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THE ROLE OF LOCAL SPACE CHARGE CONCENTRATIONS IN PRODUCING BRANCHED TREE STRUCTURES

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Electrical trees are branched damage structures produced in polymeric insulation subject to high divergent fields. The density of branching ranges from a sparse form like a tree in winter to a dense compact form like a bush. This variation in form is significant as the bush structure occurs at higher voltages but grows slower. We present here a deterministic model for the formation of electrical trees based on damage produced by charges injected into the polymer from discharges taking place within the gas-filled tubules of the tree. A number of processes within the mechanism cause the space charge fields to fluctuate chaotically, and this is held to be responsible for the branching that is observed. Different tree shapes are found depending on whether or not injected/extracted charges reach a kinetic energy high enough for damage only at a few tree tips or everywhere around the tree periphery.

Introduction

Electrical trees are initiated by defects in polymeric insulating materials such as a metallic protrusion of the electrode or a gas-filled cavity [1]. In both cases charge injection and extraction is responsible for the production of a small tubule about 4-10μm long and 1μm radius [1,2]. This tubule is large enough to support gas-discharges within it in the electric fields produced by the tree-initiating defect. These tree-discharges cause the damage to extend into the polymer, and repetition of the process leads to a step-wise advance across the insulation until eventually the electrodes are connected by a narrow gas-filled path. At typical service fields the gas in the tubular connect will break down and the insulation will short circuit, once the tubules have been eroded to a width large enough to support an arc, i.e. a width in excess of ~ 50μm. The main question regarding the formation of electrical trees is that of why they branch at all [3]. The applied field in which they propagate is largest along the axis from the defect to the counter-electrode, and any mechanism driven solely by this field would be restricted to a single straight tube with a rapidly accelerating propagation rate. Instead electrical trees bifurcate and branch out to produce a complex range of different shapes. In addition their propagation rate slows down instead of speeding up. Two main structures are distinguished. Branch – trees have the aspect of a tree in winter and are produced at low voltages around power frequencies (~ 50Hz), or high temperatures. Bush-trees are highly dense structures and are produced at high voltages and high frequencies at room temperature. The damage in bush-trees is concentrated into a small volume leading to a small length per unit of damage, and hence bush-trees slow down more rapidly than branch trees. For this reason they can be thought to be less dangerous in the short term than branch-trees.

Two alternative approaches have been taken towards the problem of branching in electrical trees. In one case [4] it is assumed that the process of stepwise addition of branches is stochastic. Here the choice of location for a new branch is made by random selection from all possible tree tips, with the local electric field and sometimes other factors weighting the various locations [4,5]. These models have had success in producing the fractal structure of branch-trees, but the structures produced are very sensitive to the value of a stochastic parameter that can not be given a physical interpretation. The other approach treats tree growth as a deterministic mechanism driven by the local electric field and charges injected or extracted from the polymer. Branching occurs because the damage mechanism is non-linear in the local electric field and is one of a number of complex feedback processes that govern the local electric field through their effect on the distribution of space charge. In consequence the electric field exhibits deterministic chaos, and its local fluctuations lead to branching as the various directions experience changes in electric field strength over time. Some experimental evidence exists to support this view [3] through the identification of deterministic chaos in the discharge-number sequence of electrical trees grown in epoxy resin. The work presented here shows that a deterministic model for electrical tree growth can indeed produce branched structures [6], and that different tree shapes can be obtained by changing the material and electrical parameters embedded in the model without any recourse to stochastic choices.

Deterministic Model for Electrical Trees

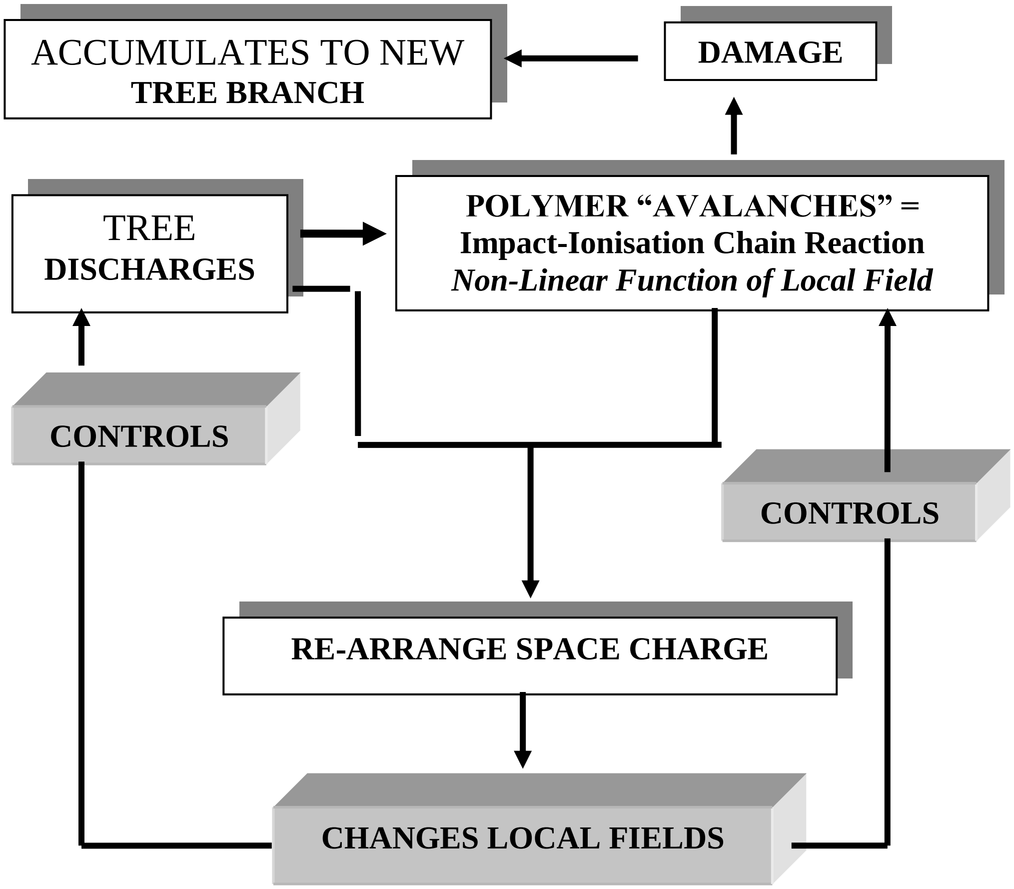


Figure 1 Diagram of construction of deterministic tree model

The schematic diagram **(*fig. 1*)** shows an overview of the construction of the deterministic tree model. The basic mechanism is that tree discharges give rise to polymer avalanches. These cause damage that accumulates to a point at which a tubular extension is formed. However, both the tree discharges and the avalanches re-arrange space charge and hence alter the local electric fields. Subsequent discharges and avalanches are therefore controlled by earlier discharge-avalanche events through this feedback mechanism. The non-linearity of the process, the dependence on the previous events in the sequence and the time lag between the effect and its consequence are the essential features required for deterministic chaos, see [3] and references therein. We would therefore expect this model to exhibit deterministic chaos in values of the local electric field and consequently in local damage formation.

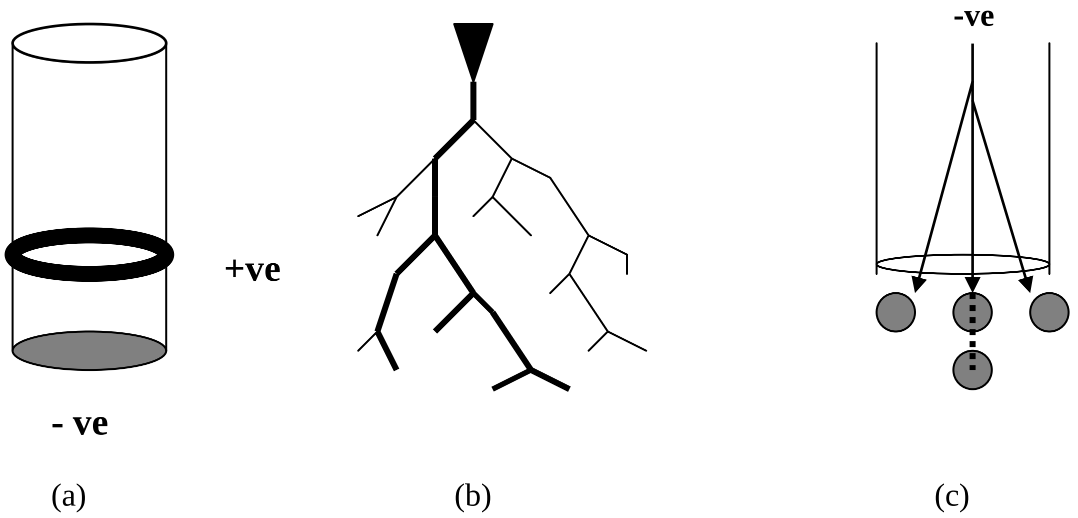


Figure 2. Model for discharges in electrical trees: (a) the effect of a discharge in single tubule showing the separation of positive ions to the tube walls and electrons to the tube end, (b) the discharge path in a tree as a sequence of single tube discharges shown by the thick line, (c) electrons from the discharge penetrate the polymer and are trapped in locations defined by the circles

When discharges in tree tubules occur they generate an electrical dipole by separating charges of opposite polarity, **(*fig. 2a*)**. The negative charges are mostly electrons that may either penetrate the polymer or be neutralised at the point electrode when this has positive polarity. The positive charges are positive ions that tend to be adsorbed onto the tube walls if the tubes have a radius of less than about 20μm. Tree discharges normally start at the electrode and extend along a path that includes only part of the tree structure, **(*fig. 2b*)** [7]. The discharge extends as long as it can maintain the field in front of the head above a defined inception level, while it reduces the field behind the head to a lower extinction level [8]. In our model this situation is replicated by treating the discharge to be a sequence of single tube discharges, each of which reduce the field behind the head to a specified level. The sequence advances into tubes only where the field is above a defined inception level. Where the discharge ends on a tree tip electrons are injected into the polymer to a representative set of centres (***fig. 2c***). The amount of charge involved depends upon the difference between inception and extinction fields within the tube and is distributed among the trapping centres according to the local fields influencing the electron movement at the tree tips. Allowance is also made for the some of the electrons to penetrate the polymer wherever there is a branch point. Again they are placed in representative trap centres according to the local field acting at the time of the discharge.

When the discharge reaches the tree tips some of the electrons have sufficient kinetic energy to initiate impact-ionisation events within the polymer. The resulting electron avalanche only acts over a limited range, L_b, which is taken from experiment to be 10μm. This length defines the bond lengths on the grid used to define the calculation. The range is limited because of the finite duration of the discharge that generates the seed electrons and high field needed for impact-ionisation. Also the immobile positive ions produced in the polymer reduce the driving field. The avalanches therefore redistribute space charge and reduce the field along the avalanche path. This is replicated in the model by placing negative charge at the end of the avalanche path and positive charge in a representative centre along the path. The possible avalanche paths are represented by bond on the grid used for calculation. A defined fraction of discharge is assumed to take part in avalanches, all possible directions participate with the amount of charge used to seed an avalanche along any given direction being assigned according to the fields acting along the avalanche paths. The higher the field driving the avalanche the more the fraction of electrons used as seeds for avalanches in that direction. The damage produced by an avalanche is assumed to be proportional to the number of ionisations that occur. It is expressed in the model as a fraction, *f*, of a critical number of ionisations, N_c, that is representative of the amount of damage required for the formation of a new tubular extension of the tree [3], with;

$$f = (N_b/N_c)[\exp\{(L_b/\lambda)\exp(-IL_b/\lambda\Delta V_b)\} - 1] \tag{1}$$

Here N_b is the number of seed electrons, determined as described above, and ΔV_b is the potential difference along the grid-bond of length L_b that corresponds to the avalanche pathlength. N_c is a material dependent parameter. As a first approximation it is taken to be the number of bonds, 10^{13}, in the volume of a new tubular extension of 10μm length and 1μm radius. I is the ionisation potential for the impact-ionisation process. The parameter λ is the minimum collision pathlength. Its value may vary from place to place within a semi-crystalline polymer such as polyethylene, and it can be expected to be larger if the avalanche path contains sub-micron voids, free volume, or incipient cavities. We have chosen to use this parameter as an indicator of the damage that is caused by the avalanche. Before avalanches occur along a given path its value is taken to be $\lambda = \lambda_o$, which in the present calculations has the same value everywhere. When an avalanche occurs λ is incremented to a new value to take account of the degradation, with

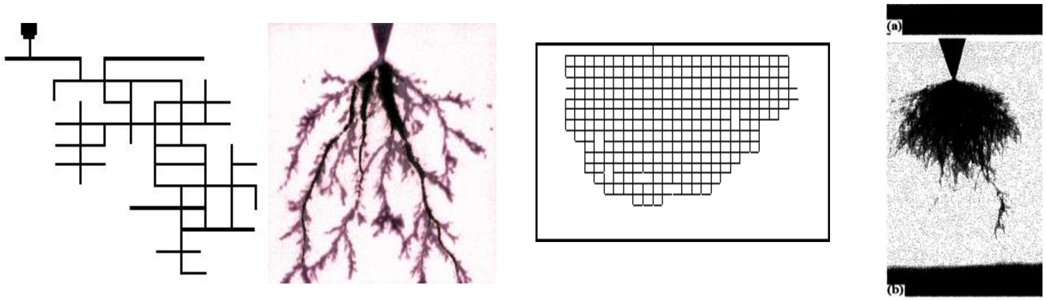
$$\lambda_{new} = \lambda_{old} + fL_b \tag{2}$$

The damage is taken to have reached a level at which a new tubule is formed when $\lambda_{new} = L_b$, i.e. when the tubule can be considered as free space. The dependence of *f* upon λ in equation (1) means that the generation of damage by one avalanche influences the size of subsequent avalanches along the same path, and hence provides yet a further feedback process in the model. If $\lambda < IL_b/\Delta V_b$ increasing λ favours bigger subsequent avalanches, reaching a maximum at $\lambda = IL_b/\Delta V_b$. In this case the damage increases strongly as the avalanches continue. On the other hand bigger values of λ cause a weak decrease in the size of subsequent avalanches and the avalanche damage tends to be nearly the same amount throughout the process.

The model is implemented by stepping through time, making calculations at sixteen positions within each cycle of the applied ac-voltage. The calculation looks for discharges initiated in the 1st tubule next to a point electrode and follows the discharge sequence thus initiated to its furthest extent. Avalanche damage is calculated and the λ associated with the various bonds affected are incremented. Undischarged tree tubules are then examined for the possibility of a discharge and the process continues until the whole tree structure has been covered. On the positive half-cycle the tree discharges move electrons towards the point-electrode where they may be neutralised. In this case the polymer avalanches transfer electrons from the polymer to the tree tips. The seed electrons are the electrons deposited by an avalanche on the previous half-cycle. Their availability depends upon the ease of release from traps within the polymer and this is described by a sigmoidal dependence upon the local field along the affected polymer grid bond. Of course all the space charges will experience a time-varying electric field during the interval between the successive calculations. They may displace in the field if that is possible, and recombine if charges of opposite polarity come into contact. The model allows for displacement of positive wall charge during the time interval, with neutralisation if it reaches the point electrode on the negative half-cycle. Recombination of a defined fraction of the charges occurs when charges of opposite polarity come into contact. Charges within the polymer are allowed to move if the local field exceeds a mobility onset value [9], and are re-arranged such that the local field reduces to the onset level. In this way charge can penetrate the polymer beyond the immediate surroundings of the tree tips. The overall situation is one of tree discharges and polymer avalanches taking place in a fluctuating space charge field combined with the ac applied field.

Trees computed by the model

Computations of electrical tree structures start from an initial state in which the first tree tubule adjoining a point electrode already exists, i.e. the tree is pre-initiated. A small amount of charge is distributed at random between the electron trapping centres shown in ***fig.2c***. This is the only point in the computation where a random feature exists and it reflects unknown variations in the details of the initial conditions. The computation gives information on space charge distributions at any time throughout the tree growth, the magnitude and number of tree discharges, the tree length and structure over the period of growth. The important result is that the model produces branched structures. Both branch trees and bush trees can be generated depending upon the choice of parameters and applied voltage. In ***fig.3a*** a typical branch-tree produced by the model is compared to an experimentally produced branch-tree. It is clear that despite the deterministic nature of



(a) (b)

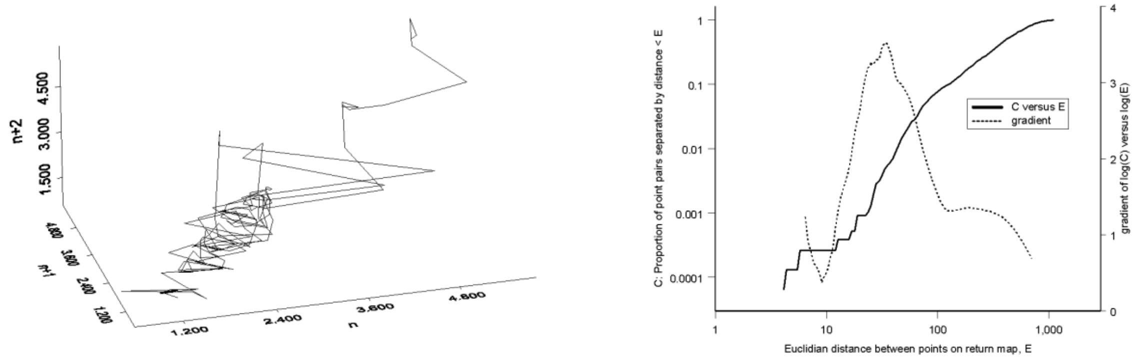
Figure 3 Comparison of tree structures from the model and experimental electric trees: (a) branched trees; (b) bush trees

the model the small amount of branching observed experimentally is reproduced. The structures are not identical but it should be noted that the experimental tree exists in a 3D-space whereas the computed tree was produced in a 2D-space. In ***fig 3b*** a bush tree produced by the model is compared to an experimental bush tree. The point to note here is that the computed bush tree does not entirely fill the 2D-space, i.e. it has a fractal dimension just less than 2 (see [3]). Real bush trees are also not completely space filling and possess fractal dimensions between ~2.2 and 2.7 [10].

The model shows that the dominating factor governing the tree shape is the availability of electrons to initiate avalanches, particularly at the branch points of the structure. When this is high then bush trees always result. A large amount of seed electrons can be caused by high applied-fields since these give big discharge magnitudes. A low recombination fraction (< 0.65) also favours bush tree growth, since this tends to leave space charges of both polarities in the region of branch points. A corollary of this is that the discharges tend to be incoherent within the tree. Bush trees are also favoured when all electrons produced in an avalanche are available for back-avalanching to the tree-tips on the positive half-cycle, i.e. in materials and situations where charges are shallow trapped. If some of these conditions are combined with a small value of λ_o (~5-10nm) bush trees result for which the damage along an avalanche path only reaches an observable size in the final one or two events. In this case it appears as though the damage is the result of single high-energy events as in the stochastic model even though it has actually accumulated over a period of time. Under these circumstances bush trees grow rapidly at first and then slow down or stop when the local fields around the tips becomes much smaller than I/λ_o and the accumulation of damage cannot reach tubule formation in a reasonable time. Such a cessation of growth is a typical feature of many experimental bush trees [1]. Branch trees are formed in the model under much more limited conditions. Seed electrons must be available, but in limited quantities. So some recombination must occur but not too much. The injection of electrons into the polymer at branch points along the discharge path must be very small. In consequence the discharge path tends to be long and restricted to just a part of the tree [7], leading to damage at only a few tree tips on the outermost part of the tree. Branch trees tend to be favoured if λ_o is large (~ 1μm) since the damage then accumulates in nearly equal amounts at the tips reached by the long discharges. A large number of such discharges are thus associated with tree extension in this case as is observed for experimentally grown branch trees [11].

Deterministic Chaos in the model

The model contains all the essential features required for deterministic chaos and it is clear that the local fields fluctuate in time to give the calculated branched structures. In order to demonstrate that the fluctuations result from deterministic chaos it is necessary to analyse the time sequence of a model-observable that is dependent upon the local field. We have chosen the tree discharge number sequence for this analysis since it was this observable that was used to demonstrate the existence of deterministic chaos during experimental tree growth [3]. Because of the limited time range we have only used a 3D return map **(*fig. 4a*)** to construct a first approximation to the strange attractor, however, the essential features are similar to the strange attractor derived in [3]. We have also computed the correlation dimension using the method of delays [3,12]] for the number sequence of the discharges, **(*fig.4b*)**. This dimension expresses one aspect of the fractal nature of the strange attractor. The value of $d_{corr} \approx 3.52 \pm 0.2$ estimated from the return map is close to that of 3.56 ± 0.1 found from experimental data on branch trees



(a) (b)

Figure 4. (a) 3-D return map for discharge number sequence; (b) Thick line shows a log-log plot of the correlation integral from the return map of (a), and the thin line its gradient for which the maximum value is an approximation to d_{corr}

[3,12]. The analysis is not comprehensive because the number of data in the computed sequence is not large. The attractor should be determined by singular value decomposition [12], and the correlation dimension should be demonstrated to saturate as a function of the number of discharge values (embedding dimension) used to form the pseudo-vector that describes the attractor. In addition we have not yet attempted to determine the Lyapunov exponent which must be positive if deterministic chaos exists (it defines the sensitivity of the sequence to initial conditions that is a characteristic of deterministic chaos). However, sufficient evidence exists to show that the fluctuations

computed by the model behave similarly to deterministic chaos, and those chaotic properties calculated are close to what has been observed experimentally.

Conclusions

We have shown that branched structures can result from a completely deterministic model of electrical trees. The reason for the branching is the presence of deterministic chaos in the local fields. This is brought about by the essential non-linearity of some of the processes controlling the time dependence of the space charge contribution to the local fields, together with a number of feedback processes and an inevitable time lag between action and effect. The time lag occurs because the discharges occur for 1-5ns and control the conditions for the next discharge that follows some microseconds later. Both bush and branch trees can be produced depending upon the availability of electrons to initiate avalanches. Bush trees occur when the availability is high especially at branch points. These conditions are particularly met under high fields. The bush trees computed tend to grow fast initially and then cease effective growth as do bush trees in experiments. Branch trees occur when the seed electron availability is moderate. Branch-tree extensions tend to be associated with discharges possessing long paths in the tree. The model only produces branch trees in a limited range of condition, which implies that a sharp crossover to bush trees will occur when the conditions move out of the required range. In particular this happens when the applied field is raised to a high value just as is observed experimentally.

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