Information theoretic description of the e-Mid interbank market: implications for systemic risk

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Abstract

In this paper we examine the temporal evolution of the e-Mid interbank market transactions and quantify topological changes of the resulting credit network at or near events that where considered pivotal in the 2007-2008 credit crisis. The main question we address is whether banks behaviour regarding the choice of counter parties in a trade changed before and during the subprime crisis. In particular, using a network based entropy measure, we assess the level of randomness in the weights distribution across the links of the credit network. We interpret this randomness as a proxy of the level of trust among credit institution. Simple analysis of fundamental properties of the time-ordered set of networks defined over non-overlapping maintenance periods indicate a shrinking market size. The number of nodes (banks) present per maintenance period, the number of edges, edge density and average degree in the system continued to shrink at a roughly constant rate. In order to compare the evolution of network metrics when the underlying network size changes over time it becomes crucial to define appropriate network null models against which the statistical significance of such metrics can be assess. Given the directed and weighted nature of our connections we construct a randomised ensemble of networks using the edge swap procedure, but conserving the vertex in-out strength sequence rather then the in-out degree sequence. We compare the entropy measure in the real networks with the one calculated in reshuffled networks and show that the interbank market moved at the beginning of the subprime crisis to a less random structure, with trading concentrated to a few selected counter parties, leading to a poor liquidity flow. Trust was only recovered after Lehman defaults, following the intervention of central banks around the world to inject liquidity into the banking system.

Keywords: e-MID Interbank Market, Financial Crisis, Network Analysis, Systemic Risk,
1. Introduction

Interbank markets play a key role in banks liquidity management by allowing credit institutions to exchange capital to overcome short-term liquidity shocks. The interest rates determined in this market represent the marginal cost of capital for credit institutions. Variations in interbank rates are rapidly transmitted to the entire term structure, affecting borrowing conditions for households and firms. Interbank dynamics thus influence the whole economic system.

In normal times, interbank markets are among the most liquid in the financial sector. Due to the short term nature of the exchanged deposits banks have accepted non collateralized loans and both liquidity and credit risks were perceived as negligible. During the 2007-2008 financial crisis though, liquidity in the interbank market has considerably dried up, even at short maturities. With the progress of the crisis credit markets experienced not only a reduction in volumes and number of trades but also an increase in volatility and in dispersion of rates paid by different institutions (see Angelini et al. (2010), Gabrielli (2011) B.Karpar et al. (2012), Gabbi et al. (2012)). Two main explanations have been suggested for the market freeze during the crisis: liquidity hoarding, and trust evaporation. The first argument suggests that banks hoard liquidity to anticipate their own unexpected liquidity shocks (Heider et al. (2005) Acharya and Merrouche (2009)). The second attributes the increase in liquidity costs to a rise in perceived counter-party risk (Freixas and Jorge (2009) Afonso et al. (2010) Gale and Yorulmazer (2011) Gabrielli (2011) Cassola et al. (2008)). Previous studies have also suggested that banks rely more extensively on relationship lending during a crisis, supporting indirectly the hypothesis that during periods of financial distress, banks may be less willing to lend and borrow indiscriminately on the interbank market. In particular Cocco et al. (1948), Affinito (2011), and Brauning (2011) confirmed the presence of long lasting relationships among credit institutions and estimated the impact of the strength of a relationship on the interest rate exchanged in a transaction. According to the authors relationship lending plays an important liquidity insurance role during a crisis and facilitates the flow of credit between counter-parties.

The suggestion that the chain of relationships developed over time by credit institutions may affect their trading decisions, and influence the efficient flow of capital in the system, is at the basis of our study. Rather then focusing on relationship lending, here we attempt to establish, with a novel approach based on network analysis, to what extent banks trust each other. As a measure of trust we take how randomly banks distribute their trades with other banks. We interpret a random choice of counter-parties as an indication that banks trust each other and perform little screening. On the other side, a concentration of trades with a few selected counter parties provides an indication that the banking system has entered a phase of trust evaporation, which is likely to generate an inefficient circulation of liquidity, and possibly a freeze of the credit market with obvious systemic consequences.

In order to compare the evolution of network metrics when the underlying network size changes over time it becomes crucial to define appropriate network null models against which
the statistical significance of such metrics can be assessed. To this extent we construct a randomised ensemble of networks using an edge swap procedure. In this way we conserve the total number (or volume) of lending and borrowing transactions performed by each bank, but, while satisfying this constraint, we allocate them randomly to counterparties. This allows us to compare the entropy measure in the real networks and in reshuffled networks with the same number of participants and trades.

The remainder of this article is organized as follows. Section 2 explains the properties of the e-MID interbank market and describes our dataset. Section 3 provides the definition of the entropy measures used for the analysis and describes the rewiring methodology used to assess the significative of our measure. Section 4 discusses the empirical findings obtained from applying these methods to our large dataset. Section 5 concludes.

2. Market mechanism and dataset

Interbank markets can be organized in different ways: physically on the floor, by telephone calls, or on electronic platforms. In Europe, interbank trades are executed in all these ways. The only electronic market for Interbank Deposits in the Euro Area and US is called e-MID. It was founded in Italy in 1990 for Italian Lira transactions and became denominated in Euros in 1999. When the financial crisis started, the market players were 246, members from 16 EU countries: Austria, Belgium, Switzerland, Germany, Denmark, Spain, France, United Kingdom, Greece, Ireland, Italy, Luxembourg, Netherlands, Norway, Poland, and Portugal.

The number of transactions and the volume increased systematically until the beginning of the financial crisis, with an average of 450 transactions each day and an exposure of about 5.5 million euros per transaction. According to the European Central Bank, [ECB (2011)], e-MID accounted, before the crisis, for 17% of total turnover in unsecured money market in the Euro Area. The last report on money markets [ECB (2011)], recorded around 10% of the total overnight turnovers. Trading in e-MID starts at 8 a.m. and ends at 6 p.m. Contracts of different maturities, from one day to a year can be traded but the overnight segment (defined as the trade for a transfer of funds to be effected on the day of the trade and to return on the subsequent business day at 9 a.m.) represents more than 90% of the transactions.

One distinctive feature of the platform is that it is fully transparent. Trades are public in terms of maturity, rate, volume, and time. Buy and sell proposals appear on the platform with the identity of the bank posting them (the quoter may choose to post a trade anonymously but this option is rarely used). Market participants can choose their counterparties. An operator willing to trade can pick a quote and manifest his wish to close the trade while the quoter has the option to reject an aggression.

The database is composed by the records of all transactions registered in the period 01/1999–12/2009. Each line contains a code labeling the quoting bank, i.e. the bank that
proposes a transaction, and the aggressor bank, i.e. the bank that accepts a proposed transaction. The rate the lending bank will receive is expressed per year; the volume of the transaction is expressed in millions of Euros. A label indicates the side of the aggressor bank, i.e. whether the latter is lending/selling (“Sell”) or borrowing/buying (“Buy”) capitals to or from the quoting bank. Other labels indicate the dates and the exact time of the transaction and the maturity of the contract. We consider only the overnight (“ON”) and the overnight long (“ONL”) contracts. The latter is the version of the ON when more than one night/day is present between two consecutive business day. The banks are reported together with a code representing their country and, for Italian banks, a label that indicates their size measured as total assets.

3. Network analysis.

The set of banks and the trades between them in a time interval $\delta t$ can be represented as a weighted directed network $G_{\delta t} = \{V_{\delta t}, E_{\delta t}\}$. In our analysis the edge weights usually represent the number of transactions or volume between pairs of banks unless otherwise stated. For a bank $i$ its degree in an interval $\delta t$ denotes its number of unique trading partners. The degree is composed of the in-degree and the out-degree, $k_i = k_{i}^{in} + k_{i}^{out}$ with the former denoting the number of banks $i$ has borrowed from and the latter the number of banks $i$ has borrowed to. The time ordered sequence of graphs $\{G_{\delta t}\}$ is then known as a temporal graph ([P.Holme and Saramaki (2011)]). The choice of $\delta t$ can have a large effect on the values of various network statistics such as the in/out-degrees and their correlation or the number of triangles in the network ([Clauset and N.Eagle (2007)]). Two ‘natural’ timescales in the e-Mid network are set by the maturity of the interbank loans (the large majority are settled on the next business day) and the monthly deposit of liquidity reserve with the central banks (around 23 business days-known as a maintenance period). For our analysis we choose the maintenance period as the time scale to aggregate the trading activity of banks. Given the high heterogeneity of the system, with some banks trading several times a day, and others only few times a month, this choice allows us to include most banks in the analysis.

Some fundamental properties of the time-ordered set of networks defined over non-overlapping maintenance periods are shown in Fig.(1). A visualisation of the network is also provided in Fig.(2). In accordance with previous work we find that most of the basic quantities indicate a shrinking market size. The number of nodes (banks) present per maintenance period seems to have exhibited three phases during the 11 year period. For the years 1999 and 2000 the number of banks decreased in number, due to several mergers, and then remained approximately constant till late 2007. Form late 2007 to late 2009 there is a third phase of rapid contraction which clearly coincides with the occurrence of the credit crisis. It is interesting that during the whole 11 years the edges, edge density and average degree in the system continued to shrink in a roughly constant slope. This indicates that even when the market had a relatively stable number of participants banks traded with fewer and fewer different partners, strengthening their relationship, or preferential, trading ([Cocco (2009)]).
3.1. Computational rewiring towards null network models.

In order to assess the statistical significance of the values of various network metrics comparisons are required with appropriate network null models. We are currently using the edge swap algorithm to generate synthetic data on which are quantities of interest are averaged and used as null models. The purpose of the edge swapping algorithm is to generate a degree preserving randomisation of graphs, usually for purposes of acting as a null model to test against empirical data. An edge swap selects two ordered pairs \((x, y), (u, v)\) and swaps the endpoints (target nodes) while keeping the sources fixed such that two new pairs will be inserted in the graph \((u, y), (x, v)\) and the original pairs deleted. Not all edges swaps are accepted during a rewiring process as some swaps can produce graphs that are not simple, i.e. contain self loops or parallel edges. If self interactions are permitted edge swapping can transform any directed matrix to any other directed matrix (see Roberts and Coolen (2011) and references therein). Such sampling bias is reduced in the limit of large or sparse graphs. To construct a randomisation a number of edge swaps \(\geq 4E\) are usually required (Squartini and Garlaschelli (2011)). Furthermore a large number of randomisations have to be performed for a given network, the quantities of interest calculated and the averaged over the sample. Although this can be a computationally expensive process in practice we were able to rewire 133 networks (one per maintenance period) 100 times (a number after which the variance in the sample remains largely constant) each and calculate a number of quantities in a matter of hours, so the time cost is not necessarily prohibitive. Quantities that are preserved in the randomised ensemble after the randomisation can then be traced back/explained as purely a consequences of the in and out degrees distributions (Squartini et al. (2011a), Squartini et al. (2011b)) and thus the degree distribution assessed in terms of its information content in the context of the real-world network.

For directed and weighted representations we can construct a randomisation using the edge swap procedure (that now conserves the vertex in-out strength sequence but not the in-out degree sequence) in the following way. Each weighted directed edge with weight \(w_{uv}\) is further inserted \(w_{uv} - 1\) times in the network and all edges have their weights set to 1. The resulting multigraph is then rewired as a directed unweighted graph where each edge now indicates a single transactions and the number of edges between \(u\) and \(v\) correspond to their number of transactions. The rewired multigraph is then collapsed to a directed weighted graph via the reverse procedure (i.e. all \(m\) directed and unweighted edges between \(u\) and \(v\) are collapsed into a single edge with weight \(m\)) and the quantities of interest are then computed in this final graph. Note that as it stands this process only works for graphs with integer weights, and the procedure does not conserve the degree of nodes. Note that this algorithm does not preserve the in and out degree of nodes but only their strengths. This is because in order to redistribute the integer weights evenly between pairs a number of edges must be created. This off course would not be the case if all weights were equal in value. In fact we use the equilibrium number of edges as a criterion to stop the rewiring algorithm. In practice we find that \(2^{17}\) edge swaps are sufficient for this to happen. Finally let us also note that in the near future we also plan to generate null network models using some other recently proposed analytic models (Squartini et al. (2011a), Squartini et al. (2011b), Pretti and Weigt (2006)).
3.2. Information-theoretic description of networks.

3.2.1. Background.

Studying ecological food webs in the 1970’s Rutledge et al. [1976] proposed an analogy between directed networks and the receiver-transmitter systems that formed the basis of Shannon’s information theory Shannon [1948]. Although other ecologists like Pahl-Wolst (1955) and Ulanowicz (1955) examined this framework, it was only recently that Wilhelm and J.Holunder (2007) generalised it to arbitrary networks. Shannon quantified the information content of a transmitter-receiver system as follows. Let \{l_i\} be the string(set) of symbols over the transmitters alphabet and \{b_i\} the set of symbols captured by some receiver. The information content of the transmitter’s signal can be quantified by its entropy \(H(L) = \sum_i p(l_i) \log p(l_i)\) where \(p(l_i)\) is the probability of symbol \(l_i\) being transmitted. The amount of information between the two parties is measured by the mutual information, i.e. the joint entropy \(I = \sum_i \sum_j p(l_i, b_j) \log p(l_i, b_j)\).

Given the above the transmitter-receiver analogy over directed networks works as follows. Let \(\tilde{w}_{ij} = w_{ij} / \sum_i \sum_j w_{ij}\) be the normalised weight, or flux, from node \(i\) to node \(j\). The network analogue of the probability of a symbol at the transmitter site is then \(p(l_i) \rightarrow p(\tilde{w}_i) = \sum_j \tilde{w}_{ij}\). Equivalently for the receiver we have \(p(b_j) \rightarrow p(\tilde{w}_j) = \sum_i \tilde{w}_{ij}\). These expressions calculate the probability of observing activity on a directed link coming out of node \(i\) and into node \(j\) respectively. The expressions \(p(l_i|b_j) = \tilde{w}_{ij}/\tilde{w}_j\) and \(p(b_j|l_i) = \tilde{w}_{ij}/\tilde{w}_i\) are the conditional probabilities of \(i\) being the source given that \(j\) is the target and \(j\) being the target given that \(i\) is the source. Finally the joint probability of \(i\) being the source and \(j\) begin the sink is \(p(l_i, b_j) \rightarrow p(\tilde{w}_i, \tilde{w}_j) = \tilde{w}_{ij}\). In the specific case of the interbank lending network senders are the providers of liquidity (lenders) and receivers are those that request liquidity (borrowers), and the \(w_{ij}\) the number of times liquidity flows along a link. Finally note that, although of little concern to us at present, unweighted or undirected networks arise as special cases of whereby the weight values are uniform across all edges or all existing edges are bidirectional. Armed with these basics we can then calculate the following:

Lender Entropy:

\[
H(L) = - \sum_i \sum_j \tilde{w}_{ij} \log \sum_j \tilde{w}_{ij} \tag{1}
\]

Borrower Entropy:

\[
H(B) = - \sum_j \sum_i \tilde{w}_{ij} \log \sum_i \tilde{w}_{ij} \tag{2}
\]

Lender Entropy given the borrower is known:
\[ H(L|B) = \sum_j p(b_j)H(L|b_j) = -\sum_i \sum_j \tilde{w}_{ij} \log \frac{\tilde{w}_{ij}}{\sum_k \tilde{w}_{kj}} \] (3)

Borrower Entropy given the lender is known:

\[ H(B|L) = \sum_i p(l_i)H(B|l_i) = -\sum_i \sum_j \tilde{w}_{ij} \log \frac{\tilde{w}_{ij}}{\sum_k \tilde{w}_{ik}} \] (4)

Joint Entropy:

\[ H(L, B) = -\sum_i \sum_j \tilde{w}_{ij} \log \tilde{w}_{ij} \] (5)

Mutual Information:

\[ I(L, B) = H(L, B) - H(L|B) - H(B|L) \] (6)

On an individual bank level it is also interesting to look at the following conditional entropies:

Lender entropy given that the borrower is node \( j \):

\[ H(L|b_j) = -\sum_i \frac{\tilde{w}_{ij}}{\sum_k \tilde{w}_{kj}} \log \frac{\tilde{w}_{ij}}{\sum_k \tilde{w}_{kj}} \] (7)

Borrower Entropy given that the lender is node \( i \):

\[ H(B|l_i) = -\sum_j \frac{\tilde{w}_{ij}}{\sum_k \tilde{w}_{ik}} \log \frac{\tilde{w}_{ij}}{\sum_k \tilde{w}_{ik}} \] (8)

The maximum values for all of the above quantities are \( \log(N) \) except for \( H(L,B)_{\text{max}} = 2 \log(N) \). The above equations give us either an individual-based (7-8) or systemic (3-6) picture of the network trading behaviour when lending or borrowing separately.

An alternative approach to quantify the randomness of individual links in a network has been proposed recently by Tumminello et al. (2011a). In Tumminello et al. (2011b) the authors use their statistical method to validate the co-occurrence of trading actions in Nokia stocks among heterogeneous investors.

3.2.2. Results: Systemic description.

Results presented below are provided for the crisis period 2006-2009. The beginning of the crisis, normally located at 2007-08-09, and the Lehman default on 2008-09-14 are indicated in the plots by dashed vertical lines. The three periods delimited by these dates are denoted as in table 1 below:

We start with the lender and borrower entropies Eqs(12-13). Low values of lender entropy indicate that loans tend to originate from a small subset of banks, while low values of borrower entropy means that borrowers prefer certain lenders. On the other hand when these entropies are high the distribution of transactions among banks approaches randomness. In
Table 1: The three periods in yyyy-mm-dd format.

<table>
<thead>
<tr>
<th>Period</th>
<th>Start</th>
<th>End</th>
</tr>
</thead>
<tbody>
<tr>
<td>pre-crisis ($p_1$)</td>
<td>2006-01-01</td>
<td>2007-08-08</td>
</tr>
<tr>
<td>subprime ($p_2$)</td>
<td>2007-08-09</td>
<td>2008-09-14</td>
</tr>
<tr>
<td>Lehman ($p_3$)</td>
<td>2008-09-15</td>
<td>2009-10-21</td>
</tr>
</tbody>
</table>

Fig(3) we see $H(L), H(B)$ for the three periods $p_1 - p_3$ and normalised by their maximum value of $H(X)_{\text{max}} = 2 \log(N)$ for $X \in \{L, B\}$. The quantities for the reshuffled ensemble are not shown here as, by the conservation of strength constraint, in the rewiring process each bank keeps its borrowing and lending number of transactions resulting in these entropies being the same. It is interesting to compare this plot with the time series for the number of nodes in the system shown in the top left panel of Fig.(1) as this is the $N$ argument in the maximum entropy expression. More specifically from the beginning of $p_1$ in 2006 to the begging of $p_3$ in 2009 both the participants in the market as well as entropy decreased indicating that during this period, in a shrinking market, loans became increasingly traceable to a subsample of banks. However, as the market continued to shrink through 2009, the randomness in the lender and borrower identities increased sharply reflecting an increased trust among the remaining lenders and borrowers. This was possibly a consequence of the implicit guarantees provided by governments, after Lehman collapse, not to let other systemically important players to default, and the introduction of quantitative easing measures.

We continue with the conditional entropies, Fig(4), for the whole system as calculated by Eqs (3-4). Firstly we can see that

$$H(L|B) > H(B|L)$$

always. This means that lenders trade less randomly than borrower than vice versa, which is to be expected given that lenders are expose dot credit risk. Secondly between the start and end of period $p_2$ both entropies fall and remain largely constant only to recover in $p_3$. Notably the sharp entropy fall is not observed in the reshuffled networks suggesting that it is not a consequence of the possibly different composition of the market, following its shrunk, but the result of a real reorganisation of the trades. Finally observe that the conditional entropies calculated in the randomised sample are always greater than those on the empirical networks. This is to be expected as the rewiring diffuses highly weight links to other parts of the system whilst conserving the strengths. For the sake of completeness we also plot the joint entropy in Fig(5) which shows a similar behaviour to the marginal and conditional entropies.

We can now look at another well known concept, the mutual information $I(X, Y) = H(X) - H(X|Y) = H(Y) - H(Y|X)$, Fig.(6), which is a measure of the mutual dependence between two random variables, in our case the identity of lender and borrower partners in an interbank transaction. When the lender and borrower identities are completely uncorrelated the mutual information takes its minimum value of 0. This quantity shows a clear peak followed by a declining trend at the $p_1 - p_2$ boundary (the beginning of the sub-prime crisis), furthermore this peak is completely absent in the randomised sample, therefore not being a consequence of the joint lending and borrowing transaction distribution. This suggests that as banks feared a crisis approaching well defined pairs emerged as stable
trading partnerships. On the contrary the rise of mutual entropy after the Lehman default, showing a similar trend in the real and reshuffled network, is likely driven by the changing joint lending and borrowing transaction distribution following the departure of several small credit institution from the market. Finally, and in contrast to all the other quantities, the mutual information for the randomised sample is bellow that of the real-world system as the randomisation destroys any correlation or preference that might exist between the trading pairs.

3.2.3. Results: Individual-based description.

We are now going to examine banks individually with the aid of equations Eq.(7) and Eq.(8). For the bank-specific quantity \( H(A|b_j) \) large values indicate that when acting as borrower the bank distributes its transactions more or less evenly between its trading partners, while small values that the borrower prefers to transact with a specific partner. When on the lending side large values of conditional entropy \( H(B|a_i) \) indicate that as a lender \( i \) distributes its loans evenly across partners and small values indicates a concentration of loans to a smaller subset of other banks.

There is large variance in individual bank behaviour both on the lending and borrowing side. Fig.(7) shows, for each bank, the averages, over the three maintenance periods separately, of Eq(8) vs Eq(7).

In Fig(8) we present time series of \( H(B|a_i) \) for a few select banks for a time period spanning \( p_1 - p_3 \). Starting with the top panel we can distinguish various different types of behaviour. The bank represented by the red line seems to have been unaffected by the crisis in its lending behaviour. Looking at the green line we see a bank that reached a minimum in uncertainty at the start of \( p_2 \) and recovered gradually to reach levels similar to its pre-crisis behaviour during the final period. Notice how this bank was already decreasing its entropy for two maintenance periods before it hit its lowest point. Finally the bank represented by the blue line did not change its lending entropy right until the end of \( p_2 \) at which point it decreased it rapidly only to partially recover at the end. In the lower panel of the same figure we see two other examples. With the green line we see a bank that gradually decreased the volatility in its lending entropy but not so much the level, while in the red line we see a bank with a rapid decrease near the \( p_2 - p_3 \) boundary followed by a similarly rapid recovery within two maintenance periods.

We can also examine the borrowing side of the bank-specific entropy in Fig(9). At the top panel we see a bank that seems to to exhibit the opposite behaviour than the system as a whole, increasing its entropy as a borrower in the middle period. Perhaps this bank was perceived as a safe borrower and was able to expand the number of its credits when borrowing during the crisis. In the bottom panel of the figure we see two different banks, one whose entropy decreases in \( p_3 \) (blue) and another bank which exhibits a decrease in entropy volatility as well as a entropy increase throughout the 3 time periods.

3.3. Discussion

In this paper we have looked at the applications of information-theoretic quantities to a systemic and individual-based description of the eMid interbank lending market for a
time span encompassing the 2007 – 2008 credit crisis. Different quantities show different behaviour during the crisis and point to different features of market reorganisation. On an individual bank level it is clear that our quantities highlight different bank behavioural trading profiles and strategies. It would be interesting to correlate these with the rates banks where able to obtain credit in the market. It would benefit our analysis to examine the information-theoretic concepts on smaller timescales, on both the individual and system levels to probe whether these quantities can be used as useful crisis early warning indicators.

Acknowledgments

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5. Figures

Figure 1: Number of nodes (top left), number of edges (top right), average degree (bottom left) and edge density (bottom right) for the set of networks defined on non-overlapping intervals of $\delta t = 1$ maintenance period.
Figure 2: Visualization of the trading network composed over a monthly period just before the collapse of Lehman brothers. Foreign banks (brown) can be seen to trade largely within themselves.
Figure 3: Entropies for lender and borrower normalised by $H(X)_{\text{max}} = \log(N)$. 
Figure 4: Conditional entropies. Left panel: entropy of borrower given lender on empirical networks and randomised sample(squares) both normalised by their maximum possible values. Right panel: same as left for entropy of lender given borrower.
Figure 5: Joint entropy.
Figure 6: Mutual information, squares (empirical), triangles (randomized sample). Information about sender if receiver is known and vice versa. $I(A, B) = H(A) - H(A|B) = H(B) - H(B|A)$
Figure 7: For each bank, the averages over all maintenance periods in $p_1, p_2, p_3$ of Eq(8) vs Eq(7). Only banks that have traded as borrowers and lenders in each maintenance period are shown.

Figure 8: Time series of entropy of borrower for five different lenders. Curves have been split into two panes for ease of view.
Figure 9: Time series of entropy of lender for three different borrowers. Curves have been split into two panes for ease of view.