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Informed Intermediation of Longevity Exposures*

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Abstract

We examine pension buyout transactions and longevity risk securitization in a common framework, emphasizing the role played by asymmetries in capital requirements and mortality forecasting technology. The results are used to develop a coherent model of intermediation of longevity exposures, between defined benefit pension schemes and capital market investors, through insurers operating in the pension buyout market. We derive several predictions consistent with the recent empirical evidence on pension buyouts, and offer insights on the role of buyout firms and regulation in the emerging market for longevity-linked securities. A multi-period version of the model is used to explore the effects of longevity risk securitization on the capacity of the pension buyout market.

1 Introduction

In the last decade, the liabilities of corporate pension schemes have reached unprecedented levels, owing to substantial increases in life expectancy, low interest rates and underperformance of backing assets. Pension trustees have addressed the deterioration of funding levels in different ways, working on the asset side, the liability side, or both. On the asset side, there has been a stronger focus on asset-liability management, which has translated into 'de-risking' strategies tilting asset allocations away from equities and toward liability hedging.[1] On the liability side, there have been closures of schemes to

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[1] In the UK, for example, liability-driven investment (LDI) has become very popular. LDI strategies often involve the use of over-the-counter derivatives and hedging programmes.

new members as well as to new accruals to cap liabilities.[2] The Global Financial Crisis (GFC) of 2008-09 has accelerated this process, and led to a number of fund terminations. Even if liabilities are locked in at the closure date, there still remains the problem of meeting the pension payments as they fall due. This is no easy task, given the typical size[3] and duration of pension liabilities. Some pension schemes have therefore opted for more radical solutions, such as the buyout of (part of) their liabilities, that is the transfer of their exposures to a counterparty, typically an insurer. The buyout market[4] took off in the UK in 2006, when a new monoline insurer, Paternoster, sealed the first deal with the Cuthbert Heath Family plan. Although the default of Lehman Brothers dampened the exuberance of buyouts in 2008-09, because of the impact on the corporate bond market, the recovery has been strong, with business volumes totalling GBP 8bn in 2010 and GBP 12bn in 2011, the highest annual levels so far.[5]

From the point of view of employers, buyouts are the most direct way to take liabilities off their balance sheet. Even if buyout costs are financed by borrowing, a regular loan replaces all the risks entailed by pension liabilities and can be more comfortably managed. Still, buyouts are (perceived as) very expensive and have prevented many schemes from adopting this solution. There are several reasons for this disconnect between the buy and sell side of pension buyouts. On the demand side, pension plan sponsors and trustees come from a tradition of lenient accounting and regulatory standards that have systematically downplayed the size and volatility of pension liabilities. On the supply side, the market has very quickly shown signs of capacity constraints, due to the strict solvency rules imposed on buyout firms and the limited success of standardized solutions (such as longevity indices and population-based longevity derivatives). For example, the GFC generated considerable mark-to-market losses in the portfolios[6] of insurers that had been particularly successful in securing large deals in the early stages of the market, preventing them from taking on further liabilities.[7]

The first contribution of this paper is to formalize the trade-offs at play in the pension buyout market, providing a rationale for why bulk buyouts have been prevalent in the

[2]According to Hewitt Associates "more than half of all [private-sector] employers surveyed at the start of this year [2009] were considering closing their final salary pension schemes to existing members, effectively freezing retirement benefits at today's levels" ('Final salary pension threat', *Financial Times*, June 5, 2009).

[3]To give an example, the liabilities of 'small' pension schemes in companies in the FTSE100 index are in the GBP100m-1bn bracket.

[4]As is common in practice, we use the term 'buyout market' to indicate the range of solutions available to a pension plan to transfer risk to another institutions. This means that we do not distinguish between transactions involving buy-outs (transfer of some of all the liabilities of a pension plan, together with the responsibility to meet them, to another institution), buy-ins (purchase of bulk-annuities to insure some of all the liabilities of the pension plan while retaining responsibility for them), or bespoke longevity swaps (swaps linked to the mortality experience of the pension plan).

[5]See, for example, "LCP Pension Buy-ins, Buy-outs and Longevity Swaps 2012", Lane Clark & Peacock LLP, April 2012: `http://www.lcp.uk.com/news--publications/publications-and-research/2012/lcp-pension-buy-ins-buy-outs-and-longevity-swaps-2012/`.

[6]Buyout firms are usually heavily invested in corporate bonds, to earn a premium on treasuries and match the duration profile of the liabilities.

[7]Paternoster had to close to new business in May 2009 after its main shareholders - Deutsche Bank and private equity firm Eton Park - refused to increase shareholder capital ('Pensions pain', *Financial Times*, July 7, 2009). Paternoster was acquired by Rothesay Life, the pensions insurance unit of Goldman Sachs, in December 2010.

early stages of the market and why buyout prices may still be perceived as very expensive by pension plans.[8] The second contribution is to understand the role that longevity risk securitization and mortality-linked securities may have in the context of pension buyouts. The reason why, until and after the GFC, buyouts attracted substantial capital from major investors is that insurers have superior expertise in forecasting and managing longevity-linked cashflows, can reap natural hedging and diversification benefits offered by their stock of exposures, and can use buyout premiums to support effective asset-liability management strategies, while earning an attractive return on capital. Still, the global net exposure to longevity risk of pension plans is well beyond the capacity of the global insurance and reinsurance markets.[9] As suggested by Blake and Burrows (2001) and Blake *et al.* (2012), there is therefore an opportunity for capital market investors to act as hedge suppliers, thus increasing capacity and contributing to a more transparent pricing of longevity risk. A wide investor base (institutional and insurance-linked securities investors, and endowment, family, sovereign wealth and hedge funds) has become interested in the longevity space, because it is thought to be virtually uncorrelated with traditional asset classes, but is still waiting for them to be packaged in investible formats delivering maximal diversification benefits. Since the bulk of global longevity exposure is carried by defined benefit pension schemes, understanding the dynamics of the pension buyout market is important, as it is an origination market that could provide the foundations for the development of a liquid market in longevity-linked securities.

Our analysis of buyout transactions and longevity risk securitization focuses on two key frictions: asymmetric information on longevity risk and differential capital requirements. We begin by looking at pension funds willing to access the buyout market to offload their liabilities. The presence of specialized firms with superior skills in forcasting and hedging mortality-linked cashflows creates an adverse selection problem for less informed insurers which need to charge more than the fair value in order to offset the expected cost of ending up with a lemon (i.e., they face a buyer's curse). This effect is amplified by the higher capital requirements in general faced by buyout firms. We show that, in this situation, it is never optimal for uninformed pension funds[10] to buy out separate components of their liabilities (e.g., different age ranges or cohorts). Rather it is optimal not only to pool longevity exposures, to prevent informed insurers from cherry picking, but also to pool longevity exposures with exposures to other sources of risk (interest rate, inflation, etc.) in order to dilute the effects of the asymmetric information associated with longevity risk.[11]

We then consider the point of view of informed holders of longevity exposures (such

[8]The following comments give an idea of market views: "There's a disconnect between what pension schemes are willing to pay to protect themselves from longevity risk and what the firms are quoting" Richard Jones (Punter Southall); "I think overwhelmingly longevity solutions are solutions in search of a problem. An awful lot are outrageously expensive." Con Keating (Brighton Rock). See 'Paying for a longer life', *Financial Times*, June 1, 2008.

[9]Recent studies suggest a figure of over USD 20tr, and quantify the net exposures to longevity risk from defined benefit pension schemes as being forty times larger than the combined net exposures of annuity providers and life assurance portfolios in the US and the UK (see Biffis and Blake, 2010b, and references therein).

[10]That is, pension funds that have not conducted an in depth quantitative analysis of their longevity exposure.

[11]See Subrahmanyam (1991), Gorton and Pennacchi (1993), DeMarzo (2005) for models demonstrating how risk pooling can reduce adverse selection costs to uninformed traders.

as (re)insurers and buyout firms) that may have originated in the buyout market or the retail market (e.g., annuities). Because of capital requirements, and the returns that can be generated by operating in the buyout market, there is an incentive to securitize the exposures to diversify risk and free up capital (see Cowley and Cummins, 2005, for an overview of insurance securitization). Issuers of longevity-linked securities, however, may face a downward sloping demand curve, as investors fear the buyer's curse when acquiring assets from more informed sellers and hence lower the quantity purchased at the equilibrium price. We use parts of the analysis carried out in Biffis and Blake (2010a) to show how issuers can mitigate the cost of adverse selection by retaining part of the longevity exposure and by suitably designing longevity-linked securities, providing links with products that have recently appeared in the market.

In the final part of the paper, we unify the two perspectives to develop a coherent story of the informed intermediation of longevity exposures. In particular, we use a multiperiod model to illustrate the limits to growth in the buyout market arising from lack of capacity on the supply side, and inefficient pricing of risk as seen from the demand side. In contrast, we show how longevity risk securitization and longevity-linked securities might offer an effective way for insurers to leverage their capital and increase returns, providing, in turn, greater capacity in the pension buyout market. Hence, there is a natural role for buyout firms to act as aggregators of the pension liabilities of small companies and to intermediate the transfer of longevity exposures originating from the pension buyout market to capital market investors.

The model also offers normative insights on the potential role of regulation in longevity space. The adverse selection problem faced by unsophisticated pension plans (a seller's curse) and less informed insurers (a buyer's curse) in the buyout market suggests that disclosure of detailed information on pension liabilities can be a double-edged sword. On the one hand, transparency could provide a more level playing field for pension plans and buyout firms. On the other hand, naive information disclosure may exacerbate the adverse selection problem by making informed buyers even more informed. There is an opportunity here for regulators to align the broad actuarial assumptions used in pension accounting with a more realistic assessment of longevity risk, while leaving the burden of more granular risk assessment on buyout firms and the choice of detailed information disclosure on pension funds and their advisors. This could favour the aggregation of liabilities and bulk buyouts while narrowing the gap between buy-side and sell-side valuations due to differences in regulatory environments. Transparency can in turn have a beneficial effect in the secondary market, as it mitigates the adverse selection problem faced by investors acquiring longevity-linked securities issued by buyout firms. Here, specialized insurers can use their information advantage to suitably pool and tranche longevity exposures in order to dilute and minimize the costs of information asymmetry. Regulators can therefore play an important role, for example by requiring rating agencies to use sufficiently granular data to assess the risk profile of securitized products, or by providing incentives to disclose and use detailed information from the very same internal models used to demonstrate the capital resilience of buyout firms in the primary market. Hence, the stricter regulation faced by insurers in the primary market, while making buyout transactions more expensive, may still provide the natural framework for promoting greater transparency in the secondary market. This could, in turn, increase buyout capacity and lead to a more efficient sharing of longevity risk.

The paper is organized as follows. In the next section, we outline a stylized model of pension liabilities and longevity exposures. In the following section, we examine the pension buyout market and outline the main characteristics of the demand side (pension funds willing to buy out their liabilities) and supply side (buyout firms willing to take on pension liabilities). We introduce a simple equilibrium model that links the salient features of buyout prices to asymmetries in information and capital requirements. In Section 4, we change perspective and look at holders of pension and insurance liabilities who are willing to securitize their exposures to diversify risk and free up capital. We use a signaling game of Walrasian equilibrium (as in DeMarzo and Duffie, 1999; DeMarzo, 2005) to understand the equilibrium securitization and the design of longevity-linked securities, providing links with current innovations in insurance-linked securities. Section 5 develops a multi-period model that brings together the insights of the previous sections and allows us to develop a coherent model of informed intermediation of longevity exposures. In particular, we demonstrate the long run beneficial effect of liability aggregation in the buyout market and in the secondary market for longevity-linked liabilities. Concluding remarks are offered in Section 6. An appendix provides further details.

2 Pension liabilities and longevity exposures

Pension liabilities include pensions in payment and deferred benefits to active members. Letting zero denote the current date, assume that payments are made at integer dates $1, 2, \ldots$, so that aggregate liabilities can be written as

$$S = \sum_{h=1}^{\infty} S_h, \tag{2.1}$$

with each S_h representing the random outflows at time h. The realization of each S_h may depend on interest rates, inflation, and the mortality experience of the pension scheme. In this paper, we focus on the latter source of uncertainty. We assume that there is a random signal Y that provides relevant information on how to correctly estimate S. For example, Y could represent the output of a superior mortality forecasting model, or a report on the health profile of the scheme members.

Conditional on the realization of the signal, an informed agent can formulate a private valuation of S by computing $p(Y) := E[S|Y]$, which we rewrite as $p(Y) = \sum_{h=1}^{\infty} E[S_h|Y] = \sum_{h=1}^{\infty} p_h(Y)$, with $p_h(Y) := E[S_h|Y]$. Although Y can belong to a generic measurable space (e.g., the space of demographic reports), we require $p(Y)$ to be continuous and to have distribution supported in a compact interval $[\underline{p}, \overline{p}]$, with $0 \le \underline{p} < \overline{p}$. We further assume that S admits the representation

$$S = p(Y) + \varepsilon = \sum_{h=1}^{\infty} (p_h(Y) + e_h), \tag{2.2}$$

with $\varepsilon = \sum_{h=1}^{\infty} e_h$ a zero-mean error term, independent of Y. We assume that there is a single riskless asset yielding an interest rate normalized to zero, so that expression (2.2) quantifies the pension liabilities in present value terms. We interpret $p(Y)$ as a demographic (systematic) trend component, and allow ε to capture other (idiosyncratic) demographic risks, as well as financial risks.

To understand the role of longevity risk in (2.2), it is useful to provide an interpretation of the liabilities in terms of survival rates. Following Biffis and Blake (2010a), consider a cohort of m members in the pension scheme at the current time. Denote their random remaining life times by $\tau^1, \ldots, \tau^m$, and suppose that members are entitled to an amount α_h at each later date $h = 1, 2, \ldots$, conditional on survival. The aggregate value of the liabilities can then be written as

$$S = \sum_{h=1}^{\infty} \sum_{i=1}^{m} \alpha_h 1_{\{\tau^i > h\}}, \tag{2.3}$$

with 1_H the indicator function, equal to unity if the event H is true, zero otherwise. Abstracting from financial risks and heterogeneity of pension accounts, let $\alpha_h = 1/m$ for all h. If death times are conditionally i.i.d. τ, given Y, then for a large enough pool of individuals we can invoke the Strong Law of Large Numbers (e.g., Schervish, 1995) and write

$$S = \sum_{h=1}^{\infty} \left(\frac{1}{m} \sum_{i=1}^{m} 1_{\{\tau^i > h\}} \right) \cong \sum_{h=1}^{\infty} E[1_{\{\tau > h\}} | Y] = \sum_{h=1}^{\infty} \mathbb{P}(\tau > h | Y). \tag{2.4}$$

We therefore obtain a representation of the exposure in terms of the survival rates of the scheme members at different time horizons. From (2.2), we can write $\mathbb{P}(\tau > h|Y) = p_h(Y) + e_h$, so that each term $p_h(Y)$ can be interpreted as a private estimate of the survival probability $\mathbb{P}(\tau > h|Y)$, while e_h can be seen as an unsystematic error term. Restricting the private signal to affect $p_h(Y)$ means that by observing Y, we can identify a trend component in survival rates, by far the most challenging source of uncertainty in mortality projections (see Cairns *et al.*, 2008, for an overview).

3 The pensions buyout market

An important feature of the pensions buyout market is that, in the early stages, buyout quotes were perceived as very expensive by pension funds and this prevented many of them from accessing the market. The buyout transactions taking place would often transfer assets and liabilities in bulk to the insurer,[12] and highly specialized firms succeeded in acquiring most of the business. Partial buyouts and pure longevity hedging solutions have initially been rarer, but have become more common as the the population of hedge suppliers has widened and pension trustees and advisors have understood the key frictions affecting the market, and have invested in data collection and disclosure.

We introduce a simple model of pension liability buyouts that is able to capture most of these features. Assume that pension funds and buyout firms are both risk neutral, but face different funding requirements. In particular, pension funds are subject to a more lenient funding regime than insurers, stands in place of formal capital requirements,[13]

[12] "We quote for mortality only buyouts [insurance against people living longer than expected], but we tend to find that when people want a quote for a mortality buyout, they end up comparing it to a bulk buyout [a complete buyout of all pension liabilities] and go for that instead", Mark Wood, CEO of Paternoster; see 'Paying for a longer life', *Financial Times* (June 1, 2008).

[13] For example, when pension liabilities are marked-to-market according to international accounting standard IAS19, assumptions on mortality improvements are not as strong as for Solvency II requirements or IFRS4 reporting. Moreover, as long as the sponsor is solvent and has a good business model, its pension scheme can run a deficit for up to 10 years in the UK.

whereas insurers need to meet restrictive solvency rules.[14] We consider a pension liability, S, admitting representation (2.2), with buyout firms required to scale up the estimate of their liability exposure by a factor $\eta > 1$, meaning that they need to hold an amount of capital equal to $\eta E[S] > E[S]$ when taking over the exposure S.[15] If the pension funds' discount factor is normalized to one, we can interpret the difference $\eta - 1$ as the additional regulatory cost per unit of liability incurred by insurers.

We can think of a buyout deal as being sealed on the basis of an auction. Buyout firms are provided with experience data for the exposure being offered for sale and are given a limited time period to produce a quote. Despite holding S, and being in a position to monitor the exposure over time, a pension fund is unlikely to have acquired the modelling capability needed to understand its longevity exposure. We therefore assume that it is uninformed. On the other hand, buyout firms have developed considerable expertise in modelling and managing longevity-linked cashflows, and are able to use the data provided to produce an informed estimate of the future liability cashflows. Moreover, these firms can aggregate smaller exposures into larger pools to make cash outflows more predictable. We let buyout firms differ in their skills in forecasting mortality improvements. We postulate that the distribution of the signal Y is available to every market participant, but its actual realization can only be observed by a subset of firms with superior forecasting technology upon examination of the data. To streamline even further, we assume that both pension funds and 'less well-informed' buyout firms are simply uninformed.[16] We denote by $\beta \in (0, 1)$ the fraction of buyout firms that are informed, and assume that they are endowed with capital $c \geq 0$ before operating in the market. Equivalently, we can think of a single informed firm participating in a fraction β of the transactions, or interpret β as the probability that an informed insurer participates in a buyout auction.

In the context of a first-price sealed-bid auction (a common buyout mechanism), suppose that the uninformed firms bid a positive insurance premium π to take over the liability S, and this premium is such that the uninformed firms recover at least the capital charges. Upon observing the realization y of Y, the informed insurers would accept taking over the liability for a price π only if they expected the buyout to be profitable, $\pi \geq p(y)$, and if they had enough capital to meet the regulatory constraint[17] $c + \pi - \eta p(y) \geq 0$. As a result, informed insurers would accept the liability for a premium π only if their private valuation $p(y)$ did not exceed a threshold $x^* \in [\underline{p}, \overline{p}]$ given by the highest private valuation x satisfying

$$\pi \geq x, \tag{3.1}$$

$$c + \pi - \eta x \geq 0. \tag{3.2}$$

In words, constraints (3.1)-(3.2) ensure that the exposures can be purchased at a non-negative discount. Constraint (3.1) will be slack for low levels of capital: if $c = 0$, for example, the informed agent would buy exposures with a private valuation at most equal

[14]See the standard IFRS4 for market-consistent valuation of insurance liabilities and the Solvency II proposal for capital requirements.

[15]The Solvency II standards would require $\eta E[S]$ to be calibrated to the 99.5^{th} percentile of the exposure over a one-year horizon.

[16]We could allow for different degrees of information, for example by working with a multi-dimensional Y and assuming access to some of its components only. The results would be qualitatively similar.

[17] In the context of Solvency II, we think of $p(y)$ as being part of an internal model, meaning that the insurer discloses its private valuation to the regulator.

to $\pi/\eta < \pi$. The larger the amount of capital supporting the premium inflow π, the greater the severity of longevity risk the insurer can bear. Anticipating that they are more likely to end up with the pension liabilities when the trend component $p(Y)$ is higher (longevity risk is more severe), the uninformed buyout firms bid more conservatively to neutralize the advantage of the informed; in other words, they ask for a higher premium to insure the pension liabilities. If they ask for less than the equilibrium price, they will make a loss on average, since the informed firms will always get the good quality exposures; the informed offer the same price as the uninformed firms (the higher the better), but will only bid when the exposure is of good quality. The minimum premium they can afford to offer is the one ensuring that they recover at least the capital charges, i.e., the minimum bid π satisfying

$$E\left[(\pi - \eta S)\left(1_{\{p(Y)>x\}} + (1-\beta)1_{\{p(Y)\leq x\}}\right)\right] = E\left[(\pi - \eta S)\left(1 - \beta 1_{\{p(Y)\leq x\}}\right)\right] = 0. \quad (3.3)$$

The second factor inside the above expectations represents the allocation to the uninformed firms, including i) the case when the informed's private valuation exceeds x and ii) the case when the private valuation does not exceed x and the informed is in the market with probability β. The equilibrium outcome of the auction is the pair (π^*, x^*) maximizing the informed firms' expected profits, while satisfying the regulatory constraint (3.2) and ensuring that the uninformed firms recover the capital charges on average. In equilibrium, the uninformed insurers earn zero profit, while pension funds transfer liabilities at a premium[18] which is sufficient to deliver the informed firms' expected profits (e.g., Milgrom and Weber, 1982; Engelbrecht-Wiggans *et al.*, 1983). The asymmetries in information and capital requirements have the following effects on equilibrium buyout prices:

Proposition 3.1. *Under the above assumptions, we have:*

(i) The equilibrium buyout price, π^*, satisfies $\eta E[S] \leq \pi^* < \overline{p}$.

(ii) π^* converges to $\eta E[S]$ as β or $|\overline{p} - \underline{p}|$ go to zero.

(iii) There is a level of capital $\hat{c}$, such that $\pi^*(c)$ is decreasing and $x^*(c)$ is increasing for $c \in [0, \hat{c}]$, and $x^*(c) = \pi^*(c) = \hat{\pi}$ for $c \geq \hat{c}$.

From (i), we see that the buyout price entails a premium for differential capital requirements (the price of an 'insurance guarantee'[19]), $(\eta - 1)E[S]$, as well as a premium for information asymmetry, $\pi^* - \eta E[S]$. This formalizes why buyout prices may be perceived as very expensive by pension funds, and just fair by more timid buyout firms. From (ii), as $|\overline{p} - \underline{p}|$ goes to zero, the informational advantage of the informed buyout firms vanishes and the buyout price is reduced to incorporate only the premium for asymmetry in capital requirements. The same effect is obtained if fewer and fewer informed buyout firms operate in the market (β goes to zero). Finally, from (iii), we see that the extent of

[18]If we had explicitly modeled the incentive of pension funds to offload longevity risks, this cost would be compensated by the value to shareholders of the resulting risk reduction and increased business flexibility.

[19]Meaning that the promise that pensions will be paid is guaranteed by the insurance regulatory framework.

pricing above the fair value is lower when informed insurers are well capitalized, as greater resources relax the solvency constraint (3.2) and allow the informed to target exposures with a higher private assessment of longevity risk. This weakens the adverse selection faced by less informed insurers and results in less severe overpricing for pension funds. For informed firms with enough capital, the equilibrium price stabilizes, as constraint (3.1) eventually binds and there is only a given fraction β of trades the informed can participate in. These comparative statics results are depicted in figure 1.

< Figure 1 about here. >

In figure 2, we provide an example based on the simplest possible stylized exposure: a survival rate S with best estimate $E[S] = 70\%$. We assume $\eta = 1.1$, meaning that 10% more than the liability best estimate needs to be set aside by the firm to comply with regulations. The plot shows that, in the absence of asymmetric information, buyout firms cannot afford to charge less than 77% to break even. As soon as asymmetric information is introduced, the equilibrium bid diverges further from 77%, the more so the higher the fraction of informed firms operating in the market.

< Figure 2 about here. >

A natural question to ask is what is the best selling strategy that a pension fund should adopt to mitigate the divergence of equilibrium buyout prices from its own valuation. In the presence of asymmetric information, it is, in general, difficult for the uninformed seller to neutralize the informational advantage of the informed buyer by disclosing any information, since doing so could reinforce overpricing.[20] The least a pension fund can do is to avoid splitting up the liabilities (for example by buying out liabilities arising from specific cohorts or age ranges of members) and rather transfer them in bulk, to prevent the informed buyout firms from cherry picking. This intuition can be formalized as follows:

Proposition 3.2. *Consider a sequence of exposures $S^1, S^2, \ldots$, with $S^i = p^i(Y) + \varepsilon^i$, and assume that the S^i's are conditionally independent, given Y. Then, as n grows larger, the buyout price per exposure converges to $\eta E[\bar{S}_n]$, where $E[\bar{S}_n] := \lim_{n\to\infty} \frac{1}{n}\sum_{i=i}^{n} E[S^i]$.*

Hence uninformed pension funds have an incentive to pool their liabilities to prevent informed insurers from using their information advantage on a systematic basis. As a special case, consider a sequence of exposures $S^1, S^2, \ldots$ that are conditionally i.i.d. S, given Y, so that $E[\bar{S}_n] = E[S]$. Then the buyout price converges to the lower bound $\eta E[S]$ of Proposition 3.1(i). We illustrate this case in figure 3, where we extend the example considered in figure 2 to show the reduction in buyout prices that can be achieved when several exposures are pooled. The higher the fraction of informed insurers operating in

[20] This occurs if what is made public happens to be complementary to what the informed party already knows (see Milgrom and Weber, 1982). As an extreme example, consider the case of a single longevity-linked cashflow in (2.2), $S = p_1(Y) + e_1$, and assume that the pension fund observes e_1 and makes it public. Then, S becomes known to the informed buyer upon observing Y, while the uninformed buyer is left with nothing relevant, as the information disclosed relates to an error term independent of the unknown $p_1(Y)$.

the market, the greater the reduction in buyout prices delivered by the pooling strategy. Of course, the situation may change if dependent exposures are pooled. In particular, the addition of specific liability segments may lead to a substantial increase in the systematic risk profile of the exposure, driving away potential insurers. This is what happens, for example, with active members and deferred liabilities, whose sensitivity to systematic longevity risk is commonly perceived as quite difficult to quantify by buyout firms. Some cherry picking is therefore to be expected in real life transactions.

< FIGURE 3 ABOUT HERE. >

4 The securitization market

Holding longevity exposures is very capital intensive for insurers, which is why they look to the capital market to offload part of their exposures, boost capacity and write more business. Capital market investors are becoming interested in insurance liabilities as an alternative asset class that is virtually uncorrelated with traditional investments. However, these investors are not yet entirely familiar with the dynamics of pensions and insurance liabilities, in particular with longevity risk. The transfer of mortality-linked cashflows creates a liquidity problem, in the sense that information asymmetries result in a downward sloping demand curve for the assets being securitized.

As in Biffis and Blake (2010a), a useful way to address this issue is to use a signaling model of market equilibrium, such as the ones developed by Gale (1992) and DeMarzo and Duffie (1999). Suppose we have a buyout firm with an exposure S originating in the buyout market and admitting representation (2.2) (alternatively, S could represent a book of annuities or other life contracts originated in the retail market). Capital requirements mean that it is expensive to hold the exposure on the balance sheet and that there is an opportunity cost, in that freeing up capital allows the insurer to exploit its informational advantage and acquire more liabilities in the pensions buyout market (or write more annuity business, for example). This provides an incentive to swap part of the liabilities for cash.

To avoid bringing credit risk into the picture, we assume that liabilities can be securitized on a fully collateralized basis.[21] Denoting by $\overline{s}$ the upper bound of the support of the distribution of S, we assume that the insurer can transfer assets $\overline{s} - S$ to the capital market (in words, the random liability S is backed by $\overline{s}$ units of capital). If no external borrowing is allowed and the insurer has capital $N \geq 0$ to put up as collateral, then only a fraction $\widetilde{\gamma} := \min(1, N/\overline{s})$ of the exposure can be securitized. We formalize the opportunity cost of the informed insurer by a discount factor $\delta \leq 1$ used for its private valuation,

$$\delta\widetilde{\gamma}E[\overline{s} - S|Y] = \delta\widetilde{\gamma}(\overline{s} - p(Y)) = \delta\widetilde{\gamma}\sum_{h=1}^{\infty}(\overline{s}^h - p_h(Y)), \tag{4.1}$$

where we have set $\overline{s} = \sum_{h=1}^{\infty} \overline{s}^h$. In the context of pensions buyouts (section 3), we can compute the opportunity cost of the informed insurer as follows. Given an equilibrium

[21]See Biffis *et al.* (2012) for an analysis of of counterparty risk mitigation tools in longevity risk transfers.

pair (π^*, x^*), a small additional amount of capital would allow the informed insurer to buy more of the highest longevity risk exposures, i.e., those with private valuation x^*. The additional purchase would yield a return $(\pi^* - x^*)/(\eta x^*)$. We could then set $\delta = (1+\frac{1}{\eta}\frac{\pi^*-x^*}{x^*})^{-1} \leq 1$. Endogenization of the discount factor δ is pursued in more detail in the multi-period model developed in section 5.

As is common in life insurance securitization and reinsurance, the issuer can retain part of the (net) exposure to signal its quality to investors. The more costly the retention of the exposure, the more credible the signal. Assume that the insurer decides the terms of the securitization before having access to the realization of Y. Once Y is observed, the net exposure is transferred to the capital markets, and only at a later stage are the cashflows from the exposure realized. Since $E[\widetilde{\gamma}(\overline{s} - S)|Y] \geq \widetilde{\gamma}(\overline{p} - p(Y)) \geq 0$, the private valuation is always nonnegative, implying that an asset rather than a liability is trasferred. We can then reason along the lines of DeMarzo and Duffie (1999) and Mailath and von Thadden (2010) to show that under rational expectations there exists a unique equilibrium separating low-longevity- from high-longevity-risk net assets, where the degree of longevity risk is determined by $p(Y)$. In equilibrium, when the optimal fraction γ^* of the exposure $\widetilde{\gamma}(\overline{s} - p(Y))$ is put up for sale, and the fraction $(1 - \gamma^*)$ is retained by the seller, investors take over the net exposure by paying the issuer its private valuation $\widetilde{\gamma}(\overline{s} - p(Y))\gamma^*$. The optimal securitization fraction admits the explicit representation

$$\gamma^*(Y) = (\overline{s} - \overline{p})^{\frac{1}{1-\delta}} (\overline{s} - p(Y))^{\frac{1}{\delta-1}} . \tag{4.2}$$

The above maximizes the securitization payoff to the insurer, which in equilibrium is equal to $(1 - \delta)\widetilde{\gamma}(\overline{s} - \overline{p})\gamma^*(Y)^{\delta}$. The expected payoff to the insurer from securitizing $\widetilde{\gamma}(\overline{s} - S)$ is therefore

$$V(\widetilde{\gamma}(\overline{s} - S)) := (1 - \delta)\widetilde{\gamma}(\overline{s} - \overline{p})E\left[\gamma^*(Y)^{\delta}\right] . \tag{4.3}$$

Note that as δ approaches unity, the incentive to securitize vanishes and full retention becomes optimal. The above results are appealing because they explicitly and uniquely characterize the equilibrium outcome, while relying only on mild regularity conditions.

If we consider a family of exposures $S^1, \ldots, S^n$, satisfying the conditions of Proposition 3.2, we are in the opposite situation to the one examined in the previous section, because the informed party is now the holder of the liabilities. It is then no surprise that it is never optimal for the insurer to pool the exposures before securitization: superior information means that each exposure can be optimally transferred on the basis of the degree of longevity risk privately assessed in each individual case. To see this, consider the event $\{Y = y\}$ and note that the payoff in (4.3) is convex in the private valuation of the net exposure, $\overline{s} - p(y)$. The expected payoff per exposure from securitizing $S^1, \ldots, S^n$ must then satisfy

$$\frac{1}{n}V\left(\widetilde{\gamma}_n \sum_{i=1}^{n}(\overline{s}^i - S^i)\right) = V\left(\frac{1}{n}\widetilde{\gamma}_n \sum_{i=1}^{n}(\overline{s}^i - S^i)\right) \leq \frac{1}{n}\widetilde{\gamma}_n \sum_{i=1}^{n} V\left(\overline{s}^i - S^i\right), \tag{4.4}$$

where $\widetilde{\gamma}_n := \min(1, N/(\sum_{i=1}^{n}\overline{s}^i))$. Hence pooling high-longevity- and low-longevity-risk exposures destroys the information advantage provided by the observation of the signal Y.

4.1 Issuance of longevity-linked securities

An alternative to selling the net exposure directly to investors is to issue a security written on the cashflows originating from $\overline{s} - S$. The idea is that a suitable payoff may reduce the sensitivity of the security to asymmetric information, thus offering better exchange opportunities. This possibility is currently being explored by a number of financial institutions and in 2010 materialized in the issuance of the first longevity bond (known as Kortis) by Swiss Re. The results we present below can also provide insights on the optimal design of longevity swaps. These allow the holder of a longevity-linked exposure to make fixed payments in exchange for floating payments depending on the mortality experience of the liabilities. For example, the results could be used to determine whether introducing suitable caps and/or floors on the floating leg of a longevity swap (and at what level) can improve the efficiency of the product. An interesting related question is the design of standardized index-based derivatives[22] with cashflows linked to publicly available indexes.[23] They would allow asymmetric information to be sidestepped, but would introduce the issue of basis risk, whose analysis is not pursued here (see Doherty and Richter, 2002, for example).

Consider a security providing a payoff $\phi(\overline{s}(T) - S(T))$ at a fixed maturity[24] $T > 0$, where ϕ is a measurable function and $S(T)$ represents the liabilities arising from an exposure S during the time interval $[0, T]$. As before, the term $\overline{s}(T) < \overline{s}$ denotes the upper bound of the support of the distribution of $S(T)$. For simplicity, in this section we assume that the issuer has enough capital to post as collateral (i.e., $\widetilde{\gamma} = 1$). We set

$$S(T) := p(Y;T) + \varepsilon(T) = \sum_{h=1}^{T} (p_h(Y) + e_h), \tag{4.5}$$

with obvious meaning of notation, and assume that $p(Y;T)$ has distribution supported in the compact interval $[\underline{p}(T), \overline{p}(T)] \subset [0, \overline{s}(T))$. We restrict our attention to contracts with nondecreasing payoffs satisfying the condition $\phi(u) \leq u$ for all $u \in [0, \overline{s}(T)]$. The timing of the transaction is as follows:

(i) the holder of the exposure S designs the contract before observing the signal Y;

(ii) after observing Y, the insurer sells the contract to investors according to the signaling game described in the previous section;

(iii) the cashflows realized from the exposure over time are observable by all agents.

Analogous to (4.2), we have γ^* units of the security which can be sold at their private information value, provided $(1 - \gamma^*)$ units of the security are retained by the issuer. The

[22] An example is represented by J.P. Morgan's q-forward contracts (Coughlan *et al.*, 2007), involving the payment of a realized mortality rate relating to a specified national population at a given future date, in exchange for a fixed mortality rate agreed at the beginning of the contract (the so called forward mortality rate).

[23] See, for example, the LifeMetrics indices (`http://www.lifemetrics.com`), which in 2011 were transferred to the Life and Longevity Markets Association (`http://www.llma.org`) with the aim of establishing a global benchmark for trading longevity and mortality risk.

[24] We take the parameter T as exogenously given. Our results can be used to optimize with respect to the contract maturity.

optimal fraction now takes the form

$$\gamma^*(Y) = \underline{\phi}^{\frac{1}{1-\delta}} \; \phi(\overline{s}(T) - S(T))^{\frac{1}{\delta-1}}, \tag{4.6}$$

where we have set $\underline{\phi} := \min_y E\left[\phi(\overline{s}(T) - S(T))|Y = y\right]$. As the next proposition shows, under mild assumptions on the error terms $\{e_h\}_{h=1}^T$, the optimal contract (i.e., the contract that maximizes the proceeds to the insurer from issuance) involves securitizing the exposure in exchange for an option that caps the longevity exposure of the issuer. . Equivalently, the contract can be in the form of a principal-at-risk bond where the principal is reduced if the realized cashflows exceed a given threshold (Blake and Biffis, 2012).

Proposition 4.1. *Assume that each e_h is log-concave,[25] for $h = 1, \ldots, T$. Then, the optimal longevity-linked security is the one providing the payoff*

$$\phi^*(\overline{s}(T) - S(T)) = (\overline{s}(T) - \kappa^*) - \max\left(S(T) - \kappa^*, 0\right), \tag{4.7}$$

where $\kappa^* \leq \overline{s}(T)$ is a strike level maximizing the payoff to the seller from issuing the contract.

The alternative representation $\phi^*(\overline{s}(T) - S(T)) = \min\left(\overline{s}(T) - S(T), \kappa\right)$, with $\kappa := \overline{s}(T) - \kappa^*$, shows that the optimal security involves 'tranching' the net exposure at level $\kappa \geq 0$. If $\overline{s}(T)$ is normalized to unity and S can be interpreted as a survival rate, the optimal contract (4.7) written on the death rate $1 - S$ embeds a survival option with strike price κ^*. See Lin and Cox (2005) Sherris and Willis (2010) for numerical examples related to the pricing of similar structures.

Proposition 4.1 shows the importance of introducing nonlinear payoffs in our setting. In particular, the idiosyncratic risk components become material for risk-neutral agents and offer valuable diversification benefits when pools of exposures are considered. In contrast with what we observed in the case of direct transfer to investors (see (4.4)), we now obtain the result that writing a security on the aggregate net exposures $\sum_{i=1}^n \left(\overline{s}^i(T) - S^i(T)\right)$ allows the insurer to overcome the loss of information arising from pooling high-longevity- and low-longevity-risk exposures. This is made explicit in the following proposition:

Proposition 4.2. *Consider a family of exposures $S^1(T), \ldots, S^n(T)$ that admit representation (4.5), are conditionally independent, given Y, and have distribution with support bounded above by $\overline{s}^i(T)$, $i = 1, \ldots, n$. If, for each i, the error terms $\{e_h^i\}_{h=1}^T$ are log-concave,[26] then it is optimal to pool the net exposures before tranching them.*

This is a fundamental result for the activity of buyout firms if we see them as aggregators of pension liabilities for later repackaging and reselling part of their exposures to the capital market. In the limit, we would expect diversification benefits to prevail over illiquidity due to information asymmetries. This is not always the case (see DeMarzo, 2005), but it does occur, for example, under the assumptions of Proposition 4.2:

[25] A random variable is log-concavely distributed it the logarithm of its probability density function is concave.

[26] We have required each term e_h^i to be log-concave to simplify the multi-period model of section 5, but we could have imposed the weaker condition of log-concavity on each term $\varepsilon^i(T)$ only (see Ibragimov, 1956; Karlin, 1968).

Proposition 4.3. *Consider a sequence* $S^1(T), S^2(T), \ldots$ *of exposures satisfying the conditions given in Proposition 4.2. Then, as the number of net exposures grows larger, the optimal strike level approaches* $\overline{p}_\infty(T)$*, where* $\overline{p}_\infty(T) := \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} \max_y E\left[S^i|Y=y\right]$ *denotes the limiting average worst-case private valuation. (Equivalently, the optimal tranching level approaches* $\overline{s}_\infty(T) - \overline{p}_\infty(T)$*, with* $\overline{s}_\infty(T) := \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\overline{s}^i(T)$*.) Moreover, the proceeds from issuance of the contract converge to* $(1-\delta)\left(\overline{s}_\infty(T) - \overline{p}_\infty(T)\right)$*.*

Proposition 4.3 shows that, in the limit, the process of pooling and tranching allows the insurer to transfer all the aggregate net exposures at their average private value, which coincides with their average worst-case private valuation. To see this, consider the special case of exposures $S^1(T), S^2(T), \ldots$ that are conditionally i.i.d. $S(T)$, given Y, with common upper bound $\overline{s}(T)$ and worst-case private valuation $\overline{p}(T)$. Then, we know from (4.6) that, as the trend component approaches the highest possible private assessment of longevity risk, $\overline{p}(T)$, the securitization fraction tends toward unity. The expected payoff to the issuer then converges toward the retention cost $(1-\delta)(\overline{s}(T) - \overline{p}(T))$, meaning that the costs of asymmetric information vanish in a large enough pool. Of course, lack of sufficient funds to post as collateral and associated borrowing costs may prevent such convergence from happening.

5 A multi-period model

In this section, we develop a coherent model of informed intermediation of longevity exposures that brings together the transactions examined so far. We consider a multi-period setting where, at each date $t = 0, 1, \ldots$, an informed buyout firm has access to pension liabilities put up for sale in the pension buyout market. During each period, the insurer decides whether to acquire more exposures in the buyout market, depending on market conditions and shareholder capacity, and meets the liability payments arising from the exposures already in the portfolio. We first examine the limits to growth imposed by this business model, and then allow the insurer to free up capital and diversify risk by issuing longevity-linked securities. The model is developed in the spirit of DeMarzo (2005) who focuses on asset-backed securities. Here, we extend the analysis to liability transfers and solvency requirements.

Between dates t and $t+1$, a continuum of pension funds indexed on $[0,1]$ put up for sale exposures $(S^i_{t,0})_{i\in[0,1]}$. Since pension funds have an incentive to pool their liabilities (Proposition 3.2), we think of each $S^i_{t,0}$ as a single liability made of smaller longevity exposures. The first index in the subscript refers to the time when the exposure is transferred, while the second index tracks the 'vintage' of the exposure after purchase. For example, $S^i_{t,z}$ represents the time-$(t+z)$ value of an exposure purchased in the buyout market at time t. As in section 2, the informed insurers use the signal Y_t observed during $(t, t+1)$ to derive their private valuations. For simplicity, we assume that the $S^i_{t,0}$'s are conditionally i.i.d. $S_{t,0}$, given Y_t, and admit the representation

$$S_{t,0} = p(Y_t) + \varepsilon_t = \sum_{h=1}^{\infty}\left(p_h(Y_t) + e_{t,h}\right). \tag{5.1}$$

By analogy with (2.2), we have set $p(Y_t) := E_t\left[S_{t,0}|Y_t\right] = \sum_{h=1}^{\infty} p_h(Y_t)$, $\varepsilon_t := \sum_{h=1}^{\infty} e_{t,h}$, the latter representing a zero-mean error term independent of Y_t. Here and in what follows, $E_t[\cdot]$ denotes conditional expectation, given the information available up to and including time t. Note that we only require $E_t[\varepsilon_t] = 0$, but are agnostic about the statistical properties of the random variable $\sum_{h=1}^{\infty} e_{t,h}$ at each date following t. As each term $e_{t,h}$ is not realized until date $t+h$, 'old' liabilities contribute to the insurer's exposure indefinitely, although for pension liabilities in run-off, one would specify the terms $p_h(Y_t)$ and $e_{t,h}$, so that they decrease on average as h grows large. From date $t+z$ onwards (with $z = 1, 2, \ldots$), the residual liabilities relative to exposure $S_{t,0}$ are given by $S_{t,z} = \sum_{h=1}^{\infty} (p_{z+h}(Y_t) + e_{t,z+h})$ and have private valuation

$$E_{t+z}[S_{t,z}] = \sum_{h=1}^{\infty} (p_{z+h}(Y_t) + E_{t+z}[e_{t,z+h}]) .$$

Hence, the private signal is only informative at the time of purchase. To simplify the analysis, we assume that the signals $\{Y_t\}$ are i.i.d. Y. Moreover, normalizing to unity the (exogenous) size of pension funds accessing the buyout market, we quantify by $F(x) := E_t\left[1_{\{p(Y_t) \leq x\}}\right] = \mathbb{P}(p(Y) \leq x)$ the mass of liability transfers with private valuation no higher than a fixed threshold x.

An informed insurer enters period $(t, t+1]$ with capital, c_t, and liabilities, L_t (the timeline is presented in figure 4). If the net worth, $N_t := c_t - \eta E_t[L_t]$, satisfies the solvency constraint $N_t \geq 0$, the insurer is solvent. An instant after time t, the buyout market opens and pension liabilities are put up for sale. There is a multitude of uninformed firms bidding for the pension liabilities according to the model of section 3. As liabilities are ex-ante identical, the uninformed firms bid a common insurance premium. The informed insurer, who participates in a fraction $\beta \in (0, 1)$ of the transactions, observes the realization of Y_t, once the data samples are made available by pension funds willing to transfer their liabilities. In equilibrium, characterized by the pair (π_t^*, x_t^*), the informed firm takes over a fraction β of exposures with private valuation not exceeding x_t^*, raising, in turn, premia totaling $\pi_t^* \beta F(x_t^*)$. The equilibrium threshold x_t^* ensures that the buyout is profitable and that the solvency requirements are satisfied (compare with (3.1)-(3.2)). We have:

$$x_t^* = \max\left\{x \in [\underline{p}, \overline{p}] \ : \ \pi_t^* \geq x, \ N_t + \beta F(x)\left(\pi_t^* - \eta E_t[p(Y)|p(Y) \leq x]\right) \geq 0\right\}. \tag{5.2}$$

As observed in Proposition 3.1, pension funds transfer liabilities at a premium, since differentials in solvency requirements and information result in an equilibrium bid satisfying $\pi_t^* \geq \eta E_t[S_{t,0}^i]$.

As soon as the buyout market closes, and before entering the next time period, the insurer observes and meets the liability payment, Δ_t, originating from the old exposures.[27] The firm also marks to market its asset and liabilities, based on information accumulated up to time $t+1$, and enters the following time period with capital

$$c_{t+1} = c_t - \Delta_t + \beta F(x_t^*)\pi_t^*, \tag{5.3}$$

and liabilities

$$L_{t+1} = L_t - \Delta_t + \beta S_{t,0}^*, \tag{5.4}$$

[27] Given the asset-liability dynamics described below, the term Δ_t summarizes the liability cashflows realized between dates t and $t+1$ for the different cohorts of exposures in the portfolio, i.e., $\Delta_t = \sum_{s=0}^{t-1} 1_{\{N_s \geq 0\}} \beta(p_{t-s}^*(Y_s) + e_{s,t-s}^*)$, where we have set $p^*(Y_t) := E_t[S_{t,0}^*|Y_t] = \sum_{h=1}^{\infty} p_h^*(Y_t)$.

which have market value $E_{t+1}[L_{t+1}]$. The random variable $S^*_{t,0}$ represents the recently acquired exposures, with private valuation $p^*(Y_t)$ not exceeding x^*_t. At the end of period $(t, t+1]$, the insurer is solvent if $N_{t+1} = c_{t+1} - \eta E_{t+1}[L_{t+1}] \geq 0$, in which case the same stages are repeated in the following period. If instead, $N_{t+1} < 0$, the insurer is prevented from taking on further liabilities and the book of liabilities is run-off until the first date $s > t+1$ when N_s is nonnegative again.

< FIGURE 4 ABOUT HERE. >

To understand the relative contribution of old and new liabilities to the solvency of the insurer, it is convenient to use (5.3)-(5.4) to write the period-by-period variation of the net worth as

$$N_{t+1}-N_t = \left\{\pi^*_t \beta F(x^*_t) - \eta\beta E_{t+1}[S^*_{t,0}]\right\}+\{\eta E_t[\Delta_t] - \Delta_t\}+\{\eta\left(E_t[L_t - \Delta_t] - E_{t+1}[L_t - \Delta_t]\right)\}\,. \tag{5.5}$$

The first term in (5.5) is a positive contribution from the premiums charged in the buyout market, net of the capital charges associated with the exposures newly acquired. The second term represents an inflow (outflow) if the amount of capital $\eta E_t[\Delta_t]$ allocated at the beginning of the period for the intraperiod outflows is sufficient (insufficient) to meet the realization of Δ_t. The last term captures any changes in the valuation of the old liabilities based on the new information gathered during the period (recall that the error terms $\{e_{t,h}\}$, for example, are not assumed to be i.i.d.). Hence the insurer has an incentive to operate in the buyout market for at least two reasons: (i) because exposures may be acquired at a discount, and (ii) because the premium incorporated into the price of the liabilities transferred in the buyout provides an extra buffer against unfavorable realizations of Δ_t and possible mark-to-market losses originating from the old liabilties. Note that only the latter two sources of uncertainty may trigger insolvency, as (5.2) ensures that the newly acquired exposures cannot lead to a violation of the solvency constraint.

5.1 Market capacity and securitization

We now add a further stage to the timing of transactions taking place in each time period. We assume that after the buyout market closes, and before meeting any liabilities arising during the period, the insurer can transfer a fraction of its liabilities to capital market investors (as in section 4), or issue a security with payoff linked to the future cashflows originating from the book of liabilities (as in section 4.1). The timeline is presented in figure 5. In both cases, we assume that the optimal securitization fraction is decided before observing Y_t. As explained in section 4, it is suboptimal to pool and securitize the new exposures, as this destroys the informational advantage of the issuer (we consider this situation to contrast it with the case of optimal security design). However, it is certainly beneficial to pool old exposures with new exposures before securitization, as the resolution of uncertainty in private signals related to the old liabilities dilutes the asymmetric information associated with the new exposures (e.g., Biffis and Blake, 2010a). Finally, recall that liabilities must be partially securitized if exposures can only be transferred on a fully collateralized basis, unless the insurer has sufficient free capital to post the required collateral.

< FIGURE 5 ABOUT HERE. >

Denote by $\bar{l}_t$ and $\bar{s}^*_{t,0}$ the upper bounds of the supports of the distribution of the old (L_t) and new liabilities ($S^*_{t,0}$). After the buyout auction closes, the insurer wishes to transfer to investors the collateralized exposure

$$D_t := \bar{l}_t - L_t + \beta\left(\bar{s}^*_{t,0} - S^*_{t,0}\right). \tag{5.6}$$

If the net assets available cannot provide for the full collateral, only a fraction $\widetilde{\gamma}_t$ of the old and new liabilities can be transferred, while a fraction $1 - \widetilde{\gamma}_t$ is retained, with

$$\widetilde{\gamma}_t := \min\left(\frac{\max(N_t, 0)}{\bar{l}_t + \beta \bar{s}^*_{t,0}}, 1\right). \tag{5.7}$$

However, we know that it would be optimal to securitize an even smaller fraction, since investors face a lemons problem. Let the informed insurer have opportunity cost $\delta_t < 1$ (we take it as given for the moment; we will endogenize it later). Then the signaling game described in section 4 results in the securitization having the following features.

Proposition 5.1. *Under our existing assumptions, the optimal securitization fraction, γ^*_t, of $\widetilde{\gamma}_t D_t$ is given by (dependence on time is dropped)*

$$\gamma^* = \left(\frac{\bar{l} - E[L] + \beta\left(\bar{s}^*_0 - p^*(Y)\right)}{\bar{l} - E[L] + \beta\left(\bar{s}^*_0 - F(x^*)x^*\right)}\right)^{\frac{1}{\delta - 1}}, \tag{5.8}$$

and the expected securitization payoff is equal to

$$\widetilde{\gamma}\,(1-\delta)\left[\bar{l} - E[L] + \beta\left(\bar{s}^*_0 - F(x^*)x^*\right)\right] E[(\gamma^*)^{\delta}]. \tag{5.9}$$

Note that the optimal securitization fraction is always bounded away from zero on the set $\{N_t \geq 0\}$, because the old liabilities in (5.8) have no residual information asymmetry. As the private valuation of the new liabilities approaches the upper bound $F(x^*)x^*$, the optimal fraction converges to unity and the exposures can be transferred at their private value.

Suppose now that the insurer writes a security on the cashflows emerging from the aggregate exposures during the time horizon $(t, t+T]$, for some maturity $T > 0$. Denote by $D_t(T)$ the fully collateralized exposure in this case,

$$D_t(T) := \bar{l}_t(T) - L_t(T) + \beta\left(\bar{s}^*_{t,0}(T) - S^*_{t,0}(T)\right), \tag{5.10}$$

with obvious meaning of the notation (compare with (5.6)). The analogue of Proposition 4.2 for the present setting is then given by:

Proposition 5.2. *Assume that the conditions of Proposition 5.1 hold, and that for all t, h, the error term $e_{t,h}$ is log-concave. Then:*

(i) The optimal security written on $\widetilde{\gamma}D(T)$ has payoff (again, dependence on time is dropped)

$$\phi^*\left(\widetilde{\gamma}D(T)\right) = \widetilde{\gamma}\left[\left(\bar{l}(T) + \beta\bar{s}^*_0(T) - \widetilde{\kappa}^*\right) - \max\left(L(T) + \beta S^*_0(T) - \widetilde{\kappa}^*, 0\right)\right],$$

with $\widetilde{\kappa}^*$ denoting the optimal strike level maximizing the payoff to the insurer from issuing the security.

(ii) *For a large enough pool of old and new exposures satisfying the conditions of Proposition 4.2, the optimal strike level approaches* $\bar{l}_t(T) - E_t[L_t] + \beta\left(\bar{s}^*_{t,0}(T) - F(x^*_t)x^*_t\right)$. *Moreover, the proceeds from issuance of the contract converge to* $\widetilde{\gamma}_t(1-\delta_t)\underline{d}^*_t(T)$, *with* $\underline{d}^*_t(T) := \bar{l}_t(T) - E_t[L_t] + \beta\left(\bar{s}^*_{t,0}(T) - F(x^*_t)x^*_t\right)$.

Propositions 5.1 and 5.2 provide a neat way of examining the scope for growth in market capacity when buyout firms transfer (part of) their exposures to the capital markets. Although we already know (by separation) that the optimal securitized fraction is always transferred at its private value, Proposition 5.2(ii) tells us more, namely that in the limit the liabilities are trasferred at their worst-case private valuation. This limiting result allows us to provide a simple dynamic characterization of the insurer's assets and liabilities when longevity-linked securities can be issued on large enough pools of old and new exposures. The only missing ingredient is the discount factor δ_t. If the insurer were to borrow extra cash to operate in the buyout market in period $(t, t+1)$, an additional unit of capital would allow her to purchase exposures with private value $x^*_t = x^*_t(N_t)$, for a premium $\pi^*_t = \pi^*_t(N_t)$, which could then be sold for their private value (before Δ_t is realized[28]) yielding a return $(\pi^*_t - x^*_t)/(\eta x^*_t)$. Hence the opportunity cost of capital is captured by a discount factor $\delta_t = \left(1 + \frac{1}{\eta}\frac{\pi^*_t - x^*_t}{x^*_t}\right)^{-1} \leq 1$.

We can now examine the effects of liability aggregation and securitization in the buyout market. We first provide the analogue of Proposition 3.1(iii) for our dynamic setting.

Proposition 5.3. *There exists a level of net worth* $\hat{N}_t$ *such that* $x^*_t(N)$ *is increasing and* $\pi^*_t(N)$ *is decreasing for* $N < \hat{N}_t$, *and* $x^*_t(N) = \pi^*_t(N) = \hat{\pi}_t$ *for* $N \geq \hat{N}_t$.

We then show that through the buyout and securitization processes, informed insurers can leverage their capital and increase returns. In particular, the informed buyout firm's net worth grows faster, on average, when pension liabilities are optimally aggregated and tranched than when they are securitized on an individual basis.

Proposition 5.4. *For given initial level of net worth* $N_0 \geq 0$, *assume that the conditions of Proposition 5.2(ii) are satisfied. Then, at each point in time* $t > 0$, *pooling and tranching is associated with an expected net worth that is no lower than in the case of individual securitization. The comparison is strict if* $\delta_s < 1$ *for some* $0 \leq s < t$.

The above results demonstrate that aggregation and securitization are beneficial for insurers that deploy their capital in the buyout market to acquire pension assets and liabilities, for later repackaging and selling on to investors. The securitization process can therefore lead to larger buyout market capacity and a more efficient pricing of buyouts. As shown by Proposition 5.3 and figure 1, however, a further reduction in mispricing can only be achieved by narrowing the difference in regulatory environments faced by pension funds and buyout firms, and by making careful use of data collection and disclosure in buyout transactions.

[28]The risk of becoming insolvent and being excluded from the buyout market in the next time period may suggest moral hazard considerations. As we assume that Y is realized before securitization, is disclosed to regulators, and does not provide any information on the law of Δ_t, we abstract from moral hazard in our model.

6 Conclusion

In this study, we have provided a stylized model of pension liability buyouts that examines the role of asymmetries in information on longevity risk and capital requirements. The implications of the model explain some of the key features of buyout transactions, such as inefficient pricing of liabilities from the point of view of pension funds and dominance of specialized firms in the market. An extension of the model allows buyout firms to securitize (part of) their exposures or to issue longevity-linked securities. We show how buyout firms can leverage their capital to increase returns, leading to greater capacity in the buyout market and more efficient pricing of buyout transactions. The results demonstrate that buyout firms, which are natural aggregators of longevity-linked liabilities in corporate pension plans, could represent an important driver of innovation in mortality-linked securities. Further research would include the introduction of learning about mortality dynamics, and the analysis of indexed longevity-linked instruments.

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A Sketch of proofs

Proposition 3.1 i) Denote by $a(x(\pi), p(Y)) := 1 - \beta 1_{\{p(Y) \leq x(\pi)\}}$ the allocation of the pension liability to the uninformed, given the signal Y, the bid π, and the informed's threshold $x(\pi)$. Point (iii) below shows that $x(\pi)$ is nondecreasing in π. One can show that under our assumptions $a(x(\pi), p)$ is continuous nonincreasing in π and continuous nondecreasing in p. Since $(\pi - \eta S)a(x(\pi), p(Y)) \leq \pi - \eta S \leq \pi$, by dominated convergence, we obtain continuity of $E\left[(\pi - \eta S)a(x(\pi), p(Y))\right]$ in π. Moreover, since $p(y) \leq \overline{p}$ and

$x(\eta\overline{p}) \leq \overline{p}$, we have $a(x(\eta\overline{p}), p) = 1 - \beta$ for all $p \in (\underline{p}, \overline{p})$, implying that for $\pi = \eta\overline{p}$ the uninformed firms make a profit on average, $E\left[\eta(\overline{p} - S)a(x(\eta\overline{p}), p)\right] > 0$. Similarly, since $p(y) \geq \underline{p}$ we have $E\left[\eta(\underline{p} - S)a(x(\eta\underline{p}), p)\right] < 0$ for all $p \in (\underline{p}, \overline{p})$, so that the uninformed firms make a loss on average for $\pi = \eta\underline{p}$. Hence there exists $\pi^* \in (\underline{p}, \overline{p})$ such that the uninformed earn zero on average, and any lower bid would result in an expected loss. Now note that from (3.3) we can write

$$0 \leq E\left[(\pi^* - \eta S)a(x(\pi^*), p(Y))\right] = E\left[(\pi^* - \eta S)\right] E\left[a(x(\pi^*), p(Y))\right] + \mathrm{Cov}\left(\pi^* - \eta S, a(x(\pi^*), p(Y))\right)$$

From the inequality above we then obtain

$$\pi^* \geq \eta E[S] - E\left[a(x(\pi^*), p(Y))\right]^{-1} \mathrm{Cov}\left(-\eta S, a(x(\pi^*), p(Y))\right) \geq \eta E[S],$$

where we have used the fact that $\mathrm{Cov}\left(-\eta p(Y), a(x(\pi), p(Y))\right) \leq 0$ because $a(x(\pi), \cdot)$ is nondecreasing.

(ii) The above arguments show that as β goes to zero, a converges to unity. By (3.3) the minimum premium the uninformed can offer is then $\eta E[S]$. As $|\overline{p} - \underline{p}|$ goes to zero, the advantage of the informed vanishes and the exposures are purchased by the informed at price $\eta E[S]$ with probability $1 - \beta$.

(iii) By total differentiation of (3.3) with respect to x, we obtain

$$\pi'(x)\left(1 - \beta\mathbb{P}(p(Y) \leq x)\right) - \left(\pi(x) - \eta x\right)\beta f(x) = 0,$$

where f denotes the density of $p(Y)$. We then have $\pi'(x) > 0$ if $\pi(x) > \eta x$, and $\pi'(x) < 0$ if $\pi(x) < \eta x$. As π is continuous in x by continuity of $p(Y)$, and $\pi \geq \eta x - c$, there exists $\hat{\pi}$ such that $\pi(x) \leq \eta x$ for $\eta x \geq \hat{\pi}$. As the solvency constraint (3.2) is relaxed by an increase in capital, c, we have x increasing and π decreasing in c until $x = \pi = \hat{\pi}$, which happens for $c = \hat{c} := (\eta - 1)\hat{\pi}$. □

Proposition 3.2 See DeMarzo (2005, Theorem 6).

Proposition 4.1 The assumption of log-concavity of the individual error terms $\{e_h\}$ implies that $\varepsilon(T) = \sum_{h=1}^{T} e_h$ is log-concave (see Ibragimov, 1956; Karlin, 1968). Since, in addition, $p(Y;T)$ is continuous, $\overline{s}(T) - S(T)$ admits a 'uniform worst case' (see DeMarzo and Duffie, 1999). It then follows that the optimal security is given by $\phi^*(\overline{s}(T) - S(T)) = \min\left(\overline{s}(T) - S(T), \kappa\right)$, for suitable a tranching level κ satisfying

$$\kappa \in \arg\max_{\rho} E\left[(1 - \delta)\underline{\Phi}(\rho)\Phi(\rho)^{\frac{1}{\delta - 1}}\right],$$

where we have set $\Phi(\rho) := \min\left(\overline{s}(T) - S(T), \rho\right)$ and $\underline{\Phi}(\rho) := \min_y E\left[\Phi(\rho)|Y = y\right]$ (see Biffis and Blake, 2010a, Proposition 5.1). Simple manipulations then yield the results, with the optimal strike level equal to $\kappa^* = \overline{s}(T) - \kappa$. □

Propositions 4.2 Under our assumptions, we can use Biffis and Blake (2010a, Proposition 6.2) to write

$$\frac{1}{n}V^{\phi^*}\left(\sum_{i=1}^{n}\left(\overline{s}^i(T) - S^i(T)\right)\right) \geq \frac{1}{n}\sum_{i=1}^{n} V^{\phi^*}\left(\overline{s}^i(T) - S^i(T)\right),$$

where $V^{\phi^*}(X)$ denotes the expected payoff from issuing the optimal security (4.7) written

on the asset X. Hence writing security ϕ^* on the pooled exposures is optimal.
Propositions 4.3 This is an immediate consequence of DeMarzo (2005, Theorem 2).
Proposition 5.1 We first note from (5.7) that $\widetilde{\gamma}_t$ does not depend on the realization of Y_t. From (5.6) we can then write the informed's private valuation as (dependence on t is dropped)

$$E[\widetilde{\gamma}D|Y] = \widetilde{\gamma}\left(\bar{l} - E[L] + \beta\left(\bar{s}_0^* - F(x^*)E[p(Y)|p(Y) \leq x^*]\right)\right),$$

and the worst-case private valuation as

$$\begin{aligned}\min_y E[\widetilde{\gamma}D|Y = y] &= \widetilde{\gamma}\left(\bar{l} - E[L] + \beta\left(\bar{s}_0^* - F(x^*)\max_y E\left[p(Y)|p(Y) \leq x^*, Y = y\right]\right)\right)\\ &= \widetilde{\gamma}\left(\bar{l} - E[L] + \beta\left(\bar{s}_0^* - F(x^*)x^*\right)\right),\end{aligned}$$

where we have used the fact that the support of $p^*(Y)$ has upper bound given by the equilibrium threshold x^*. The optimal securitization fraction (4.2) is then given by (5.8). □
Proposition 5.2 (i) Let $S_{t,0}^* = p^*(Y_t) + \varepsilon_t^* = \sum_{h=1}^{\infty}\left(p_h^*(Y_t) + e_{t,h}^*\right)$. Since the uncertainty surrounding each private signal Y_t is completely resolved at the end of each time period, the log-concavity of the terms $\{e_{t,h}\}$ and the recursive structure of L_t imply that $L_t + \beta\sum_{h=1}^{T} e_{t,h}^*$ is log-concave. Since $\beta\sum_{h=1}^{T} p_h^*(Y_t)$ is continuous, the net exposure $D_t(T)$ admits a 'uniform worst case', and this holds for each date t. Hence, the optimal security again takes the form $\phi^*(\widetilde{\gamma}_t D_t(T)) = \min\left(\widetilde{\gamma}_t D_t(T), \kappa\right)$. Simple manipulations then yield the results, with $\kappa = \widetilde{\gamma}_t\widetilde{\kappa}$, for a suitable tranching level $\widetilde{\kappa}$ and strike level $\widetilde{\kappa}^* = \bar{l}_t(T) + \beta\bar{s}_{t,0}^*(T) - \widetilde{k}$. (ii) This follows from Proposition 4.2(ii), since, as in the above proof, we have

$$\min_y E\left[D_t(T)|Y_t = y\right] = \bar{l}_t(T) - E_t[L_t(T)] + \beta\left(\bar{s}_{t,0}^*(T) - F(x_t^*)x_t^*\right),$$

where with a slight abuse of notation we have set $F(x) := \mathbb{P}(p(Y;T) \leq x)$. □
Proposition 5.3 The proof is the same as for Proposition 3.1(iii). □
Proposition 5.4 The result follows from noting that if $\delta_0 < 1$ (which requires $N_0 < \hat{N}$ by Proposition 5.3), then N_1 will be strictly higher, on average, under pooling and tranching than under straight securitization, as $\underline{d}^* > \left[\bar{l} - E[L] + \beta\left(\bar{s}_0^* - F(x^*)x^*\right)\right]E[(\gamma^*)^{\delta}]$, and Δ_t is realized before the securitization stage and is independent of Y. As $x^*(N)$ is nondecreasing, pooling and tranching allows the insurer to purchase, on average, more exposures in the buyout market, thus yielding the result. □

B Figures

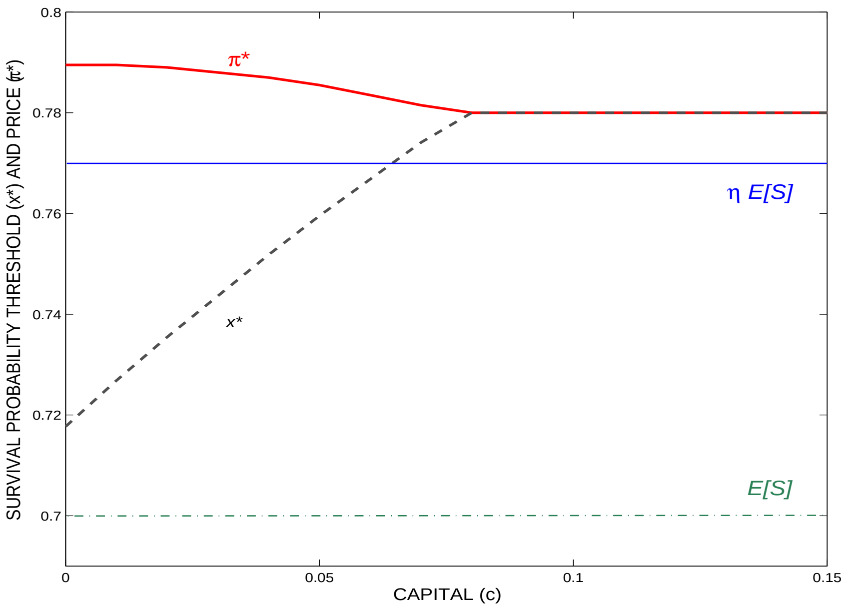

Figure 1: Comparative statics of the buyout price (π^*) and the private valuation threshold (x^*) relative to the resources of the informed (c). We set $\beta = 0.5$ and assume that $p(Y)$ is uniformly distributed on $[0.6, 0.8]$ and that ε has a truncated Normal distribution supported on $[-0.1, 0.1]$, with mean zero and variance 0.03.

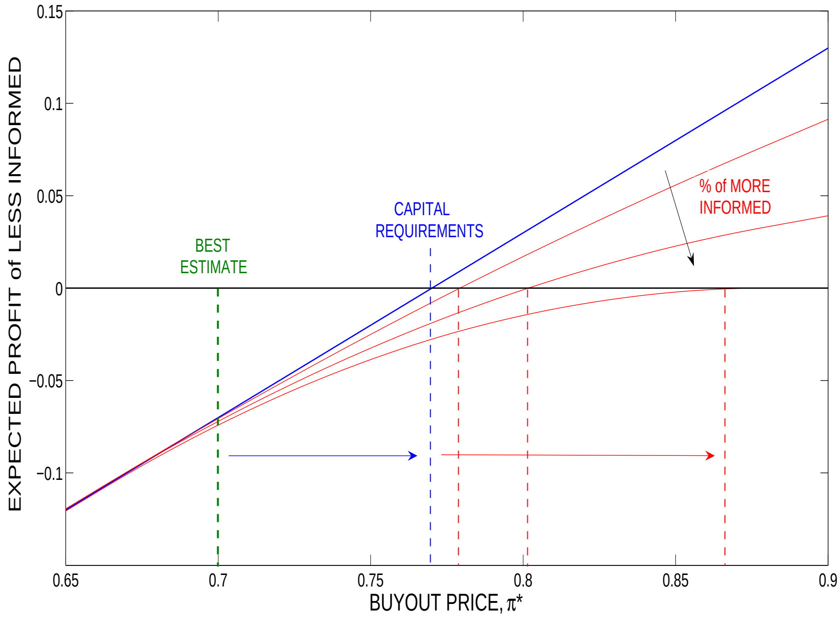

Figure 2: Divergence of equilibrium buyout prices from the uninformed's valuation $E[S]$.

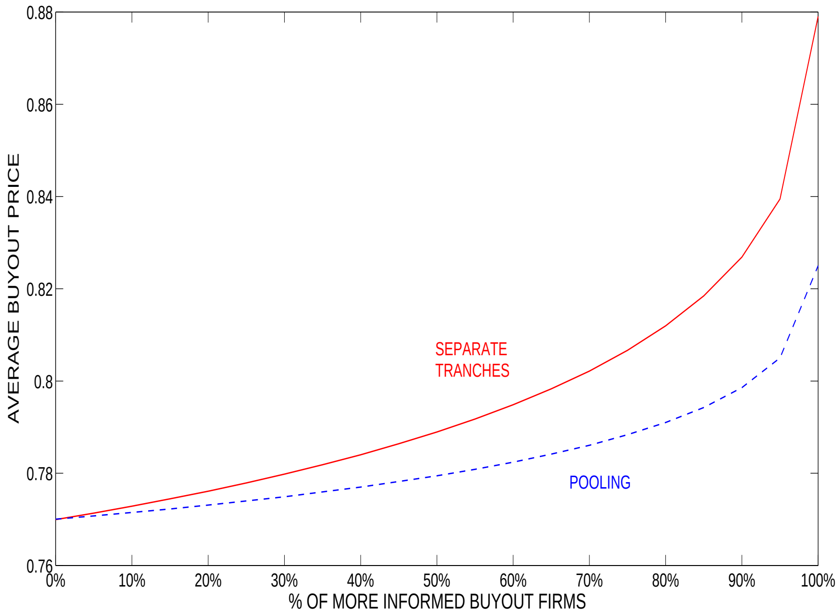

Figure 3: Buying out separate tranches versus pooling before buying out: reducing the cost of asymmetric information.

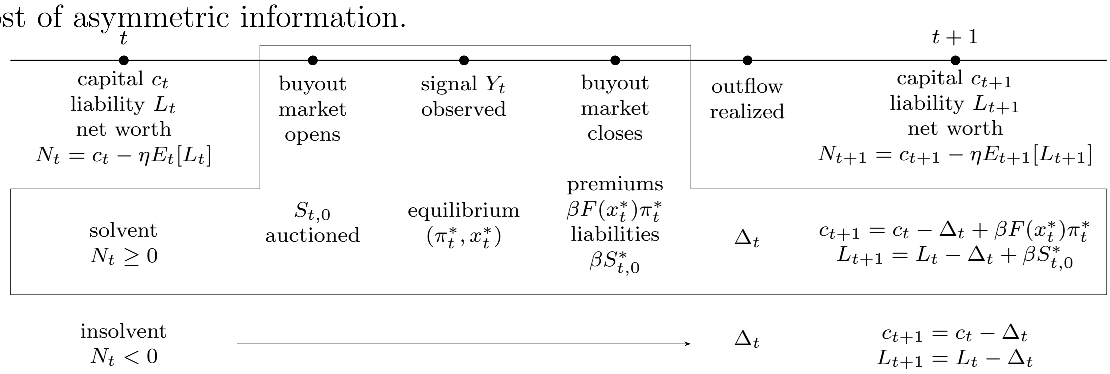

Figure 4: Transactions timeline.

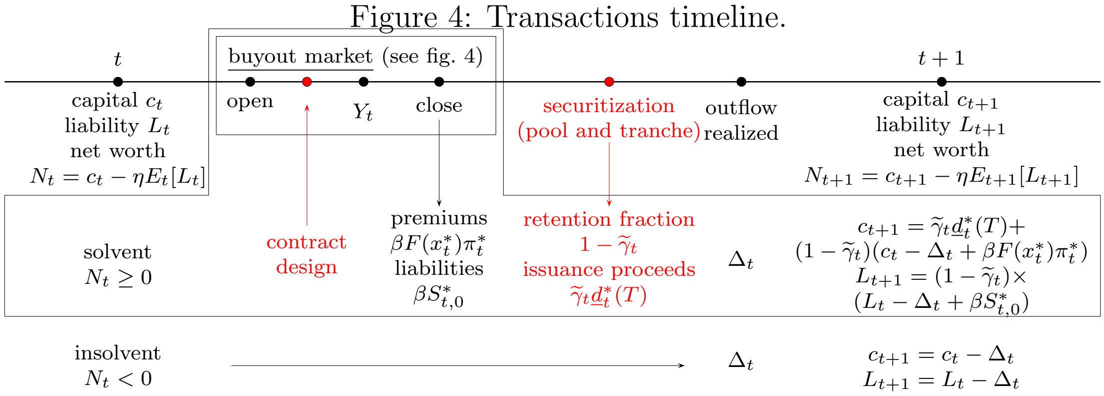

Figure 5: Timeline with issuance of longevity-linked securities.