Journal Article
Journal 38

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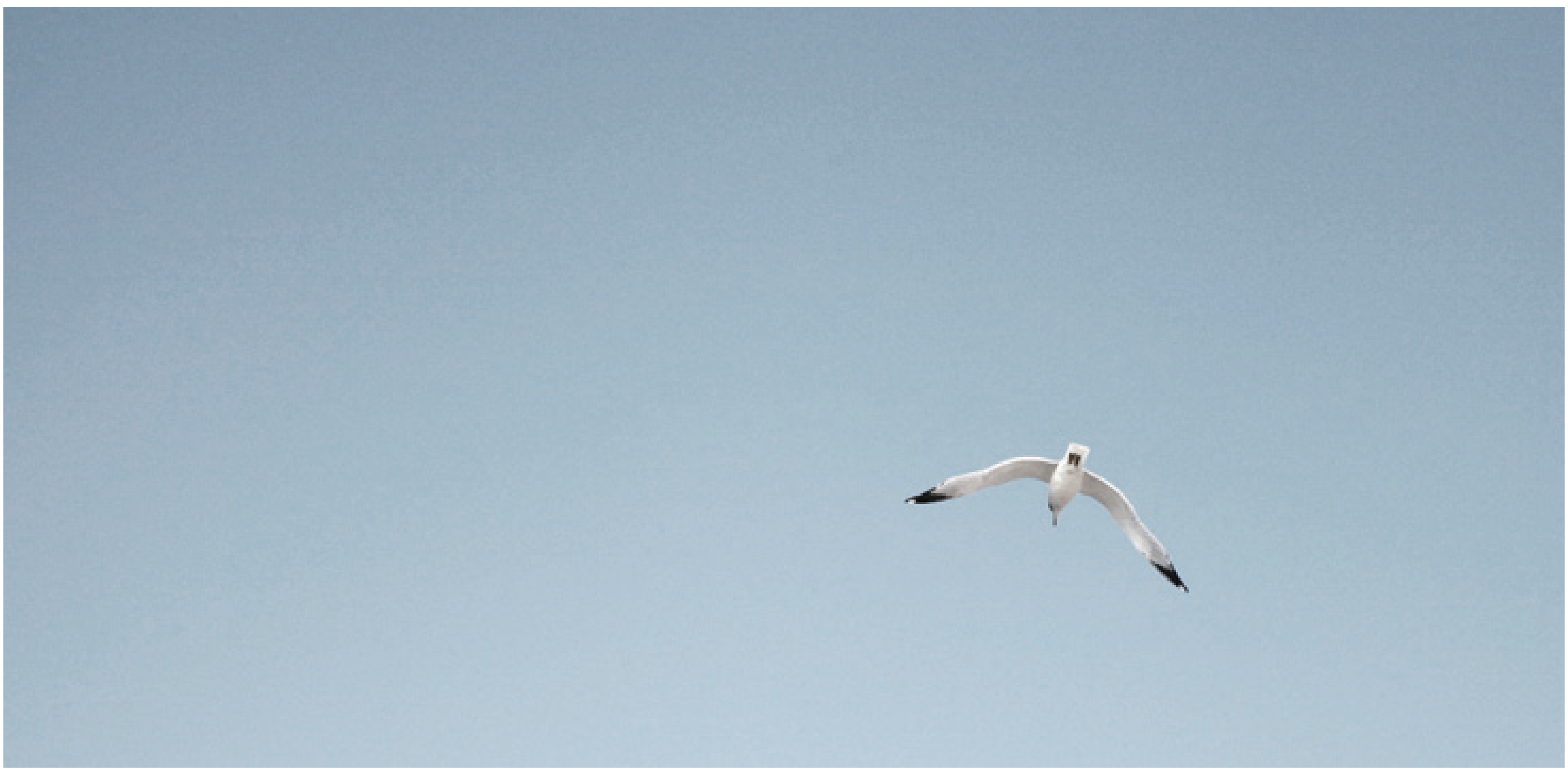
Journal
The Capco Institute Journal of Financial Transformation



Recipient of the Apex Awards for Publication Excellence 2002-2013
Zicklin-Capco Institute Paper Series in Applied Finance
#38 10.2013

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# Sovereign Credit Risk in a Hidden Markov Regime-Switching Framework. Part 2

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## Abstract

This research applies a discrete-time Markov-modulated model to default probability estimation and adapts it to Merton's contingent claims approach, backing the hypothesis that a regime-switching framework which allows for structural shifts can substantially improve the underestimation of default probabilities associated with the Merton structural model. The modeling apparatus is applied to sovereign risk proving that the methodology can be tractably extended to a contingent claims approach, and is investigated as a follow-up paper to an extensive methodology found in the previous edition of the Capco Journal of Financial Transformation (37) [Potgieter and Fusai (2013)]. CDS quotes are used to calibrate the regime switching model and are then used to estimate sovereign assets in both developed and emerging markets.

## Introduction

The recent credit crisis has raised the awareness of investors and regulators concerning the appropriate methods for valuation, trading, and risk management of sovereign debt instruments. Many existing models assume homogenous market conditions and incorporating changes in market regimes or the economic environment due to a credit event appear difficult. This motivates the application of an appropriate valuation model of sovereign debt in a regime-switching framework. We consider an extension of the Merton's contingent claims approach to the macro level, as discussed in Gray et al. (2007) and we allow for structural changes and regime-switching according to Liew and Sui (2010).

The model allows for the computation of the term structure of default probabilities (PDs) and it is calibrated to observed quotes of sovereign Credit Default Swaps (CDSs). This allows us to predict a set of observed economic balance sheet information including a sovereign's asset value (A) and local-currency liability in foreign currency term (LCL). We use the proposed model on both developed and emerging markets. The economic intuition behind the regime-switching Markov-modulated model is to incorporate the impact of suddenly changing macroeconomic conditions on the sovereign balance sheet information. This is done by assuming that the volatility of a sovereign's asset value has switching dynamics and follows a finite-state discrete Markov-chain where the states of the chain could represent the states of the economy. The final aim of the research is to establish a link between the credit market and a sovereign's balance sheet and to understand if the credit market conveys useful information to predict economic balance sheet information.

Potgieter and Fusai (2013) provides the key ideas of the model, describing the process of calculating the probability of default on a N-state hidden Markov Model, and derives the value for a sovereign's assets and a local-currency liability in foreign currency terms. The calibrated model is used to calculate the price of a standard European call option according to the contingent claim approach (CCA). The observed local-currency liability (LCL) can be compared with the valuation implied in market quotations through the switching regime model. This allows us to understand how much CDSs, conditional to the use of the proposed model, can tell us about the market estimate of the value of sovereign assets. In practice, there exist significant differences between the two. We perform a detailed empirical analysis on a set of countries, representative of advanced and emerging economies. The main conclusion it that information on LCL can be extracted from CDS market quotes across a variety of countries.

## Extracting sovereign riskiness from a Markov-modulated Merton model

Gray et al. (2007) extends the Merton's Contingent Claim Approach to the macro level to include a sovereign balance sheet analysis. The assets of a sovereign for the purpose of this approach comprise foreign reserves, net fiscal assets, and other assets minus entities too important to fail. Liabilities are defined as foreign-currency denominated debt plus a local-currency liability comprised of local-currency debt and base money. Sovereign default arises when sovereign assets cannot cover the promised payment on foreign currency debt. The default barrier, in our framework the strike of the above mentioned call option, is therefore defined as the present value of these payments. While the liabilities are known with a fair degree of certainty over any given time horizon, the sovereign assets are more uncertain as assets may change for a large number of reasons. Three factors therefore drive default: the sovereign asset value, the volatility of the assets, and the default barrier. The default barrier is defined as "senior" foreign-currency denominated debt [Crouhy et al. (2000)].

The foreign-currency debt is modeled as default-free value of debt minus an implicit put option. Local-currency liabilities are modeled as an implicit call option since such liability demonstrates "equity-like features" on a sovereign balance sheet. The local-currency multiplied by the exchange rate is considered a market cap of the sovereign in the international market.

The CCA approach relies on the relationship between balance sheet entries to extract an implied estimate of sovereign assets by a calibration procedure where the value of the local-currency liability in foreign currency terms (LCL) is a call option of the sovereign's assets (A) with the strike price as the default barrier ($B_f$) defined as foreign-currency denominated debt. Instead of using balance sheet claims to predict default probabilities according to a CCA model, we perform a reverse engineering procedure by filtering observed market data through the regime-switching model and we try to infer balance sheet information. This allows us to appreciate the implied value of a sovereign's assets relative to existing debt that is observable and could signal a looming credit event. A detailed description of the underlying mathematical framework can be found in Potgieter and Fusai (2013).

As detailed by Potgieter and Fusai (2013), once we have extracted the default probabilities from quotes on CDS spreads for different maturities, in our case 1, 2, 3, 4, 5, 7 and 10 years, the calibration consists of inferring the model parameters, iteratively adjusted to best fit the market observed default probabilities using non-linear least-squared-error minimization. Then we can use the calibrated model to infer the value of the sovereign assets that are otherwise unobservable. The basic structure of the model and formula can be found in Appendix 1.

## Application to the sovereign case

This section reports the results of the empirical analysis performed in both developed and emerging markets for various maturities. Countries include South Africa, Brazil, US, Italy, and Germany. The analysis covers a window period which differs depending on the availability of market data.

## The data

Credit Default Swaps (CDSs) are generally issued for full range of sovereign bond issues, more typically for maturities ranging from one to 10 years. Historically, the most actively traded contracts amongst various sovereign issuers are for maturities T=[1 2 3 5 7 10] years to maturity. The CDS historical market quote series vary from country to country, depending on the earliest issue in each respective market. Emerging markets generally have a longer history. Though the frequency of CDS data is obtainable on a daily basis, given that the sovereign balance sheet data is available only on a quarterly basis, the calibration considers an averaged CDS market quote on the previous quarter to have the same frequency as the balance sheet data.

For the purpose of this approach, sovereign default arises when sovereign assets cannot sufficiently cover the promised payment on foreign-currency denominated debt. The default barrier is therefore defined as the present value of these payments. The default barrier may be defined as a KMV-like measure (short-term debt plus one-half long-term debt plus interest payments up to a certain time) or "senior" foreign-currency denominated debt [Crouhy et al. (2000)]. This research adopts the latter definition. Seniority of debt is inferred from examining the behavior of policymakers during stress periods. These efforts make such debt more senior to "junior claims" on sovereign local-currency denominated debt. Local currency liabilities comprising local-currency debt and base money are modeled as an implicit call option on a sovereign's assets with the strike price as the default barrier since such liabilities demonstrate "equity-like features" on a sovereign balance sheet. The estimation of the sovereign unknown and unobserved asset value can be extracted from both the calibrated inverse leverage parameter and the observed distress barrier. We compare the estimation with an observed local-currency liability in foreign currency terms. Expression (5) given in Appendix 1 is the relevant one to obtain LCL observed. The inputs require balance sheet data such as the base money, domestic interest rates, foreign interest rates, domestic currency denominated debt as the default barrier, and forward exchange rates. We calculate the observed local-currency liability using a five year term.

Interest rate data include the Eonia (Euro Overnight Index Average) and US OIS (US Over-Night Index Swap). The Eonia is an effective overnight interest rate computed as a weighted average of all overnight unsecured lending transactions in the interbank market. It is one of the two benchmarks (the other one being Euribor) used in the money and capital markets in the Eurozone. The US OIS is an interest rate swap that allows financial institutions to swap the interest rates they are paying without having to change the terms of contracts in place with other financial institutions. The fixed rate of OIS is considered less risky than the corresponding interbank rate (LIBOR), as only the net difference in interest rates is paid at maturity of the swap. In the United States, OIS rates are calculated by reference to a daily federal funds rate. For Brazil and South Africa, interest rates have been obtained using the zero curve as computed by DataStream.

The data source for all CDS quotes and balance sheet data includes both Bloomberg and DataStream.

## A case of South Africa

The data for South Africa includes CDS market quotes averaged on a given quarter for a total of 10 observations from March 2010 to June 2012. The estimated parameters calibrated across the entire term structure on the last day of each quarter of the estimation period are presented in Table 1. The first column is the date when the calibration is performed. The second and third column present the volatility in state 1 and 2 respectively, and provide an indication of how the states are defined e.g., "good" or "bad." Columns four and five give indication of the probability of the Markov chain remaining in state 1 and probability of transitioning to state 1 from state 2 respectively. Column six presents the inverse of the leverage parameter i.e., the ratio of the sovereign asset value and the observed distress barrier. An obligor defaults when the value of its assets falls below the value of its liabilities, or equivalent when its inverse leverage ratio (the ratio of liabilities to assets) falls below one.

From the estimated transition probabilities, the probability that the country continues to stay in state 1 is high while the probability of a transition to state 2 is low. For example, consider the results on June 29, 2012, $a_{j,i}$ shows transition probability of moving from state i to state j.

| | $\sigma_1$ | $\sigma_2$ | $a_{11}$ | $a_{12}$ | S/K |
|---|---|---|---|---|---|
| 31/03/2010 | 85.01% | 2.21% | 0.9869 | 0.9891 | 1.89 |
| 30/06/2010 | 62.97% | 3.52% | 0.9790 | 0.9837 | 1.75 |
| 30/09/2010 | 71.01% | 4.15% | 0.9853 | 0.9871 | 1.92 |
| 31/12/2010 | 70.03% | 10.97% | 0.9863 | 0.9874 | 2.12 |
| 31/03/2011 | 76.19% | 3.14% | 0.9885 | 0.9883 | 1.91 |
| 30/06/2011 | 93.53% | 1.46% | 0.9704 | 0.9864 | 1.98 |
| 30/09/2011 | 86.37% | 1.20% | 0.9847 | 0.9842 | 2.04 |
| 30/12/2011 | 79.21% | 2.55% | 0.9774 | 0.9823 | 2.09 |
| 30/03/2012 | 84.58% | 2.48% | 0.9839 | 0.9863 | 1.91 |
| 29/06/2012 | 99.90% | 0.27% | 0.9990 | 0.9865 | 1.89 |

**Table 1: Estimated parameters in two-state hidden Markov regime-switching model: South Africa**

| | To state 1 | To state 2 |
|---|---|---|
| From state 1 | 0.9990 | 0.0010 |
| From state 2 | 0.9865 | 0.0135 |

**Table 2: Transition probabilities on June 29, 2012: South Africa**

| Calibration | T = 1 | T = 2 | T = 3 | T = 4 | T = 5 | T = 7 | T = 10 |
|---|---|---|---|---|---|---|---|
| 31/03/2010 | 1.90% | 2.37% | 2.19% | 2.89% | 3.68% | 4.53% | 5.43% |
| 30/06/2010 | 2.59% | 3.07% | 2.24% | 2.98% | 3.84% | 4.79% | 5.82% |
| 30/09/2010 | 2.10% | 2.51% | 2.06% | 2.73% | 3.50% | 4.34% | 5.25% |
| 31/12/2010 | 2.06% | 2.39% | 1.88% | 2.53% | 3.36% | 4.37% | 5.54% |
| 31/03/2011 | 1.40% | 1.84% | 1.95% | 2.87% | 3.83% | 4.83% | 5.85% |
| 30/06/2011 | 1.48% | 2.09% | 1.49% | 2.68% | 3.90% | 5.15% | 6.43% |
| 30/09/2011 | 2.70% | 3.29% | 3.07% | 3.97% | 4.98% | 6.09% | 7.27% |
| 30/12/2011 | 2.73% | 3.27% | 2.55% | 3.52% | 4.61% | 5.80% | 7.06% |
| 30/03/2012 | 1.82% | 2.38% | 2.35% | 3.37% | 4.46% | 5.60% | 6.77% |
| 29/06/2012 | 1.75% | 2.45% | 3.50% | 4.64% | 5.83% | 7.05% | 8.30% |

**Table 3 – Calibrated term structure of default probabilities using regime-switching model: South Africa**

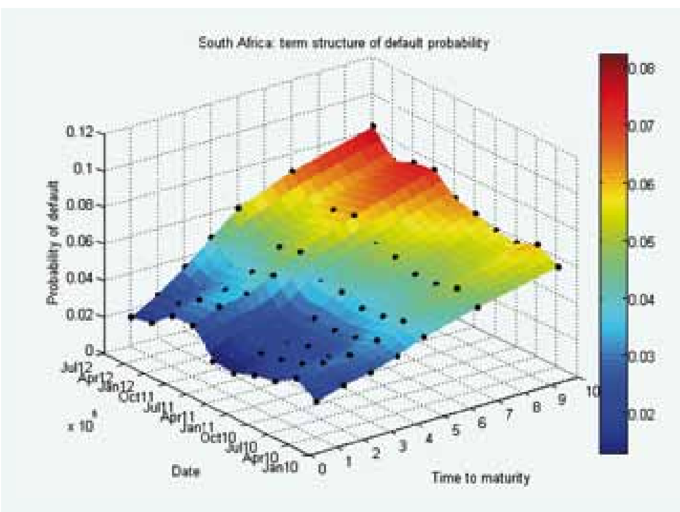


**Figure 1 – Calibrated term structure of default probability: South Africa**

| | Distress Barrier | Implied Asset Value | LCL Observed | LCL Estimate (T=5) | Average LCL Estimate |
|---|---|---|---|---|---|
| 31/03/2010 | 316,610 | 599,827 | 64,526 | 66,600 | 65,144 |
| 30/06/2010 | 334,956 | 584,993 | 62,147 | 63,232 | 62,314 |
| 30/09/2010 | 311,111 | 597,524 | 75,756 | 77,231 | 76,144 |
| 31/12/2010 | 299,561 | 634,429 | 81,653 | 82,607 | 81,807 |
| 31/03/2011 | 311,863 | 596,100 | 80,162 | 84,476 | 82,443 |
| 30/06/2011 | 318,272 | 631,165 | 83,871 | 87,120 | 85,209 |
| 30/09/2011 | 383,228 | 782,489 | 76,628 | 81,232 | 79,280 |
| 30/12/2011 | 390,675 | 814,713 | 82,327 | 85,226 | 83,672 |
| 30/03/2012 | 375,333 | 718,745 | 82,077 | 88,584 | 86,039 |
| 29/06/2012 | 409,865 | 773,502 | 81,336 | 95,011 | 90,597 |

**Table 4 – Implied sovereign asset value, distress barrier and local-currency liability (in millions USD): South Africa**



This shows that the Markov model captures the persistence in a state; in this case state 1. How is the state defined then? The two volatility estimates provide a clear indication that the level of volatility defines each state. Given that $\sigma_1 \gg \sigma_2$, state 1 and state 2 therefore could be classified as a "volatile" or "bad" economy versus a "stable" or "good" economy respectively. The large difference in the volatilities of the two states may be attributed to the fact that the historical data used in the estimation period includes a period of market downturn, several corrections and a strong rally between October 2011 and April 2012. Volatility as measured by the Johannesburg Stock Exchange's South African Volatility Index (SAVI) had increased considerably toward the end of the estimation period, albeit below the heights of the 2008 credit crisis.

The inverse leverage ratio generally fluctuates around values not too far from 2. We recall that a credit event occurs as soon as the inverse leverage ratio approaches a value of 1. The calibrated term structure of probability of default (PD) is shown in Table 3. The slope of the PD curve reflects simply a forward expectation of how the default risk is perceived.

Brigo and Tarenghi (2005) highlight that structural models often imply unrealistic short-term credit spreads. There is also empirical evidence that structural models underestimate the actual probability of default and the use of regime-switching model intends to improve the structural model [Leland (2004), Tarashev (2005), Erlwein et al. (2008)]. The calibrated results indicate that the model resolves the underestimation drawback and that the PD over short term maturities are all non-zero. Table 3 and Figure 1 illustrate how the default probability moves over time. We observe a wave-like behavior of the PD along the date axis, holding the maturity constant. The waves appear to peak around June 2010 and July 2011.

We try to understand the information conveyed by the term structure. In late April 2010, the Greek debt rating was decreased to "junk" status by Standard & Poor's amidst fears of default by the Greek Government. Thereafter, risks to global financial stability eased off as the economic recovery gained steam. In September 2011, the Global Financial Stability Report cautioned that the risks to global financial stability increased substantially in prior months. Heavy public debt burdens and weak growth prospects in many advanced economies combined with a series of shocks to the global financial system were the culprits. Despite higher growth prospects for emerging economies, markets faced the risk of sharp reversals. At this stage, the crisis moved into its fifth year, entering into a new phase in which political differences across economies were impeding progress to address the legacies of the crisis. An on-going low interest rate environment in developed markets and high uncertainty drove the asset allocations of institutional investors, with a clear shift to safety and liquidity. Net capital flows to emerging markets such as South Africa remained relatively strong during the first half of 2011 although very volatile. This reflected higher nominal interest rates, the perception that currencies would appreciate, and relatively strong fundamentals.

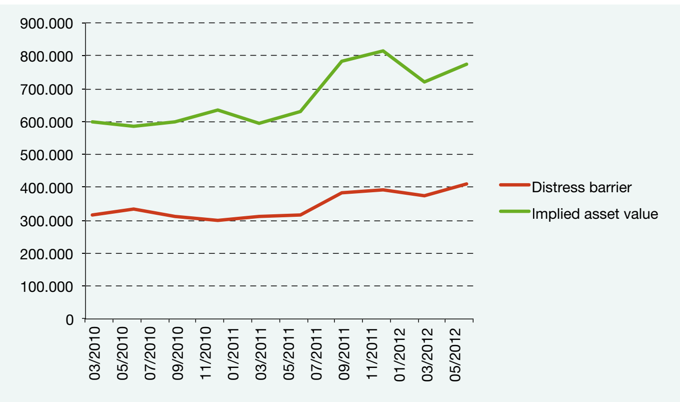


**Figure 2 – Implied sovereign asset value versus distress barrier (in millions USD) – South Africa**

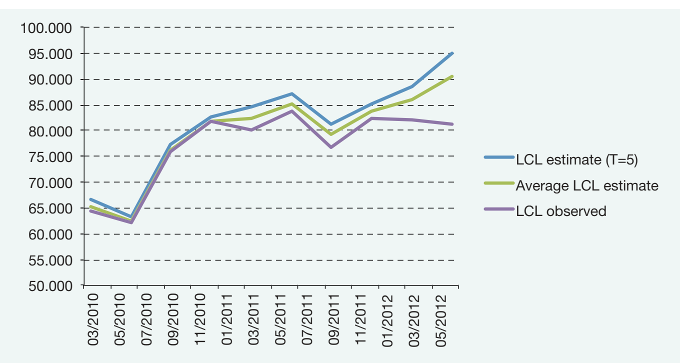


**Figure 3 – Local-currency liability observed versus estimate (in millions USD: South Africa**

Using the estimated parameters from the regime-switching model and the sovereign balance sheet data required for the contingent claim approach (CCA), an implied asset value is then computed at each estimation date. Furthermore, the local-currency liabilities (LCL) estimate is calculated using as an input the implied asset value. We compare the balance sheet estimate with the observed equivalent.

The LCL component is reported for both a T = 5 year maturity and the LCL component averaged across all maturities T= [1 2 3 5 7 10].

Figure 2 and Figure 3 provide a visualization of the results as a time series of each balance sheet component. The implied sovereign asset value lies well above the distress value for all quarters indicating that no default was imminent in the estimation period, a reasonable observation given the default probabilities are at most below 3% for a three-year horizon. This is also confirmed by the inverse leverage ratio that is well above 1 as we have seen from Table 1. The value for the LCL estimate for five-year maturity tracks the observed component closely, overestimating it within a reasonable range, signalling the reliability of the proposed reverse engineering procedure.

In order to better quantify this reliability, we next examine the explanatory power for the observed local-currency liability (LCL) over a T= 5 year maturity, using the estimated LCL obtained from the model. For this purpose, we consider the following linear regression where the goodness of the implied LCL estimate in predicting the observed LCL is tested. The relationship to be estimated between the observed and estimated LCL is as follows:

$$LCL_{Observed} = \alpha + \beta * LCL_{Estimate} + \varepsilon \qquad (1)$$

Here $\alpha$ represents the intercept of the linear regression, $\beta$ the slope and $\varepsilon$, the error term reflecting other factors that influence the observed local-currency liability. The estimated relationship turns out to be:

$$\widehat{LCL}_{Observed} = 17\,644.63 + 0.73 * LCL_{Estimate}$$

with additional outputs given in Table 5.

| | Coefficients | Standard Error | t Stat | P-value |
|---|---|---|---|---|
| $\alpha$ | 17644.63 | 8123.17 | 2.1721 | 0.061613 |
| $\beta$ | 0.7322 | 0.09948 | 7.3605 | 7.91E-05 |
| $R^2$= 0.87 | | | | |

**Table 5 – Regression analysis for observed local-currency liability (in million USD): South Africa**

A one-tailed t-statistic of 7.4 indicates significance in the estimated coefficients along with the low p-value << 0.05. Of some relevance is the high $R^2$ value, well above 0.87, which measures how much the total variation of the dependent variable $LCL_{Observed}$ is explained by the regression. This suggests that the $LCL_{Estimate}$ implied in the CDS quotes has high power in explaining the $LCL_{Observed}$. On the other hand, given the estimated value of $\beta$ lower than 1 due to the fact that $LCL_{Estimate} > LCL_{Observed}$, the prediction appears biased i.e., CDS market quotes overstate the local sovereign liabilities.

Table 6 extends the analysis using the LCL estimates implied by different points of the term structure of default probabilities. The short term maturities can provide an even greater predictive value of the observed value of LCL. In addition, the prediction extracted from the one year maturity also appears unbiased ($\hat{\alpha} = 0, \hat{\beta} = 1$). The result implies that the estimated LCL tend to capture the level of the observed balance sheet data very well.

| Maturity | T = 1 | T = 2 | T = 3 | T = 4 | T = 5 | T = 7 | T = 10 |
|---|---|---|---|---|---|---|---|
| $R^2$ | 0.99 | 0.99 | 0.92 | 0.90 | 0.87 | 0.84 | 0.80 |
| $\alpha$ | 0.00 | 275.05 | 11059.22 | 14451.81 | 17644.63 | 22212.72 | 29537.93 |
| $\beta$ | 0.99 | 0.98 | 0.86 | 0.79 | 0.73 | 0.68 | 0.61 |

**Table 6 – $R^2$ measure and regression analysis of local-currency liability estimate across term structure: South Africa**

## A case of Brazil

In the case of Brazil, the estimation period ranges from June 2005 to June 2012. The estimated parameters calibrated across the entire term structure are presented in Table 7.

The high values of $a_{11}$ indicate a clear persistence in state 1, which according to the volatility estimates, is classified as a low volatility state. The inverse leverage ratio varies between 1.5 and 2.4 throughout the estimation period, suggesting that in this period, Brazil was in quite a comfortable situation.

Figure 4 shows the term structure of estimated probability of defaults (PDs) obtained via the regime-switching model. Default probabilities have remained relatively stable for shorter maturities while the perceived risk of longer horizons has steadily decreased over the estimation period: the probability of default for the 10-year maturity steadily declined from above 15% to below 10%, fluctuating at a moderate level with the economic environment. Moody's and Fitch have viewed Brazil as showing growing economic resilience with cautious fiscal and monetary policy.

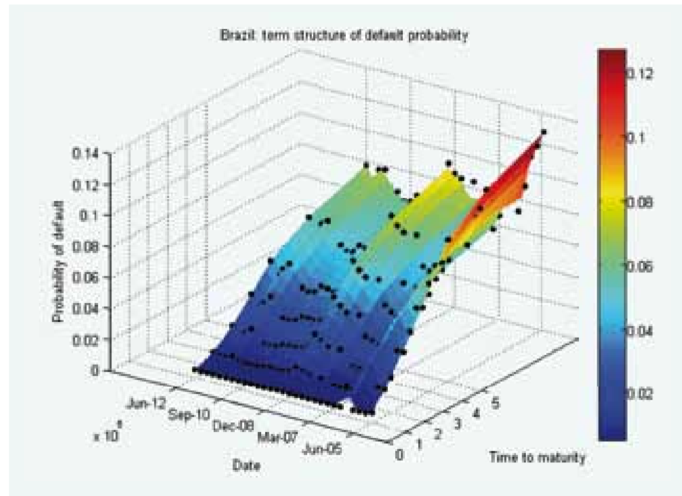


**Figure 4 – Calibrated term structure of default probability: Brazil**

| | $\sigma_1$ | $\sigma_2$ | $a_{11}$ | $a_{12}$ | S/K |
|---|---|---|---|---|---|
| **30/06/2005** | 24.21% | 18.65% | 0.9990 | 0.9990 | 1.8290 |
| **30/09/2005** | 0.81% | 7.95% | 0.9990 | 0.9674 | 1.5049 |
| **30/12/2005** | 22.65% | 19.52% | 0.9990 | 0.9990 | 2.0049 |
| **31/03/2006** | 0.81% | 8.76% | 0.9986 | 0.9859 | 1.5035 |
| **30/06/2006** | 0.80% | 11.30% | 0.9970 | 0.9990 | 1.4981 |
| **29/09/2006** | 0.80% | 9.34% | 0.9990 | 0.9899 | 1.5017 |
| **29/12/2006** | 0.80% | 9.60% | 0.9990 | 0.9990 | 1.4982 |
| **30/03/2007** | 0.81% | 8.38% | 0.9986 | 0.9926 | 1.5013 |
| **29/06/2007** | 0.81% | 9.18% | 0.9990 | 0.9990 | 1.4985 |
| **28/09/2007** | 0.81% | 7.27% | 0.9990 | 0.9883 | 1.5027 |
| **31/12/2007** | 0.80% | 8.98% | 0.9990 | 0.9919 | 1.5003 |
| **31/03/2008** | 0.80% | 11.50% | 0.9990 | 0.9981 | 1.5015 |
| **30/06/2008** | 0.80% | 7.18% | 0.9990 | 0.9831 | 1.5022 |
| **30/09/2008** | 0.68% | 6.59% | 0.9854 | 0.9668 | 1.4369 |
| **31/12/2008** | 0.75% | 21.05% | 0.9756 | 0.9700 | 1.2733 |
| **31/03/2009** | 0.80% | 22.04% | 0.9829 | 0.9829 | 1.2643 |
| **30/06/2009** | 0.80% | 11.49% | 0.9988 | 0.9989 | 1.4965 |
| **30/09/2009** | 0.82% | 8.03% | 0.9990 | 0.9835 | 1.5013 |
| **31/12/2009** | 0.82% | 7.38% | 0.9990 | 0.9827 | 1.5025 |
| **31/03/2010** | 0.86% | 7.79% | 0.9990 | 0.9828 | 1.5022 |
| **30/06/2010** | 3.38% | 8.90% | 0.9990 | 0.9803 | 1.6681 |
| **30/09/2010** | 1.25% | 10.03% | 0.9990 | 0.9846 | 1.7390 |
| **31/12/2010** | 3.88% | 11.59% | 0.9990 | 0.9879 | 1.7863 |
| **31/03/2011** | 9.24% | 14.71% | 0.9990 | 0.9912 | 2.0130 |
| **30/06/2011** | 8.12% | 19.37% | 0.9990 | 0.9990 | 2.2920 |
| **30/09/2011** | 0.81% | 19.85% | 0.9860 | 0.9859 | 1.2963 |
| **30/12/2011** | 0.80% | 11.54% | 0.9989 | 0.9985 | 1.4964 |
| **30/03/2012** | 0.81% | 8.16% | 0.9990 | 0.9851 | 1.5015 |
| **29/06/2012** | 0.85% | 8.96% | 0.9990 | 0.9838 | 1.5018 |

**Table 7 – Estimated parameters for Markov regime-switching model: Brazil**

Table 8 presents the sovereign balance sheet data required for the contingent claim approach (CCA) and the local-currency liabilities LCL component, observed and estimated for a T= 5 year maturity and the counterparts averaged across the entire term structure.

Figure 5 and Figure 6 provide a visual perspective of the quarterly time series of the sovereign balance sheet components. The growing economic conditions in Brazil are evident by the rise in the indicators. The local-currency liabilities estimate overstates the observed value for periods prior to March 2008, proceeding to understate the observed value thereafter. This implies that CDS quotations, differently from South Africa, at first overstate and then understate local sovereign liabilities. The distress barrier remains well below the implied asset value aside from March 2009, where a sharp drawdown is experienced. The financial fragility displayed here is intimately related to the probability of default which surged around this time period, when the US stock market reached its lowest point since the start of the 2008 recession.

Next, the quarterly predictive ability for the observed local-currency liability (LCL) is examined in Table 9, using the estimated local-currency liability obtained from the model. The regression analysis provides the following estimated relationship:

$$\widehat{LCL}_{Observed} = -8\ 9857.77 + 1.25 * LCL_{Estimate}$$

The case of Brazil differs from South Africa in that the implied estimate

| | Distress Barrier | Implied Asset Value | LCL Observed | LCL Estimate (T=5) | Average LCL Estimate |
|---|---|---|---|---|---|
| **30/06/2005** | 191,309 | 572,777 | 275,667 | 303,683 | 296,816 |
| **30/09/2005** | 183,151 | 648,459 | 300,826 | 339,794 | 333,364 |
| **30/12/2005** | 169,450 | 591,055 | 344,974 | 397,900 | 391,717 |
| **31/03/2006** | 166,652 | 625,321 | 349,438 | 413,395 | 407,774 |
| **30/06/2006** | 156,661 | 564,884 | 369,277 | 443,110 | 437,896 |
| **29/09/2006** | 159,560 | 598,639 | 362,863 | 440,745 | 434,799 |
| **29/12/2006** | 172,589 | 681,523 | 388,391 | 473,197 | 467,351 |
| **30/03/2007** | 182,082 | 675,851 | 402,677 | 496,021 | 490,100 |
| **29/06/2007** | 191,358 | 882,475 | 432,881 | 531,538 | 525,733 |
| **28/09/2007** | 195,331 | 749,241 | 501,279 | 594,251 | 588,641 |
| **31/12/2007** | 193,219 | 757,899 | 591,676 | 601,720 | 596,871 |
| **31/03/2008** | 201,637 | 740,365 | 613,378 | 569,518 | 564,815 |
| **30/06/2008** | 205,536 | 754,038 | 696,894 | 573,100 | 569,530 |
| **30/09/2008** | 211,381 | 732,434 | 725,278 | 549,452 | 544,967 |
| **31/12/2008** | 198,340 | 597,776 | 518,923 | 413,486 | 410,399 |
| **31/03/2009** | 192,676 | 420,082 | 484,679 | 242,094 | 238,810 |
| **30/06/2009** | 198,996 | 712,512 | 553,688 | 524,346 | 521,991 |
| **30/09/2009** | 204,934 | 786,536 | 699,871 | 591,623 | 589,518 |
| **31/12/2009** | 198,192 | 769,321 | 765,835 | 579,311 | 577,473 |
| **31/03/2010** | 211,532 | 794,666 | 728,538 | 592,566 | 590,502 |
| **30/06/2010** | 228,649 | 888,302 | 781,736 | 670,571 | 668,290 |
| **30/09/2010** | 247,812 | 933,830 | 813,936 | 696,602 | 694,517 |
| **31/12/2010** | 256,804 | 924,062 | 876,930 | 677,673 | 675,660 |
| **31/03/2011** | 275,947 | 1,086,314 | 922,448 | 822,211 | 819,955 |
| **30/06/2011** | 291,648 | 1,143,223 | 1,018,431 | 862,636 | 860,418 |
| **30/09/2011** | 298,219 | 1,055,958 | 966,011 | 773,238 | 769,524 |
| **30/12/2011** | 298,204 | 1,193,837 | 874,191 | 833,793 | 830,632 |
| **30/03/2012** | 289,606 | 1,060,937 | 900,813 | 784,285 | 781,434 |
| **29/06/2012** | 287,529 | 1,017,470 | 832,855 | 745,029 | 741,555 |

**Table 8 – Implied sovereign asset value, distress barrier and local-currency liability (in millions USD): Brazil**

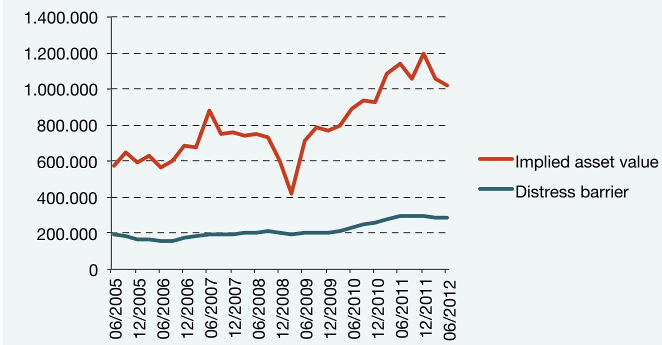


**Figure 5 – Contingent claim approach (in millions USD): Brazil**

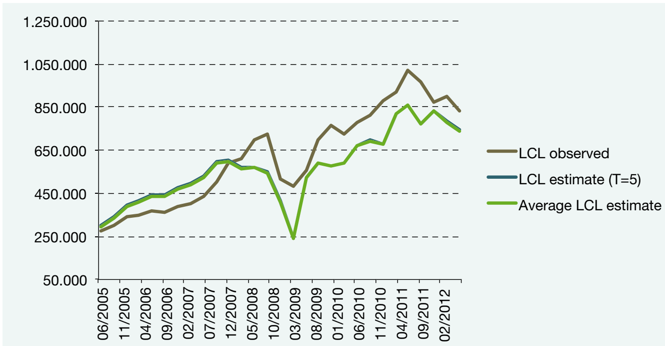


**Figure 6 – Local-currency liability observed versus estimate (in millions USD): Brazil**

| | Coefficients | Standard Error | t Stat | P-value |
|---|---|---|---|---|
| $\alpha$ | -89857.77 | 68743.39 | -1.3072 | 0.202186 |
| $LCL_{Estimate}$ | 1.2518 | 0.1161 | 10.7827 | 2.75E-05 |
| $R^2$ = 0.81 | | | | |

**Table 9 – Regression analysis for observed local-currency liability: Brazil**

| Maturity | T = 1 | T = 2 | T = 3 | T = 4 | T = 5 | T = 7 | T = 10 |
|---|---|---|---|---|---|---|---|
| R2 | 0.89 | 0.87 | 0.85 | 0.91 | 0.81 | 0.87 | 0.82 |
| $\alpha$ | 26229.31 | -4255.86 | -34375.15 | -63074.92 | -89857.77 | -136085.64 | -192699.06 |
| $\beta$ | 1.07 | 1.11 | 1.16 | 1.21 | 1.25 | 1.34 | 1.48 |

**Table 10 – $R^2$ measure and regression analysis of local-currency liability estimate across term structure: Brazil**

provides an upward bias ($\beta$>1). However, $R^2$, is still very high.

The high value of $R^2$ suggests that the regression model of the estimated local-currency liability explains the level in the observed quantity reasonably well. A one-tailed t-statistic of 10.8 indicates significance in the estimated coefficients together with a P-value < 0.05.

The explanatory power of the estimated local-currency liability as measured by the $R^2$ across the entire term structure is presented in Table 10. Its high value across maturities implies that the estimated LCL tends to capture the variation in the observed balance sheet data. For Brazil as well, predictions extracted from shorter maturities appear to have a very low bias.

## A case of Italy

Table 11 provides the calibrated parameters for Italy over a time period that extends from June 2008 to June 2012.

The high transition probability $a_{11}$ of remaining in state 1 indicates a clear persistence in state 1 which according to the volatility estimate is clearly a high volatility state. The time pattern of the ratio S/K signals that Italy went through a very critical period up to June 2011 followed by the start of a recovery. This can also be appreciated by observing the calibrated term structure of the default probabilities in Figure 7.

What is observable is how perceived risk at a shorter time horizon jumped after April 2011, easing off towards the end of the estimation period. During this time Berlusconi's government was in conflict over budget cuts and the default probability of the one-year horizon reached almost 8% from lows of 2%. Generally, the perceived riskiness of Italian sovereign debt rose over the entire maturity spectrum. Berlusconi's government resigned in November 2011 and was replaced by one led by the former European Commissioner Mario Monti. The new government contained not a single party representative of elected parliamentarians and for this reason was defined as Technocratic. However, it received the full support of The Chamber of Deputies. Financial markets hailed Monti for restoring Italy's credibility by fixing the budget, starting an ambitious reform agenda, and for steering the country off the cliff of default. This is evident in our results: the volatility in the "bad" state declined steeply from 100% to a 40% low.

| | $\sigma_1$ | $\sigma_2$ | $a_{11}$ | $a_{12}$ | S/K |
|---|---|---|---|---|---|
| **30/06/2008** | 99.90% | 1.42% | 99.90% | 99.75% | 1.108532 |
| **30/09/2008** | 99.90% | 0.40% | 99.90% | 99.70% | 1.028349 |
| **31/12/2008** | 99.90% | 0.39% | 99.90% | 99.75% | 1.018536 |
| **31/03/2009** | 99.14% | 0.11% | 99.90% | 99.43% | 1.006850 |
| **30/06/2009** | 99.61% | 0.25% | 99.90% | 99.57% | 1.020102 |
| **30/09/2009** | 99.90% | 0.10% | 99.90% | 99.70% | 1.004666 |
| **31/12/2009** | 99.88% | 0.12% | 99.90% | 99.47% | 1.007596 |
| **31/03/2010** | 99.90% | 0.10% | 99.90% | 99.66% | 1.005046 |
| **30/06/2010** | 97.77% | 1.50% | 99.90% | 99.90% | 1.067598 |
| **30/09/2010** | 95.38% | 0.37% | 99.90% | 99.46% | 1.019305 |
| **31/12/2010** | 72.75% | 0.20% | 99.90% | 98.98% | 1.011849 |
| **31/03/2011** | 99.90% | 0.10% | 99.90% | 99.71% | 1.003614 |
| **30/06/2011** | 92.44% | 0.33% | 99.90% | 99.14% | 1.024536 |
| **30/09/2011** | 85.37% | 5.80% | 99.90% | 99.26% | 1.275628 |
| **30/12/2011** | 99.90% | 11.11% | 99.06% | 99.77% | 1.553669 |
| **30/03/2012** | 41.69% | 24.12% | 67.70% | 79.37% | 2.166822 |
| **29/06/2012** | 42.33% | 35.93% | 54.34% | 66.90% | 3.086906 |

**Table 11 – Estimated parameters for Markov regime-switching model: Italy**

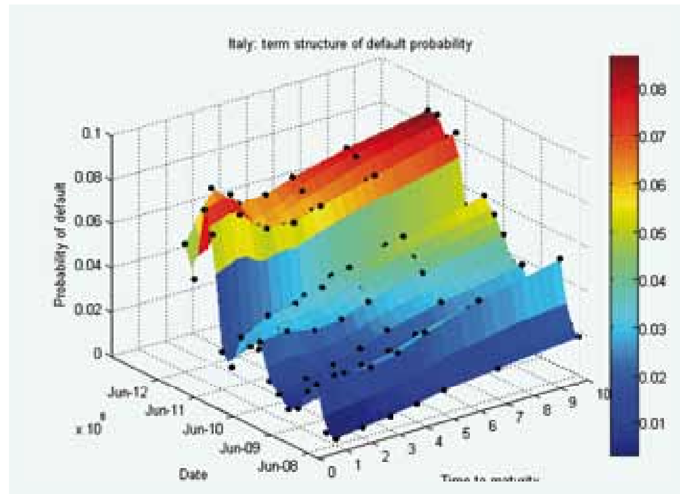


**Figure 7 – Calibrated term structure of default probability: Italy**

The most recent risk profile shows a "hump:" default probabilities first decrease (for maturities of up to five years) but then increase (conditional on no default occurring until 2017). This means that markets are expecting dampened reduced risk of default five years from June 2012 – probably due to an ease in political tensions and the reduced risk of deadlocked reform. But over the 10 year term they are anticipating greater risk. Incidentally, Moody's rating agency downgraded Italy's rating by two notches and subsequently downgraded 10 Italian banks in July 2011. Moody's cited the usual fears related to the Eurozone's debt crisis, along with the "increasing likelihood that greater collective support" will be needed for Italy. Usually an initial declining term structure denotes a high probability of default in the short run, due to a very poor economic situation.

Figure 8 and Figure 9 illustrate the quarterly time series of the sovereign balance sheet components. The distress barrier and implied asset value for periods prior to June 2006 lie precariously close, indicating an imminent default or risk thereof. Thereafter, the implied asset value experiences a sharp spike, influenced by a sharp increase in the inverse leverage parameter obtained in the regime-switching model. This in turn leads to an overestimation in the local-currency liability estimate meaning that CDS quotations overstate local sovereign liabilities. The model behavior of the observed local-currency liability mimics that observed when comparing the estimated default probability term structure to the bootstrapped equivalent.

Table 12 presents the results of the local-currency liability (LCL) estimation and observation for a T= 5 year maturity and the corresponding value averaged across the entire term structure, together with the observed distress barrier and the estimated sovereign asset.

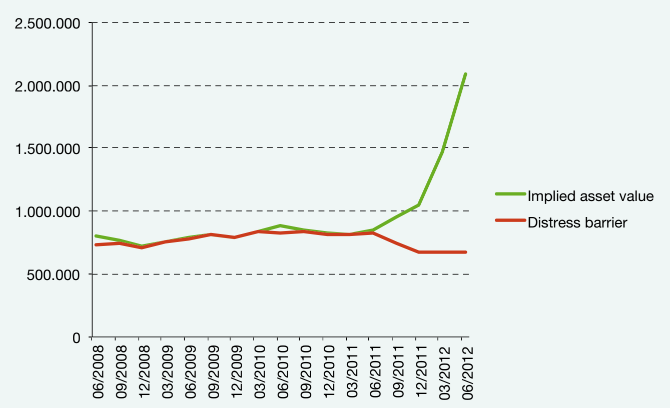


**Figure 8 – Implied sovereign asset value versus distress barrier (in millions USD): Italy**

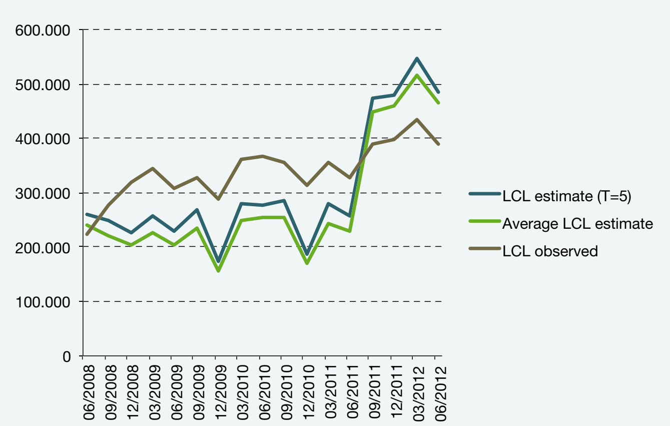


**Figure 9 – Local-currency liability observed versus estimate (in millions USD): Italy**

| | Distress Barrier | Implied Asset Value | LCL Observed | LCL Estimate (T=5) | Average LCL Estimate |
|---|---|---|---|---|---|
| **30/06/2008** | 729,202 | 808,342 | 222,849 | 260,498 | 239,305 |
| **30/09/2008** | 748,828 | 770,057 | 277,605 | 249,385 | 221,367 |
| **31/12/2008** | 708,299 | 721,431 | 318,999 | 226,554 | 202,541 |
| **31/03/2009** | 753,850 | 759,014 | 343,891 | 256,392 | 226,451 |
| **30/06/2009** | 777,003 | 792,621 | 307,652 | 228,342 | 203,217 |
| **30/09/2009** | 816,334 | 820,146 | 326,721 | 268,907 | 235,747 |
| **31/12/2009** | 790,676 | 796,685 | 287,591 | 173,806 | 154,956 |
| **31/03/2010** | 838,388 | 842,622 | 361,719 | 280,812 | 247,478 |
| **30/06/2010** | 830,750 | 886,909 | 366,995 | 277,538 | 255,028 |
| **30/09/2010** | 836,599 | 852,754 | 356,730 | 285,615 | 254,903 |
| **31/12/2010** | 811,807 | 821,427 | 312,449 | 186,951 | 169,003 |
| **31/03/2011** | 812,795 | 815,729 | 355,788 | 278,443 | 242,821 |
| **30/06/2011** | 824,494 | 844,727 | 328,732 | 257,137 | 229,105 |
| **30/09/2011** | 746,986 | 952,878 | 388,206 | 473,777 | 448,480 |
| **30/12/2011** | 673,391 | 1,046,227 | 398,073 | 478,492 | 459,768 |
| **30/03/2012** | 677,325 | 1,467,641 | 434,547 | 546,608 | 515,622 |
| **29/06/2012** | 677,325 | 2,090,841 | 387,910 | 485,534 | 464,156 |

**Table 12 – Implied sovereign asset value, distress barrier and local-currency liability (in millions USD): Italy**

| | Coefficients | Standard Error | t Stat | P-value |
|---|---|---|---|---|
| α | 233434.82 | 24030.98 | 9.7139 | 7.31E-08 |
| $LCL_{Estimate}$ | 0.3467 | 0.07373 | 4.7023 | 0.000283 |
| $R^2 = 0.60$ | | | | |

**Table 13 – Regression analysis for observed local-currency liability: Italy**

| Maturity | T = 1 | T = 2 | T = 3 | T = 4 | T = 5 | T = 7 | T = 10 |
|---|---|---|---|---|---|---|---|
| R2 | 0.47 | 0.50 | 0.53 | 0.56 | 0.60 | 0.64 | 0.69 |
| α | 238544.57 | 241620.00 | 241736.09 | 239143.01 | 233434.82 | 213021.46 | 151497.04 |
| β | 0.18 | 0.21 | 0.25 | 0.30 | 0.35 | 0.46 | 0.69 |

**Table 14 – $R^2$ measure and regression analysis of local-currency liability estimate across term structure: Italy**

Next, the quarterly predictive power for the observed local-currency liability (LCL) over a T = 5 year maturity is examined in Table 13, using the estimated local-currency liability obtained from the model. The regression analysis produces the following estimates:

$$\widehat{LCL}_{Observed} = 233434.82 + 0.35 * LCL_{Estimate}$$

The value of $R^2$, the one-tailed t-statistic of 4.7 and the p-value $< 0.05$, suggests a moderate linear relationship between the model and the observed local-currency liability, and does however conclude a significance in the estimated coefficients.

Table 14 shows the explanatory power of the estimated local-currency liability as measured by the $R^2$ across the entire term structure. The results imply that, differently from the countries considered previously, the model used to estimate the LCL tend to improve in its ability to capture the variation in the observed balance sheet data as the maturity of the term structure increases.

## A case of the United States

For the United States, the estimation period includes quarterly data from June 2008 to June 2012. The estimated parameters across the entire term structure are presented in Table 15.

Persistence in state 1 of the model is evident from the results and according to the volatility estimate, it can be classified as a high volatility state. The difference between $\sigma_1$ and $\sigma_2$ is indeed very large. The inverse leverage ratio fluctuates around 1.7, reaching levels of 2.4 over the estimation period.

From Figure 10 we observe how the default probability has remained relatively stable for shorter maturities while the perceived risk over a longer horizon has steadily increased over the estimation period. The probability of a default (PD) for the 10-year maturity has steadily risen from above 1% to near 4%, fluctuating with the economic environment. We observe a wave-like behavior of the PD along the date axis, holding maturity constant, particularly pronounced around December 2008 and July 2011.

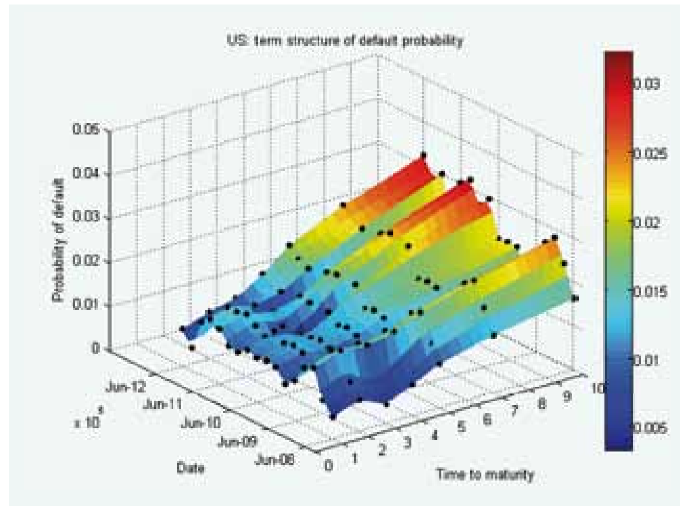


Figure 10 – Calibrated term structure of default probability: US

On 15 June 2011, Dow Jones reported that the one-year CDS spread for the United States was at 43 basis points (higher than the 41 basis points spread for Brazil), and that the cost of insuring one-year US debt against default had risen on the back of concerns related to the debt ceiling. The potential of sovereign default or restructuring and concerns that Eurozone fiscal pressures could spread is reflected in the rise in perceived risk over the longer term.

| | $\sigma_1$ | $\sigma_2$ | $a_{11}$ | $a_{12}$ | S/K |
|---|---|---|---|---|---|
| **30/06/2008** | 99.90% | 0.10% | 99.00% | 99.76% | 1.5847 |
| **30/09/2008** | 81.41% | 0.15% | 98.57% | 99.60% | 1.6992 |
| **31/12/2008** | 72.04% | 8.66% | 99.90% | 99.62% | 1.6530 |
| **31/03/2009** | 71.35% | 8.03% | 99.90% | 99.63% | 1.6572 |
| **30/06/2009** | 99.90% | 0.10% | 99.18% | 99.65% | 1.5893 |
| **30/09/2009** | 99.74% | 1.91% | 99.37% | 99.76% | 1.5648 |
| **31/12/2009** | 67.95% | 8.74% | 99.90% | 99.80% | 1.6565 |
| **31/03/2010** | 69.61% | 8.57% | 99.90% | 99.75% | 1.6529 |
| **30/06/2010** | 68.13% | 8.46% | 99.90% | 99.78% | 1.6580 |
| **30/09/2010** | 82.98% | 9.67% | 99.13% | 99.64% | 1.6361 |
| **31/12/2010** | 56.23% | 0.10% | 98.23% | 99.21% | 1.9686 |
| **31/03/2011** | 92.03% | 6.46% | 99.06% | 99.67% | 1.5511 |
| **30/06/2011** | 81.00% | 8.63% | 99.90% | 99.88% | 1.5832 |
| **30/09/2011** | 34.52% | 0.10% | 98.83% | 98.88% | 2.3708 |
| **30/12/2011** | 73.38% | 8.02% | 99.90% | 99.84% | 1.6024 |
| **30/03/2012** | 99.90% | 0.10% | 99.24% | 99.59% | 1.5931 |
| **29/06/2012** | 71.40% | 9.33% | 99.78% | 99.66% | 1.6570 |

Table 15 – Estimated parameters for Markov regime-switching model: US

The quarterly time series of the sovereign balance sheet components estimated according to the contingent claim approach for T= 5 year maturity and the LCL averaged across the term structure are reported in Table 16.

Figure 11 and Figure 12 serve to illustrate the time series presented in Table 16. The value for the local-currency liability (LCL) estimate lies above the observed value indicating that the regime-switching method tends to overestimate the observed LCL. This means that sovereign CDS quotations on US debt tend to overstate local sovereign liabilities. The implied sovereign asset value lies well above the distress value for all quarters suggesting that no default was imminent in the estimation period. The peak and sudden drop in the implied asset value in 2011 highlight the sensitivity of the approach to the inverse leverage parameters estimated in the model. At this point, the leverage declined, volatility of the model fell sharply, and the transition probability of remaining in a high volatility state declined.

Next, the quarterly predictive powers for the observed local-currency liability (LCL) over a five-year maturity is examined, using the estimated local-currency liability obtained from the model. The regression analysis estimates the following relationship:

$$\widehat{LCL}_{Observed} = -654\ 618.21 + 1.13 * LCL_{Estimate}$$

The high value of $R^2$ suggests that the regression model of the estimated local-currency liability explains the level in the observed counterpart very well. A one-tailed t-statistic of 7.4 indicates significance in the estimated coefficients. However, it appears biased: implied estimates are higher than observed.

| | Distress Barrier | Implied Asset Value | LCL Observed | LCL Estimate (T=5) | Average LCL Estimate |
|---|---|---|---|---|---|
| 30/06/2008 | 21,590,625 | 34,213,671 | 741,504 | 1,828,238 | 1,640,256 |
| 30/09/2008 | 19,204,425 | 32,633,081 | 834,246 | 1,782,926 | 1,622,492 |
| 31/12/2008 | 19,063,457 | 31,511,456 | 1,549,762 | 1,619,281 | 1,567,835 |
| 31/03/2009 | 17,755,214 | 29,423,887 | 1,580,922 | 1,878,559 | 1,818,510 |
| 30/06/2009 | 18,880,466 | 30,006,668 | 1,626,160 | 1,842,701 | 1,793,222 |
| 30/09/2009 | 20,014,068 | 31,317,414 | 1,752,930 | 1,933,729 | 1,889,990 |
| 31/12/2009 | 19,730,396 | 32,682,887 | 1,952,025 | 2,121,046 | 2,075,346 |
| 31/03/2010 | 18,802,148 | 31,077,619 | 2,033,102 | 2,287,998 | 2,237,842 |
| 30/06/2010 | 17,112,212 | 28,371,244 | 1,977,475 | 2,211,521 | 2,168,214 |
| 30/09/2010 | 19,624,284 | 32,108,233 | 1,925,069 | 2,252,161 | 2,181,921 |
| 31/12/2010 | 19,349,739 | 38,091,916 | 1,970,157 | 2,283,386 | 2,223,695 |
| 31/03/2011 | 20,987,132 | 32,552,364 | 2,269,313 | 2,492,781 | 2,430,674 |
| 30/06/2011 | 21,338,182 | 33,782,801 | 2,503,646 | 2,597,764 | 2,562,947 |
| 30/09/2011 | 20,031,240 | 47,489,583 | 2,589,661 | 2,786,039 | 2,751,102 |
| 30/12/2011 | 19,506,948 | 31,258,363 | 2,598,679 | 2,820,027 | 2,789,482 |
| 30/03/2012 | 20,220,053 | 32,211,718 | 2,647,675 | 3,038,320 | 2,948,079 |
| 29/06/2012 | 19,420,762 | 32,180,281 | 2,606,872 | 3,013,467 | 2,923,294 |

Table 16 – Implied sovereign asset value, distressed barrier and local-currency liability (in millions EUR): US

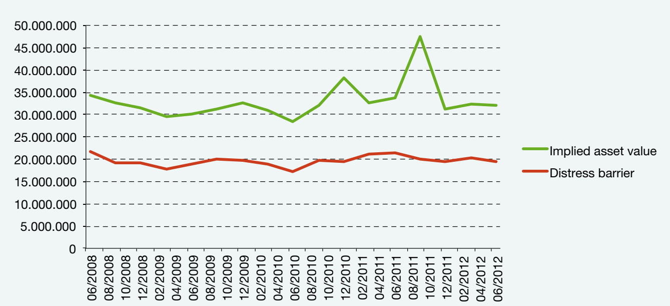


Figure 11 – Implied sovereign asset value versus distress barrier (in millions EUR): US

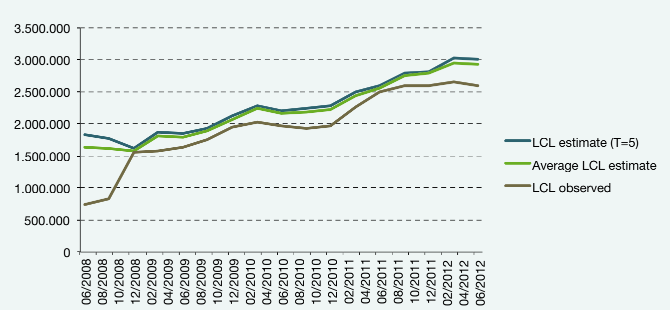


Figure 12 – Local-currency liability observed versus estimate (in millions EUR): US

| | Coefficients | Standard Error | t Stat | P-value |
|---|---|---|---|---|
| $\alpha$ | -654618.21 | 358446.60 | -1.8263 | 0.8778 |
| $LCL_{Estimate}$ | 1.1417 | 0.1543 | 7.3980 | 2.23E-06 |
| $R^2$ = 0.79 | | | | |

Table 17 – Regression analysis for observed local-currency liability: US

| Maturity | T = 1 | T = 2 | T = 3 | T = 4 | T = 5 | T = 7 | T = 10 |
|---|---|---|---|---|---|---|---|
| $R^2$ | 0.97 | 0.98 | 0.97 | 0.95 | 0.79 | 0.64 | 0.15 |
| $\alpha$ | 473684.52 | 143225.44 | -188027.20 | -489629.72 | -654618.21 | -249465.89 | 1405872.59 |
| $\beta$ | 0.82 | 0.94 | 1.06 | 1.14 | 1.14 | 0.88 | 0.18 |

Table 18 – $R^2$ measure and regression analysis of local-currency liability estimate across term structure: US

Table 18 shows the explanatory power of the estimated local-currency liability as measured by the $R^2$ across the entire term structure. The results in fact show an improvement in the predictive power of the model as we move toward the short end of the curve.

## A case of Germany

In the case of Germany, the estimation period extends from June 2008 to June 2012. The calibrated parameters across the entire term structure are presented in Table 19.

The striking difference with respect to the other countries is the low volatility levels in the two states, reaching an upper level of around 25%. The high transition probability $a_{11}$ of remaining in state 1 indicates persistence in a low risk state economy. The inverse leverage parameter ranges between 1.6 and 2.

Figure 13 shows the term structure of probability of defaults (PDs) obtained from the estimated PDs. The term structure from the start of the estimation period until around January 2009 is reasonably flat, indicating that perceived risk at the one to 10 year horizon is low and constant. After this period, the charts show how markets have changed their perception of Germany's long-term prospects over the course of the remaining period. In two years, the probability of a default over the 10 year horizon climbs from below 2% to above 6% while the one to three year level remained low. In July 2012, German 10-year debt did, in fact, reach record lows on the back of fear of a double dip recession in the US. In the same month, Moody's lowered its outlook on Germany to negative.

The quarterly time series of the sovereign balance sheet components according to the contingent claim approach for T= 5 year maturity and the LCL averaged across the term structure is reported in Table 20.

Figure 14 shows how the implied sovereign asset value lies well above the distress barrier for all quarters; analogous to a low probability less than

| | $\sigma_1$ | $\sigma_2$ | $a_{11}$ | $a_{12}$ | S/K |
|---|---|---|---|---|---|
| **30/06/2008** | 2.17% | 14.65% | 0.9987 | 0.9985 | 2.1548 |
| **30/09/2008** | 9.44% | 1.33% | 0.9982 | 0.9974 | 1.4759 |
| **31/12/2008** | 0.60% | 13.03% | 0.9962 | 0.9906 | 1.6380 |
| **31/03/2009** | 1.43% | 21.35% | 0.9990 | 0.9949 | 1.6084 |
| **30/06/2009** | 1.23% | 19.16% | 0.9990 | 0.9975 | 1.5531 |
| **30/09/2009** | 0.10% | 20.93% | 0.9990 | 0.9967 | 1.9859 |
| **31/12/2009** | 0.21% | 7.54% | 0.9987 | 0.9951 | 1.7625 |
| **31/03/2010** | 4.56% | 13.97% | 0.9988 | 0.9943 | 2.0623 |
| **30/06/2010** | 4.65% | 7.36% | 0.9978 | 0.9920 | 1.6989 |
| **30/09/2010** | 3.33% | 15.13% | 0.9979 | 0.9932 | 1.9507 |
| **31/12/2010** | 0.82% | 8.22% | 0.9974 | 0.9898 | 1.4235 |
| **31/03/2011** | 1.97% | 11.64% | 0.9969 | 0.9918 | 2.0313 |
| **30/06/2011** | 0.86% | 6.61% | 0.9951 | 0.9938 | 1.1380 |
| **30/09/2011** | 8.03% | 17.86% | 0.9986 | 0.9922 | 2.3000 |
| **30/12/2011** | 2.78% | 20.74% | 0.9950 | 0.9917 | 1.4242 |
| **30/03/2012** | 0.10% | 20.81% | 0.9972 | 0.9921 | 1.5405 |
| **29/06/2012** | 2.80% | 25.17% | 0.9933 | 0.9878 | 1.8044 |

**Table 19 – Estimated parameters for Markov regime-switching model: Germany**

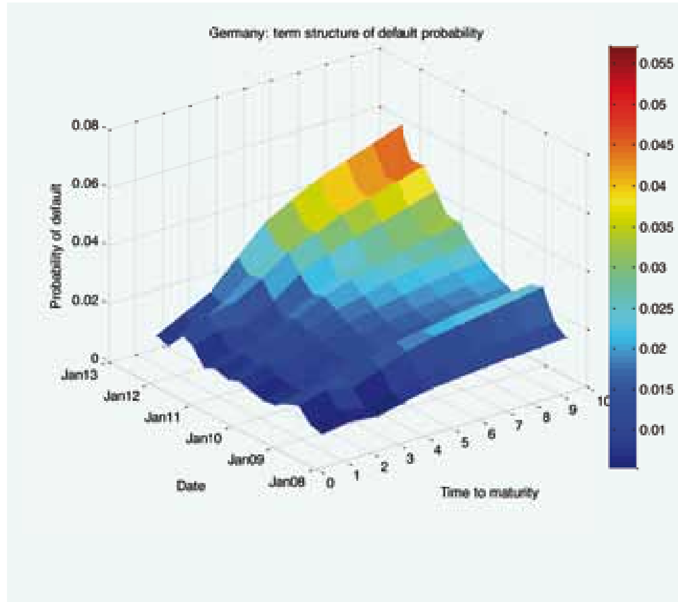


**Figure 13 – Calibrated term structure of default probability: Germany**

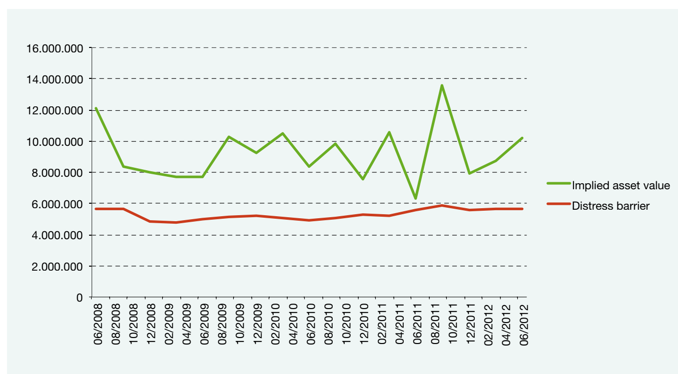


**Figure 14 – Implied sovereign asset value versus distress barrier (in millions USD): Germany**

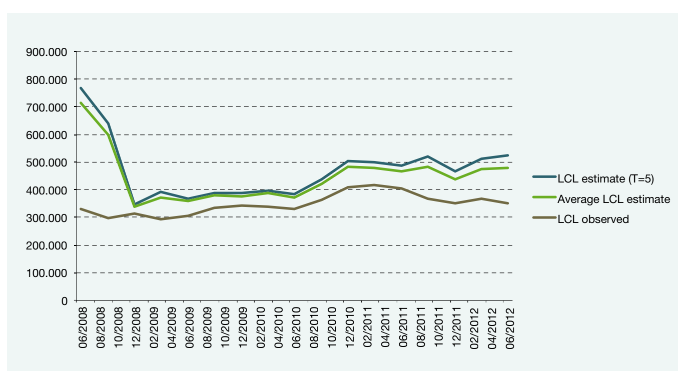


**Figure 15 – Local-currency liability observed versus estimate (in millions USD): Germany**

8% of Germany defaulting over the estimation period. The implied asset value also fluctuates considerably from one quarter to the next as observed in Figure 15, analogous to the variability observed in the calibrated inverse leverage parameter and indicative of the wave-like behavior observed in the probability of default term structure. On the other side, the observed LCL remains quite stable over time. In practice, we observe that the local-currency liability estimate overstates the observed value across the term structure. This implies that the CDS quotations overstate local sovereign liabilities.

Next, the quarterly predictive power for the observed local-currency liability (LCL) over a T= 5 year maturity is examined from the regression results obtained in Table 21, using the estimated local-currency liability obtained from the model. The following relationship is estimated:

$$\widehat{LCL}_{Observed} = -326\,508.18 + 0.05 * LCL_{Estimate}$$

The low value of $R^2$ suggests that the regression model of the estimated local-currency liability explains very little variation in the observed counterpart. A one-tailed t-statistic of 0.5 indicates that the estimated coefficients are not significant.

Table 22 shows the explanatory power of the estimated local-currency liability as measured by the $R^2$ across the entire term structure. The striking fact is that the one-year maturity already goes some way to predicting the observed LCL value. In addition, this prediction appears to be unbiased.

| | Distress Barrier | Implied Asset Value | LCL Observed | LCL Estimate (T=5) | Average LCL Estimate |
|---|---|---|---|---|---|
| 30/06/2008 | 5,629,811 | 12,131,255 | 332,105 | 766,000 | 714,543 |
| 30/09/2008 | 5,651,401 | 8,340,809 | 296,242 | 638,475 | 597,816 |
| 31/12/2008 | 4,869,564 | 7,976,342 | 314,663 | 345,243 | 337,972 |
| 31/03/2009 | 4,780,107 | 7,688,443 | 292,322 | 390,487 | 373,137 |
| 30/06/2009 | 4,986,103 | 7,744,088 | 304,066 | 369,280 | 358,438 |
| 30/09/2009 | 5,172,468 | 10,272,219 | 336,526 | 387,671 | 378,173 |
| 31/12/2009 | 5,248,690 | 9,250,694 | 341,680 | 386,837 | 376,706 |
| 31/03/2010 | 5,088,215 | 10,493,610 | 337,961 | 398,169 | 386,766 |
| 30/06/2010 | 4,915,667 | 8,351,017 | 329,289 | 383,128 | 372,209 |
| 30/09/2010 | 5,044,933 | 9,840,971 | 363,932 | 438,583 | 422,975 |
| 31/12/2010 | 5,306,088 | 7,553,110 | 408,009 | 504,486 | 482,842 |
| 31/03/2011 | 5,218,723 | 10,600,917 | 415,524 | 500,186 | 480,276 |
| 30/06/2011 | 5,556,759 | 6,323,703 | 405,900 | 486,315 | 466,626 |
| 30/09/2011 | 5,888,272 | 13,543,026 | 367,374 | 520,247 | 483,898 |
| 30/12/2011 | 5,550,577 | 7,905,355 | 352,365 | 464,870 | 437,941 |
| 30/03/2012 | 5,647,322 | 8,699,482 | 366,366 | 510,810 | 475,596 |
| 29/06/2012 | 5,662,888 | 10,218,330 | 349,963 | 524,980 | 480,244 |

Table 20 – Implied sovereign asset value, distressed barrier and local-currency liability (in millions USD): Germany

| | Coefficients | Standard Error | t Stat | P-value |
|---|---|---|---|---|
| $\alpha$ | 326508.18 | 43320.09 | 7.5371 | 1.78E-06 |
| $LCL_{Estimate}$ | 0.04479 | 0.08862 | 0.5054 | 0.6206 |
| $R^2$ = 0.02 | | | | |

Table 21 – Regression analysis for observed local-currency liability: Germany

| Maturity | T = 1 | T = 2 | T = 3 | T = 4 | T = 5 | T = 7 | T = 10 |
|---|---|---|---|---|---|---|---|
| $R^2$ | 0.99 | 0.51 | 0.16 | 0.06 | 0.02 | 0.00 | 0.05 |
| $\alpha$ | 0.00 | 115522.37 | 253870.81 | 301201.01 | 323006.38 | 350404.59 | 367717.41 |
| $\beta$ | 1.00 | 0.63 | 0.24 | 0.11 | 0.05 | -0.01 | -0.04 |

Table 22 – $R^2$ measure and regression analysis of local-currency liability estimate across term structure: Germany

## Conclusion

Merton-style structural models provide a very appealing feature that links credit risk with underlying structural variables by providing an endogenous description of credit defaults and an intuitive economic interpretation. However, they suffer the disadvantage of leading to underestimates particularly over the short term. This research offers the potential to both resolve the underestimation inherent in most standard structural models and establish a link between the credit market and a sovereign's balance sheet in an attempt to understand whether credit markets convey useful information that predicts economic stability. The methodology and application backs the hypothesis that a regime-switching framework, which allows for structural shifts, can substantially improve default risk estimators and can be tractably extended to a Contingent Claims Approach in the case of a sovereign, thereby obtaining a link between the credit market and predictions of a sovereign's balance sheet fundamentals.

The research analyses the economic impact of the estimated default probabilities on a sovereign's balance sheet. Using the calibrated regime-switching parameters extracted from the Markov-modulated model as inputs into the standard option pricing formula in the Merton Contingent Claim Approach, a sovereign asset value and local-currency liability in foreign currency terms (LCL) is extracted. The value of LCL is a call option of sovereign assets (A) with the strike price as the default barrier (B_f) defined as foreign-currency denominated debt. The LCL is observable in the market and facilitates a comparison with our estimate. We observe that under the regime-switching approach, the LCL estimate sufficiently captures the level in the observed LCL for all cases bar Germany. The regression results appear very good, albeit biased given non-zero intercepts and betas not equal to 1. However, if we consider very short maturities, predictions improve considerably, even for Germany. We speculate on the predictive ability of the model to balance sheet information shortly. First we identify that the LCL requirement according to the model is higher than the observed value for the case of South Africa, the US, and Germany. Brazil and Italy, during certain periods require a lower LCL estimate than is observed in the market. Any under- or over-estimation of the LCL estimate relative to the observed value implies that the corresponding CDS market quotes, jointly with the model used, under- or over-estimate local sovereign liabilities respectively. We also observe the relation to the estimated inverse leverage ratio and the implied asset value. Inverse leverage ratios for the case of Italy are very low, showing signs of imminent default prior to 2011, therefore resulting in a low estimated value for the LCL for the relevant sample period. A low predicted inverse leverage parameter (close to 1) signifies imminent default and given the nature of the valuation of the LCL, significantly draws the required LCL lower than its observed counterpart.

In the August 2012 monthly bulletin of the European Central Bank (ECB), the linkages between structural, financial, and fiscal imbalances are cited to have led to the sovereign debt crisis and the fragmentation of the financial markets, highlighting major weaknesses in the institutional set up of the European Monetary Union [ECB (2012)]. Structural rigidities and a build-up of imbalances resulted in more costly adjustments once the crisis erupted. The financial stability framework struggled to identify and correct systematic risk prior to the crisis and was equally challenged when containing the spread of instability across countries and markets when the risk materialised.

The model provided in this research provides a relative valuation framework for contingent claims on a sovereign's asset, estimating with a degree of caution, the balance sheet requirements necessary to predict default. In addition, Gray et al. (2007) suggests that the CCA approach could have implications on the rapidly growing sovereign wealth funds, particularly for emerging markets governments that have accumulated large reserves. Once sovereign risk exposures are calculated in a regime-switching framework, new ways of transferring sovereign risk can be explored and new instruments and risk transfer arrangements can be developed.

Overall, there are several challenges when applying a contingent claim approach to a sovereign: the lack of any single dominant model, data requirements that severely limit the scope of application for some models, and dependence of some models' results on distributional assumptions. Furthermore, that market imperfections exist implies that non-market measures may add significant power in predicting default and balance sheet information beyond that possible even with an ideal contingent claims model. The results conclude that most sovereign's observed LCL were estimated well enough by implementing the regime-switching parameters in a contingent claims approach. The regression analysis verifies this. Germany proved to be a case where the explanatory power of the model is weak and the cause is evident on inspection of the implied asset value volatility i.e., volatility implied from the leverage parameter and not calibrated explicitly. Gray et al. (2007) attribute asset volatility to high levels of foreign exchange price volatility. This highlights the need to address the relationship and causality of asset value volatility and various risk indicators.

The following issues are left for future research. The research estimates the Markov model when the number of states is two. It would be better, however, to simultaneously estimate the optimal number of states with other parameters. Standard approaches applied to samples with few defaults pose some serious drawbacks which lead to a high probability of underestimating the true default probability (PD). These include the obvious effect of a high uncertainty estimation and skewness of the sampling distribution. Given the small probability of defaults and a small sample size observed in sovereign CDS data, any default event leading to a low PD estimate under standard approaches is unlikely. The likelihood of underestimation rises with a decrease in the true PD and a decrease in the sample size [Orth (2011)]. This could affect the regulatory approaches to risk management. Basel II states that where limited data are available, a bank must adopt a conservative bias to its analysis, adding to its estimates a margin of conservatism related to the likely range of errors – this applies equally in the case of the sovereign scenarios that we have described.

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## Appendix 1

### Applying a sovereign contingent claims approach

Random fluctuations in the market prices of an entity's assets and liabilities together with changes in financial inflows and outflows constitute balance sheet risk. If the total value of the assets falls below the level of promised payment on debt, distress and/or default occurs. The value of the risky debt is calculated as a default-free value of debt less an implicit put option on the underlying assets with the strike equal to the promised payments. Equity is modeled as an implicit call option with the same underlying asset and strike. The following balance sheet identity ensues:

Asset = Equity + Liability
= Implicit Call Option + Default-Free Debt – Implicit Put Option

The assets of a sovereign for the purpose of this approach comprise foreign reserves, net fiscal assets, and other assets minus entities too important to fail. Liabilities are defined as foreign-currency denominated debt plus a local-currency liability comprised of local-currency debt and base money. Sovereign default arises when sovereign assets cannot sufficiently cover the promised payment on foreign currency debt. The default barrier is therefore defined as the present value of these payments. The default barrier may be defined as a KMV-like measure (short-term debt plus one-half long-term debt plus interest payments up to a certain time) or "senior" foreign-currency denominated debt [Crouhy, et al. (2000)]. When a lender makes a loan to a sovereign, an implicit guarantee of that loan equal to the expected loss of default is created. The action of the lender consists of pure default-free lending and bearing a risk of default by the sovereign. Risky debt can be viewed as a contingent claim on the (stochastic) sovereign assets. The foreign-currency debt can therefore be modeled as default-free value of debt minus an implicit put option. Local-currency liabilities are modeled as an implicit call option since such liability demonstrates "equity-like features" on a sovereign balance sheet. Excessive issue of both the money base and local-currency liabilities have a similar effect on inflation and price changes as the excessive issuing of corporate shares dilute shareholders' claims. The local-currency multiplied by the exchange rate is considered a market cap of the sovereign in the international market.

The main challenge is deriving an estimate for the market value and volatility of sovereign assets. Because these are not directly observable, the CCA approach relies on the relationship between balance sheet entries to extract an implied estimate of sovereign assets by a calibration procedure. The value of the local-currency liability in foreign currency terms (LCL) is a call option of the sovereign's assets (A) with the strike price as the default barrier ($B_f$) defined as foreign-currency denominated debt. The standard approach requires two equations: the first defines LCL as a call option on the asset value

$$LCL = AN(d1) - B_f e^{-r_f T} N(d2) \quad (2)$$

$$d_1 = \frac{ln\frac{A}{B_f} + \left(r + \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}} \quad (3)$$

The second equation defines the volatility of the LCL through

$$LCL^{*}\sigma_{LCL} = A\sigma_A N(d1) \quad (4)$$

while $\sigma_A$ is the volatility of the sovereign's assets.

The formula for the local currency liability in foreign currency which is observed directly from market data is defined as

$$LCL = M + B_{d,FC} = \frac{(M_{LC}e^{-r_d T} + B_d)e^{-r_f T}}{X_F} \quad (5)$$

where

| | |
|---|---|
| $M_{LC}$ | - is the base money in local-currency terms |
| M | - is the base money in foreign-currency terms |
| $r_d$ | - is the domestic interest rate |
| $r_f$ | - is the foreign interest rate |
| $B_d$ | - is the domestic currency denominated debt |
| $B_{d,FC}$ | - is the domestic currency denominated debt in foreign currency terms |
| $X_F$ | - is the forward exchange rate |
| $\sigma_A$ | - is the volatility of the sovereign assets |

Equation (2) and (4) are typically used to calculate the unknown and unobservable sovereign asset value and asset volatility. The calibrated parameters can be used to obtain sovereign risk measures such as distance-to-default and probability of default and spreads on debt.

Here we follow a different procedure. Instead of pricing CDSs or evaluating the value of balance sheet claims according to a CCA model, we use observed market data, filter it through the regime-switching model and we try to infer balance sheet information by performing a reverse engineering procedure: we use the calibrated parameters of the regime switching model to bootstrapped PDs in order to estimate the local-currency liabilities in foreign currency terms (LCL) as a call option on a sovereign's assets (A) with the strike price as the default barrier ($B_f$). An estimation of the sovereign unknown and unobserved asset value ($\hat{A}$) can be extracted from both the calibrated inverse leverage parameter (S/K) and the observed distress barrier ($B_f$) such that

$$\hat{A} = \frac{S}{K} * B_f \quad (6)$$

The underlying risky asset (S) and the strike (K) in the regime-switching framework equates to the sovereign asset value (A) and threshold barrier ($B_f$). Similarly, the inverse leverage parameter (S/K) equates to the ratio of the sovereign asset value (A) to the default threshold ($A/B_f$).

The use of the Merton-type model requires many balance sheet inputs and parameters which are not always clearly observed and can sometimes be inaccurate or difficult to obtain. By reverse engineering the valuation of a sovereign's asset value, the model requires substantially less market information and adjusts for any structural breaks in the model in an attempt to improve the fair value estimates of a sovereign's balance sheet.

Individual Selected Article from Journal 38



Layout, production and coordination: Cypres – Daniel Brandt, Kris Van de Vijver and Pieter Vereertbrugghen
Graphic design: Buro Proper – Bob Goor
Photographs: Buro Proper - Bob Goor


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