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Department of Economics
Discussion Paper Series

No. 03/08

Non-linear multivariate adjustment of the UK real exchange rate

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Abstract

Based on a multivariate non-linear model, this paper recognises an important role for the real exchange rate in affecting UK labour market conditions. The short-run real exchange rate adjusts quickly to disequilibrium deviations of the real exchange rate from its long-run level outside a rather wide interval band. When the real exchange rate is undervalued, short-run unemployment falls as firms respond to an improvement in domestic competitiveness by increasing their demand for labour. Further, there is a strong response of short-run unemployment to the disequilibrium error outside a narrow interval band. To the extent that the real exchange rate equation reflects monetary policy considerations, our results imply that unemployment can be targeted by economic policy. Furthermore, if economic authorities want to avoid large swings in unemployment then they should be prepared to intervene in exchange markets with the aim of keeping real exchange rate movements within a narrow interval band.

Keywords: Real exchange rate; Traded goods; Smooth Transition Vector Error Correction Model.

JEL classification: C32; C51; C52; F30; F41

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1. Introduction

The use of non-linear models in explaining economic phenomena is motivated by the idea that the behaviour of economic variables depends on different states of the world or regimes that prevail at any point in time. Regime switching behaviour in real exchange rates can be explained by the existence of a band in terms of the costs of trading goods; to the extent that deviations of the real exchange rate from its long-run level are small relative to the costs of trading, these deviations are left uncorrected (Dumas, 1992). Over the last few years, the Smooth Transition Autoregressive (thereafter STAR) methodology has been a popular way of introducing regime-switching behaviour in real exchange rate models, where the transition from one regime to the other occurs in a smooth way. [1] For instance, Baum *et al.* (2001) and Michael *et al*. (1997) model the real exchange rate (for a number of countries including the UK) as a stationary variable and estimate its dynamics based on Exponential STAR (ESTAR) models. Paya and Peel (2003) estimate an ESTAR model of the dollar–yen real exchange rate which incorporates a deterministic trend as a proxy for the equilibrium level. On the other hand, Sarantis (1999) finds that real exchange rates (including the UK one) are non-stationary and proceeds by estimating real exchange ESTAR and Logistic STAR (LSTAR) models in first differences. [2] A common feature of these papers is that they all estimate univariate real exchange rate models.

This marks a significant point of departure for our paper: while we follow Sarantis (1999) in treating the real exchange rate as a non-stationary variable, we find that the latter cointegrates with real wages and the real price of oil. This finding builds upon the theoretical model of Alogoskoufis (1990) who derived a linear real exchange rate equation based on the production sector of the economy (see also Chaudhuri and Daniel, 1998, who estimate a more restrictive model involving only the real exchange rate and the real price of oil). Noting that cointegration tests perform reasonably well when the adjustment process is non-linear (van Dijk and Franses 2000), we then proceed by employing a multivariate STAR framework to model the non-linear dynamics of the real exchange rate equation as part of a small system involving real wages, the unemployment rate and the real price of oil. [3] Inclusion of the unemployment rate in our system

[1] STAR models were introduced by Teräsvirta and Anderson (1992) in order to examine non-linearities over the business cycle, whereas their statistical properties were discussed in Granger and Teräsvirta (1993) and Teräsvirta (1994), among others.
[2] Empirical results in support of a stationary real exchange rate are quite mixed and depend e.g. upon the sample size selected or the definition of the price series used (for a recent survey see e.g. Baum et al., 2001).
[3] Multivariate STAR models were recently discussed in Granger and Swanson (1996), Weise (1999), Rothman *et al.* (2001) and van Dijk *et al*. (2002) among others.

could be justified in terms of an Okun's law channel; for instance, Nakagawa (2002) who looks at the relationship between real exchange rates and interest rate differentials, discusses non-linear effects in a model where an undervalued real exchange rate raises aggregate demand for output relative to its full employment level.

Modelling our system in a smooth transition framework contrasts with the Threshold Autoregressive (TAR; see e.g. Tong, 1990) and the Hamilton (1989) Markov regime-switching models, which assume that the transition between regimes occurs abruptly rather than smoothly. On economic grounds, STAR models seem to be more appropriate than TAR or Markov regime-switching models. Modelling the real exchange rate as a function of real wages and the real price of oil implies that real exchange rate movements are affected by conditions prevailing in the production sector of the economy. In this case, a smooth transition rather than a sharp switch between regimes could be justified in terms of frictions in the product market due to product heterogeneity, government imposed barriers to trade, or labour market inflexibility distorting the rapid adjustment of wages.

Our estimates suggest the existence of a long-run real exchange rate equation, which affects significantly the short-run dynamics of the system both in a linear and a non-linear way. In particular, the short-run real exchange rate adjusts quickly to disequilibrium deviations of the real exchange rate from its long-run level outside an interval band, which is estimated to be rather wide. This is not surprising as our sample period coincides with floating exchange rates being in operation. We also find that when the real exchange rate is above its long-run equilibrium level (i.e. it is undervalued), short-run unemployment falls as firms respond to an improvement in domestic competitiveness by increasing their demand for labour. Further, there is a strong response of short-run unemployment to the disequilibrium error outside an interval band, which is estimated to be rather narrow. To the extent that the real exchange rate equation reflects monetary and more generally economic policy considerations, our results imply that unemployment can be targeted by economic policy. Furthermore, if economic authorities want to avoid large swings in unemployment then they should be prepared to intervene in exchange markets with the aim of keeping real exchange rate movements within a narrow interval band. Our results also suggest that when the real exchange rate is undervalued, workers respond to an improvement in domestic competitiveness by demanding and getting higher wages. Again, this effect is non-linear. Therefore, our findings recognise an important role for the real exchange rate in affecting labour market conditions in the UK.

The structure of the paper is as follows. The next section discusses briefly the theory of linearity testing within a multivariate STAR framework. Section 3 of the paper discusses the econometric specification of a real exchange rate model, whereas Section 4 estimates the linear and non-linear versions of the model. Section 5 presents a discussion of our findings and section 6 provides some concluding remarks.

2. Specification of multivariate smooth transition models

Following Rothman *et al*. (2001), we write a k-dimensional Smooth Transition Vector Error Correction Model (STVECM) as:

$$\begin{aligned}\Delta y_t &= \left(\mu_1 + \alpha_1 z_{t-1} + \sum_{j=1}^{p-1} \Phi_{1,j}\Delta y_{t-j} + \sum_{j=0}^{p-1} \Phi_{2,j}\Delta x_{t-j}\right)(1 - G(s_t)) \\ &+ \left(\mu_2 + \alpha_2 z_{t-1} + \sum_{j=1}^{p-1} \Phi_{3,j}\Delta y_{t-j} + \sum_{j=0}^{p-1} \Phi_{4,j}\Delta x_{t-j}\right) G(s_t) + \varepsilon_t,\end{aligned} \tag{1}$$

where y_t is a (k x 1) vector of $I(1)$ endogenous variables, x_t is an (m x 1) vector of $I(1)$ exogenous variables, $\varepsilon_t \sim iid(0,\Sigma)$, α_i, $i = 1,2$, are (k x r) matrices, and $z_t = \beta'[y_t', x_t']'$ for some (q x r) matrix β denote the error correction terms, with $q = k + m$. $\Phi_{1,j}$ and $\Phi_{3,j}$, $j = 1,..., p-1$, are (k x k) matrices. $\Phi_{2,j}$ and $\Phi_{4,j}$, $j = 0,..., p-1$, are (k x m) matrices and μ_i, $i = 1,2$, are ($k \times 1$) vectors. $G(s_t)$ is the transition function, assumed to be continuous and bounded between zero and one. The STVECM framework can be considered as a regime-switching model which allows for two regimes, $G(s_t) = 0$ and $G(s_t) = 1$, respectively, where the transition from one to the other regime occurs in a smooth way. We focus our attention on the 'quadratic logistic' function (Jansen and Teräsvirta, 1996):

$$G(s_t;\gamma,c_1,c_2) = \{1 + \exp[-\gamma(s_t - c_1)(s_t - c_2)/\sigma^2(s_t)]\}^{-1}, \gamma > 0, \tag{2}$$

where $\sigma^2(s_t)$ is the sample variance of s_t. This model assumes asymmetric adjustment to deviations of s_t from an interval band (c_1, c_2). The parameter γ determines the speed of the transition from one regime to the other. If $\gamma \to 0$, the model becomes linear, whereas if $\gamma \to +\infty$,

$G(s_t)$ is equal to 1 for $s_t < c_1$ and $s_t > c_2$, and equal to 0 when $c_1 < s_t < c_2$.

In this paper we assume that the possible candidates for the transition variable s_t are the r cointegrating relationships in $z_{t-1} = \beta'[y_{t-1}', x_{t-1}']'$. More specifically, the next section estimates one cointegrating vector, which is identified as a long-run real exchange rate equation. Therefore, model (2) above is particularly attractive from an economic point of view as it implies the existence of an interval band (c_1, c_2) outside which there is a strong tendency for the real exchange rate to revert to its equilibrium value.

A test of linearity in model (1) using the transition function (2), is a test of the null hypothesis H_0: $\gamma = 0$ against the alternative H_1: $\gamma > 0$. By taking a first-order Taylor approximation of $G(s_t)$ around $\gamma = 0$, the test can be done within the reparameterised model (see e.g. the discussion in Saikkonen and Luukkonen, 1988):

$$\Delta y_t = M_0 + A_0 z_{t-1} + \sum_{j=1}^{p-1} B_{0,j} \Delta y_{t-j} + \sum_{j=0}^{p-1} B_{1,j} \Delta x_{t-j} + A_1 z_{t-1} s_t + \sum_{j=1}^{p-1} B_{2,j} \Delta y_{t-j} s_t + \sum_{j=0}^{p-1} B_{3,j} \Delta x_{t-j} s_t + A_2 z_{t-1} s_t^2 + \sum_{j=1}^{p-1} B_{4,j} \Delta y_{t-j} s_t^2 + \sum_{j=0}^{p-1} B_{5,j} \Delta x_{t-j} s_t^2 + e_t, \tag{3}$$

where e_t are the original errors ε_t plus the error arising form the Taylor approximation. Model (3) is a linear VECM augmented by additional cross-product regressors due to the Taylor expansion. Here, the null hypothesis of linearity is $H_0' : A_1 = A_2 = B_{2,j} = B_{3,j} = B_{4,j} = B_{5,j} = 0$, where $j = 1,..., p-1$ for the $B_{2,j}$ and $B_{4,j}$ matrices and $j = 0,..., p-1$ for the $B_{3,j}$ and $B_{5,j}$ matrices. For each equation in the VECM, this is a standard variable addition Lagrange Multiplier (LM) which follows asymptotically the χ^2 distribution with $2r + 2k(p-1) + 2mp$ degrees of freedom. In small samples, the χ^2 test may be heavily oversized. Therefore, it may be preferable to use an F version of the test. Both the χ^2 and F versions of the LM statistic are equation specific tests for linearity which are computed from an auxiliary regression of the residuals from each equation in the linear VECM on all variables entering model (3). To test the null hypothesis of linearity in all equations simultaneously, we need a system-wide test. Following Weise (1999), define Ω_0 and Ω_1 as the estimated variance-covariance residual matrices from the linear VECM and the augmented model (3), respectively. The appropriate log-likelihood system-wide test statistic is given by

$LR = T\{\log|\Omega_0| - \log|\Omega_1|\}$, where T is the size of the sample. Under the null hypothesis of linearity, the test follows asymptotically the χ^2 distribution with $2rk + 2k^2(p-1) + 2kmp$ degrees of freedom. The equation specific LM tests and the system wide LR test are run for all possible s_t candidates, that is, all cointegrating relationships. The decision rule is to select as the appropriate transition variable the cointegrating relationship for which the p-value of the test statistic is the smallest one. [4]

3. Econometric specification of a real exchange rate model

The theoretical framework discussed above will now be tested on a small model of the UK real exchange rate. In an earlier paper, Alogoskoufis (1990) introduces a model with traded (T) and non-traded (NT) goods. [5] The model assumes perfect competition in the T sector with firms producing according to a two-level CES production function which is separable into capital, labour and imported oil. For the NT sector, the model assumes profit maximising monopolistic competitive firms. The relative price of tradables to the price of domestic output, $p_T - p$ is derived as:

$$p_T - p = -(1-\tau)(w - p_T) + \tau\left(\frac{1-\pi_1}{\pi_1}\right)(p_O - p_T)\,, \tag{4}$$

where π_1 ($0 < \pi_1 < 1$) is the share of value added in gross output, τ is the share of tradables in total output, $(w - p_T)$ refers to real product wages in the tradables sector and $(p_O - p_T)$ is the relative price of imported oil. All variables are in logs. Assuming that the UK is a small open economy, the price of domestic tradables p_T can be proxied by $p_T = p_T^* + e$, where p_T^* is the price index of UK imports in \$, and e is the average £/\$ exchange rate. In this case, equation (4) is a measure of the real exchange rate as a negative function of $w - p_T$ and a positive function of $p_O - p_T$. An increase in $p_T - p$ is equivalent to a real depreciation or an improvement in the real competitiveness of the domestic economy.

Following the notation in Section 2 of the paper, our model uses a set of $k = 3$ endogenous

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[4] An extension of the Saikkonen and Luukkonen (1988) linearity tests involves a second-order Taylor approximation of the transition function as suggested by Escribano and Jordá (1999). This involves adding cubic and fourth power terms in model (3), which is hardly practical to implement since we are faced with a small sample size. Further, as van Dijk *et al*. (2002) point out, neither one of the tests in Saikkonen and Luukkonen (1988) or Escribano and Jordá (1999) dominates in terms of power. Given that the tests are not exact but approximations, some caution is needed when using the rule of the minimum probability value in order to determine the appropriate transition variable.

[5] For other versions of price models with traded and non-traded goods see Martin (1997).

variables:

$$y_t = [p_T - p, w - p_T, u]', \tag{5}$$

conditioning on $x_t = p_O - p_T$, that is, $m = 1$ exogenous variable. We use quarterly seasonally adjusted UK data over the period 1973(1)-2000(1). The endogenous variables refer to the real exchange rate, $p_T - p$, real product wages, $w - p_T$, in the manufacturing sector (as a proxy for tradables), and the unemployment rate, u (all variables are in logs; for more details see the Data appendix). [6] Within our multivariate system, labour market arguments suggest that real wages interact with the unemployment rate. Further, Nakagawa (2002) discusses non-linear effects in a model where an undervalued real exchange rate raises aggregate demand for output relative to its full employment level. Hence there should be a negative correlation between the real exchange rate misalignment and unemployment through an Okun's law channel; when the real exchange rate is undervalued, firms respond to an improvement in domestic competitiveness which induces shifts in aggregate demand by increasing their demand for labour. As a result, unemployment falls. In addition, firms are assumed to take the real price of oil, $p_O - p_T$, as given and therefore we impose exogeneity of this variable, which may improve the statistical properties of the system (see the discussion in Hansen and Juselius, 1994). [7]

4. Empirical results

4.1 Long-run behaviour

Figure 1 plots the logs of the levels and the first differences of the $p_T - p$, $w - p_T$, $p_O - p_T$ and u series. Preliminary analysis using the ADF unit root tests suggested that all series are $I(1)$ in levels and $I(0)$ in first differences. To test for cointegration, we estimate the linear VECM in levels using a lag length of $p = 4$ and allowing for a drift parameter to enter the VECM unrestrictedly. [8] Table 1 reports the eigenvalues, λ_i, and the λ-*max* and *trace* statistic tests for

[6] As discussed in the Appendix, we use wage costs rather than wage costs per unit of output in our empirical results. That said, use of unit wage costs does not make any qualitative difference to the results reported below.

[7] Obviously, some other variables like productivity or tax rates can affect wages or unemployment. Extending the information set to this direction is not pursued here, as we are primarily interested in discussing the non-linear behaviour of the real exchange rate equation. Furthermore, use of a larger information set is impractical because the number of estimated coefficients in the linearity tests and the STVECM rises considerably relative to the number of estimated coefficients in the linear VECM. For these reasons we settle for a relatively small baseline system.

[8] The lag length is selected by starting with 5 lags on each variable, and sequentially testing down using an F-test.

cointegration (see Johansen, 1988). The 95 percent critical values are taken from Mackinnon *et al*. (1999) in the context of the Pesaran *et al*. (2000) linear system with both endogenous and exogenous $I(1)$ variables. Both the λ-*max* and *trace* statistics indicate the existence of $r = 1$ cointegrating vector. For exact identification, we normalise the estimated vector on the real exchange rate, $p_T - p$. Then we test one over-identifying restriction, that is, long-run exclusion of the unemployment rate, u. The restriction is accepted as it calculates $\chi^2(1) = 2.73$ (p-value = 0.10) and the resulting cointegrating vector is:

$$p_T - p = \underset{(0.030)}{-0.457}\,(w - p_T) \; \underset{(0.111)}{+0.182}\,(p_O - p_T)$$

where standard errors are given in parentheses below the estimated coefficients. The estimated cointegrating relationship looks like the theoretical real exchange rate equation (4) with the share of traded goods in total output (i.e. τ) estimated at 54.3 percent and the share of value added in gross output (i.e. π_1) estimated at 74.8 percent (the latter is derived from $\tau(1 - \pi_1)/\pi_1 = 0.182$). [9] [10] Figure 2 plots the mean-corrected deviations from the estimated relationship. Movements of the disequilibrium error above (below) the zero line are associated with an undervalued (overvalued) real exchange rate. We discuss this issue further in section 5 of the paper.

4.2 Linearity testing and short-run estimates

Having estimated the long-run real exchange rate equation, we test for linearity in model (3) using the estimated cointegrating vector CV_{t-1} as the possible transition variable s_t. Linearity tests

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The three endogenous equations $y_t = [p_T - p, w - p_T, u]'$ in the linear VECM pass the Autocorrelation test (of up to order 5) and the 4th order Autoregressive Conditional Heteroscedasticity test and Heteroscedasticity. They fail Normality and Heteroscedasticity. Detailed diagnostic tests for the linear and non-linear versions of the short-run equations are reported below.

[9] Using annual data over the 1952-1985 period, Alogoskoufis (1990) estimates τ between 31 percent and 39 percent, and π_1 at 92 percent. However, he points out that his estimates for τ are implausibly low. Our estimate for π_1 at 74.8 percent is much closer to π_1 at 71.7 percent in Bruno and Sachs (1982). Their estimate is based on a system of factor-price frontier, output supply, and labour demand equations using annual data over the 1956-1978 period.

[10] Chaudhuri and Daniel (1998) adopt the Engle-Granger two-step procedure to test for cointegration in a bivariate model involving real exchange rates and real oil prices for sixteen OECD countries. Using monthly data over the 1973(1)-1996(2) period, they find that the UK real exchange rate cointegrates with the real price of oil. The coefficient on the real price of oil is estimated at 0.389.

are run for a different number of lags of the transition variable $s_{t-d} = CV_{t-d}$ (namely $d = 1, 2, 3$, and 4 lags). Then the appropriate lag is selected as the one for which the linearity test is most strongly rejected. We report bootstrapped p-values instead of asymptotic p-values although our results are not sensitive to the above choice. To compute the bootstrapped p-values of the equation specific F tests and the system wide LR test reported in Table 2, we followed closely Weise (1999). First, we estimated the linear VECM equations. To control for the presence of heteroscedasticity in the linear models, the VECM residuals were regressed on all RHS variables entering the linear VECM as well as their squares, and the original residuals were transformed using the estimated coefficients from this auxiliary regression. Draws were taken from the transformed residuals and one thousand artificial data series were constructed. For each of these artificial series, F and LR statistics were constructed and then compared to the corresponding statistics from the actual data. The bootstrapped p-values were derived as the number of times the F and LR statistics from the artificial data exceeded the corresponding statistics from the actual data, divided by one thousand.

According to the results in Table 2, linearity is mostly rejected for CV_{t-1}. Using the disequilibrium error CV_{t-1} in the 'quadratic logistic' function (2c), we therefore proceed by estimating the non-linear short-run $\Delta(p_T - p)_t$, $\Delta(w - p_T)_t$ and Δu_t equations. Before estimating the non-linear models, it is worth mentioning that Granger and Teräsvirta (1993) and Teräsvirta (1994) stress particular problems like slow convergence or overestimation associated with estimates of the γ parameter. For this reason, we follow their suggestion in scaling the 'quadratic logistic' function (2c) by dividing it by the variance of the transition variable $\sigma^2(CV_{t-1})$ (which equals 0.003), so that γ becomes a scale-free parameter. Based on this scaling, we use $\gamma = 1$ as a starting value and values of CV_{t-1} close to its minimum (which equals -0.151) and maximum (which equals 0.106) as starting values for the parameters c_1 and c_2, respectively. The estimates of the linear equations for $\Delta(p_T - p)_t$, $\Delta(w - p_T)_t$ and Δu_t are used as starting values for the remaining parameters in the STVECM equations (1). For comparison reasons, we report both the linear and the non-linear versions of the estimated equations. Tables 3A to 3C report the OLS estimates of the parsimonious linear models, whereas Tables 4 to 6 report the non-linear least squares (NLS) estimates for the parsimonious STVECM equations (1). [11]

[11] One could also argue in favour of a structural rather than a reduced form model by testing the significance of current $\Delta(p_T - p)_t$, $\Delta(w - p_T)_t$ and Δu_t effects in the estimated equations. However, these effects were insignificant.

The main parameters of interest in the non-linear models are the estimated values of the threshold parameters c_1 and c_2, and the speed of adjustment, γ. The c_1 and c_2 estimates reported in Tables 4 to 6 are statistically significant in all models. The c_1 and c_2 estimates indicate the existence of two regimes for the $\Delta(p_T - p)_t$, $\Delta(w - p_T)_t$ and Δu_t equations; one characterised by large deviations of the real exchange rate from its long-run equilibrium and an alternative one which is characterised by small real exchange rate deviations from its equilibrium level. The economic implications of these results will be discussed in the following section. The estimates of the γ parameter are rather high for all models indicating that the speed of the transition from $G(s_t;\gamma,c_1,c_2)=0$ to $G(s_t;\gamma,c_1,c_2)=1$ is rapid at the estimated thresholds c_1 and c_2. Notice, however, the rather high standard error of the γ estimates. Teräsvirta (1994) and van Dijk *et al*. (2002) point out that this should not be interpreted as evidence of weak non-linearity. Accurate estimation of γ is not always feasible, as it requires many observations in the immediate neighborhood of the threshold parameters c_1 and c_2. Further, large changes in γ have only a small effect on the shape of the transition function implying that high accuracy in estimating γ is not necessary (see the discussion in van Dijk *et al*., 2002).

From Tables 3 to 6 one can notice a large improvement in the diagnostic tests of the non-linear relative to the linear models. The error variance ratio of the non-linear relative to the linear models (i.e. s^2_{NL}/s^2_L) is less than one, indicating that the non-linear models have a better fit. In particular, the s^2_{NL}/s^2_L ratio shows a reduction in the residual variances of the non-linear compared to the linear models which ranges from around 16 percent for the $\Delta(p_T - p)_t$ and Δu_t equations in Table 4 and Table 6, respectively, to around 30 percent for the $\Delta(w - p_T)_t$ model in Table 5. In addition, the non-linear specification of all three models captures the heteroscedastic and most of the normality failures that are present in the corresponding linear models.

5. Discussion of the results

Looking at the linear short-run equations first, one can notice that the cointegrating vector ($CV_{t\text{-}1}$) enters with the correct sign in the $\Delta(p_T - p)_t$ equation (see Table 3A). The $CV_{t\text{-}1}$ effect (i.e. -0.107) suggests a slow adjustment to disequilibrium deviations of the real exchange rate from its long-run relationship. Bearing in mind that the real exchange rate equation captures aspects of the real competitiveness of the domestic economy that depend on conditions prevailing in the production sector, this sluggishness could reflect rigidities in the functioning of the product market due to product heterogeneity, government imposed barriers to trade, or labour market inflexibility distorting the adjustment of wages. [12] The disequilibrium error also affects negatively the Δu_t equation (i.e. coefficient on CV_{t-1} equals –0.236 in Table 3C). This result implies that when the real exchange rate is above its equilibrium level, that is, undervalued, unemployment falls as firms respond to an improvement in domestic competitiveness by increasing their demand for labour. We also report a short-run negative effect of past real wage growth (i.e. $\Delta(w-p_T)_{t-1}$) in the Δu_t equation. The cointegrating vector has a weak positive effect in the $\Delta(w-p_T)_t$ equation (i.e. coefficient on CV_{t-1} equals 0.094 in Table 3B). Hence, there seems to be some weak evidence that when the real exchange rate is undervalued, workers respond to an improvement in domestic competitiveness by demanding and getting higher wages. The estimates in Table 3B also suggest a significant effect from past changes in unemployment (i.e. Δu_{t-1}, Δu_{t-2}, and Δu_{t-3}) in the $\Delta(w-p_T)_t$ equation but these effects seem to cancel each other out. [13]

The NLS estimates suggest that all three $\Delta(p_T-p)_t$, $\Delta(w-p_T)_t$ and Δu_t equations exhibit a regime-switching behaviour according to the variation of the disequilibrium error. Consider first the $\Delta(p_T-p)_t$ equation in Table 4. The estimate of the disequilibrium error in the second regime (i.e. coefficient α_2 is equal to –0.143 when $G(s_t;\gamma,c_1,c_2)=1$) is higher than that of the disequilibrium error in the first regime (i.e. coefficient α_1 is equal to –0.094 when $G(s_t;\gamma,c_1,c_2)=0$). This implies that when the real exchange rate exceeds an estimated interval band of $(c_1, c_2) = (–0.096, 0.078)$, the short-run real exchange rate adjusts faster.

From Table 6 one can see that short-run unemployment Δu_t reacts fast to the disequilibrium error (i.e. coefficient α_2 is equal to –0.291) only in the second regime (i.e. when $G(s_t;\gamma,c_1,c_2)=1$), that is, when the disequilibrium error exceeds an estimated interval band of $(c_1, c_2) = (–0.040, 0.052)$. In addition, the estimated interval band (c_1, c_2) for the short-run unemployment rate is much narrower compared to that for the short-run real exchange rate. Taking into account that our sample covers a period of floating exchange rates (with the UK

[12] See also the discussion in e.g. Johansen and Juselius (1992) and Pesaran and Shin (1996) in the context of a system involving the UK effective exchange rate, UK interest rate, UK prices, foreign prices and foreign interest rate.

[13] Among other studies, Manning (1993) using annual data over the 1956-1987 period, reports a negative effect from real wages on unemployment as well as a negative effect from unemployment on real wages. However, his model uses the level of u rather than Δu which is used here.

joining the Exchange Rate Mechanism only between 1990 and 1992), it is reasonable to expect that the short-run real exchange rate $\Delta(p_T - p)_t$ will adjust faster when the cointegrating vector CV_{t-1} is outside a rather wide interval band of thresholds. To the extent that the real exchange rate equation reflects monetary and more generally economic policy considerations, the significant effect of the cointegrating vector in the short-run unemployment equation implies that unemployment can be targeted by economic policy. Further, the lower estimates of c_1 and c_2 for the Δu_t equation suggest that if economic authorities want to avoid large swings in unemployment then they should be prepared to keep real exchange rate movements within a narrow interval band of thresholds. [14]

Our results in Table 5 suggest that short-run wages $\Delta(w - p_T)_t$ are affected by real exchange rate fluctuations within an estimated interval band of $(c_1, c_2) = (–0.070, 0.103)$ (i.e. coefficient α_1 is equal to 0.102 in the first regime $G(s_t;\gamma,c_1,c_2)=0$). Contrary to our results for the $\Delta(p_T - p)_t$ and Δu_t equations, we could not find any significant effect from the disequilibrium error on short-run wages outside the estimated band of thresholds. This is rather surprising, as we would expect the impact of the disequilibrium error on the short-run dynamics to be more evident when deviations of the real exchange rate from its long-run level exceed the estimated interval band. This finding probably has to do with the rather wide interval band estimates and in particular the c_2 estimate, which is practically equal to the maximum value of CV_{t-1} (i.e. 0.106).

The relationship between the occurrence of a regime and the disequilibrium error is depicted in Figure 3, which plots the values of the transition function against CV_{t-1} for the $\Delta(p_T - p)_t$, $\Delta(w - p_T)_t$ and Δu_t equations. As discussed above, $G(s_t;\gamma,c_1,c_2)=0$ and $G(s_t;\gamma,c_1,c_2)=1$ are related to small and large deviations respectively of the real exchange rate relative to a band of thresholds. In addition, this Figure helps clarify the discussion about the speed of transition between the two regimes. One can see that the transition from one regime to the other is rapid, as the estimates of γ are rather high for all models.

[14] That said, it is worth pointing out that policy makers in the UK were never able to control the exchange rate. Describing the main characteristics of macroeconomic policies in the UK, Andrew Britton, the former director of the National Institute of Economic and Social Research, comments: “Attempts to use the exchange rate as a policy instrument misfired; attempts to control it failed; attempts to ignore it were no more successful. The authorities never really got on top of the situation at all.” (Britton, 1991, p. 298).

Figure 4 plots the estimated transition function for each model against time in order to illustrate the succession of regimes over the sample period. From Figure 4A, the 1980-1982 and 1995-1997 periods are classified into the second regime of real exchange rate deviations outside the estimated interval bands. One can notice from Figure 2, that during the 1980-1982 period, the estimated transition functions pick a highly overvalued real exchange rate. Supported by the great appreciation of the dollar *vis a vis* the currencies of most of the industrialised countries during the first half of the 1980s (see e.g. Engel and Hamilton, 1990), the real exchange rate consequently reverts to its long-run equilibrium. During the 1995-1997 period, the estimated transition functions pick a highly undervalued real exchange rate, which then began reverting to its long-run level. Figure 4C shows that switches from one regime to the other are particularly active for the unemployment equation where a much narrower interval band was estimated. It is notable that the first period, which captures the 1980-1981 economic recession, follows the second OPEC oil price hike (an increase in oil prices of around 15 percent in June 1979) and coincides with important changes in economic policies following the election of the Thatcher government in May 1979. In particular, 1979 saw the abolition of exchange rate controls, which was not aimed at any particular effect on the exchange rate, as well as public spending cuts and an increase in indirect taxation. The new government encouraged the use of cheaper labour, especially female labour, which led to more part-time employment. At the same time, a very tight monetary policy aiming at a rapid decrease in the rate of inflation, led to a more overvalued real exchange rate (see Figure 2), a rapid increase in unemployment (see Figure 1) and a severe recession (see e.g. Britton, 1991; Mizon, 1995). Taking into account the slow adjustment of the real exchange rate reported in the previous section of the paper, it is not surprising that after the UK's exit from the ERM in September 1992, the real exchange rate experienced a path of persistent depreciation, which peaked between 1995 and 1997 (see Figure 2). Indeed, this is the non-linearity in the short-run real exchange rate captured by Figure 4A.

6. Conclusions

This paper examined non-linearities in a multivariate model of the UK real exchange rate. After estimating a long-run real exchange rate equation as part of a small system involving real product wages, the unemployment rate and the real price of oil, we found evidence that deviations of the real exchange rate equation from its long-run equilibrium level affect in a non-linear way not only the short-run real exchange rate equation, but also the short-run unemployment and wage equations. According to our estimates, the short-run real exchange rate adjusts faster when the cointegrating relationship is outside a wide interval band of thresholds. This is not surprising as our sample covers a period of floating exchange rates. On the other hand, the short-run unemployment rate adjusts fast when the cointegrating relationship is outside a narrower interval band. To the extent that the real exchange rate equation reflects economic policy considerations, our results suggest that policy makers should aim at a narrow band for the real exchange rate if they want to avoid large swings in unemployment.

Data appendix

p : GDP value added price deflator (1995 = 100). *Source*: *Economic Trends Annual Supplement* (ETAS).

$p_T = p_T^* + e$: the price of domestic tradables, where p_T^* is the \$ price index of UK imports (1990 = 100) and e is the average £/\$ exchange rate. *Source*: IMF, *International Financial Statistics*.

$w = w_{NET} + t_1$: average product wages in the manufacturing sector (1990=100), where w_{NET} refers to average weekly wages in manufacturing (net of employers' taxes) and t_1 is the tax rate on labour paid by employers, constructed as: (employers' contributions) / (total wage bill). *Source*: ETAS.

p_O : price index of materials and fuels purchased by manufacturers (1995 = 100). *Source*: ETAS.

u : UK unemployment rate. *Source*: ETAS. All variables are in logs.

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Table 1

Eigenvalues, test statistics and critical values

λ_i	λ-*max*				*trace*			
0.23	H_0	H_1	Stat.	95%	H_0	H_1	Stat.	95%
0.14	r = 0	r = 1	26.82	24.87	r = 0	r ≥ 1	44.65	39.56
0.02	r ≤ 1	r = 2	15.67	18.36	r ≤ 1	r ≥ 2	17.83	23.62
0.00	r ≤ 2	r = 3	2.16	11.42	r ≤ 2	r = 3	2.16	11.42

Notes: *r* denotes the number of cointegration vectors. Critical values are from Mackinnon *et al*., (1999).

Table 2

Linearity tests

Transition variable	Lagrange Multiplier *F* statistics for: $\Delta(p_T - p)$ model	$\Delta(w - p_T)$ model	Δu model	System wide test *LR*
CV_{t-1}	1.111	1.888	2.082	103.352
	(0.375)	(0.036)	(0.008)	(0.009)
CV_{t-2}	0.855	1.361	1.833	96.414
	(0.684)	(0.180)	(0.020)	(0.029)
CV_{t-3}	0.973	1.914	1.173	89.446
	(0.511)	(0.027)	(0.311)	(0.067)
CV_{t-4}	0.933	2.103	1.044	85.701
	(0.555)	(0.016)	(0.437)	(0.094)

Notes: Bootstrapped *p*-values in parentheses. The *p*-values for the equation specific Lagrange Multiplier *F* statistics and the system wide *LR* test statistic are derived from bootstrapping with one thousand replications. CV is the transition variable: $CV = p_T - p + 0.457\ (w - p_T) - 0.182\ (p_O - p_T)$, in mean-corrected form. The null hypothesis is linearity. The alternative hypothesis is the STVECM representation.

Table 3

Estimated linear models

Panel A: Linear $\Delta(p_T - p)_t$ model

$$\Delta(p_T - p)_t = \underset{(0.002)}{-0.001} \underset{(0.044)}{-0.107\, CV_{t\text{-}1}} \underset{(0.171)}{+0.277\, \Delta(p_T - p)_{t\text{-}1}} \underset{(0.158)}{-0.210\, \Delta(w - p_T)_{t\text{-}1}}$$
$$\underset{(0.103)}{-0.178\, \Delta(p_O - p_T)_t} \underset{(0.104)}{+0.401\, \Delta(p_O - p_T)_{t\text{-}1}} \underset{(0.106)}{-0.127\, \Delta(p_O - p_T)_{t\text{-}2}}$$

s_L = 0.023, AR(5) = 0.39[0.856], ARCH(4) = 0.11[0.980], HET = 1.99[0.035], NORM(2) = 21.7[0.000]

Panel B: Linear $\Delta(w - p_T)_t$ model

$$\Delta(w - p_T)_t = \underset{(0.002)}{0.008} \underset{(0.060)}{+0.094\, CV_{t\text{-}1}} \underset{(0.097)}{-0.415\, \Delta(p_T - p)_{t\text{-}1}} \underset{(0.092)}{+0.188\, \Delta u_{t\text{-}1}}$$
$$\underset{(0.120)}{-0.263\, \Delta u_{t\text{-}2}} \underset{(0.083)}{+0.154\, \Delta u_{t\text{-}3}} \underset{(0.113)}{+0.155\, \Delta(p_O - p_T)_t} \underset{(0.116)}{-0.396\, \Delta(p_O - p_T)_{t\text{-}1}}$$

s_L = 0.025, AR(5) = 0.28[0.923], ARCH(4) = 0.60[0.662], HET = 4.60[0.000], NORM(2) = 34.1[0.000]

Panel C: Linear Δu_t model

$$\Delta u_t = \underset{(0.002)}{0.004} \underset{(0.055)}{-0.236\, CV_{t\text{-}1}} \underset{(0.187)}{-0.350\, \Delta(p_T - p)_{t\text{-}1}} \underset{(0.170)}{-0.347\, \Delta(w - p_T)_{t\text{-}1}}$$
$$\underset{(0.063)}{+0.897\, \Delta u_{t\text{-}1}} \underset{(0.058)}{-0.203\, \Delta u_{t\text{-}3}} \underset{(0.107)}{-0.256\, \Delta(p_O - p_T)_t}$$

s_L = 0.024, AR(5) = 1.10[0.364], ARCH(4) = 1.62[0.177], HET = 2.29[0.014], NORM(2) = 7.99[0.018]

Notes: Standard errors are given in parentheses below the estimates. s_L: standard error of the linear regression. AR(5): F-test for up to 5th order serial correlation. ARCH(4): 4th order Autoregressive Conditional Heteroscedasticity F-test. HET: F-test for Heteroscedasticity. NORM(2): Chi-square test for normality. Numbers in square brackets are the p-values of the test statistics.

Table 4

Estimated non-linear $\Delta(p_T - p)_t$ model

The Table reports the NLS estimates of the following STVECM equation:

$$\begin{aligned}\Delta(p_T - p)_t &= (\mu_1 + \alpha_1 CV_{t-1} + \phi_{1,1}\Delta(p_T - p)_{t-1} + \phi_{2,1}\Delta(p_O - p_T)_{t-1})(1 - G(CV_{t-1};\gamma, c_1, c_2)) \\ &\quad + (\mu_2 + \alpha_2 CV_{t-1} + \phi_{3,1}\Delta(w - p_T)_{t-1} + \phi_{4,1}\Delta(p_O - p_T)_t + \phi_{4,2}\Delta(p_O - p_T)_{t-1} \\ &\quad + \phi_{4,3}\Delta(p_O - p_T)_{t-2})G(CV_{t-1};\gamma, c_1, c_2)\end{aligned}$$

where $G(CV_{t-1}; \gamma, c_1, c_2) = \{1 + \exp[-\gamma\, (CV_{t-1} - c_1)\, (CV_{t-1} - c_2)/\, \sigma^2(CV_{t-1})]\}^{-1}$,

is the 'quadratic logistic' transition function , with CV_{t-1} as the transition variable. Values of 0 and 1 of the transition function are associated with the two alternative regimes. The $\Delta(p_T - p)_t$ dynamics in the first regime, when $G(CV_{t-1}; \gamma, c_1, c_2) = 0$, are:

$$\Delta(p_T - p)_t = (\mu_1 + \alpha_1 CV_{t-1} + \phi_{1,1}\Delta(p_T - p)_{t-1} + \phi_{2,1}\Delta(p_O - p_T)_{t-1})(1 - G(CV_{t-1};\gamma, c_1, c_2)).$$

In the second regime, when $G(CV_{t-1}; \gamma, c_1, c_2) = 1$, its dynamics are:

$$\begin{aligned}\Delta(p_T - p)_t &= (\mu_2 + \alpha_2 CV_{t-1} + \phi_{3,1}\Delta(w - p_T)_{t-1} + \phi_{4,1}\Delta(p_O - p_T)_t + \phi_{4,2}\Delta(p_O - p_T)_{t-1} \\ &\quad + \phi_{4,3}\Delta(p_O - p_T)_{t-2})G(CV_{t-1};\gamma, c_1, c_2)\end{aligned}$$

For intermediate values of $G(CV_{t-1}; \gamma, c_1, c_2)$, i.e. $0 < G(CV_{t-1}; \gamma, c_1, c_2) < 1$, $\Delta(p_T - p)_t$ dynamics are a weighted average of the two equations. The speed of transition between the two regimes is determined by the parameter γ.

$$\begin{aligned}\Delta(p_T - p)_t = \; &(\underset{(0.002)}{-0.003} \;\; \underset{(0.059)}{-0.094}\, CV_{t-1} \;\; \underset{(0.082)}{+0.387}\, \Delta(p_T - p)_{t-1} \\ &\underset{(0.096)}{+0.247}\, \Delta(p_O - p_T)_{t-1})\, (1 - G(CV_{t-1}; \gamma, c_1, c_2)) \\ &(\underset{(0.007)}{+0.010} \;\; \underset{(0.092)}{-0.143}\, CV_{t-1} \;\; \underset{(0.362)}{-1.364}\, \Delta(w - p_T)_{t-1} \;\; \underset{(0.372)}{-0.422}\, \Delta(p_O - p_T)_t \\ &\underset{(0.419)}{+0.694}\, \Delta(p_O - p_T)_{t-1} \;\; \underset{(0.432)}{-0.434}\, \Delta(p_O - p_T)_{t-2})\, G(CV_{t-1}; \gamma, c_1, c_2)\end{aligned}$$

where

$$G(CV_{t-1}; \gamma, c_1, c_2) = \{1 + \exp[\underset{(3.923)}{-7.016}(CV_{t-1} \;\; \underset{(0.003)}{+0.096})\, (CV_{t-1} \;\; \underset{(0.006)}{-0.078})/\, \sigma^2(CV_{t-1})]\}^{-1}$$

$s_{NL} = 0.021$, $s^2_{NL}/s^2_L = 0.834$, AR(5) = 2.85[0.020], ARCH(4) = 0.23[0.917], HET = 0.29[0.999],

NORM(2) = 8.24[0.016]

Notes: Standard errors are given in parentheses below the estimates. s_{NL}: standard error of the non-linear regression. The diagnostic tests are discussed in the notes of Table 3.

Table 5

Estimated non-linear $\Delta(w-p_T)_t$ model

The Table reports the NLS estimates of the following STVECM equation:

$$\Delta(w-p_T)_t = (\mu_1+\alpha_1 CV_{t-1}+\phi_{1,1}\Delta(p_T-p)_{t-1}+\phi_{2,1}\Delta(p_O-p_T)_{t-1})(1-G(CV_{t-1};\gamma,c_1,c_2))$$
$$+(\mu_2+\phi_{3,1}\Delta(p_T-p)_{t-1}+\phi_{3,2}\Delta u_{t-2}+\phi_{3,3}\Delta u_{t-3}$$
$$+\phi_{4,1}\Delta(p_O-p_T)_{t-1})G(CV_{t-1};\gamma,c_1,c_2)$$

where $G(CV_{t-1};\gamma,c_1,c_2)=\{1+\exp[-\gamma\,(CV_{t-1}-c_1)\,(CV_{t-1}-c_2)/\,\sigma^2(CV_{t-1})]\}^{-1}$,

is the 'quadratic logistic' transition function , with CV_{t-1} as the transition variable. Values of 0 and 1 of the transition function are associated with the two alternative regimes. The $\Delta(w-p_T)_t$ dynamics in the first regime, when $G(CV_{t-1};\gamma,c_1,c_2)=0$, are:

$$\Delta(w-p_T)_t = (\mu_1+\alpha_1 CV_{t-1}+\phi_{1,1}\Delta(p_T-p)_{t-1}+\phi_{2,1}\Delta(p_O-p_T)_{t-1})(1-G(CV_{t-1};\gamma,c_1,c_2)).$$

In the second regime, when $G(CV_{t-1};\gamma,c_1,c_2)=1$, its dynamics are:

$$\Delta(w-p_T)_t = (\mu_2+\phi_{3,1}\Delta(p_T-p)_{t-1}+\phi_{3,2}\Delta u_{t-2}+\phi_{3,3}\Delta u_{t-3}$$
$$+\phi_{4,1}\Delta(p_O-p_T)_{t-1})G(CV_{t-1};\gamma,c_1,c_2).$$

For intermediate values of $G(CV_{t-1};\gamma,c_1,c_2)$, i.e. $0<G(CV_{t-1};\gamma,c_1,c_2)<1$, $\Delta(w-p_T)_t$ dynamics are a weighted average of the two equations. The speed of transition between the two regimes is determined by the parameter γ.

$$\Delta(w-p_T)_t = \underset{(0.002)}{(0.008} \quad \underset{(0.062)}{+0.102}\,CV_{t-1} \quad \underset{(0.083)}{-0.343}\,\Delta(p_T-p)_{t-1}$$

$$\underset{(0.098)}{-0.331}\,\Delta(p_O-p_T)_{t-1})\;(1-G(CV_{t-1};\gamma,c_1,c_2))$$

$$\underset{(0.011)}{(+0.015} \quad \underset{(0.306)}{-1.600}\,\Delta(p_T-p)_{t-1} \quad \underset{(0.248)}{-0.933}\,\Delta u_{t-2}$$

$$\underset{(0.200)}{+1.040}\,\Delta u_{t-3} \quad \underset{(0.298)}{-0.656}\,\Delta(p_O-p_T)_{t-1})\;G(CV_{t-1};\gamma,c_1,c_2)$$

where

$$G(CV_{t-1};\gamma,c_1,c_2)=\{1+\exp[\underset{(1.657)}{-2.829}(CV_{t-1}\;\underset{(0.007)}{+0.070})\,(CV_{t-1}\;\underset{(0.014)}{-0.103})/\,\sigma^2(CV_{t-1})]\}^{-1}$$

$s_{NL}=0.021$, $s^2_{NL}/s^2_L=0.705$, AR(5) = 0.84[0.525], ARCH(4) = 0.64[0.635], HET = 0.71[0.813],

NORM(2) = 13.57[0.001]

Notes: Standard errors are given in parentheses below the estimates. s_{NL}: standard error of the non-linear regression. The diagnostic tests are discussed in the notes of Table 3.

Table 6

Estimated non-linear Δu_t model

The Table reports the NLS estimates of the following STVECM equation:

$$\Delta u_t = (\mu_1 + \phi_{1,1}\Delta(p_T - p)_{t-1} + \phi_{1,2}\Delta(w - p_T)_{t-1} + \phi_{1,3}\Delta u_{t-1} + \phi_{2,1}\Delta(p_O - p_T)_{t-1})(1 - G(CV_{t-1};\gamma,c_1,c_2)) + (\mu_2 + \alpha_2 CV_{t-1} + \phi_{3,1}\Delta u_{t-1} + \phi_{3,2}\Delta u_{t-3} + \phi_{4,1}\Delta(p_O - p_T)_t)G(CV_{t-1};\gamma,c_1,c_2)$$

where $G(CV_{t-1}; \gamma, c_1, c_2) = \{1 + \exp[-\gamma\,(CV_{t-1} - c_1)\,(CV_{t-1} - c_2)/\,\sigma^2(CV_{t-1})]\}^{-1}$,

is the 'quadratic logistic' transition function , with CV_{t-1} as the transition variable. Values of 0 and 1 of the transition function are associated with the two alternative regimes. The Δu_t dynamics in the first regime, when $G(CV_{t-1}; \gamma, c_1, c_2) = 0$, are:

$$\Delta u_t = (\mu_1 + \phi_{1,1}\Delta(p_T - p)_{t-1} + \phi_{1,2}\Delta(w - p_T)_{t-1} + \phi_{1,3}\Delta u_{t-1} + \phi_{2,1}\Delta(p_O - p_T)_{t-1})(1 - G(CV_{t-1};\gamma,c_1,c_2))$$

In the second regime, when $G(CV_{t-1}; \gamma, c_1, c_2) = 1$, its dynamics are:

$$\Delta u_t = (\mu_2 + \alpha_2 CV_{t-1} + \phi_{3,1}\Delta u_{t-1} + \phi_{3,2}\Delta u_{t-3} + \phi_{4,1}\Delta(p_O - p_T)_t)G(CV_{t-1};\gamma,c_1,c_2)\,.$$

For intermediate values of $G(CV_{t-1}; \gamma, c_1, c_2)$, i.e. $0 < G(CV_{t-1}; \gamma, c_1, c_2) < 1$, Δu_t dynamics are a weighted average of the two equations. The speed of transition between the two regimes is determined by the parameter γ.

$$\Delta u_t = \underset{(0.003)}{(0.001} \quad \underset{(0.268)}{-0.394}\,\Delta(p_T - p)_{t-1} \quad \underset{(0.246)}{-0.443}\,\Delta(w - p_T)_{t-1} \quad \underset{(0.071)}{+0.814}\,\Delta u_{t-1}$$

$$\underset{(0.127)}{-0.174}\,\Delta(p_O - p_T)_{t-1})\,(1 - G(CV_{t-1}; \gamma, c_1, c_2))$$

$$\underset{(0.004)}{(+0.009} \quad \underset{(0.063)}{-0.291}\,CV_{t-1} \quad \underset{(0.087)}{+0.991}\,\Delta u_{t-1} \quad \underset{(0.089)}{-0.473}\,\Delta u_{t-3}$$

$$\underset{(0.169)}{-0.384}\,\Delta(p_O - p_T)_t)\,G(CV_{t-1}; \gamma, c_1, c_2)$$

where

$$G(CV_{t-1}; \gamma, c_1, c_2) = \{1 + \exp[\underset{(8.196)}{-11.001}(CV_{t-1} \quad \underset{(0.003)}{+0.040})\,(CV_{t-1} \quad \underset{(0.008)}{-0.052})/\,\sigma^2(CV_{t-1})]\}^{-1}$$

$s_{NL} = 0.022$, $s^2_{NL}/s^2_L = 0.840$, AR(5) = 1.43[0.223], ARCH(4) = 3.26[0.020], HET = 1.30[0.200],

NORM(2) = 12.78[0.002]

Notes: Standard errors are given in parentheses below the estimates. s_{NL}: standard error of the non-linear regression. The diagnostic tests are discussed in the notes of Table 3.

Figure 1: Plots of the levels and the first differences of the series

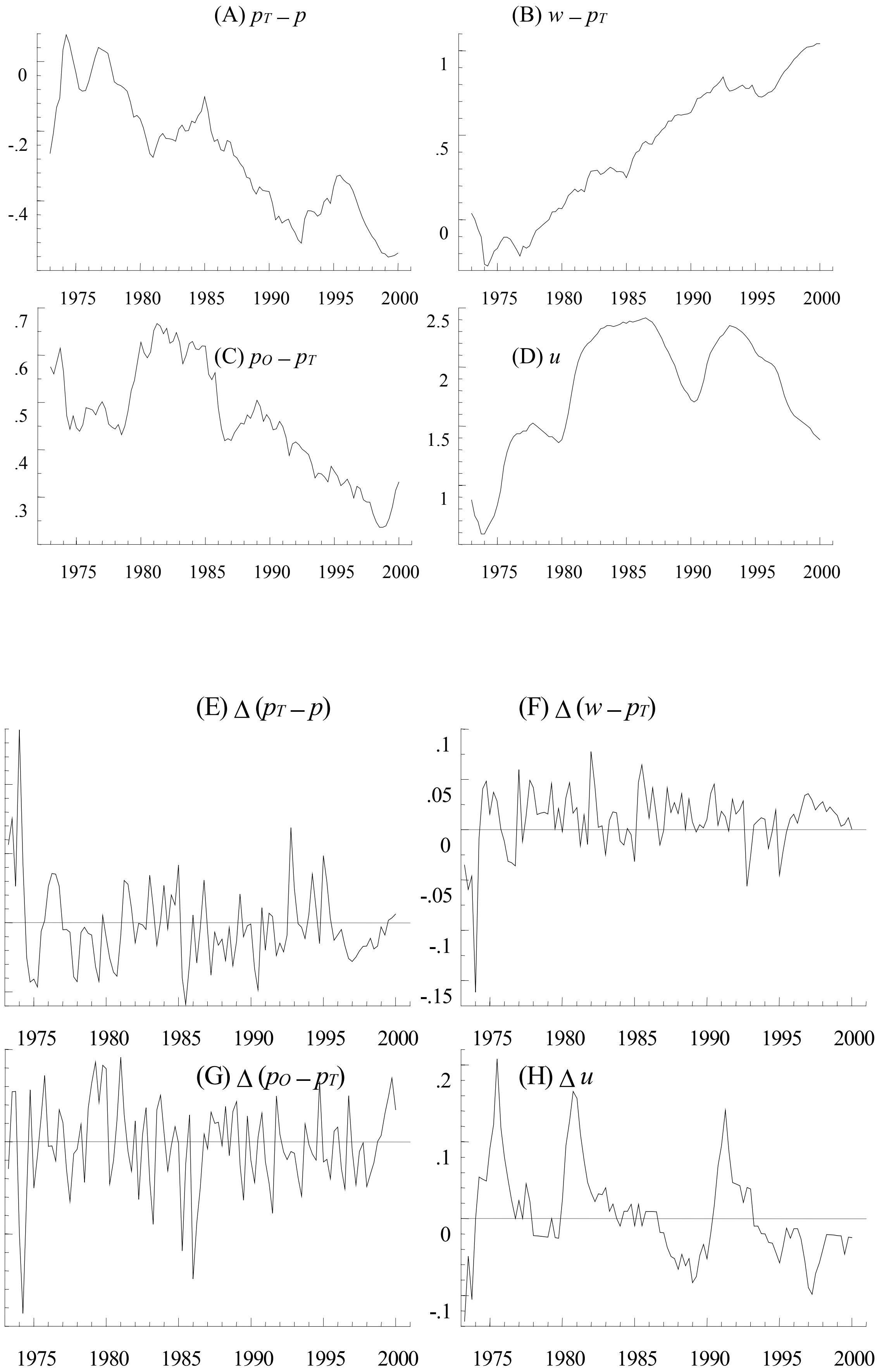


Figure 2: Deviations from the estimated long-run real exchange rate relationship

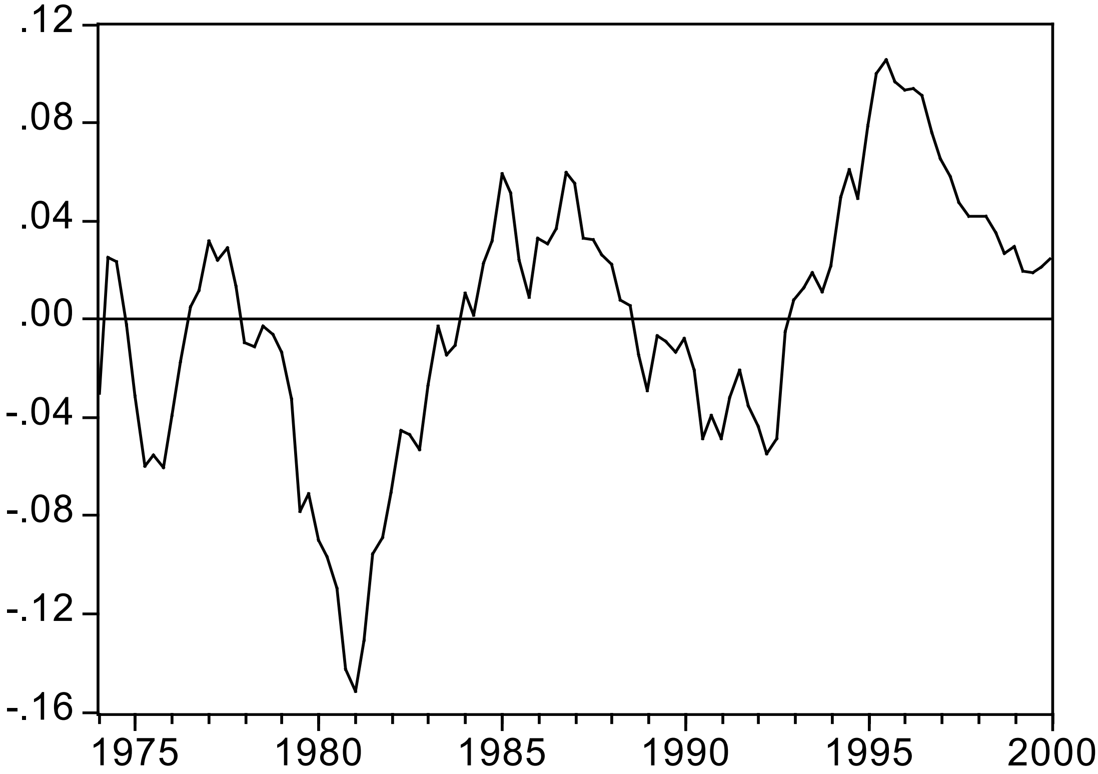
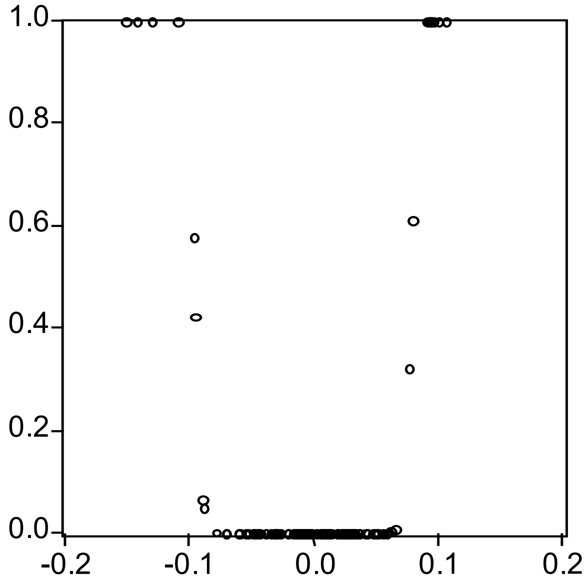
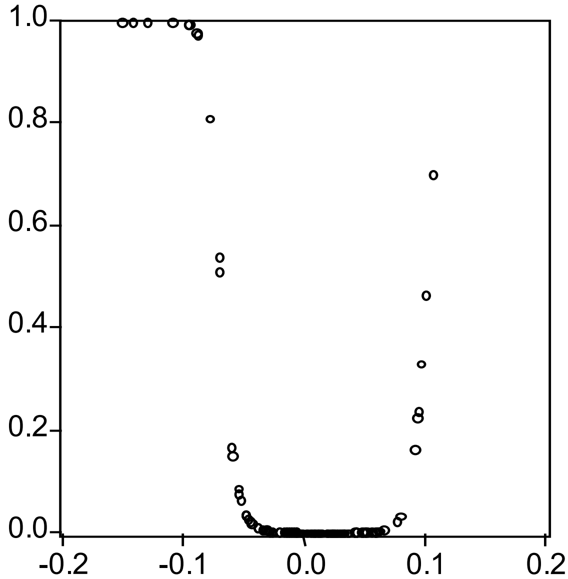


Figure 3: Estimated transition functions (vertical axis) against $CV_{t\text{-}1}$ (horizontal axis)

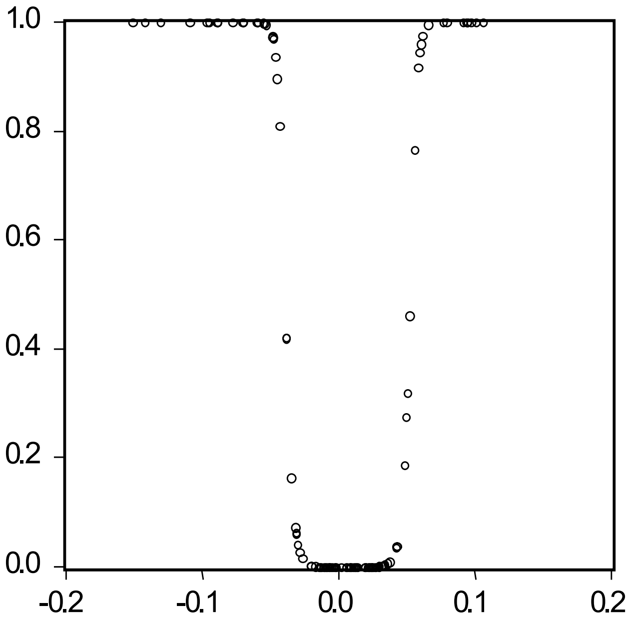
(A) $\Delta(p_T - p)_t$ model



(B) $\Delta(w - p_T)_t$ model



(C) Δu_t model

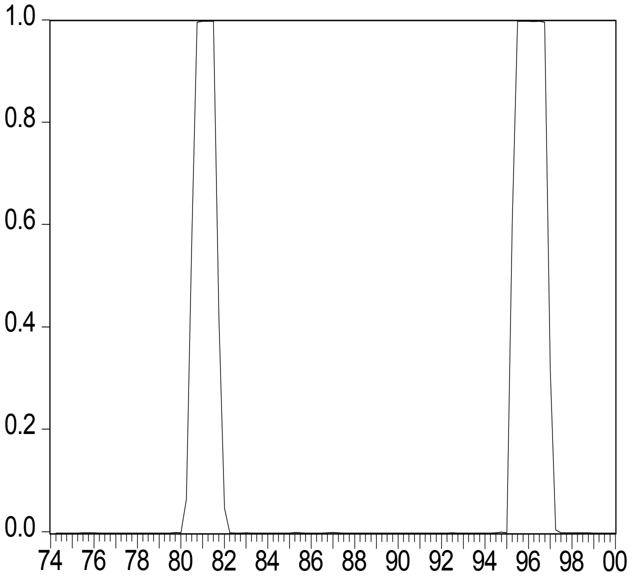


Notes: Panels (A), (B) and (C) report the estimated transition functions for $\Delta(p_T - p)_t$, $\Delta(w - p_T)_t$ and Δu_t against the transition variable $CV_{t\text{-}1}$ from the corresponding STVECM equations as reported in Tables 4 to 6. The estimated transition functions are:

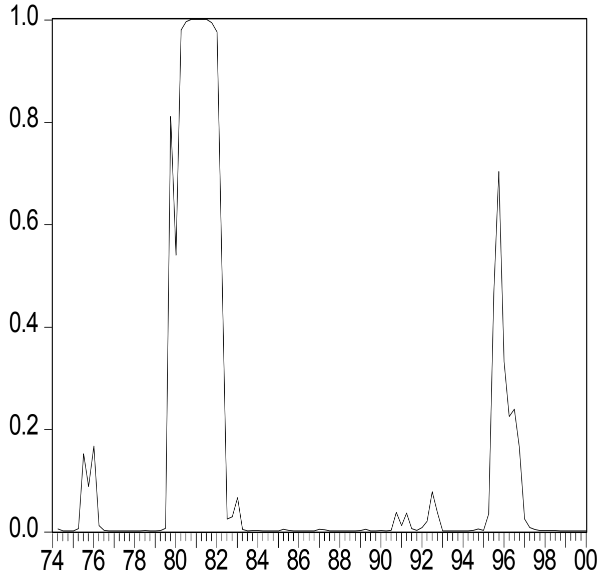
(A) $G(CV_{t\text{-}1}; \gamma, c_1, c_2) = \{1 + \exp[-7.016(CV_{t\text{-}1} + 0.096)\ (CV_{t\text{-}1} - 0.078)/\ \sigma^2(CV_{t\text{-}1})]\}^{-1}$
(B) $G(CV_{t\text{-}1}; \gamma, c_1, c_2) = \{1 + \exp[-2.829(CV_{t\text{-}1} + 0.070)\ (CV_{t\text{-}1} - 0.103)/\ \sigma^2(CV_{t\text{-}1})]\}^{-1}$
(C) $G(CV_{t\text{-}1}; \gamma, c_1, c_2) = \{1 + \exp[-11.001(CV_{t\text{-}1} + 0.040)\ (CV_{t\text{-}1} - 0.052)/\ \sigma^2(CV_{t\text{-}1})]\}^{-1}$

Figure 4: Estimated transition functions against time

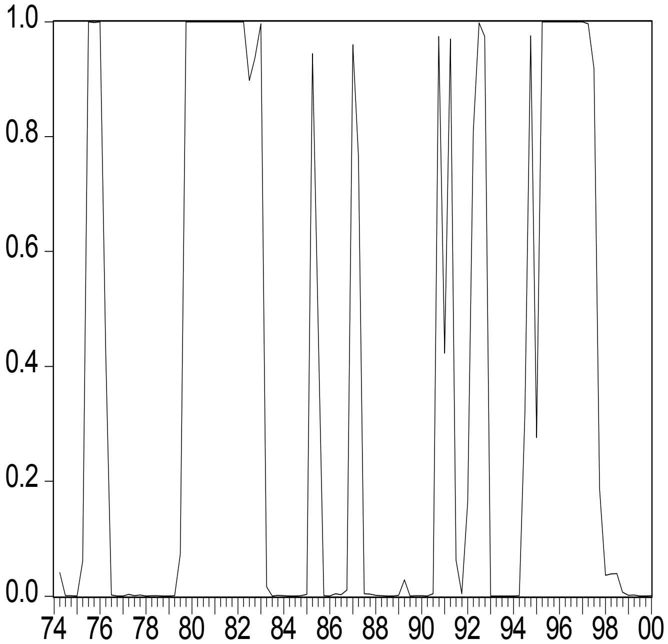
(A) $\Delta(p_T - p)_t$ model



(B) $\Delta(w - p_T)_t$ model



(C) Δu_t model



Notes: Panels (A), (B) and (C) report the estimated transition functions for $\Delta(p_T - p)_t$, $\Delta(w - p_T)_t$ and Δu_t against time from the corresponding STVECM equations as reported in Tables 4 to 6 and Figure 3. Extreme values of 0 and 1 of the transition functions are associated with the two alternative regimes.