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Herding Effects in Order Driven Markets: The Rise and Fall of Gurus

Giulia Iori
City University London

Gabriele Tedeschi
Università Politecnica delle Marche

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Abstract

We introduce an order driver market model with heterogeneous traders that imitate each other on a dynamic network structure. The communication structure evolves endogenously via a fitness mechanism based on agents performance. We assess under which assumptions imitation, among otherway noise traders, can give rise to the emergence of gurus and their rise and fall in popularity over time. We study the wealth distribution of gurus, followers and non followers and show that traders have an incentive to imitate and to be imitated since herding turns out to be profitable.

Keywords: dynamic network, herding, guru, order driver market.

1 Introduction

Mainstream economic theory does not provide a satisfactory explanation for financial market frenzies, crashes and panics. The standard reasoning, dating back to Friedman (1953), is that these phenomena, driven by irrational traders, are irrelevant in the long run since destabilizing speculators would quickly go bankrupt and be eliminated from the market. Thus, according to the mainstream literature, the study of rational speculators is enough to describe the behaviour of stock markets. Nonetheless there is a considerable empirical evidence that investors do not always act rationally and do not follow the economists’ advice. Black (1986) suggests that some traders, when they do not have access to true information, irrationally act on noise, and, following Kyle (1985), calls such investors ”noise traders”. The presence of noise traders and their impact on prices’ movement is well
documented. Some authors (Figlewski (1979), Shiller (1984), Campbell and Kyle (1987), De Long et al. (1990a)) show that if ‘rational agents’ are risk adverse, then their ability to take positions against noise traders, who drive prices away from their fundamental value, is limited.

An important mechanism that may explain the deviation of prices from their fundamental value is the formation of expectations. Expectations drive individual behaviours and individual behaviours determine the economic outcome, i.e., prices and trading. "Therefore, a market, like other social environments, may be viewed as an expectations feedback system" (Heemeijer et al. (2009)). An intuition of how expectations feedback system with 'zero intelligence agents'1 works is as follows. If noise traders share pessimistic expectations about an asset, they will sell it frantically, driving down its price. An informed trader who may want to buy the asset will update his expectations recognizing that in the near future noise traders might become even more pessimistic and drive price down even further. The informed trader may eventually conclude that it is not convenient for him to buy now. Conversely, if the informed trader wants to sell an asset about which noise traders have optimistic expectations, that would drive the price up, he may decide not to sell. Thus convergence to the rational equilibrium price becomes unlikely. In fact, because of the unpredictability of noise traders, prices can fluctuate significantly even when the fundamental price is stable.

It is generally accepted in the economic works on noise traders that ”positive feedback” traders prevail in financial markets. Theoretical models and laboratory experiments of positive feedback2 have been studied (De Long et al. (1990a), De Long et al. (1990b), Marimon and Sunder (1993), Geber et al. (2002), Hommes et al. (2005, 2007), Sutan and Willinger (2005) and Adam (2007)). All these works have shown that positive feedback traders, if sufficiently aggressive, can destabilize prices.

An important question is why traders’ expectations are often coordinated. Claude Trichet (2001) remarked: "Some operators have come to the conclusion that it is better to be wrong along with everybody else, rather than take the risk of being right or wrong alone"3. This "mass-uniform" behaviour was already present in Kaynes (1936) who called it ”animal spirits”.

Some insights into fluctuations in prices and coordination of expectations

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1Our market is populated by agents with naive trading strategies, called noise traders. In this way we are close to the tradition of Zero-Intelligence (ZI) traders as in (Becker (1962), Gode et al. (1993), Gode et al. (1997).
2Positive feedback in a stock market refer to the situation where positive (negative) expectations about the price do lead to a price increase (decrease)
3Claude Trichet, then Governor of the Banque de France.
have been provided by agent-based models. For example Lux and Marchesi (2000), Iori (2002), Chiarella et al. (2002, 2009), Kirman and Teyssière (2002), LiCalzi and Pellizzari (2003), Gaunersdorfer et al. (2008), and LeBaron et al. (2007, 2009) have analyzed how the co-ordination of traders’ strategies by market mediated interactions (for example by following chartist trading rules) or mechanisms of behavioural switching can lead to large aggregate fluctuations.

Collective behaviour nonetheless could reflect the phenomenon known as herding which occurs in situations with information externalities, when agents’ private information is swamped by the information derived from directly observing others’ actions (Bannerjee (1992, 1993); Orléan (1995); Cont and Bouchaud (1999), Stauffer and Sornette (1999), Iori (2002), Markose et al. (2004), LeBaron et al. (2009)). Most of the studies on herding effects have focused on how herding can lead to large price fluctuation but only a few papers have investigated its role on the communication network structure and on traders’ wealth, which is the focus of this paper.

We introduce a model where agents imitate the expectations of the most successful traders called the Guru. Price formation is determined by an order-driven market, as in Chiarella et al. (2009). Within their budget constraints, agents can place market orders or limit orders for arbitrary quantities. Limit orders are stored in the book and executed (partially or completely) when they find a matching market order on the opposite side. A market order is filled completely if it finds enough capacity on the book, or partially otherwise. The motivation to use an order-driven market is to avoid the limitations of the market maker approach in which there is no explicit mechanism of trading. In fact, the market maker, typically risk neutral and endowed with unbounded liquidity, absorbs excess demand and makes trading always viable, regardless of its size. In each period, the market maker adjusts the price to reduce the excess demand. Inspired by the metaphor of the Walrasian tâtonnement, this price-adjusting rule fails to recognize that in a real market trade occurs whenever two agents can match their requests at a given price. Because of the simplistic pricing rule adopted by the market maker, herding (that normally leads to a large excess demand) has an obvious and immediate impact on prices.

In an order driven market, where agents imitate the expectations of others

\footnote{Agent based models have taken many different approaches as to how strategy information could be shared outside price system (see Vriend (2000)). “Obviously, the correct model for information sharing is not identifiable, but it is clear that some imitation must take place in financial markets”-Le Baron (2009).}
and not their action, the role of imitation is less obvious. In fact even if 
the guru expects a price increase, he himself and/or the agents that imitate 
his expectations may submit limit orders instead of market orders, and the 
impact of these trades on the price may be negligible or may be delayed.

In this paper we introduce a model of herding that works as an affective 
coordination mechanism of trading actions in limit order markets and has 
an impact on prices and the wealth distribution of agents. In our model 
all agents are uninformed noise traders and directly imitate each other. The 
originality of this work respect to our previous version (Tedeschi et al. (2009)) 
is in the communication network. In Tedeschi et al. the the guru was fixed 
exogenously and each agent decided whether to imitate him or not with a 
given probability. Here we introduce an endogenous mechanism of imitation, 
by implementing a preferential attachment rule (Barabási and Albert (1999)) 
such that each trader is imitated by others with a probability proportional to 
its profit. This mechanism of links formation allows us to study under which 
assumptions a gurus endogenously rise and fall over time, and how imitation 
affects the asset price and the distribution of agents wealth.

Two models that are related to our are that of Markose et al. (2004) and that 
make a binary decision, to buy or sell a single unit of an asset, following the 
average advice of the other agents they are connected to. The interaction 
network evolves dynamically as agents adaptively modify the weights of their 
links to their neighbours by reinforcing ”good” advisors and breaking away 
from ”bad” advisors. The question Markose et al. address is whether and 
when the dynamic process of reinforced learning can lead to the creation of 
small world networks. Nonetheless trading is not explicitly modelled in their 
model and the wealth of agents is not monitored.

LeBaron et al. (2009), instead, develop a dynamic limit order model where 
traders make weighted forecasts of assets future returns by combining fund-
amental, chartist and noise rules, following Chiarella et al (2009). As time 
goes by, agents do trade and look at their own past performances (measured 
in term of their expected prices versus realized prices) and update the weights 
of their trading rules via a genetic algorithm that selects those parameters 
which have performed better. In this model there no direct imitation among 
traders and coordination arises when traders dynamically adopt the same 
rule.

Although agents in our model initially start with the same amount of stock 
and cash, when imitation is high, trading generates a fat tail distribution of
individuals’ wealth, in accordance with the empirical evidence that market participants are very heterogeneous in size (see, for example, Pareto (1897), Zipf (1949), Ijiri and Simon (1977), Axtell (2001), Pushkin and Aref (2004), Gabaix et al. (2006)). Moreover, in contrast with the prevailing economic view that informed agents need to hide their private information in order to profit from it (see Benabou and Laroque (1992), Caldentey and Stachetti (2007), Chakraborty and Yilmaz (2008)), our uninformed gurus gain the highest profits when they reveal their expectations to the highest number of followers. Furthermore, we will also show that followers, on average, gain higher profit than non-followers, thus providing a justification for herding to occur in the first place.

The rest of the paper is organized as follows. In section 2 we describe the model; in section 3 we present the results of the simulations; section 4 concludes.

2 The model

2.1 The market

This section describes our market which is based on the order-driven market used in Chiarella et al. (2009).

A population of $N$ traders can either place market orders, which are immediately executed at the current best listed price, or they can place limit orders. Limit orders are stored in the exchange’s book and executed using time priority at a given price and price priority across prices. A transaction occurs when a market order hits a quote on the opposite side of the market.

Trading happens over a number of periods $t_k$, with $k = 1, \cdots, T$. At the beginning of each period, traders make expectations about the price at the end of a given time horizon $\tau$ (that we take to be the same for all traders). The future price expected at time $t_k + \tau$ by agent $i$ is given by

$$\hat{p}_{t_k, t_k + \tau}^i = p_{t_k} e^{\hat{r}_{t_k, t_k + \tau}^i} \sqrt{\tau}$$  \hspace{1cm} (1)

where $\hat{r}_{t_k, t_k + \tau}^i$ is the agent’s expectation on the spot return which, as we will see later, may be affected by the expectation of other agents, and $p_{t_k}$ is the reference price observed by all agents at the beginning of each period. After

\footnote{In fact revealing private intentions, specially for large agents, could decrease their fitness. For this reason large investors refrain from revealing their demand, supply or their expectation (see Vaglica et al. (2008)).}
expectations are made, agents enter the market, sequentially and in a random order and place a buy or a sell order of a certain size. Orders that are not executed after a period $\tau$ are removed from the book.

The number of stocks an agent is willing to hold in its portfolio at a given price level $p$ depends on the choice of the utility function. Our agents are modeled as risk averse and maximize an exponential CARA utility function

$$U(W_t^i, \alpha^i) = -e^{-\alpha^i W_t^i},$$

where the coefficient $\alpha^i$ measures the risk aversion of trader $i$. We assume that agents’ risk aversion depends crucially on the expectation formation mechanism described below. In particular, we assume that those agents who are highly imitated, and consequently, as we will see later, are more successful are less risk averse than agents who have no impact on the expectations of others. This effect is captured by setting

$$\alpha^i_{t_k} = \alpha(1 - (1 - w)l^\%_{i,t_k})$$

where $\alpha$ is some reference level of risk aversion, $l^\%_{i,t_k}$ is the percentage of existing links that point to agent $i$ at time $t_k$ and $w$ measures the impact that agent $j$’s expectation has on the agent $i$’s expectation: smaller is $w$, higher is this impact.

We next define the portfolio wealth of each agent as

$$W_t^i = S_t^i p_t + C_t^i,$$

where $S_t^i \geq 0$ and $C_t^i \geq 0$ are respectively the stock and cash position of agent $i$ at time $t$. The optimal composition of the agent’s portfolio is determined in the usual way by trading-off expected return against expected risk. However here the agents are not allowed to engage in short-selling. When agents place a market order, their cash and stocks positions are updated accordingly. When agents place a limit order, the cash they commit to buy and the stocks they commit to sell are tentatively removed from their portfolios (even if a limit order does not comport an immediate transaction). In this way agents can not spend money or sell stocks that have already been committed in the book. If an order is cancelled, the stocks and cash that were tied down in the order are returned to the trader who had submitted it.

For the CARA utility function assumed here the optimal composition of the portfolio, that is the number of stocks the agent wishes to hold at any given time is given by

$$\pi^i(p) = \frac{\ln(\tilde{p}^i_{t+\tau}/p)}{\alpha^i V_t^i p},$$
where $V^i_t$ is the variance of returns expected by agent $i$. If the amount $\pi^i(p)$ is larger (smaller) than the number of stocks already in the portfolio of agent $i$ then the agent decides to buy (sell). Agents estimate $V^i_t$ as

$$V^i_t = \frac{1}{\tau} \sum_{j=1}^{\tau} [r_{t-j} - \bar{r}^i_t]^2; \quad (6)$$

where the average spot return $\bar{r}^i_t$ is given by

$$\bar{r}^i_t = \frac{1}{\tau} \sum_{j=1}^{\tau} r_{t-j} = \frac{1}{\tau} \sum_{j=1}^{\tau} \ln \frac{p_{t-j}}{p_{t-j-1}}, \quad (7)$$

In order to determine the buy/sell price range of a typical agent, we first estimate numerically the price level $p^*$ at which agents are satisfied with the composition of their current portfolio, which is determined by

$$\pi^i(p^*) = \ln(\hat{p}^i_t + \tau/p^*) - \alpha^i V^i_t p^* = S^i_t. \quad (8)$$

Eq. (8) admits a unique solution with $0 < p^* \leq \hat{p}^i_t + \tau$ since $S^i_t \geq 0$ (short selling is not allowed). Agents are willing to buy at any price $p < p^*$ since in this price range their demand is greater than their holding, and are willing to sell at any price $p > p^*$ since in this case their demand is less than their holding. Note that agents may thus wish to sell even if they expect a future price increase. In order to impose budget constraints we need to restrict to values of $p \leq \hat{p}^i_t + \tau = p^M$ to ensure $\pi(p) \geq 0$ and so rule out short selling. Furthermore to ensure that an agent $i$ has sufficient cash to purchase the desired stocks, the smallest value of $p$ we can allow, $p^i_m$, is determined by its cash position (see Eq. (4)), and is given by the condition

$$p^i_m \left(\pi^i(p^i_m) - S^i_t\right) = C^i_t. \quad (9)$$

Again one can easily show that this equation also admits a unique solution with $0 < p^i_m \leq \hat{p}^i_t + \tau$ since $S^i_t, C^i_t \geq 0$. Indeed, comparing Eqs. (8) and (9) it can be easily proven that $0 < p^i_m \leq p^* \leq \hat{p}^i_t + \tau$. Having determined that the possible values at which an agent can satisfactorily trade are in the interval $[p^i_m, p^M]$, we next consider how the nature of the agent’s order is determined. Suppose now that the agent chooses to trade at a price $p < p^*$, then it submits a limit order to buy an amount

$$s^i = \pi^i(p) - S^i_t,$$

where $\pi^i(p)$ can be derived on the basis of mean-variance one-period portfolio optimization.
Table 1: Summary of the trading mechanism of a typical trader $i$ with a random price level $p$ limited between the value $p_{im}$ given by Eq. (9) and the value $p_M = \hat{p}_{i,t+1}$. The current quoted best ask and best bid are $a_q$ and $b_q$ respectively.

<table>
<thead>
<tr>
<th>Position</th>
<th>Type of order</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{im} &lt; p &lt; a_q$</td>
<td>BUY</td>
<td>Limit order $s_i = \pi^i(p) - S_i^q$</td>
</tr>
<tr>
<td>$a_q \leq p &lt; p^*$</td>
<td>BUY</td>
<td>Market order $s_i = \pi^i(a_q) - S_i^q$</td>
</tr>
<tr>
<td>$p = p^*$</td>
<td>No order placement</td>
<td></td>
</tr>
<tr>
<td>$p^* &lt; p \leq b_q$</td>
<td>SELL</td>
<td>Market order $s_i = S_i^q - \pi^i(b_q)$</td>
</tr>
<tr>
<td>$b_q &lt; p \leq p_M$</td>
<td>SELL</td>
<td>Limit order $s_i = S_i^q - \pi^i(p)$</td>
</tr>
</tbody>
</table>

While if $p > p^*$ it submits a limit order to sell an amount

$$s^i = S^i - \pi^i(p).$$

However if $p < p^*$ and $p > a_q$ the buy order can be executed immediately at the ask. An agent in this case would submit a market order to buy an amount

$$s^i = \pi^i(a_q) - S_i^q.$$  

Similarly if $p > p^*$ and $p < b_q$ the agent would submit a market order to sell an amount

$$s^i = S_i^q - \pi^i(b_q).$$

If the depth at the bid (ask) is not enough to fully satisfy the order, the remaining volume is executed against limit orders in the book. The agent thus takes the next best buy (sell) order and repeats this operation as many times as necessary until the order is fully executed. This mechanism applies under the condition that sufficient quotes of these orders are above (below) price $p$. Otherwise, the remaining volume is converted into a limit order at price $p$. If the limit order is still unmatched at time $t + \tau$ it is removed from the book.

The essential details of the trading mechanism are summarized in Table 1, showing how it depends on the price level $p$, the “satisfaction level” $p^*$, the best ask $a_q$ and the best bid $b_q$.

2.2 The network

To model how agents’ decision are influenced by their mutual interaction we introduce a communication structure in which nodes represents agents
and the hedges are the connective links between them. Links are directional and go from the agent that requests advice to the agent that provides advice.

In general local interaction models agent interacts directly with a finite number of others in the population. The set of nodes with whom a node is linked is referred to as its neighbourhoods. In our model the number of out-going links is constrained to be one. The reason being that in a highly connected random network synchronisation could be achieved via indirect links. The effects of direct imitation are easier to be tested in a diluted network where indirect synchronisation is less likely to arise.

We implement an endogenous mechanism of preferential attachment based on a fitness parameter given by agent’s wealth. Agents start with the same amount of cash $C_{t=0}$ and stocks $S_{t=0}$, so that all agents have the same initial wealth $W_{t=0} = C_{t=0} + p_{t=0}S_{t=0}$. As time goes by, some traders may become richer than others. As a measure of agents’ success we define their fitness at time $t$ as their wealth relative to the wealth $W_{t}^{\text{max}}$ of the richest agent $i_{\text{max}}$:

$$f_{t}^{i} = \frac{W_{t}^{i}}{W_{t}^{\text{max}}}.$$  

(10)

Each agent $i$ starts with one outgoing link with a random agent $j$, and possibly with some incoming links from other agents. Links are rewinded at the beginning of each period, in the following way: each agent $i$ cuts his outgoing link, with agent $k$, and forms a new link, with a randomly chosen agent $j$, with a probability

$$p_{r} = \frac{1}{1 + e^{-\beta(f_{t}^{j} - f_{t}^{k})}}.$$  

The rewind algorithm is designed so that successful traders, here called gurus, gain a higher number of incoming links and thus have a higher probability of being imitated. Nonetheless the algorithm introduces a certain amount of randomness, and links with more successful agent have a finite probability to be cut in favour of links with less successful agents. In this way we model imperfect information and bounded rationality of agents. The randomness also helps unlocking the system from the situation where all agents link to the same guru.

### 2.3 The expectation formation mechanism

At the beginning of each trading period $t_{k}$, agents make idiosyncratic expectations about the spot return, $\hat{r}_{t_{k}, t_{k} + \tau}^{i}$ in the interval $(t_{k}, t_{k} + \tau)$. We assume
that agents are not informed and have random expectation of future returns. We also assume that agents are heterogeneous in that they have different forecasts of the returns’ volatility, \( \sigma_{t_k}^i \). Expected returns are thus given by

\[
\hat{r}_{t_k,t_k+\tau}^i = \sigma_{t_k}^i \epsilon_{t_k}
\]

where \( \sigma_{t_k}^i \) is a positive, agent specific, constant and \( \epsilon_t \sim N(0, 1) \) is a normal noise.

After individual expectation are generated, a consultation round starts during which agents sequentially, and in a random order, revise their expectation. The revised expected return is obtained by weighing agent \( i \)'s own expectation with that of agent \( j \) to which \( i \) is linked to

\[
r_{t_k,t_k+\tau}^i = w\hat{r}_{t_k,t_k+\tau}^i + (1 - w)\hat{r}_{t_k,t_k+\tau}^j.
\]

When \( w \) is equal to zero, \( i \) trusts completely the opinions of \( j \), while when \( w \) is equal to one \( i \) considers exclusively his own opinion and agents decisions are fully independent from each other. At the end of each period \( t_k \), after trading has taken place, agents expectations are reset to random values. We stress that in the model imitation is purely expectation based, and agents do not observe the actions of others. This choice is motivated by the fact that in a real market the order book is not normally fully visible to traders, and that the order submission is anonymous.

While our agents are noise traders, we assume that they correctly anticipate the impact of herding on asset prices. In particular if an agent has several incoming links, and \( w \) is small (in which case the agent expects to be able to influence the decisions of others), he forecast a larger price volatility. This is incorporated in the model by assuming that the volatility of returns is proportional to the number of incoming links and to the weights \( w \), such that

\[
\sigma_{t_k}^i = \sigma_0^i (A + l_{t_k}^i (1 - w))
\]

where \( l_{t_k}^i \) in the percentage of existing links that point to agent \( i \) at time \( t_k \) and \( A \) is a constant parameter. The values of \( \sigma_0^i \) are chosen, with uniform probability, in the interval \((0, \sigma_0)\).

## 3 Simulations and results

The model is studied numerically for different values of the parameter \( w \). In the first part we focus the analysis on some properties of the network such
as the in-degree and fitness distribution. Then we analyze the probability distribution of wealth and stocks and the positive feedback on prices.

In the simulations the number of traders is set at $N = 150$. Each agent is initially given the same amount of stock $S_0 = 100$ and cash $C_0 = 100$. The initial stock price is chosen at $p_0 = 1000$. We fix $\tau = 200, \alpha = 0.01$, and $\beta_i$ uniformly distributed in the interval $[5, 45]$. The results reported here are the outcome of simulations of $T = 1000$ periods and $N_t = 300$ trades per period. Simulations are repeated $M = 100$ times with a different random seed\footnote{We have tested the stability of our results and verified that the model shows a qualitatively similar behaviour for a range of values of the parameters.}

### 3.1 The network

In figure (1) we plot one shot of the configuration of the endogenous network for $w = 0.1, w = 0.5$ and $w = 1.0$. The graphs show that few gurus could co-exist and compete for popularity. As $w$ increases the network becomes less and less centralized with a higher number of smaller gurus. Moreover

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{network_configuration.png}
\caption{Network configuration for $w = 0.1$ (the guru is agent 108) (left side), for $w = 0.5$ (the guru is agent 78) (centre) and for $w = 1$ (the guru is agent 6) (right side).}
\end{figure}

the topology of the network is different from that of the random graph studied extensively by Erdos and Renyi (1960). While in an Erdos-Renyi random graph the in-degree\footnote{In directed graphs, there is the in-degree, number of edges pointed to it, and out-degree, number of edges pointing away from it. Note, the out-degree of an agent defined as} has a Binomial (or Poisson) distribution, in real world
networks some agents are found to have a disproportionately large number of incoming links while others have very few. In figure (2) we plot the com-

![Figure 2](image.png)

Figure 2: The complementary cumulative distribution (CCD) of the normalized in-degree (left side) and the complementary cumulative distribution (CCD) of the normalized fitness (right side) for $w = 0.1$ (black line), $w = 0.5$ (red line) and $w = 1.0$ (green line).

plementary cumulative distribution (CCD) of the normalized in-degree (left side) and the complementary cumulative distribution (CCD) of the normalized fitness (right side) for $w = 0.1$ (black line), $w = 0.5$ (red line) and $w = 1.0$ (green line). The distribution of in degree in our model, when imitation is large, is in keeping with that of scale-free networks and displays a 'fat tail'.

In figure (3) we plot the index of the current guru (black), the percentage of incoming link to the current guru (red) and the fitness of the current guru (green), for different $w$, as function of the time. The figure shows that agents alternate as the guru during the simulation (black line). In fact, as the guru acquires an increasing number of links (red line), one or more of his followers may become richer than the guru himself, as signalled by the fact that the fitness (green line) of the guru becomes, at times, smaller than 1. As other agents become rich they start to be imitated more and more and eventually one of them becomes the new guru.

by those edges starting from $i$ gives the number of his first order neighbours that, in our model, are constrained to be one.
The stability (or average life) of the guru becomes longer as imitation increases as shown in figure (4).

Figure 3: The index of current guru (black), the percentage of incoming link to current guru (red) and fitness of current guru (green) for \( w = 0.1 \) (left side), \( w = 0.5 \) (centre) and \( w = 1 \) (right side).

Figure 4: Average Guru’s live as a function of \( w \).

3.2 Wealth analysis

In figure. (5) we compare the different performances, in terms of wealth, of the guru (black line), his direct followers (red line) and the rest of the traders
Figure 5: Wealth time series of guru (black line), followers (red line) and rest of the system (green line) for $w = 0.1$ (left side), $w = 0.5$ (center) and $w = 1.0$ (right side).

(green line) for the same parameters as in figure (3). Comparing figure (5) and figure (3) we observe that the wealth of the guru increases with $w$ and that the gap between the wealth of the guru and the wealth of the rest of the system (both followers and non followers) widens with the level of imitation. This result is better quantified by figure (6) that shows the average wealth, over all times and all simulations, of the gurus (black line), followers (red line) and rest of the system (green line) as a function of $w$.

Figure (7) shows how, raising imitation, the model generates heterogeneity, as indicated by the fat tail distribution of agents’ wealth and stock.

3.3 Prices Analysis

Figure (8) shows prices (black lines) and average expected prices (red lines) for $w = 0.1$, $w = 0.5$ and $w = 1.0$. We can immediately notice that coordination causes wider excursions of price movement. Further, we observe that prices and expected prices follow each other closely when $w$ is small.

To better quantify this observation we calculate the mean price deviation between realized prices and expected prices defined as $|p_{tk} - \hat{p}_{tk}|$, and average it over time and the number of simulations. As figure (9) shows this deviation is smaller when imitation is high. This result is consistent with the fact that herding generates positive feedback.

Positive feedback has several implications on the correlation between as-
Figure 6: Average wealth, over all times and all simulations, of the guru (black line), followers (red line) and rest of the system (green line).

Figure 7: The decumulative distribution function (DDF) of the wealth (left side) and the decumulative distribution function (DDF) of the stocks (right side) for $w = 0.1$ (black line), $w = 0.5$ (red line) and $w = 1.0$ (green line).

set price behaviour and traders’ action. In particular, experimental investigations and theoretical models (see De Long et al. (1989), Hommes et al.
Figure 8: Prices (black line) and average expected prices (red line) with \( w = 0.1 \) (left), \( w = 0.5 \) (center) and \( w = 1.0 \) (right).

(2003), Heemeijer et al. (2007)) show that, when noise traders follow positive feedback strategies, they buy when prices increase and sell when prices fall.

Figure 9: Deviation between realized prices and expected prices as a function of \( w \)

A natural way to assess the co-movement between the increase (decrease) in prices and increase (decrease) in purchase orders is to study their correlation. Figure. (10) shows that such correlation coefficient is an decreasing function
of $w$ and reaches a value above 0.8 when imitation is significant, confirming the presence of a positive feedback.

### 3.4 Discussion

To explain the above results we first need to show that the imitation of expectations translates into imitation of trading actions. An expected price increase (decrease) in our model does not necessarily lead to a decision to buy and, even if so, buy order could be submitted as limit orders. Market orders are more likely to be submitted when agents are very optimistic or pessimistic. In fact in this case the interval $[p_m, p_M]$ over which orders can be placed is wider and it becomes more likely that a price level is chosen such that the order can be immediately executed. In our model it is the agents with many incoming links who forecast a high volatility $\sigma_i^t$ (via equation 13) and are more likely to submit market orders. In addition, if a popular agent has enough connections it can influence several others to overestimate price changes and submit market orders in turns.

In figure. (11) we plot the average fraction of the volume of market orders

\[\text{Figure 10: Average correlation coefficient between positive price changes and number of buyers as a function of } w.\]
to buy or sell, over the total volume of orders in the same direction. The average is taken over each trading period and plotted for different values of \( w \). The result shows, as anticipated, that, when increasing imitation, a higher fraction of market orders is submitted. Thus the coordination of expectation leads to a coordination of actions and the model generates an expectations feedback system.

Figure 11: Average fraction of the volume of market orders to sell (left) buy (right) over the total volume of orders in the same direction for different value of \( w \).

In turn, a series of market orders in the same direction can generate considerable price changes, as shown by figure 12. Thus, the forecasts from highly connected agents of an overall high volatility are self-fulfilling, providing an ex-post justification for equation (13).

Next we explain the distribution of agents’ wealth and stocks. First of all, as long as the guru is not the last to trade (we assume a random entrance to the market for all agents including the guru) he will consistently gain on
the trades that follow, in the same direction, his trade. Furthermore, while agents are risk adverse, highly connected agents underestimate risk, according to equation 3. Consequently these traders, when \( w \) is small and their percentage of incoming links, \( l\% \), is high, invest more (on average) in the risky asset than others, as confirmed by figure (13). Followers in turn invest on average more than non followers because they, like the guru, overestimate returns. By investing more, gurus and followers earn, on average, higher profits than no followers (as was shown in figure 6).

These results are in line with other studies on noise traders risk with positive feedback in financial markets. Particularly, De Long et al. (1990a) show that noise traders can earn higher returns solely by bearing more of the risk that they themselves create.\(^{10}\) Our results are in line with previous studies (see Haltiwanger and Waldman (1985), Heemeijer et al. (2007)) and confirm that traders have an incentive to imitate and be imitated, since predicting a price close to the predictions of other players turns out to be most profitable.

\(^{10}\) An example of this phenomenon, known under the name of market manipulation, is the 'pool' in RCA stock operated by Michael Meehan between March 7 and March 22, 1929. (see De Long et al. (1990b))
Figure 13: Mean size of orders of the guru (black) and the rest of the system (red) as a function of $w$.

4 Conclusion

Our results allow us to conclude that profit is a good mechanism of links formation, and able to generate the famous Matthew effect\textsuperscript{11}. The endogenous attachment mechanism introduced in our model allows a guru to emerge spontaneously in the system, rise and fall in popularity over time, and possibly be replaced by a new guru. A few gurus could also co-exist and compete among themselves for popularity. Nonetheless, for the endogenous attachment mechanism to be capable of creating, sustaining and destroying a guru, agents need to benefit from imitating and being imitated. In fact, if an agent profits from being imitated, he becomes richer, which induces an even larger fraction of agents points to him. Nonetheless, if only the agents who are imitated benefit from imitation, once an agent becomes the guru he

\textsuperscript{11}In 1955 Herbert Simon showed that power laws arise when ‘the rich get richer’, when the amount you get goes up with the amount you already have. In sociology this is referred to as Matthew effect (see Merton (1968)) with reference to the biblical edict. Today, this phenomenon is usually known under the name ‘preferential attachment’, coined by Barabasi and Albert (1999). Bianconi and Barabasi (2001) have proposed an extension of Barabasi and Albert. In their model each newly appearing vertex $i$ is given a ‘fitness’ that represents its attractiveness and thus its propensity to gain more links. When one considers a fitness algorithm it is true that the larger the fitness the larger the degree but a large degree is a consequence of some intrinsic quality, not the cause of the improvement of site connectivity. In our model this intrinsic quality is, precisely, the agents’ profit.
would remain the guru for ever. On the other side, if followers also profit from imitating the guru, their could eventually over-performe the guru, and become guru in turn.

The fact that our unsophisticated investors, trivially driven by imitative behaviour, can earn very high profits implies that Friedman’s hypothesis is inadequate. The assumption that noise traders quickly go bankrupt and are eliminated from the market is unrealistic in presence of herding and positive feedback. In fact we have shown that noise traders can earn very high profits and cause large price fluctuations. These results should not be underestimated, particularly in those situations when market prices exhibit large fluctuation. In these cases in fact is unlikely that prices incorporate true information and the idea of full rationality is implausible.

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