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DOCTORAL THESIS

The impact of behavioral factors on annuitisation decisions and decumulation strategies

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Abstract

Faculty of Actuarial Science and Insurance

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Doctor of Philosophy

The impact of behavioral factors on annuitisation decisions and decumulation strategies

by Anran CHEN

The ongoing shift from Defined Benefit (DB) pension plans to Defined Contribution (DC) pension plans in private sectors has transferred investment risk and longevity risk from pension providers to individuals. Professional advice on how to best generate retirement incomes from accumulated pension savings is therefore in great demand. A common solution is buying an immediate annuity; however the immediate annuity market has long been experiencing low demand. Another solution is following a safe drawdown rate during retirement; however this exposes retirees to the risk of outliving their pension savings. In recent years, behavioral factors have been successful in explaining individuals' decision-making process, this thesis is therefore devoted to the investigation of the low demand of immediate annuities by considering behavioral models; and the use of annuity products in optimal decumulation strategy designs. This thesis has two major contributions. First, both Cumulative Prospect Theory (CPT) and Hyperbolic discount model can explain the low demand of immediate annuities and suggest that people would be willing to purchase deferred annuities. This has laid a research foundation for introducing and promoting the deferred annuity product. Second, we provide an optimal partial annuitisation strategy involving deferred annuities in a utility maximisation decumulation plan. In the proposed strategy the retirement period is divided into two stages: a stage where pensioners use their savings to cover their living expenses and a second stage where a payment stream from deferred annuities is available. This strategy effectively helps retirees manage the longevity risk at advanced ages and turns the drawdown plan from accumulated savings into an easier decision than before – because of a fixed rather than unknown drawdown period.

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Contents

1

Introduction

There are two main types of pension schemes in operation across the world: Defined Benefit (DB) pension plans and Defined Contribution (DC) pension plans. A DB pension scheme is a scheme where the benefits provided on retirement are defined by a specific formula and are not directly related to the contributions payable and the investment performance of the scheme. A DC pension scheme is a scheme where pensioners have their personal pension accounts; the benefits provided to an individual member depend on the contributions paid into the account and on the investment return earned on the accumulated fund. The two pension schemes mainly differ on who bears the risks involved in saving for retirement and who needs to make up for any pension benefit shortfall. In the DB system, the employers, pension providers, are exposed to many types of risk that can lead to a lower asset value and a higher liability value. These risks include longevity risk and investment risk (especially interest rate risk). However, in the DC system, risks are transferred to individual pension scheme members.

During the past few decades, there has been a steady shift from traditional DB pension plans towards DC pension plans in many countries. Between 1980 and 2013, the number of participants in US private sector DC plans increased more than fourfold from 20 million to 92.5 million; while participants in US private sector DB plans only increased from 38 million to 39 million (US Department of Labor, 2013). Similarly in the United Kingdom (UK), active membership of DC pension schemes increased from 0.9 million in 2007 to 3.2 million in 2014; while that of DB pension schemes decreased from 2.7 million to 1.6 million during the same period (ONS, 2014). Moreover, based on data collected

in DB and DC plans from 2000 to 2015 in OECD countries, the amount of assets in DB pension plans either experienced a decline (Israel), stayed constant while assets in DC plan increased (Australia, Iceland, Mexico and Sweden); or increased but at a slower pace than assets in DC plans (e.g. Netherlands, New Zealand and the United States) (OECD, 2016).

There are a number of reasons that lead to the growing importance of DC pension arrangements and the decline in DB pension plans. First, a general trend of increased life expectancy around the world contributes to an increased longevity risk and hence increasing the cost of providing DB plans. Data from the Human Mortality Database (2016) shows that male life expectancy at birth in England and Wales increased from 70.74 years in 1980 to 79.23 years in 2013, which indicates that DB providers need to set a much larger asset base to support the same level of benefits over a longer retirement period compared with decades ago. Using actuarial and financial information provided by the U.S. Department of Labor, Kisser et al. (2012) predicts that for every year of increased life expectancy, pension liability will rise by approximately 3% - 4%. Second, we have been in a low interest rate environment in recent years, for example, the Bank of England started Quantitative Easing (QE) in 2009 by lowering long term rates after forcing short term rates down to almost zero, and this creates a large actuarial deficit. Research from the Pension Protection Fund and other academic studies shows that for every percentage point fall in long-term bond yields, a typical DB pension fund will have a 18% - 20% rise in liabilities while its assets will only increase by 4% - 10% (Altman, 2013). A poor investment performance also leads to a decrease in the overall amount of assets in DB pension plans; for example, the 2008 financial crisis led to a decrease in the total amount of pension fund assets and the funding level of DB pension plans deteriorated (OECD, 2015). Third, the regulatory changes (in the UK) have increased the cost of guarantees in DB schemes. The UK Accounting Standards Board (2000) issued an accounting standard, FRS17, requiring that companies should value their pension assets and liabilities on a mark-to-market basis. This new fair value accounting approach moves away from smoothing techniques and leads to a more volatile valuation of pension liabilities and hence more volatile incomes for sponsoring firms.

From an individual's perspective, unlike DB pension members, DC scheme members have their own personal pension accounts and they can make decisions on how to manage the account. Prior to their retirement, DC members need to decide how much to

contribute to the pension account and make investment decisions according to their own risk appetite. At the time of retirement, they have access to the accumulated DC pension account balances and need to make decisions on how to spend the wealth to support the entire retirement period. Therefore, the switch from DB to DC pension plans shifts both the investment risk and longevity risk from the pension fund to individuals, posing a big challenge to them. Advice from professionals and advisors on investment strategies and consumption plans is therefore in great demand.

In the UK, the situation has become even more challenging for DC members after the government announced a set of pension reforms to allow for greater flexibility in pension provision. For instance, in 2011, the requirement of forced annuitisation by age 75 was removed; in March 2014, the policy of capped drawdown, which allows pensioners to withdraw as much as 120% of an equivalent annuity each year in retirement, was replaced by full discretionary access to pension savings (HM Treasury, 2014).

In this PhD thesis, we focus on the longevity risk in the decumulation stage and aim at finding out the optimal annuitisation and decumulation solutions for the post-retirement period.

An immediate annuity is is commonly used to hedge increased risks bone by individuals in the retirement period. It converts a lump sum savings pot into a steady stream of income in retirement for as long as the annuitant is alive, hence effectively protecting retirees against the longevity risk of outliving their assets and the investment risk from market downturns. Yaari (1965) initially demonstrates in a life-cycle model that a risk averse individual without a bequest motive should convert all their available assets to an immediate annuity. The main reason is that those who die earlier subsidise those who live a long time. However, the empirical data from international annuity markets with flexible pension policies has long reflected the fact that retirees are reluctant to convert voluntarily any retirement savings into annuities. The disparity between the theoretical recommendations and consumers' real preferences leads to the so called "annuity puzzle" and there is a substantial literature that attempts to explain this puzzle. From a rational perspective, reasons for the low demand of annuities include fees and expenses associated with the annuity price (Brown and Warshawsky, 2001), the bequest motive (Friedman and Warshawsky, 1990), the existence of social security and pre-annuitised wealth (Dushi and Webb, 2004) and the worry about health care expenditure shocks

(Sinclair and Smetters, 2004). From an irrational behavioural perspective, possible explanations include people are loss averse rather than risk averse (Hu and Scott, 2007), individuals' behaviour depends heavily on the way in which available choices are presented (Brown et al., 2008) and regret aversion (Cannon and Tonks, 2008). To appeal to the demand of consumers for increased flexibility, better value or better liquidity, more complex features of annuity product design are proposed and the menu of annuity products available is growing. It is recommended by OECD (2016) that individuals use part of their assets accumulated for retirement to purchase a life annuity to protect themselves from longevity risk.

Another stream of literature on post-retirement strategies aims at determining safe drawdown rates so that pension savings can last for a certain number of years. The best known one is the 4% rule from Bengen (1994). This rule suggests an initial withdrawal rate of 4% from the portfolio, with the subsequent withdrawal amounts being increased annually with inflation. According to Bengen (1994), 4% is the highest spending rate that allows the portfolio to last for at least 30 years before being exhausted by withdrawals. Based on this idea, Cooley et al. (1998) conduct a "Trinity Study", where they use simulations to determine, for each spending rate, the success probability that a portfolio will last for a certain number of years. A similar approach is adopted in current practice to determine the appropriate spending rate that allows for some probability of running out of money. The 4% rule has achieved wide acceptance among retirement planners and financial advisers due to its simplicity. Nonetheless, it has been sharply criticised by some scholars. For example, Scott et al. (2009) conclude that the strategy is suboptimal since it suggests a constant spending plan while using a risky, volatile investment strategy; the retirees therefore have unspent surpluses when markets outperform and face shortfalls when markets underperform. Blanchett et al. (2016) argue that most literature is based only on historical asset returns in the United States. Based on lower expected returns in the UK, especially in the near term, Blanchett et al. (2016) suggest that the safe initial withdrawal rate should be approximately 2.5% in the UK.

In approaching the research questions of post-retirement annuitisation decisions and decumulation strategies, the expected utility maximisation approach, which takes into account an annuitant's attitude towards risk and the value to the annuitant of income payments at different points in time, has been mostly used. Consider a decision making

individual who will live for an uncertain number of years, the optimal strategy is determined by maximising the sum of discounted utility from a stream of future consumption.

The literature on this subject mainly consists of rational studies, and a standard assumption is that individuals are rational utility maximisation agents with risk-averse preferences. To be more specific, a rational framework needs to cover the following assumptions involved with the discount function and utility function. The discount function, which represents the degree of impatience to receive future utility felt by the individual, needs to capture the rational inter-temporal consistency in decision making. This is realised through geometric discount function, which assumes that the amount of discounting per period is the same regardless of the starting point. For the utility function, which converts the money amount to perceived value or level of satisfaction, three assumptions need to be satisfied. First, the individual will always prefer to have more consumption to less. Second, the individual is risk averse, which means that given a choice of a certain amount of consumption or an actuarially fair gamble, then the individual would always choose certainty. For example, a risk averse individual would prefer receiving £100 than having equal chance of receiving either nothing or £200. Third, the individual is 'prudent', which refers to how an individual respond to increases in risk. In our context, prudent individuals would optimally increase their savings in the face of future uncertainty. Economists commonly use the Constant Relative Risk Aversion (CRRA) and Constant Absolute Risk Aversion (CARA) for the choice of the utility function (Cannon and Tonks, 2008).

More recently researchers have become aware that these assumptions may not be adequate in describing actual behaviour and so current research is more devoted to consider what can be learnt form economic psychology. Ample evidence has been found showing that actual behaviour is often irrational and departs from what we assume. Therefore, this thesis look at the questions of annuitisation decision and decumulation strategy in a behavioural way. Three behavioural factors are chosen in our study: Cumulative Prospect Theory (CPT), the hyperbolic discount model and subjective mortality rates. CPT introduced by Tversky and Kahneman (1992) suggests that individuals' behaviour is modelled better by loss aversion rather than risk aversion, and individuals falsely perceive probabilities by overestimating the probability of low-probability events and underestimating the probability of high-probability events. the hyperbolic discount

model captures the fact that people tend to act impulsively in the short term but become more patient in the long term; hence the implicit discount rate varies inversely with the length of the waiting time. Subjective mortality rate model introduces an individual optimism index, reflecting that people's perception of their own life expectancies can be different from life expectancies of the average number of the general public. We choose the three behavioral models for two main reasons. First, these models have been widely discussed in literature and have been used to explain a wide range of behavioral anomalies. For example, CPT is used to explain that people purchase unfair gambles and simultaneously purchase insurance products. The hyperbolic discount model can explain the puzzle of simultaneously having large credit card debts and pre-retirement savings. Second, these three models have well deigned mathematical models to reflect their properties, which gives us the chance to build a mathematical modeling framework to approach the research questions in an analytical and quantitative way.

In the literature, the survey is also a popular methodology used to capture individuals' decisions in a very straightforward way and it is often used to study factors that cannot be easily quantified (see Brown et al. (2008) and Duxbury et al. (2013) for example). However, in this thesis we do not conduct surveys to approach the research questions, because we believe that, with a survey, it is hard to capture the impact of a single factor. For example, when we aim to understand the impact of time-inconsistent preferences on the annuitisation decision, it is hard to control for the fact that subjects may have different levels of financial education and have different levels of risk aversion. Although mathematical models experience some other problems such as using a representative agent in calibrating the model, we have minimised the impact by conducting sensitivity analyses to ensure the stability of our results.

Apart from the factors that we cover in this thesis, there are some other behavioural factors that might influence retirement decisions. Brown et al. (2008) show that a significant greater percentage of subjects prefer an annuity rather than a savings account when the choice is framed in terms of consumption rather than investment. Warner and Pleeter (2001) analyse data from the US military drawdown program of the early 1990s, when most people selected a lump sum payment rather than an annuity, and conclude that people adopt excessive discount rates when evaluating future payments. A collection of the impacts of different behavioural factors on different stages of retirement decision making is offered in Mitchell and Utkus (2004).

In behavioural economics, behavioural biases can be categorised into two types: cognitive errors and emotional biases. Cognitive errors refer to basic statistical, information-processing, or memory errors that cause the decision to deviate from the rational decisions of traditional finance; while emotional biases are a result of attitudes and feelings that can cause decisions to deviate from rational decisions (Pompian, 2011). For example, in our context, the framing effect belongs to cognitive error since it is the error in information processing; on the other hand, loss aversion belongs to the emotional bias because dislike of loss is embedded in one's personality and it varies from person to person. In order to bridge the gap between actual decisions and optimal rational decisions, financial eduction is commonly suggested to fix the cognitive errors. In the context of annuitisation decisions, financial education has two meaningful impacts. First, evidence shows that there is a lack of understanding of the annuity product among the public, due to the product's complexity (Cannon and Tonks, 2008). Financial education would help people better understand the structure of the product, hence improving the likelihood of making a purchase. Second, financial education would teach people to process information and assess probabilities in a better way, so that the cognitive errors in making an annuity purchase decision can be minimised.

This PhD thesis considers both rational framework and behavioural framework and contributes to the understanding of the annuity puzzle and the use of annuities in the retirement strategy design. More specifically, the contribution of this PhD thesis is twofold. First, in the course of analysing the annuity puzzle using behavioural models, the thesis presents evidence for a high perceived value for deferred annuities. It has laid a research foundation for introducing and promoting the deferred annuity product in the UK market. The behavioural model and techniques that are implemented in this thesis could also be adopted in the process of the annuity product design in order to test for the desirability of some additional features in annuity products. Second, it incorporates deferred annuities in the design of utility maximisation retirement strategy. The proposed strategy effectively helps retirees manage the longevity risk at advanced ages and turns the drawdown plan into an easier decision than before (because of a fixed rather than unknown drawdown period). The proposed strategy may have an implication on government policy as well (which we will elaborate in detail in the following chapters).

This thesis comprises four key parts. The first part uses Cumulative Prospect Theory (CPT) to work out the subjective values of an annuity and so it can tell whether an

annuity available in the market is desirable to purchase. The second part considers the hyperbolic discount model in the valuation of an annuity. Both behavioural models justify the low attractiveness of immediate annuities and suggest the high attractiveness for deferred annuities. Therefore in the third part, we incorporate the deferred annuities into a post-retirement decumulation plan. The proposed strategy suggests the optimal allocation in deferred annuities and optimal drawdown rates to be followed. In the final part, we test the impact of two behavioural factors, time-inconsistent preferences (as represented by the hyperbolic discount model) and having subjective opinions on their future mortality rates, on the strategy proposed in the third part.

This thesis is composed of four self-contained chapters stemming from four research papers. Being self-contained, each chapter has its own introduction, notation, conclusions and references. In the end, the last chapter presents our overall conclusions and future research perspectives.

A brief description of the contributions of each of the four key chapters follows.

Chapter 2: Cumulative prospect theory, deferred annuities and the annuity puzzle

In this chapter, we analyse the "annuity puzzle" using an economic behavioural model, Cumulative Prospect Theory (CPT). According to CPT, individuals are loss averse rather than risk averse; they also tend to overweight low-probability events and underweight high-probability events. We show that the two behavioural biases together can explain the low demand for immediate annuities at retirement age and suggest the high desirability of long-term deferred annuities purchased at retirement. By decomposing the two behavioural factors in the CPT model, we further identify that the loss aversion is the major reason that stops people from buying an annuity, while the survival rate distortion is an important factor affecting the decision of when to receive annuity incomes.

Chapter 3: Why the deferred annuity makes sense

In this chapter, we use another behavioural model, the hyperbolic discount model, to analyse annuitisation decisions. In the process of making an intertemporal choice, individuals may show time-inconsistency; for instance, the discount rate used to evaluate intertemporal benefits varies with the length of the delay period, size and signs of the benefits. This is captured in the hyperbolic discount model. We show that a typical hyperbolic discount model can explain the reason why an immediate annuity is not attractive and we find that both retirees and people at working age would be willing to pay a higher-than-market price for long-term deferred annuities. Moreover, we argue that the annuity take up rate would be higher if governments were to introduce a pre-commitment device that requires individuals to make annuitisation decisions at working age before retirement.

Chapter 4: Optimal decumulation strategy during retirement with deferred annuities

After identifying the desirability of long-term deferred annuity in the previous two chapters, in this chapter we turn our attention to a partial annuitisation retirement decision based on deferred annuities. Following the standard literature (i.e. Yaari (1965)), we aim at an optimal decumulation strategy in terms of utility maximisation. However, our model is composed of a deferred annuity rather than an immediate annuity that has been discussed in literature. Our strategy would suggest an optimal percentage of wealth to be allocated to a deferred annuity and optimal drawdown rates to be followed before the commencement of the annuity. Therefore, the investment decision during the retirement stage is simplified from an uncertain period (the remaining lifetime after retirement) to a fixed period (the deferred period of the chosen annuity product); and the majority of longevity risk is hedged because of the lifelong guaranteed annuity payments being delivered from a pre-specified age by the deferred annuity. With a set of benchmark assumptions, we have two main suggestions: (i), a retiree who would like to retain a certain level of liquidity should spend 21.6% in a 15-year deferred annuity or 9.13% in a 20-year deferred annuity; (ii), a retiree who simply wants to maximise overall utility from retirement consumption should spend 61.83% in a 6-year deferred annuity. These suggested allocations are stable relative to pricing factors.

Chapter 5: The impact of behavioural factors on retirement decumulation strategies

In order to know whether people who are exposed to behavioural biases would like to follow the same decumulation strategies as rational retirees, we extend the study in Chapter 4 by considering two behavioural factors: hyperbolic discount models and subjective mortality rates. Our modelling results suggest that hyperbolic discounters would invest a similar proportion of pension savings in deferred annuity products, however they would prefer inflation-linked payments rather than level payments. Moreover, by adding a subjective factor to mortality rates, we identify that retirees who are optimistic (pessimistic) about their life expectancies would find annuities attractive (not attractive) in general and they tend to allocate more (less) of their pension savings to annuity products.

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2

Cumulative prospect theory, deferred annuities and the annuity puzzle

Cumulative prospect theory, deferred annuities and the annuity puzzle

Abstract

Although it has been proved theoretically that annuities can provide optimal consumption during one's retirement period, retirees' reluctance to purchase annuities is a long-standing puzzle. Cumulative Prospect Theory (CPT), which considers both loss aversion and probability transformations, can explain the low demand for immediate annuities. It also shows that retirees would be willing to buy a long-term deferred annuity at retirement. By considering each component in CPT, we find that loss aversion is the major reason that stops people from buying an annuity, while the survival rate transformation is an important factor affecting the decision of when to receive annuity incomes.

Keywords: CPT, Deferred annuities, Annuity puzzle, Reservation price.

2.1 Introduction

During the past few decades, traditional defined benefit (DB) pension plans have been losing their dominance in private sector pension systems in many countries; there has been a steady shift from DB pension plans towards defined contribution (DC) pension plans. Between 1980 and 2013, the number of participants in US private sector DC plans had increased more than fourfold from 20 million to 92.5 million; while participants in US private sector DB plans only increased from 38 million to 39 million (US Department of Labor, 2013). A more extreme shift can be observed in the United Kingdom (UK) where active membership of DC pension scheme increased from 0.9 million in 2007 to 3.2 million in 2014; while that of DB pension scheme decreased from 2.7 million to 1.6 million during the same period (ONS, 2014). It is widely anticipated that pensioners will rely heavily on DC pension plans in the future.

Under DC pension plans, members have access to their accumulated individual pension account balances rather than receiving a series of regular cheques for life at retirement. In such a case, the longevity risk, the risk of outliving one's assets, is transferred from the corporate sector to DC members. Many scholars support using immediate annuities as a solution for longevity protection. An immediate annuity pays out a periodic income for as long as the annuitant is alive, in exchange for an initial premium charge. However, a low volume of premiums of voluntary annuities has been found in many international markets. Before a significant pension reform in 2011, in the UK, there used to be two markets: a compulsory one and a voluntary one. According to the sales figure reported in Cannon and Tonks (2011), the total compulsory annuity premium income grew to around £11.5 billion in 2010, while the voluntary annuity premium only amounted to £72 million.

The disparity between the theoretical optimal choice and the consumers' real preferences leads to the "annuity puzzle". Since Yaari (1965) first demonstrated the benefit of annuitisation in a life cycle model with uncertain lifetimes, the subsequent literature on annuities has provided various reasons to explain the low demand for annuities. Major reasons include the mortality risk-sharing among family members (Brown and Poterba, 2000), the existence of provision through social security and DB pension scheme membership (Dushi and Webb, 2004) and the possibility of health care expenditure

shocks at an old age (Davidoff, 2009). We provide a detailed explanation of possible reasons in next chapter.

While most conclusions are based on the assumptions that retirees are rational utility maximisers with risk-averse preferences, some recent studies have moved beyond the fully rational paradigm and proposed many behavioural factors that could play important roles in determining how retirees spend their retirement savings. For example, the decision to annuitise depends on the way in which the available choices are presented (Framing Effect). It also depends on the level of financial education or the level of understanding about annuities. In this chapter, we seek to explain the unattractiveness of annuities in the light of behavioural finance and we focus on the impact of Cumulative Prospect Theory (CPT) which addresses flaws in the expected utility hypothesis and risk aversion. This theory was initially proposed by Tversky and Kahneman (1992) to describe how individuals make choices involving risky outcomes. It states that investors are loss averse rather than risk averse and a certain transformation is performed in evaluating probabilities. Using descriptive models for CPT, many authors have explained economic anomalies that cannot be explained by rational models. For example, Benartzi and Thaler (1995) and Barberis et al. (2001) apply CPT in the explanation of the equity premium puzzle. Kaluszkaa and Krzeszowiec (2012) use CPT as a method to price insurance contracts.

Hu and Scott (2007) first adopt CPT in the analysis of annuities. They show that CPT can explain the low demand for immediate annuities purchased at retirement; and the probability transformation introduced in CPT makes people prefer deferred annuities with the first payment delaying for a few years. We build on the work by Hu and Scott (2007), extend their analysis and make the following contributions: (i) by conducting the analysis on successive age points in retirement, we conclude that immediate annuities are not attractive to purchase for retirees at all ages; however, preferences for deferred annuities increase with the deferred period. (ii) the sensitivity analysis suggests that the major reason for the unattractiveness of annuities is loss aversion. The overweighting of low probability events would shift retirees' preferences towards receiving annuity incomes at a later stage. (iii) By conducting an elasticity analysis we conclude that loss aversion is the most influential factor on the decision to purchase immediate annuities while the probability transformation determines the decision to purchase deferred annuities.

This chapter is organized as follows. In the next section, we provide a detailed introduction of CPT: the value function and the probability transformation model. Then in Section 3, we measure the perceived value of an annuity using the CPT framework. In Section 4, we present the results of relative price difference of annuities. Section 5 presents an extensive sensitivity analysis. Finally, Section 6 concludes the results and comments on the limitations of the analysis.

2.2 Introduction to Cumulative Prospect Theory (CPT)

CPT is a behavioural model that aims to capture decision making under risk and uncertainty. It states that the overall value of a risky investment is determined by three components: a reference point, a value function and a set of decision weights.

Being different from the utility function in expected utility theory, the value function $(v(\cdot))$ in CPT has three new properties. Firstly, the value function is based on the distance (y) between the investment outcome and a reference point, rather than the terminal investment outcome in the utility function. In a risky investment, the initial outlay to enter the investment is often regarded as the reference point. Investors would not simply consider the investment outcome as the gain; instead, they will deduct the initial outlay from the investment outcome and their satisfaction gained from the investment is based on this. Secondly, while the utility function describes simply a concave picture, the value function is concave above the reference point $(v''(y) < 0, y > 0)$ and convex below the reference point $(v''(y) > 0, y < 0)$. This can be illustrated by an example: the satisfaction increase between a win of £100 and a win of £200 appears to be greater than the satisfaction increase between a win of £1100 and a win of £1200. Similarly, the increment in sadness that people feel between a loss of £100 and a loss of £200 tends to be greater than the increment in sadness between a loss of £1100 and a loss of £1200, unless the large loss would compel people to lower current living standard such as moving to a less desirable neighborhood. In other words, the value function yields the property of diminishing sensitivity: the marginal value of both gains and losses generally diminish with the distance from the reference point. Furthermore, the value function captures an important characteristic of attitudes to changes in wealth: loss looms larger than gains. Thus, most people are loss averse and the satisfaction gained from a £100 win cannot erase the sadness brought by a £100 loss. From an experiment conducted

by Kahneman and Tversky (1979), people feel it unattractive to enter the symmetric bet of winning y or losing y with equal probability, which justifies the assumption that the value function for losses is steeper than that for gains $(v'(y) < v'(-y)$ for $y \geq 0)$.

Tversky and Kahneman (1992) offer an explicit form for the value function as

$$
v(y) = \begin{cases} y^{\alpha} & \text{if } y \geq 0 \\ -\lambda(-y)^{\beta} & \text{if } y < 0 \end{cases} \tag{2.1}
$$

Here, λ reflects the level of loss aversion and α and β reflect diminishing sensitivity. Tversky and Kahneman (1992) estimate that $\lambda = 2.25$ and $\alpha = \beta = 0.88$.

Whereas expected utility theory weights the utility at different states with the objective true probability of each state, experiments conducted by Tversky and Kahneman (1992) show that for both positive and negative prospects, decision makers always overweight low probability events and underweight high probability events. Therefore CPT introduces a method to transform true probabilities to decision weights which reflect perceived possibilities.

CPT introduces a capacity function w to express decision makers' opinions of the perceived likelihood of uncertain events. The capacity function w is a non-linear transformation of the real probabilities p. Two natural boundaries are certainty $(w(1) = 1)$ and impossibility $(w(0) = 0)$. The principle of diminishing sensitivity applies to the capacity function as well; it means that the influence of a given change in probability diminishes with its deviation from the boundary. For instance, the change in probability of winning a prize from 0.9 to 1 has more impact than the change in probability of winning a prize from 0.6 to 0.7. Similarly, an increase of 0.1 in probability of winning a prize has more impact when the probability changes from 0 to 0.1 than when the probability changes from 0.3 to 0.4. Therefore, the capacity function w is concave near 0 and convex near 1. In addition, the capacity function for positive and negative investment outcomes should be different because risk-seeking for a small probability of gains is more pronounced than being risk-averse for a small probability of losses. The capacity function is thus assumed to have the following form:
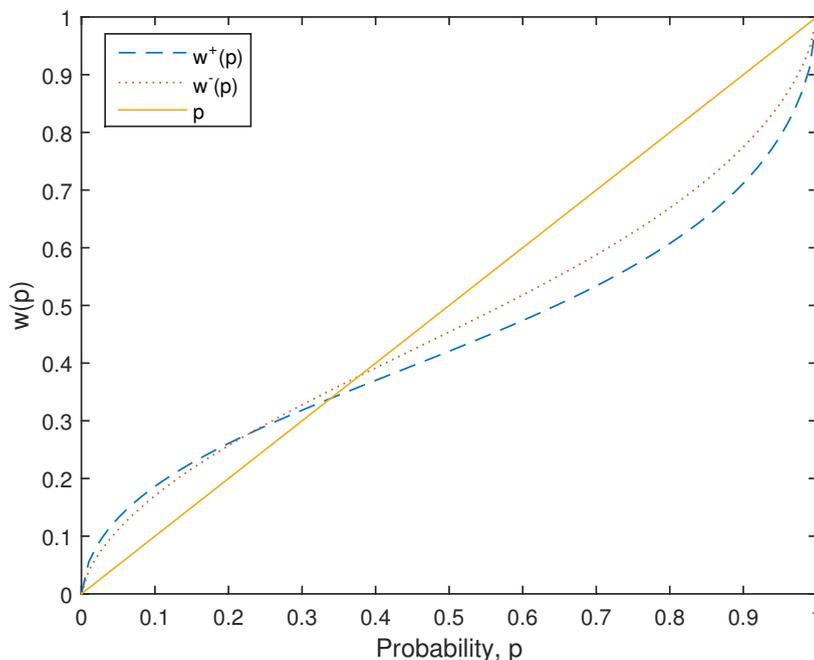
FIGURE 2.1: CPT capacity function $w(p)$

$$w^+(p) = \frac{p^\gamma}{[p^\gamma + (1-p)^\gamma]^{\frac{1}{\gamma}}};$$
$$w^-(p) = \frac{p^\delta}{[p^\delta + (1-p)^\delta]^{\frac{1}{\delta}}}. \tag{2.2}$$

Based on experimental results in Tversky and Kahneman (1992), γ is estimated to be 0.61 and δ to be 0.69. Figure 2.1 exhibits the shape of the capacity function. The inverted S-shaped capacity function shows that people tend to overweight the probability of events that are less likely to happen and underweight the probability of events that are highly likely to happen. Furthermore, we can see that the weighting function for gains and losses are quite close, although the former is slightly more curved than the latter. It reflects the point that risk aversion for gains is more pronounced for risk seeking for losses, for moderate and high probability events. Additionally, decision makers' perceptions about probabilities coincide with the true probabilities around 0.35.

Let us define p_t as the probability of having an outcome y_t. We introduce the following quantities:

$$
\begin{aligned}
\pi_1^- &= w^-(p_1), \\
\pi_t^- &= w^-(p_1 + \ldots + p_t) - w^-(p_1 + \ldots + p_{t-1}), \ 2 \leq t \leq k, \\
\pi_t^+ &= w^+(p_t + \ldots + p_T) - w^+(p_{t+1} + \ldots + p_T), \ k+1 \leq t \leq T-1, \\
\pi_T^+ &= w^+(p_T).
\end{aligned}
\tag{2.3}
$$

where the risky outcomes have been ranked as:

$$
y_1 < y_2 < \ldots < y_k < 0 \leq y_{k+1} < \ldots < y_T.
$$

The ultimate decision weight associated with an outcome is defined as the marginal value of the respective event. The decision weight π_t^+, which corresponds to positive investment outcomes, is the change in the value of w between two events: "the outcome is at least as good as y_t" and "the outcome is strictly better than y_t". The decision weight π_t^-, which corresponds to negative investment outcomes, is the change in the value of w between the events: "the outcome is at least as bad as y_t" and "the outcome is strictly worse than y_t".

The value functions and the cumulative decision weights are combined to arrive at the overall value of a risky investment under CPT:

$$
\begin{aligned}
V(f^-) &= \sum_{t=1}^{k} \pi_t^- v(y_t), \\
V(f^+) &= \sum_{t=k+1}^{T} \pi_t^+ v(y_t), \\
V(f) &= V(f^-) + V(f^+).
\end{aligned}
\tag{2.4}
$$

Letting $\pi_t = \pi_t^+$ if $t \geq k+1$ and $\pi_t = \pi_t^-$ if $t \leq k$, Equation (2.4) can be reduced to:

$$
V(f) = \sum_{t=1}^{T} \pi_t v(y_t).
\tag{2.5}
$$

2.3 Annuity Valuations under CPT

The annuity is initially designed as an insurance product that helps reduce longevity risk. However, people tend to view it as a risky investment product. One reason might be the lack of understanding of the operational details of annuity products. In a survey conducted by an American Council of Life Insurance task force, the findings of consumers' attitudes towards annuities showed that virtually no consumer fully understands how an annuity product works; the least understood aspect of annuities is how risk sharing is performed so that insurers can offer a lifelong guaranteed income (Brown and Warshawsky, 2001). Because of this, consumers are more likely to focus on the risk of dying early, while overlooking the possibility that they may live well beyond their life expectancies and receive more than they have paid. They may also believe that the odds in the gamble tend to favor insurance companies. In another similar survey, the Society of Actuaries (2004) found that 49% of workers and 44% of retirees considered protecting against loss of value from a pension or annuity investment should they die earlier than expected as very important. Therefore, within this mental accounting framework, retirees tend to equate the lifetime annuity purchase with entering a gamble on their lives. CPT can be applied here to determine the overall value of annuities.

Viewing an annuity investment as a gamble, investors gain if total discounted annuity income exceeds the annuity price; whereas investors lose if total discounted annuity income is below the annuity price. In other words, an annuitant gains if he outlives the life expectancy assumed in annuity pricing and loses if he dies before he collects as much income as he paid out. We assume that a retiree aged 65 purchases an immediate annuity at an actuarially fair price A, then the annuity investment outcome if the annuitant dies after t years (at age $65 + t$) should be:

$$y_t = -A + \sum_{i=1}^{t} \Psi \frac{1}{(1+r)^{i-1}} \tag{2.6}$$

where Ψ represents the annual annuity income that is paid in advance and is assumed to be 1 unit in our study; r represents the assumed constant interest rate; A is the actuarially fair price of an annuity that pays 1 unit per year in advance until the annuitant dies. No administrative fees or profit loadings are considered here.

(A) for a 65-year-old retiree

(B) for a 75-year-old retiree

(C) for a 85-year-old retiree

(D) for a 95-year-old retiree
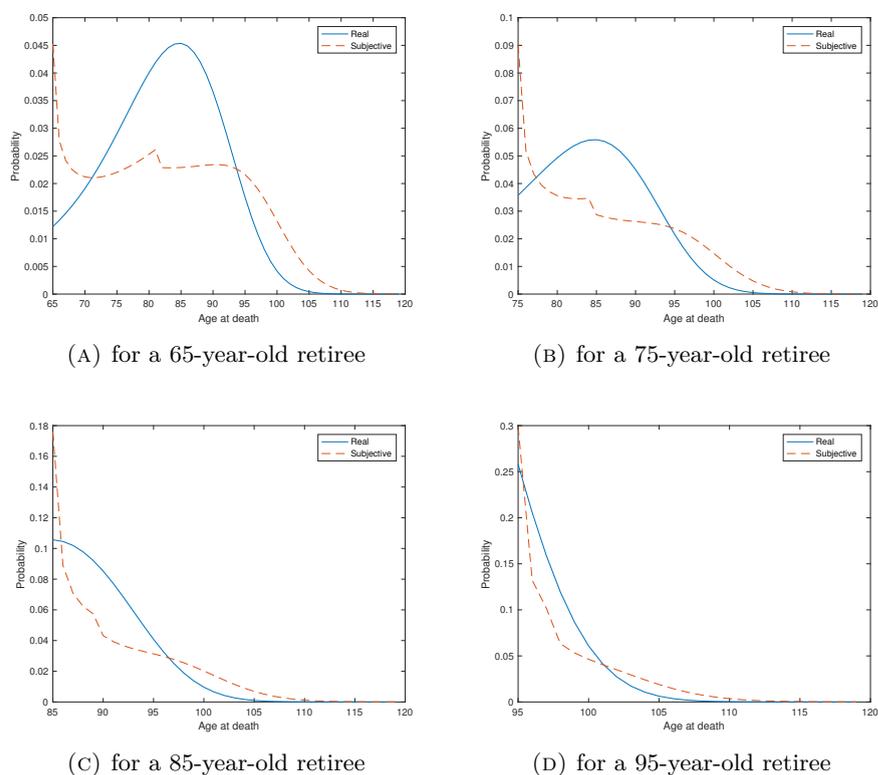
FIGURE 2.2: Probability of dying at different ages

The probability, p_t, that corresponds to each annuity outcome y_t, is the probability that the 65-year-old retiree dies in exactly t years, at age $65 + t$. With the input y_t, we can get the perceived value of the annuity investment according to Equation (2.1). Furthermore, we can transform probabilities p to decision weights π based on Equations (2.2) and (2.3). The overall value of the annuity can be calculated according to Equation (2.5).

To reveal the impact of CPT probability transformations on mortality rates, Figure 2.2 shows the distorted probability π versus the original probability p when retirees are at age 65, 75, 85 and 95. The results displayed are based on assumptions of an annual annuity payment of 1, an interest rate of 3 percent and an actuarially fair annuity price. It reflects that the ultimate decision weights will enhance the low probability of dying shortly after annuity purchase and the low probability of surviving a very long period after the annuity purchase. At the same time, it will decrease the probability corresponding to intermediate outcomes.

The transformation of the real probabilities will change people's perceptions of their life expectancies. Figure 2.3 describes the differences between subjective complete life
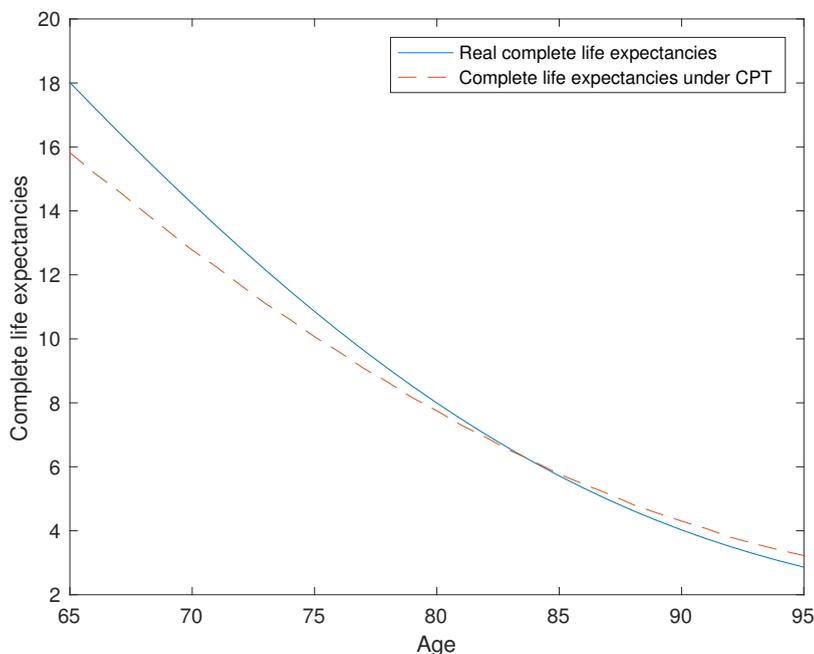
FIGURE 2.3: Complete life expectancies under the impact of CPT probability transformations

expectancies and real complete life expectancies for male individuals aged between 65 and 95. According to the figure, the distorted probability of dying at each age leads to the underestimation of life expectancies for young retirees and the overestimation of life expectancies for old retirees.

Following Hu and Scott (2007), we use the maximum acceptable price as a benchmark measure to determine if an actuarially fairly priced annuity is attractive. The maximum acceptable price, also called the "reservation price", is the highest price that a buyer is willing to pay for goods or a service. In the context of an annuity purchase, it is the price that would make an individual indifferent between purchasing an annuity and keeping the money in hand. According to Equations (2.1) to (2.6), the overall value of an annuity can be regarded as a function of annuity price. The maximum acceptable annuity price is therefore the price that makes CPT value of an annuity equal to zero.

To facilitate our analysis, we calculate the ratio R, which is the relative difference between reservation price and fair price.

$$R = \frac{Reservation\ Price - Actuarially\ fair\ price}{Actuarially\ fair\ price}$$

A positive R means people would like to pay a higher-than-market price for an annuity and thus the annuity is attractive. On the other hand, a negative R indicates unattractiveness. The modeling results in terms of R can also be interpreted as how much more or less than market price one would be prepared to pay for an annuity.

2.4 Results

Due to the fact that no closed form solution for R exists, we solve it numerically. In our analysis, we consider cumulative prospect theory with probability distortion (using π) and without probability distortion (using p). For each one, we consider the situation with loss aversion ($\lambda > 1$) and without loss aversion ($\lambda = 1$). Thus we can find out how CPT influences people's annuitisation decisions. In the following basic results, we have assumed that each parameter value in the CPT model is based on the values in Tversky and Kahneman (1992). That means $\lambda = 2.25$ and $\alpha = \beta = 0.88$ for the value function and $\gamma = 0.61$ and $\delta = 0.69$ for the probability distortion function. In terms of the annuity types, we are interested in both the immediate annuities that are purchased from age 65 to age 95 and the deferred annuities with deferred period from one year to thirty years (all purchased at age 65). We assume a constant interest rate of 3 percent. The mortality rates are calculated from the recent standard mortality table "S2PML[1]", which contains the mortality experience of male pensioners of self-administered UK pension schemes for the period 2004 to 2011.

Figures 2.4 and 2.5 respectively show the trends in the R values for immediate and deferred annuities under different versions of the CPT models. In the simplest model of linear value function with no loss aversion and no probability transformation, R remains 0 for both types of annuity. It reflects the fact that a risk-neutral individual who adopts a linear value function should be indifferent between purchasing an actuarially fairly priced annuity and keeping the money at hand. With the full CPT framework (CPT including probability distortion and with loss aversion), we identify that immediate annuities are generally not attractive for retirees between age 65 and age 95. Hence, CPT can be used as a behavioral explanation for people's not buying immediate annuities. In addition, for a 65-year-old retiree, his/her preference for a deferred annuity is always increasing

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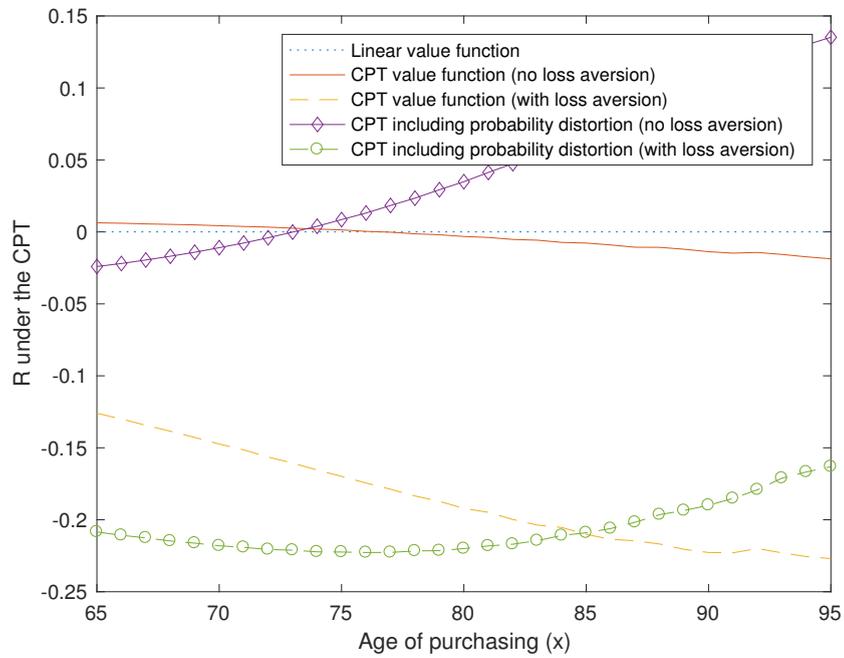[1]Source: Continuous Mortality Investigation (2013)

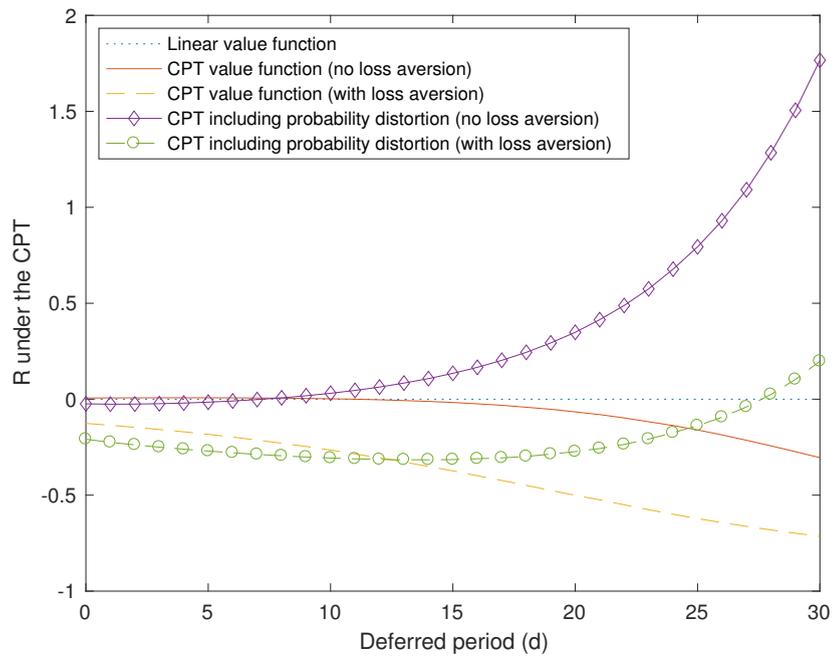FIGURE 2.4: The Relative Price Difference (R) under CPT for immediate annuities

FIGURE 2.5: The Relative Price Difference (R) under CPT for deferred annuities

with the deferred period. In the following, we will analyse each component in the CPT framework.

Comparing the values of R for CPT models with loss aversion to those without loss

aversion, we notice that in both the immediate annuity case of Figure 2.4 and the deferred annuity case of Figure 2.5, R shifts downwards significantly once the loss aversion factor is incorporated. This shifts the results in terms of the relative price difference from positive to negative for both immediate annuities and deferred annuities. Therefore, immediate annuities are not attractive for loss-averse individuals who are afraid of dying before reaching average life expectancies and deferred annuities are not attractive for loss-averse individuals who worry about dying within the deferred period. Hence, loss aversion is able to explain the low attractiveness of annuity products. Additionally, comparing the results for the CPT value function with loss aversion in Figure 2.4 with those in Figure 2.5, the magnitude of the change in R is much smaller for immediate annuities than for deferred annuities. It indicates the behavioural obstacle of loss aversion has greater impact on the choice of deferred annuities than immediate annuities. One may notice that the conclusion relies heavily on the assumed value for the loss aversion factor (which is 2.25); in the next section, we will discuss the stability of the results under a range of values of loss aversion.

In Figure 2.4 and Figure 2.5, we see the impact of probability distortion red(on immediate annuities and deferred annuities respectively) by comparing the values of R for the "CPT value function (no loss aversion)" with those for the "CPT including probability distortion (no loss aversion)". The probability distortion does not lead to the vertical shift of the Relative Price Difference (R); instead it creates a twist in the shape of R in relation to the age of purchasing (x) or the deferred period (d). The probability distortion makes immediate annuities become more preferable for older retirees than younger retirees as R is increasing smoothly with the age of purchase. Given the fact that more elderly retirees have a smaller possibility of surviving, they overestimate the survival rate to a greater degree and would be prepared to pay a higher price for immediate annuity products. The same reasoning applies for deferred annuities. Retirees who follow CPT overestimate the low probability event of surviving for a long period to a greater degree and therefore a longer deferred period is much more preferred. Overall, overweighting of low probability events suggests that people buy annuities that provide income at an older age.

Among all types of annuities discussed, a 30-year deferred annuity is predicted to be the most attractive. Since the chance of surviving from age 65 to age 95 is very low at 0.068 (based on S2PML), the 30-year deferred annuities may be perceived as a similar

product to a lottery ticket. Cumulative Prospect Theory has been used to explain why individuals love buying classical lottery tickets which have negative expected outcome (Kahneman and Tversky, 1979).

2.5 Sensitivity Analysis

In the above calculations, we have assumed that each parameter value is based on the values in Tversky and Kahneman (1992). The loss aversion factor λ equals 2.25; α and β in the value function equal 0.88; γ is estimated to be 0.61 and δ to be 0.69 in the probability distortion function. Although the results may have biases because the annuity investment decision typically involves a much larger amount of money than the gambles used in Tversky and Kahneman's psychological experiments, they still provide a good qualitative explanation for the attractiveness of different types of annuities. In this section, we conduct several sensitivity analyses to find out the relative attractiveness of annuities with regard to each parameter. The parameters we examine are: loss aversion, λ, interest rate, r, the curvature of the capacity function, δ and γ, and the probability of dying at exact age, p (by changing mortality tables).

2.5.1 Loss aversion sensitivity

The purpose of this sensitivity test is to discover whether retirees with different levels of loss aversion would like to annuitise their DC account balances. We increase the degree of loss aversion gradually from $\lambda = 1$ (no loss aversion) to $\lambda = 5$. Figure 2.6 displays the shape of R under the CPT framework with regard to different annuity types and different levels of loss aversion.

In both panels of Figure 2.6, the attractiveness indicator, R, shifts downwards significantly when the level of loss aversion increases; this confirms that loss aversion is an important driving factor that stops retirees from buying an annuity. As people become more loss averse, the losses arise from an annuity investment in the first few years will be perceived to be greater than the real values; then individuals would like to pay a lower price to enter an annuity contract. Additionally, comparing the two panels shows that immediate annuities are less attractive than deferred annuities. Immediate annuities can be attractive only when investors are not loss averse; on the contrary, a 30-year deferred

(A) immediate annuities

(B) deferred annuities
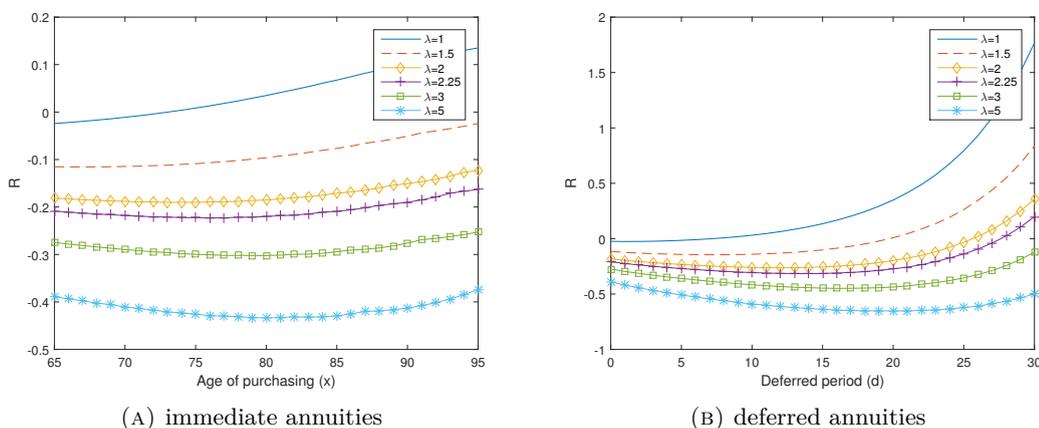
FIGURE 2.6: The sensitivity of the loss aversion factor on the Relative Price Difference (R) of annuities

annuity remains attractive even when investors' loss aversion level increases to 2.25. As explained in Section 2.4, the popularity of the 30-year deferred annuities could be due to the "lottery ticket effect". Thirdly, the sensitivity of R to the loss aversion degree is greater for deferred annuities than for immediate annuities. It is mainly because, for a deferred annuity purchaser, the annuity investment does not deliver any income until after the deferred period. When compared with investing in an immediate annuity, a deferred annuity investor needs to wait longer for the overall investment outcome to become positive.

Many empirical studies have confirmed the importance of loss aversion and estimated the degree of loss aversion. According to Benartzi and Thaler (1995), when making a material economic decision, such as investing, it is appropriate to assume a loss aversion of 2. In the annuity decision analysis, we find that the 30-year deferred annuity is attractive if retirees' loss aversion level is around 2.

2.5.2 Interest rate sensitivity

The interest rate is one of the most important factors to determine the price of an annuity and thus it may affect investment decisions. When the interest rate increases, both the annuity fair price and reservation price move downwards; it is therefore difficult to directly judge its influence on the overall attractiveness of annuities. In this section, we conduct an interest rate sensitivity analysis in order to explore whether retirees would

(A) immediate annuities

(B) deferred annuities
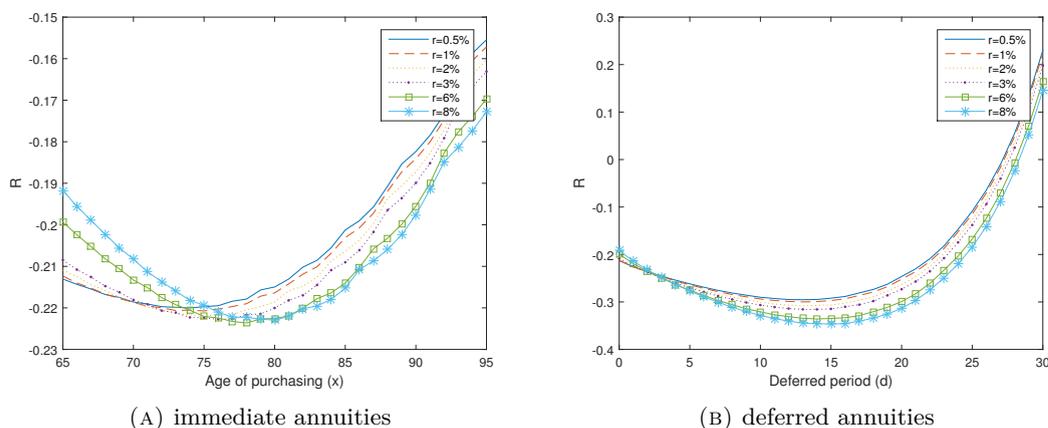
FIGURE 2.7: The sensitivity of the interest rate on the Relative Price Difference (R) of annuities

be willing to convert their DC account balances into annuities when they are exposed to a range of interest rates from as low as 0.5 percent to as high as 8 percent.

Figure 2.7 shows the trend of R for the CPT framework for immediate annuities and deferred annuities; we have the following findings. Firstly, the low vertical spreads in R results demonstrate that the influence of interest rates on annuity attractiveness is small. We expect this result since our benchmark of attractiveness is determined by relative price differences in reservation price and fair price. Secondly, with regard to immediate annuities, individuals are more prepared to make a purchase at retirement when the interest rate is very high, so that they can lock in the current high returns. As the interest rate falls, the immediate annuities become more expensive and it is better to delay the purchase. Similarly for deferred annuities, they are relatively cheap and attractive when the interest rate is very high; as the interest rate falls, deferred annuities with longer deferred periods become more preferable.

Regardless of how we change the interest rate, only the 30-year deferred annuity remains attractive among all annuity types. A major reason, as we have explained, is that deferred annuities with very long deferred periods are highly likely to be regarded as lottery tickets by retirees and thus may be attractive. Given the current low interest rate environment, immediate annuities are not an attractive purchase while the long term deferred annuities are predicted to be attractive.
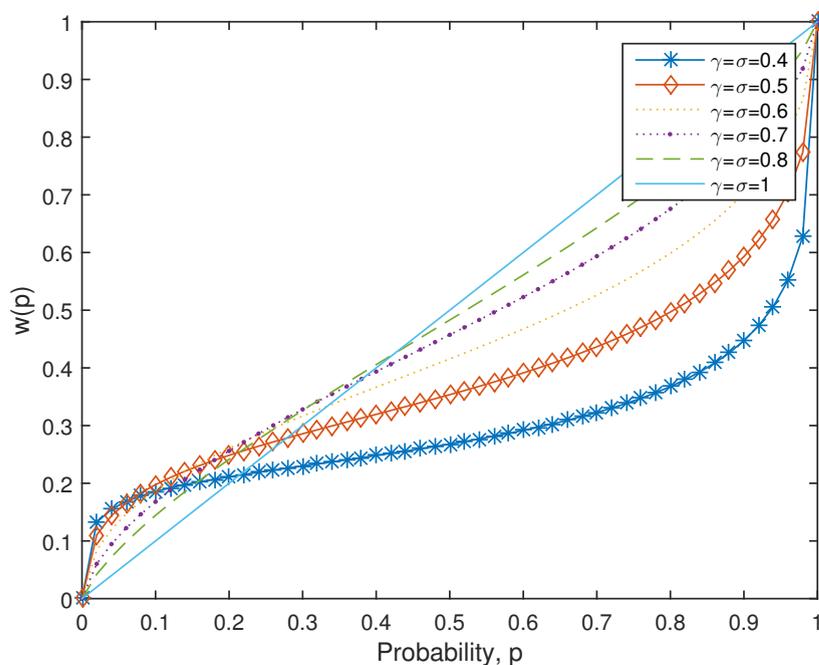
FIGURE 2.8: CPT capacity function $w(p)$ under different levels of probability distortion

2.5.3 Probability distortion sensitivity

In this section, we intend to discover the sensitivity of an annuity's attractiveness to different levels of probability distortion. Both parameters in the capacity function (see Equation (2.2)), δ and γ, which describe decision makers' opinions of perceived likelihood of uncertain events, are increased in steps of 0.1 from 0.4 to 1. Figure 2.8 shows the capacity function of the real probabilities and we can see that, when γ and δ are kept low, there is a high degree of twist in the probability values and thus one would assign higher weights to extreme outcomes and lower weights to intermediate outcomes. When both γ and δ are 1, there is no probability distortion.

Figure 2.9 illustrates how the attractiveness of immediate annuities and deferred annuities is affected by weaker or stronger probability distortions. It clearly confirms our previous conclusion that a heavier distortion in probabilities would lead to choices of annuities that start paying at an older age. For immediate annuitants, it means buying an annuity at an older age; for deferred annuitants, it means buying one with a longer deferred period. On the other hand, if decision makers do not show any biases in probability estimation, the best annuity solution is an immediate annuity at age 65.

(A) immediate annuities

(B) deferred annuities
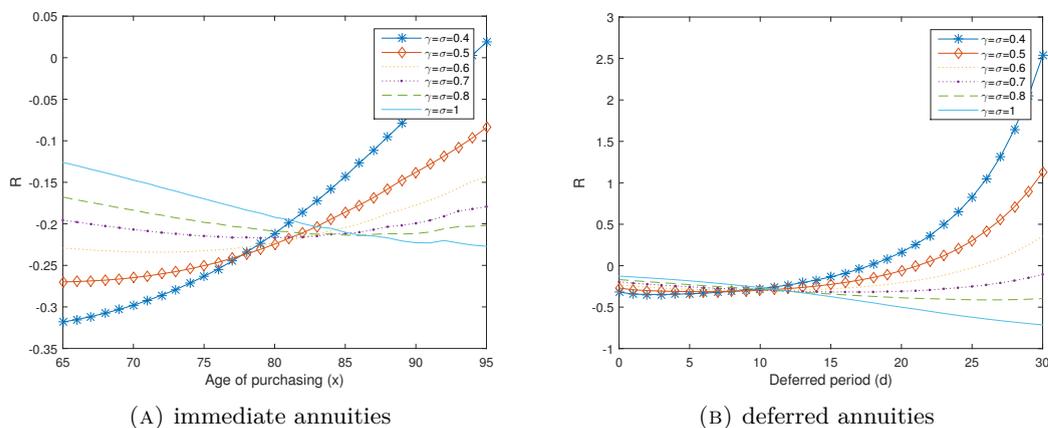
FIGURE 2.9: The sensitivity of the probability distortions on the Relative Price Difference (R) of annuities

Many studies have examined the probability transformation and provided evidence on the level of distortion which we should apply to large gambles. In the experiments conducted by Tversky and Kahneman (1992), they allowed subjects to enter gambles with final payoffs as high as twice the median monthly family income and they estimated the γ to be 0.61 and δ to be 0.69. Furthermore, Dodonova and Khoroshilov (2006) provide evidence showing that smaller gambles normally have weaker probability distortion. Therefore, both sets of empirical evidence suggest a greater probability distortion when we evaluate annuity decisions which involve large investments. When γ and δ are around 0.4, annuities that are deferred for more than 20 years are attractive for investors.

2.5.4 Mortality rates sensitivity

By changing the mortality assumptions, we can compare the attractiveness of annuities among different groups of people. In this section, we consider six different mortality groups[2]. Our baseline results are based on S2PML, the mortality experience of male pensioners of UK self-administered pension schemes for the period 2004 to 2011. S2PFL is the mortality experience of female pensioners for the same period. SPML03 captures the male pensioners' mortality experience for a different period of time: 2000-2006. Additionally, to discover whether one's pension amount would affect the decision to annuitise, the whole population dataset is divided into three subsets: S2PMA-L, S2PMA-M and S2PMA-H. S2PMA investigates the male pensioners' mortality experience during 2004

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[2]The source for the mortality table SPML03 is the Continuous Mortality Investigation (2008); all other mortality tables come from Continuous Mortality Investigation (2013)
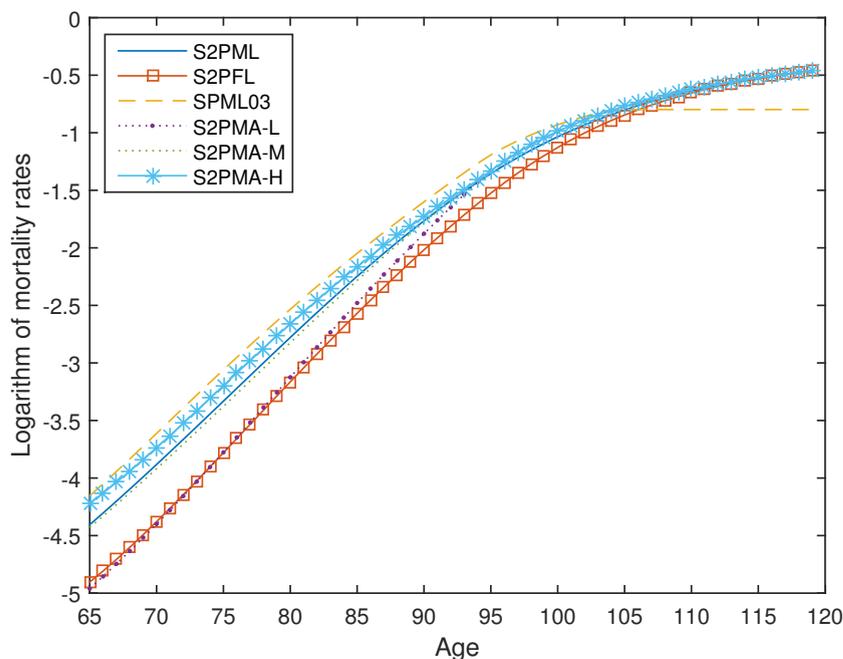
FIGURE 2.10: A comparison of Mortality rates

Notes: S2PML is the mortality experience of male pensioners of UK self-administered pension schemes for the period 2004 to 2011. S2PFL is the mortality experience of female pensioners for the period 2004 to 2011. SPML03 is the male pensioners' mortality experience between 2000 and 2006. S2PMA-L, S2PMA-M and S2PMA-H represent male pensioners' mortality experience during 2004 and 2011 by considering the size of pension savings: Light (L), Medium (M) and Heavy (H).
Source: The source for the mortality table SPML03 is the Continuous Mortality Investigation (2008); all other mortality tables come from Continuous Mortality Investigation (2013).

and 2011 by considering the size of pension savings. "Light" in S2PMA-L means that the pension size exceeds a specified amount and hence retirees in this group have relatively light mortality rates; "Heavy" in S2PMA-H means the pension size is lower than a specified amount and retirees in this group have relatively high mortality rates; "Medium" in S2PMA-M describes the mortality rate when the pension size is intermediate.

Figure 2.10 shows the differences in mortality rates q_x[3], the probability of death at age x, among the six groups of people that make up the populations for these standard life tables. The improvement in mortality rates over time can be identified since male pensioners during an earlier period 2000-2006 have the highest mortality rates. In contrast, pensioners with a high pension size and female pensioners, during the period 2004-2011, have the lowest mortality rates among the six groups. As a result, pensioners in SPML03

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[3]For clarity, we plot the logarithm of mortality rates.

(A) immediate annuities

(B) deferred annuities
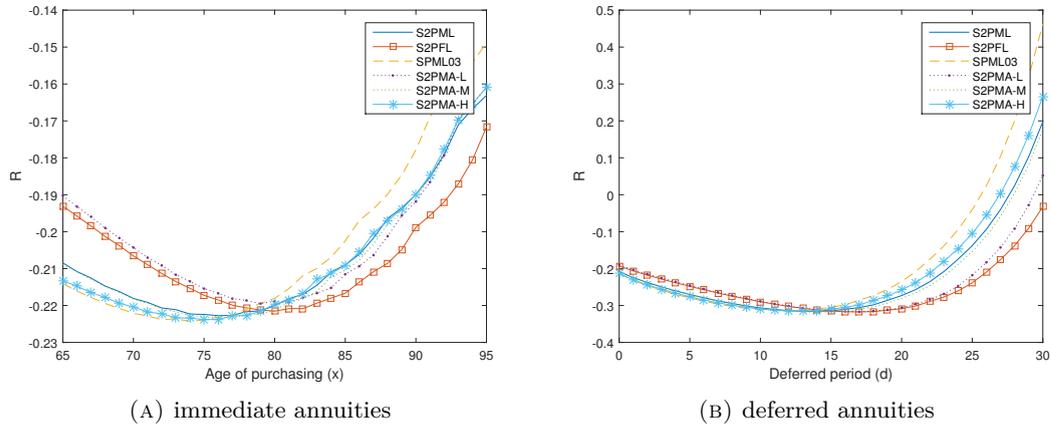
FIGURE 2.11: The sensitivity of the mortality rates on the Relative Price Difference (R) of annuities

Notes: S2PML is the mortality experience of male pensioners of UK self-administered pension schemes for the period 2004 to 2011. S2PFL is the mortality experience of female pensioners for the period 2004 to 2011. SPML03 is the male pensioners' mortality experience between 2000 and 2006. S2PMA-L, S2PMA-M and S2PMA-H represent male pensioners' mortality experience during 2004 and 2011 by considering the amount of pension: Light (L), Medium (M) and Heavy (H).
Source: The source for the mortality table SPML03 is the Continuous Mortality Investigation (2008); all other mortality tables come from Continuous Mortality Investigation (2013).

have the shortest remaining lifetime, while those in S2PFL and S2PMA-L are expected to live longer than other pensioners of the same age.

Figure 2.11 demonstrates the shape of R under these different mortality assumptions. In the figure for immediate annuities, the shape of R in relation to the age of purchase shows that individuals with better health conditions prefer buying an immediate annuity earlier in retirement while those with worse health conditions tend to delay the purchase. This is reflected by comparing S2PFL vs. S2PML and S2PML vs. SPML03. Moreover, comparing R values under S2PMA-L, S2PMA-M and S2PMA-H demonstrates that retirees with high pension benefits are more likely to purchase immediate annuities early in retirement.

In terms of the deferred annuity, all groups find the 30-year deferred annuity the most attractive to buy among all of the deferred annuity products considered. Moreover, it is worth noticing that the 30-year deferred annuity is more popular among pensioners with higher mortality rates. This is because, for pensioners who have a lower possibility of surviving to age 95, a greater level of probability distortion is involved in the mental

accounting process and the 30-year deferred annuity becomes more valuable to them. As a result, male pensioners during the period 2000-2006 would be the most interested in long-term deferred annuities while female pensioners and high-income pensioners show the least interest.

2.5.5 A comparison of parameter sensitivity

In order to measure the responsiveness of the Relative Price Difference to the change in each parameter, we calculate the elasticity of the Relative Price Difference, E_R. It addresses the percentage change in the Relative Price Difference for a given percentage change in the parameter value and the formula is as follows:

$$E_R = \frac{\text{Percentage change in } R}{\text{Percentage change in parameter value}} = \frac{\frac{\triangle R}{R_{\text{average}}}}{\frac{\triangle parameter}{parameter_{\text{average}}}} \tag{2.7}$$

In Equation (2.7), $\triangle R$ stands for the absolute change in R and $\triangle parameter$ stands for the absolute change in the considered parameter values. R_{average} stands for the absolute value of average of R under different parameter values and $parameter_{\text{average}}$ is the absolute value of the average of chosen parameter values. As the average value is used to calculate the percentage change, the elasticity of the Relative Price Difference can be regarded as a point mid-way among all of the R results. After we calculate E_R for all ages of immediate annuity purchases and for all deferred periods of deferred annuities, we average the results and obtain the elasticity of the Relative Price Difference for immediate annuities and for deferred annuities respectively. The results are presented in Table 2.1. Please note that we use life expectancy at the age of annuity purchase as an index for each mortality table.

If E_R is greater than 1, the Relative Price Difference changes proportionately more than the parameter value changes. If E_R is less than 1, the Relative Price Difference changes proportionately less than the parameter value changes, implying a less sensitive parameter. Based on the results in Table 2.1, the following conclusions can be drawn. First, loss aversion is a very sensitive factor for both immediate and deferred annuities. It confirms our finding in the previous sections that loss aversion could be the major reason for the low valuation of annuity products. Second, an interesting finding is that probability distortions, reflected in γ and σ, have a great impact on the attractiveness

TABLE 2.1: Elasticity of the Relative Price Difference (E_R)

Immediate annuities				
Parameters	λ	r	γ & δ	$\mathring{e}$
E_R	1.5434	0.0252	0.7158	0.2694
Deferred annuities				
Parameters	λ	r	γ & δ	$\mathring{e}$
E_R	5.3152	0.1784	7.0907	6.1250

Notes: λ measures the level of loss aversion, r represents the interest rate, δ and γ determines the curvature of the capacity function, $\mathring{e}$ represents the life expectancy corresponding to each mortality table.

of deferred annuities, but a small impact on that of immediate annuities. It indicates that people who have different levels of mortality rates transformation would have similar preferences for immediate annuities but distinct preferences for deferred annuities. Similarly, the deferred annuity have greater sensitivity to mortality parameters than immediate annuities, which is as we would expect. Finally, the elasticity of the Relative Price Difference for deferred annuities is in general much higher than that for immediate annuities, which justifies our previous conclusion that CPT has a greater impact on deferred annuities than immediate annuities.

2.6   Conclusions

Classical expected utility maximization theory suggests that annuities provide optimal consumption during a retiree's retirement period, and hence it does not explain why the majority of retirees do not voluntarily convert their DC pension account balances into annuities. In this chapter, we move beyond the rational paradigm that is assumed by the expected utility theory, in an attempt to discover if behavioral factors may be able to explain the way that people spend their retirement funds. We have applied cumulative prospect theory (CPT) to calculate the overall perceived value of annuities, from which we obtain the maximum price that individuals would like to pay for an annuity. By comparing this with the annuity price, we are able to conclude whether annuities in the market are attractive. We have also conducted several sensitivity analyses to study the relative attractiveness of annuities with regard to each parameter in the CPT model.

Our work is an extension of the paper by Hu and Scott (2007), who show that CPT is one of the behavioural factors that prevent people from buying an annuity. After confirming their conclusions, we have the following additional findings under CPT assumptions. First, immediate annuities are in general not attractive for retirees aged from 65 to 95; deferred annuities with a longer deferred period are more preferable. Second, loss aversion is the major reason for not purchasing annuities; and distortions in probability shift preferences towards an annuity that starts paying at an older age. Third, loss aversion is the most influential factor for the purchase decision of immediate annuities; and the mortality rate transformation determines the decision to purchase deferred annuities.

Based on our findings, some recommendations can be drawn for life insurance companies to improve the demand for annuities. One recommendation is the launch of long-term deferred annuities, which seem to have many competitive advantages. Firstly, making an early purchase decision would give a lower price. Taking a 30-year deferred annuity as an example, the price is only 0.64% of a comparable immediate annuity purchased at age 65. Secondly, with the lower price, it actually provides a similar level of longevity protection as an immediate annuity since longevity risk is concentrated in the tail. Moreover, CPT suggests individuals tend to overestimate the small probability of surviving for a long period. The popularity of lottery tickets may indicate the possible popularity of long-term deferred annuities. In recent years, deferred annuities have aroused intensive discussion in the literature as a retirement solution (Milevsky, 2005; Scott et al., 2011).

Another recommendation is to introduce some additional product features in existing products to attract loss averse customers. Our research indicates that, the major reason for low demand is that individuals are afraid of making a loss from an annuity investment. This could be improved by providing a guaranteed period of payments that does not depend on the survival of annuitants. In terms of deferred annuities, this could be a return of a certain percentage of premiums if annuitants die within a certain number of years. Of course, these additional features would make annuities more expensive; however, they also reduce the possible high losses and could make it more attractive for loss averse customers.

While we predict customer behaviors and recommend the design of possibly more attractive annuity products, our study has one limitation. The values of parameters in

the CPT model are based on the work in Tversky and Kahneman (1992). Their experimental design involves gambles rather than annuity purchases. Therefore the results presented here are not precise quantitative predictions, but can only provide qualitative explanations of the relative attractiveness of various types of annuities.

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3

Why the deferred annuity makes sense

Why the deferred annuity makes sense

Abstract

The low demand of immediate annuities at retirement has been a long-standing puzzle. We show that a hyperbolic discount model can explain this behaviour and results in attractiveness of long-term deferred annuities. We find that a 65-year-old male would pay 24 percent higher than the fair price for a 30-year deferred annuity. Moreover, if governments were to introduce a pre-commitment device which requires pensioners to make annuitisation decisions 10 years before retirement, the take up rate of annuities could become higher.

Keywords: Hyperbolic discounting, Deferred annuities, Annuity puzzle, Reservation price.

3.1   Introduction

Over the past few decades, the traditional defined benefit (DB) pension plan has been gradually losing its dominance in private sector pension systems in many countries and the defined contribution (DC) pension plan has become increasingly popular. Under the DC pension scheme, members contribute towards their personal pension savings in a way that enables them to make decisions on how to invest during the accumulation stage and how to decumulate during retirement.

In the area of retirement, a constant focus is on whether retirees receive adequate protection against longevity risk, the risk of outliving one's wealth. As an insurance product that eliminates the longevity risk, a lifetime annuity is a good option for DC pensioners. A lifetime annuity provides a stream of income payments for as long as the annuitant is alive, in exchange for an upfront premium charge. Yaari (1965) demonstrates that in a life-cycle model a risk-averse individual without a bequest motive should hold all their assets in annuities. However, empirical data has shown that retirees are reluctant to convert retirement savings into annuities. The disparity between the theoretical optimal choice and consumers' actual preferences leads to the "annuity puzzle". This can be illustrated by low levels of voluntary annuitisation in the UK market. In the past, the UK had two distinct annuity markets: a voluntary segment called the Purchased Life Annuity (PLA) market and a compulsory section called the Compulsory Purchase Annuity (CPA). Based on UK annuity sales figures for the 1994-2006, sales in the CPA market had been consistently higher than that in the PLA market. By 2010, the CPA market had grown to £11.5 billion worth of annuity premiums while the PLA market only had £72 million worth of sales (Cannon and Tonks, 2011).

Recently, the UK government implemented pension reforms to encourage free choice of the mode of pension distribution and, as a result, retirees' real preferences on annuity products could be clearly seen. The reform follows the international trend of greater pension flexibility, which has been observed in countries such as the USA, Australia and Switzerland. Prior to the 2014 UK reform, there were strict restrictions on accessing pension savings at retirement. For example, if a pensioner had overall pension savings of greater than £18,000 but could not access a guaranteed retirement income of more

than £20,000 per year [1], the only two choices that could make were to either to buy an annuity or enroll in a "capped drawdown", which allowed them to withdraw as much as 120 percent of an equivalent annuity each year during retirement. However, after the above mentioned policy change, everyone will be able to choose a lump sum (full withdrawal), an annuity or a drawdown, regardless of the size of their pension wealth (HM Treasury, 2014). With this move towards greater freedom of choice on how and when to access pension wealth, annuity sales have experienced a large decline. In Q2 2015, £990m was invested in annuities, showing a 44 percent decrease from the £1.8bn invested in Q2 2014. Moreover, 18,200 annuities had been purchased in the three months after the pension reform, showing a 61 percent decrease compared to Q2 2014 when 46,700 were purchased (ABI, 2015).

Many studies have suggested a number of reasons for the annuity puzzle, such as mortality risk-sharing among families (Brown and Poterba, 2000) and the existence of social security (Butler et al., 2016). A detailed list of reasons are covered in Section 3.2. Some research has examined the possible influences of behavioural factors such as the framing effect, cumulative prospect theory and low level of financial literacy. The findings conclude that the low demand for annuity could be simply due to irrational behaviour (Cannon and Tonks, 2008).

Since the annuity is a product that involves a series of payments at different points of time, one of the behavioural factors that affects decision making is the inconsistency of intertemporal choices. More specifically, when people assign values to future payouts, the discount rate used to evaluate intertemporal choice is not fixed, but varies in line with the length of the delay period, size and signs of the benefits. This effect is called hyperbolic discounting and is interpreted as "temporal myopia". The concept has been widely used to account for behavioural bias in savings, nutrition, healthcare, drug addictions, and other problems of willpower (Frederick et al., 2002). Laibson et al. (2003) have used the model to explain the puzzle of simultaneously having large credit card debts and pre-retirement savings.

In this chapter, we use the hyperbolic discount model derived from experimental results to analyse annuitisation decisions. We are interested in both immediate annuities and deferred annuities. The deferred annuity is a contract that is purchased today but does

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[1] A guaranteed retirement annual income of £20,000 is equivalent as a total pension savings of around £310,000 , according to stylised assumptions and calculations in HM Treasury (2014).

not pay until the annuitant survives to a pre-specified age. Compared with a conventional immediate annuity, a deferred annuity has competitive advantages of a much lower price and provides almost the same level of longevity insurance; therefore, it has aroused much discussion in the area of retirement financial planning (see Milevsky, 2005; Gong and Webb, 2010; Denuit et al., 2015). To uncover the annuitisation decisions of people at different ages, two types of deferred annuities are studied: a working age deferred annuity (WADA), which is purchased at working age and starts paying at retirement, and a retirement age deferred annuity (RADA), which is purchased at retirement and starts paying a few years later. To be more specific, we seek to explore four questions:

(a) Can we use the hyperbolic discount model to explain the low demand for immediate annuities at retirement and at a more advanced age?

(b) Are pensioners at 65 years old interested in purchasing a RADA?

(c) Would people at working age have an interest in purchasing a WADA?

(d) How would working-age members respond to a question asking them to decide today whether to buy an immediate annuity at retirement?

To seek the answers to these questions, we adopt the hyperbolic discount model to evaluate the perceived value of an annuity, which enables us to work out the reservation price. By comparing the reservation price with the theoretical market price we can determine whether an individual would choose an annuity or not. We show that time inconsistent preference is one of the factors that stops retirees from converting their DC account balances into annuities at retirement. More importantly, we identify a high willingness to purchase long-term deferred annuities for hyperbolic discounters, both at working age and in retirement. As the deferred period increases, the relative difference between reservation price and actuarial price increases considerably and at a much faster rate. Furthermore, if members are simply asked to make a decision on annuity purchase and could delay the action until the point of retirement, those with ten years until retirement value the longevity protection the most.

The rest of this chapter is organized as follows. Section 2 provides a literature review of the explanation for the annuity puzzle from both the rational and irrational framework. In section 3, a detailed introduction of the hyperbolic model is offered. In section

4, we explain how the annuities in the four questions above are evaluated and how the maximum acceptable price is derived. Major results and a sensitivity analysis are presented in sections 5 and 6. Finally, in Section 7 we conclude with major findings and limitations of this study.

3.2 Literature review

Yaari (1965) is the first to demonstrate the benefits of annuitisation in a life cycle model with an uncertain lifetime. He shows that a rational investor should invest his retirement savings in annuities rather than bonds to finance retirement. This result rests on three fundamental assumptions: a complete annuity market, a specific utility function (additive separability) and the absence of a bequest motive. The subsequent literature on annuities has relaxed one or two of these assumptions in order to assess if these factors lead to the low demand for annuities.

Annuities in a rational framework

Observing the annuity market from the supply side, a less competitive price could be the reason for low demand. Brown and Warshawsky (2001) calculate the money's worth value of an annuity using average mortality rates of the population and find that an individual could expect to receive only 85 pence per pound invested, thus justifying the existence of adverse selection in annuity pricing.

Since an annuity stops paying once the annuitant dies, people with a motive to bequeath part of their wealth obtain less welfare by purchasing a life annuity. A large literature has focused on how the bequest motive impacts the demand for annuities and shows that a strong bequest motive can eliminate the desire to purchase annuities (see Friedman and Warshawsky, 1990; Vidal-Melia and Lejarraga-Garcia, 2006; Lockwood, 2012).

Intra-family mortality sharing can also be regarded as a substitute for an annuity. Since families often share a common budget constraint, mortality risk sharing among family members can offer a substitution for risk sharing in the annuity market. To an extent, this resembles the bequest motive; an individual who dies early leaves his wealth to subsidise other family members who are alive. Brown and Poterba (2000) find evidence

showing that the utility gain from annuitisation for a couple is significantly lower than that for single people.

An alternative explanation for the low demand for additional annuitisation is the existence of social security and private DB pension plans. According to Dushi and Webb (2004), an exceptionally high proportion of a retired household's wealth has been pre-annuitised before retirement. Therefore, without purchasing an annuity in the open market, these retirees already have a minimum level of income that will last for life. Butler et al. (2016) also prove that the presence of social security reduces the value of annuitisation.

A more recent discussion relates to the worry about health care expenditure shocks at an older age and the fact that retirees may not need the smooth consumption that an annuity provides. It is true that people have a higher probability of falling ill when they become older; they may also have to make some age-specific investments in a house such as installing a stair lift. Therefore we have reason to believe that a rational retiree might want to live a very simple life in their early retirement period so that they can save for unexpected health-related expenses (Sinclair and Smetters, 2004).

Lastly, while most research focuses on a comparison of full annuitisation aged 65 with the alternative of never annuitising, in practice, a retiree can choose between annuitising now and delaying the decision until the next period. They can also annuitise only a fraction of their wealth and enter a drawdown of the rest. Gavranovic (2011) has demonstrated that the optimal annuitisation strategy for a pensioner without bequest motive is to gradually convert all pension wealth to annuities by around age 80.

Annuities in a behavioural framework

The literature mentioned above seeks to solve the annuity puzzle within a strictly rational framework. In recent years, however, there is an extensive literature on the behavioural economics of retirement savings. This moves beyond the fully rational paradigm and proposes some behavioural factors that could play important roles in determining how retirees spend their retirement savings.

One important issue is the flaws in the expected utility hypothesis that arise from risk aversion. Hu and Scott (2007) have explained the annuity puzzle by assuming that

retirees are loss-averse rather than risk-averse, and make annuity decisions based on Cumulative Prospect Theory. They also extend the application of Cumulative Prospect Theory to deferred annuities and guaranteed annuities, proving that the deferred annuity becomes optimal only when the first payment starts at or after age 93. The framing effect, which states that individuals' behaviour depends heavily on the way in which available choices are presented, is also one of the influencing factors considered by Brown et al. (2008). They have shown that 72 percent of subjects prefer an annuity rather than a savings account when the choice is framed in terms of consumption while 12 percent subjects choose an annuity when it is framed in terms of an investment. Other behavioural factors include the poor financial education of retirees and regret aversion (Cannon and Tonks, 2008).

3.3   An introduction to the hyperbolic discount model

In dealing with individuals' annuitisation decisions and other economic decisions, which involve outcomes occurring at different points in time, researchers often employ a discounted utility function to model such decisions. In a normative framework, individuals have stationary time preferences and use a constant discount rate between any two consecutive periods. However, many empirical studies on time preferences have found anomalies in the behaviour predicted by the stationary exponential discount function. Three major anomalies are discussed in the following paragraphs.

• **Decreasing Impatience** While the exponential discount function predicts that the preference between two delayed outcomes should be consistent given the same time interval, researchers have found extensive evidence showing that preferences often switch. Thaler (1981) illustrates this with a simple example. Subjects are asked to state their preferences on two questions: "Would you prefer one apple today or two apples tomorrow?" and "Would you prefer one apple in one year or two apples in one year plus one day?". According to the exponential discounting method, people who choose one apple today would make consistent choice of one apple in a year. However, empirical results show that a significant fraction of subjects that prefer one apple today would gladly wait one extra day in a year in order to receive two apples instead. Therefore, people tend to act impulsively in the short-term but become more patient in the long-term. In other

words, the implicit rate at which people discount future rewards will vary inversely with the length of waiting time.

• **The Absolute Magnitude Effect** It is universally accepted that waiting for a reward requires some mental effort. If the required mental effort does not increase proportionally with the size of rewards, the implicit discount rates with regard to different reward sizes would not stay the same. In a survey conducted by Thaler (1981), subjects are told that they would receive "a dinner worth \$15", "a trip to San Francisco worth \$250" and "a good used car worth \$3000". When they are asked what compensations they need if these rewards are delayed for three months, most subjects answer "an extra dinner", "an extra day in San Francisco" and "a fancy model of the same car" respectively. Therefore Thaler (1981) concludes that the subjects are indifferent between receiving \$15 immediately and \$60 in a year, between \$250 now and \$350 in a year, and between an immediate \$3000 and \$4000 in a year, which means large reward sizes have lower discount rates compared with small reward sizes.

• **The Gain-Loss Asymmetry** Gain-loss asymmetry refers to the empirical phenomenon that the implicit discount rates for losses are often lower than that for gains. For example, Loewenstein (1987) finds that a group of subjects, on average, are indifferent between receiving an immediate \$10 and receiving \$21 in a year; on the other hand, these subjects are indifferent between paying \$10 immediately and paying \$15 in a year. Similarly, the indifferent amount for receiving or paying an immediate \$100 were receiving \$157 or paying \$133 respectively in a year.

The anomalies introduced above can be addressed by a hyperbolic discount model, which has been widely applied to explain the problem of addiction and self control. As an example, people with low self-control often gain too much weight and find it difficult to improve their health by doing more exercise and having a diet. These people often vow to forgo all future temptations, in exchange for improved health in the future; however, when they have their next meal, they cannot resist having unhealthy fried food and sweet desserts. Presumably, they prefer this because the instant pleasure delivered by delicious food is greater than the heavily discounted future rewards of health. Therefore, the hyperbolic discount model is appropriate to describe the situation that people simultaneously require immediate satisfactions and make commitments for the future.
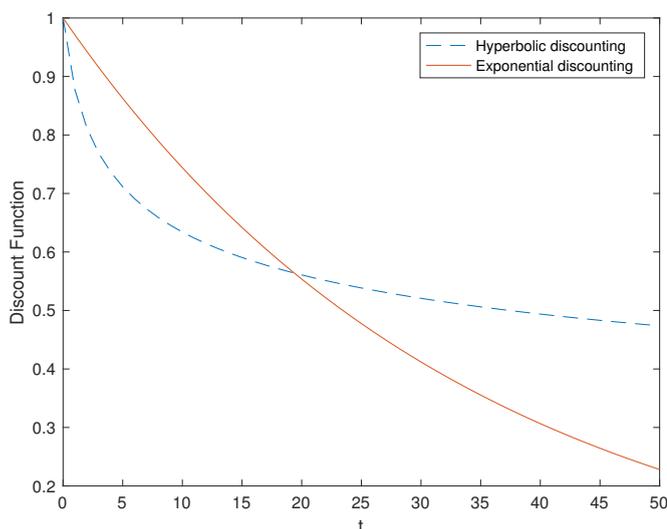
FIGURE 3.1: A comparison of hyperbolic discounting and exponential discounting

Notes: Vertical axis, discount function, represents the present value of £1 to be received at time t. We assume a constant interest rate of 3 percent for exponential discounting; $\alpha = 1$ and $\beta = 0.19$ for hyperbolic discounting.

Loewenstein and Prelec (1992) collectively present the experimental evidence and propose an explicit hyperbolic discount model to address the effect.

$$\delta(t) = (1 + \alpha t)^{-\frac{\beta}{\alpha}} \quad \text{with } \alpha > 0, \beta > 0 \tag{3.1}$$

where $\delta(t)$ is a discount function; α and β determines how much the function departs from constant exponential discounting.

To identify the parameter values that capture most people's intertemporal preferences, Abdellaoui et al. (2009) conduct a well-designed choice test and concluded that the "Power discount model" with $\delta(t) = (1 + t)^{-\beta}$ provides the best fit. Relying on this result, we use β equals 0.19 for gains and 0.11 for losses in our analysis. Please note that value of β would vary with country/cultural background of the selected group of subjects and this limitation is embedded in the experimental design. Therefore, in section 6, the sensitivity of the results to changes in the parameter β is explored, to show that the conclusions do not entirely depend on the chosen value of the parameter.

Figure 3.1 provides a comparison between the hyperbolic discounting and exponential discounting models. The horizontal axis represents the waiting time to receive £1 and the vertical axis is the present value of the £1 to be received. The present value following hyperbolic discounting decreases at a much faster rate in early years than following exponential discounting, which means that the hyperbolic discounters adopt a higher level of discounting for benefits that come in the early years than exponential discounters. However, if the benefits are to be received after 20 years (the intersection point), hyperbolic discounters believe it has a higher value than exponential discounters.

In addition to the discount function, a descriptive value function is also required in a complete discounted utility framework. Loewenstein and Prelec (1992) discuss the necessary characteristics of the value function without providing an explicit descriptive model. Abdellaoui et al. (2009) design a parameter-free measurement of utility in intertemporal choices and hence derive the value function which addresses the absolute magnitude effect and the gain-loss asymmetry. The value function $v(c_t)$ is as follows with γ being equal to 0.97 and θ being equal to 0.84. It is assumed to be separable and additive over time as recorded in literature.

$$v(c_t) = \begin{cases} -(-c_t)^\gamma & \text{if } c_t < 0 \\ c_t^\theta & \text{if } c_t \geq 0 \end{cases} \tag{3.2}$$

where c_t represents the consumption rate that would take place at a future time t, which is defined on the interval [0,T], and $v(c_t)$ represents the value of the consumption amount.

The discount rates and the value function are combined to arrive at the overall value of consumption streams.

$$V(c_0, c_1, ..., c_T) = \sum_{t=0}^{T} (\delta(t) \times v(c_t))$$

A standard approach in the literature has been to use the exponential discount model for $\delta(t)$ and Constant Relative Risk Aversion (CRRA) utility function for $v(c_t)$. In this analysis, we instead use the hyperbolic discount model in Equation (3.1) and the value function given by Equation (3.2) to analyse annuity purchase decisions by considering the effect of subjective views on the underlying consumption streams.

3.4 Annuity Valuation

In this section, we introduce four scenarios to address the questions of annuitisation decisions for people at different stages. Two types of annuities, immediate annuities and deferred annuities, are discussed in this chapter and they are priced at actuarially fair rates. In order to make a fair decision, the overall utility, V, of the investment in each scenario will be calculated. As we focus on people who show "temporal myopia", the amount of money is evaluated based on Equation (3.2) and time preference is modeled by the "power discount model", Equation (3.1). Let $_tp_x$ denote the probability that an x-year-old person can survive for t years and the maximum attainable age is set to be 120. Four scenarios are described in detail below and the corresponding valuation of the annuity investment is introduced.

a. Immediate annuities for retirees

Consider a retiree at age $x(x \geq 65)$ who needs to make a decision on whether to spend a lump sum amount A to purchase an immediate annuity which pays ψ per annum in advance. The overall value of this investment for the x-year-old is:

$$V_1(x) = v(-A) + \sum_{i=x}^{119} (\delta(i - x) \times {}_{i-x}p_x \times v(\psi))$$

b. Retirement Age Deferred Annuity (RADA) for retirees

Consider a 65-year-old pensioner $(x = 65)$ who has just retired. The individual is faced with a wide variety of RADA products which have deferred periods (d) from 1 to 30 years. By investing the pension lump sum amount A in a d-year deferred annuity, the pensioner is entitled to a lifelong guaranteed annual income of ψ in d years. However, nothing is paid back if he dies within the deferred period. The overall value of this deferred annuity investment at the time of purchase is:

$$V_2(d) = v(-A) + \sum_{i=65+d}^{119} (\delta(i - 65) \times {}_{i-65}p_{65} \times v(\psi))$$

c. Working Age Deferred Annuity (WADA) for working age individuals

An individual at age x ($25 \leq x \leq 64$) considers investing in a WADA which provides annual incomes of ψ once the annuitant survives to retirement age 65. The overall perceived value of this investment at the time of purchase is:

$$V_3(x) = v(-A) + \sum_{i=65}^{119} (\delta(i-x) \times {}_{i-x}p_x \times v(\psi))$$

d. Decision on purchasing an immediate annuity at retirement for working age individuals

In this scenario pension scheme members within the working age range ($25 \leq x \leq 64$) are asked to make decisions in advance on whether to choose a pension lump sum A at age 65 or a corresponding fair annuity starting at the same age. When evaluating this annuity, the cash flows involved are exactly the same as the immediate annuity purchased at age 65 (scenario a); however the perceived value may be different because the decision is made at an earlier age. If an individual decides to convert the lump sum A into an annuity at retirement, the overall perceived value of this investment for the individual is:

$$V_4(x) = \delta(65-x) \times {}_{65-x}p_x \times v(-A) + \sum_{i=65}^{119} (\delta(i-x) \times {}_{i-x}p_x \times v(\psi))$$

To determine whether an actuarially fairly priced annuity is attractive to purchase, we follow Hu and Scott (2007) to use the "relative difference between reservation price and fair price", R, as the benchmark measure:

$$R = \frac{Reservation\ Price - Actuarially\ fair\ price}{Actuarially\ fair\ price}$$

The "reservation price", also called the "maximum acceptable price", is the annuity price that would make an individual indifferent to buying an annuity. According to the valuation functions above, the reservation price is the initial price, A, that makes the hyperbolic present value of an annuity, V, equal to zero. If the reservation price is below the market price, the annuity would not be attractive for individuals to buy.

Therefore, a positive R means individuals are willing to purchase a fairly priced annuity, and a higher value of R implies greater willingness to purchase an annuity. R can also be interpreted as the percentage more or less than the market price that an individual would be prepared to pay for a product.

3.5 Results

In this analysis, we assume the annuity price is actuarially fair with no expenses or profit loading. The price calculation is based on the UK mortality table "S2PML[2]", which describes the mortality experience of UK male pensioners of self-administered pension schemes for the period from 2004 to 2011, and a constant interest rate of 3 percent. Annual income from annuity, ψ, is assumed to be 1 unit. Therefore, the fair market price of the annuity and the reservation price that individuals would like to pay can be calculated accordingly. In what follows, we provide results for the relative price differences, R, under the four different scenarios, analyse the attitudes of investors towards each type of annuity and discuss the trend of the relative price differences with regard to investors' age or the length of the deferred period.

a. Immediate annuities for retirees

The results of the Relative Price Differences (R) with regard to different ages of purchase are presented in Figure 3.2. Two major conclusions can be drawn from the figure. First, all the outcomes in terms of R are negative, which means that for a group of retirees who are aged between 65 and 95, fairly priced immediate annuities are unattractive to purchase. Thus, evaluating annuitisation decisions by assuming time inconsistent preferences is indeed a powerful behavioural explanation for retirees' not converting their defined contribution account balances into annuities. Secondly, as a newly retired pensioner becomes older, his preference for the immediate annuity declines at first and then increases after he reaches age 85. However, the relative difference in price is small with R lying in the range of -3% and -10%.

The results presented appear to be inconsistent with more recent research carried out by Schreiber and Weber (2015), who find that the expected present value of an immediate

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[2]Source: Continuous Mortality Investigation (2013).
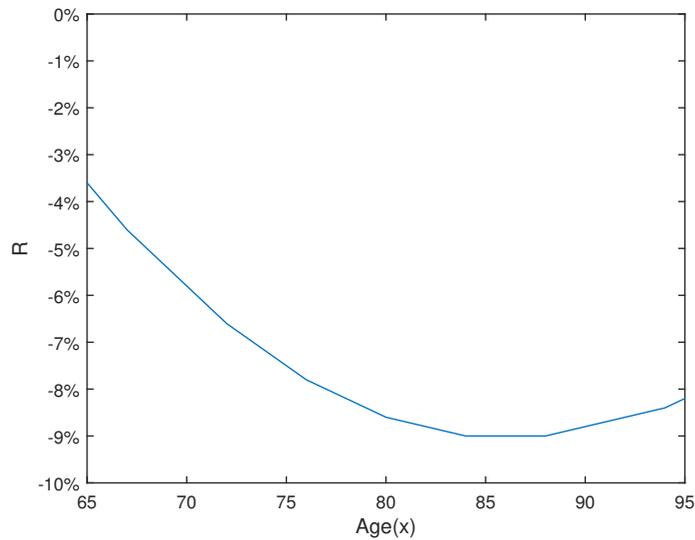
FIGURE 3.2: the Relative Price Difference (R) of immediate annuities for retirees at
age (x)

annuity declines monotonically with the age of purchase. Although both studies use the
power discounting model for annuity evaluation, different groups of people are targeted:
Schreiber and Weber (2015) survey working age individuals while we focus on retirees
above age 65; and this may explain the inconsistency.

b. Retirement Age Deferred Annuity (RADA) for retirees

Figure 3.3 shows the attractiveness of RADA with different deferred periods for a 65-
year-old retiree. It can be seen that although recently-retired individuals are reluctant
to purchase immediate annuities, they are willing to pay a higher-than-market price for
annuities with long deferred periods. From our modelling results, annuities that are
deferred for more than 10 years are generally welcomed by 65-year-old retirees. Further-
more, we identify a positive relationship between the length of the deferred period and
the attractiveness of the corresponding deferred annuity. If an annuity has a deferred pe-
riod of 30 years, a 65-year-old individual would be prepared to pay 24% greater than the
fair price[3]. This is a much higher margin than that for an immediate annuity. It implies
that such a product would have commercial potential since insurance companies could
add a greater loading in deferred annuity products without changing its attractiveness.

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[3]Please note the choice of 30-year deferred period is for the purpose of illustration. In reality, the
product may be available.
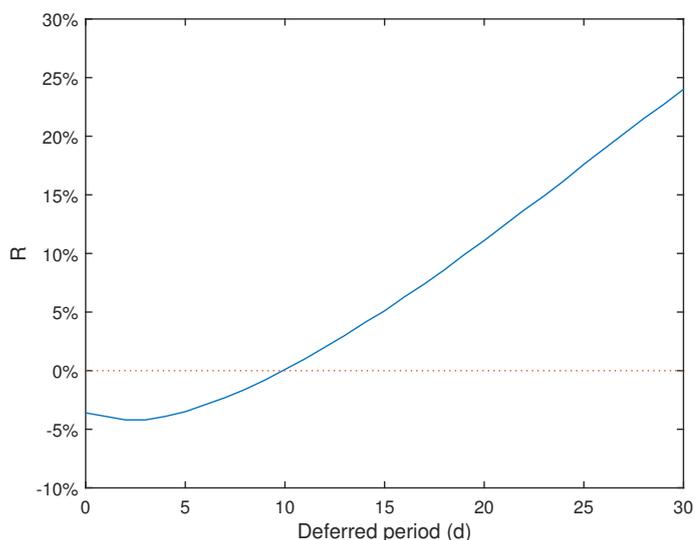
FIGURE 3.3: the Relative Price Difference (R) of d-year retirement age deferred annuities (RADA) for 65-year-old retirees

The popularity of deferred annuities have also been identified in other works. Hu and Scott (2007) adopt Cumulative Prospect Theory (CPT) to evaluate deferred annuities with deferred periods of 0, 10, 20 and 30 years and find that the 30-year deferred annuity is the most attractive to buy. In Chapter 2, we also show that the attractiveness of deferred annuity increases with the length of the deferred period, according to CPT.

c. Working Age Deferred Annuity (WADA) for working age individuals

If individuals at working age are given the opportunity to enter a deferred annuity contract that promises retirement incomes depending upon survival, their reactions are examined and reflected in Figure 3.4. It can be seen that although people who are retired are unsure of handing over a lump sum of money to insurance companies in exchange for a longevity protection, most people at working age tend to find a WADA attractive to buy. Another interesting point worthy of note is that the decision maker's age has a negative effect on the attractiveness of this type of deferred annuity. For hyperbolic discounters younger than 30-year-old, they appear even to be willing to pay double the price of the WADA.

We know that as the length of deferred period increases, the actuarially fair price of a deferred annuity which provides the same level of protection becomes cheaper; hence
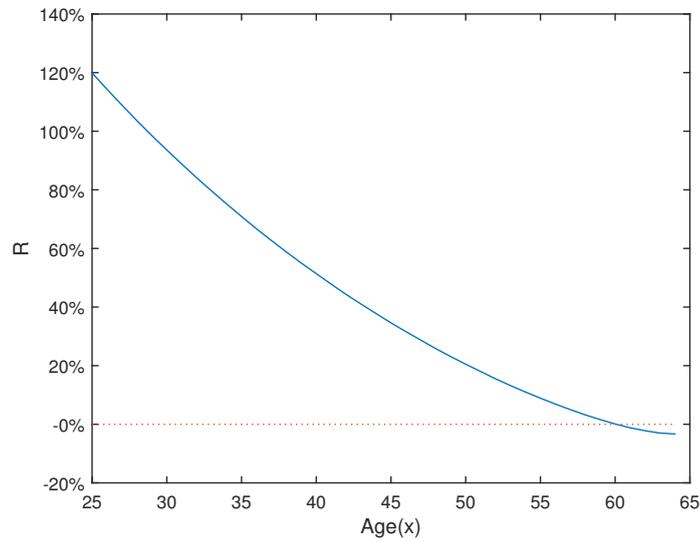
FIGURE 3.4: the Relative Price Difference (R) of d-year working age deferred annuities (WADA) for working age individuals at age (x)

younger individuals would be less hesitant to purchase a WADA which involves a smaller initial outlay. In addition, given the assumption that people have time inconsistent preferences, a young individual tends to overvalue all the annuity incomes that come in the distant future; however for an older individual, some of the deferred annuity payments are highly likely to be undervalued. The results are consistent with our conclusions in scenario b. Purchasing the pension annuities at an earlier age means a longer deferred period, and in both scenarios an annuity with a longer deferred period is more attractive. The magnitude of R is much higher in scenario c than in scenario b because a longer deferred period is considered.

Some of our findings mirror those suggested elsewhere. Shu et al. (2016) have conducted a choice-based stated-preference survey of adults aged between 45 and 65 and find that younger subjects report a higher likelihood of purchases for annuities beginning at age 65 than older subjects who are closer in age to the start date. DiCenzo et al. (2011) have also discovered that pre-retirees have stronger preferences for annuities than retirees based on online experimental research with 1,009 subjects aged between 45 and 75.
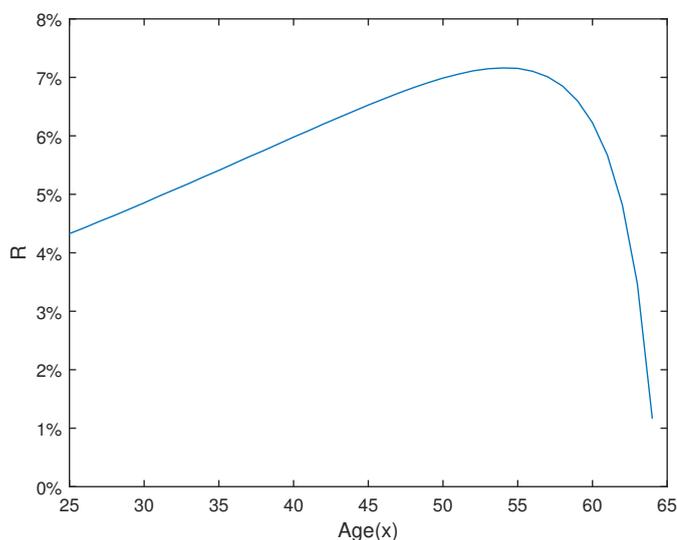
FIGURE 3.5: the Relative Price Difference (R) of an immediate annuity purchased at retirement for working age individuals at age (x)

d. Decision on purchasing an immediate annuity at retirement for working age individuals

Similar to the third scenario, we aim to discover the attitude of working age pension scheme members towards an annuity with the first payment starting when pensioners retire at age 65. Although the annuity investment payoffs are exactly the same, the purchase is made at different points. In scenario c, the price is paid now at age x while in scenario d, pensioners simply make a decision at age x but delay the purchase action until age 65. If an individual dies prior to the time of retirement, his financial status remain unchanged in scenario d but he faces an absolute loss of the price paid in scenario c. Therefore, scenario d effectively deals with the decision to buy an immediate annuity rather than a deferred annuity.

Comparing the results of R in Figure 3.5 with those in Figure 3.4, we identify a different pattern. For individuals below age 55, the attractiveness of the annuity increases slightly with age. However for individuals above age 55, the attractiveness declines sharply with age and becomes unattractive when individuals reach age 65. Therefore, we suggest that policy makers who want to promote annuitisation in public ask individuals to make a choice between lump sum and annuities 10 years before retirement. On the other hand,

one may notice that the change in R is relatively small, varying between 4% and 8%. It is similar to the results for immediate annuities in scenario a.

These findings confirm those in the survey by Schreiber and Weber (2015). In their survey, subjects are asked to predict whether they will annuitise if they were at age 66. The total sample results show that the effect of age on the decision to purchase an annuity is negative. However, observing the answers from a subsample of individuals below age 51, the effect is no longer statistically significant. To some extent, it reveals that people above age 51 have significant decreasing preferences towards annuities.

3.6 Sensitivity analysis

Previously, we assumed that each parameter value in the annuity calculations is based on Abdellaoui et al. (2009). However, questions remain on whether the behavioural biases would be stronger or weaker for people with different levels of impatience, different income levels or different health status. In this section, we test the sensitivity of the power discounting parameter, the income levels and mortality rates (by changing mortality tables).

Table 3.1 shows the results for R in Scenario a and Scenario b under different combinations of assumptions. The row HB baseline lists the standard results that are based on the benchmark assumptions in Abdellaoui et al. (2009). In the HB sensitivity analysis, we change one factor listed in each row at a time so that we can observe the impact of that factor on R. "Less" or "greater" is relative to the baseline results. Each column represents different types of annuity products with the first payment starting at a different age. For example, an annuity starts paying at age 75 represents an immediate annuity purchased at age 75 in Scenario a and a 10-year deferred annuity purchased at age 65 in Scenario b.

The first factor that is of interest is the level of impatience, measured by β. Given that the annuity pricing rate is deterministic, a higher β means that the decision maker adopts a heavier undervaluation of earlier benefits and a lighter overvaluation of later benefits. Reflecting on the curves in Figure 3.6, the intersection point between exponential discounting and hyperbolic discounting would come at a later stage as β increases.

TABLE 3.1: Sensitivity analysis of the Relative Price Difference (R) in Scenario a and Scenario b

| R	Age of first annuity payment				
	65	70	75	80	85
Scenario a					
HB baseline	-3.60%	-5.84%	-7.51%	-8.57%	-8.99%
HB sensitivity analysis					
Less impatience ($\beta = 0.15$)	4.82%	1.64%	-0.97%	-2.95%	-4.28%
Greater impatience ($\beta = 0.25$)	-14.76%	-15.83%	-16.33%	-16.21%	-15.45%
Lower income level ($\psi = 0.0721$)	34.08%	30.30%	27.06%	24.29%	21.86%
Higher income level ($\psi = 3$)	-15.81%	-17.55%	-18.72%	-19.22%	-18.99%
Lighter mortality rates (S2PFL)	-1.94%	-4.57%	-6.65%	-8.11%	-8.89%
Greater mortality rates (SPML03)	-4.65%	-6.63%	-8.03%	-8.82%	-9.00%
Scenario b					
HB baseline	-3.60%	-3.50%	0.09%	5.15%	11.10%
HB sensitivity analysis					
Less impatience ($\beta = 0.15$)	4.82%	7.22%	12.47%	19.15%	26.80%
Greater impatience ($\beta = 0.25$)	-14.76%	-17.54%	-15.94%	-12.83%	-8.87%
Lower income level ($\psi = 0.0721$)	34.08%	37.28%	42.40%	49.56%	58.06%
Higher income level ($\psi = 3$)	-15.81%	-16.71%	-13.61%	-9.25%	-4.11%
Lighter mortality rates (S2PFL)	-1.94%	-1.27%	2.61%	7.92%	14.15%
Greater mortality rates (SPML03)	-4.65%	-4.98%	-1.65%	3.17%	8.80%
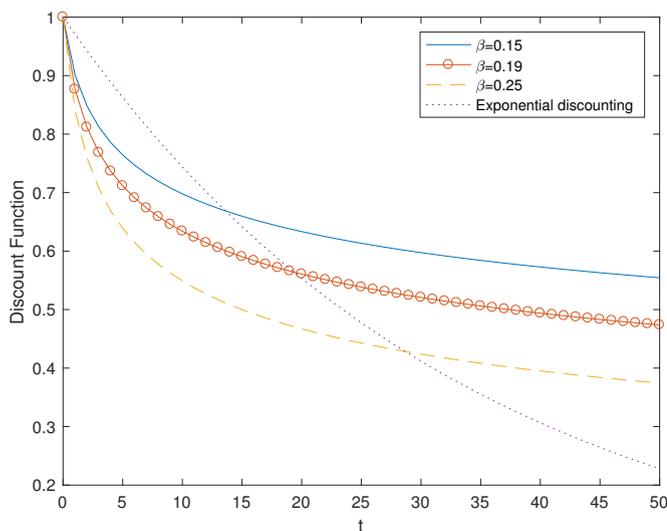
FIGURE 3.6: A comparison of hyperbolic discounting with different levels of impatience

Notes: We assume a constant interest rate of 3 percent for exponential discounting; $\alpha = 1$ for hyperbolic discounting.

Comparing our baseline results with less/greater impatience for both immediate annuities and RADA, we conclude that the attractiveness of annuity products is consistently lower in response to a greater level of impatience. This makes sense intuitively since an individual with a greater level of impatience would have stronger present bias; they would gain much higher satisfaction from consuming now rather than converting the lump sum into future cash flows and consuming regularly. According to Table 3.1, relatively patient individuals ($\beta = 0.15$) are willing to pay a slightly higher price, 4.82% and 1.64% respectively, for immediate annuities at age 65 and 70. It is because they are patient to wait and assign more weights to future incomes. Investment opportunities that convert current consumption into a future stream of cash flow are attractive to them. The same reasons lead to the attractiveness of deferred annuities for this group of people (see the row corresponding to $\beta = 0.15$ in Scenario b).

The effect of annuity income levels is examined to capture the variation in decisions of people with different wealth levels. Two levels of annual income, 0.0721[4] unit and 3 units, are adopted to represent relatively poor people and relatively rich people. Based on the value function in the hyperbolic discount model introduced above, people tend to overvalue an amount that is less than one unit and undervalue an amount that is greater than one unit. This is reasonable since people often place more values on the initial accumulation of amount of money and this portion of money is intended for the purchase of necessities such as food, utilities and rent. Therefore, the results corresponding to $\psi = 0.0721$ and $\psi = 3$ in Scenario a and Scenario b show that wealthy people who can afford an annuity providing a higher annual income are only willing to pay a lower-than-market price, while poor people are willing to pay a much higher-than-market price for annuities.

The mortality table in the calculation of baseline results is based on S2PML, the mortality experience of male pensioners from 2004 to 2011. Two other mortality tables are selected for comparison: S2PFL, the mortality experience of female pensioners during the same period, representing a group with lighter mortality, and SPML03, the mortality experience of male pensioners between 2000 and 2006, representing a group with heavier mortality. Results in Scenario a show pensioners with the highest mortality rates

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[4]The value is chosen as the annual income from converting one unit at age 65-year-old into an immediate annuity.

TABLE 3.2: Sensitivity analysis of the Relative Price Difference (R) in Scenario c and Scenario d

R	Age of decision making				
	25	35	45	55	65
Scenario c					
HB baseline	119.85%	70.90%	34.63%	8.88%	−3.60%
HB sensitivity analysis					
Less impatience ($\beta = 0.15$)	158.52%	99.11%	54.99%	23.17%	4.82%
Greater impatience ($\beta = 0.25$)	72.42%	35.90%	9.01%	−9.46%	−14.76%
Lower income level ($\psi = 0.0721$)	212.74%	143.11%	91.51%	54.89%	34.08%
Higher income level ($\psi = 3$)	89.74%	47.50%	16.20%	−6.02%	−15.81%
Lighter mortality rates (S2PFL)	127.53%	76.68%	38.95%	12.01%	−1.94%
Greater mortality rates (SPML03)	125.75%	75.34%	37.96%	11.29%	−4.65%
Scenario d					
HB baseline	4.32%	5.41%	6.52%	7.15%	−3.60%
HB sensitivity analysis					
Less impatience ($\beta = 0.15$)	40.37%	39.12%	36.96%	32.21%	4.82%
Greater impatience ($\beta = 0.25$)	37.28%	35.26%	31.82%	24.40%	−14.76%
Lower income level ($\psi = 0.0721$)	45.31%	46.85%	48.44%	49.33%	34.08%
Higher income level ($\psi = 3$)	−8.97%	−8.03%	−7.07%	−6.53%	−15.81%
Lighter mortality rates (S2PFL)	7.65%	8.68%	9.69%	10.04%	−1.94%
Greater mortality rates (SPML03)	6.84%	7.89%	8.92%	9.34%	−4.65%

(SPML03) tend to find immediate annuities the least attractive. Similarly, female pensioners with the highest life expectancies (S2PFL) show the greatest interest in RADA, as is observed in Scenario b.

Table 3.2 shows the results in terms of R in Scenario c and Scenario d. In Table 3.2, we can see the sensitivity of three factors: level of impatience, level of income and mortality rates, on the annuitisation decisions.

By comparing results corresponding to $\beta = 0.15$ and $\beta = 0.25$, we find those at working age see annuities as more valuable when they experience less impatience. In addition, for decision makers with different levels of impatience, the effect of their age on the WADA's attractiveness is consistently negative. In other words, the longer the waiting period to receive the first annuity income is, the higher the possibility of purchase will be. The intuition behind these features is as follows: incomes that arrive further in the future are more likely to be overvalued and thus the deferred annuity with a longer waiting period has a higher maximum acceptable price.

In Scenario d where the real purchase of an immediate annuity is delayed until retirement,

we have shown in the baseline results, which assume that β is 0.19 for gains and 0.11 for losses in the power discount function, that people have the greatest interest in buying an annuity around age 55. However in the sensitivity analysis when we let discount rates for gains and losses be the same, the peak in the trend of R disappears and we see a gradual decrease of value of R relative to the age of decision making. In such a case, governments may simply encourage individuals to make annuitisation decisions earlier rather than 10 years before retirement. Whether people use different discount rates for gains and losses and the resulting impact on annuitisation decisions needs future research.

Results corresponding to $\psi = 0.0721$ and $\psi = 3$ in Scenario c and Scenario d show that wealthy decision makers who can afford an annuity with a high annual income tend to find annuities less attractive. The impact of income levels on the annuity purchasing behaviour is consistent for decision makers at all ages[5].

The sensitivity of mortality rates in Table 3.2 indicates intuitively that annuities are more attractive for individuals with longer life expectancies, regardless of the age of decision making and the age of annuity purchase. Furthermore, for different mortality groups, age presents a negative influence on the attractiveness of a WADA. If the annuitisation decision needs to be made at working age and the actual purchases could be delayed until retirement, those between 50 and 55 are the most likely to choose an annuity and a strong decline in annuity preferences exists for pensioners older than 55.

3.7 Conclusions

Although purchasing an annuity at retirement can guarantee lifetime incomes, people are reluctant to spend their retirement savings on annuities voluntarily. In the UK, with fewer restrictions on accessing retirement savings, the demand for annuities has decreased and thus insurance companies are making efforts to design more attractive annuity products. This chapter discusses the implication of one behavioural factor, the hyperbolic discount model, on the annuity purchase.

Based on the analysis, we have the following primary findings. First, for an 65-year-old retiree, the reservation price of an immediate annuity is lower than the market price,

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[5]The results presented here are a reflection of our model; hence highly depending on whether the annual income is greater than one unit or not. This model does not consider other sources of income; however the state benefit is considered in Chapter 4.

and thus the hyperbolic discount method captures the low demand for annuities at retirement, seen in practice.

Second, under the hyperbolic discount model, deferred annuities are attractive for pension scheme members at all ages. The attractiveness generally increases with the deferred period. For instance, those below the age of 30 would pay more than double the market price for the WADA. However, our model does not account for factors such as affordability, a liquidity requirement and expected retirement living standards. While a 25-year-old man who wants to receive an annual annuity income of £40,000 after retirement might find a 40-year deferred annuity attractive; he will most probably not be able to afford the annuity price of £150,034.6 [6] at this young age. With time passing, he will accumulate wealth and set aside a portion for retirement protection. Often, there will be a point when accumulated retirement savings equals deferred annuity price; this is the optimal age of purchase.

We recommend using the deferred annuity contract as a retirement solution because it requires a smaller initial investment than the immediate annuity and provides similar longevity insurance. In addition, based on the fact that analytical cognitive function ability declines dramatically for older adults, it would be wise to buy a RADA to protect consumption at very advanced ages. For those in their 80s, it has been shown that 20 percent have fully diagnosed dementia and 30 percent have severe cognitive impairment; and thus, it would be difficult for these individuals to make rational withdrawal decisions if there were no income protection in place (Laibson, 2009).

In scenario d, we observe that individuals around the age of 55 are those who would most likely commit to buying an immediate annuity at the point of retirement. Therefore, a policy recommendation can be drawn. With the aim of promoting the purchase of annuities among retirees and releasing the burden from social benefit claiming, governments are advised to introduce a pre-commitment device asking people to make annuitisation decisions 10 years before retirement. When they reach retirement, their original decisions could be changed but some efforts, such as making a phone call or writing a letter, are required. In fact, in Denmark, the decisions on annuity purchases can be made during the accumulation period. As a result, about 50 percent of defined contribution

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[6] The price is the actuarially fair annuity price based on assumptions of 3% annual real rate of return, mortality table S2PML and zero profit loading.

assets are used to buy WADA type products for those aged in their 40's, 50's and 60's (Andersen and Skjodt, 2008).

Although we have shown that inconsistent time preference could be one of the reasons for the annuity puzzle, our study has one limitation. The model selected and the parameter values used are based on experiments designed by Abdellaoui et al. (2009). In their experiments, subjects were young university students who may have completely different views about money and time discounting compared to older workers and retirees. Their views may reflect a specific cultural or country background. Also, the money amount in the experimental questions is much smaller than the size of one's pension savings. Therefore, the results provide a more qualitative rather than a precise quantitative explanation of the relative attractiveness of annuities.

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4

Optimal decumulation strategies during retirement with deferred annuities

Optimal decumulation strategies during retirement with deferred annuities

Abstract

Since greater flexibility in accessing pension savings has been given to defined contribution pensioners, retirees are in need of advice on how to spend down their savings to make retirement income last throughout their lifetime. Deferred annuities have been discussed extensively in recent years as a retirement solution and have been recommended in the OECD Roadmap for the Good Design of Defined Contribution Pension Plans (OECD, 2016). Assuming a world where deferred annuities are available, we propose two utility maximising decumulation strategies comprising a deferred annuity purchased at retirement and optimal consumption and savings before the commencement of the annuity. A retiree who is concerned about longevity risk and wants to retain a certain level of liquidity is advised to spend 21.6% on a 15-year deferred annuity or 9.13% on a 20-year deferred annuity. A retiree who simply wants to use annuities to maximise overall satisfaction from retirement consumption is advised to spend 61.83% on a 6-year deferred annuity. We compare our strategies with other available decumulation strategies in the market, hence verifying the merits of the design. Moreover, the stability of our results are examined after allowing for consumption smoothness, social income benefits, a target replacement ratio and a bequest motive.

Keywords: Deferred annuities, Decumulation strategy, Defined Contribution, Retirement.

4.1 Introduction

Over the past few decades, the traditional defined benefit (DB) pension plan has been gradually losing its dominance in private sector pension systems in many countries in favour of the defined contribution (DC) pension plan. In a DC pension scheme, members need to accumulate retirement savings into their personal pension accounts and make wise decisions on how to use the money to support their life in retirement. In the process of the decumulation phase, the dilemma is that if individuals withdraw too much, they may run out of money before they die; but if they withdraw too little, they may have to bear lower living standards unnecessarily. Advice on a spending and investment strategy is in great demand from professionals such as investment advisors, pension providers, consultants and scholars, especially after the UK government recently reduced the restrictions on accessing personal pension account balances.

A great deal of effort has been made to determine a safe drawdown strategy. The best known one is the 4% rule from Bengen (1994), who suggests that an initial safe withdrawal rate from a portfolio is 4% of the assets and subsequent withdrawals increase annually with inflation rates. He described it as the highest spending rate that allows the portfolio to last for at least 30 years before being exhausted by withdrawals. Cooley et al. (1998) conduct a "Trinity Study", where they use simulations to determine, for each spending rate, the success probability that a portfolio will last for a certain number of years. A similar approach is adopted in current practice to determine the appropriate spending rate that allows for some probability of running out of money. The 4% rule has achieved wide acceptance among retirement planners and financial advisers due to its simplicity. Nonetheless, it has been sharply criticised by many scholars. For example, Scott et al. (2009) illustrate that the strategy is suboptimal since it suggests a constant spending plan while using a risky, volatile investment strategy. The retirees therefore have unspent surpluses when markets outperform and face shortfalls when markets underperform. Blanchett et al. (2016) argue that most of the literature is based on the historical returns of assets in the United States; and hence it is not applicable to other countries. According to Blanchett et al. (2016), given lower expected returns in the UK, especially in the near term, the safe initial withdrawal rate should be approximately 2.5% in the UK.

Another popular stream of retirement strategies is based on annuities and their use in retirement planning. A lifetime annuity basically provides a steady stream of income in retirement as long as the annuitant is alive, in exchange for an upfront premium charge. Therefore it is effectively protecting retirees against longevity risk, the risk of outliving one's assets which would result in a lower standard of living. Yaari (1965) initially demonstrates in a life-cycle model that a risk averse individual without a bequest motive should adopt a strategy of immediate full annuitisation at the point of retirement. This has been discussed widely in the subsequent literature. Brown et al. (2001) offer a conclusion regarding the operation of annuity markets and how annuities can assist people in retirement to allocate resources over an uncertain lifetime. Chen et al. (2006) and Ibbotson et al. (2007) demonstrate the benefits brought by an annuity as an integrated part of lifelong financial planning. Despite the popularity at the theoretical level, the empirical data from the annuity markets has shown that retirees are reluctant to purchase voluntarily immediate annuities as a retirement solution. In the UK where there were two distinct annuity markets, sales in the Compulsory Purchase Annuity (CPA) market were consistently much higher than that in the voluntary Purchased Life Annuity (PLA) market. By 2010, the CPA market had grown to £11.5 billion worth of annuity premiums while the PLA market only had £72 million worth of sales (Cannon and Tonks, 2011).

In recent years, the deferred annuity has aroused intensive discussion as an alternative retirement solution (see OECD (2016) and Sexauer et al. (2012) for example). Similar to an immediate annuity, a deferred annuity may be purchased with a lump sum payment[1] and in exchange, pays a guaranteed income as long as the annuitant is alive. The only difference is that the first payment from the deferred annuity will be delayed for a pre-determined number of years. The competitive advantages of deferred annuities have been discussed in Milevsky (2005), Gong and Webb (2010), Scott et al. (2007), and Scott (2008). First, it requires a smaller initial investment compared to immediate annuities, as its price incorporates the probability of surviving until the payment commencement date and the time value of money. Since a deferred annuity product costs only a relatively small proportion of lifetime savings, it overcomes a potential psychological barrier to annuitisation. Second, in addition to the lower price, a deferred annuity provides almost as much longevity protection as an immediate annuity product. Since the longevity risk

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[1] Deferred annuity contracts are also designed with regular periodic premiums payable during part or all of the deferment period.

is concentrated at an advanced age, deferred annuities are specially designed to protect the advanced lives. For example, a 15-year deferred annuity purchased at the age of 65 costs only 21.8% of the price of an immediate annuity purchased at the same age [2]. However, it provides the same level of income as the immediate annuity for life once the annuitant reaches age 80. Third, the use of a relatively small fraction of one's retirement savings means that retirees can preserve some liquidity by purchasing deferred annuities; hence allowing them to leave a bequest or save for unexpected health shocks. Fourth, there is the risk of increasing cognitive impairment at advanced ages – for example, Laibson (2009) shows that 20 percent of retirees in their 80s have been fully diagnosed dementia and 30 percent have severe cognitive impairment. Therefore it would be wise to purchase deferred annuities at retirement to protect consumption at very advanced ages.

In the process of retirement financial planning, one challenge comes from the uncertainty of one's lifetime. The structure of the deferred annuity improves and simplifies the process. With the deferred annuity providing income protection after a fixed period, retirees simply need to determine how to invest and consume the rest of their income or wealth across the fixed deferred period (or a shorter-than-expected period due to early death). This is a much easier task and would be less likely to expose retirees to the risk of living in poverty.

Bearing this in mind, we propose a utility-maximising decumulation strategy comprising a deferred annuity purchased at retirement and optimal consumption and savings before the commencement of the annuity. The strategy divides the entire retirement period into two stages: one fixed period with active consumption and investment decisions to be made; followed by an uncertain period with passive lifelong income from the deferred annuity. We set up two models with different breakpoints between the two stages: one in which the choice of the deferred period is exogenous (M1) and one in which it is endogenous (M2). M1 assumes that retirees already have preferences for the age from which they will be protected and they want to use that specific product to improve retirement planning. The structure of this strategy is similar as the DCDB strategy proposed by Sexauer et al. (2012), although the DCDB is a pure static model. M2 assumes retirees consider using the optimal deferred annuities to improve retirement

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[2]This annuity price is calculated based on assumptions of 3% p.a. real rate of return, mortality table S2PML and zero profit loading.

planning. This involves identifying the most favorable annuity type, which is a research question discussed in Scott et al. (2011).

Using numerical optimisation, given our chosen assumptions, we find the following strategies. (i) A retiree who is concerned about longevity risk and wants to retain a certain level of liquidity is advised to spend 21.6% in a 15-year deferred annuity or 9.13% in a 20-year deferred annuity. Prior to the commencement of the annuity, a decreasing drawdown plan is suggested. (ii) A retiree who simply wants to use annuities to maximise overall satisfaction from retirement consumption is advised to spend 61.83% in a 6-year deferred annuity and to follow a decreasing consumption path until the start of annuity payments. A sensitivity check shows that our recommended allocations are stable relative to the desire for smooth consumption, the existence of state benefits and the bequest motive. However, the existence of a target replacement ratio is an influential factor on recommended strategies. If there is a target replacement ratio, retirees would be willing to buy a deferred annuity that offers income close to the target.

In this chapter, Section 2 introduces the set up of the main models M1 and M2 and the optimal results following these two strategies. In Section 3, M1 and M2 are compared with a DCDB strategy and we comment on the major characteristics of these strategies. Section 4 provides the results of four model extensions, which discuss a smooth consumption requirement, state benefits, the target replacement ratio and the bequest motive. Section 5 offers some concluding comments on our major findings.

4.2 Basic decumulation strategy

4.2.1 Model set-up

As an individual approaches retirement, she would gain access to her personal accumulated pension account and needs to make a decision on how to allocate the lump sum of money to support living during the retirement period. In this study, we assume without loss of generality that the total wealth at retirement is $w = 1$.

Let c_x denote the annual rate of consumption that would take place at age x. x is defined on the age interval [65,119]. Suppose V is the overall expected discounted utility of the entire retirement consumption stream c_x and it measures retirees' preferences of

one strategy over another. Our objective is to find the consumption streams c_x that can maximise the value of V subject to the wealth constraint.

Following our strategy, the purchase of a d-year deferred annuity divides the process of retirement planning into two stages. In the former stage, consumption before annuity commencement, $c_{65}, \ldots, c_{65+d-1}$, is met by simple drawdowns from unannuitised savings. The investment option during this period is assumed to be a fixed savings account where wealth accumulates at the risk free interest rate. At the point when the deferred annuity commences, retirees move to the latter stage where consumption, $c_{65+d}, ..., c_{119}$, is fully secured by an annuity provider. The optimal consumption stream during the second stage enables us to identify the preferred annuity payment type and the optimal allocation of wealth into the deferred annuity, α. The model of our strategy can be expressed in the following form:

$$V = \max_{c_{65}, c_{66}, \ldots} \sum_{i=0}^{54} \delta(i) \times {}_i p_{65} \times u(c_{65+i})$$

subject to

$$\sum_{i=0}^{d-1} c_{65+i} \times (1+r)^{-i} + \sum_{i=d}^{54} c_{65+i} \times (1+r)^{-i} \times {}_i p_{65} \times (1+L) = 1$$

where

$$\alpha = \sum_{i=d}^{54} c_{65+i} \times (1+r)^{-i} \times {}_i p_{65} \times (1+L)$$

$\delta(i)$ may be interpreted as a subjective discount function for utility, ${}_i p_{65}$ as the probability of surviving for i years for a 65-year-old, $u(\cdot)$ as the utility associated with the rate of consumption, r as the real interest rate and L as the profit loading factor embedded in the annuity price.

Here, we assume that the subjective discount function for the utility is the same as the discount function for consumption rates. In addition, a constant real rate of return r is assumed for wealth accumulation and annuity pricing. We follow the literature (e.g. Gong and Webb (2010)) by assuming a time-additive constant relative risk aversion (CRRA) utility function of the following form:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

where γ measures the coefficient of risk aversion.

In this analysis, we consider two different situations regarding the deferred period. In model M1, the deferred period d is a fixed exogenous factor. This could be thought of as a public policy rule which encourages retirees to allocate a certain percentage of savings into a 15-year or a 20-year deferred annuity. In model M2, the deferred period d is an endogenous factor that is determined optimally by the retiree. In the following, we introduce the results from the two models respectively.

4.2.2 Results

The results we display are based on the following assumptions. A moderate relative risk aversion of $\gamma = 2$ is assumed and profit loading on annuity product equals $L = 10\%$. The discount rate for both utility and money amount are based on exponential discounting with a constant real interest rate of 3 percent. UK mortality table S2PML, which describes the mortality experience of UK male pensioners of self-administered pension schemes for the period from 2004 to 2011, is used to calculate the survival rate $_ip_{65}$. In what follows, we provide results for the optimal wealth allocation, α, in the deferred annuity and the optimal retirement consumption path, c_x for $65 \leq x \leq 109$. The sensitivity of the results is also explored by changing the real interest rate, profit loading and risk aversion factors.

4.2.2.1   M1: The strategy with a predetermined deferred annuity

In the case that the recommended deferred annuity contract has a fixed deferred period, we can find mathematically the closed form solution for the optimal consumption path and optimal wealth allocation in the deferred annuity. Derivation is shown in Appendix 4.A.

TABLE 4.1: Optimal investment percentage (α) in a deferred annuity (M1)

Interest rate (r)	1%	2%	3%	4%	5%
Deferred period (d)					
5	72.13%	69.88%	67.62%	65.38%	63.16%
10	47.02%	44.05%	41.19%	38.45%	35.85%
15	26.44%	23.92%	21.60%	19.47%	17.51%
20	11.97%	10.47%	9.13%	7.94%	6.90%
25	3.98%	3.36%	2.83%	2.38%	2.00%

Notes: Results are based on the following assumptions: a risk aversion factor of $\gamma = 2$, a profit loading factor of $L = 0.1$. Mortality rates are based on table S2PML.
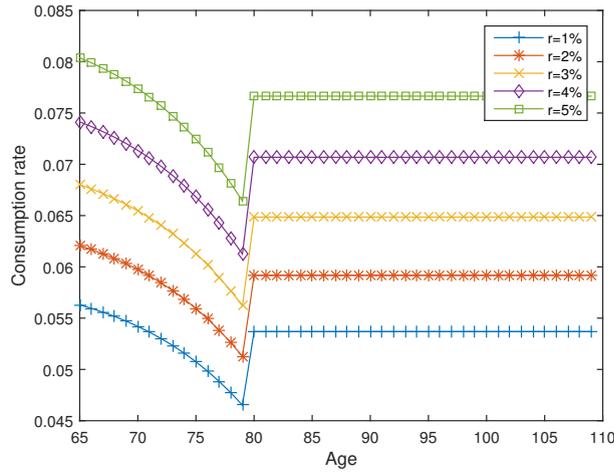
FIGURE 4.1: Optimal consumption paths with regard to different interest rates (M1)

Notes: Results are based on a 15-year deferred annuity and the following assumptions are used: a risk aversion factor of $\gamma = 2$, a profit loading factor of $L = 0.1$. Mortality rates are based on table S2PML.

$$c_{65+i} = \frac{{}_ip_{65}^{1/\gamma}}{\sum_{i=0}^{d-1}(1+r)^{-i}{}_ip_{65}^{1/\gamma} + (1+L)^{(1-1/\gamma)}\sum_{i=d}^{54}(1+r)^{-i}{}_ip_{65}} \quad \text{for} \quad i = 0,\ldots,(d-1)$$

$$c_{65+i} = \frac{(1+L)^{-1/\gamma}}{\sum_{i=0}^{d-1}(1+r)^{-i}{}_ip_{65}^{1/\gamma} + (1+L)^{(1-1/\gamma)}\sum_{i=d}^{54}(1+r)^{-i}{}_ip_{65}} \quad \text{for} \quad i = d,\ldots,54$$

(4.1)

$$\alpha = \frac{(1+L)^{(1-1/\gamma)}\sum_{i=d}^{54}(1+r)^{-i}{}_ip_{65}}{\sum_{i=0}^{d-1}(1+r)^{-i}{}_ip_{65}^{1/\gamma} + (1+L)^{(1-1/\gamma)}\sum_{i=d}^{54}(1+r)^{-i}{}_ip_{65}} \tag{4.2}$$

Table 4.1 shows the results for the optimal annuity allocation α under different possible

values of the interest rate and deferred period. It demonstrates that under a reasonable assumption of a 3 percent interest rate, retirees would choose to allocate 21.6% in a 15-year deferred annuity that starts providing income at age 80, or to allocate 9.13% in a 20-year deferred annuity that starts providing income at age 85. The results show that our strategy is a practical plan as one needs to spend only a small proportion of wealth to receive longevity protection and achieve utility maximisation at retirement. Figure 4.1 demonstrates the corresponding optimal consumption paths when the chosen deferred period is 15 years and the interest rates varies from 1 percent to 5 percent. For example, a 3 percent interest rate suggests that individuals should spend 6.8% in the first year at retirement and decrease the spending rate steadily in subsequent years. An allocation of 21.6% in a deferred annuity allows them to receive a level payment of 6.5% annually after age 80.

In terms of the sensitivity to changes in the interest rate, Table 4.1 shows that retirees tend to invest less in deferred annuities when the interest rate increases. A smaller allocation under a higher interest rate environment does not suggest that annuities become less attractive; it is rather that annuities become cheaper. As an illustration, the price of a 5-year deferred annuity would decrease by 38.53% when the interest rate increases from 1 percent to 5 percent; while the recommended investment proportion of wealth into an annuity shows only a 8.97% decrease, from 72.13% to 63.16% (in both cases, a 10% profit loading is included). Therefore, deferred annuity products are more attractive when the interest rate goes up. Figure 4.1 also shows that retirees would have an increase of 0.005% in annuity income in response to a 1 percent increase in the interest rate. From another perspective, as the interest rate goes up, retirees could get higher returns from alternative investment opportunities such as equities and bonds. Therefore, it is reasonable to see them allocate a smaller proportion of wealth in annuities and invest the unannuitised proportion on their own. Figure 4.1 shows a parallel shift of the optimal consumption path under different interest rates, which indicates that the interest rate does not influence preferred spending patterns during retirement.

Figure 4.2 shows the value of the maximum expected utility, V, achieved from the optimal consumption strategies under different levels of interest rate and deferred periods. In order to have a higher level of satisfaction, retirees may prefer to choose annuities with a shorter deferred period. However, this is at the cost of retaining low liquidity at the point of retirement by investing a high proportion of wealth in annuities. In addition,
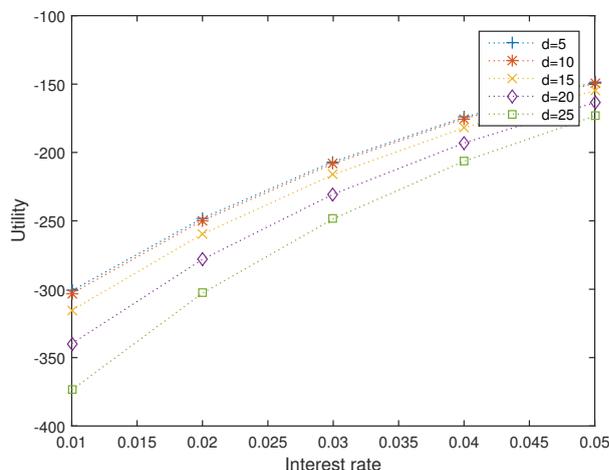
FIGURE 4.2: Maximum attainable expected utility with regard to different interest rates (M1)

Notes: d denotes the deferred period of the chosen annuity. Results are based on the following assumptions: a risk aversion factor of $\gamma = 2$ and a profit loading factor of $L = 0.1$. Mortality rates are based on table S2PML.

TABLE 4.2: Optimal investment percentage (α) in a 15-year deferred annuity (M1)

Risk aversion factor (γ)	2	3	4	5	6
Profit loading (L)					
5%	21.21%	20.99%	20.88%	20.81%	20.77%
10%	21.60%	21.51%	21.46%	21.43%	21.42%
15%	21.98%	22.02%	22.03%	22.04%	22.05%
20%	22.35%	22.51%	22.58%	22.63%	22.66%
25%	22.70%	22.98%	23.12%	23.21%	23.26%

Notes: Results are based on the following assumptions: a 15-year deferred annuity and a real rate of return of $r = 3\%$. Mortality rates are based on table S2PML.

under a high interest rate environment, retirees can achieve a higher level of satisfaction from the deferred annuity investment than under a low interest rate environment.

Table 4.2 shows the impact of the profit loading and risk-aversion on the optimal wealth allocation on a 15-year deferred annuity. Looking horizontally in the table, we note that the risk-aversion factor has a very limited influence on the optimal decision of how much to spend on a 15-year deferred annuity. Looking vertically in the table, it is optimal for retirees to allocate a higher percentage of wealth on annuity products when insurance companies charge a higher profit loading. However, the higher percentage is simply due to the higher price rather than a higher attractiveness of annuity products. The
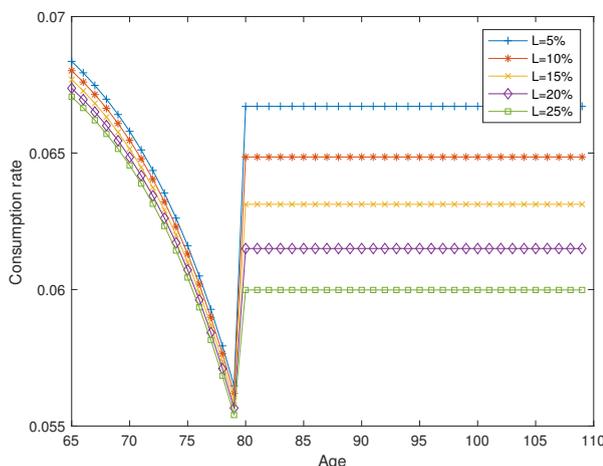
FIGURE 4.3: Optimal consumption paths with regard to different profit loading rates
(M1)

Notes: Results are based on a 15-year deferred annuity and the following assumptions are used: a risk aversion factor of $\gamma = 2$ and a real rate of return of $r = 3\%$. Mortality rates are based on table S2PML.

real wealth allocation that is used to pay for future payments, measured by $\alpha/(1 + L)$, actually decreases with profit loading; thus indicating that an annuity becomes less desirable as its price increases.

The change in the consumption path with regard to the profit loading factor, following a strategy with a 15-year deferred annuity, is illustrated in Figure 4.3. While, as expected, a higher profit loading makes the deferred annuity less desirable, it does not influence much the withdrawal rates before the commencement of an annuity. This means an annuity product which is poorer in value for money would lead to a smoother consumption path at retirement. Therefore the profit loading is a component of the consumption gap that is identified in Figure 4.1. When the profit margin required by insurance companies is relatively low, the annuity as a longevity protection product is good value for money and individuals should allocate a high proportion of wealth in it. A higher annuity allocation leads to a greater consumption gap at the point when the annuity commences. As the profit loading increases, the product becomes less attractive; an optimal plan advises investing less into the annuity, thus achieving a more balanced and smooth consumption rate during the entire retirement period.

TABLE 4.3: Greatest consumption gap (g) in each optimal consumption path with a 15-year deferred annuity (M1)

Risk aversion factor (γ)	2	3	4	5	6
Profit loading (L)					
5%	18.1%	11.7%	8.7%	6.9%	5.7%
10%	15.4%	10.0%	7.4%	5.9%	4.9%
15%	12.9%	8.4%	6.2%	5.0%	4.1%
20%	10.5%	6.9%	5.1%	4.1%	3.4%
25%	8.3%	5.4%	4.0%	3.2%	2.7%

Notes: Results are based on the following assumptions: a 15-year deferred annuity and a real rate of return of $r = 3\%$. Mortality rates are based on table S2PML.

People tend to prefer smooth consumption so that they can maintain their living standards. However, the utility maximising strategy suggests a path with an obvious gap at the transition point when the annuity payment starts. In order to understand the driving factors behind the consumption gap, we derive the mathematical expression for the greatest change between any two consecutive consumption rates during retirement, g, following a strategy with the 15-year deferred annuity. The greatest gap always occurs between the last consumption rate before the annuity starts and the annuity pay-out rate, and the formula for this gap, g, is

$$g = \frac{(1 + L)^{-1/\gamma} - {}_{14}P_{65}^{1/\gamma}}{{}_{14}P_{65}^{1/\gamma}} \tag{4.3}$$

Equation (4.3) shows that the greatest consumption gap is influenced by the profit loading factor, L, the level of risk aversion, γ, and the choice of the survival model. Table 4.3 shows the value of g for different profit loadings L and different risk aversion factors γ. As we have explained, a higher profit loading factor encourages a smaller investment in deferred annuities, leading to a smaller consumption gap during the transition point. Moreover, individuals who are more risk averse tend to prefer a more smooth consumption path during retirement by investing a similar proportion in the deferred annuities.

4.2.2.2 M2: The strategy with an optimal deferred annuity

In a perfect annuity market without any profit loading, the annuitised state should always be preferable to the non-annuitised state. In other words, the best retirement

TABLE 4.4: Optimal investment percentage (α) in a deferred annuity (M2)

Profit loading (L)	5%	10%	15%	20%	25%
Optimal deferred period	4	6	8	10	11
Risk aversion factor (γ)					
2	73.20%	61.83%	51.54%	42.25%	38.20%
3	73.30%	62.07%	51.91%	42.71%	38.77%
4	73.35%	62.19%	52.10%	42.94%	39.05%
5	73.38%	62.26%	52.21%	43.08%	39.22%
6	73.40%	62.31%	52.28%	43.17%	39.33%

Notes: Results are based on a real rate of $r = 3\%$ and the mortality table S2PML.

planning strategy can be achieved by buying an immediate annuity at age 65. This is because by buying annuities, individuals receive extra returns from mortality sharing, which is effectively a subsidy from early deaths to survivors (i.e. later deaths). In a more ideal world with profit loading charged by insurance companies, the optimal age to receive the first annuity payment is

$$T_A = min(65 + i : (1 + L) \times {}_i p_{65} \leq 1) \tag{4.4}$$

Due to this formula and with prior knowledge of the profit loading in insurance pricing and the individual mortality rates, individuals can derive their optimal deferred period, d. With this choice of d, the same analytical process will be implemented as in M1 to deduce the preferred consumption path and the investment allocation in deferred annuities.

Note that if one can find an i such that the inequality in Equation (4.4) holds with equality, then the jump size g defined in Equation (4.3) becomes zero. This means that if i can be exactly chosen, the optimal deferred period implies no gap in the consumption path.

Table 4.4 offers optimal results for model M2 with different assumptions for the profit loading, L, increasing from 5% to 25%. With the assumption of a 10% profit loading and risk aversion factor of 2, individuals would be better off by allocating 61.83% of pension wealth in a 6-year deferred annuity. If annuity providers require a higher profitability on annuity products, individuals would allocate less wealth to purchase an annuity with a longer deferred period. Moreover, an analysis of the results under different levels of risk aversion shows that retirees who are more risk averse tend to allocate a slightly higher
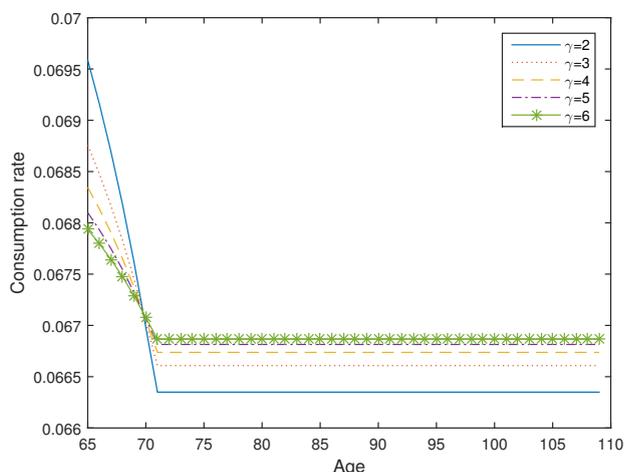
FIGURE 4.4: Optimal consumption paths with regard to different levels of risk aversion (M2)

Notes: Results are based on the following assumptions: a profit loading factor of $L = 10\%$ and a real rate of return of $r = 3\%$. Mortality rates are based on table S2PML.

percentage in deferred annuities. Since the annuity effectively helps mitigate the risk of outliving one's wealth under uncertain lifetime, it is reasonable to see that people who are more risk averse place higher values on the insurance product and hence invest more in it. They also obtain a more smooth consumption path, which is illustrated in Figure 4.4.

4.3 A comparison of three decumulation strategies

In the previous section, we proposed two strategies that involve a utility-maximising consumption withdrawal plan and the purchase of either a fixed long-term deferred annuity (M1) or a flexible deferred annuity (M2). The two strategies are to some extent similar to the DCDB decumulation strategy proposed by Sexauer et al. (2012). The DCDB strategy includes a fixed 20-year deferred annuity as well; however it suggests the purchase of laddered Treasury Inflation Protected Securities (TIPS) prior to annuity commencement in order to maintain the sustainability and purchasing power of consumption. In this section, we will give a detailed introduction of the DCDB strategy and compare it with our strategies M1 and M2.

4.3.1 DCDB benchmark

The DCDB benchmark proposed by Sexauer et al. (2012) is an investment strategy that makes a Defined Contribution plan look more like a Defined Benefit pension plan. The investment portfolio consists of two assets: a portfolio of laddered TIPS for 20 years and a 20-year deferred nominal life annuity. For the first 20 years, the income paid out of the TIPS should grow at the experienced rate of US Consumer Price Index (CPI) inflation, and the income rate has allowed for the principal payment so that the capital balance of the portfolio is zero at the end of the 20th year. Beyond the 20th year, the nominal deferred annuity starts providing income and it continues until the investor's death.

As the first payment of the annuity (and also each of its subsequent payments) is set to be equal to the last payment from the TIPS portfolio, the allocation in the deferred annuity is easily calculated, given the initial payout rate from TIPS, the expected inflation rate and the 20-year deferred annuity price. According to the market annuity price and the TIPS yield curve as of 1 August 2010, Sexauer et al. (2012) suggest investing 88.3% of wealth into the TIPS portfolio and the remaining 11.7% into the deferred annuity, for a man entering retirement at age 65.

In designing this decumulation strategy, the authors claim to achieve a compromise solution under the assumption that investors' concerns about maximising returns and minimising risk can be expressed in four "dimensions". Firstly, longevity risk, the major concern in retirement financial planning, is minimised by the inclusion of the deferred annuity in the investment portfolio. Although annuity payments come at a later stage in retirement, it provides income as long as the annuitant survives, hence converting the uncertain lifetime financial planning into a fixed period financial planning. This is how the authors make the DC plan look like a DB plan. Secondly, the liquidity risk is largely reduced since the strategy does not suggest holding the entire portfolio in annuities immediately after retirement. Much of the liquidity is preserved after investing only 11.7% in the illiquid asset. Thirdly, the investment risk and inflation risk are minimised by investing in the lowest risk US government bonds. Moreover, counterparty risk is not reduced to zero because the government can default and insurance companies may go bankrupt; however, it is the lowest that can be achieved in an investment market.

4.3.2   A comparison of three strategies

Before a comparison is made, we need to allow for strategies M1 and M2 in the context of a world with positive inflation. In the presence of an inflation rate, the remaining wealth during retirement would accumulate with a nominal rate of return, which is a combination of the real interest rate and the inflation rate. Therefore the optimal consumption stream c'_x that we are looking for is in nominal term. However, only real consumption rates contribute towards the overall utility, V, because they reflect real purchasing power. The model of our strategy allowing for inflation can therefore be expressed in the following form:

$$V = \max_{c'_{65}, c'_{66}, \dots} \sum_{i=0}^{54} (1+r)^{-i} \times {}_i p_{65} \times u\Big(\frac{c'_{65+i}}{(1+f)^i}\Big)$$

subject to

$$\sum_{i=0}^{d-1} c'_{65+i} \times (1+r_{nominal})^{-i} + \sum_{i=d}^{54} c'_{65+i} \times (1+r_{nominal})^{-i} \times {}_i p_{65} \times (1+L) = 1$$

where f represents the expected inflation rate and $r_{nominal}$ represents the nominal rate of return.

Using the Lagrangian method, we derive the closed form solution for the optimal nominal consumption rates and the optimal deferred annuity allocation as follows. For detailed derivations please see Appendix 4.B.

$$c'_{65+i} = \frac{{}_i p_{65}^{1/\gamma}(1+f)^i}{\sum_{i=0}^{d-1}(1+r)^{-i}{}_i p_{65}^{1/\gamma} + (1+L)^{(1-1/\gamma)}\sum_{i=d}^{54}(1+r)^{-i}{}_i p_{65}} \quad \text{for} \quad i = 0, \dots, (d-1)$$

$$c'_{65+i} = \frac{(1+L)^{-1/\gamma}(1+f)^i}{\sum_{i=0}^{d-1}(1+r)^{-i}{}_i p_{65}^{1/\gamma} + (1+L)^{(1-1/\gamma)}\sum_{i=d}^{54}(1+r)^{-i}{}_i p_{65}} \quad \text{for} \quad i = d, \dots, 54$$

$$\tag{4.5}$$

$$\alpha = \frac{(1+L)^{(1-1/\gamma)}\sum_{i=d}^{54}(1+r)^{-i}{}_i p_{65}}{\sum_{i=0}^{d-1}(1+r)^{-i}{}_i p_{65}^{1/\gamma} + (1+L)^{(1-1/\gamma)}\sum_{i=d}^{54}(1+r)^{-i}{}_i p_{65}} \tag{4.6}$$

Comparing Equation (4.5) and Equation (4.6) with those displayed in Equation (4.1) and Equation (4.2), we have two conclusions regarding the impact of the inflation rate. First,

TABLE 4.5: A comparison of three strategies

Strategies	Asset allocations
M1	9.13% in a 20-year deferred annuity
M2	61.83% in a 6-year deferred annuity
DCDB	92.02% in 20-year TIPS + 7.98% in a 20-year deferred annuity

Notes: Results are based on the following assumptions: a profit loading factor of $L = 10\%$, a real rate of return of $r = 3\%$ p.a., an inflation rate of $f = 2\%$ and a risk aversion factor of $\gamma = 2$. The DCDB investment percentage presented here is different from that in the paper by Sexauer et al. (2012). For comparison purposes, we recalculate the DCDB investment percentage using a different set of assumptions.

the inflation rate does not affect the real portion of optimal consumption; the nominal consumption rates would simply increase in line with the inflation rate. Second, an index-linked deferred annuity would be preferred in the presence of the inflation rate. However, the optimal wealth allocation in the deferred annuities is not affected by the inflation rate. The annuity is either priced using the real rate in a zero inflation rate environment or priced using the nominal rate in a positive inflation rate environment. In either case, the allocation would not be affected.

Now, all three strategies are allowed for in the context of a world with a positive inflation rate. Based on the descriptions of our strategies and the DCDB, the remaining fundamental difference is the target. M1 and M2 focus on maximising utility from retirement consumption; while the DCDB targets maximising returns and minimising risks. This can lead to very different strategies and retirement consumption paths. Since the DCDB benchmark has been proposed with a 20-year deferred annuity, we assume a fixed 20-year deferred period in M1. The structure of the three plans is shown in Table 4.5 and the corresponding nominal consumption paths are reflected in Figure 4.5.

Table 4.5 shows that strategies M1 and DCDB suggest investing a similar proportion of wealth in a 20-year deferred annuity product. However, the retirement consumption paths suggested by these two strategies have different shapes in the two stages, as shown in Figure 4.5. In terms of consumption from age 85 onwards, M1 suggests buying an index-linked deferred annuity while DCDB suggests buying a conventional deferred annuity with level incomes. The real purchasing power of the annuity income is therefore maintained by following M1 but it deteriorates by following DCDB. On the other hand, prior to the start of the 20-year deferred annuity, consumption rates suggested by M1
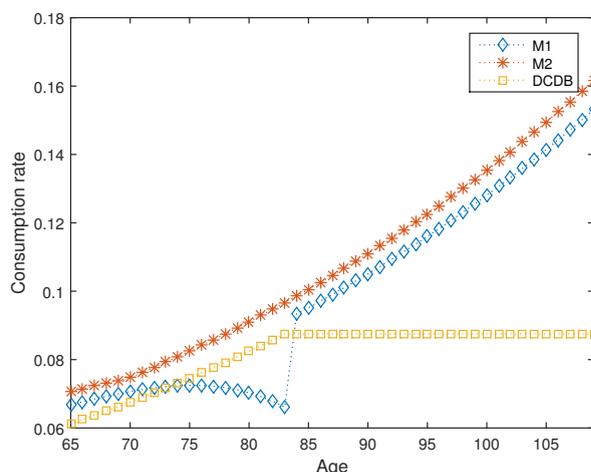
FIGURE 4.5: A comparison of optimal consumption paths under three strategies

Notes: Results are based on the following assumptions: a profit loading factor of $L = 10\%$, a real rate of return of $r = 3\%$, an inflation rate of $f = 2\%$ and a risk aversion factor of $\gamma = 2$.

fluctuate around 6.5% and have decreasing real value. The TIPS security adopted by the DCDB would ensure that purchasing power is maintained at the same level as at age 65. Overall, the DCDB strategy results in lower consumption compared with M1 for most of the time during retirement. It is because DCDB is a static model rather than an optimised model like M1.

An interesting finding is that strategy M2, which suggests spending 61.90% on an index-linked 6-year deferred annuity, achieves the highest spending rates among the three strategies at all ages during the retirement period. As can be seen from Figure 4.5, the consumption path following M2 dominates all other strategies in both stages. The utility equivalent wealth in Table 4.6 illustrates the competitive advantage of strategy M2 as well. Under the assumptions in the benchmark scenario, with the real rate of 3 percent and profit loading of 10 percent, the utility equivalent wealth of M2 is 0.8960 while that of M1 is 1 and that of DCDB is 1.0112. It means that in order to achieve the same amount of utility, following M2 one needs to spend only 89.6% of the wealth used when following strategy M1, or 88.6% (0.8960/1.0112) of the wealth used when following DCDB.

Table 4.6 also shows the impact of the real interest rate and the profit loading factor on the three strategies. In contrast with DCDB, M2 has a smaller utility equivalent wealth

TABLE 4.6: Utility equivalent wealth of three strategies

Real rate (r)	3%	1%	5%	3%	3%
Profit loading (L)	10%	10%	10%	5%	15%
Strategy					
M1	1	1	1	1	1
M2	0.8960	0.8854	0.9068	0.8725	0.9142
DCDB	1.0112	1.0128	1.0098	1.0118	1.0107

Notes: Results are based on the following assumptions. A 20-year deferred annuity, $d = 20$, for strategies M1 and DCDB. A risk aversion factor of $\gamma = 2$ and an inflation rate of $f = 2\%$. Mortality rates are based on table S2PML.

in a low interest rate environment than in a high interest rate environment. When the interest rate is 1 percent, following M2 one needs to spend only 88.54% of the wealth used when following strategy M1 to achieve the same amount of utility; however when the interest rate is 5 percent, following M2 one needs to spend 90.68% of the wealth used when following strategy M1 to achieve the same amount of utility. Therefore, the competitive advantage of M2 is greater in a lower interest rate environment. Moreover, the competitive advantage of M2 is greater in a low profit loading environment. This is because the utility equivalent wealth of M2 decreases as the profit loading factor decreases. When the profit loading factor is 5 percent, following M2 one needs to have 12.75% less wealth than if one were following M1 to achieve the same amount of utility; however, when the profit loading is 15 percent, following M2 one needs to have 8.58% less wealth than if one were following M1 to achieve same amount of utility.

We conclude that M2 is the best strategy among the three in terms of the consumption to be achieved during retirement. However, this comes at a price: a much lower liquidity would be preserved at hand in the beginning of retirement period. After the annuity purchase, M1 followers have 90.87% left as cash while M2 followers have only 38.17% left. Given that people dislike trading liquid assets for illiquid assets, deciding which strategy to follow would be affected by personal preferences such as leaving a bequest or making extra medical consumption at an advanced age.

To conclude, all three strategies have different characteristics. Table 4.7 provides a summary.

TABLE 4.7: Achievements of three strategies

	M1	M2	DCDB
Reduce longevity risk	*	*	*
Reduce liquidity risk	*		*
Reduce inflation risk	*	*	*
Achieve higher utility		*	

4.4 Model extensions

4.4.1 A decumulation strategy to ensure smooth consumption

In economics, a widely accepted concept called "consumption smoothing" states that people desire a stable path of consumption. The idea was initially introduced as Permanent Income Theory by Friedman (1957). It says that individuals choose consumption at each point in time to maximise a lifetime utility function that depends on both current income and expected income levels in the future; therefore individuals spread out the transitory changes in income over time and the predicted consumption pattern is smooth. The life cycle model introduced by Modigliani and Brumberg (1954) to address the intertemporal consumption problem suggests that the trajectory of consumption should be continuous in time; a consumption pattern with less discontinuity will increase utility without an increase in the use of resources.

While our decumulation framework is based on the maximization of discounted utility, in Model M1 there is a jump in the consumption path due to a predefined deferral period. As introduced in Section 2, the recommended consumption path with a 15-year deferred annuity contains a 15.41% jump at the point where the annuity starts paying. This is because an annuity is a cost-effective product to improve retirement income and a utility maximisation strategy recommends a high percentage of wealth to be allocated in annuity products. In order to balance the desire for smooth consumption during the retirement period and retain the benefits of annuity products, we now make an adjustment to model M1. The modified strategy can be expressed as follows.

$$V = \max_{c_{65},c_{66},\dots} \sum_{i=0}^{54} \delta(i) \times {}_i p_{65} \times u(c_{65+i}) - \kappa \sum_{i=0}^{53} (c_{65+i+1} - c_{65+i})^2 \qquad (4.7)$$

subject to

TABLE 4.8: Optimal investment percentage (α) in a 15-year deferred annuity for different rough consumption penalties κ

κ	10^2	10^3	10^4	10^5	10^6
α	21.58%	21.43%	21.06%	21.00%	21.25%

Notes: Results are based on the following assumptions: a 15-year deferred annuity, $d = 15$, a profit loading factor of $L = 10\%$, a real rate of return of $r = 3\%$ and a risk aversion factor of $\gamma = 2$. Mortality rates are based on table S2PML.

TABLE 4.9: A summary of the consumption paths presented in Figure 4.6

κ	10^2	10^3	10^4	10^5	10^6
Max consumption	6.80%	6.79%	6.74%	6.56%	6.43%
Min consumption	5.66%	5.82%	6.07%	6.28%	6.38%
Max - Min	1.14%	0.98%	0.67%	0.28%	0.05%
Smooth consumption path ?	No	No	No	Yes	Yes
Consumption gap	14.52%	9.99%	3.04%	NA	NA

- $\sum_{i=0}^{d-1} c_{65+i} \times (1+r)^{-i} + \sum_{i=d}^{54} c_{65+i} \times (1+r)^{-i} \times {}_ip_{65} \times (1+L) = 1$

- $c_{65+i} = c_a$ for $i = d, d+1, \ldots$

where κ controls the importance of smooth consumption to an individual; c_a denotes the income level from a conventional level deferred annuity.

Here we have introduced an additional term of first-order of finite differences in consumption rates as a smoothness criterion. In the literature, this is a widely adopted method to measure smoothness in the sequence of graduated values (London, 1985). It is also an effective tool to be used to compare the relative degrees of smoothness in several graduations. From Equation (4.7), we deduct the value of variations from the overall utility, indicating that a less smooth consumption path would lead to lower utility, so that the second term in Equation (4.7) can be thought of as a penalty for lack of smoothness.

Table 4.8 shows the optimal results of α under a range of possibilities of κ. An increasing level of the penalty for consumption changes does not influence the optimal results of asset allocation. In other words, an individual with a high requirement on consumption smoothness would be willing to invest a similar percentage of wealth in annuities as another individual who has no requirement on consumption smoothness. However, different attitudes towards consumption smoothness do have an impact on preferred consumption pattern, as shown in Figure 4.6. As retirees apply a greater amount of
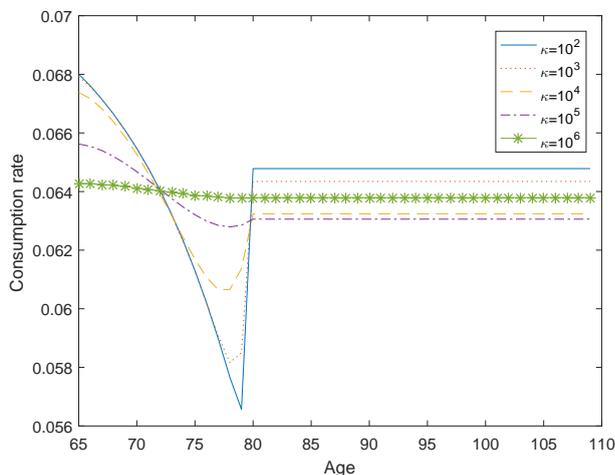
FIGURE 4.6: Optimal consumption paths with regard to different levels of penalty (κ)

Notes: Results are based on the following assumptions: a 15-year deferred annuity, $d = 15$, a profit loading factor of $L = 10\%$, a real rate of return of $r = 3\%$ and a risk aversion factor of $\gamma = 2$. Mortality rates are based on table S2PML.

penalty to changes in consumption, they follow a less volatile path. When the penalty factor κ is 10^2, optimal consumption rates during the entire retirement period vary in a range of $[5.66\%, 6.80\%]$. When the penalty factor becomes 10^6, retirees choose a constant consumption rate during the entire retirement period; an immediate annuity from age 65 would be a better product in this sense.

We have also explored three other models to ensure smooth consumption. Detailed results from these three other models are not included in this thesis but are available from the author. One model introduces a limit on the greatest percentage change in consecutive consumption rates,

$$\left| \frac{c_{i+1} - c_i}{c_i} \right| \leq g, \quad \text{for} \quad 0 \leq i \leq 54.$$

where g equalling 5% means that an individual expects the consumption level in next year to be less than 105% and greater than 95% of the consumption level in the current year. This second way of modelling smooth consumption requirement leads to the same conclusion as we discussed above. Setting an upper limit on the consumption change would not influence the wealth allocation; however, it suggests a less volatile consumption path as g becomes smaller.

The other two models dealing with the requirement on smooth consumption fail in some ways. In one model, we assume that retirees require current spending to be less than or equal to the spending in the previous year.

$$c_{65+i} \geq c_{65+i+1}, \quad \text{for} \quad i = 0, 1, \ldots, d-1.$$

This effectively removes the jump in the consumption rate; the optimal consumption rate is decreasing in the first few years and then stays constant. However, the optimal results achieved in this framework are not efficient because the annual consumption rate falls to the level of annuity payment before the start of the annuity. For example, a retiree who purchases a 15-year deferred annuity is suggested to consume the same amount as the annuity payment from age 73. In such a case, the retiree will be better off buying an annuity that commences its payments earlier. However, this breaks the assumptions of model M1.

The DCDB strategy introduced in the previous section assumes that the annuity payment is equal to the last payment of the TIPS portfolio. In light of this, we tried to set the first annuity payment equal to the inflation adjusted amount of the previous consumption level.

$$c_d = c_{d-1} \times (1 + f),$$

where f denotes the expected inflation rate. Results based on this framework show that the additional constraint does not address the problem of consumption discontinuity because the kink in consumption path remains. This is due to the fact that when the change between the final consumption in stage one and the annuity income is constrained, the gap moves one year forward. Therefore, this is not an effective framework to model the requirement for smooth consumption.

4.4.2 A decumulation strategy with consideration of state benefits

The basic state pension is a regular payment that one can receive from the government if he/she reaches state pension age. In the UK, this is part of the pension arrangement alongside workplace pension schemes. Depending on whether an individual reaches state pension age before or after 6 April 2016, he/she can claim either the basic state pension or the new state pension. The full basic state pension is £119.30 per week and the

full new state pension is £155.65 per week. The exact amount received is based on an individual's national insurance record and the rate increases every year. The increase is given as the maximum of the following three values: average wage growth, Consumer Price Index (CPI) and 2.5% (UK Government, 2016) [3].

The existence of the state pension guarantees a basic level of retirement income and lessens the longevity risk to some extent. In the worst case when an individual spends the entire pension savings, the state pension could support basic living expenses such as housing, food and utilities. Therefore, the function of longevity protection from annuity products becomes less important.

Noting that the existence of the state pension could influence demand for annuity products, we would like to explore the impact of the state pension on our proposed decumulation strategies. Although the state pension provides the same amount to everyone, people may have different views on the amount of money given their different wealth levels. In this section, we introduce a factor *sb* representing the annual state pension as a proportion of an individual's total pension wealth. For instance, we assume that the state pension is £5,000 a year. A mid-income individual has total personal pension savings of £138,650 that can be converted into an immediate annuity offering £10,000 a year [4]. Therefore the state benefit constitutes a half of annuity income level and *sb* equals 3.61% (5,000/138,650). To test the sensitivity of the factor, we assume a range of wealth levels and corresponding values of *sb* are shown in Table 4.10. The interpretation of *sb* can be in two ways. For example, a *sb* of 0.36% means either the state pension is at a very low level, or the individual has a great amount of pension wealth so that the state pension means little to him.

$$V = \max_{c_{65}, c_{66}, \dots} \sum_{i=0}^{54} \delta(i) \times {}_{i}p_{65} \times u(c_{65+i} + sb) \tag{4.8}$$

subject to

- $\sum_{i=0}^{d-1} c_{65+i} \times (1+r)^{-i} + \sum_{i=d}^{54} c_{65+i} \times (1+r)^{-i} \times {}_{i}p_{65} \times (1+L) = 1$

- $c_{65+i} \geq 0 \quad \text{for} \quad i = d, d+1, \dots, 54$

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[3]This thesis was written in February 2017, hence the information provided were the most up to date at that time.

[4]Annuity price is based on 3 percent interest rate and S2PML mortality table.

TABLE 4.10: Assumptions of sb, the state benefit as a percentage of total pension savings

State pension ($£$k)	Annuity income ($£$k)	State pension as a multiple of annuity income	Pension Wealth[a] ($£$k)	sb[b]
5	2.5	2	34.66	14.42%
5	5	1	69.33	7.21%
5	10	0.5	138.65	3.61%
5	50	0.1	693.25	0.72%
5	100	0.05	1386.50	0.36%

[a]Pension wealth is the amount of money that can be converted into an annuity that offers required annuity income. The rates are calculated based on a 3 percent interest rate and S2PML mortality table.
[b]sb is the annual state pension as a proportion of an individual's pension wealth.

TABLE 4.11: Optimal investment percentage in a deferred annuity

sb	0	0.36%	0.72%	3.61%	7.21%	14.42%
S1(d=15)	21.60%	21.62%	21.63%	21.76%	21.92%	22.25%
S2(L=10%)	61.83%	61.79%	61.75%	61.44%	61.05%	60.28%

Notes: sb represents the state benefit as a percentage of total pension savings. Results for model S1 is based on an assumption of a 15-year deferred period. Results for model S2 is based on an assumption of 10 percent profit loading, under which case a 6-year deferred annuity will be chosen. The rest assumptions are as follows: a real rate of return of $r = 3\%$, a risk aversion factor of $\gamma = 2$ and S2PML mortality table.

Since the state pension is provided annually as long as one is alive, this can be seen as a source of annual consumption and contributes to the overall utility. Before the commencement of the deferred annuity, withdrawals from pension savings, c_{65+i} for $i = 0, \ldots, d-1$, could be negative as one can make savings from the state pension if necessary. However, consumption from period d onwards, c_{65+i} for $i = d, \ldots, 119$, is supported by the annuity provider and must be positive. Based on this framework (Equation (4.8)), we explore two models, S1 and S2, depending on whether the deferred period is fixed or optimally derived. Similar to the construction of M1 and M2, S1 represents the case when the deferred period is an exogenous factor; while S2 represents the case when the deferred period is optimally derived from the mortality rates and the profit loading factor.

Table 4.11 demonstrates the optimal wealth allocation in a deferred annuity given a range of values of sb. It can be seen that investment allocations are stable with regard to different levels of sb. In other words, high-income individuals are recommended to invest a similar proportion of wealth in deferred annuities as low-income individuals.
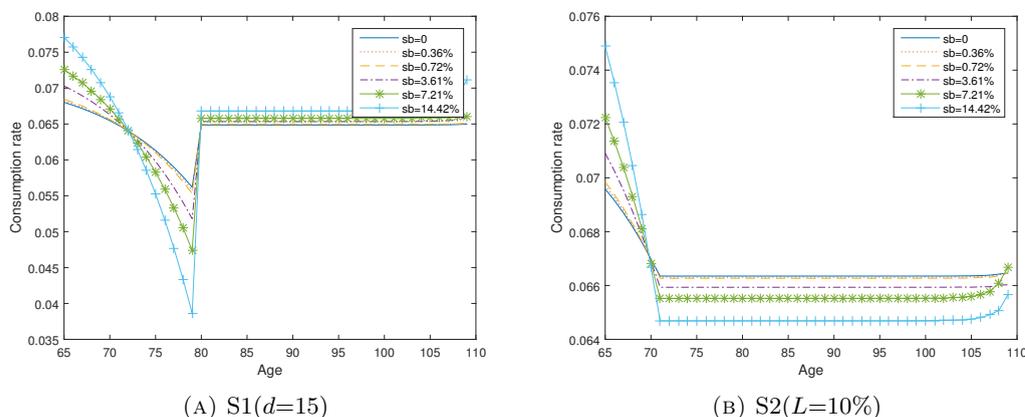
(A) S1(d=15)　　　　　　　　　　　(B) S2(L=10%)

FIGURE 4.7: Impact of state benefits on the optimal consumption path with a deferred annuity

Notes: sb represents the state benefit as a percentage of total pension savings. Results for model S1 is based on an assumption of a 15-year deferred period. Results for model S2 is based on an assumption of 10 percent profit loading, under which case a 6-year deferred annuity will be chosen. The rest assumptions are as follows: a real rate of return of $r = 3\%$, a risk aversion factor of $\gamma = 2$ and S2PML mortality table.

However, they would follow different consumption patterns, which are shown in Figure 4.7. The two panels show that high-income individuals ($sb = 0$) would prefer to have a smoother consumption path than low-income individuals. That means high-income individuals would start with a lower withdrawal rate from their total wealth than low-income individuals, and choose an annuity that offers an income close to the starting rate in order to ensure a smooth consumption path. One thing to note is that a smaller initial withdrawal rate (i.e. the optimal consumption rate at age 65) does not necessarily mean a smaller amount of consumption; a small withdrawal from high pension wealth could be more than a big withdrawal from low pension wealth.

4.4.3 A decumulation strategy with target replacement ratio

A replacement ratio is an individual's annual income during retirement divided by his annual income before retirement. Assume for example that an individual earns £50,000 per year. After retirement, his pension savings can support an annual consumption of £35,000. The replacement ratio for this individual is 70%. In general, an individual does not require the same amount of annual income after retirement. This is mainly because

of lower taxes, no need for retirement savings and reductions in work-related expenses. With a less than 100% replacement ratio, an individual might still maintain the same lifestyle after retirement. A replacement ratio study report from AON Consulting (2008) reports that an employee with annual income of $60,000 on retirement (age 65) in 2008 needs a 78% replacement ratio in order to maintain his standard of living.

In a traditional DB pension scheme, the target replacement ratio is part of the scheme design and is used to calculate projected pension liabilities. With the DC pension scheme, many studies use a target replacement ratio as the standard for assessing the adequacy of pension wealth accumulations. Therefore, individuals may have a target replacement ratio in mind when making investment and withdrawal decisions during retirement. Typical advice suggests that the replacement ratio should be between 70 and 85 percent of pre-retirement income. High income individuals normally have a relatively low replacement ratio while low income individuals are expected have a relatively high replacement ratio. This is because prior to retirement, low income individuals pay lower taxes and they save less amount of money for retirement. (Scholz and Seshadri, 2009)

We introduce a new parameter *target* representing the annual consumption target as a percentage of total wealth. Here, we assume that an individual who has a pre-retirement annual income of 1 unit has accumulated pension wealth of 13.865 units at retirement. This can be converted into an immediate annuity that pays 1 unit annually until death[5]. For example assuming that the individual requires a target replacement ratio of 0.7, the value of *target* is 0.0505 (0.7/13.865). In this framework, any deviation of consumption rates from the target will generate symmetric penalties on overall utility. The magnitude of the penalty is controlled by κ. An increasing value of κ means that the individual places a greater importance on achieving the target during retirement. The wealth constraint remains the same as in the basic model.

$$V = \max_{c_{65},c_{66},\dots} \sum_{i=0}^{54} \delta(i) \times {}_ip_{65} \times u(c_{65+i}) - \kappa \times \sum_{i=0}^{54} (c_{65+i} - target)^2 \qquad (4.9)$$

subject to

$$\sum_{i=0}^{d-1} c_{65+i} \times (1+r)^{-i} + \sum_{i=d}^{54} c_{65+i} \times (1+r)^{-i} \times {}_ip_{65} \times (1+L) = 1$$

---

[5]The price is based on 3 percent interest rate and S2PML mortality table.

TABLE 4.12: Optimal investment percentage in a 15-year deferred annuity

Replacement Ratio	0.5	0.6	0.7	0.8	1
target	0.0361	0.0433	0.0505	0.0577	0.0721
κ					
0	21.60%	21.60%	21.60%	21.60%	21.60%
10^2	20.88%	21.05%	21.22%	21.40%	21.78%
10^3	18.72%	19.38%	20.07%	20.79%	22.32%
10^4	16.43%	17.66%	18.92%	20.20%	22.81%
10^5	15.83%	17.24%	18.65%	20.07%	22.92%
10^6	15.75%	17.19%	18.62%	20.06%	22.93%

Notes: target represents the annual consumption target as a percentage of total wealth. κ denotes the magnitude of the penalty on deviations from target consumption. Results are based on the following assumptions: a 15-year deferred annuity, $d = 15$, a profit loading factor of $L = 10\%$, a real rate of return of $r = 3\%$ and a risk aversion factor of $\gamma = 2$. Mortality rates are based on table S2PML.

(A) $\kappa = 10^5$

(B) $target = 0.0505$
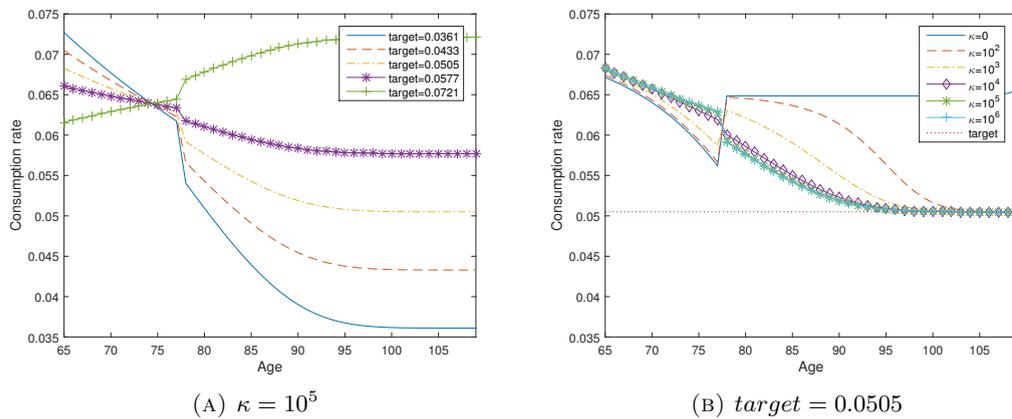
FIGURE 4.8: Impact of target replacement ratio on the optimal consumption path with a deferred annuity

Notes: target represents the annual consumption target as a percentage of total wealth. κ denotes the magnitude of the penalty on deviations from target consumption. Results are based on the following assumptions: a 15-year deferred annuity, $d = 15$, a profit loading factor of $L = 10\%$, a real rate of return of $r = 3\%$ and a risk aversion factor of $\gamma = 2$. Mortality rates are based on table S2PML.

To explore the impact of the target replacement ratio on the decumulation decisions, we test a range of replacement ratios increasing in steps of 10% from 50% to 100%. Moreover, values of κ increase from 0 (no target for retirement consumption) to 10^6 (consumption target is of high importance).

Table 4.12 shows the optimal wealth allocation in a 15-year deferred annuity following different target replacement ratios and different levels of importance. Figure 4.8 demonstrates the optimal decumulation process during retirement with a 15-year deferred annuity. Figure 4.8 (A) shows the impact of the target replacement ratio and Figure 4.8 (B) shows the impact of the penalty factor κ.

Based on the paths when following different target levels in Figure 4.8 (A), the target consumption rate determines the preferable income level from deferred annuity products. An individual who has a target replacement ratio in mind would opt for an annuity that provides the target level of income for life. This is also confirmed in Table 4.12. When retirees see the target consumption as very important (corresponding to a high level of κ), as the target consumption level is increasing, the allocation into a 15-year deferred annuity increases significantly with the target replacement ratio. In addition, Figure 4.8 (A) reflects different patterns before the start of the 15-year deferred annuity. If a low target is expected, the consumption rate is high in the beginning and decreasing as time goes by. If a high target is expected, individuals have to bear a low rate of consumption in the beginning and the situation gets better until they achieve their target.

The impact of κ is shown in Figure 4.8 (B). It again justifies our conclusion that the target replacement ratio is very influential in deciding the decumulation process since every process ultimately ends up at the target level of consumption (except for the case when κ is zero). Different values of κ determine the speed with which the consumption rate converges to the target level. An individual who sees the target as very important would achieve the target sooner than others.

Based on the same framework as shown in Equation (4.9), we also explore the optimal results when the deferred period, d, is optimally chosen rather than fixed. We find that the impact of the target replacement ratio remains the same. First, the target replacement ratio is an influential factor on the optimal consumption path. With a higher target consumption rate, individuals prefer to spend less in the beginning of retirement and then move towards the target consumption level at older ages. Second, the target replacement ratio basically drives the wealth allocation in the deferred annuity. A higher target means a higher allocation in the annuity product. Third, as people view the consumption target as more important, the consumption path will converge to the target level at a faster rate. Detailed results are available from the author.

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We also have considered other ways to model the impact of target replacement ratio. In one model, we assume asymmetric rather than symmetric penalties: individuals would have reduced utility if the consumption rate does not meet the target level; however, they have the full amount of utility increase if the consumption rate is above the target level. Modelling results show that if the target replacement ratio is set too low, it does not influence people's decisions on asset allocation and the decumulation process. On the other hand, if the target replacement ratio is set to be relatively high, the penalty on utility applies and individuals would follow the same strategy as introduced above.

In another version of the model, we assume that retirees pursue a retirement consumption target that is increasing in line with the inflation rate each year so that purchasing power is maintained. The utility maximising consumption strategy shows that they would consume at a low rate in the first few years and gradually adjust their consumption rates in line with an increasing target. As consumption after a certain number of years is supported by the deferred annuity, the preferred type of annuity for these people is the inflation protected annuity product. Again, detailed results are available from the author.

4.4.4 A decumulation strategy with consideration of the bequest motive

An annuity stops paying when the annuitant dies, therefore people with a motivation to bequeath part of their wealth to the next generation may obtain less welfare from an annuity investment. A bequest motive may give individuals an incentive to spend less on an annuity product. In the literature, there has been discussion about the impact of the bequest motive on the annuity purchasing decisions. While a bequest motive can explain why people do not annuitise the entire pension wealth, they cannot explain why people do not annuitise any pension wealth at all. Friedman and Warshawsky (1990) is an example of the literature that calculates the impact of the bequest motive on utility gains from the annuities. They show that the intention to buy an annuity can be eliminated with presence of a strong enough bequest motive. Annuities have an impact on the variance of the bequest amount; hence if people are risk averse regarding the bequest amount, annuities have a positive effect on the bequest motive (Davidoff et al.,

2005). In this section, we explore whether the bequest motive would influence optimal decumulation pattern and annuity allocation in our strategies.

We model the bequest motive as an additional contribution towards the expected utility; if the individual dies before the commencement of the annuity, all his remaining wealth will be left as a bequest at the end of that year. Let $u(c_{65+i})$ denote the utility of consumption corresponding to age $65+i$. $b(W_{i+1})$ is the utility function of the remaining wealth to bequeath if an individual dies at age $65 + i$.

q_{65+i} represents the mortality rates at age $65 + i$. Following Gerrard et al. (2006), we introduce a non-negative factor $n(\cdot)$ as a weight assigned to the importance of the ability to leave a bequest. The choice of $n(\cdot)$ can be associated with the size of the fund and also the timing of making a bequest. In addition, we assume that the discount rate of the utility of the bequest, $\delta(i)$, coincides with the discount rate of the utility of consumption. More precisely we consider the following constrained maximization.

$$V = \max_{c_{65},c_{66},\dots} \sum_{i=0}^{54} \left\{ \delta(i) \times {}_i p_{65} \times u(c_{65+i}) + (\delta(i+1) \times {}_i p_{65} q_{65+i} \times n(i+1) \times b(W_{i+1})) \right\}$$

where:

$$\alpha = \sum_{i=d}^{54} c_{65+i} \times (1+r)^{-i} \times {}_i p_{65} \times (1+L)$$

$$W_1 = (1 - \alpha - C_{65})(1+r)$$

$$W_{i+1} = (W_i - C_i)(1+r), i = 1, 2, \dots, d-2$$

$$W_i = 0, i = d, d+1, \dots$$

subject to:

- $\sum_{i=0}^{d-1} c_{65+i} \times (1+r)^{-i} + \sum_{i=d}^{54} c_{65+i} \times (1+r)^{-i} \times {}_i p_{65} \times (1+L) = 1$

- $c_{65+i} = c_a$ for $i = d, d+1, \dots, 54$

We will explore three different ways to measure the impact of the bequest motive.

B1: $n(i) = n; b(W_i) = W_i^{1-\gamma}/(1-\gamma)$

B2: $n(1) = n; n(i) = n(i-1)/(1+s); b(W_i) = W_i$

B3: $n(1) = n; n(i) = n(i-1) \times (1+s); b(W_i) = W_i$

where $s(s > 0)$ denotes the speed that the weighting factor decreases/increases with age.

In B1, the weighting of the utility of the bequest is assumed to be a constant relative to the utility of consumption over different ages at retirement. The utility function of bequest is assumed to be the same as that of consumption and we can find the optimal strategy when people are risk averse about the bequest. B2 and B3 follow the literature by assuming that the weights assigned to the possibility of leaving a bequest change during retirement period. In a life-cycle framework, Fischer (1973) and Yaari (1965) assume that the parameter that reflects the value of the bequest is a hump shaped function because individuals see the bequest as more important in the mid-years of life when family members have a greater dependence on them. Focusing on the retirement period, two different views exist in terms of the weighting of bequests. On one hand, following a hump shaped weighting function for life, the weighting function after age 65 could be decreasing, as described in B2. This type of motivation for leaving a bequest is called *Altruistic*, indicating that the pensioner simply wants to leave a bequest to his family without expecting anything in return. On the other hand, the weighting function after age 65 is increasing, as described in B3. This type of motivation is called *Strategic*, indicating that pensioners want to promise their family a bequest so they have incentives to take care of them during old age (Vidal-Melia and Lejarraga-Garcia, 2004).

4.4.4.1 B1: Constant weight of the bequest motive

Table 4.13 shows the optimal wealth allocation in a 15-year deferred annuity with different assumptions of values of n. It shows that as people place a higher level of importance on the bequest, they would invest a smaller proportion of wealth into annuities. This is because the mean value of the bequest amount is higher when the annuity allocation is smaller. The results are consistent with the literature in the sense that the bequest motive has an impact on annuity investments. However, the impact on deferred annuity investments is not as significant as that on immediate annuities because the price of a long-term deferred annuity is much smaller than that of an equivalent immediate annuity.

TABLE 4.13: Optimal investment percentage in a 15-year deferred annuity (B1)

n	0	0.1	0.3	0.5	0.7	1
α	22.17%	22.06%	21.73%	21.40%	21.06%	20.56%

Notes: Results are based on the following assumptions: a 15-year deferred annuity, $d = 15$, a profit loading factor of $L = 10\%$ and a real rate of return of $r = 3\%$. Mortality rates are based on table S2PML. Here, γ is assumed to be 0.5, which is different from what it is elsewhere. This is to avoid the problem of negative infinitive utility when wealth is decumulated towards zero.

TABLE 4.14: Optimal investment percentage in a 15-year deferred annuity under B2 and B3

s	1%	2%	3%	4%	5%
B2 - Altruistic motive	21.39%	21.40%	21.41%	21.42%	21.43%
B3 - Strategic motive	21.36%	21.35%	21.34%	21.32%	21.30%

Notes: s controls how fast the weighting of utility of bequest decreases/increases with age. Results are based on the following assumptions: the initial weighting factor of bequest of $n = 50$, a 15-year deferred annuity, $d = 15$, a profit loading factor of $L = 10\%$, a real rate of return of $r = 3\%$ and a risk aversion factor of $\gamma = 2$. Mortality rates are based on table S2PML.

Moreover, the bequest motive impacts the optimal consumption path, which is reflected in Figure 4.9. For those who see the bequest as more important (a higher n), they consume less in early years after retirement. Therefore the mean value of the possible bequest amount will be higher and this contributes positively to the expected utility that is to be maximized.

With the assumption of a fixed deferred period in the model, the consumption gap in the decumulation process still exists, after incorporating the bequest motive into the model. However, it can be seen clearly that as the factor, n, increases, the gap becomes smaller, which indicates that an individual who sees leaving a bequest as more important prefers to have a smoother consumption path during retirement. This is because people with an intention to leave a bequest would spend less in the beginning of the retirement period; they would also spend slightly less in deferred annuity products that require an advanced payment of the premium, all of which lead to a narrower consumption rate range during the retirement period.
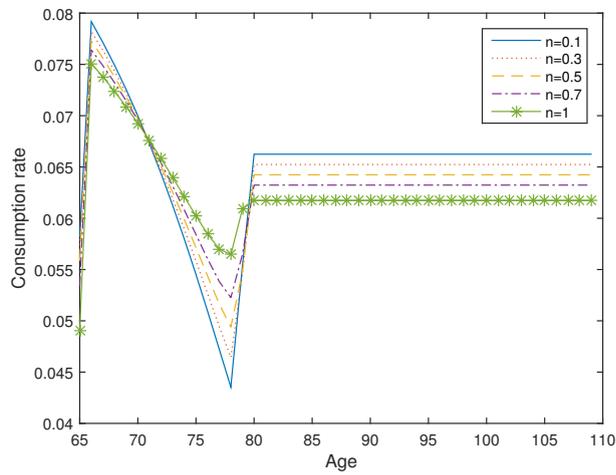
FIGURE 4.9: Impact of bequest motive on optimal consumption path with a 15-year deferred annuity (B1)

Notes: Results are based on the following assumptions: a 15-year deferred annuity, $d = 15$, a profit loading factor of $L = 10\%$ and a real rate of return of $r = 3\%$. Mortality rates are based on table S2PML. Here, γ is assumed to be 0.5, which is different from what it is elsewhere. This is to avoid the problem of negative infinitive utility when wealth is decumulated towards zero.
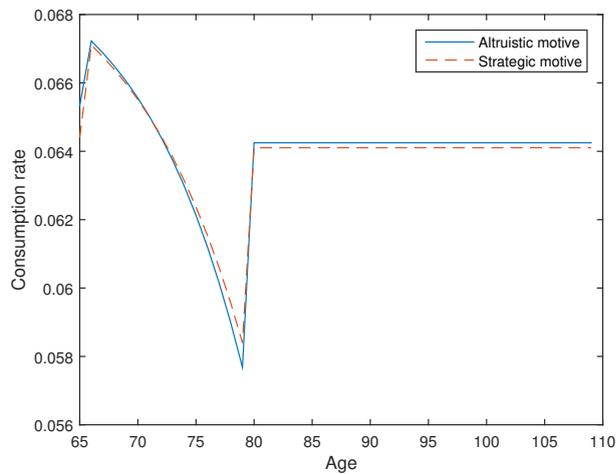
FIGURE 4.10: A comparison of optimal consumption paths under B2 and B3

Notes: Results are based on the following assumptions: the initial weighting factor of bequest of $n = 50$, the increasing/decreasing speed of the bequest weighting factor of $s = 2\%$, a 15-year deferred annuity, $d = 15$, a profit loading factor of $L = 10\%$, a real rate of return of $r = 3\%$ and a risk aversion factor of $\gamma = 2$. Mortality rates are based on table S2PML.

4.4.4.2   A comparison of B2 and B3

Table 4.14 demonstrates the optimal investment percentage in a 15-year deferred annuity, following two different models. Parameter s determines how fast the weighting of utility of bequest decreases/increases with age. We have the following findings from the table. Firstly, an individual who follows an altruistic motivation to leave a bequest tends to allocate slightly more wealth in a deferred annuity than an individual who follows a strategic motivation to leave a bequest. This occurs mainly because someone with an altruistic motivation sees the bequest as less important as time goes by; they would be more willing to spend more money upfront to ensure adequate support from deferred annuities in their advanced ages. Secondly, as s becomes greater, the weighting of the bequest increases or decreases at a faster rate, which leads to a greater difference in the optimal annuity allocations for altruistic retirees and strategic retiree. For altruistic retirees, the importance of the bequest decreases at a faster rate, and they therefore would like to pay a higher proportion in advance for deferred annuities. On the other hand, for strategic retirees, the importance of the bequest increases at a faster rate, and they would spend less on deferred annuities to allow the possibility of leaving a greater amount of bequest at a later stage.[6]

Figure 4.10 compares the decumulation processes under different motivations. Although the starting point of the weighting parameter is chosen to be very high ($n = 50$ [7], which means the bequest motive is a major factor in decision making), we cannot identify large differences between the optimal consumption paths. It suggests that the type of motivation to leave a bequest, either altruistic or strategic, is not a major factor in deciding consumption patterns.

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[6]The conclusion is based on an important assumption that the initial weight of bequest is the same for retirees with an altruistic motivation and those with a strategic motivation.

[7]One may notice that the choice of the value of n in B2 and B3 is very different from that in B1 (where n is smaller than 1). This is because the utility of bequest in B1 follows the CRRA utility function, which is the same as the utility function of consumption; however, the utility of bequest in B2 and B3 is the absolute amount of the bequest. In all cases, n is simply used to adjust the importance of bequest relative to the consumption. In B2 and B3, the choice of the utility function for bequest does not matters too much. As our objective is to compare the altruistic motive and strategic motive of leaving a bequest, we simply need to make sure the the utility functions of bequest are consistent in both models.

4.5   Conclusions

The question of how to determine investment and consumption choices in the distribution phase of a defined contribution pension plan has become more important given the greater flexibility in accessing personal pension savings. Individuals are in need of professional advice to ensure that they maximise current spending and will not live in poverty before they die. The deferred annuity has been proposed as a good retirement solution due to several competitive advantages. First, it involves a small initial outlay because the price incorporates the survival probability and the time value of money. Second, a deferred annuity is designed to provide protection to those who live to the advanced high ages, where longevity risk concentrates. Third, as the deferred annuity investment involves only a proportion of retirement savings, individuals preserve liquidity to some extent.

In this chapter, we proposed two decumulation strategies with the deferred annuities, which divides the retirement period into two phases. The first phase is the fixed deferred period during which individuals need to make active investments and consumption decisions; the second phase is supported by lifetime incomes from annuity providers. In Model M1, the choice of deferred period is exogenous. Based on the findings in Chapter 2 and Chapter 3, behavioral models such as cumulative prospect theory and the hyperbolic discount model suggest that annuities with longer deferred periods will be more desirable. Therefore, in the set-up of M1, we concentrate on the purchase of a 15-year and 20-year deferred annuity that offer protections only when individuals survive until age 80 or 85. Our results suggest that it is optimal to allocate 21.6% in a 15-year deferred annuity or 6.9% in a 20-year deferred annuity, which is an easy and realistic decision in practice. Several sensitivity studies show that our proposed strategy is stable; primarily, this is due to the cheapness of deferred annuity products.

In model M2, we aim at a more efficient strategy with the optimally chosen deferred annuity products. Equation (4.4) has shown that the choice of the optimal deferred period depends only on the cost of annuity and the mortality rates. This conclusion is also confirmed in an annuity innovation study by Scott et al. (2011), who conclude that a better annuity should contain a gap between the purchase date and the payout date when there are positive costs. With a 10% profit loading, M2 suggests investing

61.83% in a 6-year deferred annuity to maximise individual welfare. We have also shown that by following M2, one can achieve highest consumption rates during retirement in comparison with following both M1 and DCDB.

This work aims to provide optimal consumption and investment decisions for retirees in the distribution phase of the pension plan. The recommended investment allocation in deferred annuities and the consumption paths would help individuals reduce longevity risk and preserve liquidity to some extent. Moreover, the proposed strategies help achieve maximum utility within a certain set of constraints. Although we see this work as creating important foundations, there are some areas for future extensions. For instance, in the first phase, the only investment option allowed in our framework offers a fixed rate of return. In future this assumption could be relaxed to allow for stochastic returns from more risky financial asset classes.

4.A   Proof of Equation 4.1 and Equation 4.2

In this section, we show the derivation of the optimal real consumption rate and optimal wealth allocation in the deferred annuity. Recalling the proposed decumulation strategy in a world with zero inflation can be described as the following model:

$$V = \max_{c_{65}, c_{66}, \dots} \sum_{i=0}^{54} \delta(i) \times {}_i p_{65} \times u(c_{65+i})$$

subject to

$$\sum_{i=0}^{d-1} c_{65+i} \times v^i + \sum_{i=d}^{54} c_{65+i} \times v^i \times {}_i p_{65} \times (1+L) = 1$$

where:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

$$v = (1+r)^{-1}$$

$$\alpha = \sum_{i=d}^{54} c_{65+i} \times v^i \times {}_i p_{65} \times (1+L)$$

Lagrangian method is used to derive the optimal path of real consumption rate c_{65+i}

$$L = \sum_{i=0}^{54} \delta(i){}_{i}p_{65}\frac{c_{65+i}^{1-\gamma}}{1-\gamma} + \lambda\left\{1 - \sum_{i=0}^{d-1} c_{65+i}v^i - \sum_{i=d}^{54} c_{65+i}v^i{}_{i}p_{65}(1+L)\right\}$$

For i=0,1, …,d-1

$$\frac{dL}{dc_{65+i}} = \delta(i){}_{i}p_{65}c_{65+i}^{-\sigma} - \lambda v^i = 0$$

$$c_{65+i} = \left(\frac{\lambda v^i}{\delta(i){}_{i}p_{65}}\right)^{-\frac{1}{\sigma}}$$

$$c_0 = \lambda^{-\frac{1}{\sigma}}$$

$$c_{65+i} = c_0 v^{-\frac{i}{\sigma}}{}_{i}p_{65}^{\frac{1}{\sigma}}\sigma(i)^{\frac{1}{\sigma}}$$

For i=d, …,54

$$\frac{dL}{dc_{65+i}} = \delta(i){}_{i}p_{65}c_{65+i}^{-\sigma} - \lambda v^i{}_{i}p_{65}(1+L) = 0$$

$$c_{65+i} = \left(\frac{\lambda v^i(1+L)}{\delta(i)}\right)^{-\frac{1}{\sigma}}$$

$$c_{65+i} = c_0 \times v^{-\frac{i}{\sigma}}(1+L)^{-\frac{1}{\sigma}}\sigma(i)^{\frac{1}{\sigma}}$$

Taking c_{65+i} back to the wealth constraint

$$\sum_{i=0}^{d-1}\left(c_0 v^{i-\frac{i}{\gamma}}{}_{i}p_{65}^{\frac{1}{\gamma}}\right) + \sum_{i=d}^{54} c_0 v^{i-\frac{i}{\gamma}}(1+L)^{1-\frac{1}{\gamma}}\sigma(i)^{\frac{1}{\gamma}}{}_{i}p_{65} = 1$$

$$c_0 = \frac{1}{\sum_{i=0}^{d-1} v^{i-\frac{i}{\gamma}}{}_{i}p_{65}^{\frac{1}{\gamma}}\sigma(i)^{\frac{1}{\gamma}} + (1+L)^{1-\frac{1}{\gamma}}\sum_{i=d}^{54} v^{i-\frac{i}{\gamma}}\sigma(i)^{\frac{1}{\gamma}}{}_{i}p_{65}}$$

Assuming a rational individual uses an exponential discount function to discount future utilities,

$$\delta(i) = v^i = (1+r)^{-i}$$

Hence, the optimal consumption rate in real terms during retirement is:

$$c_{65+i} = \frac{{}_{i}p_{65}^{1/\gamma}}{\sum_{i=0}^{d-1}(1+r)^{-i}{}_{i}p_{65}^{1/\gamma} + (1+L)^{(1-1/\gamma)}\sum_{i=d}^{54}(1+r)^{-i}{}_{i}p_{65}} \quad \text{for} \quad i = 0,\ldots,(d-1)$$

$$c_{65+i} = \frac{(1+L)^{-1/\gamma}}{\sum_{i=0}^{d-1}(1+r)^{-i}{}_{i}p_{65}^{1/\gamma} + (1+L)^{(1-1/\gamma)}\sum_{i=d}^{54}(1+r)^{-i}{}_{i}p_{65}} \quad \text{for} \quad i = d,\ldots,54$$

The optimal wealth allocation in the deferred annuity is:

$$\alpha = \frac{(1+L)^{(1-1/\gamma)} \sum_{i=d}^{54}(1+r)^{-i}\,_ip_{65}}{\sum_{i=0}^{d-1}(1+r)^{-i}\,_ip_{65}^{1/\gamma} + (1+L)^{(1-1/\gamma)} \sum_{i=d}^{54}(1+r)^{-i}\,_ip_{65}}$$

4.B   Proof of Equation 4.5 and Equation 4.6

In this section, we show the derivation of the optimal nominal consumption rate and optimal wealth allocation in the deferred annuity. Recalling the proposed decumulation strategy in a world with positive inflation can be described as the following model:

$$V = \max_{c'_{65},c'_{66},\cdots} \sum_{i=0}^{54} \delta(i) \times \,_ip_{65} \times u\Big(\frac{c'_{65+i}}{(1+f)^i}\Big)$$

subject to

$$\sum_{i=0}^{d-1} c'_{65+i} \times v'^i + \sum_{i=d}^{54} c'_{65+i} \times v'^i \times \,_ip_{65} \times (1+L) = 1$$

where:

$$u(c') = \frac{c'^{1-\gamma}}{1-\gamma}$$

$$v' = (1+r_{nominal})^{-1} = (1+r)^{-1}(1+f)^{-1}$$

$$\alpha = \sum_{i=d}^{54} c'_{65+i} \times v'^i \times \,_ip_{65} \times (1+L)$$

Lagrangian method is used to derive the optimal path of nominal consumption rate c_{65+i}

$$L = \sum_{i=0}^{54} \delta(i)\,_ip_{65}\frac{1}{1-\gamma}\Big(\frac{c'_{65+i}}{(1+f)^i}\Big)^{1-\gamma} + \lambda\Big\{1 - \sum_{i=0}^{d-1} c'_{65+i}v'^i - \sum_{i=d}^{54} c'_{65+i}v'^i\,_ip_{65}(1+L)\Big\}$$

For i=0,1, …,d-1

$$\frac{dL}{dc'_{65+i}} = \delta(i)_ip_{65}(1+f)^{-i(1-\gamma)}c'_{65+i}{}^{-\gamma} - \lambda v'^i = 0$$

$$c'_{65+i} = \Big(\frac{\lambda v'^i}{\delta(i)_ip_{65}(1+f)^{-i(1-\gamma)}}\Big)^{-\frac{1}{\gamma}}$$

$$c'_0 = \lambda^{-\frac{1}{\gamma}}$$

$$c'_{65+i} = c'_0 v'^{-\frac{i}{\gamma}} {}_ip_{65}{}^{\frac{1}{\gamma}}\sigma(i)^{\frac{1}{\gamma}}(1+f)^{\frac{i(\gamma-1)}{\gamma}}$$

For i=d, …,54

$$\frac{dL}{dc'_{65+i}} = \delta(i)_ip_{65}(1+f)^{-i(1-\gamma)}c_i^{-\gamma} - \lambda v'^i {}_ip_{65}(1+L) = 0$$

$$c'_{65+i} = \Big(\frac{\lambda v'^i(1+L)}{\delta(i)}(1+f)^{-i(1-\gamma)}\Big)^{-\frac{1}{\gamma}}$$

$$c'_{65+i} = c'_0 v'^{-\frac{i}{\gamma}}(1+L)^{-\frac{1}{\gamma}}\sigma(i)^{\frac{1}{\gamma}}(1+f)^{\frac{i(\gamma-1)}{\gamma}}$$

Taking c'_{65+i} back to the wealth constraint

$$c'_0 = \frac{1}{\sum_{i=0}^{d-1} v'^{i-\frac{i}{\gamma}}{}_ip_{65}{}^{\frac{1}{\gamma}}\sigma(i)^{\frac{1}{\gamma}}(1+f)^{i-\frac{i}{\gamma}} + (1+L)^{1-\frac{1}{\gamma}}\sum_{i=d}^{54} v^{i-\frac{i}{\gamma}}\sigma(i)^{\frac{1}{\gamma}}{}_ip_{65}(1+f)^{i-\frac{i}{\gamma}}}$$

Assuming a rational individual uses an exponential discount function to discount future utilities,

$$\delta(i) = (1+r)^{-i}$$

Hence, the optimal consumption rate in nominal terms during retirement is:

$$c'_{65+i} = \frac{{}_ip_{65}^{1/\gamma}(1+f)^i}{\sum_{i=0}^{d-1}(1+r)^{-i}{}_ip_{65}^{1/\gamma} + (1+L)^{(1-1/\gamma)}\sum_{i=d}^{54}(1+r)^{-i}{}_ip_{65}} \quad \text{for} \quad i = 0,\ldots,(d-1)$$

$$c'_{65+i} = \frac{(1+L)^{-1/\gamma}(1+f)^i}{\sum_{i=0}^{d-1}(1+r)^{-i}{}_ip_{65}^{1/\gamma} + (1+L)^{(1-1/\gamma)}\sum_{i=d}^{54}(1+r)^{-i}{}_ip_{65}} \quad \text{for} \quad i = d,\ldots,54$$

The optimal wealth allocation in the deferred annuity is:

$$\alpha = \frac{(1+L)^{(1-1/\gamma)} \sum_{i=d}^{54}(1+r)^{-i}{}_i p_{65}}{\sum_{i=0}^{d-1}(1+r)^{-i}{}_i p_{65}^{1/\gamma} + (1+L)^{(1-1/\gamma)} \sum_{i=d}^{54}(1+r)^{-i}{}_i p_{65}}$$

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5

The impact of the hyperbolic discount model and subjective mortality rates on optimal decumulation strategies

The impact of the hyperbolic discount model and subjective mortality rates on optimal decumulation strategies

Abstract

With an ongoing shift from Defined Benefit (DB) pension plan to Defined Contribution (DC) pension plan in the private sector, investment risk and longevity risk have been transferred from pension providers to pensioners. Professional advice on how to spend down pension savings is in great demand. Recent studies have started to consider the use of deferred annuities in retirement decumulation strategies (see Sexauer et al. (2012) and OECD (2016) for example). As we know that behavioral factors play an important role in the process of annuitisation decision making, we are interested to examine whether pensioners with behavioral biases would follow different decumulation strategies compared with rational pensioners. In this chapter, we examine the impact of two factors: time-inconsistent preferences (which is captured by a hyperbolic discount model) and subjective survival rates. Using numerical optimisation techniques and a set of benchmark assumptions, we find that a hyperbolic discounter would like to allocate a similar proportion of pension savings in deferred annuities as an exponential discounter would. However, they would be interested in buying an inflation-linked annuity while exponential discounters would prefer conventional annuities. Moreover, pensioners who are optimistic (pessimistic) about their life expectancies would find annuities attractive (not attractive) in general and they would like to allocate a higher (lower) proportion of pension savings in deferred annuities.

Keywords: Deferred annuities, Decumulation strategy, Hyperbolic discount model, Subjective mortality rates.

5.1   Introduction

With an ongoing shift from Defined Benefit (DB) pension plans to Defined Contribution (DC) pension plans in the private sector, more retirees are faced with longevity risk and investment risk, and therefore they are in need of advice on how to best generate retirement incomes from accumulation pension savings. A natural solution, which can be seen as a replacement for the traditional DB pension plan, is converting the entire pension savings into the form of an immediate annuity. Retirees as annuitants would receive either fixed or inflation-linked guaranteed incomes until the end of life. Decades of economic analysis have proved that full annuitisation is an optimal strategy for retirees. The result holds even when some of the axioms of expected utility maximisation break down; Davidoff et al. (2005) point out that, when markets are complete, full annuitisation is optimal without assuming exponential discounting, the expected utility axioms, intertemporal separability, or actuarially fair annuities. However, the fact that retirees are reluctant to convert any of their savings into immediate annuities remains a puzzle in the literature and in the insurance market. Davidoff et al. (2005) therefore conclude that limited annuity purchases are plausibly due to psychological or behavioral biases.

In the previous chapters, we have introduced two behavioral models, Cumulative Prospect Theory (CPT) and the hyperbolic discount model, and illustrated their impacts on the demand for annuities. In Chapter 2, we find that being loss averse rather than risk averse is a major reason driving the undervaluation of annuities. The way that individuals interpret probabilities of an event, typically the overweighting of low-probability events and the underweighting of high-probability events, would make a deferred annuity more desirable than an immediate annuity. In Chapter 3, we find that having decreasing impatience would make people overlook the benefit of immediate annuities and prefer a deferred annuity product with a longer deferred period. Thus, it has been shown that behavioral factors in the real world would lead individuals to make a completely different decision from what is suggested in a perfectly theoretical world. It is meaningful, therefore, to examine what decumulation strategies during the retirement period people would like to pursue if they are exposed to behavioral biases.

We recall that in Chapter 4, we have proposed a utility maximising decumulation strategy comprising a deferred annuity purchased at retirement and optimal consumption

and savings before the commencement of the annuity. Following the idea, two models with different choices of deferred annuities have been set up. M1 assumes that retirees already have preferences for the age from which they would like to receive lifelong incomes; this choice of the deferred period could be guided by government policy. On the other hand, M2 assumes that retirees are able to work out an optimal deferred period using mortality rates and profit loading; and they would use this optimal deferred annuity to improve retirement planning. The merit of this design, compared to the previous literature, is that it includes a deferred annuity as a retirement asset as part of the retirement strategy, and achieves utility maximisation at the same time. The use of a deferred annuity really simplifies the retirement financial planning process by converting it from an unknown period (because of uncertain lifetime) into a fixed deferred period (after which individuals would receive guaranteed incomes for life).

In this chapter, we are interested in two behavioral factors: the hyperbolic discount mode (which has been introduced in Chapter 3) and subjective mortality rates. In brief, the hyperbolic discount model captures the fact that people have time-inconsistent preferences when they make intertemporal decisions. A subjective mortality rate model reflects people's general perception of their own life expectancies, which can be different from life expectancies of the general public. Both factors are influential in annuitisation decision making. To understand their impacts on the optimal decumulation strategies during retirement, we incorporate them separately into the construction of strategies M1 and M2. To be more specific, we seek to explore the following questions: (a) For individuals with time-inconsistent preferences / subjective mortality rates, how much they would like to allocate in a fixed / flexible deferred annuity product and what consumption pattern they would like to follow during retirement? (b) Do retirees with time-inconsistent preferences / subjective mortality rates pursue different annuity allocations and consumption patterns compared with rational retirees? (c) How would the optimal strategies change in response to retirees with different levels of time-inconsistency, or optimistic or pessimistic retirees?

Using numerical optimisation techniques and our chosen benchmark assumptions, we have the following major findings. A hyperbolic discounter (with time-inconsistent preference) at age 65 would like to allocate 22.97% of pension wealth in a 15-year deferred annuity (following strategy M1) or 60.4% of pension wealth in a 6-year deferred annuity

(following strategy M2); the allocations are similar to what we find for a rational exponential discounter. However, a hyperbolic discounter would pursue a different consumption pattern. A hyperbolic discounter prefers buying an index-linked deferred annuity and following an inverted S-shaped consumption path prior to the start of the deferred annuity; while a rational exponential discounter prefers buying a conventional deferred annuity and following a convex consumption pattern before the start of the deferred annuity.

Subjective mortality rates are key determinant factors for the desirability of annuities. Basically, people who are optimistic (pessimistic) about their life expectancies would find annuities attractive (not attractive) in general and they would like to allocate more (less) of their pension wealth in annuity products (Following M1, our analysis suggests allocating 25.64% for optimistic retirees and 19.71% for pessimistic retirees in a 15-year deferred annuity). When they are allowed to flexibly decide which deferred annuity to invest in (by following strategy M2), optimistic individuals would opt for an annuity with a longer deferred period than pessimistic individuals.

The structure of this chapter is as follows. Section 2 introduces the general hyperbolic discount model in detail and discusses the modeling results after replacing the rational exponential discount model with the irrational hyperbolic one. Section 3 offers a literature review on subjective mortality rates and discusses the impact of that on annuity desirability and decumulation strategies. Section 4 provides some concluding comments on our findings.

5.2   The impact of the hyperbolic discount model

5.2.1   An introduction of the hyperbolic discount model

In Chapter 3, we have introduced what the hyperbolic discount model is. Here we provide a recap of the main characteristics of a general hyperbolic discount model and also introduce an extension: quasi-hyperbolic discount model.

In dealing with economic decisions that involve outcomes occurring at different points in time, researchers often employ a discounted utility function to model the overall satisfaction gained from the decision; annuitisation and consumption decision is one

example of this. In order to decide the optimal consumption rates during retirement period, the discounted utility model combines a utility function that reflects perceived values of outcomes and a discount function that captures the effect of the passage of time. For future consumptions $(c_0, c_1, ...c_T)$ in periods $t = 0, ...T$, we define

$$V(c_0, c_1, ...c_T) = \sum_{t=0}^{T} \delta(t) u(c_t)$$

where $u(\cdot)$ is a real valued utility function that represents preferences over outcomes and $\delta(t)(\delta(t) \in (0, 1])$ is the discount function reflecting the weights of different points in time.

In a normative framework, individuals should have stationary time preferences, which means that the preference between two delayed outcomes does not change by going back or forth in time, given that the time interval between the two outcomes stays constant. The exponential discount function is used to model such cases. However, many empirical studies on time preferences suggest a different behavior (e.g., Strotz (1965), Thaler (1981), Benzion et al. (1989)). Thaler (1981) shows that people exhibit decreasing impatience over time with a simple experiment in which subjects are faced with the following two questions: A. Would you prefer one apple today or two apples tomorrow? B. Would you prefer one apple in one year or two apples in one year plus one day?

Assuming stationary preferences, people who choose one apple today in Question A would make a consistent choice of one apple in a year in Question B. However, the experimental results show that a significant fraction of subjects choose one apple today and gladly wait one extra day in a year in order to receive two apples instead. It reflects that people tend to act impulsively in the short-term but become more patient in the long-term.

In response to such anomalies in perceived discount rates, Loewenstein and Prelec (1992) propose a general hyperbolic discount model

$$\delta(t) = (1 + \mu t)^{-\sigma/\mu} \text{ with } \mu > 0, \sigma > 0 \tag{5.1}$$

The parameter μ reflects how much the function departs from the constant discounting. When μ approaches zero, the function leads to the limiting case of exponential discount function, $\delta(t) = e^{-\sigma t}$. Since μ remains positive, the implied discount rate between consecutive time points decreases as time passes (t increases), reflecting the finding that people generally become more patient to wait for rewards in the future.

An alternative simpler version of hyperbolic discounting that is also widely used is called "quasi-hyperbolic" discounting. In this formulation, the utility of consumption during period $t(t \neq 0)$ is discounted by the factor $\beta \tau^t$. The constant factor β reflects the time preference or myopia over the current period; while the constant factor τ represents equal amount of discounting during each period in future. The values for discrete times are

$$\left\{1, \beta\tau, \beta\tau^2, \beta\tau^3, ...\right\} \text{ with } \beta < 1, \tau < 1 \tag{5.2}$$

The quasi-hyperbolic discount model implies that people only show impulsivity in the first period when making decisions; future rewards are discounted by an exponential factor that grows at a constant rate with the length of the delay. This model has initially been adopted in Phelps and Pollak (1968) to discount utilities of consumptions of generations over time. Laibson (1997) named it "quasi-hyperbolic" and applied it to analyse consumers' consumption and savings decisions.

Figure 5.1 graphs the standard exponential discount function, the generalized hyperbolic discount function and the quasi-hyperbolic discount function. The horizontal axis represents the time delay for receiving one unit of utility and the vertical axis represents the present value of the utility to be received. The vertical gap between the three curves reflects differences in the three different ways of discounting.

Comparing the exponential and hyperbolic discounting, it could be easily identified in the figure that the exponential discount function is less convex than the hyperbolic one, which indicates that rational individuals tend to use a smaller amount of discounting in earlier periods and overlook the importance of future rewards. In addition, with our parameter choices, the hyperbolic present value coincides with the exponential present value at around time point of 20 years. It tells us that the benefits that come earlier than in 20 years tend to be undervalued by hyperbolic discounters and the benefits that come after tend to be overvalued. As the interest rate becomes relatively lower, which is
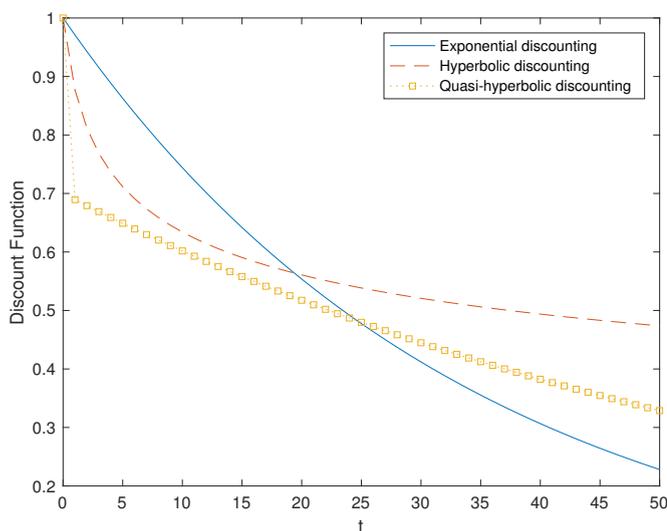
FIGURE 5.1: A comparison of three discounting curves

Notes: We assume a constant interest rate of 3 percent for exponential discounting; $\mu = 1$ and $\sigma = 0.19$ for hyperbolic discounting and $\beta = 0.7$ and $\tau = 0.985$ for quasi-hyperbolic discounting

reflected in the market today, the amount of discounting in a rational framework would be smaller and the intersection point will move rightwards along the hyperbolic discount curve. This suggests that more future benefits will be undervalued; an investment that generates immediate income will become less valuable.

Both the quasi-hyperbolic and generalized hyperbolic model include dynamic inconsistent preferences; however, this feature is constrained in the first period for the case of quasi-hyperbolic discounting. A value of β of less than 1 implies that the discount rate for the first period is greater than the long term discount rate, hence reflecting the immediate impatience effect; after the first period, quasi-hyperbolic discounting has the same discount factor between adjacent periods [1].

The hyperbolic discounting method has been widely applied to explain behavioral anomalies in various areas. As an example, people with a problem of low self-control often gain too much weight and find it difficult to improve their health by doing more exercise and having a diet. These people often vow to forgo all future temptations, in exchange

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[1]Please note that in Figure 5.1 the quasi-hyperbolic discounting curve looks linear (while it is not). This is because the per-period amount of discounting after time 0 is very small in our example and hence the convexity of the quai-hyperbolic discount curve is not as obvious as that of the exponential discount curve.

for improved health in the future; however, when they have the next meal, they cannot resist having unhealthy fried food and sweet desserts. Presumably, they prefer this because the instant pleasure brought by delicious unhealthy food is greater than the heavily discounted future rewards of health. The same defective reasoning has also been offered to account for drug addictions, procrastination, infidelity and other problems of willpower (Frederick et al., 2002). Laibson et al. (2003) have also used a hyperbolic discount function to explain the puzzle of simultaneously having large credit card debts and preretirement savings. In fact, it is not rational for individuals to make purchases using credit card and save regularly into a retirement fund because credit card debts attract a high interest rate and pre-retirement wealth only generates a low rate of return. However, if we assume that individuals are hyperbolic decision makers, the simultaneous actions seem reasonable. On the one hand, people cannot wait to do the shopping because the satisfaction gained from making a purchase today often outweighs the discounted displeasure of future payments; and as a result they build up large credit card debts. On the other hand, people tends to use a smaller discount rate for rewards in the distant future, which leads to a higher perceived value of the retirement fund and thus people would find it more attractive to accumulate wealth in retirement programs. Overall, consistent with the hyperbolic discounting method, people exhibit patience in the long term and impatience in the short term when making investment decisions.

5.2.2 Decumulation strategy

In Chapter 4, we have proposed a decumulation strategy and examined the annuity allocation rates and withdrawal rates in a completely rational framework (For detailed description please refer to Chapter 4.2). The strategy is expressed in the following form:

$$V = \max_{c_{65}, c_{66}, \dots} \sum_{i=0}^{54} \delta(i) \times {}_ip_{65} \times u(c_{65+i})$$

subject to

$$\sum_{i=0}^{d-1} c_{65+i} \times (1+r)^{-i} + \sum_{i=d}^{54} c_{65+i} \times (1+r)^{-i} \times {}_ip_{65} \times (1+L) = 1$$

where V denotes the overall expected discounted utility of the entire retirement consumption stream c_x. $\delta(i)$ may be interpreted as a subjective discount function for utility, $_ip_{65}$ as the probability of surviving for i years for a 65-year-old, $u(\cdot)$ as the utility associated with the rate of consumption, r as the real interest rate and L as the profit loading factor embedded in the annuity price.

Utility function is assumed to be a time-additive constant relative risk aversion (CRRA) function of the following form:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

where γ measures the coefficient of risk aversion.

Previously, the subjective discount function for utility, $\delta(i)$, is assumed to be the exponential discount function, the same as the discount function for consumption rates and annuity pricing. In this chapter, we explore the results when it is replaced with the hyperbolic discount model (Equation (5.1)) and the quasi-hyperbolic discount model (Equation (5.2)).

5.2.3 Modeling results following the general hyperbolic discount model

The results we display are based on the following assumptions. Relative risk aversion is moderate with $\gamma = 2$ and profit loading on annuity product equals $L = 10\%$. The discount rate for both utility and money amount are based on exponential discounting with a constant annual real interest rate of 3 percent. UK mortality table S2PML, which describes the mortality experience of UK male pensioners of self-administered pension schemes for the period from 2004 to 2011, is used to calculate the survival rate $_ip_{65}$.

The parameter values in hyperbolic functions are based on a hypothetical choices-indifferences test conducted by Abdellaoui et al. (2009), who estimate a μ of 1 and σ of 0.19 for the hyperbolic discount model. In what follows, we provide results of the optimal allocation, α, in the deferred annuity and optimal retirement consumption path, c_x for $65 \leq x \leq 119$. The sensitivity of the results is also explored.

TABLE 5.1: Optimal investment percentage in a deferred annuity for hyperbolic discounters (M1)

Profit Loading (L)	5%	10%	15%	20%	25%
Deferred period (d)					
5	67.27%	67.78%	68.26%	68.72%	69.16%
10	41.77%	42.34%	42.89%	43.41%	43.91%
15	22.56%	22.97%	23.36%	23.75%	24.12%
20	9.88%	10.09%	10.29%	10.49%	10.68%
25	3.19%	3.26%	3.33%	3.40%	3.47%

Notes: Results are based on the following assumptions: a risk aversion factor of $\gamma = 2$ and a constant real interest rate of $r = 3\%$ p.a.. Mortality rates are based on table S2PML.

5.2.3.1   M1 - the Hyperbolic discount model

In the case that retirees are guided to select a fixed deferred annuity, there are closed form solutions for the optimal consumption path and the optimal wealth allocation in the deferred annuity. A proof of the results can be derived with arguments analogue to those in Appendix 4.A in previous chapter. The results displayed below can be achieved by assuming that an irrational individual uses a hyperbolic discount model, hence $\delta(i) = (1+i)^{-\beta}$

$$c_{65+i} = \frac{{}_ip_{65}^{1/\gamma}v^{-i/\gamma}(1+i)^{-\beta/\gamma}}{\sum_{i=0}^{d-1} v^{i-i/\gamma}{}_ip_{65}^{1/\gamma}(1+i)^{-\beta/\gamma} + (1+L)^{(1-1/\gamma)}\sum_{j=d}^{54} v^{j-j/\gamma}{}_jP_{65}(1+j)^{-\beta/\gamma}} \quad \text{for } i = 0,\ldots,(d-1)$$

$$c_{65+i} = \frac{v^{-i/\gamma}(1+i)^{-\beta/\gamma}(1+L)^{-1/\gamma}}{\sum_{i=0}^{d-1} v^{i-i/\gamma}{}_ip_{65}^{1/\gamma}(1+i)^{-\beta/\gamma} + (1+L)^{(1-1/\gamma)}\sum_{j=d}^{54} v^{j-j/\gamma}{}_jP_{65}(1+j)^{-\beta/\gamma}} \quad \text{for } i = d,\ldots,54$$

$$\alpha = \frac{(1+L)\sum_{i=d}^{54} {}_ip_{65}^{1+1/\gamma}v^{i-i/\gamma}(1+i)^{-\beta/\gamma}}{\sum_{i=0}^{d-1} v^{i-i/\gamma}{}_ip_{65}^{1/\gamma}(1+i)^{-\beta/\gamma} + (1+L)^{(1-1/\gamma)}\sum_{j=d}^{54} v^{j-j/\gamma}{}_jP_{65}(1+j)^{-\beta/\gamma}}$$

where v denotes one-period exponential discount rate, $v = (1+r)^{-1}$.

Table 5.1 displays the numerical results for the optimal fraction of wealth to be spent on a deferred annuity for irrational retirees with dynamic time-inconsistent preferences. The results are provided allowing for different deferred periods and profit-loading factors. It demonstrates that it would be optimal for a hyperbolic discounter to spend 22.97% of his pension savings in a 15-year deferred annuity when an annuity provider requires

TABLE 5.2: The impact of discounting methods on the optimal investment percentage in a deferred annuity (M1)

Deferred period (d)	5	10	15	20	25
Exponential discounting	67.62%	41.19%	21.60%	9.13%	2.83%
Hyperbolic discounting	67.78%	42.34%	22.97%	10.09%	3.26%

Notes: Results are based on the following assumptions: a risk aversion factor of $\gamma = 2$ and a profit loading factor of $L = 10\%$. Exponential discounting assumes a constant real interest rate of $r = 3\%$ p.a.; Hyperbolic discounting assumes $\mu = 1$ and $\sigma = 0.19$. Mortality rates are based on table S2PML.

a profit margin of 10 percent, which is a less difficult decision to make comparing with annuitising 100% in an immediate annuity. As we mentioned in Chapter 4, this might overcome a potential psychological barrier to invest. Looking vertically in Table 5.1, the optimal allocation rates decreases significantly when retirees are guided to choose a longer-term deferred annuity: a retiree who is seeking protection after age 85 needs to spend only 10% of his retirement savings on a 20-year deferred annuity. This smaller allocation does not imply the lower popularity of deferred annuities; however, it is simply due to a much smaller actuarially fair annuity price (the survival rate becomes lower as people age).

For a certain fixed deferred annuity, the profit loading has a smaller but negative impact on the optimal annuity allocation. For example, Table 5.1 suggests that the real allocation in a 15-year deferred annuity should be 21.49% (22.56%/1.05) in the presence of a 5 percent profit loading and 19.30% (24.12%/1.25) with a 25 percent loading. The results are in line with our expectation because people in general would be willing to invest less in deferred annuities if they become more expensive.

Another interesting finding from Table 5.1 is that retirees are less price sensitive to longer term deferred annuities. As the profit loading factor increases from 5% to 25%, the optimal investment proportion in a 25-year deferred annuity increases from 3.19% to 3.47%; while that in a 5-year deferred annuity increases from 67.27% to 69.16%. It suggests that insurance companies could require a higher profit margin from writing long term deferred annuity products without affecting the demand.

Table 5.2 compares the optimal allocation in a deferred annuity for exponential discounters and hyperbolic discounters, from which we can find out the impact of the time-inconsistent preference on retirement decisions. It can be seen that people who
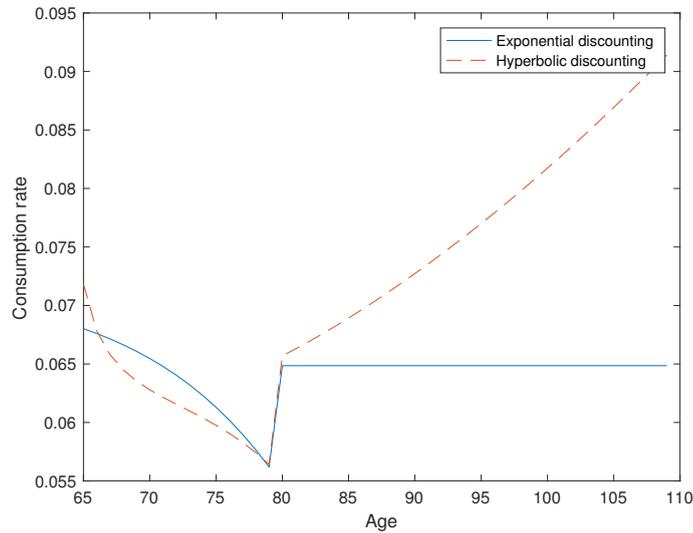
FIGURE 5.2: A comparison of optimal consumption paths for exponential discounters and hyperbolic discounters (M1)

Notes: Results are based on the following assumptions: a 15-year deferred annuity, $d = 15$, a risk aversion factor of $\gamma = 2$ and a profit loading factor of $L = 10\%$. Exponential discounting assumes a constant real interest rate of $r = 3\%$ p.a.; Hyperbolic discounting assumes $\mu = 1$ and $\sigma = 0.19$. Mortality rates are based on table S2PML.

have a decreasing patience tend to allocate slightly more in deferred annuities than rational exponential discounters. This is mostly because hyperbolic discounters tend to overweight benefits in the distant future, which we have explained in Figure 5.1.

The optimal retirement consumption paths to be followed for rational exponential discounters and irrational hyperbolic discounters are shown in Figure 5.2. A comparison of two sets of paths on the same scale allows us to make following conclusions. First, the consumption path after a d-year deferred period sheds some light on retirees' preferences towards the form of the annuity incomes. While rational investors prefer conventional annuities with level payments, irrational investors tend to prefer annuities with increasing payments. In practice, this could be an inflation linked deferred annuity whose payments increase with annual inflation rates. Second, prior to the commencement of the deferred annuity, retirees tend to follow different consumption patterns depending on the utility discounting methods. Exponential discounters can maximise overall utility if they follow a convex-shaped decreasing consumption path; while hyperbolic discounters would follow an inverted S-shaped decreasing consumption path. Hyperbolic discounters

TABLE 5.3: The optimal investment percentage in a deferred annuity with different hyperbolic discount factors (M1)

σ	0.1	0.15	0.2	0.25	0.3
α	24.01%	23.43%	22.85%	22.28%	21.70%

Notes: Results are based on the following assumptions: a risk aversion factor of $\gamma = 2$, a profit loading factor of $L = 10\%$ and a constant annual real interest rate of $r = 3\%$. One factor in hyperbolic discounting is $\mu = 1$. Mortality rates are based on table S2PML.

tend to be impatient to obtain immediate satisfaction; thus it is not surprising to see them spending higher amounts than exponential discounters in the first few years after retirement.

The kink embedded in the consumption paths remains after replacing the exponential discount model with the hyperbolic one. As we have discussed in Chapter 4, the kink is related with the risk aversion factor and the profit loading in annuity pricing. In Figure 5.2, the kink exists because the annuity is good value for money when it is priced with a 10% profit margin. If annuity providers require a higher profit loading, annuity products become less desirable; a smaller allocation in it would lead to more balanced consumption rates over the entire retirement period.

When the hyperbolic discount parameter σ increases, it implies that individuals are more impatient and so incomes in the near future are more heavily undervalued; this is illustrated in Figure 5.4. This leads to the results presented in Table 5.3 and Figure 5.3. When σ is very high at 0.3, individuals are very impatient and thus they gain higher level of satisfaction from high spending now and in the near future. On the other hand, when σ is 0.1, individuals experience less decreasing impatience, they have self-control to consume at a reasonable rate in the early period after retirement so that they can enjoy high annuity incomes at more advanced ages.

5.2.3.2 M2 - the Hyperbolic discount model

In the model M2 that we have proposed, individuals are able to work out the best deferred period with information of profit loading factor and mortality rates. The optimal age from which the annuity payment commences is
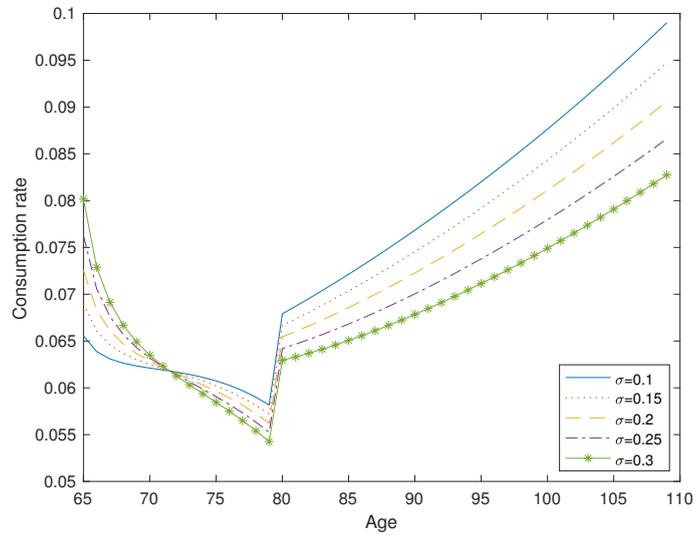
$$T_A = min(65 + i : (1 + L) \times {}_ip_{65} \leq 1)$$

FIGURE 5.3: The optimal consumption paths for retirees with different hyperbolic discount factors (M1)

Notes: Results are based on the following assumptions: a risk aversion factor of, $\gamma = 2$, a profit loading factor of $L = 10\%$ and a constant annual real interest rate of $r = 3\%$. One factor in hyperbolic discounting is $\mu = 1$. Mortality rates are based on table S2PML.

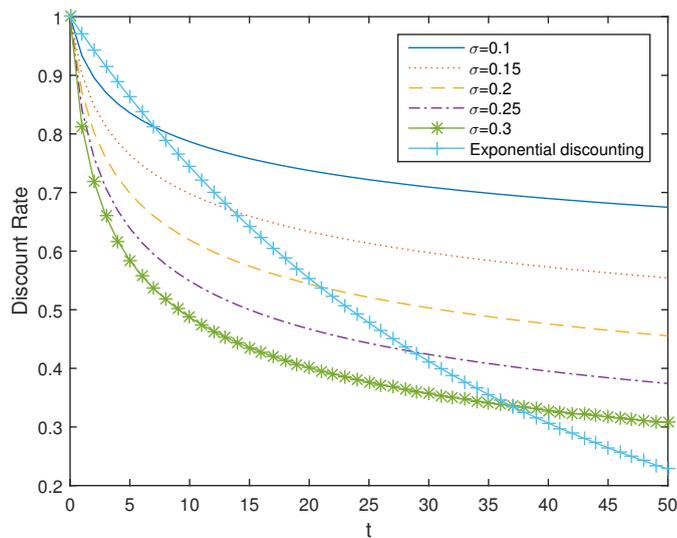FIGURE 5.4: The sensitivity of the hyperbolic discounting factor σ

Notes: Hyperbolic discounting follows Equation (5.1) with $\mu = 1$, and exponential discounting assumes an annual real interest rate of 3%.

TABLE 5.4: Optimal investment percentage (α) in a deferred annuity for hyperbolic discounters (M2)

Profit loading (L)	5%	10%	15%	20%	25%
Optimal deferred period	4	6	8	10	11
Risk aversion factor (γ)					
2	71.00%	60.41%	50.84%	42.14%	38.33%
3	71.11%	60.52%	50.93%	42.21%	38.47%
4	71.17%	60.57%	50.98%	42.25%	38.54%
5	71.21%	60.60%	51.01%	42.27%	38.58%
6	71.23%	60.62%	51.02%	42.28%	38.60%

Notes: Results are based on a real rate of return of $r = 3\%$ p.a. and the mortality table S2PML. Hyperbolic discount model assumes $\mu = 1$ and $\sigma = 0.19$.

TABLE 5.5: The impact of discounting methods on the optimal investment percentage in a deferred annuity (M2)

Profit loading (L)	5%	10%	15%	20%	25%
Optimal deferred period (d)	4	6	8	10	11
Exponential discounting	73.20%	61.83%	51.54%	42.25%	38.20%
Hyperbolic discounting	71.00%	60.41%	50.84%	42.14%	38.33%

Notes: Results are based on the following assumptions: a risk aversion factor of $\gamma = 2$ and a profit loading factor of $L = 10\%$. Exponential discounting assumes a constant annual real interest rate of $r = 3\%$; Hyperbolic discounting assumes $\mu = 1$ and $\sigma = 0.19$. Mortality rates are based on table S2PML.

and this justifies that the optimal deferred period is irrelevant with how people discount future benefits. With the choice of the optimal deferred period (d), the same analytic process will be implemented as in M1 to deduce the optimal annuity allocation and consumption pattern.

Table 5.4 presents the numerical outcomes of the optimal deferred period under different levels of profit loading and the optimal investment percentage in the corresponding deferred annuity products. First, it is better to delay receiving annuity payment when annuities are more expensive (increasing L); and the optimal investment in the deferred annuity decreases correspondingly with a longer deferred period. Second, looking vertically in the table, the risk aversion factor does not have a great impact on the optimal allocation percentage; however, it still reflects the point that people who are more risk averse would tend to allocate slightly more in a deferred annuity product that can provide a guaranteed income for advanced ages.
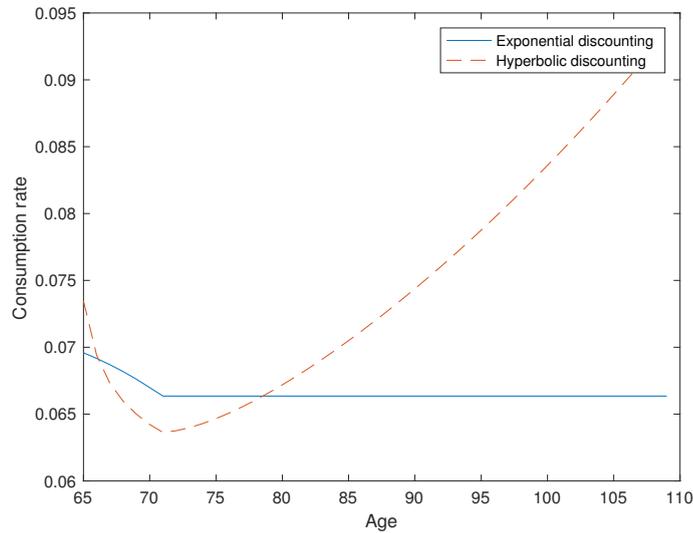
FIGURE 5.5: A comparison of optimal consumption paths for exponential discounters and hyperbolic discounters (M2)

Notes: Results are based on the following assumptions: a risk aversion factor of $\gamma = 2$ and a profit loading factor of $L = 10\%$. Exponential discounting assumes a constant annual real interest rate of $r = 3\%$; Hyperbolic discounting assumes $\mu = 1$ and $\sigma = 0.19$. Mortality rates are based on the table S2PML.

A comparison of the outcomes of α following the strategy M2 under two different methods of discounting is offered in Table 5.5. One may notice that hyperbolic discounters would not always spend more on deferred annuity products than exponential discounters. They would spend more on long-term deferred annuities but less on short-term deferred annuities; the break-even point is between 10-year and 11-year deferred period. This makes sense intuitively because individuals with time-inconsistent preferences overvalue benefits in distant future while undervalue benefits in near future. For a short-term deferred annuity, most benefits are greatly undervalued and hence are less valuable for people with irrational preferences [2].

A comparison of the optimal retirement consumption paths for rational exponential discounters and irrational hyperbolic discounters is provided in Figure 5.5. Two conclusions can be drawn. First, hyperbolic discounters prefer an annuity with increasing nominal incomes while exponential discounters prefer an annuity with level nominal incomes. This is because exponential discounters evaluate future payments using the same method as

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[2]The conclusion holds in general because hyperbolic discounters always undervalue benefits in the near future more than exponential discounters, however, the break-even point relies on the assumptions for the hyperbolic discount factor σ and interest rate r.

TABLE 5.6: The optimal investment percentage in a deferred annuity with different hyperbolic discount factors. (M2)

σ	0.1	0.15	0.2	0.25	0.3
α	61.99%	61.12%	60.23%	59.34%	58.43%

Notes: Results are based on the following assumptions: a risk aversion factor of $\gamma = 2$, a profit loading factor of $L = 10\%$ (which suggests an optimal choice of a 6-year deferred annuity) and a constant annual real rate of return of $r = 3\%$. One factor in hyperbolic discounting is $\mu = 1$. Mortality rates are based on the table S2PML.
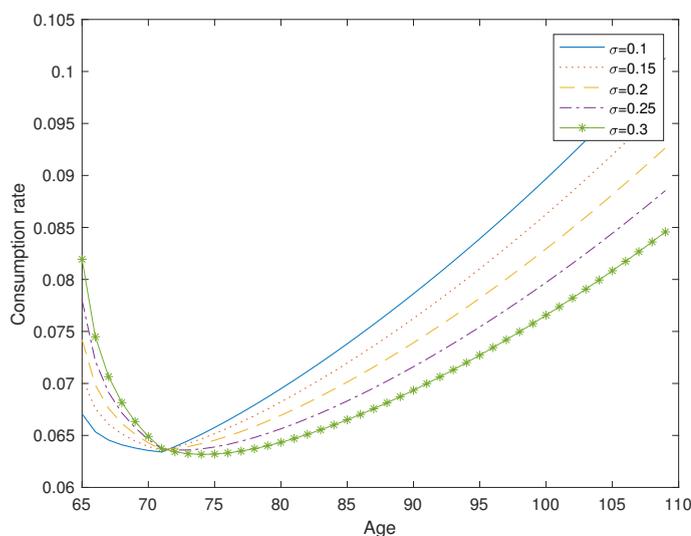

FIGURE 5.6: The optimal consumption paths for retirees with different hyperbolic discount factors (M2)

Notes: Results are based on the following assumptions: a risk aversion factor of, $\gamma = 2$, a profit loading factor of $L = 10\%$ and a constant annual real interest rate of $r = 3\%$. One factor in hyperbolic discounting is $\mu = 1$. Mortality rates are based on the table S2PML

an annuity-pricing actuary; however, hyperbolic discounters assign much higher weights to incomes in the distant future. Second, prior to the start of the deferred annuity, hyperbolic discounters tend to spend more in the first few years after retirement than exponential discounters, which leads to lower consumption rates for hyperbolic discounters in their age 70s.

Table 5.6 and Figure 5.6 show the impact of different levels of time-inconsistent preferences on the strategy M2. For those who are more impatient to receive benefits immediately (with a greater σ), they tend to allocate less money in deferred annuity products. As we have explained above, for the case of a small profit loading (10% in this example),

TABLE 5.7: Optimal investment percentage in a deferred annuity for Quasi-hyperbolic discounters (M1)

β	0.6	0.7	0.8	0.9	1
τ					
0.95	18.80%	18.93%	19.04%	19.13%	19.21%
0.96	19.92%	20.06%	20.17%	20.26%	20.34%
0.97	21.08%	21.22%	21.33%	21.42%	21.50%
0.98	22.27%	22.41%	22.52%	22.61%	22.69%
0.99	23.50%	23.64%	23.75%	23.84%	23.92%

Notes: Results are based on the following assumptions: a risk aversion factor of $\gamma = 2$, a constant annual real interest rate of $r = 3\%$, a profit loading of $L = 10\%$, and a deferred period of $d = 15$. Mortality rates are based on the table S2PML.

the optimal choice of a short-term deferred annuity (a 6-year deferred annuity in this example) is not attractive for impatient retirees. In terms of the optimal consumption path during retirement period, more impatient hyperbolic discounters would consume at a higher rate in the early years after retirement; and this will be at the cost of lower consumption rates in advanced ages in retirement.

5.2.4 Modeling results following the quasi - hyperbolic discount model

In this section, we use the quasi-hyperbolic discount model as the subjective discount model for utilities to identify the optimal decumulation strategy. Assumptions are the same as in Section 2.3. In the following, we conduct a sensitivity analysis on the quasi-hyperbolic parameter values to identify the preferences for people with different levels of impatience. In addition, we give a comparison of the impacts of quasi-hyperbolic discounting and exponential discounting. The analysis is conducted for strategy M1 and M2 respectively.

5.2.4.1 M1- the Quasi-hyperbolic discount model

Table 5.7 shows the optimal investment allocation in deferred annuities under different possible values of quasi-hyperbolic parameters. Recalling the quasi-hyperbolic function in Equation (5.2), β controls the extra amount of discounting for the first period relative to subsequent periods; while τ is the discount rate for each period. A β of 1 means the
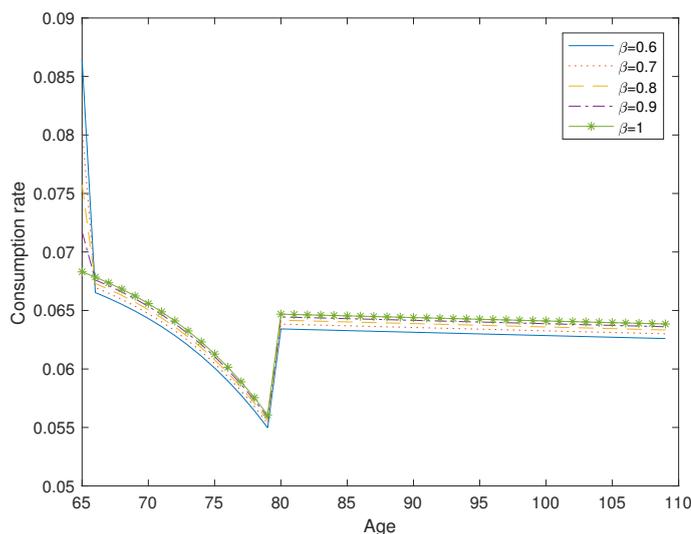
FIGURE 5.7: The optimal consumption paths with different quasi-hyperbolic discount factors (M1)

Notes: Results are based on the following assumptions: a 15-year deferred annuity, $d = 15$, a risk aversion factor of $\gamma = 2$, a profit loading factor of $L = 10\%$ and a constant annual real interest rate of $r = 3\%$. One factor in quasi-hyperbolic discounting is $\tau = 0.97$. Mortality rates are based on the table S2PML

quasi-hyperbolic discounting coincides with standard exponential discounting. Looking horizontally in the table, it can be seen that β has very limited influence on the investment in annuities, which is because deferred annuities do not involve any payments at age 66. Therefore a quasi-hyperbolic discounter would not allocate a significantly different percentage of wealth in a deferred annuity comparing with a rational exponential discounter. On the other hand, the investment allocation is very much influenced by the time-consistent discount rate τ. The increase of τ represents a smaller amount of discounting for utility in each period and so people would view future benefits as more important. This is the reason that allocation in annuities is increasing with the value of τ.

Figure 5.7 demonstrates a set of optimal consumption paths that involve with a fixed 15-year deferred annuity following different assumptions for β. Since β affects only the value of consumption in one year's time, the optimal consumption paths show differences only at the beginning of retirement. As retirees become more impatient (β decreases), they tend to consume more at age 65.
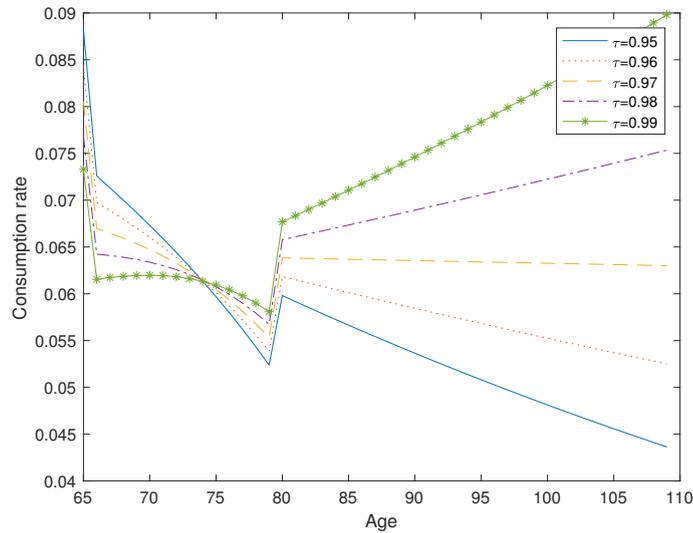
FIGURE 5.8: The optimal consumption paths with different quasi-hyperbolic discount factors (M1)

Notes: Results are based on the following assumptions: a 15-year deferred annuity, $d = 15$, a risk aversion factor of $\gamma = 2$, a profit loading factor of $L = 10\%$ and a constant annual real interest rate of $r = 3\%$. One factor in quasi-hyperbolic discounting is $\beta = 0.7$. Mortality rates are based on the table S2PML

Figure 5.8 shows the optimal consumption path for different values of τ. An interesting finding is that people require different types of annuity payments when they use different discount rates for future benefits. If τ stays at 0.97, which means people discount future utilities in the same way as annuity pricing (with the assumption of 3 percent interest rate, one period discount rate is 0.9709), they would prefer a conventional annuity with level payments. As τ increases, people adopt a smaller amount of discounting for each period and hence they place more value on future income. As a result, they would prefer annuities with increasing payments, which could be an inflation-linked annuity. However, to afford such an inflation protected annuity product, they have to sacrifice consumption in the early years after retirement. In contrast, as τ decreases, a greater amount of discounting applies and hence people tend to allocate more spending in the earlier years during retirement. When τ becomes smaller than 0.97, quasi-hyperbolic discounters would enjoy a decreasing consumption pattern; annuities that offer decreasing payments would become preferred.

TABLE 5.8: Optimal investment percentage in a deferred annuity for Quasi-hyperbolic discounters (M2)

β	0.6	0.7	0.8	0.9	1
τ					
0.95	56.09%	56.49%	56.83%	57.10%	57.34%
0.96	57.41%	57.81%	58.13%	58.40%	58.63%
0.97	58.72%	59.11%	59.43%	59.69%	59.92%
0.98	60.02%	60.40%	60.71%	60.97%	61.19%
0.99	61.31%	61.68%	61.98%	62.23%	62.45%

Notes: Results are based on the following assumptions: a risk aversion factor of $\gamma = 2$, a constant annual real interest rate of $r = 3\%$, and a profit loading of $L = 10\%$ (where a 6-year deferred annuity is optimal). Mortality rates are based on the table S2PML.
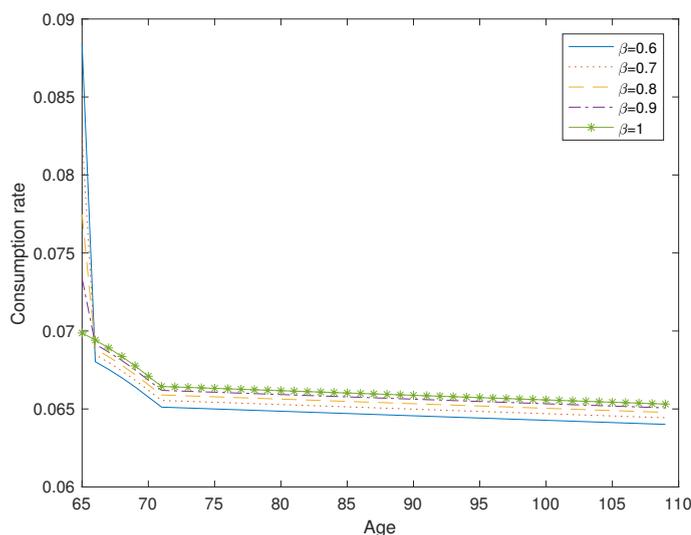

FIGURE 5.9: The optimal consumption paths with different quasi-hyperbolic discount factors (M2)

Notes: Results are based on the following assumptions: a risk aversion factor of $\gamma = 2$, a profit loading factor of $L = 10\%$ (a 6-year deferred annuity is optimal) and a constant annual real interest rate of $r = 3\%$. One factor in quasi-hyperbolic discounting is $\tau = 0.97$. Mortality rates are based on table S2PML

5.2.4.2 M2- the Quasi-hyperbolic discount model

The impact of the quasi-hyperbolic discount model on strategy M2, which is illustrated in Table 5.8, Figure 5.9 and Figure 5.10, is similar to that for strategy M1. To conclude, first, retirees following the quasi-hyperbolic way of discounting would invest a similar percentage of wealth in deferred annuities as those who follow the exponential way of
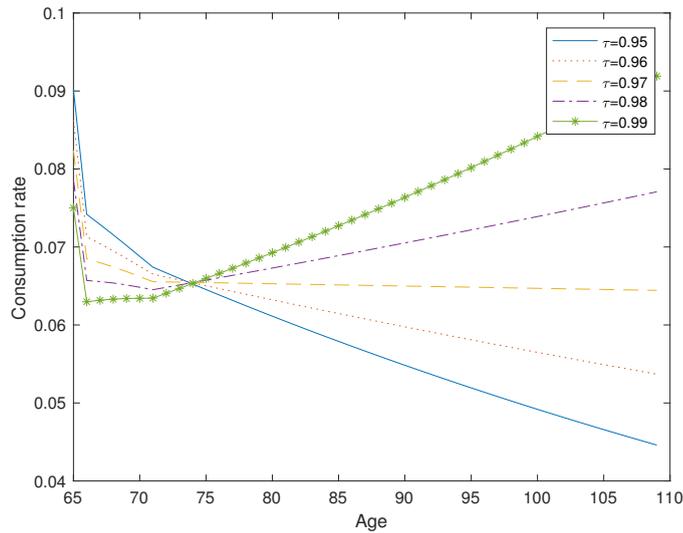
FIGURE 5.10: The optimal consumption paths with different quasi-hyperbolic discount factors (M2)

Notes: Results are based on the following assumptions: a risk aversion factor of $\gamma = 2$, a profit loading factor of $L = 10\%$ (a 6-year deferred annuity is optimal) and a constant annual real interest rate of $r = 3\%$. One factor in quasi-hyperbolic discounting is $\beta = 0.7$. Mortality rates are based on table S2PML

discounting. Second, a quasi-hyperbolic discounter tends to spend a significant higher amount at the beginning of the retirement due to high impatience for immediate benefits. Third, the type of preferred annuity depends on the relative differences between the discount rate for utilities and that for pricing. If the amount of periodic discounting for utilities is smaller than that for money amount, an inflation linked style deferred annuity will be preferred.

5.3 The impact of subjective mortality rates

5.3.1 Literature review on subjective mortality rates

As we know, the mortality rate is one of the most fundamental factors in the process of retirement planning, because every decision is based on how long a person is expected to live. At the population level, there are published mortality tables in each country for different cohorts and different groups of people (e.g. male and female); these national life tables and those produced for insured populations (for example, by the CMI, in

the UK) are the basis of insurance product pricing. However, when we move down to the individual level and talk about how an individual make a decision, the individual's opinion on his/her mortality rate might be different from published mortality data. This could be because individuals possess some private information regarding their lifestyle, health status or genetic disorders, which makes them more/less likely to die compared to the general public. It can also simply due to different personalities of being optimistic or pessimistic. In literature, some researchers (e.g. Hurd and McGarry (2002)) have concluded that subjective survival probabilities have a potential use in intertemporal decision making under uncertainty and more precisely, in forming a forward-looking retirement decision. Therefore, we intend to use the concept of subjective mortality rates to analyse optimal retirement strategies.

Previously, there have been several studies on mortality perceptions, carried out by economists, psychologists and some social scientists. Subjects tend to be faced with questions on their opinions towards survival rates. For example, typical questions in these studies are the age they expect to live and/or the probability that they will survive to a specified age. In some surveys, individuals' opinions towards desired lifespan or population lifespan will be asked as well. For example, the surveys may ask subjects the age that they wish to live to and/or the age that a person of the same age and sex would expect to live to. In addition to the opinions, lifestyle information of each individual are collected so that researchers can establish the link between subjective mortality perceptions and some risk factors such as social class, smoking, alcohol consumptions and parents' longevity. O'Brien et al. (2005) offer a summary of previous surveys, describing in detail the questions asked and the results obtained. In the following paragraphs, we review three major studies in the literature.

An early study of Hamermesh (1985) analyses responses to a questionnaire that is designed to elicit subjective expectations and probabilities of survival. The questionnaires are sent to two groups of respondents: one group is a set of 650 white male academic economists whose ages ranging from 26 to 65 and the other group is 975 people chosen randomly from the telephone directory who were aged between 20 and 70. In the questionnaire, the respondents are asked how old they expect they will live to be and the subjective probability of living to at least age 60/80.

A major finding is shown in Table 5.9. Comparing $x + e_x^s$ (the age to which people expect

TABLE 5.9: Expected age at death

	economists sample			telephone directory sample		
age	26-39	40-65	26-39	20-39	40-70	20-70
$x + e_x^s$	75.91	76.41	76.19	75.81	77.74	76.79
$x + e_x^0$	73.49	75.47	74.60	73.24	76.56	74.92

Notes: x denotes the respondents' current age. $x + e_x^s$ is the age to which they expect to live and $x + e_x^0$ is the age of death from published life tables. Source: Hamermesh (1985)

TABLE 5.10: Average probabilities of living to age 75 or 85

	Men		Women		All	
	To 75	To 85	To 75	To 85	To 75	To 85
HRS data	0.62	0.39	0.66	0.46	0.65	0.43
1990 life table	0.60	0.26	0.75	0.45	0.68	0.36

Notes: Source: Hurd and McGarry (1995)

to live) with $x + e_x^0$ (the age of death from published life tables) allows us to deduce that people expect themselves to live longer than the then current actuarial life expectancies. Hamermesh (1985) concludes that on average the subjective probabilities are consistent with published life tables and people actually extrapolated the improvement in longevity over time.

Another evaluation of the subjective probabilities of survival that is conducted by Hurd and McGarry (1995) is based on data from the Health and Retirement Survey (HRS). HRS is a panel survey on Americans over age 50. It asks respondents to indicate on a scale of 0-10 the chances that they will live to be 75/85 or more. 0 means absolutely no chance to survive till a specified age and 10 means absolutely certain to survive until a specified age. After rescaling to [0,1], the answers can be treated as the subjective probability of surviving. Table 5.10 demonstrates the probability of surviving to age 75 and age 85. From the table we can see that subjective probability of surviving to age 75 for all (both men and women) is close to the average in the 1990 life table; but the probability of surviving to age 85 for all is higher than that from the 1990 life table. The results suggest that men substantially overestimate the probability that they will survive to 85, and women underestimate the probability that they will live to 75.

An interesting longitudinal survey, which examines whether mortality experience over a period is predicted by mortality perceptions at the beginning of the period, is analysed in Hurd and McGarry (2002). In their analysis they use data from the first two waves
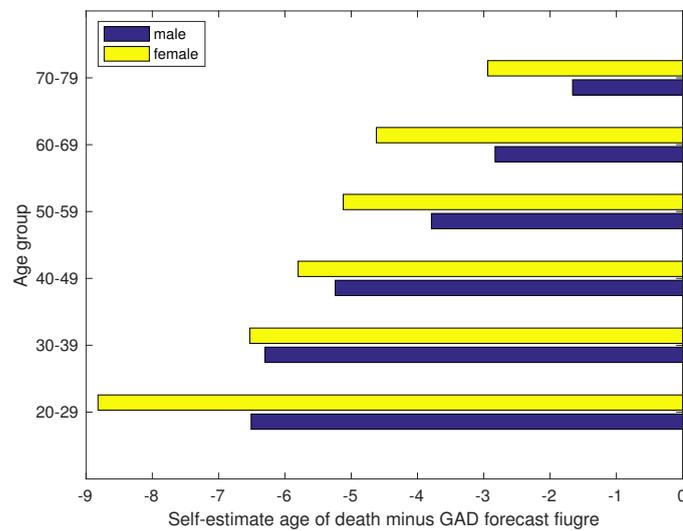
FIGURE 5.11: Mortality perceptions by age

Notes: Source: O'Brien et al. (2005)

of the HRS. They find that the subjective survival probabilities from the first wave data can predict actual survival into the second wave: the subjective survival probabilities reported in wave 1 of the HRS by those who were alive during the second wave were about 50% higher than the probabilities reported by those who died between the two waves. Therefore they conclude that the subjective survival probabilities include an expectational element rather than a simple measure of health status - since the latter showed no significant difference between the two groups .

A survey of over 3500 individuals in Great Britain was conducted by O'Brien et al. (2005) to discover the mortality perceptions of people in different age groups in the UK. In order to avoid the problem of focal-point responses: 0 and 1, in answering the questions of survival probability, this survey simply asks people the age to which they expect to live. The results are compared with the estimates from the Government Actuary Department (GAD), which include an allowance for future mortality improvement, so that we can find out whether British people are optimistic or pessimistic regarding longevity. The negative figures displayed in Figure 5.11 show that both females and males underestimate their life expectancies compared with the estimates from the GAD. On average, males underestimate by 4.62 years and females underestimate by 5.95 years. Findings from O'Brien et al. (2005) suggest that people are pessimistic about life expectancies, while

findings from Hamermesh (1985) and Hurd and McGarry (1995) suggest that people are optimistic. This is because O'Brien et al. (2005) compare subjective opinion with forecast figures, which already incorporate the mortality improvement, while Hamermesh (1985) and Hurd and McGarry (1995) compare it with current life tables from that time, which do not include any allowance for future mortality improvement.

Following Gan et al. (2005), an individual's subjective survival rate is described as

$$_tP_x = \exp\{\int_0^t -\psi\mu_{x+r}dr\} \tag{5.3}$$

where μ_{x+r} denotes the instantaneous hazard rate at age x+r and ψ denotes individual optimism index.

When ψ equals 1, the survival rate predicted by Equation (5.3) will be the same as in the published life tables. Therefore, parameter ψ basically measures whether people are optimistic or pessimistic relative to the actual mortality rates. If ψ is less than 1, the person is optimistic about his life expectancy because the perceived mortality rates are lower. On the other hand, ψ being greater than 1 indicates the person is pessimistic. In this section, we will examine two cases below to test the sensitivity of the results to the choice of ψ.

$\psi = 0.8$, the person is optimistic about his life expectancy

$\psi = 1.2$, the person is pessimistic about his life expectancy.

5.3.2 The impact of subjective mortality rates on the attractiveness of annuities

With subjective mortality rates, we can work out a revised price for annuity products. This revised price is the maximum acceptable price (reservation price) that an individual would like to pay. If an individual believes he might live for a shorter period than others, it is easily imagined that he would be willing to buy an annuity only if it has a low price. After comparing this price with the actuarially fair price, we could determine whether annuity products in the market are desirable for people with subjective mortality perceptions to purchase.
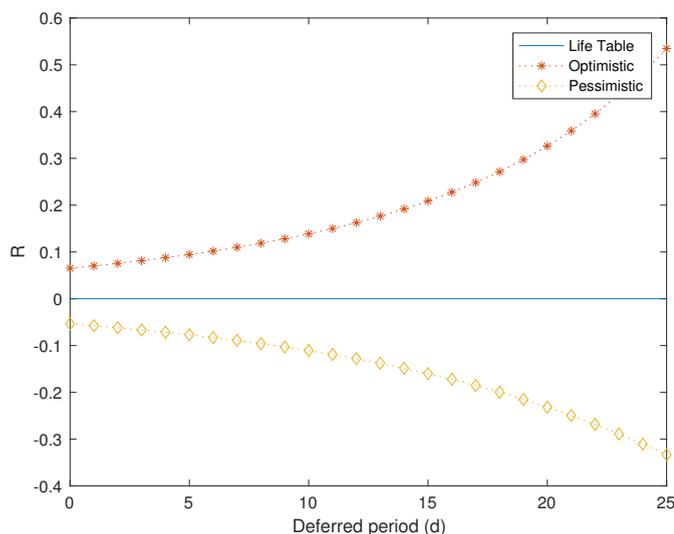
FIGURE 5.12: The Relative Price Difference (R) of immediate annuity and deferred annuities for a 65-year-old

Following Chapter 2, we calculate the ratio R, which is the relative difference between reservation price and fair price.

$$R = \frac{Reservation\ Price - Actuarially\ fair\ price}{Actuarially\ fair\ price}$$

A positive R suggests an annuity is attractive to purchase. On the other hand, a negative R suggests people would like to pay a lower-than-market price on annuities; hence annuities are not attractive. The modeling results in terms of R can also be interpreted as how much more or less than market price one would be prepared to pay for an annuity.

To produce modeling results, we assume a constant interest rate of 3 percent and the S2PML as the published mortality table. Figure 5.12 displays the attractiveness of immediate annuities ($d = 0$) and deferred annuities ($d > 0$) for retirees at age 65. It can be seen that people who are optimistic about survival rates find annuities attractive to purchase; while people who are pessimistic about their survival rates find all types of annuities not attractive at all. In addition, as the deferred period (d) increases, optimistic individuals tend to find deferred annuities more desirable while pessimistic individuals find deferred annuities less desirable. Therefore, optimistic individuals tend to invest in long-term deferred annuities. Moreover, if a person who has pessimistic

TABLE 5.11: Optimal investment percentage in a deferred annuity (M1)

	Life table $\psi = 1$	Optimistic $\psi = 0.8$	Pessimistic $\psi = 1.2$
Deferred period d			
5	68.12%	70.00%	66.41%
10	41.98%	45.01%	39.31%
15	22.41%	25.64%	19.71%
20	9.72%	12.29%	7.78%
25	3.13%	4.60%	2.17%

Notes: Results are based on the following assumptions: a risk aversion factor of $\gamma = 2$, a constant annual real interest rate of $r = 3\%$, and a profit loading of $L = 10\%$

TABLE 5.12: A summary of consumption paths in Figure 5.13

	Life table	Optimistic	Pessimistic
Max consumption	6.71%	6.35%	7.03%
Min consumption	5.60%	5.50%	5.66%
Max - Min	1.11%	0.85%	1.37%
Consumption gap	14.21%	10.16%	18.42%

views on mortality rates decides to buy an annuity, an immediate annuity starting at age 65 would be preferred.

5.3.3 The impact of subjective mortality rates on decumulation strategies

Detailed descriptions of the decumulation strategies M1 and M2 have been provided in Section 2.2. Previously, modeling results are based on the mortality rates from the published table S2PML. In this section, we will replace it with a new survival rate function (5.3) so that we can find out the impact of subjective mortality rates on our proposed retirement strategies M1 and M2. The benchmark assumptions will be the same as described in Section 2.3.

Table 5.11 demonstrates the optimal investment percentage in a fixed d-year deferred annuity following strategy M1. It offers a comparison of the results for optimistic individuals and pessimistic individuals. It is clear that people who are optimistic regarding their life expectancies would like to invest a greater percentage of wealth in deferred annuities. This is consistent with our conclusion that optimistic individuals tend to find annuities more desirable to purchase.
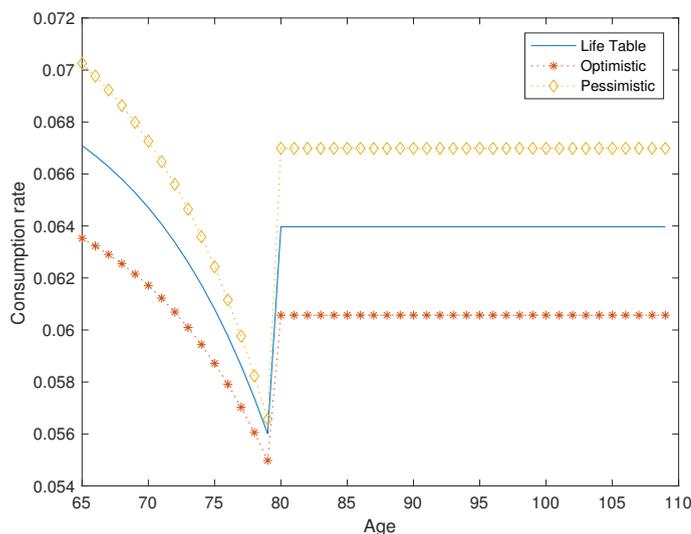
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FIGURE 5.13: The optimal consumption path following M1 for retirees with subjective mortality rate

Notes: Results are based on the following assumptions: a 15-year deferred annuity $d = 15$, a risk aversion factor of $\gamma = 2$, a constant annual real interest rate of $r = 3\%$, and a profit loading of $L = 10\%$

Figure 5.13 shows the optimal consumption paths for people with different mortality perceptions and Table 5.12 offers a summary of the consumption paths. it is interesting to note that people with optimistic views on survival rates tend to have lowest consumption rates during the entire retirement period than people with neutral or pessimistic views on survival rates. This is because those with lighter subjective mortality rates expect themselves to have a longer life expectancy; they need to consume at a lower rate and save for the future in preparation for surviving a long period during retirement. Moreover, we identify a smaller consumption gap at the transition point (when annuity payments commence) for optimistic individuals, which implies that individuals who believe they will live longer after retirement tend to follow a more stable and smooth consumption path.

Table 5.13 demonstrates the optimal investment percentage in an optimally chosen deferred annuity following strategy M2. As we have shown (in Chapter 4) that the optimal deferred period is determined by the mortality rates and the profit-loading factor, holding different opinions on mortality rates would lead to choices of different types of deferred

TABLE 5.13: Optimal investment percentage (α) in a deferred annuity (M2)

Profit loading (L)		10%	15%	20%	25%	30%
Life table	Optimal deferred period	7	9	10	11	12
	Optimal allocation	56.92%	47.26%	43.04%	39.03%	35.21%
Optimistic	Optimal deferred period	8	10	11	13	14
	Optimal allocation	54.36%	45.56%	41.72%	34.10%	30.79%
Pessimistic	Optimal deferred period	6	8	9	10	11
	Optimal allocation	60.43%	49.85%	45.23%	40.84%	36.68%

Notes: Results are based on the following assumptions: a risk aversion factor of $\gamma = 2$ and a constant annual real interest rate of $r = 3\%$.
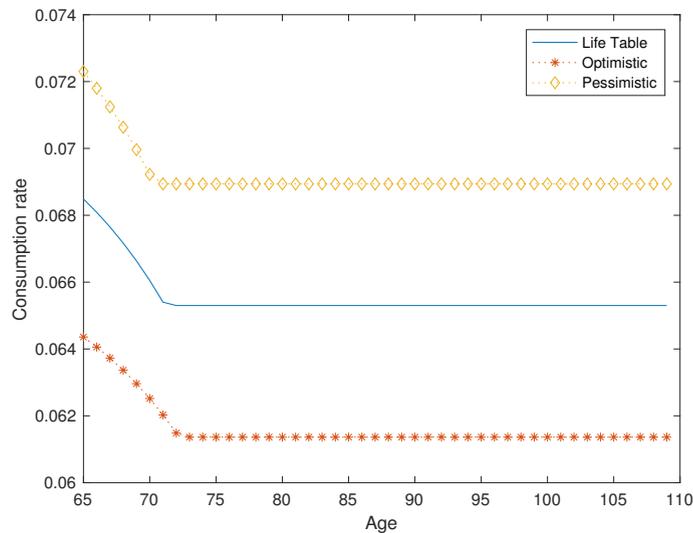

FIGURE 5.14: The optimal consumption path following M2 for retirees with subjective mortality rate

Notes: Results are based on the following assumptions: a risk aversion factor of $\gamma = 2$, a constant annual real interest rate of $r = 3\%$, and a profit loading of $L = 10\%$

TABLE 5.14: A summary of consumption paths in Figure 5.14

	Life table	Optimistic	Pessimistic
Max consumption	6.85%	6.44%	7.23%
Min consumption	6.53%	6.14%	6.89%
Max - Min	0.32%	0.30%	0.34%

annuity. With a fixed profit loading factor, Table 5.13 shows that people who are optimistic about their life expectancy tend to choose an annuity with a longer deferred period than those who are pessimistic about their life expectancy. The corresponding optimal investment allocation is linked with the deferred period. With a 10% profit loading, optimistic individuals would invest 54.36% in a 8-year deferred annuity, while pessimistic individuals would invest 60.43% in a 6-year deferred annuity.

Figure 5.14 shows the optimal consumption paths for people with different mortality perceptions and Table 5.14 offers a summary of the consumption paths. Similar conclusions can be drawn as for the model M1. Consumption rates for optimistic individuals are below those for pessimistic individuals at all ages. This is because optimistic individuals believe that they are less likely to die and they need to prepare for the long term future. In addition, optimistic individuals prefer more stable and smooth consumption path during retirement stage. As introduced in Section 5.3.1, people normally underestimate their life expectancies compared with figures that include an allowance for future mortality improvements. Therefore, an insurance company could launch a relatively short term deferred annuity; according to our modeling results, a pessimistic individual would prefer a 6-year deferred annuity when the profit loading is 10%.

5.4 Conclusions

In recent decades, data on assets and members in DB and DC pension plans confirms the increasing prominence of DC plans. In many OECD countries, almost all new schemes introduced are DC schemes. For existing DB pension schemes, they are sometimes closed to new members; or they can be closed to all members, in which cases all assets in DB plans stop accruing. In countries like Netherlands and the United States where DB plans have been running for many decades, the total value of assets in DB plans continues to grow but at a slower pace than assets in DC plans (OECD, 2016). In a pure DC scheme, individuals bear investment risk and longevity risk; therefore, professional advice is in demand so that that pensioners understand how to generate reasonable incomes from accumulated DC savings to support the entire retirement life.

This chapter focuses on optimal decumulation strategies and can be seen as an extension of Chapter 4. In this chapter, we aim to find out the implications of two behavioral

factors, time inconsistent preferences and subjective mortality rates, on the two decumulation strategies (M1 and M2) introduced in Chapter 4. In two separate sections, we have replaced the exponential discount model with the hyperbolic discount model, and replaced reported mortality rates with a subjective rates model, in the utility maximisation framework with budget constraint. Primary findings are as follows (results introduced are based on our benchmark assumptions).

First, a hyperbolic discounter with decreasing patience over time would tend to invest 22.97% of pension wealth in a 15-year deferred annuity (following M1), or invest 60.4% of pension wealth in a 6-year deferred annuity (following M2); this recommended allocation is similar to our findings for exponential discounters (see Chapter 4 for more details). Despite these similar allocations, the preferred annuity type for hyperbolic discounters and exponential discounters are different. Hyperbolic discounters, who tend to overvalue incomes in the distant future, would prefer an inflation linked annuity while rational exponential discounters would prefer a conventional annuity with level payments.

Second, a quasi-hyperbolic discounter, who simply shows impatience in the first year, tends to spend a significant higher amount at the beginning of the retirement than a exponential discounter. The preferred type of annuity depends on the relative differences between the discount rate for utilities and that for annuity pricing. If the discount rate for utilities is smaller than that for money amount, an inflation linked deferred annuity will be preferred; in the opposite case, an annuity that provides decreasing incomes will be preferred.

Third, subjective mortality rate is shown to be an influential factor in an annuity purchase decision. People who are optimistic (pessimistic) about their life expectancies would find annuities attractive (not attractive) and allocate a higher (lower) proportion of pension savings in a fixed deferred annuity. In M2, when individuals are allowed to select the optimal deferred period of an annuity, optimistic individuals would choose annuities with longer deferred periods, and hence invest a smaller proportion of savings, than pessimistic individuals.

Overall, the behavioral factors we have studied in this chapter do not greatly affect the optimal allocation in the deferred annuities; however, they do play important roles in the optimal consumption/drawdown patterns.

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6

General Conclusions

This thesis has been devoted to the understanding of the annuitisation decision and the use of annuity products in decumulation solutions for the post-retirement period. We have introduced behavioral factors based on subjective values of annuity products in order to determine whether people would be interested in purchasing an annuity with an actuarially fair price. Two behavioral factors have been studied: cumulative prospect theory (CPT) and the hyperbolic discount model. They both suggest a high demand for deferred annuity products. Hence, we propose a decumulation strategy with the involvement of deferred annuities. By following this strategy, retirees receive adequate longevity protection from a pre-determined certain age. It simplifies their investment decision-making process by turning the investment horizon from an uncertain period – because of having no prior knowledge of age of death – into a fixed period. The fixed period is the time between the retirement age and the point when the annuity payments commence. We have identified the optimal proportion of pension savings to be allocated into the deferred annuities and also the optimal consumption path (decumulation path) to be followed during retirement.

In Chapter 2, we use CPT to find out the perceived value of an annuity and hence work out the maximum price that people would like to pay for an annuity. By comparing this with the actuarially fair annuity price, we can determine whether an annuity purchase will be made. We show that being loss averse rather than risk averse is the major reason (within this framework) that stops people from buying an annuity; people would be willing to pay only a lower price because they exaggerate the expected losses from

an annuity investment in the early years. Furthermore, the behavioral tendency of overweighting low-probability events leads to the low demand for immediate annuities (because of overweighting the low probability of dying soon after age 65) and suggests a possible high demand for long-term deferred annuities (because of overweighting the low probability of surviving to an advanced old age, e.g. age 90).

In Chapter 3, we analyse the annuitisation decision using another behavioral model, the hyperbolic discount model. Using the same techniques as in Chapter 2, we find that the presence of time-inconsistent preferences in the process of making an intertemporal decision can explain the low demand for immediate annuities. Moreover, it again uncovers a possible high demand for long-term deferred annuities. Two types of deferred annuities, a working age deferred annuity (WADA) and a retirement age deferred annuity (RADA) are studied, and the results show that hyperbolic discounters, both at working age and in retirement, would find annuities with a longer deferred period more attractive.

Chapter 4 focuses on deferred annuity products and explores its best use in the DC decumulation process during retirement. We propose allocating a proportion of pension savings to either a fixed long-term deferred annuity or an optimally determined deferred annuity in order to receive protection against (tail) longevity risk. Using numerical optimisation techniques and a set of benchmark assumptions, we make the following suggestions: (i), a retiree who worries about tail longevity risk and wants to retain a certain level of liquidity is advised to spend 21.6% on a 15-year deferred annuity or 9.13% on a 20-year deferred annuity; (ii), a retiree who simply wants to use annuities to maximise overall satisfaction from retirement consumption is advised to spend 61.83% on a 6-year deferred annuity and to follow a decreasing consumption path until the start of annuity payments. A sensitivity analysis is conducted and suggests that the above recommended percentages are very stable relative to the desire for smooth consumption, the existence of state benefits and the bequest motive.

Chapter 5 extends the research questions in Chapter 4 into the area of behavioral factors, aiming to find out whether people would follow different decumulation strategies during retirement given that they are exposed to behavioral biases of having time-inconsistent preferences or having subjective opinions on their future mortality rates. The results suggest that hyperbolic discounters would invest a similar proportion of pension savings

in deferred annuity products, but prefer inflation-linked payments rather than level payments. Moreover, retirees who are optimistic (pessimistic) about their life expectancies would find annuities attractive (not attractive) in general and they tend to allocate more (less) of their pension savings in annuity products.

The advantages of deferred annuities have been discussed extensively in recent years: by delaying the payments for a few years, people access longevity protection at a fraction of the price required for an immediate annuity product. This defines a fixed period over which people need to make drawdown decisions and helps maintain liquidity to some extent. Second, planning ahead to receive protections for advanced ages could be more likely to help individuals achieve optimal retirement spending, due to the fact that people tend to experience cognitive impairment as they reach advanced ages. A study shows that 20 percent of retirees in their 80s have been fully diagnosed with dementia and 30 percent have severe cognitive impairment (Laibson, 2009). Furthermore, our proposed strategies are in line with the OECD roadmap for the good design of defined contribution pension plans (OECD, 2012); one of the recommendations is:

> For the payout phase, encourage annuitization as a protection against longevity risk. A certain level of annuitization of balances accumulated in DC pension plans should be set as the default mechanism for the payout phase, unless pay-as-you-go public pensions or the old-age safety net already provide for sufficient regular pension payments. A combination of programmed withdrawals with a deferred life annuity (e.g. starting payments at the age of 85) that offers protection against inflation could be seen as an appropriate default.

Despite the deferred annuities' strengths, there are firstly still regulatory barriers regarding offering deferred annuity products. For example, in the U.S., one concern is the Required Minimum Distributions (RMDs), which is the minimum amount that is required by law to be withdrawn annually from a retirement plan after pensioners reach age $70\frac{1}{2}$. It is only recently (2014) that deferred annuities purchased within qualified defined contribution plans[1] have been allowed to count towards the minimum distribution requirement if the annuity payments begin by age 85 and the annuity premium does not

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[1] 401(a), a 403(b) plan, a governmental 457(b) plan or a traditional IRA.

exceed the minimum of 25% of pension savings or USD 125,000 (OECD, 2016). Moreover, the EU-wide Solvency II regulation that came into effect from 1 January 2016 has introduced stringent capital requirements for insurance companies. For life insurance companies that write long-term deferred annuity business, it is difficult to predict liabilities in the distant future and hence they cannot perfectly match their liabilities. This results in life insurance companies needing to hold more capital for deferred annuities and leads to increases in the deferred annuity price. Practitioners have expressed the same view in terms of the price of buying out the DB pension liabilities, especially for deferred members. In 2015, PwC predicted that the new regulatory regime Solvency II could increase the cost of a pension buyout by 10% in a worst-case scenario (Cheong, 2015).

Another challenge for deferred annuities is the difficulty that the public has in understanding these products. The increased complexity of annuity products requires better communication of product features and risks to consumers so that consumers fully understand the products that they are purchasing. Therefore, the role of the financial advisor in helping customers to select the retirement products that are the most suitable becomes more important. Buying deferred annuities as the default option for a proportion of one's pension savings is also recommended as some pensioners may otherwise not consider buying annuities to protect themselves against outliving their resources. However, experience in the U.K. suggests that default options should be considered with caution. One concern is that the inertia to go with the default option may cause disengagement from the decision making process and stop people from shopping around for a better deferred annuity deal. Another concern is that for pensioners whose pension account balances fall below a certain amount, annuitisation would convert savings into a very small amount which may not be enough for minimum living costs and may affect their subsequent entitlement to welfare benefits (OECD, 2016).

There are several areas where the work of this thesis can be extended. First, the optimal retirement strategy proposed in Chapter 4 is based on an assumption that the only available investment choices are annuities and risk-free savings. In the real world, some retirees may like to access some riskier asset class such as equities to benefit from higher expected return; this is because the investment horizon for retirees is medium to long term and wealthier retirees have the ability to take higher risk. Therefore, the proposed retirement decumulation strategy could be extended to allow for a risky asset

class and we could then work out the optimal allocations and withdrawal strategies. Second, our current analysis is based on an assumption of fixed interest rates. These could be replaced by a stochastic interest rate model to achieve a more realistic retirement solution. Third, the proposed strategy is a static one-time decision at retirement without further rebalancing of the portfolio. Due to the movements in insurance and financial markets, the recommended retirement strategy could involve several rebalances of the portfolio, which could either be triggered by market movements or the elapsing of specific time periods, e.g. 5 years. A rebalanced portfolio could be helpful in adjusting risks and receiving benefits; for example, retirees would be suggested to underweight equity allocations in market downturns; and they can purchase additional annuities when interest rate is high and annuities are cheap. However, transaction costs that come with rebalancing will also need to be considered.

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