-hand side of Equation (42) does not depend on the agents' signals and so the integral equation defined over $p^-(s)$ has many solutions. This proves that the seller could find an optimal pricing function where a uniform price is applied to all investors. Note finally that the seller may choose among different pricing rules that may be applied for retail investors when s belongs to Ω^-. This ends the proof of Proposition 2. ■

Proof of Proposition 3. In the proof of Proposition 2, we defined $s^{\circ}{}_i(v_{-i}) = \frac{\alpha\overline{s}-(\alpha-1)v_{-i}}{(2\alpha-1)}$. Define the following sets for all s

$$\begin{aligned} N^-(s) &= \{i \in N \mid s_i \leq s^{\circ}{}_i(v_{-i})\} \\ N^+(s) &= \{i \in N \mid s_i > s^{\circ}{}_i(v_{-i})\} \\ \Omega^+(s) &= \{i \in N \mid q_i(s) > 0\} \end{aligned}$$

and let $\varpi^+ = Card(\Omega^+(s))$. Note that $\Omega^+(s)$ will be defined endogenously. The solution of the relaxed problem is very similar to that of Proposition 2. Pointwise maximization leads to the following Kuhn-Tucker conditions:

$$\begin{cases} (\alpha-1)v(s) - \delta q_i - \frac{\alpha}{n}(\overline{s}-s_i) + \lambda_i(s) - \beta(s) + \gamma(s) = 0, & \text{for all } i \\ \lambda_i(s) q_i(s) = 0 \\ \beta(s)[1 - \sum_i q_i(s)] = 0 \\ \gamma(s)[1 - \sum_i q_i(s) - K] = 0 \end{cases} \tag{46}$$

where λ_i, β and γ are the Kuhn-Tucker multipliers associated to the three remaining constraints. Note that β and γ cannot be different from zero at the same time. If we consider that $\gamma = 0$, then we obtain the same problem as in the case without budget constraints which proves the first, second and fourth points in the allocation part of the proposition, and the first point in the pricing rule part. For the third point of the allocation rule, suppose that $\sum_{i \in N^+(s)} \widetilde{q}_i(s) \leq 1-K$. In this case $\beta(s) = 0$ and the seller will choose the allocation rule that minimizes the cost associated to the allocation constraint, i.e., the parameter $\gamma(s)$. In this case, retail investors receive K shares and the optimal allocation rule for informed investors is that satisfying the following condition:

$$(\alpha-1)v(s) - \delta q_i - \frac{\alpha}{n}(\overline{s}-s_i) + \gamma(s) = 0 \tag{47}$$

for all $i \in \Omega^+(s)$, with $\gamma(s)$ solving the equation below:

$$\gamma(s) = \frac{\delta(1-K)}{\varpi^+} + \frac{\alpha}{n}\overline{s} - \frac{\alpha}{n\varpi^+} \sum_{j \in \Omega^+(s)} s_j - (\alpha-1)v(s) \tag{48}$$

By replacing $\gamma(s)$ in the previous FOC, we finally obtain that the optimal quantity for informed investors is

$$\widetilde{q}_i^K(s) = \frac{1}{\delta\varpi^+}\left\{\delta(1-K) + \frac{\alpha}{n}(\varpi^+ - 1)s_i - \frac{\alpha}{n} \sum_{j \in \Omega^+(s),\ j \neq i} s_j \right\} \tag{49}$$

which must be positive. So the threshold s_i^K is the highest value for which $\widetilde{q}_i^K(s) < 0$. We omit the proof of the optimal pricing function as it is the same as that of Proposition 2. ■

Proof of Proposition 4. We omit the proof of the optimal allocation rule as it is the same as for Proposition 1. For the pricing rule, note that the uniform price schedule must satisfy retail investors' participation constraints and budget constraints. This yields the following conditions

$$\begin{aligned}\int_{\Omega} v(s)\,(1-q(s))\,f(s)ds &= K \\ p(s)\,(1-q(s)) &= K \qquad \text{for all } s\end{aligned} \tag{50}$$

where p, the unitary price paid by the institutional agent, satisfies Equation (7). The substitution of p from the retail investors' budget constraint in Equation (7) gives the following condition

$$\int_{[\underline{s},s_m]^{n-1}} \left\{ \left[\frac{K}{(1-q(s))}\right] q(s) + \frac{1}{n}\left[\int_{\underline{s}}^{s_m} q(\widetilde{s}_m, s_{-m})d\widetilde{s}_m\right]\right\} f_{-m}(s_{-m})ds_{-m} = 0. \tag{51}$$

Note that in Equation (51), integration is only considered for the values of s_{-m} for which $q(s)$ assigned to agent m is positive. Proving the existence of a uniform price function is then equivalent to proving the existence of a function $q(s)$ that is the solution to equations (51) and (50). Finally denote by $z(s_{-m})$ a function such that:

$$\int_{[\underline{s},s_m]^{n-1}} z(s_{-m})f_{-m}(s_{-m})ds_{-m} = 0.$$

We can thus re-write Equation (51) as follows

$$\left[\frac{K}{(1-q(s))}\right] q(s) + \frac{1}{n}\left[\int_{\underline{s}}^{s_m} q(\widetilde{s}_m, s_{-m})d\widetilde{s}_m\right] = z(s_{-m}). \tag{52}$$

Denoting $X(s_m, s_{-m}) = \int_{\underline{s}}^{s_m} q(\widetilde{s}_m, s_{-m})d\widetilde{s}_m$,[26] we can also write Equation (52) as follows

$$[K - \frac{1}{n}X(s_m,s_{-m}) + z(s_{-m})]X'(s_m,s_{-m}) + \frac{1}{n}X(s_m,s_{-m}) - z(s_{-m}) = 0 \tag{53}$$

where $X'(s_m,s_{-m})) = \frac{\partial X(s_m,s_{-m})}{\partial s_m} = q(s_m,s_{-m})$.

We have so far shown that establishing the existence of a uniform price for all investors is equivalent to proving the existence of a function $X(s_m, s_{-m})$ that solves the non-linear first-order differential equation presented in Equation (53) with Equation (50) as a final condition.

[26] More precisely this is $X(s_m,s_{-m}) = \int_{s_l}^{s_m} q(\widetilde{s}_m, s_{-m})ds_m$ where s_l is the highest signal in s_{-m}. To keep notation simple, we rewrite the integral as indicated, using the fact that $q(\widetilde{s}_m, s_{-m}) = 0$ for some values of $\widetilde{s}_m$ in the interval $[\underline{s}, s_m]$.

This differential equation has at least one solution for each chosen function $z(\cdot)$. This gives the seller enough leeway to construct the desired uniform price consistent with the optimal mechanism. ■

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