Imperfect Inspection Characterization for Gamma Process Structural Deterioration Model

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ABSTRACT: The deterioration of infrastructure facilities such as bridges has raised concerns over objective methodology to quantify the changes in their safety levels during the service life. In this paper the novel modeling of existing reinforced concrete structures likely future deterioration of strength is of interest. It is assumed that inspection outcomes are the source of data about the deterioration process and should provide help with the updating of the deterioration model with respect to the current structural condition. However, the inspection outcomes are associated with uncertainties that need to be taken into account for deterioration modeling. Sample reinforced concrete structure deterioration process is characterized as a time-dependent, non-negative and incremental process. In this paper we follow recent developments and the continuous gamma process has been adopted to represent the mathematical model of the deterioration process. In the current study two data sources were considered, the expert opinion, which is considered to reflect 'perfect inspection' and data obtained through scheduled inspections as 'imperfect inspection'. This paper reports on early development of the model to quantify the measurement error as inspection uncertainty and to establish continuous gamma process parameters for future deterioration prediction.

1 INTRODUCTION

The deterioration of infrastructure facilities such as highway bridges built in 50s and 60s has raised concerns over objective methodology to quantify the change in their safety level during the service life.

Highway structures are always subject to destructive effects of material ageing, harsh weather conditions, extensive corrosion etc. These factors, accompanied with uncertainty of design and construction, influence the deterioration of highway structures and result in the loss of serviceability and load carrying capacity over the long term, (Dong et al., 2010). The rate of strength loss is dependent on the deterioration mechanism, the aggressive environment, the degree of protection of structure against environment attack etc. (Ohadi & Micic, 2011). In many engineering problems but in particular for infrastructure such as highway bridges, there is very low availability of data i.e. sometimes only one or two observations.

Extensive data is needed to estimate the current safety level of existing structures and identify the degradation process. In recent years, modern technology has enabled greater variety of monitoring techniques and therefore availability of data from sensors, video imaging, etc. is increasing. It is identified that long established infrastructure inspection processes can be reviewed to reconcile quality and diversity of site-specific data, physical behavior models and technology. However, the non-destructive inspection techniques can bring in additional uncertainty in the deterioration model due to the uncertainty of inspection techniques (Ohadi & Micic, 2011). If the current status of deterioration is to be established on the basis of inspections it has become evident that the quality and consistency of the of the data acquired needs to be taken into account.

The deterioration of structures can be represented using deterministic or probabilistic approach. However, considering that the current and the future status of the structure are associated with many sources of uncertainty the deterministic approach cannot provide an appropriate mathematical model. Instead, probabilistic approach should be considered as more appropriate alternative (Frangopol et al., 2004). In order to develop a probabilistic deterioration model, all uncertainties associated with the deterioration process need to be identified (JCSS, 2008).

The random variable and stochastic processes are two alternative probabilisitc models to represent the deterioration process. In the last decades, researchers have focused on the random variable approach (Frangopol et al., 2004). However, it has been established that for civil engineering structures in particular available data is often insufficient to establish appropriate random variable models. Highway structures are often unique in design and site exposure so the lack of data for random variable models is even more pronounced. Availability of data is often poor and process of inspection or site testing very expensive. That is why, increasingly, stochastic process models have emerged as an alternative.

2 DETERIORATION REPRESENTATION

In the first decades after the Second World War, when much of developed world experienced rapid investment in infrastructure, highway bridge maintenance, repair, rehabilitation, and replacement activities were decided on an as-needed basis, employing the best existing practice (Frangopol et al., 2001). Such decision system has no mathematical model to predict the structural condition of the structure in the future. Thus, in such circumstances, decisions were often made using a deterministic model that is effectively relying on the expert judgment. During this time, the bridge engineering community focused on new construction, and the reactive strategies appeared sufficient to address any potential bridge safety issues (Frangopol et al., 2001). If aim is to ensure that the deterioration model is practical, the expert judgment model could be acceptable. However, the expert judgment model is discrete, as it cannot result in explicit formulation for future deterioration that includes the new inspection outcomes as they are obtained.

Increasingly, distinct deterioration patterns have emerged and a variety of techniques have been proposed to model processes associated with structural loads (Thoft-Christensen & Sorensen, 1987), and materials (Li, 2003, Biondini et al., 2006).

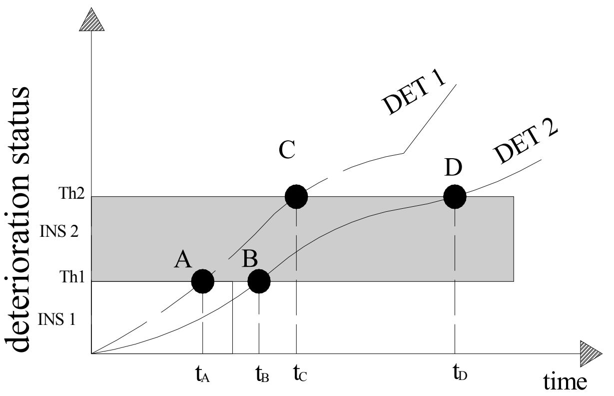

Figure 1. Illustration of progress of deterioration and suitability of inspection technique

Many factors such as the structural condition at inspection time, the sensitivity of inspection technique, environmental conditions, etc. can influence the inspection outcomes (Li, 2003). Figure 1 illustrates two alternative pathways for progress of deterioration over time and application of alternative inspection techniques. It is evident that the scale of deterioration and inspection technique cannot be independent as it would be rational to select the inspection technique on the basis of likely deterioration. Current practice specifies rather strict forms of inspection at pre-defined intervals and therefore over the lifecycle there will be an increase in uncertainty in respect to the true structural condition.

2.1 *Probabilistic model of deterioration*

Recently, many different probabilistic models have been considered to represent the deterioration process. In the probabilistic representation, deterioration model has to systematically combine subjective source of data such as the expert judgment and experiences from observations to obtain a balanced estimate of the bridge condition (JCSS, 2008). This is often difficult due to lack of data.

The applications of some of the probabilistic models of deterioration representation are given in (Thoft-Christensen & Sorensen, 1987, Mori & Ellingwood, 1992, Cheung et al., 1996, Yang et al., 2006, Cheung et al., 2010).

The probabilistic model for deterioration can be established using random variable or stochastic process representation. For random variable models see (Yang et al., 2006, Cheung et al., 2010, Marsh & Frangopol, 2008, Li, 2003). However, owing to the lack of failure data a random variable approach solely to represent deterioration process is unsatisfactory. Pandey et al. (2009) concluded that random variable model cannot reflect temporal variability associated with deterioration process (Pandey et al., 2009) because:

- A sample for component deterioration is set at the start and does not change over the lifetime.
- COV of deterioration model is constant over the lifetime.
- After the first inspection, the deterioration modelling is effectively deterministic.

Aforementioned concerns, and in particular that the deterioration throughout a specific sample path is deterministic in the random variable model, have brought introduction of stochastic process models (Pandey et al., 2009) as an alternative approach.

2.2 *Stochastic processes models for deterioration*

A sample deterioration process such as the loss of the section capacity of highway bridge can be identified as a time-dependent process with a stochastic damage accumulation model so a variety of stochastic process models have been considered to evaluate the structural condition. In recent years, diverse stochastic processes have been utilized to model the bridge deterioration.

In order to model the structural deterioration process using the stochastic process approach, random deterioration rate, Markov process, Brownian motion with drift and Gamma process can be applied (Van Noortwijk, 2009). Campoli & Ellingwood (2002) applied a Markov process to determine the damage accumulation of the structural component subject to aging (Campoli & Ellingwood, 2002). They have considered the average condition rate of the bridge's component as a random parameter. It should be noted that the Markov process can be classified as discrete Markov process i.e. Markov chain and continuous processes i.e. Brownian motion with drift.

Van Noortwijk & Frangopol (2004) have identified the failure rate model where the deterioration process is considered as Markov chain to describe and compare the structural condition in terms of a limited number of condition states (Van Noortwijk & Frangopol, 2004). Due to the characteristic of deterioration process that is a non-negative, independent and monotonic incremental process over the time in sequence of small intervals, this can lead to rather expensive subsequent actions such as additional repair, maintenance, etc.

Orcesi and Frangopol (2011) developed a model using the lifetime function to evaluate the probability of failure of bridge components. The possible outcomes with non-destructive inspections were incorporated in an event-tree model. The probability function of failure was assumed to be Weibull. It has been shown that for poor-quality inspections, there is a significant risk to overestimate the probability of safe performance, with such model.

3 CONTINOUS GAMMA PROCESS

Since seventies, continuous gamma process models were successfully used to model data on creep of concrete (Cinlar et al., 1979), fatigue crack growth (Li, 2003), corroded steel gates (Frangopol et al., 2004), thinning due to corrosion (Kallen & Van Noortwijk, 2005), chloride ingress into concrete (Bakker, 2004), and degradation of flexural moment capacity of concrete slab subject to the corrosion (Marsh & Frangopol, 2008).

In mathematical terms for the continuous gamma process modelling, we first consider a random variable X that has a gamma distribution with the shape parameter α>0 and scale parameter β>0. Its probability density function is given by:

$$Ga(x|\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} \exp(-\beta x) \qquad (1)$$

$$\Gamma(a) = \int_{z=0}^{\infty} z^{a-1} e^{-z} dz$$

where Γ( ) is the gamma function for $a > 0$.

Since gamma processes with different parameters are effectively independent we could obtain the conditional distribution for a variable only on the basis of the current observation. It is immediately noticeable that such representation would be appropriate for deterioration processes for standard structural applications.

Now, we can assume that α(t) is a non-decreasing, right-continuous, real valued function for t≥0, with α(0)=0. The gamma process with shape function α(t)>0 and scale parameter β>0 is a continuous-time stochastic process {X(t), t≥0} with the following properties:

$$\begin{Bmatrix} X(0) = 0 & \text{with probability one} \\ X(\eta) - X(t) \approx Ga(\alpha(\eta) - \alpha(t), \beta) & \text{for all } \eta > t \geq 0 \\ X(t) & \text{has independent increments} \end{Bmatrix} \qquad (2)$$

For more information of the time-dependent stationary gamma processes see Ohadi & Micic (2011).

3.1 *Continuous Gamma process parameter estimation*

Once it is identified that the continuopus gamma process is an appropriate model for deterioration, it is necessary to define its unique shape parameter α>0 and scale parameter β>0. These parameters are obtained on the basis of observational data and the three most common estimation methods are:

- Method of moments
- Maximum likelihood
- Bayesian statistics

More information about the estimation methods can be found in (Van Noortwijk, 2009). Here we implement for illustrative purpose the Method of Moments as the simplest of all.

3.1.1 *Method of moments*

In statistics, the method of moments is a very simple approach to estimate some population parameters, by equating sample moments with unobservable population moments and then solving the equations for the quantities to be estimated (Haldar & Mahadevan, 1999). It has to be acknowledged that estimates by method of moments can be used as the first approximation for gamma process parameters.

Once the gamma parameters have been estimated, the deterioration process can be presented as a gamma process over the time. Assuming that perfect inspection outcomes are considered to estimate the gamma process parameters.

Figure 2 illustrates the gamma process density functions for resistance degradation of a concrete bridge deck slab under the corrosion for three different time horizons. It is evident that using the functions from Figure 2 one can identify likelyhood of target resistance at certain time in the future.

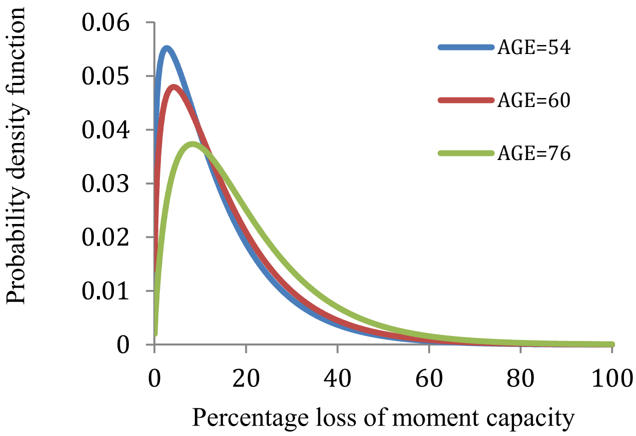

Figure 2. Sample probability density functions of the resistance degradation process

4 IMPERFECT INSPECTION

It is well known that unfortunately, available inspection techniques aren't perfect. In order to be able to characterize the deterioration using inspection outcomes the uncertainty associated with inspection outcomes needs to be characterized. These uncertainties reflect the inspection technique's features and several parameters can be used to quantify them, (Zhang & Mahadevan, 2001), namely:

- The probability of detection, POD(x), evaluates the capability of inspection technique to detect a given defect size. Practically, an inspection technique can't detect all sizes of a defect with certainty.
- The probability of false alarm, PFA(x), is a measure that determines the probability of reporting a defect that does not exist. This measure actually is the value of POD when the defect size is equal zero.
- The report ability threshold is another measure that represents the lower defect size, which can be detected by a particular inspection technique. This measure characterizes the inspection equipment accuracy and divides the defect's population into two groups; detected and undetected.
- The measurement error that represents the factor that is associated with the observed defect size. This factor is generally considered as normal distributed function and the actual defect size of observed established as linear function of the observed size and error.

$$x_t^a = x_t^m + \varepsilon \tag{3a}$$

Where the x_t^a is the actual defect size, ε denotes the error and x_t^m is the observed defect size.

All uncertainties that are described above should be taken into account for the comprehensive deterioration model characterization. However, in order to establish a realistic deterioration model, each parameter needs to be characterized separately.

4.1 *Review of imperfect inspection models*

Zhang and Mahadevan (2001) developed a comprehensive approach to integrate computational reliability methods and nondestructive inspection for fatigue reliability evaluation. They used three measures as POD, measurement error and PFA to quantify the inspection outcomes uncertainty. The POD has been modeled in form of an exponential function of actual fatigue crack depth while the relationship between the actual and measured crack depth size is expressed with a linear function. The PFA is obtained as POD when the actual defect size equals zero (Zhang & Mahadevan, 2001).

Pandey (1997) presented a probabilistic analysis framework to estimate the pipeline reliability incorporating the impact of inspection and repair activities planned over the service life. Two measures, POD and measurement error, have been taken into account to evaluate the uncertainty of in-line inspection outcomes. The POD has been determined by a parametric exponential function. Using the Bayes theorem, the probability density function of detectable defect size has been calculated from the overall defect size distribution (Pandey, 1997).

A Bayesian decision model by Kallen and Van Noortwijk (2005) proposed to determine the optimal inspection plans under uncertain deterioration. The measurement error has been considered as imperfect inspection parameter, which has been represented as a normal distribution (Kallen & Van Noortwijk, 2005).

Maes and Dann (2011) used a Bayesian approach in respect to pipelines in-line inspection data. In order to evaluate the inspection uncertainties, they used POD, PFA, measurement error and reportability. These measures were modeled similarly to Zhang & Mahadevan (2001) models, however the hierarchical Bayes model was employed to upgrade the deterioration model (Maes & Dann, 2011).

Frangopol et al. (1997) used the cumulative normal distribution function to calculate the probability of detection (Frangopol et al., 1997).

In this paper, the measurement error as one of the most significant and well-known inspection uncertainties is considered to demonstrate the inclusion of imperfect inspection outcomes for gamma process bridge deterioration characterization.

4.2 *Measurement error representation*

The measurement error is a known inspection uncertainty, which is often presented, as a normally distributed random variable with known variance and specific mean value (Zhang & Mahadevan, 2001). In order to take into account the measurement error at inspection with deterioration model, the actual defect size $X_{t\,insp}^{a}$ is presented in form of:

$$X_{t_{insp}}^{a} = X_{t_{insp}}^{m} + X_{\varepsilon} \tag{3b}$$

where $X_{t\,insp}^{a}$ is the actual defect size, $X_{t\,insp}^{m}$ is the measured defect size, which is modelled as a normally distributed random variable in this paper, and X_{ε} is the measurement error. The measurement error distribution definitions reflect the inspection technique features.

4.3 *Gamma process deterioration model with inspection measurement error*

Once the definitions of measurement error distribution function are provided through the empirical model, for the purpose of estimation of the gamma process parameters through the method of moments, the characteristic value of the actual defect size is needed. This is determined for a certain confidence level (*cl*).

This characteristic actual defect size represents the imperfect inspection outcome and will be used within the method of moments estimation. The following equation explains that how the actual defect size with certain confidence level can be determined.

$$X_{t_{insp}}^{a,cl} = x_{t_{insp}}^{i} \quad \text{for } P\left(0 < X_{t_{insp}}^{a} \le x_{t_{insp}}^{i}\right) = P_{cl} \tag{4}$$

where, P_{cl} , reflects agreed confidence level and $X_{t\,insp}^{i}$ is the value of the actual defect size at inspection time with certain probability. The parameter for gamma process model are then evaluated in a usual way, (Ohadi & Micic, 2011)

5 NUMERICAL EXAMPLE

In this section, a bridge slab deck is considered when it is subjected to reinforcement corrosion as most common defect of RC Highway Bridges. The updated gamma process is applied with respect to the imperfect inspection outcomes for this bridge to identify the percentage loss of moment capacity over the lifetime of this bridge. Firstly the degradation process of moment capacity of such slab is presented in Figure 3 assuming that the inspection outcomes are perfect (i.e. no measurement error).

However, the inspection outcomes that are used to estimate the gamma process parameters are often associated with uncertainties, as stated above. Here two different inspection techniques will be considered to characterize the inspection uncertainties.

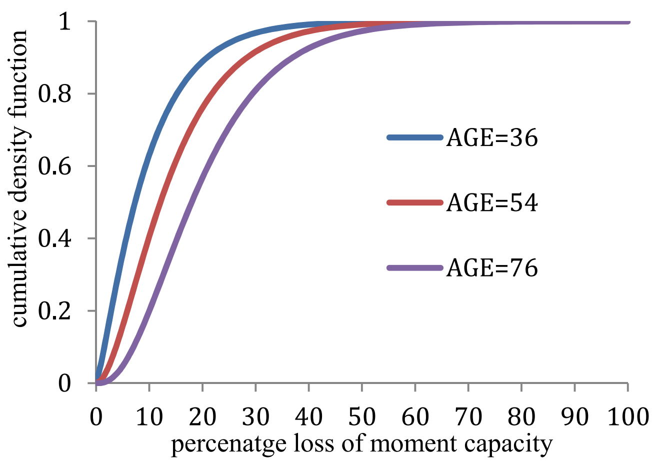

Figure 3 Cumulative distribution functions for moment capacity loss

Firstly, we consider the sample of perfect inspection outcomes for the constant (effectively deterministic) corrosion rate of 0.076 mm/year and the Gamma process parameters are demonstrated in Table 1 and 2 respectively.

Table 1 Sample inspection outcome for corrosion (deterministic outcomes of inspections)

Time (year)	M Before inspec. (kN.m)	M after inspec. (kN.m)	X_t^{im} (%)	γ_i(%)	ω_i (year)
0	62.21	62.21	0	-	-
18	62.21	61.22	1.59	1.59	18
24	61.22	58.13	6.56	4.97	6
30	58.13	55.16	11.33	4.77	6
36	55.16	52.18	16.12	4.79	6

where γ_ι is the deterioration incerement over the interval and ω_ι is the inspection interval and the other information in the table includes that deterioration initiation has occurred before year 18. Subsequently the parameters of the gamma process are obtained for this case of perfect inspection and shown in Table 2.

Table 2 Gamma process parameters (deterministic outcomes of inspections)

Time (year)	β	c
24	0.11	0.0314
30	0.16	0.06
36	0.19	0.086

For selected time horizons, the cumulative density function for X_t^m is obtained:

$$F(x_t^m|\alpha(t),\beta) = \int_0^x f(u_t^m|\alpha(t),\beta)du = \frac{\gamma(\alpha(t),\beta x)}{\Gamma(\alpha(t))}$$

where $\gamma(\alpha(t),\beta x)$ is the lower incomplete gamma function and $f(x_t^i\,|\alpha(t),\beta)=Ga(x|\alpha(t),\beta)$.

Subsequently we consider uncertainties due to imperfect inspection techniques (measurement error in this case) to estimate the Gamma process parameters. For measured defect size we assume normal distribution function with age dependent standard deviation and mean value equal to the 0.076 (Marsh & Frangopol, 2008). Two inspection techniques are assumed to provide the corrosion information. The measurement error definitions of each technique are demonstrated in Table 3.

Table 3 Inspection technique measurement error definitions (AASHTO, 2001)

Technique	$x_{0.5}$(%)	σ
INS1	0.0714	1.14
INS2	0.1714	2.58

The measured defect size function and the measurement error function are substituted in Equation (3b) to determine the actual defect function. It should be noted that the measured defect and measurement error are assumed independent functions. Then the characteristic actual defect size at certain level of probability $(P_{cl}=0.9)$ is considered to estimate the actual deteriorarion at the time of inspection. Using this information Gamma process parameters are obtained and it is possible to characterize the actual degradation process. The gamma process parameters with respect to the actual inspection outcomes are demonstrated in Table 4 and 5, respectively.

Table 4 Sample of inspection (INS1) technique outcomes

Time (year)	M Before inspec. (KN.m)	M after inspec. (KN.m)	X_t^{ia} (%)	γ_i(%)	ω_i (year)
0	62.21	62.21	0	-	-
18	62.21	60.94	2.04	2.04	18
24	60.94	57.00	8.37	6.33	6
30	57.00	53.21	14.46	6.09	6
36	53.21	49.41	20.57	6.11	6

In the same way as for the perfect inspection outcomes we obtain the gamma process parameters using the method of moments. These parameters are demonstrated for both inspection techniques in Table 5

Table 5 Gamma process parameters using inspection 1 (INS1) and inspection 2 (INS2) outcomes

	Inspection Technique 1		Inspection Technique 2	
Time (year)	β_1	c_1	β_2	c_2
24	0.089	0.03	0.054	0.031
30	0.138	0.066	0.083	0.068
36	0.15	0.085	0.1	0.086

Once parameters in Table 5 are obtained it is possible to establish capacity loss profiles for selected time horizons. It is evident that with inclusion of further inspection technique characteristics a realistic model would emerge and enable a well informed modelling of deterioration projections. This model is therefor reflecting the current status of the structure and the current technique quality, therefore accounting for multiple sources of temporal variablility.

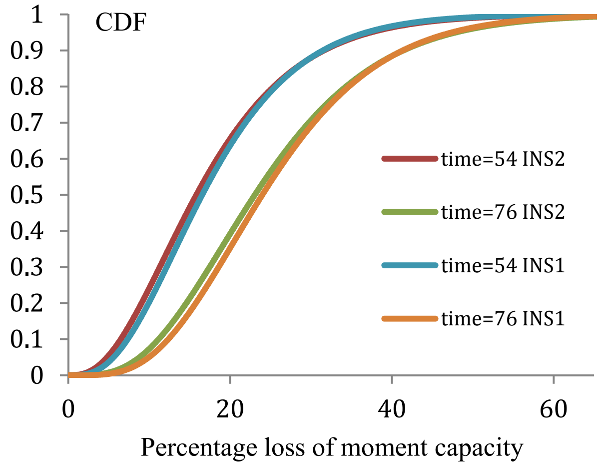

Figure 4 Comparison of cumulative density function of actual percentage loss of moment capacity with respect to inspection techniques INS1 & INS2

Figure 4 demonstrates the comparison of the cumulative density functions for degradation process of concrete slab bridge when two different inspection techniques have been implemented. Thus, it has been demonstrated that there is a pronounced effect of the inspection technique features. Once further inspection imperfections are considered the gamma process model would represent a rather versatile tool for planning of maintenance and/or repair.

6 CONCLUSION

This paper has considered the structural deterioration process as a time-dependent stochastic process. In particular, the continuous gamma process is identified that can represent the deterioration process in an efficient manner. The gamma process parameters are estimated based on the inspection outcomes, which are associated with uncertainties.

Due to the imperfect detection and measurement capabilities of current inspection techniques, the inspection data is represented to reflect the uncertainties associated with the actual defect size. This paper presents the measurement error as the inspection data uncertainty. As a result the descriptors for the deterioration process are more realistic over the lifetime.

Benefits of using this method to produce the deterioration profile for a bridge are:

- Required mathematical calculations are relatively straightforward.
- Continuous gamma process model for deterioration is more realistic in comparison with standard random variable models.
- The inspection intervals can be increased and cost interventions such as additional maintenance and repair reduced.
- The best available inspection techniques can be selected at specific times.
- An optimization technique could be used to define the optimum inspection regime.

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REFRENCES

Bakker, J.D. 2004, 'Inspection validation model for life-cycle analysis', Bridge Maintenance, Safety, Management and Cost. Proceedings of the second International Conference on Bridge Maintenance, Safety and Management.

Biondini, F., Bontempi, F., Frangopol, D.M., and Malerba, P.G., 2006, 'Probabilistic service life assessment and maintenance planning of concrete structures', Journal of Structural Engineering, ASCE, 132(5), 810-825.

Campoli, M. and Ellingwood, B.R., 2002, 'Probabilistic methods for assessing current and future performance of concrete structures in nuclear power plants', Journal of Materials and Structures,35,3-14.

Cheung, M.S., and Kyle, B.R., 1996, 'Service life prediction of concrete structures by reliability analysis', Journal of Construction and Building Materials, 10(1),45-55.

Cheung, M.S., Kevin, K.L., and Xeueqing, Z., 2010, 'Life cycle management of concrete structures relative to chloride-induced reinforcement corrosion', Journal of Structure and Infrastructure Engineering, Taylor& Francis,6(3),1-15

Cinlar, E. Baznat, Z.P., and Osman, E., 1979, 'Stochastic process for extrapolating concrete creep', Journal of Engineering Mechanical Division, 103, 1069-1088.

Dong, Y., Song, R., and Liu, H., 2010, 'Bridges structural health monitoring and deterioration detection synthesis of knowledge and technology', Alaska University Transportation Center, Final Report.

Frangopol, D.M., Li, K.Y., and Estes, A.C., 1997, 'Life-cycle cost design of deteriorating structures', Journal of Structural Engineering, ASCE, 123(10), 1375-1381.

Frangopol, D.M., Kong, J.S., and Gharaibeh, E.S., 2001,' Reliability-based life-cycle management of Highway bridges', Journal of Computing In Civil Engineering, 15(1), 27-34.

Frangopol, D.M., Maarten, M., Kallen, J., and Van Noortwijk, J.M., 2004, 'Probabilistic models for life-cycle performance of deteriorating structures: Review and Future directions', Journal of Structural Engineering Material, 6, 197-212.

Haldar, A., and Mahadevan, S., 1999, 'Probability, reliability and statistical methods in engineering design', John Wiley& Sons. New York.

Joint Committee structural safety, 2008, JCSS,' Interpretation of uncertainties and probabilities in civil engineering decision analysis', *Netherland, JCSS Publishing.*

Kallen, M.J., and Van Noortwijk, J.M., 2005,' Optimal maintenance decisions under imperfect inspection', *Journal of Reliability Engineering and System Safety, 90(2-3), 177-185.*

Li, C.Q., 2003,' Life-cycle modeling of corrosion affected concrete structures: propagation', *Journal of Structural Engineering, 129(6), 753-761.*

Maes, M.A., and Dann, M.R., 2011, 'A unified probabilistic treatment for in-line inspection with respect to detect ability, report ability, false call potential', *International Conference of Applications of Statistics and Probability in Civil Engineering, Proceeding of ICASP11, Taylor & Francis Group.*

Marsh, P., and Frangopol, D.M., 2008,'Reinforced concrete bridge deck reliability model incorporating temporal and spatial variations of probabilistic corrosion rate sensor data', *Journal of Reliability Engineering and System Safety, 93, 394-409.*

Mori, Y., and Elingwood, B.R., 1992,'Reliability-based service-life assessment of aging concrete structures', *Journal of Structural Engineering, ASCE, 199(5), 1600-1621.*

Ohadi, A.R, Micic, T., 2011,'Stochastic process deterioration modeling for adaptive inspection', *International Conference of Applications of Statistics and Probability in Civil Engineering, Proceeding of ICASP11, Taylor & Francis Group.*

Orcesi, A.D., and Frangopol, D.M., 2011, 'Use of lifetime functions in the optimization of nondestructive inspection strategies of bridges', *Journal of Structural Engineering, ASCE, 137(4), 531-539.*

Pandey, M.D., 1997,'Probabilistic models for condition assessment of oil and gas pipelines', *Journal of NDT & E International, 31(5), 349-358*.

Pandey, M.D., Yuan, X.X., and Van Noortwijk, J.M., 2009, 'The influence of temporal uncertainty of deterioration on life-cycle management of structures', *Journal of Structure and Infrastructure Engineering, 15, 145-156.*

Thoft-Christensen, P., and Baker, M.J., 1987, 'Optimal strategy for inspection and repair of structural systems', *Journal of Civil Engineering Systems, 4(2), 94-100.*

Van Noortwijk, J.M., and Frangopol, D.M., 2004, 'Two probabilistic life-cycle maintenance models for deteriorating civil infrastructures', *Journal of Engineering Mechanics, 19, 345-359.*

Van Noortwijk, J.M., 2009, 'A survey of the application of gamma processes in maintenance', *Journal of Reliability Engineering and System Safety, 94, 2-21.*

Yang, S.I., Frangopol, D.M., and Neves, L.C., 2006, 'Optimum maintenance strategy for deteriorating bridge structures based on lifetime functions', *Journal of Engineering Structures, 28(2), 196-206.*

Zhang, R., Mahadevan, S., 2001, ' Fatigue reliability analysis using nondestructive inspection', Journal of Structural Engineering, ASCE, 127, 957-965