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CASS BUSINESS SCHOOL, CITY UNIVERSITY LONDON

PHD THESIS

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**Theoretical Essays on  
Bank Risk-Taking  
and Financial Stability**

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Ka Kei CHAN

THIS THESIS IS SUBMITTED FOR THE DEGREE OF  
DOCTOR OF PHILOSOPHY TO CASS BUSINESS SCHOOL,  
CITY UNIVERSITY LONDON

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# Declaration

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# Abstract

This thesis proposes theoretical models to study bank risk-taking and financial stability. Three issues are explored: (1) the moral-hazard incentive for securitisation, (2) the socially optimal banking structure for the economy, and (3) the relationship between bank competition and financial stability, based on bank funding structures and fire-sale risks.

Chapter 2 proposes a model to study how bank securitisation affects the value of bank equity, and hence what leads a bank to securitise its assets. The proposed model shows that moral hazard (which is induced by the deposit insurance scheme), can be one essential motive for the securitisation of deposit-taking commercial banks. This chapter also discusses some factors that can restrain the moral-hazard and risk-taking behaviour in bank securitisation.

Chapter 3 investigates the social value of different banking structures. The proposed model finds that total separation is not the optimal banking structure for an economy, because it forbids the liquidity transfer between subsidiary banks, which is socially valuable. The comparison between ring-fencing and universal banking is more complicated; Chapter 3 shows that whether ring-fencing or universal banking is the best banking structure for an economy depends on the returns to the different subsidiary banking sectors.

Chapter 4 studies how asset fire-sales risks and bank funding structures can affect the relationship between bank competition and financial stability. The proposed model finds that the funding-structure risks of the banks can create an incentive for excess risk-taking in a multi-bank economy. Moreover, the model shows that the excessive risk taking increases with the number of banks in the economy. This result is similar in spirit to the Cournot equilibrium in standard microeconomic theory.

# Abbreviations

<b>BHC</b>	Bank Holding Company
<b>BIS</b>	Bank of International Settlements
<b>CDO</b>	Collateralised Debt Obligation
<b>CDS</b>	Credit Default Swap
<b>CFI</b>	Capitalised Financial Intermediation
<b>CLO</b>	Collateralised Loan Obligation
<b>CRRA</b>	Constant Relative Risk Aversion
<b>CRT</b>	Credit Risk Transfer
<b>DFM</b>	Deposit Funding Mode
<b>FLP</b>	First Loss Piece
<b>FOC</b>	First Order Condition
<b>FSB</b>	Financial Stability Board
<b>G20</b>	Group of 20
<b>GMAC</b>	General Motors Acceptance Corporation
<b>HHI</b>	Herfindahl-Hirschmann Index
<b>ICB</b>	Independent Commission on Banking
<b>IMF</b>	International Monetary Fund
<b>NPL</b>	Non-Performing Loans Ratio
<b>SCLO</b>	Synthetic Collateralised Loan Obligation
<b>SFM</b>	Securitisation Funding Mode
<b>SOC</b>	Second Order Condition

# Chapter 1

## Thesis Introduction

### 1.1 Nature of This Study

This thesis proposes theoretical models to study bank risk-taking and financial stability. Three issues are explored: (1) the moral-hazard incentive for securitisation, (2) the socially optimal banking structure for the economy, and (3) the relationship between bank competition and financial stability, based on fire-sale risks and bank funding structures.

A common feature of these three issues is that they are all closely-related to the recent 2007-2009 global financial crisis and the following financial reforms.

This thesis contains three distinct research studies. Although the three studies are all motivated by the financial crisis and the regulatory response, the model framework is not closely related. Therefore, there is no separate chapter for literature review. Instead, each of the three studies contains its own literature review.

### 1.2 Purpose and Contribution

This thesis aims to provide some insights into bank risk-taking and financial stability. Three main questions are addressed in this thesis.

- How does the value of deposit insurance affect the motives for bank securitisation? Does it result in moral hazard, with securitisation increasing bank risk-taking? And how can this moral hazard be controlled?
- What is the socially optimal banking structure for the economy: is this separation of different banking activities or should they be combined within universal banks or under ring fencing? What are the factors that determine the optimal banking structure?

- Taking into consideration of the fire-sale loss, how does bank competition affect the banks' choice on their funding structures, which in turn affect the financial stability? What policy interventions can be used to improve financial stability?

Although several previous studies have explored related issues, these questions are still not yet fully answered. This thesis adds to the literature by focussing on some unexplored aspects. The contribution of each study in this thesis is further explained in Section 1.4 and in the introduction in each chapter. The implications for future research are discussed in the last chapter of this thesis.

### 1.3 Motivation

After the 2007-2009 financial crisis, one important aspect that have been widely discussed in literature is the credit expansion from bank securitisation. Prior to the crisis, many banks (such as Northern Rock) used the *originate and securitise* process to support their lending expansion. This type of business models allowed them to keep on expanding their asset portfolios and market shares.

The larger lending capacity in the banking system led to the emergence and growth of poor-quality loans, such as the sub-prime mortgage lending in the United States (Calomiris (2009) [23], Spiegel (2011) [74]). As a result, bank risk increased.

However, the increased bank risk was not reflected completely on bank funding costs due to the existence of deposit insurance scheme. Specifically, retail depositors are insensitive to bank risk due to the protection from deposit insurance. This advantage generates moral hazard in bank securitisation; the banks maximised their benefits during bank securitisation at the price of the enormous burden to the deposit insurance scheme. Unambiguously, there is a close relationship between bank securitisation and the moral hazard induced by deposit insurance. Finding out this relationship is the motivation for the first study in this thesis.

Another widely-discussed aspect is the rapid growth of the shadow banking sector. According to FRBNY staff report (2010) [39], the shadow bank liabilities in the US already exceeded the traditional bank liabilities in 1995, and continued to increase dramatically to almost 20 trillion US dollar before the 2007-2009 crisis. This is almost twice as large as traditional banking liabilities at that time (about 13 trillion US dollars). Governors and politicians have pointed out the shadow banking sector engages in banking activities, that are very different from the traditional banking sector. The Governor of the Bank of England, Mervyn King has distinguished the utility functions of the banking system from its riskier activities:

*"The banking system provides two crucial services to the rest of the econ-*

*omy: providing companies and households a ready means by which they can make payments for goods and services and intermediating flows of savings to finance investment. Those are the utility aspects of banking... they are quite different in nature from some of the riskier financial activities that banks undertake.” [56]*

UK Business Minister Vince Cable has made even pointed criticism, and described the issue as follows

*”Investment banking has, in recent years, resembled a casino, and the massive scale of gambling losses has dragged down traditional business and retail lending activities as banks try to rebuild their balance sheets.” [21]*

Following the crisis, new financial reforms, such as BASEL III,<sup>1</sup> were proposed. Some countries also started to consider structural reforms in the banking sector to promote financial stability. In the United Kingdom, the Independent Commission on Banking (ICB) suggests in the ICB final report [48] that ring-fencing of subsidiary banks with different banking activities should be introduced. ICB also suggests that promoting effective competition, in which banks compete to serve customers well rather than exploiting lack of customer awareness or poor regulation.<sup>2</sup> The European Commission also formed a *High-Level Expert Group* to consider structural reforms of EU banks [38]. The discussion in the European Commission is still on-going, and no suggestion has been made before the completion of this thesis.

The recent suggestions for the structural reform of the banking sector motivate the second and third studies in this thesis. The second study focussing specifically on the recommendations of the ICB final report, aims to explore the social value of different types of banking structures. In particular, it addresses the question of whether ring-fencing of the kind proposed by the ICB is the socially optimal banking structures.

The third study examines another question, the relationship between bank competition and financial stability. Focussing on a different aspect from the existing literature, the third study takes into the consideration of asset fire-sales in financial distresses, and from this alternative aspect, revisits the long-debated question of the relationship between bank competition and financial stability.

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<sup>1</sup>Allen et al (2012) [6] provides an overview and some discussions on the proposal of BASEL III.

<sup>2</sup>The government’s response to the ICB final report (Armstrong (2012) [8]) accepts ICB competitive proposal, but the response also points out that some suggestions in the ICB final report (including holding prudential buffers and excess capital to avoid excess risks) are inconsistent concepts in a competitive market.

## 1.4 Thesis Organisation

The rest of this thesis is organised into four chapters.

Chapter 2 proposes a model to study how bank securitisation affects the value of bank equity, and hence what leads a bank to securitise its assets. The proposed model shows that moral hazard (which is induced by the deposit insurance scheme), can be one essential motive for the securitisation of deposit-taking commercial banks. Specifically, bank securitisation, driven by moral hazard, can be used as a tool to increase the insolvency risk of a bank, and this in turn increases the value of deposit insurance, which can be considered as a component of the value of bank equity.

Yet, the excessive risk-taking behaviour can endanger the deposit insurance scheme and financial stability, and this needs to be restrained. Therefore, this chapter also used the proposed model to analyse how best to control this moral hazard in securitisation. The analysis shows that some commonly suggested policy interventions, such as increasing the minimum size of the retained tranche(s), can be ineffective in controlling the moral hazard induced by deposit insurance.

This chapter suggests two factors to restrain the risk-maximising behaviour: (1) a carefully-designed and risk-sensitive deposit insurance premium can be an efficient policy to control the risk-taking behaviour in securitisation, and (2) franchise value of banks is valuable to the equity holders; banks with high franchise value are less-likely to be seduced to maximise the value of deposit insurance (moral hazard); this result implies that policies that promotes the franchise value of banks can also help control the risk-taking behaviour in securitisation.

What is new in this chapter is the study of moral hazard (induced by deposit insurance) as a motive for securitisation. This is different from most previous studies which explore the motive originated from arbitrage profit, which can be a result of asymmetric information, market friction or incomplete information from credit rating. This chapter proposes a model and finds that moral hazard induced by deposit insurance can be another motive for securitisation. The model also shows how bank insolvency risk can substantially vary, depending on how the proceeds from securitisation are used.

Chapter 3 studies the social value of different banking structures. This chapter begins with the construction of a simple model to characterise a banking group that consists two different subsidiary banks. The model distinguishes safe *utility* subsidiary bank from riskier *casino* subsidiary bank. Under these model specifications, three types of banking structures: (1) total separation, (2) ring fencing, and (3) universal banking are studied.

In this chapter, banking structures are defined mainly based on the restriction of capital and liquidity transfer between subsidiary banks. Under each type of banking

structure, the model derives the required return demanded by the consumers, the expected net return to each subsidiary bank and to the banking group, and the cost of deposit insurance. The model then compares the social values under the three banking structures.

The proposed model suggests that, total separation is always suboptimal to both the banking group and the economy as a whole. Whether ring fencing or universal banking is the best banking structure depends on other factors. This model conclusion suggests an insightful view on the choice of banking structures: whether it is socially beneficial to protect utility banking sector with ring fencing should include careful assessment on (1) the social value and cost of liquidity transfer, and (2) the return to utility and casino subsidiary banking sector.

What is new in this study is the modelling of the trade-off under different banking structures. It investigates how the trade-off affects the risk and the return to subsidiary banks and banking group, the required return demanded by the consumers, the cost of deposit insurance and the social value.

Chapter 4 explores how banks' funding structure and fire-sale risks affect the relationship between bank competition and financial stability. This chapter applies a simple liquidity modelling framework and shows that fire-sale, which has rarely been included in the discussion of the debated topic, plays an important role.

An important finding in this chapter is that, the existence of fire-sale can create an incentive for banks' excessive risk-taking. This incentive is originated from the fact that in a multi-bank economy, a bank can take advantage of other banks in fire-sale by choosing a riskier funding structure.

I also show that, in equilibrium, the magnitude of the risk-taking incentive increases with the number of banks in the economy (the measure of bank competition in this chapter). This result shows that banking competition can weaken financial stability. The chapter then discusses the efficiency of some policy interventions as the controls of the excessive risk-taking due to bank competition.

What is new in this chapter is how fire-sale risks and bank funding structure affect the relationship between bank competition and financial stability. Although numerous studies have been done on this topic, these studies mainly focus on the asset risks of financial institutions: the risks come from the choice of investment portfolio, the profit margin from asset returns, and the loan defaults. However, these asset risks only cover the risks that are originated from one side of the banks' balance sheet. The risks from the other side, the bank funding structures, have rarely been discussed. This chapter adds to the literature by exploring how fire-sale risks and bank funding structures can affect the debated topic.

Chapter 5 summarises and concludes this thesis, identifies the limitations of the proposed models, and suggests possible directions for future research.

# Chapter 2

## Bank Securitisation, Moral Hazard and Efficient Controls

### 2.1 Introduction

Prior to the current credit crisis, securitisation was a crucial part in the credit market; it has been used to raise more than US\$12.9 trillion in the global system (estimated by Milne (2009) [62], table 2.2, page 22). In 2007, more than half of the money borrowed in the US credit markets was financed through asset-backed securities. Yet, in just a few months, such securities caused trillion dollars of loss and paralysed the credit market.

Many people believe that this is the end to securitisation and of the market for loan-backed securities. However, a recent report of the International Monetary Fund has argued otherwise (IMF (2009) [49]). The IMF report suggested that restarting bank securitisation is crucial to limiting the real sector fallout from the credit crisis. Its views were supported by the Bank of International Settlements (BIS) and Group of 20 (G20) (FSB (2009) [40]). Economist, Gary Gorton, also pointed out in a recent document to U.S. Financial Crisis Inquiry Commission (Gorton (2010) [43]) that *“Securitization is an efficient, cheaper way to fund the traditional banking system.”*

There is a small theoretical literature on securitisation (reviewed in Section 2.2), but surprisingly this does not address one of the most obvious reasons for securitisation: the risk transferring onto bank safety net (deposit insurance scheme). Much previous research has focussed on the weaker incentives for monitoring loan qualities when banks securitise, and analyses different retention rules. However, this literature does not address moral hazard.

The objective of this paper is to disentangle different motives for securitisation; specifically, I distinguish a risk-taking and a funding motivations.

- Under the risk-taking motivation, a bank uses the proceeds from securitisation

for further lending, i.e. more bank loans are originated. This increases the bank insolvency risk even if the new loans have the same quality as the securitised ones; this is because the retained first loss piece (FLP) or equity tranche is embedded with high default risk. Therefore, securitisation can increase the insolvency risk of a bank.

- Under the funding motivation, securitisation is used as a substitute for the existing debts (for example, the proceeds can be used to replace the costly short-term wholesale funding); this reduces the leverage ratio of the bank, resulting in a decrease in the insolvency risk of a bank.

Under both motivations, securitisation is used as a financing tool, but the effects on the bank risk are opposite.

This chapter proposes a model to study how securitisation affects the value of equity, and hence what leads a bank to securitise its assets. The proposed model shows that moral hazard (which is induced by the deposit insurance scheme), can be one essential motive for the securitisation of deposit-taking commercial banks. Specifically, bank securitisation, driven by moral hazard, can be used as a tool to increase the insolvency risk of a bank, and this in turn increases the value of deposit insurance, which can be considered as a component of the value of equity.

Yet, the excessive risk-taking behaviour can endanger the deposit insurance scheme and financial stability, and this needs to be controlled. Therefore, this chapter also used the proposed model to analyse how best to control this moral hazard in securitisation. The analysis shows that some commonly suggested policy interventions, such as increasing the minimum size of the retained tranche(s), can be ineffective.

This chapter suggests two alternative factors to restrain the risk-maximising behaviour: (1) a carefully-designed and risk-sensitive deposit insurance premium can be an efficient policy to control the risk-taking behaviour in securitisation, and (2) franchise value of banks is valuable to the equity holders; banks with high franchise value are less-likely to be seduced to maximise the value of deposit insurance (moral hazard); this result implies that policies that increase the franchise value of banks can also help control the risk-taking behaviour in securitisation.

The numerical results in this chapter show that these two policy interventions can both restrain the moral hazard in securitisation, but in two different ways. Deposit insurance premium controls moral hazard by reducing the size of securitisation; franchise value encourages using securitisation as a tool for replacing costly wholesale funding (which leads to a lower bank insolvency risk), rather than as a tool for loan expansion. These two controls, if properly used, can help support the revival of a healthier securitisation market.

To summarise, this chapter contributes to the literature in two ways. First, it demonstrates how bank moral hazard can act as an incentive for securitisation. And second, it analyses the impact of different factors to restrain the moral hazard in securitisation. To the best of my knowledge, this is the first theoretical paper which explores these relationships among securitisation motives, deposit insurance and franchise value.

The chapter is organised in seven sections. Section 2.2 provides a brief literature review in theoretical modelling and other related studies for securitisation. Section 2.3 presents the model specifications. Section 2.4 derives the impact of moral hazard on bank securitisation decision, and introduces the efficient controls for moral hazard. Section 2.5 studies an alternative proceeds allocation, the provision of wholesale lending. Section 2.6 presents numerical results of the model, examining the effects of both deposit insurance premium and franchise value to bank securitisation. Section 2.7 concludes.

## **2.2 Literature Review**

There has been a large empirical literature on securitisation, but relatively little theoretical modelling. In this section, I review the literature as follows: (1) first, I look at the related literature on loan sales, a financing technique with similar implication for monitoring as securitisation; (2) then, I review some important theoretical models of securitisation; and finally (3) related empirical studies and other relevant works are discussed.

### **2.2.1 Review of Literature on Loan Sales**

The main features which differentiate securitisation from loan sales are the integrity of underlying loans. In the process of loan sales, an originator bank usually sells a certain portion of each loans (e.g. 80% of each loan in the pool), the remaining portion is retained on the balance sheet of the originator bank. When the underlying loans default, the originator bank and the loan buyers share the default loss according to their holding proportions. The aim for loan retention is to signal the quality of loans being sold so that the cost of asymmetric information is reduced.

On the other hand, the process of securitisation usually aggregates a pool of non-subdivided loans (in order to satisfy the definition of a true sale which is required by laws). The originator bank retains the FLP (or equity tranche) which absorbs the initial losses; therefore the other investors suffer from default loss only when the loss exceeds the retained portion. If the proceeds are used for liquidity purposes instead of reinvestment, a loan sale reduces the overall insolvency risk of a bank,

while a securitisation reduces some, but not all, of the tail risk of a bank. Krahen and Wilde (2008) [57] develop a Monte-Carlo simulation model to show that senior tranches bear some of the tail risk; yet a significant portion of tail risk is still remained in the equity tranche.

Pennacchi (1988) [67] develops a model to study loan sales of banks. His model is a single-period and continuous-state model which maximises the gain of a bank by choosing optimal level of loans, monitoring efforts, deposit and equity investment. His model assumes that there is a capital requirement for banks, and the bank depositors are protected by a fairly-charged deposit insurance. Pennacchi's model suggests that loan sales reduce the banks' capital requirement (minimum equity investment), and therefore decrease the cost of capital.

Gorton and Pennacchi (1995) [42] also study loan sales. Their paper constructs a model which assumes a bank sells a portion of a single loan, and at the same time guarantees to buy back the sold loan at a pre-specified price if it defaults. Their model assumes that the loan return depends on a continuous distribution which is a function of bank monitoring effort. The loan return is also assumed to be independent of the bank's insolvency probability. Their paper shows that a bank sells a larger portion (i.e. retains a smaller portion) of the loan if (1) the deposit rate is high, (2) the loan is less risky, and (3) the bank's probability of solvency is high. Their model also shows that bank chooses less-than-efficient levels of credit screening when portions of loans are sold and not fully guaranteed.

Early literature on loan sales, although some of them consider the return on the retained first loss piece (FLP) (which is equivalent to equity tranche), do not derive the required return for the remaining sold portion (which has a similar structure as a senior tranche). This is crucial in the determination of the pay-off to the FLP (or equity tranche) under different states of the economy. This problem has been addressed in later works on securitisation.

### **2.2.2 Review of Theoretical Literature on Securitisation**

Jiangli, Pritsker and Raupach (2007) [51] introduce a monitoring-based model which studies both loan sales and securitisation. Their model is constructed under a one-period and finite-state framework, in which a bank maximises its equity value by choosing some optimal decision variables, under a monitoring-incentive constraint. Their model concludes that securitisation increases bank profit; it also shows that the effect on the risk level varies according to different circumstances.

However, there is a minor problem in their definition of securitisation as a synthetic collateralised loan obligation (SCLLO). They define securitisation in the same way as the definition of loan sales defined in Gorton and Pennacchi (1995): a pro-

cedure of retaining a portion of every sold (securitised) loan and at the same time providing credit protection to the investors. This is different from a typical definition of SCLO which is defined as the securitisation by pooling a portfolio of credit default swap (CDS).

The core difference between Jiangli, Pritsker and Raupach (2007) and the proposed model in this chapter is that I relate the insolvency risk of bank (which is assumed to be exogenous in Jiangli et al's paper) with the loan defaults; this enables the proposed model to study how loan defaults affect the insolvency of a bank, which is crucial in determining the moral hazard induced by deposit insurance.

Shin (2009) [71] does not model bank securitisation explicitly; instead, he proposes a accounting-based model to capture the borrowing and lending within the banking sector and between banking sector and non-banking sector. Based on the assumption that banks choose their face value of equity according to a certain value-at-risk level, the model shows that a decrease in loan default rate creates excessive equity on the balance sheet of banks. Based on further assumptions that excessive equity are used to support more debts and the new debts comes from non-banking sector (through the channel of issuing securities backed by assets, or securitisation), the model finds that the aggregate lending to the non-banking sector increases. Shin also suggests that lending boom can be the result of a feedback loop between the loan default rate and aggregate lending. Shin argues that securitisation plays a role to facilitate the origination of new bank debts from the excessive borrowing from non-bank sectors; moreover, he also argues that lending standards decrease is a natural result when the number of good prime borrowers is relatively small.

Wagner and Marsh (2006) [81] study the credit risk transfer (CRT) within the banking sector, and across sectors (from the fragile banking sector to the less fragile non-banking sector). They find that (1) an increase in both type of CRT enhances efficiency; (2) an increase of CRT within the banking sector reduces financial stability, but the increase of CRT across sectors can improve stability under a specific condition. They suggest a stricter capital requirement in the banking sector (compared with the non-banking sector) encourages a socially optimal allocation of risk across sector; a level playing field for regulation is therefore sub-optimal according to their model result.

A brief outline of Wagner and Marsh's model is described as follows. The proposed model has a two-date time horizon; on date 0, entrepreneurs decide how much capital and effort to be invested in their productions, which produce a random output on date 1. The randomness of the output consists of an idiosyncratic (region-specific) component and an aggregate component. The model assumes that entrepreneurs only live on date 0; therefore, they sell their production to banks and non-banks. Banks has a monitoring technology to observe entrepreneurial effort,

but bank financing is more costly due to the fragility of banks; non-banks only have a pure channelling function. Fragility of banks is captured by higher private and social costs during bankruptcy for banks than for non-banks. As the banks do not internalise the social cost of bankruptcy, the equilibrium allocations in the economy is inefficient.

CRT within the banking sector is defined as allowing banks to swap their regional-specific claims for the diversification of firm risk. Wagner and Marsh find that an increase in CRT encourages additional bank risk-taking. Since aggregate risk cannot be diversified, the increased aggregate risk from the additional risk-taking can outweigh the stabilising impact of the diversification of idiosyncratic risk, leading to weaker financial stability. CRT across sectors is defined as the selling of a fraction of asset from banks to non-banks. Wagner and Marsh show that for CRT to be stability-improving, the risk should flow from the socially more fragile banking sector to the less fragile non-banking sector; moreover, the social fragility (social cost of bankruptcy) of the banking sector relative to that of the non-banking sector has to be higher than the relative private fragilities (private cost of bankruptcy).

Chiesa (2008) [29] presents a static model to study the optimal credit risk transfer (CRT), under an asymmetric information framework. Chiesa's model assumes that there are two states of economy at the end of period and a bank can choose whether or not to monitor a continuum portfolio of loans. The only situation in which a bank becomes insolvent is a combination of no monitoring and bad economy; otherwise the bank survives. The model shows that debt financing always leads to a suboptimal level of lending compared with equity financing (the author admitted that this can lead to distorted conclusion, last paragraph on page 470).

If CRT is allowed, banks can restore to an optimal level of lending under debt financing. However, there is no natural incentive for CRT under the specification of Chiesa's model. Chiesa's paper suggests that a prudential capital requirement is a solution for the motivation problem.

The model suggested in Greenbaum and Thakor (1987) [44] is most closely related to the proposed model in this chapter. Their paper proposes a theoretical model to study deposit funding mode (DFM) and securitisation funding mode (SFM) for commercial banks. Their model is based on a two-dated (single period) framework and assumes that there are two possible outcomes on date 1 (end of period): the bank survives (if the bank loan does not default), and the bank fails (if the bank loan defaults). Under their framework, a bank can choose either DFM or SFM on date 0. Their model begins with an economy without asymmetric information, deposit insurance and bank regulation, under which the model shows that the two funding modes are equivalent (this is consistent with Proposition 2 in this chapter). And under asymmetric information, the model assumes that the quality of a loan

is signalled by an insurance coverage. This leads to a conclusion that good loans are funded by securitisation (because loan borrowers can lower the funding cost by choosing a high insurance coverage) and the bad loans are funded by deposit (because the cost of insurance coverage is too high and outstrips the reduction in funding cost).

The proposed model in this chapter extends the first part (symmetric-information economy) of Greenbaum and Thakor (1987) by incorporating continuous states of economy; this allows the model to specify the default rates of the loans in the form of a factor model, under which the problem of loan correlation are addressed by the property of conditional independence. The numerical illustration in this chapter also relaxes the restriction on funding modes to study the optimal level of securitisation. I also distinguish the different roles between retail depositors, wholesale investors, senior-tranche investors to further explores different sources of funding available to the banks.

Unlike most of the previous research, the proposed model in this chapter does not focus on the monitoring incentive constraint. I argue that this constraint may not be an appropriate reflection of the market practices. The aim for introducing a monitoring incentive constraint is to lower the cost of asymmetric information, so that the securitised assets are less under-priced by the investors. However, from previous studies, these securities are more often over-priced due to the asymmetric information and market friction; this provides a bank with an arbitrage profit during the process of securitisation.

Odenbach (2002) [65] suggests that some financial institutions, such as pension funds, cannot originate loans in the same way as banks; this market friction creates some rooms for an arbitrage profit. Brennan, Hein and Poon (2009) [19] studies arbitrage securitisation and suggests a model to capture this arbitrage profit, which is a mismatch between the yields of securitised assets and the yields of the securitised tranches. Their paper shows that there are positive gains to the investment bankers if the investors rely solely on the rating information about the securities. In particular, the gains are significantly higher if the investors choose to use a default-probability based rating system (Standard & Poor's and Fitch) rather than a default-loss based system (Moody's). Their model also shows that the arbitrage gains are higher if the investment bankers securitise with multi-tranching.

The above studies support the fact that senior tranches sold in bank securitisation may not be under-priced due to asymmetric information. In contrast, this chapter, which aims to study the moral hazard induced by deposit insurance in bank securitisation, assumes that symmetric information is available; this rules out the arbitrage-profit related motives in bank securitisation and allows the model to apply standard asset pricing method in a no-arbitrage environment.

### 2.2.3 Empirical Studies and Other Related Works

There is a large literature on the empirical studies of bank securitisation; most of these studies attempt to find out the motives of securitisation and how it affects the profit, risk, and other characteristics of banks.

Bannier and Hansel (2007) [9] aims to find out which types of banks are more likely to perform securitisation. They apply firm-specific data and macroeconomic data from 1997 to 2004, with a sample of 316 European banks and find that a bank is more likely to securitise if it has a larger amount of asset, a higher credit exposure, lower liquidity, and lower performance (measured by cost-income ratio). Moreover, some macroeconomic data (such as GDP growth and interest rates) also seem to be positively correlated to the banks' securitisation decisions.

Hansel and Krahnert (2007) [45] find evidence from the European CDO market that securitisation (in the form of CDOs) increases the systematic risk of banks, especially for the banks which are financially weak (low profitability and high leverage).

Cardone-Risportella et al (2010) [25] study the motivations for bank securitisation in Spain. They collect 408 observations from Spanish commercial banks, savings banks and credit cooperatives between 2000-2007; using logistic regression, they find that Spanish banks securitise mainly to provide liquidity and to improve efficiency. From their collected data, they find no evidence to support that securitisation is used for the transfer of credit risk and for regulatory capital arbitrage; this is an important evidence to support the suggestion in this chapter, that bank securitisation does not necessary reduce bank risk; this chapter suggests that the moral hazard from deposit insurance can be a possible reason.

Rosenthal and Ocampo (1988) [68] discusses the benefits of balance sheet securitisation, with a case study of a commercial finance group, General Motors Acceptance Corporation (GMAC). Their paper suggests that balance sheet securitisation is more efficient compared with conventional lending, mainly in the perspective of how securitisation reduces the risk by isolating loans from balance sheet. However, the claims of loans being more transparent and the investors' better understanding of risk both lack empirical evidence.

Odenbach (2002) [65] reviews the background for securitisation and suggests that there are six motives for securitisation, which includes economic risk transfer, cheaper funding, diversification of funding sources, market arbitrage, improvement of capital ratios and solvency ratios. Odenbach's paper also suggests that it is less likely for banks with a high rating and with access to cheap funding sources (such as debts with high credit ratings and deposits with low interest rates) to benefit largely from securitisation, which can be more expensive than the traditional sources. However,

due to the regulations, banks are not permitted to finance all the loans with the cheap and unsecured funding sources; therefore there is still room for securitisation in the high-rating banks to support their funding requirement. On the other hand, financial institutions with low credit rating are more likely to securitise because they can construct securitised pools with higher credit ratings than their own ratings. In doing so, they benefit from securitisation which provides a cheaper source of funding than the traditional sources.

## 2.3 Model Specifications

In this model, the proceeds from securitisation are assumed to be used in *one* of two ways: (1) originating new loans and (2) providing wholesale lending to other banks. Although the two ways of proceeds allocation have very different effects on the bank insolvency risk, their modellings are very similar. To avoid cumbersome explanation, this chapter focuses on the explanation of the first proceeds allocation (originating new loans) throughout Section 2.3 and 2.4; the second proceeds allocation (providing wholesale lending) is explained in a more compact way in Section 2.5.<sup>1</sup>

Table 2.1 summarises all notations that are used in the model.

### 2.3.1 The Economy and Asset Pricing

This model is based on a 2-dated time horizon. All decisions are made under uncertainty on date 0, and the uncertainty is resolved on date 1.

The economy condition is the unique source of uncertainty in the model; it is characterised by a continuously-distributed random variable  $\theta$ , which follows the probability density function  $f(\theta)$ .

The model assumes that every agent in this model has symmetric information, which means that everyone is well aware of the risk of investments; therefore, no arbitrage opportunity exists in the economy. The investors determine the required rate of return for an asset according to the risk they have to bear, such that the present value of the expected return equals the cost of the initial investment.

The assumption of symmetric information has one major advantage; it produces a no-arbitrage environment so that the model can price bank equity and other assets based on standard asset pricing method. It is this method that supports the propositions derived in the coming sections.<sup>2</sup>

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<sup>1</sup>In Section 2.6, numerical examples are used to study bank decision when a bank can allocate the securitisation proceeds in both proceeds allocation.

<sup>2</sup>For the readers that need renewal of this standard asset pricing method, Cochrane (2005) [33] provides an excellent introduction to the method applied in this chapter.

Notations	Definitions
$V$	Value of equity
$A$	Book value of bank asset
$S$	Book value of the securitised loans
$L$	Book value of non-securitised bank loans
$L_n$	Book value of new bank loans
$W$	Book value of wholesale borrowing
$W_n$	Book value of wholesale lending
$D$	Book value of retail deposit
$E$	Book value of bank equity
$\phi$	Width of equity tranche
$\tilde{D}$	Value of deposit insurance
$\tilde{C}$	Deposit insurance premium
$\Phi_1$	Date-1 Franchise Value
$\Phi_0$	Date-0 (present value of) franchise value
$r_{min}$	Minimum required rate of return for original loans
$r$	Loan rate for original loans
$r_n$	Loan rate for new loans
$r^D$	Retail deposit rate
$r^W$	Wholesale borrowing/lending rate
$r^S$	Senior tranche rate
$m(\theta)$	Conditional return from the two loan portfolios
$n(\theta)$	Conditional return from the equity tranche
$\alpha$	Random variable that characterises loan default
$\bar{\alpha}$	Default threshold for non-securitised loans
$\bar{\alpha}_n$	Default threshold for new loans
$a$	Factor loading
$\beta$	Subjective discount factor
$\delta$	CRRA Coefficient
$\varepsilon$	Specific risk variable
$\theta$	Economy state variable
$c_0$	Consumption level on date 0
$c_1(\theta)$	Consumption level on date 1
$f(\theta)$	Probability density function for $\theta$
$Z(\theta)$	Stochastic discount factor
$\gamma(\theta)$	Default rate of existing bank loans
$\gamma_n(\theta)$	Default rate of new bank loans
$u(\cdot)$	Utility function

Table 2.1: Table of Notation in Chapter 2

The stochastic discount factor  $Z(\theta)$  used in the pricing of asset is defined by the marginal utility of consumption according to classical asset pricing approach,

$$Z(\theta) = \beta \frac{u'(c_1(\theta))}{u'(c_0)}$$

where  $\beta$  is the subjective discount factor,  $u(\cdot)$  is the utility function,  $c_0$  is the date-0 consumption level which is a known parameter, and  $c_1(\theta)$  is the date-1 consumption level whose realisation depends on the future economy condition  $\theta$ .

### 2.3.2 Bank and Bank Loans

This model studies bank decisions on securitisation in the economy. On date 0, a bank has an existing portfolio of bank loans (bank asset), which is funded by retail deposit, wholesale borrowing and bank equity. The objective of the bank is to maximise its value of equity, by choosing an optimal level of securitisation of bank loans on date 0.

On date 1, depending on the realised economy condition, some bank loans default. The bank receives the return from non-defaulted bank loans, and repay the cost of funding; the remaining profit after the funding-cost repayment is the net return to the bank. In some extreme economy conditions in which the loan returns are insufficient to cover the funding costs, the bank becomes insolvent and goes bankrupt.

The bank loans are assumed to be continuum, with a mass measure of  $A$ . All bank loans are identical and are single-period simple loans which sell at par value.<sup>3</sup> The model assumes that the default probability for bank loans is identical, and is dependent on the economy condition on date 0. The default probability is denoted as  $\gamma(\theta)$ . For simplicity, I also assume that the defaulted loans have *no recovery value*.

In this model, I employ a standard tool of portfolio credit risk analysis, the factor-copula model, for the specification of default probability. The factor-copula model was suggested by Vasiek (2002) [76], and was used also in BASEL II (to determine risk weighting of assets) and the pricing of credit derivatives (Hull and White (2004) [47]). The advantage of applying one-factor-copula model in this model is that it allows the model to use a single factor to model both the default rate of the loan portfolio and the pricing of bank equity and other assets within a simple model framework. The specifications are given as follows.

Assume that there exists a random variable  $\alpha$  which characterises the default of a bank loan; this random variable is given by

$$\alpha = a\theta + \sqrt{1 - a^2}\varepsilon$$

where  $a$  is the factor loading for the common factor  $\theta$ , and  $\varepsilon$  is another random variable which represents the specific factor to the bank loan; both  $\theta$  and  $\varepsilon$  are

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<sup>3</sup>The bank lends the book value of loans to the borrowers on date 0, and receives the book value together with an pre-specified interest on date 1 if the loans do not default.

assumed to follow standard normal distribution  $N(0, 1)$ .

The bank loan is assumed to default if  $\alpha < \bar{\alpha}$ . Therefore, the default probability  $\gamma(\theta)$  can be specified by

$$\gamma(\theta) = \Pr(\alpha < \bar{\alpha}|\theta) = N\left(\frac{\bar{\alpha} - a\theta}{\sqrt{1 - a^2}}\right)$$

where  $N(\cdot)$  is the cumulative density function of standard normal distribution.

It is worth mentioning that due to the specification of the factor-copula model, the default of each bank loan, conditional on  $\theta$ , is independent to each other. And, due to the continuum nature of bank loans, the (conditional) default probability  $\gamma(\theta)$  is equal to the (conditional) proportion of default.

With the default probability, the bank can determine its required rate of return to the bank loans. Under a no-arbitrage environment, the expectation of the discounted date-1 return should equal to the book value of the investment. Recall that the book value of bank loans is denoted as  $A$ , and let  $r$  be the required minimum rate of return for the loans,  $r$  is determined by solving

$$A = \int_{-\infty}^{\infty} r_{min} A (1 - \gamma(\theta)) Z(\theta) f(\theta) d\theta$$

or simply

$$1 = r_{min} \int_{-\infty}^{\infty} (1 - \gamma(\theta)) Z(\theta) f(\theta) d\theta$$

The term  $r_{min}(1 - \gamma(\theta))Z(\theta)$  represents the discounted return from one unit of bank loan, conditional on the economy condition  $\theta$ . Integrating the discounted returns with the corresponding probability density function  $f(\theta)$  derives the expectation of the discounted future returns. In the remainder of this chapter I will assume that the lending rate of loans  $r = r_{min}$ . In a static model of this kind this is simply a convenience, I could instead assume that banks have market power in their loan market and  $r > r_{min}$ , this would increase their profits but not affect the incentives to securitise.<sup>4</sup>

### 2.3.3 Bank Securitisation

Bank securitisation of bank loans is specified in this subsection. As mentioned, this chapter studies two types of proceeds allocation: (1) originating new loans and (2) providing wholesale lending to other banks. The securitisation specified in the following characterises the former type of proceeds allocation; the latter type is specified in Section 2.5.

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<sup>4</sup>Similar assumptions are also made to the lending rate of new loans and the rate of return to senior tranche.

Before the proper mathematical expression for securitisation is introduced, it is helpful to understand how securitisation affects the balance sheet of a bank. The following is a figure which summaries asset portfolio of a bank before and after securitisation (if the proceeds from securitisation are used to provide new loans).

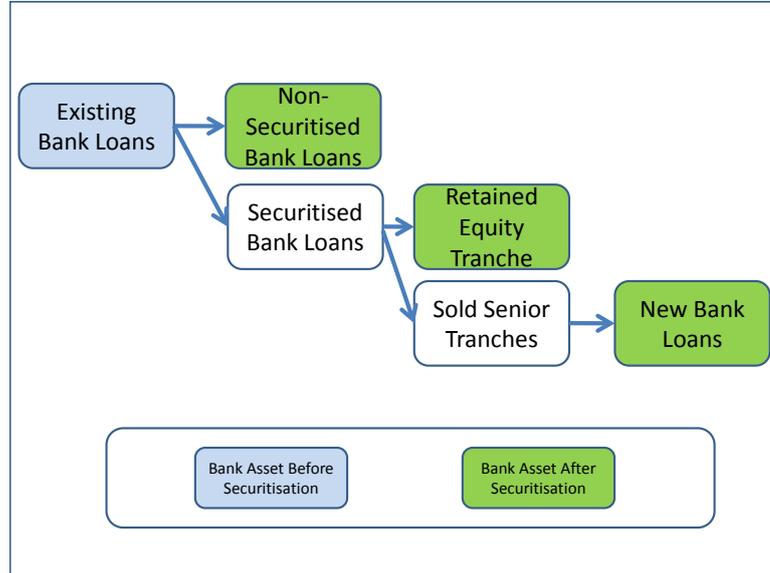


Figure 2.1: Bank asset before and after securitisation, with securitisation proceeds used for the provision of new loans.

In this model, bank securitisation is specified as the pooling of bank loans into a collateralised debt obligation (CDO), which is divided into different tranches according to the risk levels. The junior most tranche, called the equity tranche, absorbs the initial loss to the pool of bank loans and is therefore the riskiest tranche of all. It is assumed that the equity tranche is retained in the bank. Other senior tranches is sold to the public. Due to the different risk levels, equity tranche has the highest rate of return for the compensation of the high default risk. For simplicity, the senior tranches are considered as one senior tranche for the derivation of the senior-tranche return.

Securitisation has three important features on the asset side of a balance sheet. First, it reduces the amount of the existing bank loans; this is because some existing loans are securitised. The size of securitisation is limited by the size of the loan portfolio. This constraint is given by

$$S = A - L \leq A$$

where  $L$  denotes the book value of non-securitised loans that remains on the balance sheet of the bank, and  $S$  is the book value for the securitised loans, which is also the book value of the securitised loan pool. The maximum amount of securitisation is  $S = A$ , which implies all existing loans are securitised, and therefore  $L = 0$ .

The second feature is that a retained portion (equity tranche) of securitisation is kept on the bank balance sheet. This retained portion functions as a buffer to absorb the initial loss of the securitised loan pool. The non-retained portion (senior tranche) is bundled into securities and is sold to the public. In the following, the book value of the equity tranche is denoted as  $\phi S$  where  $\phi$  is the proportion of retention in the securitisation  $S$  (in other words, the width of equity tranche). The senior tranche, which is the complement of equity tranche, is denoted as  $(1 - \phi)S$ .

Since an equity tranche absorbs the initial loss from the securitised loans, the equity-tranche holder (the bank) receives its return after the required return for the senior-tranche holders is fully repaid. In the cases where the senior-tranche holders do not obtain their promised return, the equity-tranche holder receives nothing. The pay-off to the equity-tranche ( $\phi S$ ) is expressed by

$$\max[0, rS(1 - \gamma(\theta)) - r^S S(1 - \phi)]$$

where  $r^S$  is the required rate of return for the senior-tranche  $((1 - \phi)S)$ .

Another way for expression is to apply an indicator function. Let

$$\mathbb{I}_S = \begin{cases} 1 & \text{if } rS(1 - \gamma(\theta)) - r^S S(1 - \phi) < 0 \\ 0 & \text{Otherwise} \end{cases}$$

This indicator function divides the conditions under which the equity tranche has a positive return from those in which the equity tranche receives nothing. The expression for the equity-tranche pay-off can be rewritten as

$$[rS(1 - \gamma(\theta)) - r^S S(1 - \phi)](1 - \mathbb{I}_S)$$

The value of  $\phi$  is exogenous. Typically it will be very small. The required return to the senior tranche will be determined by the following equation

$$(1 - \phi)S = \int_{-\infty}^{\infty} [r^S S(1 - \phi)(1 - \mathbb{I}_S) + rS(1 - \gamma(\theta))\mathbb{I}_S] Z(\theta)f(\theta)d\theta$$

The first term on the right hand side stands for the full repayment to senior-tranche holders when the equity tranche survives  $((1 - \mathbb{I}_S) = 1)$ . The second term refers to the situation in which senior-tranche holders receive all proceeds from securitisation ( $S$ ) because their promised return is not fulfilled; i.e., equity tranche is *dead* ( $\mathbb{I}_S = 1$ ). Again, under the no-arbitrage environment, the expected value of the discounted pay-off should equal to the book value of senior tranche,  $(1 - \phi)S$ . Similarly, the

return to the equity tranche can be expressed by

$$\phi S = \int_{-\infty}^{\infty} [rS(1 - \gamma(\theta)) - r^S S(1 - \phi)] (1 - \mathbb{I}_S) Z(\theta) f(\theta) d\theta$$

which represents the residual return from the securitised loans after the promised return to senior tranche has been fulfilled.

The last feature is the proceeds allocation. According to the first type of proceeds allocation, the proceeds from securitisation are used to originate new bank loans. I assume that the new bank loans are riskier than the securitised bank loans. The aim of this assumption is to capture the fact that the loans with good quality are finite; when the banks keep originating new loans, the quality of loans should deteriorate, this is consistent with the increase in sub-prime loans and other low-quality loans in the 2007-2009 financial crisis. The book value of new loans is denoted as  $L_n$ .

The book value of new loans is determined by the proceeds from securitisation. Note that the bank needs to retain the equity tranche; therefore the proceeds should be equal to the book value of securitised loans, minus the book value of equity tranche (or simply, the book value of the senior tranche). This is given by

$$L_n = (1 - \phi)S$$

The default rate and the loan rate for new loans are denoted as  $\gamma_n(\theta)$  and  $r_n$  respectively. They are obtained in a similar way as those of the old loans ( $\gamma(\theta)$  and  $r$ ), but with a higher default threshold ( $\bar{\alpha}_n$ ).

It is worth mentioning that the size of the balance sheet does not change due to securitisation.

$$\begin{aligned} A &= L + S \\ &= L + (1 - \phi)S + \phi S \\ &= L + L_n + \phi S \end{aligned}$$

This is consistent with the definition of securitisation given by Jeffrey (2006) [50], which defines securitisation as the process of monetising the bank loans.

After securitisation, the bank assets ( $A$ ) can be subdivided into three parts: two loan portfolios ( $L$  and  $L_n$ ) and one equity tranche ( $\phi S$ ). These assets have different expected returns. To simplify the expressions, I abbreviate them with the following notations. Let the conditional return from the two loan portfolios be

$$m(\theta) = rL(1 - \gamma(\theta)) + r_n L_n(1 - \gamma_n(\theta))$$

and the conditional return from the equity tranche  $\phi S$  be

$$n(\theta) = rS(1 - \gamma(\theta)) - r^S S(1 - \phi)$$

respectively.

With these notations, I can now separate the different realisations of asset returns easily. The different realisations can be divided into two situations. The first one is

$$n(\theta) > 0$$

which refers to the situations in which all assets (old non-securitised loans, new loans, and equity tranche) provide non-zero returns; this happens when the realised economy condition is good. The second situation is

$$n(\theta) = 0$$

which refers to the situations in which the equity tranche is *dead*; i.e. the indicator function  $\mathbb{I}_S = 1$  and the equity tranche offers no return. Note that the total loan return  $m(\theta)$  is always positive, although its value can be extremely small. This is because according to the model specification, default rates depend on normal-distributed economy condition, and since  $\theta \in (-\infty, \infty)$ , default rates cannot be 1 (although it can be very close to 1), therefore the loan return is always positive.

Incorporating the indicator function, the total asset return can be defined as

$$R(\theta) = [m(\theta) + n(\theta)](1 - \mathbb{I}_S) + m(\theta)\mathbb{I}_S$$

Rearranging the terms, one can get

$$R(\theta) = m(\theta) + n(\theta)(1 - \mathbb{I}_S)$$

This expression means that the total return from asset portfolio is simply a combination of the return from bank loans and surviving equity tranche.

### 2.3.4 Wholesale Borrowing and Retail Deposit

This model assumes that there are two traditional funding sources for which a bank uses to finance its assets. The two sources are wholesale borrowing and retail deposit. Wholesale borrowing is not a risk-free investment because the wholesale investors may suffer losses if the bank goes bankrupt. Therefore they demand a required rate of return (or wholesale rate), which is higher than the risk-free rate, to compensate for the risk they bear. The book value of wholesale borrowing on date 0 and the

wholesale rate are denoted respectively as  $W$  and  $r^W$ ; the bank promises to return  $r^W W$  for its wholesale borrowing if the bank remains solvent on date 1.

Retail deposit is the other source of funding to the bank. Unlike wholesale borrowing, I assume the retail deposit is protected by deposit insurance. I assume that on date 0, the book value of retail deposit is  $D$  and the required rate of return demanded by the retail depositors (deposit rate) is  $r^D$ . On date 1, the retail depositors receive the promised return  $r^D D$  if the bank survives.<sup>5</sup>

If the total asset return cannot cover all the funding costs, the bank becomes insolvent and goes bankrupt. The following indicator function is used to define a bankruptcy.

$$\mathbb{I}_B = \begin{cases} 1 & \text{if } m(\theta) + n(\theta)(1 - \mathbb{I}_S) - (r^W W + r^D D) < 0 \\ 0 & \text{Otherwise} \end{cases}$$

During a bankruptcy, wholesale investors and retail depositors take hold of all asset returns of the bank; they are assumed to jointly share the asset returns according to the proportion of their investments.

For simplicity of model specifications, I assume that  $\phi$  is sufficiently small that  $\mathbb{I}_S \geq \mathbb{I}_B, \forall \theta$ . That such exists is clear by considering the special case of a loan sale ( $\phi = 0$ ), in which case the equity tranche by definition offers no return, and so the securitisation always defaults at a higher value of  $\theta$  than that at which the bank defaults. I am assuming that  $\phi$  is sufficiently close to zero that when the bank defaults the securitisation also always defaults. The meaning of this assumption is that the probability of having a *dead* equity tranche is higher than the probability of the bankruptcy of the bank. This assumption is very realistic in practice, because the risk of equity tranche is indeed very high (equity tranche usually covers only the first 2% to 3% of the securitisation pool, whereas minimum Tier 1 regulatory capital requirements are 4% equity and other Tier 1 are 8% total capital). Therefore the banks are usually much safer due to the capital requirement demanded by the regulators. Therefore, since, the values of  $\phi$  which are used in real world securitisations are smaller than regulatory capital requirements, it is quite reasonable to assume that when the bank defaults the securitisation also always defaults.

As retail deposit is protected by deposit insurance, the specification of  $r^D$  requires a proper definition of deposit insurance. In this subsection, I ignore the provision of deposit insurance temporarily and determine  $r^W$  and  $r^D$  in an economy without deposit insurance. In the next section, deposit insurance is properly explained, and the problem of a different  $r^D$  will be revisited.

Similarly to other assets, the returns to wholesale borrowing and retail deposit

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<sup>5</sup>Note that  $r^W$  and  $r^D$  are both gross rates of return.

are determined respectively by solving

$$W = \int_{-\infty}^{\infty} \left\{ r^W W (1 - \mathbb{I}_B) + R(\theta) \mathbb{I}_B \cdot \frac{r^W W}{r^W W + r^D D} \right\} Z(\theta) f(\theta) d\theta$$

and

$$D = \int_{-\infty}^{\infty} \left\{ r^D D (1 - \mathbb{I}_B) + R(\theta) \mathbb{I}_B \cdot \frac{r^D D}{r^W W + r^D D} \right\} Z(\theta) f(\theta) d\theta$$

The first terms on the right hand side of both equations refer to repayment of promised return when the bank survives, and the second terms refer to the sharing of asset returns in a bankruptcy. After cancelling the common terms on both sides and some algebra rearrangements, the two equations can be expressed as

$$(r^W)^{-1} = \int_{-\infty}^{\infty} \left\{ (1 - \mathbb{I}_B) + R(\theta) \mathbb{I}_B \cdot \frac{1}{r^W W + r^D D} \right\} Z(\theta) f(\theta) d\theta$$

and

$$(r^D)^{-1} = \int_{-\infty}^{\infty} \left\{ (1 - \mathbb{I}_B) + R(\theta) \mathbb{I}_B \cdot \frac{1}{r^W W + r^D D} \right\} Z(\theta) f(\theta) d\theta$$

It is obvious that  $r^W = r^D$ , which is not a surprising result because without deposit insurance wholesale investors and retail depositors are both sensitive to the risk of bankruptcy, and therefore they both demand the same required rate of return which can exactly compensate the insolvency risk.

## 2.4 Value of Equity, Deposit Insurance, and Moral-Hazard Controls

### 2.4.1 Value of Deposit Insurance

In reality, deposit insurance protects retail depositors in bank failures. It secures some or all of the retail deposit. For this reason, retail deposit becomes less risky (or even risk-free) compared with wholesale borrowing. Therefore, the retail depositors are willing to accept a deposit rate which is lower than the wholesale rate, and this reduces the funding cost of banks. As the cost of deposit insurance is usually risk-insensitive (some countries even impose a flat rate for deposit insurance premium) and is smaller than its expected return. It can be considered as a subsidy from the deposit insurance scheme to the banks, and this is usually called *the value of deposit insurance*.

The value of deposit insurance is a major source of moral hazard. This is because this value increases with the insolvency risk of banks. One important aim of this chapter is to show that, due to the existence of deposit insurance, a bank increases

its risk during loan securitisation, so that the value of deposit insurance (which can be considered as a measure for the insolvency risk of bank) also increases. For simplicity, I assume in this model that deposit insurance fully secures the return to retail depositors and therefore the retail deposit is risk-free.

One approach for modelling the value of deposit insurance is to assume that an insured bank is subject to a lower deposit rate compared with a non-insured bank. To be precise, if  $r^D$  is the deposit rate for a non-insured bank, then the rate for a fully-insured bank should be  $r^D - r^P = r^f$ , where  $r^P$  is the risk premium and  $r^f$  is the risk-free rate. However, I am going to apply an alternative approach for the derivation of the value of deposit insurance, with the value of deposit insurance being expressed as an explicit compensation instead of a reduction in deposit rate.

This alternative approach has an important advantage for the proposed model; it allows the model to specify the value of deposit insurance in an explicit form, which provides a clearer expression when the model compares the magnitude of moral hazard and of the efficient controls. The following story explains the concept of this alternative approach.

Suppose there exists a non-insured bank, the retail depositors are therefore risk-sensitive and they demand a deposit rate  $r^D$  according to the bank insolvency risk (as described in the last subsection). And in order to protect their investment, the depositors buy *personal insurance* which protects their deposit when the non-insured bank becomes insolvent. In a no-arbitrage economy, the cost of personal insurance should exactly be equal to the expectation of its discounted pay-off. Theoretically, (1) depositing in a non-insured bank (with a higher deposit rate) and at the same time buying a personal insurance and (2) depositing in a insured bank (with a lower deposit rate) are indifferent to the depositors.

Using the same logic, from the bank's point of view, the value of deposit insurance can be considered as an explicit form (which should be equal to the cost of personal insurance in the story) to compensate for the risk-sensitive deposit rate (determined under the assumption of no deposit insurance).

I apply this logic to the model by assuming that deposit rate is determined under a risk-sensitive basis as described in Section 2.3.4, but the bank is compensated by the value of deposit insurance, which is expressed as an explicit form.

As the value of deposit insurance, denoted as  $\tilde{D}$ , is equivalent to the expectation of its discounted pay-off, it can be expressed as

$$\tilde{D} = \int_{-\infty}^{\infty} \left[ r^D D - R(\theta) \cdot \frac{r^D D}{r^W W + r^D D} \right] \mathbb{I}_B Z(\theta) f(\theta) d\theta$$

In Section 2.3.4, I showed that if both retail deposit and wholesale borrowing are risk-sensitive, then  $r^W = r^D$ . Therefore, the expression for  $\tilde{D}$  can be further simplified

to

$$\tilde{D} = \int_{-\infty}^{\infty} \left[ r^D D - R(\theta) \cdot \frac{D}{W + D} \right] \mathbb{I}_B Z(\theta) f(\theta) d\theta$$

It is worth mentioning that the value of deposit insurance can also be considered as the expected shortfall to the depositors during bank insolvency.

To show that the value of deposit insurance is equivalent to the risk premium, I prove the following proposition in the appendix.<sup>6</sup>

**Proposition 1** *The value of deposit insurance is positive and is equivalent to the risk premium demanded by retail depositors of a non-insured bank. That is,  $\tilde{D} = D(r^D/r^f - 1) > 0$ .*

This proposition shows three important features. First of all, the value of deposit insurance can be expressed as an explicit term as described above. Second, the value of deposit insurance is the risk premium demanded by retail depositors in order to compensate for the bank insolvency risk. Third, the value of deposit insurance is always positive.

## 2.4.2 Value of Equity

The bank equity is funded by the investment from bank equity holders. The *book value of equity* on date 0 is denoted as  $E$ . This book value, similar to the other external funding sources, is used to finance bank assets on date 0.

As mentioned, the objective of the bank in this model is to maximise the value of equity ( $V$ ),<sup>7</sup> by choosing an optimal amount of loan securitisation  $S$ . In this subsection, two scenarios are discussed. In the first scenario, there is no deposit insurance; the model shows that without deposit insurance the bank has no preference on securitisation. In the second scenario, deposit insurance is available; the model shows that the bank chooses a corner solution, which is to securitise all existing loans in order to maximise the value of deposit insurance (i.e. the moral hazard). The factors for restraining moral hazard are discussed in the next subsection.

### An Economy without Deposit Insurance

In the first scenario, there is no deposit insurance available to retail depositors. The value of equity is purely the expectation of a bank's discounted net return.

$$V = \int_{-\infty}^{\infty} [R(\theta) - (r^W W + r^D D)] (1 - \mathbb{I}_B) Z(\theta) f(\theta) d\theta$$

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<sup>6</sup>Recall that  $r^f$  is the risk-free rate.

<sup>7</sup>The specification of the value of equity depends on the existence of deposit insurance and the moral-hazard controls.

Note that during bank insolvency ( $\mathbb{I}_B = 1$ ), all asset returns are used to repay funding costs and equity holders have zero return.

Based on these specifications, I prove the following proposition in the appendix.

**Proposition 2** *In a frictionless and no-arbitrage economy, where deposit insurance ( $\tilde{D}$ ) is unavailable and the proceeds from securitisation is being used in originating new loans ( $L_n$ ), the size of loan securitisation ( $S$ ) does not affect the expected value of equity ( $V$ ); i.e.  $V$  is independent of  $S$ .*

This proposition is not surprising, because the model specifications satisfy the Modigliani-Miller theorem. This means that securitisation is nothing special but an alternative source of funding. Since all funding sources (securitisation, wholesale funding and retail deposit) do not affect the value of equity, the bank has no particular preference on how its assets are being financed. In fact, the proof in the appendix shows that  $V = E$ ; this means that the discounted net returns to bank (or the (discounted) expected value of equity) is equivalent to the book value of equity, i.e. the equity is also fairly priced in the no-arbitrage economy.

### An Economy with Deposit Insurance

In the second scenario, retail depositors are fully protected by deposit insurance. This is equivalent to an explicit subsidy (which equals the value of deposit insurance ( $\tilde{D}$ )) be given to the bank. According to proposition 1, the value of equity can be expressed by

$$V_1 = E + \tilde{D}$$

in which the decision on the amount of securitisation  $S$  are embedded in  $r^D$ , which is used to determine  $\tilde{D}$ .

The following proposition is proved in the appendix.

**Proposition 3** *In a frictionless and no-arbitrage economy, where deposit insurance ( $\tilde{D}$ ) is available and the proceeds from securitisation is being used in originating new loans ( $L_n$ ), a bank securitises all available loans to maximise the value of equity; i.e.  $\partial V_1 / \partial S > 0$  for all values of  $S$ .*

The bank maximises the value of equity by maximising the value of deposit insurance; this follows from the mechanism that (1) originating new loans using the proceeds from securitisation increases bank insolvency risk and (2) an increase in insolvency risk raises the value of deposit insurance. The bank therefore maximises the value of equity by choosing the decision variable such that bank insolvency risk is maximised: securitising all existing loans to originate new loans. Therefore, the optimal choice for securitisation is a corner solution.

According to this result, one can see that securitisation is being used to maximise the value of deposit insurance (moral hazard). Since there is no factor restraining moral hazard, maximising the insolvency risk of a bank becomes the bank's best choice.

## **A Discussion on Risk Aversion**

In this model, I assume that all agents have the same degree of risk aversion and all required returns of assets are derived according to this fundamental assumption. Therefore, according to this framework, the (ex-ante) expected return from the bank loan portfolio is equivalent to the expected return from the asset-backed tranches formed by this portfolio. As a result, it is impossible for the senior-tranche investors (the risk-buyers) to require a higher return for their investment than the return offered by the originator bank.

Yet, it is interesting to discuss the situations in which the risk-buyers have a different degree of risk aversion compared with the originator bank. The source of the differences between the degree of risk aversion can be a result of the existence of other market frictions which are not included in the proposed model; for example: the existence of liquidity risk in the senior-tranche market. I will begin the discussion in an economy without deposit insurance.

Recall that in an economy without deposit insurance, the proposed model tells us that the originator bank has no preference on any type of funding sources (Proposition 2); in other words, the originator bank may securitise any amount of its loan portfolio without affecting its value of equity. However, this conclusion can change if the risk aversion of the risk-buyers is different from that of the bank. If the risk-buyers have a higher risk aversion, they require a higher return for the senior tranche than the return offered by the originator bank. This implies the expected return to the equity tranche retained in the bank provides a less-than-required return to the originator bank. This destroys the motivation for the originator bank to securitise, resulting in a bank decision for not securitising its loan portfolio. On the contrary, if the risk-buyers have a lower risk aversion, the equity tranche should then have a higher-than-required expected return to the originator bank, which can be considered as a profit arbitrage in securitisation; this creates an incentive for the originator bank to securitise as much as possible to maximise this arbitrage profit.

The same logic applies to an economy with deposit insurance. However, recall that there exists the value of deposit insurance for the originator bank, the analysis is a bit more complicated. If the risk-buyers have a higher degree of risk aversion, the originator bank will then compare the trade-off between the lower-than-required expected return from equity tranche and the value of deposit insurance. If the risk-aversion of risk-buyer is just slightly higher than that of the originator bank, it can

be expected that the value of deposit insurance dominates in the trade-off, and the bank still chooses to securitise to a maximum; otherwise, the low return from equity tranche destroys the incentive for bank securitisation.<sup>8</sup>

### 2.4.3 Controlling Moral Hazard

In the last subsections, one can see that securitisation is used entirely as a tool for risk maximisation. This is not a good practice as it increases the burden of deposit insurance scheme, causing financial instability in banking sector when the economy condition is bad. Therefore, in this subsection, I introduce two factors which can help restrain the moral hazard induced by deposit insurance. These two factors are (1) a carefully-designed and risk-sensitive deposit insurance premium and (2) the franchise value of banks.

#### Deposit Insurance Premium

Deposit insurance premium is a fee that a bank needs to pay in order to have its depositors protected in a bankruptcy. According to the policies of different countries, the calculation for deposit insurance premium varies significantly. In this chapter, I do not aim to calculate a fair value of deposit insurance premium, instead I would like to see how the choice on securitisation changes based on different specifications of deposit insurance premium. The deposit insurance premium is denoted as  $\tilde{C}$ , and it is assumed to be a function of the value of deposit insurance (which can be considered as a measure of the bank insolvency risk).

$$\tilde{C} = H(\tilde{D})$$

where  $H(\cdot)$  is a strictly increasing function with a non-negative range. For simplicity, this model assumes that the premium is paid on date 1, regardless of the bank insolvency; therefore there is no need to finance the premium on date 0.

In Section 2.6, the numerical simulations show that according to different specifications of  $H(\cdot)$ , the effects on the choice of securitisation can vary significantly. Further discussion is included in the next section, but meanwhile I would like to briefly describe some possible results.

Provided that the proceeds are used to originate new loans, Proposition 3 shows that insolvency risk (which is captured by  $\tilde{D}$ ) increases with the amount of securitisation  $S$ . There are four possible outcomes according to the specification of  $H(\cdot)$ . First of all, if deposit insurance premium is insensitive to bank insolvency risk, it

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<sup>8</sup>Note that if the risk-buyers have lower risk aversion than that of the originator bank, then the incentive for bank securitisation is further strengthened due to both the value of deposit insurance and the arbitrage profit; in such cases, the bank chooses to securitise to a maximum.

is likely to observe that banks choose to securitise all existing loans, because when insolvency risk raises, the increase in benefit ( $\tilde{D}$ ) is larger than the increase in cost ( $\tilde{C}$ ).

The second specification is the opposite of the first one, with deposit insurance premium being hyper-sensitive to insolvency risk. In this case a bank gives up the opportunity to securitise. The reason is that the increase in cost is larger than the increase in benefit from securitisation.

The third specification assumes that the deposit premium is fairly priced, such that it exactly offsets the value of deposit insurance. The model will then be boiled down to the scenario under which there is no deposit insurance. Proposition 2 has proved that the bank has no preference on any source of funding.

So far, none of the above specifications satisfy the intentions of the policy-makers. As mentioned in the chapter introduction, the policy-makers (IMF and G20) want to restart securitisation, but it should be under control. The above specifications do not create the effect that policy-makers would like to see. Therefore, this paper suggests the fourth specification: the deposit insurance premium being less sensitive to low insolvency risk but being more sensitive to high insolvency risk, this creates an effect to motivate the banks to securitise a certain portion, but not all or none, of its assets. The illustration is shown in Section 2.6.

## Franchise Value

Previous literatures, such as Keeley (1990) [55], suggest that banks with high franchise value are likely to have smaller risk compared with the ones with low franchise value.<sup>9</sup> The reason is that if a bank becomes insolvent, the equity holders are likely to lose some, if not all, of the bank franchise value. Therefore, equity holders of a high-franchise-value bank have a stronger intention to avoid insolvency, by limiting insolvency risk to an acceptable level.

Therefore, franchise value can also be a possible factor that restrain moral hazard in securitisation. Although franchise value is not a measure that policy-makers can impose directly to control moral hazard, policy-makers can affect the franchise value of banks indirectly by regulating bank competition.

In this model, franchise value is defined as the market value of a bank in excess of its book value. I assume that date-1 franchise value is a positive constant when the bank survives, and zero when the bank becomes insolvent. The date-0 franchise value is defined as the expectation of the discounted date-1 franchise value.

How does securitisation affect the franchise value of a bank? By definition, date-1 franchise value only exists when the bank survives; if the insolvency risk of a bank

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<sup>9</sup>Please refer to Chapter 4 for more literature reviews on the franchise value of banks.

increases, the probability of having a positive franchise value decreases as a result, which in turn decreases the date-0 franchise value.

The above specifications can be expressed mathematically as follows. Let  $\Phi_1$  be the date-1 franchise value. The date-0 discounted franchise value is given by

$$\Phi_0 = \int_{-\infty}^{\infty} \Phi_1(1 - \mathbb{I}_B)Z(\theta)f(\theta)d\theta$$

where  $\Phi_0$  is the date-0 franchise value on date 0.

When the insolvency of bank increases due to the securitisation of bank loans, the probability of having  $\mathbb{I}_B = 1$  increases, resulting in a smaller  $\Phi_0$ .

The difference between the  $\Phi_0$ 's generated by different levels of insolvency risk represents the change in the date-0 discounted franchise value due to the change in insolvency risk in bank securitisation. This difference can offset the value of deposit insurance and can therefore limit moral hazard.

## A Discussion on Capital Requirement

In this chapter, I do not include the numerical analysis of capital requirements as a control for moral hazard. This is because according to the model specifications in this chapter, capital requirement should have no effect on controlling moral hazard. This is because the risk of bank equity (as well as all other assets) are fully compensated by its returns. Specifically, if the bank is required to hold more equity, the equity-holders will then bear a larger burden if the bank fails, and they will require a higher return for their equity investment; however, due to more bank equity, the bank risk reduces; as a result, the cost of other funding (wholesale borrowing and retail deposit) also reduces. The reduction of the cost of funding will then fully compensate the increase of the cost of equity, resulting in a zero-sum effect (as shown in Proposition 2). Therefore, the requirement for holding more bank equity does not necessary create an incentive for a bank to change its decisions.

However, it is worth pointing out that the model specifications in this chapter do not consider the demand and supply of equity funding which play important roles in reality. When the demand for equity funding increases, the required return to equity-holders also increases; this increase comes from the shortage of equity funding in the economy, not from bank risk. Therefore, in reality, when a bank needs to increase its equity, it needs to pay for a cost of equity which consists of (1) the risk premium and (2) the market price for equity. Taking into consideration of the demand and supply of equity, an increase in bank equity becomes more costly, and therefore capital requirement can still be an effective control for the moral hazard in bank securitisation.

Another aspect that is worth discussing is the relationship between capital re-

quirement and the quality of loans to be securitised. In this chapter, all initial loans are assumed to be identical; this rules out the choice of which loans to securitise. In reality, the quality of loans to be securitised is closely related to the capital requirement of the originator bank. Ignoring the existence of implicit guarantee between the originator banks and the senior-tranche investors, one can expect that in an economy in which equity funding is costly, a bank chooses to securitise the more risky loans in order to transfer credit risk to outside investors; this reduces the level of required capital, and in turn reduces the costly equity funding. Under this situation, a higher capital requirement reduces the insolvency risk of banks (i.e. the value of deposit insurance), and this may help control moral hazard in securitisation.

## 2.5 Alternative Proceeds Allocation: Wholesale Lending

In the two previous sections, the derived result from the model is based on the assumption that the proceeds from securitisation is used to originate new loans ( $L_n$ ). However, recall that this chapter also introduces another proceeds allocation: the wholesale lending. In the following, bank securitisation for the provision of wholesale lending is specified and discussed.

Providing wholesale lending has very similar effects to banks as reducing current wholesale borrowing, but using the former approach has certain advantages over the latter. The first advantage is to avoid negative position on balance sheet; if the amount of securitisation is larger than the current wholesale borrowing, reducing the wholesale borrowing with the proceeds may create negative position on the liability side, which can cause confusions to the model results. The second advantage is to keep the size of the balance sheet constant; if the proceeds are used to reduce wholesale funding, both the assets and the liabilities on the balance sheet are reduced. This creates a ‘smaller’ balance sheet which is unfavourable for the comparison between different securitisation decisions.

In this subsection, I assume that the proceeds are used to lend to other banks with the same insolvency risk as the modelled bank. Due to the identical insolvency risk, the wholesale borrowing and lending rates should be the same, and the effect of lending and borrowing should therefore offset each other, achieving a similar effect as reducing wholesale borrowing.

First of all, let look at the change in the bank asset portfolio before and after securitisation (if the proceeds from securitisation are used to provide wholesale lending) in Figure 2.2.

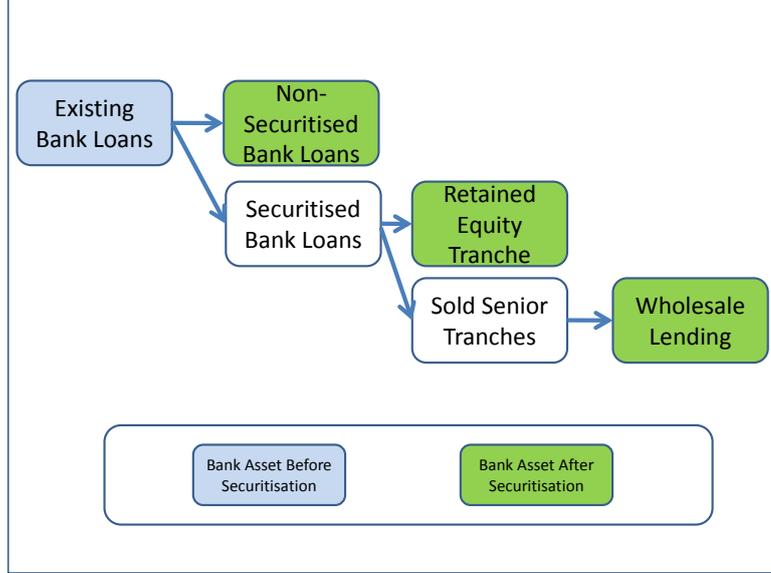


Figure 2.2: Bank asset before and after securitisation, with securitisation proceeds used for the provision of wholesale lending.

The book value of wholesale lending is given by,

$$W_n = (1 - \phi)S$$

In the situation in which there is no deposit insurance and other policy interventions, a conclusion similar to Proposition 2 can be derived: the bank has no preference on securitisation, as well as the other funding sources.

Based on the above specifications, the following proposition is derived. The proof of the proposition is given in the appendix.

**Proposition 4** *In a frictionless and no-arbitrage economy, where deposit insurance ( $\tilde{D}$ ) is unavailable and the proceeds from securitisation is being used in wholesale lending ( $W_n$ ), the size of securitisation ( $S$ ) does not affect the value of equity ( $V$ ); i.e.  $V$  is independent of  $S$ .*

In an economy with deposit insurance, the provision of wholesale lending has the opposite effect of originating new loans. As the wholesale borrowing and wholesale lending have an offsetting effect to each other, the leverage ratio and funding cost of a bank decrease, leading to smaller insolvency risk. The more assets a bank chooses to securitise, the smaller the insolvency risk is. This means that securitisation for wholesale lending reduces the value of deposit insurance. The bank therefore maximises its value of equity ( $V_1$ ) by choosing not to securitise.

Due to the recursive structure in the specification, an analytical proposition, similar to the Proposition 3, to show that  $\partial V_1 / \partial S < 0$  for all  $S$  (based on the proceeds allocation on wholesale lending) cannot be properly derived. Therefore, instead of

giving a formal proof, this result is demonstrated by the numerical simulations in Section 2.6.

Under the alternative proceeds allocation, it is obvious that deposit insurance premium decreases when a bank securitises. Recall the specification of deposit insurance premium is given by  $\tilde{C} = H(\tilde{D})$ ; Since  $H$  is an increasing function, and securitisation reduces the value of deposit insurance ( $\tilde{D}$ ), the premium ( $\tilde{C}$ ) decreases with the amount of securitisation.

Similarly, for the franchise value, a smaller insolvency risk leads to a higher date-0 franchise value; this implies if the proceeds are used for wholesale lending, securitisation increases the date-0 franchise value. In Section 2.6, these results are explored more thoroughly and a mixture of the two proceeds allocation is also considered.

### **Discussion on Debt Priority**

In this chapter, I assume that (1) both wholesale borrowing and retail deposit have the same priority to claim bank assets when the bank fails to pay their promised returns, and (2) the asset recovery rate is zero. I would like to point out that relaxing these assumptions can affect the conclusions of the model.

In the case that the retail deposit has a higher priority than wholesale borrowing in bank failures, and given a sufficiently high asset recovery rate, it is possible that the asset return to the bank can satisfy the promised payments to the retail depositors. Therefore, under these alternative specifications, a change in the size of new loan creation or in the amount of wholesale lending provision may have no impact on the expected loss on deposits; this also means that the value of deposit insurance remains unchanged. This further implies the moral-hazard motivation for bank securitisation will no longer exist.

Therefore, the conclusion of this chapter relies strongly on the specifications that characterise the possibility of retail depositors to suffer losses under some bad economic scenarios.

## **2.6 Numerical Results**

In this section, several numerical examples are simulated to give a clear image on the effects of proceeds allocation, value of deposit insurance and its premium, franchise value and other parameters on securitisation. The most important objective of this section is to show that a carefully-designed deposit insurance and a decent level of franchise value can put the distorted usage of securitisation back to the right track.

## 2.6.1 Parameter Specification and Graph Interpretation

For the numerical simulations, I assume the utility function is defined as

$$u(c) = \frac{c^{1-\delta}}{1-\delta}$$

which is the commonly-used constant-relative-risk-aversion (CRRA) function, where  $\delta$  is the risk aversion coefficient.

The economy condition ( $\theta$ ) which follows a standard normal distribution is discretised evenly into small intervals between the range -4 and 4 for numerical programming. The consumption level on date 1, denoted as  $c_1(\theta)$ , is also discretised evenly between the range 0.95 to 1.15 to correspond to each state of the economy. All the agents in the model have a constant relative risk aversion (CRRA) utility function, with a risk aversion ( $\delta$ ) of 2 and subjective discount factor ( $\beta$ ) of 0.99.

The balance sheet figures and the loan parameters applied for the numerical examples are listed in the following table.

### Bank Parameters

Book value of the existing bank loans ( $A$ )	100
Retained equity tranche per unit of loans ( $\phi$ )	0.1
Book value of retail deposit ( $D$ )	50
Book value of wholesale borrowing ( $W$ )	30
Book value of equity ( $E$ )	20
Date-1 franchise value of bank ( $\Phi_1$ )	7.5

### Loan Parameters

Factor loading for $L$ ( $a$ )	0.6
Factor loading for $L_n$ ( $a_n$ )	0.9
Threshold level for $L$ ( $\alpha$ )	-1.5
Threshold level for $L_n$ ( $\alpha_n$ )	-0.8

Table 2.2: Parameters Used in Numerical Examples

All the graphs are generated by MatLab and are three-dimensional, with x-axis as the percentage of securitisation relative to the initial bank loans ( $A$ ), with y-axis as the percentage of proceeds allocation on new bank loans ( $L_n$ ), and with z-axis as the value of the subject of the graph. It is worth mentioning that the y-axis captures the mixture of the two proceeds-allocation approaches. For example, the point  $x = 30\%$  and  $y = 60\%$  refers to the situation in which (1) a bank securitises 30% of the existing loans and the remaining 70% is left on the balance sheet, and (2) out of the 30% loan securitisation, 60% of the proceeds is used to originate new

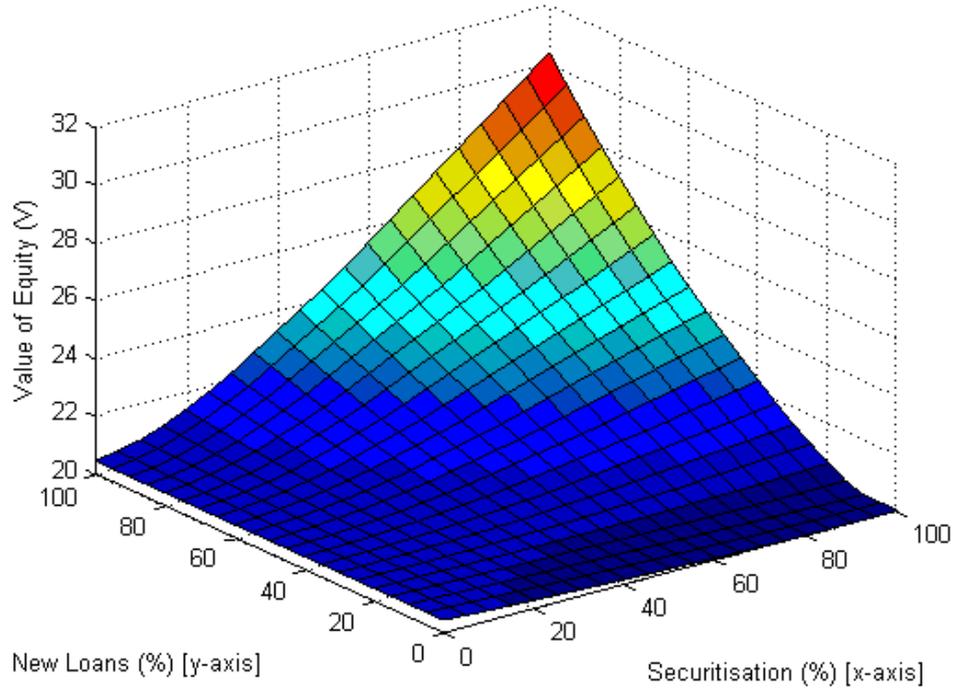


Figure 2.3: The value of equity ( $V = E + \tilde{D}$ ) under different combinations of securitisation and proceeds allocation, without any moral-hazard control.

bank loans and the remaining 40% is used for wholesale lending.

All the figures generated in this section are for illustration purpose and have not been calibrated for realism.

## 2.6.2 Moral Hazard: Value of Deposit Insurance

The first numerical example shows how moral hazard (the value of deposit insurance) affects securitisation and proceeds allocation. Without considering any controls for moral hazard, the value of equity is expressed by  $V_1 = E + \tilde{D}$ . In fact, Proposition 3 has already suggested the numerical result.

Figure 2.3 shows that the optimal decision is to securitise all the existing assets ( $x = 100\%$ ) and to allocate all the proceeds to originate new loan ( $y = 100\%$ ); this is a corner solution and refers to a risk-maximising behaviour of a bank because the optimal strategy is also the riskiest possible strategy that a bank can choose in this model. This result is consistent to Proposition 3, which suggests that a bank maximises its risk to maximise the value of deposit insurance.

Another conclusion that one can obtain from this numerical example is that, if a bank is only allowed to allocate the proceeds in wholesale lending ( $y = 0\%$ ),

the optimal strategy for a bank is to securitise nothing ( $x = 0\%$ ). This can be observed from Figure 2.3 that the value of equity decreases with securitisation given  $y = 0\%$ . This implies a bank has no intention to use securitisation as a funding tool, because providing wholesale lending (which offsets the wholesale borrowing) reduces the insolvency risk and the value of deposit insurance. In particular, the least desirable decision for a bank under this numerical example is  $x = 100\%$  and  $y = 0\%$  (securitising everything and use all the proceeds for wholesale lending) because this decision has the lowest insolvency risk and therefore minimises the value of deposit insurance.

### 2.6.3 Restraining Moral Hazard in Bank Securitisation

Due to the existence of moral hazard (from the value of deposit insurance), some factors that may restrain moral hazard are studied in this subsection. The first one that is discussed is the size of the retention (or the size of equity tranche). Many researchers and policy-makers believe that a larger retention can help restrain the moral hazard in bank securitisation because the banks are responsible for a larger FLP. In the following, the numerical simulation suggests that this may not be true. This chapter suggests that two other controls: (1) a carefully-designed deposit insurance premium and (2) the franchise value of bank, can be two better controls for the moral hazard in securitisation.

#### Larger Retention

Applying a similar setting as the first numerical example, except that the size of equity tranche ( $\phi$ ) is increased from 0.1 to 0.2 to generate Figure 2.4. One can see that Figure 2.4 and Figure 2.3 look very much alike. In fact, the optimal strategy in both numerical examples are the same ( $x = 100\%$  and  $y = 100\%$ ); this means that increasing the size of retention may not be a useful control for the risk-taking behaviour in securitisation.

The reason is that although a larger retention can increase the monitoring efforts of the banks (not discussed in this chapter), it has no effect on the moral hazard from deposit insurance. This is because in an no-arbitrage economy, the equity tranche is fairly priced, and therefore a change in risk is fully compensated by a change in its return; therefore, the equity holders have no concern for the size of equity tranche; i.e. requiring a larger equity tranche may not be an effective control for the moral hazard (induced by deposit insurance) in bank securitisation.

It is worth mentioning that moral hazard induced by the existence of deposit insurance is different from the moral hazard that comes from bank monitoring. As mentioned in the literature review of this chapter, when there exists asymmetric

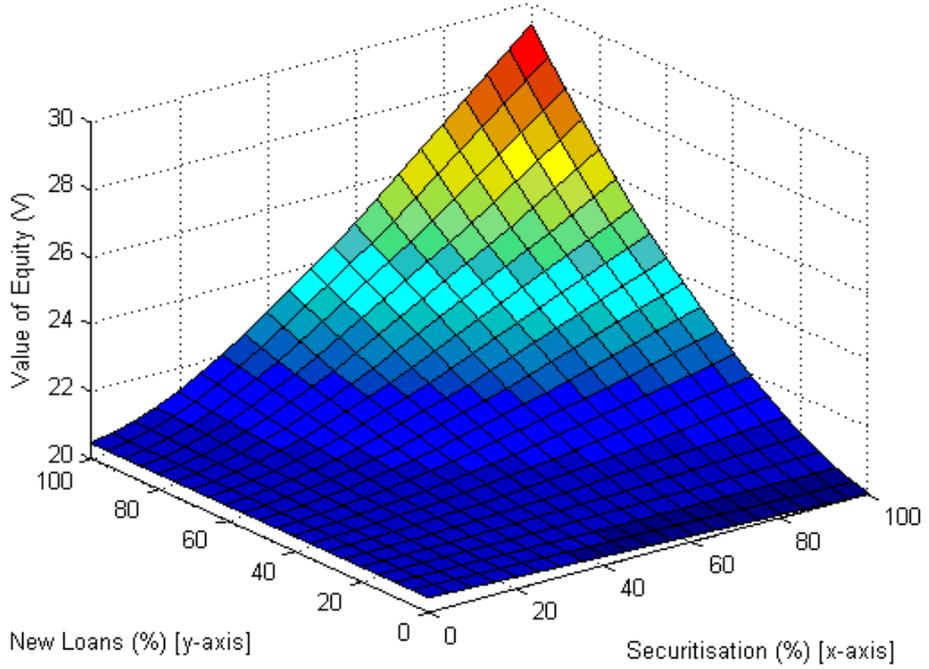


Figure 2.4: The value of equity ( $V = E + \tilde{D}$ ) under different combinations of securitisation and proceeds allocation, with a larger equity tranche.

information on banks' monitoring effort, the banks may have an incentive to reduce its monitoring effort (private benefit for the bank); this socially-undesirable private benefit can also be considered as a type of moral hazard. Therefore, I should stress here that this numerical example only shows that a larger retention has no impact on the moral hazard induced by deposit insurance. It does not say anything about the relationship between larger retention of equity tranche and the moral hazard that comes from asymmetric information on banks' monitoring effort.

### Deposit Insurance Premium

In the following numerical examples, I apply a normalised quadratic equation with a minimum value of zero to specify deposit insurance premium.

$$\tilde{C} = H(\tilde{D}) = \max \left[ 0, \Psi(\tilde{D}) \right]$$

where

$$\Psi(\tilde{D}) = \left[ b_1 \left( \frac{\tilde{D}}{\tilde{D}^*} \right)^2 - b_2 \left( \frac{\tilde{D}}{\tilde{D}^*} \right) \right] \cdot \tilde{D}^*$$

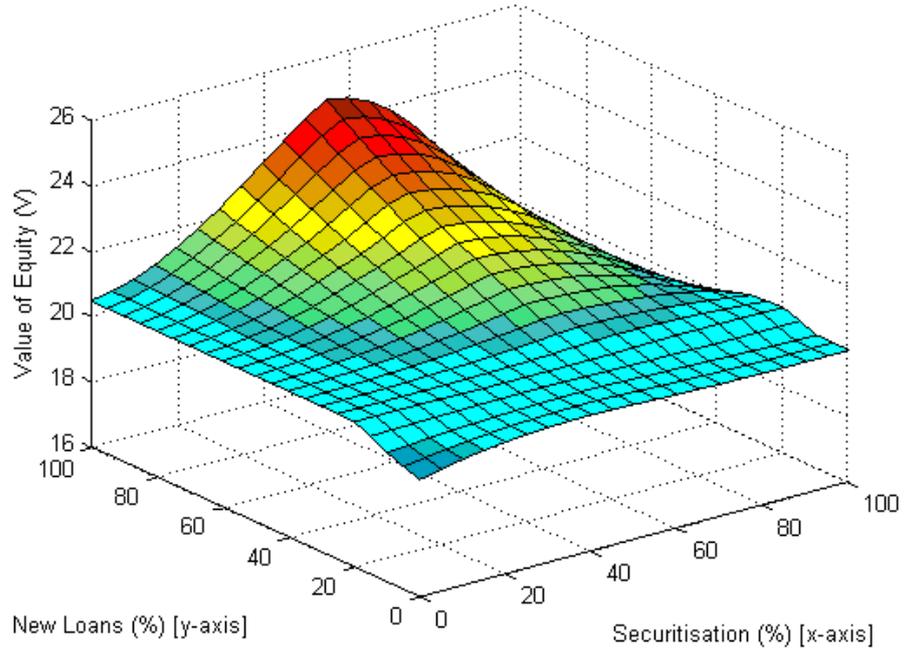


Figure 2.5: The value of equity ( $V = E + \tilde{D} - \tilde{C}$ ) under different combinations of securitisation and proceeds allocation, with the cost of deposit insurance premium.

$\tilde{D}^*$  is the arithmetic average of all values of deposit insurance depending on different amount of securitisation, given a particular proceeds allocation. In other words, subject to a fixed  $y$ , the average of  $\tilde{D}$  for all values of  $x$ . The two parameters  $b_1$  and  $b_2$  determine the shape of the function.

**Deposit insurance premium as an efficient control** As mentioned early, deposit insurance premium can be an effective control for securitisation if it has different risk sensitivity to different level of insolvency risk. To be effective, the deposit insurance premium should be less sensitive to low insolvency risk and more sensitive to high insolvency risk. This creates a motive for banks to begin using securitisation at a low cost, but at the same time controls the size of securitisation. An example for this is to let  $b_1 = 1$  and  $b_2 = -1$ .

Figure 2.5 shows the numerical result to this specification. The optimal decision is ( $x = 55\%$  ,  $y = 100\%$ ). This means that the bank only securitises about half of its assets. However, one can still observe that this is a corner solution to the proceeds allocation (all proceeds are used to originate new loans). From the numerical example, it seems that deposit insurance premium can only control the size of securitisation, but not how the proceeds are allocated.

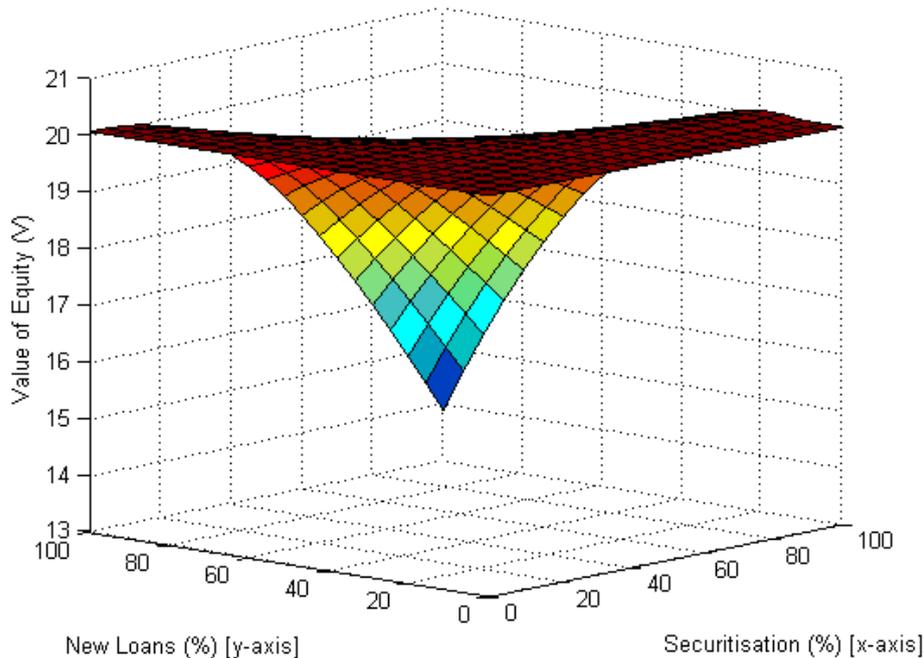


Figure 2.6: The value of equity ( $V = E + \tilde{D} - \tilde{C}$ ) under different combinations of securitisation and proceeds allocation, with a hyper-risk-sensitive cost of deposit insurance premium.

In the following, some numerical examples are generated to show other specifications for deposit insurance premium which are not desirable.

**Undesirable specifications for deposit insurance premium** In Section 2.4.3, other specifications of deposit insurance premium that are not desirable are discussed. To illustrate a deposit insurance premium that is over sensitive to all level of insolvency risk, I apply the following specification:  $\tilde{C} = H(\tilde{D}) = \tilde{D}^b$ , where  $b > 1$ . Figure 2.6 shows the numerical result for  $b = 1.2$ ; one can observe that the optimal strategy is  $x = 0\%$ . Note that when the amount of securitisation is zero, the proceeds allocation is no longer relevant because there is no proceeds for allocation. This result implies that charging an over-sensitive deposit insurance premium undermines the motivation for bank securitisation.

Another undesirable specification is a risk-insensitive premium for all level of insolvency risk. An extreme case is a flat rate applied on any amount of retail deposit ( $\tilde{C} = H(D) = bD$ , where  $b > 0$ ). One can imagine if all the ‘points’ in a graph are deducted by a fixed value, the shape of the graph remains unchanged; this implies the optimal strategy of the bank is identical to the scenario in which there

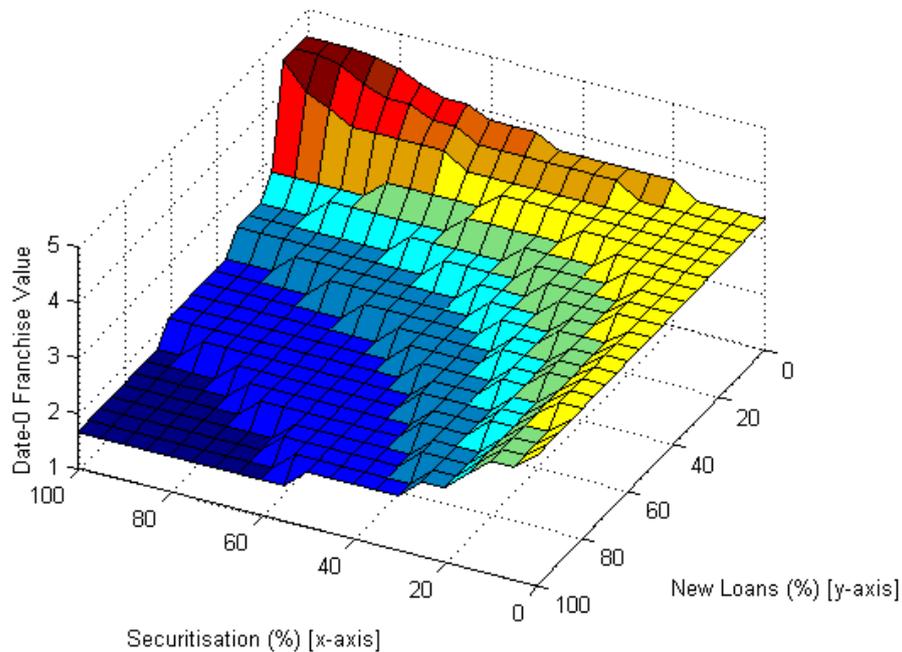


Figure 2.7: The date-0 franchise value under different combinations of securitisation and proceeds allocation.

is no deposit insurance premium.

The last specification is a fair premium which exactly offsets the value of deposit insurance ( $\tilde{C} = H(\tilde{D}) = \tilde{D}$ ). In this case,  $V = E$ , implying that the bank has no preference on any sources of funding, and the size of securitisation does not affect the value of equity.<sup>10</sup> Therefore, the fair deposit insurance premium also rules out the motivation for securitisation, and is not a desirable control for restarting the securitisation market.

## Franchise Value

The final numerical example shows how the franchise value of bank can be a factor that can restrain moral hazard in bank securitisation.

Figure 2.7 shows the pattern of franchise value *alone*. One can see that the pattern of franchise value looks like stairs. The areas that have the same value refer to bank choices that have the same insolvency risk. Choices with smaller insolvency risk have higher franchise value, and vice versa. It is obvious that the decision for securitisation which generates the highest date-0 franchise value is to securitise

<sup>10</sup>This scenario is consistent to the one described in Proposition 2.

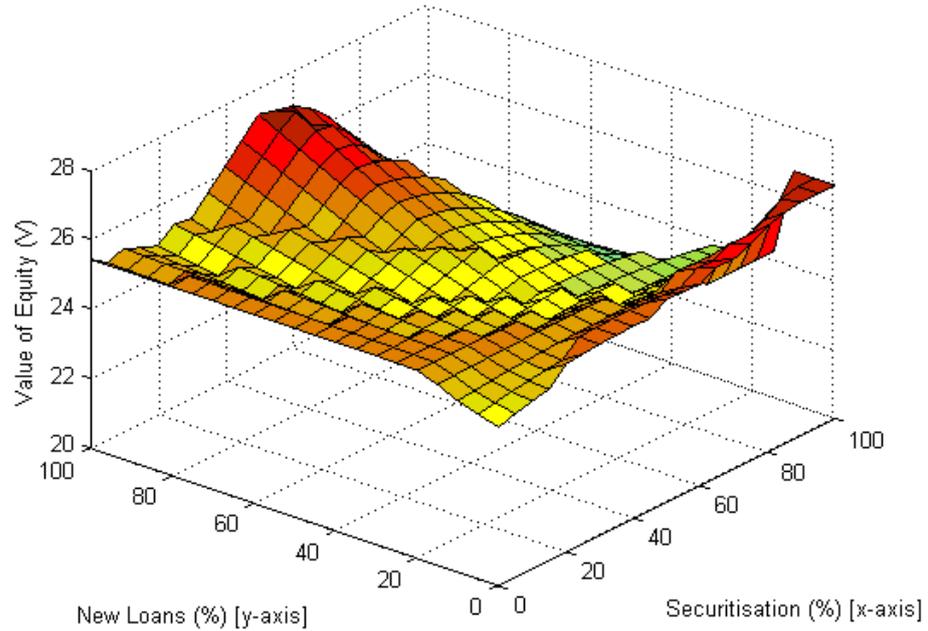


Figure 2.8: The value of equity ( $V = E + \tilde{D} - \tilde{C} + \Phi_0$ ) under different combinations of securitisation and proceeds allocation, with the cost of deposit insurance premium and franchise value as moral-hazard controls.

everything ( $x = 100\%$ ) and use all the proceeds for wholesale lending ( $y = 0\%$ ).

If the value of deposit insurance and the controls of both deposit insurance premium and franchise value are considered, an interesting result is obtained. From Figure 2.8, one can see there are two peaks with similar value of equity. The peak on the left refers to an optimal strategy that 55% of the loans are securitised and all proceeds are used to originate new loans; the peak on the right refers to another optimal strategy that all loans are securitised and are used for wholesale lending.

From Figure 2.8, one can observe that although insurance premium and franchise value are both factors that can restrain moral hazard, they have very different impacts on securitisation decision. The former limits insolvency risk by restricting the amount of loans that are securitised; the latter reduces the insolvency risk by providing wholesale lending to reduce funding costs.

An insightful conclusion can be drawn from Figure 2.8. One can expect that if a bank has a high franchise value, the bank is likely to use securitisation is a safe way (securitisation for the reduction of insolvency risk) regardless of the deposit insurance premium. On the other hand, if a bank has low franchise value, the bank is likely to use securitisation in a risky way (securitisation for the maximisation of

insolvency risk); if this is the case, the cost of deposit insurance premium plays its role to limit the size of securitisation so that the insolvency risk of the bank is under control.

## **2.7 Conclusion**

This chapter proposes a theoretical model of bank securitisation and bank risk-taking. The model suggests that without deposit insurance, banks have no preference on securitisation as a funding tool. However, if deposit insurance is available, it can create moral hazard which has a strong impact on the bank decision on securitisation. In order to maximise the value of deposit insurance, a bank securitises in to maximise its risk-taking.

To restrain this moral hazard, this chapter suggests two possible factors: (1) a carefully designed and risk-sensitive deposit insurance, and (2) franchise value of the bank. The numerical simulations suggest that the two controls have very different impact on moral hazard. The former limits the size of securitisation, and the latter encourages banks to securitise for wholesale lending to reduce bank insolvency risk.

## 2.8 Appendix

### 2.8.1 Proof of Proposition 1

In Section 2.3.4, the model shows that

$$D = \int_{-\infty}^{\infty} \left\{ r^D D (1 - \mathbb{I}_B) + R(\theta) \mathbb{I}_B \cdot \frac{r^D D}{r^W W + r^D D} \right\} Z(\theta) f(\theta) d\theta$$

which can be rewritten as

$$\int_{-\infty}^{\infty} r^D D \mathbb{I}_B Z(\theta) f(\theta) d\theta = \int_{-\infty}^{\infty} \left\{ r^D D + R(\theta) \mathbb{I}_B \cdot \frac{r^D D}{r^W W + r^D D} \right\} Z(\theta) f(\theta) d\theta - D \quad (2.1)$$

And from Section 2.4.1,  $\tilde{D}$  can be specified as

$$\tilde{D} = \int_{-\infty}^{\infty} \left[ r^D D - R(\theta) \cdot \frac{r^D D}{r^W W + r^D D} \right] \mathbb{I}_B Z(\theta) f(\theta) d\theta$$

which can also be rewritten as

$$\tilde{D} = \int_{-\infty}^{\infty} r^D D \mathbb{I}_B Z(\theta) f(\theta) d\theta - \int_{-\infty}^{\infty} R(\theta) \mathbb{I}_B \cdot \frac{r^D D}{r^W W + r^D D} Z(\theta) f(\theta) d\theta \quad (2.2)$$

Substitute equation (2.1) into equation (2.2), one can get

$$\begin{aligned} \tilde{D} &= \int_{-\infty}^{\infty} r^D D Z(\theta) f(\theta) d\theta - D \\ \tilde{D} &= D \left( r^D \int_{-\infty}^{\infty} Z(\theta) f(\theta) d\theta - 1 \right) \end{aligned}$$

By definition, the expectation of the stochastic discount factor is the risk-free rate discount factor. This is expressed by

$$\frac{1}{r^f} = \int_{-\infty}^{\infty} Z(\theta) f(\theta) d\theta$$

Substitute this result into  $\tilde{D}$  and one can get

$$\tilde{D} = D \left( \frac{r^D}{r^f} - 1 \right)$$

Since  $r^D$  is bigger than  $r^f$  by definition

$$\tilde{D} = D \left( \frac{r^D}{r^f} - 1 \right) > 0$$

■

## 2.8.2 Proof of Proposition 2

In Section 2.3.4, the model shows that

$$W = \int_{-\infty}^{\infty} \left\{ r^W W (1 - \mathbb{I}_B) + R(\theta) \mathbb{I}_B \cdot \frac{r^W W}{r^W W + r^D D} \right\} Z(\theta) f(\theta) d\theta$$

$$D = \int_{-\infty}^{\infty} \left\{ r^D D (1 - \mathbb{I}_B) + R(\theta) \mathbb{I}_B \cdot \frac{r^D D}{r^W W + r^D D} \right\} Z(\theta) f(\theta) d\theta$$

By summing up the two equations, one can get

$$W + D = \int_{-\infty}^{\infty} \{ r^W W (1 - \mathbb{I}_B) + r^D D (1 - \mathbb{I}_B) + R(\theta) \mathbb{I}_B \} Z(\theta) f(\theta) d\theta$$

which can be further rewritten as

$$- \int_{-\infty}^{\infty} (r^W W + r^D D) (1 - \mathbb{I}_B) Z(\theta) f(\theta) d\theta = \int_{-\infty}^{\infty} R(\theta) \mathbb{I}_B Z(\theta) f(\theta) d\theta - (W + D) \quad (2.3)$$

In Section 2.4.2, it has been shown that

$$V = \int_{-\infty}^{\infty} [R(\theta) - (r^W W + r^D D)] (1 - \mathbb{I}_B) Z(\theta) f(\theta) d\theta$$

which can be can also be rewritten as

$$V = \int_{-\infty}^{\infty} R(\theta) (1 - \mathbb{I}_B) Z(\theta) f(\theta) d\theta - \int_{-\infty}^{\infty} (r^W W + r^D D) (1 - \mathbb{I}_B) Z(\theta) f(\theta) d\theta \quad (2.4)$$

Substitute equation (2.3) into equation (2.4)

$$V = \int_{-\infty}^{\infty} R(\theta) Z(\theta) f(\theta) d\theta - (W + D)$$

Replacing  $R(\theta)$  with its definition

$$V = \int_{-\infty}^{\infty} [m(\theta) + n(\theta) (1 - \mathbb{I}_S)] Z(\theta) f(\theta) d\theta - (W + D)$$

Replacing  $n(\theta)$  with its definition

$$V = \int_{-\infty}^{\infty} \left[ \begin{array}{c} m(\theta) \\ + [rS(1 - \gamma(\theta)) - r^S S(1 - \phi)] (1 - \mathbb{I}_S) \end{array} \right] Z(\theta) f(\theta) d\theta - (W + D)$$

$$\begin{aligned}
V &= \int_{-\infty}^{\infty} \left[ \begin{array}{c} m(\theta) \\ +rS(1-\gamma(\theta))(1-\mathbb{I}_S) \end{array} \right] Z(\theta)f(\theta)d\theta - (W+D) \\
&\quad - \int_{-\infty}^{\infty} [r^S S(1-\phi)(1-\mathbb{I}_S)] Z(\theta)f(\theta)d\theta
\end{aligned} \tag{2.5}$$

In Section 2.3.3, it has been shown that

$$(1-\phi)S = \int_{-\infty}^{\infty} [r^S S(1-\phi)(1-\mathbb{I}_S) + rS(1-\gamma(\theta))\mathbb{I}_S] Z(\theta)f(\theta)d\theta$$

which can be rewritten as

$$- \int_{-\infty}^{\infty} r^S S(1-\phi)(1-\mathbb{I}_S)Z(\theta)f(\theta)d\theta = \int_{-\infty}^{\infty} rS(1-\gamma(\theta))\mathbb{I}_S Z(\theta)f(\theta)d\theta - (1-\phi)S \tag{2.6}$$

Substitute equation (2.6) into equation (2.5),

$$\begin{aligned}
V &= \int_{-\infty}^{\infty} [m(\theta) + rS(1-\gamma(\theta))] Z(\theta)f(\theta)d\theta \\
&\quad - [W+D+(1-\phi)S]
\end{aligned}$$

Replacing  $m(\theta)$  with its definition

$$\begin{aligned}
V &= \int_{-\infty}^{\infty} r(L+S)(1-\gamma(\theta))Z(\theta)f(\theta)d\theta \\
&\quad + \int_{-\infty}^{\infty} r_n L_n(1-\gamma_n(\theta))Z(\theta)f(\theta)d\theta - [W+D+(1-\phi)S]
\end{aligned}$$

Note that  $A = L + S$  and  $L_n = \int_{-\infty}^{\infty} r_n L_n(1-\gamma_n(\theta))Z(\theta)f(\theta)d\theta$

$$\begin{aligned}
V &= \int_{-\infty}^{\infty} rA(1-\gamma(\theta))Z(\theta)f(\theta)d\theta \\
&\quad + L_n - [W+D+(1-\phi)S]
\end{aligned}$$

And because  $L_n = (1-\phi)S$  and  $\int_{-\infty}^{\infty} r(1-\gamma(\theta))Z(\theta)f(\theta)d\theta = 1$

$$V = A - (W + D)$$

Due to balance sheet constraint  $A = W + D + E$ ,

$$V = E$$

■

### 2.8.3 Proof of Proposition 3

Note that in an economy with deposit insurance  $\tilde{D}$ , the value of equity is expressed as  $V_1 = E + \tilde{D}$ . The differentiation of  $V_1$  with respect to  $S$  is simply differentiating  $\tilde{D}$  with respect to  $S$ . In Proposition 1, it has been shown that

$$\tilde{D} = D \left( \frac{r^D}{r^f} - 1 \right) > 0$$

Because  $S$  is only embedded in  $r^D$ ; therefore, to prove this proposition, I have to show that  $\partial r^D / \partial S_1 > 0$ . Please refer to Section 2.4.1 for the explanation of the relationship between deposit rate, risk-free rate, and the value of deposit insurance. From Section 2.3.4,

$$D = \int_{-\infty}^{\infty} \left\{ r^D D (1 - \mathbb{I}_B) + R(\theta) \mathbb{I}_B \cdot \frac{r^D D}{r^W W + r^D D} \right\} Z(\theta) f(\theta) d\theta$$

and because  $r^W = r^D$

$$D = \int_{-\infty}^{\infty} \left\{ r^D D (1 - \mathbb{I}_B) + R(\theta) \mathbb{I}_B \cdot \frac{D}{W + D} \right\} Z(\theta) f(\theta) d\theta$$

Let  $\theta_B^*$  be the threshold of a bankruptcy event, I can drop out the indicator function by expressing the equation as

$$D = \int_{-\infty}^{\theta_B^*} \left[ R(\theta) \cdot \frac{D}{W + D} \right] Z(\theta) f(\theta) d\theta + \int_{\theta_B^*}^{\infty} [r^D D] Z(\theta) f(\theta) d\theta$$

Taking total differential with respect to  $S$ , one can get

$$\begin{aligned} 0 = \frac{\partial \theta_B^*}{\partial S} \left[ R(\theta_B^*) \cdot \frac{D}{W + D} \right] Z(\theta_B^*) f(\theta_B^*) + \int_{-\infty}^{\theta_B^*} \left[ \frac{\partial R(\theta)}{\partial S} \cdot \frac{D}{W + D} \right] Z(\theta) f(\theta) d\theta \\ - \frac{\partial \theta_B^*}{\partial S} [r^D D] Z(\theta_B^*) f(\theta_B^*) + \int_{\theta_B^*}^{\infty} \left[ \frac{\partial r^D}{\partial S} \cdot D \right] Z(\theta) f(\theta) d\theta \end{aligned}$$

Note that by definition,  $R(\theta_B^*)$  represents the gross return to bank at the bankruptcy threshold, which is equal to the promised returns to all debts (retail deposit and wholesale borrowing); therefore  $R(\theta_B^*) = r^D (W + D)$  and  $R(\theta_B^*) \cdot \frac{D}{W + D} = r^D D$ . For this reason, the first and third terms on the right hand side of the equation cancel each other. And after some algebraic arrangement, one can get

$$\frac{\partial r^D}{\partial S} = \frac{-\frac{1}{W + D} \int_{-\infty}^{\theta_B^*} \left[ \frac{\partial R(\theta)}{\partial S} \right] Z(\theta) f(\theta) d\theta}{\int_{\theta_B^*}^{\infty} Z(\theta) f(\theta) d\theta}$$

Note that the denominator is always positive by definition. Therefore, I only need to show that the numerator is positive to complete the proof. By definition,

$$R(\theta) = r(A - S)(1 - \gamma(\theta)) + r_n S(1 - \phi)(1 - \gamma_n(\theta)) + [rS(1 - \gamma(\theta)) - r^S S(1 - \phi)](1 - \mathbb{I}_S)$$

Therefore,

$$-\int_{-\infty}^{\theta_B^*} \left[ \frac{\partial R(\theta)}{\partial S} \right] Z(\theta) f(\theta) d\theta = \int_{-\infty}^{\theta_B^*} \left[ \begin{array}{c} r(1 - \gamma(\theta)) \\ -r_n(1 - \phi)(1 - \gamma_n(\theta)) \\ -[r(1 - \gamma(\theta)) - r^S(1 - \phi)](1 - \mathbb{I}_S) \end{array} \right] Z(\theta) f(\theta) d\theta \quad (2.7)$$

Taking a closer look at the above equation, one should be aware that the three terms within the square bracket are in fact three different assets described in this chapter. The first term is the future value of one unit of initial bank loan; the second term is the future value of  $(1 - \phi)$  unit of new bank loan; the last term is the future value of  $\phi$  unit of equity tranche.

In the following Lemma 1 (see below), I show that the value of equation (2.7) is positive. Therefore,  $\frac{r^D}{\partial S} > 0$ , which means  $\frac{\partial V_1}{\partial S} > 0$ . ■

**Lemma 1:**

$$\int_{-\infty}^{\theta_B^*} \left[ \begin{array}{c} r(1 - \gamma(\theta)) \\ -r_n(1 - \phi)(1 - \gamma_n(\theta)) \\ -[r(1 - \gamma(\theta)) - r^S(1 - \phi)](1 - \mathbb{I}_S) \end{array} \right] Z(\theta) f(\theta) d\theta > 0$$

**Proof:**

To prove this lemma, one has to be aware of the following properties by the definitions in the model:

Property (1): Both  $\gamma(\theta)$  and  $\gamma_n(\theta)$  are strictly decreasing in  $\theta$ ;

Property (2):  $\gamma(\theta) < \gamma_n(\theta)$ ,  $\forall \theta$ ;

Property (3):  $r < r_n$ ;

Property (4):  $\int_{-\infty}^{\infty} r(1 - \gamma(\theta))Z(\theta)f(\theta)d\theta = 1$ ;

Property (5):  $\int_{-\infty}^{\infty} r_n(1 - \gamma_n(\theta))Z(\theta)f(\theta)d\theta = 1$ .

Property (6):  $\int_{-\infty}^{\infty} [r(1 - \gamma(\theta)) - r^S(1 - \phi)](1 - \mathbb{I}_S)Z(\theta)f(\theta)d\theta = \phi$ .

The expression in this lemma can be separated into two parts:

$$\int_{-\infty}^{\theta_B^*} r(1 - \phi)(1 - \gamma(\theta))Z(\theta)f(\theta)d\theta - \int_{-\infty}^{\theta_B^*} r_n(1 - \phi)(1 - \gamma_n(\theta))Z(\theta)f(\theta)d\theta$$

and

$$\int_{-\infty}^{\theta_B^*} r\phi(1 - \gamma(\theta))Z(\theta)f(\theta)d\theta - \int_{-\infty}^{\theta_B^*} [r(1 - \gamma(\theta)) - r^S(1 - \phi)](1 - \mathbb{I}_S)Z(\theta)f(\theta)d\theta$$

In the following, I prove that both of these two parts are positive to complete the proof in Lemma 1.

For the first part, we know from the model specifications that

$$\lim_{\theta \rightarrow \infty} r(1 - \phi)(1 - \gamma(\theta)) = (1 - \phi)r$$

and

$$\lim_{\theta \rightarrow \infty} r_n(1 - \phi)(1 - \gamma_n(\theta)) = (1 - \phi)r_n$$

Due to the fact that  $r < r_n$  (Property 3), and that both  $r(1 - \phi)(1 - \gamma(\theta))$  and  $r_n(1 - \phi)(1 - \gamma_n(\theta))$  decreases strictly when  $\theta$  decreases (Property 1), and also Property (4) and (5), we must have

$$\int_a^\infty r(1 - \phi)(1 - \gamma(\theta))Z(\theta)f(\theta)d\theta < \int_a^\infty r_n(1 - \phi)(1 - \gamma_n(\theta))Z(\theta)f(\theta)d\theta$$

for all  $a \in (-\infty, \infty)$ . And from Property (4) and (5), the following expression must also be true

$$\int_\infty^a r(1 - \phi)(1 - \gamma(\theta))Z(\theta)f(\theta)d\theta > \int_\infty^a r_n(1 - \phi)(1 - \gamma_n(\theta))Z(\theta)f(\theta)d\theta$$

for all  $a \in (-\infty, \infty)$ . As  $a$  can be any real number, this shows that the first expression

$$\int_{-\infty}^{\theta_B^*} r(1 - \phi)(1 - \gamma(\theta))Z(\theta)f(\theta)d\theta - \int_{-\infty}^{\theta_B^*} r_n(1 - \phi)(1 - \gamma_n(\theta))Z(\theta)f(\theta)d\theta$$

must be positive.

For the second part, from Property (4) and (6), and from the assumption that  $\mathbb{I}_S \geq \mathbb{I}_B, \forall \theta$ , we must have

$$\int_a^\infty r\phi(1 - \gamma(\theta))Z(\theta)f(\theta)d\theta < \phi$$

and

$$\int_a^\infty [r(1 - \gamma(\theta)) - r^S(1 - \phi)](1 - \mathbb{I}_S)Z(\theta)f(\theta)d\theta = \phi$$

for all  $a \in (-\infty, \theta_S^*)$ , where  $\theta_S^*$  is the threshold for the survival of equity tranche

(i.e. for  $\theta > \theta_S^*$ ,  $\mathbb{I}_S = 0$ ; for  $\theta < \theta_S^*$ ,  $\mathbb{I}_S = 1$ ).<sup>11</sup> Therefore, we have

$$\int_a^\infty r\phi(1 - \gamma(\theta))Z(\theta)f(\theta)d\theta < \int_a^\infty [r(1 - \gamma(\theta)) - r^S(1 - \phi)](1 - \mathbb{I}_S)Z(\theta)f(\theta)d\theta$$

for all  $a \in (-\infty, \theta_S^*)$ . And from Property (4) and (6), the following expression must also be true

$$\int_{-\infty}^a r\phi(1 - \gamma(\theta))Z(\theta)f(\theta)d\theta > \int_{-\infty}^a [r(1 - \gamma(\theta)) - r^S(1 - \phi)](1 - \mathbb{I}_S)Z(\theta)f(\theta)d\theta$$

for all  $a \in (-\infty, \theta_S^*)$ .

As  $\theta_B^* < \theta_S^*$ , the second expression

$$\int_{-\infty}^{\theta_B^*} r\phi(1 - \gamma(\theta))Z(\theta)f(\theta)d\theta - \int_{-\infty}^{\theta_B^*} [r(1 - \gamma(\theta)) - r^S(1 - \phi)](1 - \mathbb{I}_S)Z(\theta)f(\theta)d\theta$$

must also be positive. ■

---

<sup>11</sup>Also note that

$$\int_{\theta_S^*}^\infty [r(1 - \gamma(\theta)) - r^S(1 - \phi)](1 - \mathbb{I}_S)Z(\theta)f(\theta)d\theta$$

is equivalent to the definition of the return to equity tranche (Property 6)

$$\int_{-\infty}^\infty [r(1 - \gamma(\theta)) - r^S(1 - \phi)](1 - \mathbb{I}_S)Z(\theta)f(\theta)d\theta = \phi$$

## 2.8.4 Proof of Proposition 4

In Section 2.3.4, it has been shown that

$$W = \int_{-\infty}^{\infty} \left\{ r^W W (1 - \mathbb{I}_B) + R(\theta) \mathbb{I}_B \cdot \frac{r^W W}{r^W W + r^D D} \right\} Z(\theta) f(\theta) d\theta$$

$$D = \int_{-\infty}^{\infty} \left\{ r^D D (1 - \mathbb{I}_B) + R(\theta) \mathbb{I}_B \cdot \frac{r^D D}{r^W W + r^D D} \right\} Z(\theta) f(\theta) d\theta$$

By summing up the two equations, one can get

$$W + D = \int_{-\infty}^{\infty} \{ r^W W (1 - \mathbb{I}_B) + r^D D (1 - \mathbb{I}_B) + R(\theta) \mathbb{I}_B \} Z(\theta) f(\theta) d\theta$$

which can be further rewritten as

$$- \int_{-\infty}^{\infty} (r^W W + r^D D) (1 - \mathbb{I}_B) Z(\theta) f(\theta) d\theta = \int_{-\infty}^{\infty} R(\theta) \mathbb{I}_B Z(\theta) f(\theta) d\theta - (W + D) \quad (2.8)$$

In Section 2.4.2, it has been shown that

$$V = \int_{-\infty}^{\infty} [R(\theta) - (r^W W + r^D D)] (1 - \mathbb{I}_B) Z(\theta) f(\theta) d\theta$$

which can be can also be rewritten as

$$V = \int_{-\infty}^{\infty} R(\theta) (1 - \mathbb{I}_B) Z(\theta) f(\theta) d\theta - \int_{-\infty}^{\infty} (r^W W + r^D D) (1 - \mathbb{I}_B) Z(\theta) f(\theta) d\theta \quad (2.9)$$

Substitute equation (2.8) into equation (2.9)

$$V = \int_{-\infty}^{\infty} R(\theta) Z(\theta) f(\theta) d\theta - (W + D)$$

Replacing  $R(\theta)$  with its definition

$$V = \int_{-\infty}^{\infty} [m(\theta) + n(\theta) (1 - \mathbb{I}_S)] Z(\theta) f(\theta) d\theta - (W + D)$$

Replacing  $n(\theta)$  with its definition

$$V = \int_{-\infty}^{\infty} \left[ \begin{array}{c} m(\theta) \\ + [rS(1 - \gamma(\theta)) - r^S S(1 - \phi)] (1 - \mathbb{I}_S) \end{array} \right] Z(\theta) f(\theta) d\theta - (W + D)$$

$$\begin{aligned}
V &= \int_{-\infty}^{\infty} \left[ \begin{array}{c} m(\theta) \\ +rS(1-\gamma(\theta))(1-\mathbb{I}_S) \end{array} \right] Z(\theta)f(\theta)d\theta - (W+D) \\
&\quad - \int_{-\infty}^{\infty} [r^S S(1-\phi)(1-\mathbb{I}_S)] Z(\theta)f(\theta)d\theta
\end{aligned} \tag{2.10}$$

In Section 2.3.3, it has been shown that

$$(1-\phi)S = \int_{-\infty}^{\infty} [r^S S(1-\phi)(1-\mathbb{I}_S) + rS(1-\gamma(\theta))\mathbb{I}_S] Z(\theta)f(\theta)d\theta$$

which can be rewritten as

$$- \int_{-\infty}^{\infty} r^S S(1-\phi)(1-\mathbb{I}_S)Z(\theta)f(\theta)d\theta = \int_{-\infty}^{\infty} rS(1-\gamma(\theta))\mathbb{I}_S Z(\theta)f(\theta)d\theta - (1-\phi)S \tag{2.11}$$

Substitute equation (2.11) into equation (2.10),

$$\begin{aligned}
V &= \int_{-\infty}^{\infty} [m(\theta) + rS(1-\gamma(\theta))] Z(\theta)f(\theta)d\theta \\
&\quad - [W+D+(1-\phi)S]
\end{aligned}$$

Substituting  $m(\theta) = rL(1-\gamma(\theta)) + r^W W_n(1-\mathbb{I}_B) + R(\theta)\mathbb{I}_B \cdot \frac{r^W W_n}{r^W W + r^D D}$

$$\begin{aligned}
V &= \int_{-\infty}^{\infty} r(L+S)(1-\gamma(\theta))Z(\theta)f(\theta)d\theta \\
&\quad + \int_{-\infty}^{\infty} \left\{ r^W W_n(1-\mathbb{I}_B) + R(\theta)\mathbb{I}_B \cdot \frac{r^W W_n}{r^W W + r^D D} \right\} Z(\theta)f(\theta)d\theta \\
&\quad - [W+D+(1-\phi)S]
\end{aligned}$$

Substitute  $A = L + S$  and

$$W_n = \int_{-\infty}^{\infty} \left\{ r^W W_n(1-\mathbb{I}_B) + R(\theta)\mathbb{I}_B \cdot \frac{r^W W_n}{r^W W + r^D D} \right\} Z(\theta)f(\theta)d\theta$$

into  $V$ ,

$$V = \int_{-\infty}^{\infty} rA(1-\gamma(\theta))Z(\theta)f(\theta)d\theta + W_n - [W+D+(1-\phi)S]$$

And because  $W_n = (1-\phi)S$  and  $\int_{-\infty}^{\infty} r(1-\gamma(\theta))Z(\theta)f(\theta)d\theta = 1$

$$V = A - (W+D)$$

Due to balance sheet constraint  $A = W + D + E$ , one can get  $V = E$ .

■

# Chapter 3

## Banking Structures and Social Value

### 3.1 Introduction

Following the 2007-2009 global financial crisis, the Independent Commission on Banking (ICB) in the United Kingdom, chaired by Sir John Vickers, was asked to consider structural and related non-structural reforms to promote financial stability and competition in UK banking sector. And in its final report [48], released on 12 September 2011, the key suggestion on financial stability is to reduce the taxpayer subsidies to bank risk-taking by the ring fencing of different subsidiary banks and banking activities. In short, the report suggests that, without prohibiting banking groups from providing both commercial and investment banking services, the UK-domestic retail banking subsidiaries should be legally insulated (ring-fenced), but not separated, from other investment-banking and global-banking activities.

From the ICB final report, it is clear that an important purpose of ring fencing of subsidiary banks is to ensure that capital and liquidity transfers from ring-fenced subsidiary banks to non-ring-fenced activities are restricted: *“one case where the ring-fence would constrain capital flows... is when the group did not have sufficient capital to maintain appropriate safeguard levels in the ring-fenced bank... at times of financial distress when the safety and continuity of the retail banking operations could be jeopardised by transferring capital across the ring-fence.”* (paragraph 5.52, page 138).

However, restricting liquidity and capital transfer between ring-fenced and non-ring-fenced subsidiary banks does not mean it is forbidden to do so. In fact, one key advantage of ring fencing, according to the ICB final report, is that *“different parts of the group would be able to recapitalise each other subject to meeting regulatory minima – when the transfer of capital is likely to be socially valuable.”* (paragraph

A3.20, page 274).

It is also obvious from the ICB final report that the UK policy makers are not in favour of total separation of subsidiary banks: “*the Commission does not accept the conclusion that only total separation will work.*” (paragraph 2.14, page 26). This is because “*in reality, the ability to transfer excess capital around the group in normal conditions should provide substantial advantages to creditors and capital providers compared to full separation*” (paragraph A3.71, page 292).

Thus the reform of the UK banking sector will be focused on the control of universal banking. A lot of research has shown that universal bank affiliations create positive value (Vennet (2002) [77], Overfelt et al (2009) [66] and Chronopoulos et al (2011) [31]). The objective of this chapter is to compare, in a theoretical setting, the social value of different banking structures. Is ring fencing the best banking structure to an economy as a whole?

This chapter constructs a simple model to characterise a banking group that consists two different subsidiary banks. The model distinguishes safe *utility* subsidiary bank from riskier *casino* subsidiary bank, and derives the expected return to the two subsidiary banks under different banking structures. The model assumes that utility subsidiary bank is protected by the government deposit insurance and its risk is restricted. The risk restriction ensures that utility bank operates a safe business model which guarantees its survival at different economy states. On the other hand, casino subsidiary bank is not protected by deposit insurance, and has no risk restriction on the choice of investment. Therefore, casino subsidiary bank operates a risk-taking business model to achieve a high expected return, subject to the possibilities of bank runs when the economy is at bad state(s).<sup>1</sup>

Under these model specifications, three types of banking structures: (1) total separation, (2) ring fencing, and (3) universal banking are studied. In this chapter, banking structures are defined mainly based on the restriction of capital and liquidity transfer between bank subsidiaries. Total separation of subsidiary banks is defined as a bank structure under which subsidiary banks (utility and casino banks) are not allowed to transfer capital and liquidity to each other, regardless of the economy states. Ring fencing of subsidiary banks is defined as a bank structure under which subsidiary banks are allowed to transfer capital and liquidity to each other, provided that the ring-fenced utility bank has fulfilled its promise return to the consumers. Universal banking is defined as a bank structure under which subsidiary banks are committed to transfer capital and liquidity to each other, whenever there is a shortfall in one of the subsidiary bank.

Under each type of banking structure, the model derives the required return

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<sup>1</sup>To avoid cumbersome expression, utility (casino) subsidiary bank are simply called utility (casino) bank in the following of this chapter.

demanded by the consumers, the expected net return to each subsidiary bank and to the banking group, and the cost of deposit insurance. The model then compares the social values under the three banking structures.

The proposed model suggests that, among the three banking structures, total separation is always suboptimal to both the banking group and to the economy as a whole. The key reason is that the forbidden liquidity transfer restricts the banking group from preventing the occurrence of bank run in casino bank (even when this can avoid unnecessary asset liquidation which is socially-wasteful).

The model also suggests that, whether ring fencing or universal banking is the best banking structure depends on other factors. The proposed model suggests that if the expected return to casino bank is sufficiently high relative to utility bank, universal banking creates higher social value than ring fencing. This conclusion suggests an insightful view on the choice of banking structures: whether it is socially beneficial to protect utility banking sector with ring fencing should include careful assessment on (1) the social value and cost of liquidity transfer and (2) the return to utility and casino banking sector. If the policy makers fail to do so, ring fencing may not be beneficial to the economy, or even worse, it simply becomes a policy to isolate some riskier banking activities and takes away the responsibility of the policy makers on regulating those riskier activities.

What is new in this study is the modelling of the trade-off under different banking structures. It investigates how the trade-off affects the risk and the return to the subsidiary banks and to the banking group, the required return demanded by the consumers, the cost of deposit insurance and the social value. To analyse these issues, this chapter applies the standard framework used in the modelling of liquidity risk in banking sector to capture the uncertainties to the subsidiary banks. Specifically, the specifications of utility bank and casino bank are in spirit similar to two standard models, Diamond and Dybvig (1983) [36] and Allen and Gale (1998) [2].

To the best of my knowledge, this is the first study to construct a formal theoretical model to compare the banking structures of total separation, ring-fencing, and universal banking, and to evaluate the social value of different banking structures. To summarise, this study has two main contributions. First, I develop a theoretical model to capture utility banking and casino banking in the economy; under this framework, the three types of banking structures, defined by the different restrictions on liquidity transfer, are specified. Second, the trade-off and the social value of the three types of banking structures are derived and compared; the proposed model suggests that total separation is always suboptimal, and the choice between ring fencing and universal banking depends on the expected return to the subsidiary banks.

The rest of this chapter is organised as follows: Section 2 provides a brief liter-

ature review on ring-fencing and universal banking. Section 3 presents the model, including the specification of consumers, utility bank, casino bank, and the banking group. Section 4 defines the three banking structures, and compares the social value of the banking structures. Section 5 provides some numerical examples to better illustrate the theoretical results. Section 6 concludes.

## 3.2 Literature Review

### 3.2.1 Studies on Ring-Fencing

The studies on ring-fencing in banking sector is still in its infancy. Exploring ring-fencing in banking as a major objective is scarce in previous research; most of the previous papers discuss ring fence only as a minor policy suggestion. Acharya (2011) [1] comments that the introduction of ring fence according to the ICB final report may fail because banks may be encouraged to take excessive risks with activities that are inside the fence. A major source for the 2007-2009 crisis is a direct result from the risky mortgages and mortgage backed securities, which were held in the commercial banking exposures (within the fence). And ring fencing, by itself, would not necessarily have controlled the risky exposures within the fence. Acharya suggests the determination of risk weights for bank assets may need further attention, particularly on the very low risk weights on residential mortgage backed securities.

Chow and Surti (2011) [30] discuss the motivation, content, operational challenges and potential costs of three major proposals for financial reform after the crisis; the three proposals include narrow utility banking (a reversion of deposit-funded banks into traditional payment function outfits without lending and investment banking activities), the Volcker Rule (forbidding deposit-funded banks to carry out certain types of investment banking activities), and Vickers' Ring-Fence. The paper concludes that all proposals fail to consider the shadow banking sector, and since regulated banking institutions are likely to maintain links with the shadow banking sector, the systemic risk that is shifted to the unregulated sector can still affect the regulated banks. Their paper also concludes that retail ring-fence has a smaller loss of diversification benefits than the other two proposals. Nonetheless, whether the loss of diversification benefits can be balanced by the gains of the proposals remains unclear. Their paper also points out that operational challenges specific to each proposal may limit their effectiveness.

Song (2004) [73] briefly introduces and discusses ring-fencing as a mean for supervision of foreign banks in emerging markets. The ring-fence discussed in Song's paper focuses on the insulation of domestic bank subsidiaries from foreign parent

banks. Song comments that there may exist concerns for transparency because market participants may not be fully aware of the ring-fencing contracts, and markets could operate in a very different way during crises due to the ring-fencing regulations.

Empirical studies for ring-fencing retail banking subsidiaries from other activities are hardly possible because of the lack of data. The existing empirical studies on ring fence in banking industry focus on cross-border ring fencing (insulating domestic subsidiaries from foreign parent banks). Cerutti et al(2007) [26] find evidence that subsidiary operations are preferred by foreign banks seeking to penetrate host markets by establishing large retail operations, while bank branches are more commonly found in countries that have higher taxes and lower regulatory restrictions. Cerutti et al(2010) [27] focus on the costs of ring-fencing (measured in terms of the amount of external capital that is required to cover capital shortfalls faced by the affiliates of these groups as a result of a credit shock) for cross-border banking groups under three different forms of ring-fencing (Partial, Nearly Complete, and Full Ring Fencing). With the data from European bank groups and markets, they find evidence that under stricter forms of ring-fencing, sample banking groups have substantially larger needs for capital buffers at the parent and/or subsidiary level than under less strict or in the absence of any ring-fencing.

These empirical studies provide very limited insights for the type of ring fencing (of retail banking subsidiaries from other activities) suggested in the ICB final report. To the best of my knowledge, no formal theoretical modelling for ring fencing in banking industry has been done in previous studies.

### **3.2.2 Studies on Universal Banking**

Compared with those of ring fence, the studies of universal banking is much better developed.

Saunders (1999) [70] comments on how different types of universal banking can have different impacts on bank competition. He points out that there are two ways to achieve the consolidation of financial services in the Unites States. One is by establishing *Section 20 subsidiary banks* which are allowed to underwrite corporate securities; the other is by merger and acquisition of securities firms. Although the two ways of achieving universal banking can both achieve the goal of the consolidation of financial services, their impacts on bank competition are very different. The establishment of subsidiary banks has a pro-competitive effect bank competition from the evidence that it lowers the yields for underwriting debts. On the contrary, merger and acquisition reduces the number of competitors in the market for investment banking services, and therefore has an anti-competitive effect. He reminds the regulators that monitoring the form of entry is necessary for a precise

evaluation of the overall benefits in financial consolidation.

Wagner (2008) [79] has a strong implication to this chapter. In his paper, Wagner studies how more homogenised large banks affect financial stability. Wagner defines homogenised large banks as banks that extend beyond their traditional activities to combine investment banking and insurance activities in one organisation; this definition is very similar to the concept of universal banking in this chapter.

Wagner (2008) proposes a 3-dated model which has a liquidity risk framework. On date 0, banks invest on behalf of the households in both a storage technology and an risky asset. The risky asset produces a random return on date 1 and also requires an injection of liquidity at the same time; if the risky asset receives the injection of liquidity, it produces a constant return on date 2; otherwise, the return on date 2 is zero. The random date-1 return depends on an aggregate factor and an idiosyncratic factor. On date 1, if the aggregate liquidity available in the banking system is higher than the liquidity demand, all risky assets receive the required injection of liquidity (through interbank market); otherwise, liquidity crisis occurs, under which some risky assets are sold to banks with liquidity surplus and the rest are discontinued.

Wagner shows that the choice of investment made by individual banks are socially inefficient (too much investment is made in risky asset and not enough investment in liquidity), because individual banks do not internalise the impact of their investment choice on the aggregate liquidity level in the banking system. Moreover, homogenisation, which is defined as the possibility of banks to invest in an *aggregate asset* that has no idiosyncratic risk due to diversification, makes the problem worse; this is because homogenisation, through the reduction of inefficiencies, lowers a bank's cost of investing in the risky asset; this encourages a bank to invest more in risky asset and less in liquidity, and this in turn reduces financial stability. An important implication from Wagner's paper is that, although universal banking usually enjoys the benefits from risk diversification, it does not necessarily mean that it is a socially optimal banking structure because diversification of idiosyncratic risk does not affect the aggregate risk in banking system as a whole.

Although universal banking shares a few similarities as ring-fenced banking (mainly in efficiency and customer synergies), these studies can hardly be applied to the analysis of ring-fenced banks, due to some major differences between the two banking structures. One major difference is the limit on risk contagion: under ring-fencing, the ring-fenced bank subsidiaries have a strict limit on the transfer of funding to the non-ring-fenced subsidiaries. This is very different from the free transfer of liquid assets and other resources between the different branches in the universal banks. For this reason, the efficiency of asset allocation in universal banking may be largely reduced in the ring-fenced banking. The suggestions for potential benefits of

universal banking, such as the diversification benefits suggested by Allen and Jagtiani (2000) [5] and the de-specialisation efficiency suggested by Vennet (2002) [77] and Overfelt et al (2009) [66] and the strong evidence of being more cost- and profit efficient in diversified institutions found in Chronopoulos et al (2011) [31], may not be applicable in ring-fenced banking.

Another major difference is the existence of moral hazard; under the banking structure of universal banking there exist incentives for excess risk-taking due to the safety net (as suggested by Boyd (1999) [14]) or other forms of moral hazards and conflicts of interest (as suggested by John et al (1994) [53] and Boyd et al (1998) [15]).<sup>2</sup> These incentives may no longer exist in ring-fenced banking because the non-ring-fenced subsidiaries may not be protected or subsidised by the authorities, and the risk-sensitivity from the creditors is expected to increase and acts as a major source of control for the risk-taking of non-ring-fenced subsidiaries. Due to these major differences, it is obvious that, although there exist similarities between universal banking and ring fencing, the conclusions from the studies on universal banking should not be applied directly to the banking structure of ring-fencing.

For more general discussions and a wider scope of literature reviews on universal banking. Please refer to Calomiris (1995) [22] for an excellent introduction for universal banking, and a comparison of the universal banking systems between the United States and Germany on a historical basis.

The proposed model in this study is related to a number of studies on bank liquidity models and bank failures, including Byrant (1980) [20], Diamond and Dybvig (1983) [36], Chari and Jagannathan (1988) [28], Allen and Gale (1998 [2], 2004 [3], 2007 [4]), Zhu (2005) [83], Samartin (2005) [69], Diamond and Rajan (2005) [37], Calomiris and Kahn (1991) [24] and Marini (2008) [60]. However, because bank liquidity models are not the focus in this study, a thorough literature review on these models is not included here. For the readers who are interested in bank liquidity models applied in this chapter, please refer to the mentioned papers for details.

### 3.3 Model Specifications

The proposed model modifies the frameworks suggested in Diamond and Dybvig (1983) and Allen and Gale (1998) to capture the characteristics of utility bank and casino bank under different banking structures. With this framework, the model aims to study how different banking structures affect the risk and profit of the subsidiary banks, which in turn affect the social value of the banking structures. In this

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<sup>2</sup>Kroszner and Rajan (1994) [58] suggest otherwise. They find no evidence of conflicts of interest with their US dataset. Mishkin (1999) [63] also argues that the diversification benefit from financial consolidation are less prone to failure, making deposit insurance less necessary.

section, the model specifications and assumptions for the economy, the consumers, the banks and the deposit insurance are explained. Note that these specifications are not affected by the choice of banking structures, which is explained in Section 3.4.

This section is divided into four subsections: the first subsection specifies the time horizon and the assets (production technologies) in the economy; the second subsection describes the characteristics of the consumers; the third subsection explains the specifications for the banking group which consists of utility bank and casino bank; the fourth subsection defines the deposit insurance in the economy.

The story of the model is summarised in the following time line.

Date 0	Date 1	Date 2
<p>Consumers receive an endowment of goods. They invest their endowment in one of the two subsidiary banks (utility bank and casino bank).</p> <p>Subsidiary banks absorb the endowment from the consumers, and invest in storage technology and different long-term assets.</p> <p>The date-2 economy state, which affects the return to casino-bank investment, is uncertain at this time.</p>	<p>The economy state on date 2 is announced to all agents; there are three possible economy states, <math>S = H, M, L</math></p> <p>Early consumers withdraw their investment from their subsidiary banks.</p> <p>If the late consumers know that they will receive their promised return on date 2, no bank run occurs. If not, bank run is triggered.</p> <p>Different combinations of banking structure and the economy state cause different bank-run scenarios.</p>	<p>Depending on the banking structures and the liquidity shortfall, liquidity transfer may take place.</p> <p>The late consumers withdraw their investment from their subsidiary banks, if no bank run occurs on date 1.</p> <p>After all consumer withdrawals are satisfied, the remaining profit is the net return to the subsidiary banks.</p>

Figure 3.1: The time line of the model.

All notations used in this model are listed in Table 3.1.

Notations	Definitions
$\lambda$	Proportion of early consumers
$p_S \in \{p_H, p_M, p_L\}$	Probability of having economy state $S$
$R_S \in \{R_H, R_M, R_L\}$	Return to risky asset at economy state $S$
$R$	Return to safe asset
$r$	Liquidation value for risky asset, per unit of realised return
$C_j \in \{C_t, C_r, C_u\}$	Required return for the casino-bank consumers, under banking structure $j$
$V_j \in \{V_t, V_r, V_u\}$	Expected net return to the banking group, under banking structure $j$
$D$	Cost of deposit insurance

Table 3.1: Table of Notation in Chapter 3

### 3.3.1 The Economy

There are three dates (denoted by 0, 1, 2) in the model. On date 0, bank contracts are drawn and investments are made; on date 1, uncertainties are resolved and early liquidity demand is settled; on date 2, residual resources are distributed to the remaining claimants.

There are four types of agents in the economy: (1) consumers, (2) utility subsidiary bank, (3) casino subsidiary bank, and (4) banking group. All agents are assumed to be risk-neutral; therefore, their utility function is linear and can be expressed simply by

$$U(x) = x$$

where  $x \geq 0$ . The specifications of the agents are explained in detail in the next subsection.

In the economy, there is one type of good which can be used for either consumption or production. There are also three types of production technologies: (1) storage technology, (2) safe asset, and (3) risky asset. The return to the assets are summarised in Table 3.2.

<b>Date-2 Return (No Liquidation)</b>			
Economy State	Storage	Safe Asset	Risky Asset
$S = H$	1	R	$R_H > 1$
$S = M$	1	R	$R_M = 1$
$S = L$	1	R	$R_L < 1$

<b>Date-1 Return (Liquidation)</b>			
Economy State	Storage	Safe Asset	Risky Asset
$S = H$	1	1	$rR_H$
$S = M$	1	1	$rR_M$
$S = L$	1	1	$rR_L$

Table 3.2: Rate of Return to Production Technologies

The storage technology preserves the good between any two adjacent dates; one unit of good stored on date  $t$  produces one unit of good on date  $t + 1$ ,  $t = 0, 1$ , with probability one.

The safe asset is a risk-free and moderately-productive long-term investment. It requires an investment on date 0 and produces a constant return on date 2; each unit of good invested on date 0 produces  $R > 1$  units of good on date 2, with probability one. The safe asset can be liquidated on date 1 if needed. The liquidation value for the safe asset is assumed to be one, per unit of safe asset invested on date 0.

The risky asset is also a long-term investment, which requires an investment on date 0, and produces a return on date 2. However, the return to risky asset is

random. The random return takes three values,  $R_H > 1$ ,  $R_M = 1$ , and  $R_L < 1$ , per unit of good invested on date 0. The realisation of the random return depends on the economy state ( $S = H, M, L$ ). The probability of having state  $S$  is denoted by  $p_S$ , and three probabilities sum up to one ( $p_H + p_M + p_L = 1$ ). Although the return to risky asset is generated on date 2, this model assumes that the information of the economy state is resolved on date 1, and this information is available to all agents in the model.<sup>3</sup>

The risky asset can also be liquidated on date 1, but there is a liquidation discount  $r$  on the realised returns. Specifically, the liquidation value for the risky asset is given by  $rR_S$  at economy state  $S$ , per unit of good invested on date 0. Recall that the information of  $R_S$  is resolved on date 1 and therefore the liquidation value  $rR_S$  is deterministic on date 1.

### 3.3.2 Consumers

Consumers are assumed to be risk-neutral, perfect-competitive and continuum, with a mass measure normalised to one. Each consumer is assumed to have an endowment of good on date 0 and nothing else on date 1 and 2. The consumers have a random consumption pattern: those who prefer to consume on date 1 are called *early consumers* and they only value consumption on date 1; those who choose to consume on date 2 are called *late consumers* and they only value consumption on date 2. A consumer's ex-ante (risk-neutral) utility function is given by

$$u(c_1, c_2) = \begin{cases} U(c_1) = c_1 & \text{for early consumer} \\ U(c_2) = c_2 & \text{for late consumer} \end{cases}$$

where  $c_1$  and  $c_2$  are the consumption on date 1 and date 2 respectively.

The consumers do not know their consumption pattern on date 0; therefore, all consumers are date-0 identical. They are assumed to be aware of their consumption pattern on date 1. The consumption pattern of any individual consumer is a private information and is only available to the individual consumer only; in other words, no agent (other than the consumer himself (herself)) can distinguish whether a consumer is an early consumer or a late consumer at all times.

The probabilities of being an early consumer and a late consumer are given by  $\lambda$  and  $1 - \lambda$  respectively, where  $\lambda < 1$  is an exogenous constant. Due to the law of large number, the aggregate proportion of early and late consumers is non-random to an economy; the aggregate proportions of early and late consumers equal the corresponding probabilities respectively.<sup>4</sup>

<sup>3</sup>This specification is similar to Allen and Gale (1998).

<sup>4</sup>This specification is similar to Diamond and Dybvig (1983).

The consumers are assumed to have storage technology, but they cannot invest in the two assets directly. Instead, they can invest their endowment in *one* subsidiary bank. As the consumers are risk-neutral, the subsidiary banks must have to provide the same expected return to the consumers (otherwise, the one with lower return is dominated). Because the subsidiary banks have the same return, the consumers are indifferent between the choice of subsidiary bank. In this model, I assume that the consumers invest their endowment randomly in the subsidiary banks; due to the continuum nature of consumers, each bank receives the same amount of endowment; this amount is normalised to  $\frac{1}{2}$  unit of good for each subsidiary bank.

Due to the perfect-competitive nature of the consumers, they are willing to invest in the subsidiary banks if the expected returns from banks is the same as the storage technology that they have.<sup>5</sup> For simplicity, I assume that the consumers do not store their endowment on date 0, because the banks provide an expected return which is equal to the storage technology.<sup>6</sup>

### 3.3.3 Banking Group

There is a representative banking group in the economy. The banking group consists of two subsidiary banks: *utility bank* and *casino bank*. In this model, the representative banking group is assumed to have no equity, but it has a financial-intermediation technique which can be passed to its subsidiary banks. This technique allows the subsidiary banks to collect endowment from the public and to invest in the long-term assets. In this model, the banking group is assumed to be risk-neutral. The objective of the banking group is to maximise its expected net returns (the sum of the expected net returns from the two subsidiary banks), subject to the risk of potential subsidiary-bank failures.

This model assumes sequential service constraint (or first-come-first-serve principle) as in Diamond and Dybvig (1983): in a bank run, consumers get their full promised return before the depletion of bank resources, and the rest get nothing. However, unlike in Diamond and Dybvig (1983) model, this chapter focuses on bank failures that are absolutely unavoidable, or simply, *fundamental bank runs*. An essential bank run happens when the late consumers are aware that they will not be able to receive their promised return from their bank on date 2. Due to the indistinguishable nature of the consumers, the late consumers withdraw on date 1; this causes an unavoidable bank run. The Diamond-and-Dybvig type of bank failure, which is caused by pure consumer panicking (or *sunspot bank run*), is not discussed

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<sup>5</sup>Another way to interpret the return to consumers is that the subsidiary banks offer a small premium ( $\varepsilon$ ) over the return to storage technology to attract the investment from consumers, where  $\varepsilon \approx 0$ .

<sup>6</sup>Storage technology is only applied during bank run, which is explained in the next subsections.

in this chapter.

Attempting to satisfy the withdrawal in a bank run, the subsidiary bank liquidates the long-term asset. The proceeds from asset liquidation, together with the liquid asset preserved in storage technology, are paid to the withdrawing consumers according to the first-come-first-serve manner. The average rate of return to the consumers, conditional on a bank-run state, is equal to the total return (which comes from liquidated asset and the good preserved in storage technology) divided by the initial investment.

Under different banking structures, there are different combinations of bank-run scenarios in the two subsidiary banks. Detailed discussion of these combinations is discussed in Section 3.4.

In the following, the specifications of utility bank and casino bank are explained.

### Utility Bank

The key feature of the utility bank is that, it is assumed to be protected by the government deposit insurance; this guarantees the returns to the consumers (who invest in utility bank on date 0) when utility bank fails to pay its promised returns. However, due to the protection of deposit insurance, the risk of utility bank is restricted. In this model, this is represented by the forbidden investment in risky asset. In other words, utility bank is only allowed to invest the consumer endowment in storage technology and safe asset.

By definition, there is no uncertainty to utility bank: both the proportion of early (late) consumers and the investment returns are known on date 0. This information allows utility bank to provide a constant promised return to the consumers, while remains safe and not subject to any bank run (except in the banking structure of universal banking<sup>7</sup>). Due to the perfect-competitive nature of the consumers, utility bank can offer the minimum return to the consumers (which is the same as the return provided from the storage technology). Specifically, utility bank promises to pay one unit of good on either date 1 or date 2 for an investment of one unit of good on date 0.

Due to the law of large number, utility bank can accurately predict the date-1 withdrawal from consumers, which is equal to  $\frac{1}{2}\lambda$  unit of good.<sup>8</sup> Therefore, the investment decision for utility bank is straightforward. Utility bank invests  $\frac{1}{2}\lambda$  unit of good in storage technology for the date-1 withdrawal of early consumers, and  $\frac{1}{2}(1 - \lambda)$  unit of good in safe asset to generate a gross return of  $\frac{R}{2}(1 - \lambda)$  unit of good

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<sup>7</sup>In the next section, the model specifies how a bank run can occur to utility bank under the banking structure of universal banking.

<sup>8</sup>Recall that the amount of good invested in each subsidiary bank on date 0 is  $\frac{1}{2}$ ; therefore the withdrawal on date 1 is equal to the size of initial investment ( $\frac{1}{2}$ ) multiplies by the proportion of early consumers ( $\lambda$ ).

on date 1. The net return to utility bank is  $\frac{1}{2}[R(1 - \lambda) - (1 - \lambda)] = \frac{1}{2}(R - 1)(1 - \lambda)$  unit of good.<sup>9</sup>

### Casino Bank

Unlike utility bank, casino bank is not protected by the government deposit insurance. This creates two important consequences. First, casino bank has no restriction on its choice of investment; therefore, casino bank can invest in the storage technology and both types of long-term assets. However, due to the risk-neutral nature of casino bank, casino bank chooses to invest only in the asset which can generate a higher (expected) net return to the bank.

For casino bank to invest in risky asset, this model assumes that the expected return to risky asset is high enough such that, the (expected) net return to casino bank from the investment of risky asset is higher than the net return from the investment of safe asset. As the (expected) net return from the risky asset dominates the safe asset, casino bank does not have an incentive to invest in safe asset. In other words, casino bank only invests in storage technology and risky asset.

The second consequence is that, casino bank is subject to potential bank run. When there is a bank run in casino bank, the average rate of return to the consumers is smaller than one by definition; in other words, the average rate of return at bank-run state(s) is lower than the return from storage technology. For this reason, the promised return to consumers (when there is no bank run) has to be bigger than the return to the storage technology, such that the *expected return* offered by casino bank is indifferent from the storage technology.

Recall that at the two economy states  $S = M, L$ , the gross rates of return to risky asset are  $R_M = 1$  and  $R_L < 1$ . As the promised rate of return to the casino-bank consumers has to be higher than one, this implies that the late consumers will not be able to receive the promised return from casino bank at the two mentioned states, if casino bank does not obtain liquidity transfer from utility bank.<sup>10</sup> As the early consumers can still receive their promised return on date 1 regardless of the economy states, this motivates the late consumers to withdraw on date 1 due to the first-come-first-serve principle. As a result, a bank run occurs in casino bank.

It is worth mentioning that the promised return from casino bank must be the same for both date-1 and date-2 withdrawals: if the return to date-1 withdrawal is higher, a bank run is always triggered; if the return to date-2 return is higher, casino bank pays more than the required return and is not profit-maximising.

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<sup>9</sup>Note that utility bank can also invest all endowment in safe asset and liquidate a portion of safe asset to satisfy the date-1 withdrawal. This choice of investment is equivalent to the mentioned investment choice.

<sup>10</sup>In the next section, the possibilities of liquidity transfer from utility bank to casino bank under the different banking structures are discussed.

As mentioned, if there is no liquidity transfer, bank run occurs in casino bank at the economy state  $S = M, L$ . Let  $C$  be the promised rate of return when there is no bank run. The amount of good casino bank needs to preserve in storage technology is given by  $\frac{1}{2}\lambda C$ , and the amount of endowment invested in risky asset is given by  $\frac{1}{2}(1 - \lambda C)$ . The average return to the consumers at the economy states  $S = M$  and  $S = L$  (bank-run states) are given respectively by

$$\frac{1}{2}(\lambda C + r(1 - \lambda C))$$

$$\frac{1}{2}(\lambda C + rR_L(1 - \lambda C))$$

With these specifications,  $C$  can be determined by solving the following equation<sup>11</sup>

$$U(1) = p_H U(C) + p_M U(\lambda C + r(1 - \lambda C)) + p_L U(\lambda C + rR_L(1 - \lambda C))$$

Recall that  $U(x) = x$ , the expression can be simplified to

$$1 = p_H C + p_M [\lambda C + r(1 - \lambda C)] + p_L [\lambda C + rR_L(1 - \lambda C)]$$

After some algebraic rearrangement, one can get

$$C = \frac{p_H + p_M(1 - r) + p_L(1 - rR_L)}{p_H + \lambda \cdot [p_M(1 - r) + p_L(1 - rR_L)]} > 1$$

At the bank-run states ( $S = M, L$ ), the net return to casino bank is zero. Therefore, the expected net return to casino bank is given by

$$p_H \cdot \frac{1}{2} [R_H(1 - \lambda C) - (1 - \lambda)C]$$

which is the difference between the return from risky asset and the promised return to late consumers at the state  $S = H$ .

It is worth pointing out that the type of deposit contract offered by casino bank in this model is not an optimal financing contract because the return to depositors are not state-contingent. If the deposit contract can be state-contingent, then casino bank can offer different returns to its depositors at different economy states; theoretically, this avoids the occurrence of bank run. However, as the state-contingent

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<sup>11</sup>It is worth mentioning that, due to the risk-neutral nature of consumers, the consumers care only for the expected return. Therefore, it is not necessary to specify which consumers receive the full promised return and which consumers get nothing under the sequential service constraint, as long as each consumer has an equal chance of receiving full return in a bank run. For this reason, the average return is applied in the derivation of  $C$  in the following.

contract does not reflect the possibility of casino-bank failure, it is a less interesting framework and therefore this chapter does not consider the assumption of state-contingent contract in the model.

### 3.3.4 Deposit Insurance

Deposit insurance protects the consumers of utility bank. It guarantees that all consumers who invest in utility bank can *eventually* get their promised return, by compensating the gap between the promised return and the actual return to the utility-bank consumers.

In this model, I assume that to claim the compensation from deposit insurance, the consumers need to spend effort on the application procedure, which generates inconvenience to the consumers. This inconvenience reduces the consumer utility by a very small amount  $\epsilon \approx 0$ .

Even though the reduction in consumer utility is so small that it is almost zero, this assumption is very important to the specifications of banking structures in the next section. In the next section, the model shows that due to the inconvenience  $\epsilon$ , a bank run in utility bank is possible under the banking structure of universal banking.

The cost of deposit insurance, denoted by  $D$ , is defined as the total amount of good compensated to the consumers when there is a bank run in utility bank. The mathematical expression for  $D$  is derived in the Section 3.4.4.

## 3.4 Banking Structures

In this section, three types of banking structures (total separation, ring fencing, and universal banking) are specified, studied, and compared. This section is divided into six subsections. The first subsection defines the three types of banking structures, and explains how economy states affect the returns to subsidiary banks based on the different banking structures. The second, third and fourth subsections derive the returns to consumers and to subsidiary banks, based on the banking structure of total separation, ring fencing, and universal banking respectively. The fifth subsection compares the social value under different banking structures and summarises this section. The sixth subsection further discusses the related issues of transfer pricing, and of the different types of profit maximisation (subsidiary banks verse banking group).

### 3.4.1 Banking Structures and States of Economy

In this model, banking structures are defined based on the restriction of liquidity transfer between the two subsidiary banks. The definitions of the three types of banking structures are as follows.

- *Total separation* of subsidiary banks is defined as a bank structure under which the subsidiary banks (utility bank and casino bank) are not allowed to transfer liquidity (in the form of net return) to each other, regardless of the economy states.
- *Ring fencing* of subsidiary banks is defined as a bank structure under which a subsidiary bank is allowed to transfer liquidity to the other bank, provided that the promised returns to its consumers has already been fulfilled.
- *Universal banking* is defined as a bank structure under which the subsidiary banks are committed to transfer liquidity to each other, whenever there is a shortfall in one of the subsidiary bank.

In this model, a liquidity transfer is defined as transferring a part of the net return from one bank subsidiary to the other *on date 2*, such that the other bank can fulfil the promised return to the late consumers. As the late consumers is guaranteed on date 1 that they will receive their promised return on date 2, no bank run is triggered and no asset liquidation is needed. Note that if the late consumers do not run a bank, the bank always has enough liquidity (from storage technology) to satisfy early-consumer withdrawal.

In order to have a meaningful specification for the economy states, the three economy states satisfy the following assumptions.

- When  $S = H$ , both subsidiary banks have positive net returns and no liquidity transfer is needed, and no bank run occurs in the economy.
- When  $S = M$ , there is a small net loss in casino bank, causing a bank run in casino bank; however, if liquidity transfer is allowed, utility bank can provide enough liquidity to casino bank to avoid the bank run, without affecting the promised returns to the utility-bank consumers.
- When  $S = L$ , there is a huge net loss in casino bank, causing a bank run in casino bank; utility bank is unable to provide enough liquidity to avoid the bank run in casino bank.

The condition that ensures the above specifications are satisfied is

$$(1 - \lambda)C - (1 - \lambda C) < (R - 1)(1 - \lambda) < (1 - \lambda)C - R_L(1 - \lambda C) \quad (3.1)$$

This condition specifies that utility bank has enough profit to provide a liquidity transfer to casino bank at the state  $S = M$ , but not at the state  $S = L$ .

It is worth mentioning that although utility bank is safe due to the non-random return from safe asset and the predictable date-1 withdrawal, a bank run is still possible under the banking structure of universal banking; this happens when the economy state is at the state  $S = L$ . By definition, utility bank is committed to help casino bank whenever there is a shortfall in casino bank; however, the consumers invested in utility bank know that utility bank is unable to transfer enough liquidity to casino bank without affecting their promised return. Due to the assumption of sequential service constraint and the inconvenience of deposit insurance ( $\epsilon$ ), the late utility-bank consumers attempt to withdraw on date 1 to avoid the utility reduction of  $\epsilon$ ; this causes a bank run to occur. Note that, no matter how small  $\epsilon$  is, the bank run in utility bank is triggered as long as  $\epsilon$  is positive.

Table 3.3 summaries the relationship between the banking structures and the economy states, where a  $\checkmark$  represents no bank run occurs and a  $X$  represents bank run occurs.

Structures	Total	Separation	Ring	Fencing	Universal	Banking
Subsidiary	Utility	Casino	Utility	Casino	Utility	Casino
$S = H$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
$S = M$	$\checkmark$	$X$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
$S = L$	$\checkmark$	$X$	$\checkmark$	$X$	$X$	$X$

Table 3.3: Bank Runs under Different Banking Structures

In the following three subsections, under different banking structures I derive (1) the required return to the consumers, (2) the expected returns to the two subsidiary banks, and in Section 3.4.4 (3) the cost of deposit insurance. In the fifth subsection, the social values under different banking structures are compared.

Note that deposit insurance is not required under total separation and ring fencing by definition; i.e. the cost of deposit insurance is zero under these two banking structures. Therefore, the net expected return to the banking group can be interpreted as a measure of social value.<sup>12</sup> Under universal banking structure, the social value is defined as the expected return to the banking group *minus* the expected cost of deposit insurance.

To avoid notation confusion under the different banking structures, I re-notation  $C$  and  $V$  with the subscripts  $t, r, u$ , which stands for the banking structures of total separation, ring fencing, and universal banking respectively. For example,  $C_t$

<sup>12</sup>By definition, the expected return to the consumers does not affect the social value in this model because their expected return is always the same (a gross rate of return of one) regardless of the banking structures.

represents the required return by the consumers invested in casino bank (in the no-bank-run state) under the banking structure of total separation.

### 3.4.2 Total Separation

As mentioned, the banking structure of total separation is defined as the forbidden transfer of liquidity between subsidiary banks. Therefore, utility bank always survives and its expected net return is not affected by the economy states; casino bank survives at the economy state  $S = H$ , and suffers bank runs at the states  $S = M, L$ . In fact, the derivation of the required return by the consumers invested in casino bank ( $C_t$ ) and the expected net return to the subsidiary banks have already been derived in Section 3.3.3. The expressions are reproduced in the following.

The net return to utility bank is

$$\frac{1}{2}(R-1)(1-\lambda)$$

and the expected net return to casino bank is

$$p_H \cdot \frac{1}{2}[R_H(1-\lambda C_t) - (1-\lambda)C_t]$$

where

$$C_t = \frac{p_H + p_M(1-r) + p_L(1-rR_L)}{p_H + \lambda \cdot [p_M(1-r) + p_L(1-rR_L)]} > 1$$

Substituting  $C_t$  into the net return of casino bank, one can get

$$p_H \cdot \frac{1-\lambda}{2} \left[ \frac{p_H(R_H-1) - p_M(1-r) - p_L(1-rR_L)}{p_H + \lambda[p_M(1-r) + p_L(1-rR_L)]} \right]$$

Note that if the expected net return to casino bank is positive, the following expression must be true:

$$p_H(R_H-1) - p_M(1-r) - p_L(1-rR_L) > 0 \quad (3.2)$$

Under the banking structure of total separation, the total net return to the banking group  $V_t$  is expressed by

$$V_t = \frac{1-\lambda}{2} \left[ (R-1) + p_H \left[ \frac{p_H(R_H-1) - p_M(1-r) - p_L(1-rR_L)}{p_H + \lambda(p_M(1-r) + p_L(1-rR_L))} \right] \right]$$

which is the the sum of the net returns from utility bank and casino bank.

### 3.4.3 Ring Fencing

Ring fencing is defined as a bank structure under which utility bank is allowed to transfer liquidity to casino bank, provided that it has fulfilled the promised return to the utility-bank consumers. From equation (3.1), it is clear that under ring fencing, utility bank is only allowed to transfer liquidity to casino bank at the economy state  $S = M$ . The amount of liquidity transfer from utility bank to casino bank on date 2 is the gap between the promised return to the late casino-bank consumers and the return from the casino bank's investment on risky asset at economy state  $S = M$ ; this is given by

$$\frac{1}{2}[(1 - \lambda)C_r - (1 - \lambda C_r)] = \frac{1}{2}(C_r - 1) > 0$$

The liquidity transfer ensures that the late casino-bank consumers receive their promised return on date 2; therefore, the late consumers do not run casino bank on date 1 at the economy state  $M$ . However, as utility bank cannot transfer liquidity to casino bank at the economy state  $L$ , a bank run in casino bank is inevitable.

Taking into consideration of the liquidity transfer, the expected net return to utility bank is given by

$$\frac{1}{2}[(R - 1)(1 - \lambda) - p_M(C_r - 1)]$$

The required rate of return to the casino-bank consumers can be derived by solving

$$1 = p_H C_r + p_M C_r + p_L [\lambda C_r + r R_L (1 - \lambda C_r)]$$

Simple algebraic calculation produces the following solution

$$C_r = \frac{p_H + p_M + p_L(1 - r R_L)}{p_H + p_M + \lambda p_L(1 - r R_L)} > 1$$

Substituting  $C_r$  into the amount of liquidity transfer ( $\frac{1}{2}(C_r - 1)$ ), one can get

$$\frac{1}{2} \cdot \frac{p_L(1 - r R_L)(1 - \lambda)}{p_H + p_M + \lambda p_L(1 - r R_L)}$$

It is worth mentioning that although casino bank receives liquidity transfer from utility bank at the economy state  $M$ , the transfer does not change the fact that casino bank has zero net return at that state; this is because the liquidity transfer is just enough to satisfy the withdrawal of late consumers, leaving casino bank with zero profit. Therefore, the expression for the expected net return to casino bank is very similar as in the previous subsection

$$p_H \cdot \frac{1}{2} [R_H(1 - \lambda C_r) - (1 - \lambda)C_r]$$

Substituting the expression of  $C_r$  into the expected net return of casino bank, one can get

$$p_H \cdot \frac{1 - \lambda}{2} \left[ \frac{(p_H + p_M)(R_H - 1) - p_L(1 - rR_L)}{p_H + p_M + \lambda p_L(1 - rR_L)} \right]$$

Again, if the expected net return to casino bank is positive, the following expression must be true:

$$(p_H + p_M)(R_H - 1) - p_L(1 - rR_L) > 0 \quad (3.3)$$

Under the banking structure of ring fencing, the total net return to the banking group  $V_r$  is expressed by

$$V_r = \frac{1 - \lambda}{2} \left[ (R - 1) + \frac{p_H [(p_H + p_M)(R_H - 1) - p_L(1 - rR_L)] - p_M [p_L(1 - rR_L)]}{p_H + p_M + \lambda p_L(1 - rR_L)} \right]$$

which is the sum of the net returns of utility bank and casino bank under the banking structure of ring fencing.

### 3.4.4 Universal Banking

Universal banking is defined as a banking structure under which the subsidiary banks are committed to transfer liquidity to each other, whenever there is a shortfall in one of the subsidiary banks. According to the model specifications, this means that utility bank needs to transfer liquidity to casino bank at two economy states,  $S = M, L$ .

At the economy state  $S = M$ , the specifications under the universal-banking structure is very similar to those of the ring-fencing structure. Specifically, utility bank transfers enough liquidity to avoid bank run in casino bank. The amount of liquidity transfer required is given by

$$\frac{1}{2}[(1 - \lambda)C_u - (1 - \lambda C_u)] = \frac{1}{2}(C_u - 1) > 0$$

However, at the economy state  $S = L$ , utility bank does not have enough net profit to guarantee the return to the consumers in casino bank without affecting the promised return to utility-bank consumers (equation 3.1).

As mentioned, the commitment of liquidity transfer creates an incentive for the utility-bank consumers to run utility bank at the state  $S = L$ . Under this situation, both utility bank and casino bank liquidate their assets in an attempt to satisfy the withdrawals.

In such a joint bank run, I assume that all consumers (regardless of the subsidiary bank they invested in) equally share the proceeds from the banking group. The assumption captures the fact that, the subsidiary banks are neither separated nor

insulated under the banking structure of universal banking, and the banking group can be considered as a single entity; therefore, the creditors (the consumers) have equal right to claim all assets in the banking group.

As the consumers in utility bank is protected by government deposit insurance, their loss is compensated by the government. Therefore, the utility-bank consumers always receive their promised return (one unit of good per unit of initial investment). Therefore, they do not require a higher return at the no-bank-run states in the same way as the casino-bank consumers.

Note that if there is a bank run in utility bank, the net return to utility bank is zero. Therefore, the expected net return to utility bank is given by

$$\frac{1}{2}[(p_H + p_M)(R - 1)(1 - \lambda) - p_M(C_u - 1)]$$

The required rate of return to the casino-bank consumers can be derived by solving

$$1 = p_H C_u + p_M C_u + p_L \left[ \frac{\lambda C_u + r R_L (1 - \lambda C_u) + 1}{2} \right]$$

Recall that when utility bank liquidates the safe asset, the return from asset liquidation is one unit of good (per unit of initial investment). Therefore, the total amount of good that utility bank can generate in a bank run is  $\lambda + (1 - \lambda) = 1$  (per unit of good invested on date 1). According to the equal-share assumption in the joint bank run, at state  $L$  the rate of return to all consumers is  $\frac{\lambda C_u + r R_L (1 - \lambda C_u) + 1}{2}$ . Rearranging the expression, one can get

$$C_u = \frac{p_H + p_M + \frac{1}{2} p_L (1 - r R_L)}{p_H + p_M + \frac{\lambda}{2} p_L (1 - r R_L)} > 1$$

Substituting  $C_u$  into the liquidity transfer  $\frac{1}{2}(C_u - 1)$ , one can get

$$\frac{1}{2} \cdot \frac{\frac{1}{2} p_L (1 - r R_L) (1 - \lambda)}{p_H + p_M + \frac{\lambda}{2} p_L (1 - r R_L)}$$

The expected net return to casino bank is again very similar to the previous two banking structures

$$p_H \cdot \frac{1}{2} [R_H (1 - \lambda C_u) - (1 - \lambda) C_u]$$

Substituting the expression of  $C_u$  into the expected net return of casino bank, one can get

$$p_H \cdot \frac{1 - \lambda}{2} \left[ \frac{(p_H + p_M)(R_H - 1) - \frac{1}{2} p_L (1 - r R_L)}{p_H + p_M + \frac{\lambda}{2} p_L (1 - r R_L)} \right]$$

If the expected net return to casino bank is positive, the following expression must

be true:

$$(p_H + p_M)(R_H - 1) - \frac{1}{2}p_L(1 - rR_L) > 0 \quad (3.4)$$

Under the banking structure of universal banking, the total net return to the banking group  $V_u$  is expressed by

$$V_u = \frac{1 - \lambda}{2} \left[ (p_H + p_M)(R - 1) + \frac{p_H [(p_H + p_M)(R_H - 1) - \frac{1}{2}p_L(1 - rR_L)]}{p_H + p_M + \frac{\lambda}{2}p_L(1 - rR_L)} - \frac{p_M [\frac{1}{2}p_L(1 - rR_L)]}{p_H + p_M + \frac{\lambda}{2}p_L(1 - rR_L)} \right]$$

which is the sum of the net returns of utility bank and casino bank, minus the liquidity transfer.

What happens if the model considers the cost of deposit insurance in universal banking? As mentioned, the expected net return to the banking group is no longer a measure of social value under universal banking, because it fails to consider the cost of deposit insurance. The social value under the banking structure of universal banking should therefore be defined as

$$V_u^* = V_u - p_L D$$

where  $D$  is the cost of deposit insurance, and can be expressed as

$$D = \frac{1}{2} \left[ 1 - \frac{\lambda C_u + rR_L(1 - \lambda C_u) + 1}{2} \right] = \frac{1}{4} \left[ \frac{(1 - \lambda)(p_H + p_M)(1 - rR_L)}{p_H + p_M + \frac{\lambda}{2}p_L(1 - rR_L)} \right] > 0$$

which is the gap between the promised return and the bank-run share to the utility-bank consumers.

### 3.4.5 Comparison and Summary

In this subsection, the required returns to the casino-bank consumers ( $C_t$ ,  $C_r$ , and  $C_u$ ) and social values ( $V_t, V_r, V_u$  and  $V_u^*$ ) under different banking structures are compared.

I begin with the comparison of the required returns. It is obvious that  $C_r$  must be smaller than  $C_t$ , because bank-run happens at two states under total separation and only one state under ring fencing. Mathematically, one can easily show that

$$C_t - C_r = \frac{(1 - \lambda)[p_H p_M(1 - r) + p_M^2(1 - r) + p_M p_L(1 - rR_L)]}{[p_H + \lambda(p_M(1 - r) + p_L(1 - rR_L))][p_H + p_M + \lambda p_L(1 - rR_L)]} > 0$$

Again, it is also obvious that  $C_u$  must be smaller than  $C_r$ , because at state  $L$ , the

casino-bank consumers receive a higher return (due to the equal share with the utility-bank consumers). It can be easily shown that

$$C_r - C_u = \frac{(1 - \lambda) \cdot (p_H + p_M) [\frac{1}{2} p_L (1 - r R_L)]}{[p_H + p_M + \lambda p_L (1 - r R_L)] [p_H + p_M + \frac{\lambda}{2} p_L (1 - r R_L)]} > 0$$

From the above expressions, one can conclude that

$$C_u < C_r < C_t$$

This result has an important implication. Recall that the casino-bank investment in risky asset is given by

$$\frac{1}{2}(1 - \lambda C_j)$$

where  $j = t, r, u$ . This implies the size of investment in risky asset and the asset return is smallest under total separation, and largest under universal banking, with ring fencing in between the two. This implication is important for the interpretation of the following propositions, and will be revisited shortly in this subsection.

After the discussion of required returns, I now compare the social values of the three banking structures in the following. The formal proofs of the following propositions are given in the appendix of this chapter.

Compare  $V_r$  with  $V_t$ , the following proposition can be concluded.

**Proposition 1** *Both the social value and the expected return to the banking group is higher under the banking structure of ring fencing than under total separation.*

Proposition 1 tells us that ring fencing has two advantages compared with total separation: (1) allowing the ring-fencing liquidity transfer can help reduce the loss from asset liquidation; (2) due to the smaller risk to casino-bank consumers under ring fencing, the casino-bank consumers are willing to accept a smaller return ( $C_r < C_t$ ); this contributes to a higher net return to the casino bank and to the banking group.

Compare  $V_u$  with  $V_r$ , the following proposition can be concluded.

**Proposition 2** *The expected return to the banking group is higher under the banking structure of ring fencing than under universal banking if and only if  $R > f(R_H)$ , where*

$$f(R_H) = 1 + \frac{\frac{1}{2}(1 - r R_L)(p_H + p_M)[p_H(1 + \lambda(R_H - 1)) + p_M]}{[p_H + p_M + \lambda p_L(1 - r R_L)][p_H + p_M + \frac{\lambda}{2} p_L(1 - r R_L)]}$$

Proposition 2 tells us that whether the banking structure of ring fencing or universal banking provides a higher expected return to the banking group has no definite conclusion. This is because, although ring fencing prevents bank runs in utility bank, the required return to casino-bank consumers ( $C_r$ ) under ring-fencing is higher than the required return ( $C_u$ ) under universal banking; therefore, casino bank invests less in the risky asset under ring-fencing. If the return on risky asset at the state  $S = H$  is sufficiently high, it is possible that the reduction of risky-asset investment under ring fencing creates a larger loss compared with the loss of bank run in utility bank under universal banking. Therefore, whether ring fencing or universal banking is a better banking structure to the banking group depends on the returns to both the safe asset and the risky asset.

Finally, compare  $V_u^*$  with  $V_r$ , the following proposition can be concluded.

**Proposition 3** *The social value is higher under the banking structure of ring fencing than under universal banking if and only if  $R > g(R_H)$ , where*

$$g(R_H) = 1 + \frac{\frac{1}{2}\lambda(1-rR_L)(p_H+p_M)[p_H(R_H-1)-p_L(1-rR_L)]}{[p_H+p_M+\lambda p_L(1-rR_L)][p_H+p_M+\frac{\lambda}{2}p_L(1-rR_L)]}.$$

Proposition 3 is very similar to Proposition 2. One can realise that, again, there is no definite conclusion in the comparison of the banking structures of ring fencing and universal banking, even if the cost of deposit insurance is considered. However, this result is in fact not entirely surprising.

It can be observed that the cost of deposit insurance is neither a function of return to safe asset  $R$  nor the return to risky asset  $R_H$ , and it is bounded above by the minimum value of  $R_L$ . Note that the minimum value of  $R_L$  is zero, the maximum value for  $D$  is  $\frac{1}{4} \left( \frac{(1-\lambda)(p_H+p_M)}{p_H+p_M+\frac{\lambda}{2}p_L} \right)$ ; this implies it is possible for the return to risky asset  $R_H$  to be large enough (relative to the return to safe asset  $R$ ) to dominate the cost of deposit insurance.

However, it can also be observed from Proposition 2 and 3 that, it is less likely for universal banking to achieve a higher social value than ring fencing, because it requires an extremely high return to risky asset relative to safe asset.

In the following, I summarise the comparison under the three different banking structures. Two important conclusions can be made. First of all, the model suggests that total separation is not the best banking structures. The reason is that liquidity transfer is a valuable activity between subsidiary banks. The banking structure of total separation destroys the value of liquidity transfer, resulting in suboptimal social value. What is the value of liquidity transfer? From the proposed model, one can observe that liquidity transfer generates value in two ways: (1) Liquidity transfer prevents the avoidable bank run in casino bank; this reduces the loss from

asset liquidation. (2) Due to a smaller expected loss, the consumers require a lower return from casino bank; this allows casino bank to invest more endowment in the (productive) risky asset, generating a higher expected return.

The second conclusion is that, whether liquidity transfer should be restricted depends on other factors. From the definition of the banking structures, this chapter defines two types of liquidity transfer. The first type is the restricted liquidity transfer under ring fencing, and the second type is the committed liquidity transfer under universal banking. The trade-off of these two types of banking structures is between the survival of utility bank and the amount of investment in casino bank.

At the first glance on Table 3.3, the readers might think that the model assumptions should lead to a conclusion that supports ring fencing as the best banking structure, simply because there are less bank-run scenarios under ring fencing than under universal banking. However, this is not true. The model shows that if the expected return to casino bank is sufficiently high, it can dominate the expected loss caused by the failure of utility bank and the expected cost of deposit insurance.

This conclusion suggests an insightful view on the choice of banking structures: whether it is socially beneficial to protect utility banking sector with ring fencing should include careful assessment on (1) the social value and the cost of liquidity transfer and (2) the return to utility and casino banking sector. If the policy makers fail to do so, ring fencing may not be beneficial to the economy.

### **3.4.6 Further Discussion**

#### **Transfer Pricing**

In this chapter, I have not modelled the possibility of transfer pricing between utility bank and casino bank. However, this is an important issue to the choice of banking structure; therefore in the following, I discuss how the existence of transfer pricing can affect the model conclusion.

Transfer pricing happens when a firm has two or more internal units that can generate profits on their own, and each of the units can maximise its own profit subject to its constraints; the internal units are allowed to negotiate the price to buy/sell the products (or services) provided by other internal units. Under classical economic theory (Hirshleifer, 1956 [46]), if the product market is perfectly competitive, the internal price should be the market price; if the market is imperfectly competitive, the internal price can be between marginal cost and market price. In this model, there is no competitive market price for the liquidity support; however, it is still possible to determine an internal price such that liquidity transfer between the subsidiary banks occurs.

To address the transfer pricing in this model properly, the key issue is to realise

the fact that the cost of funding is cheaper in utility bank, but the expected return to casino bank is higher. Therefore, there exists a motivation for utility bank to transfer liquidity to casino bank at the initial time (date 0), so that casino bank can invest this cheaper source of funding in risky asset for a higher return.

Due to the fact that liquidity transfer is allowed in the ring-fencing structure (conditional on the promised return to utility-bank depositors being satisfied), utility bank can reduce its investment in safe asset, so that it can transfer a part of its endowment (funding from consumers) to casino bank on date 0. In fact, if there exists an attractive price for the transfer (discussed below), utility bank will only invest the minimum amount of endowment in the safe asset in order to generate enough return to repay the late (utility-bank) depositors. According to the specifications of the model, this minimum amount of safe-asset investment is given by  $\frac{1}{R} \left(\frac{1-\lambda}{2}\right)$ .<sup>13</sup> This means that utility bank will transfer  $\left(1 - \frac{1}{R}\right) \left(\frac{1-\lambda}{2}\right)$  unit of funding to casino bank.

It is interesting to ask, in order for the above transfer pricing to occur, what should be the (manipulated) internal price for the liquidity transfer? Recall that according to the model, both utility bank and casino bank have to be indifferent to the risk-neutral consumers to prevent being dominated by each other; under this specification, the endowment is therefore distributed equally between the subsidiary banks. As there is no way for casino bank to obtain extra endowment except from the utility-bank transfer, casino bank should be willing to accept the transfer even if it has to surrender all (state-contingent) risky-asset return generated from the liquidity transfer to utility bank.

On the other hand, utility bank is willing to accept an expected return from casino bank as long as it is higher than the safe-asset return. Therefore, the minimum price for utility bank to transfer endowment to casino bank is any expected rate of return that is larger than  $R$ . It is obvious that there is some room for negotiation of the internal price between utility bank and casino bank. However, as long as the internal price is between the acceptable range of the subsidiary banks, both subsidiary banks will be motivated to transfer liquidity on date 0.

However, although this transfer pricing can be beneficial to both subsidiary banks, it may also have a negative impact on the effectiveness of ring-fencing. There are two issues to be discussed here. First, for financial stability, this transfer destroys the benefits of ring fencing. This is because after this date-0 transfer, utility bank will no longer have excess liquidity to support the promised payments to the late casino-bank depositors at the state  $S = M$ ; therefore, a bank run in casino bank, which can be avoidable under the ring-fencing structure, becomes unavoidable as in

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<sup>13</sup>Recall that the promised payment to the late (utility-bank) consumers is  $\left(\frac{1-\lambda}{2}\right)$  and the return to safe asset is  $R$ .

the total-separation structure. One can consider this transfer pricing as a strategic default due to moral hazard, because it is a deliberate choice of default chosen by the subsidiary banks themselves, caused by a moral-hazard type of profit maximisation.

The other issue on the effectiveness of ring fencing is the under-priced rate of return to the casino-bank depositors. This happens when the internal transfer is unobservable and unexpected by the consumers; as the casino-bank consumers expect utility bank to support casino bank at the state  $S = M$ , they are willing to accept a lower rate of return for their investment in casino bank; however, this rate of return becomes under-priced if unexpected liquidity transfer on date 0 takes place. The result is a loss in the consumer welfare of casino-bank depositors.

From the above discussion, it is obvious that transfer pricing can invalidate the motivations and benefits for the ring-fencing banking structure. In fact, the consumer welfare can be worse under ring fencing than under total separation. Therefore, it is possible that there is no particular attractiveness for the choice of ring-fencing banking structure over total-separation banking structure.

Is this socially-undesirable transfer pricing avoidable? The answer is yes, at least theoretically. If the policy-makers can monitor the the transfer between subsidiary banks, so that the transfer cannot be used for increasing investment in risky assets, this type of transfer pricing can be banned, and the benefits of ring fencing can be restored. However, it should also be acknowledged that this can be complicated in practice, because it is difficult to track how the liquidity transfer between subsidiary banks is used, from a huge amount of transactions that modern banks undertake everyday. Therefore, an effective ring-fencing also requires many well-designed supplementary policies to avoid the suggested potential problem.

### **Profit Maximisation Assumptions**

In the following, I discuss an interesting question: whether the profit maximisation decision is made by individual subsidiary banks (maximising profits of individual subsidiary bank) or by the entity as a whole (maximising profits as a whole) makes a difference to the derived model conclusion?

The answer is no, because according to the model specifications in this chapter, maximising the profit of individual subsidiary banks is equivalent to maximising the profit to banking group. One should be aware that the key issue of this question is, whether the investment decisions (storage technology verse assets) of the subsidiary banks will change if the decisions are made 'centrally' by the banking group (instead of the subsidiary banks individually). The answer is no even when the decisions are made centrally by the banking group. First of all, one needs to know that keeping more liquid storage does not reduce or improve the bank-run scenarios: (1) if utility bank keeps more liquid storage, it reduces its date-2 return, and this further reduces

the amount of liquidity that can be transferred to casino bank; therefore utility bank has no intention to change its investment decisions; (2) if casino bank keeps more liquid storage, it does not change the results that it will fail at bad economy states; this is because no matter how much liquid storage is kept by casino bank, as long as there exists a positive investment in risky asset, casino bank can never fulfil the required return to its depositors due to  $C > 1$ . Therefore, keeping more liquid reserve in casino bank can only reduce the total profit to the banking group. For these reasons, it can be concluded that the investment decisions made by individual subsidiary banks should be the same as the decisions made by the banking group 'centrally'.

### 3.5 Numerical Results

In this section, some numerical examples are generated to give a better image of the model results and conclusions.

**Numerical Example 1:** In the first numerical example, I apply the following model parameters.

$$p_H = 0.5, \quad p_M = 0.3, \quad p_L = 0.2, \quad \lambda = 0.5$$

$$R_H = 2.5, \quad R_M = 1, \quad R_L = 0, \quad R = 1.8, \quad r = 0.5$$

The following results are obtained.

$$V_t = 0.5481, \quad V_r = 0.6444, \quad V_u = 0.6259, \quad V_u^* = 0.5788$$

From this numerical example, one can observe that ring fencing is the socially optimal banking structure; universal banking (with consideration of the cost of deposit insurance) is the second best; total separation is the worst. Moreover, as  $V_u < V_r$ , ring-fencing is also the best banking structure for the banking group.

**Numerical Example 2:** However, the proposed model tells us that as the return to risky asset  $R_H$  increases, the optimal banking structure may change. I apply the same parameters as above, except that the value of  $R_H$  is replaced by 5. The following results are obtained.

$$V_t = 1.0111, \quad V_r = 1.2, \quad V_u = 1.2141, \quad V_u^* = 1.1671$$

One can observe that with a higher  $R_H$ ,  $V_u > V_r$ . This implies that the best banking structure for the banking group becomes universal banking. Yet, the socially optimal

banking structure is still ring fencing, because  $V_r > V_u^*$ .

**Numerical Example 3:** In the last example, a very high return to risky asset is applied. Again, with the values of all other parameters unchanged, the value of  $R_H$  is replaced by 10. The following results are obtained.

$$V_t = 1.9370, \quad V_r = 2.3111, \quad V_u = 2.3906, \quad V_u^* = 2.3435$$

With the extremely high  $R_H$ , universal banking is both the socially optimal choice of banking structure and the best banking structure to the banking group. As mentioned in the last section, universal banking has the lowest required return by consumers, which implies the largest amount of risky-asset investment. Given a sufficiently high risky-asset return, the expected return can dominate the cost of deposit insurance (in other words, the social loss to the bank run in utility bank).

Although it may seem unrealistic for risky asset to have an extremely high return relative to the safe-asset return, the three numerical examples show that the socially optimal banking structure do not solely depend on loss incurred in the bank runs; it also depends on the net returns to both utility bank and casino bank.

### 3.6 Conclusion

This chapter introduces a framework that characterises the utility and casino banking activities in a banking group. Under this framework, this chapter studies and compares the social value of three banking structures: total separation, ring fencing and universal banking. The proposed model suggests that the liquidity transfer between subsidiary banks has a positive social value. Forbidding this transfer (total separation) is socially suboptimal and is therefore not recommended in this chapter. The comparison of social value between ring fencing and universal banking is more complicated. The model suggests that whether ring fencing or universal banking is the best banking structure depends on the investment returns to the bank subsidiaries. Specifically, the model shows that if the asset return to casino bank is sufficiently high relative to the asset return to utility bank, universal banking produces a higher social value than ring fencing; in the extreme cases in which the asset return to casino bank is extremely high, the social value of this high return can even dominate the cost of deposit insurance. Otherwise, ring fencing is a better choice for the banking structure for an economy.

## 3.7 Appendix

### 3.7.1 Proof of Proposition 1

**Proof.** As mentioned, the net return to the banking group can be considered as a measure of social value under the banking structure of total separation and ring fencing, because the cost of deposit insurance is zero under both banking structures, regardless of the economy states.

Therefore, to show that ring fencing is better than total separation, I simply need to show that

$$V_r - V_t > 0$$

From the specifications of  $V_r$  and  $V_t$ ,  $V_r - V_t$  is equal to

$$\begin{aligned} & \frac{1 - \lambda}{2} \left[ \frac{1}{[p_H + \lambda(p_M z_M + p_L z_L)][p_H + p_M + \lambda p_L z_L]} \right] \\ & \cdot \{p_H [y_1(p_H + \lambda(p_M z_M + p_L z_L)) - y_2(p_H + p_M + \lambda p_L z_L)] \\ & \quad - p_M [p_L z_L (p_H + \lambda(p_M z_M + p_L z_L))]\} \end{aligned}$$

where

$$z_M = 1 - r$$

$$z_L = 1 - r R_L$$

$$y_1 = (p_H + p_M)(R_H - 1) - p_L z_L$$

$$y_2 = p_H(R_H - 1) - p_M z_M - p_L z_L$$

As the first term of the expression (the fraction) is positive by definition, I need to show that the second term (the one within the curly brackets) is also positive to complete the proof. After some algebraic expansion and rearrangement, the second term can be separated as the following three terms

$$\begin{aligned} & \lambda p_M p_L z_L [p_H(R_H - 1) - p_M z_M - p_L z_L] \\ & + \lambda p_H p_M z_M [p_H(R_H - 1) + p_M(R_H - 1) - p_L z_L] \\ & + [p_H^2 p_M z_M + p_H p_M^2 z_M + \lambda p_H p_M p_L z_M z_L] \end{aligned}$$

Equation (3.2) and (3.3) show that the first and two terms are positive, and the third term is positive by definition. ■

### 3.7.2 Proof of Proposition 2

**Proof.** Substitute the specifications of  $V_r$  and  $V_u$  into the expression  $V_r - V_u$ , one can get

$$\frac{1-\lambda}{2} \left\{ p_L(R-1) + p_H \left[ \frac{y_1}{p_H + p_M + \lambda p_L z_L} - \frac{y_3}{p_H + p_M + \frac{\lambda}{2} p_L z_L} \right] - p_M \left[ \frac{p_L z_L}{p_H + p_M + \lambda p_L z_L} - \frac{\frac{1}{2} p_L z_L}{p_H + p_M + \frac{\lambda}{2} p_L z_L} \right] \right\}$$

where

$$z_L = 1 - rR_L$$

$$y_1 = (p_H + p_M)(R_H - 1) - p_L z_L$$

$$y_3 = (p_H + p_M)(R_H - 1) - \frac{1}{2} p_L z_L$$

After some algebraic rearrangements, one can get

$$\begin{aligned} & \frac{1}{2} \cdot \frac{p_L(1-\lambda)}{(p_H + p_M + \lambda p_L z_L)(p_H + p_M + \frac{\lambda}{2} p_L z_L)} \\ & \cdot \left\{ (R-1)(p_H + p_M + \lambda p_L z_L)(p_H + p_M + \frac{\lambda}{2} p_L z_L) \right. \\ & \left. - \frac{1}{2} z_L (p_H + p_M) [p_H(1 + \lambda(R_H - 1)) + p_M] \right\} \end{aligned}$$

Again, the first term (fraction) is positive by definition. The second term (the curly bracket) is positive if

$$(R-1)(p_H + p_M + \lambda p_L z_L)(p_H + p_M + \frac{\lambda}{2} p_L z_L) > \frac{1}{2} p_L z_L (p_H + p_M) [p_H(1 + \lambda(R_H - 1)) + p_M]$$

Rearranging the terms, one can derive the following condition

$$R > 1 + \frac{\frac{1}{2} z_L (p_H + p_M) [p_H(1 + \lambda(R_H - 1)) + p_M]}{(p_H + p_M + \lambda p_L z_L)(p_H + p_M + \frac{\lambda}{2} p_L z_L)} \equiv f(R_H)$$

■

### 3.7.3 Proof of Proposition 3

**Proof.** From the proof of Proposition 2 and the specification of  $D$ ,  $V_r - V_u^*$  can be expressed as

$$\begin{aligned} & \frac{p_L(1-\lambda)}{(p_H + p_M + \lambda p_L z_L)(p_H + p_M + \frac{\lambda}{2} p_L z_L)} \\ & \cdot \left\{ (R-1)(p_H + p_M + \lambda p_L z_L)(p_H + p_M + \frac{\lambda}{2} p_L z_L) \right. \\ & \left. - \frac{1}{2} \lambda z_L (p_H + p_M) [p_H(R_H - 1) - p_L z_L] \right\} \end{aligned}$$

where

$$z_L = 1 - rR_L$$

Again, the first term (fraction) is positive by definition. The second term (curly bracket) is positive if

$$(R-1)(p_H + p_M + \lambda p_L z_L)(p_H + p_M + \frac{\lambda}{2} p_L z_L) > \frac{1}{2} \lambda z_L (p_H + p_M) [p_H(R_H - 1) - p_L z_L]$$

Rearranging the terms, one can derive the following condition

$$R > 1 + \frac{\frac{1}{2} \lambda z_L (p_H + p_M) [p_H(R_H - 1) - p_L z_L]}{(p_H + p_M + \lambda p_L z_L)(p_H + p_M + \frac{\lambda}{2} p_L z_L)} \equiv g(R_H)$$

■

# Chapter 4

## Bank Competition, Fire-sale and Financial Stability

### 4.1 Introduction

For more than two decades, researchers have been trying to figure out how bank competition is related to financial stability. However, their results are inconsistent and ambiguous, in both theoretical and empirical studies. The traditional view of the literature suggests the competition-fragility view (also called "franchise-value" paradigm) that bank competition has a negative impact on financial stability. The argument lies in the fact that bank competition leads to smaller franchise value (Keeley, 1990 [55]), this motivates banks to take excessive risk, resulting in higher bank losses when there are economic or financial distresses. Empirically, the competition-fragility view is supported by the evidence that, when bank competition increases there are a larger proportion of non-performing loans (NPLs), a smaller capital-to-asset ratio, and/or a higher frequency of financial crisis (Beck, Demirgus-Kunt and Levine, 2006 [10]; Jimenez, Lopez and Saurina, 2007 [52]).

Recently, some literature suggests otherwise. The opposite view (competition-stability view) argues that banks, when facing little competition in the market, are usually inefficient and demands higher loan rates; this creates a risk-shifting effect (Boyd and De Nicolo, 2005 [16]) to the borrowers and causes a higher probability of loan defaults, which can also be detrimental to financial stability. The competition-stability view is also supported by some empirical evidences (Boyd, De Nicolo, and Jalal, 2006 [17]; Amidu and Wolfe, 2011 [7]).

To add confusion to the debated topic, some studies suggest more diverse and complicated results. For example, Berger, Klapper and Turk-Ariss (2009) [11] find evidence that, when market power of banks grows, although loan risk of banks increases, overall bank risk decreases. Boyd and Runkle (1993) [18] suggest that

failure probabilities are essentially unrelated to bank size. Molyneux and Nguyen-Linh (2008) [64], based on the data of South-East Asian banking industry, find no evidence to support bank competition can lead to risk-taking behaviour. Martinez-Miera and Repullo (2010) [61] predict with their model that a U-shaped pattern can also be a possible answer to the debated question; however, soon after that, Jimenez, Lopez and Saurina (2007) [52] show that there is no evidence to support the U-shaped pattern.<sup>1</sup>

Although the conclusions of previous studies are very diverse, most of the mentioned papers have a common feature. These previous studies mainly focus on the asset risks of financial institutions. These risks come from the choice of investment portfolio, the profit margin from asset returns, and the probability of defaults. However, these asset risks only cover the risks that are originated from one side of the banks' balance sheet. How the risks that come from the other side, the funding structure of banks, affect the relationship between bank competition and financial stability has been rarely discussed. One key difference between the asset risks and funding-structure risks is that these risks belong to very different risk categories. Most of the asset risks are market risks, they rely on the market prices, asset returns, and business cycles. These are different from the funding-structure risks which comes more often from the supplies of funding and liquidity risks.

The objective of this chapter is to explore how the funding structure of banks affect the relationship between bank competition and financial stability. This chapter applies a simple liquidity modelling framework and shows that fire-sale, which has rarely been included in the discussion of the debated topic, plays an important role. One key feature of fire-sale is that it is very often based on a systemic basis.<sup>2</sup> For this reason, it is difficult for individuals to accurately evaluate fire-sale costs given the incomplete information that they have. In the latest paper of Shleifer and Vishny (2011) [72], they explain that asset *fire-sales* can deplete the balance sheets of financial institutions and aggravate the fragility of the financial system; they also point out the problem of fire-sales in the 2007-2009 financial crisis in their paper. This supports the necessity to include the role of fire-sale in the studies of financial stability. And this is why this chapter attempts to extend the discussion of the debated topic to characterise fire-sale in the proposed model.

An important finding in this chapter is that, the existence of fire-sale can create an incentive for banks' excessive risk-taking. This incentive is originated from the fact that in a multi-bank economy, a bank can take advantage of other banks in fire-sale by choosing riskier funding structure.

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<sup>1</sup>The criticism of Jimenez, Lopez and Saurina (2007) is based on the working paper in 2007 of Martinez-Miera and Repullo (2010).

<sup>2</sup>Besar et al. (2011) [12] examines how systemic risk can impact the entire financial system and explores its disturbances in the banking sector.

This is because fire-sale price depends on the aggregate amount of asset sold in the economy; it decreases when the amount of asset sold in the economy increases. When a bank chooses a riskier funding structure than the other banks, it needs to sell more asset than the other banks when the economy is at bad states. However, since all other banks are selling a smaller amount of asset, the fire-sale price is relatively high to the riskier bank. On the contrary, the fire-sale price is relatively low to the safer banks. This can be interpreted as a subsidy from safer banks to the riskier bank.

In this chapter, I show that this result holds even when the economy is in equilibrium. Moreover, I show that this excessive risk-taking incentive increases with the number of banks in the economy (which is the measure for bank competition in the proposed model). With this result, I can conclude that, based on the model framework of this chapter, banking competition leads to financial instability; in other words, my results support the traditional view of the mentioned topic (that bank competition weakens financial stability), with a different aspect on the source of risks.

This chapter also discusses policy interventions to control the excessive risk-taking in funding structures. I show that capital requirement is a good way to restore the banks' funding structure to the socially optimal level. However, since capital requirement has been widely used for the control of asset risks in banks, it may be difficult to apply the same policy to the liquidity risks in funding structure. Therefore, I also discuss two other policies: reducing the gap between the costs of deposit and equity, and applying a fire-sale penalty. The numerical results of this chapter show that reducing the gap between the costs of different sources of funding can effectively restore the optimal funding structure. But fire-sale penalty seems to have limited effect and can be outrun by the excessive risk-taking incentive.

This chapter builds on the recent studies of Stein (2011) [75], and Kashyap and Stein (2011) [54]. In their papers, they construct a model to show that banks have an intention to create excessive short-term debt, because the loss from asset fire sales is not fully internalised by individual banks. Although the motivations are different, this chapter is similar in spirit to Stein and Kashyap's papers.

In this chapter, the banks are assumed to be able to raise funding by issuing deposit contract and equity stock. The deposit contract is guaranteed to be risk-free by banks, and therefore is a cheaper source of funding to banks compared with the risky equity stock. This chapter extends Kashyap and Stein's papers by introducing liquidity risk to banks. This liquidity risk comes from the randomness of the proportion of early (late) production (similar to Diamond and Rajan, 2005 [37]), and the liquidity demand that comes from pre-deterministic deposit withdrawals from the households (similar in spirit to Diamond and Dybvig, 1983 [36]). If banks

cannot satisfy the withdrawal of deposit contract with the output from production on an interim date, the banks will have to sell their assets (bank loans) to external investors at a discount (fire-sale price). The discount depends on the aggregate size of asset sold in the fire-sale; the more bank assets need to be sold, the lower is the fire sale price.

The aim of the banks is to maximise their net expected return to their equity holders. In this chapter, I begin with the monopoly-bank economy, the construction of the monopoly-bank decision problem is straightforward because the decision of the monopoly bank does not depend on others. The funding structure chosen by the monopoly bank has the highest level of equity stock (which implies lowest level of liquidity risk) compared with the funding structure chosen in the multi-bank economy under symmetric equilibrium, and is therefore considered as the socially optimal choice.

I then construct the decision problems in a two-bank economy, which is more complicated because a bank decision needs to depend on the conjectures of the decision made by the other bank. The model shows that there exists a unique equilibrium in which the two banks choose the same funding structure. The choice of funding structure under this symmetric equilibrium acts as a basis for the later analysis and comparison in the model.

The model is then further extended from the two-bank economy to a n-bank economy for generalisation. With these specifications, I prove that banks choose riskier funding structures in economies with more banks.

This chapter contributes to the literature in several aspects. First, to the best of my knowledge, this is the first study that incorporates fire-sale to the discussion of the relationship between bank competition and financial stability. Second, this study applies a liquidity framework for the modelling; this is rare in the literature, because most literature focuses on the credit-risk or market-risk aspect (return randomness) for their model construction. The advantage of the liquidity framework is that it can easily capture the existence of liquidity shortfalls and characterise the cost of fire-sale. Third, most literature defines financial instability by measuring loan risk and/or bank overall risk; this neglects the existence of liquidity risk and systemic risk, which can have significant impact in financial crises. This study introduces liquidity shortfall as a source of liquidity risk and fire-sale as a source of systemic risk to study the impact of these risks in financial distresses.

The rest of the chapter is divided into six sections. Section 4.2 reviews important literature which discusses bank competition and financial stability. Section 4.3 presents the model specifications, in which the role of each agent is carefully explained and the model background is constructed. Section 4.4 discusses the decision problems of banks step by step, beginning from a monopoly-bank economy, and then

a two-bank economy, and finally a n-bank economy. Section 4.5 illustrates the results from a numerical example based on the proposed model. Section 4.6 discusses the policy interventions that can help control the funding structure of banks based on the proposed model framework. Section 4.7 concludes.

## 4.2 Literature Review

Numerous research has tried to determine how bank competition can affect financial stability. This section reviews some of these important studies. However, due to the extensive previous research, this literature review is incapable to include all the works that have been done. For a better understanding of previous works, please refer to Vives (2010) [78] and Jimenez, Lopez and Saurina (2007) [52] for excellent and comprehensive reviews on the topic.

In this section, theoretical models and empirical works are discussed separately. The reviewed literature is categorised into two groups according to their results: studies that support *competition fragility*, and those that support *competition stability*.

### 4.2.1 Theoretical Models

#### Competition-Fragility

The traditional view of the debated topic suggests that excessive bank competition has a negative impact on financial stability. Following Furlong and Keeley (1987) [41] and Marcus (1984) [59], Keeley (1990) [55] proposes a 2-dated model, in which a bank faces a random asset return that can take two values (two states). The model shows that if the expected charter (franchise) value is high, the bank chooses a high capital level and a low asset risk to guarantee solvency in both states. Otherwise, the bank makes a riskier decision and bankruptcy is observed in the bad state. The model shows the importance of franchise value in determining bank (overall) risk.

Wagner (2010) [80] proposes a generalised version of the Boyd and De Nicolo (2005) model (reviewed in the following subsection of competition-stability models) and argues that their conclusion may not hold. Wagner shows that the conclusion of Boyd and De Nicolo's model is strongly based on the assumption that bank risk is entirely determined by the borrowers, and the bank has no control over its own risk.

Wagner relaxes the assumption and proposes the following 2-dated model: On date 1, there exist a continuum of entrepreneurs that have different risk-return portfolios; the entrepreneurs are given the right to decide the riskiness of its own project;

however, it is the modelled bank who chooses the entrepreneur to be financed. Therefore, it can be interpreted that the bank chooses its own preferred level of risk in the model. Bank competition is measured by the cost of switching; Specifically, the cost of switching reduces when bank competition gets more intense; this limits the maximum loan rate determined by the modelled bank. Wagner proves the following mechanism: when bank competition gets more intense, loan rate decreases; this encourages the financed entrepreneur to reduce its project risk.<sup>3</sup> However, this lower level of risk is below the preferred level of risk chosen by the bank. As a result, the bank chooses another entrepreneur (instead of the one that it picks in the less competitive banking industry) who has a higher level of risk than the original one. This unambiguously leads to the weakening of financial stability.

### Competition-Stability

Recently, Boyd and De Nicolo (2005) [16] challenges the traditional competition-fragility view by comparing two simple models. In the first *base* model, banks compete only in deposit market, and they maximise their expected return by choosing an optimal level of asset risk and deposit taking. Their paper shows that as the number of bank increases, the asset risk chosen by banks strictly increases. This result supports the traditional view of competition fragility. However, in the *extended* second model, when banks extend their competition to both loan market and deposit market, the role of borrowers is taken into consideration; borrowers are given the right to decide the asset risk they prefer, leaving the banks with only the decision on the amount of deposit taking (which is equal to the amount of loans the banks choose to grant). The authors show that the asset risk is decreasing in the number of banks; this is because higher bank competition leads to lower loan rates, and in turn lower the moral hazard of borrowers, resulting in a choice of lower asset risk. This result supports the alternative view of competition stability. Their paper argues that previous studies fail to consider the risk-shifting effect from banks to borrowers, and the loan market channel is as important as the deposit market channel.

Following Boyd and De Nicolo's work, Martinez-Miera and Repullo (2010) [61] point out that the earlier work fails to consider that lower loan rates can also reduce bank returns. And if this effect is taken into account, the relationship between bank competition and bank risk can be U-shaped. In the latter model, the probability of default is determined endogenously by the borrowers, whose investments are imperfectly correlated. Their paper finds that there is a margin effect together with Boyd and De Nicolo's risk-shifting effect. The margin effect originated from the fact

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<sup>3</sup>A lower loan rate increases the return to the entrepreneurs, leading to a choice of a lower level of risk to prevent the failure of the project.

that more competition leads to lower loan rates, which in turn lower the return of non-defaulting loans to banks; this reduces the buffer against loan losses, resulting in the existence of riskier banks.

Boot and Thakor (2000) [13] studies intra-bank competition and also competition from capital market, based on an alternative aspect: the roles of relationship banking and the welfare of borrowers. They propose a four-dated model and specify the roles of borrowers, depositors, banks and underwriters (capital market). The model characterises intra-bank competition in the form of competitive bidding of loan offers between the banks; relationship banking is defined as a costly sector specialisation investment by the banks, which increases the probability of having successful borrowers' projects. Boot and Thakor show that more intensive intra-bank competition increases the welfare of borrowers with good-quality (high probability of success) projects; however, the welfare of the other borrowers with poor-quality (low probability of success) projects are ambiguous. The empirical study of Degryse and Ongena (2005) [34] finds evidence to support the prediction of Boot and Thakor.

## 4.2.2 Empirical Studies

### Competition-Fragility

There are a lot of empirical studies supporting the traditional competition-fragility view; however, the measures that these papers have applied for quantifying bank competition and bank risk are quite different.

Keeley (1990) [55] aims to show that deregulation in the United States in mid-1960's and the expanded powers of thrifts in the early 1980's increased the competition in banking industry, which in turn eroded banks' charter (franchise) values. This encouraged the U.S. banks to take excessive risks, causing the sharp increase of bank failures after 1980's. Using the data of 85 largest bank holding companies (BHCs) in the U.S. between 1970-1986, Keeley finds that banks with greater market power (measured by market-to-book asset ratio) have lower overall bank risk (larger capital-to-asset ratios) and lower default risk (measured by the loan risk premium on certificate of deposit).

Demsetz, Saidenberg and Strahan (1996) [35] discuss the relationship between franchise value and bank risks, based on the market value and accounting data of 100 bank holding companies (BHC) during 1986-1994. They define franchise value as the difference between market value and replacement cost of a bank, which after normalisation is in the form of Tobin's q ratio. Their paper shows significant results of (1) a negative relationship between franchise value and bank risks (all-in risk, systematic risk and firm-specific risk), and (2) the high-franchise-value BHCs reduce their risk by increasing the capital-to-asset ratio and shifting to a less risky

and more diversified asset portfolio.

Beck, Demirguc-Kunt and Levine (2006) [10] conduct the first empirical test that uses cross-country data. In their paper, they use bank data across 60 countries between 1980-1997. They measure financial stability using the frequency of crises (which is defined by non-performing loans of at least 10 percent, and/or requiring government interventions to restore market order). The measure for market concentration is the share of assets of the three largest banks in the countries. They find that crises are less likely in economies with more concentrated systems. However, their paper has been criticised by some latter studies that market concentration is not a proper measure for competition; Classens and Laeven (2004) [32] point out that there is no evidence for a negative relationship between bank concentration and bank competitiveness.

Jimenez, Lopez and Saurine (2007) [52] also find evidence to support competition-fragility view. They use unique dataset from Spanish banking system, which allows them to calculate Lerner index (their measure for market power) from the marginal interest rates charged by each bank for several banking products. They also extract the risk premium from the marginal interest rates and obtain the non-performing loan ratios (NPLs) as their measures for bank risk. They find a negative relationship between market power and bank risk. Another important contribution from their paper is that the authors have used some standard proxies of market concentration, including Herfindahl-Hirschmann indexes (HHI) and the number of banks operating in the market, to conduct the empirical tests; and they find that the measures of market concentration do not affect the bank risk measure. This finding supports Classens and Laeven (2004) [32] that bank concentration is not a proper measure for bank competitiveness. However, their paper fails to consider the deposit market of banking industry, and this weakens their evidence on competition-fragility view.

### **Competition-Stability**

In contrast, to support their risk-shifting model, Boyd, De Nicolo and Jalal (2006) [17] use z-score and the ratio of equity to total assets as their measures of overall bank risk, and Herfindahl-Hirschmann index (HHI) as their measure of market concentration, to study two data samples: a cross sectional sample of 2500 banks in the United States, and a panel sample of 2700 banks in 134 countries. The results from both samples are consistent, and suggest more concentrated banking industries are associated with more bank failures. However, this empirical literature has the same shortcoming as in Beck, Demirguc-Kunt and Levine (2006): market concentration (HHI) is not a good measure for the degree of bank competition.

Yaldiz and Bazzana (2010) [82] study the role of market power in both the loan risk and overall bank risk, using the data from Turkish banks during 2001-2009 to

conduct their empirical tests. They use the non-performing loans to total loans ratio (NPL) as a measure of loan risk for the Turkish banks, and Z-score as the measure of bank overall risk. For the measures of market competitiveness, they use Lerner index and the ratio of the difference between the total revenues and total costs to the total revenues. Their results suggest that (1) as market power of a bank increases, bank risk increases, and (2) as market power decreases, competition creates less risky banks, which in turn contributes to the stability of the whole banking system.

Amidu and Wolfe (2011) [7] empirically investigate the significance of diversification in the relationship between bank competition and financial stability. They employ three-stage-least-squares-estimate techniques to a panel dataset of 978 banks during the period 2000-2007. They use a number of measures for bank risk (Z-score, capital ratio, and non-performing loans ratio (NPL)) and for proxies of market power (H statistics and Lerner index). For the measure of revenue diversification, they calculate Herfindahl-Hirschmann index (HHI) for each bank. The core finding for their paper is that as bank competition increases, diversification across and within both interest and non-interest income generating activities increases, and this increases financial stability. Their paper also points out that funding structure plays an important role on financial stability, which is consistent to the suggestion of this chapter.

## 4.3 Model Specifications

### 4.3.1 Model Framework

The framework of the model is similar to the one proposed in Diamond and Rajan (2005) [37]. The model begins with an economy with a three-dated time horizon, date 0, 1, and 2. All contracts are drawn under uncertainties on date 0, and all uncertainties are resolved on date 1. The framework of the model characterises the choice of funding structure (deposit contract verse equity stock) when a bank maximises its shareholders' value, taking into consideration of a random liquidity risk and fire-sale loss. Under this framework I determine how the choice of funding structure changes as the number of banks in the economy increases.

There are four types of agents in this model: firms, households, banks and external investors. The following time line summarises their relationship and the story of the model. All notations that are used in this model is tabulated in the following Table 4.1 for reference.

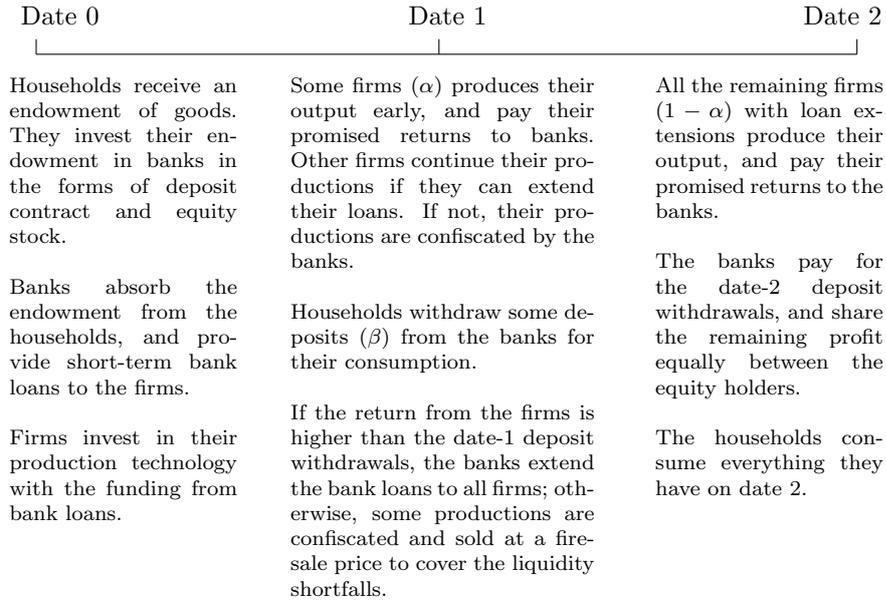


Figure 4.1: The time line of the model.

Notations	Definitions
$n$	Number of banks in the economy
$\alpha$	Random proportion of firms that produces on date 1
$\beta$	Proportion of deposits withdrawn on date 1
$\rho$	Constant return to production technology
$r_t$	Return to deposit withdrawn on date $t$ , $t = 1, 2$
$d_k$	Proportion of deposit in Bank $k$
$1 - d_k$	Proportion of equity in Bank $k$
$q_n$	Amount of endowment invested in <i>one bank</i> in a $n$ -bank economy
$h$	Fire-sale-price coefficient
$x_{jk}$	Per unit production sold by Bank $k$ in a $j$ -bank fire-sale
$\alpha_k^*$	Liquidity-shortfall threshold for Bank $k$

Table 4.1: Table of Notation in Chapter 4

### 4.3.2 Firms

There are a large number (continuum) of perfectly competitive and independent firms in the economy. They are assumed to have no endowments, but all of them have an identical constant-return-to-scale production technology, which requires an initial input of endowment (good) on date 0, and produces  $\rho$  units of goods on either date 1 or date 2;  $\rho > 1$  represents the exogenous gross rate of return from production. The randomness of the production maturity depends on a uniform-distributed random variable  $\alpha \in (0, 1)$ : a proportion of  $\alpha$  of the firms produces output on date 1 and the rest  $(1 - \alpha)$  produces output on date 2;  $\alpha$  is realised on

date 1, and this is a public information available to all agents.<sup>4</sup>

As the firms have no endowment, they need to obtain funding for their production from the banks on date 0. This model assumes that the firms lack the technology to collect funding from the households; therefore, financing is only possible through the financial intermediation services provided by the banks (which are assumed to have the technology to collect households' endowment). The banks are assumed to provide only short-term bank loans (with maturity on date 1) to the firms. These bank loans can be extended to date 2 for the firms with late production, subject to banks' approval. If the banks do not agree to extend the bank loans, the on-going production on date 1 can be fully or partially confiscated by the banks as a repayment for the bank loans. On-going production that is confiscated does not produce any output on date 2.

### 4.3.3 Households

Households are assumed to be a group of identical and continuum individuals. They have a total of one unit of endowment (good) on date 0. This good can be used for either production or consumption. Each household needs to consume on both date 1 and date 2. This model assumes that the households withdraw a constant proportion ( $\beta$ ) of the date-1 withdraw-able investment (deposit contract) for their consumption on date 1, and the returns to the remaining investment (non-withdrawn deposit and equity stock) are left until date 2 for consumption. The utility function for a consumer (household) can be represented by

$$U(C_1, C_2) = u(C_1) + u(C_2)$$

where  $C_1$  and  $C_2$  are the consumption on date 1 and date 2 respectively, and  $u(\cdot)$  is a neoclassical utility function (increasing, concave, and twice continuously differentiable).

There are two possible investments available to the households. The first one is to deposit their endowment in banks; the deposit is risk-free. The second one is to invest in equity stock issued by the banks; these investments in banks are described in details in section 4.3.4. For simplicity, I assume that the households do not store the endowments themselves because the banks provide a higher expected return to both the deposit contract and the equity stock compared with the storage technology. I also assume that the households' endowment are equally invested in the banks; therefore, the endowment invested in a bank in a  $n$ -bank economy is given by  $q_n = 1/n$ .

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<sup>4</sup>It is worth pointing out that although the production maturity is random on date 0, there is no uncertainty in the production return ( $\rho$ ) over time.

### 4.3.4 Banks

Banks are financial intermediaries between firms and households. They are assumed to have a banking technology which allows them to collect endowment from the households, so that bank loans can be created to finance the firms' production. As the firms are perfectly competitive, they are willing to produce with zero profit. Therefore, the returns to the banks are  $\alpha\rho$  on date 1 and  $(1 - \alpha)\rho$  on date 2, if no on-going production is confiscated on date 1.<sup>5</sup>

As mentioned, the banks provide only short-term bank loans (with maturity on date 1) to the firms on date 0; the maturity for the short-term loans can be extended to date 2 for the firms with on-going production on date 1 (late-producing firms). However, whether the short-term loans can be extended depends on the banks' liquidity on date 1. If the proportion of early-producing firms ( $\alpha$ ) is sufficiently large, such that the banks have enough liquidity to satisfy their date-1 deposit withdrawals, all bank loans to late-producing firms are extended. Otherwise, the banks have liquidity shortfalls; some short-term loans for the on-going production have to be discontinued, and their productions are confiscated and sold to external investors in fire-sale; the proceeds from fire-sale is used to fill up the liquidity shortfall. The banks try to avoid a fire-sale if possible because they always suffer some losses due to the low fire-sale price.

To obtain the household endowment on date 0, the banks issue both deposit contract and equity stock to raise funding for the bank loans. Deposit contract is a risk-free debt contract which can be withdrawn any time by the depositors. The returns to deposit contract depend on the date of withdrawal: the per-unit gross returns to the deposit withdrawn on date 1 and 2 are exogenous in this model, and are denoted as  $r_1$  and  $r_2$  respectively, where  $1 \leq r_1 \leq r_2$ .<sup>6</sup> Equity stock provides a return to its holders only on date 2, and it is not risk-free. The risk to equity stock comes from the uncertain proportion of early production ( $\alpha$ ). If the return from early production turns out to be too small to satisfy the withdrawal of short-term debt, there is fire-sale loss which affects the return to the equity holders. For this reason, the banks are not able to provide promised returns for equity stock; instead, the equity holders equally share the value of the banks on date 2. The proportion of funding structure in Bank  $k$  is characterised by the decision variable  $d_k$ :  $d_k$  represents the proportion of deposit contract in Bank  $k$  and  $1 - d_k$  represents the proportion of equity stock.<sup>7</sup>

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<sup>5</sup>It can also be assumed that the return to production technology is  $\rho + y$ , where  $y$  is an exogenous rent given to the firms. This specification does not affect the following model.

<sup>6</sup>Note that due to the risk-free nature of deposit contract, the returns to deposit contract have to be the same for all banks; otherwise, households will only deposit in the bank(s) with the highest returns.

<sup>7</sup>In this model, I do not assume that the banks possess storage technology because this can be

In this model, I assume that in a multi-bank economy, the banks are not aware of the actual funding structures of each other. Therefore, their decisions are based on some reaction functions that depend on the conjectures of others' decision variables. With these reaction functions, I aim to find out the optimal funding structure of the banks under a symmetric equilibrium, which is in nature very similar to the standard Cournot equilibrium. I then use this symmetric equilibrium as a foundation to analyse how the number of banks can affect the banks' funding structure under symmetric equilibrium, which in turn affects financial stability. Further details for the reaction functions and the symmetric equilibrium are explained in Section 4.4.

### 4.3.5 External Investors and Fire-sale

When banks have liquidity shortfalls on date 1, they need to sell their confiscated production to external investors. The external investors determine the fire-sale price based on the aggregate amount of asset sold in the economy. The fire-sale price of a  $j$ -bank fire-sale in a  $n$ -bank economy is assumed to be  $1 - h(\sum_{k=1}^j q_n x_{jk})$ , where  $q_n x_{jk}$  is the amount of on-going production sold by Bank  $k$  to the external investors in a  $j$ -bank fire-sale, in a  $n$ -bank economy;  $h < 1$  is an exogenous coefficient for the fire-sale price. By definition, the fire-sale price is smaller than one whenever there is a fire-sale; therefore, there must be a loss to the banks as the gross rate of return is smaller than one.

In a  $n$ -bank economy, the total proceeds for Bank  $k$  in a  $j$ -bank fire-sale is given by

$$q_n x_{jk} \left( 1 - h \left( \sum_{k=1}^j q_n x_{jk} \right) \right)$$

The proceeds must be equivalent to the liquidity shortfall of Bank  $k$ , because no bank is willing to liquidate more assets than necessary due to the fire-sale loss. This is given by

$$q_n x_{jk} \left( 1 - h \left( \sum_{k=1}^j q_n x_{jk} \right) \right) = q_n (\beta r_1 d_k - \alpha \rho)$$

or simply

$$x_{jk} \left( 1 - h \left( \sum_{k=1}^j q_n x_{jk} \right) \right) = \beta r_1 d_k - \alpha \rho$$

The right-hand side of the equation represents the liquidity shortfall in Bank  $k$ ; the

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redundant. One can easily figure out that the only purpose for banks to store liquid asset (good) is to increase their liquidity on date 1; this reduces the possibility of liquidity shortage in the fire-sale. However, this approach is not necessary because the same result can be obtained by choosing a less risky funding structure. Moreover, raising extra equity stock is also costly (the cost of equity is explained in section 4.4.) Therefore, storage technology is not necessary for banks to achieve their preferred risk level. This further implies that in this model, the total amount of endowment collected from the households is equal to the amount of bank loans provided to the firms.

first term refers to the deposit withdrawal on date 1, and the second term refers to the return to bank from early production. It is worth mentioning that fire-sale does not necessarily imply a huge loss to the banks. In fact, the loss in a small-scale fire-sale is minimal. In this model, I assume that the loss in the fire-sale is not big enough to affect the guaranteed returns to the deposit contract; therefore, the deposit contract is risk-free and the fire-sale loss is absorbed by the equity holders of the distressed banks.

Whether Bank  $k$  has a liquidity shortfall on date 1 depends on a threshold  $\alpha_k^*$ ; this is given by

$$q_n \alpha_k^* \rho = q_n \beta r_1 d_k,$$

or

$$\alpha_k^* = \frac{\beta r_1 d_k}{\rho}$$

When  $\alpha \in [\alpha_k^*, 1]$ , Bank  $k$  has no liquidity shortfall and does not need to liquidate its asset in fire-sale; otherwise, when  $\alpha \in [0, \alpha_k^*]$ , Bank  $k$  has a positive liquidity shortfall and some on-going production needs to be discontinued and sold in a fire-sale at a loss. It is worth mentioning a smaller  $\alpha_k^*$  (lower threshold) implies that Bank  $k$  has a smaller liquidity risk, because it is less likely for  $\alpha$  to be smaller than the (lower) threshold.

## 4.4 Decision Problems

In this section, I construct the decision problems for banks. The objective for a bank is to choose a funding structure in order to maximise the expected *net* return to their equity holders. For Bank  $k$ , this is given by

$$q_n(1 - d_k)([E_\alpha[R(\alpha)] - c)$$

where  $R(\alpha)$  is the rate of return to the equity holders (after all returns to deposit contract are deducted), and  $c$  is the (per unit) cost of equity. I assume that the cost of equity is higher than the cost of deposit ( $c^*$ ) due to the existence of market friction<sup>8</sup>; this is represented by

$$c > \beta r_1 + (1 - \beta)r_2 \equiv c^*$$

Note that both  $R(\alpha)$  and  $c$  are the values based on *per unit* of equity; therefore, both of them have to be multiplied by the actual size of equity stock in Bank  $k$ , which is  $q_n(1 - d_k)$ , to generate the absolute values in Bank  $k$ .

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<sup>8</sup>A common example of market friction is the taxation benefit of debt (deposit).

In the following subsections, I begin with the construction of the decision problem of a monopoly bank. This is the simplest model because the decision of a monopoly bank does not rely the conjectures of other banks. Then, I construct the decision problems under the more complicated two-bank economy, in which conjectures and reaction functions are required to determine the optimal funding structure. After that, I extend the framework of the two-bank economy to a generalised framework of a n-bank economy.

#### 4.4.1 Monopoly-Bank Economy

In an economy with one (monopoly) bank, the decision problem for the bank is quite straightforward. It needs to choose its funding structure by maintaining a balance in the trade-off of the (lower) cost and the (higher) risk of deposit contract. Recall that due to the existence of market friction, the cost of equity is higher than the cost of deposit. For this reason, it is relatively cheaper for a bank to fund its asset (bank loans) by deposit contract. However, a higher proportion of deposit also means that a bank has to face a higher liquidity risk. The higher risk comes from the higher level of deposit withdrawal on date 1, which increases the threshold for liquidity shortfall (a higher probability of having liquidity shortfall) and the fire-sale losses.

In the following, the returns to the monopoly bank are specified under two scenarios: (Scenario 0) there is no liquidity shortfall; the return to bank does not depend on  $\alpha$ ; and (Scenario 1) there is a positive liquidity shortfall; the return to bank decreases in  $\alpha$ . It is worth mentioning that the fire-sale in a monopoly-bank economy is not an externality to the monopoly bank because the bank fully internalises the cost of fire-sale within its decision problem.

**Scenario (0): No Fire-Sale.** When there is no fire-sale, the gross return to the bank consists of the returns from early and late production, which sum up to  $\rho$ . The total cost of deposit is  $c^* = \beta r_1 + (1 - \beta)r_2$ . Therefore, the rate of return to equity is simply

$$R_0 = \frac{q_1 [\rho - (\beta r_1 + (1 - \beta)r_2)d_1]}{q_1(1 - d_1)}$$

or equivalently

$$R_0(1 - d_1) = \rho - c^*d_1$$

where the subscript of  $R_0$  corresponds to Scenario (0). Note that the Scenario (0) equity return is independent of  $\alpha$ . The condition for having Scenario (0) is given by  $\alpha \geq \alpha_1^* = \frac{\beta r_1 d_1}{\rho}$ .

**Scenario (1): Fire-Sale.** When there is a liquidity shortfall on date 1, fire-sale is necessary. The amount of asset that the monopoly bank (or Bank 1) needs to sell

in the one-bank fire-sale is denoted as  $x_{11}$ ; this value is obtained by solving

$$x_{11}(1 - hq_1x_{11}) = \beta r_1 d_1 - \alpha \rho$$

As part of the on-going production is confiscated and sold in the fire-sale, the date-2 return is reduced. The date-2 return (after fire-sale) is expressed by  $(1 - \alpha - x_{11})\rho$ .

The rate of equity return, conditional on a positive liquidity shortfall on date 1, is

$$R_1 = \frac{q_1 [\alpha \rho + x_{11}(1 - hq_1x_{11}) - \beta r_1 d_1 + (1 - \alpha - x_{11})\rho - (1 - \beta)r_2 d_1]}{q_1(1 - d_1)}$$

The first three numerator terms within the square bracket on the right-hand side are the pay-off to the bank on date 1: the first term is the return from early production; the second term is the proceeds from fire-sale; the third term is the payment to date-1 deposit withdrawal. These three terms, by definition, sum up to zero; however, I keep this terms in the equation for further simplification of algebraic expression. The last two numerator terms correspond to the date-2 pay-off: the fourth term is the return from *remaining* late production after the fire-sale; the fifth term is the payment to date-2 deposit withdrawal. The above expression can be simplified as

$$R_1(1 - d_1) = R_0(1 - d_1) - hq_1x_{11}^2 - (\rho - 1)x_{11}$$

The term  $hq_1x_{11}^2$  represents the loss in the fire-sale due to the low fire-sale price (captured by the fire-sale price coefficient  $h$ ); the term  $(\rho - 1)x_{11}$  represents the reduction in date-2 return due to the fire-sale of on-going production on date 1. The condition for having Scenario (1) is given by  $\alpha < \alpha_1^*$ .

## Decision Problem

The decision problem of the monopoly bank is given by<sup>9</sup>

$$\max_{d_1} q_1(1 - d_1)(E_\alpha[R] - c)$$

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<sup>9</sup>Note that  $(R_0 - c)(1 - d_1) = \rho - c^*d_1 - c(1 - d_1) = (\rho - c) - (c - c^*)(1 - d_1)$

Substituting  $q_1 = 1$ , and the expressions of  $R_0$  and  $R_1$  into  $R$ , one can get the following expression.

$$\begin{aligned} & \max_{d_1} \int_{\alpha_1^*}^1 (R_0 - c)(1 - d_1)d\alpha + \int_0^{\alpha_1^*} (R_1 - c)(1 - d_1)d\alpha \\ &= \max_{d_1} (R_0 - c)(1 - d_1) - h \int_0^{\alpha_1^*} x_{11}^2 d\alpha - (\rho - 1) \int_0^{\alpha_1^*} x_{11} d\alpha \\ &= \max_{d_1} (\rho - c) - (c - c^*)(1 - d_1) - h \int_0^{\alpha_1^*} x_{11}^2 d\alpha - (\rho - 1) \int_0^{\alpha_1^*} x_{11} d\alpha \end{aligned}$$

The first-order condition (FOC) for the decision problem with respect to  $d_1$  is given by

$$(c - c^*) - 2h \int_0^{\alpha_1^*} x_{11} \cdot \frac{\partial x_{11}}{\partial d_1} d\alpha - (\rho - 1) \int_0^{\alpha_1^*} \frac{\partial x_{11}}{\partial d_1} d\alpha = 0$$

where

$$\frac{\partial x_{11}}{\partial d_1} = \frac{\beta r_1}{1 - 2hx_{11}} > 0$$

The second-order condition (SOC) is given by

$$- \int_0^{\alpha_1^*} \left[ 2h \left( x_{11} \frac{\partial^2 x_{11}}{\partial d_1^2} + \left( \frac{\partial x_{11}}{\partial d_1} \right)^2 \right) + (\rho - 1) \frac{\partial^2 x_{11}}{\partial d_1^2} \right] d\alpha$$

where

$$\frac{\partial^2 x_{11}}{\partial d_1^2} = \frac{2h(\beta r_1)^2}{(1 - 2hx_{11})^3}$$

To have a stable FOC that maximises the objective function, the SOC has to be negative for all values of  $\alpha$ . The sufficient condition for negative SOC is  $h < \frac{1}{2}$ . This ensures that  $\frac{\partial^2 x_{11}}{\partial d_1^2} > 0$  even when  $x_{11}$  is at its maximum value of one<sup>10</sup>.

To avoid unstable optimisation results, this model assumes that  $h < \frac{1}{2}$  throughout the chapter.<sup>11</sup>

#### 4.4.2 Two-Bank Economy

The two-bank economy is more complicated. Since the two banks do not know each other's decision, they have to choose their funding structure based on the conjectures of the funding structure of the other bank. In this model, I assume that the two banks make their decision based on a reaction function. In order to specify the reaction function correctly, one needs to know that the specification for the

<sup>10</sup>The derivations of  $\frac{\partial x_{11}}{\partial d_1}$  and  $\frac{\partial^2 x_{11}}{\partial d_1^2}$  are shown in the appendix.

<sup>11</sup>I show in the coming subsections that this sufficient condition is also sufficient to ensure the stableness of all FOC's in the multi-bank economies.

banks' return depends on the risk levels of the two banks. Specifically, the return specification for Bank 1 being the riskier bank (based on the conjectures of a safer funding structure in Bank 2) is different from the return specification for Bank 1 being the safer bank (based on the conjectures of a riskier funding structure in Bank 2). In the following, I discuss these two types of return specifications separately. To avoid cumbersome, I discuss the decision making and the scenario analysis only from the point of view of Bank 1; the discussion for Bank 2 is exactly the same, and is therefore omitted.

### Being a Riskier Bank

Recall that the liquidity risk from the random firm production is a common shock to all banks in the economy; in other words, all banks have the same level of liquidity shock, which is given by  $\alpha\rho$  and  $(1-\alpha)\rho$ . Therefore, the condition for Bank 1 being riskier than Bank 2 comes from the funding structure,  $d_1 \geq d_2$ . If Bank 1 has a higher proportion of deposit funding, it also has a higher date-1 deposit withdrawal, and therefore it has a higher liquidity risk compared with Bank 2. Equivalently, this is represented by the thresholds that  $\alpha_1^* \geq \alpha_2^*$ .<sup>12</sup>

With Bank 1 being the riskier bank, its return specification is subdivided into three scenarios: (Scenario 0) there is no liquidity shortfall in both banks; (Scenario 1) there is a positive liquidity shortfall in Bank 1, but not in Bank 2, causing a one-bank fire-sale; (Scenario 2) there are positive liquidity shortfalls in both banks, causing a two-bank fire-sale.

**Scenario (0): No Fire-Sale.** Similar to the monopoly-bank economy, when there is no fire-sale, the rate of return to Bank-1 equity holders is simply

$$R_0 = \frac{q_2 [\rho - (\beta r_1 + (1 - \beta)r_2)d_1]}{q_2(1 - d_1)}$$

Or equivalently

$$R_0(1 - d_1) = \rho - (\beta r_1 + (1 - \beta)r_2)d_1$$

Again, the return to Bank 1 is not affected by the random proportion of early production  $\alpha$ . The condition for having Scenario (0) is given by  $\alpha \geq \alpha_1^*$ .

**Scenario (1): One-Bank Fire-Sale.** If there is a liquidity shortfall in Bank 1 but not in Bank 2, there is a one-bank fire-sale. The amount of asset needs to be sold by Bank 1 is determined by the following equation.

$$x_{11}(1 - hq_2x_{11}) = \beta r_1 d_1 - \alpha \rho$$

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<sup>12</sup>Recall that in this model, a higher threshold corresponds to a higher probability of having liquidity shortfall.

where  $x_{11}$  is the amount of asset liquidation for Bank 1 in a one-bank fire-sale. The reduced date-2 return for Bank 1 (after the fire-sale) is specified by  $(1 - \alpha - x_{11})\rho$ .

The rate of equity return for Bank 1 in Scenario (1) is specified by,

$$R_1 = \frac{q_2 [\alpha\rho + x_{11}(1 - hq_2x_{11}) - \beta r_1 d_1 + (1 - \alpha - x_{11})\rho - (1 - \beta)r_2 d_1]}{q_2(1 - d_1)}$$

or in a simpler way

$$R_1(1 - d_1) = R_0(1 - d_1) - hq_2x_{11}^2 - (\rho - 1)x_{11}$$

The condition for having Scenario (1) is given by  $\alpha_2^* < \alpha \leq \alpha_1^*$ .

**Scenario (2): Two-Bank Fire-Sale.** If there are liquidity shortfalls in both banks, there is a two-bank fire-sale. The amount of asset needs to be sold by Bank 1 and Bank 2 in a two-bank fire-sale are determined by the following equations respectively.

$$x_{21}(1 - hq_2(x_{21} + x_{22})) = \beta r_1 d_1 - \alpha\rho$$

$$x_{22}(1 - hq_2(x_{21} + x_{22})) = \beta r_1 d_2 - \alpha\rho$$

The reduced date-2 return for Bank 1 is specified by  $(1 - \alpha - x_{21})\rho$ .

The rate of equity return for Bank 1 in Scenario (2) is specified by,

$$R_2 = \frac{q_2 [\alpha\rho + x_{21}(1 - hq_2(x_{21} + x_{22})) - \beta r_1 d_1 + (1 - \alpha - x_{21})\rho - (1 - \beta)r_2 d_1]}{q_2(1 - d_1)}$$

or in a simpler way

$$R_2(1 - d_1) = R_0(1 - d_1) - hq_2(x_{21}^2 + x_{21}x_{22}) - (\rho - 1)x_{21}$$

The condition for having Scenario (2) is given by  $\alpha < \alpha_2^*$ .

### Decision Problem for Riskier Bank

The decision problem for Bank 1 being the riskier bank in the two-bank economy can be expressed by (with substitution of  $q_2 = 1/2$ )

$$\begin{aligned} \max_{d_1} \int_{\alpha_1^*}^1 (R_0 - c)(1 - d_1)d\alpha + \int_{\alpha_2^*}^{\alpha_1^*} (R_1 - c)(1 - d_1)d\alpha + \int_0^{\alpha_2^*} (R_2 - c)(1 - d_1)d\alpha \quad (4.1) \\ = \max_{d_1} (\rho - c) - (c - c^*)(1 - d_1) - \frac{h}{2} \int_{\alpha_2^*}^{\alpha_1^*} x_{11}^2 d\alpha - (\rho - 1) \int_{\alpha_2^*}^{\alpha_1^*} x_{11} d\alpha \\ - \frac{h}{2} \int_0^{\alpha_2^*} (x_{21}^2 + x_{21}x_{22})d\alpha - (\rho - 1) \int_0^{\alpha_2^*} x_{21} d\alpha \end{aligned}$$

Note that given different values for  $d_2$  (conjectures on Bank 2), which affects the values for  $x_{22}$  and  $\alpha_2^*$  in the expression, there are different values for the optimal decision variable  $d_1$ . Therefore, the above optimisation problem is in fact a reaction function (based on the value of  $d_2$ ).

The first-order condition (FOC) for the decision problem with respect to  $d_1$  is given by

$$(c - c^*) - h \int_{\alpha_2^*}^{\alpha_1^*} x_{11} \cdot \frac{\partial x_{11}}{\partial d_1} d\alpha - (\rho - 1) \int_{\alpha_2^*}^{\alpha_1^*} \frac{\partial x_{11}}{\partial d_1} d\alpha \\ - h \int_0^{\alpha_2^*} (x_{21} + \frac{1}{2}x_{22}) \cdot \frac{\partial x_{21}}{\partial d_1} d\alpha - (\rho - 1) \int_0^{\alpha_2^*} \frac{\partial x_{21}}{\partial d_1} d\alpha = 0$$

where

$$\frac{\partial x_{11}}{\partial d_1} = \frac{\beta r_1}{1 - hx_{11}} > 0 \\ \frac{\partial x_{21}}{\partial d_1} = \frac{\beta r_1}{1 - h(x_{21} + \frac{1}{2}x_{22})} > 0$$

The second-order condition (SOC) is given by

$$- \int_{\alpha_2^*}^{\alpha_1^*} \left[ h \left( x_{11} \frac{\partial^2 x_{11}}{\partial d_1^2} + \left( \frac{\partial x_{11}}{\partial d_1} \right)^2 \right) + (\rho - 1) \frac{\partial^2 x_{11}}{\partial d_1^2} \right] d\alpha \\ - \int_0^{\alpha_2^*} \left[ h \left( (x_{21} + \frac{1}{2}x_{22}) \frac{\partial^2 x_{21}}{\partial d_1^2} + \left( \frac{\partial x_{21}}{\partial d_1} \right)^2 \right) + (\rho - 1) \frac{\partial^2 x_{21}}{\partial d_1^2} \right] d\alpha$$

where

$$\frac{\partial^2 x_{11}}{\partial d_1^2} = \frac{h(\beta r_1)^2}{(1 - hx_{11})^3}$$

and

$$\frac{\partial^2 x_{21}}{\partial d_1^2} = \frac{h(\beta r_1)^2}{(1 - h(x_{21} + \frac{1}{2}x_{22}))^3}$$

To ensure that SOC is negative such that FOC always maximises the objective function, the sufficient condition is  $h < \frac{2}{3}$ . This ensures that both  $\frac{\partial^2 x_{11}}{\partial d_1^2}$  and  $\frac{\partial^2 x_{21}}{\partial d_1^2}$  are positive even when  $x_{11}$ ,  $x_{21}$ , and  $x_{22}$  are equal to their maximum value of one.<sup>13</sup> Note that the sufficient condition in the two-bank economy is weaker than the one in one-bank economy because  $h < 1/2$  is a stronger condition than  $h < 2/3$ .

### Being a Safer Bank

If Bank 1 chooses a safer funding structure than Bank 2, then the model must have  $d_1 \leq d_2$ . Or equivalently,  $\alpha_1^* \leq \alpha_2^*$ . There are only two scenarios for Bank 1: (Scenario 0) there is no liquidity shortfall for Bank 1; and (Scenario 1) there are

<sup>13</sup>The derivations of  $\frac{\partial x_{11}}{\partial d_1}$ ,  $\frac{\partial x_{21}}{\partial d_1}$ ,  $\frac{\partial^2 x_{11}}{\partial d_1^2}$ , and  $\frac{\partial^2 x_{21}}{\partial d_1^2}$  are shown in the appendix.

liquidity shortfalls in both banks. The reason that one-bank (Bank 2) fire-sale is irrelevant is because the return to Bank 1 is not affected by the liquidity shortfall of Bank 2 in a one-bank fire-sale.

**Scenario (0): No Fire-Sale needed by Bank 1.** When there is no fire-sale, the rate of return to Bank-1 equity holders is again

$$R_0 = \frac{q_2 [\rho - (\beta r_1 + (1 - \beta)r_2)d_1]}{q_2(1 - d_1)}$$

Or equivalently

$$R_0(1 - d_1) = \rho - (\beta r_1 + (1 - \beta)r_2)d_1$$

The condition for having Scenario (0) is given by  $\alpha \geq \alpha_1^*$ .

**Scenario (1): Two-Bank Fire-Sale.** Note that when there is a liquidity shortfall in Bank 1, there must also be a liquidity shortfall in Bank 2, due to the definition of  $\alpha_1^* \leq \alpha_2^*$ . In a two-bank fire-sale, the rate of equity return for Bank 1 is specified by,

$$R_1 = \frac{q_2 [\alpha\rho + x_{21}(1 - hq_2(x_{21} + x_{22})) - \beta r_1 d_1 + (1 - \alpha - x_{21})\rho - (1 - \beta)r_2 d_1]}{q_2(1 - d_1)}$$

or in a simpler way

$$R_1(1 - d_1) = R_0(1 - d_1) - hq_2(x_{21}^2 + x_{21}x_{22}) - (\rho - 1)x_{21}$$

The condition for having Scenario (1) is given by  $\alpha < \alpha_1^*$ .

### Decision Problem for Safer Bank

The decision problem for Bank 1 being the safer bank in the two-bank economy can be expressed by

$$\begin{aligned} & \max_{d_1} \int_{\alpha_1^*}^1 (R_0 - c)(1 - d_1)d\alpha + \int_0^{\alpha_1^*} (R_1 - c)(1 - d_1)d\alpha \quad (4.2) \\ & = \max_{d_1} (\rho - c) - (c - c^*)(1 - d_1) - \frac{h}{2} \int_0^{\alpha_1^*} (x_{21}^2 + x_{21}x_{22})d\alpha - (\rho - 1) \int_0^{\alpha_1^*} x_{21}d\alpha \end{aligned}$$

Again, given different values for  $d_2$  (conjectures on Bank 2's funding structure) which affect the values for  $x_{22}$  in the expression, there will be different optimal values for  $d_1$ . Therefore, the above optimisation problem is a reaction function based on the value of  $d_2$ .

The first-order condition (FOC) for the decision problem with respect to  $d_1$  is

given by

$$(c - c^*) - h \int_0^{\alpha_1^*} (x_{21} + \frac{1}{2}x_{22}) \cdot \frac{\partial x_{21}}{\partial d_1} d\alpha - (\rho - 1) \int_0^{\alpha_1^*} \frac{\partial x_{21}}{\partial d_1} d\alpha = 0$$

The second-order condition (SOC) is given by

$$- \int_0^{\alpha_1^*} \left[ h \left( (x_{21} + \frac{1}{2}x_{22}) \frac{\partial^2 x_{21}}{\partial d_1^2} + \left( \frac{\partial x_{21}}{\partial d_1} \right)^2 \right) + (\rho - 1) \frac{\partial^2 x_{21}}{\partial d_1^2} \right] d\alpha$$

The expression for  $\frac{\partial x_{21}}{\partial d_1}$  and  $\frac{\partial^2 x_{21}}{\partial d_1^2}$ , and the sufficient conditions are the same as in Section 4.4.2.

## Equilibrium

As there can be infinite conjectures on the funding structure, this model focuses on the analysis of funding structure in symmetric equilibrium. I define *symmetric equilibrium* in the following.

**Definition 1** *In a two-bank economy, given one bank chooses the funding structure  $d_s$ , the other bank has no intention to deviate from this funding structure and also chooses  $d_s$  as its funding structure,  $d_s$  is said to be the funding structure in symmetric equilibrium.*

Mathematically, this is expressed as follows. Given  $d_2 = d_s$ , the optimal decision variable  $d_1$  derived from both objective functions, eq.(4.1) and eq.(4.2), are equivalent and are both equal to  $d_s$ . Then the funding structure of Bank 1 and Bank 2 are said to be in symmetric equilibrium. The proof of the following proposition is given in the appendix of this chapter.

**Proposition 1** *In symmetric equilibrium, the proportion of deposit funding chosen in a two-bank economy is higher than the proportion chosen in a one-bank (monopoly) economy.*

This proposition is very similar to the Cournot equilibrium in standard microeconomics in which a firm decides how much to produce based on the conjectures of the other firm in duopoly. In this model, the choice of the monopoly bank can be interpreted as the socially optimal funding structure for bank(s), because this choice generates the lowest fire-sale loss in the banking industry (equivalently, highest net expected returns to the equity holders) compared with all other symmetric choices. Why do the banks in the two-bank economy choose a riskier funding structure? The reason is that when there are two banks, there exists a motivation for one bank to

take advantage of the other bank by choosing a riskier funding structure; by doing so, the safer bank subsidises the riskier bank in the two-bank fire-sale. Specifically, the riskier bank is selling more asset than the safer bank in the two-bank fire-sale; however, the fire-sale price is not too low for the riskier bank because the safer bank is selling less asset. On the other hand, the fire-price is too low for the safer bank because the riskier bank is selling more asset. This can be interpreted as a subsidy from the safer bank to the riskier bank. Due to this motivation, both banks choose a riskier funding structure under symmetric equilibrium.

The above phenomenon shows that, even under symmetric equilibrium, the externality from fire-sale in a two-bank economy cannot be fully internalised as in the monopoly-bank economy, because the funding structure chosen by one bank does not take into account of the fire-sale cost of the choice of its funding structure imposed on the other bank.

### 4.4.3 N-Bank Economy

Based on the framework of the two-bank economy, the model can be easily extended to a n-bank economy. As the model analysis is studied based on the optimal decision under symmetric equilibrium, without loss of generality, I simplify the model by assuming that when Bank 1 makes its decision, it assumes that all other banks have a symmetric funding structure; in other words, the conjectures on all other banks are the same. In the following, I use Bank 2 as a representative for the other (n-1) banks. Therefore, the variables for Bank 2 is the same as the corresponding variables of the other banks (except for Bank 1). For example, the liquidity-shortfall threshold of Bank 2 (denoted by  $\alpha_2^*$ ) is also the threshold for all other banks except for Bank 1.

As the explanation for the scenarios and decision problems in the n-bank economy is very similar to those in the two-bank economy, I present the following subsections in a briefer way to avoid cumbersome repeats in explanation.

#### Being a Riskier Bank

When Bank 1 is the riskier bank, compared with all other (n-1) banks in the economy, its return specification is subdivided into three scenarios: (Scenario 0) there is no liquidity shortfall in all banks; (Scenario 1) there is a positive liquidity shortfall in Bank 1, but not in the other (n-1) banks (i.e. a one-bank fire-sale); (Scenario 2) there are positive liquidity shortfalls in all banks (i.e. a n-bank fire-sale).

**Scenario (0): No Fire-Sale.** When there is no fire-sale, the rate of return to

Bank-1 equity holders is

$$R_0 = \frac{q_n [\rho - (\beta r_1 + (1 - \beta)r_2)d_1]}{q_n(1 - d_1)}$$

Or equivalently

$$R_0(1 - d_1) = \rho - (\beta r_1 + (1 - \beta)r_2)d_1$$

The condition for having Scenario (0) is given by  $\alpha \geq \alpha_1^*$ .

**Scenario (1): One-Bank Fire-Sale.** If there is a liquidity shortfall in Bank 1 only, there is a one-bank fire-sale. The amount of asset needs to be sold by Bank 1 is determined by the following equation.

$$x_{11}(1 - hq_n x_{11}) = \beta r_1 d_1 - \alpha \rho$$

where  $x_{11}$  is the amount of asset liquidation for Bank 1 in a one-bank fire-sale.

The rate of equity return for Bank 1 in Scenario (1) is specified by,

$$R_1 = \frac{q_n [\alpha \rho + x_{11}(1 - hq_n x_{11}) - \beta r_1 d_1 + (1 - \alpha - x_{11})\rho - (1 - \beta)r_2 d_1]}{q_n(1 - d_1)}$$

or in a simpler way

$$R_1(1 - d_1) = R_0(1 - d_1) - hq_n x_{11}^2 - (\rho - 1)x_{11}$$

The condition for having Scenario (1) is given by  $\alpha_2^* < \alpha \leq \alpha_1^*$ .

**Scenario (2): n-Bank Fire-Sale.** If there are liquidity shortfalls in all banks, there is a n-bank fire-sale. The amount of asset needs to be sold by Bank 1 and Bank 2 in a n-bank fire-sale are determined by the following equations respectively.

$$x_{n1}(1 - hq_n(x_{n1} + (n - 1)x_{n2})) = \beta r_1 d_1 - \alpha \rho$$

$$x_{n2}(1 - hq_n(x_{n1} + (n - 1)x_{n2})) = \beta r_1 d_2 - \alpha \rho$$

The rate of equity return for Bank 1 in Scenario (2) is specified by,

$$R_2 = \frac{q_n [\alpha \rho + x_{n1}(1 - hq_n(x_{n1} + (n - 1)x_{n2})) - \beta r_1 d_1 + (1 - \alpha - x_{n1})\rho - (1 - \beta)r_2 d_1]}{q_n(1 - d_1)}$$

or in a simpler way

$$R_2(1 - d_1) = R_0(1 - d_1) - hq_n(x_{n1}^2 + (n - 1)x_{n1}x_{n2}) - (\rho - 1)x_{n1}$$

The condition for having Scenario (2) is given by  $\alpha < \alpha_2^*$ .

## Decision Problem for Riskier Bank

The decision problem for Bank 1 being the riskier bank in the n-bank economy can be expressed by

$$\begin{aligned} \max_{d_1} \int_{\alpha_1^*}^1 (R_0 - c)(1 - d_1)d\alpha + \int_{\alpha_2^*}^{\alpha_1^*} (R_1 - c)(1 - d_1)d\alpha + \int_0^{\alpha_2^*} (R_2 - c)(1 - d_1)d\alpha \quad (4.3) \\ = \max_{d_1} (\rho - c) - (c - c^*)(1 - d_1) - hq_n \int_{\alpha_2^*}^{\alpha_1^*} x_{11}^2 d\alpha - (\rho - 1) \int_{\alpha_2^*}^{\alpha_1^*} x_{11} d\alpha \\ - hq_n \int_0^{\alpha_2^*} (x_{n1}^2 + (n - 1)x_{n1}x_{n2})d\alpha - (\rho - 1) \int_0^{\alpha_2^*} x_{n1} d\alpha \end{aligned}$$

The first-order condition (FOC) for the decision problem with respect to  $d_1$  is given by

$$\begin{aligned} (c - c^*) - 2hq_n \int_{\alpha_2^*}^{\alpha_1^*} x_{11} \cdot \frac{\partial x_{11}}{\partial d_1} d\alpha - (\rho - 1) \int_{\alpha_2^*}^{\alpha_1^*} \frac{\partial x_{11}}{\partial d_1} d\alpha \\ - hq_n \int_0^{\alpha_2^*} (2x_{n1} + (n - 1)x_{n2}) \cdot \frac{\partial x_{n1}}{\partial d_1} d\alpha - (\rho - 1) \int_0^{\alpha_2^*} \frac{\partial x_{n1}}{\partial d_1} d\alpha = 0 \end{aligned}$$

where

$$\begin{aligned} \frac{\partial x_{11}}{\partial d_1} &= \frac{\beta r_1}{1 - 2hq_n x_{11}} > 0 \\ \frac{\partial x_{n1}}{\partial d_1} &= \frac{\beta r_1}{1 - hq_n(2x_{n1} + (n - 1)x_{n2})} > 0 \end{aligned}$$

The second-order condition (SOC) is given by

$$\begin{aligned} - \int_{\alpha_2^*}^{\alpha_1^*} \left[ 2hq_n \left( x_{11} \frac{\partial^2 x_{11}}{\partial d_1^2} + \left( \frac{\partial x_{11}}{\partial d_1} \right)^2 \right) + (\rho - 1) \frac{\partial^2 x_{11}}{\partial d_1^2} \right] d\alpha \\ - \int_0^{\alpha_2^*} \left[ hq_n \left( (2x_{n1} + (n - 1)x_{n2}) \frac{\partial^2 x_{n1}}{\partial d_1^2} + 2 \left( \frac{\partial x_{n1}}{\partial d_1} \right)^2 \right) + (\rho - 1) \frac{\partial^2 x_{n1}}{\partial d_1^2} \right] d\alpha \end{aligned}$$

where

$$\frac{\partial^2 x_{11}}{\partial d_1^2} = \frac{2hq_n(\beta r_1)^2}{(1 - 2hq_n x_{11})^3}$$

and

$$\frac{\partial^2 x_{n1}}{\partial d_1^2} = \frac{2hq_n(\beta r_1)^2}{(1 - hq_n(2x_{n1} + (n - 1)x_{n2}))^3}$$

To ensure that SOC is negative such that FOC always maximises the objective function, the sufficient condition is  $h < \frac{n}{n+1}$ . This ensures that both SOC's are positive even when  $x_{11}$ ,  $x_{n1}$ , and  $x_{n2}$  are equal to their maximum value of one. Note that the sufficient condition in the n-bank economy is weaker than the one in one-bank economy, because  $\frac{1}{2} < \frac{n}{n+1}$  for  $n \geq 2$ ; therefore, the assumption of  $h < 1/2$

is sufficient to guarantee the stableness of all FOC's, regardless of the number of banks in the economy.<sup>14</sup>

### Being a Safer Bank

If Bank 1 chooses a safer funding structure than all other (n-1) banks, then the model must have  $d_1 \leq d_2$ . Or equivalently,  $\alpha_1^* \leq \alpha_2^*$ . There are only two scenarios in the decision-making for Bank 1: (Scenario 0) there is no liquidity shortfall for Bank 1; and (Scenario 1) there are liquidity shortfalls in all banks.

**Scenario (0): No Fire-Sale needed by Bank 1.** The rate of return to Bank-1 equity holders when there is no fire-sale is

$$R_0 = \frac{q_n [\rho - (\beta r_1 + (1 - \beta)r_2)d_1]}{q_n(1 - d_1)}$$

Or equivalently

$$R_0(1 - d_1) = \rho - (\beta r_1 + (1 - \beta)r_2)d_1$$

The condition for having Scenario (0) is given by  $\alpha \geq \alpha_1^*$ .

**Scenario (1): n-Bank Fire-Sale.** In a n-bank fire-sale, the rate of equity return for Bank 1 is specified by,

$$R_1 = \frac{q_n [\alpha\rho + x_{n1}(1 - hq_n(x_{n1} + (n - 1)x_{n2})) - \beta r_1 d_1 + (1 - \alpha - x_{n1})\rho - (1 - \beta)r_2 d_1]}{q_n(1 - d_1)}$$

or in a simpler way

$$R_1(1 - d_1) = R_0(1 - d_1) - hq_n(x_{n1}^2 + (n - 1)x_{n1}x_{n2}) - (\rho - 1)x_{n1}$$

The condition for having Scenario (1) is given by  $\alpha < \alpha_1^*$ .

### Decision Problem for Safer Bank

The decision problem for Bank 1 being the safer bank in the n-bank economy can be expressed by

$$\begin{aligned} & \max_{d_1} \int_{\alpha_1^*}^1 (R_0 - c)(1 - d_1)d\alpha + \int_0^{\alpha_1^*} (R_1 - c)(1 - d_1)d\alpha \quad (4.4) \\ & = \max_{d_1} (\rho - c) - (c - c^*)(1 - d_1) - hq_n \int_0^{\alpha_1^*} (x_{n1}^2 + (n - 1)x_{n1}x_{n2})d\alpha - (\rho - 1) \int_0^{\alpha_1^*} x_{n1}d\alpha \end{aligned}$$

<sup>14</sup>The derivations of  $\frac{\partial x_{11}}{\partial d_1}$ ,  $\frac{\partial x_{n1}}{\partial d_1}$ ,  $\frac{\partial^2 x_{11}}{\partial d_1^2}$ , and  $\frac{\partial^2 x_{n1}}{\partial d_1^2}$  are shown in the appendix.

The first-order condition (FOC) for the decision problem with respect to  $d_1$  is given by

$$(c - c^*) - hq_n \int_0^{\alpha_1^*} (2x_{n1} + (n-1)x_{n2}) \cdot \frac{\partial x_{n1}}{\partial d_1} d\alpha - (\rho - 1) \int_0^{\alpha_1^*} \frac{\partial x_{n1}}{\partial d_1} d\alpha = 0$$

The second-order condition (SOC) is given by

$$- \int_0^{\alpha_1^*} \left[ hq_n \left( (2x_{n1} + (n-1)x_{n2}) \frac{\partial^2 x_{n1}}{\partial d_1^2} + 2 \left( \frac{\partial x_{n1}}{\partial d_1} \right)^2 \right) + (\rho - 1) \frac{\partial^2 x_{n1}}{\partial d_1^2} \right] d\alpha$$

The expression for  $\frac{\partial x_{n1}}{\partial d_1}$  and  $\frac{\partial^2 x_{n1}}{\partial d_1^2}$ , and the sufficient conditions are the same as in Section 4.4.3.

## Equilibrium

I define *symmetric equilibrium* for the n-bank economy in the following.

**Definition 2** *In a n-bank economy, given (n-1) banks choose the funding structure  $d_s$ , the remaining (one) bank has no intention to deviate from this funding structure and also chooses  $d_s$  as its funding structure,  $d_s$  is said to be the funding structure in symmetric equilibrium.*

Mathematically, this is expressed as follows. Given  $d_2 = d_s$ , the optimal decision variable  $d_1$  derived from the two objective functions, eq.(4.3) and eq.(4.4), are equivalent and are both equal to  $d_s$ . Then the funding structure of all banks are said to be in symmetric equilibrium. The proof for the following proposition is given in the appendix of this chapter.

**Proposition 2** *In symmetric equilibrium, the proportion of deposit funding chosen in a n-bank economy is higher than the proportion chosen in a (n-1)-bank economy, for  $n \geq 2$ .*

From Proposition 2, one can observe that the liquidity risk for the banks is higher in an economy with more banks. The reason is that each bank has a stronger incentive to take excessive risk due to the larger subsidy from the larger number of *other banks*. Under symmetric equilibrium, the banks choose a higher proportion of deposit (higher liquidity risk) to attempt to obtain this subsidy from each other. The result leads to weaker financial stability. As this chapter uses the number of banks in the economy as a measure for bank competition. Proposition 2 implies that bank competition leads to financial instability.

#### 4.4.4 Further Discussion

In this model, I assume that the banks cannot acquire fire-sale assets from each others. However, as this alternative assumption can affect the model conclusion, particularly on its effect in the risk-taking incentives in the banking system. Therefore, it would be interesting to discuss this alternative setting briefly in this chapter.

There are two ways to address this issue. The first one is based on the original framework of the model suggested in this chapter, and studies how the symmetric equilibrium could be affected if the banks can acquire fire-sale assets from each other. The other way is to consider an asymmetric equilibrium, under which banks can have different choices of their funding structure under equilibrium. Under this asymmetric equilibrium, some banks may choose to be safer than the others in order to reserve excess liquidity to acquire fire-sale assets from other distressed banks. In the following, I address the issue based on these two different frameworks separately.

##### **Symmetric Equilibrium**

To address the mentioned issue using the symmetric equilibrium specified in the model, the best way is to begin with the original symmetric equilibrium that has been constructed in this chapter. The original symmetric equilibrium is defined as a situation under which a bank has no intention to derive from the equilibrium choice of funding structure chosen by other banks. Using this original equilibrium as a starting point, I now consider what happens when the banks are allowed to acquire fire-sale assets from other distressed banks. For the ease of the following discussion, I assume that all other model specifications remain unchanged.

From the point of view of a particular bank, this alternative assumption creates a new motivation for it to choose a safer funding structure than the original equilibrium choice, because a safer funding structure (i.e. a higher (lower) proportion of equity (deposit) funding) increases the probability of the bank to have excess liquidity, while other banks are in need of asset fire-sale; the excess liquidity can be used to acquire fire-sale asset at the profitable fire-sale price; this increases the expected return for the bank.

When every bank thinks in the same way as this particular bank, the result will be that all banks choose a safer funding structure than the original symmetric equilibrium. And a new symmetric is formed with every bank being safer than before.

Based on this new symmetric equilibrium, it is interesting to ask again the key question of this chapter: *How does bank competition affect financial stability?* Or, more precisely, how does the number of banks in the economy affects the equilibrium choice of banks' funding structure?

Recall that in the original symmetric equilibrium, the increase of the number

of banks increases the banks' incentive to take advantage of each other during fire-sale; therefore, the more banks exist in the economy, the stronger is this socially-undesirable incentive, and the riskier is the choice of banks' funding structure. In this new symmetric equilibrium, it is obvious that there exists a new incentive with an opposite effect on the choice of funding structure: the benefits from being safer to acquire fire-sale asset.

Therefore, whether the original conclusion of this chapter still holds depends on the magnitude of these two opposite forces. The observation of the numerical example (Section 4.5) provides some insightful hint for the comparison of the magnitude of the two opposite forces. From Figure 4.2, one can observe that as the number of banks increase, the choice of funding structure converges to a single point; the marginal effect of the increase in the number of banks is therefore decreasing. On the other hand, one can expect the incentive from acquiring fire-sale asset should be marginal increasing, because being *the last bank standing* in an economy with numerous banks can result in a huge fire-sale profit.

From this analysis, the most likely situations that can happen are (1) the asset-acquiring incentive is dominated by the taking-advantage-of-other-banks incentive when the number of banks is small in the economy, but vice versa when the number of banks is large; this results in a U-shaped relationship between the relationship of bank competition and financial stability: the financial stability is weakened when the number of banks increases in an economy with a small amount of banks, but if the number of banks keep on increasing, bank competition eventually leads to a stronger financial stability; (2) the asset-acquiring incentive dominates the other force at the first place, leading to a simple conclusion of bank competition leads to financial stability.

### **Asymmetric Equilibrium**

The other framework that is also worth discussing is the existence of an asymmetric (coordinated) equilibrium. The asymmetric (coordinated) equilibrium can be defined as a situation in which the banks are divided into two types, with one type having a safer funding structure than the other, and both types have no intention to deviate from the choice of funding structure that they have chosen, because the expected returns to the safer banks and the riskier banks are the same.

Under this asymmetric equilibrium, it is possible, under some economy states, that the safer banks will have excess liquidity when the riskier banks are in need of liquidity. The safer group can compete with the external investors to acquire the fire-sale assets from the riskier banks. For simplicity of the analysis, I assume that the safer type of banks are price-takers and therefore they do not have market power to alter the fire-sale price.

Based on this framework, there exists a trade-off when the banks choose their

types: (1) being the riskier banks allows them to take advantage of other banks when all banks require asset fire-sale at the very bad economy states; (2) being the safer banks allows them to acquire assets from the riskier banks when the economy is bad but not too bad (riskier banks have liquidity shortage while safer banks still have excess liquidity), at the cost of the expensive equity funding.

It is obvious that in order for (2) to be a possible choice, the expected profit from acquiring assets has to be larger than the higher cost of equity. I would like to point out that it is quite unlikely, because the cost of equity is independent from the economy states, but the profit from fire-sales only occurs at a narrow range of (bad-but-not-too-bad) economy states. If the cost of equity dominates the fire-sale profit, then (2) is not an optimal option, and the asymmetric framework boils down to the symmetric framework introduced in this chapter.

What if the cost of equity is lower than the expected profit from fire-sales? The situation will then become more complicated. The magnitude of two forces becomes the key issue for determining bank decisions. Recall that the main factor that affects both (1) and (2) is the fire-sale price, which is determined by the fire-sale coefficient  $h$  in this model. A high value of this coefficient implies higher loss in fire-sale (lower fire-sale price). If  $h = 0$  (meaning that there is no loss in fire-sale), the profit from acquiring assets during fire-sale is zero; in this case no bank will choose to be safer than others. If  $h$  is relatively high (meaning that the fire-sale loss is relatively high), the profit from acquiring assets is high; in such case some banks may choose to be safer to earn a profit from acquiring assets in fire-sale. The relationship between bank competition and financial stability is therefore ambiguous under this asymmetric (coordinated) framework.

## 4.5 Numerical Results

To generate numerical results, I apply the following parameters for the numerical calculations.

$$\rho = 1.5, \quad \beta = 0.5, \quad r_1 = r_2 = 1, \quad c = 1.05, \quad h = 0.25 \quad n = 1, 2, 4, 6, 8, 10, \infty$$

The results are shown in the Figure 4.2<sup>15</sup>. One can see that as the number of banks in the economy increases, the (symmetric) proportion of deposit contract increases. This observation is consistent with Proposition 1 and 2; one can conclude that due to the motivation to take advantage of other banks in the fire-sale, a larger number of banks in the economy lead to further weakening of financial stability, due to the

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<sup>15</sup>Due to the numerical limit, it is impossible to input  $n = \infty$  into the coding; instead I use  $n = 1,000,000$  to generate the result for perfect competition.

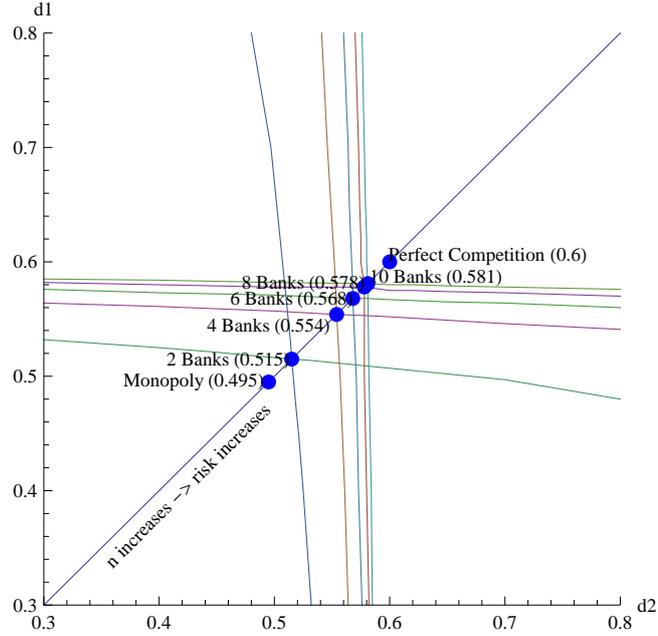


Figure 4.2: Proportion of deposit funding in an economy with different number of banks.

riskier funding structure chosen by the banks.

To guarantee the risk-free nature for deposit contract in the numerical results, the proportion of deposit funding has to be constrained. I make sure that the numerical results satisfy the risk-free nature of deposit contract by adding the following constraint to the programming.

$$(1 - \beta)r_2d_1 \leq \rho(1 - x_{n1})$$

This constraint ensures that even in the worst scenario ( $\alpha = 0$ ), the return from fire-sale is enough to repay all deposit contracts on date 2 ( $(1 - \beta)r_2d_1$ ); this, of course, also implies all deposit contracts on date 1 are fully repaid. Other minor details for the coding are explained in the appendix of this chapter.

## 4.6 Policy Discussion

In this section, some policy interventions that may help restoring the socially optimal funding structure are discussed. As mentioned, the socially optimal funding structure in this chapter is the structure chosen by the monopoly bank. As the number of banks increases, the (symmetric) funding structure becomes more risky and are socially suboptimal. Three policies are discussed in this section: (1) restricting the minimum capital requirement; (2) bridging the gap between the costs of equity and deposit; and (3) fire-sale penalty. In the following, I apply the numerical parameters used in Section 4.5 to generate supporting results for the policy discussion.

### 4.6.1 Minimum Capital Requirement

Setting a minimum capital requirement is the most straightforward policy according to this model. From the numerical results, it is clear that the socially optimal funding structure is characterised by  $d_s = 0.495$ , meaning that the optimal proportions for deposit and equity are 0.495 and 0.505 respectively. If the central authority can set a capital requirement to force all banks in the multi-bank economy to hold a proportion of equity of at least 50.5 percent of the total assets, then the socially optimal funding structure is restored.

However, capital requirement has already been commonly used in reality as a control for the asset risk of banks. Specifically, many central authorities set a minimum capital requirement such that bank equity has to be at least equal to a certain proportion of the risk-weighted assets, in order to protect the depositors. For this reason, it is hardly possible to adjust the minimum capital requirement to control for both the risks from asset portfolio and funding structure. Therefore, two other policies which can also control the funding structure of banks are studied.

### 4.6.2 Bridging the Gap of the Costs of Funding

According to the model specifications, the motivation for banks to accept deposit comes from the assumption that the cost of deposit is lower than the cost of equity. This difference between the two costs is defined as a market friction. A common example for this market friction is the taxation benefit from deposit (debt): as the interest expense from deposit is tax-deductible, funding bank loans with deposit is comparatively cheaper than funding with equity stock.

If the central authority can reduce the gap between the two sources of funding, it motivates the banks to choose a higher proportion of equity. Yet, according to this model, it is not a good idea to remove the gap between these two sources of funding entirely, because by doing so the banks will have no motivation to accept any deposit from households. How much subsidy should the central authority provide such that the banks can choose a funding structure that is socially optimal? I use the parameters from the numerical results in Section 4.5 to generate the following table.

The following table shows the cost of equity that can restore the symmetric funding structure to  $d_s = 0.495$ . One can see that the maximum subsidy is 0.00875 per unit of equity in an economy with infinite number of banks.<sup>16</sup> This subsidy is not particularly high, but it is very effective for restoring socially optimal funding structure.

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<sup>16</sup> $1.05 - 1.04125 = 0.00875$

Number of Banks (n)	Cost of Equity (c)
1	1.05
2	1.0477
4	1.0443
6	1.0432
8	1.0427
10	1.0424
$\infty$	1.04125

Table 4.2: Cost of Equity for Restoring Optimal Funding Structure.

In reality, this explicit subsidy can be implemented by limiting the tax-deductible interest expense in deposits; this also reduces the market friction between the costs of deposit and equity, and helps control the bank risk to the central authority's target.

### 4.6.3 Fire-Sale Penalty

Is it possible for the central authority to punish the banks for asset fire-sale, in order to control the funding structure of banks? According to the model specifications in this chapter, penalty is not an efficient tool to control bank risk (or funding structure) because the incentive for excessive risk-taking can outrun the penalty in an economy with many banks. Again, I use the parameters from the numerical results to generate the following table.

Number of Banks (n)	Fire-Sale Price Coefficient (h)
2	0.325
4	0.65
6	0.97
8	> 1
10	> 1
$\infty$	$\infty$

Table 4.3: Fire-Sale Penalty for Restoring Optimal Funding Structure.

A penalty to fire-sale can be interpreted as an increase in the fire-sale-price coefficient ( $h$ ). When  $h$  increases, the fire-sale price drops, causing larger loss to the banks. These extra losses can be interpreted as the penalty from the central authority for banks who have liquidity shortfalls. The table shows the value of  $h$  that can restore the funding structure to  $d_s = 0.495$ .

One can observe from the table that,  $h$  has to increase a lot in order to restore the socially optimal funding structure. In fact, some of the  $h$  have already violated the model constraint of  $h < \frac{n}{n+1}$ . What do these results imply? It implies that,

according to this model, penalty can be outrun by the incentive for excessive risk-taking in a multi-bank economy. In extreme cases, the central authority needs to set penalty to infinite when the number of banks approaches infinite; this is neither realistic nor possible. In fact, from the numerical results shown in the table, one can see that it is hardly possible for the central authority to restore optimal funding structure even in a four-bank economy, because the central authority will almost need to triple  $h$  to achieve the goal (increase  $h$  from 0.25 to 0.65). Therefore, according to the numerical results, it is inefficient or even impossible to use penalty to restore optimal funding structure.

## 4.7 Conclusion

This chapter studies the long-debated question, does bank competition lead to the weakening of financial stability? I examine this question from with a different perspective compared with the existing literature. Instead of the commonly-discussed asset risks, I apply a liquidity-risk framework to study how the interaction of competition and funding structure of banks can affect financial stability. I show that the existence of fire-sale plays an important role in the determination of funding structure of banks. In this chapter, fire-sale creates an incentive for a bank to take advantage of other banks by choosing a riskier funding structure. Based on this, I prove that banks choose riskier funding structures in an economy with more banks in symmetric equilibrium. This implies that higher bank competition (characterised by more banks in the economy) leads to higher financial instability.

In the numerical simulations, I show that capital requirement and subsidising for the difference between the costs of different funding are efficient policies to restore the banks' funding structure to the socially optimal level, but fire-sale penalty seems to have limited effect and can be outrun by the excessive risk-taking incentive. These results suggest some room for further studies in both empirical and theoretical research, not on the widely-discussed asset risk aspects, but on the liquidity risk aspects for the relationship between bank competition and financial stability.

## 4.8 Appendix

### 4.8.1 Proof of Proposition 1

**Proof.** Step 1: In symmetric equilibrium,  $d_1 = d_2 = d_s$  in the two-bank economy; therefore,  $\alpha_1^* = \alpha_2^*$  and  $x_{21} = x_{22}$  also hold. Substitute these results into the two FOC's in the two-bank economy, one can find that the two FOC's converge to the same result, which is

$$(c - c^*) - \left[ \frac{3h}{2} \int_0^{\alpha_1^*} x_{21} \frac{\partial x_{21}}{\partial d_1} d\alpha + (\rho - 1) \int_0^{\alpha_1^*} \frac{\partial x_{21}}{\partial d_1} d\alpha \right] = 0 \quad (4.5)$$

Rewrite the FOC of the one-bank economy in the following.

$$(c - c^*) - \left[ 2h \int_0^{\alpha_1^*} x_{11} \cdot \frac{\partial x_{11}}{\partial d_1} d\alpha + (\rho - 1) \int_0^{\alpha_1^*} \frac{\partial x_{11}}{\partial d_1} d\alpha \right] = 0 \quad (4.6)$$

Step 2: I prove that there is a contradiction if  $x_{11} \geq x_{21}$  (which implies the  $d_1$  in the one-bank economy is greater than or equal to the  $d_1$  in the two-bank economy); therefore, the model must have  $x_{11} < x_{21}$  (which implies the  $d_1$  in the one-bank economy is less than the  $d_1$  in the two-bank economy).

If  $x_{11} \geq x_{21}$ , then

$$\frac{\partial x_{11}}{\partial d_1} = \frac{\beta r_1}{1 - 2hx_{11}} > \frac{\beta r_1}{1 - \frac{3h}{2}x_{21}} = \frac{\partial x_{21}}{\partial d_1}$$

Therefore, the second term of eq.(4.6) must be bigger than the second term of eq.(4.5).

As both FOC's equal zero, the threshold of eq.(4.6) must be smaller than the threshold of eq.(4.5) for the zero sum to be true. However, this is a contraction because if  $x_{11} \geq x_{21}$ , the threshold for eq.(4.6) must be bigger than the threshold for eq.(4.5).

Therefore, the model must have  $x_{11} < x_{21}$ , implying that the proportion of deposit funding in the one-bank economy is smaller than the proportion in the two-bank economy. ■

## 4.8.2 Proof of Proposition 2

**Proof.** Step 1: In symmetric equilibrium,  $d_1 = d_2 = d_s$  in the n-bank economy; therefore,  $\alpha_1^* = \alpha_2^*$  and  $x_{n1} = x_{n2}$  also hold. Substitute these results into the two FOC's in the n-bank economy, one can find that the two FOC's converge to the same result, which is (with substitution of  $q_n = 1/n$ )

$$(c - c^*) - \left[ \frac{h(n+1)}{n} \int_0^{\alpha_1^*} x_{n1} \frac{\partial x_{n1}}{\partial d_1} d\alpha + (\rho - 1) \int_0^{\alpha_1^*} \frac{\partial x_{n1}}{\partial d_1} d\alpha \right] = 0 \quad (4.7)$$

The FOC of the (n-1)-bank economy is

$$(c - c^*) - \left[ \frac{hn}{n-1} \int_0^{\alpha_1^*} x_{(n-1)1} \frac{\partial x_{(n-1)1}}{\partial d_1} d\alpha + (\rho - 1) \int_0^{\alpha_1^*} \frac{\partial x_{(n-1)1}}{\partial d_1} d\alpha \right] = 0 \quad (4.8)$$

Step 2: I prove that there is a contradiction if  $x_{n-1,1} \geq x_{n1}$  (which implies  $d_1$  in the (n-1)-bank economy is greater than or equal to  $d_1$  in the n-bank economy); therefore, the model must have  $x_{n-1,1} < x_{n1}$  (which implies  $d_1$  in the (n-1)-bank economy is less than  $d_1$  in the n-bank economy).

If  $x_{n-1,1} \geq x_{n1}$ , then

$$\frac{\partial x_{n-1,1}}{\partial d_1} = \frac{\beta r_1}{1 - hq_{n-1}(2x_{n-1,1} + (n-2)x_{n-1,2})} > \frac{\beta r_1}{1 - hq_n(2x_{n1} + (n-1)x_{n2})} = \frac{\partial x_{n1}}{\partial d_1}$$

The second term of eq.(4.8) must be bigger than the second term of eq.(4.7).

As both FOC's equal zero, the threshold of eq.(4.8) must be smaller than the threshold of eq.(4.7) for the zero sum to be true. However, this is a contraction because if  $x_{n-1,1} \geq x_{n1}$ , the threshold for eq.(4.8) must be bigger than the threshold for eq.(4.7).

Therefore, the model must have  $x_{(n-1)1} < x_{n1}$ , implying that the proportion of deposit funding in the (n-1)-bank economy is smaller than the proportion in the n-bank economy. ■

### 4.8.3 The Differentiation of $x_{jk}$ with Respect to $d_1$

In the following, I show the differentiation of  $x_{jk}$  with respect to  $d_1$ .

#### One-Bank Fire-Sale

In a n-bank economy, the liquidity shortfalls of Bank 1 in a one-bank fire-sale is given by

$$x_{11}(1 - hq_n x_{11}) = \beta r_1 d_1 - \alpha \rho$$

The change of  $x_{11}$  with respect to  $d_1$  is therefore

$$(1 - 2hq_n x_{11}) \frac{\partial x_{11}}{\partial d_1} = \beta r_1$$

or

$$\frac{\partial x_{11}}{\partial d_1} = \frac{\beta r_1}{1 - 2hq_n x_{11}}$$

The second derivative can be calculated from the following chain rule.

$$\frac{\partial^2 x_{11}}{\partial d_1^2} = \frac{\partial \frac{\partial x_{11}}{\partial d_1}}{\partial x_{11}} \cdot \frac{\partial x_{11}}{\partial d_1} = \frac{2hq_n \beta r_1}{(1 - 2hq_n x_{11})^2} \cdot \frac{\beta r_1}{1 - 2hq_n x_{11}} = \frac{2hq_n (\beta r_1)^2}{(1 - 2hq_n x_{11})^3}$$

Substituting  $n = 2$  and  $q_n = 1/2$ , one can obtain the differentials in the two-bank economy, which are

$$\begin{aligned} \frac{\partial x_{11}}{\partial d_1} &= \frac{\beta r_1}{1 - hx_{11}} \\ \frac{\partial^2 x_{11}}{\partial d_1^2} &= \frac{h(\beta r_1)^2}{(1 - hx_{11})^3} \end{aligned}$$

Substituting  $n = 1$  and  $q_n = 1$ , one can obtain the differentials in the monopoly-bank economy, which are

$$\begin{aligned} \frac{\partial x_{11}}{\partial d_1} &= \frac{\beta r_1}{1 - 2hx_{11}} \\ \frac{\partial^2 x_{11}}{\partial d_1^2} &= \frac{2h(\beta r_1)^2}{(1 - 2hx_{11})^3} \end{aligned}$$

#### N-Bank Fire-Sale

Similarly, the liquidity shortfalls of Bank 1 in a n-bank fire-sale is given by

$$x_{n1}[1 - hq_n(x_{n1} + (n - 1)x_{n2})] = \beta r_1 d_1 - \alpha \rho$$

The change of  $x_{n1}$  with respect to  $d_1$  is therefore

$$[1 - hq_n(2x_{n1} + (n - 1)x_{n2})] \frac{\partial x_{n1}}{\partial d_1} = \beta r_1$$

or

$$\frac{\partial x_{n1}}{\partial d_1} = \frac{\beta r_1}{1 - hq_n[2x_{n1} + (n-1)x_{n2}]}$$

The second derivative calculated from the following chain rule is given by

$$\begin{aligned} \frac{\partial^2 x_{n1}}{\partial d_1^2} &= \frac{2hq_n\beta r_1}{[1 - hq_n(2x_{n1} + (n-1)x_{n2})]^2} \cdot \frac{\beta r_1}{1 - hq_n[2x_{n1} + (n-1)x_{n2}]} \\ &= \frac{2hq_n(\beta r_1)^2}{[1 - hq_n[2x_{n1} + (n-1)x_{n2}]]^3} \end{aligned}$$

Substituting  $n = 2$  and  $q_n = 1/2$ , one can obtain the differentials in the two-bank economy, which are

$$\begin{aligned} \frac{\partial x_{21}}{\partial d_1} &= \frac{\beta r_1}{1 - h(x_{21} + \frac{1}{2}x_{22})} \\ \frac{\partial^2 x_{21}}{\partial d_1^2} &= \frac{h(\beta r_1)^2}{[1 - h(x_{21} + \frac{1}{2}x_{22})]^3} \end{aligned}$$

as shown in Section 4.4.

#### 4.8.4 Minor Details in Numerical Programming

In order to generate the numerical programming for the model, better expressions for  $x_{11}$ ,  $x_{n1}$ , and  $x_{n2}$  are needed. Recall that the definition of  $x_{11}$  is given by

$$x_{11}(1 - hq_n x_{11}) = \beta r_1 d_1 - \alpha \rho$$

or

$$hq_n x_{11}^2 - x_{11} + (\beta r_1 d_1 - \alpha \rho) = 0$$

The solutions for  $x_{11}$  are

$$x_{11} = \frac{1 \pm \sqrt{1 - 4hq_n(\beta r_1 d_1 - \alpha \rho)}}{2hq_n}$$

I need to rule out one solution in the numerical programming. I do so by showing that there is a contradiction if  $x_{11} = \frac{1 + \sqrt{1 - 4hq_n(\beta r_1 d_1 - \alpha \rho)}}{2hq_n}$ ; therefore, the unique solution should be  $x_{11} = \frac{1 - \sqrt{1 - 4hq_n(\beta r_1 d_1 - \alpha \rho)}}{2hq_n}$ .

By definition,  $x_{11} \geq 0$ . If  $x_{11} = \frac{1 + \sqrt{1 - 4hq_n(\beta r_1 d_1 - \alpha \rho)}}{2hq_n}$  is the solution, then

$$\sqrt{1 - 4hq_n(\beta r_1 d_1 - \alpha \rho)} \geq -1$$

which results in a complex number for  $x_{11}$ . This violates the definition of  $x_{11}$  of being a positive real number. Therefore,  $x_{11} = \frac{1 + \sqrt{1 - 4hq_n(\beta r_1 d_1 - \alpha \rho)}}{2hq_n}$  is not a real-number solution.

If  $x_{11} = \frac{1 - \sqrt{1 - 4hq_n(\beta r_1 d_1 - \alpha \rho)}}{2hq_n}$  is the solution, then

$$\sqrt{1 - 4hq_n(\beta r_1 d_1 - \alpha \rho)} \leq 1$$

which shows that  $x_{11} = \frac{1 - \sqrt{1 - 4hq_n(\beta r_1 d_1 - \alpha \rho)}}{2hq_n}$  is the real-number solution, and I apply this solution to the numerical programming for the model.

Similarly, using the same technique, one can rule out a complex-number solution for  $x_{n1}$  and  $x_{n2}$ , and get the following unique real-number solution.

$$x_{n1} = \frac{(1 - hq_n(n-1)x_{n2}) - \sqrt{(1 - hq_n(n-1)x_{n2})^2 - 4hq_n(\beta r_1 d_1 - \alpha \rho)}}{2hq_n}$$

$$x_{n2} = \frac{(1 - hq_n x_{n1}) - \sqrt{(1 - hq_n x_{n1})^2 - 4hq_n(n-1)(\beta r_1 d_2 - \alpha \rho)}}{2hq_n(n-1)}$$

I apply these solutions to the numerical programming for the model.

# Chapter 5

## Thesis Conclusion

### 5.1 Summary

This thesis investigates several issues concerning bank risk-taking and financial stability. Specifically, this thesis explores (1) how moral hazard induced by the deposit insurance systems can encourage risky securitisation in deposit-taking commercial banks; (2) the social value of different banking structures (total separation, ring-fencing and universal banking); (3) how banks' funding structure and fire-sale risks affect the relationship between bank competition and financial stability.

Chapter 2 studies the first issue and finds that, in an economy with symmetric information, the existence of risk-insensitive deposit insurance can create an incentive for banks to securitise for the purpose of exploiting the deposit insurance systems. Chapter 2 also discusses some moral-hazard controls to restrain the risk-taking behaviour in securitisation.

In the numerical simulations, I show that both deposit insurance premium and franchise value can be efficient factors to restrain the magnitude of moral hazard. The former limits the size of securitisation that is used to provide new loans to borrowers. The latter encourages banks to securitise to reduce the banks' funding costs and insolvency risk.

Chapter 3 studies the second issue and finds that total separation is not a socially optimal banking structure for an economy, because it forbids the liquidity transfer between subsidiary banks, which is socially valuable. The comparison between ring-fencing and universal banking is more complicated; Chapter 3 shows that whether ring-fencing or universal banking is the best banking structure for an economy depends on the returns to the different subsidiary banking sectors.

If the returns to the utility banking sector is relatively low compared with the returns to casino banking sector, universal banking is a better banking structure; otherwise, ring-fencing has a higher social value than universal banking. The nu-

merical illustrations of Chapter 3 further support this model result.

Chapter 4 studies the third issue and finds that the fire-sale risks and banks' funding structure can create an incentive for excess risk-taking in a multi-bank economy. Moreover, the model shows that the excessive risk taking increases with the number of banks in the economy. This result is in spirit similar to the Cournot equilibrium in standard microeconomic theory.

To limit the excessive risk taking, the numerical simulations in Chapter 4 suggest that capital requirement and government subsidy (to narrow the difference between the costs of different types of funding for banks) can be effective policy interventions. Fire-sale penalty seems to be less efficient from the illustrated numerical results, as it can be dominated by the benefit from excess risk-taking easily.

## 5.2 Thesis Limitation

As other theoretical studies, this thesis has its limits. The specifications and assumptions in each chapter provide a tractable framework for the derivations of analytical results. However, the explored issues are far more complicated in the real world.

The key limit to Chapter 2 is the assumption of symmetric information. Although this assumption provides an advantage for the proposed model to apply standard asset pricing method, it also limits the model to incorporate other motives of securitisation that are caused by asymmetric information and market friction; this leads to a model framework that is less realistic and insightful.

However, I would like to argue that the model conclusion in Chapter 2 is unaffected even if an asymmetric-information environment is applied, because under asymmetric information, the key difference is that there also exists other motives for *more* securitisation, and it does not change the fact that the value of deposit insurance is a source of moral hazard to the banks.

The key limit to Chapter 3 is the specifications on banking structures. The proposed specification focuses solely on the restriction of liquidity transfer between subsidiary banks. However, in reality, there are other differences in the three compared banking structures, such as the effect of risk diversification between the subsidiaries, the benefit from customer synergies, the share of valuable information, and the government rescue due to the nature of *Too-Important-To-Fail*.

However, I also believe that incorporating these specifications do not change the key conclusion derived in this chapter. This is because most of these benefits are only available in the banking structure of ring-fencing and universal banking, incorporating these factors will only strengthen the suggested result that total separation is suboptimal. Moreover, these benefits will also strengthen the model result that the comparison between ring fencing and universal banking depends on other

factors. It can be expected that, incorporating these factors, the social value of universal banking should increase, because not all of these benefits are available in ring-fencing. In this case, the optimal choice between universal banking and ring fencing will depend on the magnitude of the trade-offs between the two banking structures.

The key limit to Chapter 4 is that the proposed model focuses only on banks' funding structure, and is incapable to incorporate banks' asset risks.

Yet, I would like to argue that incorporating the asset risk should not alter the model conclusion; from previous literature, it is obvious that most of the suggested models support that banking competition leads to higher asset risk (lower profit margin leading to smaller capital buffer); therefore, the conclusion in Chapter 4 is consistent to the predictions of previous literature that bank competition leads to financial instability.

## 5.3 Suggestion for Future Research

### Theoretical Research

This thesis also suggests two possible directions for further research in theoretical modelling.

The first suggestion for theoretical research is an extension of Wagner and Marsh (2006) [81], by incorporating the model suggested in Chapter 4. In Wagner and Marsh's work, private and social costs of bankruptcy are exogenous variables, and whether CRT across banking sector is beneficial to financial stability remains unclear due to these exogenous variables. In Chapter 4, I suggest that bank competition leads to higher bankruptcy cost due to the lower fire-sale price; this can be interpreted as the private cost of bankruptcy. The social cost can be characterised by the government cost of (risk-insensitive) deposit insurance, and it can be predicted that the cost of deposit insurance increases due to bank competition. Applying these endogenised costs of bankruptcy on CRT can explore, how bank competition can affect the efficiency of CRT within the banking sector and across the banking and non-banking sectors. This may also provide a new insight to the 2007-2009 crisis: based on the intense competition before the crisis, is CRT efficient and how does it affect financial stability? This suggested research may help provide an answer.

The second proposed research is to extend the model in Chapter 4 to study banks' choice of funding structure under asymmetric equilibrium. Recall that in Chapter 4, the choice of banks' funding structure is analysed based on symmetric equilibrium; this framework can be extended to study asymmetric equilibrium by allowing banks to acquire the assets of other banks if the former have excess liquidity. In a similar

way as Wagner (2008), the inefficient cost can be defined as the originating banks being able to make better use of the investment projects. It can be expected that this proposed model can have two possible conclusions: (1) all banks still choose the same funding structure as in the symmetric equilibrium if the benefit for risky funding structure is sufficiently high; (2) some banks choose safer funding structures by keeping excess liquidity to acquire the assets sold by the risky banks in fire-sales. It can be assumed that due to the existence of a high amount of outside investors, the fire-sale price is bounded below; this limits the fire-sale price that the safe banks can charge for acquiring assets. The contribution of this proposed research is that, it enhances the understanding on how to reduce the taxpayers' subsidies by encouraging the banks to absorb the liquidity risk of each other in an efficient way.

### **Empirical Research**

Based on the theoretical foundations built in this thesis, I would also like to suggest three potential areas for future empirical works.

The first suggestion is the empirical works to investigate how banks' funding structure and fire-sales affect financial stability. Previously, most regulators believe that asset-backed lending is less risky than credit lending. However, as suggested in Chapter 4, the asset mortgages may have very low liquidation values during fire-sales, especially when the aggregate size of asset sold in fire-sales is large in the economy. Therefore, it is worth exploring whether encouraging asset-backed lending is beneficial to an economy.

Moreover, I expect that the mechanism proposed in Chapter 4 (that the incentive for banks to take advantage of each other in fire-sales can result in excessive risk taking in a multi-bank economy) can be observed from empirical data. Specifically, I expect that there is a higher increase in the expansion of asset-backed lending compared with other credit lending (less affected by fire-sales), in an economy with fiercer bank competition. If this is true, this chapter helps explain the underlying reason for this empirical result.

The second suggestion is the empirical studies on the comparison of different banking structures, particularly on the comparison between universal banking and ring fencing. As it is hard to quantify the magnitude of the trade-off between different banking structures under theoretical framework, empirical studies can fill this gap by providing evidence for the social value of the different benefits according to different banking structures. In a few years, ring fencing will be implemented in some countries, and data will be available at that time for the empirical studies for the banking structures.

The third suggestion is the empirical study on bank securitisation, to explore how banks with different franchise values can have different motives for securitisation.

Based on the model specification in Chapter 2, it is clear that bank securitisation can increase or decrease the insolvency risk of banks, depending on how the proceeds from securitisation are used. The numerical results from Chapter 2 show that banks with different franchise values can have different motives for securitisation. Specifically, banks with high franchise values securitise to reduce their funding costs and insolvency risk, while banks with low franchise values securitise for pure moral hazard purpose. Therefore, finding evidence for the different motives of securitisation, based on the banks with different franchise values, could provide important insights for regulators to set up more efficient policies for the control of risks induced by securitisation.

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