Application of polymer ageing models to power cables

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Abstract: Ageing models have been developed to predict the lifetime of polymeric insulation subject to electro-thermal stresses. We present here a method for applying the models to situations in which the field is not constant over the whole specimen, as for cable geometry. The method has been applied to characteristic lifetime data from AC ageing experiments on cables. The results are presented, and the effect of insulation volume upon the model parameters is discussed.

Introduction

The aim of polymer lifetime models is to predict the working lifetime of polymeric insulation subject to thermal and electrical stresses. Such theories have been developed by a number of authors [1-4]. The model used in this work is that developed by Dissado et al – abbreviated here to the DMM model. For this model [1] the lifetime equation is given in terms of applied electrical field, E and temperature, T, by (1)

\[ L(E,T) = \frac{h}{2kT} \exp \left( \frac{-s_d}{k} \right) \exp \left( \frac{H_{dE}}{2T} \right) \left( -\ln \left( \frac{A_d(E,T)}{A'_d(E,T)} \right) \right) \cos \left( \frac{K_{EB} - CE_{dE}}{2T} \right) \]

L is predicted lifetime, k is Boltzmann’s constant, h is Planck’s constant. The model describes the ageing process in terms of moieties within the polymer undergoing a reversible reaction from a reactant state, to a less energetically favourable product state leading to the initiation of local degradation. The difference in free energy between the reactant and product states is called \( K_d \). The reaction is characterised by an activation energy per moiety, \#G, which is made up of an enthalpy part, \( H_{dE} \), and an entropy part, \( S_d \) with \#G (=\( kH_{dE} - TS_d \)). As a polymer specimen evolves from the as-prepared state towards thermodynamic equilibrium, the concentration of moieties in each of these states alters. If the fraction of moieties in the product state, \( A \), exceeds a critical fraction, \( A^* \) in any localised area, then the insulator is considered to have broken down, since failure in that area becomes inevitable. The \( C_d \) and \( b \) parameters in (1) describe the effect of an applied field on \#G, which is reduced by an amount equal to \( C_dE^{4b} \) on application of a field, \( E \).

Equation (1) can be fitted to data from ageing experiments to give values for the parameters in the model, and from these values, information can be drawn about the ageing process. In fact in any polymer specimen, each reacting moiety must have its own value of \( H_{dE}, S_d \) and \( K_d \), and \( A^* \), \( C \) and \( b \) may vary with location in the specimen. Fitting (1) to the characteristic lifetime from any given set of ageing experiments gives the characteristic value for each parameter, and these must be thought of as typical of the specimens that have been aged. In the case of thin film ageing experiments, the fitting is straightforward. Ageing tests on sets of identical specimens under various E and T conditions can be analysed using Weibull statistics to obtain the characteristic lifetime, B63, for each experimental condition. The RHS of (1) containing the relevant E and T values can then be set directly equal to each B63 to yield parameter values. In fact, according to the model, the parameters should be independent of E and T, so this should be done simultaneously for all available E, T and B63 data.

Any dependence of the failure times on polymer specimen volume or size must naturally be reflected in the parameter values obtained using (1), through B63. A volume effect on insulation lifetime is often assumed by e.g. cable manufacturers, but little is known about the way in which volume affects the ageing process and polymer lifetime.

For the thin film case, the temperature, \( T \), and field, \( E \), experienced by each film can be considered spatially constant. In systems such as power cables this assumption cannot be made. The insulation of a power cable under load experiences a radially varying temperature distribution due to Ohmic heating in the core as shown [5].

\[ T(r) = T_1 + \left( \frac{W \times T_h}{2\pi} \ln \left( \frac{R_d}{r} \right) \right) \]

\( T(r) \) is the temperature at radius \( r \), \( T_1 \) is the temperature outside the cable and \( R_d \) is the cross sectional radius of the cable. \( W \) is the power dissipated per unit length along the cable core by the current, and \( T_h \) is the thermal resistivity of the cable insulation. The insulation also experiences a radially varying electrical stress distribution, as shown in (3) for an AC case [6].
In (3) \( E(r) \) is the electrical stress at radius \( r \). \( R_O \) is the cable radius as before, and \( R_i \) is the cross sectional radius of the cable core. \( V \) is the voltage of the core relative to the outer edge of the cable, which is generally earthed. This radial variation in \( E \) and \( T \) makes fitting the ageing models to lifetime data from cable ageing experiments more difficult than in the thin film case, and a method is presented here for doing this.

**Fitting method**

Substituting (2) and (3) into (1) gives a lifetime prediction for cable insulation that is radius dependent. In order to fit this radially dependent lifetime prediction to the B63 value from cable ageing experiments, the insulation is first be split into \( N \) concentric shells. Each shell is at radius \( r_i \) from the centre of the cable core, Figure 1. These shells should be thin enough that \( E \) and \( T \) can be considered constant over their volume. Equal volume shells are used in this investigation, so that the ratio of the cable insulation volume, \( Vc \), to the volume of each shell, \( VS \), is \( N \).

![Figure 1 - Example shell at radius \( r_i \).](image)

Each shell experiences \( E_i \) and \( T_i \) according to its position \( r_i \) and (2) and (3). Substituting \( E_i \) and \( T_i \) into equation (1) then provides a lifetime expression for each shell in terms of \( r_i \) and the DMM parameters. These \( N \) lifetime expressions then need to be fitted to the B63 value for each experimental ageing condition to give parameter values. An expression linking the lifetime expressions of the shells and the experimental B63 value is therefore required, and this can be obtained via probability equations.

Assuming that any insulation system can be considered as made up of many smaller insulation volumes, and that failure in any one of the volumes will cause the entire insulation to fail, the following equation can be used.

\[
PS(L) = \prod_{i=1}^{N} PS(S)_i \tag{4}
\]

Where \( PS(L) \) is the probability of survival at time \( t \) of a large volume of insulation, made up of \( N \) components. \( PS(S)_i \) is the probability of survival at time \( t \) of the \( i \)th, smaller component. If the values of \( PS(S) \) are all the same, this leads to

\[
PS(L) = (PS(S))^N \tag{5}
\]

Assuming that \( PS(L) \) and \( PS(S) \) are Weibull distributions with the same \( \beta \) values, (4) can be used to derive an expression for the characteristic lifetime of a large volume of insulation, B63 in terms of the characteristic lifetimes of a set of smaller volumes of insulation, \( L_i \).

\[
\frac{1}{B63^\beta} = \frac{1}{\sum_i \left( \frac{1}{L_i} \right) ^\beta} \tag{6}
\]

In this case, B63 is the characteristic lifetime of a cable set, and \( L_i \) is an expression for the lifetime of the ‘\( i \)’th shell. Parameter values obtained from fitting this equation will necessarily depend on the volume of the cable insulation through B63 in the same way as for thin films. However, they will also have a dependence on the shell volume (or equivalently a dependence on \( N \)), since the probabilities in (4) are volume dependent.

Parameters that depend on both \( VC \) and \( N \) have the disadvantage that direct comparisons between cable and film experiments are then difficult, since parameters from film experiments will only depend on the total insulation volume. To get parameter values from cable experiments that only depend on \( VC \), it is necessary to ’scale up’ the probability of failure of each shell to the total insulation volume. In other words, it is necessary to work out the probability of failure that each shell would have if it had the volume of the whole insulation. This can be obtained using (7).

\[
PS(SS) = PS(SH)^N \tag{7}
\]

Where \( PS(SS) \) is the probability of survival of the scaled up shell, and \( PS(SH) \) is the probability of survival of the shell. Equation (4) shows that taking the product of the \( PS(SS) \) values would then give the probability of survival of a volume of insulation \( N \) times bigger than \( VC \). Using (4) and (7) therefore gives

\[
\prod_{i=1}^{N} PS(SS)_i = \prod_{i=1}^{N} (PS(SH))_i^N = PS(NV) \tag{8}
\]

Where \( PS(NV) \) is the probability of survival of an insulation specimen with volume \( N \) times bigger than \( VC \). To get the probability of survival of insulation of volume \( VC \) (i.e. of the total cable insulation), (5) can be used again with (8) to give
Where $PS(VC)$ is the probability of survival of the cable. Using this equation, and assuming again that the probabilities of survival are all Weibull distributions with the same shape parameter, the following equation is derived

$$\frac{1}{B63^\beta} = \frac{1}{N} \sum_{i} \left( \frac{1}{L_i^\beta} \right)$$

(10)

$L_i$ is now an expression for the lifetime of a scaled up shell – i.e. an expression for the lifetime that a shell would have if it had volume $VC$. Substituting (1) for each $L_i$ and fitting the equation to experimental B63 data, results in parameter values that have no dependence on $N$, and depend only on the total volume of insulation, $VC$ through B63.

As in the thin film case, the parameters should have no dependence on $E$ or $T$, so the function to be minimised is

$$\sum_j \left[ \log(B63_j) - \log \left( \frac{1}{N} \sum_{i} \left( \frac{1}{L_i^\beta} \right) \right) \right]^2$$

(11)

$B63_j$ is the $J$th experimental characteristic lifetime value. In (11), logs are used due to the extreme non-linearity of (1). Squares are used to ensure a good fit for all B63 values.

**Application of the method**

The above fitting method was applied to data from sets of cables aged under nine different AC voltages and temperature conditions. Twelve cables were aged under each of the nine experimental conditions. The cables had core radii of 5.9mm surrounded by XLPE insulation of 4.4mm thick. Equation (11) was minimised using a grid search method implemented in FORTRAN, the output of which was the error function magnitude with values for each of the DMM parameters.

**Results**

The parameters obtained above can be used to plot lifelines – i.e. lines showing predicted cable lifetime with applied voltage. The lifelines for the optimal parameter set obtained are shown in figure 2 with the experimental B63 data from the cable tests. The experimental B63 data points are shown as crosses with their 90% confidence limits shown as error bars. Lifelines corresponding to the temperatures under which experiments were conducted are shown as lines. The error function for this parameter set has a value of 0.85.

The parameters obtained are shown in table 1, along with the parameter values found from fitting the model to PET film samples and XLPE insulated minicables [1]. These other fittings were carried out using the Levenberg-Marquardt algorithm to minimise the difference between experimental data and the DMM lifetime predictions, which is a more sophisticated method than a grid search. A grid search was used here due to the higher level of complexity of the equation to be minimised.

**Discussion**

The match between the data and the lifelines obtained using the parameters from the fitting method can be seen to be fairly good, with the model being within the 90% confidence limits for four of the nine data points. The parameters were allowed to vary over wide ranges in the grid search fitting, and the magnitudes of the model parameters obtained are all similar in magnitude to those obtained in previous fittings for XLPE minicables and PET thin films. This suggests that the method works well, and supports the theory that the
ageing process is the same for each of the materials studied.

Any dependence of specimen lifetime on volume must be reflected in the magnitudes of the DMM parameter values obtained from fitting to the data. The question of how specimen volume affects parameter values in table 1, however, is not clear, since the parameters obtained are all for different materials, as well as for different volumes. It is therefore not possible to separate out differences due to volume from differences due to material morphology and chemical composition. The volume of insulation used in this investigation, however, is considerably larger than in the other two cases.

It is possible to speculate on which of the DMM parameter values might be affected by volume. The idea that a larger volume of insulation will fail faster than a smaller volume is essentially based on the fact that a larger volume must contain more moieties that can take part in the ageing process. Thus the likelihood of finding moieties that are in some way more susceptible to ageing, or can age faster is increased.

It is possible that a larger volume of polymer may contain more moieties with very low $\#G$ values, and this may be responsible for the shorter lifetimes observed in larger polymer specimens. The values obtained show however, that the characteristic value of $\#G$ is actually very similar for all three of the parameter sets. Over a range of temperatures from 20°C to 100°C the ratio of the energy barrier magnitudes is never more than 1.2:1. This would imply that the ageing process is very similar in each of the specimens regardless of material or volume. The differences in the magnitudes of the parameters between this investigation and the others may be due more to the different fitting methods used than any physical reason. However it should be noted that because $\#G$ appears in an exponential form in (1) a small decrease in its value will reduce the characteristic lifetime substantially.

It is also possible that $A^*$ could vary from area to area of specimen. If so, in a larger volume of polymer there must be an increased likelihood of finding areas where fewer moieties need to be in the aged state for breakdown to be initiated. As a result the sample will require less local energy concentration ($\propto KdA^*$) for the initiation of failure. The differences in characteristic $A^*$ found so far imply that the samples with larger volumes require lower energies and hence probably a lower threshold field with a concomitant reduction in lifetime.

$C$ and $b$ describe the effect of a field on the barrier to ageing, $\#G$. On the application of an electrical field of magnitude $E$, $\#G$ is reduced by an amount equal to $CE^{b0}$, and this acts to accelerate the ageing reaction. Large values of $C$ and $b$ for a set of specimens therefore indicate that the ageing reaction is accelerated strongly by the electrical field. A greater volume of polymer is more likely to contain sites at which this is the case – i.e. sites at which the field can have a strong influence on the ageing process. According to the model such sites will be those that have greater ability to trap charge and store electro-mechanical energy. They may be expected to have a bigger electrostriction coefficient, and/or smaller bulk modulus and relative permittivity than the average. Variations in these material characteristics seem likely on a microscopic scale, which makes these two parameters the most likely to have a volume dependency. The data in table 1 seems to support this to some extent, with the largest polymer volume showing the largest values of $C$ and $b$. These parameters are also likely to be material dependent, however, so this is by no means conclusive.

Conclusions

The method outlined above for fitting the DMM model to data from ageing experiments involving cables has been shown to produce results that match experimental data well. The parameter values so obtained are consistent with values obtained from fitting the model to data from ageing tests involving other polymeric materials, supporting the theory that the ageing process is the same in each. The question of how volume affects ageing in polymers is still poorly understood, and requires further research, but it seems that the DMM parameters $C$ and $b$, which describe the effect of the field on ageing, are the ones most likely to have volume dependence.

References