Optimal Sequential Auctions*

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Abstract

Sequential sealed first-price and open descending-price procurement-auctions are studied. We examine which procurement-auction rule achieves the low procurement cost. We show that the answer to this policy question depends on whether the items are complements or substitutes. With substitutes, the first-price procurement-auction is preferred, while with complements, the open descending-price procurement-auction is preferred. We also illustrate the procurement cost minimizing auction and the auction rule preferred by the bidders. With substitutes, bidders prefer the open descending-price procurement-auction, while with complements bidders prefer the first-price procurement-auction.

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An important result in the theoretical auction literature concerns the revenue equivalence of first-price and open ascending bid auctions, see Vickrey (1961), Myerson (1981) and Riley and Samuelson (1981). A number of papers have studied the robustness of the equivalence result to departures from the basic assumptions including risk-aversion, see Riley and Samuelson (1981), Matthews (1983), Maskin and Riley (1984), budget constraints, see Che and Gale (1998), positively correlated valuations, see Milgrom and Weber (1982), and bidder asymmetry, see Maskin and Riley (2000a). We relax the assumption of a single period auction and study sequential auctions. We take into account that winning an item may affect the winning bidder's values in the next auction.

A sequential auction game is a selling mechanism commonly used when a seller has a number of related items for sale. Typically, an individual item is allocated to a bidder at each round by means of either a sealed bid first-price or an open English auction. Usually the same auction format is used for early and late items, and there is no change in the auction format over time. As the auction proceeds sequentially, a bidder's valuation for an additional item may depend on the number of items acquired so far. Substitutes arise if the value of an additional item falls in the number of acquired items, while complements arise if the value increases in the number of acquired items. This paper explores the relationship between substitutes and complements, and the choice of auction format both from the bidders' and the auctioneer's point of view.

Substitutability is pervasive in a number of settings including sequential real-estate auctions, sequential eBay auctions for used durables, and livestock auctions. What these auctions have in common is that the incremental value of owning a second unit is lower than it was for the first. A private house buyer is interested in the purchase of a single house only, an eBay bidder may wish to buy a single durable good. Similarly, a farmer that wishes to purchase one bull for breeding will value a second one much less than the first.[1] Substitutes also arise in sequential procurement contracting when the technology exhibits decreasing returns to scale: the cost of the marginal contract is higher when the bidder is already committed to a previously won and uncompleted contract than when the bidder is uncommitted.

Complements arise when the value of an additional item increases with the number of items acquired so far: a complete cycle of paintings or a complete china placesetting may have a higher value than the sum of the individual item values. Complements may arise for procurement contracts when there are learning-by-doing effects or experience effects. Additionally, if an up-front investment is required to undertake a project, then this may induce complementarities: the first period winner has already sunk the investment so that she is more competitive in the second period auction.

Empirical studies documenting the importance of substitutes and complements in sequential auctions are abound: substitutes are found in industries in which bidders' capacity is limited, as shown in papers such as Jofre-Bonet and Pesendorfer (2003), Balat (2012) for sequential highway-paving procurement-auctions; and List, Millimet and Price (2007) for sequential timber auctions. Jofre-Bonet and Pesendorfer (2003) devise an empirical technique to measure consistently the effect of substitutes and show that the bid mark-up increase due to the existence of the substitution effect is

[1] Bidder behavior at sequential cattle auctions is nicely described in Zulehner [2009].

substantial. Zulehner (2009) describes a negative correlation between the initial and the subsequent bids by the same bidder in sequential cattle auctions.

Wolfram (1998) documents that bids at sequential electricity auctions include a startup price and a no-load price, which enables bidders to indicate complementarities in electricity generation between adjacent time periods. Anton and Yao (1987) show that complementarities arise in sequential competition for defense contracts as the incumbent firm may achieve a higher experience level and thus a lower cost. Gandal (1997) documents complementarities in sequential cable television license auctions. Incumbency advantage in sequential procurement auctions for school milk contracts may arise due to sunk investments by dairies, see Pesendorfer (2000). There are also complementarities between adjacent school milk contracts, see Marshall, Raiff, Richard and Schulenberg (2006).

Motivated by some of these empirical studies, we consider a buyer's procurement auction model in which bidders (sellers) have private information about their costs. We consider a two period procurement auction game in which every period a single contract is offered for sale. There are two bidders who become privately informed about their contract costs at the beginning of each period. We assume that the identity of the winner of the first auction is publicly observed before the second auction starts, and we consider situations in which winning the first contract may affect the distribution of the winning bidder's costs at the next auction. We shall say that the items are substitutes if at the second auction the first period winning bidder has on average a higher cost than a losing bidder, and the items are complements if instead the first period winning bidder has on average a lower cost than a losing bidder. The asymmetry in the second period arises endogenously as it depends on the first period's auction outcome. We study the payoff and procurement cost ranking of sealed first-price and open descending bid or second-price procurement-auctions.

As most of the empirical auction evidence on substitutes and complements arises in procurement-auctions, in this paper we state our results in terms of a buyer's procurement auction. An alternative model formulation exists for a seller's auction in which bidders have private information about their willingness to pay and the seller awards the item to the high bidder. This alternative model formulation has the same mathematical structure than the one we have chosen, and therefore, our subsequent results can be restated in terms of a seller's auction with the appropriate changes in place.

There is prior work on the relationship between sequential auctions and the substitutes or complements property of the items auctioned. Jeitschko and Wolfstetter (2002) show that the English auction extracts more rent than the first-price auction with complements and the rent being equal with substitutes. The finding is based on the restrictive assumption of binary valuations. It uses the argument that a single shot English auction extracts more rent than first-price auctions with asymmetric binary valuations. Yet, this binary valuations revenue ranking result is fragile and does not hold in the general class of continuously distributed valuations as was shown in Maskin and Riley (2000a). In the continuous case, the revenue ranking can go either way. We follow Maskin and Riley and consider the general class of continuously distributed valuations. Our revenue ranking results confirm that the binary valuation set-up leads to misleading revenue conclusions. The Jeitschko and Wolfstetter revenue equivalence results for substitute items holds only with the degenerate binary valuations assumption. We show that in the general case of continuously distributed valuations there is in fact a clear revenue ranking in which the first-price auction is strictly preferred to the English auction for the case of substitutes. Our continuous valuations setting is general and enables us to draw intuitive parallels with the mechanism design literature. We characterize the optimal sequential auction which to our knowledge has not been studied before.

Most of the theoretical literature on sequential auctions has focused on the martingale property of sequential auction prices and deviations thereof, see Weber (1983). Empirical evidence on declining prices is documented in Ashenfelter (1989) for wine auctions. McAfee and Vincent (1993) explain declining prices with a model in which items are perfect substitutes, each bidder acquires at most one item, and bidders are risk averse. Pitchik and Schotter (1988) study the effect of bidder budget-constraint on the second period auction outcome. Benoit and Krishna (2001) study whether it is better to sell the more valuable item first or second when bidders face budget constraints and information is complete. Branco (1998) shows that with complements auction prices decline. Saini (2009) studies sequential first-price auctions with substitutes and examines the optimal timing of auctions using numerical methods.

Simultaneous multi-unit auctions are studied recently by Milgrom (2000) and Ausubel (2004). When goods are complements, then selling the items in a bundle can increase sellers' revenues, as is already shown in Palfrey (1983), Levin (1997) and Armstrong (2000). Grimm (2007) finds that bundle auctions are preferred to sequential auctions. These papers differ from our setting in that we do not consider simultaneous sales, but consider sequential auctions. In our setting, bidders can condition their behavior on past auction outcomes which are publicly observed. Our setting arises naturally in highway procurement as the need for repairs and maintenance arises periodically and needs to be fulfilled immediately. A simultaneous auction is not feasible in such settings.

The paper is organized as follows: The next section illustrates the effect of auctioned items being complements or substitutes on procurement costs using an example. Section II describes the two period model. We assume that the first period winner draws the second period cost from a distinct cost distribution than a losing bidder. Section III illustrates the bidding equilibrium in second-price and first-price procurement-auctions. Section IV describes our main results. It compares the first-price and second-price equilibrium in terms of procurement cost and bidders' rent. Section V illustrates the procurement cost minimizing auction rule. Section VI concludes.

I. An Example

We illustrate up-front the intuitive effect of items being complements or substitutes on procurement costs using a simple parametric example. Consider a buyer who wishes to procure two items over two periods. In every period a single item is procured. The buyer imposes no reserve price. Two risk-neutral bidders are willing to provide the items. The game payoff for a bidder equals the sum of period payoffs. The bidders' costs for an item are privately known and independently drawn every period anew from an exponential cost distribution function. Both bidders in the first period, and the bidder in the second period that has not supplied an item yet, draw their costs independently from the exponential $F(c) = 1 - \exp[-(c-1)]$ with $1 < c < \infty$. The bidder w, who has supplied the item in the first period, draws the second period cost from the distribution function $F_w(c) = 1 - \exp[-\alpha(c - 1/\alpha)]$, with $1/\alpha < c < \infty$ and $0 < \alpha < \infty$. Here, the parameter α measures whether items are complements or substitutes because it indicates the expected effect of supplying the item in the first period on the second period cost draw. Substitutes emerge for $0 < \alpha < 1$, while complements emerge for $1 < \alpha < \infty$. The items are neither substitutes nor complements for $\alpha = 1$. To see this, observe that the cost of the winner in the second period is drawn from F_w with an expected cost equal to $2/\alpha$. The cost of the other bidder is drawn from F with an expected cost equal to 2.

We next illustrate the bidding equilibrium and the procurement cost ranking of the two standard auction formats: First-price procurement-auction and second-price procurement-auction.

First-price procurement-auction: Suppose that in both periods the bidder submitting the low bid sells the item and receives his bid. We shall illustrate that the Bayesian Nash equilibrium bidding strategies. In the second period, bidder i with cost c chooses bid b to maximize the expected payoff $(b - c)\Pr(\text{i wins})$. The solution to the optimization problem has the following features: The first-period winner bids $b_w^{FP}(c) = c + 1$, while the first period loser bids $b_l^{FP}(c) = c + 1/\alpha$. The expected second-period payoffs equal $\pi_w^{FP} = \alpha/(\alpha+1)$ and $\pi_l^{FP} = 1/(\alpha(\alpha+1))$ for the first-period winner and loser, respectively.

The parameter α has the following effect on second period profits: With substitutes, when $\alpha < 1$, the winner's cost disadvantage results in a payoff disadvantage, that is $\pi_w^{FP} < \pi_l^{FP}$. With complements, when $\alpha > 1$, the winner's cost advantage translates into a payoff advantage, that is $\pi_w^{FP} > \pi_l^{FP}$. Additionally, observe that bidders mark-ups over costs in their bidding functions differ. The loser charges a mark-up over costs of $1/\alpha$ while the winner charges 1. The differential bid shading implies that the first-price auction outcome is not efficient.

In the first period bidders will anticipate the cost or benefit conferred on the winner in the second period. This results in an additional mark-up term added onto the first period bid. To see this, observe that bidder i's objective is to chose a bid b to maximize expected payoffs $(b - c + \pi_w^{FP})\Pr(i \text{ wins}) + \pi_l^{FP}[1 - \Pr(i \text{ wins})]$. The optimization problem can be equivalently written as $\max(b - c - [\pi_l^{FP} - \pi_w^{FP}])\Pr(i \text{ wins}) + \pi_l^{FP}$. Thus, there is an additional mark-up equal to the payoff difference $\pi_l^{FP} - \pi_w^{FP}$ reflecting the opportunity cost of winning. In the first period there is a unique Bayesian Nash equilibrium in which the first-period bidding strategies are symmetric and given by $b^{FP}(c) = c + 1 + \left[\pi_l^{FP} - \pi_w^{FP}\right]$. The outcome in the first period is efficient. The opportunity cost of winning is passed onto the auctioneer. The total ex ante expected payoff to a bidder equals one half plus the expected second period payoff of the losing bidder, $1/2 + 1/(\alpha(\alpha + 1))$.

Second-price procurement auction: Suppose that in both periods the bidder submitting the low bid wins the item and pays the bid of the other bidder. To illustrate the equilibrium we start in the second period. The total second period game has a dominant strategy equilibrium in which bidders bid their cost draw. The resulting outcome is efficient. The payoff of the first-period loser is given by the expression $\pi_l^{SP} = 1/(\alpha(\alpha + 1)) \cdot \exp[-(\alpha + 1) + 2]$, which differs from the above first-price procurement auction outcome by the factor $\exp[-(\alpha+1)+2]$.

In the first period bidders anticipate the additional benefit from winning which leads to the additional mark-up term, now consisting of the difference $\pi_l^{SP} - \pi_w^{SP}$. The first-period bidding strategy becomes $b^{SP}(c) = c + \left[\pi_l^{SP} - \pi_w^{SP}\right]$ where the term in square brackets measures the opportunity cost of winning. The outcome in the first period is efficient. The opportunity cost of winning is passed onto the auctioneer. The total ex ante expected payoff to a bidder equals one half plus the expected second period payoff to a losing bidder, $1/2 + 1/(\alpha(\alpha+1)) \cdot \exp[-(\alpha+1)+2]$, which differs from the corresponding expression under the first-price procurement auction.

Having characterized the bidding equilibrium in the two standard auctions, we next illustrate that there is an unambiguous payoff ranking between these auction formats.

Payoff ranking. With complements, when $\alpha > 1$, bidders prefer the first-price procurement auction. With substitutes, when $\alpha < 1$, bidders prefer the second-price procurement auction. The payoff ranking arises due to the asymmetry in the second period. Bidders in the first-price procurement-auction shade their bids strategically resulting in a lower mark-up for the bidder with the higher expected cost. With complements, when $\alpha > 1$, the first period loser is disadvantaged in the second period. The losing bidder shades the bid up by less than the winner, as $1/\alpha < 1$. The lower mark-up results in an increased winning probability relative to the second-price auction where the strategic bid shading is absent as both bidders bid their cost. In turn, the increased winning probability implies an increased expected second period payoff in the first-price procurement auction relative to the second-price procurement auction. The total difference in ex ante payoffs between first and second-price procurement auctions amounts to $1/(\alpha(\alpha+1)) \cdot (1 - \exp[-(\alpha + 1) + 2])$. With complements, when $\alpha > 1$, the difference is positive. With substitutes, when $\alpha < 1$, the opposite effects arise, and the difference becomes negative.

The procurement cost ranking of these two standard auction rules is illustrated in Figure 1. The figure reports the procurement cost as a percentage of the first-best outcome with publicly observed costs. The solid line measures the second-price procurement auction. The dashed line measures the first-price procurement auction.

[Figure 1 about here]

Figure 1 shows that the first-price procurement-auction achieves the low procurement cost when the items are substitutes, $\alpha < 1$, while the second-price procurement-auction achieve the low procurement cost when the items are complements, $\alpha > 1$. When α is 0.5 the procurement cost difference between the two auction formats amounts to 40%. As α increases, the procurement costs under both auction formats decrease.[2] The difference in procurement costs between the two auction format is about 10% when $\alpha = 2$.

The intuition for the procurement cost ranking is closely related to the payoff ranking. With complements, when $\alpha > 1$, bidders' expected rents are higher in the

[2] The decrease is related to the functional form assumption. The winner's expected cost equals $\frac{2}{\alpha}$, which decreases as α increase. In the limit, as α approaches ∞, the winner has a cost of 0 with probability one.

first-price procurement-auction than the second-price procurement-auction. Therefore, the procurement cost is lower under the efficient second-price procurement auction.

With substitutes, when $\alpha < 1$, the intuition is more involved. The payoff ranking effect suggests a lower procurement cost under the first-price auction. However, there is now an additionally effect that the first-price auction is inefficient which increases procurement costs. Overall, the first effect dominates. The intuition for this dominance is described later on, in section V, where we show that the first-price procurement allocation rule is closer to the optimal auction rule derived from the corresponding mechanism design problem.

Next we introduce our general set-up that does not rely on the functional form assumptions. We shall demonstrate that the ideas illustrated in the example hold in a general set-up.

II. Model

A two period game is considered.[3] Every period a single item is procured. For simplicity of exposition, we assume that there is no reserve price. There are two bidders which are denoted by $i = 1, 2$.[4] We sometimes refer to the bidder that won (lost) the first period auction as the "winner" ("loser"). A bid in the procurement-auction indicates a price at which the bidder is willing to provide the project. The

[3] The restriction to two periods simplifies the exposition, but is not needed. The subsequent analysis and results extend to a multi-period setting in which all pairs of adjacent periods exhibit the substitutes property or all pairs exhibit complements property.

[4] The restriction to two bidders allows us to adopt equilibrium characterization and uniqueness results for asymmetric auctions, see Maskin and Riley [1996, 2000a, 2000b].

price may depend on the bidder's cost for the project, the perception about the cost of the other bidder, and on the rules of the auction game. We make the following assumptions on the bidders' costs and the procurement-auction game:

Private cost: Each bidder observes privately his/her own cost draw at the beginning of every period. The second period's cost draw is not known in the first period.[5] The assumption arises when time elapses between periods, or when the properties of the second contract become known at the beginning of the second period only.[6] The period cost draw is private information and not observed by other bidders or the auctioneer. The first period cost is drawn from the distribution function F with associated probability density function f. The second period cost draw depends on the outcome of the first period procurement-auction game. The winner draws from the distribution function F_w and the loser from F_l. The distributions are continuous, differentiable, and have common interval support $S = \left(\underline{C}, \overline{C}\right) \subset \Re_+$. We denote with $f_i(c)$ for $i = l, w$ the associated probability density function.

We assume that the cost distributions satisfy the (strict) monotone likelihood ratio property, see Milgrom (1981). Based on this property, we define items as substitutes or complements using the two conditions below.

Condition 1: We shall say the items are *substitutes* if in the second period the first period winner is more likely to have a higher cost than a loser in the likelihood

[5] A model in which second period cost draws are known in period one would be qualitatively similar, but would entail the additional feature that bidders update their beliefs about second period costs based on the observed first period bid. See Budish and Zeithammer (2011).

[6] In highway paving contracts, the auctioneer reveals upcoming contracts a short period before the letting date only.

ratio sense,

$$\text{(1)} \qquad \frac{f_w(c)}{f_w(c')} > \frac{f_l(c)}{f_l(c')} \text{ for all } c, c' \in S \text{ with } c > c'.$$

The substitutes property (1) has the following intuitive implications on the cost distribution functions: (i) $F_l(c) > F_w(c)$ for all $c \in S$; (ii) $F_l(c)/f_l(c) > F_w(c)/f_w(c)$ for all $c \in S$; and (iii) $[1 - F_l(c)]/f_l(c) < [1 - F_w(c)]/f_w(c)$ for all $c \in S$. A proof of these properties is given in the appendix.

Condition 2: We shall say the items are *complements* if the first period winner is more likely to have a lower second period cost than a loser in the likelihood ratio sense,

$$\text{(2)} \qquad \frac{f_w(c)}{f_w(c')} < \frac{f_l(c)}{f_l(c')} \text{ for all } c, c' \in S \text{ with } c > c'.$$

Procurement-auction game: We shall consider two distinct procurement-auction games in the period game: (i) a first-price sealed-bid procurement-auction in which the low bidder wins and pays his bid; and (ii) a second-price sealed-bid procurement-auction in which the low bidder wins and pays the bid of the other bidder, which under our assumptions is strategically equivalent to an open descending-price procurement-auction. We shall ignore ties, as the probability of a tie is zero with continuous probability distributions.

Bidders are risk neutral. They discount future payoffs with the common discount factor $\beta \in (0, 1)$. Bidders' objective is to maximize the sum of first period and discounted second period payoffs.

A strategy in the first period specifies a bid as a function of the cost, $b_f(c)$. A strategy in the second period specifies a bid in the second period for the winning and losing bidder as a function of the period cost, $b_w(c), b_l(c)$. We omit the dependence of the second period strategy on the first period privately observed cost draw and publicly observed bids as these variables are not payoff relevant in the second period, and will not affect the outcome.

We are interested in symmetric *Perfect Bayesian Nash Equilibria,* PBNE.

Definition: A PBNE is a tuple (b_f, b_w, b_l) such that (i) players play best-responses, given their beliefs and their opponent's strategy; and (ii) the beliefs are consistent with Bayes' rule.

The next section examines bidding behavior in standard procurement-auctions. We examine the second-price and first-price procurement-auction. Then, we compare these two procurement-auctions' outcomes and identify the procurement-auction rule that minimizes procurement costs in the presence of a complementarity and a substitutability in the items sequentially auctioned.

III. Standard procurement-auctions

This section examines bidding behavior in standard procurement-auctions. We start with the second-price procurement-auction, and establish that there exists an equilibrium which is efficient. Then, we examine the first-price procurement-auction.

III.A. Second-Price procurement-auction

In a second-price procurement-auction the low bidder wins. The price paid equals the opponent's bid and does not depend on the bidder's own bid.

In a one period model, the second-price procurement-auction has a dominant strategy equilibrium in which bidders submit a bid equal to their cost, $b = c$. In the dynamic two-period procurement-auction game, with positive discounting $\beta > 0$ and when the items are substitutes or complements, it is no longer an equilibrium to bid the cost. The reason is that winning confers an opportunity cost (benefit) at the next procurement-auction, which will influence optimal bidding and render bidding of the own cost unprofitable. An optimal bid choice will take into account both, the cost of the project and the opportunity cost. We shall begin with a discussion of the second period payoffs, then quantify the opportunity cost, and finally examine the first period bid choice.

The second-price procurement-auction has a dominant strategy equilibrium in the second period in which bidders submit a bid equal to their cost, $b = c$.[7] The dominant strategy equilibrium yields the efficient outcome. Ignoring the zero probability event of ties, the second-price allocation rule is given by:

$$q_w^{SP}(c_w, c_l) = \begin{cases} 1 & \text{if } c_w < c_l; \\ 0 & \text{otherwise.} \end{cases}$$

and $q_l^{SP} = 1 - q_w^{SP}$. Let Q_w^{SP}, Q_l^{SP} denote the interim winning probabilities, $Q_w^{SP}(c_w) = \int_{c_w}^{\overline{C}} q_w^{SP}(c_w, c_l) f_l(c_l) dc_l$ and $Q_l^{SP}(c_l) = \int_{c_l}^{\overline{C}} q_l^{SP}(c_w, c_l) f_w(c_w) dc_w$, and let Π_w^{SP}, Π_l^{SP} denote the ex ante expected period rent for the winner and loser associated with the second-price allocation rule. Following Myerson (1981), the ex ante expected second period profits reduces to the expected virtual rent, $\Pi_l^{SP} = \int_S F_l(c) Q_l^{SP}(c) dc$ and $\Pi_w^{SP} = \int_S F_w(c) Q_w^{SP}(c) dc$. The expression is obtained by using the envelope

[7] When the cost support is bounded, $\overline{C} < \infty$, then there exist also pooling equilibria, for example $b_w = 0$ and $b_l = \overline{C}$. The described pooling equilibrium involves weakly dominated strategies and it is not efficient. As customary, we shall ignore pooling equilibria and focus our analysis on the unique equilibrium surviving the iterated elminination of weakly dominated strategies.

theorem and integration by parts.

In the first period of the game, the period's gain plus the discounted expected second period payoff equals $b_{(2)} - c + \beta \Pi_w^{SP}$ if the bidder wins at price $b_{(2)}$, and it equals $\beta \Pi_l^{SP}$ if the bidder loses. As the bidder is risk neutral, the rent increment between winning and losing, $\left[b_{(2)} - c + \beta \left(\Pi_w^{SP} - \Pi_l^{SP}\right)\right]$, determines the first period bid choice. The first term in the rent difference equals the usual expression of the bid minus the period cost. The second term, $\beta \left[\Pi_w^{SP} - \Pi_l^{SP}\right]$, denotes the opportunity benefit of winning, and enters as an additive constant when the bidder wins the item. As illustrated in the following proposition, the symmetric first period equilibrium bidding strategy will take the added constant into account.

Proposition 1 *The symmetric first period equilibrium bid function in the second-price procurement-auction equals:*

$$b_f^{SP}(c) = c + \beta \left[\Pi_l^{SP} - \Pi_w^{SP}\right]. \tag{3}$$

The proof follows from standard arguments for second-price procurement-auctions by which bidders bid their cost and therefore there is no static mark-up component. The argument is based on the second-price procurement-auctions' property that the bid does not affect the price paid, and affects the winning probability only.

The equilibrium bidding strategy in (3) has an intuitive explanation. With complements, the opportunity benefit of winning equals the discounted payoff difference between winning and losing in the second period procurement-auction game, $\beta \left[\Pi_w^{SP} - \Pi_l^{SP}\right]$. It will be passed on to the auctioneer as an additive mark-down independent of the cost realization in the first period. The range of optimal bids in the first period is thus not the cost range $\left[\underline{C}, \overline{C}\right]$. Indeed, bidders may be willing to pay for the first contract when the degree of complementarity is sufficiently strong (and $\underline{C}$ is close to zero).

The bidding strategy in (3) is the unique equilibrium surviving the iterated elimination of weakly dominated strategies, see Fudenberg and Tirole (1991).

Observe also that the PBNE in the second-price procurement-auction retains the efficiency property of the static second-price procurement-auction.

Corollary 1 *The PBNE in the second-price procurement-auction is efficient.*

Corollary 1 follows from two properties of the equilibrium bid functions. These properties are: (i) that the mark-up is independent of the cost realization; and (ii) that the mark-up is identical for both bidders. These two properties imply that the low cost bidder will submit the low bid.

So far, we have characterized the bidding equilibrium in the second-price procurement-auction. Next, we consider the bidding equilibrium in the first-price procurement-auction.

III.B. First-Price procurement-auction

In a first-price procurement-auction, the low bidder wins and receives his bid. The period payoff of the winner is given by the bid minus the cost, while the loser receives zero.

In period 2 under both cases, substitutes and complements, there will be one strong and one weak bidder. With substitutes, the winner will be weak in the second period in the sense of condition (1), while with complements, the loser will be weak in the second period in the sense of condition (2). The existence and the uniqueness of an equilibrium in first-price asymmetric auctions has been established by a number of authors, including Maskin and Riley (1996, 2000b), Athey (2001), and Jackson, Simon, Swinkels and Zame (2002). Based on these results we shall proceed with the knowledge that a unique equilibrium exists with equilibrium bid functions (b_w, b_l).

The second period equilibrium allocation rule, ignoring the zero probability event of ties, is given by,

$$q_w^{FP}(c_w, c_l) = \begin{cases} 1 & \text{if } b_w(c_w) < b_l(c_l); \\ 0 & \text{otherwise.} \end{cases}$$

and $q_l^{FP} = 1 - q_w^{FP}$. It says that the bidder i, with $i = w, l$, wins the second contract when his bid is low. Let Q_w^{FP}, Q_l^{FP} denote the interim expected winning probabilities, $Q_w^{FP}(c_w) = \int_{c_w}^{\overline{C}} q_w^{FP}(c_w, c_l) f_l(c_l) dc_l$ and $Q_l^{FP}(c_l) = \int_{c_l}^{\overline{C}} q_l^{FP}(c_w, c_l) f_w(c_w) dc_w$. Let Π_l^{FP}, Π_w^{FP} denote the ex ante expected second period rent for the first period losing and winning bidder associated with the first-price allocation rule, $\Pi_l^{FP} = \int_S F_l(c) Q_l^{FP}(c) dc$ and $\Pi_w^{FP} = \int_S F_w(c) Q_w^{FP}(c) dc$.

The following Lemma describes properties of the equilibrium bid strategies and expected payoffs that will be essential in the subsequent arguments (the appendix contains formal proofs of lemmas, propositions and theorems).[8]

[8] These properties have also been employed in the context of mergers, Waehrer [1999], and bidder collusion, Pesendorfer [2000].

Lemma 1 *For any c in S:*

(i) Under condition (1) (when the contracts are substitutes), $b_w(c) < b_l(c)$, and $\Pi_w^{SP} < \Pi_w^{FP} < \Pi_l^{FP} < \Pi_l^{SP}$.

(ii) Under condition (2) (when the contracts are complements), $b_w(c) > b_l(c)$, and $\Pi_w^{SP} > \Pi_w^{FP} > \Pi_l^{FP} > \Pi_l^{SP}$.

The Lemma illustrates intuitive properties of asymmetric first-price procurement-auctions. The weak bidder bids more aggressively than the strong bidder. The reason is that the weak bidder expects tougher competition in the procurement-auction than the strong bidder. The strategic effect has the following implications: When contracts are substitutes, the first period winning bidder knows she is weaker in the second period, and charges a smaller mark-up over costs than a losing bidder. The reduction in the mark-up implies that she wins more frequently in the second period than is efficient, i.e. winning despite of having a higher cost draw than the opponent. In turn, this implies that she makes an expected profit larger than in the second-price procurement-auction.[9] When contracts are complements, the first period winning bidder charges a higher mark-up over costs than a losing bidder, and thus she wins less frequently and makes less rent than in the second-price procurement-auction.

Next, we consider the remaining element in the equilibrium construction: The first period bid strategy. Winning the item results in the payoff of $b - c + \beta\Pi_w^{FP}$ while losing results in the payoff of $\beta\Pi_l^{FP}$. The optimal bid b to maximizes the expected payoff of $(b - c + \pi_w^{FP})\,\Pr(i \text{ wins}) + \pi_l^{FP}[1 - \Pr(i \text{ wins})]$. As the opportunity cost

[9] This follows from the statement of profits based on the envelope theorem: if $Q_w^{FP}(c_w) > \Pr(c_l \geq c_w) = Q_w^{SP}(c_w)$, then $\Pi_w^{FP} = \int_S F_w(c)Q_w^{FP}(c)dc > \int_S F_w(c)Q_w^{SP}(c)dc = \Pi_w^{SP}$.

of winning does not depend on the period cost realization, it simply shifts the cost by a constant term, $\beta \left[\Pi_l^{FP} - \Pi_w^{FP}\right]$, and the equilibrium bid function takes the well known form.

Proposition 2 *The first period equilibrium bid function in the first-price procurement-auction equals:*

$$b_f^{FP}(c) = c + \frac{\int_c^{\overline{C}} [1 - F(x)]\, dx}{1 - F(c)} + \beta \left[\Pi_l^{FP} - \Pi_w^{FP}\right]. \tag{4}$$

The proof follows from standard arguments for first-price auctions, see for example Proposition 2 in Riley and Samuelson (1981). The equilibrium bid in equation (4) has an intuitive explanation. It equals the cost plus a mark-up. The mark-up has two components: (i) the static mark-up equal to the expected opponent's cost conditional on the opponent's cost exceeding the own cost, and (ii) a dynamic mark-up equal to the opportunity cost of winning.

There are two features of the equilibrium worth emphasizing: First, the sign of the dynamic mark-up under the first-price procurement-auction coincides with the sign of the dynamic mark-up under the second-price procurement-auction rule. It is positive when the contracts are substitutes, and negative when the goods are complements.

Second, when contracts are substitutes, the dynamic mark-up is smaller under the first-price procurement-auction rule than under the second-price procurement-auction rule:

$$\left[\Pi_l^{FP} - \Pi_w^{FP}\right] < \left[\Pi_l^{SP} - \Pi_w^{SP}\right].$$

When contracts are complements, the dynamic mark-up is larger under the first-price procurement-auction rule than under the second-price procurement-auction rule:

$$\left[\Pi_l^{FP} - \Pi_w^{FP}\right] > \left[\Pi_l^{SP} - \Pi_w^{SP}\right].$$

These features follow from the payoff inequalities in Lemma 1 and are explained by the strategic bid shading in first-price procurement-auctions.

We shall see next that the first period mark-up ranking plays a central role in determining the bidder's rent and procurement cost ranking.

IV. Bidders' Rents and Procurement Cost in Sequential procurement-auctions

This section describes our main results. We compare the outcome under the first-price procurement-auction and the second-price procurement-auction. Subsection IV.A considers this issue from the perspective of the bidders and compares the bidders' rents. Subsection IV.B compares the total procurement cost associated to the first-price and second-price procurement-auctions.

IV.A. Bidders' Rent

The equilibrium characterization in section III allows us to determine a bidder's rent under the first-price and second-price procurement-auction rule. By using the envelope theorem and integration by parts, the ex ante expected equilibrium game payoff in the first-price procurement-auction, Π^{FP}, and in the second-price procurement-auction, Π^{SP}, equal:

$$\Pi^j = \int_S F(c)\left[1 - F(c)\right] dc + \beta \Pi_l^j \qquad \text{for } j = FP, SP. \tag{5}$$

Equation (5) consists of the usual expression for the bidder's information rent in the first period, and a modified expression in the second period that captures the expected payoff of a losing bidder. The modification arises as competition in the first period diminishes any expected payoff advantages (or disadvantages) of the winning bidder. This can be seen most clearly in the analysis in section III, where the first period bid passes any subsequent payoff losses (gains) of the winner on to the seller by adding the opportunity cost (benefit) of winning to the bid. Thus both, the winning and the losing bidder, expect to receive the losing bidder's second period rent only.

The following Theorem compares bidders' rents between the two procurement-auction formats.

Theorem 1 (Payoff Ranking)

(i) Under condition (1) (when the contracts are substitutes), $\Pi^{FP} < \Pi^{SP}$*.*

(ii) Under condition (2) (when the contracts are complements), $\Pi^{FP} > \Pi^{SP}$*.*

This Theorem establishes that bidders prefer the second-price procurement-auction when the contracts are substitutes, while they prefer the first-price procurement-auction when contracts are complements. The result is already apparent in the differential bid shading behavior illustrated at the end of section III, as with substitutes, bidders' first period dynamic mark-ups and thus first period payoffs are lower with the first-price procurement-auction than in the second-price procurement-auction. Analogously, with complements, bidders' dynamic mark-ups and thus first period payoffs are higher with the first-price procurement-auction than in the second-price procurement-auction.

Equation (5) also shows that the payoff comparison across procurement-auction formats reduces to a comparison of the second period expected rent to the losing bidder. The result in Theorem 1 is then easily explained as with substitutes, the second period losing bidder's rent is lowest under the first-price procurement-auction rule as is shown in Lemma 1 in section III. As described earlier, the intuition for this result lies in the fact that with substitutes the losing bidder bids less aggressively than the winning bidder in the first-price procurement-auction resulting in a lower winning probability and thus a lower rent than that associated with the socially efficient outcome in the second-price procurement-auction. The opposite result holds in the case of complements by an analogous argument.

Next, we consider the procurement cost.

IV.B. Procurement Cost

The total procurement cost of the first-price and the second-price procurement-auction, PC^{FP} and PC^{SP}, equals the cost of the winning bidder plus the bidder's rent:

$$\begin{aligned} PC^j &= 2\int_S c\left[1-F(c)\right]f(c)dc+\beta\int_S\int_S\left[c_l q_l^j\left(c_w,c_l\right)+c_w q_w^j\left(c_w,c_l\right)\right]f_l(c_l)f_w(c_w)dc_w dc_l \\ &\quad +2\int_S F(c)\left[1-F(c)\right]dc+2\beta\int_S F_l(c)Q_l^j(c)dc \qquad \text{for } j=FP,SP \end{aligned}$$

where the first period procurement cost can be attributed to the usual virtual cost of a bidder, $c+F/f$. The second period procurement cost differs from the usual virtual cost expression. It equals the cost of the winning bidder, $c_l q_l^j + c_w q_w^j$, plus twice the second period rent of the losing bidder, $2 \cdot F_l/f_l$, for $j = FP, SP$. The expression involving twice the losing bidder's rent arises as expected payoff advantages (disadvantages) of the winning bidder in the second period are passed on to the auctioneer with the first period bid choice, and both, winning and losing, bidders expect to receive the losing bidder's rent in the second period.

The magnitude of the procurement cost difference between the second-price and the first-price procurement-auction amounts to:

$$
\begin{aligned}
PC^{SP} - PC^{FP} &= \beta \int_S \int_S [c_l - c_w] \left[q_l^{SP}(c_w, c_l) - q_l^{FP}(c_w, c_l) \right] f_l(c_l) f_w(c_w) dc_w dc_l \\
&\quad + 2\beta \int_S \int_S F_l(c) \left[Q_l^{SP}(c) - Q_l^{FP}(c) \right] dc. \qquad (6)
\end{aligned}
$$

To explain the above equation, please observe that from Corollary 1 the second price procurement-auction induces the efficient allocation rule. Thus, the first term on the right hand side measures the efficiency loss of the first-price procurement-auction. This term is always negative. The second term on the right hand side reflects the difference in second period payoff of the losing bidder between the efficient second-price allocation rule and the allocation rule of the first-price procurement-auction. Lemma 1 shows that with substitutes, the term is positive, while with complements it is negative.

The following Theorem states our central result.

Theorem 2 (Procurement Cost Ranking)

(i) Under condition (1) (when the contracts are substitutes), $PC^{SP} > PC^{FP}$.

(ii) Under condition (2) (when the contracts are complements), $PC^{SP} < PC^{FP}$.

Theorem 2 gives a clear policy recommendation: The efficient second-price procurement-auction is optimal when contracts are complements, while the first-price procurement-auction is optimal when contracts are substitutes. Observe also that the ranking in

Theorem 2 is the reverse ranking of Theorem 1, which illustrated the bidders' rent ranking. The intuition is again based on the feature that bidders bid more aggressively in the first-price procurement-auction than in the efficient procurement-auction when contracts are substitutes. This implies that when the items are substitutes a lower procurement cost under the first-price than under the efficient second-price procurement-auction rule. Analogously bidders bid less aggressively in the first-price procurement-auction than in the efficient procurement-auction when contracts are complements resulting in a higher procurement cost under the first-price than under the efficient second-price procurement-auction rule.

Theorem 2 may seem surprising in light of a result in Maskin and Riley (2000a), which establishes that the procurement cost (or revenue) ranking between first-price and open procurement-auctions *is ambiguous* when bidders are asymmetric.[10] The ambiguity result in Maskin and Riley is obtained under the assumption of a single period procurement-auction game in which the bidders' asymmetry is taken as exogenously given. In our model, this scenario is equivalent to considering the second period procurement-auction game in isolation only. In contrast, Theorem 2 shows that when bidders are ex ante symmetric and the asymmetries arise endogenously due to the first period procurement-auction outcome, then the ambiguity disappears and the total procurement cost, consisting of the first and the second periods' procurement

[10] Maskin and Riley [2000a] show that there is a class of distribution functions such that the first-price auction is preferred. The class has the feature that asymmetries arise due to a shift (or stretch) in the distribution. They also show that there is a second class of distribution functions such that the open auction is preferred. The asymmetry in the second class is based on a shift of probability mass to the upper end point in the support.

cost, has a clear and *unambiguous* ranking across procurement-auction formats.

In order to illuminate the ranking result in more detail, we shall use the techniques developed in Myerson (1981) to illustrate the procurement cost minimizing allocation rule. Doing so, will allow us to interpret the procurement cost ranking more intuitively. This is done in the next section.

V. Procurement Cost Minimization

We conclude the discussion with a brief illustration of the procurement-auction rule that minimizes the procurement cost. The illustration will enable us to interpret the procurement cost ranking of the first-price and efficient second-price procurement-auction intuitively. We explore the commitment solution in which the auctioneer fixes the procurement-auction rule for periods one and two before the bidding starts, and we do not permit the auctioneer to modify the procurement-auction rule after period one.

The techniques developed in Myerson (1981) allow us to address this problem. We consider the set of incentive compatible procurement-auction rules that satisfy the voluntary participation constraints and incentive constraints in every period. Let $q_i^t(c_i, c_j)$ denote the probability that bidder i receives the object in period t when bidder i announces cost c_i and bidder j announces cost c_j. We shall assume that the procurement agency needs to award the contract to one of the two bidders, $q_i^t(c_i, c_j) + q_j^t(c_i, c_j) = 1$, in every period. This requirement corresponds to our earlier assumption of no reserve price. Let T_i^t denote the expected transfer payment of bidder i (to the seller) when the bidder announces cost c_i in period t and let Q_i^t denote the expected winning probability, $Q_i^t(c_i) = \int q_i^t(c_i, c_j)\, f_j(c_j) dc_j$. The expected payoff of bidder i in period t, for $t = 1,2$ and $i = 1,2$, when the bidder with cost c_i reports cost c_i' equals:

$$\begin{aligned}
\Pi_i^2(c_i, c_i') &= T_i^2(c_i') - c_i Q_i^2(c_i') . \\
\Pi_i^1(c_i, c_i') &= T_i^1(c_i') - c_i Q_i^1(c_i') + \beta Q_i^1(c_i') \int_S \Pi_w^2(x,x) f_w(x) dx \\
&\quad + \beta \left[1 - Q_i^1(c_i')\right] \int_S \Pi_l^2(x,x) f_l(x) dx.
\end{aligned}$$

The incentive constraints take the form,

$$\text{(IC)} \quad \Pi_i^t(c_i, c_i) \geq \Pi_i^t(c_i, c_i') \text{ for all } c_i, c_i' \in S \text{ and for } i = 1,2,\ t = 1,2.$$

and the voluntary participation constraint take the form,

$$\begin{aligned}
\text{(VP)} \quad \Pi_i^1(c_i, c_i) &\geq \int_S \Pi_l^2(x,x) f_l(x) dx \text{ for all } c_i \in S \text{ and for } i = 1,2; \\
\Pi_i^2(c_i, c_i) &\geq 0 \qquad \text{for all } c_i \in S \text{ and for } i = w, l;
\end{aligned}$$

where the participation payoff in the first period equals at least the expected payoff of a bidder that participates in the second period only, $\int_S \Pi_l^2(x,x) f_l(x) dx$. The (VP) constraint assumes that a bidder that refrains from bidding in the first period cannot be prevented from participating in the second period procurement-auction. This formulation of the (VP) constraint comes closest to the (implicit) assumption in the sequential first-price and second-price procurement-auction, analyzed earlier, in which a bidder cannot be prevented from participating in the second period procurement-auction.

A weaker (VP) constraint arises if a non-participating bidder is banned from the second procurement-auction. With the weaker constraint, the first period reservation value becomes zero, $\Pi_i^1(c,c) \geq 0$ for all $c \in S$ and for $i = 1,2$, and the auctioneer can extract all the rent in the second period by charging bidders a fee in period one equal to the expected second period's rent and by using the efficient second-price procurement-auction in the second period. As the fee is collected in period one, before the second period private information is observed, it will not affect subsequent behavior and enable the auctioneer to collect all the (expected) rent. We shall not consider the weaker (VP) constraint further and instead consider the (VP) constraint defined above.

The following Lemma states an expression for the procurement cost. We show in the appendix, by using the techniques developed in Myerson (1981), that this expression applies under (VP) and (IC).

Lemma 2 *In any incentive compatible procurement-auction rule that satisfies (VP) and (IC), the functions* $Q_i^t(c)$ *for* $i, t = 1, 2$*, are monotone decreasing and the procurement cost equals:*

$$\begin{aligned} PC &= \int_S \int_S \left[\sum_{i=1}^{2} \left(c_i + \frac{F(c_i)}{f(c_i)} \right) q_i^1(c_1, c_2) \right] f(c_1) f(c_2) \, dc_1 dc_2 \\ &+ \sum_{i=1,2} \left[\Pi_i^1(\overline{C}, \overline{C}) - \beta \int F_l(x) Q_l^2(x) \, dx \right] \\ &+ \beta \int_S \int_S \left[c_w q_w(c_w, c_l) + c_l q_l(c_w, c_l) + 2 \frac{F_l(c_l)}{f_l(c_l)} q_l(c_w, c_l) \right] f_w(c_w) f_l(c_l) \, dc_w dc_l, \end{aligned}$$

where the constraints $\Pi_i^1(\overline{C}, \overline{C}) \geq \beta \int F_l(x) Q_l^2(x) \, dx$ *for* $i = 1, 2$ *must hold.*

The first term in the procurement cost accounts for the virtual cost in the first period; the second term reflects the voluntary participation constraint; and, the third term accounts for the second period virtual cost.

The optimal procurement-auction rule minimizes the above expression. Observe that the first expression is the usual procurement cost expression, which is maximized with a first-price or second-price procurement-auction. The second term reflects the voluntary participation constraint. The third expression differs as it takes the dynamic bidding effect into account. Pointwise minimization of the third expression yields the optimal rule (we ignore again the zero probability event of a tie).

Proposition 3 *The procurement cost minimizing solution is a first-price (or second-price) procurement-auction followed by an procurement-auction with the following allocation rule:*

$$q_w(c_w, c_l) = \begin{cases} 1 & \text{if } c_w < c_l + 2\frac{F_l(c_l)}{f_l(c_l)}; \\ 0 & \text{otherwise.} \end{cases}$$

and $q_l(c_w, c_l) = 1 - q_w(c_w, c_l)$.

The optimal second period allocation rule assigns an increased winning probability to the bidder who won the first period item under both, complements and substitutes. The amount of the increase relative to the efficient rule equals twice the virtual rent of the losing bidder. The optimal choice balances two opposing effects: On the one hand, an increase in the second period winning probability leads to an increase in second period rent differential between the winning and the losing bidder. In turn, the increased rent differential implies more aggressive bidding and thus induces the benefit of lower procurement cost in the first period. On the other hand, the increase in the second period winning probability comes at the cost of an increased inefficiency in the second period. At the optimum, the marginal benefit of the reduced first period procurement cost equals the marginal cost of the second period efficiency loss, and the usual marginal condition holds.

The result in Proposition 3 allows us to illustrate the ranking obtained in Theorem 2 intuitively. A graphical illustration is given in Figures 2 and 3. The Figures assume a uniform cost distribution for the losing bidder, $F_l(c) = c$, for $0 < c < 1$, and plot the losing bidder's costs, c_l, on the horizontal axis and the winning bidder's cost, c_w, on the vertical axis. Figure 2 assumes that the winning bidders' cost are drawn from the distribution function $F_w(c) = c^{3/2}$, which implies substitutes, and Figure 3 assumes the distribution function $F_w(c) = c^{1/4}$, which reflects complements. Line I^* describes the optimal awarding rule characterized in Proposition 3 and given by the line $c_w = c_l + 2F_l(c_l)/f_l(c_l)$. To the northwest of line I^*, the second period contract is awarded to the losing bidder, and to the southeast of line I^*, the second period contract is awarded to the winning bidder. Line I^e describes the awarding rule under the efficient second-price procurement-auction, which coincides with the 45 degree line. To the northwest of line I^e, the second-price procurement-auction awards the second period contract to the losing bidder, while to the southeast of line I^*, it awards the second period contract to the winning bidder.

(Figures 2 and 3 about here)

Figures 2 and 3 also illustrate the outcome under the first-price procurement-auction. In both cases, the asymmetric first-price equilibrium can be calculated numerically[11] and the resulting optimal first-price allocation rule is described by line I^{FP}. To the northwest of line I^{FP} the first-price procurement-auction awards the

[11] Marshall, Meurer, Richard and Stromquist [1994] describe numerical methods to calculate the asymmetric first-price auction equillibrium b_l, b_w. The boundary is then the set of points, such that $b_l(c_l) = b_w(c_w)$.

second period contract to the losing bidder, while to the southeast of line I^{FP} the first-price procurement-auction awards the second period contract to the winning bidder. We are now in a position to compare all three procurement-auction rules, and highlight the key features of the comparison:

The efficient line I^e is to the right of the procurement cost minimizing line I^*. The reason is that the winning bidder receives the item less frequently under the efficient second-price procurement-auction than under the procurement cost minimizing rule.

The first-price awarding rule I^{FP} lies entirely either to the left or to the right of the efficient rule I^e. With substitutes, the first-price awarding rule I^{FP} is to the left, while with complements it is to the right of I^e.

Now, consider the case of substitutes, as illustrated in Figure 2. With substitutes, the first-price awarding rule I^{FP} is to the left of the efficient rule I^e, as it assigns the item to the winning bidder more frequent than is socially efficient. As a result, the first-price cut-off rule is closer to the optimal rule I^* than the efficient rule. We can conclude that the first-price procurement-auction dominates the efficient second-price procurement-auction.

Finally, consider the case of complements, as illustrated in Figure 3. With complements, the first-price rule I^{FP} lies to the right of the efficient rule I^e, and is thus further away from the procurement cost minimizing rule I^* than the efficient rule. So, in this case, we can conclude that the first-price procurement-auction is dominated by the second-price procurement-auction in terms of efficiency and also in terms of reduced procurement costs.

V. Conclusions

In this paper, we have examined optimal sequential procurement-auctions when items are complements or substitutes in the sense that an item's value increases or decreases with the number of items acquired already. We have found that the existence of complementarity or substitutability between sequentially auctioned items has consequences on the procurement costs associated with different procurement-auction rules. Our analysis has definite policy recommendations for an auctioneer that wants to minimize procurement costs:

(i) If the items are substitutes, then it is preferable to use a sealed-bid first-price procurement-auction rather than an open descending-price procurement-auction (or sealed-bid second-price auction).

(ii) If the items are complements, then it is preferable to use an open descending-price procurement-auction (or sealed-bid second-price procurement-auction) rather than a sealed-bid first-price procurement-auction.

The explanation is intuitive: Enhancing the winning probability of the first round winner in the second period leads to increased competition in the first period, and thus to lower procurement costs. With substitutes, the first-price procurement-auction correctly favors the first period winning bidder yielding lower procurement costs than the socially efficient second-price procurement-auction, which does not favor any bidder. In contrast, when items are complements, the first-price procurement-auction incorrectly favors the first period losing bidder resulting in an increased procurement cost vis-a-vis the efficient second-price procurement-auction.

It is tempting to try to explain observed procurement-auction rules and relate them to our results on the complementarity and the substitutability of the items for sale: Casual empiricism suggests that job contract bidding for governmental institutions tends to be conducted in a sealed bid format, while fine art, antiques, wine, and livestock are mostly conducted openly.

The empirical evidence from procurement-auctions for highway paving jobs and forest timber sales described earlier confirms the existence of substitutability between the items. To the extent that these procurement jobs do have a technology with decreasing returns to scale (or the firms supplying them have limited capacities), the auctioneers' chosen first-price sealed procurement-auction format is adequate in order to minimize procurement costs.

The empirical evidence on art auctions is largely anecdotal and there is no conclusive evidence on complementarities.[12] Yet, the purpose behind the purchase may be indicative of complementarities or substitutabilities between the items from the bidder's point of view. Thus, knowing the purpose motivating most of bidders' bids may be important for the auctioneer in order minimize procurement costs: When facing a bidders that are mostly trying to complete a collection, complementarities may exist and an open descending procurement-auction should be preferred. When faced with bidders that desire to acquire at most one item each, the auctioneer should anticipate the existence of substitutes and a sealed first-price procurement-auction should be chosen. For example, some of the empirical evidence on livestock auctions suggests the existence of substitutes and, according to our findings, it may be beneficial to the auctioneer to switch to a sealed bid format in those instances.

Although there is some evidence that the cost minimizing auction format is chosen

[12] See Ashenfelter and Graddy (2006) for a survey on art auctions.

in a number of settings, a more throughout empirical investigation of the auctioneer's choice of auction format is required to answer this question in more detail and to understand its implications in each case.

Appendix

Properties implied by condition (1):

(i) $F_l(c) > F_w(c)$ *for all* $c \in S$*;*

(ii) $F_l(c)/f_l(c) > F_w(c)/f_w(c)$ *for all* $c \in S$*; and*

(iii) $[1 - F_l(c)]/f_l(c) < [1 - F_w(c)]/f_w(c)$ *for all* $c \in S$*.*

Proof. Condition (1), the monotone likelihood ratio property, implies

$$\text{(A1)} \qquad f_l(c')f_w(c) > f_l(c)f_w(c') \text{ for all } c, c' \in S \text{ with } c > c'.$$

Integrating both sides of the inequality over c' from the lower endpoint of the support S to c, yields

$$\text{(A2)} \qquad \frac{f_w(c)}{f_l(c)} > \frac{F_w(c)}{F_l(c)} \text{ for all } c \in S,$$

which implies property (ii).

Next, integrate both sides of (A1) over c from c' to the upper endpoint of the support S, yields

$$\text{(A3)} \qquad \frac{1 - F_w(c')}{1 - F_l(c')} > \frac{f_w(c')}{f_l(c')} \text{ for all } c' \in S,$$

which implies property (iii).

Combining (A3) and (A2) gives

$$\frac{1 - F_w(c)}{1 - F_l(c)} > \frac{F_w(c)}{F_l(c)} \text{ for all } c \in S,$$

which implies property (i). ■

Proof of Lemma 1. We consider the case of substitutes (1). The case of complements follows by mimicking the steps in the argument with permuted bidder identity.

First, we prove that $b_l(c) > b_w(c)$. Let $\phi_i(b)$ denote the inverse of the bid function for $i = w, l$. Theorem 1 in Lebrun (1999) establishes that bid functions are strictly increasing in costs and Theorem 2 in Lebrun (1999) establishes the existence of equilibrium with common bid support. Thus, the inverse of the bid function exists and is strictly increasing. We can write the payoff of bidder w as

$$\max_b [b - c][1 - F_l(\phi_l(b))],$$

and it's associated necessary first order condition implies

$$\text{(A4)} \qquad \frac{1}{b - \phi_w(b)} = \frac{f_l(\phi_l(b))\, \phi_l'(b)}{1 - F_l(\phi_l(b))}$$

Similarly, bidder l's payoff is given by

$$\max_b [b - c][1 - F_w(\phi_w(b))],$$

and, the associated necessary first order condition implies

$$\text{(A5)} \qquad \frac{1}{b - \phi_l(b)} = \frac{f_w(\phi_w(b))\, \phi_w'(b)}{1 - F_w(\phi_w(b))}.$$

Consider a point $c \in S$ such that $b_w(c) = b_l(c)$. The first order condition and condition (1) imply that $b_l'(c) > b_w'(c)$. As the support of bids is identical, this implies $b_l(c) > b_w(c)$.

Second, we show that $Q_w^{SP}(c) < Q_w^{FP}(c)$ and $Q_l^{FP}(c) < Q_l^{SP}(c)$: From the first part above, we can deduce that the inverse bid functions satisfy, $\phi_l(b) < \phi_w(b)$

for all b contained in the interior of the support of bids. The winning probability $Q_l^{FP}(c) = 1 - F_w(\phi_w(b_l(c)))$ is less than the efficient probability $Q_l^{SP}(c) = 1 - F_w(c)$ as $c < \phi_w(b_l(c))$, and $Q_w^{FP}(c) = 1 - F_l(\phi_l(b_w(c)))$ exceeds the efficient probability $Q_w^{SP}(c) = 1 - F_l(c)$.

Third, we establish the payoff inequalities $\Pi_w^{SP} < \Pi_w^{FP} < \Pi_l^{FP} < \Pi_l^{SP}$. The interim expected equilibrium payoff of a bidder of type $i = l, w$ with cost c equals $\Pi_i^{FP}(c) = (b_i(c) - c)\, Q_i^{FP}(b_i(c))$. Now, the envelope theorem implies that

$$\frac{d}{dc}\Pi_i^{FP}(c) = Q_i^{FP}(b_i(c)).$$

As $\Pi_i^{FP}\left(\overline{C}\right) = 0$, the interim expected payoff can be written as $\Pi_i^{FP}(c) = \int_c^{\overline{C}} Q_i^{FP}(x)dx$ and, by definition, $\Pi_i^{SP}(c) = \int_c^{\overline{C}} Q_i^{SP}(x)dx$. Now, the payoff inequalities $\Pi_w^{SP}(c) < \Pi_w^{FP}(c)$ and $\Pi_l^{FP}(c) < \Pi_l^{SP}(c)$ follow from the probability inequalities in the second part, $Q_w^{SP}(c) < Q_w^{FP}(c)$ and $Q_l^{FP}(c) < Q_l^{SP}(c)$. As the payoff payoff inequalities hold for all $c \in S$, they hold also ex ante, before costs are observed, which establishes the claim.

The final inequality that we need to establish is $\Pi_w^{FP} < \Pi_l^{FP}$: Let $G_i(b)$ denote the probability that a bid b wins the procurement-auction for bidder $i = l, w$. We begin by showing that $G_w(b) < G_l(b)$ and then establish the inequality on profits. Let $\underline{b}$ denote the lower endpoint of the support of bids. As $\phi_l(b) < \phi_w(b)$ for all b contained in the interior of the support of bids, conditions (A4) and (A5) imply that

$$f_w(\phi_w(b))\, \phi_w'(b) \,/\, [1 - F_w(\phi_w(b))] < f_l(\phi_l(b))\, \phi_l'(b) \,/\, [1 - F_l(\phi_l(b))]\,.$$

This can be written as $-(d/db)\ln[1 - F_w(\phi_w(b))] < -(d/db)\ln[1 - F_l(\phi_l(b))]$. Since $[1 - F_w(\phi_w(\underline{b}))] = [1 - F_l(\phi_l(\underline{b}))]$, this implies $\ln[1 - F_w(\phi_w(b))] > \ln[1 - F_l(\phi_l(b))]$, or equivalently, $G_l(b) = 1 - F_w(\phi_w(b)) > 1 - F_l(\phi_l(b)) = G_w(b)$. Now,

$$\begin{aligned}
\Pi_w^{FP}(c) &= [b_w(c) - c]\, G_w(b_w(c)) \\
&< [b_w(c) - c]\, G_l(b_w(c)) \\
&\leq [b_l(c) - c]\, G_l(b_l(c)) \\
&= \Pi_l^{FP}(c),
\end{aligned}$$

which establishes that $\Pi_w^{FP}(c) < \Pi_l^{FP}(c)$ for all $c \in S$.

Now, the ex ante payoff difference can be written as:

$$\begin{aligned}
\Pi_w^{FP} - \Pi_l^{FP} &= \int_S \Pi_w^{FP}(c)\, f_w(c) dc - \int_S \Pi_l^{FP}(c)\, f_l(c) dc \\
&< \int_S \Pi_l^{FP}(c)\, [f_w(c) - f_l(c)]\, dc \\
&= \int_S \left(-\frac{\partial \Pi_l^{FP}(x)}{\partial c} \right) [F_w(x) - F_l(x)]\, dx \\
&\leq 0
\end{aligned}$$

The first inequality uses $\Pi_w^{FP}(c) < \Pi_l^{FP}(c)$ for all $c \in S$. The second equality follows from integration by parts. The final inequality is based on two observations: First, the term in square brackets is negative from property (i) of condition (1). Second, the term in round brackets is positive as $-\frac{\partial \Pi_l^{FP}(c)}{\partial c} = -Q_l^{FP}(c) < 0$. This completes the proof. ■

Proof of Theorem 1. By using equation (5), the difference between the second-price game payoff and the first-price game payoff, equals,

$$\Pi^{SP} - \Pi^{FP} = \beta \left[\Pi_l^{SP} - \Pi_l^{FP} \right].$$

By Lemma 1, the right hand side is positive under condition (1), and negative under condition (2). ■

Proof of Theorem 2. The efficient second-price winning probabilities are given by $Q_l^{SP}(c) = 1 - F_w(c)$ and $Q_w^{SP}(c) = 1 - F_l(c)$. As in the proof of Lemma 1, we denote the inverse bid functions in the first-price procurement-auction equilibrium by ϕ_w and ϕ_l. The first-price winning probabilities are then given by $Q_l^{FP}(c) = 1 - F_w(\phi_w(b_l(c)))$ and $Q_w^{FP}(c) = 1 - F_l(\phi_l(b_l(c)))$. The procurement cost difference, $D = \left[PC^{SP} - PC^{FP}\right]/\beta$, can be written as:

$$\begin{aligned}
D &= \int_S c\left[F_w\left(\phi_w\left(b_l(c)\right)\right) - F_w(c)\right] f_l(c)\, dc \\
&\quad + \int_S c\left[F_l\left(\phi_l\left(b_w(c)\right)\right) - F_l(c)\right] f_w(c)\, dc \\
&\quad + 2\int_S F_l(c)\left[F_w\left(\phi_w\left(b_l(c)\right)\right) - F_w(c)\right] dc \\
&= \int_S c\left[F_w\left(\phi_w\left(b_l(c)\right)\right) - F_w(c)\right] f_l(c)\, dc \\
&\quad + \int_S c\left[F_l\left(\phi_l\left(b_w(c)\right)\right) - F_l(c)\right] f_w(c)\, dc \\
&\quad + \int_S F_l(c)\left[F_w\left(\phi_w\left(b_l(c)\right)\right) - F_w(c)\right] dc \\
&\quad - \int_S c\left[F_w\left(\phi_w\left(b_l(c)\right)\right) - F_w(c)\right] f_l(c)\, dc \\
&\quad - \int_S cF_l(c)\left[f_w\left(\phi_w\left(b_l(c)\right)\right)\phi_w'\left(b_l(c)\right) b_l'(c) - f_w(c)\right] dc \\
&= \int_S F_l(c)\left[F_w\left(\phi_w\left(b_l(c)\right)\right) - F_w(c)\right] dc \\
&\quad + \int_S \left[c - \phi_l\left(b_w(c)\right)\right] F_l\left(\phi_l\left(b_w(c)\right)\right) f_w(c)\, dc.
\end{aligned}$$

The second equality follows from integration by parts of the expression,

$$\begin{aligned}
\int_S F_l(c)\left[F_w\left(\phi_w\left(b_l(c)\right)\right) - F_w(c)\right] dc &= -\int_S c\left[F_w\left(\phi_w\left(b_l(c)\right)\right) - F_w(c)\right] f_l(c)\, dc \\
&\quad - \int_S cF_l(c)\left[f_w\left(\phi_w\left(b_l(c)\right)\right)\phi_w'\left(b_l(c)\right) b_l'(c) - f_w(c)\right] dc.
\end{aligned}$$

The third equality cancels terms and uses the substitution $u = \phi_w(b_l(c))$ which yields:

$$\int_S cF_l(c) f_w(\phi_w(b_l(c))) \phi_w'(b_l(c)) b_l'(c)dc = \int_S \phi_l(b_w(c)) F_l(\phi_l(b_w(c))) f_w(c) dc$$

as the function $\phi_w(b_l(c))$ is from S onto S.

By Lemma 1, condition (1) implies that $\phi_w(b_l(c)) > c$ and $c > \phi_l(b_w(c))$. Thus, $PC^{SP} - PC^{FP} > 0$ which establishes part (i). By Lemma 1, condition (2) implies that $\phi_w(b_l(c)) < c$ and $c < \phi_l(b_w(c))$. Thus, $PC^{SP} - PC^{FP} < 0$ which establishes part (ii). ■

Proof of Lemma 2. It is well known - Mas-Colell, Whinston and Greene (1995), Proposition 23.D.2 - that the allocation (Q_i^t, T_i^t) is Bayesian incentive compatible if and only if, for all $i, t = 1, 2$,

(i) Q_i^t is monotone decreasing, and

(ii) $\Pi_i^t(c,c) = \int_c^{\overline{C}} Q_i^t(x)\,dx + \Pi_i^t(\overline{C},\overline{C})$ for all $c \in S$.

Notice, that integration by parts yields:

$$\int_S \Pi_i^2(x,x) f_i(x) dx = \int_S F_i(c) Q_i^2(c)\,dc + \Pi_i^2(\overline{C},\overline{C}) \quad \text{for } i = l, w.$$

In turn this implies that the expected transfer payment of a bidder with cost c equals:

$$\begin{aligned}
T_i^2(c) &= cQ_i^2(c) + \int_c^{\overline{C}} Q_i^2(x)\,dx + \Pi_i^2(\overline{C},\overline{C}); \\
T_i^1(c) &= cQ_i^1(c) + \int_c^{\overline{C}} Q_i^1(x)\,dx - \beta\left[\int F_l(x) Q_l^2(x)\,dx + \Pi_l^2(\overline{C},\overline{C})\right] \\
&\quad + \beta Q_i^1(c)\left[\int F_l(x) Q_l^2(x)\,dx + \Pi_l^2(\overline{C},\overline{C}) - \int F_w(x) Q_w^2(x)\,dx - \Pi_w^2(\overline{C},\overline{C})\right] \\
&\quad + \Pi_i^1(\overline{C},\overline{C});
\end{aligned}$$

and then, as $Q_1^1 + Q_2^1 = 1$, the expected sum of discounted transfer payments equals,

$$\sum_{i=1,2} \int_S \left[\left(c + \frac{F(c)}{f(c)} \right) Q_i^1(c) \right] f(c)\, dc + \sum_{i=1,2} \Pi_i^1(\overline{C}, \overline{C})$$
$$+\beta \int_S c Q_l^2(c)\, f_l(c)\, dc + \beta \int_S c Q_w^2(c)\, f_w(c)\, dc.$$

To obtain the final expression stated in the Lemma, we add and subtract the term $2\beta \int F_l(x) Q_l^2(x)\, dx$. Observe also, that the voluntary participation constraint in period one requires that $\Pi_i^1(\overline{C}, \overline{C}) \geq \beta \int F_l(x) Q_l^2(x)\, dx$. ■

Proof of Proposition 3. Pointwise maximization of the procurement cost expression implies the stated allocation rule. The cost minimizing first period expected continuation payoff equals $\Pi_i^1(\overline{C}, \overline{C}) = \beta \int F_l(x) Q_l^2(x)\, dx$. ■

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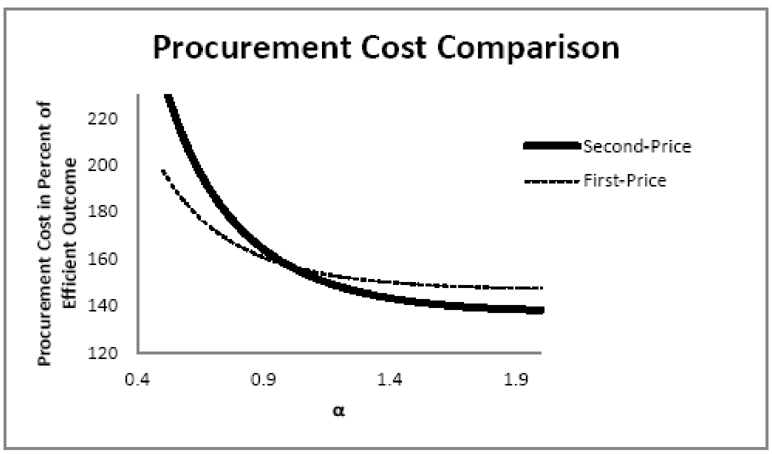
Procurement Cost Comparison
Procurement Cost in Percent of Efficient Outcome
220
200
180
160
140
120
0.4
0.9
1.4
1.9
α
Second-Price
First-Price

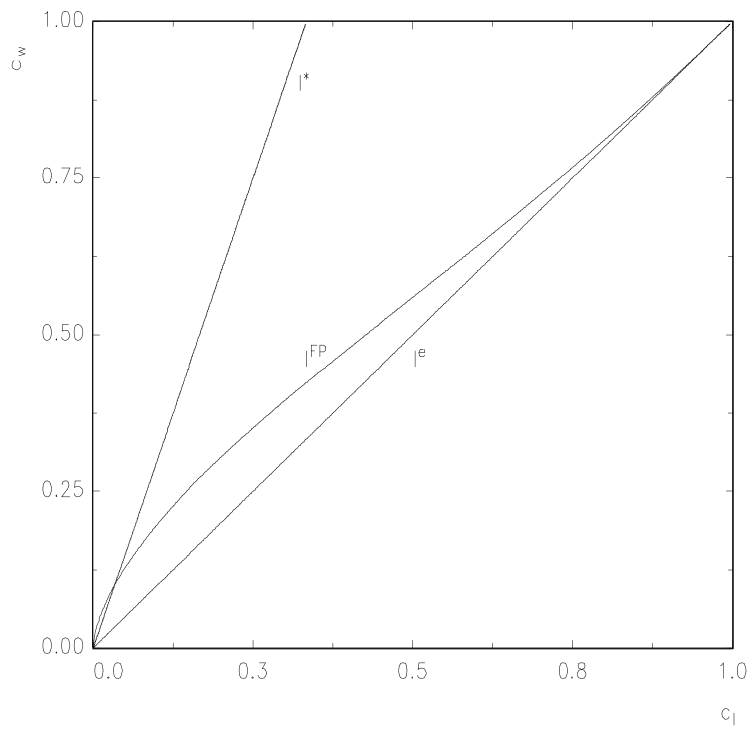

Figure 1: Substitutes

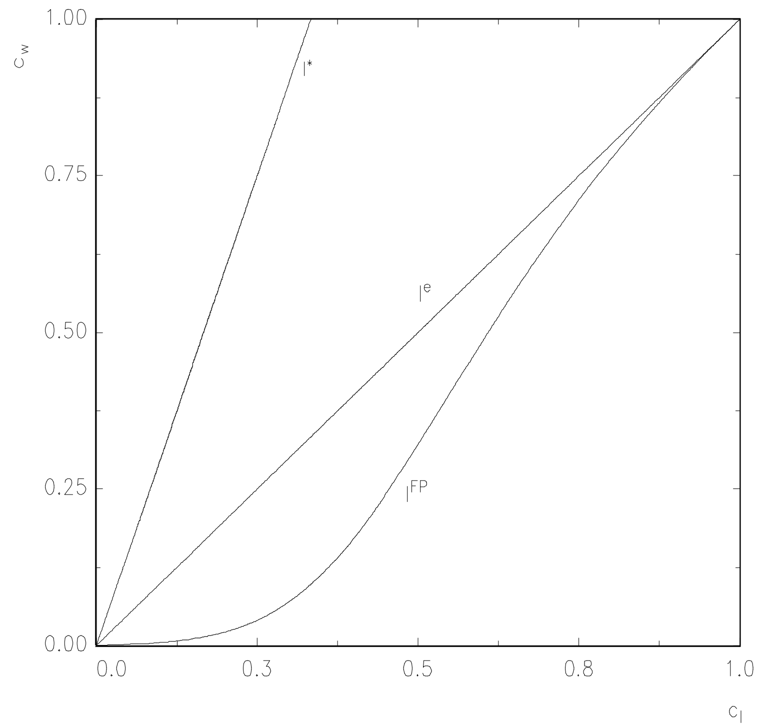

Figure 2: Complements