WZ COUPLINGS OF D-BRANES AND O-PLANES

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Abstract In this short note we review the construction and role of Wess-Zumino couplings of Dirichlet branes and Orientifold planes, and show how these combine to give the Green-Schwarz anomaly cancelling terms.

The low energy effective theory of a D-brane has two types of couplings, the Dirac-Born-Infeld (DBI) and Wess-Zumino (WZ) terms. The former describes the coupling of the vector potential and scalars coupling to the NS⊗NS fields, while the latter gives the coupling of the vector potential and the pullback of the curvature fields to the R⊗R potentials. The first D-brane WZ term [1] simply states that a Dp-brane is charged under $C^{(p+1)}$, the $p+1$-form R⊗R potential

$$S^D_{WZ} = \mu_p \int_{\mathcal{B}_p} C^{(p+1)}\,, \qquad (1.1)$$

where μ_p is the charge density of a D-brane and $\mathcal{B}_p$ its worldvolume. It was then shown that D-branes couple to lower dimensional R⊗R potentials [2] and that a 'brane within brane' picture emerged

$$S^D_{WZ} = \mu_p \int_{\mathcal{B}_p} C \wedge \mathrm{tr}\exp(i\mathcal{F}/2\pi)\,. \qquad (1.2)$$

In the above C represents a formal sum of R⊗R potentials, $\mathcal{F} = F - B$, F is the worldvolume gauge field and B the NS⊗NS two form.[1]

An R^4 term found in [3] required, as a consequence of duality, the presence of a gravitational WZ coupling [4]. Later the entire WZ action

[1] $\mathcal{F}$, rather than F, is present in the action as the latter quantity is not gauge invariant.

was found using the anomaly inflow mechanism [5, 6] and reads

$$S^D_{WZ} = \mu_p \int_{\mathcal{B}_p} C \wedge \mathrm{tr}\exp(i\mathcal{F}/2\pi) \wedge \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}}\,, \tag{1.3}$$

where $\hat{A}$ is the Dirac or A-roof genus and $R_T,\ R_N$ are the pull-backs of the tangent and normal bundle curvatures to the D-brane world-volume, respectively.

This coupling has a natural interpretation within K-theory [7]. In K-theory there is a natural bilinear pairing of bundles given by the index of the Dirac operator on the tensor product of the two bundles. For E a bundle over a manifold X, with TX the tangent space of X, the map

$$E \to \mathrm{ch}(E)\sqrt{\hat{A}(TX)}\,, \tag{1.4}$$

is an isometry with respect to this pairing and the DeRham pairing in $H^*(X)$, the cohomology of X. In fact,following [13], it was shown [14] that D-brane charges are indeed classified by K-theory.

The D-brane WZ couplings were confirmed by string amplitude calculations. In [9] the four-form couplings were computed at tree level. A one-loop amplitude in [10] and tree-level amplitude in [11] confirmed the presence of all the couplings, while in [12] extra, non-anomalous couplings, as well as the normal bundle contributions were determined.

In [8], it was first observed that O-planes too carry gravitational WZ couplings. For consider Type I theory, in which 16 D9-branes, their images and an O9-plane, fill the spacetime. The WZ coupling of the theory is the Green-Schwarz (GS) coupling [15]. This differs from the WZ couplings of 32 D9-branes, indicating that O9-planes too have WZ couplings. By studying one-loop [10], and tree-level [11] scattering amplitudes it was found that the WZ coupling of Op-planes is

$$S^O_{WZ} = -2^{p-4}\mu_p \int_{\mathcal{B}_p} C \wedge \sqrt{\frac{L(R_T/4)}{L(R_N/4)}}\,, \tag{1.5}$$

where L is the Hirzebruch polynomial. To see that the O9-plane and D9-brane WZ couplings match the GS term consider the following. The massless chiral fields of Type I string theory are a neutral gravitino, a neutral fermion of opposite chirality and SO(32) fermions. The total anomaly for this theory follows by descent from[2]

$$I \;=\; \frac{1}{2}2\pi \left(\hat{A}(R)(\mathrm{tr}e^{iR/2\pi} - 2) + \hat{A}(R)\mathrm{tr}e^{iF/2\pi}\right)\Big|_{\text{12-form}}$$

[2]Note that all of these are real fields.

$$= \quad \pi \frac{1}{(4\pi)^2}(\mathrm{tr}R^2 - \mathrm{tr}F^2)_\wedge X_8\,, \tag{1.6}$$

where

$$X_8 = \frac{1}{(4\pi)^4}\left(\frac{2}{3}\mathrm{tr}F^4 + \frac{1}{12}\mathrm{tr}R^4 + \frac{1}{48}(\mathrm{tr}R^2)^2 - \frac{1}{12}\mathrm{tr}R^2\mathrm{tr}F^2\right)\,. \tag{1.7}$$

In the units of [16] the action extracted from the string theory amplitudes [10, 11] is

$$\begin{aligned} S \quad &= \quad -\frac{1}{4\kappa_{10}^2}\int \mathrm{d}C^{(2)}{}_\wedge * \mathrm{d}C^{(2)} \\ &\quad + \ \mu_9 \int \left(\frac{2}{(4\pi)^2}C^{(6)}{}_\wedge(\mathrm{tr}R^2 - \mathrm{tr}F^2) + C^{(2)}{}_\wedge X_8\right)\,. \end{aligned} \tag{1.8}$$

Since $H = \mathrm{d}C^{(2)} + \ldots$ is gauge invariant and $\mathrm{d}C^{(6)} = * \mathrm{d}C^{(2)}$ the gauge transformation for $C^{(2)}$ is

$$\delta C^{(2)} = 4\mu_9\kappa_{10}^2(\omega_{2,Y}^1 - \omega_{2,L}^1)\,, \tag{1.9}$$

hence (1.8) has an anomalous variation which follows by descent from

$$I_{WZ} = 4\mu_9^2\kappa_{10}^2(\mathrm{tr}F^2 - \mathrm{tr}R^2)_\wedge X_8\,. \tag{1.10}$$

The Type I charge density satisfies

$$(\mu_9\kappa_{10})^2 = \frac{\pi}{2}\,, \tag{1.11}$$

and hence $I_{WZ} = I$ as required.

O-planes cannot couple to gauge fields so their WZ couplings have to be purely gravitational. Hence it is a consistency check for the D9-brane and GS gauge and mixed couplings to agree.

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