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Poly: A Reliable and Energy Efficient Topology Control Protocol for Wireless Sensor Networks

Hassaan Khaliq Qureshi*[†], Sajjad Rizvi*, Muhammad Saleem[‡], Syed Ali Khayam*,
Veselin Rakocevic[†] and Muttukrishnan Rajarajan[†]
*School of Electrical Engineering & Computer Science (SEECS)
National University of Sciences & Technology (NUST), Islamabad, Pakistan.
Email: {hassaan.khaliq, sajjad.rizvi, ali.khayam}@seecs.edu.pk
[†]School of Engineering and Mathematical Sciences, City University, London, UK.
Email: {hassaan.qureshi.1, V.Rakocevic, R.Muttukrishnan}@city.ac.uk
[‡]Center for Advanced Studies in Engineering (CASE), Islamabad, Pakistan.
Email: msaleem@case.edu.pk

***Abstract*—Energy efficiency and reliability are the two important requirements for mission-critical wireless sensor networks. In the context of sensor topology control for routing and dissemination, Connected Dominating Set (CDS) based techniques proposed in prior literature provide the most promising efficiency and reliability. In a CDS-based topology control technique, a backbone – comprising a set of highly connected nodes – is formed which allows communication between any arbitrary pair of nodes in the network. In this paper, we show that formation of a polygon in the network provides a reliable and energy-efficient topology. Based on this observation, we propose Poly, a novel topology construction protocol based on the idea of polygons. We compare the performance of Poly with three prominent CDS-based topology construction protocols namely CDS-Rule K, Energy-efficient CDS (EECDS) and A3. Our simulation results demonstrate that Poly performs consistently better in terms of message overhead and other selected metrics. We also model the reliability of Poly and compare it with other CDS-based techniques to show that it achieves better connectivity under highly dynamic network topologies.**

I. INTRODUCTION

Wireless Sensor Networks (WSNs) are envisioned as an enabling technology for a broad class of mission-critical applications. It is generally assumed that nodes in a WSN are connected to their neighbors with a certain probability of packet loss. Since wireless links are inherently unreliable, these packet losses are not acceptable for many mission-critical WSN applications (e.g., forest fire detection, battle field monitoring) which require the network topology to provide a certain desired level of reliability. This reliability should however be achieved while keeping in mind the fundamental energy consumption constraint of a WSN. In this context, the graph-theoretic Connected Dominating Set (CDS) principle has emerged as the most popular method for energy-efficient topology control (TC) in WSNs [2], [3], [4], [5], [6], [7].

TC consists of two phases: *topology construction* and *topology maintenance*. In the topology construction phase, a desired topological property is established in the network while maintaining connectivity. Once the topology is constructed, topology maintenance phase starts in which nodes switch their roles to cater for topological changes. In CDS-based TC schemes, some nodes are a part of the virtual backbone which is responsible for relaying packets in the WSN. Non-CDS nodes conserve energy by turning off their transceivers. CDS size is a critical parameter which controls the compromise between reliability and energy efficiency.For instance, for small CDSs, fewer nodes handle the bulk of the network traffic and consequently deplete their batteries quickly [5]. The positive side of a small CDS is that more nodes can go to sleep mode. While both of these metrics – energy efficiency and reliability – are equally important for mission-critical WSNs, existing CDS-based routing protocols cannot simultaneously cater both metrics [5], [6], [7].

In our earlier work, we analyzed the performance of maintaining a cycle in a 10 node network [17]. However, to understand practical limitations, it is important to analyze the performance on larger networks against other widely available protocols. In this paper, we propose a semi distributed graph-theoretic topology control protocol for wireless sensor networks. The protocol, referred to as the Poly protocol, models the network as a connected graph and finds the number of polygons present in the network. Based on the duplicate node IDs of different nodes, Poly adaptively finds a polygenic backbone to turn-off the unnecessary nodes while keeping the network connected and covered. To achieve energy efficiency, the protocol forms a CDS like polygenic network which in turn provides reliability in the case of random link failures. Moreover, it adapts to topological changes in the network based on the remaining energy of the nodes. This allows topology maintenance among different set of nodes to increase the network lifetime.

The Poly protocol has a low message complexity which allows the protocol to run multiple times during topology construction and maintenance phases. It can also be applied to different data reporting models which aim to find rendezvous point's (RPs) and can provide polygenic redundancy to RPs [10]. In addition, Poly achieves energy efficiency while considering network reliability.

The protocol is compared through simulations with A3 [5], Energy Efficient CDS (EECDS) [6] and CDS-Rule K [7] pro-

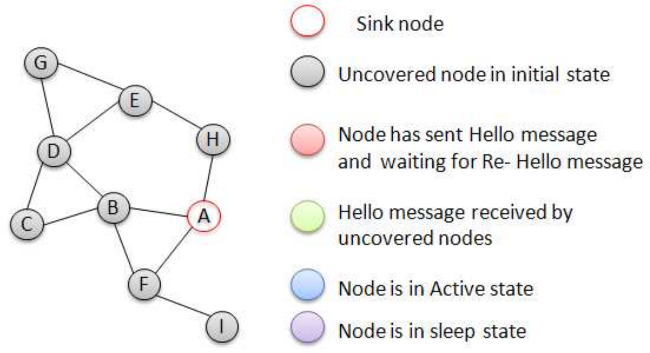


Fig. 1. A sample Network

tocols. Simulations are performed under different underlying topologies, varying node densities to analyze message overhead, energy overhead, residual energy and network connectivity. Simulation results show that the proposed Poly protocol has low energy overhead and it has 19% better residual energy when compared with CDS-Rule K protocol. Similarly, it has 32% and 34% better residual energy versus EECDS and A3 protocol while performing better under topology maintenance techniques. In addition, the results also demonstrate that Poly is 120% more reliable under varying link probabilities than rest of the competitor protocols.

The rest of this paper is organized as follows. Section II summarizes the background and related work in this area. Section III contains the description of the Poly protocol. We describe the empirical evaluation framework utilized for the performance analysis of Poly in Section IV. Simulation results are provided in Section V. Reliability analysis of Poly is presented in Section VI. We summarize the salient findings of this paper in Section VII.

II. BACKGROUND AND RELATED WORK

In this section, we first describe the prominent topology construction protocols. In the second subsection, we summarize the topology maintenance techniques which are later utilized to evaluate the performances of different topology construction protocols.

A. *Topology Construction Protocols*

To achieve energy efficiency, [2] and [11] construct topologies by controlling the transmission power of WSN nodes. Another approach is to make use of geographical location of the nodes [12]. The down side of these approaches is the fact that power control and location awareness are difficult to realize in practical WSN deployments.

An alternative mechanism is proposed in [13] in which a vertex dominating itself and all the adjacent vertices forms a cluster in the graph. A similar Dominating Set (DS) based solution is proposed in [4] which uses the concept of independent dominating sets. Both of these protocols have led to the concept of Connected Dominating Set (CDS) based topology construction protocols for the generation of energy-efficient topology in WSNs. The authors of [5] have proposed a topology construction protocol that produces an approximate solution to form a sub-optimal CDS. A3 selects active nodes which are at the farthest distance from the parent based on the signal strength and remaining energy. This allows fewer nodes to be selected in the CDS tree which in turn leads to an overhead of long distance communication.

The authors of [6] have proposed an Energy-Efficient CDS (EECDS) protocol that computes a sub-optimal CDS in an arbitrary connected graph. EECDS uses two phase strategy to find a CDS. In the first phase, a node elects itself as a cluster-head and then all its neighbors are marked as covered in order to find a Maximal Independent Set (MIS). In the second phase, all the covered nodes except the cluster-heads compete to become gateways to form a CDS. In EECDS, nodes maintain the cluster-head role by gathering neighbor information which allows uniform distribution of energy resources. CDS-Rule K, proposed in [7], uses marking and pruning rules to exchange the neighbors lists among a set of nodes. A node remains marked if there is at least one pair of unconnected neighbors and unmarks itself if it determines that all of its neighbors are covered with higher priority. The node's higher priority is indicated by its level in the tree.

Interestingly, the authors of [8] and [9] have shown that CDS backbones are more vulnerable to node and link failures in WSNs. To this end, they have proposed two approximation algorithms – Connecting Dominating Set Augmentation (CDSA) and k-connected m-dominating set (k, mCDS) – to construct a k-connected virtual backbone which can accommodate the failure of one wireless node. However, they do not analyze the impact of having k-connected virtual backbone on the energy efficiency of the network.

B. *Topology Maintenance Techniques*

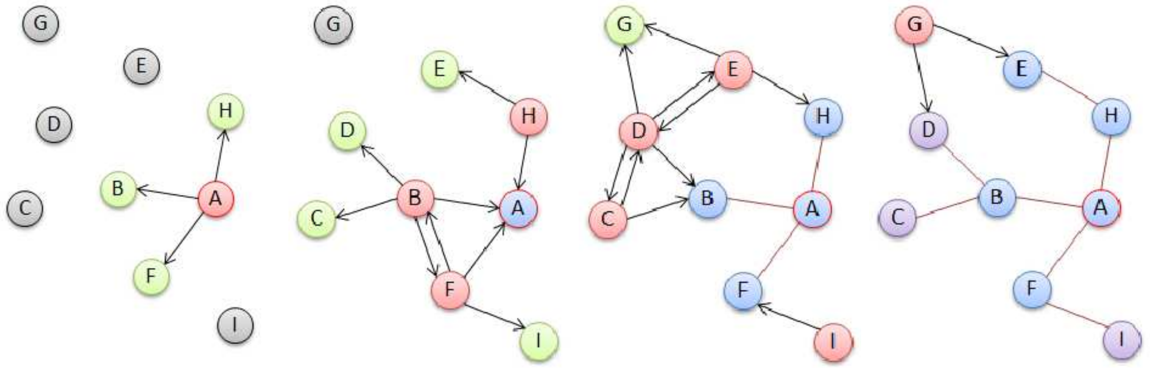
Topology maintenance is a process in which the network topology is changed / maintained during the lifetime of a network. There are various classes of topology maintenance techniques which can be broadly classified into two categories: static and dynamic. As the name suggests, in static maintenance procedures, all possible sets of topologies are computed off-line / during the initial topology construction process. These topologies are then rotated in a desired fashion. On the other hand, dynamic topology maintenance techniques form a new topology based on the present condition of the network, e.g. as an energy threshold is reached.

Topology maintenance procedures may also be classified on the basis of time and energy triggering mechanisms. In time-triggered methods, topology is rebuilt after a specific period of time. However, these mechanisms are generally expensive in terms of message and energy overhead. Therefore, in this paper, we only focus on topology maintenance based on energy thresholds.

We now explain the working of the Poly protocol in the next section.

III. THE POLY PROTOCOL

Due to this paper's focus on mission-critical applications, two fundamental design constraints that we impose on a

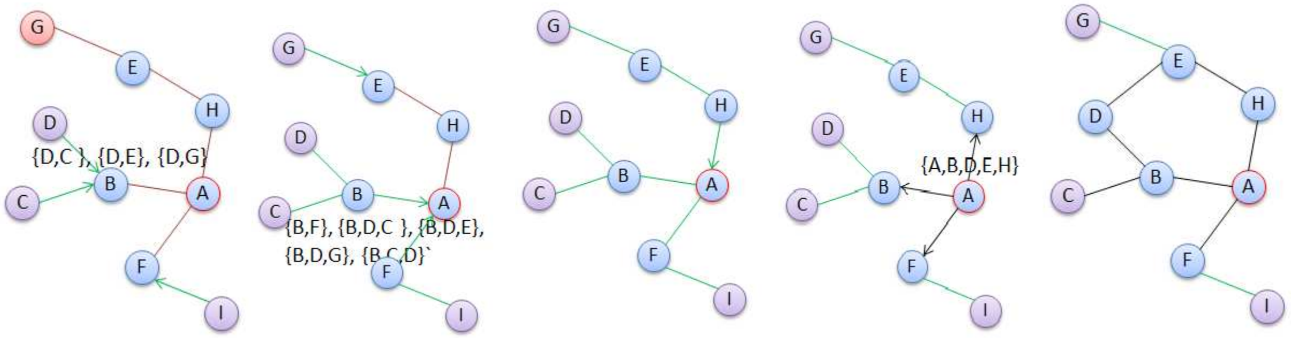


(a) Sink node, A, broadcasts *hello* message - received by nodes B, F and H - and sets a timeout to receive Hello message in response from its children. Nodes B, F and H recognize sender node A as their parent.

(b) B, H and F further broadcast *hello* message with parent ID set to A. Covered nodes, B and F, also recognizes one another as neighbors. When a node recognizes its children, it waits for *Finish discovery* message. Therefore, A is now waiting for Finish message from B, F and H.

(c) Next level nodes again broadcast the Hello message after changing the parent IDs to their respective parent . Lets say G chooses E as its parent . Moreover, C, D and E recognize their neighbors through *hello* message exchange.

(d) G broadcasts *hello* message. Timeout for *hello* message from children expires at C, D and I in which these nodes do not receive any *hello* message with their own IDs as parent ID. Therefore, these nodes consider them as leaf nodes.



(e) Each leaf node sends *finish discovery* message to its parent node and immediate neighbor after the timeout. These sets of nodes are called message paths. Message paths sent by D are shown in the figure.

(f) Node B extends the message paths with its own ID and send it to its parent node. In this way, all message paths, in the form of branches, reaches the sink node. Message paths sent by B are shown in the figure.

(g) After receiving *finish discovery* message from all children, sink node adds it own ID to message paths and figures out a polygon.

(h) Sink node then broadcasts the *create topology* message for the chosen polygon. Nodes in the polygon set turns them as active nodes.

(i) Final topology- a polygon with redundant paths.

Fig. 2. The Poly Protocol

topology construction protocol are: 1) its resultant topology should provide a desired level of packet delivery reliability, and 2) its energy efficiency should be comparable to or less than existing CDS-based topology construction protocols. To satisfy these constraints, the Poly protocol arranges the nodes in such a way that they form a closed path among a set of nodes. The closed path provides a reliable and energy efficient topology because: 1) the sink node gets polygenic redundancy with its neighbors which allows the nodes to use an alternative path in case of random link failures, and 2) it forms an active node set – nodes comprising a polygon – allowing leaf nodes to enter into the dormant / sleep mode. An additional advantage of polygenic is that the topology construction protocol does not need to position or orientation information of the nodes.

In the following two subsections, we describe the polygon formation process. In the first subsection, we define the type of control messages that are used during the topology construction. Subsequently, we illustrate the mechanism that leads to the formation of polygons in the network.

A. Description of control messages

The Poly protocol uses three types of messages which are involved in the polygon formation process. A *hello* message which contains the parent ID of the sender. A *finish discovery* message which is used by the parent node to announce the end of the topology discovery process. In the finish discovery message, each node sends a list of its discovered neighbors. Finally, a *create topology* message containing the IDs of active node set is propagated in the network.

B. Topology construction protocol

Topology construction phase of the Poly protocol is divided into three phases. In the first phase, a CDS is created during which the nodes discover their neighbors. The neighbor discovery process is initiated by a pre-defined node (e.g., the sink node) and terminates at the leaf nodes. In the second phase, each leaf node sends its neighbor list through the upstream neighbor – the so-called parent node – to the sink node. In the third phase, the sink node discovers polygons in the graph. Subsequently, the polygon nodes are informed that they are part of the active node set. In this way, a closed path is formed with connecting paths to the branches.

We describe the formation of polygon with the help of an example network shown in Figure 1. The Poly protocol starts with an initiator node which in our case is node A. Node A broadcasts a *hello* message and starts a timer to receive a *hello* response from its children (see Figure 2(a)). As described earlier, the *hello* message contains the parent ID of the sending node. In the case of the initiator node, this field is empty.

The *hello* of node A is received by B, F and H nodes located within its transmission radius. These nodes are *uncovered* nodes which means that they are in the initial state and have not yet chosen any parent node. Therefore, nodes B, F and H – after receiving the message – choose A as their parent node. The uncovered nodes further rebroadcast the *hello* message to discover their children, and also start their respective timers to receive their children nodes' responses. Every rebroadcasting node, before forwarding the message, updates the parent ID field by replacing it with its own parent ID; for instance, nodes B, F and H in Figure 2(b) update the parent field to the ID of node A.

The rebroadcast *hello* message is also received by the parent node A. Consequently, node A identifies the sender of the *hello* message as one of its children. Once identified, nodes B, F and H are considered as *covered* nodes. Furthermore, when a node identifies a child node, it switches to an active state and starts to wait for *finish discovery* message from the children. When the *hello* message is received by a covered non-parent node, the receiving node identifies the sender as one of its neighbors. For instance, in the given example, the *hello* message from B is also received by node F – a non-parent node – leading to the identification of node B as a neighbor of node F. In this manner, the nodes discover their neighbors during CDS creation. and the process is repeated until the network is completely covered (see Figure 2(c)).

The rebroadcast of *hello* messages continues until they reach the *leaf nodes* e.g. nodes C, D and I in Figure 2. The leaf nodes follow the same process but their timeout expires as these nodes do not have any child node. When timeout expires at leaf nodes, they send *finish discovery* messages to their parent nodes thereby initiating the second phase of the protocol as shown in Figure 2(d). After sending a *finish discovery* message, a leaf node enters the sleep mode and turns off its transceivers to conserve energy. Note that the neighbors of node A do not send any explicit response. Instead, they simply rebroadcast the *hello* message which also functions as a response message for node A. Consequently, nodes avoid the use of any explicit response message and reduce the number of control messages exchanged during the topology discovery.

In the *finish discovery* message, each node sends the list of its neighbors to its parent node. In the example, node C and node E are neighbors of node D. Therefore, node D sends $\{D, C\}, \{D, E\}, \{D, G\}$ to its parent (node B) in the *finish discovery* message (see Figure 2(e)). These sets of nodes create message paths which can then be used for polygon formation among a set of nodes. A node wait for the *finish discovery* messages from all its children. When it receives all the expected messages, it creates its own *finish discovery* message and forwards it to its parent node. Node B in Figure 2(f) extends the received set of message paths with its own message ID. In this way, *finish discovery* message converges towards the sink node.

When the sink node has received *finish discovery* message from all of its children, it combines different message paths to discover polygons in the network. If there are at least two common nodes in the two message paths, then a polygon exists in the network. For instance, $\{A, B, D, G\}$ and $\{A, H, E, D\}$ are two message paths and have two common nodes A and D. Hence, it can be inferred that there exists a polygon in the network comprising nodes $\{A, B, D, E, H, A\}$ (see Figure 2(g)).

Figure 2(h) shows the selected polygon by a sink node. Once selected, sink node broadcasts the *create topology* message which contains the list of nodes that are part of the polygon. The size of polygon is dependent on the needed level of reliability as a large size allows more nodes to be connected with the sink node. On the other hand, a small size provides minimum reliability among nodes forming a polygon. When a node receives the *create topology* message from a sink node, it searches its ID in the polygon. If it finds its ID in the polygon, it marks itself as an active node. At the end of this process, each node is either in *active* state or in *sleep* state as shown in Figure 2(i). The active nodes forms a backbone responsible for data communication in the whole network.

This completes the description of the Poly protocol. We now provide our experimental setup which is used for the evaluation of the Poly protocol. It is then followed by a detailed discussion on simulation results.

IV. EMPIRICAL EVALUATION FRAMEWORK

In this section, we describe the empirical evaluation framework which is utilized for the evaluation of the Poly protocol and three other prominent CDS protocols, namely A3, EECDS, and CDS-Rule K. We explain the empirical setup which contains the description of various network topologies and simulation parameters. We then provide the definitions of the metrics used for the performance analysis of the four protocols. In the subsequent section, we discuss the simulation results.

A. Simulation Setup

To evaluate the protocols under consideration, we used the Atarraya simulator which has been designed specifically for WSN topology control protocols [14]. In our experiments, we assume that the sensor nodes are randomly deployed in an area of $600m \times 600m$. The experiments are performed in different network topologies ranging from 50 to 250 nodes. The transmission radius and initial energy level of each node are set to $42m$ and $1J$, respectively. The nodes communicate with each other using full duplex wireless radios. The actuation energy equals $50nJ/bit$ while the communication energy is $100PJ/bit/m^2$.

As described in Section II, we only consider energy-based topology maintenance technique. To this end, we set the energy threshold to 10% i.e. topology maintenance process is triggered when the network energy falls by 10%. Data packet size of 25 bytes is used in the experiments and we assume an ideal Medium Access Control (MAC) layer; i.e. there is no packet loss due to channel contention / collisions.

The reported values of the selected metrics are averaged over 50 simulation runs. In static techniques, performance is mainly dependent on efficient topology construction. Therefore, we only report the results for dynamic topology maintenance techniques based on energy-threshold. Finally, we reemphasize that size of the polygon in the Poly protocol is a critical parameter, characterizing the tradeoff between reliability and energy efficiency. In all our experiments, the size of polygon is varied approximately between 10 to 50 nodes with the increase in the network size.

We now provide definitions of the metrics used in the evaluation process.

- Message overhead: Message overhead is defined as *the total number of packets – sent or received – generated in the whole network during an experiment.* Message overhead is an extremely important parameter as it directly affects the energy consumed in the network. Higher message overhead consumes higher energy and, in general, also needs significant processing overhead. Therefore, any protocol designed for WSNs must try to minimize this metric.
- Energy overhead: Energy overhead is defined as *the fraction of the network energy expended during construction of the topology.* In case of topology maintenance, this metric calculates the overhead during the re-construction of the topology under dynamic conditions.
- Residual energy: Residual energy *is defined as the ratio of energy in the active set of nodes to the total network energy at the end of an experiment*. Residual energy is a measure of network lifetime. As the residual energy falls below a certain threshold value, the probability of network partitioning increases.
- Connectivity: Connectivity *refers to the number of nodes which are disconnected from the sink node after the activation of topology maintenance technique.* This parameter measures the effectiveness of a topology construction protocol. If connectivity values equals zero, the protocol is at its best. Higher values of connectivity shows that the protocol is unable to provide a backbone which is capable of collecting data from the sensor nodes in the network.

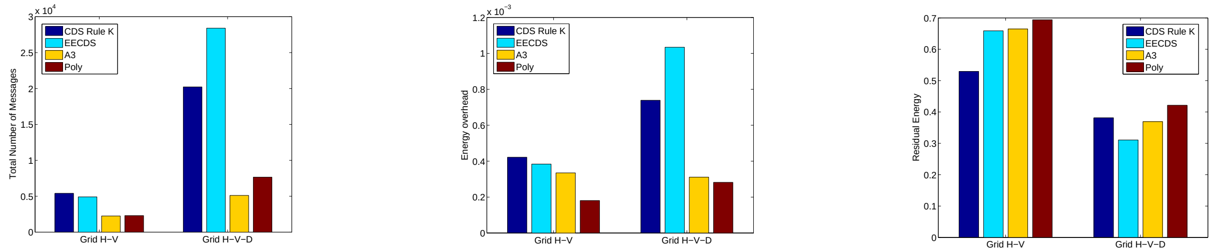
V. SIMULATION RESULTS

Simulation results are described in three subsections. First, we evaluate all the four protocols in two ideal grid environments observed in controlled indoor deployments: the Grid H-V and the Grid H-V-D topologies. In the Grid H-V topology, nodes can communicate with their horizontal and vertical neighbors, while in the Grid H-V-D topology, nodes can communicate with their diagonal neighbors as well. Subsequently, we compare the protocols' performances under varying node densities assuming that: (1) the nodes are randomly deployed, and (2) the protocols only construct the topology. We then discuss the performance of the protocols under a dynamic topology maintenance technique triggered by energy thresholds. In the next section, we model and compare the reliability of these protocols.

A. Grid Topology

In the case of Grid H-V topology, we assumed a network of 169 nodes while restricting the transmission range to $28m$. For Grid H-V-D, we increased the network size to 324 nodes.

The message overhead, energy overhead and residual energy results are shown in Figure 3. The message and energy overhead of EECDS and CDS-Rule K protocol increases due to the two phase topology creation mechanism used by both protocols. On the other hand, A3 has low message overhead due to its three-way handshake process which allows nodes to have less energy overhead to form a reduced topology. The proposed Poly protocols has low energy overhead despite the fact that its message overhead is greater than A3 protocol. This is because A3 uses a selection metric based on signal strength which allows distant nodes to be selected in the CDS. However, in grid topologies the neighbors of the sink node are at equal distances which introduces more energy overhead for the A3 protocol. The Poly protocol uses a broadcast mechanism to select nodes in proportion with the size of the network, hence yielding better residual energy as compared to CDS-Rule K, EECDS, and A3 Protocols.

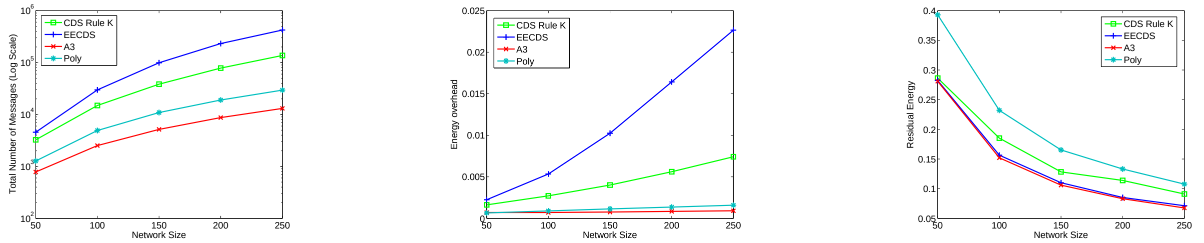


(a) Message overhead

(b) Energy overhead

(c) Residual energy

Fig. 3. Performance comparison under Grid H-V and Grid H-V-D topologies.



(a) Message overhead

(b) Energy overhead

(c) Residual energy

Fig. 4. Impact of varying node densities.

B. *Impact of Node Density*

Figure 4 shows the message overhead, energy overhead and residual energy of all the four protocols. As the network size grows, the number of exchanged messages rises exponentially for all the four protocols. The increase in the node density results in a proportional increase in the node degree which ultimately leads to an increase in the number of messages exchanged. This trend is noticeable in the results shown in Figure 4(a). Number of exchanged messages for EECDS and CDS-Rule K is significantly higher than the Poly protocol. This is caused by the two-phase topology construction process utilized by EECDS and CDS-Rule K protocols.

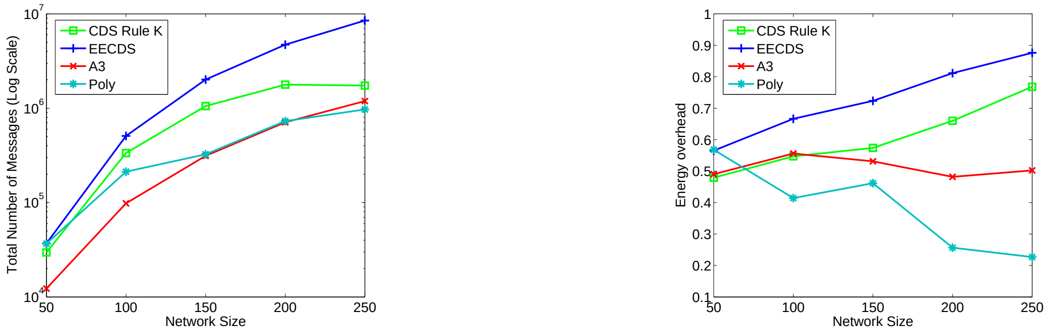
In comparison, A3 generates fewer messages because it chooses the distant nodes using signal strength. This allows fewer nodes to become part of the CDS, thus leading to quick convergence of the protocol. On the other hand, the Poly protocol forms a polygon in which all the nodes send their IDs back to the parent node. This process incurs higher message overhead than the A3 protocol.

Energy overheads of EECDS and CDS-Rule K are significantly higher than A3 and Poly, as shown in Figure 4(b). As can be intuitively argued, an increasing node density leads to higher energy consumption due to an increase in the number of received packets. However, Poly still has lower energy consumption due to its rebroadcast strategy for topology discovery. As mentioned earlier, Poly does not use any messages explicitly sent to a parent node by its children. Instead, it overhears the broadcast at the parent node to get aware of its children. A3 protocol has less message overhead when compared with all the other three protocols. The energy overhead curve flattens for A3 and Poly protocols because both protocols do not use a two-phase strategy like EECDS and CDS-Rule K.

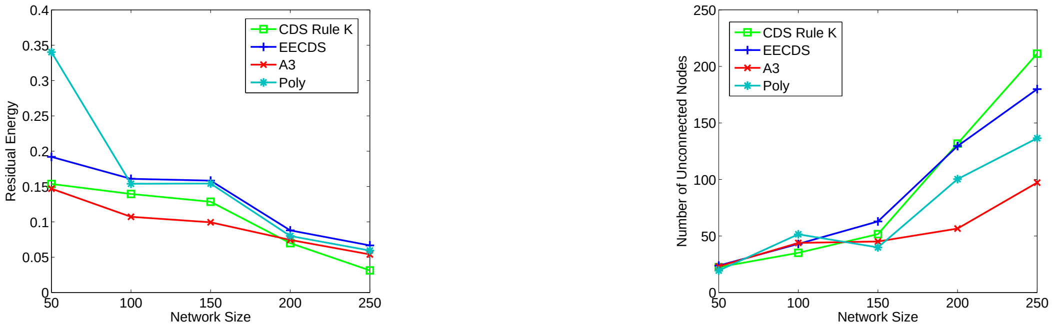
Figure 4(c) shows the residual energy of all the four protocols. Usually, high energy overheads lead to lower residual energies. However, we observe that A3 which has low message and energy overheads, has significantly less residual energy. This is due to non-uniform distribution of communication overhead which drains the battery of fewer nodes resulting in lower residual energy levels among nodes in the network. On the other hand, EECDS and CDS-Rule K protocols have less residual energy due to high energy overhead. Poly provides better residual energy when compared with all the three protocols. This is because: 1) It forms the active node set in proportion to the network size, and 2) it uses a rebroadcast mechanism which inherently consumes battery of nodes at an equal rate.

C. *Dynamic Topology Maintenance*

Figure 5 shows the metric values of all the four protocols under dynamic topology maintenance. Formation of Maximal Independent Set (MIS) and the formation of CDS in EECDS



(a) Message overhead (b) Energy overhead



(c) Residual energy (d) Connectivity

Fig. 5. Performance comparison under dynamic topology maintenance.

contribute to large number of exchanged messages as the network size is increased. This is shown in Figure 5(a). However, the number of exchanged messages decreases slightly in case of CDS-Rule K protocol. This is due to less number of connected nodes as the node density is increased (see Fig. 5(d)). For A3 and Poly protocols, the number of exchanged messages increases exponentially due to higher number of connected nodes.

Similarly, consumed energy also increases linearly in case of EECDS and CDS-Rule K protocols (Figure 5(b)). However, EECDS allows uniform distribution of energy resources which results in better residual energy (Fig. 5(c)). On the other hand, CDS-Rule K uses a pruning process in which every node updates its two hop neighbors when it is not marked and the process gradually increases as the node density is changed. Therefore, CDS-Rule K has less residual energy as shown in Figure 5(c). A3 protocol shows consistent behavior in terms of energy overhead. However, it has less residual energy due to its three way message exchange and distant node selection metric (see Fig. 5(c)). The energy overhead for Poly protocol decreases when the number of connected nodes gets lower. However, it has better residual energy when compared with all the three protocols as shown in Figure 5(c) for the reasons mentioned in the previous subsection.

Figure 5(d) shows the number of unconnected nodes under dynamic topology maintenance. In CDS-Rule K, nodes remained marked if there is at least one pair of unconnected neighbors. The energy depletion of the marked node leads to higher number of unconnected nodes as compared with the other three protocols. On the other hand, A3 has less number of unconnected nodes due to its node selection process based on signal strength metric. Performance of Poly in larger networks is better than EECDS and CDS-Rule K. However, it is alarming to note that the number of unconnected nodes increases as the network size gets bigger.

Before we conclude this paper, in the following section, we analyze and compare the reliability of all the four protocols.

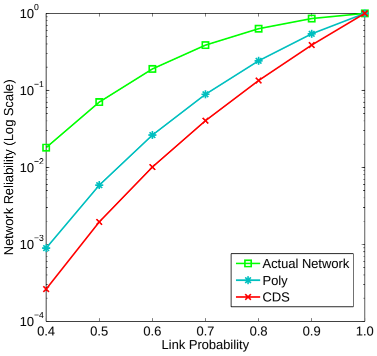


Fig. 6. Network Reliability

VI. NETWORK RELIABILITY

In graph theory, redundancy is defined as *the expected number of functional spanning trees in a graph*. Removal of an edge from a spanning tree partitions the graph. Therefore, every edge may be considered as a bridge in a spanning tree. The number of spanning trees measures the network performance under highly dynamic conditions, e.g. frequent link failures. Reliability, on the other hand, is the probability that there is at least one spanning tree or the probability that the sensor nodes can communicate with each other in case of random link failures. Hence, reliability is another critical parameter that measures the redundancy of the protocol [15].

We used linalg (Linear Algebra) package available in Maple [16] to analyze the performance of Poly protocol by assuming the network shown in Figure 1. We generated different random topologies of different size and averaged the reliability results. However, Poly provides similar results regardless of the underlying topology and network size due to polygenic nature of the protocol. Moreover, CDS protocols also provides similar behavior as they form a CDS tree.

Let $A = (a_{i.j})_{n.n}$ denote the adjacency matrix of graph G, then

$$a_{i.j} = \begin{cases} 1 & \text{if vertices } v_i \text{ and } v_j \text{ are adjacent,} \\ 0 & \text{otherwise.} \end{cases}$$

The degrees of the vertices are represented by a diagonal matrix. If $D = (d_{i.j})_{n.n}$ denote the diagonal matrix of graph G, then

$$d_{i.j} = \begin{cases} deg(v_i), & \text{for } i = j, \\ 0 & i \neq j. \end{cases}$$

We used the matrix tree theorem [15] to find the number of non-identical spanning trees for the network shown in Figure 1. According to the theorem, the spanning trees of graph G is the value of any cofactor of the matrix, i.e. $T = D - A$. Therefore, the matrix T for the assumed network equals

$$T = \begin{bmatrix} 3 & -1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 \\ -1 & 4 & -1 & -1 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 4 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 3 & 0 & -1 & -1 & 0 \\ -1 & -1 & 0 & 0 & 0 & 3 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & -1 & 0 & 2 & 0 & 0 \\ -1 & 0 & 0 & 0 & -1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

The cofactor of matrix T equals 108. Hence, there are 108 non-identical spanning trees which represent the total redundancy of the network. We are interested in measuring the probability that at least one of the spanning trees is working or the reliability that the network will be functional in events of random edge failures. To compute this, we must represent all spanning trees as a disjoint product as given below:

$$P(t_1 \vee t_2 \vee t_3 \vee ...... \vee t_{108}) = P(t_1) + P(t_2\overline{t_1}) + P(t_3\overline{t_2t_1}) + \\ \ldots + P(t_{108}\overline{t_{107}t_{106}}....\overline{t_1}),$$

where t is a spanning tree in the network.

We computed the reliability for CDS (CDS RuleK, EECDS and A3) protocols and compared them with the proposed Poly protocol. The adjacency matrix for CDS-based protocols remains the same. Therefore, all the three protocols have the same reliability. Consequently, these existing protocols maintain a CDS tree in which every edge serves as a bridge edge. If we suppose that all edges have the same reliability $P_1 = P_2 = ... = P_n = P$ then the reliability of the network shown in Figure 1 is given by: $108p^8 - 315p^9 + 348p^{10} - 172p^{11} + 32p^{12}$.

Figure 6 compares the reliability of poly protocol with CDS based protocols. The decrease in the link probability causes a proportional decrease in the network reliability. However, the Poly protocol provides better network reliability as the link probability is decreased. Existing CDS-based protocols have considerably lower network reliability because each edge (link) in these topologies serves as a bridge edge, and therefore does not provide any redundancy in the network.

VII. CONCLUSIONS

In this paper, we proposed a topology control protocol – Poly – which forms a CDS by finding polygons present in a WSN. We performed simulations to compare the performance of Poly with other prominent topology construction protocols– CDS-Rule K, A3 and EECDS–over a large operational spectrum. Simulation results demonstrated that Poly has low message overhead and energy consumption, and can provide higher network reliability. The Poly protocol also works well with dynamic topology maintenance techniques.

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