
This is the draft version of the paper.

This version of the publication may differ from the final published version.

Permanent repository link: http://openaccess.city.ac.uk/16843/

Copyright and reuse: City Research Online aims to make research outputs of City, University of London available to a wider audience. Copyright and Moral Rights remain with the author(s) and/or copyright holders. URLs from City Research Online may be freely distributed and linked to.
An evaluation of alternative equity indices

Part 2: Fundamental weighting schemes

Andrew Clare, Nick Motson & Steve Thomas

March 2013

---

1 We would like to thank John Belgrove and his colleagues at AON Hewitt for encouraging and sponsoring the research in this paper. All of the authors are members of Cass Business School’s Centre for Asset Management Research. Postal address: 106, Bunhill Row, London. EC1Y 8TZ. Email addresses: a.clare@city.ac.uk; n.motson@city.ac.uk; stephen.thomas.1@city.ac.uk.
Abstract

In this paper we explore an alternative approach for determining constituent weights for equity indices. This approach makes use of alternative definitions of company size, and is referred to as *Fundamental Indexation* (Arnott et al (2005)). Based upon a data set that comprises the largest 1,000 US stocks for each year in our sample, our results show that between 1968 and 2011 the fundamental index alternatives that we consider have outperformed a comparable index constructed on the basis of the market capitalisation of the index constituents in risk-adjusted terms. Our Monte Carlo experiments show that this superior risk-adjusted performance cannot be attributed easily to luck. We also find that although the superior performance is achieved with higher constituent turnover than required using the Market-cap approach to index construction, the turnover is lower, and in some cases much lower, than required by some of the heuristic and optimised index construction techniques that we explored in our last paper. Finally, we find that although the application of a simple market-timing rule does not enhance the returns on these fundamentally-weighted indices very significantly, it does reduce the volatility of their returns and their maximum drawdown quite considerably.

**Keywords:** Fundamentally-weighted equity indices; risk-adjusted performance; Monte Carlo simulation
1. Introduction
It has been common practice for a number of years for both institutional and retail investors to use equity index benchmarks constructed on the basis of market capitalisation, where the company with the largest market capitalisation is assigned the highest constituent weight. The arguments in favour of adopting this approach to indexing equities and for benchmarking equity portfolios have been well rehearsed in the past so we will not repeat them here. Instead, in this paper we examine approaches to equity index construction that adopt alternative definitions of company size to determine index weights. This approach is referred to as *Fundamental Indexation*, as first proposed by Arnott et al (2005).²

In our empirical work we determine the index weights of US equities every year from 1968 to 2011, by using one of the following alternative measures of company size or ‘scale’:

- total annual dividend;
- total annual cashflow;
- book value; and
- total annual sales.

Each of these criteria produce a different weight for each company and therefore a different index. So, for example, when we use total dividends as the indicator of company scale, the company that made the largest dividend payment over a particular period would have the highest weight in the fundamental, dividend-weighted index, while the company that paid out the smallest dividend over the same period would have the smallest weight in this index. We applied the same criteria to the other metrics of scale to produce four fundamentally-weighted indices. Following Arnott et al we also produced a fifth, composite index that was made up of a combination of the weights derived from the first four indices.

But what is the potential appeal of constructing an equity index based on an alternative definition of company size, or scale? To answer this question we need to cast our minds back to the high tech bubble of the late 1990s and early Noughties. The clamour for Dot.Com stocks in those heady, equity bull market days drove up the market capitalisation of many stocks that had little or worse, no history of profitable sales. As their market capitalisations grew, investors tracking market-cap indices were effectively forced to buy more and more of these stocks at the expense of stocks that had a long and proud history of earnings and dividend growth. This caused Dot.Com stock prices to rise further still, driving up their market capitalisations. As their market capitalisations rose, so their index weights increased to meaning that investors benchmarked against market-cap based indices effectively increased their weightings in these stocks too. Ultimately many of these stocks proved to be massively overvalued, and when their values plummeted so did the wealth of those investors that had invested in them. In the aftermath of the collapse of the Dot.Com bubble some researchers and investors wondered whether more ‘fundamental’ measures of size might help to avoid a re-run of the Dot.Com experience.

The rest of this report is organised as follows: in section 2 we describe the data and index construction methodology; in Sections 3 and 4 we present the main results of our work; in sections 5 we try to determine how much of the performance of the indices was due to design and how much to luck; in section 6 we consider whether a simple market timing indicator can improve the performance of the indices; in section 7 we consider the possible impact of transactions costs on the index returns; and finally, we conclude the paper in Section 8.

2. Index construction methodology and data
Arnott et al. (2005) argue that alternative measures of the size or scale of a company may be just as appropriate a basis for determining constituent weights as the more commonly used metric of market capitalisation. One of the advantages of the alternative approaches is that the fads and bubbles that can affect share prices and therefore the market capitalisation of companies may be avoided with a ‘fundamental’ approach. We use four alternative measures of the size of a company to construct alternative indices of US equities. The methodology that we use is based upon the one outlined in more detail in Arnott et al.

In this section of the paper we will begin by describing the index construction methodology for each alternative and then move on to describe the universe of stocks that we use to build the indices.

2.1 The benchmark: Market Capitalisation weights
Since each of these alternative indices are effectively alternatives to a market capitalisation-weighted approach to the same problem, we calculated our own index using the market capitalisation of the same stocks in our universe that we used to construct the various alternatives. The weight of each stock in the Market-cap index is calculated in the conventional manner: the weight of each stock is equal to its market capitalisation divided by the sum of the market capitalisation of all of the other stocks in our chosen universe of 1,000 US stocks. This is the same benchmark index that we analysed in the previous paper in this series.

2.2 Fundamental index weights
We investigated the value in constructing an index of US equities using four alternative measures of scale. These were:

- total annual dividend;
- total annual cashflow;
- book value; and
- total annual sales.

Although most trustees will understand what is meant by dividends, cashflow and sales, the term “book value” may be unfamiliar. Book value is an accounting term. In accounting speak, the book value of any asset is recorded on a company’s balance sheet as the original cost of the asset minus depreciation and subject to some other accounting adjustments. A company’s book value is the total value of its booked assets minus its liabilities and also minus intangible assets like ‘goodwill’. Importantly, the book value of a company can often be very different from its market value. For example, the book value of Dot.Com firms in the late 1990s was often very, very small relative to the market value of the same firms. This is because these companies unlike, for example, large manufacturing firms had few assets, but they still had a market price that had been inflated by the irrational exuberance of the time. Overall, the fact that the dividends, cashflow, book value and sales of a company are not directly determined by irrational exuberance, makes them all ideal alternative metrics of company scale and, in turn, good candidates for the basis of determining equity index weights.

To calculate the Dividend-weighted index we ranked each stock in the first year of our sample according to its average total dividend payout over the previous five years. We then summed this average total dividend payout of each stock over the previous five years, to obtain the total five-year average total dividend payout of all the stocks in the universe. To obtain the weight of each stock, we then divided the five-year average total dividend payout
of each stock by the total five-year average total dividend payout of all stocks. In this way the company whose average total dividend payout had been the highest over the previous five years would be assigned the largest weight, and so on. At the end of each year in our sample we repeated this process. The end result of this procedure is an annually rebalanced, Dividend-weighted index\(^3\).

We used the same procedure for the other three metrics of scale, total annual cashflow, book value and total annual sales.

2.3 **A composite index**

In their work Arnott *et al* also construct a simple composite of the alternative measures of company size that they consider. The composite index in their work comprises the four fundamental, alternative measures of size described above. We approximated their approach as follows: the index weight of each constituent stock at the end of each year was determined as the average of the weights of the other four metrics. So, for example, if at the end of a particular year a stock had a dividend weight of 2%, a cashflow weight of 2%, a book value weight of 1% and a sales weight of 1% then its weight in the composite index would be an average of these, that is, a weight of 1.5%, \([(2\%+2\%+1\%+1\%)/4]\). We repeated this process at the end of each year in our sample.

2.4 **Data**

To investigate the performance of the five fundamentally-weighted indices of US equities we made use of two data bases. For the monthly, total return stock price data we used the CRSP\(^4\) data files, that is, the same data that we used to investigate the performance of the heuristic and optimised alternative indices in our last paper. This dataset contains the end month, total returns of all US equities quoted on the NYSE, Amex and NASDAQ stock exchanges. In keeping with previous work in this area\(^5\), we excluded Exchange Traded Funds (ETFs) and American Depository Receipts (ADRs). The sample period that we use spans the period from January 1963 to December 2011. For the fundamental data needed to construct the index weights we collected data from Computstat\(^6\), making use of the CRSP/Compustat merged list of US stocks.

In December 1968 we applied the fundamental weights described to a universe of 1,000 stocks. These 1,000 stocks represented the largest 1,000 stocks by market capitalisation that had the five full years of return and fundamental data needed to determine the index weights. The performance of the relevant index, and any relevant sub-components were then collated. This process was repeated in the following year, where we again identified the 1,000 largest US stocks in December 1969 that had five full years of return and fundamental history, repeating both the index calculation exercise and the collation of relevant return data. This process eventually produces a continuous portfolio, representative of each index construction methodology, plus sub-components of this index, from January 1969 to December 2011. As well as calculating the performance of these alternative indices using the usual performance statistics, such as the Sharpe ratio etc, we also collect data on the characteristics of the index constituents.

---

\(^3\) If a stock pays no dividends over any five year period, it receives a weight of zero in the index in the following year. In addition, we do not allow negative weights so, for example, whenever a firm’s cashflow is negative we set that observation equal to zero.

\(^4\) The Chicago Booth Centre for Research in Securities Prices (CRSP) historic database provide US daily corporate actions, price, volume, return, and shares outstanding data for securities with primary listings on the NYSE, NASDAQ, Amex, and ARCA exchanges.

\(^5\) In particular see Arnott *et al* (2005) and Chow *et al* (2011).

\(^6\) Compustat is a database that comprises financial information on both active and inactive companies from equity markets around the world. Within the database the US equity market has the most comprehensive coverage stretching back to 1962.
3. Main Results
In Table 1 we have presented some familiar performance statistics for the five fundamentally-weighted indices described in Section 2 along with the index constructed using the market capitalisation of stocks from the same stock universe.

3.1 Annualised returns and standard deviation of returns
The second column of Panel A in Table 1 presents the annualised returns of the Market-cap and fundamentally-weighted indices. In each case the fundamentally-weighted indices outperform the Market-cap benchmark. Over this period the best performing index was found to be the Sales-weighted index, which produced an annualised return that was two percentage points higher than that produced by the Market-cap index. These results represent a clean sweep for the alternative indices considered here and in our previous paper in this series. In return terms the Market-cap index was the worst performer over this sample period. However, as with the results for the heuristic and optimised indices considered earlier a decomposition of the return performance by decade, shown in Panel B, reveals once again that it was in the Noughties where the Market-cap index really underperformed the alternatives and, conversely, that the Market-cap index was the best performer in return terms over the 1990s. However, the Book Value and Sales-weighted indices also performed well in the 1990s, where both produced an annualised return of 17.0%. Finally, the table also shows that the Market-cap index underperformed all of the fundamentally-weighted indices over the 1970s by around 3 percentage points.

<table>
<thead>
<tr>
<th>Table 1: Main results (1969 to 2011)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Full sample results (1969 to 2011)</strong></td>
</tr>
<tr>
<td><strong>Return</strong></td>
</tr>
<tr>
<td>Market cap-weighted</td>
</tr>
<tr>
<td>Dividend-weighted</td>
</tr>
<tr>
<td>Cashflow-weighted</td>
</tr>
<tr>
<td>Book Value-weighted</td>
</tr>
<tr>
<td>Sales-weighted</td>
</tr>
<tr>
<td>Fundamentals composite-weighted</td>
</tr>
</tbody>
</table>

**Panel B: Annualised Returns and volatility by decade**

| **Return** | **st. dev.** | **Return** | **st. dev.** | **Return** | **st. dev.** | **Return** | **st. dev.** |
| Market cap-weighted | 6.1% | 16.2% | 16.9% | 16.1% | 17.6% | 13.1% | 0.4% | 15.2% |
| Dividend-weighted | 8.7% | 15.4% | 19.1% | 14.3% | 15.4% | 11.7% | 4.0% | 15.5% |
| Cashflow-weighted | 9.2% | 16.1% | 18.6% | 15.4% | 16.4% | 12.0% | 3.2% | 16.4% |
| Book Value-weighted | 9.1% | 16.4% | 18.3% | 15.4% | 17.0% | 12.9% | 2.9% | 17.0% |
| Sales-weighted | 9.1% | 17.6% | 19.4% | 16.2% | 17.0% | 13.2% | 4.2% | 16.9% |
| Fundamentals composite-weighted | 9.0% | 16.3% | 18.8% | 15.3% | 16.5% | 12.4% | 3.6% | 16.3% |

Notes: The returns presented in column 2 of the table are compound annual returns (APR). The alphas presented in column 8 are monthly.

The third column in the table in Panel A shows the annualised standard deviation of the index returns. The Dividend-weighted index produced the lowest volatility over the full sample period according to this measure (14.5%), while the Sales-weighted index produced the highest (16.2%). However, the differences do not seem to be particularly pronounced. The annualised standard deviation results by decade, shown in Panel B of the table, also suggest that these indices generally produce similar levels of volatility.

3.2 Risk ratios
3.2.1 Sharpe, Sortino and Information ratios
Table 1 also presents three reward to risk metrics: the Sharpe ratio, the Sortino ratio and the information ratio. The Sharpe ratio for each index (Si) was calculated as:
\[ S_i = \frac{R_i - R_f}{\sigma_i} \]

where \( R_i \) is the average return on the index, \( R_f \) is a proxy for the risk free rate of interest, in this case the average return on US T-bills; and \( \sigma_i \) is the standard deviation of the returns on index \( i \). So the higher this ratio, the higher has been the return relative to each unit of risk. The higher the Sharpe ratio the better. The less familiar Sortino ratio is based upon the semi-deviation, \( \sigma_{s-d,i} \), of index \( i \)’s returns, rather than on the full range of returns, in other words it is only based upon negative returns and disregards all positive returns:

\[ S_i = \frac{R_i - R_f}{\sigma_{s-d,i}} \]

once again, the higher the Sortino ratio the better the performance of the index.

The Market-cap index produced the lowest Sharpe ratio (0.32). The Sharpe ratios of the fundamentally-weighted indices are all remarkably similar, ranging only from 0.39 (Book Value-weighted) to 0.42 (Sales-weighted). The spread of Sortino ratios for these alternative indices is also very narrow, ranging from 0.48 (Book Value-weighted) to 0.53 (Sales-weighted).

3.2.2 Sharpe ratio differences

The reward to risk characteristics of these alternatives presented in Table 1, are very similar, although they are superior to those of the Market-cap index. To try to determine whether the differences in the Sharpe ratios were significant in a statistical sense we performed a set of tests. The results are shown in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>Cashflow-weighted</th>
<th>Dividend-weighted</th>
<th>Book Value-weighted</th>
<th>Sales-weighted</th>
<th>Composite-weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market cap-weighted</td>
<td>4.0%</td>
<td>9.8%</td>
<td>12.5%</td>
<td>4.2%</td>
<td>4.3%</td>
</tr>
<tr>
<td>Cashflow-weighted</td>
<td>0</td>
<td>0</td>
<td>4.2%</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Dividend-weighted</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Book Value-weighted</td>
<td>80.7%</td>
<td>23.3%</td>
<td>66.4%</td>
<td>93.4%</td>
<td></td>
</tr>
<tr>
<td>Sales-weighted</td>
<td>33.8%</td>
<td>97.0%</td>
<td>76.7%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Composite-weighted</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: A value of 1 in a cell means that the null is rejected at the 95% level of confidence; the associated p-value is also shown in each cell.

Each cell in Table 2 shows the results of the test of the difference between the full sample Sharpe ratios of each pair of indices. A value of 1 in the cell is a rejection of the null hypothesis that the Sharpe ratios are the same at the 95% level of confidence. In each cell we have also presented the relevant p-values. The results indicate that we can be at least 95% sure that the Sharpe ratio produced by the Market-cap index is different from that produced by the Cashflow, Sales and Composite-weighted indices. With regard to the Dividend-weighted index we can be just over 90% sure that its Sharpe ratio is different from the one produced by the Market-cap benchmark. The fundamental indices do seem to

\[ 7 \text{See Ledoit, O., & Wolf, M. (2008), Robust performance hypothesis testing with the Sharpe ratio. Journal of Empirical Finance, 15 (5), 850-859. To perform these tests we used the authors’ Matlab code that can be downloaded from Michael Wolf’s website: http://www.econ.uzh.ch/faculty/wolf/publications.html.} \]
produce a risk-return trade-off that is different than that which is produced by the Market-cap index. The Table also shows that the Sharpe ratios of some fundamental index pairs are different. For example, we reject the hypothesis that the Sharpe’s of the Sales and Book-Value weighted indices are the same with statistical confidence of 97%.

3.2.3 Summary
Taken together, the risk ratios indicate that over this sample period investors would have received superior risk-adjusted returns from any of the fundamentally-weighted indices considered here compared to the Market-cap index. The best risk-adjusted performer was the Sales-weighted index, which also produced a Sharpe ratio that was significantly different to that of the Market-cap benchmark.

3.3 Systematic risk
Unsurprisingly each of the fundamentally-weighted indices produce positive monthly alphas. In this sense then the indices might be said to have ‘demonstrated skill’ relative to the Market-cap index. The Dividend-weighted index produced the highest monthly alpha of 0.19%. We also find that the betas of the fundamentally-weighted indices are all close to one (with the exception of the beta for the Dividend-weighted index) which indicates that on average the fundamentally-weighted indices embody a level of systematic risk that is quite similar to that embodied in the Market-cap index.

3.4 Tail Risk
Reward to risk ratios are one way of calibrating risk for investors, or at least a way of calibrating the reward they receive relative to the risks that they bear. However, many investors, including trustees of DB schemes, or individuals that may be in the final phase of the pension accumulation journey, are also very concerned about ‘tail risk’. Interestingly Table 1 shows that the Market-cap index has the lowest maximum drawdown over the sample period considered – 48.5%. Unlike the heuristic and optimised indices considered in the previous paper, where the maximum drawdown statistics were lower than those of the Market-cap index, if anything, the fundamentally-weighted indices seem to exacerbate tail risk. The maximum drawdown statistics for the fundamental indices are between four and seven percentage points higher than for the Market-cap index. The Book Value-weighted index experienced the highest maximum drawdown of all the indices of 55.1%. However, the annualised return, Sharpe and Sortino ratio statistics discussed earlier, show that investors were ultimately rewarded for bearing this higher maximum drawdown risk.

Nevertheless, for investors concerned about large drawdowns and tails risk, fundamentally-weighted indices may not ‘solve’ this particular concern for investors.

3.6 Summary
The fundamentally-weighted indices considered here, including the composite index, have all been designed to allocate higher index weights to the ‘larger’ companies in our universe, where size is defined in a variety of ways. It should not be surprising then that in many respects the performance of the indices in terms of maximum drawdown, systematic risk and turnover is quite similar to the performance generated by the Market-cap index. The annualised returns on the fundamentally-weighted indices have been, however, higher than those generated by the Market-cap equivalent and, when we consider the decade by decade return performance of the indices, more consistent than those produced by the benchmark; this is particularly true of the Sales-weighted index.

In the next section of this paper we will take a more disaggregated view of the fundamentally-weighted indices, so that we can understand more about the source of their performance.
4. A further decomposition of the index returns
In section 3 we saw that the fundamentally-weighted indices tended to produce very similar performance statistics. The results in this section broadly confirm that the underlying characteristics of the indices are also very similar. In other words, the weighting schemes tend to be assigning similar weights to the universe of stocks.

4.1 The Market-cap index
For the purposes of comparison, we began by decomposing the returns of the Market-cap index. These results are presented in Appendix 1.

Figure 1A in this appendix shows the three year rolling returns of the index. Figure 1B shows the average weight of the Market-cap index by Beta decile. This figure shows that the Market-cap index has an average 10% weighting in those 10% of stocks with the smallest betas (decile 1), and has the lowest average weighting of 3% to the 10% of stocks with the highest betas (decile 10). Figure 1C presents analogous results with regard to the average weight of the Market-cap index by size. The figure shows that on average just under 60% of its market capitalisation is concentrated in the largest decile of stocks. Figure 1D shows the exposure of the index by book-to-market. In accounting speak, the book value of any asset is recorded on a company’s balance sheet as the original cost of the asset minus depreciation and subject to some other accounting adjustments. A company’s book value is the total value of its booked assets minus its liabilities and also minus intangible assets like ‘goodwill’. We will say more about this exposure in section 4.3.3, but for the moment we can simply note that the Market-cap weighted index is tilted towards stocks with low book values relative to their market values. Figure 1E shows the Market-cap weighted index’s exposure to high and low momentum stocks. The first momentum decile is comprised of those stocks whose prices have been the worst performing ten percent of stocks over the previous 12 months. Conversely decile 10 is made up of the stocks who have been the top ten percent of stock price performers over the previous 12 months. Again we will say more about the meaning of these momentum exposures in section 4.3.3, but for now we can see that there is no obvious momentum bias in the Market-cap index. Figure 1F presents the index’s weight by volatility decile. The figure shows, for example, that the Market-cap index has an average 20% weighting to the decile of stocks with the lowest volatility and a 3% weighting towards those stocks with the highest volatility. Figure 1G shows the average monthly return on the Market-cap index in months when the market rises and falls. Figure 1H shows the average weight of the Market-cap based index by volume decile. On average, and unsurprisingly, the Market-cap index has a weighting of 53% in the top decile of stocks by traded volume, and a combined average of 12% in the five deciles of stocks with the lowest traded volume.

Taken together these results show that a Market-cap index is heavily weighted towards large cap stocks (by design), to the most highly liquid stocks (almost by design), but also to low stocks with low return volatilities.

Appendices 2 to 6 present the same results for the fundamentally-weighted indices, but where the results are expressed relative to the Market-cap index. So, for example, Figure 2A in Appendix 2 shows the three year rolling returns of the Dividend-weighted index relative to those generated by the Market-cap index. We have produced identical analysis for each of the other fundamentally-weighted indices which we present in appendices 3 to 6.

4.2 The three year rolling returns of the alternative indices
When we consider rolling three-year performance periods there are a number of occasions when the fundamentally-weighted indices underperform the Market-cap index. As mentioned above, the most significant period of underperformance was in the 1990s: all of
the fundamentally-weighted indices underperform significantly between 1997 and 2000. There are also other periods of underperformance, for example, between 1991 and 1993, though not for the Dividend-weighted index. The Dividend-weighted index also underperforms between 1980 and 1981.

Perhaps the most interesting period is the post-Lehmans period. The Dividend and Book Value-weighted indices both underperform between 2008 and 2011. However, the Cashflow and Sales-weighted indices outperform. Over the crisis period investors seem to have favoured companies with stronger cashflow and sales, than those companies with higher book values that made greater dividend payments. The composite index, being a weighted average of the four therefore has tended to match the performance of the Market-cap index over this period.

4.3 Index factor exposures

The analysis of the performance of equity portfolios often begins by decomposing returns into four, systematic sources of return, that is, estimating how much of the fund’s return was due to its exposure to the ‘market’, to small cap stocks, to stocks with high book-to-market values, or to stocks with relatively high price momentum. The appendices of Figures for each index construction methodology contain information on their exposures to these four factors; Figures B, C, D and E show the index tilts towards market risk (represented by beta), size, book-to-market value and momentum respectively.

4.3.1 The market

According to the Capital Asset Pricing Model (CAPM) the “universe of all tradable assets” is the only source of undiversifiable, systematic risk. According to the theory over long periods of time, investors should earn a return over and above a cash return, known as a risk premium, simply from being passively exposed to this source of risk. When analysing equity market risk to capture this source of risk, finance researchers and industry practitioners generally use the return on a broad index of equities as a proxy for market risk.

Figures 2B and 3B in the appendices show respectively how both the Dividend and Cashflow-weighted indices are overweight the two bottom decile of stocks with the lowest betas, and are underweight the two to three deciles of stocks comprised of higher beta stocks. But compared to the under and overweights calculated for some of the heuristic and optimised indices in our last paper, the deviations from the Market-cap benchmark are generally quite small. The Book Value index is also underweight the bottom decile of stocks by beta but only by 2.4%, the remaining beta decile under and overweights are relatively small. The composite index is overweight the two deciles comprising the lowest beta stocks and underweight the two top beta deciles.

Although the CAPM remains the benchmark model of risk and return, dissatisfaction with the empirical performance of this model led Fama and French (1993)\(^8\) to propose that systematic returns were comprised of two further factors: size and book-to-market value. We now discuss the exposure of the indices to these two risk factors.

4.3.2 Size

A number of researchers had previously found that small cap stocks tended to outperform large cap stocks even though the identifiable market risk in the small cap stocks was lower than was evident in the large cap stocks. In other words, there seemed to be a risk premium that could be earned from investing in small cap stocks that was independent of the premium that could be earned from being only exposed to ‘market’ risk.

The figures in the appendices show that the Dividend and Cashflow-weighted indices all have a very similar exposure to large cap stocks as the Market-cap index. The largest variations occur in the deciles of large cap stocks (deciles 8 to 10) but the under and overweights are never greater than 0.4%. Overall then it would be fair to say that these two indices have the same ‘bias’ towards large cap stocks as the Market-cap index. The over and underweight market cap positions for the size deciles of the Book Value and Sales-weighted indices are all relatively small, with the exception of decile ten, which comprises the largest stocks. The Book Value index is 7.0% underweight the decile of large stocks, while the Sales index is almost 10% underweight this decile of stocks. The Composite index, which is a combination of all four indices, is 4.2% underweight the largest decile and marginally overweight the other nine deciles.

Another way of looking at the differences in the stocks that comprise each index is by calculating the average size of the stocks within each index. Figure 1 shows the weighted average size of each stock in each index over 2011. The index with the largest average constituents over 2011 was the Dividend-weighted index (just over $96bn), while the index that comprised the lowest average was the Sales-weighted index (just over $67bn). The Market-cap and composite indices had an average constituent weight that was almost identical over this period (just under $80bn)

Compared with most of the heuristic and optimised indices explored in our last paper, these results confirm that fundamentally-weighted indices allocate greater weight to larger stocks. This should not come as a surprise, since each of the weighting criteria are alternative proxies for size.
4.3.3 Book-to-market value
Researchers found that companies with a high book-to-market value tended to outperform those with lower book-to-market value. Fama and French (1993) went further and suggested that book-to-market value was a systematic risk factor. They argued that there was a risk premium that could be earned from passive exposure to stocks with a high book-to-market value. Such stocks are often described as ‘value stocks’ because the price being ‘asked’ for their shares in the market is low relative to their recorded book value, by contrast ‘growth’ stocks tend have very low book-to-market values. Importantly, the book value of a company can often be very different from its market value. For example, the book value of Dot.Com firms in the late 1990s was often very, very small relative to the market value of the same firms. This is because these companies unlike, for example, large manufacturing firms had few assets, but they still had a market price that had been inflated by the irrational exuberance of the time.

Figure 2D in appendix 2 shows the relative decile exposures by book-to-market value for the Dividend-weighted index. It shows that the index is 8% underweight the ten percent of stocks with the lowest book to market value and 6% overweight the ten per cent of stocks with the highest book-to-market values. This index is also underweight deciles 2, 3 and 4 and overweight deciles 5 to 9. The other Fundamental indices display very similar exposures by book-to-market value. The index with the largest tilt to high book-to-market value stocks, unsurprisingly, is the Book Value-weighted index, which is around 12% underweight the decile of stocks with the lowest book-to-market values and 11% overweight the decile of stocks with the highest book-to-market values.

Overall, all of the Fundamentally-weighted indices are tilted towards stocks with high book-to-market values, often referred to as value stocks.

4.3.4 Momentum
Fama and French (1993) first proposed size and book-to-market value as additional systematic sources of risk and return; in further work in the same spirit, Carhart (1997) identified yet another factor: momentum. Carhart argued that there was another premium available to investors that could be derived from tilting portfolios towards stocks with high price momentum, that is, stocks that had performed relatively well compared with other stocks. Essentially Carhart’s argument was that an additional and independent risk premium could be earned by equity portfolio managers over time by tilting their portfolios towards stocks that had performed relatively well in the past (usually over the previous 12 months).

Figure 2E in appendix 2 shows the relative decile exposures by momentum for the Dividend-weighted index. This index is marginally overweight momentum deciles 1 to 6, that is, those stocks with relatively low price momentum. The index is 2.8% and 4.0% underweight the stocks that comprise momentum deciles 9 and 10, that is, those stocks with relatively high price momentum. However, overall the momentum tilts are relatively small. This is true of all the other Fundamentally-weighted indices too, which display very similar under and overweight momentum exposures.

Overall, the Fundamentally-weighted indices considered here have a slight bias towards low momentum stocks.

4.3.5 The impact of factor tilts on index returns
The return produced by a tilt in any equity portfolio or index depends not only on the size of the tilt, as discussed above, but also the return to that tilt. In other words, if there were no additional return available to investors from overweighting small cap stocks over time, it

---

would make little difference to overall performance. To determine the impact of the tilts to
the four risk factors for each index and for each factor we calculated the average value of,
for example, being overweight small cap stocks relative to the Market-cap weighted index.
These results are shown in Table 3.

<table>
<thead>
<tr>
<th>Table 3: The ‘returns’ to factor exposures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Dividend-weighted</td>
</tr>
<tr>
<td>Cashflow-weighted</td>
</tr>
<tr>
<td>Book Value-weighted</td>
</tr>
<tr>
<td>Sales-weighted</td>
</tr>
<tr>
<td>Fundamentals composite-weighted</td>
</tr>
</tbody>
</table>

Notes: The return value in the table represent the annualised difference between the return to the factor exposure of the alternative index relative to the Market-cap index.

The results in table 3 indicate that the Book-to-market value tilt produces by far the greatest marginal positive return for the five fundamentally-weighted indices. It is highest – naturally – for the Book Value-weighted index at 2.02%, but is well above 1.5% for the other four indices. The returns derived from the other tilts are all relatively small, and with regard to the momentum factor, consistently negative.

Taken together these results indicate that much of the difference in the performance of these indices is due to the overweight position that these indices have in stocks with a high book-to-market value and associated underweight towards stocks with a low book-to-market value.

4.4 Index compositions by total volatility
Some investors are uninterested in the factor decomposition of returns, and instead are more concerned with total volatility, so in addition to decomposing the indices into factor risk buckets, we also decomposed them by total volatility. Each of the appendices for the fundamental indices show the decile weights relative to the Market-cap index by total volatility. For example, we can see from appendix 2 Figure 2F that the Dividend-weighted index is 10% overweight the decile of stocks with the lowest return volatility, and generally has small underweights to the remaining volatility deciles. The Cashflow and Composite-weighted indices have a similar exposure to the volatility deciles. The Book Value-weighted index has only very small under and overweight to the volatility deciles. In contrast to the other indices, the Sales-weighted index is slightly underweight (4%) the decile of stocks with the lowest volatility and decile 2 as well (2%); it is slightly overweight volatility deciles 3 to 7 and has no under or overweight position in volatility deciles 8 to 10.

Overall, these indices tend to be either underweight low volatility stocks, or at least not significantly overweight high volatility stocks.

4.5 Summary
Overall, the results in this section of the paper show that when compared to the over and underweight positions of the heuristic and optimised index alternatives discussed in our last paper these positions are generally much smaller for the fundamentally-weighted indices. The fundamental indices generally allocate greater weight to larger, more liquid stocks than the optimised alternative indices in particular. However, they do tend to be overweight small cap stocks and stocks with a high book-to-market value.
5. Fooled by randomness\textsuperscript{10}

The results in Section 3 indicate that all of the fundamental indices produced higher risk-adjusted returns than the Market-cap benchmark. But in our last paper we emphasised that many financial market phenomena, which may appear to have been generated by superior skill, or by intelligent design, can often be the result of chance. In that paper we outlined a process for distinguishing between luck and skill. Effectively we programmed the computer to simulate the stock picking abilities of ten million monkeys.

More precisely, at the end of each year the computer chose, at random, a stock from the 1,000 available. This stock was then assigned a weight of 0.1\%, and was then placed back in to the pool of 1,000. The computer then choose, randomly the next stock and this stock was also assigned a weight of 0.1\% and placed back into the pool and so on. This process was repeated 1,000 times until the weights of all of the chosen stocks summed to 100\%. If the computer had chosen the same stock twice its weight in the index that year would have been 0.2\%, if it had chosen the same stock three times, its weight in the index that year would have been 0.3\%, and so on. If the computer did not choose the stock once in the 1,000 draws, its weight in the index would have been zero. At the other extreme, if the computer had chosen exactly the same stock 1,000 times (though the odds against this are almost incomprehensible), then that year, this stock would have comprised 100\% of the index. This process was repeated ten million times for every year in our sample. In other words the computer generated ten million indices where the weights might as well have been chosen by monkeys. The results of this process are shown in Figures 2 and 3.

Figure 2 presents the value at the end of 2011 of each $100 invested by the ten million monkeys in 1968. The black line shows the frequency of the terminal values (left hand vertical axis) while the grey lines represent the cumulative frequency of these values (right hand vertical axis). The vertical lines on the Figure indicate the terminal value that would have been achieved by the various fundamental indices considered above along with the terminal value that would have been produced by the Market-cap index. The vast majority of the monkeys produced a better result than was achieved by the Market-cap index; we explored this result in our last paper. With regard to the fundamentally-weighted indices: the Book Value index outperformed just under 10\% of the monkeys; the Dividend index outperformed just under 30\% of the monkeys; the Cashflow index outperformed almost exactly half of the monkeys; while the Composite index outperformed 60\% of the monkeys. The best index was the Sales Index. In terms of terminal value, this index outperformed 99\% of the randomly constructed indices.

\textsuperscript{10} Fooled by Randomness: The Hidden Role of Chance in Life and in the Markets, Nassim Nicholas Taleb, Penguin.
Figure 3 shows the results of the same experiment, but where we have chosen to show the frequency of the Sharpe ratios produced by each monkey. All of the fundamentally-weighted indices outperform more than half of the monkey fund managers. The Book Value index outperforms around 75% of the randomly chosen indices, while the other fundamental indices outperform nearly 100% of the monkeys in terms of Sharpe ratio. These results suggest strongly that the fundamental indices did not achieve their superior risk-adjusted returns by luck.\footnote{We experimented with a number of other variations of the original random experiment, including picking from sets of actual market cap weights and reassigning these weights randomly each year, etc. But ultimately the results were largely unaffected.}

The results also suggest that over the sample period considered the fundamental indices have produced a vastly superior risk-adjusted return than was produced by the Market-cap index.
6. Can timing indicators improve performance?
In their paper of last summer Sengupta et al proposed the possible use of ‘timing indicators’ as a way of improving the risk return outcome for long only equity investors. We explored one such technique in our last paper in this series that has found support in the academic literature. These papers all find evidence to suggest that applying trend following rules at a relatively low, monthly frequency can improve the risk-return properties of broad-based indices, including market capitalisation-weighted indices such as the S&P 500 Composite index.

We applied a simple trend following rule suggested by these authors to all of the indices explored in Table 1. The rule that we applied was very simple:

- at the end of the month if the index value was greater than its ten month moving average we ‘invested’ 100% in the equity index and earned the return on that index in the following month;

---


13 There is of course an infinite number of rules that we could have used here. However, the main purpose of this section of the paper was not to experiment with different rules. Instead its purpose was to address the proposal in Sengupta et al (2012) which was that market timing rules might be able to add value to the performance of an equity index over time. The evidence presented in Table 4 suggests that they might. Furthermore, in their work Clare et al (2012a) present results that are largely invariant to a wide range moving average, trend following rules. They found that their results were robust to the use of 6, 8, 10, 12 and 14 month moving average rules, as were the original results reported by Faber (2007).
but if at the end of the month the index value was lower than its ten month moving average, we ‘invested’ 100% in US T-bills and earned the T-bill return in the following month.

The results are shown in Table 4.

The results presented in Table 4 are broadly consistent with those published in academic journals: compared to a passive alternative, annualised returns are either enhanced or unaffected by a low frequency trend following filter while maximum drawdowns are significantly reduced. The table shows that annualised returns are marginally enhanced for all but the Sales-weighted index. The annualised standard deviations of index returns shown in column 3 of Table 4 are around three quarters of the levels of their equivalent values shown in Table 1. For example, the annualised standard deviation of the Dividend-weighted index return is 14.5% without the trend following filter, but this figure falls to 10.8% with the application of the low frequency filter. The improvements (that is, reductions) in index volatility shown in Table 4 compared to Table 1 are of a similar scale to those that we documented in our previous paper when we applied the same filter to the heuristic and optimised indices. The Sharpe ratios also improve, but only marginally, from an increase of 0.06 for the Sales-weighted index and 0.13 for the Dividend-weighted index. The alphas for the fundamental indices with the trend following filter are all enhanced by around 0.30% per month and the betas all fall from close to one to between 0.51 to 0.57.

Table 4: Adding a “timing indicator”

<table>
<thead>
<tr>
<th>Panel A: Full sample results (1969 to 2011)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Returns</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>Market cap-weighted</td>
</tr>
<tr>
<td>Dividend-weighted</td>
</tr>
<tr>
<td>Cashflow-weighted</td>
</tr>
<tr>
<td>Book Value-weighted</td>
</tr>
<tr>
<td>Sales-weighted</td>
</tr>
<tr>
<td>Fundamentals composite-weighted</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Annualised Returns and volatility by decade</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970s</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>Market cap-weighted</td>
</tr>
<tr>
<td>Dividend-weighted</td>
</tr>
<tr>
<td>Cashflow-weighted</td>
</tr>
<tr>
<td>Book Value-weighted</td>
</tr>
<tr>
<td>Sales-weighted</td>
</tr>
<tr>
<td>Fundamentals composite-weighted</td>
</tr>
</tbody>
</table>

Notes: The returns presented in column 2 of the table are compound annual returns (APR). The alphas presented in column 7 are monthly.

Finally, the trend following filter appears to have a very significant impact on the maximum drawdown of the indices. The filter reduces maximum drawdowns dramatically for all of the fundamentally-weighted indices. For example, the maximum drawdown falls by 28 percentage points for the Sales-weighted index and by 33 percentage points for the Dividend-weighted index with the application of this filter. A significant amount for any risk averse investor.

Panel B of Table 4 shows the annualised returns and volatilities of each index where we have applied the trend following market timing rule. One obvious concern about applying a trend following rule like this might be “what happens when the market isn’t trending?” The Noughties were a difficult time for global equities. They peaked at around the start of this
period and were still well below this peak by the end of the sample period. If anything they trended down over this period. But when we compare the annualised returns for the Noughties in Panel B of Table 1, with those in Panel B of Table 4 we can see that in all cases annualised returns are higher in the Noughties with the trend following filter than without it. In particular the Market-cap index has an annualised return of 0.4% before the application of the trend following filter, but an annualised return of 7.5% with the addition of this filter.

Once again then the trend following versions of these alternative indices do enhance their risk-adjusted performances, particular with regard to volatility, tail risk and with regard to the challenging environment of the Noughties.

7. Trading volumes and transactions costs

7.1 Trading volumes

The last figure in appendices 2 to 6 shows the relative exposures of the indices to deciles of stocks ordered by trading volume. These under and overweights for the Dividend and Cashflow indices are all small. However, the Book Value index has a fairly large underweight of 7% to the top decile of stocks by traded volume while the equivalent figure for the Sales-weighted index is 8%. The composite index therefore has a lower underweight to this decile of 4.3%. Generally speaking then, the proxies for company size used to construct these fundamental indices do not come at the expense of focusing on less liquid stocks.

This result is reinforced by Figure 4. Each bar in the Figure represents the weighted-average traded volume of the index constituents over 2011. This chart shows that over this year, the Dividend-weighted index comprised those stocks with the highest average traded volume.
volume, and that the average traded volume of the Cashflow, Book Value and Composite indices were almost identical to that of the Market-cap index.

7.2 Transaction costs

The average trading volumes shown in Figure 4 and in Figure H in each of the appendices give an idea of the liquidity of the underlying stocks of each index. Liquidity is an important issue of course. However, a second, related issue is transactions costs. For any investor wishing to hold a US equity portfolio with one of these indices as the benchmark, one of the key questions is whether the trading costs of mimicking the index weights could outweigh any potential risk-return benefits that might exist. We have tried to give an indication of the scale of the likely costs of mimicking these indices by calculating an average annual turnover statistic for each index. We calculate the annual one-way turnover statistic for all of the indices, including the Market-cap index. In Table 5 we have provided a simple example of how the statistic has been calculated. In this example, the annual turnover of the portfolio that consisting of these five stocks between year 1 and year 2 is 50%.

<table>
<thead>
<tr>
<th>Stock</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Turnover</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>50%</td>
<td>0%</td>
<td>50%</td>
</tr>
<tr>
<td>B</td>
<td>25%</td>
<td>30%</td>
<td>5%</td>
</tr>
<tr>
<td>C</td>
<td>15%</td>
<td>20%</td>
<td>5%</td>
</tr>
<tr>
<td>D</td>
<td>10%</td>
<td>10%</td>
<td>0%</td>
</tr>
<tr>
<td>E</td>
<td>40%</td>
<td>40%</td>
<td>50%</td>
</tr>
</tbody>
</table>

In our last paper we identified the very low turnover of the Market-cap index as one of its major advantages over the heuristic and optimised alternative indices.

In Figure 5 we have presented the average, annual one-way turnover statistics for each index. The Figure shows that the index with the lowest turnover is the Market-cap based index, which turns over approximately 5% on average every year. The Fundamentally-weighted indices have annual turnover of between 11 and 13% per year, higher than that of the Market-cap index, but lower than most of the heuristic indices that we considered in our last paper, and significantly lower than most of the optimised indices considered in the same paper.
Of course turnover is only half of the story – if the costs of buying and selling stocks were zero, turnover would matter little. Could trading costs have reduced the performance advantages of the fundamentally-weighted indices? The answer is yes, but it seems unlikely to us that it would be enough to eliminate the return outperformance. It is impossible to know the trading costs that would have been involved in mimicking any of these index strategies in the past, particular in the 1970s and 1980s. However, if an alternative index outperformed a completely costless Market-cap index by 2.0% per year with additional turnover of 10% per year, then the average bid-ask spread on the stocks would need to be 20.0% to eliminate the performance difference.

To put this into perspective, the Sales-weighted index outperformed the Market-cap index by two percentage point per annum, and had additional turnover of seven percentage points, meaning that spreads would have had to average 29% to eliminate the performance differential – an implausibly large number.

8. Summary
In this paper we have explored the properties of fundamentally-weighted indices, and compared their performance with that of one constructed on the basis of market capitalisation. Overall, the results show that between 1968 and 2011 the fundamental index alternatives that we consider have, in risk-adjusted terms, out-performed a comparable index constructed on the basis of market capitalisation constituent weights. We find that this superior risk-adjusted performance cannot be attributed easily to luck. We also find that although the superior performance is achieved with higher constituent turnover than required using the market cap approach to index construction, the turnover is lower, and in some cases much lower, than required by some of the heuristic and optimised index construction techniques that we explored in our last paper. This in turn means that there is probably a negligible difference between the transactions costs involved in constructing a market-cap based equity portfolio and those incurred by constructing any one of the Fundamentally-weighted indices considered here. Finally, we find that although the application of a simple
market-timing rule does not enhance the returns on these fundamentally-weighted indices very significantly, it does reduce the volatility of index returns and the maximum drawdown of the indices quite considerably.
Appendix 1: Market cap weights

**Notes:** In Figure 1B: decile 1 contains the 10% of stocks with the lowest beta, decile 10 contains the 10% of stocks with highest beta. In Figure 1C: decile 1 contains the smallest 10% of stocks, decile 10 the largest 10% of stocks. In Figure 1D: decile 1 contains the 10% of stocks with the lowest book-to-market-value, decile 10 contains the 10% of stocks with highest book-to-market-value. In Figure 1E: decile 1 contains the 10% of stocks with the lowest return momentum, decile 10 contains the 10% of stocks with greatest return momentum. In Figure 1F: decile 1 contains the 10% of stocks with the lowest volatility, decile 10 contains the 10% of stocks with greatest volatility. In Figure 1H: decile 1 contains the 10% of stocks with the lowest traded volume, decile 10 contains the 10% of stocks with greatest traded volume.
Appendix 2: Dividend weight – relative to market cap

Notes: In Figure 2B: decile 1 contains the 10% of stocks with the lowest beta, decile 10 contains the 10% of stocks with highest beta. In Figure 2C: decile 1 contains the smallest 10% of stocks, decile 10 the largest 10% of stocks. In Figure 2D: decile 1 contains the 10% of stocks with the lowest book-to-market-value, decile 10 contains the 10% of stocks with highest book-to-market-value. In Figure 2E: decile 1 contains the 10% of stocks with the lowest return momentum, decile 10 contains the 10% of stocks with greatest return momentum. In Figure 2F: decile 1 contains the 10% of stocks with the lowest volatility, decile 10 contains the 10% of stocks with greatest volatility. In Figure 2H: decile 1 contains the 10% of stocks with the lowest traded volume, decile 10 contains the 10% of stocks with greatest traded volume.
Appendix 3: Cashflow weight – relative to market cap

Notes: In Figure 3B: decile 1 contains the 10% of stocks with the lowest beta, decile 10 contains the 10% of stocks with highest beta. In Figure 3C: decile 1 contains the smallest 10% of stocks, decile 10 the largest 10% of stocks. In Figure 3D: decile 1 contains the 10% of stocks with the lowest book-to-market-value, decile 10 contains the 10% of stocks with highest book-to-market-value. In Figure 3E: decile 1 contains the 10% of stocks with the lowest return momentum, decile 10 contains the 10% of stocks with greatest return momentum. In Figure 3F: decile 1 contains the 10% of stocks with the lowest volatility, decile 10 contains the 10% of stocks with greatest volatility. In Figure 3H: decile 1 contains the 10% of stocks with the lowest traded volume, decile 10 contains the 10% of stocks with greatest traded volume.
Appendix 4: Book Value weight – relative to market cap

**Notes:** In Figure 4B: decile 1 contains the 10% of stocks with the lowest beta, decile 10 contains the 10% of stocks with highest beta. In Figure 4C: decile 1 contains the smallest 10% of stocks, decile 10 the largest 10% of stocks. In Figure 4D: decile 1 contains the 10% of stocks with the lowest book-to-market-value, decile 10 contains the 10% of stocks with highest book-to-market-value. In Figure 4E: decile 1 contains the 10% of stocks with the lowest return momentum, decile 10 contains the 10% of stocks with greatest return momentum. In Figure 4F: decile 1 contains the 10% of stocks with the lowest volatility, decile 10 contains the 10% of stocks with greatest volatility. In Figure 4H: decile 1 contains the 10% of stocks with the lowest traded volume, decile 10 contains the 10% of stocks with greatest traded volume.
Appendix 5: Sales weight – relative to market cap

Notes: In Figure 5B: decile 1 contains the 10% of stocks with the lowest beta, decile 10 contains the 10% of stocks with highest beta. In Figure 5C: decile 1 contains the smallest 10% of stocks, decile 10 the largest 10% of stocks. In Figure 5D: decile 1 contains the 10% of stocks with the lowest book-to-market-value, decile 10 contains the 10% of stocks with highest book-to-market-value. In Figure 5E: decile 1 contains the 10% of stocks with the lowest return momentum, decile 10 contains the 10% of stocks with greatest return momentum. In Figure 5F: decile 1 contains the 10% of stocks with the lowest volatility, decile 10 contains the 10% of stocks with greatest volatility. In Figure 5H: decile 1 contains the 10% of stocks with the lowest traded volume, decile 10 contains the 10% of stocks with greatest traded volume.
Appendix 6: Fundamental Composite weight – relative to market cap

Figure 6A: 3-Year Relative Rolling Performance

Figure 6B: Mean Weight by Beta Decile

Figure 6C: Mean weight by size decile

Figure 6D: Mean weight by BTMV Decile

Figure 6E: Mean Weight by Momentum Decile

Figure 6F: Mean Weight by Volatility Decile

Figure 6G: Up/Down capture

Figure 6H: Mean Weight by Volume Decile

**Notes:** In Figure 6B: decile 1 contains the 10% of stocks with the lowest beta, decile 10 contains the 10% of stocks with highest beta. In Figure 6C: decile 1 contains the smallest 10% of stocks, decile 10 the largest 10% of stocks. In Figure 6D: decile 1 contains the 10% of stocks with the lowest book-to-market-value, decile 10 contains the 10% of stocks with highest book-to-market-value. In Figure 6E: decile 1 contains the 10% of stocks with the lowest return momentum, decile 10 contains the 10% of stocks with greatest return momentum. In Figure 6F: decile 1 contains the 10% of stocks with the lowest volatility, decile 10 contains the 10% of stocks with greatest volatility. In Figure 6H: decile 1 contains the 10% of stocks with the lowest traded volume, decile 10 contains the 10% of stocks with greatest traded volume.