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Judgements with errors lead to behavioral heuristics

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Abstract

A decision process robust to errors in the estimation of values, probabilities and times will employ heuristics that generate consistent apparent biases like loss aversion, nonlinear probability weighting with discontinuities and present bias.

KEYWORDS: heuristics, endowment effect, prospect theory, probability weighting function, loss aversion, hyperbolic discounting.
JEL CLASSIFICATION: D03.

1 Introduction

Suppose that we have a risk averse decision maker who is considering whether to exchange good 1 for another good, 2. Further, assume that the goods are different, and that one of the goods is already in possession. If the value of the good is something that can be determined only with error, and if that error is reduced by possession, or over time, then there is a natural explanation based solely on risk aversion of why the decision maker can exhibit loss aversion. Let the value of the good currently possessed be w, and the value of the good that can be gained in the exchange also w, but assume that the decision maker thinks that the good has possible values $\{w-\epsilon, w+\epsilon\}$ with equal probabilities. For now, the risk that the decision maker is facing is described in very simple terms and ϵ is an unspecified constant. Then, the expected utility of the choice to exchange is:

$$\begin{aligned} U(1) &= U(w); \\ U(2) &= \frac{1}{2}U(w-\epsilon) + \frac{1}{2}U(w+\epsilon). \end{aligned} \tag{1}$$

If $U(\cdot)$ is strictly concave, assuming risk aversion, then $U(2) > U(1)$. If the decision maker is asked to exchange his current endowment of good 1 for good 2, like in the famous

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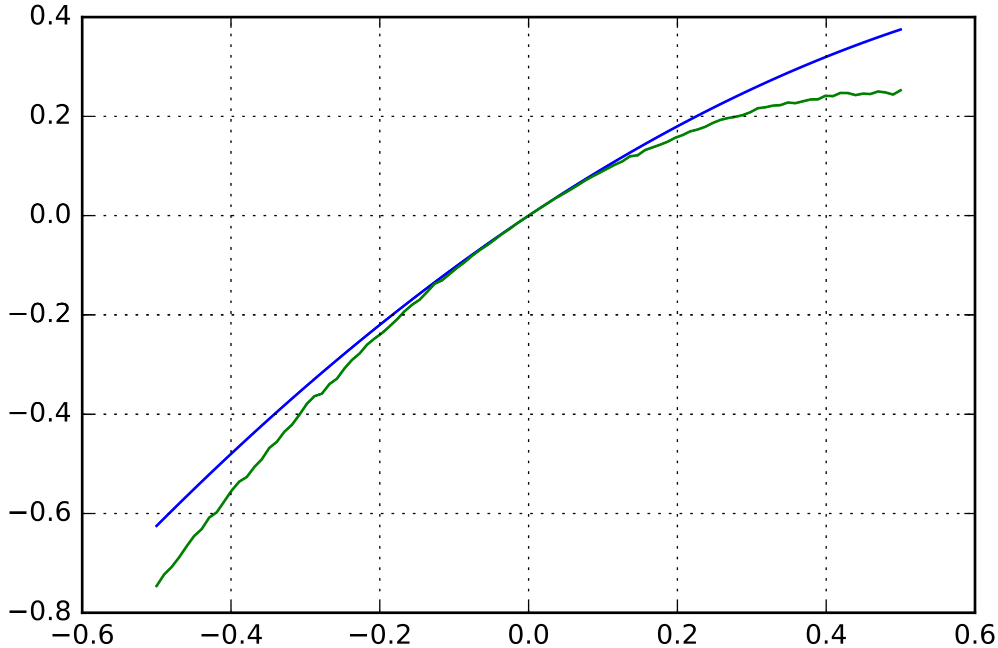

Figure 1: A numerical evaluation of an example of a utility function, $U(x) := x - \frac{x^2}{2}$, without (blue) and with (green) errors in the value.

chocolate-mug experiment [Kahneman et al., 1990], he will show a preference for the good that he already owns. This is not unlike the effect of risk aversion when an object to be received with uncertain worth is evaluated. The difference here is entirely one of interpretation, because I posit that the value risk derived from the valuation error is not consciously considered; rather, a bias coming from some heuristic mental valuation process affects choice, even when the possibility of risk is not entertained. I have assumed here that the individual considers the value of the good owned to be determined with certainty (Figure 1). In Section 2, it is shown that reducing the uncertainty in the value of good 1 relative to good 2 would also lead to the same conclusion.

1.1 Errors in the probabilities of a lottery

Now suppose that the risk averse decision maker is giving his WTP value for very simple prospects, where he can gain a prize, w, with a certain probability, p, and nothing with probability $1-p$. By varying p, we can get a sense of his treatment of various probabilities. Let us further assume that p is know with error, so that instead of p, the decision maker is thinking that he will receive the prize with either probability $p-\epsilon$, or $p+\epsilon$, and these two possibilities are equally likely *ex ante*, before the lottery is resolved. For meaningful probabilities, say $\epsilon < 1/2$, and when $p-\epsilon < 0$, the two possibilities are to get the prize with probabilities 0 or $p+\epsilon$, and when $p > 1-\epsilon$, the two probabilities are $p-\epsilon$ and

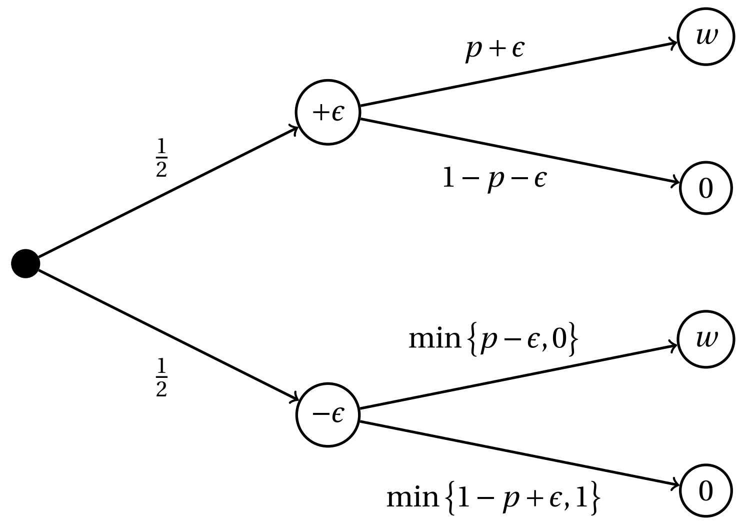

Figure 2: A representation of the two stage process of resolving uncertainty in the probabilities of a lottery for $\epsilon, p \leq 1/2$.

1 instead. Another way of representing the set-up is with a compound lottery, where in the first stage, the error values for the probabilities of the outcomes are drawn from a set of distributions, and in the second stage, the outcome is determined according to the lottery odds adjusted by the errors determined in the first stage (Figure 2).

The value of the lottery is:

$$\begin{aligned}
U(w,p) &= \frac{1-p-\epsilon}{2}U(0) + \frac{1-p+\epsilon}{2}U(0) + \frac{p-\epsilon}{2}U(w) + \frac{p+\epsilon}{2}U(w) = \\
&= (1-p)U(0) + pU(w), && p \in [\epsilon, 1-\epsilon], \\
U(w,p) &= \frac{1-p-\epsilon}{2}U(0) + \frac{1}{2}U(0) + \frac{0}{2}U(w) + \frac{p+\epsilon}{2}U(w) = \\
&= \left(1 - \frac{p+\epsilon}{2}\right)U(0) + \frac{p+\epsilon}{2}U(w), && p \in [0, \epsilon), \\
U(w,p) &= \frac{0}{2}U(0) + \frac{1-p+\epsilon}{2}U(0) + \frac{p-\epsilon}{2}U(w) + \frac{1}{2}U(w) = \\
&= \frac{1-p+\epsilon}{2}U(0) + \frac{1+p-\epsilon}{2}U(w), && p \in (1-\epsilon, 1].
\end{aligned} \tag{2}$$

If $\pi(\cdot)$ is set to be the probability weighting function that leads to the valuation of the prospect, then

$$U(w,p) = \pi(1-p)U(0) + \pi(p)U(w). \tag{3}$$

Assuming ϵ small, and comparing the two sets of expressions, we must have that for $p \in [\epsilon, 1-\epsilon]$, $\pi(p) = p$. For $p < \epsilon$, $\pi(p) = \frac{p+\epsilon}{2} > p$, so the low probability p is overvalued; $\pi(1-p) = \left(1 - \frac{p+\epsilon}{2}\right) < 1-p$, so the large probability $1-p$ is undervalued. For $p > 1-\epsilon$, $\pi(p) = \frac{1+p-\epsilon}{2} < \frac{p+p}{2} = p$, so the large probability p is undervalued; $\pi(1-p) = \frac{1-p+\epsilon}{2} > \frac{1-p+1-p}{2} = 1-p$, so the small probability $1-p$ is overvalued (see Figures 3 and 4).

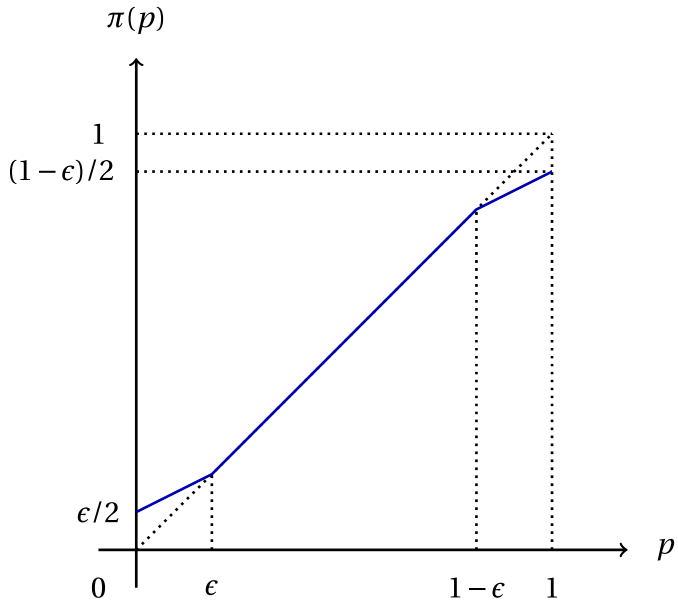

Figure 3: The probability weighting function $\pi(p)$.

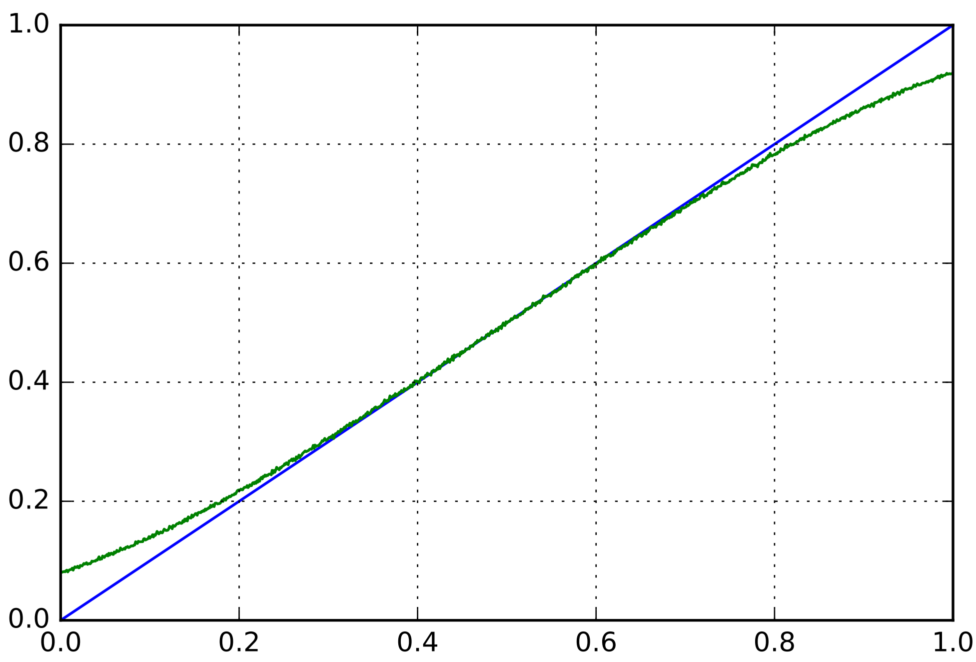

Figure 4: The probability weighting function numerically evaluated, if the probability is a truncated value, between 0 and 1, and the error is normal with $\sigma = 0.2$.

If the decision maker evaluates the odds of certain rewards without any error, redefine $\pi(0) = 0$ and $\pi(1) = 1$, and this can account for the certainty effect.

If $q = 1-p$, and $p < \epsilon$, we see that $q > 1-\epsilon$. According to the expressions above $\pi(q) = \frac{1+q-\epsilon}{2} = \frac{1+1-p-\epsilon}{2} = 1 - \frac{p+\epsilon}{2} = 1 - \pi(p)$, so we have the property that $\pi(1-p) = 1 - \pi(p)$.

1.2 Errors in the timing of payoffs

Assume that a decision maker is faced with a choice over the time a specific reward is received. Specifically, assume that she is an expected utility maximizer that discounts future payoffs exponentially, as in classical economic theory. Again, consider the situation where the WTP in the present is elicited for a reward w in the future, at time t. The decision maker assumes that the timing of the payoff is uncertain, and it could be either $t-\epsilon$ or $t+\epsilon$, with equal *ex ante* odds. If $0 < t < \epsilon$, assume that the timing of the payoffs can be 0 or $t+\epsilon$, and if $t = 0$, assume that the timing is certain. Given the flow utility function $U(w)$ and the exponential time discount rate $\rho > 0$, the present utility of the delayed reward is then:

$$U(w,t) = \begin{cases} = \frac{1}{2}U(w)e^{-\rho(t-\epsilon)} + \frac{1}{2}U(w)e^{-\rho(t+\epsilon)}, & t \in [\epsilon, \infty), \\ = \frac{1}{2}U(w) + \frac{1}{2}U(w)e^{-\rho(t+\epsilon)}, & t \in (0, \epsilon), \\ = U(w). & \end{cases} \tag{4}$$

Implicitly define the time discounting function by $D(t)$, given that the value of the delayed reward is:

$$U(w,t) =: D(t)U(w).$$

This allows the determination of the time discounting function for the two ranges:

$$D(t) = \begin{cases} \frac{e^{-\rho(t-\epsilon)} + e^{-\rho(t+\epsilon)}}{2} = e^{-\rho t} \cdot \frac{e^{\rho\epsilon} + e^{-\rho\epsilon}}{2}, & t \in [\epsilon, \infty), \\ \frac{1+e^{-\rho(t+\epsilon)}}{2} = e^{-\rho t} \cdot \frac{e^{\rho t} + e^{-\rho\epsilon}}{2}, & t \in (0, \epsilon), \\ 1, & t = 0. \end{cases} \tag{5}$$

Using the arithmetic-mean-geometric-mean inequality, it is easy to see that:

$$\frac{e^{\rho\epsilon} + e^{-\rho\epsilon}}{2} > \sqrt{e^{\rho\epsilon}e^{-\rho\epsilon}} = 1,$$

and that:

$$\frac{e^{\rho t} + e^{-\rho\epsilon}}{2} < 1, \quad \text{if } t < \frac{\log(2 - e^{-\rho\epsilon})}{\rho}.$$

We can verify the relationship above by noting that, for $t = \log(2 - e^{-\rho\epsilon})/\rho$:

$$\frac{e^{\rho t} + e^{-\rho\epsilon}}{2} = \frac{2 - e^{-\rho\epsilon} + e^{-\rho\epsilon}}{2} = 1,$$

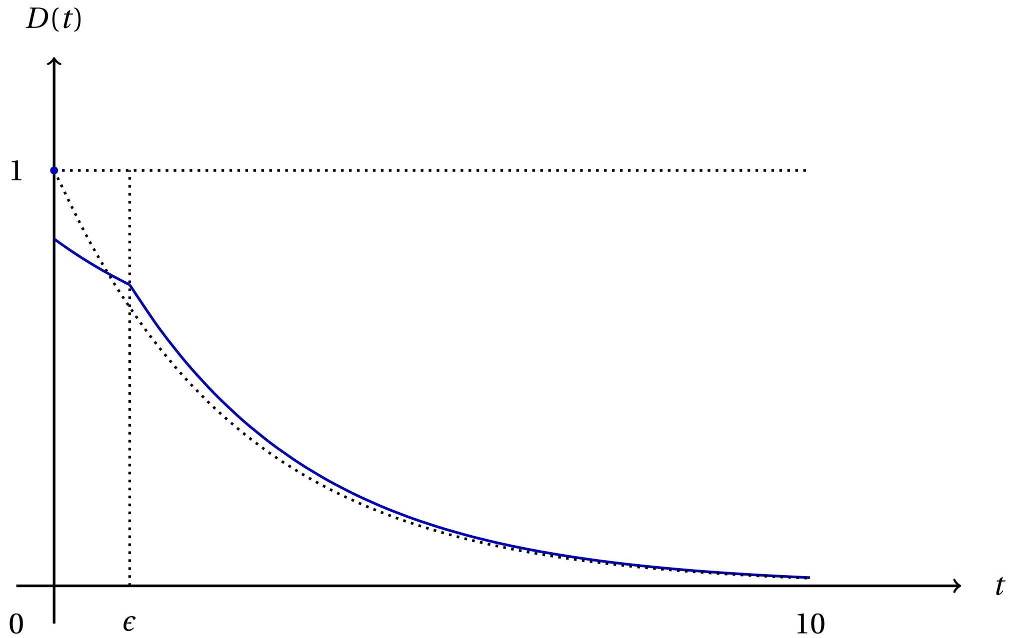

Figure 5: The time discounting function $D(t)$, plotted for $\rho = 0.4$, $\epsilon = 1$.

and observing that $(e^{\rho t} + e^{-\rho\epsilon})/2$ is an increasing function in t.

This implies that $D(t) > e^{-\rho t}$, for $t > \epsilon$, but it is proportional. For $t < \log(2 - e^{\rho\epsilon})/\rho$, $D(t) < e^{-\rho t}$, and moreover it decreases at a lower rate than $e^{-\rho t}$ (see Figures 5 and 6). Assuming that ϵ is small, this means that the time discounting of rewards that are significantly delayed is not as large as exponential discounting with ρ would predict, while the discounting of rewards that are supposed to arrive soon is, in fact, larger than it would be expected. $\lim_{t\to 0} D(t) = (1 + e^{-\rho\epsilon})/2 < 1$. As t becomes very small, the discount factor $D(t)$ converges to a value less than 1. Assuming that rewards that are known to be immediate are evaluated as immediate, a heuristic for an error in the estimated time of a delayed reward can account for present bias, and for a slower-than-exponential time discounting function.

Another simple way of presenting the result is to derive the instantaneous discount factor $\beta(t)$ corresponding to $D(t)$. In the classic discounted utility setting, $\beta := e^{-\rho}$ is a constant. With a variable discount rate, the equivalent definition is

$$\beta(t) := \lim_{\delta\to 0^+} \left(\frac{D(t+\delta)}{D(t)} \right)^{\frac{1}{\delta}}. \tag{6}$$

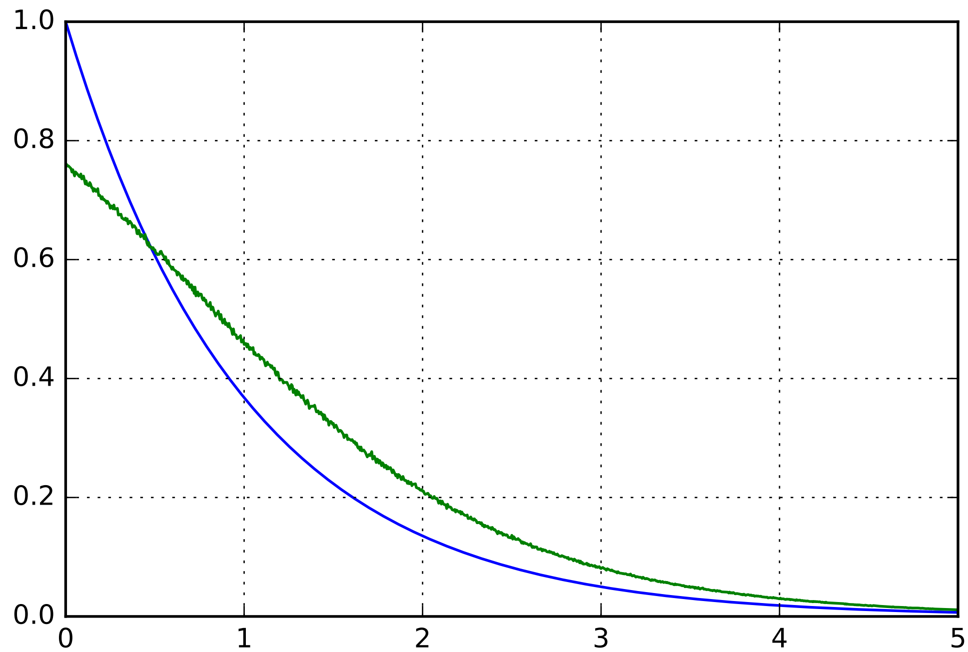

Figure 6: A numerical evaluation of a discounting function, if a normal error is added to the time parameter, and the values are truncated at 0.

Using the mean value theorem, $\forall \delta > 0, \exists \xi \in [0,\delta]$ s.t. $D(t+\delta) = D(t) + \delta D'(t+\xi)$, so

$$\begin{aligned}\beta(t) &= \lim_{\delta\to 0^+}\left(\frac{D(t)+\delta D'(t+\xi)}{D(t)}\right)^{\frac{1}{\delta}} \\ &= \left(\lim_{\delta\to 0^+}\left(1+\frac{\delta D'(t+\xi)}{D(t)}\right)^{\frac{D(t)}{\delta D'(t+\xi)}}\right)^{\lim_{\delta\to 0^+}\frac{D'(t+\xi)}{D(t)}} \\ &= e^{\frac{D'(t)}{D(t)}},\end{aligned} \tag{7}$$

and the equivalent instantaneous discount rate is then $r(t) := -\frac{D'(t)}{D(t)}$. Finally,

$$\beta(t) = \begin{cases} e^{-\rho}, & t \in [\epsilon,\infty), \\ e^{-\rho+\frac{\rho}{2D(t)}}, & t \in (0,\epsilon), \end{cases} \tag{8}$$

and

$$r(t) = \begin{cases} \rho, & t \in [\epsilon,\infty), \\ \rho - \frac{\rho}{2D(t)}, & t \in (0,\epsilon). \end{cases} \tag{9}$$

It is now clear that, in the interval $[0,\epsilon]$, the instantaneous discount factor is progressively increased from $\beta(\epsilon) = e^{-\rho}$ to $\beta(\epsilon)\cdot e^{\frac{\rho}{1+e^{-\rho\epsilon}}}$ as t goes to 0. Instantaneous β and r are undefined at 0 because $D(t)$ is discontinuous. Moving from the present to immediately afterwards requires infinite values. Hence, values in the present are qualitatively different according to the simple model described, in concordance with experimental present bias. Another point of observation is that the model above has an exponentially decaying discount function for $t \geq \epsilon$, so it does not introduce time inconsistency if present, or near-present, rewards are not evaluated. This feature is shared with the well know quasi-hyperbolic discount model [Laibson, 1997].

1.3 Scaled errors in the timing of payoffs

One problem of the model above is that its conclusion may not be robust to the form of the timing errors, and specifically if the scale of the error depends on the delay. This is important since it is more realistic to assume that sooner rewards can be timed more precisely than later rewards. For maximum simplicity, assume again that ϵ is a parameter that describes the possible error in the delay, but this time the timings are $t + t\epsilon$ or $t - t\epsilon$. Of course, there are many other specifications, but this suffices for intuitively describing the effects. Repeating the procedure in the previous subsection,

$$U(w,t) = \frac{1}{2}U(w)e^{-\rho t(1-\epsilon)} + \frac{1}{2}U(w)e^{-\rho t(1+\epsilon)}, \quad t \in [0,\infty),$$

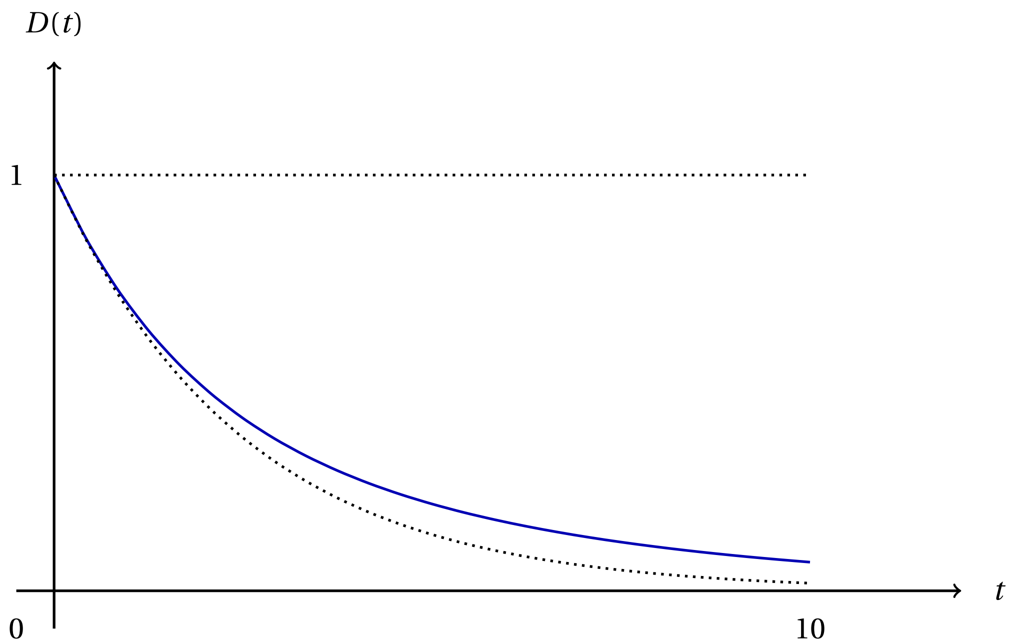

Figure 7: The time discounting function $D(t)$ with scaled errors, plotted for $\rho = 0.4$, $\epsilon = 0.5$.

which leads to

$$D(t) = e^{-\rho t}\frac{e^{\rho t\epsilon} + e^{-\rho t\epsilon}}{2}, \quad r(t) = -\rho + \rho\epsilon\frac{1 - e^{-2\rho t\epsilon}}{1 + e^{-2\rho t\epsilon}}.$$

From Figure 7, it is clear that the discounting function decreases at a slower rate than the exponential. Intuitively, as the delay grows the convexity of the discount function, coupled with the increasing time spread because of the scaled error, create a growing factor which is multiplied to the main exponential discounting effect. This captures, at least in part, some of the experimental observations that justify hyperbolic discounting and the time inconsistency.

2 Modelling General Concurrent Errors

This section introduces concurrent errors in the choice relevant parameters, and such errors are assumed to be derived from a more general distribution.

2.1 A link to loss aversion

In the following, let utility be represented by a measurable increasing and strictly concave real function $U : \mathbb{R} \to \mathbb{R}$. The simplest case is that of an error in the value of an object offered in exchange. Following the definition in Tversky and Kahneman [1991], loss aversion is defined to be the higher valuation in absolute terms of an object seen as a loss, when compared to the value of the same object seen as a gain. Of course, marking an outcome as a loss or a gain requires the fixation of a zero level, usually the status quo. The canonical test for the existence of loss aversion is then the exchange of one good for another with equal value, but of a different kind [Kahneman et al., 1990]. Since the loss of the owned good weighs more heavily than the gain of the new good, the choice should be biased against the exchange. Here it is assumed that when evaluating the value of the exchange the benefit of the new good is not certain, but this lack of certainty is not consciously appraised, but shows up as a bias in the final decision. A random variable (r.v.) $\epsilon \sim F_\epsilon$ representing the valuation error is added, from some cdf F such that $\mathbb{E}[\epsilon] = 0$ and ϵ not almost surely 0 – therefore non-trivial. The value of the new object accounting of the error in the valuation, $U(w+\epsilon)$, is evaluated as follows:

$$U(w+\epsilon) := \int U(w+x)\,dF_\epsilon(x). \tag{10}$$

A simple application of Jensen's inequality [Rudin, 1987] shows that, for a strictly concave utility and a non-trivial error with mean 0, $\int U(w+x)dF_\epsilon(x) < U(\int(w+x)dF_\epsilon(x)) = U(w)$, therefore the new good is valued less than the status quo good. This discussion is precisely the same as for risk, but the point here is that a bias against changing the status quo exists even if such a misvaluation risk is not consciously considered. This hypothesis also fits with the general observation that familiarity and expertise tend to reduce the loss aversion effect [List, 2003]. Furthermore, if this bias is supposed to work as a general heuristic for the most relevant choices – invariably choices with large changes in utility – then it would explain why the loss aversion effect is unusually strong even for insignificant choices.

There is one further issue to consider, and that is the coexistence of the bias described with the conscious evaluation of risk. It is clear, at least to introspection, that individuals are capable of explicitly considering risk and therefore discounting the value of risky options, as when deciding that it is better to forego a premium than to accept the gamble of not being health insured. Therefore, if risk can be explicitly considered, a decision bias may serve no purpose, or it can even be counter-productive by unnecessarily overweighting risk. In the spirit of the previous example, consider now a risky choice where the outcomes and the probabilities are known, but the specific values of the outcomes are again subject to valuation error. In such a situation, a decision biased towards the status quo is useful if it is superior to one in which only the explicit risk is considered.

Let G be the cdf of the r.v. w, which represents a lottery outcome of the choice, subject to risk that is explicitly considered. To this value, a possibly dependent error r.v. $\epsilon \sim F_\epsilon(\cdot|w)$ is added, which represents the error in the valuation of the outcome w. With the conditional ϵ again centered at 0 for each value w, the new good is evaluated according to:

$$\begin{aligned} U(w+\epsilon) &:= \int\int U(w+x)\,dF_\epsilon(x)\,dG(w) \\ &= \int U\left(\int (w+x)\,dF_\epsilon(x)\right) dG(w) \\ &> \int U(w)\,dG(w), \end{aligned} \tag{11}$$

which is the value of the good without errors. In particular, based on the argument above, reducing the error in the value of a good leads to a lower value.

3 Conclusion

The analysis of decision making presented above showed the effect of errors on in the valuation of outcomes, subjective probabilities and delays. A decision that is robust to such errors requires a bias against the exchange of equally valued goods – hence status quo bias, a bias towards small probabilities greater than 0 – the small probability effect, and against large probabilities less than 1 – the certainty effect, and a bias towards the present together with a slower than exponential discount of future outcomes – explaining present bias and hyperbolic discounting.

This shows that the need for robustness to an error prone decision process is a possible unifying explanation for observed deviations from normative decision making. The discussion above does not exclude other possible explanations, and indeed it is unlikely that strong effects like loss aversion or status quo bias will have only one source. Moreover, there are many other sources of error not discussed here, which can have their own biasing influence, as well as typical decision problems that use specific heuristics.

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