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A note on the nature of utility in time and health and implications for cost utility analysis

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A note on the nature of utility in time and health and implications for cost utility analysis

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Summary

Time Trade-Off valuations of health are widely used in economic evaluation of health care. Current approaches to eliciting TTO values, and their use in economic evaluation, rests on specific assumptions about the way utility relates to time and health. Both the assumptions themselves and evidence of violations of them are discussed in the literature - yet the issues appear not to be widely appreciated by those using and applying TTO. This paper adds to that literature by demonstrating both the requirements of TTO and violations of these assumptions in terms of the underlying indifference curve maps and utility functions. The advantage of this approach is that is demonstrates very clearly a number of fundamental problems for the way TTO values are currently elicited and used in Cost Utility Analysis. In essence, it is extremely unwise to assume that the current 'tariffs' of TTO values, such as those routinely used by NICE and other organisations, can be applied irrespective of the duration of the health states to which they are assigned. The estimates of QALYs that result will, quite often, simply be wrong. We suggest a number of solutions, including the provision of multiple value sets, for a range of durations.

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Introduction

Economic evaluation of health care is commonly undertaken by estimating the incremental cost per quality adjusted life year (QALY) gained. QALYs are estimated by multiplying length of life in a given state with the value placed on that state, on a scale from 1 (full health) to 0 (dead), with health states worse than dead represented by negative values. A widely used method for eliciting these values is the Time Trade Off TTO). First proposed by Torrance and Zapursky (1972), TTO values are sought by asking participants to consider living for some fixed period of time *t* in the state to be valued, in comparison with some shorter period of time *x* in full health. The latter is varied until the participant is indifferent between the two options, at which point the value for the state is yielded by (*x/t*). While variations on this procedure exist (for example, for temporary rather than chronic states, and for states worse rather than better than dead) this remains the basic approach (Drummond et al 2005).

The TTO is, together with other, less widely used alternatives to the valuation of health states – the Standard Gamble (SG), Visual Analogue Scale (VAS) and Discrete Choice Experiments (DCE) – a *stated preference* procedure. In each case, preferences are obtained by statements and responses made by individuals presented with hypothetical scenarios, rather than obtained from budget- (or other resource-) constrained choices they have been observed to make in 'real life'. TTO was developed initially as a more pragmatic way of eliciting valuations that approximate SG utilities, but has since become a valuation method of interest in its own right. Mehrez and Gafni (1990) note that, despite its widespread use, the TTO "has not been related in a general way to any existing behavioural theory". More generally, Arnesen and Norheim (2003) note their opinion that, in the field of health state valuation research, there is

> "…a disproportion between the abundance of empirical and technical papers and a lack of a more conceptual debate….Discussions within the same paradigmatic framework tend to turn around technical solutions, rather than questioning the underlying assumptions."

There have been some attempts to place the TTO in the context of economic theory. For example, Mehrez and Gafni (1990) represent TTO valuations of health-related quality of life using indifference curves occupying utility space in health and length of life. Buckingham and Devlin (2006) also use indifference curves to demonstrate the TTO, and use these to represent the TTO (as conventionally used) as a Hicksian 'compensating variation' for a gain (improvement in health).

However, neither of these papers considers in any detail the *nature* of the indifference curve maps for health and length of life, or the utility functions underpinning them, and how these might relate to particular requirements of these assumed in the application of TTO values in economic evaluation

This paper considers the shape and characteristics of the indifference curves in health and time that are suggested by generally accepted beliefs and empirical evidence about preferences regarding health and length of life. Specifically, we show what the indifference curves for health and time would be like if the basic requirements of TTO valuations were to hold. We then consider the indifference curves suggested by a number of circumstances where these requirements will *not* hold, including - in the simplest case - the possibility of diminishing marginal utility in states of poor health and (more fundamentally) the existence of maximum endurable time (MET) (Sutherland et al., 1982; Dolan and Stalmeier, 2003; Tsuchiya and Dolan, 2005) and states worse than being dead (SWD) (Patrick et al., 1994; Macran and Kind P, 2001; Robinson and Spencer 2006).

In doing so, we demonstrate fundamental and non-trivial problems with the way TTO values are currently used to estimate QALYs that we think merit wider debate among health economists. We contend that the use of a single 'tariff' of TTO values for health states, such as that used by NICE (NICE 2004) and other similar decision-making bodies internationally, will yield estimates of QALYs that are, in many cases, quite simply wrong.

Constant proportionality and the requirements for "well behaved" numeraires and valuands

The key assumption required for the current TTO approach to yield value tariffs that, for any given state, may be applied for any duration is *constant proportionality*

(Bleichrodt and Johansson 1996; Dolan and Stalmeier 2003). As described in the previous section, a TTO value is given by the ratio (x/t), achieved as the point of indifference between t in the health state of interest and x ($< t$) in full health. Constant proportionality requires that participants indicate that they are prepared to give up the same *proportion* of time (eg. 20%) in perfect health (x), regardless of the length of time in poor health (t) being considered. So, for example, if a person is indifferent between a given state for $t = 10$ years and $x = 6$ years in perfect health, for the resulting TTO value (0.6) to be invariant to duration would require that the same state considered for $t = 20$ would yield $x = 12$; and $t = 5$ with $x = 3$ and so on – a sacrifice of 40% of t in each case.

Constant proportionality arises from a relationship between two utility functions – the utility of additional years in perfect health, and the utility of additional years in the (less than perfect) health state of interest. Under conditions where the marginal utility functions of each have similar characteristics, constant proportionality will result. For example, if the marginal utility of additional years in full health and the marginal utility of additional years in some other health state of interest are both constant, constant proportionality will result. That is, the same TTO value for that state will be produced regardless of whether the 'reference point' duration t is 10 years (as it was in the TTO value set currently used by NICE in the UK – see Dolan 1997) or 10 weeks. However, constant proportionality *in itself* does not require that the marginal utility of time in a given (less than perfect) health state is constant. Constant proportionality might arise where non-linearities in the utility of time, in the less than perfect health being valued, are exactly matched by non-linearities of time in perfect health. Evidence of violations of constant proportionality suggests this is not the case.

Suppose that each year in perfect health is valued at exactly 1 util - that is, each additional year in perfect health confers a constant addition to total utility [1]. If this were the case, time in perfect health might provide a 'well behaved' measure of value or 'numeraire'. (Recall that the TTO values a specified state of health for a specified period of time as a number of years in perfect health which is equivalent in utility terms).

The thing being valued (the *valuand*) is time spent in poor health. In this instance, a well behaved valuand would also be one in which utility was proportional to the time spent in the relevant state. The nature of this relationship would not be of concern providing that we are only attempting to value a specific combination of length of life and health state - for example, if our aim were to value 10 years in a particular state of poor health. If we determine that the utility of 10 years in poor health is equivalent to 5 years in perfect health, we would value that state at 5 utils. However, the relationship between utility and time in poor health does become material if we attempt to determine the *average* flow of utility through time, as we do when we construct an index or 'tariff' of values such as those commonly used in CUA. In the example above, we observe that the 5 utils are experienced over the 10 years of poor health, so the average flow of utility is 0.5 utils *per year*, or simply, as in the sort of value sets that are published and widely used in CUA, a value of 0.5 for the health state of interest.

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[1] We use the term 'util' to focus the readers' attention on the underlying concepts of utility and consumer choice. However, since one year of full health = 1 QALY, it may help readers to instead think of these utils as QALY estimates.

This can create a serious problem if we were to apply the average flow of utility, elicited from some specified time period, to value the experience of living in that state of health for a *different* period of time; as we subsequently show.

An example of a utility map with a well behaved numeraire and well behaved valuands is illustrated by Figure 1 which can be interpreted as follows. The indifference curves (which can be thought of as analogous to the contour lines on a map) indicate specific levels of utility. To continue the analogy, closely packed indifference curves indicate where the utility increases most rapidly. Increasing utility can result from increasing levels of health. Under 'normal circumstances' we might also expect utility to increase with length of life (as when we gain extra pleasure by living longer), however, we see below that this need not necessarily be the case. The corresponding length of life to utility functions (derived from the same mathematical model[2]) are shown in Figure 2 and the corresponding marginal utility schedules in Figure 3. Figure 1 suggests that utility is always increasing in both health and in length of life.

FIGURES 1, 2 AND 3 ABOUT HERE

In the hypothetical examples used here, time (T) is scaled from 0 years to 10 years. Health (H) is scaled from 0 to 1000. This latter scaling is chosen to avoid potential confusion with the utility index (more properly, the average utility flow) associated with states of health. The interpretation of 'health' is best thought of as some physical

[2] The mathematical model used to produce Figures 1 to 3 is $U = H/1000*T/10$.

measure or description of 'healthiness'. If it helps, the reader is encouraged to imagine this measure of health to be something like the maximum distance a person can walk before their pulse rate reaches some specified value.

Sources of non-proportionality and non-monotonicity in the relationship between time and utility

First we consider diminishing marginal utility in time. This is plausible *a priori*, particularly for states of health that are less than perfect[3]. As length of life is increased, the addition to utility from later years in a poor health state may be less than the addition to utility from earlier years in that state[4]. The possibility of diminishing marginal utility of health in time is illustrated for the intermediate state of health labelled 'H = 500' in Figures 4 and 5[5].

FIGURES 4, 5 AND 6 ABOUT HERE

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[3] A related issue, which we do not discuss here, is whether there is also a satiation point for perfect health. If we were guaranteed to remain in perfect health, would be wish to live forever?

[4] Note that marginal utility, in the context of stated preferences, is determined by 'affective forecasts' made by members of the general public being asked to contemplate a given health state now and in the future, as opposed to the marginal utility experienced by those who live in that state year on year. The literature on experienced utility suggests that patients with chronic conditions tend to assign higher values to their health than members of the general public – a finding generally attributed to adaptation (Brazier et al 2005). The effect of adaptation works in the opposite direction to diminishing marginal utility. We do not address experienced utility in this paper, although our conceptual framework could readily be adapted to accommodate it.

[5] Figures 4, 5 and 6 are produced using the expression U =H/1000*T/10-0.4*((1-H/1000)*(T/10)^2)

Now consider the possibility of MET. The intuition is appealing: namely, that there exist states of health so severe that people would only willingly endure them for a limited time (for example, to allow them to 'put their affairs in order'). One might speculate that the MET would be longer for states of health that are less bad, and shorter for states of health that are more so.

The health state labelled 'H = 250' provides an example of MET. We observe that, in the example shown in Figure 4, the total utility of years of life in that state increases up to approximately 4 years, after which further increases in length of life in that state lead to lower utility. In other words, the marginal utility of time after approximately 4 years becomes negative as shown in Figure 5. Further, in this example if the length of life experienced in this state exceeds 8 years, *total* utility becomes negative[6].

Now consider states worse than dead[7]. The health state labelled as 'H = 0' in Figure 4 provides an example of a state worse than dead for all lengths of life. We see that the utility associated with living in such a state is negative over the entire domain of longevity.

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[6] There are two levels of time which may have some bearing on MET: the *t* at which MU becomes negative (and TU starts to decline), and the *t* at which TU becomes negative. We proceed on the basis that it is the latter which is relevant to identifying MET, on the grounds that so long as TU remains positive, surviving will remain attractive.

[7] A weakness of this empirical literature is that states are considered to be either *always* better than, or *always* worse than, dead – irrespective of the time those states are experienced. Further,, the TTO methods used to value states worse than dead in TTO often rely on quite different elicitation procedures than states better than dead at any given duration (Drummond *et al* 2005). Aggregating values < 0 and > 0 in the construction of value tariffs therefore has questionable validity; this issue potentially affects the values for *all* states - and the reliability of estimates of changes in utility resulting from an improvements in health.

Implications for the utility map

We now consider the implications for the indifference mapping (Figure 6) suggested by non-proportionality and non-monotonicity in the relationship between time and utility.

Let utility be a function of time (T) and health (H):

$$U = U(T, H) \qquad [1]$$

We will assume that utility is always a positive function of health ($\delta U/ \delta H > 0$), so that we always prefer more health to less health for any life expectancy. However, given diminishing returns and MET, the marginal utility of T may be either >, equal to, or < 0.

The change in utility arising from any change in the amount of H and T available is given by:

$$dU = dT * \delta U/ \delta T + dH * \delta U/ \delta H \qquad [2]$$

As one moves along an indifference curve, the increase in utility resulting from a given increase in H must be exactly offset by a corresponding reduction in utility arising from the reduction in T since, by definition, utility is identical at every point along the indifference curve. Because utility is equal at all points along the indifference curve, $dU = 0$, hence

$$dH/dT = - \delta U/ \delta T * \delta H/ \delta U \quad [3]$$

Three results follow:

i. The marginal rate of substitution ($MRS_{H,T}$) of health for time (dH/ dT) is always of opposite sign to the marginal utility of time, $\delta U/ \delta T$.

ii. (dH/ dT) equals zero when $\delta U/\delta T = 0$.

iii. As the marginal utility of time ($\delta U/ \delta T$) assumes larger negative values, so the gradient of the indifference curves (dH/ dT) assumes larger positive values and vice versa.

We can now infer the shape of the indifference curves in Figure 6 from the relationships portrayed in Figures 4 and 5.

For the intermediate state of health H = 500, Figure 5 shows that marginal utility with respect to time to be universally positive but falling. Correspondingly, in Figure 6, as we move horizontally across at H = 500, we see that the indifference curves are downward sloping (in accordance with result i), but with reducing slope (in accordance with result iii).

For health state H = 250 we see the effect of MET on the utility map. Moving across Figure 6 at H = 250, we begin with positive marginal utility of time (as shown in

Figure 5) and corresponding downward sloping indifference curves (($MRS_{H,T}$ is negative, in accordance with result i). We see in Figure 5 that the marginal utility of time = 0 at approximately T = 4. This corresponds to the MET. In Figure 6 we see that the indifference curves are horizontal ($\delta U / \delta T = 0$ in accordance with result ii). For H = 250 and T > 4 (approximately) the marginal utility of time is seen in Figure 5 to be negative and increasingly so with increases in time. Correspondingly, we see in Figure 6 that the indifference curves slope upwards ($MRS_{H,T}$ is positive, in accordance with result i), and that the slope increases as time increases (in accordance with result iii).

Observe that in Figure 6 the minima on the indifference curves shift to the right as health increases, representing a plausible situation in which MET becomes longer as health states improve.

For health state H = 0, Figure 5 shows that the marginal utility with respect to time is universally negative and decreasing with increasing time. This corresponds to the situation in Figure 6, where indifference curves are always upward sloping ($MRS_{H,T}$ is positive in accordance with result i) and increasingly so (in accordance with result iii).

We have shown, in terms of rudimentary consumer choice theory, the characteristics of the utility function underlying the TTO which would be compatible with observed non-proportionalities in the utility:time relation. An advantage of this approach is that it allows non-proportionalities, including MET, and states worse than dead, to be defined in the precise language of utility theory.

However, more importantly, speculating on the characteristics of the utility map reveals some quite fundamental problems with the way health state value sets are currently used to calculate QALYs.

The implication for QALYs

With non proportionalities generally, but particularly in the case of poor states of health, where MET is more likely, it would be highly inadvisable to use average utility flows (indices or 'tariffs') to estimate QALYs for periods *other* than the period used in their construction. Consider the case of H = 250 in Figure 6. To begin, observe that we could not apply a conventional TTO with a 10 year time horizon. To value 10 years in that state of health as so many years in perfect health we need to say how many years in perfect health, the 10 years in health state 250 are equivalent to. Observe that the indifference curve passing through point health state 250 and length of life 10 years does not correspond with *any* length of life in perfect health.

Considering Figure 6 again, it would be feasible to value health over a 7 year time horizon. Seven years in H = 250 is approximately of equal utility (is on the same indifference curve) as 6 months in perfect health. Unfortunately we would have derived approximately the same equivalent time in perfect health if we had asked respondents to value H = 250 using a 1 year time horizon. Converting the results into *average* utility flows, the mean utility flow over the 7 year time horizon = 0.5/7.0 (approximately 0.07

utils per year). The mean utility flow over a 1 year time horizon is 0.5/1.0 (0.5 utils per year). Neither of these results is wrong. They differ simply because they apply to different periods of time.

The consequence of applying an index derived from one time horizon to produce QALYs for a different time horizon is clearly illustrated in the example. If we estimated the utility of spending 1 year in the health state using the index derived from 7 years spent in that state, we would estimate the utility as 0.07 utils (or QALYs) instead of the 0.5 utils (or QALYs) we 'know' it to be.

Conversely if we had used the utility established from a 1 year time horizon to estimate the utility of 7 years in that state of health, we would have estimated it to be 3.5 utils (QALYs) rather than the 0.5 utils (QALYs) we 'know' it to be.

Accordingly, it makes little sense to advocate particular durations when estimating average utility flows (indices or 'tariffs') if the aim is to avoid problems that arise from MET.

A theoretically acceptable solution to the problems that effectively arise from the use of average utility flows would be to sum the marginal utility of time spent in various health states. This suggests that, in turn, the availability of *multiple* tariffs of health state values, corresponding to a range of alternative durations. In economic modelling, very severe states lasting for short periods – for example, resulting from treatment side effects or acute exacerbations – could then be valued using a correspondingly short-duration tariff, rather than the value corresponding to that state lasting for 10 years, as at present (NICE 2004).

There are a number of challenges associated with such an approach. Pragmatically, the production of multiple tariffs requires more research effort both to elicit and model value sets for various durations. More evidence is required to understand how many tariffs are required and for what durations. More fundamentally, understanding how the marginal utility of time in one state may be related to the duration and utility of a preceding state is important. We are exploring these issues in our ongoing research.

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Figure 1 Utility as a function of health and length of life - an 'ideal' case

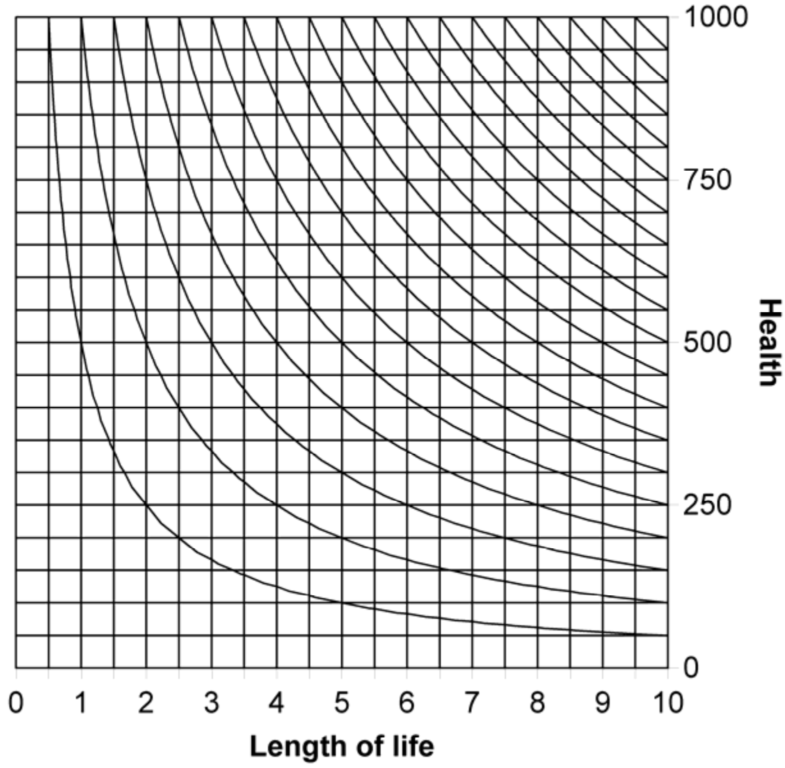

Figure 2 Utility as a function of length of life illustrating proportionality at all health states

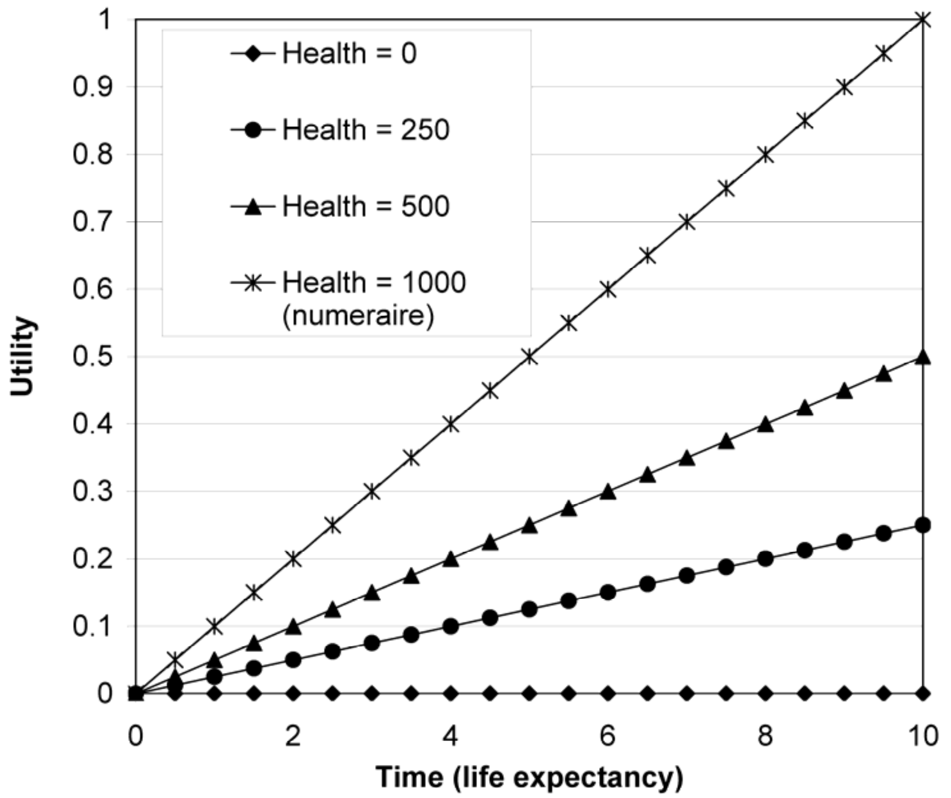

Figure 3 Marginal utility in length of life illustrating proportionality for all health states

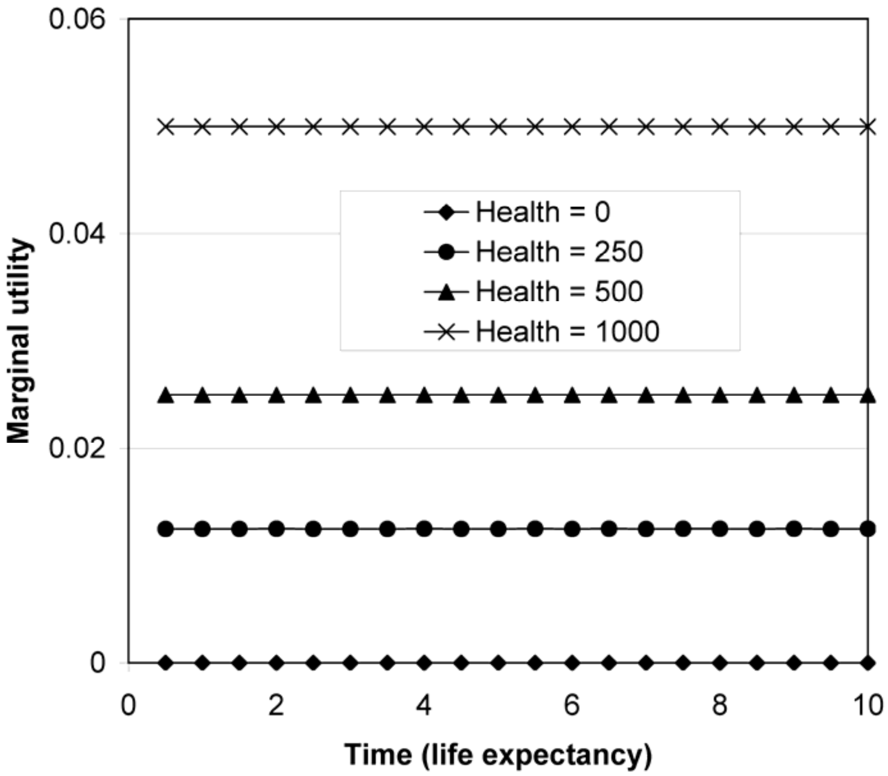

Figure 4 Utility as a function of length of life illustrating non-proportionalities

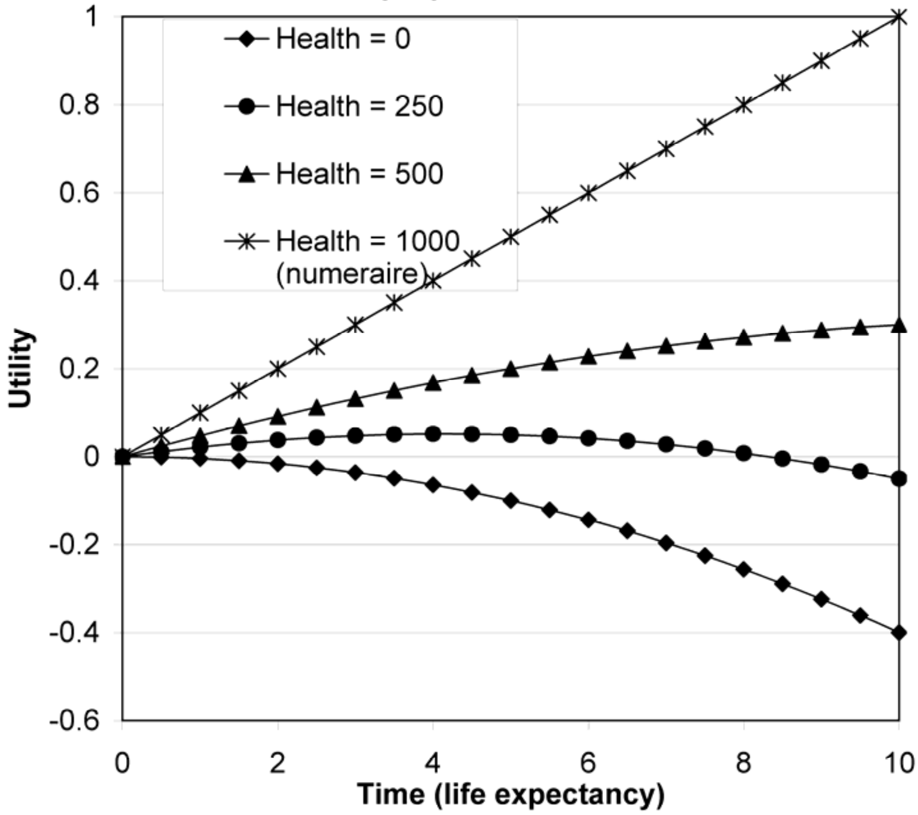

Figure 5 Marginal utility in length of life illustrating non- proportionalities

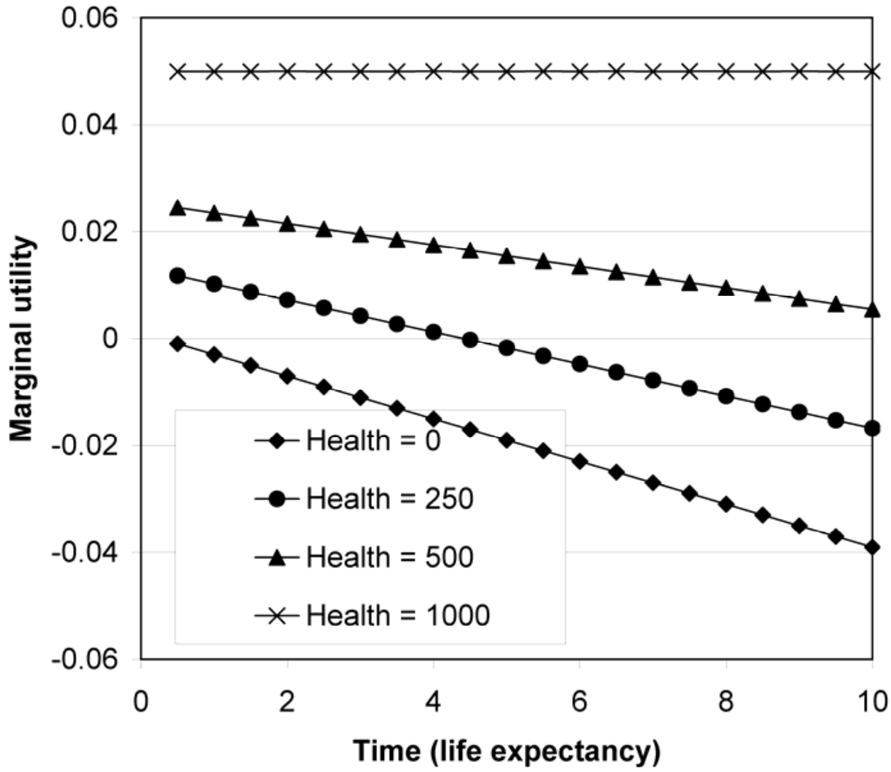

Figure 6 Utility as a function of health and length of life showing MET