Fixed Income Portfolio Construction:

A Bayesian Approach for the Allocation of Risk Factors

by

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To my wife and son

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Declaration

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Abstract

Active portfolio management is driven by the trade-off between the expected return and the associated risks. In light of the most recent extensions of Black-Litterman model, we stick to a Bayesian approach for the construction of active fixed income portfolios. Within the investment grade universe, the equilibrium returns are approximated by the yield levels implied by the market prices and these are blended together with investment views. In parallel, risk factors are preferred over asset class risk modelling. Affinity towards risk factors rather than asset classes is primarily linked with two elements; the reduction of the dimensionality of the risk estimation problem and the intuitive way in which portfolio exposures per risk factor can be expressed as performance drivers. The first empirical part of the thesis deals with the optimisation of a relative to an index portfolio where the centre of gravity is the chosen benchmark. The first ingredient of the optimisation is the blend of the yield advantage over the index and the expectations for excess returns over the index emanating from the investment views. The second ingredient is the risk estimated by a multifactor risk model. Then, a set of relative to the index investment grade portfolios is constructed. The second empirical part investigates whether there is scope to blend the multifactor risk framework with more sophisticated risk estimation techniques such as resampling. Tail risk estimated by block bootstrapping on the risk exposures of real actively managed portfolio exposures vs. the Barclays Capital US Aggregate index is compared with the parametric and exponentially weighted moving average risk model findings. The multifactor risk estimate using block bootstrapping exhibits better performance than the alternatives tested but struggles to capture the out of sample extremes. Finally, the third empirical part aims to enhance the allocation model by taking advantage of the findings of the second empirical part. The blending mechanism of equilibrium returns and investment views, which are expressed as optimisation constraints, is performed with the aid of a numerically approximated returns' distribution. The resampled distribution deviates from the normality assumption imposed initially in the Black-Litterman model and forms a more realistic basis for the evaluation of investment views and for the portfolio construction against tail risk measures such as value at risk and conditional value at risk.

1 Introduction

1.1 Motivation and Objectives

The aim of this PhD thesis is to bring to surface problems in fixed income asset allocation which cannot be addressed via the existing toolset designed for different asset classes, and propose feasible solutions for the identification of both the equilibrium returns and the associated risks. Markowitz (1952) and his mean-variance approach revolutionized the portfolio management world and set the stage for further elaboration on the main themes of asset allocation: evaluation of the expected returns, evaluation of the risk and combination of the two into a consolidated portfolio. Failure to successfully respond to the asset pricing question gave birth to the Black-Litterman model (1992) which extracts the market equilibrium returns from the CAPM framework and allows the investment manager to incorporate his own views on the allocation procedure. Tempting as several of its features may be, the Black-Litterman model was abandoned by most practitioners because of disliking the notion of "CAPM Equilibrium". Since then a decent amount of research has been conducted to enhance the original model.

This thesis aims to fill in the gap in the literature of the lack of focus on fixed income markets, the dynamics of which differ from the other asset classes. The bond market exhibits very different characteristics both in terms of return profile and risk profile. Would it be possible for an allocation tool such as the Black-Litterman model to be used in fixed income markets? To what extent are the model assumptions realistic when it comes to bond portfolios risk modelling? Could we improve the existing framework to better accommodate the tail behaviour and dependence structure of financial data? And finally, what lesson could be drawn out of the bond market regarding the estimation of equilibrium returns of the other asset classes?

1.2 Outline of the Thesis

As mentioned, the purpose of this PhD thesis is to assist in the portfolio construction process specific to fixed income. Each of the Chapters 3, 4 and 5 in this PhD thesis can be regarded as a standalone research paper dealing with a set of research questions which are though part of the broader asset allocation problem.

Chapter 2 concentrates on the main concepts that this PhD thesis touches upon in relation to fixed income portfolio construction decisions. Namely, how the starting point of the portfolio allocation can be formed and what is a proper representation of the market portfolio. Given the amount of investable assets, portfolio construction vacillates between more passive or more active investment strategies. A way of implementing and measuring the efficiency of outstanding portfolios is by the use of a benchmark index. Additionally, a critical point from a modelling and investment perspective is the identification of the expected returns. Their demystification entails an enormous amount of resources employed into the financial system and determines the success of institutions and individuals alike. That is the reason why the way of measuring and monitoring the associated risks is further elaborated and has a major role in the allocation process. All in all Chapter 2 focuses on the role of the utility function which may alter the mix and the analogy of the assets within a formulated portfolio.

Specifically in Chapter 3 we propose a re-evaluation of the Black-Litterman model both in terms of equilibrium returns and in terms of risk estimation. More specifically, we focus on the fixed income investment grade universe. The choice for this investment universe enables to approximate the equilibrium returns by the bonds' yield to maturity which is juxtaposed against the equilibrium returns, implied by the Black-Litterman model and the observed market capitalisations. The second step is to challenge the (equities compatible) risk model that Black-Litterman originally employs. Using time series data for specific bonds would not make much sense as the yield curve dynamics change across different tenors and the bonds' tenors change as the time to maturity decays. Instead, we propose using a multifactor risk model which is more representative of the structure of fixed income markets.

The multifactor model is not only useful in the estimation of the aggregated risk and risk exposures but also serves as a tool to express investment views. The investment manager may express his views in terms of expected changes in yield and spread levels. Then this is converted through a risk factor loadings matrix into estimated asset classes' returns. The final step is to start forming portfolios. The initial allocation is the benchmark index and then the relative portfolio is optimized for several risk levels.

In Chapter 4 several variations of multifactor risk models are tested for the monitoring and risk estimation of actively managed portfolios against their index. In detail, a set of broad risk factor exposures is used for twelve real actively managed portfolios vs. the Barclays Capital US Aggregate Index. The Value at Risk estimates are back tested and compared using a Variance covariance matrix, exponentially weighted moving average, ex post tracking error and resampling via block bootstrapping. Given the risk factors available there is scope for improving the risk model by relaxing the normality assumption implied by the parametric Value at risk framework.

In Chapter 5 the evidence in the literature for the existence of excess kurtosis and skewness in financial data is taken into account. In light of this we test how the Black-Litterman assumption can be relaxed in practice by introducing resampling techniques for the estimation of the portfolio risk profile. It is illustrated how investment views can be blended together with the equilibrium returns, which are extracted directly from the bond valuations to assess the expected performance of assets relative to the chosen benchmark. It also enables the financial analyst to evaluate the effect of his views on the major segments of the market and how this would potentially affect the portfolio's performance and risk profile. Once the investment views are established, and the posterior distributions finalized, the portfolio construction process can kick off either it terms of return to volatility base or in terms of return to tail risk base.

In this thesis we seek to cast light on the three elements which drive the portfolio construction process, the expected returns, the risk and the dependence structure between portfolio components.

1.3 Summary of findings and contribution to the literature

This PhD thesis contributed to several segments of the literature. Initially, it elaborates on how the blending of equilibrium results and investment views can be formulated to assist in the construction of investment grade fixed income portfolios. Second, it evaluates the accuracy of a series of risk models for the estimation of portfolios' risk profile. Third, it takes into account the asymmetric and fat tailed distribution of financial data to improve the allocation process while using a multifactor model and allowing for investment views to be taken into account.

One major issue the Black-Litterman model tried to address is the making sense of the final allocation vs. the market portfolio. Provisional on our ability to specify the market portfolio in its full breadth and on the accuracy of the underlying asset pricing model, an allocation tool such as the Black-Litterman model would provide accurate estimates of the equilibrium returns and also a reasonable final allocation. In practice, it is difficult both to capture the entire market portfolio and to pick up an asset pricing model which would generate the true equilibrium returns. In reality, the starting point has been the market portfolio or a subset of it in guise of a chosen benchmark index, which reflects the strategy to which the investor is willing to be tied. For that purpose, Black-Litterman uses the initial benchmark portfolio as the market portfolio and from there the equilibrium returns can be modelled. In fact, this would, by definition, lead to the benchmark allocation but would not necessarily correspond to the true equilibrium returns. As such, equilibrium returns do not strictly represent the actual consensus of the market on future returns but rather have instrumental value in that they lead to the allocation of the selected portfolio.

The first contribution of this PhD thesis is that the CAPM equilibrium returns are challenged for fixed income portfolios. The occurring yield to maturity is an accurate representation of the expected return on each bond provided a credit event does not happen. The yields implied by market valuations are not compatible with the CAPM equilibrium returns. Additionally, a multifactor risk model is used to capture the risk dynamics of fixed income portfolios. In that sense, the epicentre of the risk analysis is moved from asset classes to risk factors, where the overall risk of the portfolio, becomes a function of the overall risk factor loadings, the riskiness of each risk factor and the way the risk factors are correlated. In order to implement the Black-Litterman model in the investment grade universe, the market portfolio is set to be the (any) chosen benchmark. This constitutes the starting point for the allocation of the portfolio. Then the actual portfolio is optimized on a relative to the index basis. The main drivers of the optimization are the yield advantage over the yield of the benchmark, the investment views and the associated risk. If no excess risk is undertaken the portfolio will bear no difference to the benchmark. Depending on the risk budget, the allocation is shifted towards the higher yielding assets, and those with the highest investment conviction.

Furthermore, a goal of this thesis is to test the validity of risk factor risk modelling and the scope for deviating away from the normality assumption as part of the portfolio construction process. For that purpose, several alternatives are attempted to judge the tail behaviour of twelve real portfolios, actively managed against the Barclays Capital US Aggregate Index. The data are sourced from a leading investment management institution. The findings are supportive of the idea to deviate away from Black-Litterman's normality assumption and incorporate resampling techniques to better capture the risk dynamics of those portfolios.

In light of the above findings, and given the evidence provided in the literature that financial data exhibit skewness and excess kurtosis, risk factor modelling is employed using simulated marginal distributions per risk factor. The simulated risk profile of the portfolio is then tweaked after the inclusion of investment views and that enables restarting the allocation process not only against standard deviation of relative to the index returns, but tail risk measures as well, such as the value at risk and the expected shortfall. The process per se of blending the views into the resampled distributions is also insightful for scenario analysis and stress testing.

2 Background Material: Portfolio Theory and Portfolio Risk Management

2.1 Introduction

In this chapter we introduce some of the main concepts of portfolio theory and portfolio risk management. In section 2.2 we describe the barbell relationship between active and passive portfolio management depending on the level of trading activity as a reflection of active implementation of investment outlook. In sections 2.3, 2.4 and 2.5 we focus on the indexation and how this would determine the key allocation decisions of actively managed portfolios. Section 2.6 elaborates on the main risk sources, driving the uncertainty in a fixed income portfolio. Finally, section 2.7 highlights in brief the role of benchmarking in the portfolio construction process, the quantification of fixed income risk via specific risk measures, their aggregation on portfolio level and their interdependence.

2.2 Active vs. Passive Portfolio Management

Two options are available for a fixed income investor; to invest in passive or active portfolio management. Passive portfolio management relies on investing in an index, with specified risk characteristics whose particular weights should be rebalanced regularly so that they remain relatively unchanged over time. The advantage of such a decision is that the investor can allocate money to a diversified portfolio in order to capture a particular opportunity identified in the market. The main drawback is that the passive manager as implied by the word “passive” is not actively trading the portfolio but rather maintaining it, even if the market conditions are changing.

On the other hand side, active portfolio management offers the manager the flexibility to continuously and actively trade the portfolio in order to add some extra value. Needless to say that active portfolio management requires extra skill from the portfolio manager in order to generate performance. The manager is not only responsible for the strategic allocation i.e. investing in Investment Grade Credit on the belief that credit spreads will tighten, or in a short duration portfolio expecting that central banks will ease monetary policy and yield curves will become steeper but for tactical moves as well. Tactical moves include all opportunistic trades, intended to benefit from a particular dislocation of the market, with the strategic goal of the portfolio being unaffected i.e. sector allocation within credit portfolios on the back of expectations that financials will outperform utilities.

2.3 Benchmarking: the Relative to Index Portfolio

It is not hard to imagine that active portfolio management is more demanding than passive because more resources are required to continuously monitor the portfolio and the market. In addition, higher discretion is required as the role of the manager is active. That is the reason why clients transitioning to active management pay higher fees vs. the fees charged by passive management. But what matters at the end of the day is whether active management can deliver superior performance to passive investment management therefore justifying higher fees.

Bond portfolios are benchmarked against their relative index portfolios. For example an Investment Grade Credit portfolio may be benchmarked against the Barclays Capital or the Merrill Lynch Global Investment Grade Index. These indices are largely diversified and contain a big number of securities. Hence, the portfolio should be designed to outperform its benchmark in a way that it is both adequately diversified and keeps transaction costs to the lowest possible level. The relative performance of an actively managed portfolio vs. a benchmark portfolio is known as alpha. And the portfolio defined as the difference between the managed portfolio and its benchmark will be referred to from now onwards as the relative portfolio.

Portfolio weights:

$$\sum_{i=1}^{n} w_{iPTF} = 1 \qquad \textbf{2.1}$$

Benchmark portfolio weights:

$$\sum_{i=1}^{n} w_{iBMK} = 1 \qquad \textbf{2.2}$$

Relative Portfolio weights:

$$\sum_{i=1}^{n} (w_{iPTF} - w_{iBMK}) = 0 \qquad \textbf{2.3}$$

As the portfolio and the benchmark are fully invested the sum of weights is the unity. In terms of relative positioning if the portfolio bears positive weight on a particular security this means that the portfolio's excess investment to this security against the benchmark is equal to the weight. On aggregate level, the relative portfolio weights are zero implying that in order to finance an over-weighted position relative to benchmark there should be at least one underweighting position. These deviations from the benchmark are what can generate alpha. Performance wise the above relations are becoming:

Portfolio return:

$$r_{PTF} = \sum_{i=1}^{n} w_{iPTF} r_i \qquad \textbf{2.4}$$

Benchmark portfolio return:

$$r_{BMK} = \sum_{i=1}^{n} w_{iBMK} r_i \qquad \textbf{2.5}$$

Relative Portfolio return:

$$alpha = r_{PTF} - r_{BMK} = \sum_{i=1}^{n} (w_{iPTF} - w_{iBMK})(r_i - r_{BMK}) \qquad \textbf{2.6}$$

Where:

r_{PTF} is the return of the portfolio

w_{iPTF} is the weight of security i in the portfolio

r_{BMK} is the return of the benchmark

w_{iBMK} is the weight of security i in the portfolio

$alpha$ is the alpha return

Proof:

$$\sum_{i=1}^{n}(w_{iPTF} - w_{iBMK})(r_i - r_{BMK}) = \\ = \sum_{i=1}^{n} w_{iptf} r_i - \sum_{i=1}^{n} w_{iptf} r_{BMK} - \sum_{i=1}^{n} w_{iBMK} r_r + \sum_{i=1}^{n} w_{iBMK} r_{BMK} \quad \textbf{2.7}$$

But $\sum_{i=1}^{n} w_{iBMK} r_{BMK} = r_{BMK} \sum_{i=1}^{n} w_{iBMK} = r_{BMK}$, because the benchmark is a fully invested portfolio i.e. $\sum_{i=1}^{n} w_{iBMK} = 1$, therefore:

$$= r_{PTF} - r_{BMK} - r_{BMK} + r_{BMK} \\ = r_{PTF} - r_{BMK} \\ = Alpha \quad \textbf{2.8}$$

This is the main building block to decompose performance on a security level. The intuition behind this is that the relative to the benchmark portfolio performance is attributed to each security. Each security held within the portfolio contributes positively or negatively to the overall portfolio performance. This depends on the performance of the security itself relative to the performance of the entire index and also on the relative weighting of the security within the portfolio against the index. For instance, if the security is over-weighted by 1% in the portfolio versus the benchmark and it has outperformed the benchmark by 100bps, the overall contribution of this particular i[th] security to the alpha is:

$$\begin{aligned} &Contribution-to-alpha = (w_{iPTF} - w_{iBMK})(r_i - r_{BMK}) \\ &= 100bps * 1\% \\ &= 1bps \end{aligned} \qquad \mathbf{2.9}$$

By summing up all n securities held within the portfolio and the benchmark portfolio (i.e. all the over weights and all the under weights), we should have a clear view on how performance is attributed per each line item held within the portfolio. It is noteworthy that for all securities that are off-benchmark investments, the overweight is equal to the weight of the security in the portfolio. Equivalently, the underweight for all the securities that the portfolio is not invested in but are held in the benchmark is equal to the weight of the position in the benchmark. As explained later on it is meaningful that the benchmark is not replicated on a security by security basis by the portfolio manager for various reasons. Due to this fact it is important to understand the two cases of securities which are not being part of the portfolio however are included in the benchmark and also the off-benchmark positions.

Equivalently the volatility of the relative portfolio, also known as Tracking Error is measured as follows:

$$\sigma_{REL}^2 = (w_{PTF} - w_{BMK})\Sigma(w_{PTF} - w_{BMK})^T \qquad \mathbf{2.10}$$

Where:

σ_{REL}^2	is the relative portfolio variance
$w_{PTF} - w_{BMK}$	is the vector of relative to the index weights
Σ	is the variance covariance matrix

Here the tracking error or the relative to the benchmark portfolio risk is an ex post measure as it is calculated based on relative weights and on realised vol.
At one side of spectrum there are passive portfolios that are targeted to match the benchmark risk profile as closely as possible. At the other end there are very active portfolios that can largely deviate from benchmark and use it only as a nonbinding reference point. In most cases the portfolios managed fall somewhere in the area between these two extremes. A portfolio identical to the benchmark bears no risk from a portfolio manager's point of view. The goodness of the portfolio manager is evaluated on the dual basis of assessing both performance and risk. The measure designed to distinguish the lucky manager from the skilful one is very similar to the Sharpe Ratio and is called Information Ratio. Information Ratio measures the alpha delivered per unit of risk undertaken relative to the benchmark.

$$Information\_Ratio = \frac{alpha}{TrackingError} \qquad \textbf{2.11}$$

2.4 Choosing the right Benchmark

Unless there is a bias in a given sector leading to significant portfolio deviation from the benchmark, portfolio performance should be away from benchmark performance. That is the reason why the choice of the right benchmark is so vital for managing a portfolio. There is a very wide variety of benchmarks available in the market place. To make sure that the index is reflective of the investment opportunity and the discretion provided to the manager by the client, an existing index may need to be tailored. In some cases there are some very highly customised indices that are constructed. The choice of a good benchmark is crucial because it establishes the risk and return profile for managing the portfolio.

Typically, a benchmark should tie out to the following criteria:

- Unambiguous and transparent- securities held in the index should be clearly defined.
- Investable- the securities that a benchmark is invested in should be tradable so that the index can be potentially replicated by an investor.
- Priced daily- to allow for daily monitoring.
- Availability of historical data- to allow for statistical analysis on returns' history.
- Low turnover- benchmarking against an index that is constantly changing would prove both difficult and costly.
- Specified in advance- the benchmark should be set up before a portfolio is invested.
- Published risk characteristics- Risk metrics of the benchmark should be available allowing the portfolio manager to actively manage the portfolio on a relative to index basis.
- Reflect liabilities- Match closely the liabilities that should be met during the life of the portfolio if the investment is in a liability driven space.
- Constraints imposed on the portfolio reducing its opportunity set should also be reflected in the benchmark.

2.5 Benchmark Replication

Many indices hold hundreds or even thousands of securities making it tough for the investment manager to replicate the benchmark. It would not be feasible to match the benchmark on security by security basis as many securities might be highly illiquid or even non available. In the treasury market the security by security match might be feasible but is not necessarily desirable. However, full replication would be reasonably hard to implement in agency, mortgage, or corporate bond markets. The existence of transaction costs makes it extremely costly to attempt such a type of benchmark replication even if it is theoretically feasible. These facts are pushing portfolio management towards a new approach; the extensive use of quantitative methods. Passive portfolio managers, or "indexers," seek to replicate the returns of a broad market index.

They use risk models to help keep the portfolio closely aligned to the index across all risk dimensions. Active portfolio managers attempt to outperform the benchmark by positioning the portfolio to capitalize on market views.
For the economy of this PhD thesis the investment universe will be considered to be the Barclays Capital US Aggregate Index. The potential benchmarks which can be used here are the Barclays Capital US Aggregate Index or a subset of it. The reason is that the benchmark cannot be broader than the investment universe used to form the portfolio.

2.6 Multifactor Risk Models

In the plethora of data available in today's marketplace, an investment manager might be tempted to build a risk model directly from the historical return characteristics of individual securities. The standard deviation of a security's return in the period to come can be projected to match its past volatility; the correlation between any two securities can be determined from their historical performance. Despite the simplicity of this scheme, the multifactor approach bears important advantages.

First of all, the number of risk factors in the model is much smaller than the number of securities in a typical investment universe. This greatly reduces the matrix operations needed to calculate portfolio risk. This increases the speed of computation (which is becoming less important with gains in processing power) and, more importantly, improves the numerical stability of the calculations. A large covariance matrix of individual security volatilities and correlations is likely to cause numerical instability. This is especially true in the fixed income world, where returns of many securities are very highly correlated. Risk factors may also exhibit moderately high correlations with each other, but much less so than for individual securities. Some practitioners insist on a set of risk factors that are uncorrelated to each other. Dynkin et al. (2005) have found it more useful to select risk factors that are intuitively clear to investors, even at the expense of allowing positive correlations among the factors.

A more fundamental problem arising from relying on individual security data is that not all securities can be modelled adequately in this way. For illiquid securities, pricing histories are either unavailable or unreliable; for new securities, histories do not exist. For still other securities, there may be plenty of reliable historical data, but changes in security characteristics make this data irrelevant to future results. For instance, a ratings upgrade of an issuer would make future returns less volatile than those of the past. Moreover a change in interest rates can significantly alter the effective duration of a callable bond.
As any bond ages, its duration shortens, making its price less sensitive to interest rates. A multi-factor model estimates the risk from owning a particular bond based not on the historical performance of that bond, but on the historical returns of all bonds with characteristics similar to those currently pertaining to the bond.

Bhansali (2009) stresses that beyond simplification from the reduced dimensionality of the problem a major contribution of factor analysis is that factors' behaviour can be mapped to economic variables. It is more intuitive to the asset allocator to make forecasts and express his views on economic variables that will determine portfolio performance rather than having particular views on each asset class. In that sense factor analysis goes one step ahead in comparison to only focusing on asset classes, because it goes straight to causalities identification. Apart from helping to analyse portfolios from an allocation and management perspective, factor based framework enables the investment managers to articulate their stories vis-à-vis their clients and make arguments on the particular views that are driving portfolio construction and portfolio performance.

Quantitative analysis tried to fill in the gap in portfolio construction in terms of identifying portfolio risks and replicating benchmarks at a much lower cost. The idea is to move away from asset classes and stress on risk analysis. The relative portfolio total risk should be approximated via a multifactor model on ex ante basis. This approach has gained ground in the market place against ex post risk analysis because it allows to judge what the risk of a relative portfolio should be based on how it is exposed to the various risk factors. The methodology is fairly straightforward. A variance covariance matrix is calculated for the set of risk factors specified in the model. Then according to the way the relative portfolio is positioned against them the total portfolio risk is derived. This is stated to be an ex ante analysis which can be extended to the analysis of scenarios as well.

The notion of applying a multifactor model goes back to the basics of portfolio theory. Portfolio theory suggests that when a number of securities are added in a portfolio non-systematic or specific risk is eliminated due to the diversification effect. That means practically that there would be no risk left in the portfolio associated to specific issues. In this case, idiosyncratic risk emanating from high concentrations and lack of diversification would be eliminated. What would be left is only market risk which should be explained by a set of risk factors driving the entire market. According to Dynkin et al. (2009) these risk factors for bond portfolios can be movements of the key rates, credit sector spreads or volatility. Bhansali (2009) indicates that a reasonable set of fundamental risk factors for fixed income can be level, curve, spread durations and convexity risk.

2.6.1 Interest Rate Risk

The prices of bonds in the secondary market are determined by supply and demand dynamics. They are not priced using a formula. Of course, brokers might use the below formula if the market is not very liquid, but in general they set their prices by supply and demand. The present value of a bond is the sum of the discounted future cash flows.

$$PV = \sum_{i=1}^{n} C_{T_i} (1 + R_{T_i})^{-T_i} \qquad \textbf{2.12}$$

Where:

T_i is the maturity of i-th cash flow

C_{T_i} is the cash flow paid at T_i

R_{T_i} is the discount rate, the yield

As the yield is not observable in the market, prices only are observed as determined by trading activity i.e. supply and demand. The yield can be implied or backward engineered by the market price using the above formula. R_{T_i} is the yield that the market requires in order to invest in this particular security. The market tends to price all the risk characteristics of a bond into its yield. If the market consensus on the yield of a bond changes so does its price. The first order sensitivity of a bond price to yield changes is measured by duration. Convexity is the second order sensitivity of a bond price to yield changes, or the rate of change of duration itself when yield changes. Duration is a more robust figure for small changes in yield and should be better adjusted by convexity when jumps in yield occur. Taylor Series expansion should be used to approximate change in price. As a general rule, regular fixed coupon bonds have positive duration (expressed in years because duration is a time function). In most cases the minus sign is added in front of the duration figure to indicate that for any change in yield price is moving to the opposite direction. In terms of convexity it refers to the convex nature of the cash flow discounting function.

Convexity is positive for all “plain vanilla” bonds. But it is becoming negative when optionality is introduced into the bond. Typical examples are mortgage securities. These securities are both callable and extendible bearing both prepayment and extension risk. That is the reason why convexity is negative across any possible yield level.

2.6.2 Spread Risk

Several types of securities fall under the umbrella of Fixed Income instruments: Treasuries, Corporate bonds, Mortgages, Agencies or derivatives such as Interest rate Swaps or Credit Default Swaps. Expanding too much on the details of each instrument is beyond the scope of this study. It is however essential to note that different types of securities despite having exactly the same cash flow structure may exhibit substantial discrepancies in yields. The reason is that, as mentioned before, the market tends to price all types of risks. Different issuers may be of completely different risk profiles. This is causing corporate bonds to trade at a higher yield vs. treasuries or corporates. The lower the credit quality of the issuer the higher the reward required by the market in order to invest in a particular security and not in the risk-free alternative. According to Blanco et al. (2005) the market consensus about the risk free rate of interest is to take either Treasury or Swap rates.

This implies that for all the securities, not only corporates, which trade at a spread over the risk free rate, there is one new form of risk introduced. This is spread risk. Spread risk reflects the excess yield required by the market due to Credit risk, Prepayment and Extension risk, Liquidity risk, Counterparty risk etc. A rational investor would never undertake more risks without expecting higher reward. As shown earlier, the sensitivity of each security to yield changes is measured by duration. From now onwards the yield of each security will be broken down in two main constituents. Firstly, an interest rate component corresponding to the changes of the risk free rate i.e. treasuries curve. Secondly, a spread risk component is used to accommodate the excess yield required by the market per asset class, as highlighted by Leibowitz et al. (1990). Duration refers to the interest rate risk component and an additional risk measure, the spread duration, refers to the spread risk component.

2.6.3 Risk Factor Sensitivities on Portfolio Level

Yields normally incorporate a term premium. Bonds of different maturities have a different yield. This is associated with interest rates' term structure. When aggregating duration to portfolio level the weighted average is used.

$$Dur_{PTF} = \sum_{i=1}^{n} w_i Dur_i \qquad (2.13)$$

Where

w_i is the weight of security i

Dur_i is the Duration of security i

Dur_{PTF} is the portfolio Duration

Obviously, to evaluate the change in yields at portfolio level by solely counting on duration might be misleading. Different maturity bonds have different underlying yields. Hence considering only one yield change for the entire portfolio and ignoring different yield changes occurring across the yield curve may cause severe problems. The aggregation mechanism is the same for spread duration measures and other risk measures as well. Consolidating the key risk exposures into a handful of sensitivities measures for the entire portfolio is intuitive but can disguise the actual risk profile of the portfolio. This is the problem of using averages at a time when different maturities, countries, sectors and credit quality exposures are crucial to the behaviour of financial assets.

2.6.4 Other risk types

Apart from interest rate, yield curve risk and credit risk, there are other forms of risk which may occur in a fixed income portfolio. These include but are not limited to sector risk linked to the exposure towards different industry groups of securities, volatility risk linked to the convexity profile of the instruments, prepayment risk linked to callable bonds and mortgage backed securities and currency risk linked to various currencies of issues which differ from the portfolio's base currency. Finally, issue specific risk can be another form of risk which may appear into the portfolio if it is not adequately diversified.

2.7 Conclusion

Two basic options are available for the fixed income investor, passive and active portfolio management. The degree and magnitude of activity may differ from one portfolio to another but a measure of success for active implementation of trading ideas is by using a chosen benchmark index. An adequate benchmark index, should describe the nature of the strategy to be followed and the risk appetite as well. In a sense, the benchmark is the opportunity cost for being invested in active portfolio management and sets the bar for the desired initial allocation. In order for the investment manager to outperform the index, relative risk should be undertaken vs. the benchmark, in line with the investor's risk tolerance. If the strategy is correct the investor should be rewarded for the relative risk undertaken by excess returns.

In order to achieve proximity to the benchmark, in terms of returns and risk, multifactor risk models are often employed for both benchmark replication and active risk management. A key component of the allocation process is the understanding of the risk factors driving portfolio performance and how they are aggregated on portfolio level. The impact of diversification should also not be neglected when it comes to the combination of all individual risks into a portfolio structure. In particular, when monitoring a portfolio, attention should be paid into the effect of averaging up risk exposures, which are similar but not identical in nature.

3 Fixed Income Portfolio Construction: A Bayesian Approach for the Allocation of Risk Factors

3.1 Introduction

A main theme in every portfolio construction problem is the estimation of the set of expected returns and risk. How exactly the expected returns and the risk are formulated is the differentiating factor from both a modelling and an asset allocation perspective. After the returns and risk are estimated it comes down to put the two together in order to construct an optimum portfolio.

Black-Litterman pioneering work has been pivotal in a sense of i) providing an intuitive starting point based on CAPM and ii) allowing for investment views to be used as part of the allocation process. Black-Litterman model backward engineers the equilibrium returns based on CAPM and the calculated variance covariance so that the chosen portfolio/benchmark becomes optimum and corner solutions are avoided. When choosing a different benchmark as an allocation starting point the equilibrium returns do change as well. Therefore, CAPM equilibrium returns are of instrumental value rather than true representations of the market expected returns as both the validity of CAPM and the extent to which the market portfolio is observable are arguable. In this doctoral thesis, the equilibrium returns are approximated by the occurring yields to maturity for investment grade bonds and the risk is estimated through a multifactor model which is essential for capturing the dynamics of the fixed income market.

Fixed Income markets are by nature different to equities markets. Different bonds issued by a particular issuer are likely to display different risk characteristics from each other as the time to maturity and the coupons may differ. For example a 30 year bond is likely to have little in common with a 2 year bond issued by the same issuer. Additionally as opposed to equities, in the fixed income space most transactions take place OTC (over the counter) which implies less price transparency and limited data availability. Considering the case of a newly issued bond; there will be not enough price history to feed into the variance covariance calculator as this bond has only in existence from issuance to date. On the other hand side, even when history is available this might not be enough to calculate the variance covariance matrix. Every day there is a decline in the time left to maturity of a bond, implying that a 10 year bond will become eventually a two year bond after eight years. Using the price, yield or return history of this bond will be misleading in what the risk profile of the 2 year bond is. The reasoning is that the term of the bond changes with the passage of time and so does its risk profile. This is why data providers such as Bloomberg have constructed generic yield indices with fixed maturity to better reflect the risk dynamics of a particular point on the yield curve.

In fact, fixed income securities exhibit high correlations, implying that there are underlying factors driving their behaviour. Bonds are exposed to various risk factors such as interest rate, spread and currency risks. A bond’s price is a function of the yield and the spread which account for interest rate, credit and liquidity risk respectively. Individual exposures to those risks are coming straight from the bond pricing models. The use of multifactor risk models as an investment management tool is widely used in the industry as it reduces the dimensionality of the risk estimation problem and provides more stable results. It is also highly intuitive as it can be linked to a quite straightforward economic interpretation. The roots of using multifactor models for portfolio management go back to the Arbitrage Price Theory. Once the portfolio can be mapped onto a set of risk factors we are well positioned to start thinking of the allocation decision.

As mentioned, the construction of a portfolio depends on the expected returns and risk. Largely speaking using a multifactor model should resolve most of the problems linked to the risk estimation in the fixed income space. It is time therefore to focus on the other pillar driving the allocation process: the returns. In Black-Litterman world the equilibrium returns are implied by the CAPM from the market capitalizations i.e. the weights of the market portfolio and the estimated volatility. This is a smart way to construct better diversified portfolios, more aligned to the market portfolio than what the mean variance optimization would result to. Then the equilibrium returns will be blended together with the portfolio manager's view to form the Black-Litterman returns and variance covariance matrix.

Our goal here is to provide a framework for active fixed income portfolio construction. This goal is setting a dual target: (i) to construct a portfolio against an index ensuring the performance and risks are measured vs. a benchmark which represents the market and as such offers a high level of diversification and (ii) to construct the portfolio taking into account the investment manager's views in order to outperform the index portfolio for a given level of risk undertaken relative to the index.

As mentioned earlier, instead of using the Black-Litterman equilibrium returns as best proxy for the bonds' expected total return, the selected total return will be the one implied by the current price levels; the yield to maturity. In order to make this statement valid this analysis is strictly limited to the investment grade universe, which is not prone to heavy default losses, if vulnerable to default losses at all. Moreover, the allocation will only be on a relative to index basis, implying that even if there are some names defaulting in the portfolio the same names will be part of the benchmark portfolio muting out the total effect in relative terms. Using the yields as a starting point for the returns' estimation is linked to the market efficiency hypothesis which should hold in its semi-strong form in order for the Black-Litterman model to be meaningful.

That told, the equilibrium returns can be extracted and the risk can be estimated as long as there is access to an index provider where this type of data is normally available. The next step is to incorporate the investment views by applying the Black-Litterman model in order to estimate the blended returns and the new variance covariance matrix.

The views to be specified are in the form of risk factors. Fixed income portfolio managers express their views on the main macroeconomic indicators affecting the returns of their portfolios. Therefore the views are on risk factor terms rather than on asset class terms.

The bond portfolio should consist of a mix of the sub-indices of Barclays Capital US Aggregate Index modelled in POINT. The breakdown of the index is designed to capture (i) the main key rate durations and (ii) the main credit sectors. The risk factors consist of the Duration and Convexity measures for the interest rate risk and the spread Duration and Convexity measures for the spread risk.

The yields are used to form the market implied return vector and the Variance Covariance Matrix. The risk factor sensitivities per sub-index are loaded in a matrix F that enables the transfer from asset class space to risk factor space and vice versa. Once the views are set based on matrix F the views on risk factors are converted into returns of the asset classes. The calculations are on a relative to the benchmark basis. The relative to the index weights (the overweight and the underweight) is what is targeted. If the portfolio is invested exactly as the benchmark the relative weights would be zero, there would be no excess yield, no performance caused of market movements and no risk relative to the benchmark.

The index can be the Bar Cap US Aggregate or any subset of it. It is essential that the index is not broader than the investment universe set up in the beginning. Here the investment universe is the US bond market.

3.2 Literature Review

A milestone to address the portfolio construction problem was the Markowitz (1952) Mean Variance approach. The concept of the model is that the investor prefers higher expected return than lower and dislikes risk. The Prior knowledge of the expected return is required for the mean variance optimiser to work. The calculation of expected return might be a problematic area as its computation, based on historic prices, takes as granted that the historic means are the best future estimate. That is the reason why the Mean-Variance approach, although very intuitive, has been characterized by Michaud (1998) as "Error maximisation". The extent to which Markowitz efficiency is in line with the expected utility maximization should be questioned. If the answer is no, then optimisation of specific utility functions should replace Markowitz efficiency.

The capital asset pricing model (CAPM) presented by Sharpe (1964) can be seen as a consequence of mean-variance portfolio theory. It defines the required return level by a rational investor in order to hold any particular risky asset. Markowitz Mean Variance efficiency and CAPM is a single period model.

To accommodate investment decisions made on longer time horizon, Merton (1973) published an "Intertemporal Capital Asset Pricing Model" showing how to generalize the capital asset pricing model to a comprehensive intertemporal equilibrium model. Merton's intertemporal CAPM with stochastic investment opportunities indicated that the expected excess return on any asset is given by a multi-beta version of the CAPM.

According to Fama (1996) the ICAPM generalizes the idea of the CAPM. In practice, if borrowing and lending is free and if short selling of risky asset is allowed, market prices imply that market portfolio is multifactor efficient. Moreover, multifactor efficiency establishes a link between expected return and beta risks but it requires additional betas apart from the market beta for the explanation of expected returns.

Fama and French (1993) state that size and book-to-market equity are not state variables but the higher average returns are on small stocks and high book-to-market stocks should be unidentified state variables responsible for non-diversifiable risks in returns. These risks are priced separately from market betas.

Frankel and Lee (1998), Dechow, Hutton and Sloan (1999), Piotroski (2000) highlighted that in portfolios constructed on price ratios such as book to market equity, stocks with higher expected cash flows have higher average returns that are not captured by the three-factor model or the CAPM. The conclusion reached there is that prices are irrational to the degree they do not incorporate available information about profitability expectations.

Lew and Vassalou (2000) state that annual returns on the SMB and HML hedge portfolios forecast growth in several countries. Vassalou (2002) states that a portfolio set to track news on the future growth of GDP captures much of the explanatory power of the Fama and French portfolios.

Studies such as Berk, Green and Naik (1999) and Gomes, Kogan and Zhang (2000) develop models explaining the Fama and French results in problems related to the measurement of beta.

The market portfolio should in theory include all types of assets that are held by any investor as an investment. In practice, such a market portfolio is unobservable and usually a stock index is used as a proxy for the true market portfolio. According to Roll (1977) CAPM might not be empirically testable due to the true market portfolio not being observable.

Goltz and Le Sourd (2011) highlighted that CAPM is frequently used to advocate passive index investing. Other studies including Roxburgh et al (2011) and Doeswijk et al (2012) have further explored the investable assets universe forming the market portfolio.

Black and Litterman (1992) considered market equilibrium return as the starting point for the allocation problem. For the estimation of the excess equilibrium returns, reverse optimisation was used, under the CAPM assumption. Then equilibrium returns were blended with the investment views to form the posterior set of expected returns.

Regarding the blending process Black and Litterman (1992), He and Litterman (2002), Idzorek (2004) considered the set of equilibrium returns as the prior; whilst Satchel and Scowcroft (2000) considered the investor views as the prior distribution and the equilibrium returns as the likelihood.

Krishman and Mains (2005) re-derived the Black-Litterman model on the basis of a two factor risk framework. They have added a recession risk factor to the traditional single factor model to better capture the actual risk dynamics into the utility function.

According to Giacometti (2007) the Black-Litterman model generates those equilibrium returns to replicate the allocation of the chosen market portfolio, and the equilibrium returns differ across different investors with different initial allocations.

In this chapter the Black-Litterman model is revisited in order to permit the construction of active fixed income portfolios, where the return and risk dynamics are substantially different from the equities markets.

3.3 Methodology

The scope of this chapter is to enhance the Black and Litterman model so that it can be applicable to fixed income portfolios. For that purpose both equilibrium returns and portfolio risk are revisited. The CAPM equilibrium returns are juxtaposed against the occurring yield to maturity set for investment grade securities and a multifactor risk model is used. The bonds are mapped onto risk factor space, making it feasible to express both the risk and the views based on a risk factor framework.

3.3.1 The Equilibrium Returns

Black-Litterman model (1992) proposed a framework for portfolio construction. Its two major contributions can be summarized into the following. First, it uses the CAPM equilibrium market portfolio as a starting point to generate the expected return. Only historic prices and returns can be observed in the market place. In the Black-Litterman model, the equilibrium return is backward engineered by the market volatility which is occurring in the market.

Second, it combines the portfolio manager's particular view with the market equilibrium return. The latter constitutes a centre of gravity which is adjusted by the view depending on the confidence of the investor on it. Nothing similar had been published prior to the Black-Litterman model. It offers the quantitative platform to specify the investor views and to blend them together with the market implied equilibrium return and form a new combined distribution.

The Black-Litterman model takes the market equilibrium return as a starting point. As mentioned above due to the nature of the data available in the market the model is using a reverse optimisation method to derive the excess equilibrium returns.

The equations for reverse optimisation are derived. The starting point is the quadratic utility function:

$$U = W^T \Pi - \left(\frac{\lambda}{2} \right) W^T \Sigma W \qquad \textbf{3.1}$$

Where:

U is the Investors utility, the objective function during portfolio optimisation

Π is the Excess Equilibrium Return Vector (N x 1 column vector)

λ is the risk aversion parameter of the market

Σ is the covariance matrix of returns (N x N matrix)

W is the weight invested in each asset (N x 1 column vector)

As a concave function U is having one single global maximum. By maximising the utility function without any constraints, a closed form solution is derived. The first order derivative with respect to the weights (W) is calculated and is then set to 0.

$$\frac{dU}{dw} = \Pi - \lambda \Sigma W = 0 \qquad \textbf{3.2}$$

Solving the above equation for Π:

$$\Pi = \lambda \Sigma W \qquad \textbf{3.3}$$

The risk aversion coefficient lambda corresponds to the risk premium required by the market in order to undertake one more unit of risk. This parameter needs to be known in order to use formula (3). In most cases in the literature the value of parameter λ is defined prior to using the model. The process of calibrating returns of Bevan and Winkelmann (1998) was to input a Sharpe ratio based on their experience. Black and Litterman (1992) use a Sharpe ratio closer to 0.5 as part of their analysis.

More specifically, λ can be derived when equation (3) is multiplied at both sides by W^T and when vector terms are replaced by scalar terms.

$$(E(R) - r_{free}) = \lambda \sigma^2 \tag{3.4}$$

$$\lambda = \frac{E(R) - r_{free}}{\sigma^2} \tag{3.5}$$

Where:

E(R)	is the return expectation
r	is the risk free rate
σ^2	is the variance

As part of the recent analysis, formula (5) should be used. E(R), r_{free} and σ^2 are inputs in order to calculate λ. Once we have a value for λ then we feed W, λ and Σ into formula (3) to compute the set of equilibrium asset returns. Formula (3) is nothing but the closed form solution to the reverse optimisation problem for the calculation of asset returns given an optimal mean-variance portfolio in the absence of constraints. Formula (3) can be rearranged for the computation of optimal portfolio weights in the absence of constraints.

$$w = (\lambda \Sigma)^{-1} \Pi \tag{3.6}$$

By plugging Π, λ and Σ back into the formula (6), we can get the weights (w). Using historical excess returns rather than equilibrium returns would make the results extremely sensitive to changes in Π. In Black-Litterman model the weight vector is less sensitive to the Π vector. One of the pros of Black-

Litterman framework is the stability of the optimisation process. The cons are linked with the validity of the CAPM assumptions and the so called 'CAPM equilibrium returns'.

When focusing on fixed income, a good representation of the promised return is the yield to maturity. However, this may not be an accurate estimate of the expected returns given that bonds may suffer default losses. When focusing on investment grade fixed income universe, where the default risk is very low, and the main source of uncertainty is price/yields volatility, the yield to maturity is a good proxy of the expected return to be achieved if a security is held to maturity and no credit events occur.

The yields are stripped out of the occurring valuations and constitute, by nature, a representation of the market consensus on future returns, under the no default condition. Whereas yield to maturity is a reasonable measure for expected returns of a 'buy and hold' strategy within investment grade space, it would not be applicable to the high yield universe, where the probability of default is not negligible.

As such, in this chapter a comparison is made between the CAPM implied equilibrium returns and the yield to maturity set. That tests the compatibility of Black- Litterman equilibrium returns against the market consensus and also tests if the risk aversion coefficient can be calibrated so as to reconcile the two.

3.3.2 Specification of the Views

This section focused on how to specify views on the estimated mean excess returns. Views can be absolute i.e. for only one asset class or relative i.e. measuring the relative expected performance of two or more asset classes. This step will allow the investment managers to express their particular views which will be incorporated in the model into a new excess returns distribution, conditional on the market equilibrium findings. This new conditional distribution is often referred in the literature as posterior distribution. Two conditions are met by construction:

1. All views should be unique and uncorrelated with each other.
2. Views should be "fully invested". The sum of their weights should be either one for absolute views or zero for relative views.
3. It is not necessary to impose views on all assets. In the extreme case that there are no views at all, the model will use by default the market implied equilibrium excess return.

The investment manager's k views on n assets will be represented as follows:

- P is a $(k \times n)$ matrix of the assets' weights corresponding to each view. For all relative views the sum of the weights is zero and for all the absolute views the weight is one. Satchell and Scowcroft (2000) use an equal weighted scheme, whilst He and Litterman (1999) and Idzorek (2005) use a market capitalization weighted scheme.
- Q is a $(k \times 1)$ matrix of the returns per view.
- Ω is a $(k \times k)$ matrix corresponding to the variance-covariance matrix of the views. This is by construction a diagonal matrix due to the requirement of the views being independent and uncorrelated. Ω is symmetric and zero on all non-diagonal elements.

Having set the views specification it is now feasible to express the conditional distribution mean and variance in views space as:

$$P(B|A) \sim N(Q, \Omega) \qquad \textbf{3.7}$$

And in asset space as:

$$P(B|A) \sim N(P^{-1}Q, [P^T \Omega^{-1} P]^{-1}) \qquad \textbf{3.8}$$

Though interesting to see how views are translated in asset space, there is no need to evaluate the above expression in order to implement the Black-Litterman model.

In a risk factor space all return input parameters for the Black-Litterman model are replaced by changes in underlying yields and spreads. A number of steps have been followed in order to bring the views to a Black-Litterman compatible form. The market views are expressed in a similar way to Satchell and Scowcroft (2000). A main modification is the replacement of views on returns by views on yield and spread changes. In risk factor space only absolute views can be used as input.

The input of views on risk factors is feasible through a vector V. The length of vector V is the number of risk factors. The first 13 elements of vector V correspond to the expected changes in yield and OAS which will affect the portfolio via duration exposures. The last 13 elements of vector V correspond to the squared changes of yields and OAS which will affect the portfolio via the convexity exposures. An adjustment vector A is created to help mimicking the Taylor series expansion for a given level of yield and spread change. The first 13 elements of vector A are equal to -1, whereas the last 13 elements of vector A are equal to ½. When multiplying, element by element, the views vector V and the adjustment vector A, we end up with the adjusted views vector $\hat{V}$. Q vector is becoming:

$$Q = F\hat{V} \quad \textbf{3.9}$$

The Black-Litterman $\hat{Q}$ vector is finalized after the zero rows are deleted. $\hat{Q}$ vector is expressed on asset class basis thanks to multiplying by matrix F.

The final number of views is the number of all asset classes which will be affected by views in terms of yield and spread level changes. This number equals to the number of asset classes with non-zero exposure to this risk particular risk factors. This is what controls the dimensions of P matrix i.e. the number of views per asset classes by the number of all asset classes.

3.3.3 The Risk Estimation

One of the main assumptions of the Black-Litterman model is that the input returns are normally distributed with mean equal to the market equilibrium return. Then we expand on the variance calculation. Black and Litterman made the simplifying assumption that the covariance of the mean estimate is proportional to the covariance structure of the returns Σ. The constant of proportionality created is τ so that:

$$\Sigma_{\Pi} = \tau\Sigma \quad (3.10)$$

By putting all components together, the prior distribution is formed. It shows the estimate of the mean and variance of excess returns.

$$P(A) \sim N(\Pi, \tau\Sigma) \quad (3.11)$$

In the estimation of the mean of a distribution, the uncertainty will be proportional to the number of samples. As suggested by Walters (2009) τ can be calibrated on the basis of the maximum likelihood estimator:

$$\tau = \frac{1}{T} \quad (3.12)$$

Or on the basis of the best quadratic unbiased estimator

$$\tau = \frac{1}{T-k} \tag{3.13}$$

Where:

T is the number of samples

k is the number of assets

The parameter τ is one of the most obscure elements around the Black-Litterman model. In fact, it is nothing but a scaling parameter to reflect the variance of the mean as opposed to the variance of the population. In a Black-Litterman environment a higher τ would result in a higher variance for the model and a lower τ would result in a lower variance. For some practitioners this is a way of calibrating the overall risk estimate which is input to the optimization process.

A number of papers use a τ within the range (0.025, 0.05) such as Black and Litterman (1992), He and Litterman (1999) and Idzorek (2005). Satchell and Scowcroft (2000) use a τ at around 1 that fits to their reference model. The value for τ used is this thesis is 0.025 which in line with the first class of papers.

3.3.4 The estimation model

Before advancing, it is important to introduce the Black-Litterman formula and provide a brief description of each of its elements. Throughout this article, k is used to represent the number of views and n is used to express the number of assets in the formula.

$$E[R] = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right] \tag{3.14}$$

Where:

E[R] is the new (posterior) Combined Return Vector (n x 1 column vector) τ is a Scalar

Σ is the Covariance Matrix of Returns (n x n matrix)

P refers to the assets involved in the views (k x n matrix or 1 x n row vector in the special case of 1 view)

Ω is the diagonal covariance matrix of error terms in expressed views representing the level of confidence in each view (k x k matrix)

Π is the implied Equilibrium Return Vector (n x 1 column vector)

Q is the Views Vector (k x 1 column vector)

And the variance is equal to:

$$M = [(\tau\Sigma)^{-1} + P^{T}\Omega^{-1}P]^{-1} \quad \textbf{3.15}$$

Here the posterior variance is the variance of the posterior estimate of the mean and not the variance of the returns.

Bayes theorem can be applied to the fusion of prior and conditional distributions to generate the posterior distribution of asset returns. The derivation is in both Walters (2009) paper and in Satchell and Scowcroft (2000).

The posterior distribution formed is:

$$P(A|B) \sim N([(\tau\Sigma)^{-1}\Pi + P^{T}\Omega^{-1}Q][(\tau\Sigma)^{-1} + P^{T}\Omega^{-1}P]^{-1}, [(\tau\Sigma)^{-1} + P^{T}\Omega^{-1}P]^{-1}) \quad \textbf{3.16}$$

Conceptually, the Black-Litterman model is nothing but a weighted average of the Market Equilibrium Return Vector (Π) and the Views Vector (Q), where weights depend on the scalar (τ) and the uncertainty of the views (Ω).

3.3.5 Calculation of the variance covariance matrix

An exponentially weighted moving average (EWMA) is used for the calculation of the variance covariance matrix. This technique suggested among others by Jorrion (2000) is giving extra weight to the most recent observations so that a high volatility period is more likely to be followed by a high volatility period. Weights are adjusted by a decay rate of 2%, implying that as we move back in time each weight is 2% less than the next one.

$$w_i = \delta(1-\delta)^{i-1} \qquad \textbf{3.17}$$

Where:

w_i is the weight assigned to the i^{th} observation

δ is the decay rate

3.3.6 The risk factor loadings F matrix

In order to facilitate the calculation of both the variance covariance matrix and the views, a matrix F is introduced. The variance covariance is initially calculated on 13 risk factors as a 2% decaying EWMA on 5year of weekly changes in yields and spreads. In order to for this variance and covariance estimate to liaise with the portfolio under construction we introduce matrix F. The dimensions of F are determined by the number of asset classes and the number of the risk factors. Litterman (2003) describes the portfolio risk as:

$$\sigma^2 = WF\Sigma F^T W^T \qquad \textbf{3.18}$$

Where:

W is the portfolio weights vector

F is the risk factor loading matrix representing the exposures of each asset class to each one of the risk factors

∑ is the variance covariance matrix on risk factor level

$F\Sigma F^T$ is the variance covariance matrix on asset class level

It is noteworthy that in the above formula there is no term for the idiosyncratic risk. This is because in the context of a diversified portfolio the specific risk should be eliminated. In the long only space portfolios are managed against an index which is a broad market representation of the strategy followed and offers a high degree of diversification. The diversification benefit of indexation is analysed in more detail in chapter 2.

3.3.7 The Multi Factor Reference Model

Highly customized solutions may be applied in portfolio management such as defining the appropriate benchmark and setting the appropriate set of risk factors depending on the nature of the risks that will be part of the portfolio. Most factor models that portfolio managers use comprise a very big number of risk factors, usually exceeding one hundred, to accommodate most types of global fixed income portfolios. If a particular type of risk is not included in a portfolio, the sensitivity assigned to this risk factor is zero.

The choice of the multifactor risk model made here is relatively easy and intuitive. As long as the investment universe is the Barclays Capital US Aggregate Index the risk factors are separated into interest rate and credit risk ones. This is in line with Litterman (2003) and Dynkin (2005).

As part of this paper we are going to focus interchangeably on 13 or 26 sources of uncertainty. Only the “durations” will be used for the risk estimation. We also tried to incorporate the convexities into the risk estimation but this has not added much to the overall risk whilst it dramatically increased the complexity of the calculation. As such, convexities are not used for the risk calculation. On the other hand side, the convexities will be used into the derivation of the Black-Litterman views through Taylor series expansion. The full set of risk factors containing both first and second order sensitivities is displayed below.

Table 1: Set of risk factors

Duration 1-3yrs	Convexity 1-3yrs
Duration 3-5yrs	Convexity 3-5yrs
Duration 5-7yrs	Convexity 5-7yrs
Duration 7-10yrs	Convexity 7-10yrs
Duration 10-15yrs	Convexity 10-15yrs
Duration 15-25yrs	Convexity 15-25yrs
Duration 25+yrs	Convexity 25+yrs
OAS Duration US Agg Gvt Rtd	OAS Convexity US Agg Gvt Rtd
OAS Duration US Agg Securitized MBS	OAS Convexity US Agg Securitized MBS
OAS Duration US Agg Securitized CMBS_ABS	OAS Convexity US Agg Securitized CMBS_ABS
OAS Duration US Agg Corp Ind	OAS Convexity US Agg Corp Ind
OAS Duration US Agg Corp Utility	OAS Convexity US Agg Corp Utility
OAS Duration US Agg Corp Fin	OAS Convexity US Agg Corp Fin

These risk factors are designed to explain the price fluctuations per asset class based on the changes of the underlying yields and spreads. Based on the way the factor model has been specified, the interest rate risk will be captured by duration measures and the spread risk by spread duration measures. The key difference vs. an econometric model is that the interest rate and spread duration sensitivities are generated by Barclays Capital bond pricing models rather than resulting from a regression analysis.

The degree of accuracy of the model is described for individual risk measures in the literature in Chapter 4. Two ways of evaluating the error term are by either factor based performance attribution which is resulting in an unexplained part of performance generated or by back testing the multifactor risk model. The later one is further elaborated in chapter 4. Introduction of high concentration risk to the portfolio would undermine the accuracy of the multifactor model chosen.

3.3.8 The Optimization

At this point it is noteworthy to comment on the optimization process. To begin with, optimization here refers to the relative to the index portfolio. Where index, potentially any index can be used. Secondly, it refers to its risk profile. From these two points we can infer that the normal full investment constraint (weights adding up to one) which is valid in the optimization of absolute portfolios per asset class, does not make sense any more. Instead, the restriction imposed is that the sum of weights for the relative portfolio should be zero. That is to say that if the portfolio manager decides to overweight one sector, he must underweight one or multiple other sectors to finance this position.

In addition, the short selling restriction holds in a sense of allowing the portfolio to have any exposure in a particular asset class in absolute terms but never have negative exposure. Relative weights can be negative but should be aligned with the short selling limitation. The reason is that the optimization is transitioned from the absolute portfolio to the relative portfolio. Therefore, any negative figures generated as part of the optimum solution imply that exposure to these risk factors is under weighted relative to the benchmark. Risk factors with positive weights assigned are over weighted respectively.

Now that it has been clarified that the model aims at the relative portfolio it is important to stress what the benchmark is. The answer is the benchmark's choice is upon investor's discretion. Here the benchmark is the Barclays Capital US aggregate index. The optimiser aims to the relative portfolio as a standalone portfolio, as a portfolio over and under – weighted vs. a particular benchmark. Once the over/under weights are derived they can be applied then to the benchmark to generate the risk factor positioning of the managed portfolio.

This approach is quite flexible in allowing the allocator to define the positioning of the portfolio, namely the relative portfolio, against the benchmark. Any benchmark that is suitable can be used and once adjusted by the relative portfolio optimum exposures should give the absolute portfolio to be held.

3.4 Data and Estimation

As specified in the section referring to the choice of risk factors the model is based on changes in yields and spreads. Specifically, all of the time series used over 5 years of weekly data up until August 31st, 2012. Daily data has not been selected due to the great deal of noise embedded in daily time series. In addition, weekly observations were preferred over monthly because they offers relatively more information about the time series variability without the noise accompanying daily prices.

The following sub-indices have been created using BarCap POINT. They have been selected so that each sub-index represents a combination of a maturity bucket and a credit sector. Based on this a duration/convexity and a spread duration/convexity are assigned to each asset class so that each asset class can only have exposure to four of the risk factors.

Table 2: Sector by maturity sub-indices

Credit Sector

Maturity Bucket

US Agg Tsy 1-3	US Agg Securitized MBS 1-3	US Agg Gvt Rtd 1-3	US Agg Securitized CMBS_ABS 1-3	US Agg Corp Ind 1-3	US Agg Corp Utility 1-3	US Agg Corp Fin 1-3
US Agg Tsy 3-5	US Agg Securitized MBS 3-5	US Agg Gvt Rtd 3-5	US Agg Securitized CMBS_ABS 3-5	US Agg Corp Ind 3-5	US Agg Corp Utility 3-5	US Agg Corp Fin 3-5
US Agg Tsy 5-7	US Agg Securitized MBS 5-7	US Agg Gvt Rtd 5-7	US Agg Securitized CMBS_ABS 5-7	US Agg Corp Ind 5-7	US Agg Corp Utility 5-7	US Agg Corp Fin 5-7
US Agg Tsy 7-10	US Agg Securitized MBS 7-10	US Agg Gvt Rtd 7-10	US Agg Securitized CMBS_ABS 7-10	US Agg Corp Ind 7-10	US Agg Corp Utility 7-10	US Agg Corp Fin 7-10
US Agg Tsy 10-15		US Agg Gvt Rtd 10-15	US Agg Securitized CMBS_ABS 10-15	US Agg Corp Ind 10-15	US Agg Corp Utility 10-15	US Agg Corp Fin 10-15
US Agg Tsy 15-25		US Agg Gvt Rtd 15-25	US Agg Securitized CMBS_ABS 15-25	US Agg Corp Ind 15-25	US Agg Corp Utility 15-25	US Agg Corp Fin 15-25
US Agg Tsy 25+		US Agg Gvt Rtd 25+		US Agg Corp Ind 25+	US Agg Corp Utility 25+	US Agg Corp Fin 25+

The yield and spread levels, the duration and convexity sensitivities and the market capitalization weights are sourced form Barclays Capital POINT.

3.5 Empirical Results

Once all elements are explicitly defined the Black-Litterman posterior relative yields and spread movements expectations are coupled with the new variance covariance matrix to create the efficient frontier. The input to the Optimiser is the vector of expected changes in yields and spreads and the variance and covariance matrix. As per the set of views presented above, with which the

Black-Litterman model was fed, we got the following risk and return combinations. The risk calculation is the so-called ex ante Tracking Error in the literature because it projects the risk profile of the portfolio based on the exposure to the various risk factors. The target variables here are the relative weights.

Scenario I: Reflationary pressures, the US economy warms up, interest rates go up by 20bps and spreads of risky assets do not react. The input of views into vector V is as follows.

Table 3: First scenario changes in rates and spreads

	Change in Yield
Duration 1-3yrs	0.20%
Duration 3-5yrs	0.20%
Duration 5-7yrs	0.20%
Duration 7-10yrs	0.20%
Duration 10-15yrs	0.20%
Duration 15-25yrs	0.20%
Duration 25+yrs	0.20%

The portfolio combinations are reported below in table 4.

Table 4: First scenario allocation per tracking error target

Tracking Error	Alpha	Duration	Spread Duration
0.00%	0.00%	4.91	4.43
0.10%	0.78%	4.94	6.28
0.20%	1.20%	4.63	6.43
0.30%	1.23%	4.49	6.36
0.40%	1.35%	4.52	5.35
0.50%	1.38%	4.39	4.79
0.70%	1.36%	5.36	5.58
0.80%	1.35%	5.55	5.76
0.90%	1.34%	5.72	5.93
1.00%	1.34%	5.89	6.09
1.10%	1.33%	6.05	6.24
1.20%	1.32%	6.21	6.39
1.30%	1.32%	6.36	6.54

As the risk budget increases so does the concentration of the portfolio towards the higher yielding financial sector. It is relatively easier to increase the expected alpha when deviating from the benchmark. The results per risk factor allocation are presented below.

Table 5: First scenario relative to the index allocation

1-3yrs	3-5yrs	5-7yrs	7-10yrs	10-15yrs	15-25yrs	25+yrs	US Agg Gvt Rtd	US Agg Securitized MBS	US Agg Securitized CMBS_ABS	US Agg Corp Ind	US Agg Corp Utility	US Agg Corp Fin
0.46	1.04	0.76	0.73	0.13	0.52	1.27	0.51	1.14	0.07	0.86	0.20	0.38
0.35	1.53	0.45	0.65	0.40	0.36	1.19	0.67	0.49	0.22	0.75	0.69	2.26
0.00	2.01	0.44	0.33	0.04	0.52	1.29	-	2.06	0.14	0.00	0.00	2.93
-	2.13	0.42	0.09	0.03	0.48	1.33	-	2.11	0.17	-	0.00	2.74
-	2.72	-	-	1.32	-	0.48	-	0.61	0.00	-	-	4.25
-	2.70	-	-	1.66	-	0.03	-	0.59	-	-	-	4.16
-	2.31	-	-	3.04	-	-	-	-	-	-	-	5.58
-	2.18	-	-	3.37	-	-	-	-	-	-	-	5.76
-	2.07	-	-	3.66	-	-	-	-	-	-	-	5.93
-	1.95	-	-	3.94	-	-	-	-	-	-	-	6.09
-	1.85	-	-	4.20	-	-	-	-	-	-	-	6.24
-	1.74	-	-	4.46	-	-	-	-	-	-	-	6.39
-	1.64	-	-	4.72	-	-	-	-	-	-	-	6.54

Scenario II: Reflationary scenario, the Quantitative Easing programs start bearing fruit; the yield curve shifts up by 20bps and credit spreads start to compress. The views vector V becomes as follows:

Table 6: Second scenario changes in rates and spreads

	Change in Yield/OAS
Duration 1-3yrs	0.20%
Duration 3-5yrs	0.20%
Duration 5-7yrs	0.20%
Duration 7-10yrs	0.20%
Duration 10-15yrs	0.20%
Duration 15-25yrs	0.20%
Duration 25+yrs	0.20%
Spread Duration US Agg Gvt Rtd	0.00%
Spread Duration US Agg Securitized MBS	0.00%
Spread Duration US Agg Securitized CMBS_ABS	-0.80%
Spread Duration US Agg Corp Ind	-0.80%
Spread Duration US Agg Corp Utility	-0.20%
Spread Duration US Agg Corp Fin	-0.50%

The different mix of overall duration and spread duration exposures are displayed below:

Table 7: Second scenario allocation per tracking error target

Tracking Error	Expected Alpha	Duration	Spread
0.00%	0.00%	4.91	4.43
0.10%	1.07%	4.80	6.32
0.20%	1.33%	4.99	6.61
0.30%	1.45%	5.21	6.81
0.40%	1.57%	5.40	6.50
0.50%	1.64%	5.56	6.18
0.60%	1.69%	5.73	6.11
0.70%	1.76%	5.58	5.56
0.80%	1.81%	5.72	5.72
0.90%	1.86%	5.84	5.85
1.00%	1.91%	6.21	6.84
1.10%	1.94%	6.32	6.85
1.20%	1.98%	6.42	6.85
1.30%	2.02%	6.53	6.85

The more detailed risk exposures are as follows:

Table 8: Second scenario relative to the index allocation

1-3yrs	3-5yrs	5-7yrs	7-10yrs	10-15yrs	15-25yrs	25+yrs	US Agg Gvt Rtd	US Agg Securitized MBS	US Agg Securitized CMBS_ABS	US Agg Corp Ind	US Agg Corp Utility	US Agg Corp Fin
0.46	1.04	0.76	0.73	0.13	0.52	1.27	0.51	1.14	0.07	0.86	0.20	0.38
0.00	1.90	0.20	0.63	0.07	1.02	0.98	0.29	2.30	0.10	1.36	0.23	1.05
-	2.17	0.49	0.25	0.04	0.72	1.32	-	1.22	0.57	0.01	0.00	3.49
-	2.53	0.39	-	0.05	0.75	1.48	-	0.42	1.00	-	-	3.91
-	2.82	-	-	0.12	1.34	1.12	-	-	1.74	-	-	3.64
-	2.72	-	-	0.21	1.89	0.73	-	-	2.73	-	-	2.72
-	2.66	-	-	0.20	2.36	0.51	-	-	2.66	-	-	2.94
-	2.33	-	-	2.32	0.93	0.00	-	-	2.34	-	-	3.23
-	2.18	-	-	2.89	0.64	0.00	-	-	2.19	-	-	3.53
-	2.04	-	-	3.45	0.34	0.00	-	-	2.05	-	-	3.81
-	1.89	-	-	3.69	-	0.63	-	-	1.89	-	-	4.32
-	1.79	-	-	4.01	-	0.52	-	-	1.79	-	-	4.54
-	1.69	-	-	4.33	-	0.41	-	-	1.69	-	-	4.75
-	1.59	-	-	4.64	-	0.30	-	-	1.59	-	-	4.96

Scenario III: Reflation of US market, Federal bank starts hiking rates, bear flattening of the curve and rally in industrial credits.

The views vector V becomes as follows:

Table 9: Third scenario changes in rates and spreads

	Change in Yield/OAS
Duration 1-3yrs	1.00%
Duration 3-5yrs	0.80%
Duration 5-7yrs	0.50%
Duration 7-10yrs	0.20%
Duration 10-15yrs	0.05%
Duration 15-25yrs	0.02%
Duration 25+yrs	0.01%
Spread Duration US Agg Gvt Rtd	0.00%
Spread Duration US Agg Securitized MBS	0.00%
Spread Duration US Agg Securitized CMBS_ABS	-0.80%
Spread Duration US Agg Corp Ind	-1.50%
Spread Duration US Agg Corp Utility	-0.05%
Spread Duration US Agg Corp Fin	-0.05%

The different mix of overall duration and spread duration exposures are displayed below:

Table 10: Third scenario allocation per tracking error target

Tracking Error	Expected Alpha	Duration	Spread
0.00%	0.00%	4.91	4.43
0.10%	1.10%	4.94	6.63
0.20%	1.23%	5.13	6.93
0.30%	1.36%	5.32	7.40
0.40%	1.45%	5.49	7.78
0.50%	1.54%	5.67	8.02
0.60%	1.62%	5.83	8.37
0.70%	1.70%	6.00	8.65
0.80%	1.79%	6.18	8.98
0.90%	1.88%	6.36	9.28
1.00%	1.97%	6.53	9.59
1.10%	2.06%	6.70	9.91
1.20%	2.15%	6.87	10.22
1.30%	2.24%	7.04	10.53
1.40%	2.32%	7.21	10.83
1.50%	2.41%	7.39	11.15
1.60%	2.50%	7.56	11.45
1.70%	2.63%	7.72	12.27
1.90%	2.80%	8.10	12.62
2.00%	2.84%	8.23	12.52
2.10%	2.93%	8.39	12.86
2.20%	3.04%	8.53	13.55
2.30%	3.11%	8.67	13.79
2.40%	3.19%	8.91	13.88
2.50%	3.28%	9.03	14.58
2.60%	3.36%	9.18	14.66
2.70%	3.44%	9.35	15.00
2.80%	3.54%	9.60	15.52

The detailed risk factor exposures are displayed in the table below:

Table 11: Third scenario relative to the index allocation

1-3yrs	3-5yrs	5-7yrs	7-10yrs	10-15yrs	15-25yrs	25+yrs	US Agg Gvt Rtd	US Agg Securitized MBS	US Agg Securitized CMBS_ABS	US Agg Corp Ind	US Agg Corp Utility	US Agg Corp Fin
0.46	1.04	0.76	0.73	0.13	0.52	1.27	0.51	1.14	0.07	0.86	0.20	0.38
0.15	1.47	0.21	1.09	0.06	0.70	1.25	0.29	2.04	0.10	0.64	0.00	2.30
0.19	1.32	0.17	1.38	0.06	0.58	1.42	0.05	1.81	0.15	0.12	0.00	3.38
0.28	1.19	0.02	1.52	0.08	0.47	1.76	-	1.63	0.29	-	0.00	3.72
0.30	1.13	-	1.50	0.08	0.49	2.00	-	1.55	0.31	-	0.00	3.92
0.33	1.02	0.01	1.58	0.08	0.55	2.11	-	1.40	0.28	-	0.00	4.23
0.35	0.98	-	1.48	0.07	0.62	2.33	-	1.34	0.26	-	0.00	4.44
0.37	0.91	-	1.48	0.08	0.70	2.47	-	1.25	0.27	-	0.00	4.66
0.43	0.78	-	1.53	0.08	0.68	2.69	-	1.07	0.30	-	0.00	4.93
0.45	0.70	-	1.57	0.08	0.71	2.85	-	0.95	0.31	-	0.00	5.16
0.49	0.59	-	1.58	0.09	0.74	3.04	-	0.81	0.34	-	0.00	5.40
0.54	0.49	-	1.59	0.09	0.76	3.23	-	0.67	0.36	-	0.00	5.65
0.58	0.39	-	1.60	0.09	0.79	3.41	-	0.53	0.38	-	0.00	5.88
0.60	0.31	-	1.62	0.10	0.82	3.60	-	0.42	0.40	-	0.00	6.11
0.64	0.21	-	1.64	0.10	0.85	3.77	-	0.29	0.42	-	0.00	6.35
0.68	0.11	-	1.65	0.11	0.87	3.96	-	0.15	0.44	-	0.00	6.59
0.72	0.01	-	1.68	0.11	0.90	4.14	-	0.02	0.46	-	0.00	6.83
0.71	-	-	1.50	0.21	0.49	4.80	-	-	0.75	-	-	6.72
0.60	-	-	1.82	0.21	0.69	4.79	-	-	0.63	-	-	7.20
0.63	-	-	1.56	0.13	1.34	4.58	-	-	0.48	-	0.00	7.46
0.61	-	-	1.51	0.13	1.38	4.77	-	-	0.50	-	0.00	7.60
0.63	-	-	1.16	0.17	1.24	5.32	-	-	0.66	-	-	7.56
0.64	-	-	0.93	0.18	1.48	5.44	-	-	0.68	-	0.00	7.67
0.50	0.02	-	1.61	0.14	1.36	5.29	-	0.02	0.52	-	0.00	8.05
0.58	-0.00	-	1.03	0.19	1.33	5.90	-	-	0.61	-	-	8.08

Scenario IV: Replication of a Lehman Brothers crash like scenario where all credit sectors suffered and there was a big bull steepening of the curve. The difference of the view set and what happened back in October 2008 is that the drop at the very short end of the curve i.e. 1-3yrs maturities were reduced to only 24bps as opposed to the 80bps realized drop as there is a natural floor for the yields at 0 and they cannot go any further down at least on the long run.

The views vector V associated to this scenario is described below:

Table 12: Forth scenario changes in rates and spreads

	Change in Yield/OAS
Duration 1-3yrs	-0.24%
Duration 3-5yrs	-0.40%
Duration 5-7yrs	-0.20%
Duration 7-10yrs	-0.05%
Duration 10-15yrs	0.12%
Duration 15-25yrs	0.18%
Duration 25+yrs	0.01%
Spread Duration US Agg Gvt Rtd	0.90%
Spread Duration US Agg Securitized MBS	0.65%
Spread Duration US Agg Securitized CMBS_ABS	4.00%
Spread Duration US Agg Corp Ind	2.50%
Spread Duration US Agg Corp Utility	2.50%
Spread Duration US Agg Corp Fin	2.60%

Table 13: Forth scenario allocation per tracking error target

Tracking Error	Expected Alpha	Duration	Spread
0.00%	0.00%	4.91	4.43
0.10%	1.18%	4.68	6.70
0.20%	1.56%	4.51	6.63
0.30%	1.87%	4.33	6.37
0.40%	2.14%	4.16	6.09
0.50%	2.45%	3.99	5.74
0.60%	2.73%	3.82	5.40
0.70%	2.95%	3.64	5.20
0.80%	3.23%	3.46	4.96
1.00%	3.83%	3.13	4.19
1.10%	4.10%	2.97	3.94
1.20%	4.36%	2.80	3.70
1.30%	4.63%	2.65	3.58

And the overall risk factor positioning is presented in the below table:

Table 14: Fourth scenario relative to the index allocation

1-3yrs	3-5yrs	5-7yrs	7-10yrs	10-15yrs	15-25yrs	25+yrs	US Agg Gvt Rtd	US Agg Securitized MBS	US Agg Securitized CMBS_ABS	US Agg Corp Ind	US Agg Corp Utility	US Agg Corp Fin
0.46	1.04	0.76	0.73	0.13	0.52	1.27	0.51	1.14	0.07	0.86	0.20	0.38
0.07	1.41	0.64	0.81	0.07	0.26	1.42	0.37	2.76	0.02	1.38	0.20	0.56
0.03	1.53	0.62	0.83	0.03	0.00	1.47	0.44	2.93	0.00	0.83	0.05	0.92
0.02	1.61	0.60	0.74	0.02	0.00	1.35	0.45	3.04	0.00	0.53	0.03	0.97
0.02	1.67	0.62	0.62	0.02	0.00	1.22	0.36	3.13	0.00	0.56	0.03	0.80
0.00	1.93	0.22	0.81	0.02	0.00	1.02	0.60	3.02	0.00	-	0.01	1.09
0.00	2.00	0.18	0.79	0.02	0.00	0.84	0.47	3.05	0.00	0.00	0.01	1.04
0.03	1.85	0.59	0.39	0.03	0.00	0.76	0.16	3.28	0.01	0.54	0.04	0.41
0.07	1.64	0.92	0.17	0.01	0.00	0.65	0.13	3.47	0.01	0.44	0.01	0.26
0.04	2.20	-	0.66	0.01	0.00	0.22	-	3.16	0.00	-	0.00	0.81
0.04	2.29	-	0.53	0.01	0.00	0.10	-	3.24	0.00	-	0.00	0.59
0.06	2.30	-	0.43	0.01	0.00	-	-	3.26	0.00	-	0.00	0.43
0.05	2.40	0.00	0.19	0.00	0.00	-	0.00	3.39	0.00	0.00	0.00	0.19

The above scenarios are representations of different states of the world regarding rates and spreads. This is a reflection of possible economic outlooks an investor would like to analyse. In practice, numerous scenarios could be described depending on the investment process, the views of economists and investment boards, the input from credit analysts and the output of financial models.

3.5.1 Factor Based Black-Litterman Optimization and the Normality Condition

Trying to move away from equities to fixed income, the main drawback of Black-Litterman model is its generic assumption that expected returns are normally distributed. Fixed income securities have the peculiarity that they cannot be modelled using the standard equity tool set because in the absence of credit events the investor knows that at maturity the price of the bond will be equal to its par value. This is why Black Scholes formula is never used to price options on bonds. Moreover, the basic geometric Brownian motion assumption to model returns does not hold as the bond prices pull to par. Alternatively Black's model (1976) is used where forward price is modelled instead.

In order to overcome this problem and be able to apply the Black-Litterman this paper shifts gears from asset classes to risk factors. Instead of normally distributed expected returns, normally distributed changes in yields and spreads are used which is a less unreasonable assumption to impose. It has been largely debated to what extent the normality condition holds, and if so, how well can this fit to turbulent periods when the entire market is under extreme credit and liquidity pressure.

In order to test this, a data visualisation has been used, called normal probability plot. This plot which is generated by a set of readings illustrates whether the fluctuations in the readings can be assumed to be statistically normal. Then the order statistics of normally distributed readings (norm-scores) are graphically depicted against the standardised values of the readings (z-scores). If the readings are effectively normal, the points drawn in regards to the two scores should more or less lie onto a straight line. Departures from this pattern indicate deviations from normality and may be due to several reasons.

In the graphs below, one can identify deviations from normality, especially for the corporate sector spreads. Namely, financial, utility and industrial companies' spreads behaved in a completely different manner than what normal distribution would forecast. Yields and mortgage spreads where more in line with the normal pattern. To demystify the severe mismatch observed along the left tail of the distribution we should consider the market conditions affecting our 5- year data set.

The deleveraging process that kicked off in the market after the failure of Lehman Brothers is the reason of this massive market dislocation. Most of the levered players tried to pull leverage and effectively cut their risk exposures down. Almost all participants in the market tried to de-lever, de-risk their balance sheets provoking a credit and liquidity squeeze. Spread changes of several market segments were not captured by normal distribution, due to the systemic events occurred. As mentioned already, this is plotted graphically as deviations from a straight line pattern.

Figure 3-1: US Government 2 year rates

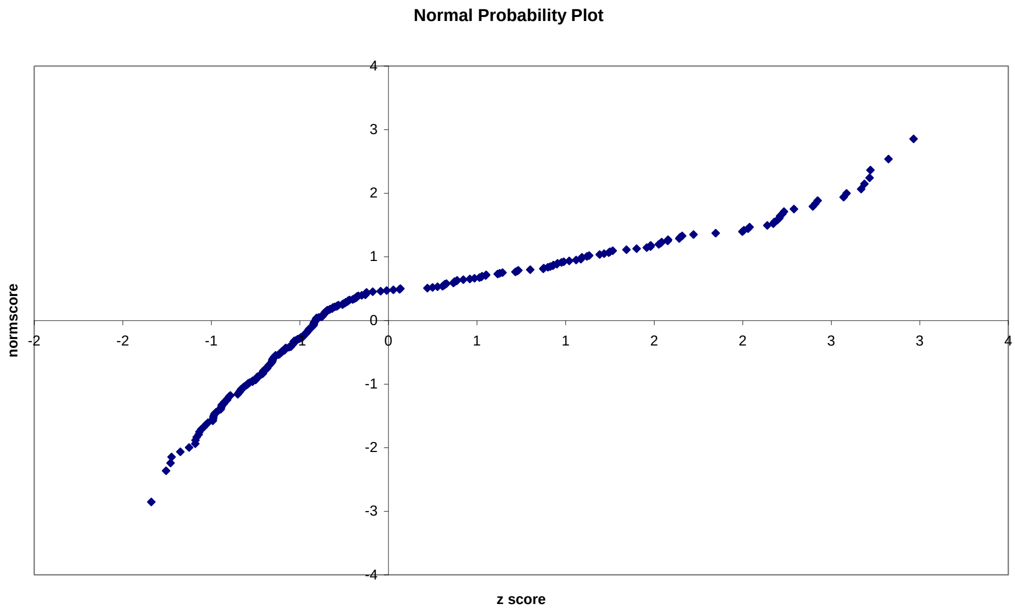


Figure 3-2: US Government 5 year rates

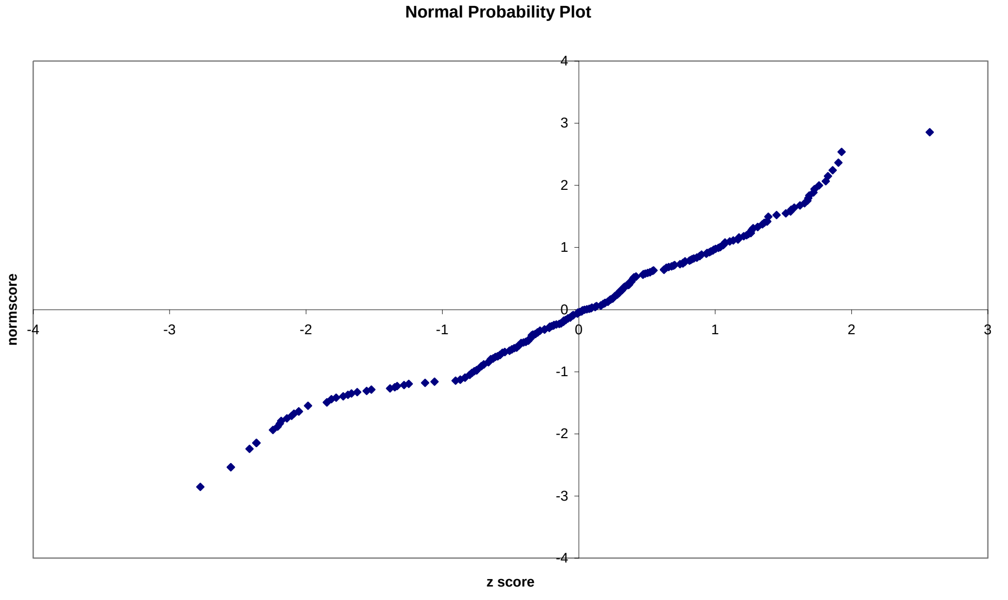


Figure 3-3: US Government 10 year rates

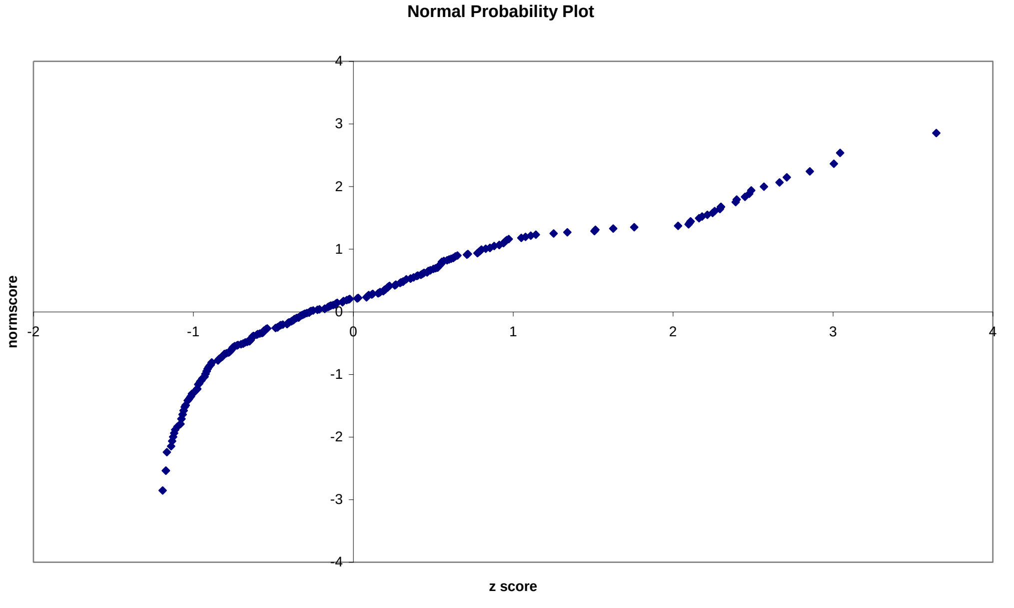


Figure 3-4: US Government 30 year rates

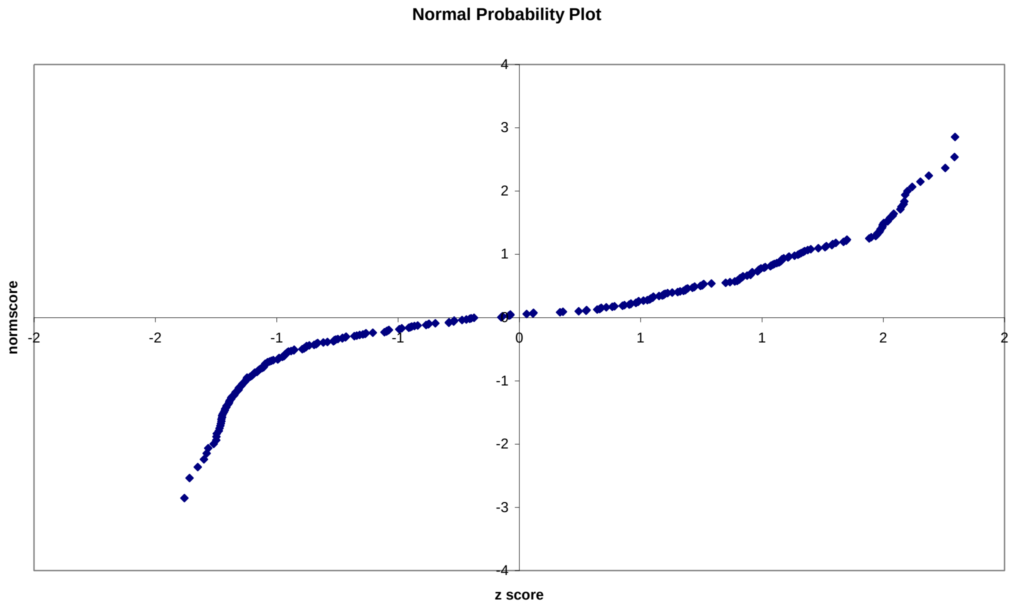


Figure 3-5: US Aggregate Government Related spread

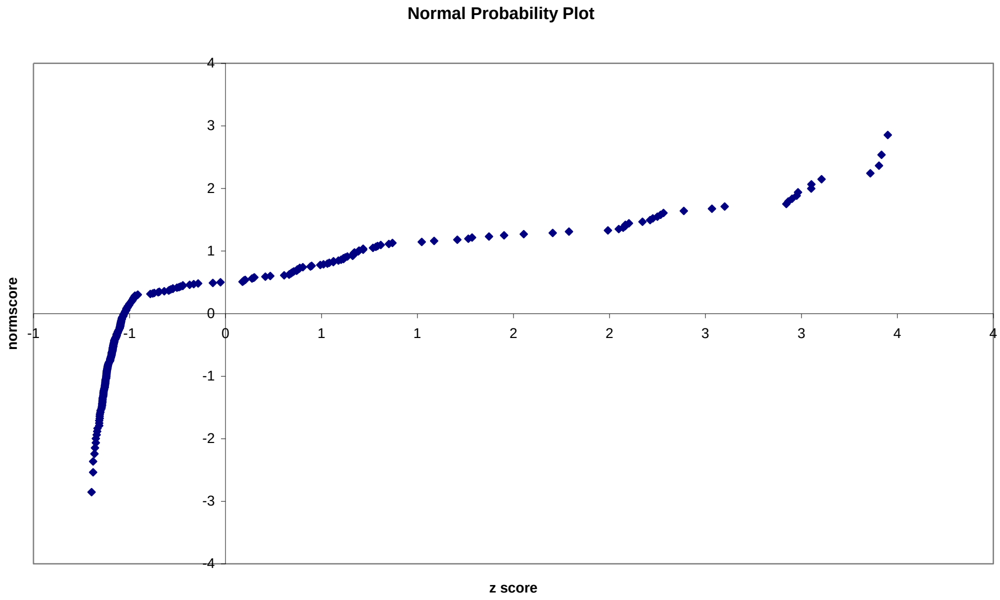


Figure 3-6: US Aggregate Securitised CMBS_ABS spread

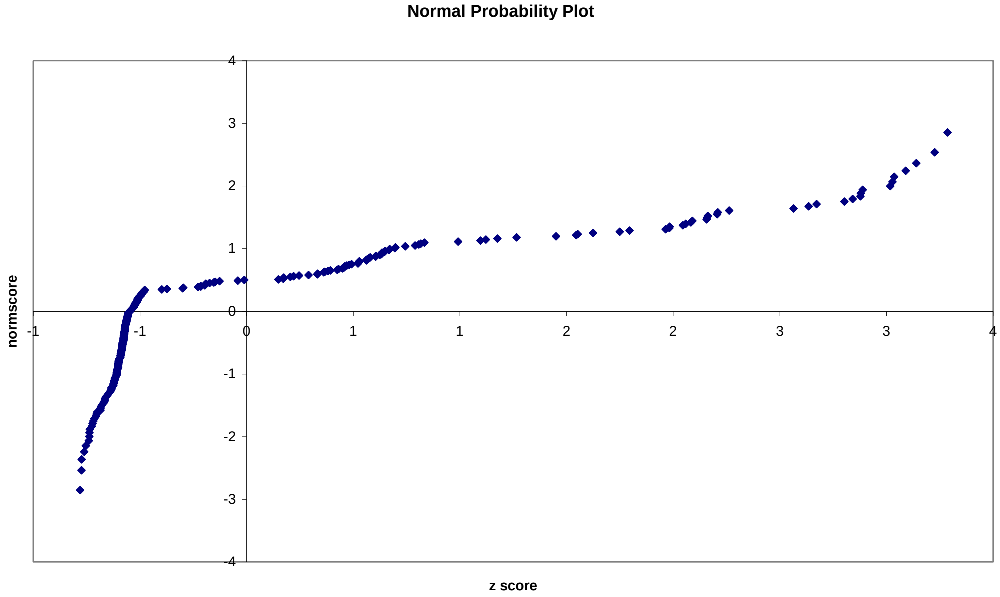


Figure 3-7: US Aggregate Securitised MBS spread

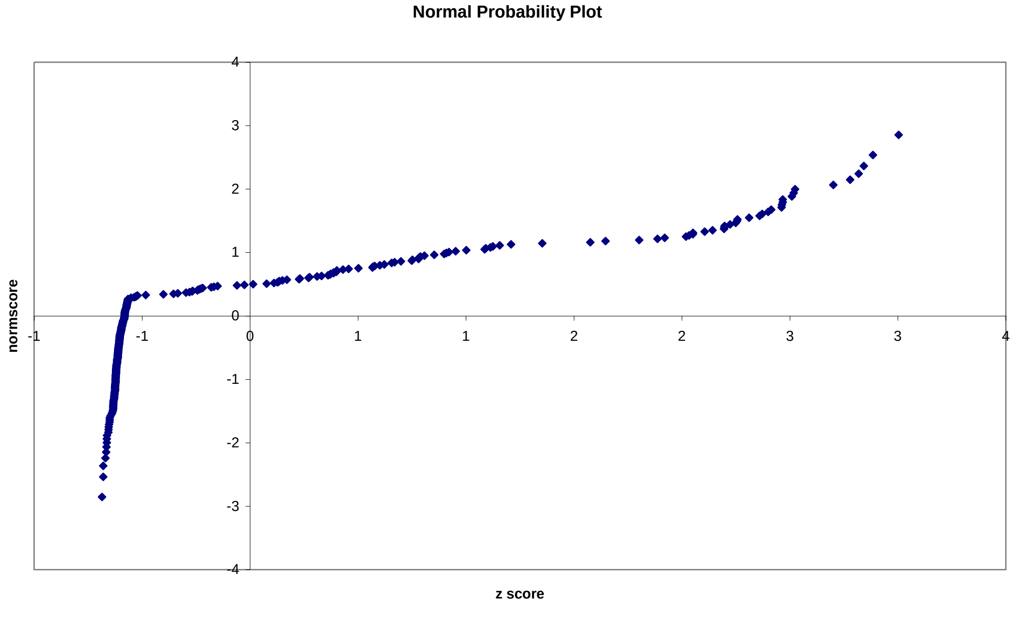


Figure 3-8: US Aggregate Corporate Industrial spread

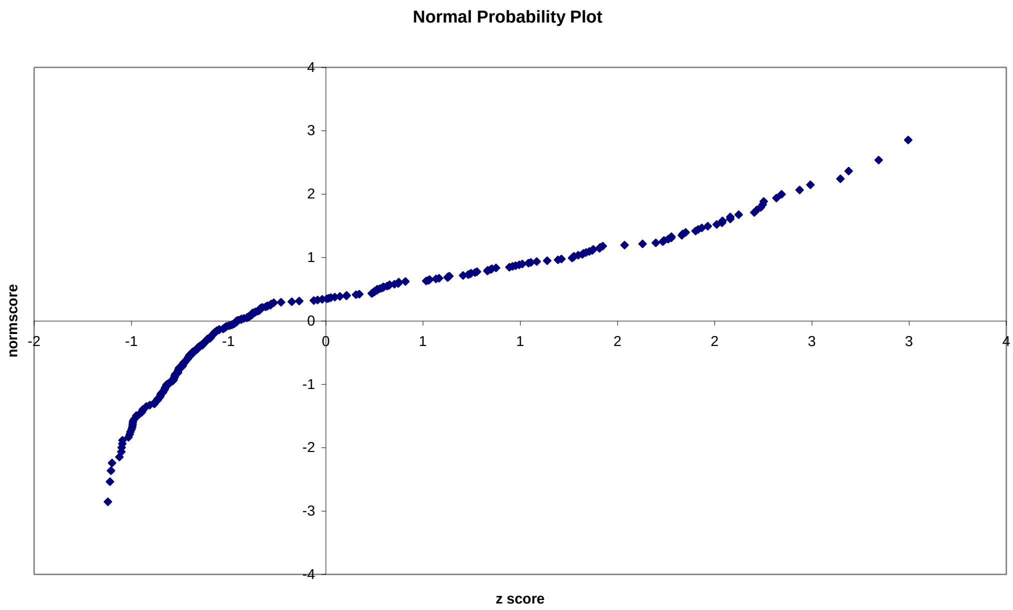


Figure 3-9: US Aggregate Corporate Utility spread

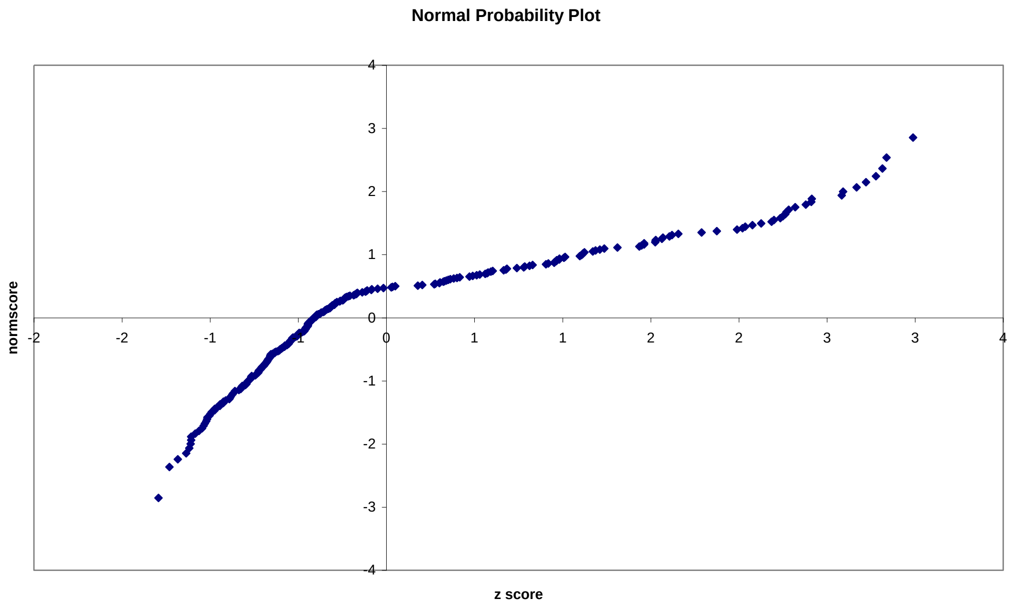
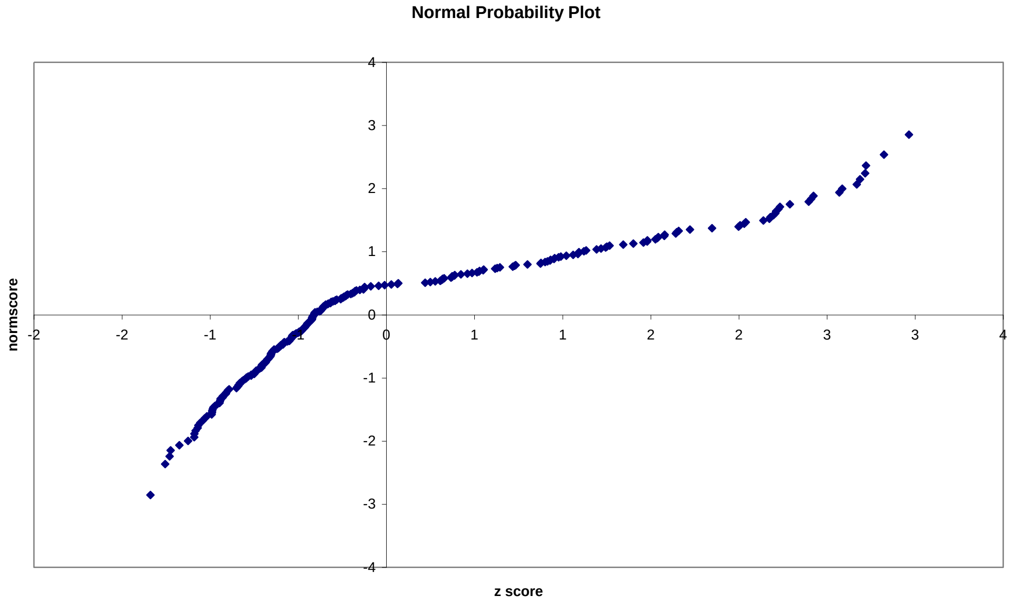


Figure 3-10: US Aggregate Corporate Financial spread



The focus is still on the relative portfolio implying that all risk factor sensitivities are on a relative to the benchmark basis. The risk factors' risk and return calculations for the relative portfolio are identical to the regular Black-Litterman ones. The main difference is that instead of weights per asset classes, the model provides sensitivities per risk factors. The variance covariance matrix is computed on yields and spreads changes. The construction of the efficient frontier from the Black-Litterman model will now refer to optimum combinations of exposures to risk factors rather than securities weights.

3.6 Conclusion

Starting from the portfolio theory cornerstones such as the Mean Variance and the CAPM we have stepped up to the Black-Litterman framework. This approach has been adjusted to meet the modern fixed income portfolio construction requirements. For that purpose, a multi factor risk model has been used to approximate the systematic risk of the portfolio. The issuer specific risk is eliminated through diversification in adequately large portfolios. This is in line with real-life fixed income portfolios that usually include no less than a hundred securities. In fixed income space, the portfolios are managed against a benchmark to ensure sufficient diversification and decent returns. The goal of the active investment manager is to beat the index in terms of performance without taking too much risk. The introduction of a risk factor framework makes feasible the use of the Black-Litterman model for fixed income relative portfolios. What is also new to the way that the model has been implemented is that the yield has replaced the equilibrium returns. The former is the market's best proxy for bond returns in the investment grade universe.

The advantage of the model is that it brings the economics back to the equation by using the drivers of fixed income volatility. In order to set a portfolio strategy and communicate it to the broader audience the discussion was focused on these risk factors. This chapter establishes an intuitive link between market wisdom and the latest academic developments on the portfolio construction front. Another positive characteristic of the model is that it is flexible enough to incorporate different types of risk factors that reflect the risk profile of each portfolio in question, depending on the types of risk that the investor is keen to gain exposure to. The model is also accommodative to different types of benchmarks.

A drawback is that in order to be implemented it requires high level of analytical and technical support. However, in most cases active fixed income portfolio management is carried out by institutions that have the required infrastructure. Another disadvantage is that this framework is limited to investment grade portfolios as the inclusion of high yield would introduce default risk and as such would prevent us from using yields as a proxy for equilibrium returns. Furthermore, the model cannot handle currency risk. Nonetheless, it can estimate currency hedged portfolios.

All in all, the fusion of the multi-factor analysis and the Black-Litterman model is a step forward in portfolio construction which enables the portfolio manager to better express his views in a factor based language.

4 Exploring the Tail Risk of Fixed Income Portfolios via Multifactor Risk Models

4.1 Introduction

Active investment managers seek to add extra value to their portfolios versus the benchmark. The track record of performance is one of the first things an investor would consider prior to allocating any funds to a manager. The second element an investment manager is judged for is the amount of risk undertaken per unit of return. Then other qualitative characteristics such as the investment process are taken into consideration.

The benchmark composition should speak volumes about the portfolio set up, as it represents the starting point of the asset allocation. Given the risk appetite of investors, each portfolio should offer excess return vs. its benchmark but remain aligned in terms of risk. There is a limited amount of risk which should be undertaken versus the benchmark and this is measured by the tracking error, or in its simpler form the standard deviation of the relative to the index returns over time (alpha). A set of tools is developed to assist the investment managers in these tasks.

There has been a lot of academic and practical interest in disentangling fixed income volatility. There has also been interest in decomposing the performance of fixed income portfolios into their main return drivers. Both the performance attribution and the risk decomposition, which are the two different sides of the same coin, are performed with the help of a factor model. The reason behind the risk factor focus becomes clear when we think of bonds. A bond is the most generic component of a fixed income portfolio and, has by nature a dynamically changing risk profile as its maturity diminishes over time. Additionally, newly issued bonds have no price history associated to them, from which their risk profile could be deducted. Finally, fixed income asset classes display high correlations implying the existence of underlying risk factors driving their performances.

In this chapter, the effort is to cast light on the tail risk measurement of fixed income portfolios which remains largely speaking unexplored by solely counting on a multifactor risk model. Fixed income portfolios are driven by yield and credit spread changes and this is what forms the basis for the risk calculation. For this purpose, the relative to the index portfolio exposures towards sources of risk are coupled with market data namely an estimated variance covariance matrix. Then under the normality assumption, exposures to tail events can be drawn. These results are compared with the tail risk exposure as computed by using an exponentially weighted variance - covariance matrix or by using block bootstrapping for dependent data. Our findings are tested on twelve real actively managed portfolios which are benchmarked against the Barclays Capital US Aggregate Index and exhibit more than a thousand monthly return observations in total.

The outline of this paper is as follows. In section 2, we review the related literature. Section 3 describes the methodology followed and section 4 refers to the dataset used for the estimation of results which are presented in section 5. At last, section 6 summarizes the conclusions reached.

4.2 Literature Survey

Arbitrage Pricing Theory (APT), introduced by Ross (1976) is the most generic theoretical framework that recognizes multiple risk sources as drivers of asset returns. Chen et al. (1986) identified a set of macro-economic factors as significant in explaining stock returns.

In the fixed income space Chambers et al. (1988), Prisman and Shores (1988) and Bierwag et al. (1988) stress that changes in the term structure are combinations of level, slope and curvature changes of the yield curve.

Additionally, Litterman and Scheinkman (1991) proposed a three factor model, explaining on average 98.4% of the total yield curve variance. Namely, the three factors used are “level”, “steepness” and “curvature”. Furthermore, they made a distinction between the yield and the specific factor i.e. the spread, as the main components to determine bonds’ returns. Accordingly, Jones (1991) also suggested a three-factor model to explain the return of treasuries. In fact, he used a similar set of risk factors named differently as “shift”, “twist” and “butterfly”. These three factors combined together accounted for approximately 95% of the total return of a government bonds portfolio.

A slightly different approach was followed by Willner (1996), who introduced a new way of measuring risk sensitivities towards these three risk factors, which emanate from each type of change of the yield curve. The sensitivity measures introduced are level duration, slope duration and curvature duration. This framework has been widely used by academics and market practitioners for the monitoring, performance attribution and risk analysis of the fixed income portfolios, as it helped to better accommodate the dynamics across different maturities of the term structure.

On the other hand, Ho (1992) set a number of maturities on the yield curve as being the key rate durations, with typical values of 3 months, 1, 2, 3, 5, 7, 10, 15, 20, 25 and 30 years. Duration was estimated to measure interest-rate sensitivity, to a movement of the yield, at each of the above points in isolation. In other words, key rate duration estimates the effect of a change in the term structure which is localized at a particular maturity point, and restricted to the immediate proximity of this maturity point, usually by having the change drop linearly to zero at adjacent points.

This is an alternative representation of the term structure which is commercially available by most index and fixed income data providers. Crack and Nawalkha (2000) presented how the bond risk measures evolve when the shape of the term structure is changing and found that durations and convexities of barbell portfolios are more sensitive to the changes of level and shape of the yield curve than durations and convexities of bullet portfolios.

However, interest rate exposure is not enough to fully capture the dynamics of the fixed income market. Several bonds and fixed income securities, which are trading at a spread over the treasury curve, exhibit a credit and/or a liquidity premium. According to Litterman (1991) the fixed income market returns not only are analysed to the yield component but also to the idiosyncratic component, which is priced in the spread set by the market. Leibowitz et al. (1990) introduced a sensitivity measure, equivalent to duration, for spread risk. This new measure, which was named, spread duration, aimed to accommodate the credit and liquidity risk, which was neglected by only focusing on the term structure, especially when considering credit portfolios.

This has become a very popular risk measure, and gained ground in the industry as it helped to fully translate a fixed income portfolio into yield and spread exposures. This methodology works better for individual securities, but becomes problematic when the spread duration exposures are aggregated to portfolio level. This is because individual securities spread movements may decouple, slightly or largely, from each other, due to the issue/sector specific characteristics. A possible solution to this problem is to use different spreads as reference points for groups of securities falling into different instrument type, industry and rating category.

A different approach to address the aggregation issue was presented by Fabozzi et al. (2006); a beta adjustment mechanism initially applied across the spread durations of various countries. This technique became popular amongst market practitioners, who used it across sectors, to make the adjusted spread duration reflect the different reaction levels of different market segments, to the arrival of the new information. As explained by Ambastha et al. (2010) the spread dynamics are very different between higher and lower rated securities. Accordingly, investment grade universe is more dominated by interest rate risk whilst high yield spreads are more reactive in absorbing market shocks.

Recently, Dor et al. (2007) introduced a new solution to the aggregation problem, revolutionizing the spread exposure measurement. They named the new metric Duration Times Spread (DTS). The notion behind DTS is that the volatility of spread changes is linearly proportional to spread level. Spread duration measures the sensitivity of a portfolio to the changes of a reference spread in absolute terms. TS instead focuses on the sensitivity to the relative (percentage) spread change, practically by scaling up or down the spread duration exposure, based on the spread level of each security.

This chapter is also related to the literature that focuses on the risk estimation of a portfolio as a whole. Litterman (2003) makes explicit reference to the multifactor model specification for equities portfolio management. There are two components that make up the total risk of the portfolio: 1) the market/non-diversifiable risk approximated by a multifactor model, and 2) a set of uncorrelated issue specific risks which can eventually be diversified away for adequately big portfolios. According to Markowitz (1952) if the number of securities turns out to be big the specific/idiosyncratic risk can be eliminated thanks to the diversification benefit. Thankfully all the real portfolios we are going to test our results against hold at least a thousand securities and as such neglecting the security specific component and solely focusing on the multifactor model is robust.

Furthermore, Dynkin et al. (2006) proposed a model which used three components for the total risk estimation. The systematic risk, the idiosyncratic risk and the credit default risk. The systematic risk is what was explained by the multifactor risk model. Equivalently, the idiosyncratic risk can be analysed into issuer-specific/ issue specific risk and the credit default risk which stems from any exposure to the default risk of high-yield securities. The multifactor model consists of a key rate durations based model for the yield curve using the 6 month, and 2, 5, 10, 20 and 30 year and a credit spread component where spread durations for AAA/AA, A and BBB rated securities are used across different industry sectors. A similar technique, spread durations based, is used for agency and non-agency MBS and ABS securities. The duration measure used is Option Adjusted Spread Duration (OASD).

According to Barra (2007), using shift, twist and butterfly risk measures captures between 90%-98% of the total volatility of the yield curve as measured by an 8-factor key rate model. Key rate models use a bigger number of risk factors than necessary which exhibit a high degree of dependency. In terms of spread risk, Barra uses individual “sector-by-rating” factors, rather than having each bond exposed both to a sector factor and a rating factor. The rationale behind it is that spreads of different rating classes in different sectors, behave independently.

As far as the idiosyncratic risk is concerned, it is not included in the multifactor risk expression, used here, because:

1. Relatively big size portfolios ensure adequate level of diversification and the specific risk is eliminated
2. Litterman’s specification for the idiosyncratic risk refers to equities, and would most likely not be feasible in fixed income space, due to the lack of long enough data histories
3. Drilling down to security level exposures is beyond the scope of this paper. Instead, the focus is on identifying the liaison between the portfolio performance and the main market risk drivers

Additionally, the exposure to credit default risk is not further examined as part of the present analysis, given that the benchmark exposure is limited to the investment grade space.

This class of multifactor models is used in the literature for both risk estimation and the performance attribution. We instead follow a different approach using the multifactor models as a starting point to explore the potential of providing better ex ante tail risk estimates. This is done by relaxing the normality assumption and incorporating block bootstrapping algorithms.

4.3 Methodology

The aim of this paper is to examine whether the integration of block bootstrapping into the multifactor framework is beneficial for the tail risk estimation of fixed income portfolios compared to a variance-covariance based methodology.

According to Litterman (2003), two different expressions can be used for the analysis of portfolio performance; the factor based representation and the asset grouping representation. However, in the risk space we are restricted to take the risk factor route. As stated in the introduction due to data unavailability an asset class based risk estimation would neither be feasible nor meaningful.

The multi-factor risk approach instead has the flexibility of translating all the risks into exposures towards risk factors. Then based on publicly available market information the ex-ante risk of the portfolio relative to the benchmark, i.e. the ex-ante tracking error, can be computed. The notion is that the best estimate for the future volatility of the relative to the index portfolio should be based on the most recent risk exposures of the portfolio. An additional reason, why the asset class alternative is not followed, has to do with the performance history of an actively traded portfolio being incapable of predicting ex ante risk, as the actual risk exposure of the portfolio is likely to differ substantially over time alongside with the evolution of the active views of the investment manager. In this paper the latest available risk exposures form the basis to estimate the ex-ante risk.

According to Litterman (2003) the return of a portfolio can be analysed into a linear factor model accounting for the systematic part of risk and an idiosyncratic component for the asset specific risk. This is summarized into the below equation:

$$r_{i,t} = a_i + \sum_{i=1}^{K} f_{i,t} dy_{i,t} + \varepsilon_{i,t} \qquad \textbf{4.1}$$

Where,

fi	the risk factor coefficients or the factor loadings
dy i	the change of the underlying yield or spread
εi	the idiosyncratic term

Idiosyncratic risk has zero mean as it incorporates, by assumption, unforeseen changes in the return of asset i. In addition because the error term is asset specific it is assumed to be uncorrelated with the systematic factors. The relevant risk representation becomes as follows:

$$\Sigma_R = F\Sigma_F F^T + \Sigma_\varepsilon \qquad \textbf{4.2}$$

Where,

F	the risk factor coefficients or the factor loadings per asset
∑F	the variance-covariance matrix of the yield and spread changes
∑ε	the diagonal idiosyncratic risk matrix per asset

Accordingly, a typical multifactor model for fixed income portfolios as described by Dynkin (2006) is composed of three parts; the systematic risk, the idiosyncratic term and the default risk. The latter two comprise all the remaining forms of risk not captured by the multifactor model. Various techniques described earlier, such as inserting an idiosyncratic risk component or a credit default component and introducing DTS as a risk factor into the risk model try to eliminate the specific risk terms.

$$\sigma_{Portfolio} = \sqrt{(\sigma^2_{Systematic} + \sigma^2_{Default} + \sigma^2_{Idiosyncratic})} \quad \textbf{4.3}$$

At this stage it is worth noting that two assumptions are imposed regarding the idiosyncratic and default risk. The portfolios examined include more than 1000 securities ensuring a high degree of diversification. As such, the idiosyncratic risk component will be neglected going forward. Idiosyncratic risk as a reflection of concentration risk is unlikely to arise for properly risk managed funds. However, if for some reason allocation leads to heavily loaded positions specific risk can potentially, though unlikely, become an issue. What is more possible is that the correlation risk goes up, rather than the concentration risk, which may have a similar effect in the overall riskiness of the portfolio, but can be captured as changing covariance is the variance matrix.

The default risk is also not further considered in this paper because the portfolios used are benchmarked against an investment grade index, the Barclays Capital US Aggregate Index. As the portfolios are actively managed a minor allocation was directed towards high yield in order to increase the return potential. The overall high yield exposure is limited to less than 0.1yrs of spread duration and the securities included are cherry picked so that over the course of our analysis there has not been any high yield default reported.

The starting point is the risk exposures available across twelve real portfolios as provided by a leading financial institution. This is a set of durations and spread durations across asset types. The risk factors used are split into two categories, those designed to capture the interest rate risk and those designed to capture the various sectors' spread changes. US duration, swap spread duration, US mortgage spread duration, US corporate spread duration, high yield spread duration and emerging markets spread duration are the risk measures which form the factor model used.

In this way two things can be tested: firstly the impact of the variations on interest rate level on portfolio performance and secondly the effect of credit spreads' variations on top of the US treasury curve on the bond portfolios' performance.

All in all, from a multifactor modelling perspective the performance equation becomes as per below:

$$Ptf\_Perf = D^T Y_d + \varepsilon \tag{4.4}$$

Where:

DKR is the (6x1) vector containing the duration and spread duration loadings

Yd is the (6x1) vector containing the changes in the US rate and various spreads

ε is the residual term

When referring to the relative to the benchmark exposures, the above expression becomes:

$$alpha = \tilde{D}^T Y_d + \varepsilon \tag{4.5}$$

Where:

$\tilde{D}$ is the (6x1) vector containing the relative duration and spread duration loadings

Y_d is the (6x1) vector containing the changes in the US rate and various spreads ε is the residual term

In order for the present analysis to be meaningful, the following assumptions are imposed:

- The systematic risk is fully described by the set of sectors chosen. If broader portfolios are examined, the underlying multifactor model needs expansion to accommodate the risks added. It is commonplace that market practitioners use models with hundreds or thousands of risk factors. As described in the literature though, a handful of risk factors should be enough for most fixed income portfolios.
- There are no price differences between the returns calculated by the investment manager and the index provider. Otherwise, the returns of the portfolio and the benchmark would not be comparable.

Since the multifactor model has been specified based on the available set of risk factors for the real portfolios used, the portfolios are mapped onto the risk factor space. The second step in order to proceed with the ex-ante tail risk estimation is to specify what the actual risk is for this combination of risk factor loadings. In order to do this we need to use an underlying risk model. The simplest option is to calculate the variance and covariance and thereafter deduct the tail exposure. Specifically, following Dynkin et al. (2006) and Litterman (2003) the total tracking error (TE) based on risk factor loadings is:

$$TE = \tilde{D}^T \Sigma \tilde{D} \qquad \textbf{4.6}$$

Where:

$\tilde{D}$ is the (6x1) vector containing the relative duration and spread duration loadings

$\sum$ is the (6x6) variance-covariance matrix on the changes of US rate and various spreads

In terms of calculating the variance-covariance matrix both the standard method has been followed as well as the Exponentially Weighted Moving Average as described by Jorrion (2002). The more recent observations are weighted more than the older ones, with the weight assigned to each observation fading down as we move back in time. The time decay factor λ is set at 1.5% and the forecast volatility for time t is defined as:

$$h_t = (1-\lambda)(h_{t-1}^2 + \lambda h_{t-2} + \lambda^2 h_{t-3} + ...) \qquad \textbf{4.7}$$

Where:

h_t is the estimated variance

h_{t-n} is the deviation of the observation at time t-n from the mean

After the calculation of the variance-covariance matrix, the estimation of the tail risk in the form of the Value at Risk (VaR) for a given confidence level is performed on the basis of the normality assumption. The VaR is calculated as follows:

$$VaR_{conf,t} = a_{conf} TE_t \qquad \textbf{4.8}$$

Where:

a_{conf} is the multiplicative factor depending on the confidence level and the distribution

TE_t is the tracking error, or the ex-ante estimate at time t of the standard deviation of alpha

An alternative approach to estimate the VaR is via resampling. Bootstrapping techniques have been introduced by Efron (1979) with the working assumption that the underlying data is independently and identically distributed. Bootstrapping is the effort to estimate a parameter $\hat{\theta} = S(\tilde{X})$ which is a function of a random variable x drawn by an unknown distribution F, where the only known is a sample of N observations (X1, X2, X3,..., XN). By regenerating a big number of pseudo samples $X^* = (X_4, X_1, X_N ... X_4)$ with replacement from the original sample an estimate of the parameter $\hat{\theta} = S(X^*)$ can be generated. When the process is repeated for a number of times the statistical properties for S can be deducted. As shown by Efron and Tibshirany (1993) and Shao and Tsu (1995) the bigger the number of simulations, the better is the approximation of S. For the special case of N being large the distribution of S converges to the normal distribution from the Central Limit Theorem.

Korajczyk (1985) made one of the early attempts to use bootstrap techniques for the analysis of financial problems. Because the bootstrap was originally developed on the data independence assumption, the bootstrap inference of several early analyses suffered from lack of the desired properties when the bootstrap was applied directly to raw returns, as presented in Hsieh and Miller (1990) and Levich and Thomas (1993).

One solution to this problem proposed by Kunsch (1989) was the moving block bootstrapping with blocks of fixed length. As stated by Ruiz and Pasqual (2002) the results of this type of bootstrap are not stationary even when the original time series are stationary. To address this Politis and Romano (1994) introduced the stationary bootstrap method which is using overlapping blocks of random length.

Given that the time series used in this paper are stationary and weak dependent, the block bootstrapping framework with blocks of random length introduced by Politis and Romano (1994) is followed. In order to approximate the distribution of the portfolio returns the yield and spread changes are bootstrapped based on the below algorithm:

$$B_{i,b} = \{X_i, X_{i+1}, ..., X_{i+b-1})$$ **4.9**

Where:

$B_{i,b}$ represents the block consisting of

b observations and

Xi is the first element of the block

Let the starting point of the block i be drawn from a discrete uniform distribution In~ {1,2,...,N}. The length of the block is drawn from a geometric distribution using the inverse of the geometric distribution cumulative density function:

$$L_k = \frac{\ln(1-X)}{\ln(1-p)}$$ **4.10**

Where,

X is drawn from a uniform distribution (0,1) and

p denoting a fixed number in [0,1]

The choice of the probability parameter P is critical as it controls the size of the block and affects the variance estimate of the model. Politis and Romano (1994) capped the block size to N so that this could be contained into the sample. In this paper, P=1/36 which stands for one month over 3 years of observations. The frequency of the performance we are trying to evaluate is monthly and this is how the choice of the numerator was made. The denominator represents that the rolling three year data used is analysed into 36 time periods. Setting P at 1/36 makes the block length contained within the sample size without having to impose the cap N restriction which would give a high concentration of blocks with length N. In this way the block length varies significantly over the different runs.

At this point it is useful to clarify that our set of original data (X1, X2, X3,... XN) is rearranged prior to performing the bootstrap. An extended sample where the data is placed end to end with itself (X1, X2, X3,... XN, X1, X2,..., XN) is used so as to ensure that the bootstrapping procedure is still feasible when In + Lk > N.

The parameter $\hat{\theta} = S(\tilde{X})$ which needs to be estimated here is the alpha estimate (see expression 4.5) based on the most recently available factor loadings of the portfolio. As shown in the next section the data used is weekly and in order to derive the monthly return impact we aggregate the change in the yields and spreads over the length of each block and then rescale it by a parameter $\xi = \frac{52}{12L_k}$ to convert the measure to its monthly equivalent.

Then using the multifactor expression 4.5 we estimate the alpha and store this number. The same process is run for 800 times and the value at risk is calculated. In order to achieve more robust results the above VaR calculation is repeated for 30 times and the average is computed. This technique is used to overcome computational limitations whilst incorporating a decent number of simulations leading to more stable results.

Once the VaR estimate is calculated the results of each method, the parametric VaR, the exponentially weighted moving average VaR and the Bootstrapped VaR, are back tested against the monthly alphas delivered across 12 real portfolios which provide more than 1,100 test points out of which the accuracy of the VaR estimates is challenged.

4.4 Data

The aim of this paper is to incorporate different models into a multifactor risk framework for the VaR estimation and examine to what extend the use of block bootstrapping is a step forward in the tail risk evaluation. In order to do so, the data set used is divided into two broad categories; the portfolio information, and the market information. This is the rationale of the multifactor risk models; that a well-diversified portfolio bears only market risk, which is explained by the set risk factors underlying the risk model. In that sense the risk is determined by how risky the market is and how much the portfolio is exposed to the risk factors that drive the market.

The portfolio information refers to the portfolio and benchmark monthly performance and to the resulting alpha. It also comprises the relative to the benchmark monthly risk factor loadings toward each risk factor i.e. the US duration, the swap spread duration, the mortgage spread duration, the corporate spread duration, the high yield spread duration and the emerging markets spread duration for the period February 2003 to July 2003. This data is provided by a leading investment management financial institution for twelve real actively managed portfolios against the Barclays Capital US Aggregate index.

In terms of the market data, which is more widely available, the 10 year US treasury yields were downloaded from Bloomberg, on weekly basis for the period February 1999 to July 2013. The weekly history of the 10 year US swap spreads was also pulled from Bloomberg for the same period of time. In order to measure the credit risk exposure of the portfolios the weekly history of option adjusted spread (OAS) and option adjusted spread changes has been pulled from Bank of America Merrill Lynch as shown in the table below:

Table 15: Data set of indices

Risk Factor	Index	Source	Yield	Spread	OAS	OAS weekly change
1	US Treasury 10yrs	Bloomberg	x			
2	US Swap Spread 10yrs	Bloomberg		x		
3	US Mortgage Master	BoA MLX			x	x
4	US Corporate Master	BoA MLX			x	x

5	US High Yield Master II	BoA MLX	x	x
6	Global EM Sovereign & Corporate	BoA MLX	x	x

When not readily available, the yield and spread changes are calculated in order to construct the variance covariance matrix and set the stage for the block bootstrapping. Finally, the track record of alpha is used to back test the findings of the model each month and test their accuracy.

At this point it is useful to analyse the properties of the data used. First of all, the risk is derived from changes in yields and spreads by using the difference of each measure and its first differences. Graphically this is shown in the below tables:

Graph 2

Figure 4-1: Changes in ten year yield

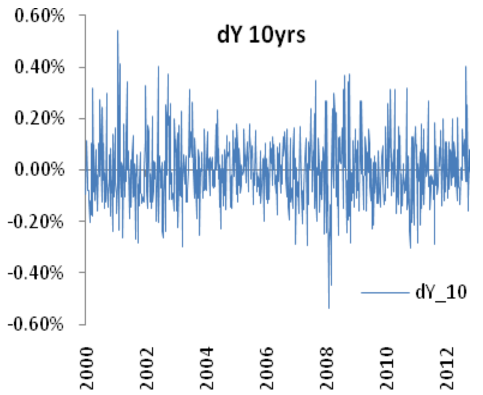


Figure 4-2: Changes in swap spread

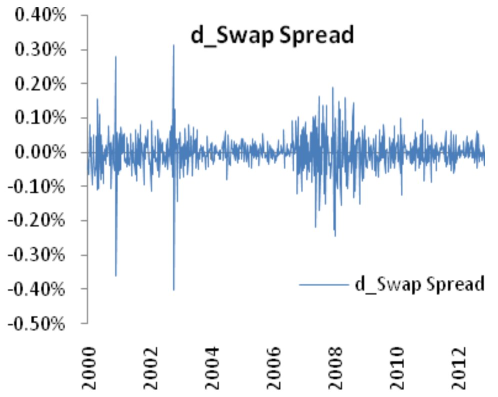


Figure 4-3: Changes in corporate spread

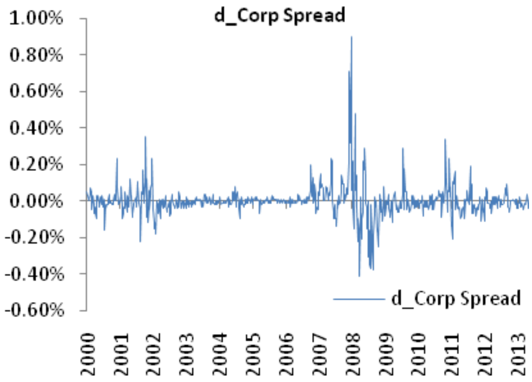


Figure 4-4: Changes in high yield spread

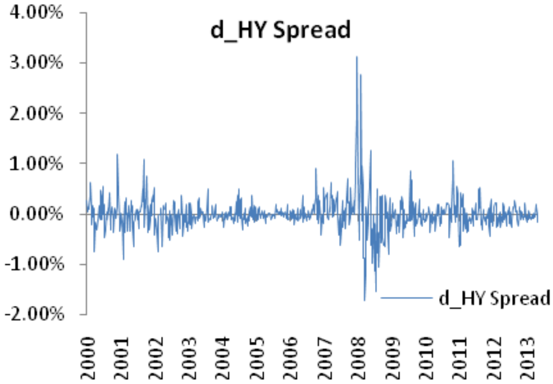


Figure 4-5: Changes in mortgage backed securities spread

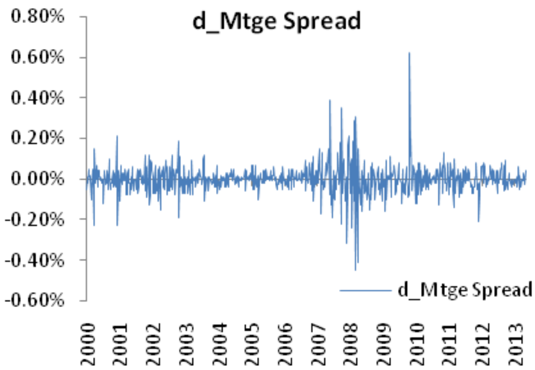
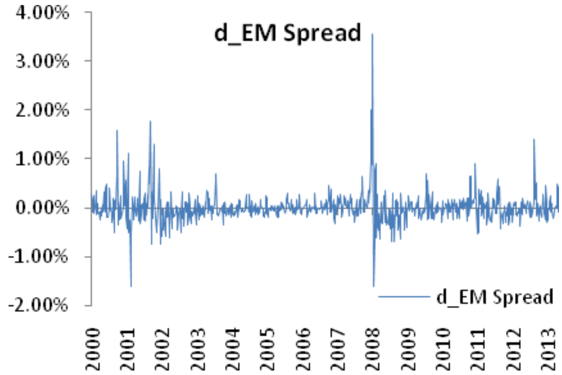


Figure 4-6: Changes emerging market spread

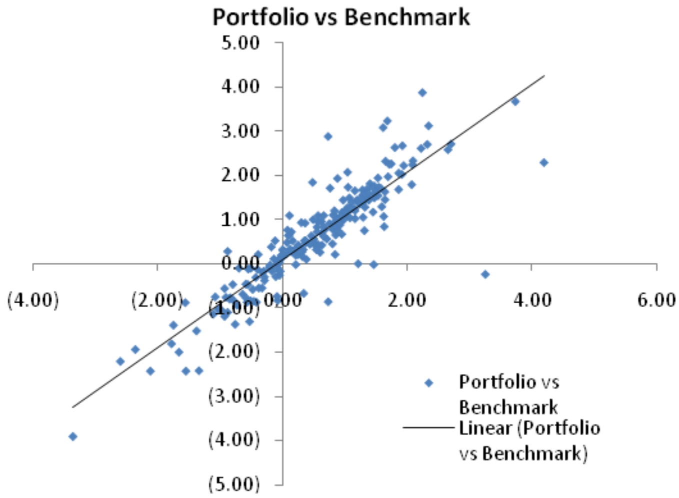


We have run an augmented Dickey-Fuller test for each of the above time series and the null hypothesis that there is a unit root has been rejected. This implies that the time series of the yields and spread differences are stationary. Chan et al. (1992) provide evidence on stationary interest rate changes which is supportive of what is illustrated in the first graph. The data has also been tested for serial correlation and has been found weekly dependent. This one of the reasons why a stationary bootstrapping that fits the properties of the data is chosen.

4.5 Empirical Results

The commitment of indexed active portfolio managers towards the investor is to deliver excess return over the chosen benchmark. The alpha generated over a period of time should ideally be located mostly in the positive territory but should remain well managed in terms of risk. In the below chart the returns of a real portfolio relative to the benchmark (US Aggregate) are depicted. The dispersion of the portfolio performance away from the benchmark performance is examined in this section.

Figure 4-7: Portfolio vs. benchmark return plot



At first, the risk associated with the volatility of alpha can be approximated by the information contained into the variance covariance matrix and a distribution assumption. Given the normality assumption several alternative methods are attempted for the estimation of the portfolio tracking error. Then the estimated VaR for 95% and 99% confidence levels are tested against the realised alpha over time. The alternative approach is linked with the approximation of the entire distribution of alpha via block bootstrapping instead of focusing on the tracking error calculation. Similarly, once the VaR for 95% and 99% confidence levels are computed, the results are tested against the realised alpha. The four methods tested in this paper are:

1. Calculation of ex post tracking error based on the monthly alpha and then the VaR estimation
2. Computation of the variance covariance matrix and deduction of the portfolio ex ante tracking error through the multifactor model specified in order to estimate the VaR
3. Calculation of an EWMA variance covariance matrix to extract the ex-ante tracking error and then estimate the VaR
4. Use of block bootstrap to generate the entire distribution for the alpha and from there estimate the VaR

The results for the twelve real portfolios tested for the period 2003 to 2013 for each of the above methods is presented in the below table:

Table 16: VaR estimates per method

Portfolio	VaR - 95%				VaR - 99%			
	Covariance - 3yrs data	EWMA- 1.5% decay	Ex Post Tracking Error	Block Bootstrap	Covariance - 3yrs data	EWMA- 1.5% decay	Ex Post Tracking Error	Block Bootstrap
1	4.2%	8.3%	4.2%	8.3%	4.2%	8.3%	0.0%	8.3%
2	6.4%	10.6%	4.3%	14.9%	4.3%	8.5%	4.3%	6.4%
3	0.0%	6.7%	6.7%	2.2%	0.0%	6.7%	4.4%	0.0%
4	6.7%	9.0%	7.9%	10.1%	4.5%	7.9%	3.4%	5.6%
5	0.0%	2.7%	4.5%	0.0%	0.0%	0.9%	1.8%	0.0%
6	2.4%	6.4%	6.4%	3.2%	1.6%	5.6%	3.2%	1.6%
7	3.2%	5.6%	4.8%	4.0%	0.0%	3.2%	4.0%	1.6%
8	2.9%	2.9%	6.7%	3.8%	1.0%	2.9%	4.8%	1.0%
9	3.2%	7.2%	5.6%	4.0%	3.2%	6.4%	2.4%	3.2%
10	1.6%	3.2%	4.8%	3.2%	0.8%	2.4%	4.8%	0.8%
11	1.9%	5.7%	6.7%	2.9%	1.0%	3.8%	4.8%	1.9%
12	8.7%	12.6%	5.5%	10.2%	7.9%	11.0%	3.1%	7.9%
Total:	**3.4%**	**6.4%**	**5.7%**	**4.9%**	**2.3%**	**5.2%**	**3.6%**	**2.8%**

Unlike the performance which is reported monthly and forms the basis for testing the accuracy of an attribution model, the accuracy of risk models cannot be that easily tested. A good way to back test the ex-ante estimates of a risk model is to rely on the ex post performance in order to challenge its accuracy. The focus is on the total VaR estimates as these are computed for more than a thousand observations.

Using the historic covariance matrix we have calculated both the 95% and the 99% VaR. On average 3.4% of VaR breaches occurred instead of the expected 5%, which were implied by the confidence level of the 95% VaR. Instead, 2.3% of VaR breaches occurred when confidence level was set at 99%. This means that this method overestimates the probability of a tail exceeding the 95% of the total distribution mass and underestimates the probability of a tail exceeding the 99% of the total distribution mass.

On the other hand side, the EWMA with decay rate at 1.5% displays breaches exceeding those implied by the confidence level. That is a clear sign of tail risk being underestimated. This risk estimation method has the property of adjusting the risk based on the most recent observations. When there are periods of low volatility being succeeded by periods of higher volatility, the risk calculated may suffer from neglecting the information carried in the more distant observations. A potential remedy could be to calibrate the decay parameter so that the optimum fit is achieved.

The third method which has been tested in this chapter is to simply count on the ex post tracking error to assess the total risk of the relative portfolio. This method delivered better risk estimates from the variance-covariance based alternatives but these results should be analysed further. Specifically, these results reflect the fact that the portfolios managed had the same benchmark and the same investment philosophy throughout the time period analysed. This is the reason why the ex post alpha has been relevant in risk forecasting. Such a method, would suffer tremendously, if no track record of alpha is available, in the case of newly set up portfolios and when there is a strategy or benchmark shift which would make the realized returns irrelevant to the risks associated with the most recent portfolio positioning.

Finally, the block bootstrapping method with overlapping blocks of random length has led to better results compared to the other methods. By imposing no distribution assumption and potentially allowing for excess skewness and kurtosis, the 95% VaR has been better approximated. However 2.8% of VaR breaches instead of the expected 1% of VaR for 99% confidence level are a clear sign that on the extreme the risk has been underestimated. This is understandable when we think of two different elements; the way that the bootstrapping works, and the behaviour of the markets during the last decade.
Even though bootstrapping is offering more flexibility in accommodating different shapes of distributions, it may suffer when estimating the extreme points. The minimum and maximum points are limited to those appearing in the sample used and there is no probability at all assigned to outcomes exceeding the upper and lower bound. This had been especially true in the results presented in this paper which account the crash of Lehman Brothers

and the unprecedented banking crises of 2008 which could not be fully captured by a bootstrap framework.

The following graphs depict the back test for the 95% and 99% VaR on the alpha of the 12th portfolio per each approach.

Figure 4-8: Back testing of parametric approach

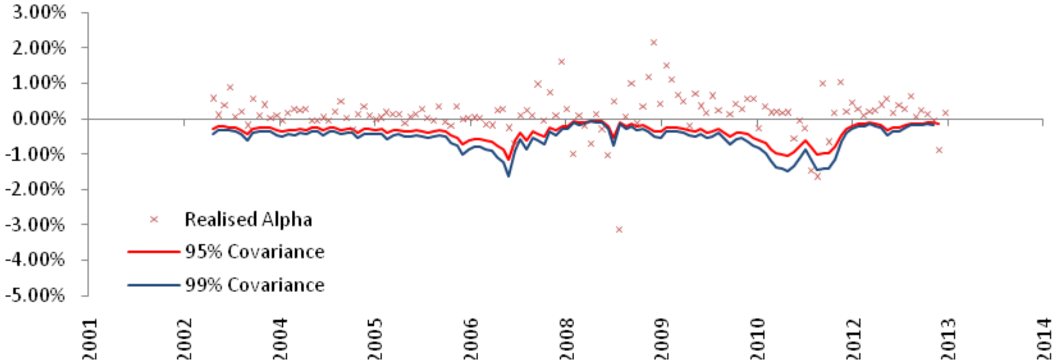


Figure 4-9: Back testing of exponentially weighted moving average approach

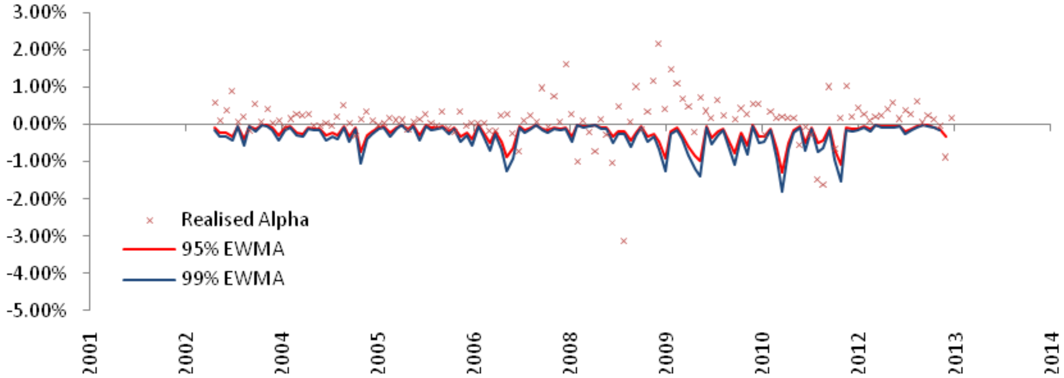


Figure 4-10: Back testing of ex post tracking error approach

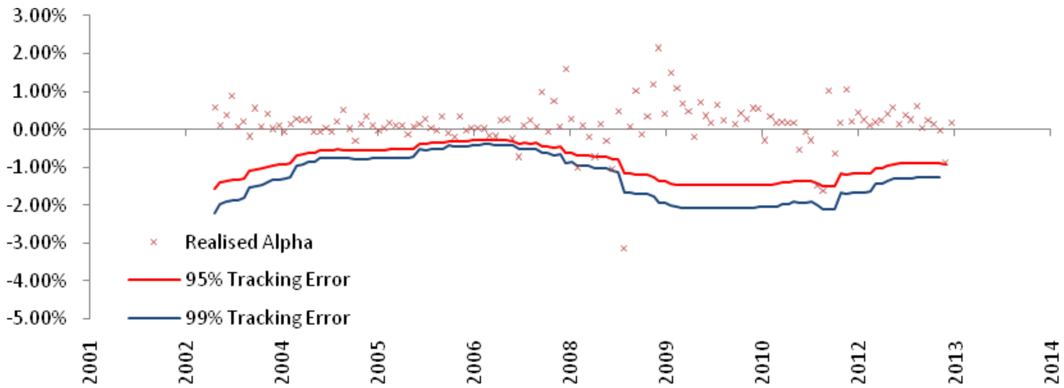
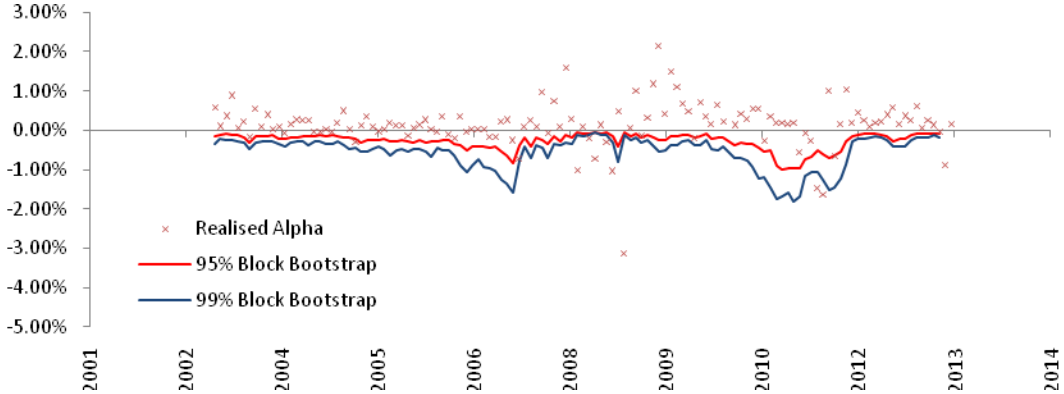


Figure 4-11: Back testing of block bootstrap approach



The value at risk used in this paper as a measure of risk is driven by two elements: the portfolio positioning and the market volatility. When the VaR increases this is because: i) the portfolio is more aggressively exposed vs. the benchmark, ii) the market volatility (including the correlations) has increased or iii) both of the above.

The smoothest risk estimates, displayed in graph 10, are generated by the ex post tracking error method, where the most recent relative to the index positioning of the portfolio is ignored. The market spikes are also very modestly affecting the risk calculation as the rolling tracking error tends to average up the dispersion around the alpha.

The VaR estimation method via the historic variance covariance matrix and via the block bootstrapping incorporates the latest available risk factor loadings relative to the index and this is why graph 8 and graph 11 are showing a more volatile VaR estimate when compared to the ex post tracking error results in graph 10. The EWMA (graph 9) is giving the most responsive results both to the changes of portfolio positioning and the changes of the overall market riskiness.

It is noteworthy that most of the breaches occurred during the period between 2008 and 2009 when the banking crises occurred. The worst performing month has been November 2008 both return wise and risk model wise. The poor performance was due to the investment view that the authorities would never let a major bank to go bankrupt. The investment view proved wrong and those portfolios which were in risk on mode substantially underperformed their index. This had an effect on the ex post tracking error which rose substantially thereafter.

One limitation of the various risk models became evident during the Lehman crisis; they are backward looking to some extent in a sense of relying to the available observations to predict the future. These models failed in predicting the turning point which was the Lehman crash. Some of the models adapt to the higher risk regime faster than some others but still failing to capture the turning point. Regime switching models or models (such as Black-Litterman), which incorporate investment views in the risk estimation can better accommodate extreme events like Lehman. This would be easier to use though as a portfolio construction tool rather than as a risk management tool given that the performance of the model would be dependent on the validity of the views.

4.6 Conclusion

Four different approaches are tested for the tail risk estimation of twelve real actively managed portfolios against the Barclays Capital US Aggregate Index. These methods are based on a multifactor risk framework where the portfolio exposures are mapped to the risk space. Working in a risk factor environment offers the dual benefit of feasibility for fixed income portfolios and high intuition at the same time. The value at risk for both the 95% and 99% confidence levels has been calculated under the normality assumption by estimating both a covariance matrix and an exponentially weighted covariance matrix. The ex post tracking error has also been used for the value at risk estimation. Finally, block bootstrapping with overlapping blocks of random length has been used for the same purpose allowing for the normality assumption to be relaxed.

Bootstrapping has given better estimates than the other methods and especially for the tail risk estimation with confidence level at 95%. This is indicative of the benefits emanating from blending together a fixed income multifactor risk framework and resampling techniques. This leads to better risk estimates than the traditional variance-covariance based approach and can be used both in risk budgeting and asset allocation processes. The main limitation of combining risk factors and bootstrapping has been illustrated by back testing the results against the realized alpha for the 99% confidence level. Even though the block bootstrapping performed better than the alternatives tested, it failed to fully capture the extremes occurred in 2008 as the estimated distribution was bounded by the maxima and minima occurred in the sample using information prior to the distressed period.

Amending the portfolio risk estimation not only can increase the efficiency of risk monitoring but can lead potentially to more resistant portfolios to tail risk events when incorporated into portfolio construction.

5 Bayesian Fixed Income Portfolio construction vs. Tail Risk Exposure: A Multifactor Risk Modeling Approach

5.1 Introduction

During the recent years the fixed markets have been dominated by extreme events such as the crisis in the U.S. mortgage subprime market, the collapse of Lehman Brothers and the resulting banking crisis, the dovish monetary policy which has led interest rates to historic lows and the extension of the central banks' balance sheets in the form of extensive quantitative easing programs at a time when conventional monetary stimulus tools had been exhausted.

Against this backdrop, where the valuation levels are important and on the other hand the financial and political risks are elevated, it is crucial for investors to be able to both quantify what the market consensus is and to input their own outlook into the investment process. The Black-Litterman framework and its various extensions available form a platform that can potentially accommodate both the market sentiment and the investor's outlook.

In this chapter, the market valuations are the starting point for estimating the expected returns. The second step is the consideration of investment views on any potential market shifts or political implications, which can drive market shifts. Blending the two together into a set of expected returns is feasible through Bayesian inference. The risk is considered as the entire returns distribution rather than the portfolio variance. This allows excess kurtosis and fat tail behaviour of the asset returns to be taken into consideration when forming the portfolios.

The outline of this chapter is as follows. In section 2, we review the related literature. Section 3 describes the methodology followed and section 4 refers to the dataset used for the estimation of results which are presented in section 5. Finally, section 6 summarizes the conclusions reached.

5.2 Literature Survey

Portfolio construction, in a nutshell, is the art of combining the expecting returns and the associated risks onto portfolio level. There are various ways of estimating the expected returns and various ways of estimating the portfolio risk. The most generic idea of combining return expectations and risk goes back to the mean variance optimization introduced by Markowitz (1952). Even though the shortfalls of mean variance optimization have been criticized in the literature, Markowitz set the stage for the asset allocation problem.

The main criticism was concentrated to the estimation error of risk and expected returns' parameters per se. According to Michaud (1989) the mean variance is in fact an estimation error optimization due to the nature of the process which magnifies the errors linked with the input estimates. These findings were also supported by Britten-Jones (1999). Additionally, the mean variance has been contested by Green and Hollifield (1992) for its ability to produce adequately diversified portfolios.

In order to resolve this problem, Michaud (1998) and Michaud (2008) proposed resampling techniques as a remedy of the "error maximization" problem. Accordingly, the five step solution proposed could be summarized in (1) sampling a mean vector and a covariance matrix for the returns, (2) computing the efficient frontier based on these risk and return estimates, (3) repeating the above process several times (4) averaging up the results of step two and (5) take into consideration any investment restrictions.

On the other hand side Ledoit and Wolf (2004) and Ledoit and Wolf (2006) introduced the concept of shrinkage in the estimation of the sample covariance matrix. They presented an alternative risk estimation which smoothed the correlation coefficients towards a common constant correlation and reduces the dimensionality of the covariance matrix. The merit of this technique is the computational simplicity and the unbiased outcome. According to the authors the shrinkage framework is in line with the notion of multifactor risk models which are the market standard.

A third way to avoid corner solutions in portfolio optimization is to use the model introduced by Black and Litterman (1992). The Black-Litterman model is using a Bayesian framework to blend together the implied equilibrium returns and the investor's views. In a CAPM environment, the equilibrium returns are implied by the market portfolio weights, the realized volatility and an assumed risk aversion coefficient. That is to say that the equilibrium returns are those making the market portfolio optimum. In terms of the blending process Black and Litterman (1992), He and Litterman (2002), Idzorek (2004) consider the set of equilibrium returns as the prior; whilst Satchel and Scowcroft (2000) consider the investor views as the prior distribution and the equilibrium returns as the likelihood.

The main advantage of the Black-Litterman model is that it leads to a more reasonable allocation which is aligned, to some extent, with the market portfolio. Furthermore, it allows the investment manager to input his views, if any, both in absolute and in relative terms. Even though BL has clearly been a step forward in the asset allocation process, it is linked to the shortfalls of CAPM. The reference Black-Litterman model was nicely presented by Walters (2009).

Following Black and Litterman (1992), He and Litterman (1999) provided a more detailed analysis on the way the reference model works. Satchel and Scowcroft (2000) presented an alternative solution to the Black-Litterman equation, where the prior distribution was formed by the investor views and the market equilibrium returns were used as the likelihood. Mankert (2006) described a different way to derive the Black-Litterman model via sampling theory. Idzorek (2004) extended the BL model so that confidence levels can be used as input parameters together with the investment views.

Fusai and Meucci (2003) provided a framework for the assessment of the input views so that corner solutions are avoided. This involved fine tuning of the views, in a manner that the model would not get disruptive and the resulting allocation would remain reasonable. Krishman and Mains (2005) re-derived the BL model on the basis of a two factor risk framework. The notion is that the single risk factor, traditionally used as part of the utility function is not enough to fully represent the overall portfolio risk. Therefore a recession risk factor has been added to better represent the actual risk dynamics into the utility function.

Giacometti et al. (2007) extended the Black-Litterman model to accommodate other distributions in addition to the normal distribution. Their findings are generated using a multivariate Gaussian distribution, a symmetric t-student, and a-stable distributions. Their second goal has been to ameliorate the BL model by using different risk measures such as dispersion-based risk measures, value-at-risk and conditional value-at-risk in line with Gaivoronski and Pflug (2005).

Qian and Gorman (2001) proposed a way to obtain a conditional mean vector and a conditional covariance matrix given the investment views. Through their analytical derivation of the conditional covariance matrix, they ensured that the results generated by mean-variance analysis tend to be stabilized.

Almgren and Chriss (2006) proposed an allocation formulated on three ingredients; ordering information which leads to a cone of consistent results, a probability density and a constraint set to which the portfolio is limited to. According to the authors, this method has led to more robust results as opposed to the original Black-Litterman model.

Pezier (2007) used a distance measure between return forecasts based on investment views and return forecasts based on market equilibrium. By minimizing this distance they tried to examine the impact of the investment views on the optimization process and allowed them to be revised until a satisfactory combination of forecast and optimal allocation was reached.

Meucci (2006) proposed a “copula opinion pooling” approach to accommodate deviations from normality when combining assets to portfolio level. In this framework the posterior was numerically derived through Monte Carlo Simulations. Meucci (2008) revisited the proposed framework to allow for correlation stress testing and non-linear views. To do so, he introduced the so called entropy pooling approach which delivered superior results over the other BL extensions, presented above, as stated in Meucci (2010). Meucci (2009) highlighted that the BL is likely to be restricted by the normality assumption for the markets exhibiting a skewed, non-normal returns profile. However, he showed that when the randomness in the underlying risk factors is normal, the BL model can still be used. The case study used in this paper focused on European call options, where the risk could be approximated by delta and gamma, especially over a short term time horizon.

Martellini and Ziewann (2007) illustrated a BL extension which incorporated higher moments of returns distribution as part of the portfolio construction process. They contested the accuracy of the standard CAPM and preferred instead a four-moment asset pricing model to derive the equilibrium return.

Cheung (2007) proposed an extension of BL to accommodate equity risk factor models. He utilized a linear factor model based on economic and financial risk factors to capture the variability of equity returns. Conner et al. (2010) presented a similar factor model. The “Augmented Black Litterman” is a smooth process to blend portfolio views and equilibrium returns for stock portfolios as part of a factor based portfolio construction process. One of the main concerns regarding the model is the normality assumption which is imposed.

Several hedge fund related studies, focused on the fat tail features of return distributions which cannot be captured under the normality assumption umbrella. Such studies include Favre and Galeano (2002), DeSouza and Gokcan (2004), Agarwal and Naik (2004) and Harris and Mazzibas (2010). A common theme among them is constructing portfolios against tail risk measures such as value at risk and conditional value at risk.

Aguilar and West (2012) used Bayesian inference focusing on the dynamic factor nature of spot foreign exchange rates with a goal of forming an international currencies portfolio. They have used a k factor dynamic risk model consisted of a systematic and an idiosyncratic component. The systematic risk was approximated by a factor loadings matrix and a factor covariance matrix.

In this chapter, the focus is on the fixed income portfolio construction and as a result a different multifactor model is used. The scope of this study is to fill in the gap in the literature regarding the construction of fixed income portfolios using multifactor risk models, Bayesian inference to blend equilibrium returns with investment views and block bootstrapping technology for the estimation of the tail risk. Thanks to the nature of fixed income markets, a different formulation of the equilibrium returns is used, as opposed to a CAPM based methodology followed, largely speaking, in the literature.

5.3 Methodology

We consider the Barclays Capital US Aggregate index as the investment universe for the construction of active portfolios. The optimization process is centred to the benchmark selected, which is the starting point for the allocation selected. Additionally we work in the context of a multi-factor risk model which is suitable for the risk representation of fixed income portfolios. Following Meucci (2008), we assume that a risk model for the joint distribution of risk factors exists and is described by its probability density function (pdf). In a Black-Litterman framework this would form the prior distribution of risk factors.

$$X \sim f_x \tag{5.1}$$

The above representation is used in order to estimate the volatility, the relative to the index volatility or tracking error, the value at risk etc. From an investment management perspective this model is used for the optimization of the portfolio positions. The final allocation is dependent on the utility function linking the weights w of the portfolio with the underlying distribution. In light of the above, the optimum portfolio of weights w* is described by the following:

$$w^* = \arg\max_{w \in c}\{U(w; f_x)\} \tag{5.2}$$

Where:

C represents the set of investment constraints

U represents the total utility

w represents the positioning of the portfolio and

f_x represents the probability density function

The views which are used as input are expressed on generic functions $g_1(X),\ldots,g_K(X)$. This is how a K-dimensional random variable is formed with joint distribution:

$$V = g(x) \sim f_v \quad (5.3)$$

Where,

V represents the expressed view

g(x) represents the generic functions of the market

f_V represents the probability density function associated with the view

Following Meucci (2008) we make use of the entropy between a generic distribution $\tilde{f}_x$ approximated by block bootstrapping and the reference model f_x as follows:

$$\varepsilon(\tilde{p}, p) = \sum_1^N \tilde{p}\,(ln\tilde{p} - lnp) \quad (5.4)$$

Where:

ε is the entropy between the generic and the reference distribution

$\tilde{p}$ is the reference probability of bootstrapped distribution function

p is the generic probability of bootstrapped distribution function

N is the number of simulations defining the probability distribution

Practically, this is a measure of how distorted the generic distribution is as opposed to the reference distribution. In case that the two distributions are identical, the entropy is zero. As long as constraints are imposed on the generic distribution, the relative entropy goes up. That told, the posterior distribution is defined as:

$$\varepsilon = arg\,\min_{f \in V}\{\varepsilon(\tilde{p}, p)\} \quad (5.5)$$

Where $f \in V$ indicates the distributions which are aligned with the views statements. In case that the investor has no views, V is becoming an empty set and the posterior distribution equals the reference distribution f_x. On the other hand side, when the view statements fully describe the joint distribution the minimization process presented above is not needed. In this case the posterior distribution would become:

$$\tilde{f}_x(x) = \int f_{x|v}(x)\,\tilde{f}_V(v)dv \quad \textbf{5.6}$$

If the investment manager is not fully confident on the views imposed in the equation (5) as a set of restrictions then the posterior distribution is becoming as follows:

$$p_C = (1 - C)p + C\tilde{p} \quad \textbf{5.7}$$

Where:

p_C	represents the bootstrapped probability
$\tilde{p}$	represents the probability after taking into account the views
C	represents the confidence of the investor to the expressed views

At this stage it is essential to specify the parameters which are going to be used in the framework described above. First it is crucial to elaborate more on the specification of the equilibrium returns used. Consistent with the semi-strong form of market efficiency as presented in Fama (1970) and used in Cheung (2007), based on the publically available information the market view should be the best available view. The market views are typically incorporated in the asset prices. This is valid for the fixed income markets where the market defines the bond prices. Bearing in mind that the investment universe is an investment grade bond index, the yields implied by the price levels form the most reliable set of expected returns reflecting the market consensus. As a result, in this paper the equilibrium returns are not provided by any type of CAPM framework, or any extension of it, but rather sticks to what the market dictates via price levels.

That said, it is time to elaborate more on the reference multifactor risk model. In this paper we follow Ho (1992) who set a number of maturities on the yield curve as being the key rate durations, with typical values of 3 months, 1, 2, 3, 5, 7, 10, 15, 20, 25 and 30 years. Duration is estimated to measure interest-rate sensitivity, to a movement of the yield, at each of the above points in isolation. In other words, key rate duration estimates the effect of a change in the term structure which is localized at a particular maturity point, and restricted to the immediate proximity of this maturity point, usually by having the change drop linearly to zero at adjacent points. In this paper we are restricted to use slightly different key maturities due to the data availability by the index provider as presented later on. Additionally, in order to avoid making the reference model too congested with risk factors a duration risk measure against the ten year maturity is used as a single risk factor for all the non-US issuers.

Recently, Dor et al. (2007) introduced Duration Times Spread (DTS) as a solution to the spread risk aggregation problem, revolutionizing the spread exposure measurement. The notion behind DTS is that the volatility of spread changes is linearly proportional to spread level. Spread duration measures the sensitivity of a portfolio to the changes of a reference spread in absolute terms. DTS instead focuses on the sensitivity to the relative (percentage) spread change, practically by scaling up or down the spread duration exposure, based on the spread level of each security.

Given that the time series used in this paper are stationary and weak dependent, the block bootstrapping framework with blocks of random length introduced by Politis and Romano (1994) is followed. In order to approximate the distribution of the portfolio returns the yield and spread changes are bootstrapped based on the below algorithm:

$$B_{i,b} = \{X_i, X_{i+1}, ..., X_{i+b-1})$$ **5.8**

Where:

$B_{i,b}$ represents the block consisting of

b observations and

X_i is the first element of the block

Let the starting point of the block i be drawn from a discrete uniform distribution I_n~ {1,2,...,N}. The length of the block is drawn from a geometric distribution using the inverse of the geometric distribution cumulative density function. The above process is repeated for thousand times so as to draw the reference distribution non-parametrically. This technique has the advantage of keeping the correlation structure between the risk factors over time and not restricting the results by any distribution assumption that would be required in a parametric environment. In this way, the risk profile of the risk factors is derived via the block bootstrapping and the entropy pooling processes.

The expected changes of yields and spreads are also given by the model at this stage by incorporating both the equilibrium returns and the views. Equilibrium returns refer to the yield advantage of each portfolio component versus the yield of the chosen benchmark, under the assumption, that there are no defaults and the current state of the world does not change (including shape of yield curves and spread levels). This would be the starting point for designing "carry" strategies to benefit from a static rather than a changing investment landscape over a specific time horizon. The portfolio returns against the benchmark index (alpha) are:

$$alpha = \sum_1^N (W_{Pi} - W_{Bi})(r_i - r_B)$$ **5.9**

Where:

W_{pi} represents the weight of the i-th component of the portfolio

W_{Bi}	represents the weight of the i-th component of the benchmark
r_i	represents the return of the i-th component of the portfolio
r_B	represents the overall return of the benchmark

In this way, two factors drive the excess return of a portfolio; the over-weight or under-weight of each instrument and the relative performance of the instrument against the benchmark. i.e. overweighting an instrument which outperforms the benchmark is a contributor to return and so on so forth.

Let us now set aside the equilibrium returns/excess returns, and focus on what is happening when the market moves. The skill required by an active manager, is to deliver excess return, as a result of actively taking advantage of the changing investment environment. A way to measure the effect of "directional" trades as opposed to "carry" ones is by considering the views on market changes and the exposure of the portfolios to them:

$$alpha = (W_p - W_B)FE \qquad \textbf{5.10}$$

Where:

W_p	is the vector with weights of the portfolio
W_B	is the vector with weights of the benchmark
F	is the matrix showing the risk factor loadings per component of the portfolio and the benchmark
E	is the vector containing the expected changes in yield and spread levels based on the views

$$E = B^T P_C^T \qquad \textbf{5.11}$$

Where:

B is the matrix with the block bootstrapped scenario over the changes in yields and spreads

P_C is the vector with the posterior probabilities over the block bootstrapped scenario

By combining (9), (10) and (11) the overall excess return of the portfolio becomes:

$$alpha = (1 - C)(\widetilde{W} - W)Y^T + C(\widetilde{W} - W)FB^T P_C^T \tag{5.12}$$

Where:

C represents the investor's confidence in the views

W is the vector with the initial weights of the portfolio which are set to be equal to the benchmark's weights $W_p = W_B$

$\widetilde{W}$ is the vector of the targeted weights of the portfolio to be optimized

Y represents the equilibrium returns vector

F is the matrix showing the risk factor loadings per component of the portfolio and the benchmark

B is the matrix with the block bootstrapped scenario over the changes in yields and spreads

P_C^T is the vector with the posterior probabilities over the block bootstrapped scenario

The Y vector (where the equilibrium returns are approximated by the yields of investment grade bonds) is in use under two assumptions. The no default assumption and the assumption of even allocation of yield over the time to maturity. In reality the bonds appreciate more at those maturity points where the yield curve is steeper (and upward sloping). Expectations towards modifications of the yield curve shape are captured by the second part of equation 5.12.

The associated portfolio risk/returns distribution, based on the distance $\widetilde{W} - W$ is taking the form below:

$$R = (\widetilde{W} - W)FB^T \qquad \textbf{5.13}$$

Where:

W is the vector with the initial weights of the portfolio which are set to be equal to the benchmark's weights $W_p = W_B$

$\widetilde{W}$ is the vector of the targeted weights of the portfolio to be optimized

F is the matrix showing the risk factor loadings per component of the portfolio and the benchmark

B is the matrix with the block bootstrapped scenario over the changes in yields and spreads

By taking into account the representation (13), which forms the set of block bootstrapped portfolio relative returns, and the probability vector P_C we can calculate the standard deviation, the VaR and the CVaR of the portfolio.

The aim of this framework is to provide a set of tools for each part of the investment process. Those are the consideration of the current valuations in the form of equilibrium returns, and the estimated impact of views onto both the return and risk profile of the constructed portfolio. The blending mechanism can either be used to form the expected returns on a set of pre-defined views or even help identifying the views per se by incorporating the dependence structure of the data into the analysis.

5.4 Data

First of all, it is necessary to stress, that the nature of the benchmark used determines the structure of the multifactor model chosen. In this paper, a set of managed fixed income portfolios, benchmarked against the Barclays Capital US Aggregate Index, were selected. Such portfolios do not bear any currency risk as all the securities included are denominated in USD. However this benchmark allows for issuers from various countries. In fact, more than 91% of the index is comprised of issuers in the United States, and the remainder reflects issuers from outside the United States. To avoid the extra layer of complexity, in order to accommodate the various countries' interest rates and spread exposures and for the sake of keeping the dimensionality of the problem lower than higher, the benchmark is restricted to US Aggregate US Only Index.

The portfolio information refers to the monthly performance and the resulting alpha of the portfolio and benchmark. Other basic statistics may also become available but they are not going to be further elaborated.

The benchmark information required was split into two categories: the risk factor exposures and the asset groups. As for the multifactor risk model, a series of US Aggregate US Only rating-by-sector and by-maturity sub-indices have been customized in Barclays Capital POINT. Additionally, the duration and spread duration exposures per country of issue were pulled from Barclays Capital POINT, for the subset of Barclays Capital US Aggregate US only Index corresponding to non-US issuers. Equivalently, US key rate durations were used for the interest rate exposure.

The data sourced for the multifactor model are the US key rate durations of the following maturity points: 0.5yr, 2yrs, 5yrs, 10yrs, 20yrs and 30yrs for each of the above indices. Additionally the option adjusted spread (OAS) and the option adjusted spread duration (OASD) has been sourced for each of the reference indices presented in table 17. The data frequency is weekly and the time period is from 31/12/2012 to 31/10/2014.

The underlying US treasury yields per key maturity were downloaded from Bloomberg. Specifically, the key rates' history was sourced on a monthly basis for the period 31/12/2012 to 31/10/2014. The OAS levels were sourced from Barclays Capital POINT. A summary of the market data used is displayed in table 17 overleaf.

A special note should be made for high yield and emerging market debt which are not included in the following tables at all. As stated in the methodology section, the equilibrium returns described are meaningful under the no default assumption. High yield debt and emerging market debt are prone to default and because of this their yield to maturity may decouple substantially from their expected return if held to maturity. Moreover, the emerging market bonds require special treatment as some are issued in local currency and some in hard currency. Introducing them in the dataset would add an extra layer of complexity in the specification of the factor model without adding much to what the core scope of the present chapter is.

Table 17: Data set per sub-indices and risk measures I

Index	Market Value in USD	ISMA Mod Duration	ISMA Mod Convexity	KRD 0.5	KRD 2yr	KRD 5yr	KRD 10yr	KRD 20yr	KRD 30yr	YTM	OASD	OASC	DTS	OAS
US Aggregate Treasury 1-3yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Treasury 3-5yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Treasury 5-7yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Treasury 7-10yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Treasury 10+yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Financial AA 1-3yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Financial AA 3-5yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Financial AA 5-7yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Financial AA 7-10yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Financial AA 10+yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Financial A 1-3yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Financial A 3-5yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Financial A 5-7yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Financial A 7-10yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Financial A 10+yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Financial BBB 1-3yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Financial BBB 3-5yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Financial BBB 5-7yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Financial BBB 7-10yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Financial BBB 10+yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Industrial AAA 1-3yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Industrial AAA 3-5yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Industrial AAA 5-7yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Industrial AAA 7-10yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Industrial AAA 10+yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Industrial AA 1-3yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Industrial AA 3-5yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Industrial AA 5-7yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Industrial AA 7-10yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x

Table 18: Data set per sub-index and risk measures II

Index	Market Value in USD	ISMA Mod Duration	ISMA Mod Convexity	KRD 0.5	KRD 2yr	KRD 5yr	KRD 10yr	KRD 20yr	KRD 30yr	YTM	OASD	OASC	DTS	OAS
US Aggregate Corporate Industrial AA 10+yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Industrial A 1-3yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Industrial A 3-5yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Industrial A 5-7yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Industrial A 7-10yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Industrial A 10+yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Industrial BBB 1-3yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Industrial BBB 3-5yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Industrial BBB 5-7yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Industrial BBB 7-10yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Industrial BBB 10+yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Utility AA 1-3yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Utility AA 3-5yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Utility AA 5-7yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Utility AA 7-10yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Utility AA 10+yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Utility A 1-3yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Utility A 3-5yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Utility A 5-7yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Utility A 7-10yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Utility A 10+yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Utility BBB 1-3yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Utility BBB 3-5yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Utility BBB 5-7yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Utility BBB 7-10yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Corporate Utility BBB 10+yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Govt Related AAA 1-3yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Govt Related AAA 3-5yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Govt Related AAA 5-7yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x

Table 19: Data set per sub-index and risk measures III

Index	Market Value in USD	ISMA Mod Duration	ISMA Mod Convexity	KRD 0.5	KRD 2yr	KRD 5yr	KRD 10yr	KRD 20yr	KRD 30yr	YTM	OASD	OASC	DTS	OAS
US Aggregate Govt Related AAA 7-10yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Govt Related AAA 10+yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Govt Related AA 1-3yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Govt Related AA 3-5yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Govt Related AA 5-7yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Govt Related AA 7-10yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Govt Related AA 10+yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Govt Related A 1-3yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Govt Related A 3-5yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Govt Related A 5-7yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Govt Related A 7-10yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Govt Related A 10+yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Govt Related BBB 10+yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Securitised CMBS_ABS AAA 1-3yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Securitised CMBS_ABS AAA 3-5yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Securitised CMBS_ABS AAA 5-7yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Securitised CMBS_ABS AAA 7-10yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Securitised CMBS_ABS AAA 10+yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Securitised CMBS_ABS AA 1-3yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Securitised CMBS_ABS AA 3-5yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Securitised CMBS_ABS AA 7-10yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Securitised CMBS_ABS A 1-3yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Securitised CMBS_ABS A 3-5yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Securitised CMBS_ABS A 7-10yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Securitised CMBS_ABS BBB 1-3yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Securitised CMBS_ABS BBB 3-5yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Securitised CMBS_ABS BBB 7-10yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Securitised MBS AAA 1-3yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Securitised MBS AAA 3-5yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x

Table 20: Data set per sub-index and risk measures IV

Index	Market Value in USD	ISMA Mod Duration	ISMA Mod Convexity	KRD 0.5	KRD 2yr	KRD 5yr	KRD 10yr	KRD 20yr	KRD 30yr	YTM	OASD	OASC	DTS	OAS
US Aggregate Securitised MBS AAA 5-7yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Securitised MBS AAA 7-10yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Securitised MBS AAA 5-7yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x
US Aggregate Securitised MBS AAA 7-10yrs	x	x	x	x	x	x	x	x	x	x	x	x	x	x

Table 21: Data set of rates

	Bloomberg Data - Rates					
	0.5yr	2yr	5yr	10yr	20yr	30yr
US Government Bond Generic Index	x	x	x	x		x
US Interest Rate Swap Generic Index					x	
US Interest Rate Swap Spread Generic Index					x	

5.5 Results

The purpose of this framework is to assist in the estimation of both the expected returns and the underlying total risk. In order to identify the set of expected returns an intuitive prior is the starting point, the yield to maturity for investment grade bonds. In this case the optimum portfolio would be a combination of the yield advantage each asset class offers for a given risk budget. In case the investment manager has a market view, this can be used as input in the model. The effect of incorporating a set of views is dual. The set of the expected returns will be changed and so will the underlying distribution. This process can prove informative in terms of better understanding the market dynamics as it is generates representations of the most likely performance profile of the asset classes, part of the defined investment universe, for which no view is expressed. The views can be either absolute or relative.

According to one of the scenarios analysed the US economy shows signs of recovery and this is translated into a view that the very short end of the yield curve i.e. the 6 month point will eventually increase by 50bps. When no additional restrictions are imposed, the set of expected returns incorporating the rising rates' view is as follows:

Table 22: Scenarios description

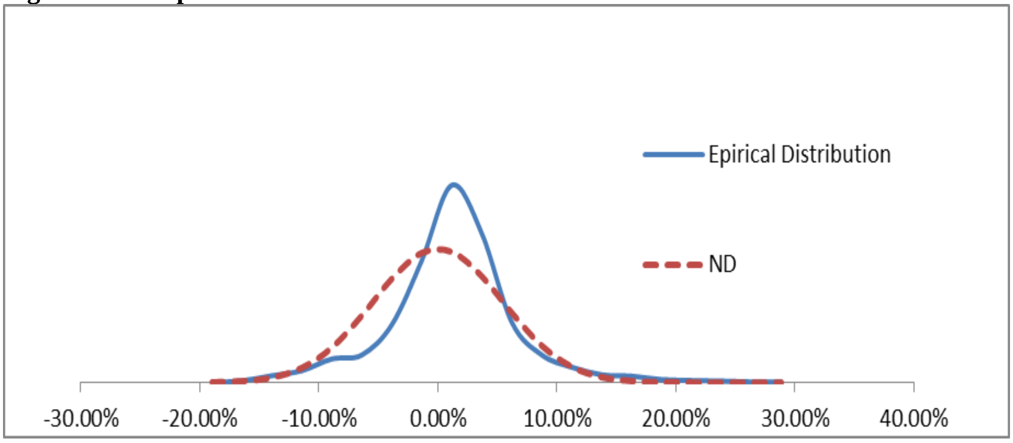
Expected change (in bps)	5000 Obs (Case 1: E[X1]=+50bps)	5000 Obs (Case 2: same as case 1 but allow all E[Xi] to change)	5000 Obs (Case 3: E[X3]-E[X5]=+50 bps)	5000 Obs (Case 3: E[X3]-E[X5]=+25 bps)	5000 Obs (Case 4: E[X11]-E[X12]=+50 bps)
d_USGG6M Index	50	50	64	30	-10
d_USGG2YR Index	0	42	67	33	-10
d_USGG5YR Index	0	28	42	22	-9
d_USGG10YR Index	0	15	7	4	-7
d_USSW20 Curncy - d_USSS20 Curncy	0	10	-8	-3	-6
d_USGG30YR Index	0	6	-21	-10	-6
dOAS_US Aggregate Corporate Utility	0	-13	-10	-7	17
dOAS_US Aggregate Corporate Financial	0	-17	-10	-8	21
dOAS_US Aggregate Corporate Industrial	0	-13	-10	-7	17
dOAS_US Aggregate Govt Related	0	-3	-6	-3	3
dOAS_US Aggregate Securitised CMBS_ABS	0	-35	-33	-21	53
dOAS_US Aggregate Securitised MBS	0	-3	-2	-2	3
dOAS_US Aggregate Treasury	0	-1	-2	-1	1

What is implied by the above table is that based on the bootstrapped data, the risky assets tend to perform well in hawkish interest rates environment. Alternatively this can be an indication that elevating levels of interest rates comes as a response to a warm up of the economy which normally affects risky assets positively and the bond market valuations are adjusted accordingly.

However, given the interesting investment landscape we are experiencing, characterised by both yields and spreads at historic lows, one needs to be cautious when using these results. What would make sense in other market conditions is not necessarily valid; namely, the further compression of the spread levels.

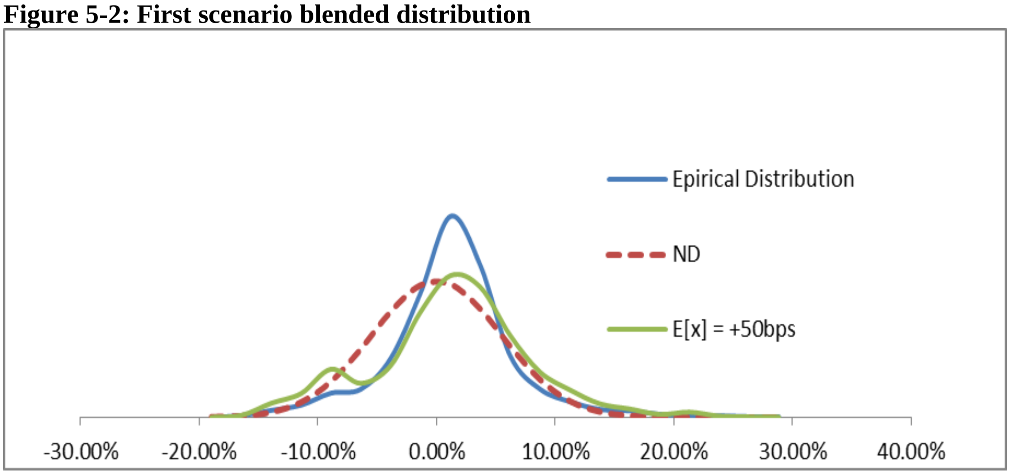
The first graph shown below is a depiction of the aggregated bootstrapped distribution against the normal distribution. There is evidence that the actual distribution exhibits excess kurtosis as opposed to normal distribution.

Figure 5-1: Empirical distribution



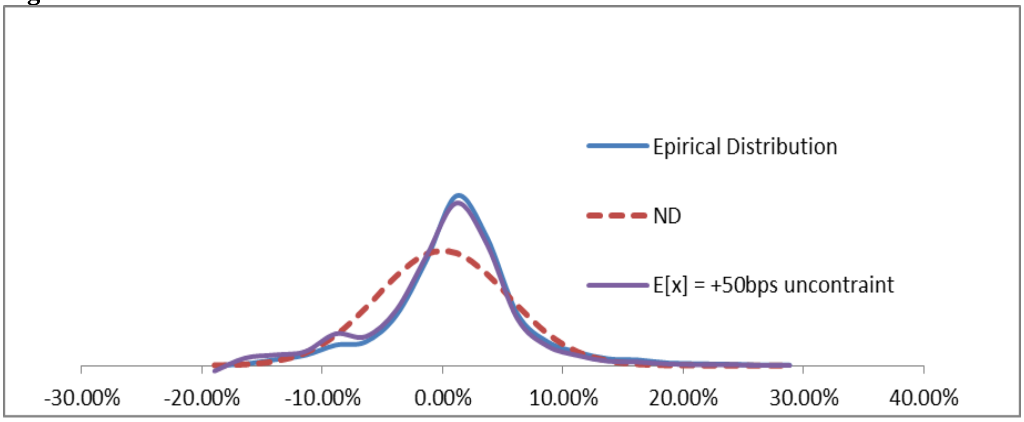
The first scenario examined, reflects a rates rise which would only affect the front end of the curve with every other maturity and all risky assets remaining unchanged. We observe that the optimum solution reached is closer to the normal distribution but is still leptokurtic aligned with the prior distribution.

Figure 5-2: First scenario blended distribution



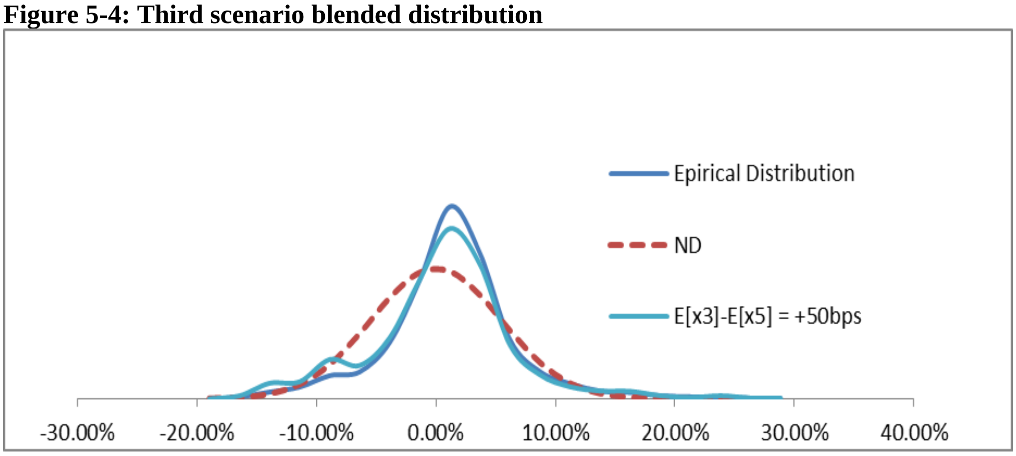
Equivalently, as described above, the following graph illustrates the differentiation when there is a 50 basis points rise to the level of short dated rates and all other maturity points and the corresponding spreads of risky assets are left unconstrained. The risk profile of the aggregated distribution is closer to the prior with a heavier left tail. As mentioned earlier, cautious interpretation of the linkage between raising rates and spread compression is advised given the current spread levels of major investment grades indices.

Figure 5-3: Second scenario blended distribution



In the following example a relative rather than absolute view is analysed. One specific point of the curve is going to be more affected by the rising rated than a slightly later maturity. The model still derives a solution which is linked to the statistical properties of the prior.

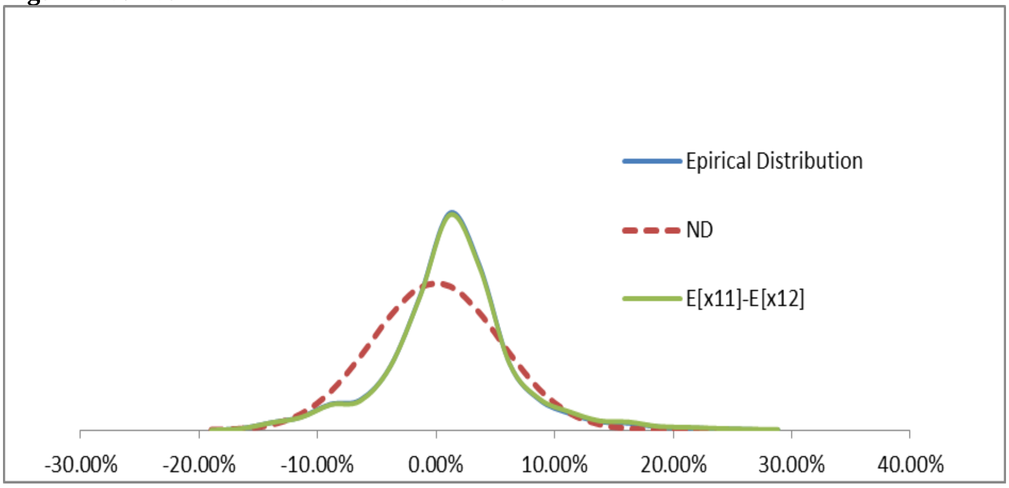
Figure 5-4: Third scenario blended distribution



The model is quite flexible to accommodate potentially of any type of view, absolute or relative. Then a full joint distribution based on the input view is provided which incorporates both the views and the resulting expected changes in the unconstrained variables.

In the graph to follow, the assumption imposed is that Securitized CMBS and ABS will underperform Securitized MBS, with CMBS and ABS underlying spreads widening by 50bps more than those of their MBS peers.

Figure 5-5: Fourth scenario blended distribution



The fully confident posterior distribution is largely speaking aligned with the prior distribution. The reason why this is happening is the relatively low exposure of the index portfolio to those asset classes. Given the index allocation there is limited scope of the overall risk being affected due to any relative changes on that front. How those views could affect the allocation decision is a different matter which should be examined separately.

The statistics of the probability distributions, estimated through the blending of the block bootstrapped resampling and the investment views are summarized in table I. We observe that, driven by the prior, the posterior distribution exhibits excess kurtosis which corresponds to the leptokurtic shape of the probability mass, depicted in the above graphs.

This kind of evidence has been presented in the literature. Szego (2002) presented a case of leptokurtic distribution for emerging market credits. Premaratne and Bera (2000) showed that financial data exhibit excess kurtosis and asymmetry whilst Christie-David and Chaudhry (2001) presented similar findings for future returns. Other studies observing the presence of leptokurtic distributions in finance include Kon (1984), Mills (1995), Peiro (1999), Premaratne and Bera (2005) and Patton (2004). Additionally Jefferson, Longstaff and Yu (2007) highlighted that arbitrage strategies in fixed income display excess kurtosis and so did Kat and Miffre (2006) and Bal, Brown and Demirtas (2013) for hedge fund strategies and Enrique, Christodoulakis and Poon (2013) for credit loss distributions.

Table 23: Distributions tail behaviour

	P Equilibrium	5000 Obs (Case 1: E[X1]=+50bps)	5000 Obs (Case 2: same as case 1 but allow all E[Xi] to change)	5000 Obs (Case 3: E[X3]-E[X5]=+50 bps)	5000 Obs (Case 3: E[X3]-E[X5]=+25 bps)	5000 Obs (Case 4: E[X11]-E[X12]=+50 bps)
Min						
Max						
Mean	0.00%	-0.05%	-0.89%	-0.74%	-0.38%	0.14%
σ	5.44%	6.61%	5.42%	6.08%	5.61%	5.59%
Skew	0.33	0.06	-0.07	0.26	0.22	0.46
Kurt	5.50	3.60	4.56	4.83	5.20	5.61
VaR (95%)	-9.35%	-12.12%	-11.24%	-11.40%	-10.51%	-9.31%
VaR (97.5%)	-12.01%	-13.58%	-13.12%	-14.10%	-12.90%	-12.01%
VaR (99%)	-14.34%	-15.18%	-15.00%	-15.00%	-14.77%	-14.34%
CVaR (95%)	-12.26%	-13.95%	-13.44%	-13.90%	-13.15%	-12.22%
CVaR (97.5%)	-14.02%	-15.12%	-14.85%	-15.10%	-14.71%	-14.02%
CVaR (99%)	-15.70%	-16.07%	-16.13%	-16.48%	-16.05%	-15.68%

5.6 Conclusion

This chapter has analysed the issue with fixed income asset allocation and portfolio construction with its main components: expected returns and portfolio risk being examined separately. Starting from the pioneering work of Black and Litterman (1992) and navigating through the various extensions available in the literature - this chapter is different in its use of bonds yields to define market equilibrium returns, instead of a CAPM framework. Moreover, the views represent changes in yield and spread level which is highly intuitive.

On the other hand side, a multifactor model has been blended with resampling techniques and an entropy pooling approach to jointly capture the risk profile of the individual risk factors driving the behaviour of investment grade bonds. This sets the groundwork for a realistic estimation of expected returns as the prior per se is realistic. The yields extracted from market valuations is nothing but the consensus on future returns of a particular bond. As such, blending the yield advantage which forms the prior and the views on market movements constituted a very reasonable process. In contrast, the models using a utility function to extract the equilibrium returns provide a set of equilibrium returns which have rather instrumental value in a sense of leading to a reasonable allocation. However it is questionable to what extent the blend of actual views and instrumental equilibrium returns would be meaningful for the identification of expected returns.

As a result the main advantage of this framework is the requirement of fewer assumptions to be imposed regarding both the equilibrium returns and the associated risk. The main weakness is that the yield levels are a meaningful approximation of equilibrium returns in a strictly default free environment. Thus the benefits of it are limited to the investment grade universe. The second drawback is linked with the limitations of resampling which does not assign any probability to out of sample extreme events even though it better reflects the tail behaviour and correlation structure between time series.

All told, the framework proposed here is a step forward for the expression of views given the latest market valuations and the estimation of blended distributions without imposing a normality constraint or any other restriction. That forms the basis for the asset allocation discussion.

6 Conclusion and Directions for Future Return

6.1 Conclusion

This PhD thesis deals with the asset allocation problem for actively managed fixed income portfolios; the equilibrium returns, the investment views, the risk dynamics, the correlation structure between asset classes and risk factors and the optimization process are all revisited in order to address the multiple issues that arise from the portfolio construction process. In this respect, each Chapter of the current PhD thesis explores alternative research questions in regards to the above topics. Chapter 3 juxtaposes the CAPM implied equilibrium returns with the occurring yield to maturities in the investment grade universe and uses the yield advantage of each component of the portfolio over a benchmark index to determine the relative to the benchmark allocation. This is performed using Black-Litterman model, but tweaked to allow for the representation of the investment views and more importantly of portfolio views onto a risk factor space. Chapter 4 relates to the examination of the risk behaviour of twelve real portfolios of a leading investment institution actively managed against the Barclays Capital US Aggregate Index. The tail risk dynamics of these portfolios have been explored given the set of available risk factor sensitivities over time. Chapter 5 takes into account the latest developments in the literature regarding Bayesian portfolio allocation and risk factor specification, to propose an allocation risk factor framework which allows for leptokurtic and skewed distributions.

Specifically, in chapter 3 the equilibrium returns of the Black-Litterman model are compared with the yield to maturity for investment grade bond indices. The results show that not only the two are not in line but modification of the level of the risk aversion parameter, which controls the reverse optimization process in the Black-Litterman model cannot lead to a solution. The notion is that yield to maturity is a good proxy of future return of a buy and hold investment strategy under the assumption of no defaults. This assumption is compatible with the fixed income investment grade universe. As such, the yield advantage of each asset over the chosen benchmark index is used as a measure of its ability to generate excess returns. Incorporating views into the model, allows captivating the scenario of a changing investment landscape in terms of both yields and spreads. It is noteworthy, that there is foundation to blend the yield differential over each asset class and the index and the relative returns due to market moves according to investment views. In contrast, the CAPM equilibrium returns are more of instrumental value and it is doubtful how to intuitively blend them with the expressed investment views as they may differ substantially. The risk estimation is still performed on the basis of a variance covariance matrix but this is on risk factor level rather than on asset class level. One of the main advantages of the Black-Litterman model is that the optimization process leads, by construction to a more aligned allocation with what has initially been specified as the market portfolio even though in practice some of the underlying assumptions may not hold. To overcome this difficulty, the starting point of the allocation in chapter 3 is the chosen benchmark. Thereafter the optimizer focuses on the relative to the index portfolio based on the relative returns calculated and the risk budget in relative terms.

Chapter 4 relates to the evaluation of the risk profile of twelve real fixed income portfolios, actively managed against Barclays Capital US Aggregate Index. The available set of sensitivity exposures of these portfolios against the main interest rate and spread risk factors are used to test their tail risk behaviour. The focus in this chapter remains tied to the relative to the index portfolios. As a result, the relative to the index risk factor loadings are used for the assessment of the relative to the benchmark risk. The goal is to investigate if there is scope to improve a multifactor risk model for active investment management by using techniques such as resampling and block bootstrapping. The alternative risk methodologies tested are the parametric approach, the parametric approach where the covariance matrix is estimated by exponentially weighted moving average, ex post tracking error and block bootstrapping with blocks of random length. The Value at risk estimations, corresponding to each of the above methodologies have been back tested for more than five hundred points of realized performances. In fact, the performance of the multifactor risk model improves when moving from the parametric approach to resampling. The main empirical results highlight the importance of tail characteristics of relative to the index returns which could not be captured under the normality assumption of the parametric approach.

In light of chapters 3 and 4, chapter 5 elaborates on how to improve the portfolio construction mechanism when the normality assumption is in practice violated. The multifactor risk model is adjusted to incorporate the most recent developments in the measurement of interest rate and spread risk exposures. Then block bootstrapping is used to mimic the actual marginal distributions per risk factor. Block bootstrapping has the property of incorporating the correlation structure of the corresponding marginal distributions which is useful for the allocation process. A risk factor loadings vector makes it possible to translate the resampled scenarios into portfolio returns. The findings are in line with the main bulk of the literature showing that financial data exhibit asymmetric and fat tail distributions. The next step is to re-estimate the resampled probability distribution on the basis of entropy minimization, when investment views are included, in the form of restrictions. This is giving the new probability space that satisfies the investment views. The contribution of this chapter is the mix of risk factor modelling with the most recent portfolio construction Bayesian techniques to allow the assessment of fixed income portfolio construction in a non – normal risk factor environment. One other main advantage of this framework is the ability to run scenario analysis and to stress test the portfolios against different investment outlooks.

6.2 Directions for Future Research

Without any doubt, the research ideas and methodologies presented in each Chapter of this PhD thesis can be further improved and extended in multiple directions. Possible improvements could include the exploration of more utility functions in search of compatibility between model implied expected returns and yield to maturity. Also, alternative ways to determine equilibrium returns could be evaluated and compared, including the roll down effect of the yield curve, the yield advantage over an index, the probability of default which would allow expanding to other segments of the fixed income market such as high yield. The adoption of the utility function as a tool to backward engineer the equilibrium returns and to construct the portfolio is pivotal. It has to lead to both a realistic set of equilibrium returns and a realistic allocation. Incorporating alternatives which use higher moments and more than one risk factors would be a step forward. Other areas of further research would include the impact of increased concentration risk onto portfolio level, when the investment convictions are high, which would potentially introduce some form of idiosyncratic risk into the portfolio.

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