Economic Ordering and Payment Policies Under Progressive Payment Schemes and Time-Value of Money

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Abstract: Trade credits have received considerable attention in recent years and have become one of the most important sources of short-term funding for many companies. The paper at hand studies the optimal ordering and payment policies of a buyer assuming that the supplier offers a progressive interest scheme. The contribution to the literature is twofold. First, the different financial conditions of the companies involved are taken into account by assuming that the credit interest rate of the buyer may, but not necessarily has to, exceed the interest rate charged by the supplier. In addition, the time-value of money is considered in this scenario which is relevant when trade credit terms are valid for a long period of time and payment flows need to be evaluated by their net present value to ensure long-term profitability. The models proposed enable decision makers to improve ordering and payment decisions and the results reveal that taking into account the temporal allocation of payments, the prevailing interest relation influences replenishment policies significantly.

Keywords: Economic Order Quantity, Trade Credit, Progressive Credit Period, Discounted Cash Flow, Net Present Value, Inventory Management

Introduction

The classical economic order quantity (EOQ) model implicitly assumes that the buyer pays the supplier immediately after receiving the order into inventory. In recent years, more and more authors have started to relax this assumption by studying scenarios where the supplier grants the buyer a temporary or permanent payment delay (see, for overviews, Seifert et al., 2013; Glock et al., 2014a). This trend in the literature on inventory management reflects a tendency that can be observed in many business transactions today, where 'permissible delays in payments' or 'trade credits' have become one of the most important means of short-term debt financing (cf. Summer and Wilson, 2002). This trend will likely gain further momentum in the near future, as access to finance is currently one of the most important problems especially of small and medium-sized companies (ECB, 2013).

One aspect that has received relatively little attention in the literature on trade credit inventory management is the time-value of money. Especially in situations where trade credit agreements are used over a long period of time and where discount rates are too large to be ignored, explicitly considering the time-value of money in inventory models helps to make them more realistic. As decisions on the working capital structure of the company defined by an appropriate inventory and payment policy significantly influence future cash-flows and thus the temporal allocation of payments, they always should be evaluated in terms of long-term profitability by considering their net present value or equivalent measures (cf. Grubbström, 1980, Beullens and Janssens, 2014). A second aspect our study of the literature revealed is that inventory models under trade credit, with a few exceptions, had a one-sided focus on situations where the interest rate that is charged by the supplier exceeds the credit interest rate of the buyer. It is clear that in practice, interest rates may differ from company to company, such that the credit interest rate of the buyer, which could be the interest rate the buyer is charged from its bank, for example (see Summers and Wilson, 2002), could exceed the interest charged by the supplier. In such a scenario, the buyer would not be interested in settling the unpaid balance as soon as interest is charged on the outstanding balance, as was assumed in the literature so far. Instead, the buyer could maximize its interest earnings and thus achieve substantial savings in total inventory expenses by keeping the sales revenue invested, and by settling the unpaid balance not before the interest charged by the supplier exceeds the incomes from the investment (cf. Glock et al., 2014b, 2015), or just before the next order is issued (cf. Cheng et al., 2012).

Both time-value of money and varying interest conditions have only been addressed insufficiently in the literature so far. Cheng et al. (2012) analyzed the impact of different financial environments on optimal ordering and payment policies and illustrated the benefits of prolonged payment intervals for a retailer given that the supplier offers a single credit period. Similarly, Glock et al. (2014b) and (2015) studied optimal ordering and payment policies in the presence of progressive payment schemes. However, the analysis was based on the average cost approach, which is widely accepted as an approximation for the present-value approach when interest rates are low and the temporal allocation of payments has only minor influences on optimal ordering and payment decisions. On the other hand, Soni et al. (2006) considered the time-value of money in a trade credit inventory model with progressive payment schemes, but neglected that the credit interest rate of the buyer may exceed the interest charges of the supplier, which might be the case for large buyers investing into the stock market or into developing new products. In such a case it would be more rational to prolong the payment and maximize interest earnings.

Consequently, the paper at hand aims at generalizing trade credit inventory models with progressive interest schemes and time-value of money by considering the case where the credit interest rate of the buyer may (but not necessarily has to) exceed the interest rate charged by the supplier. This enables the buyer to realize arbitrage gains by implementing efficient inventory and payment policies that are evaluated by their present value. In addition, some inaccuracies contained in earlier formulations of the effective interest cost are corrected in this paper to increase the applicability of the models.

The remainder of the paper is structured as follows: The next section provides an overview of the related literature. Section 3 outlines assumptions and notations that are used throughout the paper, and Section 4 develops formal models for determining the optimal order quantity and payment policies for different interest conditions considering time-value of money. Section 5 illustrates the behavior of the models with the help of a numerical study, and Section 6 concludes the article.

Literature Review

In his seminal work on EOQ models with trade credits, Goyal (1985) showed that the economic replenishment interval and the order quantity generally increase if a delay in paying is permitted, as compared to the classical EOQ model. The total costs of the buyer, in turn, decrease. Until today, Goyal’s model has frequently been extended. Recent works consider shortages (Salameh et al., 2003; Su, 2012), stock-dependent demand rates (e.g., Soni and Shah, 2008; Sarker, 2012), or an order quantity-dependent length of the credit period (e.g., Shinn and Hwang, 2003; Ouyang et al., 2009), for example.

A related stream of research has focused on so-called progressive interest schemes, where the supplier offers two or more credit periods to the buyer, with the interest rate increasing from period to period. One of the first works on progressive interest schemes is the one of Goyal et al. (2007), who assumed that the supplier offers three credit periods. If the supplier settles the balance in the first period, then no interest is charged on the outstanding balance. In the subsequent two credit periods, interest is charged on the outstanding balance, where the interest rate that is charged in the third period exceeds the interest rate of the second period. Chung (2009) revisited Goyal et al.'s work and improved their solution procedure. Other extensions are those of Soni and Shah (2008, 2009), who took account of stock-dependent demand rates. Teng et al. (2011) then showed that in the case of stock-dependent demand, inventory should not be depleted entirely to stimulate customer demand. Glock et al. (2014b, 2015) further generalized the case of progressive interest schemes and studied a situation where the buyer's own interest rate may exceed the interest rate charged by the supplier in the second credit period. In this case, the buyer has no incentive to settle the unpaid balance before the start of the third credit period.

An early work on trade credit inventory models considering time-value of money was presented by Chung (1989), who used the discounted cash flow (DCF) approach to study ordering decisions under different trade credit scenarios. The author showed that the discount rate and the credit period length may have a significant influence on inventory replenishment decisions, and that their impact is moderated by the type of credit and the way the credit is settled. Chung and Liao (2009) extended this model and assumed that the trade credit is only granted if a minimum order quantity is issued. A similar setting was considered in Chung and Liao (2006) and Chang et al. (2010), who assumed in addition that the product deteriorates. Optimal ordering policies for deteriorating items have also been studied by Liao and Huang (2010) and Guchhait (2014), who considered two-level trade credits in addition. Teng (2006) developed an EOQ model with a two-period payment scheme and DCF analysis by assuming that if the buyer settles the outstanding balance in the first period, then the supplier grants a cash discount. If the buyer pays during the second period, then no cash discount is granted, but also no interest has to be paid. At the end of the second period, the entire balance has to be settled at the latest. A related model is the one of Soni et al. (2006), who investigated ordering decisions under progressive payment schemes and the DCF approach. Balkhi (2011) developed a dynamic EOQ model with multiple order cycles considering the time-value of money as well as cycle-dependent demand and product deterioration rates. The author assumed that the supplier offers a trade credit in each order cycle, and that the trade credit period may be longer than the order cycle itself.

The analysis of the literature revealed that optimal ordering and payment policies in the presence of progressive interest schemes have, with the exception of Soni et al. (2006), not been analyzed by the help of the discounted cash flow approach that takes into account the time-value of money. In addition, previous research has widely neglected the case where the interest rate of the buyer exceeds the interest rate charged by the supplier. To gain further insights, the paper at hand generalizes trade credit inventory models with progressive interest schemes by considering the impact of varying financial environments and time-value of money.

Assumptions and Notation

The following conditions will be assumed in this paper:

1. We consider a buyer sourcing a single product at a supplier for an infinite planning horizon.
2. Shortages are not allowed and the demand rate for the item is constant and deterministic.
3. Lead times are zero and replenishments are made instantaneously.
4. All payments are considered by their present value, and the discount factor is assumed constant for the period under study. The comparison of discount rates corresponding to different period lengths requires the transformation $\rho = Log[1 + r]$, where r denotes

the effective annual discount rate and ρ the continuous discount rate corresponding to the limit length zero (cf. Grubbström, 1980).

5. The supplier provides a trade credit to the buyer with progressive interest rates. If the buyer pays before time M, the supplier does not charge any interest, whereas in case the buyer pays between times M and N with $M < N$, the supplier charges an interest at the rate of Ic_1. In case the buyer pays after time N, the supplier charges an interest at the rate of Ic_2, with $Ic_2 > Ic_1$.
6. The buyer has the option to deposit money in an interest bearing account with a fixed interest rate of Ie. Thus, the buyer may use sales revenues to earn interest until the account is completely settled. Other investment decisions that are not related to the lot sizing problem are not considered.

In addition, the following terminology is used throughout the paper:

A	cost of placing an order
C	unit purchasing cost with $C < P$
D	demand rate per unit of time
δ	percentage mark-up of the buyer
h	physical unit holding cost per unit per unit of time
Ic_1	interest rate per unit of time charged during the first credit period
Ic_2	interest rate per unit of time charged during the second credit period
Ie	interest rate on deposits per unit of time
M	first credit period offered by the supplier
N	second credit period offered by the supplier
P	selling price per unit
Q	order quantity
r	annual discount rate
ρ	continuous discount rate
T	replenishment time interval
V	net present value of the cash flows

Model Development

This paper considers a buyer facing a constant customer demand rate D. Thus, inventory continuously decreases, and the inventory level at time t, $I(t)$, can be described by the following differential equation

$$\frac{dI(t)}{dt} = -D, \quad 0 \leq t \leq T, \tag{1}$$

with the boundary conditions $I(0) = Q$ and $I(T) = 0$. The solution of the differential equation is

$$I(t) = D(T - t), \quad 0 \leq t \leq T. \tag{2}$$

The economic order quantity per replenishment, which equals the inventory level at $t = 0$, equals $Q = DT$.

All cash flows are considered with regard to their time of occurrence, i.e. they are evaluated by their corresponding time-value of money. This can be done by taking the net present value of the costs and earnings that occur in all replenishment cycles starting from time $t = 0$. In a first step, all costs and earnings that occur within a replenishment cycle have to be discounted to take account of their net present value at the beginning of the respective cycle. Subsequently,

all the cyclic cash flows have to be discounted again to consider their value at the beginning of the planning horizon, i.e. at time $t = 0$. The resulting net present values of all relevant future costs and earnings are consequently used to determine the economic replenishment interval.
The total relevant costs per replenishment cycle consist of the sum of ordering, inventory carrying and interest costs, reduced by interest earnings. The cost for placing an order at the supplier at time $t = 0$ equals

$$OC = A. \tag{3}$$

The net present value of inventory holding cost per replenishment cycle amounts to:

$$IHC = h\int_0^T I(t)e^{-\rho t}dt = hD(e^{-\rho T} + \rho T - 1)/\rho^2. \tag{4}$$

Depending on the length of the replenishment cycle T, the ratio of the interest rates of the buyer and the supplier (i.e., the ratio of Ie to Ic_1 and to Ic_2) and the lengths of the credit periods, M and N–M, the buyer incurs interest costs and/or realizes interest earnings. Accordingly, different cases for determining the net present value of the total costs have to be distinguished. The relevant cases will be discussed in more detail in the following.

Case 1: $Ie \leq Ic_1 < Ic_2$

Case 1.1: $T \leq M$
In this case, the buyer sells off the entire batch of $Q = DT$ units at time T with $T \leq M$, and is able to settle the account completely before the supplier starts charging interest at time M. During the period $[0,M]$, the buyer deposits sales revenues in an interest bearing account to generate interest earnings at the rate Ie. Between times 0 and T, sales revenues accumulate until the total revenue, PDT, is available at time T. The present value of interest earned can be written as:

$$IE_{1.1} = IeP\left(\int_0^T Dte^{-\rho t}dt + DT(M-T)e^{-\rho M}\right) = IePD\big((M-T)Te^{-\rho M} + (1-(1+\rho T)e^{-\rho T})/\rho^2\big) \tag{5}$$

To avoid interest payments to the supplier, the buyer settles the balance at time M, such that $IC_{1.1} = 0$. The net present value of all the relevant future costs and earnings thus amounts to

$$V_{1.1} = \frac{1}{1-e^{-\rho T}}\left(A + hD(e^{-\rho T} + \rho T - 1)/\rho^2 - IePD\big((M-T)Te^{-\rho M} + (1-(1+\rho T)e^{-\rho T})/\rho^2\big)\right) \tag{6}$$

The optimal solution to Eq. (6) is the solution of the following non-linear equation (provided that the second derivation of Eq. (6) with respect to T is positive for all $T > 0$, see also Appendix Figure a) for an illustration of the derivative function):

$$\frac{dV_{1.1}}{dT} = -\frac{e^{\rho T}}{\rho(e^{\rho T}-1)^2}\left(\rho^2 A - hD(e^{\rho T} - \rho T - 1) + IePD\Big(e^{-\rho T} + \rho T - 1 + \rho\big((M-2T)e^{\rho T} + (\rho T + 2)T - (\rho T + 1)M\big)e^{-\rho M}\Big)\right) = 0 \tag{7}$$

Case 1.2: $M < T \leq N$

In the case where $Ie < Ic_1$ and $M < T \leq N$, the buyer settles as much of the unpaid balance as possible at time M to minimize interest payments. In the period $[0,M]$, the buyer sells DM products and generates direct revenues in the amount of PDM dollars. Until time M, sales revenues that are realized are again continuously deposited in an interest bearing account that earns interest at the rate of Ie per unit of time. At time M, the buyer then uses the sum of revenues and interest earnings to pay the supplier. Depending on the ratio of the total purchase cost, which amount to CDT dollars for a lot of size DT, to the sum of earnings from sales and interest received at time M, two different subcases may arise that will be discussed in the following

Case 1.2-1: $CDT \leq PDM(1 + IeM/2)$

In subcase 1, the sum of sales revenues and interest earned at time M exceeds the unpaid balance, i.e. $CDT \leq PDM(1 + IeM/2)$, such that the buyer is able to settle the entire balance. The present value of interest earned until time M is calculated as

$$IE_{1.2-1} = IeP \int_0^M Dte^{-\rho t} dt = IePD(1 - (1 + \rho M)e^{-\rho M})/\rho^2 \tag{8}$$

Since there are no outstanding payments at time M, the buyer does not have to pay any interest to the supplier in this subcase (i.e., $IC_{1.2-1} = 0$). The net present value of all relevant future costs and earnings thus amounts to:

$$V_{1.2-1} = \frac{1}{1-e^{-\rho T}} \left(A + hD(e^{-\rho T} + \rho T - 1)/\rho^2 - IePD(1 - (1 + \rho M)e^{-\rho M})/\rho^2\right) \tag{9}$$

The optimal solution to Eq. (9) is the solution of the following non-linear equation (provided that the second derivation of Eq. (9) with respect to T is positive for all $T > 0$, see also Appendix Figure b) for an illustration of the derivative function):

$$\frac{dV_{1.2-1}}{dT} = -\frac{e^{\rho T}}{\rho(e^{\rho T}-1)^2} \left(\rho^2 A - hD(e^{\rho T} - \rho T - 1) - IePD\big((1 + \rho M)e^{-\rho M} - 1\big)\right) = 0 \tag{10}$$

Case 1.2-2: $CDT > PDM(1 + IeM/2)$

In contrast to the previous subcase, the buyer is now unable to settle the balance completely at time M, which occurs in case $CDT > PDM(1 + IeM/2)$. As a result, the supplier starts charging interest on the unpaid balance at the rate Ic_1 at time M. Considering sales revenues realized and interest earned in the period $[0,M]$, the open account at time M amounts to $CDT - PDM(1 + IeM/2)$, and the present value of the interest earned until time M is the one given in Eq. (8). To minimize interest payments, the buyer transfers each dollar earned after time M directly to the supplier, which leads to a constantly decreasing outstanding balance. For the case where the unpaid balance cannot be settled at time M, but before time N, the present value of the interest cost can be formulated as follows:

$$\begin{aligned} IC_{1.2-2} &= Ic_1 \int_M^{M+z_1} \Big(\big(CDT - PDM(1 + IeM/2)\big) - PD(t - M)\Big) e^{-\rho t} dt = \\ &Ic_1 PD\left(e^{-\rho\left(TC/P - IeM^2/2\right)} + e^{-\rho M}(\rho T\, C/P + \rho M(1 + Ie\, M/2) - 1)\right)/\rho^2 \end{aligned} \tag{11}$$

where $M + z_1$ denotes the point in time when the unpaid balance has been completely settled, with $z_1 = \big(CDT - PDM(1 + IeM/2)\big)/PD$. Thus, the net present value of all relevant future costs and earnings amounts to:

$$V_{1.2-2} = \frac{1}{1-e^{-\rho T}}\Big(A + hD(e^{-\rho T} + \rho T - 1)/\rho^2 +$$
$$Ic_1 PD\left(e^{-\rho(TC/P - IeM^2/2)} + e^{-\rho M}(\rho T\, C/P + \rho M(1 + Ie\, M/2) - 1)\right)/\rho^2 -$$
$$IePD(1 - (1 + \rho M)e^{-\rho M})/\rho^2\Big) \tag{12}$$

The optimal solution to Eq. (12) is the solution of the following non-linear equation (provided that the second derivation of Eq. (12) with respect to T is positive for all $T > 0$, see also Appendix Figure c) for an illustration of the derivative function):

$$\frac{dV_{1.2-2}}{dT} = -\frac{e^{\rho T}}{\rho(e^{\rho T}-1)^2}\Bigg(\rho^2 A - hD(e^{\rho T} - \rho T - 1) - IcPD\Bigg(\frac{C}{P}e^{\rho(T-M)} + \Big(\frac{C}{P}(1 - e^{\rho T}) -$$
$$1\Big)e^{-\rho\left(\frac{C}{P}T - \frac{IeM^2}{2}\right)} - \left(\frac{C}{P}(1 + T\rho) - \rho\left(M + \frac{IeM^2}{2}\right) - 1\right)e^{-M\rho}\Bigg) + IePD\big((1 + \rho M)e^{-\rho M} -$$
$$1\big)\Bigg) = 0 \tag{13}$$

Case 1.3: $N < T$
The case where $Ie < Ic_1$ and $T > N$ is similar to Case 1.2. Again, the buyer uses the sum of revenues and interest earned to pay the supplier. To minimize interest payments, he/she settles as much of the outstanding balance as possible at time M, and afterwards reduces the outstanding amount continuously by transferring each dollar earned directly to the supplier's account. The unpaid balance at time N can be calculated by considering interest charges that accrue between times M and N which leads to $CDT - PDM(1 + IeM/2) - PD(N - M) + Ic_1 \int_M^N \left(\big(CDT - PDM(1 + IeM/2)\big) - PD(t - M)\right) dt$. Integrating over the limits and rearranging leads to an unpaid balance at time N of $\big(CDT - PDM(1 + IeM/2)\big)\big(1 + Ic_1(N - M)\big) - PD(N - M)(1 + Ic_1(N - M)/2)$. Accor-ding to the ratio of the total purchase cost to the sum of sales revenues and interest earnings, three possible subcases may arise that differ according to the balance of the buyer's account at times M and N.

Case 1.3-1: $CDT \leq PDM(1 + IeM/2)$
The case where $Ie < Ic_1$ and $T > N$ is identical to Subcase 1.2-1. Thus, the buyer settles the open account completely at time M without paying any interest to the supplier.

Case 1.3-2: $CDT > PDM(1 + IeM/2)$ and $\big(CDT - PDM(1 + IeM/2)\big)\big(1 + Ic_1(N - M)\big) \leq PD(N - M)(1 + Ic_1(N - M)/2)$

The case where $Ie < Ic_1$ and $T > N$ is identical to Subcase 1.2-2. Consequently, the buyer is unable to settle the entire account at time M, but reduces the open account as much as possible at time M. Afterwards, the open account is again continuously reduced by transferring each dollar earned from sales to the supplier. The account is entirely settled at time $M + z_1$ with $M + z_1 < N$, and the supplier charges interest on the unpaid balance between times M and $M + z_1$.

Case 1.3-3: $CDT > PDM(1 + IeM/2)$ and $\big(CDT - PDM(1 + IeM/2)\big)\big(1 + Ic_1(N - M)\big) > PD(N - M)(1 + Ic_1(N - M)/2)$

In the case where $Ie < Ic_1$ and $T > N$, the buyer is not able to pay off the total purchase and interest costs at times M or N. Thus, he/she settles as much of the balance as possible at time M. Afterwards, the open account is continuously reduced by transferring each dollar earned from sales to the supplier. The supplier, in turn, charges interest on the unpaid balance at the rate Ic_1 between times M and N and at the rate Ic_2 after time N. Before time M, the buyer realizes interest earnings at the rate Ie equal to those of Eq. (8). Considering both the interest charges that accumulate between times M and N and the transfer payments the buyer makes between times M and N to reduce the debt, the present value of the interest cost for this subcase, $IC_{1.3\text{-}3}$, amounts to

$$IC_{1.3-3} = Ic_1 \int_M^N \Big(\big(CDT - PDM(1 + IeM/2)\big) - PD(t - M) \Big) e^{-\rho t} dt + Ic_2 \int_N^{N+z_2} \Big(\big(CDT - PDM(1 + IeM/2)\big)\big(1 + Ic1(N - M)\big) - PD(N - M)(1 + Ic1(N - M)/2) - PD(t - N) \Big) e^{-\rho t} dt = \frac{Ic_1 PD}{\rho^2} e^{-\rho M} (\rho T\, C/P - \rho M(1 + IeM/2) - 1) - \frac{Ic_1 PD}{\rho^2} e^{-\rho N} \big(\rho T\big(1 - Ic_2(N - M)\big)\, C/P - \rho\big(1 - Ic_2(N - M)\big)\, IeM^2/2 + \rho\, Ic_2(N^2 - M^2)/2 - \rho N - 1\big) + \frac{Ic_2 PD}{\rho^2} \Big(e^{-\rho N} (\rho T\, C/P - \rho M(N/M + IeM/2) - 1) + e^{-\rho\left(T\left(1 - Ic_1(M-N)\right) C/P - \left(1 + Ic_1(N-M)\right) IeM^2/2 - Ic_1\left(N^2 - M^2\right)/2\right)} \Big) \tag{14}$$

where $N + z_2$ is the point in time when the unpaid balance has been completely settled, with $z_2 = \Big(\big(CDT - PDM(1 + IeM/2) - PD(N - M)\big)\big(1 + Ic_1(N - M)\big) + Ic_1 PD(N - M)^2/2 \Big) / PD$. The net present value of all relevant future costs and earnings for this case amounts to:

$$V_{1.3-3} = \frac{1}{1 - e^{-\rho T}} \Big(A + \frac{hD\left(\rho T + e^{-\rho T} - 1\right)}{\rho^2} + \frac{Ic_1 PD}{\rho^2} e^{-\rho M} (\rho T\, C/P - \rho M(1 + IeM/2) - 1) - \frac{Ic_1 PD}{\rho^2} e^{-\rho N} \big(\rho T\big(1 - Ic_2(N - M)\big)\, C/P - \rho(1 - Ic_2(N - M))\, IeM^2/2 + \rho\, Ic_2(N^2 - M^2)/2 - \rho N - 1\big) + \frac{Ic_2 PD}{\rho^2} \Big(e^{-\rho N} (\rho T\, C/P - \rho M(N/M + IeM/2) - 1) + e^{-\rho\left(T\left(1 - Ic_1(M-N)\right) C/P - \left(1 + Ic_1(N-M)\right) IeM^2/2 - Ic_1\left(N^2 - M^2\right)/2\right)} \Big) - IePD(1 - (1 + \rho M) e^{-\rho M})/\rho^2 \Big) \tag{15}$$

The optimal solution to Eq. (15) is the solution of the following non-linear equation (provided that the second derivation of Eq. (15) with respect to T is positive for all $T > 0$, see also Appendix Figure d) for an illustration of the derivative function)

$$\frac{dV_{1.3-3}}{dT} = -\frac{e^{\rho T}}{\rho (e^{\rho T} - 1)^2} \Big(\rho^2 A - hD(e^{\rho T} - \rho T - 1) - Ic_1 PD e^{-\rho M} (1 + \rho M(1 + IeM/2) + (e^{\rho T} - \rho T - 1)\, C/P) + Ic_1 PD e^{-\rho N} \big(1 + \rho(N + IeM^2/2) - \rho\big(N + M(1 + IeM)\big)\, Ic_2(N - M)/2 + \big(1 - Ic_2(N - M)\big)(e^{\rho T} - \rho T - 1)\, C/P\big) - Ic_2 PD e^{-\rho N} (1 + \rho(N + IeM^2/2) + (e^{\rho T} - \rho T - 1)\, C/P) + Ic_2 PD e^{-\rho\left(T\left(1 + Ic1(n-m)\right) C/P - IeM^2/2 - \left(N + M(1 + IeM)\right) Ic_1(N-M)/2\right)} \big(1 + (e^{\rho T} - 1)\big(1 + Ic_1(N - M)\big)\, C/P\big) + IePD\big((1 + \rho M) e^{-\rho M} - 1\big) \Big) = 0 \tag{16}$$

Case 2: $Ic_1 < Ie < Ic_2$

Case 2.1: $T \leq M$

For $Ie > Ic_1$ and $T \leq M$, the buyer may realize a profit from keeping the sales revenue in an interest bearing account until time N. Between times M and N, he/she has to pay interest to the supplier. However, since $Ie > Ic_1$, the interest earned exceeds the interest paid during this period. Similar to Subcase 1.1, the present value of the interest earned can be calculated as

$$IE_{2.1} = IeP\left(\int_0^T Dte^{-\rho t}dt + DT(N-T)e^{-\rho N}\right) = IePD\big((N-T)Te^{-\rho N} + (1-(1+\rho T)e^{-\rho T})/\rho^2\big) \quad (17)$$

The present value of the interest cost that accrues between times M and N amounts to

$$IC_{2.1} = Ic_1 CDT(N-M)e^{-\rho N} \quad (18)$$

The net present value of all relevant future costs and earnings thus equals

$$V_{2.1} = \frac{1}{1-e^{-\rho T}}\Big(A + h\,D(e^{-\rho T} + \rho T - 1)/\rho^2 + Ic_1 CDT(N-M)e^{-\rho N} - IePD\big((N-T)Te^{-\rho N} + (1-(1+\rho T)e^{-\rho T})/\rho^2\big)\Big) \quad (19)$$

The optimal solution to Eq. (19) is the solution of the following non-linear equation (provided that the second derivation of Eq. (19) with respect to T is positive for all $T > 0$, see also Appendix Figure e) for an illustration of the derivative function)

$$\frac{dV_{2.1}}{dT} = -\frac{e^{\rho T}}{\rho(e^{\rho T}-1)^2}\Big(\rho^2 A - hD(e^{\rho T} - \rho T - 1) - \rho Ic_1 CD(N-M)e^{-\rho N}(e^{\rho T} - T\rho - 1) + IePD\big(e^{-\rho T} + \rho T - 1 + \rho\big((N-2T)e^{\rho T} + (\rho T + 2)T - (\rho T + 1)N\big)e^{-\rho N}\big)\Big) = 0 \quad (20)$$

Case 2.2: $M < T \leq N$

The case where $M < T < N$ and $Ic_1 < Ie$ is identical to Case 2.1. The buyer accepts interest charges between times M and N and realizes interest earning by depositing sales revenues in an interest bearing account. The account is settled completely at time N.

Case 2.3: $N < T$

In the case where $Ic_1 < Ie$ and $N < T$, the buyer settles as much of the unpaid balance as possible at time N to minimize interest payments. Until time N, the buyer sells a total quantity of DN units and generates revenues totaling PDN dollars. The buyer deposits sales revenues in an interest bearing account that earns interest at the rate Ie. At time N, the buyer may again use the sum of sales revenues and interest earnings to settle the open account. According to the ratio of the total purchase cost to the total sales and interest earnings, two possible subcases may arise that can be distinguished according to the buyer's balance at time N, which equals $CDT\big(1 + Ic_1(N-M)\big) - PDN(1 + IeN/2)$.

Case 2.3-1: $CDT\big(1 + Ic_1(N-M)\big) \leq PDN(1 + IeN/2)$

In this subcase, the earnings from interest and sales at time N exceed the purchase cost wherefore the account is completely settled at time N. The present value of the interest earned is then given as:

$$IE_{2.3-1} = IeP\int_0^N Dte^{-\rho t}dt = IePD(1-(1+\rho N)e^{-\rho N})/\rho^2 \quad (21)$$

The present value of the interest charged by the supplier is in this case the same as those given in Eq. (18). Thus, the net present value of all relevant future costs and earnings can be calculated as

$$V_{2.3-1} = \frac{1}{1-e^{-\rho T}}(A + hD(e^{-\rho T} + \rho T - 1)/\rho^2 + Ic_1 CDT(N-M)e^{-\rho N} - IePD(1-(1+\rho N)e^{-\rho N})/\rho^2) \tag{22}$$

The optimal solution to Eq. (22) is the solution of the following non-linear equation (provided that the second derivation of Eq. (22) with respect to T is positive for all $T > 0$ see also Appendix Figure f) for an illustration of the derivative function):

$$\frac{dV_{2.3-1}}{dT} = -\frac{e^{\rho T}}{\rho(e^{\rho T}-1)^2}\Big(\rho^2 A - hD(e^{\rho T} - \rho T - 1) - \rho Ic_1 CD(N-M)e^{-\rho N}(e^{\rho T} - T\rho - 1) + IePD\big((1+\rho N)e^{-\rho N} - 1\big)\Big) = 0 \tag{23}$$

Case 2.3-2: $CDT\big(1 + Ic_1(N-M)\big) > PDN(1 + IeN/2)$

For the case where $Ie > Ic_1$ and where the buyer is unable to settle the balance entirely at time N, the buyer settles as much of the unpaid balance as possible at time N. Subsequently, the open account is continuously reduced by transferring each dollar earned from sales to the supplier's account until the balance has been completely settled. The net present value of the interest earned until time N is again the same as the one given in Eq. (21), and the present value of the interest charges can be formulated as:

$$IC_{2.3-2} = \frac{Ic_1}{T}CDT(N-M)e^{-\rho N} + \frac{Ic_2}{T}\int_N^{N+z_3}\Big(\Big(CDT\big(1 + Ic_1(N-M)\big) - PDN(1 + IeN/2)\Big) - PD(t-N)\Big)e^{-\rho t}dt = Ic_1 CDT(N-M)e^{-\rho N} + Ic_2 PD\Big(e^{-\rho N}\big(\rho T\big(1 + Ic_1(N-M)\big)C/P - \rho N(1 + IeN/2) - 1\big) + e^{-\rho\left(T\left(1-Ic_1(M-N)\right)C/P - IeN^2/2\right)}\Big)/\rho^2 \tag{24}$$

where $N+z_3$ denotes the point in time when the unpaid balance has been settled completely, with $z_3 = \Big(CDT\big(1 + Ic_1(N-M)\big) - PDN(1 + IeN/2)\Big)/PD$. The net present value of all relevant future costs and earnings for this case amounts to:

$$V_{2.3-2} = \frac{1}{1-e^{-\rho T}}\Big(A + hD(e^{-\rho T} + \rho T - 1)/\rho^2 + Ic_1 CDT(N-M)e^{-\rho N} + Ic_2 PD\Big(e^{-\rho N}\big(\rho T\big(1 + Ic_1(N-M)\big)C/P - \rho N(1 + IeN/2) - 1\big) + e^{-\rho\left(T\left(1-Ic_1(M-N)\right)C/P - IeN^2/2\right)}\Big)/\rho^2 - IePD(1-(1+\rho N)e^{-\rho N})/\rho^2\Big) \tag{25}$$

The optimal solution to Eq. (25) is the solution of the following non-linear equation (provided that the second derivation of Eq. (25) with respect to T is positive for all $T > 0$, see also Appendix Figure g) for an illustration of the derivative function):

$$\frac{dV_{2.3-2}}{dT} = -\frac{e^{\rho T}}{\rho(e^{\rho T}-1)^2}\Bigg(\rho^2 A - hD(e^{\rho T} - \rho T - 1) - \rho Ic_1 CD(N-M)e^{-\rho N}(e^{\rho T} - T\rho - 1) + Ic_2 PD\Big(e^{-\rho N}\Big((1 + T\rho - e^{\rho T})\big(1 + Ic_1(N-M)\big)C/P - \big(1 + \rho(N + IeN^2/2)\big)\Big) +$$

$$e^{-\rho\left(T\left(1+Ic_1(N-M)\right)C/P-IeN^2/2\right)}\left(1+(e^{\rho T}-1)\left(1+Ic_1(N-M)\right)C/P\right)+IePD\left((1+\rho N)e^{-\rho N}-1\right)\right)=0 \quad (26)$$

Although the objective functions are too complex for formal proofs of convexity, a good, but not necessarily optimal, solution for the replenishment interval in the presence of a progressive payment scheme and time-value of money can be found using the following iterative procedure (see Soni and Shah, 2008 or Soni and Shah, 2009, for a similar approach)

Step1: If $Ie \leq Ic_1$, then calculate $\hat{T}$ from Eq. (7) and go to Step2, else calculate $\hat{T}$ from Eq. (20) and go to Step5.
Step2: If $\hat{T} \leq M$, then $T^* = \hat{T}$ and stop, else if $\hat{T} \leq N$ then calculate $\hat{T}$ from Eq. (10) and go to Step 3, else calculate $\hat{T}$ from Eq. (10) and go to Step4.
Step3: If $CD\hat{T} \leq PDM(1 + IeM/2)$, then $T^* = \hat{T}$ and stop, else calculate T^* from Eq. (13) and stop.
Step4: If $CD\hat{T} \leq PDM(1 + IeM/2)$, then $T^* = \hat{T}$ and stop, else if $CD\hat{T} > PDM(1 + IeM/2)$ and $\left(CD\hat{T} - PDM(1 + IeM/2)\right)\left(1 + Ic_1(N - M)\right) \leq PD(N - M)(1 + Ic_1(N - M)/2)$, then calculate T^* from Eq. (13) and stop, else calculate T^* from Eq. (16) and stop.
Step5: If $\hat{T} \leq N$, then $T^* = \hat{T}$ and stop, else calculate $\hat{T}$ from Eq. (23) and go to Step6.
Step6: If $CD\hat{T}\left(1 + Ic_1(N - M)\right) \leq PDN(1 + IeN/2)$, then $T^* = \hat{T}$ and stop, else calculate T^* from Eq. (26) and stop.

Based on the two different interest scenarios (i.e. $Ie < Ic_1 < Ic_2$ and $Ic_1 < Ie < Ic_2$), the implemented heuristic procedure consecutively examines the relevant subcases to identify their cost-efficient replenishment cycles and returns the cycle time that falls within the respective range of validity. Even though there is no guarantee that this procedure results in the optimal solution for the cycle time, numerical evaluations indicated that the objective function is piecewise-convex for the considered parameter settings. In this case, our procedure would find the optimal solution for T, otherwise the result might only be a local minimum.

Computational Results
To illustrate the behavior of the models developed in Section 3, different numerical experiments are performed based on the parametric values shown in Table 1. In the first instance, comparing the buyer's replenishment decisions for different trade credit conditions offered by the supplier in terms of interest relations and lengths of the credit periods (cf. Tables 2 and 3) illustrates the impact of the financial conditions on the optimal ordering and payment behavior. Subsequently, a sensitivity analysis indicates the influence of the prevailing cost structure and the economic conditions on the ordering decision and total discounted cost (cf. Table 4). In the course of this, a one-at-a-time method is employed to study the change in the model output followed on a change of a single input parameter (see Borgonovo, 2010, for a discussion of sensitivity analysis in inventory management and Sarkar, 2012, for a similar approach).

Table 1 *Model Parameters*

A	=	100	Ordering Cost Per Order
C	=	20	Unit Purchase Cost
D	=	1000	Annual Customer Demand
h	=	4	Inventory Holding Cost Per Unit And Year
Ic_1	=	0.06	Interest Rate Per Year For The First Credit Period
Ic_2	=	0.12	Interest Rate Per Year For The Second Credit Period
Ie	=	{0.04,0.08}	Interest Rate On Deposits For The Retailer
M	=	30	First Permissible Credit Period
N	=	60	Second Permissible Credit Period
P	=	30	Unit Selling Price
r	=	0.10	Annual Discount Rate

The numerical examples (cf. Table 2) indicate that for a low interest rate on deposits of 4%, an increase in the length of the first credit period (*M*) has only minor influences on the quantity ordered and the length of the replenishment cycles. The buyer, however, has an incentive to increase his/her order quantities slightly. The net present value of the total costs, in turn, is reduced as *M* adopts higher values due to the saving of interest cost that result from deferring the payment to the supplier. An increase in the length of the second credit period (*M*–*N*) neither influences the ordering policy nor the present value of the total costs (the fact that the length of the second credit period has no influence on the ordering policy and total cost is caused by the specific setting considered here, where the balance is completely settled before time *N*).
For high interest rates on deposits (cf. Table 3), in contrast, the first credit period (*M*) again does not influence the buyer's ordering policy significantly, whereas an increase of the second credit period length (*N*-*M*) induces smaller order quantities. This seems not very intuitive as trade credits are intended to enable buyers to increase their order quantities as the time-value of money effectively lowers the price as frequently assumed in the literature (cf. Seifert et al., 2013). However, in the present case a relaxed trade credit policy that offers more generous payment cycles may induce contrary effects given that the financial conditions allow the buyer to gain interests by depositing money in an interest bearing account or by investing it elsewhere that exceed the interest charged by the supplier on the outstanding balance. This is obviously a reaction of the buyer to maximize annual interest earnings. In the case where $Ic_1 < Ie < Ic_2$, the interest gains exceed the interest cost between time M and time N. If N is increased, the buyer may have an incentive to take advantage of potentially higher interest gains and by reducing the length of the replenishment cycles, the annual interest earnings will be increased. Finally, it can be seen that an increase in both credit periods reduces the present value of the total costs.

Table 2 *Effect of M and N on ordering decision with Ie = 4%*

M→ / N↓	**20/365**	**30/365**	**40/365**
50/365	T = 0.2044 Q = 204.44 V = 9700.1	T = 0.2056 Q = 205.61 V = 9411.6	T = 0.2071 Q = 207.08 V = 9146.7
60/365	T = 0.2044 Q = 204.44 V = 9700.1	T = 0.2056 Q = 205.65 V = 9411.6	T = 0.2073 Q = 207.30 V = 9146.6
70/365	T = 0.2044 Q = 204.44	T = 0.2056 Q = 205.65	T = 0.2073 Q = 207.30

	V = 9700.1	V = 9411.6	V = 9146.6

Table 3 *Effect of M and N on ordering decision with Ie = 8%*

M→ / ***N***↓	**20/365**	**30/365**	**40/365**
50/365	T = 0.1962 Q = 196.23 V = 9353.7	T = 0.1963 Q = 196.31 V = 9010.1	T = 0.1964 Q = 196.38 V = 8666.4
60/365	T = 0.1835 Q = 183.45 V = 9149.0	T = 0.1835 Q = 183.52 V = 8806.5	T = 0.1836 Q = 183.59 V = 8463.9
70/365	T = 0.1768 Q = 176.76 V = 8819.7	T = 0.1768 Q = 176.81 V = 8477.9	T = 0.1768 Q = 176.85 V = 8136.4

A comparison of Tables 2 and 3 also reveals that a high interest rate Ie with $Ic_1 < Ie < Ic_2$ leads to a reduction of the replenishment quantities up to 15% and a significantly reduced present value of the total costs for all considered scenarios. Consequently, taking into account the temporal allocation of payments, the prevailing interest relation significantly influences the replenishment decisions and the present value of the total costs. This becomes especially clear when comparing our results to those of average cost approaches (e.g., Glock et al. 2014b; 2015), where the replenishment quantities were found to be quite insensitive to changes in the interest relation.

Table 4 *Effect of Cost and Economic Parameters on Ordering Decision*

Parameters change		**Scenario 1 with *Ie* = 4%**			**Scenario 2 with *Ie* = 8%**		
		Δ*T*	**Δ*Q***	**Δ*V***	**Δ*T***	**Δ*Q***	**Δ*V***
A	**-50%**	-28.50%	-28.51%	-31.88%	-31.77%	-31.78%	-39.57%
	-25%	-13.04%	-13.07%	-14.63%	-16.57%	-16.56%	-18.02%
	+25%	11.58%	11.53%	12.95%	16.95%	16.91%	15.11%
	+50%	21.69%	21.64%	24.69%	31.66%	31.65%	28.31%
h	**-50%**	28.45%	28.43%	-26.01%	38.26%	38.25%	-25.65%
	-25%	12.40%	12.36%	-12.17%	15.37%	15.37%	-11.75%
	+25%	-9.00%	-9.01%	10.96%	-10.46%	-10.48%	10.35%
	+50%	-15.95%	-15.95%	21.00%	-15.97%	-15.96%	19.86%
δ	**-50%**	-1.75%	-1.78%	1.63%	3.87%	3.85%	3.44%
	-25%	-0.83%	-0.83%	0.74%	1.96%	1.95%	1.73%
	+25%	0.73%	0.72%	-0.62%	-1.96%	-1.98%	-1.77%
	+50%	1.41%	1.37%	-1.14%	-3.98%	-4.01%	-3.57%
D	**-50%**	37.74%	-31.14%	-27.07%	50.95%	-24.52%	-24.25%
	-25%	15.13%	-13.66%	-12.27%	22.02%	-8.48%	-10.24%
	+25%	-10.26%	12.12%	10.58%	-13.84%	7.68%	7.29%
	+50%	-17.85%	23.16%	19.97%	-21.31%	18.05%	12.89%
r	**-50%**	0.15%	0.13%	94.70%	0.11%	0.08%	94.72%
	-25%	0.10%	0.06%	31.57%	0.05%	0.04%	31.58%
	+25%	-0.05%	-0.07%	-18.95%	0.00%	-0.04%	-18.96%
	+50%	-0.10%	-0.13%	-31.59%	-0.05%	-0.08%	-31.60%

Table 4, in addition, illustrates the sensitivity of the ordering policy and the related present value of the total cost regarding changes in the prevailing cost structure and the economic conditions (note that input parameters were changed -50%, -25%, +25% and +50% according to the base case). An increase in the buyer's ordering and holding costs obviously lead to an increase of the total discounted cost for both interest scenarios. The effect on the replenishment interval and the order quantity, however, is different. Whereas increasing order cost induce larger order quantities, larger holding costs lead to smaller order quantities. Both effects tend to be slightly more pronounced in the second scenario. An increase in the buyer's mark-up leads to a lower net present value of the total cost, which is a result of higher interest earnings at the buyer. It can be observed that this effect is slightly stronger for a high relative interest rate at the buyer, i.e. for a high difference $Ie - Ic_1$. This may be explained by the fact that the net profit from interest raises in this case. As to the order quantity, it could be observed that Q increases in the buyer's mark-up for low interest rates, while the opposite effect was observed for the case where interest rates are high. An increasing demand rate induces increasing order quantities and higher total discounted cost, whereas the replenishment interval is reduced at the same time. Finally, an increase in the discount rate, in turn, obviously leads to a lower net present value of the total costs. The influence of r on the replenishment quantity, however, is ambiguous. If the interest rate on deposits is low, then the order quantity is reduced as r increases, while there is nearly no effect observable for the case of high interest rates. Moreover, whereas changes in the annual discount rate have significant influence on the net present value of the total cost, the impact on the replenishment policy is rather negligible for both scenarios.

Conclusion

The paper at hand studied the optimal ordering and payment behavior in the presence of trade credits with a progressive interest scheme. In this case, a buyer is not required to pay immediately after the receipt of an order, but is instead allowed to postpone the payment to its supplier. The supplier, in turn, offers a sequence of three credit periods, where the interest rate that is charged on the outstanding balance increases from period to period. In such a scenario, there exist various options for settling the open account, where the financial impact of each option depends on the current credit interest structure and the alternative investment conditions. The contribution to the literature is twofold. First, the different financial conditions of the companies involved are taken into account by assuming that the credit interest rate of the buyer may, but not necessarily has to, exceed the interest rate charged by the supplier. In such a scenario, it would not be rational from the buyer's point of view to settle the unpaid balance as soon as interest is charged on the outstanding balance. Instead, the buyer should keep the sales revenue invested until the interest charged by the supplier exceeds the incomes from the investment, or just before the next order is issued. In addition, the paper extended prior research on trade credits with progressive interest rates by considering the time-value of money in this scenario which is relevant when trade credit terms are valid for a long period of time and payment flows should be evaluated by their net present value. The results of the paper indicate that taking into account the time-value of money and the interest relation significantly influences the replenishment policy of the buyer and his/her financial performance. Whereas the net present value of the total cost decreases as the discount rate increases, the influence on the order quantities is ambiguous regarding the prevailing interest structure. In addition, taking into account the prevailing ratio of interest rates on deposits and liabilities significantly influences the ordering and payment behavior of the buyer and may lead to an improved financial performance in the long run.

From a managerial perspective, considering the prevailing interest structure that governs the payment and replenishment policy of the buyer is indispensable for minimizing the present value of the total cost. Neglecting characteristics of financial conditions in determining order sizes and payment intervals may lead to inferior order and payment policies, which unnecessarily increases the present value of the total costs. However, the current discount rates only influence the level of the net present value rather than the optimal payment and replenishment policy. The results of this paper illustrate the close linkage between operational and financial aspects, which should be fostered by employing integrated planning approaches. Moreover, the results reveal that economic conditions also influence the configuration of trade credits offered by the supplier. Ignoring the prevailing interest structure, the credit-related incentives set by the supplier may induce contrary effects and worsen the supplier's financial position.

The model presented in this paper could be extended into various directions. Future research could study alternative demand functions, for example functions that assume that demand is dependent on the inventory level on hand. Earlier research has shown that in the presence of stock-dependent demand, orders should be placed earlier, such that a positive inventory level occurs at the end of each cycle (e.g., Teng et al., 2011). In addition, we note that our analysis concentrated on investigating the impact of financial regulations on the buyer's replenishment policy. Other aspects that are also of high importance for inventory replenishment decisions, such as product quality (see, e.g., Lo Sorto 2015), for example, were not considered in our model. Clearly, integrating other product and contract characteristics that influence inventory replenishment decisions at the buyer into our model could lead to additional interesting insights. Another research opportunity is that multi-product scenarios and the resulting scheduling problems could be studied in a trade-credit scenario, as this topic has not received much attention in the literature so far (a related work would be the one of Savino et al. 2010). The same applies to environmental issues (see, e.g., Savino and Apolloni 2007; Manzini et al. 2015) that also have not been studied in trade credit inventory models frequently in the past. Finally, the presented approach could be extended by considering different product characteristics, for example by introducing a limited shelf live (cf. Chang et al, 2010) or different types of trade credit contracts offered by the supplier (cf. Chung and Liao, 2009).

References

Balkhi, Z.T. (2011): Optimal economic ordering policy with deteriorating items under different supplier trade credits for finite horizon case. International Journal of Production Economics, 133 (1), 216-223.

Beullens, P.; Janssens, G.K. (2014): Adapting inventory models for handling various payment structures using net present value equivalence analysis. International Journal of Production Economics, 157, 190-200.

Borgonovo, E. (2010): Sensitivity analysis with finite changes: An application to modified EOQ models. European Journal of Operational Research, 200 (1), 127-138.

Chang, C.-T. (2002): Extended economic order quantity model under cash discount and payment delay. Information and Management Sciences, 13 (3), 57-69.

Chang, C.-T.; Teng, J.-T. (2004): Retailer's optimal ordering policy under supplier credits. Mathematical Methods of Operations Research, 60 (3), 471-483.

Chang, C.-T.; Ouyang, L.-Y.; Teng, J.-T.; Cheng, M.-C. (2010): Optimal ordering policies for deteriorating items using a discounted cash-flow analysis when a trade credit is linked to order quantity. Computers & Industrial Engineering, 59 (4), 770-777.

Chung, K.H. (1989): Inventory control and trade credit revisited. Journal of the Operational Research Society, 40 (5), 495-498.

Chung, K.-J. (2009): A note on optimal ordering policies when the supplier provides a progressive interest scheme. European Journal of Operational Research, 199 (2), 611-617.

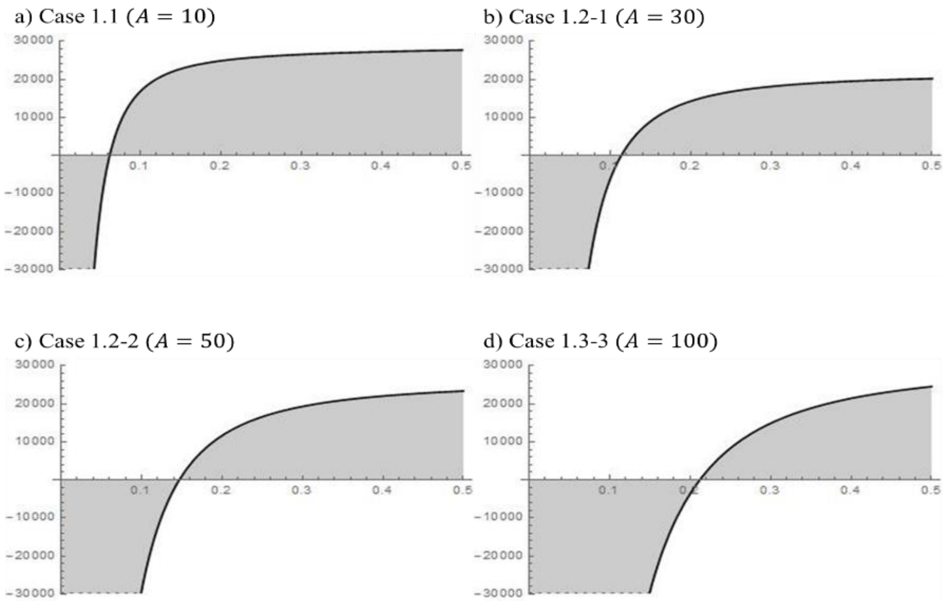
Chung, K.-J.; Liao, J.-J. (2006): The optimal ordering policy in a DCF analysis for deteriorating items when trade credit depends on the order quantity. International Journal of Production Economics, 100 (1), 116-130.
Chung, K.-J.; Liao, J.-J. (2009): The optimal ordering policy of the EOQ model under trade credit depending on the ordering quantity from the DCF approach. European Journal of Operational Research, 196 (2), 563-568.
Chung, K.-J.; Goyal, S.K.; Huang, Y.-F. (2005): The optimal inventory policies under permissible delay in payments depending on the order quantity. International Journal of Production Economics, 95 (2), 203-213.
Chung, K.-J.; Lin, S.-D. (2011): A complete solution procedure for the economic order quantity under conditions of a one-time-only extended permissible delay period in payments from the viewpoint of logic. Journal of Information and Optimization Sciences, 32 (1), 205-211.
European Central Bank (2013): Survey on the access to finance of small and medium-sized enterprises in the euro area: October 2012 to March 2013. April 2013, https://www.ecb.int/pub/pdf/other/accesstofinancesmallmediumsizedenterprises201304en.pdf?51cefb959e634f433ee48cfd80a10a13.
Gomm, M.L. (2010): Supply chain finance: applying finance theory to supply chain management to enhance finance in supply chains. International Journal of Logistics: Research and Applications, 13 (2), 133-142.
Glock, C.H.; Grosse, E.H.; Ries, J.M. (2014a): The lot sizing problem: A tertiary study. International Journal of Production Economics, 155 (1-3), 39-51.
Glock, C.H.; Ries, J.M.; Schwindl, K. (2014b): A note on: Optimal ordering policy for stock-dependent demand under progressive payment scheme. European Journal of Operational Research, 232 (2), 423-426.
Glock, C.H.; Ries, J.M.; Schwindl, K. (2015): Ordering policy for stock-dependent demand rate under progressive payment scheme: a comment. International Journal of Systems Science, 46 (5), 872-877.
Goyal, S.K. (1985a): Economic order quantity under conditions of permissible delays in payments. Journal of the Operational Research Society, 36 (4), 335-338.
Goyal, S.K. (1985b): On “Economic order quantity under conditions of permissible delays in payments”. Journal of the Operational Research Society, 36 (11), 1069-1070.
Goyal, S.K.; Teng, J.-T.; Chang, C.-T. (2007): Optimal ordering policies when the supplier provides a progressive interest scheme. European Journal of Operational Research, 179 (2), 404-413.
Goyal, S.K.; Chang, C.-T. (2008): Economic order quantity under conditions of a one-time-only extended permissible delay period in payments. Asia-Pacific Journal of Operational Research, 25 (2), 267-277.
Grubbström, R.W, 1980. A principle for determining the correct capital costs of work-in-progress and inventory. International Journal of Production Research, 259-271.
Guchhait, P.; Maiti, M.K.; Maiti, M. (2014): Inventory policy of a deteriorating item with variable demand under trade credit period. Computers & Industrial Engineering, 76, 75-88.
Huang, Y.-F. (2002): Optimal buyer’s replenishment policy for the EPQ model under the supplier’s trade credit policy. Production Planning & Control, 15 (1), 27-33.
Huang, Y.-F. (2007): Economic order quantity under conditionally permissible delay in payments. European Journal of Operational Research, 176 (2), 911-924.
Huang, Y.-F.; Chung, K.-J. (2003): Optimal replenishment and payment policies in the EOQ model under cash discount and trade credit. Asia-Pacific Journal of Operational Research, 20 (2), 177-190.
Klapper, L.; Laeven, L.; Rajan, R. (2012): Trade Credit Contracts. Review of Financial Studies, 25 (3), 838–867.

Liao, J.-J.; Huang, K.-N. (2010): An inventory model for deteriorating items with two levels of trade credit taking account of time discounting. Acta Applicandae Mathematicae, 110 (1), 313-326.
Lo Sorto, C. (2015): Is quality still an important determinant of small suppliers' competitive strategy? An empirical cross-industry comparison. International Journal of Management and Enterprise Development, 14 (4), 345-384.
Mentzer, J.T.; DeWitt, W.; Keebler, J.S.; Min, S.; Nix, N.W.; Smith, C.D.; Zacharia, Z.G. (2001): Defining Supply Chain Management, Journal of Business Logistics, 22 (2), 1-25.
Ng, C.K.; Smith, J.K.; Smith, R.L. (1999): Evidence on the Determinants of Credit Terms Used in Interfirm Trade. Journal of Finance, 54 (3), 1109–1129.
Ouyang, L.-Y.; Ho, C.-H.; Su, C.-H. (2009): An optimization approach for joint pricing and ordering problem in an integrated inventory system with order-size dependent trade credit. Computers & Industrial Engineering, 57 (3), 920-930.
Pfohl, H.-C.; Gomm, M. (2009): Supply chain finance: optimizing financial flows in supply chains. Logistics Research, 1 (3-4), 149-161.
Randall, W.S.; Farris, M.T. (2009): Supply chain financing: using cash-to-cash variables to strengthen the supply chain. International Journal of Physical Distribution & Logistics Management, 39 (8), 669-689.
Salameh, M.K.; Abboud, N.E.; El-Kassar, A.N.; Ghattas, R.E. (2003): Continuous review inventory model with delay in payments. International Journal of Production Economics, 85(1), 91-95.
Sarkar, B. (2012): An EOQ model with delay in payments and stock dependent demand in the presence of imperfect production. Applied Mathematics and Computation, 218 (17), 8295-8308.
Savino, M.M.; Apolloni, S. (2007): Environmental plant optimization in small sized enterprises through an operative framework. International Journal of Operations and Quantitative Management, 13 (2), 1-19.
Savino, M.M.; Meoli, E.; Luo, M.; Wong, M.M. (2010): Dynamic batch scheduling in a continuous cycle-constrained production system. International Journal of Services Operations and Informatics, 5 (4), 313-329.
Savino, M.M.; Manzini, R.; Mazza, A. (2015): Environmental and economic assessment of fresh fruit supply chain through value chain analysis. A case study in chestnuts industry. Production Planning & Control, 26 (1), 1-18.
Seifert, D.; Seifert, R.W.; Protopappa-Sieke, M. (2013): A review of trade credit literature: Opportunities for research in operations. European Journal of Operational Research, 231 (2), 245-256.
Shi, X.; Zhang, S. (2010): An incentive-compatible solution for trade credit term incorporating default risk. European Journal of Operational Research, 206 (1), 178-196.
Shin, S.W.; Hwang, H. (2003): Optimal pricing and ordering policies for retailers under order-size-dependent delay in payments. Computers & Operations Research, 30 (1), 35-50.
Soni, H.; Gor, A.S.; Shah, N.T. (2006): An EOQ model for progressive payment scheme under DCF approach. Asia-Pacific Journal of Operational Research, 23 (4), 509-524.
Soni, H.; Shah, N.H. (2008): Optimal ordering policy for stock-dependent demand under progressive payment scheme. European Journal of Operational Research, 184 (1), 91-100.
Soni, H.; Shah, N.H. (2009): Optimal policy for stock-dependent demand rate under progressive payment scheme. International Journal of Systems Science, 40 (1), 81-89.
Su, C.H. (2012): Optimal replenishment policy for an integrated inventory system with defective items and allowable shortage under trade credit. International Journal of Production Economics, 139(1), 247-256.

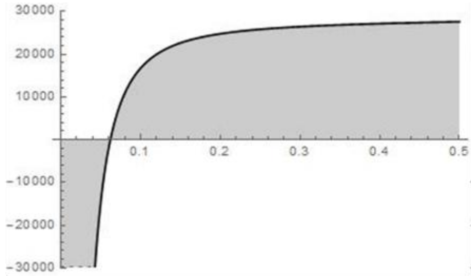
Summers, B.; Wilson, N. (2002): An Empirical Investigation of Trade Credit Demand. International Journal of the Economics of Business, 9 (2), 257-270.
Teng, J.-T. (2002): On the economic order quantity under conditions of permissible delays in payments. Journal of the Operational Research Society, 53 (8), 915-918.
Teng, J.-T. (2006): Discount cash-flow analysis on inventory control under various supplier's trade credits. International Journal of Operations Research, 3 (1), 23-29.
Teng, J.-T. (2009): Optimal ordering policies for a buyer who offers distinct trade credits to its good and bad credit customers. International Journal of Production Economics, 119 (2), 415-423.
Teng, J.-T.; Krommyda, I.-P.; Skouri, K.; Lou, K.-R. (2011): A comprehensive extension of optimal ordering policy for stock-dependent demand under progressive payment scheme. European Journal of Operational Research, 215 (1), 97-104.
Teng, J.-T.; Ouyang, L.-Y.; Chen, L.-H. (2006): Optimal manufacturer's pricing and lot-sizing policies under trade credit financing. International Transactions in Operational Research, 13 (6), 515-528.

Appendix

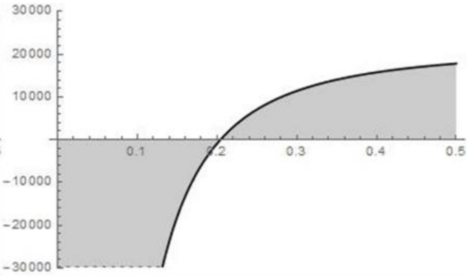
Illustrations of the Derivative Functions



e) Case 2.1 ($A = 10$)



f) Case 2.3-1 ($A = 100$)



g) Case 2.3-2 ($A = 150$)