Criticality in a model of banking crises.

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(February 1, 2008)

An interbank market lets participants pool the risk arising from the combination of illiquid investments and random withdrawals by depositors. But it also creates the potential for one bank's failure to trigger off avalanches of further failures. We simulate a model of interbank lending to study the interplay of these two effects. We show that when banks are similar in size and exposure to risk, avalanche effects are small so that widening the interbank market leads to more stability. But as heterogeneity increases, avalanche effects become more important. By varying the heterogeneity and connectivity across banks, the system enters a critical regime with a power law distribution of avalanche sizes.

I. INTRODUCTION

Systemic failure in banking arises when one bank's failure triggers off further failures. The history of modern banking is full of examples of systemic failure at both moderate and large scales, with the 1997 East Asian crisis being the most recent example of large scale bank failure.

Several channels through which systemic failure may arise have been explored [1–7]. In this paper, we focus on one channel, namely interbank lending. Interbank lending allows banks which face temporary shortfalls in funds to borrow from banks which have surpluses. Such lending creates a network of credit and debt relationships within the banking system. If one debtor bank fails, it adversely affects the balance sheet of its creditors and can trigger off their subsequent failures.

At the same time, interbank lending allows for the pooling of risk arising from the stochastic pattern of deposits and withdrawals by each bank's customers. Without interbank lending, banks might become even more vulnerable to failure, although such failure would be idiosyncratic and free of the symptoms of systemic collapse. Judgment on the effects of interbank lending for a banking system's stability should therefore take into account both the *ex ante* risk-sharing effect and the *ex post* systemic failure effect. In this paper, we study these effects by developing and simulating a dynamic model of a banking system, linked together through an interbank credit market.

Our approach abstracts from the rationality of individual behavior and resembles the approach taken in the statistical mechanics of disordered systems. Episodes of systemic failure in banking appear analogous to the periodic collapses or 'avalanches' which arise in natural systems in the form of earthquakes, microfracturing, epidemics, magnetization of ferro-magnetic systems, etc. The capacity of a natural system to generate avalanches of various sizes seems to depend on certain general features, such as the heterogeneity of the elementary constituents of a system and the nature of interaction between them. By analogy in a banking system, we focus on how the individual characteristics of banks and their mutual interaction on an interbank market can affect the stability of the system.

II. THE MODEL.

Time is discrete. At the initial time, $t = 0$, the system starts with N_0 banks. At any subsequent time, there are N_t banks operating in the system. The number of banks may go down as a result of failures, but failing banks are not replaced by new entrants.

Credit linkages between banks are defined by a connectivity matrix, J_{ij}. J_{ij} is either one or zero; a value of one indicates that a credit linkage exists between banks i and j and zero indicates no relationship. J_{ij} are randomly chosen at the beginning of the simulation. c denotes the probability that J_{ij} is one for any two banks. At one extreme, $c = 0$ represents the case of no interbank lending, while $c = 1$ represents a situation in which all banks can potentially borrow and lend from each other.

The primary purpose of a bank is to channel funds received from depositors towards productive investment. Deposits, A^k_t, and investment opportunities, ω^k_t, are randomly and independently drawn for each bank k at each time t from Gaussian distributions with means $\bar{A}^k$ and $\bar{\omega}^k$ respectively, and variances $\bar{A}^k\sigma_A$ and $\bar{\omega}^k\sigma_\omega$ respectively:

$$A_t^k = |\bar{A}^k(1+\sigma_A\epsilon_t^1)|$$

$$\omega_t^k = |\bar{\omega}^k(1+\sigma_\omega\epsilon_t^2)|$$

$$\epsilon^{1,2} \sim N(0,1)$$

Once tied up in investment, resources become relatively illiquid; any investment made at time t fully matures only at some time $t+\tau$. A return, ρ, is realized at each point, $t+1$, $t+2$, $\cdots t+\tau$. ρ is exogenous and risk-free. However, since withdrawals are unpredictable, a bank may find it itself unable to repay its depositors due to the illiquidity of its investment.

Each bank k starts a period, t, with cash holdings inherited from the past, M_{t-1}^k. These are constituted by

$$M_{t-1}^k = A_{t-1}^k + B_{t-1}^k + V_{t-1}^k - \sum_{s=1}^{\tau} I_{t-s}^k,$$

where A^k are its deposits, V^k represents its equity, I^k represents past investments and B^k represents borrowings. B^k can be negative or positive and satisfies: $\sum_{k=1}^N B^k = 0$. Borrowing consists of one-period loans, which require repayment in full in the period after which they are undertaken.

Each bank receives income, $\rho\sum_{s=1}^{\tau-1} I_{t-s}^k$ and $(1+\rho)I_{t-\tau}^k$ from investments made over the last τ periods. It then receives a new level of A_t^k, as defined above. Its cash holdings then adjust to reflect $(A_t^k - A_{t-1}^k)$, the net change in its deposits and $\rho\sum_{s=1}^{\tau} I_{t-s}^k + I_{t-\tau}^k$, the receipt of incomes from past investment. If the adjusted cash holdings become negative, the bank can issue negotiable debt certificates to cover any excess of payments over its cash reserves. However, the certificates have to be redeemed at the end of the period through borrowing from other banks. If this is not done, the bank fails and its debt certificates become worthless.

If a bank has borrowed in the past period, its priority is to repay its creditors. Such payments are made in cash. Hence, banks with cash holdings in excess of their debt obligations, which equal $(1+r_b)B_{t-1}^j$ for bank j, make payments to their respective creditors. Here, r_b is the exogenous interest rate on interbank borrowing.

At this point, two types of banks can be distinguished, those with positive cash and those with negative cash. Accordingly, they get classified as potential lenders and potential borrowers respectively. Borrowing banks issue demands for loans equaling their debt obligations (interest plus principal) minus their current cash (which if negative adds to their demand).

Lending banks give priority to dividend payments (to shareholders) and investment. Dividend payments are made for maintaining a target equity:deposit ratio, χ, and for preventing excessive levels of capitalization. Only banks whose adjusted cash holdings plus illiquid assets exceed a certain fraction of their total deposits make dividend payments. After dividends have been paid, the bank is assumed to undertake investment on the basis of its available liquid resources on the one hand, and its stochastic investment opportunity on the other. Available liquid resources comprise the bank's current cash minus any statutory reserve requirement imposed on it by a regulator. If the available investment opportunity exceeds this limit, the bank cannot exploit the full opportunity. It then invests upto the extent of its available cash resources.

After investment, any excess left over is made available to those borrowing banks with whom a credit linkage exists. Each borrowing bank contacts each lending bank with which it is linked in a random order. The two banks exchange an amount of credit equal to the minimum of the two banks' respective demand and supply. If the borrowing bank is left with an unfulfilled trade it contacts another lender with which it is linked.

A borrowing bank does not receive actual funds until it has lined up enough credit to ensure that it will not fail during the current period. Once a bank has obtained sufficient credit, funds are transferred and the cash positions of all banks involved is updated. This continues until either all loanable funds are exhausted or all demands for credit are satisfied.

At this point, the process reiterates itself through the following steps: banks which had not repaid creditors in the first round but now have borrowed enough cash to pay off past debt entirely, do so; these payments go to their creditors and money holdings get updated accordingly; potential borrowers and lenders are determined for the next round; if lenders had underutilized investment opportunities from the previous round, they make further investment. Finally, a fresh round of borrowing and lending takes place. The process repeats itself until reiteration produces no further exchange of credit.

All banks which are left with negative cash holdings or cash holdings which fall short of their remaining debt obligations are deemed to be in default. These banks are removed from the system. If, at the time of closure, a failing bank has illiquid assets in the form of investments made previously, these are liquidated at a fraction, γ (which is exogenous), of their true value. The proceeds are distributed, first to the depositors, then, if there is still some value left, this goes to creditors from the previous period and finally to the shareholders.

III. SIMULATIONS AND RESULTS.

In all the simulations to be presented, the return on investment, ρ, was risk-free and equal to 1.0% , the interest rate on interbank borrowing, r_b, was 0.5, χ, the equity:deposit ratio was 30%, the period of maturity τ was set at 3 and the recovery rate γ was set at 0. Parameters which vary across the reported simulations are identified separately.

At $t = 0$ each bank holds deposits, A_0, worth 1000 units and initial equity, V_0, equal to 0.3 times the initial deposit.

Initially, $\bar{A}_k$ and $\bar{\omega}_k$ were chosen as identical for each bank. This led to homogeneity in average size of deposits and investment opportunities across banks.

Figure 1 displays results on the main question of the paper. It compares bank failures with different degrees of linkage, c, in the interbank market. Increasing linkage adds stability, in the sense that in any period, there are more surviving banks the greater the degree of linkage. This pattern was very robust to changing the parameters of the simulation.

A policy issue raised by the results of Figure 1 concerns the role of statutory reserve requirements. By preventing banks from investing more than a certain fraction of the deposits placed by customers in the form of illiquid and/or risky assets, reserve requirements reduce the exposure of individual banks to the risk of failure. But they also inhibit interbank lending activity. This can reduce the risk sharing provided by interbank credit and destabilise the system. We conducted experiments along the lines of those reported in Figure 1 and found that, indeed, while without any interbank credit higher reserve requirements always led to fewer bank failures, with interbank credit linkages, similar increases in reserve requirements could increase the incidence of bank failures (see [8]).

In the homogeneous case depicted in Figure 1, episodes of bank failure affect only a small number of banks at a time and there appears to be no correlation across the episodes. This is because there is relatively little borrowing and lending taking place at any time. Even this small amount appears to save banks which face short run liquidity problems, but is not enough to generate contagion effects.

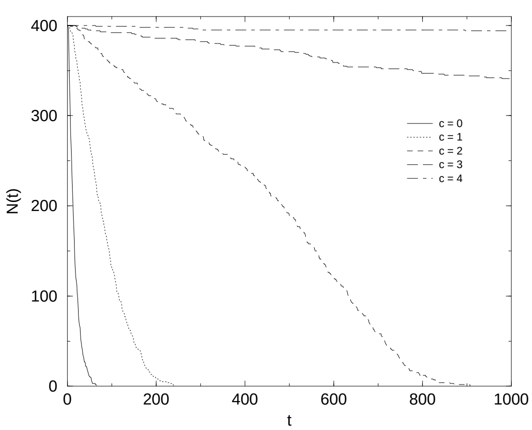

FIG. 1. Surviving banks with different interbank linkages.

To allow for systematically larger volumes of interbank activity, we try to differentiate banks in a way that some banks tend to be lenders and others to be borrowers. We therefore make banks differ according to investment opportunities, by choosing: $\bar{\omega}^k = \bar{\omega}|z^k|$ with $z^k \sim N(0, \sigma_{\bar{\omega}})$. Given that banks remained identical in terms of their deposit fluctuations, a large $\bar{\omega}_k$ meant that a bank would invest large amounts and consequently face a greater exposure to liquidity risk. Such a bank would be more likely to act as a large borrower on the credit market, while for analogous reasons, a bank with small $\bar{\omega}_k$ would be more likely to act as a large lender.

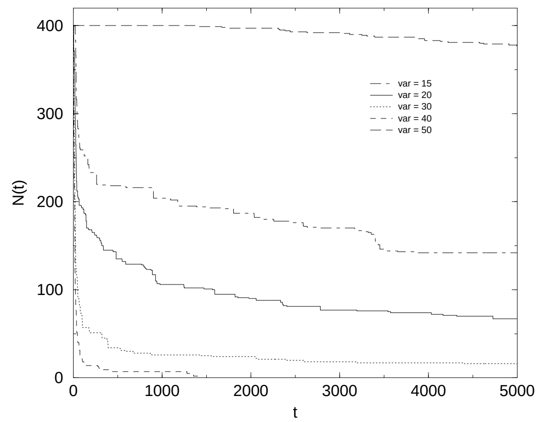

FIG. 2. Effect of variance in opportunities on failure: 100 percent linkage.

Figures 2 and 3 show some of the results from this experiment. Figure 2 shows the effect of greater heterogeneity on the incidence of bank failures when banks are 100 percent connected to each other. As the variance of types increases from 5 to 40, the incidence of failures goes up. Note that now the time path shows 'avalanches', *i.e.* many banks collapse together over brief periods of time. Since the aggregate environment is on average constant, avalanches suggest that knock-on effects are taking place.

Figure 3 shows that, fixing the variance of types at 10, increasing connectivity from 5 to 50 percent leads to less failures during the interval studied, but increasing connectivity to 80 percent leads to more failures than at 50 percent. Hence, in contrast to the case of homogeneous banks, increasing the extent of the interbank market need not increase overall stability.

We repeated the experiments discussed in Figures 2 and 3 by differentiating banks both in terms of size and in terms of investment opportunity, as follows: $\bar{A}^k = \bar{A}|z^k|$, and $\bar{\omega}^k = \bar{\omega}|z^k|$, with $z^k \sim N(0, \sigma_{\bar{\omega}})$. With this formulation, all banks face the same liquidity risk, but heterogeneity exists in the volume of borrowing and lending by different banks. A big borrowing bank might need to contact many small lending banks and a big lending bank might be a source of liquidity to many small banks. In both cases, the collapse of a big bank can create ripples through the system. The simulation results verified that this indeed does happen [8]. These results are also consistent with empirical observations made in the literature (see, *e.g.*, [1,5]).

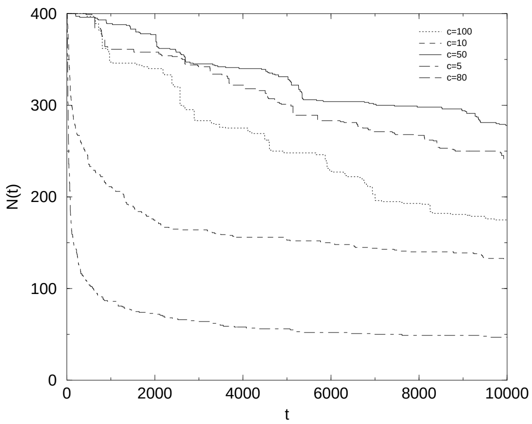

FIG. 3. Effect of increasing linkages on failure; heterogeneous investment opportunities.

The results suggest that when banks are heterogeneous in the volume of their activity on the interbank market, contagion effects can arise which can undermine the overall risk sharing influence of interbank credit. It is interesting to observe that heterogeneity plays a similar role in a well known physical model, the Random Field Ising Model (RFIM). In particular, the RFIM shows a 'disorder' induced phase transition between a regime displaying only finite size avalanches and a regime corresponding to avalanches which cover the whole system. At a critical level of disorder, avalanches of all sizes, up to and including the system as a whole, may occur. The statistical distribution of avalanches takes the form of a power-law at the critical value [9].

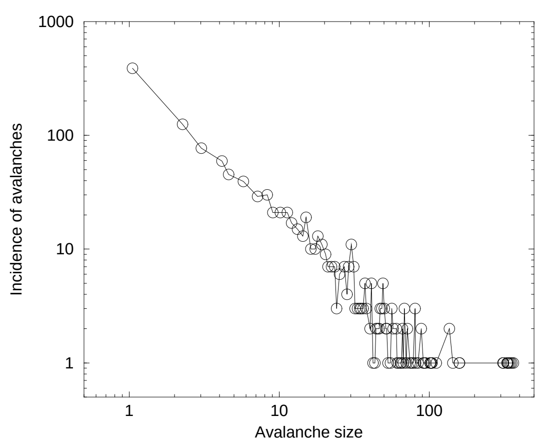

FIG. 4. Log-log plot of the statistical distribution of avalanche size.

A similar phase transition is found in our model. For example, Figure 4 shows that when linkages are fixed at 100 percent and setting $\sigma_{\bar{\omega}} = 40$, the system reaches a critical state where the size of avalanches becomes distributed according to a power-law. In order to better identify this power-law behaviour, we increased the number of banks to 1600 and repeated the simulations several times, resampling the investment opportunities each time. A preliminary investigation indicates that for each value of linkage across banks, it is possible to find a critical value of heterogeneity in the system which leads to power-law distributions of avalanches sizes. The variance of sizes needed to attain the power-law distribution increases as the connectivity decreases.

IV. CONCLUSIONS:

Our simulations have identified certain characteristics which can lead interbank lending to be associated with complex forms of instability in a banking system. Whether or not the crises which periodically grip actual banking systems represent such complex behaviour is difficult to establish empirically. The data needs for such measurement outstrip the recorded evidence on the subject. It is nonetheless useful for policy makers to be able to distinguish circumstances under which interbank credit can destabilise a system from those under which it stabilises. In this respect, our simulations suggest that when bank lending is extended across banks which are heterogeneous in size and risk exposure, it can exarcebate the potential for avalanche effects and destabilise the system.

[1] Angelini, P., G. Maresca and D. Russo (1996), Systemic risk in the netting system, *Journal of Banking and Finance*, 20, 853.

[2] Calomiris, C.W. and C.M. Kahn (1996), The efficiency of self-regulated payment systems: Learning from the Suffolk system, *Journal of Money, Credit and Banking*, 28, 766.

[3] Diamond, D.W. and P.H. Dybvig T. (1983), Bank runs, deposit insurance and liquidity, *Journal of Political Economy*, 91, 401.

[4] Edison, H.J., P. Luangaram and M. Miller (2000), Asset bubbles, domino effects and "lifeboats": Elements of the East Asian crisis', *Economic Journal*, 110.

[5] Furfine, C. (1999), Interbank exposures: Quantifying the risk of contagion, *mimeo*, Bank for International Settlements.

[6] Jacklin, C.J. and S. Bhattacharya (1988), Distinguishing panics and information-based bank runs: Welfare and policy implications, *Journal of Political Economy*, 96, 568.

[7] Rochet, J-C and J. Tirole (1996), Interbank lending and systemic risk, *Journal of Money, Credit and Banking*, 28, 733.

[8] Iori, G. and S. Jafarey (2000), Interbank lending, reserve requirements and systemic risk, King's College preprint KCL-MTH-0060.

[9] Sethna, J.P., K.A. Dahmen, S. Kartha, J.A. Krumhansl, B. W. Roberts, and J.D. Shore (1993), *Physical Review Letters*, 70, 3347.