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## “Structure Evolving Systems and Control in Integrated Design”

Nicos Karcanias

Control Engineering Centre,  
School of Engineering and Mathematical Sciences,  
City University, Northampton Square, London EC1V 0HB, UK

e.mail: [N.Karcanias@city.ac.uk](mailto:N.Karcanias@city.ac.uk)

### Abstract

Existing methods in Systems and Control deal predominantly with *Fixed Systems*, that have been designed in the past, and for which the control design has to be performed. The new paradigm of *Structure Evolving Systems (SES)*, expresses a new form of system complexity where the components, interconnection topology, measurement-actuation schemes may not be fixed, the control scheme also may vary within the system-lifecycle and different views of the system of varying complexity may be required by the designer. Such systems emerge in many application domains and in the engineering context in problems such as integrated system design, integrated operations, re-engineering, lifecycle design issues, networks etc. The paper focuses on the *Integrated Engineering Design (IED)*, which is revealed as a typical *structure evolution process* that is strongly linked to Control Theory and Design type problems. It is shown, that the formation of the system, which is finally used for control design evolves during the earlier design stages and that process synthesis and overall instrumentation are critical stages of this evolutionary process that shapes the final system structure and thus the potential for control design. The paper aims at revealing the control theory context of the evolutionary mechanism in overall system design by defining a number of generic clusters of system structure evolution problems and by establishing links with existing areas of control theory. Different aspects of *model evolution* during the overall design are identified which include cases such as: (i) Time dependent evolution of system models from “early” to “late” stages of design. (ii) Design stage dependent evolution from conceptualisation to process synthesis and to overall instrumentation. (iii) Re-design of given systems and constrained system evolution. Within each cluster a number of well defined new Control Theory problems are introduced, which may be studied within the structural methodologies framework of Linear Systems. The problems posed have a general systems character, but the emphasis here is on Linear Systems; an overview of relevant results is given and links with existing research topics are established. The paper defines the Structural Control Theoretic context of an important family of complex systems emerging in engineering design and defines a new research agenda for structural methods of Control Theory.

## 1. Introduction

Complex Systems emerge in many disciplines and domains and have many interpretations and implications. Different communities view complexity from their domain specifics and frequently the dialogue between communities such as biologists, physicist, economists, sociologists, computer scientists and engineers becomes difficult, or impossible. Mathematical Systems and Control Theory have the potential to provide the unifying framework (language and concepts) and the required tools (analysis, synthesis) for studying such problems, as long as it develops to handle some of the new challenging paradigms emerging from the widening field of applications. The paper deals with the form of complexity inherent in the new paradigm of *Structure Evolving Systems (SES)*, as this emerges in the context of system, or integrated design. Such forms of systems emerge in many applications and are characterised by a variability of the system structure, its components and possibly its environment, in a way that defines an evolution of the system structure and the associated properties. Integrated Design (ID) (Karcnias, 1994a, 2000) is a challenging task in many application areas (aeronautics, process systems (Stephanopoulos, 1984) etc) and defines the focus of our study of the *SES* paradigm by providing a number of mostly open structural systems and control problems. The case of process systems (Perkins, 1990), (Rijnsdorp, 1991) provides the additional focus of our current study. The aim of this paper is to identify the open issues which have a clear systems and control character and establish the links with existing areas in control.

Existing methods in Systems and Control deal predominantly with “fixed systems”, that is those where the components, interconnection topology, measurement-actuation schemes, systems environment and control structures are fixed. The process of overall design of a system (process synthesis - global instrumentation - control) has a cascade nature (Karcnias, 2000) and this introduces a notion of “shaping”, evolution of the model and thus of the resulting system properties. In fact, as we go through the different design stages we have an evolution of structure, topology, properties and behaviour of the overall system. Design is an iterative process and thus it is characterised by “early” and “late” stages. “Early design” requires evaluation of many alternatives using simple models and methodology (EPIC, 1989), (Rijnsdorp, 1991) whereas “late design” uses models of greater complexity and accuracy and requires more detailed evaluation of performance. Decision making at each stage is largely based on local criteria; this is a consequence of overspecialisation and lack of a holistic co-ordination of design methodology. Similar nature problems arise in the re-engineering of existing systems/networks in their upgrading to meet new requirements and performance demands. This may involve physical addition (growth), or removal (death) of parts of the system and represents evolution of a given system shell along a number of possible paths by intervening on the subsystem components, process synthesis/topology of interconnections, and overall instrumentation. Key questions that arise relate to modelling such forms of evolution, and then express model evolution in terms of the structural features and properties of the respective models. The major challenge is managing complexity involving the control, or direction of such an evolutionary process along “paths” with desirable properties. This requires a methodology that is based on results characterising the potential for evolution of system properties (genetic selection of alternatives), explain the link of structure to invariants and performance indicators, characterise model uncertainty within a given system structure, define the “good” or “bad” potential for design, and provide means for addressing “structure assignment” problems. Responding to such tasks requires a new and richer form of Control Theory that is empowered to deal with the management of structure evolution.

The need for integrating process and control design has been recognised in the Process, Aerospace and other areas of applications, but with a few exceptions (EU Project SEDIP (1994)), little attention has been given in the development of an integrated Systems and Control Theory Based Framework that may integrate the traditional design stages, such as *Process Synthesis (PS)*, *Global Instrumentation (GS)* and finally *Control Design (CD)*. Design is a cascade and complex process that is characterised by different forms of system evolution. This evolution has three main features: The first is a natural evolution of the system structure as this is shaped through the design stages from conceptualisation, to process synthesis, global instrumentation and finally control design and it is referred to as *cascade structural evolution*. The second stems from the need to address design and decision problems at “early” and “late” stages of system design (as part of an iterative design cycle) using models with a variability in their complexity and it is referred to as *design time evolution*. The third deals with the type of evolution linked to re-engineering of a given structure and will be referred to as *structural growth-death evolution*. The paper aims to describe those three forms of system evolution from a control theoretic viewpoint, review existing approaches and define a research agenda for the structural system methodologies that can deal with such issues. The proposed structural approach is based on defining and studying a number of partial problems, which when combined may provide the essentials for a control theoretic framework for systems integration. The paper addresses control theory issues linked to overall system design, which have a structural nature and express aspects and stages of the evolutionary design process. Amongst the issues considered are:

- (i) Modelling issues in Early Process Design and model structure evolution (conditioning of “progenitor models”, variable complexity modelling etc).
- (ii) Composite systems properties as functions of the interconnection graph (completeness of the interconnection and deviations from completeness).

- (iii) System and Control problems in Global Instrumentation (Orientation, Model Projection, Local-Global Model Enhancement)
- (iv) Generalised Structural Design (assignment by design, or modification of the interconnection graph, selection of the effective set of inputs, outputs).
- (v) “Growth-Death” system evolution and Life-cycle Design.

The above areas introduce major challenges for Control Theory in the context of the new paradigm represented by the *Structure Evolving Systems*. This family of systems departs considerably from the traditional assumption that the system is fixed and its dominant features are: **(i)** The topology of interconnections is not fixed but may vary through the life-cycle of the system (*Variability of Interconnection Topology*). **(ii)** The overall system may evolve through the early-late stages of the design process (*Evolution through the Design Process*). **(iii)** There may be Variability and/or uncertainty on the system’s environment during the lifecycle requiring flexibility in organisation and control strategies (*Lifecycle Complexity*). **(iv)** The system may be large scale, multi-component and this may impact on methodologies and computations (*Large Scale – Multi-component Complexity*). **(v)** There may be variability in the Organisational Structures of the information and decision making (control) in response to changes in goals and operational requirements (*Organisational Complexity Variability*).

Some of the issues emerging here are related to structure assignment and have been considered in some particular form in Control theory in the study of: (i) zero assignment by squaring down (Rosenbrock & Rower, 1970), (Kouvaritakis & Macfarlane, 1976), (Karcnias & Giannakopoulos, 1989), (Saber & Sannuti, 1990); (ii) the dynamic cover problem of geometric theory (Wonham, 1979), (Karcnias & Vafiadis, 1993); (iii) the Morgan problem in control design (Descusse etc, 1988); (iv) structure assignment of matrix pencils (Loiseau etc, 2004), (Leventides etc 2000); (v) selection of decentralisation structure (Karcnias etc 1997) etc. Such results contribute to the shaping of this new framework, but they are of partial nature and no effort to link them in a unifying framework has been made so far. Here, we introduce a number of challenging new problems for structural system theory linked to system evolution. This provides a framework for studying structure evolution in a systematic way, by defining key partial problems, discussing their fundamentals, reviewing existing approaches and finally defining a research agenda with a clear structural perspective for this new systems paradigm. The study for this new paradigm uses results of the classical structural methodologies for linear systems (Kalman, 1962, 1963, 1972), (Popov, 1969) (Rosenbrock, 1970), (Brunovsky, 1970), (Morse, 1973), (Wolovich, 1974) (Forney, 1975), Warren & Eckberg, 1975), (Karcnias & MacBean, 1981), (Ozcaldiran, 1986), (Siljak, 1991), (Loiseau etc 1991b), (Lewis, 1992), (Karcnias & Leventides, 1995), (Reinschke, 1998) etc and introduces new challenging problems.

The paper is organised as follows: Section (2) introduces the problem of integrated design as an evolutionary process. In Section (3) we discuss the general modelling issues in early-late system design the associated problem of “model embedding”, the evaluation of properties at early stages, the conditioning of “early” models and the problem of structural identification. Section (4) examines the area of process synthesis as a feedback design problem and the related issues of completeness and deviations from completeness. The system problems associated with the overall selection of inputs and outputs, referred to as “global instrumentation” are considered in section (5), where issues of model orientation, model projection, model expansion and local-global model enhancement are considered. Finally, in section (6) we introduce the problem of “growth-death” system evolution in problems of Life-cycle Design with particular emphasis on networks.

## 2. The Problem of Integrated Design as an Evolutionary Process

Complex Systems is a generic term used to describe some of the major challenges in Science and its applications, Engineering, Business, Society, Environment, etc. The term refers to problems which may be of large or small scale, centralised or distributed, have a composite nature (in terms of simpler sub-problems), high degree of interaction between subsystems, manifest a multi-facet behaviour (in terms of particular aspects), have possibly an internal organisation and require a multidisciplinary approach for their study. It is thus clear that complexity has many different dimensions and gaining understanding for each of these dimensions is critical in developing approaches for complex systems. The nature of complexity implies that there is need for division of the overall problem into sub-problems which may be more easily handled by teams of specialists. Such solutions are usually worked out by teams of experts with little knowledge on the issues of the other areas; furthermore, there is no global co-ordination and understanding of the interactions of the alternative aspects of complexity and this makes the development of acceptable global solutions a major challenge. Systems Integration emerges as the general task that can co-ordinate the activities in the particular sub-problem areas to produce solutions which are meaningful and optimal (in some sense) for the whole. The development of a systemic, holistic approach for integration requires ability to specialise the set of global objectives to the level of the subsystem, methods to work out solutions which are locally and globally feasible and in a sense optimal, as well as understanding of interactions between the subsystems and alternative aspects of the overall problem.

Systems integration is a typical form of a complex engineering that has a multidisciplinary character and deals with the integrating (Rijnsdorp, 1991), (Karcnias, 2000): **(i)** Engineering Design Stages, **(ii)** Process Operations, **(iii)**

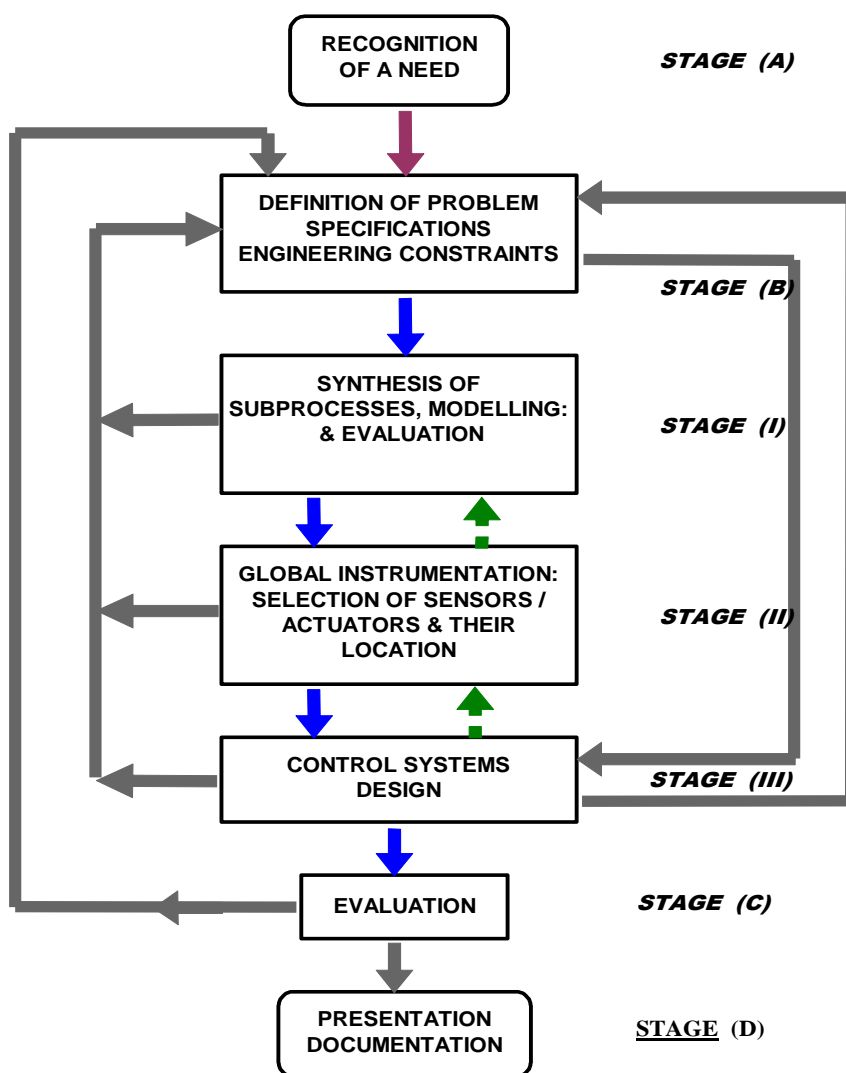
Engineering Design and Process Operations and (iv) Process Operations and Business Aspects. Each of the above areas has distinct dimensions relating to: (i) Physical Process, (ii) Signals, Operations and (iii) Data, IT, Software. The Physical Process Dimension deals with issues of design-redesign of the engineering system and predominantly relates to our driving paradigm the Integrated Design. Clearly, design has also a signals and data dimension which is not considered here. The area of integrated Process and Control Design has been recognised as very important, especially in the Chemical Processes (Perkins, 1990), (Rijnsdorp,1991), (Morari, 1992); however, the existing approaches largely depend on the specifics of the application area, rather than providing a general framework that may be used in different application areas. The ESPRIT II Project EPIC (1989), “Early Process Design Integrated with Control”, has been an effort to provide a control based framework for the Early Process Design Stages of Continuous Chemical Processes; a first attempt to develop a system based framework for control and instrumentation integration has been the ESPRIT Project SEDIP (1994) and a description of the overall integration philosophy described as an evolutionary process introduced in (Karcianas, 1995a, 1996, 2000) is elaborated here.

The general features of the technological stages of the overall system design are briefly considered first and are summarised by the diagram of Figure (1). The exchange of information illustrated in the above diagram between the different design stages has a short prediction horizon, as far as the impact on the subsequent design stages, and it is of a local character. This local character is dominated by the specialised skills, theory and techniques needed for a given engineering task. The ability to translate local decisions as actions assigning certain structure to the stage model is currently missing. The common engineering practice is dominated by heuristics, simulations, trial and error and final testing on a pilot plant. Accelerating the design process is crucial and this may be helped by developing a Global Coordination Theory for the design process. Our attention is focused on the technological stages of design, that is:

**STAGE (I) :** Process Synthesis

**STAGE (II):** Overall System Instrumentation (Global Instrumentation)

**STAGE (III):** Control Design.



**Figure (1):** Simplified form of Engineering Design Process

Such a procedure is iterative and it is the result of the technological complexity of the engineering task that requires specialisation. The process synthesis – global instrumentation - control design stages have a cascade nature with feedback loops between the various sub-stages. The cascade nature of design is underlying the evolutionary process of model shaping, that drives the integrated design paradigm (Karcnias,1995a, 2000). The main inputs at every design stage are the special skills, knowledge, local objectives and specification and the model of the previous design stage. Secondary inputs are provided by the exchange of information between the given stage and the other design stages, whenever they exist. The cascade design process is dynamic in the sense that what it is feasible to achieve at a given stage is influenced by the decisions taken at the previous design stages. It is thus a characteristic feature of the cascade design process that decisions taken at one stage, which may be seen as technically reasonable and economically sound according to local criteria, may, not necessarily be good as far as the overall design process. In fact, the overall system tends to display behaviour that is not an aggregate of partial behaviours, but a gradual evolution, expressed in terms of model, properties and respective behaviour. Understanding this evolution of the system and its properties as we go through the successive design stages is crucial in developing the tools for managing this evolution and thus contribute to the development of a *Global Coordination Theory (GCT)* (Karcnias, 2000) that is crucial for addressing *systems integration* of complex modern designs.

The formation of structural characteristics of the overall process is reminiscent of an evolution process. The first stage, the process synthesis, acts as the parent gene and thus predetermines a possible range of key characteristics of the final process. Decisions on the successive design stages, contribute to the gradual shaping of the final model structure, however, within a range of possible options and correspond to a sequence of successive mutations. Structural properties evolve, but not in a simple manner. Ideally, we would like to have assigned certain desirable characteristics to the model of every single design stage and thus finally guarantee the shaping of a process with specified properties. This requires perfect control of the model evolution process; however, not all activities may be modelled and what is desirable as final design is impossible to predict at the beginning of each design stage. A more feasible design philosophy is to direct the model evolution process towards final designs that may possess some desirable properties and avoid the formation of undesirable features that may penalize the final control design. This methodological framework is specialised to the design stage of process synthesis and global instrumentation considered below.

**Process Synthesis:** This is an act of determining the optimal interconnection of processing units, as well as the optimal type and design of the units within a process system. The structure of the system and the performance of the process units are not determined uniquely by the performance specifications. The task is then to select a particular system out of the large number of alternatives which meet the specific performance specifications. Some of the basic problems in Process synthesis are (Morari & Stephanopoulos, 1980), (Morari, 1992): **(i)** The Representation Problem, **(ii)** The Evaluation Problem, **(iii)** The Strategy Problem. The first deals with the question of whether a representation can be developed, which is rich enough to allow all alternatives to be included. The second deals with the question of whether the design alternatives can be evaluated effectively, so they may be compared. The final problem deals with whether it is possible to locate quickly the better alternatives without totally enumerating and testing all options. Problems **(i)** and **(iii)** heavily depend on the specific applications domain. Systems and Control provide generic results which can be used to formulate alternative approaches based on generic concepts and these will be considered subsequently.

**Global Instrumentation:** This deals with the selection of the set and the distribution of inputs and outputs and its study revolves around the investigation of a number of fundamental system type problems. The traditional instrumentation of a process (referred to as “micro”, local aspect) deals with the problem of measurement, or implementation of action upon given physical variables. When however we move from the single physical variable to the classification of internal variables and then the selection of sets of measurement variables (outputs), and actuation variables (inputs), this role changes; we then move from the physical layer to a systems level that is referred to as a “macro” (global) aspect of the instrumentation. The “micro” role of instrumentation (Finkelstein & Grattan, 1994) has been well developed and deals with instrumentation theory and practice. The “macro” aspects (Karcnias, 1994a) of instrumentation stem from that the selection and classification of system variables into inputs and outputs expresses the attempt of the “observer” (designer) to build bridges with the “internal mechanism” of the process in order to observe it and/or act upon it. What is considered as the final system, on which Control System Design is to be performed, is the object obtained by the interaction of the “internal mechanism” and the specification of the overall instrumentation scheme, which plays a critical role in specifying many of structural characteristics of the final system model that determine the easy, or difficult nature of the control design problem. The model evolution at this stage and the implications on the shaping of structural characteristics is an issue with a distinct control theoretic context and it will be examined subsequently.

The problem of evolution of model structure through the successive design stages has not been considered before in any systematic way with the exception of the work in the projects EPIC, 1989, and SESDIP (1994). Over specialisation in engineering has resulted that process and instrumentation engineers have no understanding of the effects of their decisions on model structure shaping and control engineers have assumed that the system is already formed and thus with few exceptions (squaring down problem (Kouvaritakis & MacFarlane, 1976), Morgan problem (Descusse etc, 1988)), they have not addressed the problem of studying the mechanisms of model structure formation. Understanding

the mechanisms of model structure formation in the early stages of design is challenging and introduces new problems. The following forms of evolution in design are considered:

- (a) *Design Time Evolution* from “early” to “late” design stages
- (b) *Numerical Dependent Evolution* and model accuracy.
- (c) *Cascade Design* from Process Synthesis to Global Instrumentation.
- (d) *Physical Growth, or Lifecycle Evolution.*

The first notion of model evolution is linked to the general procedure in design, where we have a fixed interconnection structure, but at the early stages we require *simple modelling* for sub-processes and physical interconnections; at the late stages of design *more detailed*, full dynamics models are required for sub-processes and physical interconnection structures. Here, we observe an evolution of the given structure of the system in the *design stage time* axis and this problem expresses the *Early-Late Design Variability of Model Complexity* and corresponding accuracy (Karcianas, 1995b). The second type is linked to the level of numerical accuracy of the model that is used and it refers to the corresponding evolution of predicted properties for the respective model families. The third form of evolution is clearly connected to the cascade nature of the design process. The fourth form is linked to the physical growth, reshaping of the system during some re-engineering in response to different demands and it is a form linked to lifecycle issues.

### 3. Modelling in Early-Late Integrated Design as an Evolutionary Process

The problems posed here arise in different fields of engineering, and most predominantly in chemical processes (Douglas, 1988), (Rijnsdorp, 1991), (Stephanopoulos, 1984) where there is a need to visualise a complete design of the system, or many possible alternative designs at very early stages, evaluate them in some way and then select the most promising one for further elaboration. These imply that we need: (i) Methods for generating simple models from specifications and appropriate conceptualisations (*Representation*); (ii) Ability to create nests of models as we proceed from early to late stages that evolve within a physical structure and increase in complexity (*Structured Variable Complexity Nesting*) and accuracy; (iii) Methods to evaluate full model features and properties on simple early forms (*Early Property Prediction*); (iv) Theory to explain evolution of system properties as a function of model complexity (*Property Evolution*). The current practice is dominated by heuristics and little theory. There is need to formalise these unstructured problems in a way that will enable formal methods to be used and appropriate concepts and tools to be developed. These issues are considered in this section and representative problems will be defined.

#### 3.1 Early Models and the Design Time Evolution Nesting

A special form of system evolution is linked to the need for *variable complexity modelling* (VCM) in the design process as we move from early to late design. In fact, by assuming that we have a fixed interconnection structure throughout the design, then at the Early Stages we require *simple modelling* for sub-processes and physical interconnections, whereas at the Late Stages of design *more detailed*, full dynamic models are required for both sub-processes and physical interconnection structures. The study of such problems requires the development of a framework that permits the transition from simple graphs to full dynamic models and allows study of Systems and Control properties in a unifying way. In the following, Process systems are used as the motivating example. For such systems a fundamental stage in the design process is the problem of Conceptual Modelling. This transforms Requirements and Objectives to sets of Preliminary Designs referred to here as *conceptual process flow-sheets*  $\mathbf{M}_i^c$  and the procedure is described in (Douglas, 1988), (EPIC, 1989). For Chemical processes, this is done by experienced Chemical Engineers and the overall set of such models is denoted by:  $\mathbf{M} = \{\mathbf{M}_i^c, i=1,2,\dots,k\}$  where the basic elements in modelling are:

- (i) The general interconnection rule defining the associated graph.
- (ii) The early description of sub-processes in terms of simple models.

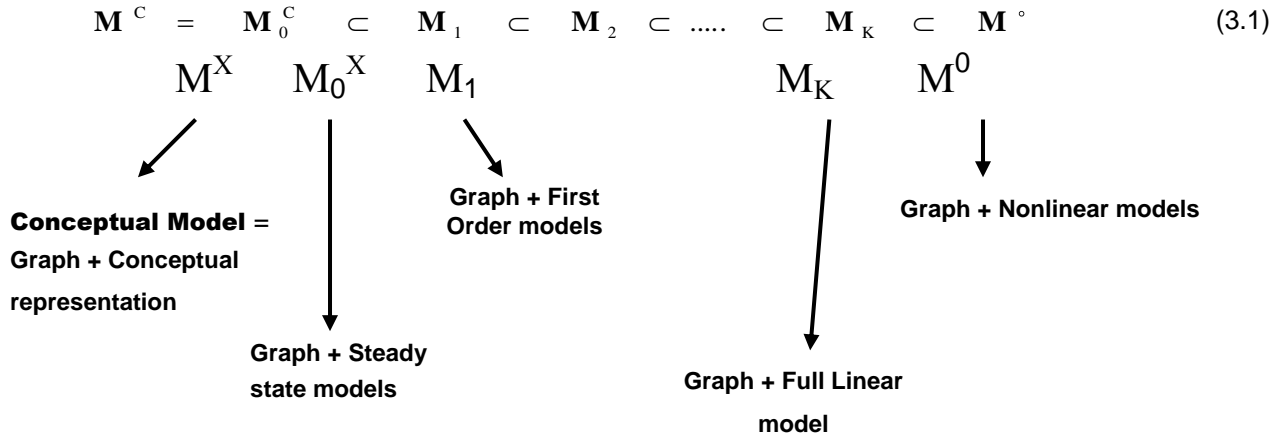
The exact nature of the graph depends on the stage of the design (*early, late*) and this is affected by the nature of models for local processes and the description of the physical interconnection streams. We may define the following notion of a graph associated with the system:

**Definition (3.1):** Let us denote every subsystem  $\Sigma_i$  by a pair of vertices  $(\underline{e}_i, \underline{w}_i)$ , denoting inputs and outputs, and an edge  $\underline{g}_i$  providing an input-output description of  $\Sigma_i$ . If we denote by  $f_{ik}$  the physical (information) streams connecting the  $\underline{w}_i$  output and the  $\underline{e}_k$  input, the set  $\{(\underline{e}_i, \underline{w}_i), \underline{g}_i, f_{ik} \forall i, k=1,2,\dots,\mu\}$  will be called the *kernel graph* of the system. ■

This graph model is the simplest representation of the flow-sheet, it is denoted as  $\mathbf{M}^c$  and it is referred to as the *kernel model*.  $\mathbf{M}^c$  contains the basic information linked to subsystems and physical streams, defines a primitive form of structure that stems from the conceptual model of the system and provides the minimal information on the physical interconnection topology. At later stages the dimensionality of physical interconnection streams may change, if more than one variable is associated with the physical streams, as we increase our requirements for modelling. This

variability from 1-dimensional vertices, edges to many dimension vertices, edges respectively describes a form of evolution defined as *Dimensional Variability of Graphs*. Fundamental issues related to the dimensional variability of the graph relate to the classification of the properties of the directed graph, which are independent/dependent on the dimensionality of the corresponding nodes.

Starting from the kernel model, we may develop models of increasing complexity, generated from the same  $M^c$  model. This is done by preserving the generic structure of the interconnection rule, the *kernel graph*, and successively using models with increasing complexity for the sub-process. This leads to the following nested set of models:



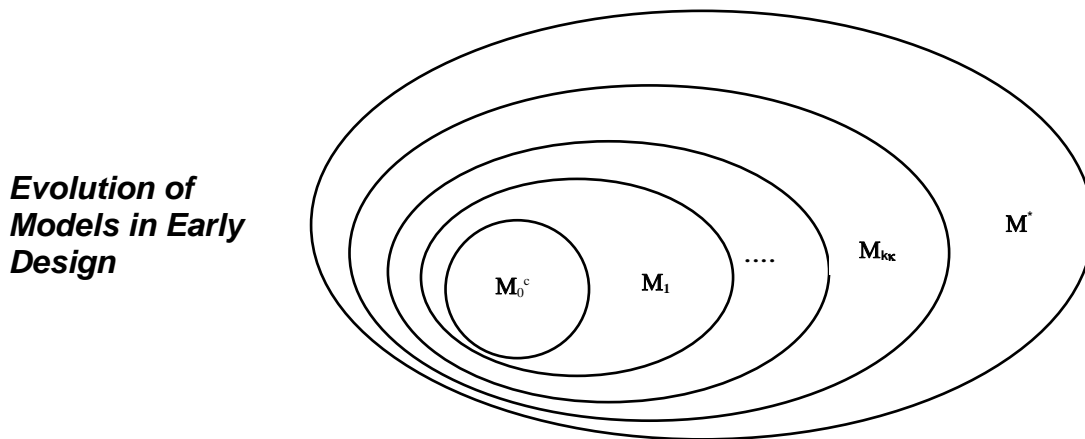
**Figure (2):** Nested set of models of variable complexity

Clearly, the process of model building continues beyond the construction of  $M^o$ , which is the simplest nonlinear model. This nesting described expresses an evolution of the overall system model, parameterised by complexity (McMillan degree for linear systems) which is due to the evolution of *dynamic richness* of the subsystem models and it is due to the time dimension (*Early-Late*) of the design process referred to as *Design Time Evolution*.

If  $\mathfrak{I}$  denotes the Graph of  $M^c$  and we denote by  $\{M_i^a, i=1, \dots, \mu\}$  the aggregate of the simple models of the a-stage, we can denote by  $M^a$ , the model defined as:  $M^a = \mathfrak{I} * \text{diag}\{M_i^a; i=1, \dots, \mu\}$ . As model complexity for subsystems increases, we may also consider issues of dimensional expansion and/or evolutionary expansion of the corresponding graph (vertices, edges expansion). The latter implies that scalar nodes and edges in a graph may become vector nodes and vector edges and this represents a *Dimensional Graph Evolution* form. Instead of assuming a fixed  $\mathfrak{I}$  as above we may assume a set  $\{\mathfrak{I}\}$  and for every  $\mathfrak{I} \in \{\mathfrak{I}\}$  we define  $M^a = \mathfrak{I} * \text{diag}\{M_i^a; i=1, \dots, \mu\}$  with  $\mathfrak{I}$  satisfying the graph evolution:

$$\mathfrak{I}_0 \subseteq \mathfrak{I}_1 \subseteq \mathfrak{I}_2 \subseteq \dots \subseteq \mathfrak{I}_k \subseteq \mathfrak{I}^* \quad (3.2)$$

The above nesting expresses the progressive enrichment of the initial graph that may be due either to increased local model complexity, and/or due to enhancement of description of the physical interconnection streams (dimensional expansion of graph branches). Such changes express distinct forms of evolution in the overall model and raise important new issues referred to as *graph evolution*; the resulting nesting of models is denoted in Figure (3).



**Figure (3):** Model Embedding Process



This problem has two different variations: **(i)** We adopt a procedure for simplification of description of subsystem models by using Model Reduction, while we preserve the Graph Structure. **(ii)** We assume a fixed input, output structure for the subsystems and examine the graph variability by using input-output models for subsystems while preserving the kernel graph structure of interconnections. Either of these two processes generates the sequences of models:

$$\mathbf{M}_0^a \subset \mathbf{M}_1^a \subset \mathbf{M}_2^a \subset \dots \subset \mathbf{M}_k^a \subset \mathbf{M}_{k+1}^a \subset \mathbf{M}_{k+2}^a \subset \dots \quad (3.3)$$

where  $\mathbf{M}_0^a$  is the kernel model,  $\mathbf{M}_1^a$  is the linear steady state model,  $\mathbf{M}_2^a$  corresponds to first order dynamics,  $\mathbf{M}_{k+1}^a$  may be a nonlinear model with simple Volterra description,  $\mathbf{M}_{k+2}^a$  the nonlinear model with double Volterra description (Sastry, 1999) etc. Note that there is a reversibility of the *model complexity evolution* and *model simplification approach*. Model Evolution and Model Reduction may become completely reverse processes, if we use fixed input, output subsystem structures and interconnection graphs. This expresses a form of *duality* between model reduction and model complexity evolution. Important research tasks that emerge are:

**Complexity Nesting Problems:** For the problems of Early Design models the following problems are open:

**(a)** Describe the mechanisms of nesting by developing appropriate representations for graph evolution, measures for model complexity, and appropriate representation for overall model embedding.

**(b)** Study the evolution of properties and structural characteristics in the evolutionary ordering of the Composite System Models and classify system properties according to:

- Invariance, or dependence of the stage model and on early, or late appearance of property in the nesting.
- Study and clarify the evolution of properties and structural characteristics in the evolutionary process. ■

Challenging tasks are the representation of such forms of evolution, the simplification of graphs (in some sense), defining general measures of model complexity beyond the family of linear models, and linking complexity, and genericity of system properties etc.

### 3.2 The Model Environment of Integrated Design

The characteristics and nature of Process Synthesis and Global Instrumentation depend on the type of available models and this is referred to as the “Model Environment” of the problem. Note that Instrumentation follows the Process synthesis, but it may be considered at the early stages, when the models are simple and rough, but also at the late stages, when detailed models are available. We may have models where some of the internal variables are classified into potential inputs, outputs, internal variables and referred to as *oriented models*, or models where no classification has been made of the internal variables and are called *implicit models*. All such models may be used for selection of effective sets of inputs and outputs, they are referred to as *progenitor models* and they may be classified as:

- Internal Models (IM)
- External Models (EM)
- Internal-External Models (IEM)

**(i) Internal Models** (Lewis, 1989): These are described in terms of first order ordinary nonlinear equations and they are the standard state space descriptions of the implicit type  $\underline{g}(\underline{\zeta}, \dot{\underline{\zeta}}) = 0$ , where  $\underline{\zeta}$  is the vector of all internal model variables. In the linear case, the above reduces to *matrix pencil* model (Karcaniyas & Hayton, 1982) defined by

$$F \dot{\underline{\zeta}} = G \underline{\zeta} \quad (3.4)$$

When the inputs  $\underline{u}$ , outputs  $\underline{y}$  have been defined, then  $\underline{y} = \underline{h}(\underline{\zeta}, \underline{u})$ ,  $\underline{q}(\underline{\zeta}, \dot{\underline{\zeta}}, \underline{u}) = 0$  is the nonlinear model, which in the linear case becomes the singular model

$$E \dot{\underline{\zeta}} = A \underline{\zeta} + B \underline{u}, \quad \underline{y} = C \underline{\zeta} \quad (3.5)$$

When we have higher order derivatives, then autoregressive descriptions are used (Willems, 1989).

**(ii) External Models** (Vidyasagar, 1981): If  $\mathcal{C}$ ,  $\mathcal{Z}$  denote the spaces of all potential inputs, measurements, referred to as extended input, output spaces respectively and  $\underline{v}$ ,  $\underline{z}$  are the corresponding p, q-dimensional vectors, then the external, or input-output map  $f$  is a function  $f: \mathcal{V} \rightarrow \mathcal{Z}$  where  $\underline{z} = f(\underline{u})$ . For the case of linear, time invariant systems  $f$  is a convolution function, or it is represented by the  $q \times p$  rational transfer function matrix  $F(s)$ , for which

$$\underline{z}(s) = F(s) \underline{v}(s) \quad (3.6)$$

Note that  $\underline{\zeta}$ ,  $\underline{Z}$  denote the potential input, output spaces and not the effective ones, which are denoted by  $\underline{Y}$ ,  $\underline{\Psi}$  and have corresponding dimensions  $\lambda$ ,  $m$ . If  $\underline{u}$ ,  $\underline{y}$  are the effective input, output vectors, and if  $H$ ,  $Q$  are representations of the sensor, actuator maps, then  $\underline{y}(s) = H\underline{z}(s)$ ,  $\underline{v}(s) = Q\underline{u}(s)$  and the *effective transfer function* is:

$$W(s) = H F(s) Q \quad (3.8)$$

(iii) **Internal-External Models:** A large process is always synthesised by connecting sub-processes and the two fundamental ingredients of the composite system model are: **(a)** The topology (graph) of system interconnections  $\mathbf{F}$ , represented by a matrix  $F$ , and **(b)** The family  $\{\mathbf{M}\}$  of subsystem models which may be of any of the types discussed before. If  $\Sigma_a$  denotes the *aggregate* (direct sum) of the sub-processes and  $\mathbf{F}$  the graph interconnection rule, then

$$\Sigma_c = \mathbf{F} * \Sigma_a \quad (3.9)$$

represents the composite system model and a feedback type representation will be given in the next section. For the linear case  $\Sigma_a$  may be represented as a diagonal of transfer functions, and thus  $\Sigma_c$  becomes also a transfer function with a specific structure.

### 3.3 Prediction and Evaluation of Overall System Properties in Early Models

The development of the family of *early design* models is integral part of the need to evaluate properties of the final system at early stages and thus avoid taking roots that may lead to bad designs. In the context of Chemical Processes this is usually referred to as *process controllability* and *operability studies* (Douglas, 1988), (Perkins, 1990), (Morari, 1992). Process Controllability is a generic term which does not carry the same meaning as that used in mainstream system and control; usually, it is interpreted as the ability to operate despite the affect of disturbances, ability of the designer to achieve performance requirements after implementation of a suitable control scheme etc. As such, Process Controllability (contrary to the linear systems notion of State Controllability) is a term that requires an exact definition of its meaning. Operability also has a quite diverse meaning and it is linked to issues such as: Product, Input Driven Flexibility (material variability), Recoverability (Ability to drive the process back to safety after faults), maintainability etc (Perkins, 1990), (Morari, 1992). The variety of alternative structures (process flowsheets) generated at the early stages of design have to be evaluated with a variety of criteria. There have been two main schools of thought (Rijnsdorp, 1991):

**Ideal Evaluation:** Assess the behaviour of a system with a controller of specified complexity, which is tuned optimally.  
**Realistic Evaluation:** Development of low effort analysis tools giving a reasonable indication of the quality of closed-loop behaviour allowing the designer at least to rank and order alternatives according to controllability, operability etc.

The first requires a complete system (with instrumentation and Control) and it is rather unrealistic (although scenarios based on models may be deployed). The following properties are important in such evaluations:

**Flexibility:** Is defined as the ability of the system to handle a new situation at steady-state and thus express the ability to operate at different steady states.

**Switchability:** Considers ability of a plant to be moved from one steady state operating point to another. This also involves start up and shut down of the process.

**Controllability:** Is the “best” dynamic performance (set point following and disturbance rejection) achievable for a system under closed loop control.

**Safety:** Examines the hazards that may be involved with particular designs and using process dependent heuristics provides a classification.

The above key properties for the evaluation are *emergent system properties* and express aggregation of structural system and controller dependent features of the family of models. Process Controllability is a much more general notion than the traditional system controllability, Flexibility depends mainly on the structure of the process, whereas “Switchability” and Controllability depend on the system structure and the selected control structure (Morari, 1992). The available tools for such studies are mostly heuristic and there is need for a Systems and Control framework for characterising these properties.

**Control Based Characterisation of Design Emergent Properties:** Develop a systems and control type framework for:

- (i) Interpretation of Evaluation Criteria for Process Synthesis;
- (ii) Prediction of Full Model System Properties based on early simple models.

■

The exact System and Control context of these emergent properties is not specified, and thus they are not structurally interpretable in terms of values, properties of design indicators, invariants etc. Specifying exactly the meaning of all such properties is essential prerequisite for the Control theoretic evaluation of the alternative process structures. The prediction of full model properties benefits from the knowledge gained in (i) and the understanding coming from the model structure evolution. So far, there is no control theoretic framework for such problems with few exceptions the conditioning of early design models (Karcianas & Vafiadis, 2001) considered subsequently. A feasible approach is to use simple models, assume that global instrumentation delivers the best possible final structure and then try to establish criteria predicting the fundamental system properties on the final system that will emerge.

### 3.4 Well conditioning of Early Design Models

The development of models, which may be used for evaluation of alternatives is an integral part of the Early Process Design of process plants (Douglas, 1988). Such models are usually developed for the entire plant, are based on the selected process flow-sheet and involve the use of simple models of the sub-processes. They are large dimension models and their final structure is determined when the control structure is decided. This problem involves a number of key issues of system theoretic nature and they are considered here. It is assumed that a linear model of a system is given with given inputs and outputs. At the early stages of design it is desirable to include as inputs, and outputs all possible variables that can be used which may play the corresponding role; such sets are defined as *potential inputs*, *potential outputs* respectively. The model that corresponds to the potential inputs, outputs provides the basis for deriving all subsequent models based on *effective* input, output sets and it is thus referred to as the *progenitor model*. For such models, all inputs and outputs are physical variables that can be controlled and measured. At a later stage, when we proceed to control design, the number of effective inputs and outputs requires reduction. This reduction implies an appropriate selection of rows, columns of the transfer function matrix, or the corresponding system matrix such that the corresponding subsystem has desirable properties.

Progenitor models involve all potential inputs and outputs and they may have structural undesirable properties that do not allow their effective use in design. In fact, a progenitor model may be structurally degenerate (Rosenbrock, 1974), might have redundancy in the input, output schemes, may be uncontrollable/ unobservable and may have high order infinite zeros. A progenitor model represents all our knowledge about the system at a given stage of early design and the McMillan degree of this transfer function represents the *natural order*  $n$  of the system. System models, which are degenerate, do not satisfy the basic condition of the output function controllability (Rosenbrock, 1970). It is thus desirable to select subsets of the potential inputs and outputs, such that the resulting transfer function is "well-conditioned" in some sense. Systems which are well-behaved are referred to as well-conditioned system. Amongst the basic criteria used, are the properties of non-degeneracy, controllability and observability of the system model and non-redundancy of the input and output scheme. The problem considered here, is the use of the characterisation of the above system properties to develop a parameterisation of all possible systems corresponding to *effective inputs and effective outputs* by selecting appropriate sub-sets of the potential variable sets (Karcianas & Vafiadis, 2001) and which are well behaved in some specified sense.

It is assumed that the progenitor model is described by the minimal state space equations for  $S(A, B, C, D)$  :

$$\dot{\underline{x}} = A \underline{x} + B \underline{u} , A \in \mathbb{R}^{n \times n} , B \in \mathbb{R}^{n \times r} \quad \underline{y} = C \underline{x} + D \underline{u} , C \in \mathbb{R}^{q \times n} , D \in \mathbb{R}^{q \times r} \quad (3.10)$$

with a transfer function  $H(s) \in \mathbb{R}^{q \times r}(s)$  and  $\rho = \text{rank}_{\mathbb{P}(s)} \{H(s)\}$ . Clearly (Karcianas, 2002), if  $\rho < \min(q, r)$ , then the system is *degenerate*, and if  $\rho = \min(q, r)$ , it is *non-degenerate*.

**Remark (3.1)**(Rosenbrock, 1974):  $\rho$  defines the maximal number of output variables that may be controlled independently (output function controllability criterion) and the minimal number of independent inputs required to control  $\rho$  outputs. ■

If  $r, q \leq n$ , and define:  $\tau_r = \text{rank}\{[B^T, D^T]\} \leq r$ ,  $\tau_q = \text{rank}\{[C, D]\} \leq q$ , then if  $\tau_r < r$ , ( $\tau_q < q$ ) the system will be said to have *input (output) redundancy*; otherwise, i.e. if  $\tau_r = r$ , ( $\tau_q = q$ ) , it will be called *regular*. Regularity of the model is equivalent to non-redundancy of both sensor and actuator schemes. The *well conditioning problem* corresponds to selection of subsets of potential inputs and outputs; thus, it is equivalent to deriving a smaller transfer function  $H(s)'$  from  $H(s)$  by eliminating sets of rows and columns, such that  $H(s)'$  has certain properties. Important issues are:

**Well Conditioning of Progenitor Models Problems:** Given the progenitor model described by  $H(s)$ , or with a minimal realization  $S(A, B, C, D)$ , define subsystems  $S'(A', B', C', D')$  by selection of subsets of inputs and outputs such that:

- (i)  $S'(A', B', C', D')$  have the maximal cardinality subset of the potential input and output sets, the resulting transfer function is non-degenerate, it has the maximal possible normal rank and it is also proper.
- (ii) Amongst the above solutions, determine whether there exist solutions, which have McMillan degree equal to that of  $H(s)$ . Parameterise all solutions  $S'(A', B', C', D')$  with these properties.

■

The solution of problem (i) is referred to, as *well-conditioning of Progenitor models* and part (ii) describes the property that the resulting model is both controllable and observable. Note that controllability and observability are notions defined on  $S(A, B; C, D)$  where  $A$  corresponds to the minimal realisation of  $H(s)$ . The latter problem will be referred to as *normal-conditioning of Progenitor Models*. Note that the above problems involve the study of properties of the sub-matrices of the rational transfer function matrix  $H(s)$  obtained by elimination of certain sets of columns, rows.

The notion of degeneracy has been classified into a *simple* form when there is redundancy of the input, output map of the progenitor model and to a structural type referred to as *strong* which is linked to internal properties of the system. The distinction between the two types is based on the zero- nonzero values of associated minimal indices of the transfer function. Criteria have been given for the presence of input, output redundancy, strong degeneracy, lack of minimality (based on Hankel matrix tests) (Antsaklis & Michel, 1997), and high order infinite zeros (Karcaniyas, 2002). Procedures have been suggested on how such properties can be avoided and searching techniques for defining sub-systems with maximal input, output cardinality which are also well conditioned, have been given (Karcaniyas & Vafiadis, 2001), based on the notion of natural bases of matrices (Karcaniyas and Mitrouli, 1998) and use of Grammians (Gantmacher, 1959). The results have led to parameterisation of all subsystems, which are input-output regular, nondegenerate, minimal and have maximal cardinality  $(\tilde{r}, \tilde{q})$  (Karcaniyas & Vafiadis, 2001).

### 3.5 Structural Identification

The study of system properties based on well defined models is a well established activity. However, at the early stages of design, models are characterized by structural and parametric uncertainty and this makes the study of system problems on so called “ill-defined” models a challenging problem (Karcaniyas et al, 1996). Features such as high dimensionality, uncertainty in system parameters and constraints of information structure often lead to problems, which cannot be solved using traditional methods developed for well-defined models. At the early stages of design we would like to have some insight into the structure of the system, and more precisely into general properties, such as possible value of McMillan degree, controllability, observability, existence of fixed modes, high order infinite zeros etc, even on models which are not precisely defined. These properties may be regarded as “potential” system properties, characterize, or enter the solvability of control synthesis problems. Ideally we would like to be able to predict the true system properties of the full model from the properties of ill-defined models, which are available at the early process design stage; this is equivalent to predicting properties and structure on the fully evolved model. Special classes of system models for which some results have been defined are those characterized by:

- (a) Certain general parameters, such as the number of inputs, outputs, states are fixed, but otherwise having generic values for their parameters and referred to as *Fixed Order Generic Systems (FGOS)*.
- (b) The interconnection graph is known and the subsystems are represented by fixed dominant dynamics, or fixed relative order of the entries of the transfer functions, but still have uncertainty in the remaining parameters; these are referred to as *Structured Dominant Dynamics Systems (SDDS)*.

Representative models in the *SDDS* class are the *Structured Transfer Functions (STF)* models which have certain elements fixed to zero, some elements being constant and other elements expressing some identified dominant dynamics of the system, or having fixed relative order of the entries of the transfer functions. Structural properties are generically possessed by all systems that may have different parametric values, but share the same underlying graph structure, or fixed structural features. Computing structural invariants on such families of uncertain models by using genericity arguments and exploiting the underlying structure, is referred to as *Structural Identification*; these problems include the evaluation of structural characteristics on state-space or transfer function models, such as minimal indices (controllability, observability, Forney indices, etc.) and invariant zeros. For such structured models the evaluation of certain system properties (Karcaniyas, 2002), (Karcaniyas & Vafiadis, 2002a) involves graph theory (Reinschke, 1998) and computations may be reduced to optimization problems on integer matrices (Karcaniyas et al, 2007). The properties that have attracted most of the attention for such systems have been the evaluation of the generic McMillan degree (Karcaniyas et al, 1996), (Van der Woude, 1995) and the generic infinite zero structure (Vardulakis et al, 1982), (Hovelaque et al., 1997). A promising approach for such computations is to use the genericity argument and this reduces the problem to study of properties of “weight” of integer matrices (Karcaniyas et al, 2007).

**Definition (3.2):** Given a matrix  $A \in \mathbb{R}^{m \times n}$  we define: (i) A  $k$ -length independent path  $\{a_{i_1 j_1}, a_{i_2 j_2}, \dots, a_{i_k j_k}\}$ , as a set of elements from the matrix such that  $a_{i_v j_v} \neq 0, \forall v = 1, \dots, k$  and there is no common index in the sets  $\{i_1, i_2, \dots, i_k\}$  and  $\{j_1, j_2, \dots, j_k\}$ . (ii) The weight of a path is the sum of the elements of the matrix that belong to the chosen path. (iii) The maximal weight of all the independent paths of a matrix  $A$  is denoted by  $\gamma(A)$  and it is simply referred to as the *weight* of the matrix

■

Determining the “weight” of integer matrices is equivalent to “optimal assignment problems” in operational research (Bertsekas, 1981), (Sagianos, 2008). The development of fast and reliable computations for the generic values of structural characteristics on the family of early models is a challenging task. An efficient computational procedures using the notion of reducibility of integer matrices for finding solutions to “optimal assignment problems” has been developed in (Karcianas et al, 2007).

### 3.6 Model Nesting and the Issue of Small Numbers

For linear state-space, or transfer function models, the issue that often arises, is how to handle numbers, which are very small and what is the impact of rounding off certain numbers of order less than a given order on the structural properties. It is essential to make a distinction between numbers which are small enough to be assumed equal to zero, and thus do not affect the overall structure of the system, or any of its properties, and numbers which are small, but represent a coupling, which has to be preserved. The small numbers, appearing in the  $A, B, C$  and  $D$  matrices, or in the set of Markov parameters  $\{D, CB, CAB, CA^2B, \dots\}$  may be classified into *structural* which affect structural properties and *non-structural*, the removal of which has no effect on system properties. This classification introduces a numerical form of model nesting (Karcianas & Sagianos, 2008). Removing small numbers is a form of Robust Structural Simplification, given that the structural properties of the original system have to be close to those of the reduced system.

Consider a state-space model  $(A, B, C)$  and let  $r$  be the element of maximal absolute value in  $(A, B, C)$ . We can define the scaled model  $(A', B', C')$  as:

$$P' = \frac{1}{r} \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & 0 \end{bmatrix} \quad (3.11)$$

For the elements of  $P'$ , we have clearly  $0 \leq |p_{ij}| \leq 1$ . If  $\varepsilon > 0$  is any small number, then all elements of  $P'$ , for which  $|p_{ij}| \leq \varepsilon$  defines a set, that will be called the  $\varepsilon$  – power of the original system. According to the value we select for  $\varepsilon$  we have a new model, the  $\varepsilon$  – simplified model, which is defined by the matrices  $A_\varepsilon, B_\varepsilon, C_\varepsilon$ , or  $A_\varepsilon = r \cdot A_\varepsilon, B_\varepsilon = r \cdot B_\varepsilon$  and  $C_\varepsilon = r \cdot C_\varepsilon$ . The Boolean matrices associated with the  $\varepsilon$  – simplified model, will define the  $\varepsilon$  – structured model denoted by  $\{P\}_\varepsilon$ .

**Robust Structural Simplification Problem:** Different methods can be used to decide on the significance of the value of  $\varepsilon$  and the final form of the  $\varepsilon$  – structured model  $\{P\}_\varepsilon$ . Amongst the possible methodologies we can use to analyse the effect of small numbers on the system properties are (Sagianos, 2008): **(i)** Sensitivity Analysis Approaches; **(ii)** Robustness of Graph Structures; **(iii)** Degree of Controllability and Observability; **(iv)** System Based Metrics and Properties; **(v)** Matrix Perturbation Theory. All these methodologies adopt the same philosophy, which is to evaluate the effects of the different  $\varepsilon$  we choose on the structural properties of the system. ■

Consider a linear system represented by a state-space description  $S(A, B, C, D)$ , or by the set of *Markov Parameters*  $\Sigma(H_0, H_1, H_2, \dots, H_{n-1}, \dots)$ . We assume that the elements in the matrices involved are known only in terms of their relative order, but they are otherwise generic. This may be defined precisely as follows: Consider the set of positive real numbers  $\{a_0, a_1, a_2, \dots, a_\mu\}$  such that  $a_0 > a_1 > a_2 > \dots > a_\mu$  and define the following intervals:

$$E_0 = (\infty, a_0], E_1 = (a_0, a_1], E_2 = (a_1, a_2], \dots, E_\mu = (a_{\mu-1}, a_\mu], E_{\mu+1} = (a_\mu, 0] \quad (3.12)$$

**Definition (3.3):** Let  $M \in R^{m \times n}$  and assume that the order of its elements (absolute values) are known only in terms of their membership of the sets  $E_0, E_1, E_\mu, \dots, E_{\mu+1}$ , but they are otherwise generic. Such a matrix  $M$  will be called  $\{a_0, a_1, \dots, a_\mu\}$  – structured generic matrix (ie  $a_0 = 10^{-2}, a_1 = 10^{-3}, \dots, a_\mu = 10^{-6}$ ) and the set of such matrices will be denoted by  $R_{a_0, \dots, a_\mu}^{m \times n}$ . For any matrix  $M \in R_{a_0, a_1, \dots, a_\mu}^{m \times n}$  we may:

$M_0$ : is obtained from  $M$  by setting all elements which are not in  $E_0$  equal to zero.

$M_1$ : is obtained from  $M$  by setting all elements which are not in  $E_0 \cup E_1$  equal to zero.

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$M_\mu$  : is obtained from  $M$  by setting all elements which are in  $E_{\mu+1}$  equal to zero, or equivalently all elements not in  $E_0 \cup E_1 \cup \dots \cup E_\mu$  equal to zero. ■

The above process creates from  $M$  a set of matrices  $M_0, M_1, M_2, \dots, M_\mu$  which together with  $M$  define a nesting condition denoted by

$$M_0 \subset M_1 \subset M_2 \subset \dots \subset M_\mu \subseteq M \quad (3.13)$$

The relation “ $\subset$ ” means that  $M_\nu$  is obtained from  $M_{\nu-1}$  those elements of  $M$  which have an absolute value in the interval  $(a_{\nu-1}, a_\nu]$ . This ordering on the  $M$  matrix will be called an  $\{a_0, a_1, a_2, \dots, a_\mu\}$ -induced nesting and this numerical nesting leads to families of State Space and Markov Parameter Nested Models which may be denoted by

$$\{S\}_\varepsilon = S_j(A^j, B^j, C^j, D^j), \quad j=1, 2, \dots, \mu \quad (3.14)$$

$$\{\Sigma\}_\varepsilon = \Sigma_j(H^j_0, H^j_1, H^j_2, \dots, H^j_{n-1}, \dots), \quad j=1, 2, \dots, \mu \quad (3.15)$$

**Structural Numerical Model Embedding Problem:** For the families  $\{S\}_\varepsilon, \{\Sigma\}_\varepsilon$  systems investigate:

- (i) The minimal value of the order  $\varepsilon$  required for the emergence of different system properties, such as controllability, observability, stability etc. in either of the two families.
- (ii) How the McMillan degree varies as a function of the in the family  $\{\Sigma\}_\varepsilon$  in terms of  $\varepsilon$
- (iii) Whether properties established for a certain  $\varepsilon$  are preserved in models of higher accuracy  $\varepsilon$ .
- (iv) The existence of properties which are dependent, or independent on the selected value of  $\varepsilon$ . ■

Note that the state space modelling and the input-output modelling based on the Markov parameters provides different approaches for the study of robustness and sensitivity. The Markov parameters approach is naturally linked to the partial realization problem (Kalman, 1979), (Antoulas etc, 1991) and this in turn implies some further evolution of structural properties based on the predicted McMillan degree of the partial realization (Sagianos, 2008).

#### 4. Process Synthesis as a Generalised Feedback Design Problem

The problem of process synthesis is usually addressed using methodologies linked to the specifics of the application area. The development of a generic synthesis framework that transcends the different application areas is a significant challenge. The modelling of composite systems using energy considerations, or behaviours (Willems, 1997) and the use of the traditional network synthesis together with the completion of the analogy between electrical and mechanical domains (Smith, 1995) are important contributions. In this section, we explore an alternative idea that relates to the reduction of process synthesis to an equivalent feedback design problem using the standard composite system description (Callier & Desoer, 1982) and its particular characteristics based on the nature of the physical interconnection streams and the selection of the local input and output structure (Karcianas, 1995a). This work introduces an important notion of *completeness* (Karcianas, 1996) and provides a representation of the synthesis as generalised feedback design problem. Such representation provides the means to intervene with systems and control tools in a design area which is dominated by the specifics of the application area. If  $\{\Sigma_i, i \in \mu\}$  is a set of subsystems with models  $\{M_i, i \in \mu\}$  of a certain type and if  $\Phi$  is the interconnection rule (described by a graph), then  $\Sigma_a = \Sigma_1 \oplus \Sigma_2 \oplus \dots \oplus \Sigma_\mu$  denotes the *aggregate system* with a model  $M_a = \text{block-diag}\{M_1, M_2, \dots, M_\mu\}$ . The *Composite System* is denoted by  $\Sigma_c = \mathfrak{T} * \Sigma_a$ , where  $*$  denotes the action of  $\mathfrak{T}$  on  $\Sigma_a$  which will be subsequently defined.

The definition of Composite Systems involves the specification of the physical input and output streams and the selection of inputs and outputs at the subsystem level. Sub-processes enter the composite structure, by interconnecting local variables (subsystem connecting inputs, outputs and effective control inputs and measured outputs) and this affects drastically the overall properties of the composite system. Previous work (Saeks & DeCarlo, 1981), examines composite systems properties without seeing the interconnection scheme and the selection of local input, output structure as design parameters. A first attempt to link model composition to feedback was made in (Callier & Desoer, 1982) and subsequently developed in (Karcianas, 1994a). The definition of the composite from the aggregate by the action of the interconnection topology raises important questions, which are linked to: (i) The representation of the composite system; (ii) The relationships between the structure and properties of the aggregate and the composite in terms of the characteristics of the interconnection topology (Karcianas, 1996). The general scheme that is considered satisfies certain assumptions which are described below:

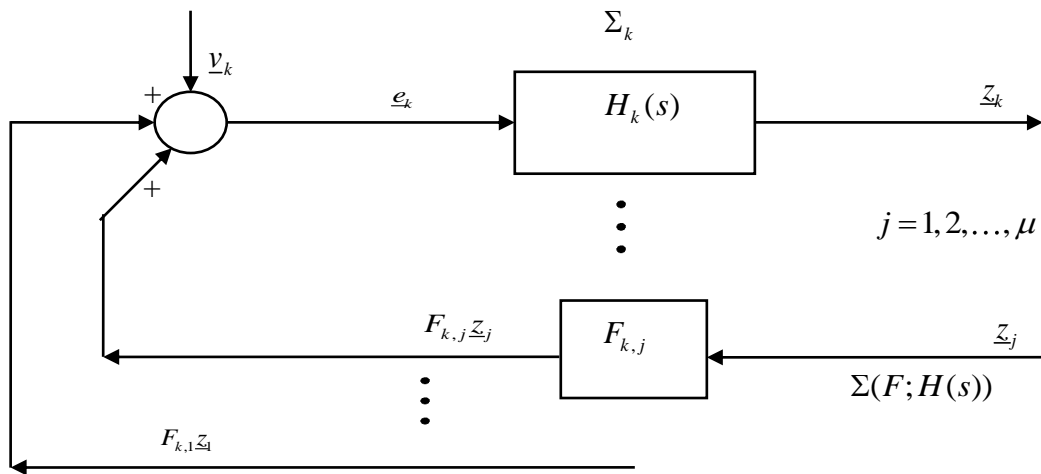
**(a) Local Well Connectedness Assumption (LWCA):** The physical linking of a subsystem  $\Sigma_k$  to the rest of the subsystems implies that there is a connecting input vector  $\underline{e}_k$  having as coordinates all variables connected directly to at least one subsystem output, or external variable (manipulated, or disturbance) and having as connecting output vector  $\underline{z}_k$  the vector with coordinates all variables which feed to at least one of the subsystems or measured variables. We assume that the transfer functions  $H_k(s): \underline{e}_k \rightarrow \underline{z}_k$  are well defined and they are proper. These assumptions are referred to as *Local Well Connectedness (LWC)* and  $H_k(s)$  is the  $k$ -th connecting transfer function; furthermore, if  $H_k(s)$  represents a minimal system, then the system satisfies the *Minimal LWC (MLWC)* assumption. The aggregate system  $\Sigma_a$  is represented by the transfer function matrix  $H(s)=\text{block-diag}\{H_k(s), k=1, \dots, \mu\}$ .

**(b) Local Well Structured Assumption (LWSA):** For every subsystem with  $\underline{e}_k, \underline{z}_k$  physical inputs and outputs we shall denote by  $\underline{v}_k, \underline{y}_k$  the effective input, output vectors. We shall assume that  $\underline{v}_k$  is a sub-vector of  $\underline{z}_k$  in the sense that  $\underline{y}_k = K_k \underline{z}_k, K_k \in \mathbb{R}^{p_k \times q_k}, p_k \leq q_k$  and that  $\underline{e}_k$  is expressed as

$$\underline{e}_k = \underline{f}_k + L_k \underline{u}_k = \underline{f}_k + \underline{v}_k \in \mathbb{R}^{p_k \times q_k} \quad p_k \leq q_k \quad (4.1)$$

where  $\underline{f}_k$  is some vector of dependent variables, defined by the interconnections and  $\underline{v}_k = L_k \underline{u}_k$  has independently assignable (control, or disturbance) variables, defined as a combination of a larger potential vector  $\underline{u}_k$ ; thus  $\underline{u}_k, \underline{z}_k$  emerge as potential inputs, outputs. This assumption is referred to as *Local Well Structured (LWS)* assumption.

**(c) Global Well Formedness Assumption (GWFA) (Callier & Desoer, 1982):** Consider the system aggregate  $\Sigma_a (\Sigma_k, k \in \mu)$  under the LWC and LWS assumptions. The composite system will be called *Globally Well Formed (GWF)*, if the interconnection rule  $F: \underline{e}_1 \times \dots \times \underline{e}_\mu \rightarrow \underline{z}_1 \times \dots \times \underline{z}_\mu$  represented by the diagram of Fig.(4) satisfies:



**Figure (4):** The global well formedness assumption

(i) Its output is  $[\underline{z}_1^t, \dots, \underline{z}_\mu^t] = \underline{z}$  and if  $\underline{v}_k$  are external vectors, its inputs  $\underline{e}_k$  are  $\underline{e}_k = \sum_{j=1}^{\mu} F_{kj} \underline{z}_j + \underline{v}_k, F_{kj}$  real.

(ii) The transfer function from  $\underline{v} = [\underline{v}_1^t, \dots, \underline{v}_\mu^t]^t \rightarrow \underline{e} = [\underline{e}_1^t, \dots, \underline{e}_\mu^t]^t$  is defined.

■

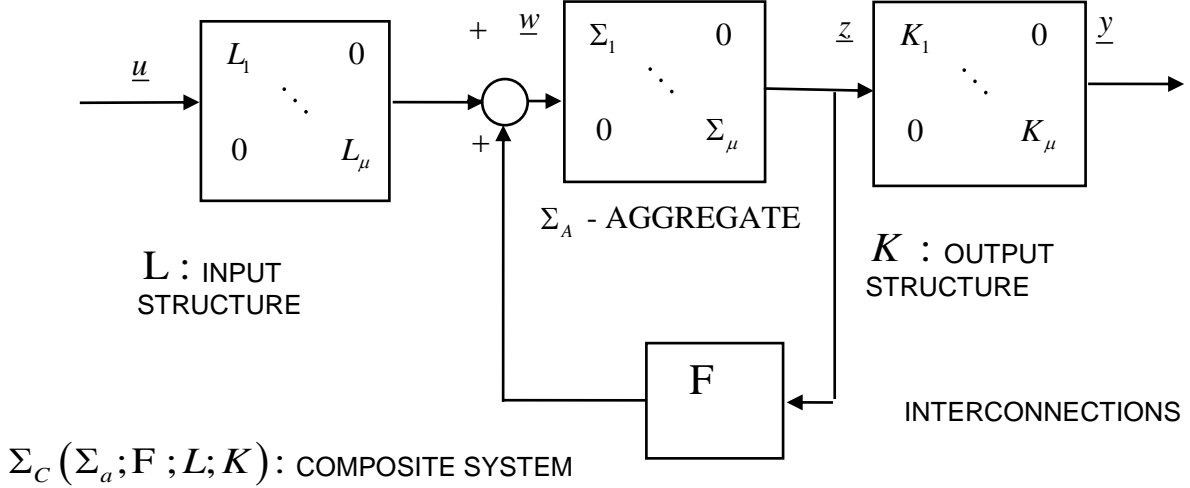
If  $F = [F_{kj}]_{k,j \in \mu}, K = \text{bl.diag}\{K_i, i \in \mu\}, L = \text{bl.diag}\{L_i, i \in \mu\} \underline{u} = [\dots, \underline{u}_i^t, \dots]^t, \underline{y} = [\dots, \underline{y}_i^\mu, \dots]$ , then

$$\underline{e} = \underline{v} + F \underline{z}, \quad \underline{z} = H(s) \underline{e}, \quad \underline{v} = L \underline{u}, \quad \underline{y} = K \underline{z} \quad (4.2)$$

and the composite configuration is represented as the feedback configuration of Fig. (5). Note that condition (c.ii) implies that  $I - FH(s)$  is an invertible matrix. Clearly, the interconnection graph acts as feedback and the selection of effective inputs, outputs is represented as input, output constant compensators and the composite transfer function is

$$G(s) = K \cdot \hat{H}(s) \cdot L \quad \text{where} \quad \hat{H}(s) = H(s) \cdot (I - F \cdot H(s))^{-1} \quad (4.3)$$

The above expresses the composite system as the action of decentralised input and output reduction (squaring down operation), represented by the input, output transformations  $K, L$  respectively and of an internal feedback  $F$ , representing the topology of the interconnections. The matrix  $\hat{H}(s)$  is referred to as *progenitor model* of the composite system. The actions of  $K, L$  are usually referred to as *Model Projection (MP)* operations and are forms of “squaring down” (Karcnias & Giannakopoulos, 1989). The representation of the interconnecting topology as internal feedback provides the means for linking the properties of aggregate and the composite system and allows our intervention in the synthesis problem using results from the feedback theory. An important special case of the above configuration is described below (Karcnias, 1996):



**Figure (5):** Equivalent Feedback Configuration

**(d) Completeness Assumption:** The well formed composite system of Fig.(4) will be said to be *complete*, if the following two further conditions hold true: **(i)** Every effective subsystem output  $\underline{y}_k$  satisfies the condition  $\underline{y}_k = Q_k \underline{z}_k$ ,  $Q_k$  square invertible. **(ii)** Every external subsystem vector  $\underline{y}_k$  has as many independent coordinates as the dimension of  $\underline{e}_k$  input vector, i.e.  $\underline{e}_k = R_k \underline{y}_k$  with  $R_k$  square and invertible. ■

**Remark (4.1):** As a result of completeness the composite and the aggregate are output feedback and input, output coordinate transformation equivalent and thus they have the same basic structural characteristics. ■

Guaranteeing the validity of the above assumptions, is both a matter of modelling and selection of input, output schemes; in fact, each assumption is linked to the following problems:

**Local Well Connectedness Problem (LWCP):** The properties of well connectedness at subsystem level are closely linked to the design of individual sub-processes. Deviation from such assumptions may be handled by system redesign at the sub-process level. ■

**Local Well Structuring Problem (LWSP):** The essence of local well structuring is that we have to identify the effective connecting inputs, outputs  $\underline{e}_k, \underline{z}_k$  respectively the potential control variables and outputs  $\underline{u}_k, \underline{y}_k$  respectively and we have to guarantee the corresponding transfer function matrix properties. It may be frequently the case that  $\underline{y}_k$  is not a subvector of  $\underline{z}_k$  and that  $\underline{u}_k$  is not related to  $\underline{e}_k$ . In this case, it is always possible to expand the vectors  $\underline{e}_k, \underline{z}_k$  to  $\underline{e}'_k, \underline{z}'_k$  vectors such that the respective conditions are satisfied. ■

**Global Well Formedness Problem (GWFP):** Under the assumptions LWFP, LWSP, then GWFP is a modelling problem, since it requires definition of  $F_{kj}$  matrices such that  $I - FH(s)$  is non-singular. When there is flexibility in the design of  $F$ , the objective may be extended in designing  $F$  such that the resulting progenitor model  $\hat{H}(s)$  is also stable. ■

**Decentralised Model Projection Problems (DMPP):** The general configuration of Fig.(5) suggests that the final selection of inputs, outputs are reduced MPP where  $L$  and  $K$  are block diagonal. This is then addressed on the model  $\hat{H}(s)$  with model projection criteria (see following section) and under the  $L, K$  block diagonal conditions. ■



The above cluster of problems are problems of process synthesis, but they are addressed in a control theoretic setup. They are referred to as *Model Composition Problems (MCP)*; their study is linked to the design of the interconnection scheme ( $F$  matrix design) and are connected to the local input, output structure selection ( $L, K$  matrices design).

## 5. The Fundamental System Problems in Global Instrumentation

Global instrumentation is a problem of selection of inputs and outputs and its study revolves around the study of four fundamental problems which are: **(i)** Model Orientation Problems (MOP), **(ii)** Model Projection Problems (MPP) **(iii)** Model Expansion Problems (MEP) and **(iv)** Local- Global Structure Problems (LGSP). These problems have a clear model shaping role, each one of them expresses a form of system evolution and their study is reduced to problems of Control Theory and Design. These problems are essential in the effort to develop conceptual and design tools for assisting the “good” shaping of the system model as a result of the instrumentation process. The distinguishing feature of instrumentation as far as model shaping is that it acts on the shaping of the input-output structure, rather than the interconnection graph, shaped by process synthesis as described previously. The problem cluster (iv) has already been addressed and expresses the interaction between the global instrumentation and process synthesis. An overview of the overall instrumentation that includes traditional (macro) and systems aspects is given in (Karcianas, 1994a).

### 5.1 Model Orientation Problems

Physical modelling based on basic laws and use of interconnection topology may be used for large families of systems. If all important variables are included and there is no effort to guarantee their minimality, and their classification into inputs, internal variables is made, the emerging descriptions are referred to as *implicit* (Lewis, 1989), (Malabre, 1989) and in the case of first order differential descriptions they correspond to the *matrix pencil*, or generalised autonomous description (Karcianas & Hayton, 1982), (Karcianas & Kalogeropoulos, 1989):

$$S(F,G): Fp \underline{\xi} = G \underline{\xi}, \quad F, G \in \mathbb{R}^{r \times v} \quad (5.1)$$

where  $p$  is the differentiation, or shift operator and  $\underline{\xi}$  is the vector of all problem variables. The natural operator associated with such descriptions is the matrix pencil  $pF-G$  and thus their study relies on the structure of  $pF-G$ . For control, as well as handling issues of creating composite structures, it is important to classify the variables in  $\underline{\xi}$  into internal variables, or states  $\underline{x}$ , assignable, or control variables  $\underline{u}$ , and measurement, or dependent variables  $\underline{y}$ . The general problem of the classification of systems variables as inputs and outputs is referred to as *model orientation problem (MOP)*. In many systems, the orientation is not known, or that depending on the use of the system the orientation changes. Questions such as, when is a set of variables implied, or not anticipated by another, or when is it free, have to be answered, if model orientation criteria based on the nature of the process are to be derived; the specific use of the system may provide additional model orientation criteria. It may happen, that the above two types of criteria do not provide a unique solution to model orientation; note that for each alternative orientation we have a different i/o model and thus criteria based on the resulting model characteristics have to be used for the final evaluation.

We consider implicit descriptions of state space type, or more generally autoregressive forms of the type  $H(p)\underline{\zeta} = 0$ ,  $H(p)$  polynomial matrix,  $p = d/dt$ , where the *implicit* vectors  $\underline{\zeta}$  contain all variables of importance to the structure of the system without making a distinction between control, observation, internal dynamic variables and without making any assumption on their independence. Although the study of dynamics may be carried out on implicit non-oriented forms (Karcianas & Hayton, 1982), (Willems, 1983), when it comes to observing, controlling, or connecting the process as part of a composite structure, the orientation of the model is essential. The unconstrained MOP is defined as:

**Definition (5.1):** Given the model  $S(F,G): F \underline{\zeta} = G \underline{\zeta}$  define a transformation  $Q: \underline{\zeta} = Q \hat{\underline{\zeta}}$ ,  $Q \in \mathbb{R}^{r \times r}$ ,  $|Q| \neq 0$

$$\underline{\zeta} = Q \hat{\underline{\zeta}}, \text{ where } \hat{\underline{\zeta}} = \begin{bmatrix} \underline{x}^t \\ \underline{u}^t \\ \underline{y}^t \end{bmatrix} \text{ and } Q \in \mathbb{R}^{v \times v}, |Q| \neq 0 \quad (5.2)$$

where  $\underline{x} \in \mathbb{R}^n$ ,  $\underline{u} \in \mathbb{R}^t$ ,  $\underline{y} \in \mathbb{R}^l$  such that  $S(F,G)$  is equivalent to

$$\begin{bmatrix} p\hat{E} - \hat{A} & -\hat{B} \\ -\hat{C} & 0 \end{bmatrix} \begin{bmatrix} \underline{x}(t) \\ \underline{u}(t) \end{bmatrix} = \begin{bmatrix} 0 \\ -\underline{y}(t) \end{bmatrix} \quad (5.3)$$

■

The system  $S(E, A, B, C)$  is called an *orientation* of  $pF-G$ , in general, it is a singular (Kuijper & Schumacher, 1991) and  $\Sigma(F, G)$  denotes the family of all such systems. Determining the conditions under which  $S(F, G)$  may be reduced to singular,  $S(E, A, B, C, D)$ , or regular  $S(A, B, C, D)$  descriptions is part of the study of model orientation and it is in a way a problem of partitioning of the Kronecker set of invariants of the matrix pencil  $pF-G$  (Karcnias & Vafiadis, 2002a). Defining subsystems of  $S(A, B, C, D)$  by reduction of the input, output structure such that the reduced system  $S(A, B', C', D')$  has desirable properties is referred to as *input-output structure reduction problem* (I-ORP) which includes problems such as the squaring down and are special forms of model projection problems considered in the following section. A similar treatment may be given for the autoregressive form (Willems, 1989) and this leads to Rosenbrock type representation (Rosenbrock, 1970). The family  $\Sigma(F, G)$  has more than one elements and these may be classified according to the structural characteristics and the input, output type properties of the resulting oriented model. An important issue in selecting oriented models is the issue of model minimality (Kuijper & Schumacher, 1991, 1992), (Willems, 1983) which is equivalent to selecting a minimal number of internal variables.

A more general implicit description is the polynomial, or the *autoregressive representation* (AR), which is specified by a polynomial matrix  $R(p)$ , having as many columns as there are external variables expressed by the vector  $\underline{w}$ . The external behaviour (Willems, 1989) is simply the set of all external-variable trajectories  $\underline{w}$  satisfying

$$R(p)\underline{w} = 0 \quad (5.4)$$

A first effort to introduce orientation is expressed by the introduction of some internal variables, expressed by the vector  $\underline{\xi}$  and this leads to the AR/MA representation which is specified by two polynomial matrices  $P(s)$  and  $Q(s)$ ; these determine the external behaviour consisting of all trajectories  $\underline{w}$  of the external variables for which there exists a trajectory  $\underline{\xi}$  of the internal variables such that

$$H(p)\underline{\xi} = \underline{0}, \quad \underline{w} = Q(p)\underline{\xi} \quad (5.5)$$

For systems with an explicit input/output structure (splitting of  $\underline{w}$  into  $\underline{u}$  and  $\underline{y}$ ), the Rosenbrock's system matrix (Rosenbrock, 1970) in the s-domain, provides a natural model, ie:

$$T(s)\underline{\xi} = U(s)\underline{u}, \quad \underline{y} = V(s)\underline{\xi} + W(s)\underline{u} \quad (5.6)$$

where all matrices are polynomial, with  $T(s)$  square and invertible. The corresponding transfer function matrix  $G(s)$  is represented as  $V(s)T^{-1}(s)U(s) + W(s)$ . The orientation problem may now be addressed in a more general setup where first we model the system behaviour, in terms of outputs, then we introduce the internal variables and then we consider the orientation. This procedure involves a realization of  $R(p)$ , which may be in a matrix pencil form that retains  $\underline{w}$  as an output vector and includes physical variables that may act as inputs. In this context,  $R(p)$  leads to a family of transfer functions  $\{G(s)\}$  with properties that evolve from those of  $R(p)$ .

## 5.2 Model Projection Problems

For many systems the number of potential control variables and potential measurements, which ideally may be used, can become very large. In an ideal design, unconstrained by resources and effort all possible inputs and outputs should be used; economic and technical reasons however, force us frequently to select a subset of the potential inputs, outputs as effective, operational inputs, outputs. Developing criteria and techniques for selection of an effective input, output scheme, as projections of the extended input, output vectors respectively, is what we call *Model Projection Problems* (MPP). For linear systems, where orientation has already been decided, and represented by a progenitor  $q \times p$  rational matrix  $H(s)$  the MPP is equivalent to selecting the sensor, actuator maps  $h, g$  (or  $m \times q, p \times l$  rational matrices  $H(s), Q(s)$   $m \leq q, l \leq p$ ) such that the transfer function  $W(s) = H(s)F(s)Q(s)$  has certain desirable properties. Clearly, the problem as stated above is in the form of a generalised *two parameter Model Matching*. The design matrices  $H(s), Q(s)$  may be assumed in the first instance to be constant. Note that the  $H, Q$  maps are not completely free, but they are constrained by the nature of the specific problem and the need to use certain physical variables.

The family of MPPs expresses forms of model structure evolution linked to the process of obtaining new models by reducing the original larger input, or output sets. In this sense, projection tends to aggregate, reduce an original model to a smaller dimension with desirable properties. A special problem in this area, is the zero assignment by squaring down (Rosenbrock and Rower, 1970) (Karcnias and Giannakoploulos, 1989), (Saberi and Sannuti, 1990). Selecting effective input, output schemes is reduced to finding  $Q, H$  matrices by using criteria characterising the effectiveness of the actuator, sensor schemes represented by the  $Q, H$  respectively. Some key issues in this process are:

**(a) Desired Generic Dimensions Problem (DGDP):** Defining desirable general characteristics, such as number of inputs, outputs on a system model, with some assumed internal structure (graph, or generic model with a given McMillan degree) is referred to as the *generic dimensionality problem*. Such investigations are carried out on progenitor models and use conditions, for generic solvability, or generic system properties to define the least required numbers of effective inputs, outputs needed for certain structural properties. Early results are based on indicators, such as the Segre index (Karcnias, 2002). Using existing results on generic solvability of control problems leads to constraint integer optimization problems aiming to define generic families of systems for which a range of problems may be solved. The solvability of control problems introduces alternative criteria which involve the McMillan degree, and/or the generic infinite zero structure. Use of such invariants requires the development of methodologies for their robust computation and this is what leads to the study of *structural identification* (Karcnias, et al, 2007) on early models.

**(b) Input-Output Structure Reduction Problem (I-ORP) and Well Conditioning of Early Models:** Frequently, the resulting model from the process of model orientation has physical input, output variables; this is a desired property to preserve, but the overall system is not well-conditioned in terms of its properties. Then a special form of model projection takes place, where only an  $\alpha$  subset of inputs and a  $\beta$  subset of outputs is used, leading to an  $S_{\alpha,\beta} = S(A, B_{\alpha}, C_{\beta}, D_{\alpha,\beta})$  subsystem with transfer function  $H_{\alpha,\beta}(s)$ . The objective is to select the  $\alpha$ ,  $\beta$  sets such that the resulting  $S_{\alpha,\beta}, H_{\alpha,\beta}(s)$  is well structured as far as certain properties, which may include input, output regularity, nondegeneracy, minimality etc. This problem has been considered in section 3.4 and is referred to as *well conditioning by input-output reduction* (WCP). Note that in a transfer function matrix setup, WCP is equivalent to defining sub-matrices of  $H(s)$  by eliminating certain columns and rows and which have desirable properties. An integral part of the study is the parameterisation of the maximal input, output cardinality solutions (Karcnias & Vafiadis, 2001).

**(c) Graph Structural MPPs (GS-MPP):** On early design models of the interconnected type there is frequently the need to define subsets of inputs, outputs at the local, or global level, or appropriate structural combinations of them to guarantee structural properties such as controllability, observability, rejection of disturbances etc. Issues related to robustness under fault conditions may also be used as criteria here. In terms of the two parameter scheme associated with MPP, the problem here is to expand the tasks described in part (b) by defining the required Boolean structure of  $Q$ , and  $H$  transformations on an internal model, or the modification to the internal graph that can guarantee such properties. This area involves the examination of the Kronecker structure of graph structured pencil models, which are linked to the presence, or absence of certain system properties, and the study of properties for systems with closely related graphs. Amongst the interesting problems is the interpretation of “small numbers” on early process models and their impact on the resulting structure and properties.

**(d) Structure Transformation/Assignment Problems (ST-MPP):** On linear progenitor model  $F(s)$  the selection of given dimension and structure constant  $Q$  or  $H$ , matrices,  $W(s) = HF(s)Q$  leads to new models where the invariant structure of  $W(s)$  is obtained by transformation of the invariants of  $F(s)$  structure. Apart from the study of generic properties and their link to discrete types of invariants, there is also the need to investigate the effects of input, output reduction transformation on well defined models with fixed parameters. The transformation of one set of invariants to another is a problem not fully understood; certain results in relationship to decoupling have appeared in (Descusse et al, 1988), whereas in (Karcnias & Vafiadis, 1993) it has been shown that the one-sided MPPs are equivalent to generalised cover problems of geometric theory. A special case of IS-MPP is the zero assignment by squaring down (Karcnias & Giannakopoulos, 1989), (Karcnias & Leventides, 2007) which is well developed and linked to the general formulation of Determinantal Assignment Problems (Karcnias & Giannakopoulos, 1984). The two parameter version of squaring down aims for a transfer function  $W(s)$  which is square and has a given zero structure; similar problems may be defined for the case of graph structured state-space models instead of the transfer function formulations. For all such problems, the overall philosophy is to design  $Q, H$  such that the resulting model has a given desirable invariant structure or avoids having undesirable structural characteristics. The study of the Morgan’s problem may be seen within this class as a transformation of the input structure. The two parameter design problem is open, as far as structure assignment; here, the use of the results of the investigations in (a)-(c) are key elements in the development of a general design process. In this class, evolution of structure is due to projection of input, output structures to smaller dimensions. The dimensional reduction transforms one type of invariants to another and this mechanism is not well understood yet.

**(d) Structure Re-interpretation Problems (SR-MPP):** The structure of a linear system is defined in terms of invariants and the underlying graph structure of the interconnection topology. Different descriptions may be associated with a system (such as implicit, regular, rational etc) and for a given system the relations between such descriptions may be expressed in terms of their structural invariants; in fact, these relations may be seen as forms of re-organising the system structure. We distinguish two important classes of problems:

- The Feedback Simulation Problem, (Heymann, 1976), (Loiseau and Zagalak, 1994)
- Proper Invariant Realizations of Autonomous Descriptions (Karcnias, 1990)

**Feedback Simulation Problem:** This class of problems involves the transformation of invariants from internal to external descriptions and may be expressed as follows: Consider the linear system of (5.7) and a strictly proper transfer function matrix  $T_m(s) \in \mathbf{P}^{m \times q}(s)$ , which is referred to as a model.

$$S(A, B) : \quad \dot{\underline{x}} = A \underline{x} + B \underline{u} \quad , \quad A \in \mathbb{R}^{n \times n} \quad , \quad B \in \mathbb{R}^{n \times r} \quad , \quad \text{rank}(B) = r \quad (5.7)$$

Let us assume that there exist matrices  $C \in \mathbb{R}^{m \times n}$  ,  $G \in \mathbb{R}^{r \times q}$  ,  $F \in \mathbb{R}^{r \times n}$  , where  $\text{rank}(C) = m$  ,  $\text{rank}(G) = q$  such that the following condition holds

$$T_{C,F,G}(s) = C(sI - A - BF)^{-1}BG = T_m(s) \quad (5.8)$$

Then we say that the system given by  $T_m(s)$  is simulated by the system  $S(A, B)$ , the matrix  $C$  and the state feedback law  $\underline{u} = F\underline{x} + G\underline{y}$ . This is referred to as *Simulation Problem* (Loiseau and Zagalak, 1994) which is an extension of the *Feedback Simulation* (Heymann, 1976), that provides conditions for the realization of proper rational matrices as state feedbacks, or output injection laws applied on systems. The current version is more general than the classical version, since the introduction of  $C$  provides extra degrees of freedom. Forms of feedback which are non-regular as far as the input transformation  $G$ , change aspects of the structure; such transformations of invariants are significant in the study of the Morgan's problem (row-by-row decoupling problem) (Descusse et al, 1988), (Koumboulis et al, 2004).

**Proper Invariant Realizations:** The other form of problems in this class is linked to the interpretation of problems of implicit systems (geometric theory etc) as regular state space theory problems and vice-versa. This establishes an equivalence between the trajectories of the autonomous first order description  $S(F, G) : F \dot{\underline{\xi}} = G \underline{\xi}$  and the forced solutions of the regular state space descriptions  $S_r(A, B, C)$ , referred to as *proper invariant realizations of  $S(F, G)$*  (Karcnias, 1990). This equivalence uses the explicit links between the Kronecker structures of the pencils associated with the two systems. The Kronecker structure of  $S_r(A, B, C)$  evolves from that of  $S(F, G)$  in a prescribed manner for the different types of invariants, and this evolution is expressed as a simple form of re-ordering and partitioning of the implicit model.

**(e) Performance Optimisation:** For a linear progenitor model  $H(s)$  and the structure transformation problems defined by  $W(s) = HF(s)Q$ , we may pose problems where we avoid formation of certain undesirable structural characteristics (such as right half plane zeros, high order infinite zeros etc) and at the same time optimise the values of certain key indicators, such as singular value properties of controllability, observability Grammians, condition number, etc. Within this class we may also consider the problems where the selection of  $Q$ ,  $H$  aims at minimising some form of uncertainty of the progenitor model. The overall approach here is to utilise the degrees of freedom in  $Q$ ,  $H$  matrices, which exist when avoidance of structural features rather than assignment of them is the central objective, to optimise certain key performance, or control structure indicator. Such problems are essential in the development of the system aspects of instrumentation, which is an open research area with very few generic results available. Key problems in this area are:

**Performance Optimisation MPPs:** Structuring an effective instrumentation scheme requires:

- Defining the lowest bounds for the number of effective inputs, outputs, which are needed for a control scheme, or family of alternative control schemes.
- Determining the best location of effective inputs, outputs, as well as, the structure of actuator, sensor maps, which may guarantee structural controllability, and observability.
- Evaluating the degree of dependence, independence of given input, output instrumentation schemes and its implications on process controllability, observability.
- Assessing the effect of a selected input, output scheme on the control quality, characteristics of the final system and selection of "best" schemes for easy, reliable control.
- Defining the "optimal" sensor-actuator distribution to achieve certain control objectives on the resulting model. ■

### 5.3 Model Expansion Problems

Defining input test signals and corresponding output measurements, is an integral part of the identification exercise. Defining input output schemes with the aim to identify, (or improve) a system model, or reconstruct an unmeasured internal variable, characterises the family of *Model Expansion Problems* (MEP). Questions related to the nature of test

signals, or properties of the measured signals are also important, on top of the more general questions related to the structure of the i/o scheme; the latter gives a distinct signal processing flavour to MEP. Some distinct problem areas are:

**(a) Additional Measurements for Estimation of Variables:** Frequently in applications, some important variables are not available for measurement. Secondary measurements have to be selected and used in conjunction with estimators to infer the value of unmeasurable variables. The proper selection of secondary measurements is a task of paramount importance for the synthesis of control schemes. The various aspects for the problem are discussed within the well developed area of state estimation (Kwakernaak & Sivan, 1972), (Lewis, 1986).

**(b) Input, Output schemes for System identification:** The selection of input test signals and output measurements is an integral part of the setting up of model identification experiments. In fact, the identified model is always a function of the way the system is excited and observed, i.e., of the way the system is embedded in its experimental environment. Most of the work so far has concentrated on SISO identification techniques and on the effect of test signal characteristics on the identification aspects of the model (Johanson, 1993), (Eykhoff, 1994). The study of effect of location of the group of excitation signals and corresponding group of extracted measurements on the identification problem has received less attention and its proper study is long overdue. Issues such as how and whether additional excitation signals and extracted measurements may enhance the scope and accuracy of identifiable models are challenging problems. This area of work is closely related to the problem of identifiability of models (Grewal & Glover, 1976), (Ljung, 1987).

**(c) Model Completion Problems:** This class of problems, deals with the problem of augmentation of a system operator, like the matrix pencil and has a dual nature to that of model projection, since now we deal with dimensional expansion of the relevant operator. Such problems are defined as: Let  $sE-H$  be an  $rxq$  pencil, which is a sub-pencil of the  $(r+t)x(q+v)$  pencil  $sE'-H'$ , where

$$sE-H = \begin{bmatrix} sE'-H' & X \\ X & X \end{bmatrix} \quad (5.6)$$

and the  $X$ 's stand for unspecified pencils of compatible dimensions. It is of interest to study the relationships between the sets of invariants of  $sE-H$  and  $sE'-H'$  pencils and in particular examine the conditions under which we may assign arbitrarily the structure of  $sE-H$ . This problem is known as *Matrix Pencil Completion Problem* (MPCP) (Loiseau et al, 2004), (Zaballa, 1987, 1999), (Cabral & Silva, 1991), and includes as special case the problem of invariant polynomial assignment by state feedback (Loiseau & Zagalak, 1994). The above formulation may be also extended to that of expansion of polynomial or rational models.

Model expansion are once more examples of model structure evolution, where additional inputs, outputs help a system model to grow to a more full representation of the existing system. The problems in this area express an alternative form of evolution of structure and properties of the model by manipulation of the input-output, external structure. We may refer to such systems with variability in the external structure as *Externally Evolving Systems* (EES).

## 6. Physical Growth-Death System Evolution and Re-engineering

### 6.1 Introduction

A new and challenging class of problems that expresses a different form of system evolution emerge within the framework of the general passive network system synthesis, as well as redesign of general systems and it is linked to lifecycle analysis of systems. Such problems aim to alter the dynamic behaviour by changing the system structure, without resorting to design of a controller. This is achieved by a number of actions distinct from control design aiming to change the overall system behaviour and involve:

- (i) Changing the values of the components of the system
- (ii) Altering the nature of components without changing the topology
- (iii) Modifying the topology and possibly reducing the system by removing components/subsystems.
- (iv) Augmenting the system by adding subsystems and extending the existing topology.

The problems we are addressing here are questions of open-loop design of the system on which control is eventually applied. Such problems are distinct from control, but their study involves system and control concepts and their formulation may be reduced to problems that may be identified as problems of the structural system framework; in particular, some of these problems may be formulated as problems of frequency assignment (Karcianas &

Giannakopoulos, 1984). A clearer presentation of the issues may be achieved by restricting ourselves to the case of passive RLC type networks (Vlach & Singhal, 1983). A summary of the key issues is considered next.

## 6.2 The Electrical Networks Problem

Electrical network theory is formulated in terms of two variables, the *current*  $i(t)$  and *voltage*  $v(t)$ , associated with each network element called a (*network*) *branch*. The voltages  $v(t)$  may be thought of as "across-variables", while currents may be thought of as "through variables". Thus, the voltage  $v(t)$  and the current  $i(t)$  are both oriented variables, and we use the edge-orientation arrows of a directed graph to define the positive orientations of the voltages and currents, which are usually called the voltage and current *references*. An *electrical network* is a directed graph  $G$  (Seshu & Reed, 1961) (of  $p$ -circuits,  $n$ -nodes and  $b$ -edges) with two functions  $v(s)$  and  $i(s)$  of a complex variable  $s$  associated with each edge of  $G$ , satisfying the following three postulates:

$$\text{Kirchhoff's current law:} \quad A_a I(s) = 0 \quad (6.1)$$

$$\text{Kirchhoff's voltage law:} \quad B_a V(s) = 0 \quad (6.2)$$

$$\text{Generalised Ohm's law:} \quad V(s) = E(s) + Z(s)I(s), \quad I(s) = J(s) + Y(s)V(s) \quad (6.3,4)$$

Where:  $I(s)$  is a  $b$ -vector denoting the currents of the edges, and is called the branch-current vector of  $G$  and  $A_a$  is the node-edge incidence matrix of  $G$ ;  $V(s)$  is a  $b$ -vector denoting the voltages of the edges, and is called the branch-voltage vector of  $G$  and  $B_a$  is the circuit-edge incidence matrix or the circuit matrix.  $Z(s)$  and  $Y(s)$  are given matrices of order  $b$ , and called the branch impedance and the branch admittance matrices, respectively; The matrices  $Z(s)$  and  $Y(s)$  are diagonal matrices containing the impedances and the admittances of all the branches of the network. On the other hand,  $E(s)$  and  $J(s)$  are given  $b$ -vectors denoting the sources and the initial conditions, and are called the branch voltage-source and the branch current-source vectors of  $G$ , respectively. The *electrical network fundamental problem* is to solve equations (6.1-6.4) with respect to the vectors  $I(s)$ ,  $V(s)$ .

The analysis of dynamics and behaviour of the network, involves removing the redundant equations and then solve the resulting set of equations. If  $A$ ,  $B$  are maximal sets of linearly independent rows of  $A_a$ ,  $B_a$  respectively, then the system of network equations can be expressed as (Vlach & Singhal, 1983)

$$I = -B^t (BZB^t)^{-1}BE \quad \text{and} \quad V = E - ZB^t (BZB^t)^{-1}BE \quad (6.5)$$

or equivalently as:

$$V = -A^t (AYA^t)^{-1}AJ \quad \text{and} \quad I = J - YA^t (AYA^t)^{-1}AJ \quad (6.6)$$

The matrices  $BZB^t$  and  $AYA^t$  are rational and they are called *loop-impedance* and *node-admittance* matrices respectively and their zeros define the natural frequencies of the network. It is well known that the natural frequencies characterize the network's natural response and they are invariants of the network since they do not depend on the choice of the basis matrices  $A$ ,  $B$  nor on the choice of circuits or cuts which have been used for their derivation. Loop Impedance and Node Admittance models are alternative system representations which have not been properly studied from the control viewpoint. For such models standard frequency assignment problems may be defined.

## 6.3 The Problem of tuning the Natural Frequencies of a Network

The natural frequencies of an electrical network depend on the topology of the network and on the nature and the values of the elements of the network. In assigning the characteristic frequencies the designer can exploit the available degrees of freedom as described below:

**Case1. Free topology and elements:** This is the general problem of the network synthesis, where both the topology of the network and the elements are design parameters and can be formulated as: Given a rational matrix, determine the conditions under which it can be realised as an RLC network. This is the classical problem of network synthesis (Kim & Meadows, 1971) and it is not considered here. When the topology is fixed and the nature of the elements is given, but not their values, then we have a general problem of assignment of impedance and admittance matrices which is not a standard network theory problem, since the topology and location of elements are not any more free. Such classes of problems are relevant to different areas where network models play a key role.

**Case 2. Fixed topology, nature of elements, but free values of parameters:** These network problems are essentially characteristic frequency assignment problems, where the free parameters are used to assign the network characteristic frequencies. A more restricted version is let free only the values of certain type of elements. The latter case represents typical problems of network redesign, which involve changing the least number of elements to improve the zero structure of the resulting system. Two special cases of this version that can be readily handled with the Determinantal Assignment Framework (Leventides, etc, 2000) are:

**(i) Determining the resistors in an RL network:** Assume that the branch impedance matrix is given by

$$Z(s) = \begin{bmatrix} sL + R & 0 \\ 0 & D \end{bmatrix} \quad (6.7)$$

where  $sL+R$  is a known diagonal matrix,  $D$  is diagonal static matrix to be determined (characterising variable resistances) and  $B$  is the network matrix that can be partitioned as  $B=[B_1, B_2]$ . Then the loop impedance matrix is  $BZ(s)B^t = B_1(sL+R)B_1^t + B_2D$ . If  $B_2$  is non singular, then the zeros of  $BZ(s)B^t$  are defined by those of the matrix

$$(B_2)^{-1}B_1(sL+R)B_1^t(B_2^t)^{-1} + D. \quad (6.8)$$

The problem then is to construct a diagonal perturbation  $D$  such that the above matrix has predefined zeros. This is dual to the problem of determining the resistor values for tuning the zeros of the admittance matrix  $AY(s)A^t$  in an RC network, which may be expressed as  $(A_2)^{-1}A_1(sC+G)A_1^t(A_2^t)^{-1} + D$ .

**(ii) Determining the resistors in an RLC network:** Assume that the branch impedance matrix has fixed inductances, capacitances and some resistances, but some other resistances are free. Then the branch impedance matrix is

$$Z(s) = \begin{bmatrix} sL + R + \frac{1}{sC} & 0 \\ 0 & D \end{bmatrix} \quad (6.9)$$

Where  $sL+R$  is a known diagonal matrix,  $D$  is diagonal static matrix to be determined. Then  $B$  can be partitioned as  $B=[B_1, B_2]$  and the loop impedance matrix is given by  $BZ(s)B^t = B_1(sL+R+1/sC)B_1^t + B_2D$ . If  $B_2$  is non singular then the zeros of  $BZ(s)B^t$  are defined by those of

$$(B_2)^{-1}B_1(sL+R+1/sC)B_1^t(B_2^t)^{-1} + D. \quad (6.10)$$

This leads to the problem: Select a diagonal matrix  $D$  such that the zeros of the matrix  $sF+G+1/sH+D$  where  $(F,G,H)$  is a given triple are arbitrarily assignable, This problem may be treated using an algebraic framework. In fact, if  $N_1(s)D_1(s)^{-1}$  is a right MFD of  $(B_2)^{-1}B_1(sL+R+1/sC)B_1^t(B_2^t)^{-1}$  then the problem is equivalent to solving with respect to the static diagonal matrix  $D$ , the equation (6.10), which is a DAP formulation with diagonal controllers (Leventides et al, 2000)

$$\det\left(\begin{bmatrix} I & D \end{bmatrix} \begin{bmatrix} N_1(s) \\ D_1(s) \end{bmatrix}\right) = p(s) \quad (6.11)$$

**Case3: Re-engineering of a network:** An initial topology and values for the elements of the network are given, but these have to be modified according to some new specifications. Such problems can be formulated as: Given an impedance matrix  $BZ(s)B^t$  with a given zero polynomial  $q(s)$ , derive a new desired polynomial zero polynomial  $p(s)$  by actions such as: **(i)** modification of some of the rows of  $B$ ; **(ii)** addition of some new rows to the matrix  $B$ ; **(iii)** Augmentation of  $Z(s)$  as well as tuning of some of its elements. These are more general problems of redesign, where the topology is also subject to modification and they are challenging open issues.

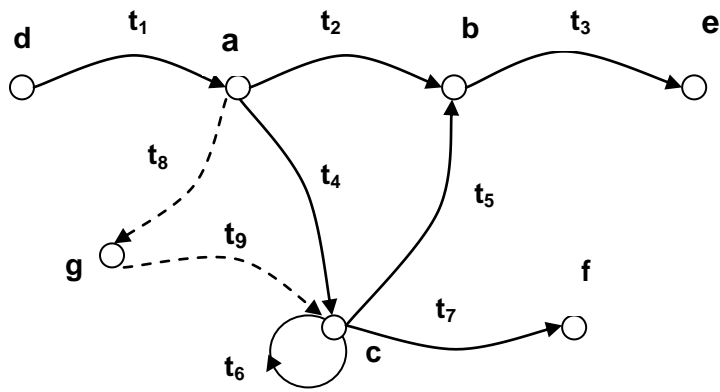
## 6.4 The Graph Growth Problem

The study of the above redesign problems involves the physical growth of a system graph and such problems are referred to as *graph structural evolution problems*. Some important problems in this area involve:

- (a) The representation problem:** Define an appropriate modelling framework for describing the graph augmentation and graph reduction/death problems.
- (b) The graph structural growth problem:** Investigate how the properties of a directed graph are changing by addition of new nodes, or elimination of existing ones.
- (c) Control of graph evolution problem:** Develop a theory for modification of the associated system properties and parametric structure as functions of such transformations.

For the graph structural growth problem, the main interest is the study of evolution of structural and non-structural properties under such transformations. An illustration of the above, is provided by the following example.

**Example (6.1):** Consider the directed graph below represented by the a, b, c, d, e, f nodes and the solid edges. This is modified by adding a new node g and the new dotted line edges and produces an evolution of the previous graph.



**Figure (6):** Example of Structural Graph Growth problem

The *Structural Graph Growth Problem* introduced here, may be combined with the signal, dimensional growth, or *Dimensional Graph Evolution* discussed in Section (3.1). Clearly, combinations of the two may be considered and this may be referred to as the *General Graph Growth Problem*. The above represent open areas for research on the fundamentals of *Graph Growth-Death*. This research aims at general system synthesis procedures:

**General System Synthesis Problems:** Define a methodology that leads to the synthesis of composite systems, which have a set of desirable properties, or avoid the formulation of undesirable properties for the resulting composite system. This includes as special cases: **(i)** The graph to be designed is entirely free. **(ii)** The graph to be designed is subject to a set of constraints imposed by other considerations. **(iii)** The graph to be designed has to evolve from an existing one by dimensional, or structural Growth.

## 7. Conclusions: Control Theory in the Context of Integrated Design

Control Theory and Design have developed around the classical servomechanism paradigm. The area of Systems Integration for large Complex Systems introduces many new challenges and a number of new paradigms which generate new requirements for future developments. In the case of design, the challenges come from the cascade and design-time dependent evolutionary nature of the process. Major challenges relate to the following two areas:

- (i) Controlling the development of the Evolutionary Design Process and thus development of Control Theory and Design for Evolutionary Systems
- (ii) Addressing special requirements for Control Design of Large Complex Systems

In the first area, the overall philosophy, which is adopted, is that each particular design stage, in the overall design process, shapes a local model; the structure of this local model has important implications on what can be achieved at the next design stage, and it thus determines overall cost, operability, safety and performance of the final process. Each design stage starts with a model and decisions taken there contribute to the gradual shaping of the final structural characteristics; however, this happens within a range of possible options. Structural properties and thus performance, operability, etc. characteristics evolve, but not in a simple manner. This evolution of structure and related potential for delivering certain level of performance are not well understood. We would like to drive the model evolution along paths avoiding the formation of undesirable structural features and where possible to assign desirable characteristics and values. In the effort to formulate a generic system/control based framework it is essential to address issues such as:

- (P.1) Characterisation of desirable, undesirable performance characteristics and the limits of what can be achieved.
- (P.2) Relate the best achievable performance characteristics to system model structure.
- (P.3) Establish functional relations between model structure and characteristics and model parameters.

These are traditional Control Theory tasks which provide the cornerstone of the philosophy required for integration based around the control of the evolutionary mechanism in design. Further aspects of the evolutionary process are:

- (P.4) Interpretation of Evaluation Criteria for Process Synthesis in Control Design terms.
- (P.5) Prediction of full model System properties in nested system model chains characterised forms of variability in the model Complexity.
- (P.6) Input-Output Based and Variable dimensionality structural analysis of composite systems.
- (P.7) Representation and Properties of graph structure evolving systems.
- (P.8) Graph impact on composite system properties.

The above problems refer to generalised process synthesis and the interconnection graph is central in the characterisation of structure and related properties. Global Instrumentation deals with the model shaping role of the



selection of inputs, outputs and it is here that structure is predominantly expressed in terms of structural invariants (algebraic, geometric, graph etc.). Key problems for this area are:

- (P.9)** Evolution of properties and structure in Model Orientation Problems.
- (P.10)** Effects of Completeness and deviations from Completeness on Composite System structure and Properties.
- (P.11)** Evolution of Structure and Properties under Model Projection.
- (P.12)** Evolution of Structure and Properties under Model Expansion.
- (P.13)** Structure assignment, or “optimal” structure design for the above problem areas.

Each one of the above areas has also a design dimension which involves the shaping of the system model structure (invariants and/or graph), system properties and related property indicators (measuring degree of presence of properties). Ideally, we would like to assign desirable properties, but in reality it would be more relevant to avoid the formation of undesirable characteristics. Versions of the above families of problems may be formulated when the redesign problem is considered, where either we want to modify the graph, the input, output structure, or the controller to achieve new requirements and objectives.

For large dimension problems additional challenges emerge from the computations, as well as handling many design objectives simultaneously. The large dimensions and the geographical separation of process units may also require decentralisation of the control scheme. Two of the main design problems in this area are:

- (a)** Design Problem Decomposition
- (b)** Selection and Design of Decentralised Control Schemes

The area of Design Problem Decomposition has as key issues (Morari & Stephanopoulos, 1980): **(i)** Process decomposition; **(ii)** Decomposition into unit goal; **(iii)** Sequencing of the design process. Process decomposition is the reduction of a large problem into a sequence of smaller problems at the expense of having to deal with the co-ordination of the sequence of these sub-problems. In (ii) the decomposition of operations of each subsystem (unit) into Specific Unit Goals is considered, which in turn have to be co-ordinated. Interactions between process units introduce additional complications. The sequencing of the design is the result of the Process design decomposition and involves the co-ordination of the individual goals into a sequence that refers to the plant as a whole.

The selection and design of Decentralised Control Schemes is based on: **(i)** Application Dependent Methodologies and **(ii)** Control Theory and Design based Approaches. Synthesis of control structures has long been practised by experienced control engineers, who relied on intuition, insight and judgement to pick a feasible solution from the vast number of alternatives that were possible. In practice, this problem is solved by considering the issues such as (Morari, 1992): (a) economics, safety and reliability goals of a given process; (b) steady-state and dynamic behaviour of the complete process; (c) failure modes of the components within the process; (d) Possible changes in the process to improve control. Experienced engineers have produced empirical rules and procedures for developing control structures starting from loosely defined flow-sheets and goals and proceeding to well-defined systems. Most of the procedures do not involve the use of detailed dynamic models of the process. Such methodologies may be considerably assisted by developing approaches that tackle open issues such as:

- (P.16)** Selection of Decentralisation Structures and their required dynamics by exploiting the system structure.
- (P.17)** Design of given structure robust and flexible Decentralisation Schemes.

The selection of Decentralisation Structure and required dynamics may use a number of results which explore system structure from a number of viewpoints (Karcianas et al, 1997). Graph theory, together with the results on process decomposition may be used to provide a first screening of possible alternatives (a first set defined by the process heuristics); advanced structural diagnostics based on solvability of different types of control synthesis problems may be used to provide a list of possible alternatives. By reducing the problem to smaller dimensions, interaction analysis may be deployed to specify the required decentralisation at the local level and the dynamic complexity of local controllers. Having decided on the required decentralisation structure, the design of robust decentralised controllers has to be addressed. The lack of parameterisation of all decentralised stabilising controllers, essential for the development of  $H_\infty$  type robust methodologies for decentralised control is a major gap in this area. The requirements for flexibility in operations imply that selection of decentralisation, and of the decentralised controller, have to be valid for more than one operating points. This raises the issues of simultaneous decentralized design, which is a further major challenge.

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