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Citation: Castillo-Rivera, S. & Tomas-Rodriguez, M. (2016). Helicopter nonlinear aerodynamics modelling using VehicleSim. Advances in Engineering Software, 100, pp. 252-265. doi: 10.1016/j.advengsoft.2016.08.001

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Helicopter Nonlinear Aerodynamics Modelling using VehicleSim

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Abstract

This work describes a model developed to analyze the aerodynamic loads on a helicopter model on conventional configuration implemented with VehicleSim, a multibody software specialized in modelling mechanical systems composed by rigid bodies. The rotors are articulated and the main rotor implementation takes into account flap, lag and feather degrees of freedom for each of the equispaced blades as well as their dynamic couplings. This article presents an aerodynamic model that allows to simulate hover, climb, descent and forward flight as well as trajectories under the action of several aerodynamics loads. The aerodynamic model has been built up using blade element theory. All the dynamics, aerodynamic forces and control action are embedded in a single code, being this an advantage as the compilation time is greatly reduced. The software used in this work, VehicleSim does not need external connection to other software. This new tool may be used to develop robust control methods. The nonlinear equations of the system which can be very complex, are obtained, in particular, this article presents the equations for flap and lag degrees of freedom in hover flight. The control approach used in here consists of PID controllers (Proportional, Integral, Derivative), which allow to use VehicleSim command exclusively to simulate several helicopter flight conditions. The results obtained are shown to agree with the expected behaviour.

Keywords: Helicopter, Aerodynamic, Blade, Element

1 Introduction

Full helicopter behaviour modelling is still a far fetched task as the aerodynamic environment that surrounds a helicopter is highly complex and taking into account the nonlinear dynamics that would represent a realistic and high fidelity helicopter model is a cumbersome task. The availability of simplified aerodynamic models for the response of the rotors aerodynamic loads in the control inputs is very helpful for robust control design.

In helicopter study, experimental data related to several flight conditions are significantly limited due to practical and economical reasons. This difficulty can be overcome using simulations in order to evaluate the applicability of a model identification scheme and to validate the test of robust control systems for the helicopter. The design codes should cover a wide range of rotorcraft configurations and rotor types, as well as dealing with the entire rotorcraft. The code should be flexible to adapt or extend new problems for all operating conditions and all helicopter configurations. The main goal of this paper is to present a first approach to helicopter modelling using VehicleSim, this sets the path to build up a complete rotorcraft model that takes into account the nonlinear dynamics and aerodynamics couplings.

Since the 1990's, several multibody dynamics software have been implemented and developed in the rotorcraft field. The multibody dynamics approach is needed in order to deal with complex mechanisms with great variety of shapes as the mechanisms found in rotors. For example, a simulation program for the dynamics of the main rotor of the AGUSTA A109c helicopter was developed (based on Automatic Dynamic Analysis of Mechanical Systems (ADAMS) general-purpose multibody simulation code). Due to the complexity of the dynamics and aerodynamics of the helicopter rotor, extensive use was made of particular features of this code, such as the possibility to import data from finite elements codes in order to model flexible bodies, and to link user-written subroutines to the main body of the program for the simulation of applied aerodynamic loads and nonmechanical phenomena [1].

Comprehensive Analytical Model of Rotorcraft Aerodynamics and Dynamics (CAMRAD) II was used to research performance enhancements to large rotorcraft. The rotor models were allocated to be as basic as possible to separate the effects of each advanced concept. The rotors contained rigid blades, rigid control systems and they were shaped as isolated rotors in a wind tunnel. The tilt rotor was implemented as a single rotor. The effects of improved airfoils and active controls were studied. Airfoils with higher maximum lift and with reduced drag were also analyzed. The results presented an improvement in the maximum lift capability for the helicopter, however, they showed a large improvement for the tilt rotor [2].

DYMORE was used to predict the aeroelastic response and stability of the helicopter rotors [3]. However, this software provides a simple two-dimensional unsteady airfoil theory and a finite state dynamic inflow model to calculate the inflow velocity field over the rotor [4], [5]. On the other hand, modelling and control of a laboratory helicopter was carried out by Zupancic et al. [6] using Dymola/Modelica environment. The model consisted of three inputs, the corresponding voltages drove to the rotors and the servomotor. The function of the last one, was to position the weight. So, the model displayed a nonlinear multivariable system. Two different areas such as mechanical systems and control systems were combined as a multi-domain approach. The overall scheme of the model consists of the coordinate system definition, the stand model, the helicopter body, the tail and main rotor model and the controller with two reference signals for the pitch and the rotation angle. The control system was designed and optimised using the Matlab/Simulink environment. In Simulink the overall

mechanical model was presented with the Dymola block.

In addition to this, MBDyn (MultiBody Dynamics) is a multibody code that provides a framework for integrated simulation of complex multi-physics problems. It has been used in [7] to implement a nonlinear multibody aeroelastic model of the SA330 Puma helicopter. The main rotor was modelled using the multibody approach. The relative motion between rigid bodies were described using kinematically exact constraints, applied by means of Lagrange multipliers, allowing to describe the relative motion between rigid bodies; structural dynamics were dealt with by using a finite element approach, based on an original formulation of Ghiringhelli et al. [8], while inertia was represented by lumping masses at the nodes. A simple inflow model based on momentum theory was used on the blade element (2D) aerodynamics. The results presented the soundness of the co-simulation approach, which could provide a tool to add some frictionless and frictional contact capabilities to multibody formulations, in this manner, it is not necessary the complete reformulation of the dynamics in a nonsmooth dynamics simulation framework.

Multibody software packages can be used for control purposes, in fact, Masarati [9] used the general multibody solver MBDyn, to present an algorithm for the real time solution of inverse kinematics and inverse dynamics of redundant manipulators formulated in redundant coordinates. In order to show the capability of the approach, three inverse kinematics problems were displayed such as three link arm, feedforward control of a PA10 like robot, feedforward and feedback control of a bioinspired robot. They allowed to estimate positions, velocities, and accelerations. Furthermore, an inverse dynamics problem that calculates feedforward generalized driving forces was also considered. Shen et al. [10] showed the development and validation of a stiff-inplane tiltrotor model using two multibody helicopter codes, the two dynamics codes used DYMORE and MBDyn. The dynamics models included the gimballed hub, rotor blades, feather links, swashplate, conversion actuators which were linked to the pylon, and the elastic wing. The rotating system was implemented with a clamped blade model. It consisted of a clamped beam. This clamped blade model was connected to the gimbal hub through a flexbeam, torque tube and control system (feather link, feather horn and swashplate), allowing to build the single blade model. The experimental data derived, in the wind tunnel and in the ground vibration tests, were used to validate the models. In addition to this, these models were used to obtain the whirl-flutter stability boundary. The results provided good agreement with the experimental measurements.

In [11], the author derived a helicopter mathematical model consisting of main and tail rotors, with six nonlinear equations. Some simulations were carried out under specific flight conditions and each induced velocities were also used according to these. Perturbations such as wind crossing or vibration engine were considered in order to build up a robust model. Simulations of lateral and ascend flight could be found using the model. This approach was suitable to simulate and to control a helicopter, as a result the model could be used to control autonomous radio helicopter or automatic helicopter pilot as well as to build a flying computer training program. Nowadays, the industry of miniaturized is focused in low power embedded systems as they provide accelerated 3D graph-

ics that allow complex visualization for embedded applications. Frantis et al. [12] dealt with development of a simple flight model suitable to be implemented as a C++ algorithm for real time usage. The model was used for embedded flight simulator in a synthetic vision system. Siva et al. [13] studied the effect of uncertainties on performance predictions of a helicopter. They considered uncertain variables, the main rotor angular velocity, main rotor radius, air density, blade chord and blade profile drag coefficient. The propagation of these uncertainties in the performance parameters such as thrust coefficient, induced velocity amongst others, were studied. In this work, different flight regimes and their aerodynamic environment were considered.

Blade element theory is commonly used in helicopter flight simulation models. The theory consists on dividing a blade into various blade elements orientated in the chord wise direction; each element is aerodynamically independent from the neighbouring elements. This allows the use of 2D aerofoil characteristics in order to determine the forces and moments produced on the blade [14]. Other approaches can also be considered such as Brown's Vorticity Transport Model (VTM) [15] that allows to analyse aerodynamic and dynamic performance on several rotor helicopter configurations under both steady and manoeuvring flight conditions. The aerodynamical module uses a computational solution of the Navier-Stokes equations. It is expressed in vorticity velocity form in order to simulate the evolution of the wake of a helicopter rotor. The key to the method is the use of a Computational Fluid Dynamics (CFD) type vorticity conserving algorithm to evolve the governing equations through time. Standard CFD methods that rely on a primitive variable formulation of the Navier-Stokes equations, that is, in terms of velocity and pressure, are susceptible to excessive numerical dissipation of vorticity [16]. CFD is an extensive method of research, this approaches can solve issues such as rotor wake prediction, compressible aerodynamics, interaction problems, amongst others [17], [18].

The aerodynamic helicopter model presented in this paper, is a new implementation that aims to connect the necessity of a stable rotorcraft able to operate in several flight conditions to the robust control community. This allows to test the approach presented by Carrillo-Ahumada et al. [19], in this work the authors improved the performance of a helicopter with two degrees of freedom by using the tuning of Pareto-optimal robust controllers. The tuning procedure was established on the simultaneous minimization of the integral of square sum of errors and the integral of square sum of control action. Thus, the helicopter model presented in this article, provides a new platform to validate the tuning procedure as a reliable tool. On the other hand, a controller based on eigenstructure assignment could be applied to this model [20] too, this approach was applied to an unmanned helicopter and the flexibility of eigenstructure assignment should improve the rotorcraft response if a recurrent algorithm was used to select new eigenvalues and/or eigenvectors for decoupling purposes. This technique proved its efficiency for that type of system.

This work simulates the nonlinear dynamics coupling and the rotors' aerodynamic loads in a single code, being an advantage with respect to the work presented by Bertogalli et al. [1], Zupancic et al. [6]. In addition, it allows some additional advantages such as the reduction of the compilation time and

portability. Furthermore, this work goes in the line of Frantis et al. [12], being this an alternative to embedded codes in the aeronautical field. On the other hand, small perturbations theory is applied to equations to ease their analytical solution. However, for more advanced applications, the fully descriptive nonlinear form of the equations should be obtained. These equations are difficult to solve analytically. VehicleSim provides three dimensional nonlinear aerodynamic equations which is a novel characteristic with respect to work presented by Fancello et al. [7]. These equations should allow to design and test robust control systems for helicopters in future works, being this a significant improvement to the works present by Bertogalli et al. [1], Peters et al. ([4], [5]), Zupancic et al. [6], Salazar [11]. The code and the control approaches have been carried out by using VehicleSim exclusively. This is an aspect that the authors would like to emphasize, as this contribution provides a new tool for rotorcraft robust control. The implementation of a more elaborated model could be carried out and it is in the author's future plans. However, the work here presented suffices to highlight VehicleSim capabilities as aerodynamic helicopter modelling tool. Thus, its features as autonomous software are tested, without requesting any connection with other type of program which would complicate the modelling process enormously.

In the view of these considerations, the main contributions of this paper can be summarized as: a) To present a simulation approach for the main and tail rotors aerodynamics of a helicopter model using blade element. This task has been carried out using VehicleSim. The helicopter dynamic model in here has been applied already to rotorcraft analysis problems (see [21], [22]). b) To describe and discuss the results obtained in a series of simulated flying conditions as hover, climb, descent and forward flight. This is accomplished with the objective of facilitating the study of trajectories and flying characteristics.

The outline of the paper is as follows; Section 2 presents the helicopter aeromechanics. Section 3 describes VehicleSim main features and computing characteristics. Section 4 provides a detailed description of the modelling process carried out in this work. Section 5 contains applications of the helicopter model in order to study their aerodynamics responses. Finally, Section 6 summarizes the main conclusions.

2 Helicopter Aeromechanics

Rotor modelling should be structured in different steps, following the fundamental and convenient distinction between mechanical and aerodynamic phenomena. The first one includes the dynamic response of the blades to a generic loading. A multibody formulation is particularly attractive, allowing to retain a representation of the blade displacement and inertial properties as well as the kinematic nonlinearities. Each aerodynamic phenomena should be considered in a specific manner for the computation of blade loadings. At present, the aerodynamical aspects of this model do not contemplate complex environments such as appearing vorticities, turbulences, etc. This implementation is carried out in this way, in order to develop a single code without external sources, us-

ing exclusively the VehicleSim commands such as forces, moments, constrains, amongst others. Other type of approaches could be carried out and these would change the modelling procedure shown in here. However, the intention of the authors is to study VehicleSim as pure helicopter simulation tool, allowing to test the capabilities of this multibody software in the rotorcraft field without the external code in form of turbulence approach, for example.

Helicopter blade element theory is applied in form of forces on the blades. This theory provides the proper environment for the purpose of this work, i.e., to design a single code where the helicopter dynamic model and their corresponding aerodynamic loads are embedded, using VehicleSim commands such as forces or moments exclusively.

2.1 Hinges and Blade Motions

The role of the main rotor is to support the aircraft's weight, as it generates the lift force. It allows to keep the helicopter suspended in the air and provides the control that allows to follow a prescribed trajectory by changing altitude and executing turns. It transfers prevailingly aerodynamic forces and moments from the rotating blades to the non rotating frame (fuselage). The blades are kept in uniform rotational motion (rotational speed Ω), by a shaft torque from the engine. A common design solution adopted in the development of the helicopter is to use hinges at the blades roots that allow free motion of the blade normal to and in the plane of the disc (see Figure 1). The most common of these hinges is the flap hinge that allows the blade to flap, this is, to move in a plane containing the blade and the shaft, of the disc plane, about either the actual flap hinge or in some other cases, the flap hinge is substituted by a region of structural flexibility at the root of the blade. The flap hinge is more frequently designed to be a short distance from the centre line. This is termed an "offset" (eR) and it offers the designer a number of important advantages [3], [23].

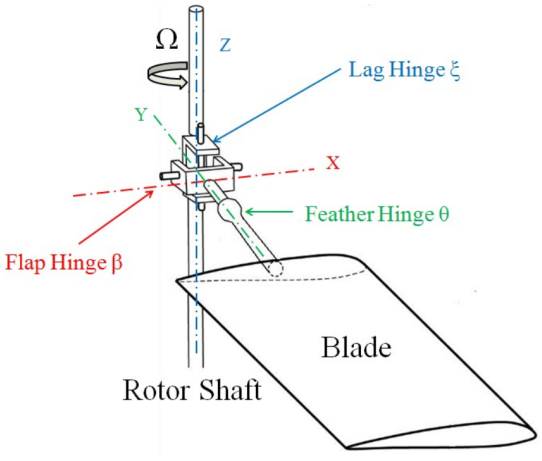

Figure 1: Schematic diagram of main rotor's hub and hinges system.

A blade that is free to flap, experiences large Coriolis moments in the plane of rotation. A lag hinge is introduced in order to relieve these moments. This degree of freedom allows the blade motion to be parallel to the disc plane [14], [24].

A blade can also feather around an axis parallel to the blade span. Blade feather motions are necessary to control the aerodynamic lift developed and, in forward motion of the helicopter, to allow the advancing blade to reach a lower angle of incidence than the retreating blade and thereby to balance the lift across the craft. In order for the helicopter to climb up, the feather angle needs to be increased. On the other hand, in order to descend, the blade's feather angle is decreased. Because all blades are acting simultaneously in this case, this is known as collective feather and allows the rotorcraft to rise/fall vertically. Additionally to this control, in order to achieve forward, backward and sideways flight, a different additional change of feather is required. The feather on each individual blade is increased at the same selected point on its circular pathway. This is known as cyclic feather or cyclic control. Blade feather control is performed through a linkage of the blade to a swashplate [25], [26].

2.2 Aeromechanics and Simulations

In simulations of rotor dynamics, the blade elasticity should be taken into account, although certain studies can be conducted using a rigid blade assumption [2]. Even in such computations with three angular degrees of freedom for each blade, the large motion amplitudes must be properly modelled. The geometrical non linearity of the problem should be considered to produce the lag motion. The flap motion is inherently well damped because of the large related lift changes, but drag changes in the plane of rotation are small. In certain conditions on the ground, the lag motion may even become unstable. From the discussion above, it is evident that the simulation of a helicopter rotor in flight should combine a non linear structural or mechanical model with a time-dependent aerodynamic loads capable of modelling different flight conditions. Since the blade feather angles at their roots are controlled and the blade motions determine the interaction with aerodynamic forces, see Figure 2.

3 Modelling

Rotating systems represent a class of mechanisms, characterized by the non negligible angular motion they are subjected to. These systems have been analyzed by formulations and software tools that intrinsically considered the reference rotation motion of the system. Such approaches may be efficient and effective, but may suffer from lack of generality.

The main aim of this section is to describe the modelling developed for implementing the aerodynamic on the helicopter model. An aerodynamic model is presented for both rotors (main and tail) by using blade element theory. The general considerations of the model are described in the following lines.

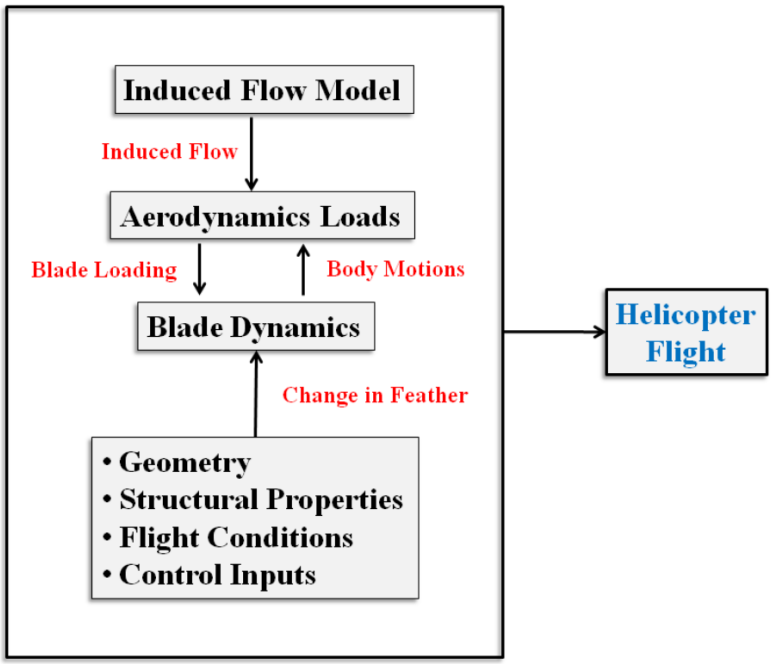

Figure 2: *Functional block diagram of helicopter flight.*

3.1 Helicopter Dynamic Model

A helicopter can be modelled in different ways; one of them is the combination of several interacting subsystems. For the purposes of this work, a full dynamic model of the helicopter is considered as multibody system with several subsystems as well as constraints between the different degrees of freedom. VehicleSim allows to develop other technical considerations in the modelling process, for example, the coupling between the flap-feather on the tail rotor by using the various restrictions and constraints that the software allows to implement.

The following considerations are made regarding the physical structure of the helicopter and the general structure of the helicopter is given (see [21]):

- The helicopter has the conventional configuration i.e., main rotor in combination with a tail rotor. Both systems are mounted on the fuselage. The model has been set up without empennage.
- The main rotor consists of four equally spaced blades joined to a central hub. The blades have free motion in and out of the plane of the disc; this is allowed by the inclusion of hinges.
- The tail rotor consists of two equally spaced blades joined to a secondary hub.
- The blades are rigid in both rotors.

- The rotor's angular speeds are constant, and a proportional ratio exists between them. The axis of rotation of the tail rotor is transverse to the main rotor's axis.
- The main rotor hinges allow for three degrees of freedom: flap, lag and feather motions.
- The tail rotor hinges allow for two degrees of freedom: flap and feather.
- Feather-flap coupling is considered in the tail rotor analysis.
- The helicopter's main body has six degrees of freedom. Three translations along the (X, Y, Z) axes and three rotations around the same axes.
- The fuselage loads are obtained as the outputs of their corresponding translational and rotational degrees of freedom.

VehicleSim and its methodology have also been used to develop UAV helicopter models from a dynamical perspective only (see [21], [27]). In those cases, the rotorcraft geometry and features were different. In addition to this, VehicleSim has been also used to model a quadrotor UAV (see [28]), in this case, the VehicleSim code was connected to an external Matlab/Simulink environment, showing other aspects of VehicleSim as UAV modelling tool.

3.2 Aerodynamic Model

A helicopter rotor experiences a complex aerodynamic phenomenon that does not occur in fixed-wing airplanes. The rotor blades produce lift by rotating and induce their own airspeed over the airfoils. The lift generated by an airfoil depends on various factors: airflow speed, air density, total area of the segment or airfoil and angle of attack between the air and the airfoil. The angle of attack is the angle at which the airfoil meets the oncoming airflow or vice versa. In the case of a helicopter, the object is the rotor blade (airfoil) and the fluid is the air. Lift is produced when a mass of air is deflected, and it always acts perpendicular to the resultant relative wind. A symmetric airfoil must have a positive angle of attack to generate positive lift. At a zero angle of attack, no lift is produced. At a negative angle of attack, negative lift is generated [29].

3.2.1 Blade Element Theory

Blade element theory forms the basis of most modern analyses of helicopter rotor aerodynamics as it estimates the radial and azimuthal distributions of blade aerodynamic forces (and moments). In addition to this, the rotor performance can be obtained by integrating the sectional airloads at each blade's elements over the length of the blade and averaging the result over a rotor revolution [14].

The blade is the mechanical part of the rotor which requires the greatest attention in the modelling process: rotor blades are light, subject to a distributed and time-varying external loading and characterized by non uniform structural

and inertial properties. Blade element theory implicates split the blade into a number of elements and estimate out the flow at each one. In this theory the forces on the blade elements are obtained by the lift and drag coefficients.

3.2.2 Blade Element Subdivision

In order to apply blade element theory the blades are divided into three discrete number of sections, as it is shown in Figure 3.

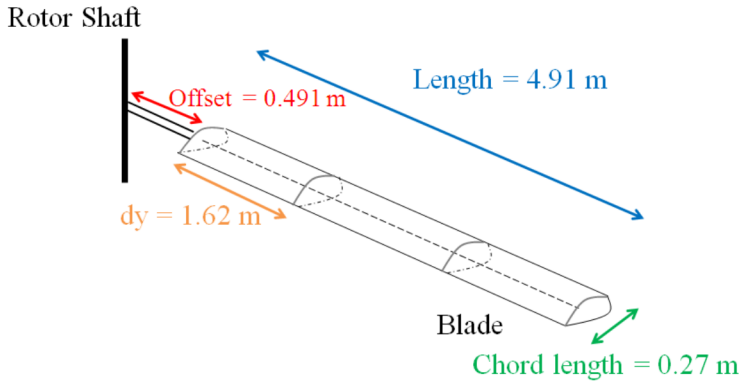

Figure 3: *The blade element model.*

To carry out a correct implementation according to VehicleSim methodology, the helicopter geometry should be considered. As a consequence of this, various points on the helicopter nominal configuration need to be defined within the coordinate system of the inertial reference frame. Three points should be allocated as centre of pressure for each subdivision considered on the blade. Their corresponding coordinates should be defined in connection with the inertial reference frame. In addition, the blade offset (eR), the height of the blade (h), the distance along the axis blade (y) and the blade chord (c) should be taken into account to obtain a blade element environment, as Figure 4 shown.

3.2.3 Lift and Drag Forces

To model the aerodynamic model, the lift and drag coefficients must be considered to derive the corresponding forces. The lift coefficient (C_l) is obtained by using a constant lift curve slope i.e., this proportional to the angle of attack (α) [25], [30]:

$$C_l = a\alpha \tag{1}$$

where "a" is the lift slope and it is equal to 2π when the angle α is measured in radians [30].

If the feather angle at the blade element is θ, then the angle of attack is:

$$\alpha = \theta - \phi \tag{2}$$

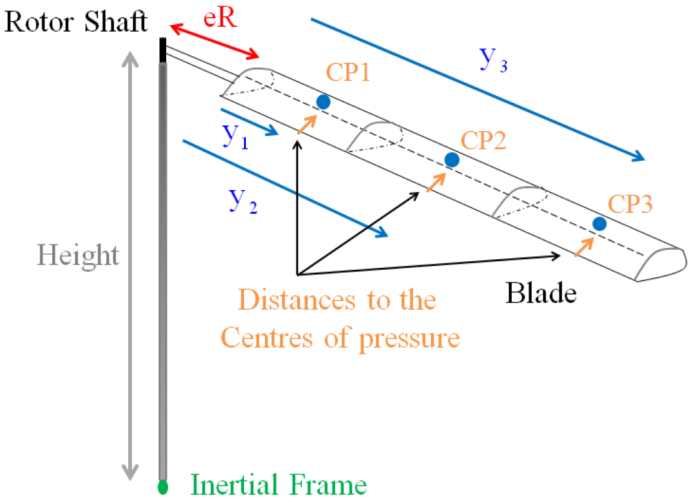

Figure 4: *Blade and centre of pressure geometry, showing the spatial configuration in connection with inertial frame.*

where ϕ is the inflow angle. This angle can be figured out using blade element analysis for the different flight conditions.

- Induced Flow Model: Blade Element Analysis in Hover and Axial Flight

  The inflow angle ϕ, is given as [31]:

  $$\phi = tan^{-1}\left(\frac{V_c + v_i}{\Omega y}\right) \tag{3}$$

  y is the distance along the blade's axis where the angle of incidence is measured, Ω is the rotor angular speed, v_i is the induced velocity V_c is the upward velocity [31].

- Induced Flow Model: Blade Element Analysis in Forward Flight

  For helicopter rotors the following assumptions can be made:

  a) The out of plane velocity U_P (perpendicular component of the velocity) is smaller than the in plane velocity U_T (component due to the blade rotation about the rotor shaft), so that $U = \sqrt{U_T^2 + U_P^2} \approx U_T$. This is a valid approximation except near the blade root, but the aerodynamic forces are small here anyway.

  b) The induced angle ϕ, is small, so that $\phi = tan^{-1}(U_P/U_T) \approx U_P/U_T$ [14].

As a result of these approximations, the blade angle of incidence may now be written as:

$$\alpha = \theta - \phi = \theta - \frac{U_P}{U_T} \tag{4}$$

Helicopter performance calculations use a drag coefficient (C_d) as a function of the angle of attack. Sissingh [32] used a general drag expression of the form:

$$C_d = d_0 + d_1\alpha + d_2\alpha^2 \tag{5}$$

Bailey [33] developed a method to calculate the coefficients and assumed that $d_0 = 0.0087$, $d_1 = -0.021$ and $d_2 = 0.400$. This approach is frequently used for the blade drag in rotor analyses, as it has been indicated by Leishman [14], Johnson [25] and Bramwell et al. [34].

The resultant incremental lift, dL, and drag dD per unit span on a blade element are:

$$dL = \frac{1}{2}\rho U^2 c C_l dy \tag{6}$$

$$dD = \frac{1}{2}\rho U^2 c C_d dy \tag{7}$$

ρ is the air density, C_l and C_d are the lift and drag coefficients, c is the local blade chord. The lift dL and drag dD act perpendicular and parallel respectively to the resultant flow velocity.

The airfoil section is shown in Figure 5. As it can be seen, the chord length is equal to the chord line and the centre of pressure is located at 1/4 of the chord.

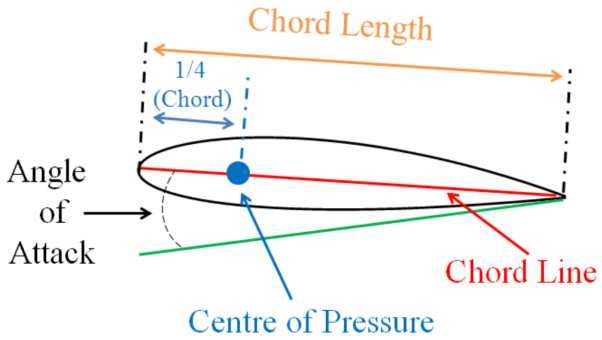

Figure 5: *Blade aerofoil section.*

Taking into account the previous consideration as well as the subsections 3.2.1, 3.2.2. The VehicleSim command `add-line-force` introduces the lift and drag forces on the rotor blades. Each of the blade elements display a different force as they have a different angular speed, as it is plotted in Figure 6.

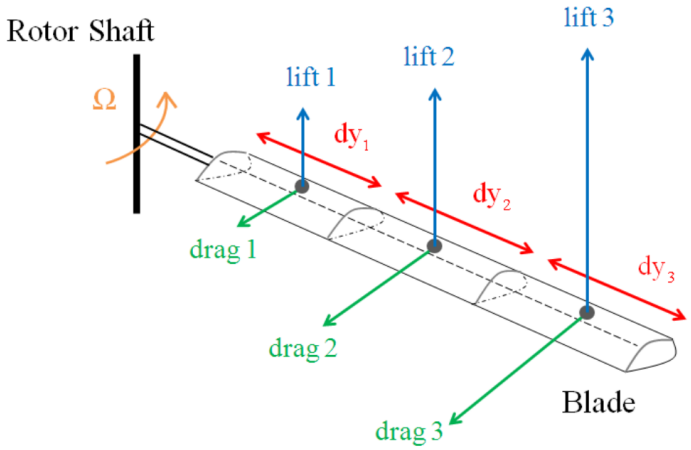

Figure 6: *Lift and drag forces representation for the corresponding points on the main rotor.*

3.2.4 Induced Velocity

In hovering flight, the induced velocity is: $v_i = v_{ih}$, where v_{ih} is the hover induced velocity, it can be considered constant in hover. The traction force, T, becomes equal to the disc loading (weight of the helicopter) [14]:

$$v_{ih} = \sqrt{\frac{T}{2\rho\pi R^2}} \tag{8}$$

where ρ is the air density and R is the rotor radius.

The induced climb, descent and forward velocities will be obtained by using their corresponding relations with the induced hover velocity. These will be obtained from different approaches in form of graph for each case under study. This procedure has been selected by the authors in order to insert these values as simple ratio with the induced hover velocity, which eases the calibration of the controller's gains for each flight condition.

3.2.5 Flight Model

The flight model here presented is based on the following hypotheses:

- The helicopter model is defined on the inertial reference frame. As the bodies are added, the points which belong to them can be located with respect to this reference frame.
- Gravity is constant relative to the altitude. Thus is a constant vector with magnitude $g = 9.81$ m/s^2
- The air density is given as: $\rho = \rho_0 e^{-0.0296h/304.8}$, h is expressed in meters and $\rho_0 = 1.225$ kg/m^3 (air density at sea level) [14].

- The aerodynamic points are defined in the corresponding blade. These are fixed points in the bodies to which they belong, but they may be moved with their locations specified by coordinates in a defined reference system.

4 VehicleSim as Modelling Tool

Over the years, immense efforts have been devoted to develop the helicopter simulation field. Nowadays, several computer packages for assisted mechanical modelling are available. These should be separated in two categories: numerical or symbolic. Numerical codes prepare and solve equations in number form and post-process the results to provide the output in graphical form or as animations. On the other hand, symbolic codes derive equations of motion using symbols instead of numbers. They require numerical substitution and further processing before any output can be obtained (linear analysis, time histories via numerical integration, etc). It is well known that symbolic equations are more difficult to obtain than numerical results. On the contrary, once obtained for a system they do not need to be generated again. Indeed, they are better suited for real time simulations that require fast code execution. A symbolic software is VehicleSim.

VehicleSim Lisp is a computer program that allows to model, simulate as well as to derive symbolic equations of motion for mechanical systems composed of multiple rigid bodies. The VehicleSim code is used to generate a C simulation program, capable of computing general motions corresponding to specified cases with initial conditions and external forcing inputs.

The geometric and inertial properties of the rigid bodies that will conform the model are provided as inputs. Forces and torques among the various model's component can be added acting on the corresponding bodies.

VehicleSim solvers on Windows are compiled as dynamically linked library (DLL) files with a standard VehicleSim application program interface (API) that works with several possible simulation environments, including the VehicleSim browser and Matlab. The source code generated by VehicleSim Lisp is accessible by the user.

VehicleSim Lisp generates the equations of motion for a multibody system in symbolic form. Then a C source code is generated such that this program will solve the equations numerically to simulate the behaviour of the system represented by the model.

The information needed to generate the source code in C for the VehicleSim solver is assembled in VehicleSim Lisp as it processes the model description that the user provides.

The symbolic equations generated by VehicleSim Lisp can be viewed and used with other software packages such as Word or Matlab. VehicleSim Lisp is not a complete simulation system as it can generate equations but it does not solve them. Therefore, a C compiler is needed to compile the source code generated for a VehicleSim solver program and the DLL solvers are built up [35].

The main purpose of each VehicleSim solver is to calculate time histories of the system's variables, this is, the positions, speeds and accelerations of the bodies conforming the system and all user-created variables. These time histories are stored in a binary data file with the extension BIN. The data in a BIN file are organized by variable name and sample number. A companion file, with extension ERD, documents the layout of the BIN file. The ERD header file also contains labelling information for each variable.

Data processing programs for ERD and BIN files obtain the information needed from the ERD file. For example, the high level of automation in the animator and plotter is possible because both are designed to extract the information from ERD files [36].

On the other hand, VehicleSim generates the model's linearised and nonlinearised equations of motion in form of differential equations. In addition to this, VehicleSim also provides in form of a Matlab file the linearised state-space model in symbolic form, see Figure 7.

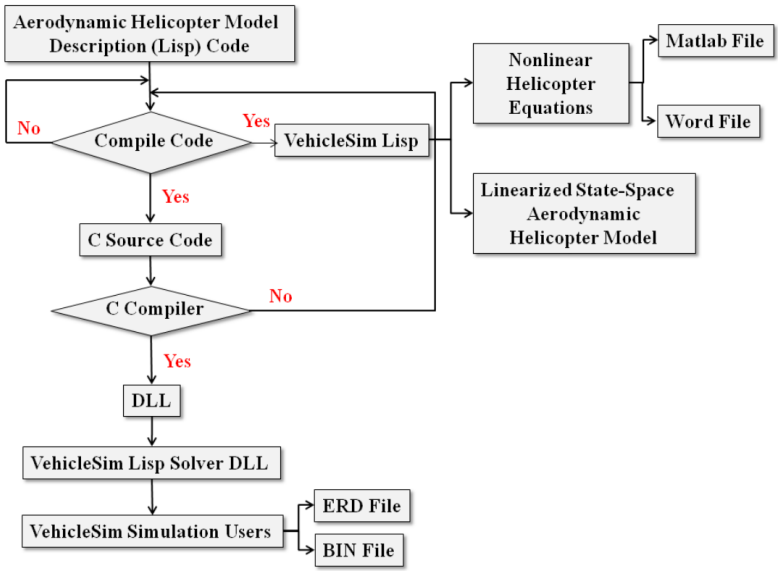

Figure 7: *Flow chart of VehicleSim simulation procedure.*

4.1 State Variables

VehicleSim Lisp introduces the state variables needed to describe the moving reference frames, and uses them as needed to develop mathematical expressions of variables that represent the equations of motion of the system under study. The state variables for a VehicleSim multibody system are divided into two sets: generalized coordinates and generalized speeds.

The generalized coordinates are the variables involving angular and translational displacements. The set of generalized coordinates is defined such that it is possible to reconstruct the position of any point in any part in the system. VehicleSim Lisp can derive expressions for the absolute coordinates of any point located on any mechanical part using the generalized coordinates and dimensional parameters. On the other hand, a set of generalized speeds complements the set of generalized coordinates, by adding the capability for VehicleSim Lisp to determine the velocity vector of any point located on any part in the multibody system.

From the user point of view, the set of state variables also includes variables that are not defined by differential equations, but whose values are, nevertheless, required to fully define the current state of the model. However, in VehicleSim environment, the term "state variables" usually refers only to variables that are defined by differential equations [35].

4.1.1 Controlling the Definition of the State Variables

State variables are introduced automatically when the command `add-body` is used to define a body that can move relative to its parent. One generalized coordinate is introduced for each degree of freedom of the new body relative to its parent, and the coordinates are the amplitudes of the permitted movements. The equations of motion for a system involve a minimum number of coordinates and a speed variable is introduced automatically with each coordinate. The speed variable is defined as the derivative of the coordinate, except for the following three cases [35]:

- A body with three rotational degrees of freedom relative to its parent has three rotational speed variables defined as scalar measures of the absolute rotational velocity of the body.
- A body with three translational degrees of freedom relative to its parent has three translational speed variables defined as scalar measures of the absolute velocity of the body mass centre.
- A body restricted to planar motions, with two translational degrees of freedom has the translational speeds defined as scalar measures of the absolute velocity of the body mass centre.

4.2 VehicleSim Methodology

When writing up the code for building a VehicleSim model certain order in which the commands appear is usually followed. In general, at the head of a VehicleSim program there are certain commands that are used to reset the system, define the gravitational field, select the unit system to be used, the possible linearization of equations of motion, etc. When the analyst does not exercise choice overtly in these matters, default settings come from a configuration file that the analyst is free to access.

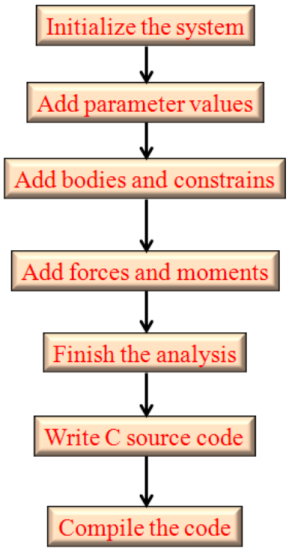

Figure 8: *VehicleSim commands main sequence.*

VehicleSim Lisp in general follows the modelling sequence as shown in Figure 8. VehicleSim commands are used to describe the components of a multibody system in a parent/child relationship according to their physical constraints and joints. As it can be seen in Figure 8, the commands describe in sequential manner the bodies that conform the system. The corresponding physical values are included.

A VehicleSim program begins with an inertial reference frame with a fixed origin selected by the user, and a trihedron with their directions is defined. As new bodies are added to the system, having freedoms relative to the inertial reference frame, local origins and axes are defined.

The code can transform from local coordinates to global ones and vice versa. Points specified globally are conveniently used to define points in bodies. The body fixed points coincide with the corresponding global points, fixed in the inertial reference frame, when the system is on its nominal configuration. Most points are fixed in bodies but a point may be defined as moving with its location in a body determined by its specified coordinates. This is especially useful for describing time varying contact points between two bodies [35].

4.2.1 Helicopter Modelling

The system to be modelled consists of three subsystems: fuselage, main rotor and tail rotor. The multibody system is subdivided into its constituent bodies for the purpose of writing the VehicleSim code. The bodies are arranged as a parent/child relationship as shown in Figure 9.

This is useful for describing time varying points of contact between one body

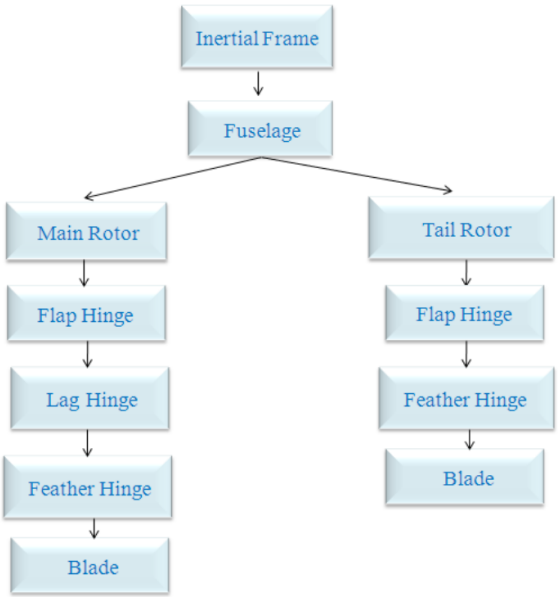

Figure 9: *Body structure diagram of the conventional model helicopter. Both main and tail rotor in this diagram contain one blade element only due to space restrictions, but the model can contain as many blades as needed.*

and another one. The first body included in the code is the fuselage, it is implemented as the child of the nominal reference frame. The fuselage is located at the origin of the inertial coordinates system and it is the parent of both the main and tail rotors. The main rotor rotates around its vertical axis, Z axis. The main rotor is the parent of the flap hinges that rotates around the corresponding X axis, each lag hinge is the child of the corresponding flap hinge. The lag hinges rotate around the Z axis. The feather hinges are the child of the lag hinges. Each feather hinge rotates around the Y axis. Finally, a blade is added to the program structure as the child of each feather hinge. The tail rotor is built up following this same parent/child structure.

The remaining commands in the program describe in a sequential manner the set of rigid bodies that conform the dynamical system to be modelled. This also includes data parameters such as masses, inertia values, position of gravity centres, the allowed rotations and translations of each body.

4.2.2 Flight Control

Aerodynamic forces and loads do have an impact on the vehicle's trajectory and therefore there exist a clear need for paying special attention to the flight control. This is a fundamental aspect of the aerodynamic model here presented. The helicopter model includes rigid body dynamics supplemented with forces and moments induced by the main and tail rotors. The coordinates in the inertial frame are (x, y, z), Euler angles (θ, ϕ, ψ) (roll, pitch, yaw), linear velocities $(\dot{x}, \dot{y}, \dot{z})$ and angular velocities $(\dot{\theta}, \dot{\phi}, \dot{\psi})$ in the body coordinate frame.

In order to guarantee realistic system's behaviour and trajectory tracking capabilities, the model is implemented with various PID controllers due to their simplicity to be modelled in VehicleSim, not being necessary in this way to use further complex approaches. PID controllers allow to achieve the goal set by the authors. Their corresponding proportional, derivatives and integral gains (different for each controller) are manually tuned for each flying condition/trajectory. The program compilation conditions determine the corresponding gains calibration, and therefore, the authors did not consider previous knowledge to choose the controller's gains.

For tracking purposes, the error between the reference and the actual states must be measured at each time step of the simulation (Figure 10). The outputs of these controllers need to be applied between the fuselage and the inertial frame to achieve satisfactory action control. For example, in order to control the helicopter's position and displacement on the X axis, the variable x_{rf} is defined to represent the longitudinal trajectory prescription for the fuselage's position. It provides the reference value for the longitudinal position; it allows to describe the trajectory in the space:

$$x_{rf} = x_i + V_x t \tag{9}$$

where x_i is the longitudinal initial position and V_x is the speed in forward flight [31]:

$$V_x = \frac{\mu \Omega R}{cos\,(\alpha_r)} \tag{10}$$

μ is the advance ratio, Ω is the rotor angular speed, R is the rotor radius and α_r is assumed positive downwards since that is the natural direction of the tilt needed to obtain a forward component of the thrust.

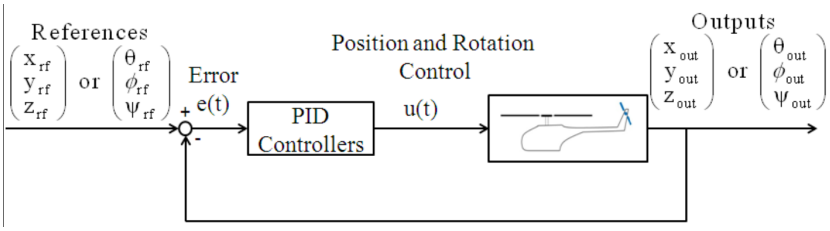

Figure 10: *PID helicopter control*

To control the height of the helicopter, the variable z_{rf} is considered as the helicopter's vertical trajectory prescribed for the vertical position:

$$z_{rf} = z_i + V_z t \tag{11}$$

where z_i is the vertical initial position and V_z is the helicopter's vertical velocity. This velocity has two different expression, if the helicopter climbs $V_z = V_c$ where V_c is the helicopter's climb velocity. On the contrary, if the rotorcraft descends $V_z = V_d$, being V_d the rotorcraft descent velocity.

The pilot action is carried out by selecting the feather angles for the main and tail rotors as well as taking into account the controllers variables required for the corresponding flight condition.

4.2.3 Equation's Numerical Solution

The solver program included in the VehicleSim package is named as VehicleSim-Browser, it computes the output variables at intervals of time as the simulation is being carried out. The time history of the output variables is calculated by solving the dynamical equations of motion containing the state variables.

There are four classes of computation methods that might be used by in a VehicleSim solver program.

- Simple arithmetic statements.
- Numerical integration of a set of ordinary differential equations.
- Solution of a set of simultaneous linear algebraic equations.
- Solution of a set of simultaneous nonlinear algebraic equations.

The reader is referred to VehicleSim Manual for more details on the integration methods used and computation explanation.

5 Results: Helicopter Model Response

Helicopter flight usually involves four types of aerodynamic conditions: hover, climb, descent and forward flight as well as take off and landing. The two last conditions are not considered in this work, because they need a specific and detailed study outside the theme of this work. The following sections present in three dimensions the helicopter model motion as a whole. They are shown as a sequence of separate trajectories in order to study each one separately and a combination of them are also presented.

5.1 Aerodynamic Load Equations

The nonlinear equations of motion for the aerodynamic model can be obtained from VehicleSim. The results for the case of uncoupled flap and lag degrees of freedom are presented in here. It is convenient to bear in mind that the case under study is for rigid blades, being flexibility and torsion two effects not considered in this article. The aerodynamic equations are obtained for one blade in hover flight conditions and the main rotor angular speed is given by restriction in order to obtain simpler equations.

5.1.1 Flap Equation with Aerodynamical Load Validation

Flap is the first degree of freedom on the main rotor blade under the influence of the aerodynamic forces to be considered. A restoring moment is provided by the lift force and this induces flap. As a result, a vertical force appears on the helicopter. Taking all these into account the equations of motion are obtained after a simulation is run.

The software provides the nonlinear equation of motion corresponding to the dynamic and aerodynamic loads and the rotor's provided geometry. The aerodynamic equation obtained with VehicleSim for a blade with flap degree of freedom, it is shown in equation (12).

$$\ddot{\beta}\left(I_{blx}+m_{bl}\cdot y_{bl}^2\right)+\Omega^2\cdot(((I_{blz}-I_{bly})+m_{bl}\cdot y_{bl}^2)\cdot cos\beta+(m_{bl}\cdot eR\cdot y_{bl}))\cdot sin\beta=dy_a\cdot\tfrac{1}{2}\cdot\Omega^2\cdot\rho\cdot chord\cdot(-(-2\cdot\pi\cdot x_a\cdot(y_a^2+eR^2)\cdot sin(2\cdot\pi\cdot q_{bl})-2\cdot\pi\cdot(cos(2\cdot\pi\cdot q_{bl})\cdot(y_a^3+eR^3\cdot cos(2\cdot\pi\cdot q_{bl}))+eR^3\cdot sin(2\cdot\pi\cdot q_{bl})^2)+eR\cdot(y_a\cdot(-4\cdot\pi\cdot x_a\cdot sin(2\cdot\pi\cdot q_{bl})+eR\cdot(4\cdot\pi-4\cdot\pi\cdot cos(2\cdot\pi\cdot q_{bl})^2)+y_a\cdot(2\cdot\pi-cos(2\cdot\pi\cdot q_{bl})\cdot(4\cdot\pi+2\cdot\pi\cdot cos(2\cdot\pi\cdot q_{bl}))-2\cdot\pi\cdot sin(2\cdot\pi\cdot q_{bl})^2))+eR\cdot(2\cdot\pi\cdot eR-y_a\cdot(2\cdot\pi\cdot cos(2\cdot\pi\cdot q_{bl})+4\cdot\pi\cdot sin(2\cdot\pi\cdot q_{bl})^2))))\cdot atan(v_{ih}/\Omega\cdot(y_a+eR))) \tag{12}$$

$q_{bl}=(nblades-blade_number_1)/nblades$, ρ is the air density, $chord$ is the chord of the blade, dy_a is the length differential interval, and v_{ih} is the hover induced velocity.

5.1.2 Lag Equation with Aerodynamical Load Validation

The nonlinear lag aerodynamic equation can be obtained in a similar way using VehicleSim. The VehicleSim aerodynamic equation for a blade with lag is given by equation (13).

$$\ddot{\xi}(I_{blz}+m_{bl}\cdot y_{bl}^2)+\Omega^2\cdot eR\cdot m_{bl}\cdot y_{bl}\cdot sin\xi=dy_a\cdot\tfrac{1}{2}\cdot\Omega^2\cdot\rho\cdot chord\cdot(-x_a\cdot sin(2\cdot\pi\cdot q_{bl})\cdot(eR\cdot(2\cdot y_a\cdot(\delta_0)+eR\cdot(\delta_0))+y_a\cdot(y_a\cdot(\delta_0)-atan(v_{ih}/\Omega\cdot(y_a+eR))\cdot(y_a\cdot(\delta_1+\delta_2\cdot(-atan(v_{ih}/\Omega\cdot(y_a+eR))))+eR\cdot(2\cdot\delta_1+\delta_2\cdot(-2\cdot atan(v_{ih}/\Omega\cdot(y_a+eR)))))))+eR\cdot(eR\cdot(y_a\cdot(2\cdot\delta_0-2\cdot(\delta_0)\cdot sin(2\cdot\pi\cdot q_{bl})^2+2\cdot atan(v_{ih}/\Omega\cdot(y_a+eR))\cdot(\delta_1\cdot sin(2\cdot\pi\cdot q_{bl})^2+\delta_2\cdot atan(v_{ih}/\Omega\cdot(y_a+eR))))+x_a\cdot sin(2\cdot\pi\cdot q_{bl})\cdot atan(v_{ih}/\Omega\cdot(y_a+eR))\cdot(\delta_1+\delta_2\cdot(-atan(v_{ih}/\Omega\cdot(y_a+eR))))+eR\cdot(\delta_0-sin(2\cdot\pi\cdot q_{bl})^2\cdot(\delta_0)))+y_a\cdot(-eR\cdot(atan(v_{ih}/\Omega\cdot(y_a+eR))\cdot(2\cdot\delta_1+\delta_2\cdot(2\cdot sin(2\cdot\pi\cdot q_{bl})^2\cdot atan(v_{ih}/\Omega\cdot(y_a+eR)))))+y_a\cdot(\delta_0-atan(v_{ih}/\Omega\cdot(y_a+eR))\cdot(\delta_1\cdot cos(2\cdot\pi\cdot q_{bl})^2+\delta_2\cdot(-cos(2\cdot\pi\cdot q_{bl})^2\cdot atan(v_{ih}/\Omega\cdot(y_a+eR))))-sin(2\cdot\pi\cdot q_{bl})^2\cdot(\delta_0))))-cos(2\cdot\pi\cdot q_{bl})\cdot(y_a\cdot(eR^2\cdot(2\cdot(\delta_0)\cdot cos(2\cdot\pi\cdot q_{bl})+\delta_2\cdot(atan(v_{ih}/\Omega\cdot(y_a+eR))^2))+y_a\cdot(y_a\cdot(\delta_0-atan(v_{ih}/\Omega\cdot(y_a+eR))\cdot(\delta_1+\delta_2\cdot(-atan(v_{ih}/\Omega\cdot(y_a+eR)))))+eR\cdot(2\cdot\delta_0+atan(v_{ih}/\Omega\cdot(y_a+eR))\cdot(-2\cdot\delta_1-\delta_2\cdot(-2\cdot atan(v_{ih}/\Omega\cdot(y_a+eR)))))))+eR\cdot(y_a\cdot eR\cdot(\delta_0-atan(v_{ih}/\Omega\cdot(y_a+eR))\cdot(\delta_1\cdot(1+2\cdot cos(2\cdot\pi\cdot q_{bl}))+\delta_2\cdot(-2\cdot cos(2\cdot\pi\cdot q_{bl})\cdot atan(v_{ih}/\Omega\cdot(y_a+eR)))))+cos(2\cdot\pi\cdot q_{bl})\cdot(eR^2\cdot(\delta_0)+y_a^2\cdot(\delta_0-atan(v_{ih}/\Omega\cdot(y_a+eR))\cdot(\delta_1+\delta_2\cdot(-atan(v_{ih}/\Omega\cdot(y_a+eR))))))))) \tag{13}$$

$q_{bl}=(nblades-blade_number_1)/nblades$, ρ is the air density, $chord$ is the blade's chord, dy_a is the length differential interval, v_{ih} is the induced velocity

in hover and δ_0, δ_1 and δ_2 are the corresponding drag coefficients values.

These analytical expressions are here provided in order to show how VehicleSim works. Expressions like these are suitable to carry out robust control approach in between several other research potential uses. These equations have been derived under particular conditions and geometry configurations, therefore, they should be validated with experiment data or other method proposed on each particular case.

Various helicopter model parameters are shown in Table 1, establishing the system behaviour and the numerical values for the previous equations, which make difficult the validation of these equations with standard theoretic expressions as terms such as the moments of inertia are often not well described and the validation process becomes imprecise.

Parameters	**Magnitude**	**Units**
Helicopter mass	2200	kg
Main rotor blade mass	31.06	kg
Tail rotor blade mass	6.21	kg
Fuselage-main rotor vertical distance	1.48	m
Fuselage-tail rotor longitudinal distance	6	m
Fuselage-tail rotor vertical distance	1.72	m
Main rotor blade length	4.91	m
Tail rotor blade length	0.982	m
Main rotor lag hinge damping coefficient	349.58	Nms/rad
Delta three angle	-0.785	rad
Main rotor angular speed	44.4	rad/s
Tail rotor gearing	5.25	–

Table 1: Helicopter model parameters [37], [38].

5.2 Response of the Helicopter Model in Hover Flight

Hover flight is the first response to be simulated and tested. Although this flight condition may seem straight forward or not relevant, it is necessary in order to test the capability of the model to reproduce this flying condition.

In order to select the main rotor collective feather in hover flight, reference [31] is considered, as a consequence of this, the collective feather angle is selected approximately 10 degrees. As the simulation is carried out for 50 s at helicopter's height 250 m, providing VehicleSim the parameter $C_T/\sigma = 0.099$, the expected ratio matches with Seddon [31]. Once the main rotor feather angles is chosen, the tail rotor collective feather is also selected as approximately 10 degrees in order to calculate C_{Ttl}/σ_{tl}, being $C_{Ttl}/\sigma_{tl} = 0.045$, the tail rotor feather angle maintains the expected ratio according to Wiesner et al. [39]. These values, for the main and tail rotors collective feather angles allow to simulate hover flight conditions. The helicopter keeps the initial orientation around

Z axis, i.e., the yaw remains constant. Hover conditions are simulated at a height of 250 m and for 50 seconds. The helicopter's position under these flight conditions is represented, the expected behaviour should be a constant position at 250 m height during the simulation. This behaviour can be seen in Figure 11.

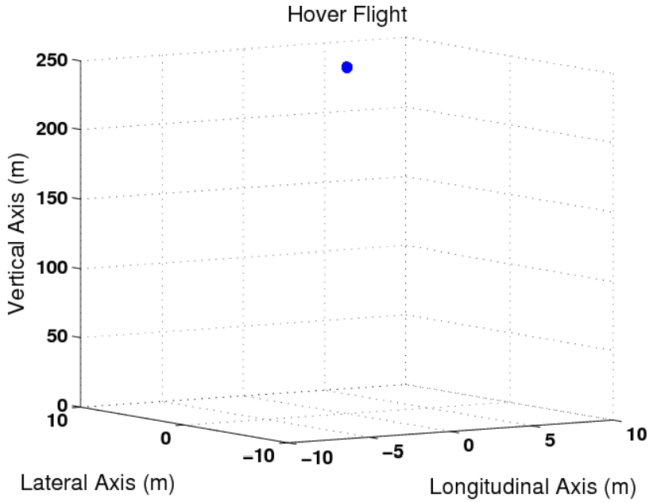

Figure 11: *3-D representation of hover flight conditions. Blue point represents the helicopter's position. Vertical axis value is 250 m.*

The main rotor blades' flap amplitudes are shown in hover flight, see Figure 12. As it can be seen, four subfigures display the corresponding flap amplitudes for each blade: blue line (blade 1), green line (blade 2), orange line (blade 3) and red line (blade 4). The flap amplitudes decrease, being damped by the aerodynamic loads existing around the main rotor. Zoom of these are jointly shown in Figure 13, in order to see clearly the behaviour. By using a Fast Fourier Transform (FFT) Matlab algorithm, the main rotor flap frequencies are calculated for the four blades. Due to the aerodynamic load helps to increase the frequency, these are approximately 47.7 rad/s i.e., 1.07 the main rotor frequency.

The tail rotor blades' flap amplitudes are shown in Figure 14, two subfigures display the flap dynamics for each blade: dotted red line (blade 1), solid blue line (blade 2). Zoom of these are jointly plotted in the last second, in order to see the behaviour clearly, see Figure 15. As it can be seen, the flap amplitudes are approximately constant due to the action of delta-three angle in the tail rotor, and the flap frequencies are near the tail rotor's frequency i.e., 233 rad/s.

In the following sections, the maneuvers are the result of prescribing the desired trajectory and the corresponding collective and cyclic control inputs are selected according to the model's demands. The speeds are assumed to be constant in order to ease the choice of the different flight parameters as well as to obtain a not too complicated calibration for the controller's manual tuning procedure. In addition to this, it is important to bear in mind that signal processing such as noise removal (amongst others) is not considered in this work.

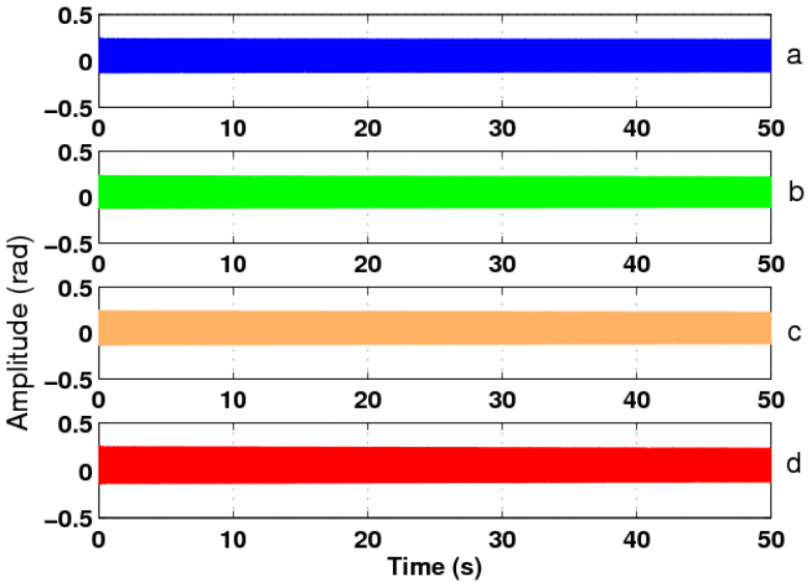

Figure 12: *Flap amplitude for the four main rotor blades in hover flight ((a) blue (blade 1), (b) green (blade 2), (c) orange (blade 3) and (d) red (blade 4)).*

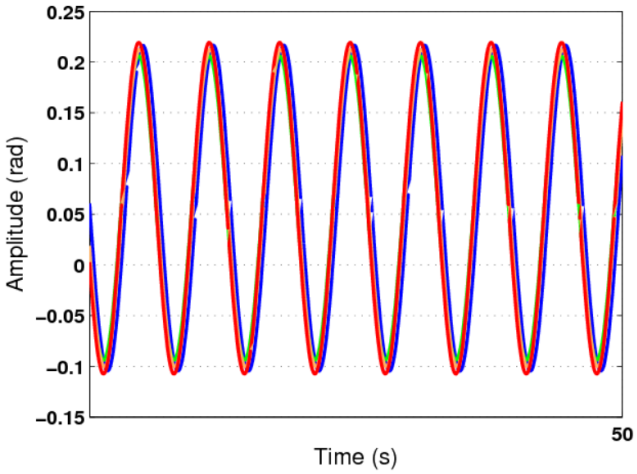

Figure 13: *Zoom and comparison of the flap amplitude for the four main rotor blades in hover flight shown in Figure 12 (blue (blade 1), green (blade 2), orange (blade 3) and red (blade 4)).*

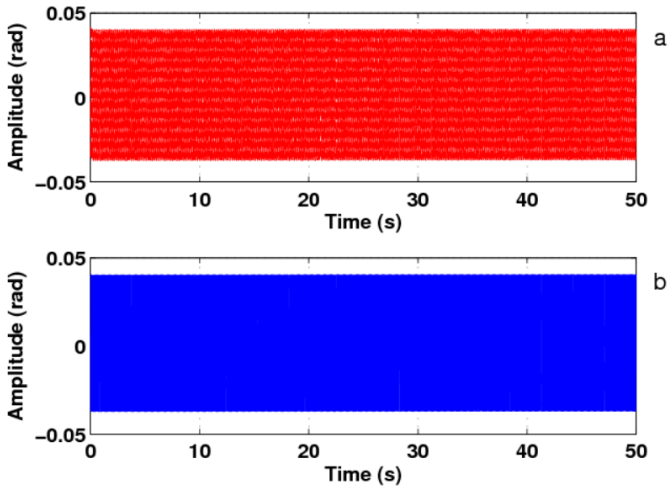

Figure 14: *Tail rotor blades' flap amplitudes in hover flight ((a) dotted red line (blade 1), (b) solid blue line (blade 2)).*

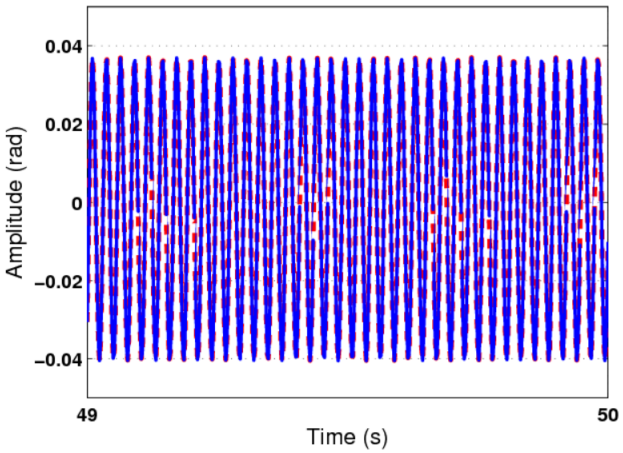

Figure 15: *Zoom and comparison of the tail rotor blades' flap amplitudes in hover flight displayed in Figure 14 (dotted red line (blade 1), solid blue line (blade 2)).*

5.3 Response of the Helicopter Model in Climb Flight

In this section, climb flight is simulated under the following conditions: the climb velocity and induced climb velocity are chosen according to the model proposed by Leishman [14]. The climb velocity is chosen to be: $v_c = 0.9 \cdot v_h$, so the induced velocity is $v_{ic} = 0.65 \cdot v_h$. This climb velocity is enough to carry out a climb simulation in VehicleSim. The main rotor collective feather is $\theta \cong 5.20$ degrees. The tail rotor blades' collective angle is taken as 11.45 degrees. Other combinations can be done, but this just was carried out as an example.

In order to simulate the climb flight a simulation is carried out in VehicleSim using the previous considerations. The initial helicopter's height is 250 m. Figure 16 shows the helicopter's position under climb conditions. As it can be seen, the helicopter climbs from 250 m to 744 m, it covers 494 m in 50 s, its climb speed is 9.88 m/s. According to Newman [40] a typical value for maximum climb speed is $5 - 10$ m/s, so the simulated climb speed agrees with the theoretical values.

The helicopter model under climb flight conditions has shown its capability to reproduce the desired behaviour.

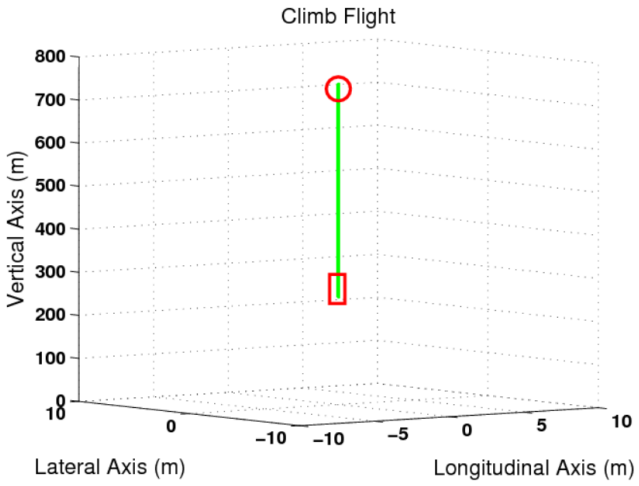

Figure 16: *Climb flight simulation. Green line shows the helicopter displacement from $h_i = 250$ m to $h_f = 744$ m. Red '□', represents the initial position. Red 'o', final position.*

Flap amplitudes for the four main rotor blades are damped by the main rotor aerodynamic loads, showing each blade similar performance and their frequencies are around the main rotor frequency. In addition to this, the delta-three angle action makes more uniform the corresponding flap amplitudes in the tail rotor. The tail blades' flap frequency is approximately equal to the tail rotor

frequency. These behaviours on the tail rotor will be similar, in the following flight conditions.

5.4 Response of the Helicopter Model in Descent Flight

Descent flight is also simulated in VehicleSim under the following conditions: the descent velocity and induced descent velocity are selected according to the data [14]. As a consequence, the descent velocity is chosen as: $-2.2 \cdot v_h$ and the induced velocity is $0.6 \cdot v_h$. It is known that in vertical descent flight the climb velocity is negative whilst the induced velocity remains positive as the main rotor maintains lift [31]. The main rotor blades' collective feather angle is chosen as $\theta \cong -5.20$ rad. Other configurations are possible and this one in here is just an example. The tail rotor blades' collective angle is 11.45 degrees.

In order to validate the descent flight, a simulation is carried out in VehicleSim. In this case, the initial helicopter's height is 250 m and the final height is 78 m (see Figure 17).

The rotorcraft model under descent flight conditions has also shown its corresponding capability to follow the theoretical approach.

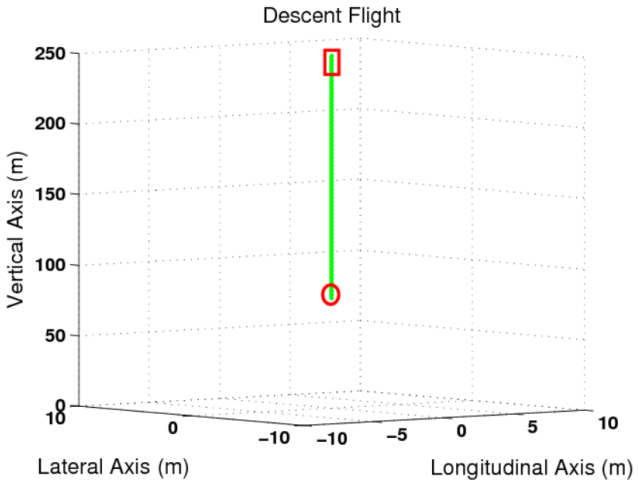

Figure 17: *Representation in three dimensions in descent flight. Green line shows the helicopter's position. Red '* $\square$*', represents the initial position. Red '*$\circ$*', final position.*

The main rotor flap displacements are analogous for each blade and the flap amplitudes decrease as a consequence of the main rotor aerodynamic loads in axial flight. Their corresponding frequencies are near the main rotor's frequency. In the tail rotor, the flap amplitudes are approximately constant due to the action of delta-three angle and the flap frequencies are in the order of the tail

rotor frequency.

5.5 Response of the Helicopter Model in Forward Flight

In forward flight the following conditions are considered in order to carry out the simulations:

- The forward flight speed is given by $V cos\,(\alpha_r) = \mu \Omega R$. In this work, the corresponding advance ratio is $\mu = 0.1$, because the estimated tail rotor contribution is for low speed flight. The standard values for the advance ratio are taken to be in the range from 0.1 to $0.35 - 0.40$ [25].
- The induced forward velocity is given by the relationship between the forward flight velocity and the hover velocity (v/v_h) as well as the induced forward velocity and hover velocity (v_i/v_h). This allows to obtain the induced forward velocity for $\mu = 0.1$ as $v_{ifw} = 0.4 \cdot v_h$. According to Seddon [31], $v_h = v_{i0}$.

The collective feather controls the averaged blade force, and hence the rotor thrust magnitude. The main rotor feather is given by: $\theta = \theta_0 + \theta_{1C} cos\psi + \theta_{1S} sin\psi + ..$ where $\psi = \Omega t$ being Ω the constant rotational speed of the main rotor. The mean angle θ_0 is the collective feather, the harmonics θ_{1C} and θ_{1S} correspond to the cyclic feather angles. The collective feather angle is chosen to be about $\theta_0 \cong 8.50$ degrees, $\theta_{1S} \cong -4.50$ degrees and $\theta_{1C} \cong 2.75$ degrees. The tail collective feather angle is 5 degrees.

The forward flight condition is simulated in VehicleSim. The simulated forward flight is represented, in Figure 18 by plotting in a 3D graph the corresponding vehicle's X, Y and Z coordinates. It can be seen that the trajectory shown in Figure 18 coincides with an expected behaviour of forward flight.

The main rotor blades' flap amplitudes remain approximately constant and their frequencies are near the main rotor's frequency, as a consequence of the aerodynamic loads in forward flight. In the tail rotor, the flap amplitudes are constant due to the action of delta-three angle and the flap frequencies are in the order of the tail rotor frequency.

5.6 Response of the Helicopter Model in a Trajectory

Several combinations of various flight conditions are considered in order to simulate a helicopter trajectory as Figure 19 shows. The trajectory starts with hover flight at a height of h = 250 m, then a climb flight from h = 250 m up to h = 279 m. After this, forward flight with a constant rate climb is performed (see Figure 20), its initial height is h = 279 m and its final height is 336 m. Then, hover flight is simulated at a constant height h = 336 m. Finally, a descent flight manoeuvre from h = 336 m down to h = 254 m is simulated. The transition between the different flight conditions has been performed by linking

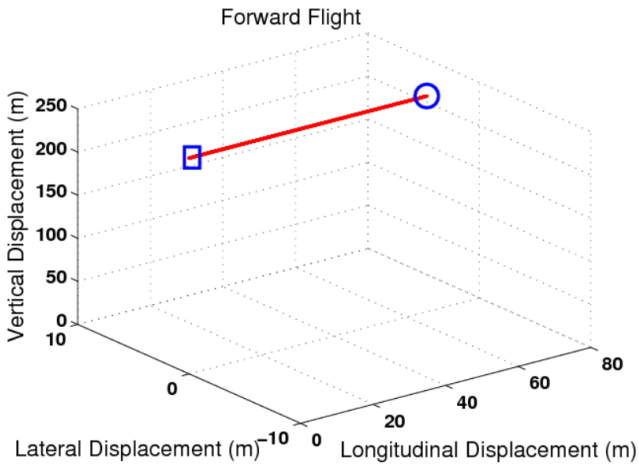

Figure 18: *Forward fight. Red line shows the helicopter's position. Blue '* $\square$ *', represents the initial position. Blue '*$\circ$*', final position.*

each of the variables' final conditions of each flight to the corresponding initial conditions for the following flight. This was done on each maneuver transition, the controllers were calibrated according to the corresponding model demands.

This trajectory simulation was just an example on the achievable manoeuvring that can be performed by the model here presented.

6 Conclusions

This paper introduces a new aerodynamic model for a helicopter on conventional configuration. The model reproduces the dynamic behaviour of a helicopter, which is capable of interacting with external aerodynamic loads. The model has been implemented in VehicleSim, a program that allows to define the systems as a composition of several bodies and ligatures by using a parent/child structure.

The helicopter aerodynamic model has been modelled using blade element theory, allowing to establish analogies between the rotor and experimental behaviour such as aerodynamic loads, etc. The implementation issues related to the use of VehicleSim multibody software have been discussed and their advantages as symbolic software have been contrasted with respect to numerical software. The multibody dynamics approaches offer challenging problems for rotorcraft modelling due to its inherent nonlinearities as it was highlighted. This implementation is a rotorcraft able to operate in several flight conditions and simulates nonlinear dynamics coupling and rotors' aerodynamic loads in a single code. PID controllers have been used in order to control the helicopter model because it is the easier control that it can be developed, and it is effective in order to achieve the required objectives. The controllers have been modelled in

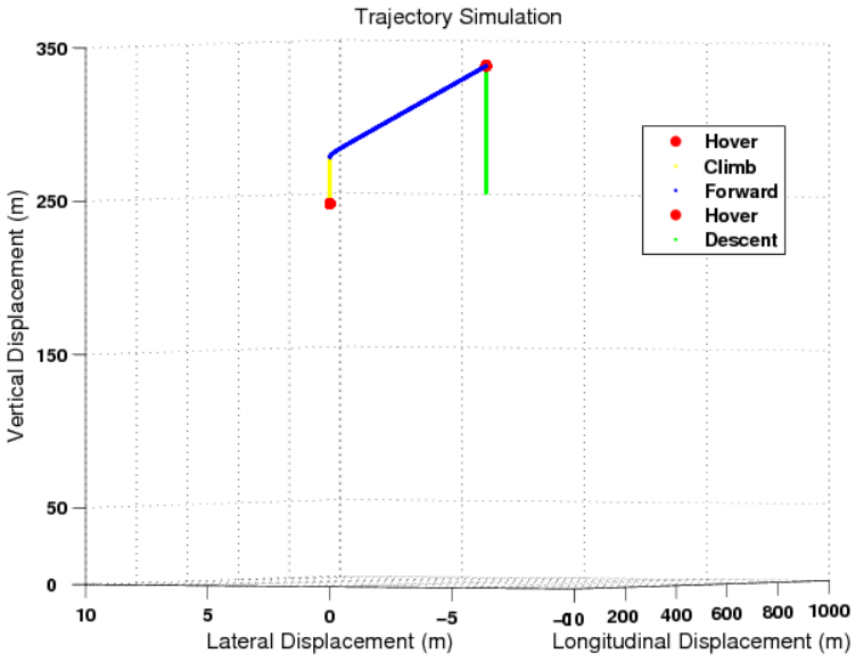

Figure 19: *Helicopter trajectory.*

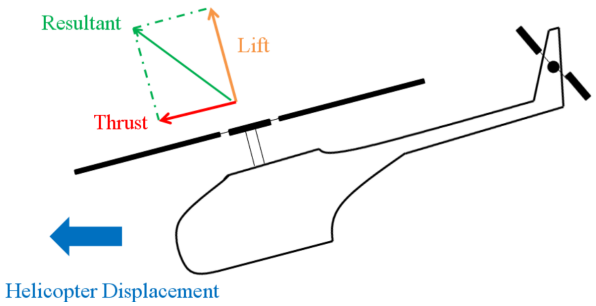

Figure 20: *Forward flight.*

VehicleSim, being not necessary to use any external code. This provides some advantages such as to reduce the compilation and portability. In addition, the nonlinear aerodynamic equations in hover flight have been obtained in order to validate these with the existing theory in the specialist literature. The flying and handling qualities of a rotorcraft are intimately dependent on the stability and control characteristics of the airframe including the flight control. The model has shown that the different flight conditions satisfy the expected behaviour following the control systems commands.

Four types of flight regimes for helicopters such as hover, climb, descent and forward flight were simulated as well as different flight conditions were combined to allow the model to follow certain trajectories. The obtained results match those predicted by theoretical approaches. This work provides a new modelling tool to cover the cumbersome task to represent a realistic and high fidelity helicopter model taking into account the nonlinear dynamics and the aerodynamic environment that surrounds a rotorcraft. The simplified aerodynamic models for the response of the rotors aerodynamic loads in the control inputs have been proved. At present, the aerodynamical aspects of this model do not contemplate complex environments such as appearing vorticities, turbulences, etc. The authors expect to increase the complexity of the aerodynamical model in further works, this task should involve connecting VehicleSim to other external sources such as CFD based codes. However, the main goal of the work here presented was to set up the basis of an embedded helicopter model code, by using VehicleSim only.

This work opens the door for future helicopter studies to develop robust control design as this model provides the basis for not too complicated approach to these type of vehicles. This model, in which nonlinearities and couplings are captured and maintained, increases the fidelity of future research. Validation using experimental data instead of theoretical approaches is explored at present. The behaviour of the system depending on the PID parameters values should be analysed in order to optimize the model's behaviour.

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