Metadata of the chapter that will be visualized in SpringerLink

Book Title	Advances in Fluid-Structure Interaction
Series Title	
Chapter Title	Fluid Structure Interaction of Multiple Flapping Filaments Using Lattice Boltzmann and Immersed Boundary Methods
Copyright Year	2016
Copyright HolderName	Springer International Publishing Switzerland

Corresponding Author	Family Name	**Favier**
	Particle	
	Given Name	**Julien**
	Prefix	
	Suffix	
	Division	
	Organization	Laboratoire de Mécanique, Modélisation et Procédés Propres (M2P2) Aix Marseille Université, CNRS UMR 7340
	Address	Centrale Marseille, France
	Email	Julien.Favier@univ-amu.fr
Author	Family Name	**Revell**
	Particle	
	Given Name	**Alistair**
	Prefix	
	Suffix	
	Division	
	Organization	School of Mechanical, Aerospace and Civil Engineering (MACE) University of Manchester
	Address	Manchester, UK
	Email	
Author	Family Name	**Pinelli**
	Particle	
	Given Name	**Alfredo**
	Prefix	
	Suffix	
	Division	School of Engineering and Mathematical Sciences
	Organization	City University
	Address	London, UK
	Email	

Abstract

The problem of flapping filaments in an uniform incoming flow is tackled using a Lattice Boltzmann—Immersed Boundary method. The fluid momentum equations are solved on a Cartesian uniform lattice while the beating filaments are tracked through a series of markers, whose dynamics are functions of the forces exerted by the fluid, the filament flexural rigidity and the tension. The instantaneous wall conditions on the filament are imposed via a system of singular body forces, consistently discretised on the lattice of the Boltzmann equation. We first consider the case of a single beating filament, and then the case of multiple beating filaments in a side-by-side configuration, focussing on the modal behaviour of the whole dynamical systems.

Keywords (separated by '-') Beating filaments - Immersed boundary - Lattice Boltzma - Flapping modes

Fluid Structure Interaction of Multiple Flapping Filaments Using Lattice Boltzmann and Immersed Boundary Methods

Julien Favier, Alistair Revell and Alfredo Pinelli

Abstract The problem of flapping filaments in an uniform incoming flow is tackled using a Lattice Boltzmann—Immersed Boundary method. The fluid momentum equations are solved on a Cartesian uniform lattice while the beating filaments are tracked through a series of markers, whose dynamics are functions of the forces exerted by the fluid, the filament flexural rigidity and the tension. The instantaneous wall conditions on the filament are imposed via a system of singular body forces, consistently discretised on the lattice of the Boltzmann equation. We first consider the case of a single beating filament, and then the case of multiple beating filaments in a side-by-side configuration, focussing on the modal behaviour of the whole dynamical systems.

Keywords Beating filaments · Immersed boundary · Lattice Boltzma · Flapping modes

1 Introduction

The dynamics of flapping filaments in a streaming ambient fluid covers a broadband range of applications (aeronautics, civil engineering, biological flows, etc.) and constitutes a challenging problem, from the theoretical and numerical point of view [1, 2]. In particular, the experiments in soap films performed by [3, 4] are very interesting in this context, as they can be considered as a reasonable approximation of

J. Favier (✉)
Laboratoire de Mécanique, Modélisation et Procédés Propres (M2P2)
Aix Marseille Université, CNRS UMR 7340, Centrale Marseille, France
e-mail: Julien.Favier@univ-amu.fr

A. Revell
School of Mechanical, Aerospace and Civil Engineering (MACE)
University of Manchester, Manchester, UK

A. Pinelli
School of Engineering and Mathematical Sciences, City University, London, UK

M. Braza et al. (eds.), *Advances in Fluid-Structure Interaction*,
Notes on Numerical Fluid Mechanics and Multidisciplinary Design 133,
DOI 10.1007/978-3-319-27386-0_10

331174_1_En_10_Chapter ☑ TYPESET ☐ DISK ☐ LE ☑ CP Disp.:23/12/2015 Pages: 13 Layout: T1-Standard

2D fluid structure interaction scenarios, thus suitable for the validation of the results obtained with our numerical approach. In our simulations, we consider a 2D incoming incompressible flow modeled through a Lattice Boltzmann method, coupled to a model of infinitely thin and inextensible filaments experiencing tension, gravity, fluid forces and flexural rigidity (i.e. a bending term in the form of a 4th derivative with respect to the curvilinear coordinate describing the filament). Also, at all time instants tension forces are determined to maintain the inextensibility of the structure. In this simple model the energy balance of the system is driven by the bending forces and fluid forces, as the structure is controlled by an inextensibility constraint which prohibits stretching or elongation motions that would dissipate energy. This system encompasses all the essential ingredients of a complex fluid-structure interaction problem: large deformations, slender flexible body, competition between bending versus fluid forces, inextensibility and effect of the filament tips on the surrounding flow as vorticity generators.

To enforce the presence of the solid on the fluid lattice, we use a variant of the immersed boundary method previously developed by the authors [5]. This approach is efficient, accurate, computationally cheap and directly provides for the forces exerted on the fluid by the filaments without the introduction of any empirical parameter. Using the Lattice Boltzmann method in conjunction with an Immersed Boundary technique to solve the motion of an incompressible fluid also allows for a clean imposition of the boundary conditions on the solid since it does not suffer from errors originating from the projection step associated with unsteady incompressible Navier Stokes solvers [6].

Making use of the outlined Lattice Boltzmann—Immersed Boundary approach, we consider the coupled dynamics of systems made of flapping filaments placed side-by-side in an uniform incoming flow. No artificial contact force is introduced between the filaments to keep a purely hydrodynamical interaction between filaments. The ultimate aim of the simulations concerns the dynamical characterisation of the collective behavior of a set of filaments and their potential use as a deforming actuator for the control of fluid flows. In particular, the modal behavior of the system, that mainly depends on the filament spacings [7, 8], could be envisaged either as an *unsteady* generator of vortical structures, able to energize locally boundary layers on the verge of separation (thus delaying their detachment), or to control the wake behind bluff bodies [9].

2 Coupled Lattice Boltzmann—Immersed Boundary Method

The fluid-structure problem involving the mutual interaction between moving flexible objects and a surrounding fluid flow is tackled using an Immersed Boundary method coupled with a Lattice Boltzmann solver. In the following we will just present brief highlights on the numerical techniques entering in the whole numerical formulation. More details about the numerical methodology can be found in [10].

331174_1_En_10_Chapter ☑ TYPESET ☐ DISK ☐ LE ☑ CP Disp.:23/12/2015 Pages: 13 Layout: T1-Standard

The fluid flow is modeled by advancing in time the Lattice Boltzmann equation which governs the transport of particles density distribution f (probability of finding a particle in a certain location with a certain velocity). It is often classified as a mesoscopic method, where the macroscopic variables, namely mass and momentum, are derived from the distribution functions f. An excellent review of the method can be found in [11].

Using the classical BGK approach [12], the Boltzmann transport equation for the distribution function $f = f(\mathbf{x}, \mathbf{e}, t)$ at a node $\mathbf{x}$ and at time t with particle velocity vector $\mathbf{e}$ is given as follows:

$$f_i\left(\mathbf{x} + \mathbf{e_i}\Delta t, t + \Delta t\right) - f_i\left(\mathbf{x}, t\right) = -\frac{\Delta t}{\tau}\left(f\left(\mathbf{x}, t\right) - f^{(eq)}\left(\mathbf{x}, t\right)\right) + \Delta t F_i \qquad (1)$$

In this formulation, $\mathbf{x}$ are the space coordinates, $\mathbf{e_i}$ is the particle velocity in the ith direction of the lattice and F_i accounts for the body force applied to the fluid, which conveys the information between the fluid and the flexible structure. The local particles distributions relax to an equilibrium state $f^{(eq)}$ in a single time τ. Equation 1 governs the collision of particles relaxing toward equilibrium (first term of the r.h.s.) together with their streaming which drives the data shifting between lattice cells (l.h.s of the equation). The rate of approach to equilibrium is controlled by the relaxation time τ, which is related to the kinematic viscosity of the fluid by $\nu = (\tau - 1/2)/3$. This equation is solved on a cartesian uniform lattice. To each particle of each cell of the lattice a finite number of discrete velocity vectors are assigned. In particular, we use the D2Q9 model, which refers to two-dimensional and nine discrete velocities per lattice node (which corresponds to the directions east, west, north, south, center, and the 4 diagonal directions). In Eq. 1 the subscript i refers to these discrete particle directions. As it is usually done, a convenient normalization is used so that the spatial and temporal discretization in the lattice are set to unity, and thus the discrete velocities are defined as follows:

$$\mathbf{e_i} = c\begin{pmatrix} 0 & 1 & -1 & 0 & 0 & 1 & -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & -1 & -1 & 1 & 0 \end{pmatrix} \quad (i = 0, 1, \ldots, 8) \qquad (2)$$

where c is the lattice speed which defined by $c = \Delta x/\Delta t = 1$ with the current normalization. The equilibrium function $f^{(eq)}\left(\mathbf{x}, t\right)$ can be obtained by Taylor series expansion of the Maxwell-Boltzmann equilibrium distribution [13]:

$$f_i^{(eq)} = \rho\omega_i\left[1 + \frac{\mathbf{e_i}\cdot\mathbf{u}}{c_s^2} + \frac{(\mathbf{e_i}\cdot\mathbf{u})^2}{2c_s^4} - \frac{\mathbf{u}^2}{2c_s^2}\right] \qquad (3)$$

In Eq. 3, c_s is the speed of sound $c_s = 1/\sqrt{3}$ and the weight coefficient ω_i are $\omega_0 = 4/9$, $\omega_i = 1/9$, $i = 1, \ldots 4$ and $\omega_5 = 1/36$, $i = 5, \ldots 8$ according to the current normalization. The macroscopic velocity $\mathbf{u}$ in Eq. 3 must satisfy the requirement for low Mach number, M, i.e. that $\mid \mathbf{u} \mid /c_s \approx M << 1$. This stands as the equivalent of the CFL number for classical Navier Stokes solvers. The force F_i in Eq. 1 is

331174_1_En_10_Chapter ☑ TYPESET ☐ DISK ☐ LE ☑ CP Disp.:23/12/2015 Pages: 13 Layout: T1-Standard

computed using a power series in the particle velocity with coefficients that depend on the actual volume force $\mathbf{f_{ib}}$ applied on the fluid. The latter is determined using the Immersed Boundary method, following the formulation described in [10]. In this approach, the flexible filaments are discretised by a set of markers $\mathbf{X}_k$, that in general do not correspond with the lattice nodes $\mathbf{x}_{i,j}$. The role of $\mathbf{f_{ib}}$ is to restore the desired velocity boundary values on the immersed surfaces at each time step.

The global algorithm is decomposed as follows. The Lattice-Boltzmann equations for the fluid are first advanced to the next time step without immersed object ($F_i = 0$), which provides the distribution functions f_i needed to build a predictive velocity $\mathbf{u^p}$ by $\rho\mathbf{u^p} = \sum_i \mathbf{e_i} f_i$ and $\rho = \sum_i f_i$. The predictive velocity is then interpolated on the structure markers, which allows to derive the forcing required to impose the desired boundary condition at each marker using:

$$\mathbf{F_{ib}}(\mathbf{X}_k) = \frac{\mathbf{U^d}^{n+1}(\mathbf{X}_k) - \mathcal{I}[\mathbf{u^p}](\mathbf{X_k})}{\Delta t} \tag{4}$$

In Eq. 4, the term $\mathbf{U^d}^{n+1}(\mathbf{X}_k)$ denotes the velocity value at the location $\mathbf{X}_k$ we wish to obtain at time step completion. It is determined by the motion equation of the filaments given by:

$$\frac{d\mathbf{U^d}^{n+1}}{dt} = \frac{\partial}{\partial s}\left(T\frac{\partial \mathbf{X}_k}{\partial s}\right) - K_B\frac{\partial^4 \mathbf{X}_k}{\partial s^4} + Ri\frac{\mathbf{g}}{g} - \mathbf{F}_{ib} \tag{5}$$

Here, Ri is the Richardson number $Ri = gL/U_\infty^2$, T is the tension of the filament and K_B is the flexural rigidity. The closure of Eq. 5 is provided by the inextensibility condition that reads:

$$\frac{\partial \mathbf{X}_k}{\partial s} \cdot \frac{\partial \mathbf{X}_k}{\partial s} = 1 \tag{6}$$

This condition basically ensures that the filament does not stretch, and thus its length remains constant. The boundary conditions are $\mathbf{X} = \mathbf{X}_0$, $\dfrac{\partial^2 \mathbf{X}_k}{\partial s^2} = 0$ for the fixed end and $T = 0$, $\dfrac{\partial^2 \mathbf{X}_k}{\partial s^2} = 0$ for the free end.

Coming back to Eq. 4, the term $\mathcal{I}[\mathbf{u^p}](\mathbf{X}_k)$ refers to the value of the predictive velocity field interpolated at $\mathbf{X}_k$. This basically provides the kinematic compatibility between solid and fluid motion, i.e. zero relative velocity on the solid boundary. At this stage, the required forcing is known at each marker by Eq. 4, and needs to be spread onto the lattice neighbours by: $\mathbf{f_{ib}}(\mathbf{x}) = \mathcal{S}(\mathbf{F_{ib}}(\mathbf{X}_k))$. More details on the interpolation operator $\mathcal{I}$, spreading operator $\mathcal{S}$ and the filaments equations of motion can be found in [10]. The forcing $\mathbf{f_{ib}}$ is finally discretised on the lattice directions using the series expansion suggested in [14]:

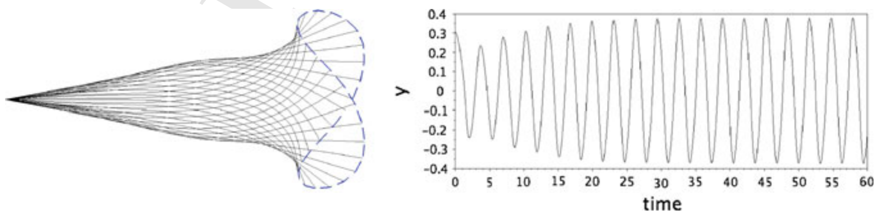
$$F_i = \left(1 - \frac{1}{2\tau}\right)\omega_i \left[\frac{\mathbf{e_i} - \mathbf{u}}{c_s^2} + \frac{\mathbf{e_i} \cdot \mathbf{u}}{c_s^4}\mathbf{e_i}\right] \cdot \mathbf{f_{ib}} \tag{7}$$

Equation 1 is then solved once again with the forcing F_i which impose the correct boundary condition at each marker $\mathbf{X}_k$. The macroscopic quantities are then derived from the obtained distribution functions f by $\rho\mathbf{u} = \sum_i \mathbf{e_i} f_i + \frac{\Delta t}{2}\mathbf{F}$ and $\rho = \sum_i f_i$, which closes one time step of the solver.

3 One Single Flapping Filament in an Incoming Fluid Flow

Following the experiments of [15], and the numerical study of [16], we start by considering the beating of a single filament fixed at one end, and subject to gravity and hydrodynamics forces. We fix the density difference between solid and fluid to $\Delta\rho = 1.5$, the non-dimensional bending rigidity to $K_B = 0.001$, and the value of the Richardson number to $Ri = 0.5$. The inlet velocity imposed in the Lattice-Boltzmann normalization is set to $U_\infty = 0.04$ (aligned with gravity direction), with a relaxation time of $\tau = 0.524$ and a filament length of $L = 40$. With these values, the simulation is run at a Reynolds number $Re = U_\infty L/\nu$ equal to 200. The size of the computational domain is set to $10L \times 15L$, in the transverse and streamwise direction respectively. The lattice discretization (600×400 nodes) has been determined as the result of a preliminary grid convergence study. The initial angle of the filament is set to $\theta = 18^o$ with respect to the gravity direction, and its fixed end is placed at the centerline of the domain, at a distance of $4L$ from the inlet. The L2 norm of the inextensibility error is kept below 10^{-12} systematically at all times.

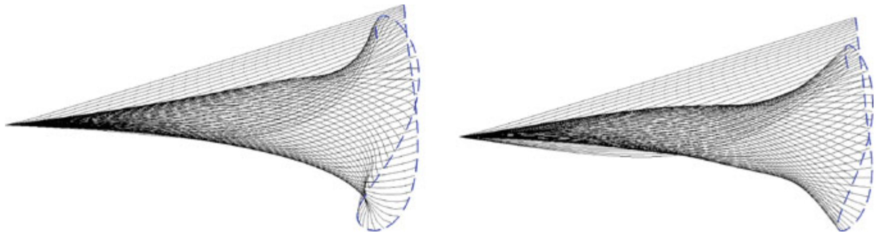
Figure 1a shows the periodic pattern of the beating in the established regime, characterised by sinuous traveling waves moving and amplifying downstream from the fixed end. The same behavior has been observed both in the simulations of [17] and in the experiments of [2]. Figure 1b shows the time evolution of the y-coordinate (transverse direction) of the free end of the filament. After six beating cycles, a periodic orbit is established, with a period of 3 time units (the same value as the one



(a) Periodic flapping pattern **(b)** y-coordinate of the free end

Fig. 1 Flapping motion of a single filament immersed in fluid at $Re = 200$, $Ri = 0.5$, $\Delta\rho = 1.5$. Fluid flows from left to right. **a** Beating pattern visualised by superimposed positions of the filament over one beating cycle. **b** Periodic time evolution of the y-coordinate of the free end

331174_1_En_10_Chapter ☑ TYPESET ☐ DISK ☐ LE ☑ CP Disp.:23/12/2015 Pages: 13 Layout: T1-Standard



(a) Totally flexible filament ($K_B = 0$) **(b)** Rigid filament ($K_B = 0.001$)

Fig. 2 Comparison between instantaneous snapshots of the flapping filament without bending (**a**) and with bending (**b**) starting from a straight initial configuration at an angle of $\theta_0 = 18^{\text{o}}$. The trajectory of the free end is shown in dashed line

found by [16]). The predicted amplitude of the beating compares well: the difference with reference data on the maximal excursion of the free end is less than 5 %. Also, the peculiar trajectory of the free end exhibiting a characteristic *figure-eight* orbit (dashed line in Fig. 1a) is recovered, in agreement with the findings of the soap film experiments carried out by Zhang et al. [3].

Figure 2a, b show the effect of the bending rigidity coefficient on the beating pattern. Without bending rigidity (Fig. 2a), the filament is totally flexible and a rolling up of the free extremity is observed. This effect has been termed as *kick* after the works of [18]. On the other hand, when the filament has a finite flexural rigidity ($K_B = 0.001$ in this simulation), the rolling up of the free end is inhibited, the *kick* disappears and the flapping amplitude is reduced. Thus, the proposed slender structure model, incorporating both bending terms and tension, computed to enforce inextensibility, reproduce thus successfully the same phenomena as the ones observed in experiments.

4 Multiple Flapping Filaments in an Incoming Fluid Flow

We consider here the case of two filaments in a *side-by-side* configuration. The non-dimensional values, the domain size and the initial angles ($\theta = 18^{\text{o}}$) are kept the same as in the case of the single beating filament. According to the experiments of [3] varying the spacing between filaments d/L leads to the appearance of different filaments beating regimes. In particular, a symmetrical flapping is observed for distances $d/L < 0.21$. For higher values, a bifurcation towards a regime characterised by an out-of-phase flapping is detected. Additionally, the linear stability analyses carried out in [7, 8] have put forward the existence of three different modes for such configurations. Therefore, in this context we have considered various scenarios corresponding to different values of the spacing d/L.

331174_1_En_10_Chapter ✓ TYPESET DISK LE ✓ CP Disp.:23/12/2015 Pages: 13 Layout: T1-Standard

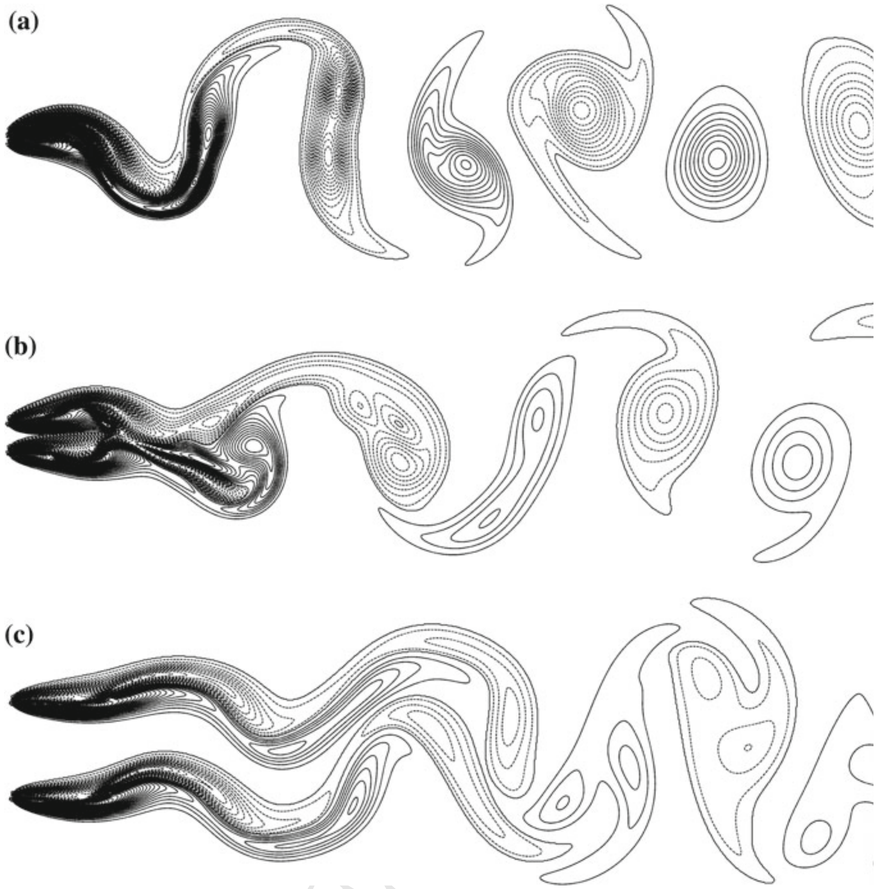


Fig. 3 Snapshots of iso-vorticity for the case of two beating filaments using $\rho = 1.5$, $K_B = 0.001$, $Re = 300$, $Ri = 0.5$ and two different spacings. **a** mode M1 at $d/L = 0.1$, **b** mode M2 at $d/L = 0.3$, **c** mode M2 at $d/L = 1.0$

Figure 3 displays the snapshots of iso-vorticity that we predict when considering three different spacings. The wakes are characterised by a periodic vortex shedding and by a flapping motion of the filaments (shown in Fig. 4 for the three cases).

- When the spacing is small ($d/L = 0.1$), we observe the mode M1, where the filaments are very close to each other and they behave almost as a single thick filament (see Fig. 3a), resulting in an in-phase beating of the filaments, as displayed in Fig. 4a.
- In contrast, when increasing the distance to $d/L = 0.3$, a different behaviour is observed. This mode (mode M2) is characterised by symmetrical out-of-phase oscillations, occurring after a transient period going on between $t = 20$ and $t = 60$ (see Fig. 4b). By increasing the filament spacing, the lock-in effect weakens but the

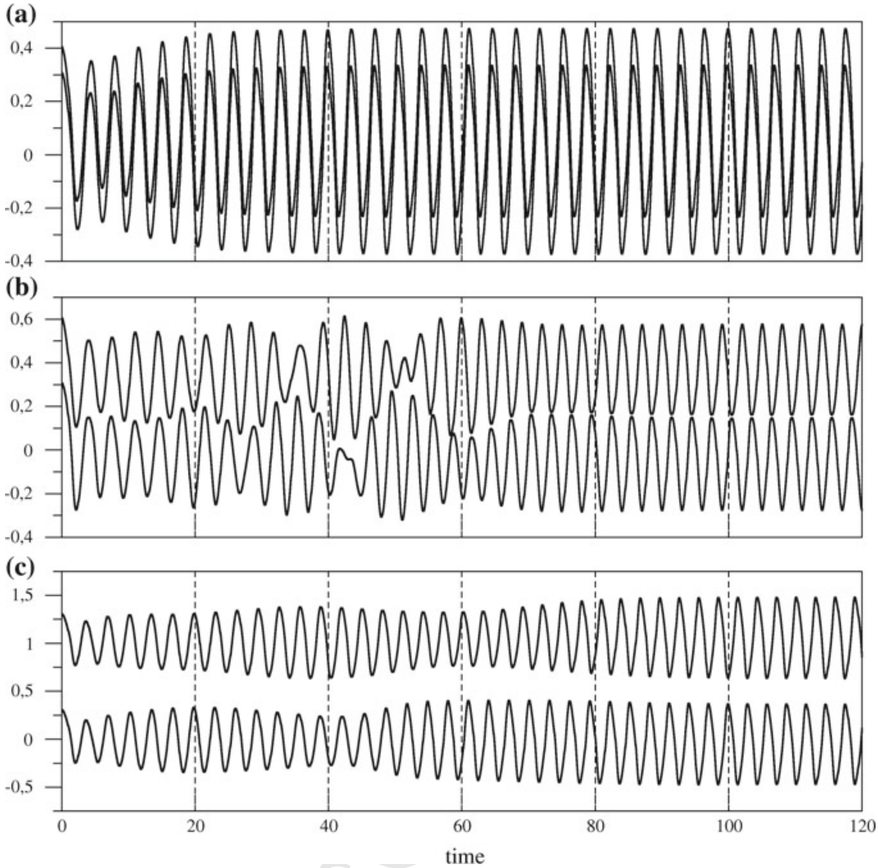


Fig. 4 Time evolution of the y-coordinates of the free extremity of a system of two beating filaments using $\rho = 1.5$, $K_B = 0.001$, $Re = 300$ and $Ri = 0.5$. **a** Mode M1 at $d/L = 0.1$, **b** Mode M2 at $d/L = 0.3$, **c** Mode M2 at $d/L = 1.0$

interaction between the wakes generated by each filament still plays a dominant role, as shown in Fig. 3b. In this regime, the enclosed fluid between both filaments behaves like a flow generated by a pump due to the out-of-phase flapping, being compressed when the two free ends approach (which is the case of the snapshot displayed in Fig. 3b), and released when they move apart.

- Further increasing the spacing to $d/L = 1$, the wake interaction weakens even more and the vortex streets behind the filaments decouple (see Fig. 3c). However, beyond 5L downstream of the filaments, the vortices merge into a unique wake and the filaments reach the mode M2 characterised by an out-of-phase flapping (see Fig. 4c).
- If the spacing d/L is further increased, the two filaments eventually reach a totally decoupled dynamics with an in-phase flapping of the filaments (mode M1).

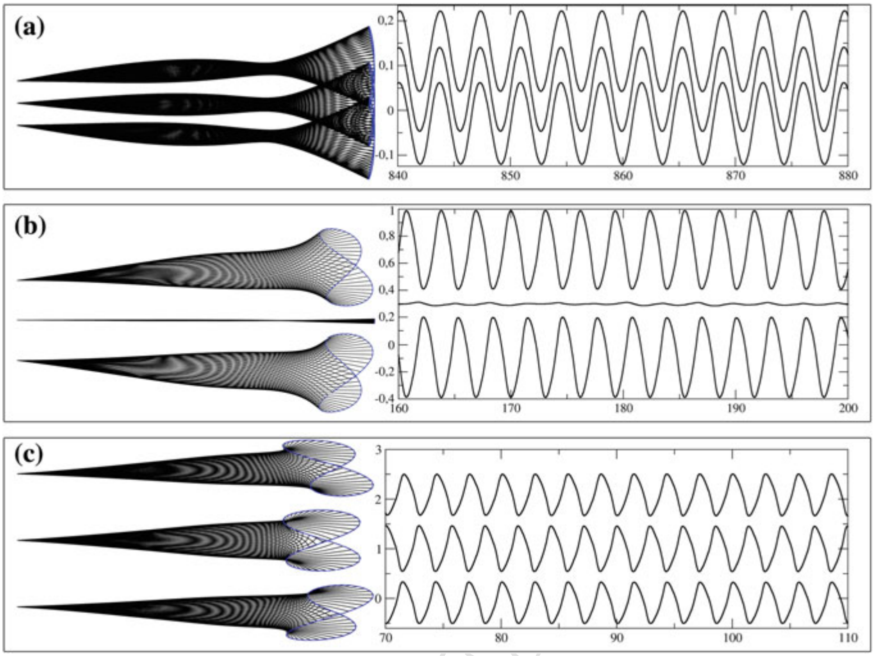


Fig. 5 Flapping patterns in the established regime for the beating of three filaments in an uniform flow for various spacing. **a** Mode M1 at $d/L = 0.05$, **b** mode M2 at $d/L = 0.3$, **c** mode M3 at $d/L = 1.0$. The *solid lines* represent the time evolution of the y-coordinates of the free extremity of each filament, for $\rho = 1.5$, $K_B = 0.001$, $Re = 300$ and $Ri = 0.5$

The modal behaviour is consistent with the experimental observations of [3] that report the onset of the out of phase regime at $d/L = 0.21$, compared to our numerical predictions indicating a transitory regime occurring between $d/L = 0.21$ and $d/L = 0.24$.

By keeping the same Reynolds number $Re = 300$, the configuration of three filaments placed side-by-side at an initial angle of 0° is investigated. Figure 5 summarizes the different coupled dynamics obtained with the present simulations. The system follows the same behaviour as for the case of two filaments, except that a different beating mode appears:

- for small spacings ($d/L < 0.1$), the mode M1 is observed, as in the case of two filaments, where the three filaments are in-phase (mode M1 in Fig. 5a);
- for $d/L = 0.3$, the two outer filaments flap out of phase while the inner filament stays almost at rest (mode M2 in Fig. 5b);
- for large spacing ($d/L = 1.0$) the outer filaments flap in-phase and the inner filament is out of phase (mode M3 in Fig. 5c);
- as for the case of two filaments, mode M1 is observed for very large spacing ($d/L > 4.0$) with an in-phase flapping of the three filaments.

331174_1_En_10_Chapter ☑ TYPESET ☐ DISK ☐ LE ☑ CP Disp.:23/12/2015 Pages: 13 Layout: T1-Standard

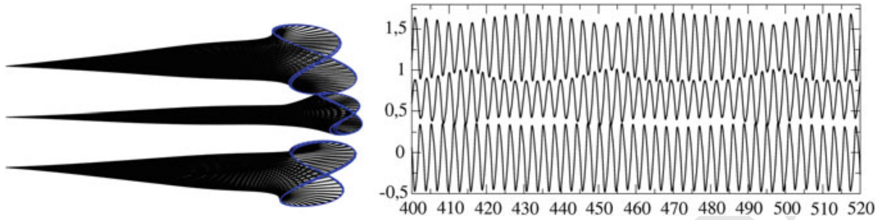


Fig. 6 Transition mode observed between M2 and M3 for $d/L = 0.6$. The *solid lines* represent the time evolution of the y-coordinates of the free extremity of each filament, for $\rho = 1.5$, $K_B = 0.001$, $Re = 300$ and $Ri = 0.5$

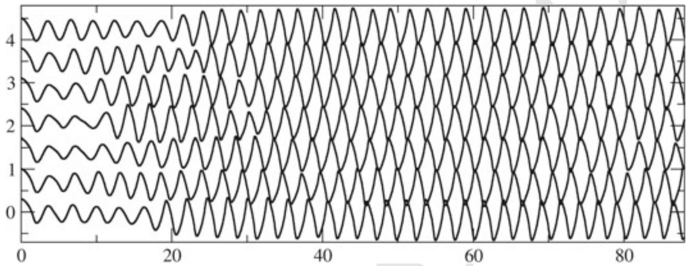


Fig. 7 Time evolution of the y-coordinates of the free extremity of a system of seven beating filaments using $\rho = 1.5$, $K_B = 0.001$, $Re = 300$ and $Ri = 0.5$

Additionally, we observed for $d/L = 0.6$ a transition mode characterised by the same behaviour as mode M3 but with a low frequency modulation in the amplitude of the flapping of the filaments, as shown in Fig. 6.

This transition mode has also been reported in the numerical study of [19]. In their simulations at $Re = 100$, they also point out another transitional mode where the inner filament is flapping at a frequency reduced by half compared to outer filaments, which we don't observe in our simulations at $Re = 300$.

When more than three filaments are considered, the system is expected to exhibit more transitory modes resulting from the coupling between the described baseline modes (M1, M2 and M3). Figure 7 displays for instance a mode similar to mode M3 obtained for seven flapping filaments with a spacing of $d/L = 0.7$.

5 Concluding Remarks

We have shown that a simple structural model of a flexible slender structure including its flexural rigidity, the tension (enforcing inextensibility) and the added mass can successfully capture numerically the dynamics of a flapping filament immersed in an uniform incoming flow. When considering two filaments placed side-by-side, the wake interactions and the modal behaviour of the system have been captured correctly, in agreement with the predictions of linear stability analysis and experiments.

331174_1_En_10_Chapter ☑ TYPESET ☐ DISK ☐ LE ☑ CP Disp.:23/12/2015 Pages: 13 Layout: T1-Standard

However, we have only considered here the influence of the filament spacing, but as pointed out by the study of [8], the influence of the added mass $\Delta\rho$ plays also a significant role.

For the case of three filaments, a set of three baseline modes have been highlighted: in-phase flapping (M1), out-of-phase flapping with the inner filament at rest (M2), in-phase flapping with the inner filament flapping out of phase (M3). For the general case of a layer made of *N* filaments, one would expect the system to be characterised by the appearance of *N* baseline modes originating from the combination of the M1, M2 and M3 baseline ones consistently with the theoretical prediction of [8].

Close-term perspectives of this work will be focussed on the shape adaptation properties and modal behavior of a layer of filaments flapping in three dimensions, within the scope of flow control applications.

Acknowledgments The authors acknowledge the financial help of the *PELskin* European project (FP7 AAT.2012.6.3-1). This work was partially supported by the Spanish Ministry of Economics through the grant DPI2010-20746-C03-02.

References

1. Païdoussis, M.P.: Fluid-Structure Interactions: Slender Structures and Axial Flow, vol. 2. Elsevier Academic Press, Cambridge (2004)
2. Shelley, M.J., Zhang, J.: Flapping and bending bodies interacting with fluid flows. Ann. Rev. Fluid Mech. **43**(1), 449–465 (2011)
3. Zhang, J., Childress, S., Libchaber, A., Shelley, M.: Flexible filaments in a flowing soap film as a model for one-dimensional flags in a two-dimensional wind. Nature **408**, 835–839 (2000)
4. Zhu, L., Peskin, C.S.: Interaction of two flapping filaments in a flowing soap film. Phys. Fluids **15**, 1954–1960 (2000)
5. Pinelli, A., Naqavi, I.Z., Piomelli, U., Favier, J.: Immersed-boundary methods for general finite-difference and finite-volume navier-stokes solvers. J. Comput. Phys. **229**(24), 9073–9091 (2010)
6. Domenichini, F.: On the consistency of the direct forcing method in the fractional step solution of the navier-stokes equations. J. Comput. Phys. **227**(12), 6372–6384 (2008)
7. Schouweiler, L., Eloy, C.: Coupled flutter of parallel plates. Phys. Fluids **21**, 081703 (2009)
8. Michelin, S., Llewellyn Smith, S.G.: Linear stability analysis of coupled parallel flexible plates in an axial flow. J. Fluids Struct. **25**(7), 1136–1157 (2009)
9. Favier, J., Dauptain, A., Basso, D., Bottaro, A.: Passive separation control using a self-adaptive hairy coating. J. Fluid Mech. **627**, 451 (2009)
10. Favier, J., Revell, A., Pinelli, A.: A lattice boltzmann—immersed boundary method to simulate the fluid interaction with moving and slender flexible objects. HAL, hal(00822044) (2013)
11. Succi, S.: The Lattice Boltzmann Equation. Oxford University Press, New York (2001)
12. Bhatnagar, P., Gross, E., Krook, M.: A model for collision processes in gases. i: small amplitude processes in charged and neutral one-component system. Phys. Rev. **94**, 511–525 (1954)
13. Qian, Y., D'Humieres, D., Lallemand, P.: Lattice bgk models for navier-stokes equation. Europhys. Lett. **17**(6), 479–484 (1992)
14. Guo, Z., Zheng, C., Shi, B.: Discrete lattice effects on the forcing term in the lattice boltzmann method. Phys. Rev. E **65**, 046308 (2002)
15. Zhu, L., Peskin, C.S.: Simulation of a flapping flexible filament in a flowing soap film by the immersed boundary method. Phys. Fluids **179**, 452–468 (2002)

16. Huang, W.-X., Shin, S.J., Sung, H.J.: Simulation of flexible filaments in a uniform flow by the immersed boundary method. J. Comput. Phys. **226**(2), 2206–2228 (2007)
17. Bagheri, Shervin, Mazzino, Andrea, Bottaro, Alessandro: Spontaneous symmetry breaking of a hinged flapping filament generates lift. Phys. Rev. Lett. **109**, 154502 (2012)
18. Bailey, H.: Motion of a hanging chain after the free end is given an initial velocity. Am. J. Phys. **68**, 764–767 (2000)
19. Tian, F.-B., Luo, H., Zhu, L., Lu, X.-Y.: Coupling modes of three filaments in side-by-side arrangement. Phys. Fluids **23**(11), 111903 (2011)

Author Queries

Chapter 10

Query Refs.	Details Required	Author's response
	No queries.	

MARKED PROOF

Please correct and return this set

Please use the proof correction marks shown below for all alterations and corrections. If you wish to return your proof by fax you should ensure that all amendments are written clearly in dark ink and are made well within the page margins.

Instruction to printer	*Textual mark*	*Marginal mark*
Leave unchanged	• • • under matter to remain	
Insert in text the matter indicated in the margin		New matter followed by or
Delete	through single character, rule or underline or through all characters to be deleted	or
Substitute character or substitute part of one or more word(s)	through letter or through characters	new character or new characters
Change to italics	under matter to be changed	
Change to capitals	under matter to be changed	
Change to small capitals	under matter to be changed	
Change to bold type	under matter to be changed	
Change to bold italic	under matter to be changed	
Change to lower case	Encircle matter to be changed	
Change italic to upright type	(As above)	
Change bold to non-bold type	(As above)	
Insert 'superior' character	through character or where required	or under character e.g. or
Insert 'inferior' character	(As above)	over character e.g.
Insert full stop	(As above)	
Insert comma	(As above)	,
Insert single quotation marks	(As above)	or and/or or
Insert double quotation marks	(As above)	or and/or or
Insert hyphen	(As above)	
Start new paragraph		
No new paragraph		
Transpose		
Close up	linking characters	
Insert or substitute space between characters or words	through character or where required	
Reduce space between characters or words	between characters or words affected	