The Effects of Interbank Networks on Efficiency and Stability in a Macroeconomic Agent-Based Model

Andrea Gurgone[a], Giulia Iori[b], and Saqib Jafarey[b]

[a] *The Complexity Lab in Economics (CLE), Catholic University of Milan, Italy*
[b] *Department of Economics, City, University of London, United Kingdom*

Abstract

We develop a macroeconomic agent-based model that consists of firms, banks, unions and households who interact on labour, goods, credit and interbank markets. The model endogenises pricing decisions by firms, wage setting by unions and interest rate setting by banks on both firm and interbank lending. Banks also set leverage targets and precautionary liquidity buffers on the basis of internal risk models. Our model produces endogenous fluctuations driven by the pricing behaviour of firms and the wage setting behaviour of unions. Fluctuations lead to loan defaults which are exacerbated as lenders reduce lending and charge higher interest rates, inducing a credit crunch. We also study how making the inter-banking network more connected affects the key outcomes of the economy and find that while the flow of funds from surplus banks to firms can be increased, the latter effect is soon dominated by increasing instability in the real sector as firms default at higher rates. While the banking sector experiences fewer defaults as a whole, losses on the interbank market increase as a source of bank defaults.

Keywords: Financial fragility, liquidity hoarding, macroeconomic stability, Agent-Based macroeconomics, interbank market
JEL Classification: G21, G28, E32

We are grateful to Domenico Delli Gatti and Tiziana Assenza for their valuable and constructive suggestions and their useful critiques of this research work. We are also grateful to two anonymous referees whose criticisms and comments have greatly improved the robustness of our results. The authors are solely responsible for any remaining errors.

1 Introduction

Following the 2007-2008 crisis, it has come to be widely accepted that price flexibility and monetary stability are not enough to ensure macroeconomic stability, and that financial stability needs to be targeted both for its own sake and as a key component of any macroeconomic framework. In particular, it is now believed that financial institutions, particularly large banks, have the potential to generate systemic risk that spills over into the real economy.

An early contribution by Carl Chiarella highlighted precisely these insights: in (*Chiarella et al. (2012)*), he showed that the expansion of banking activities into non-traditional areas such as stock trading can expose both financial markets and the real economy to instability, even when the central bank undertakes appropriate monetary policy; by contrast a Fisherian system in which banks hold 100% reserves against demand deposits and are excluded from stock trading can support macroeconomic stability while guaranteeing a sufficient loan supply to businesses.

A key mechanism through which systemic risk operates is balance sheet contagion, *i.e.* losses arising in the balance sheet of one institution have the tendency to spread to other institutions. In recent years agent-based models (ABMs) have been widely employed to study the channels through which balance sheet contagion spreads within the financial system and beyond. Here too, Carl Chiarella has played a pioneering role as one of the first group of economists to work within this framework, *e.g. (Chiarella et al., 2009, 2002).*

Three channels for balance sheet contagion have been identified: (1) the direct 'knock-on' effect, as default by one bank creates losses on the balance sheets of its creditors; (2) an indirect 'fire sale' effect, as banks that suffer losses deleverage by selling off assets, leading to collapsing asset prices that undermine otherwise liquid and solvent banks; (3) an indirect 'liquidity hoarding' effect, as healthy institutions react to the accumulation of losses in the banking system by withholding liquidity on the interbank market to their distressed counterparts.

Of these three channels, the last is probably the most important one through which the interbank market contributes to the spread of financial crises. Indeed, liquidity hoarding was identified by several authors as a key trigger of the 2007-8 financial and economic crisis, (*Allen and Carletti (2008); Heider et al. (2009); Acharya and Merrouche (2010)*). One can argue that precisely because the interbank market is normally so important in redistributing liquidity within the banking sector and on to the real economy, when crisis hits it quickly becomes subject to the reversal of liquidity provision by otherwise sound banks. Yet, in our opinion, liquidity hoarding has been one of the least studied channels of contagion, at least within the framework of ABMs.

In this paper, we develop a macroeconomic ABM that incorporates households and firms who interact with each other on labour and goods markets; banks that take in deposits from other sectors, lend to firms on a credit market and to each other on an interbank market, a government which collects taxes, makes transfer payments and issues debt and a Central Bank that buys government debt and acts as lender of last resort to the banking system. Our purpose is to model how interbank markets affect the performance of the real sector over the course of a business cycle and indeed, how they affect the properties of the cycle itself.

The economy is closed. Credit flows constitute the only feasible mechanism for exchange, with all transactions concluded via transfers between the bank balances of agents. Firms and banks are assumed to be price setters. Imperfect competition arises by the assumption that both goods and lending markets are subject to matching frictions that limit opportunities for price arbitrage. Agents are boundedly rational in that they use simple rules-of-thumb to make decisions, but these rules are updated in light of experience. Another feature of the model is stock-flow consistency in the sense of *Godley (2007)*, ensuring that value is not accidentally created or destroyed. Disequilibrium is a possibility in the model: markets do not clear at every time step, thus rationing might occur and the *short side rule (Bénassy, 2002)* is applied.

The main contribution of the model is in the way it treats the banking sector. We endogenise banks' strategies as to how much lending they wish to undertake at any given time, who they lend to and at what interest rates. To be precise, banks determine a target leverage ratio on the basis of an expected shortfall measure which varies with financial market conditions, they prioritise counter-parties for lending to on the basis of their perceived risk of defaulting and charge interest rates to those whom they lend to on the basis of both that perceived risk and their own sense of economic vulnerability, as proxied by their expected shortfall.

To our knowledge this is the first attempt to determine all three aspects of bank strategy in a single, unified framework. We find that the model is capable of generating endogenous business cycles without imposing any external disturbance. Moreover we find evidence that when financial downturns occur, banks contribute to them by withholding liquidity from the interbank and credit markets and seeking higher interest rates on the funds which they choose to make available. These effects occur via a decrease in the maximum leverage that banks are willing to undertake and an increase in the interest rates that they charge on the funds that they do offer for lending. Finally we find that an increase in interbank connectivity on one hand improves credit to the real economy but on the other exacerbate liquidity hoarding.

To our knowledge, the ABMs that have been developed thus far can be broadly categorised into three groups: those that are mainly concerned with the macroeconomic role of the banking sector as a whole and thus lack the interbank market as an additional channel for contagion; those that focus primarily on the interbank market and finally, those that combine elements of the two.

In the first group are the seminal papers of *Delli Gatti et al. (2009, 2011); De Masi and Gallegati (2012); Ashraf et al. (2017)*. Other papers in this vein include *Assenza et al. (2015); Caiani et al. (2016); van der Hoog and Dawid (2017); Dosi et al. (2010); Popoyan et al. (2017)*. These models study contagion among firms that operate within the real sector as well as between the firms and banks but do not study the role of the interbank market in magnifying contagious effects.

By contrast, papers in the second group either ignore the real macroeconomy entirely or treat it as an exogenous source of shocks to the banking sector. For instance in *Gabbi et al. (2015)* the real sector is considered as a black-box that demands bank loans and creates exogenous shocks to banks' deposits, while the banking sector reacts to these external factors by its own optimal behaviour on the credit and interbank markets. A similar approach is adopted in *Iori et al. (2006)*, while *Georg (2013); Allen et al. (2009); Montagna and Kok (2013); Lux (2015); Berardi and Tedeschi (2017)* are pure models of interbank networks.

In the last group, *Tedeschi et al. (2012)* modelled a three sector economy with goods, credit and interbank markets in order to study the correlation between bankruptcy cascades and endogenous business cycles. Other contributions along these lines include *Grilli et al. (2014)*. However, while these papers do consider both credit and interbank markets they do not endogenise banks' strategies as we do, neither do they take up the issue of liquidity hoarding as a specific phenomenon that amplifies losses on the interbank market.

It should also be noted that while several of the papers mentioned above do assume that lenders follow strategies for screening potential borrowers on grounds of perceived risk and setting interest rates accordingly, none of them relates these strategies to the lender's own perceived vulnerability. An exception is that of *(Delli Gatti et al., 2011)* who assume that, controlling for borrower risk, more financially sound lenders offer lower interest rates as a competitive strategy. Our own formulation makes this concept more precise, via linking financial soundness to the concept of expected shortfall which is well known as both a tool of risk management and as a benchmark for determining capital adequacy under Basel regulations.

The rest of the paper is structured as follows: Sect. 2 contains the model and its underlying assumptions. Results from the simulations are described in Sect. 3, while Sect. 4 is about the effects of growing interbank connectivity. Conclusions are in Sect. 5.

2 The Model

Our macroeconomic model represents a simplified version of the one used in *(Delli Gatti et al., 2011)* as far as the structure of final goods production is concerned, but extends the latter by introducing a detailed model of the interbank market in which loan supplies, selective rationing and interest rates are all determined endogenously. Additionally we build on *Godley (2007)* to model the sectoral structure of the economy.

2.1 Overview

The economy is composed of five types of agents: households, firms, banks, a government and a central bank (hereafter, CB). The (discrete) numbers of households, firms and banks are N^H, N^F, N^B respectively. Interactions take place on different markets: firms and households meet on markets for goods and for labour, while firms borrow from banks on the credit market and banks exchange liquidity on the interbank market.[1] The CB buys government-issued bills on the bond market.

The sole role of the government is to make transfer payments to the household sector, funding these by issuing bills and collecting taxes. The CB generates liquidity by buying government bills and providing advances to those banks that require them; it furthermore holds banks' reserve deposits in its reserve account.

Households work and spend their income, which is made up of wage income, asset income and transfers on buying consumption goods, adding to their assets and paying taxes. In the labour market, households are represented by unions in their wage negotiations with firms, while on the capital market, they own firms and banks, receiving a share of profits as part of their asset income.

Firms borrow from banks in order to pay their wage bills in advance, hire workers, produce and sell their output on the goods market.

The banking sector provides credit to firms, subject to regulatory constraints. In each period every bank tries to anticipate its liquidity needs and accesses the interbank market as a lender or a borrower, thus the interbank market works as a mechanism to ensure the proper flow of credit to the real economy. If a bank is short of liquidity, it seeks an advance from the CB.

This section contains some general specifications of the model, *i.e.* the timing, matching mechanisms in the goods, credit and interbank markets, and the maturity structure of loans to the firm sector. Sections 2.2-2.7 describe in detail the behaviour of each class of agents and their respective balance sheets.

[1] There is no market for deposits. We assume that each bank has a fixed and equal number of depositors.

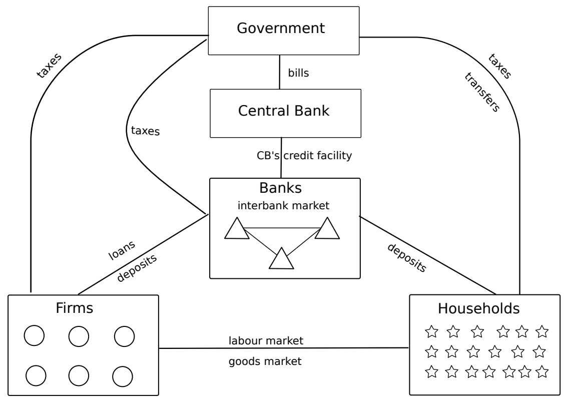


Figure 1: Diagram of the macro-financial framework.

2.1.1 Timing

The sequence of events in the model is described below.

1. The interbank market opens: demand and supply are respectively determined by the difference between a bank's expected liquidity target and its actual liquidity.

2. Banks compute the expected shortfall based on observed losses and choose their maximum credit supply. Firms decide their planned hiring and production levels, and use these to compute their credit demand.

3. The credit market opens: each bank computes the interest rate charged to each possible borrower. Firms enter the market and seek out potential lenders, sorting banks in ascending order of the interest rates that they charge.

4. The labour market operates and production takes place. Firms compute their labour demand in line with their planned output levels. They hire workers on the basis of a frictional matching process and all employed workers are paid the same wage, which is set each period by a union.

5. Households spend their consumption budget, starting from sellers that charge lower prices.

6. Firms and banks that obtain positive profits pay taxes and distribute dividends. They update the dividend share.

7. A loop cycle accounts for potential cascades of bankruptcies in the firms and banks sectors.

8. The credit and the interbank markets close. Firms and banks settle their obligations.

9. The government collects tax revenues and issues bills, which are bought by the Central Bank. Unions update their required wage rate following a Phillips rule.

10. Bankrupt firms are replaced with newborn start-ups. Banks are recapitalized by their creditors.

2.1.2 The matching mechanisms, network structure and maturity of loans

The matching mechanism Consumers and firms interact on the goods market, firms and banks on the credit market, while banks exchange liquidity with each other on the interbank market. A network structure of linkages between buyers and sellers determines the interaction in each market.

A simple matching process operates in the goods market, where each household observes a subset of firms in a random order, sorts them in ascending order and spends its consumption budget, starting from the cheapest ones. The process ends once the budget is exhausted or the household has visited all the firms in its subset. The potential out-degree of the consumer nodes in the network is equal to the number of firms in the system, since each household is linked to all the firms, but can visit only a fraction of them at each round of interactions. This friction is introduced in order to model search costs.

The parameter $Fh \in (0, 1]$ determines the share of sellers that can be visited by each household at the opening of the market. If $Fh = 1$, meaning that each household can visit all the firms, they would spend their entire consumption budget at the cheapest sellers, while the most expensive firms would be unable to sell all their goods. On the other hand, for low values of Fh, when just a small sample of firms can be visited, the buyers are likely to end up not exhausting their budgets, although the most expensive sellers could sell more than in the previous case. Thus there is a tradeoff between demand rationing and unsold output, depending on the value of Fh.

Interbank and credit markets work with a simple matching mechanism, where each borrower observes a subset of potential partners, choosing the lender according to the interest rate asked. Borrowers in turn are sorted in ascending order with respect to their default probabilities, so that the more risky banks are the first to be rationed in case the loan supply is not enough to meet total demand. The rationale of this simple rule is to allow banks to limit their exposure by rationing those agents that are more likely to be insolvent, when there is contraction in credit supply. The number of potential partners is determined by the network topology, which is fixed during each simulation.

The network topology The financial topology of the system is composed by a set of interrelated networks. We model the firm-bank credit network, deposit networks and the interbank market. The former two are bipartite graphs: the firm-bank credit network allows firms to link to banks in the credit market, while the deposit networks determine in which banks households and firms hold their deposits, subject to the condition that each depositor can have at most one link. Banks are coupled with other banks in the interbank network.

In order to match the empirical evidence on interbank networks *(see for instance Iori et al., 2008)*, we assume that few banks are connected to many depositors, i.e. they have available loanable funds, but at the same time they experience low credit demand because of scarce investment opportunities, i.e. low out-degree on the credit market. On the other side banks with a high out-degree on the credit market face large credit demand from firms, but they have low funds. For the first group it is profitable to lend liquidity to the other one through the interbank market, while the second group prefers to borrow funds in order to lend to the real sector. Moreover the interbank network has a core-periphery structure, similar to those observed in real world networks: nodes in high supply of liquidity are the peripheral ones, while the core of the network is composed by net lenders that are also characterized by greater dimensions in terms of equity. The generation process and detailed networks statistics can be found in Appendix. 6.D. Although the overall structure is static, it reduces the complexity of the system for the purposes of this article; a more realistic set-up of the network structure and its implications for the economy will be examined in further research.

The structure of maturities The introduction of a heterogeneous structure of maturities of firm-bank loans is a new feature in this family of ABM. It assumes that firms have only intermittent access to the credit market at intervals of time that differ in length across firms. This leads to heterogeneity in bank portfolios, even though banks operate in the credit market at each time. We assume a random maturity length of loans for each firm, ranging from a minimum to a maximum possible duration (see Appendix 6A), which is assigned each time a firm enters the credit market to seek credit or when the firm is revived in case of default. Furthermore we add a couple of simplifying assumptions. First, in case a firm borrows multiple loans during a session of the credit market, the maturity is equal across all of them, since the maturity depends on the next time the firm can access the market. Second, the negotiated interest rates between banks and firms are unrelated to the length of loans.[2]

[2] The *expectations* theory of term structure implies this outcome so long as creditors expect short-term interest rates to remain constant or change unpredictably. By contrast, the *liquidity preference* theory argues that interest rates should increase with maturity even if expected short-term rates are stationary. This is because long maturity loans bear greater market risk arising from volatility of short-term interest rates and market risk reduces the value of debt securities on secondary loan markets. While acknowledging this debate, our main interest is in how banks take into account their own fragility and the credit risk imposed by their borrowers, rather than the market-based risks that liquidity preference theory is concerned with. In our model there are no secondary markets for debt (or for that matter, equity) so all loans are held till maturity. For this reason we opt for the above simplifying assumption.

2.2 Households

The household sector consists of N^H units indexed by i. Households work, buy consumption goods and save. All households supply equal amounts of labour and own equal shares in banks and firms. Each household also has a deposit account at some bank, initialised to be identical across households, whereas each bank has an equal number of households with accounts in it. In case of default, households use their deposits to recapitalize firms or banks, while as depositors they may lose a part of their savings after the default of their creditor bank.[3]

The net worth of the i-th household is defined as the value of deposits kept in a bank account.[4]

$$nw_{i,t}^H = D_{i,t}^H \tag{1}$$

Households receive their income from wages, interest on deposits and dividends. The law of motion of deposits is given by the accounting equation (2), which states that the variation in the deposits from $t-1$ to t, defined $\Delta D_t \equiv D_t - D_{t-1}$, is given by the interest rate r^D on deposits at time $t-1$, plus worked hours N^H times the wage rate W, net of the tax rate θ, plus the constant dividend share δ^k on net profits of owned firms and banks $(1-\theta)\Pi^k$, with $k = f, b$, minus consumption. Moreover there is an exogenous fiscal component G, which consists of transfers to the household sector, such that everyone receives the same amount $\frac{G}{N^H}$, which adds to the disposable income.

Thus, in any period t, household $i's$ savings are equal to

$$S_{i,t}^H = D_{i,t-1}^H r^D + (1-\theta)\left(W_{t-1}N_{i,t-1}^H + \sum_{k=f,b} \delta^k \Pi_{i,t-1}^k\right) - C_{i,t-1} + \frac{G}{N^H} \tag{2}$$

Savings are identically equal to the change in the value of bank deposits, since these are the only stores of wealth in this model.

$$S_{i,t}^H = \Delta D_{i,t}^H$$

The equation of consumption resembles a permanent income rule *(see Modigliani and Brumberg, 1954)* that states that households consume a fraction c_1 of their current labour income and a fraction c_2 of their wealth. In nominal terms:[5]

[3] Households play a secondary role in this paper; unlike firms and banks we do not endow them with strategic decision-making. At the same time, households are needed in order to complete the payment and expenditure cycle of a macro-economy and it is in this spirit that we assume that they collectively and equally own profit-making entities, and that they share in the costs of capitalising new entities, as these represent flows of income and expenditure within the system.

[4] Households' net wealth also includes the values of shares in firms and banks. However, these components of net wealth cannot be explicitly valued since there are no secondary stock markets in the present generation of this model, nor are credit markets open to borrowing by households. Shares in businesses therefore only carry implicit value and are excluded from the computation of net wealth.

[5] The assumption that consumption follows a linear rule in relation to available resources is

$$C^d_{i,t} = c_1 \left[(1-\theta)W_t N^H_{i,t} + \frac{G}{N^H}\right] + c_2 D^H_{i,t} \tag{3}$$

It is worth noting that (3) represents a consumption budget that each household wishes to spend during a given period. If a household is rationed on the goods market, such involuntary saving increases its stock of deposits.

2.3 The labour market

Each household supplies one unit of labour inelastically, making the total labor force equal to N^H. However, due to hiring frictions that will be described later, there are at any time t only $\hat{N}_t \leq N^H$ *employable* workers. All employable workers are available to work at a wage determined by unions so that the actual number of employed will be determined by the demand for labour by firms, which in turn is in proportion to their available liquidity.

The reason that not all workers are employable at any given time arises because worker productivity depends on both a fixed productivity parameter, α and a multiplicative match-specific shock $\epsilon_{i,j}$ that arises when worker i meets firm j that is independently and identically distributed across all worker-firm pairs, with $\epsilon_{i,j} = 1$ with probability p and $\epsilon_{i,j} = 0$ with probability $(1-p)$. Thus

$$\alpha_{i,j} = \alpha\epsilon_{i,j} = \left\{ \begin{array}{l} \alpha \text{ with probability } p \\ 0 \text{ with probability } (1-p) \end{array} \right\}$$

This friction generates persistence in the time path of unemployment. Unemployment arises as firms fire their workers, and is not immediately eliminated despite the emergence of new firms, due to matching friction. Each job seeker can meet a subset n of prospective employers in a given period but meetings need not result in a vacancy being filled.[6]

made to approximate household behaviour via a simple rule of thumb. Bank-to-bank and firm-to-bank interactions are the main concern of this paper. We acknowledge that both theory and empirical evidence suggests that the consumption function might be far more complex and subject to discontinuities and non-linearities (see *Carroll (2001)* for a review article). At the same time, it has been argued that simple linear rules of thumb might closely approximate optimal consumption behaviour (*Allen and Carroll (2001)*). In future work when household behaviour becomes more the focus of analysis, we shall explore more sophisticated formulations for generating consumption choices: for example, the marginal propensity to consume out of available resources could go up in booms and down in recessions as precautionary savings adapt to the business cycle.

[6] Match-specific frictions are widely used in the job search literature to create frictional unemployment. *Postel-Vinay and Turon (2010)*, for example, assume that worker productivity is the product of a fixed parameter that is identical across workers and a match-specific shock that varies across worker-employer pairs. One way to conceive of a match-specific shock is that it measures mutual compatibility between the employer and his/her workers. In order for a worker to achieve their potential productivity they must 'get along' with their employer.

Given that the (simple) probability of "success" in any given interview is p, the probability of k successful interviews in n trials is given by

$$p(s) = \frac{n!}{k!(n-k)!}p^k(1-p)^{n-k};$$

where s stands for success. Setting $n = 2$, $k = 1$ this boils down to

$$p(s) = 2p(1-p)$$

Furthermore we arbitrarily assume that $p = 0.6$ so the probability of "success" is $p(s) = 0.48$.

A candidate who is compatible might be hired depending on the firm's demand for labour. If aggregate labour demand is lower than the available number of workers $\hat{N}_t$, each firm can hire its desired labour demand. Otherwise it is assumed that firms hire in proportion to their demands with respect to the total labour demand, so that full employment cannot be exceeded.[7]

We assume that the union adjusts the wage rate sluggishly, based on an adaptive mechanism, in order to prevent the wage time series jumping up or down sharply. The adjustment takes into account a simple moving average of past realized values of inflation ($\hat{\pi}^p$) and unemployment($\hat{u}$) over the last τ^w periods. The wage inflation rate $\pi^w \equiv \frac{W_t}{W_{t-1}}$ is *(see Fazzari et al., 2008)*

$$\pi_t^w = \hat{\pi}_t^p - \sigma_1(\hat{u}_t - u^*) - \sigma_2(\hat{u}_t - \hat{u}_{t-1}) \tag{4}$$

with $\hat{\pi}^p \equiv \frac{P_t}{P_{t-1}}$ the price inflation rate computed on the average price level, $\hat{u}_t$ is the unemployment rate, u^* is the average unemployment rate that would prevail if the economy were to evolve without cycles. The term $\sigma_2(\hat{u}_t - \hat{u}_{t-1})$ represents the effect of the change of unemployment on the wage inflation rate. It entails a trend rise or a decline in the wage inflation, thus affecting the persistence of the adjustment process. This effect has been backed by empirical and theoretical works *(Layard et al., 2005; McMorrow, 1996)*.

2.4 Government and the central bank

The roles of government and the CB are crucial to understand the logic of the model. Every agent tries to achieve a positive net worth, except the government and CB. The former is always in debt with the CB and the latter has zero net worth. According to the aggregate balance sheet identity for the whole economy, the negative net worth of the government is balanced by the positive net worth

[7] For instance, if $\hat{N}_t = 100$ and $\sum_{j=1}^{N^F} N_j^d = 120$, with $N_1^d = 30$, then firm 1 can hire $\frac{N_1^d}{\sum_j N_j^d}\hat{N}_t = 25$ units of labour. Otherwise, if $\sum_j N_j^d < \hat{N}_t$, the constraint is not binding and firm 1 can hire all its labour demand.

of the private sectors so that aggregate net worth is zero (see Appendix 6.B.1 for further details).

$$\sum_{i \in N^H} nw^H_{i,t} + \sum_{j \in N^F} nw^F_{j,t} + \sum_{h \in N^B} nw^B_{h,t} + nw^G_t = 0$$

Government bills are sold directly to the CB. They have one-period maturity and pay an interest rate r^B. The government's budget constraint can be expressed in the single equation

$$\Delta B_t = B_{t+1} - B_t = r^B B_t + G - T_t - \Pi^{CB}_t \tag{5}$$

where B_t is the outstanding stock of government bills at time t, G are (constant) transfers to households, T_t are tax revenues and Π^{CB}_t are the profits of the CB, repatriated to the government. The expenditure on transfers is assumed to be exogenous. In each period a part of the aggregate demand consists of G, which in turn is financed with the money created by the CB. This particular mechanism is known as a *pure auto-economy* in *Hicks et al. (1974, p. 51)* and is detailed in *Godley (2007, chap. 2)*.

The circuit starts when government issues bills that are bought by the CB. The funds raised from this sale are transferred to the household sector's bank accounts. The firm sector borrows funds from banks, pays workers to produce and then sells the goods to households. Firms then deposit revenues from sales in banks. Notice that banks are creditors of the CB, since the high powered money on their account can be asked by firms at any time and in turn can be claimed by them from the CB. The circuit closes once taxes are collected by the government and firms repay loans.

The governmental deficit is countercyclical, acting as a stabilizer. During bad times tax collections are low, so the amount of liquidity in the system increases by the budget deficit due to the constant level of public expenditure. In good times, when $T_t > G + r^B B_t$, the stock of bills is lowered and the excess liquidity is destroyed. In this simple setting we do not assume any limit to the stock of public debt or debt/GDP ratio. The event of a spiral driven by interest on outstanding debt is ruled-out by Eq. *5*, as profit of the CB are transferred to the government. Additionally the stock of bills cannot grow indefinitely because public transfers increase the budget of households, which in turn is spent in consumption goods, thus governmental deficit helps the economy to recover from the crisis. Finally when the crisis is beyond, money in the balance of agents collected with taxes and the stock of debt decreases.

The profits of the CB are given by the interest payment on advances A, bills B and reserves R:

$$\Pi^{CB}_t = A_{t-1} r^H + B_{t-1} r^B - R_{t-1} r^L \tag{6}$$

Interest rates on advances r^H and reserves r^L are fixed and form the corridor through which lending to firms and banks takes place.

Furthermore the CB acts as a lender of last resort, providing liquidity to the banking sector. Its behaviour is described with respect to bank h by

$$\Delta A_{h,t} = -\min(R_{h,t} + I^l_{h,t} - rrD_{h,t} - I^b_{h,t}, A_{h,t-1}) \tag{7}$$

When a solvent bank h has liquidity (taking into account interbank lending I^l and interbank borrowing I^b) that exceeds the advances due to the CB, i.e. $R_{h,t} + I^l_{h,t} - rrD_{h,t} - I^b_{h,t} \geq A_{h,t-1}$ it extinguishes the debt $A_{h,t-1}$. If instead $A_{t-1,h}$ is greater or equal than $R_{h,t} + I^l_{h,t} - rrD_{h,t} - I^b_{h,t}$ the bank either refunds a part of the debt ($R_{h,t} + I^l_{h,t} - rrD_{h,t} - I^b_{h,t} > 0$) or borrows ($R_{h,t} + I^l_{h,t} - rrD_{h,t} - I^b_{h,t} < 0$).

2.5 The business sector

There are N^F firms indexed by j that produce an homogeneous good using labour alone. In order to hire workers, firms need to pay the wage bill in advance. We assume that this cash-in-advance constraint always binds so that firms can only hire up to the point where available liquidity allows.[8]

The balance sheet of a firm is composed of bank deposits D^F on the asset side while liabilities consist of loans L.[9]

$$nw^F_t = D^F_t - L^F_t \tag{8}$$

Each period, firms make their decisions in the following sequence: (1) set a target output level from which they calculate a labour target; (2) seek financing by borrowing if needed (subject to access to the credit market in that period) in order to meet the expected wage bill; (3) hire workers until the wage bill has been met or no further employable workers can be found, then produce; (4) set a price for their output and attempt to sell it.

2.5.1 Quantity:

Firm j's output and labour demand choices at each time step t are determined by the following set of equations:

$$Y^{target}_{j,t} = \begin{cases} Y^s_{j,t-1}(1-\chi_j) & \text{if } Y_{j,t-1} < Y^s_{j,t-1} \\ Y^s_{j,t-1}(1+\chi_j) & \text{if } Y_{j,t-1} = Y^s_{j,t-1} \end{cases} \tag{9}$$

[8]Absent this constraint, firms could have an indeterminate demand for labour because of (i) the linearity of the production technology, (ii) the exogeneity of the wage rate to their hiring decisions and (iii) their ability to manipulate the price of their own product.

[9] Inventories are not included as they are assumed to fully depreciate each period. These assumptions, which are in line with *Delli Gatti et al. (2011)*, rule out business cycle driven by the accumulation of unsold inventories. Rather they allow for business cycles arising from variations in business expectations driven by variations in sales.

where:
$Y^{target} \equiv$ target output; $Y^s \equiv$ actual output, $Y \equiv$ actual sales and $\chi \equiv$ optimism/pessimism coefficient regarding future sales. In other words, if firm j manages to sell all its output in a given period it adjusts its target output for the next period above its output in the given period and if it has unsold output it adjusts its target output to be less than its output in the given period.

The firm's labour target directly follows from the output target.

$$N_{j,t}^{target} = \frac{1}{\alpha} Y_{j,t}^{target} \tag{10}$$

where $N^{target} \equiv$ labour target calculated at the output target. The question is whether and how this demand will be financed under the cash-in-advance constraint on the labour market.

2.5.2 Credit demand and the evolution of liquidity

On the basis of the target output and employment, firms calculate a target wage bill and try to ensure that enough liquidity is available to finance it. They first use their own available resources and go to the credit market to borrow any excess. Since they can access the credit market only intermittently, on each visit they have to calculate a loan target which covers financing needs over the entire maturity of the loan.

Whether or not a firm can enter the credit market at some time t, it calculates a loan target for that t, as a weighted average of a cumulative loan target and a current financing need in that period t. The loan target accumulates according to

$$L_{j,t}^{target} = \beta L_{j,t-1}^{target} + (1-\beta)\left(W_t N_{j,t}^{target} - \zeta n w_{j,t}^F\right) \tag{11}$$

where L^{target} is a loan target, nw^F is the net worth of the firm and β is a smoothing factor. Note that the self-financing portion of the wage bill comprises the firm's net worth at any time t and not its bank deposits $D_{j,t}^F$. This is because it has to ensure enough liquidity to pay off any liabilities from the last period of borrowing. $\zeta \in [0,1]$ is a parameter that weighs the relative priority given to internal finance ($\zeta = 1$) over borrowing ($\zeta = 0$) for meeting operational needs.[10]

If the firm enters the market at some $t = \{s, 2s, 3s \ldots\}$, where s is the interval of time between visits to the credit market, its loan demand is given by

$$L_{j,t}^d = L_{j,t}^{target}$$

To see how the loan target evolves in between visits to the credit market, consider what happens at $t+1$, i.e. the time period immediately following the

[10] The choice of ζ depends on striking a balance between the lower cost of internal finance (as suggested by *pecking-order* theory) and the tax advantages of debt finance (as argued by *trade-off theory*) plus the need to maintain positive net worth as collateral in case of future borrowing. In the simulation we set $\zeta = 0.8$ which is close to the benchmark of pecking-order theory. Lower values of ζ were found to affect outcomes in the financial sector without changing the dynamics of the real economy very significantly.

visit to the credit market. Suppose that $s = 1$, where s is the interval between visits to the credit market. In that case, the firms loan target would be given by

$$L_{j,t+1}^{target} = (1-\beta)\left(W_{t+1}N_{j,t+1}^{target} - \zeta nw_{j,t+1}^{F}\right);$$

Now suppose that $s = 2$, so that the firm can borrow every other period. In that case, it would carry over its loan target from $t+1$ and add to that its liquidity need for $t+2$.

$$L_{j,t+2}^{target} = \beta L_{j,t+1}^{target} + (1-\beta)\left(W_{t+2}N_{j,t+2}^{target} - \zeta nw_{j,t+2}^{F}\right)$$

Iterating this forward for any integer value of s we arrive at the formulation above. Note that the next time the firm enters the credit market, its loan demand will be

$$L_{j,t+s}^{d} = L_{j,t+s}^{target}$$

and $L_{j,t+s+1}^{target} = 0$ and the process repeats itself.

Since a firm might be rationed on the credit market, its actual loan might be less than its loan demand

$$L_{j,t} \leq L_{j,t}^{d}$$

Once a loan has been obtained, it is added to the firm's deposit account, which is updated as

$$D_{j,t}^{F} = nw_{j,t}^{F} + L_{j,t} \tag{12}$$

implying that the funds are immediately available to spend. Note that in any period in which the firm is in the credit market its past liabilities must be cleared before new loans can be used to finance its wage bill. Thus at the time a new loan is taken out. In addition, between t and $t+s$ the outgoings are on wage payments, taxes, dividends and interest payments on the loan.

Thus, between $t+1$ and $t+s$, D_j^F evolves as

$$\begin{aligned} D_{j,t+i}^{F} \quad = \quad & D_{j,t+i-1}^{F}(1+r^{D}) - W_t N_{j,t+i-1} + P_{j,t}Y_{j,t+i-1} - [\text{taxes} + \text{dividends} + \\ & \text{interest payments at time } t+i-1] \\ & i = \{1, \ldots s\} \end{aligned}$$

where N and Y are labour hired and output sold while P is the price of output. These are addressed in the following subsections as are the tax and dividend payments.

2.5.3 Hiring and Production

Once the firm has secured a loan and updated its liquidity, it determines an expected wage bill, Ω by balancing its planned outputs against its available funds.

$$\Omega_{j,t} = \min\left[D_{j,t}^{F}, W_t N_{j,t}^{target}]\right] \tag{13}$$

Because of frictions on the labour market, a firm might not manage to hire all its budgeted workers:

$$N_{j,t} \leq \frac{\Omega_{j,t}}{W_t}$$

where $N_{j,t}$ is the actual employment of firm j. From here we get the actual output supplied.

$$Y^s_{j,t} = \alpha N_{j,t} \tag{14}$$

2.5.4 Pricing

Each firm has some monopoly power since search costs prevent all consumers to sample all firms each time. This implies that a firm can charge a price higher than its marginal costs. However, firms that charge lower prices increase their market share. The pricing mechanism is a delicate job that may affect the outcome of the model: in principle a good pricing rule would clear the market at each time, but this ideal condition cannot be always satisfied, indeed shifts in the consumption budget of households, the cost structure and bounded rationality prevent firms from setting the market clearing price.

As in *Dosi et al. (2013)*, we assume that firms use their previous market share to compute the price they charge by adding a mark-up on their costs of production. The mark-up charged by firm j at time t, $\mu_{j,t}$, follows the rule

$$\mu_{j,t} = \mu_{j,t-1}\left(1 + \Delta y_{j,t-1}\right) \tag{15}$$

where y_j represents firm j's market share, expressed as the ratio of j's sales to the market sales from the previous period. The change in the market share between the previous two periods is denoted by $\Delta y_{j,t-1} = y_{j,t-1} - y_{j,t-2}$. According to the rule, each firm computes the most recently observed change in market share, then forms an expectation about the mark-up it can apply: if the difference is positive, the past price could be raised by an amount proportional to the size of the increase, otherwise price is too high with respect to competitors and should be reduced. Those firms that went bankrupt during the previous period start with a mark-up equal to the initial one and reset their memory with respect to past market shares.

Individual price is then determined as:

$$P_{j,t} = (1 + \mu_{j,t})uc_{j,t} \tag{16}$$

The cost of producing one unit is $uc_{j,t}$ is defined as the ratio of wage bill to j's output.

$$uc_{j,t} = \frac{W_t N_{j,t}}{Y^s_{j,t}} \tag{17}$$

Following the time-line of the model, after production and pricing took place, the goods market opens and consumers spend their consumption budget following the matching mechanism described in Sect. 2.1.2. The output sold by firm j is denoted by Y_j. The firm's gross profits Π^F are given by

$$\Pi^F_{j,t} = P_{j,t}Y_{j,t} - W_t N_{j,t} + D^F_{j,t-1}r^D - \sum_{h=1}^{N^b} r^f_{jh,t-k} L_{jh,t-k} \tag{18}$$

where $t-k$ refers to a loan taken k periods before $k = 1 \ldots s$. In words, gross profits equal sales revenues minus wage costs and interest charges. If $\Pi^F_{j,t} > 0$ the firm pays taxes and dividends, otherwise it absorbs the losses. Net profits equal gross profits minus taxes imposed at the rate θ. A share δ^f_t of net profits is distributed as dividends, with this share composed of two parts: a direct component δ^f and a component that depends on the net worth of the firm relative to its after-tax profits.

$$\delta^f_t = \delta^f + \partial^F \frac{nw^F_{j.t}}{(1-\theta)\Pi^F_{j,t}}$$

This formulation prevents firms becoming too large on the basis of retained earnings, as it tends to increase overall dividends when net worth is rising and lower them when net worth is falling. With this dividend policy the firm's net worth evolves according to

$$nw^F_{j,t} = (1-\partial^f)nw^F_{j,t-1} + (1-\theta)(1-\delta^f)\Pi^F_{j,t-1} \tag{19}$$

If $nw^F_{j,t} \geq 0$ the firm's debt can be serviced, otherwise the firm is insolvent at the end of the period and bankruptcy occurs.

2.6 The banking sector

There are N^B banks, indexed by h. They finance themselves with short-term liabilities and provide loans with longer maturities to the real sector. In case they have an excess or a shortage of liquidity, they exchange it on the interbank market or borrow from the CB in case of rationing.[11]

The asset side of their balance sheet includes outstanding loans to firms, indexed by j and banks, indexed by q, denoted respectively by L and I^l, plus

[11] It is worth noticing that the banking sector cannot finance itself without limits just by creating new deposits by lending to firms. Rather money is controlled by the CB that finances government's expenditure. Banks can provisionally anticipate liquidity to firms, then they access the interbank market to retrieve funds and comply with prudential regulation. The overall liquidity of the banking system correspond to the money supply of CB, hence if the interbank market is frictionless, money is never created by banks but they lend the existing funds to firms. If there are frictions on the interbank market, an illiquid bank can obtain a loan from the CB, but it will pay it back as soon as it finds a cheaper source of liquidity, for instance substituting CB's funds with interbank loans.

liquidity R. Liabilities could include banks' own funds and external liabilities, such as interbank borrowing I^b towards creditors, indexed by z, deposits D^B and advances A from the CB.

Bank h's net worth is given by:

$$nw^B_{h,t} = R_{h,t} + \sum_{j=1}^{J} L_{hj,t-k_j} + \sum_{q=1}^{Q} I^l_{hq,t-1} - D^B_{h,t} - A_{h,t} - \sum_{z=1}^{Z} I^b_{zh,t-1} \qquad (20)$$

where k_j is the maturity of outstanding loans held by firm j.

At the beginning of each period banks face credit requests from firms and try to serve these in full, while respecting the regulatory constraints. A bank h can supply up to the maximum amount allowed by the regulator net of the outstanding stock of loans.[12]

$$L^s_{h,t} = \lambda nw^B_{h,t} - \sum_{j=1}^{J} L_{hj,t-k_j} - \sum_{q=1}^{Q} I^l_{hq,t-1} \qquad (21)$$

The credit market matches firms with a predetermined number of banks (see Sect. 2.1.2). A generic firm chooses to take the loan out from that bank in its own subset that offers the lowest interest rate. Each bank charges an interest rate, taking into account their counter-party risk and their own cost of funds, leading to heterogeneous interest rates. Furthermore, banks prefer to extend loans to borrowers in order of increasing default probabilities, so that risky firms are more likely to be rationed in case the supply is insufficient.

Eq. (22) describes the default risk $\rho^f_{t,hj}$ perceived by bank h for firm j:

$$\rho^f_{t,hj} = 1 - e^{-v(\ell_j ES_{h,t})} \qquad (22)$$

The default probability is an increasing function of borrower $j's$ leverage rate ℓ_j, corrected for the financial vulnerability perceived by bank h at time t, in terms of its own expected shortfall, $ES_{h,t}$. The latter accounts for the fact that an increase in bank h's vulnerability due to greater expected losses induces it to place greater weight on the risk created by a given default. As will be clearer in the next paragraph, this mechanism amplifies negative sentiments regarding the economy by raising interest rates when non performing loans increase, therefore deteriorating credit conditions and lowering the net worth of firms, increasing the likelihood of further defaults. Although constructed differently, this mechanism resembles in a broad sense the *network-based financial accelerator* as a mechanism to amplify shocks *(Delli Gatti et al., 2010)*.

[12] The prudential constraint states that the stock of loans to net worth ratio cannot exceed a parameter λ:

$$\frac{L_t}{nw^B_t} \leq \lambda$$

Define the loan supply as the change in the stock of loans

$$L_t = L_{t-1} + L^s_t$$

then by substitution: $L^s_t = \lambda nw^B_t - L_{t-1}$.

The interest rate at which bank h offers to lend funds to firm j is denoted by $r^f_{t,hj}$. It is a function of the cost of funds and j's specific probability of default.

$$r^f_{t,hj} = \frac{1 + cf_{h,t}}{1 - \rho^f_{j,t}} - 1 \tag{23}$$

where $cf_{h,t}$ is bank $h's$ cost of funds, given by

$$cf_{h,t} = \omega^D_{h,t} r^D + \omega^A_{h,t} r^H + \omega^I_{h,t} r^b_{t-k,h} \tag{24}$$

The cost of funds depends on the composition of the bank's liabilities, with $\omega^i_{h,t}$ representing the share of each source of liquidity (deposits, advances, interbank borrowing) over liabilities.

$$\omega^i_{h,t} = \frac{i_{h,t}}{D^B_{h,t} + A_{h,t} + I^b_{h,t}} \quad i = D^B, A, I^b \tag{25}$$

Gross profits $\Pi^B_{h,t}$ are

$$\Pi^B_{h,t} = R_{h,t-1} r^L + \sum_{j=1}^{J} L^F_{hj,t-k_j} r^f_{hj,t-k_j} + \sum_{q=1}^{Q} I^l_{hq,t-1} r^b_{hq,t-1} - A_{h,t-1} r^H - D_{h,t-1} r^D \tag{26}$$

If positive, these are subject to taxes at the rate θ, then the fixed share δ^b is distributed to shareholders.

The loss for a bank in case of a default by firm j at time t is equal to the stock of loans outstanding to the firm, minus the debt that can be serviced in case of default. In other words it is the difference between the stock of loans and the firm's own deposits, i.e. the (negative) net worth of the defaulted firm.

$$loss^F_{t,hj} = L^F_{j,t-k_j} - D^F_{j,t} = -nw^F_{j,t};$$

with $nw^F_{j,t} \leq 0$.

Things are slightly more complex in case of an interbank default, indeed a bank has more than one creditor, including depositors, the CB and other banks. Moreover it is assumed that the asset portfolios of defaulting banks remain intact, as they are recapitalised after default. In other words, there are no fire sales of assets. Each creditor of the defaulting bank claims its share of residual assets in proportion to its claims on the defaulter's aggregate liabilities $\mathcal{L}_{t,q}$, as will be detailed in sect. 2.7. A creditor bank h then loses a part of its loan to the failed bank q according to:

$$loss^B_{t,hq} = \frac{I^l_{hq}}{\mathcal{L}_{t,q}} nw^B_{t,q}$$

It is worth noting that contagion can arise. If a borrower defaults, the creditor bank may become insolvent and go into bankruptcy as well, triggering a series of bankruptcies or losses on the interbank and credit markets.

The net worth of bank h updates with the retained profits minus the losses:

$$\Delta nw^B_{h,t} = (1-\theta)(1-\delta^b)\Pi^B_{h,t} - \sum_{j=1}^{J} loss^F_{t,hj} - \sum_{q=1}^{Q} loss^B_{t,hq} \tag{27}$$

Finally, in order to understand the change in liquidity, the accounting equation describing the law of motion of R is reported below. Here $\Delta L_{h,t}$ indicates the change between $t-1$ and t of loans in both credit and interbank markets.

$$\Delta R_{h,t} = \Delta D^B_{h,t} + \Delta A_{h,t} - \Delta L_{h,t} + (1-\theta)(1-\delta)\Pi^B_{h,t} - \sum_{j=1}^{J} loss^F_{t,hj} - \sum_{q=1}^{Q} loss^B_{t,hq} \tag{28}$$

2.6.1 Minimum capital requirement and financial leverage

We adopt a simplified version of Basel III regulatory constraints. In detail we suppose that banks must comply with a minimum capital requirement (solvency ratio) and a financial leverage ratio.

The minimum capital requirement (MCR) implies that the ratio of net worth:weighted assets must be greater than a parameter ϕ, where the weight is given by the *expected shortfall* (ES) computed by each bank depending upon past losses in line with the Basel framework. The ES is computed as the average of the value of losses on the overall portfolio of the bank, exceeding the historical VAR over the last n periods at 97.5% confidence level. In other words the ES represents the expected percentage loss on the portfolio in worst case scenarios (occurring with probability 2.5% or less), over the last n observed periods.[13] Thus

$$nw^B_{h,t} \geq \phi_t ES_{h,t} \left(\sum_{j=1}^{J} L^F_{hj,t-k_j} + \sum_{z=1}^{Z} I^l_{t-1,hz} \right)$$

If the net capital of a bank h is lower than the potential losses on the loan portfolio, h is in violation of its MCR. This means that its portfolio is too risky, or its financial leverage rate is too high. As a response, h attempts to move to a safer position by de-leveraging until its capital complies with the

[13] The losses over loans ratio is preferred to the absolute losses approach, as the latter would be a wrong signal if the relative size of the actual portfolio respect to the one used in the computation of ES changes over time. If this is the case, the bank should deleverage even when its stock of loans is lower than ES.

prudential rules. This is done by reducing its credit supply and by not renewing the outstanding loans.

The condition above can be rewritten in form of a leverage rate:

$$\frac{\sum_{j=1}^{J} L^F_{hj,t-k} + \sum_{z=1}^{Z} I^l_{t-1,hz}}{nw^B_{h,t}} \leq \frac{1}{\phi_t ES_{h,t}}$$

In parallel with minimum capital requirements, banks must comply with a maximum financial leverage, by which they cannot exceed a threshold λ, set by the regulator uniformly across all banks.

$$\frac{\sum_{j=1}^{J} L^F_{hj,t-k} + \sum_{z=1}^{Z} I^l_{t-1,hz}}{nw^B_{h,t}} \leq \lambda$$

It follows that a bank may increase its leverage up to a maximum threshold λ^{max}, depending on which constraint is stricter:

$$\lambda^{max}_{h,t} = \min\left(\frac{1}{\phi_t ES_{h,t}}, \lambda\right)$$

So long as the statutory constraint is not binding, the maximum leverage ratio will change in response to ES, meaning that when expected losses are high, λ^{max} will be low and so on.

The loan supply in eq. (21) can be rewritten with λ^{max} in place of λ

$$L^s_{h,t} = \lambda^{max}_{h,t} nw^B_{h,t} - \sum_{j=1}^{J} L_{hj,t-k_j} - \sum_{q=1}^{Q} I^l_{hq,t-1} \tag{29}$$

2.6.2 The interbank market

The interbank market is the place where banks exchange liquidity, mutually protecting themselves against the risk of shortages. At the opening of the market banks exchange funds in order to have a buffer of liquidity large enough to face outflows during the period, that originate from changes in the balance sheets of other agents, for instance defaults of firms, withdrawals etc.[14]

Fig. 2 shows a stylized time-line of the interbank market. During each period there are three different sessions of the interbank market, each one takes place in conjunction with the main changes in banks' deposits occurring in the model. At each interbank session banks try to anticipate how much liquidity they need to avoid shortages until the closing of the market, thus they form a *liquidity target* that is determined on the basis of each bank's specific characteristics. After a session it might be the case that a bank has not enough liquidity to reach its target. In such circumstances it asks an advance at the CB (see Eq.

[14]In our approach banks form an internal liquidity coverage ratio based on their individual characteristics. As an alternative banks might enter the interbank market to comply with a prudential liquidity coverage ratio, as under Basel III *(see Popoyan et al., 2017)*.

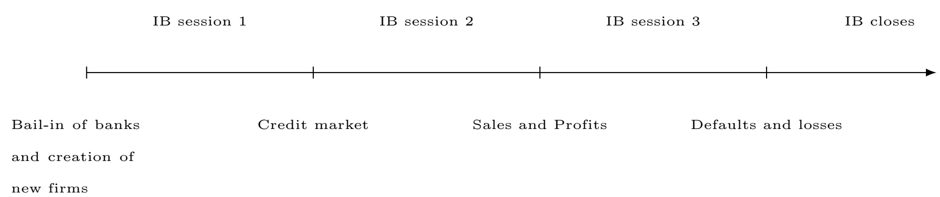


Figure 2: Timeline of the interbank market (IB).

7), which exposes it to the highest interest rate costs of all the possible sources of funds.

At the end the market closes and banks settle their positions.

To assess the needed liquidity, a bank h computes a liquidity ratio (LR), defined as

$$LR_{h,t} = \frac{R_{h,t} - rrD^B_{h,t}}{liq^{tag}_{h,t}} \geq 1$$

The numerator is the liquidity held at the CB net of the compulsory reserve ratio on deposits. In the denominator liq^{tag} represents the liquidity buffer, that is the difference between expected cash outflows out^E and inflows in^E of a bank during a single period. LR must be greater or equal than one.

The liquidity buffer of bank h is defined as

$$liq^{tag}_{h,t} = out^E_{t+1,h} - in^E_{t+1,h} \tag{30}$$

The expected cash outflows consist of the payment of interest rates on deposits plus their run-off rate, that is assumed to be the standard deviation to deposits ratio over the last 50 periods, plus the advances borrowed from the CB and the relative interest rate[15].

$$out^E_{h,t+1} = (r^D + ror_{h,t})D^B_{h,t} + (1 + r^H)A_{h,t}$$

The expected cash inflows are given by the sum of interest payments on loans by the subset of firms j, plus the principal of loans that will be paid back at the end of t by borrowers V weighted by their default probabilities, plus the interest paid by the CB on reserves. It is worth noticing that banks form an expectation about their liquidity need, based on the state of the economy, which is reflected by the default probabilities of borrowers. When losses are large, their desired liquidity is larger than during periods of stability.

$$in^E_{h,t+1} = \sum_{j \in J} L_{hj,t} r^f_{hj,t-k_j} + \sum_{v \in V} (1 - \rho^f_{hv,t}) L_{hv,t} + r^L R_{h,t}$$

[15] Given the sequence of events in the interbank market, a bank that borrowed from the CB prefers to pay back the advance and to resort to interbank liquidity, rather than to roll over the loan at unfavourable interest rate

At each session of the interbank market, if the liquidity is below the buffer, bank h enters the interbank market as a borrower; otherwise it enters as a lender. Interbank demand and supply are described in Eq.s (31)-(32).[16]

$$I^d_{h,t} = liq^{tag}_{h,t} - (R_{h,t} - rrD^B_{h,t}) \tag{31}$$

$$I^s_{h,t} = \min\left[R_{h,t} - rrD^B_{h,t} - liq^{tag}_{h,t},\ \lambda^{max}_{h,t} nw^B_{h,t} - \left(\sum_{j\in J} L^F_{hj,t-k} \sum_{z\in Z} I^l_{hz,t-k}\right)\right] \tag{32}$$

The interbank rate r^b is the minimum rate at which h is willing to lend interbank funds, if not met it keeps them at the CB, where they are remunerated at the set rate r^L; r^b is adjusted for the default probability of the counterparty, ρ^b. For an hypothetical borrower z it is:

$$r^b_{hz,t} = \frac{1 + r^L}{1 - \rho^b_{hz,t}} - 1 \tag{33}$$

The default probability computed by h for z is a function of the observed financial leverage of z. Moreover, as in eq. (22), it takes into account the vulnerability perceived by h via its own expected shortfall.

$$\rho^b_{hz,t} = 1 - e^{-s(lev^{obs}_{z,t} ES_{h,t})} \tag{34}$$

As discussed in sect. 2.1.2, riskier borrowers are more likely to be rationed in case liquidity is scarce. It is also worth stressing that the model allows for multiple lending, thus a bank can borrow from several lenders until its desired borrowing is satisfied, but a bank cannot be a borrower and lender at the same time.

2.7 Bankruptcies and new entrants

If the net worth turns negative firms or banks go bankrupt. Their losses are absorbed by the balance sheets of their creditors, that could fail as well. Each defaulted agent is then recapitalized and enters the system again. Households are shareholders of firms and banks, so they participate to profits receiving dividends. For the sake of simplicity, each firm or bank is assumed to be owned by an equal number of households, which coincide with depositors for the bank.

[16] I^s and I^d represent total loan supply and demand on the interbank market, which do not equal actual borrowing or lending due to the failure of market clearing.

2.7.1 Firms

When a firm defaults on loans, banks may not lose the entire amount of the loan because they can seize the defaulting firm's deposits. This results in a loss equal to the net worth of the failed firm. The credit market permits for multiple lenders, hence if a bankrupted firm has more than one creditor its default affects all outstanding loans, with each creditor suffering a loss proportional to the size of its loan with respect to the net worth of the borrower. After the default, firms leave the market and are replaced in the next period by new start-ups, which are initialized without liabilities and with positive deposits obtained from a random share of shareholders' wealth.

2.7.2 Banks

A bankrupt bank may also have multiple credit relationships, indeed the liabilities side of banks' balance sheet includes deposits from firms and households, interbank funds and advances from the CB. In case of default each creditor suffer a loss proportional to its credit with respect to the net worth of the bank. There is just one creditor that is always guaranteed, namely the CB. This assumption responds to the fact that in the real world advances or open market operations are fully collateralized, but since the model does not include collateral, it is assumed that the CB cannot make losses.

A bank in default does not leave the market, but it cannot operate in the financial markets until it is recapitalized with a *bail-in*. For conciseness we treat all banks in the same way, without considering the *too-big-to-fail* or *too-interconnected-to-fail issues*. A *bail-in* consists in the conversion of liabilities ($\mathcal{L}$) in assets ($\mathcal{A}$), in other words creditors turn a part of their deposits into bank capital, such that:[17]

$$\mathcal{A} \geq \frac{1}{1-0.03}\mathcal{L}$$

We assume that the minimum time needed to complete the bankruptcy procedure and recapitalization cannot be lower than t^{recap} periods. In any case the bail-in is successful only if creditors have enough capital to reach the required asset/liabilities target, otherwise the bank remains in default until creditors can afford such operation.

Finally is worth noting that the default of a bank may trigger the defaults of its creditors, namely firms and banks. Households respond only with their deposits, so that they end up without net worth in the worst case, while if a firm loses a part of its deposits, it may not be able to repay loans and go bankrupt. A similar reasoning applies for banks, whose balance sheet includes interbank loans.

[17] The assets-liabilities ratio is set above the the minimum leverage rate defined by Basel III since $\frac{capital(tierI)}{\mathcal{A}} \geq 3\%$, then $\mathcal{A} \geq \frac{1}{1-3\%}\mathcal{L}$.

3 Results

The dynamics of the model are explored through numerical simulations. Baseline parameters are listed in Appendix 6A. For each parametrisation of the economy, Monte Carlo runs were set at 40. Subsection 3.1 discusses the macrodynamics of the model and investigates whether it can reproduce known empirical regularities.

3.1 Baseline dynamics

The complex dynamics generated by the model do not admit closed form solutions; rather they are analysed by means of numerical simulation. The following set of figures demonstrate the baseline dynamics of the model. The displayed time series in this subsection represent a single Monte Carlo run but they are representative of the dynamics generated over the entire set of runs (see Sect. 3.3).

3.1.1 Business cycles

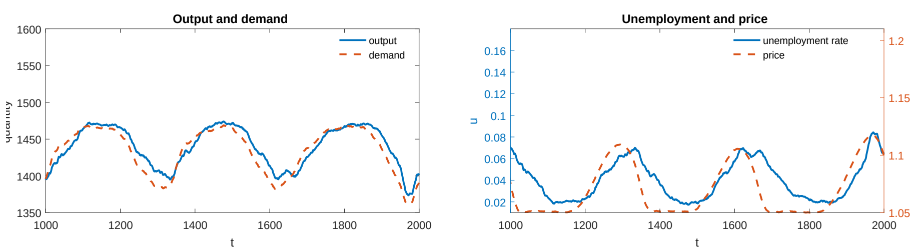


Figure 3: *Left:* aggregate output and aggregate demand. *Right:* unemployment rate and price index. The displayed series are 30-periods moving averages to improve legibility.

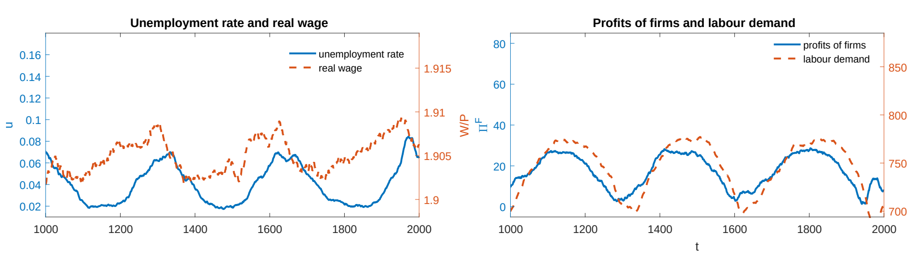


Figure 4: *Left:* Unemployment rate and real wage. *Right:* profits in the firms sector and labour demand. The displayed series are 30-periods moving averages to improve legibility.

The model exhibits endogenous business cycles. Firms compete to sell on the goods market. They hire and pay wages to households, who in turn visit sellers in order to buy goods at the most favourable price. As long as each firm is able to sell all its output, it revises upwards its production targets. Thus in good times, unemployment is low and falling. This induces pressure on nominal wages, as set by unions, resulting in rising costs of production. In comparing the top right panel with the bottom left one, it can be observed that real wages rise even when the price level is fairly constant (compare movements in real wages with those in prices at times 1050, 1400 and 1750). Indeed rising nominal and real wages induce cost-push inflation (via the mark-up over unit costs pricing strategy of firms) which in turn leads to declining demand, falling profits and increasing losses due to the combined effects of rising wage costs and unsold output, and eventually rising unemployment and recession. Note that because inflation is cost-push rather than demand-pull, we do not observe the negative synchronicity between inflation and unemployment.[18]

An increase in unemployment reduces aggregate demand even further, prolonging the recession over several periods. Note that labour incomes account for about 50% of the aggregate consumption budget of households. Moreover new entrants into the business sector are limited in production by their size, while the surviving firms cannot immediately absorb the unemployed workers, due to the matching friction discussed in Sect. 2.3. During the recessionary phase, firms losses build up as do defaults. From Fig. 5 it can be seen how close the patterns of movement in unemployment and firm defaults are.

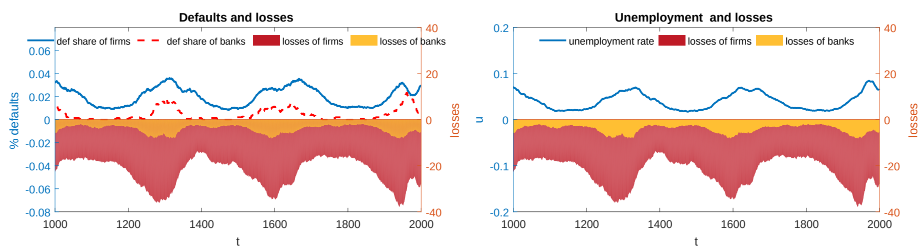


Figure 5: *Left:* share of defaults and losses of firms and banks. *Right:* unemployment rate and losses of firms and banks. The displayed series are 30-periods moving averages to improve legibility.

Turning to banks, the dynamic pattern appears to coincide with those of firms. A peak in firm defaults is associated with a peak in bank defaults. Fig. 5 offers a description of what happens in both sectors. It is worth noting that a *self-reinforcing process* exists: defaults in the business sector weaken the balance sheets of banks, triggering defaults of the most fragile ones. In turn these bankruptcies affect the banks' creditors through deposits and interbank loans, and may further weaken the net worth of the involved agents.

Recovery begins as the firm sector produces less and charges lower prices, both because mark-ups have shrunk during the recession and nominal wages

[18] A further indication that inflation is cost-push is that rising prices precede by a brief interval of time declining demand (note the movements of the two series at approximately time 1200, 1500 and 1830 respectively). If inflation was demand-pull declines in demand would precede declining prices.

have reduced due to lower inflation and high rates of unemployment. As prices fall and unemployment decreases, aggregate demand picks up again. Under these conditions firms can make positive profits. As profits increase, labour demand moves upward (Fig. 4).

3.1.2 The financial side

We described above the dynamics of the real sector. We now consider the dynamics of the financial sector and how these interact with those of firms.

The financial accelerator is evident in our model, with variations in interest rates and the availability of funds on the credit market being the main transmission mechanisms. Fig. 6 shows how the average interest rate on credit markets evolves along with unemployment over the course of a typical business cycle.

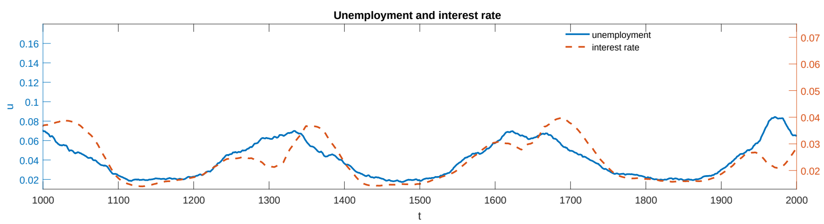


Figure 6: Unemployment rate and interest rate to firms. The displayed series are 30-periods moving averages to improve legibility.

The pattern is counter-cyclical (note that unemployment is perfectly counter-cyclical in our model so a positive co-movement with unemployment means a counter-cyclical variable). As the business cycle turns downwards, lenders increase the interest rate charged to firms. This is both because of the increase in counter-party risk and because they in turn face increasing fragility and higher cost of funds.

To see the above mechanisms in greater detail, consider Fig. 7. The left panel shows the co-movement between the interest rate charged to firms, the interest rate on interbank loans and the expected shortfall of banks. Each of these goes up when the macroeconomy heads towards recession. The right panel shows that the maximum leverage banks allow themselves varies pro-cyclically, increasing in good times (when expected shortfall ES goes down) and decreasing in bad ones (when ES goes up).

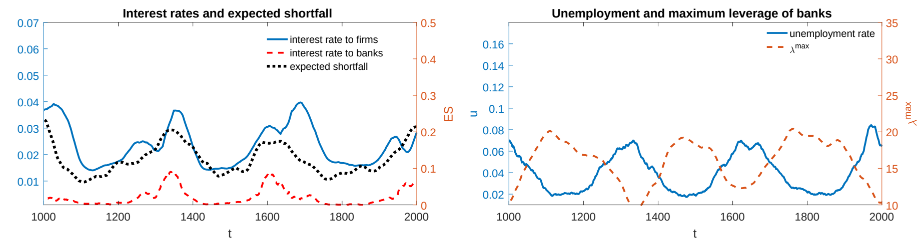


Figure 7: *Left:* interest rates and expected shortfall. *Right:* unemployment rate and maximum allowed leverage of banks. The displayed series are 30-periods moving averages to improve legibility.

In Fig. 8 we display the behaviour of credit supply to firms and credit demand from firms in relation to the business cycle. The latter is counter-cyclical with a lag while the former is pro-cyclical. The pro-cyclicality of credit supply is based on their leverage strategy and almost mirrors the behaviour of maximum leverage. The behaviour of credit demand follows from the financing strategy of firms, which reflects the prediction of *pecking-order* theory that firms use internal funds as a first recourse to meeting their operating costs and borrow only to fill the gap. We have modelled this by calculating the liquidity need of firms as equal to their planned wage bill minus their available net worth (minus a precautionary liquidity buffer equal to 20% of net worth). During a downturn the planned wage bill goes down but so does the net worth. In principle the credit demand goes either way but the simulations resulted in lagged counter-cyclicality.

Figure 8: *Left:* credit demand by firms and unemployment. *Right:* credit supply to firms and unemployment. The displayed series are 30-periods moving averages to improve legibility.

Comparing the two series, it can be seen that aggregate credit supply exceeds aggregate credit demand throughout the business cycle. A couple of points need further explanation at this stage. First, because of market segmentation, aggregate supply exceeding aggregate demand does not mean that each individual firm receives its entire demand as some firms can still be rationed within their local network. We therefore plot actual credit exchanged (as a proportion of output) in relation to the business cycle in Fig. 9. This verifies that the credit cycle is mainly demand-driven as it follows the same pattern as credit demand.

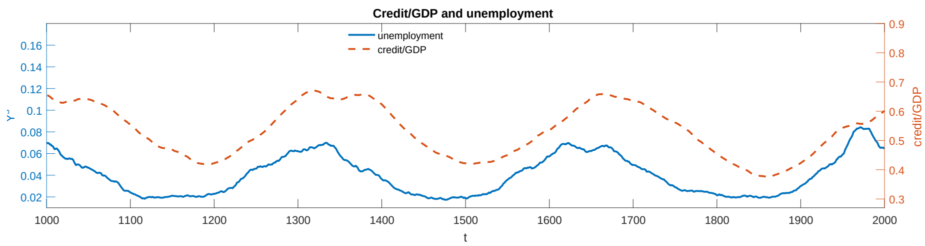


Figure 9: Credit to GDP ratio and unemployment. The displayed series are 30-periods moving averages to improve legibility.

Second, the credit market is counter-cyclical and lags the business cycle.[19] This is a result of the mechanism underlying credit demand, which in turn determines overall lending by the short-side rule: firms' borrow more as a recession unfolds, even though the market, or average, interest rate goes up. This apparent relationship between credit demand and interest rates arises not because of any direct effect of the market interest rate; in fact individual borrowers can only observe the interest rates offered to them by banks within their own networks and are unable to react to average rates. Rather it arises because credit demand is calculated as a residual that bears a negative relationship to firms' net worth which is a pro-cyclical variable.[20]

3.1.3 Interbank lending

The model also reproduces declines in interbank lending volumes and increased interest rates during periods of financial turmoil.

Interbank interest rates (Eq. 33) depends upon the counter-party risk of default, represented by banks' financial leverage, and the vulnerability perceived by each bank given its expected shortfall. In general r^b follows a similar pattern as ES, as already seen in Fig. 7, but its local dynamics is also determined by the default probability of borrowers, computed as a function of financial leverage. In accordance with stylised facts, during a downturn the interbank rate reaches a peak, as displayed in Fig. 10. At the same time there is a reduction in traded volumes, mainly led by the supply side. Recall that banks are subject

[19] This contrasts with *Jordà et al. (2011)*, *i.e.* who report excess credit leading the amplitude and the severity of the subsequent recession.

[20] Of course firms are interest-rate sensitive in that they borrow from the cheapest available lender. In a previous version of this paper, we have used a different approach where firms first choose their financial structure via a leverage target and then base their labour demand on the success at obtaining loans towards that target. That model displayed a different relationship between credit demand and the market interest rate and thus replicated the stylised fact of credit market booms leading the business cycle. However in acknowledgement of a referee's comment that firm behaviour is not conventionally driven by leverage targets we have changed the manner in which firms base their output and financing decisions.

to regulatory constraints on their loanable funds, so they lend in proportion to their net worth (see Eq. 21). As a result, there are disruptions in interbank activity along with credit rationing, which in turn exacerbates reductions in loan supply to firms and might give rise to rationing on the credit market.

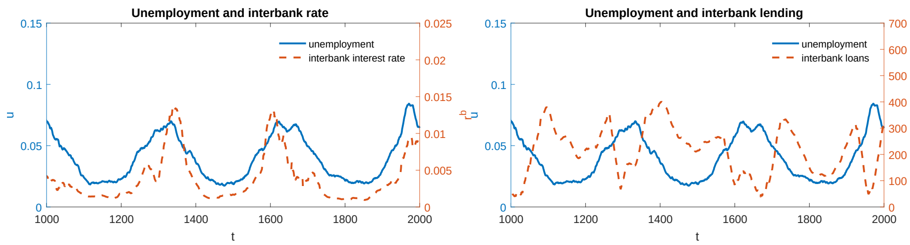


Figure 10: During a crisis the interbank rate increases, while the traded volume diminishes. The displayed series are 30-periods moving averages to improve legibility.

However emergence of interbank rationing is more complex than the simple supply-demand mismatch. Of course there is rationing when the supply is less than demand, but there are other channels as well. In particular the incomplete nature of the interbank network might prevent potential borrowers from connecting with lenders, which results in interbank rationing even if supply is greater than demand at the aggregate level. Moreover banks choose the order of borrowers on the basis of the counter-party's perceived default probability, so that the riskier ones are placed the last in queue even though they might be in most immediate need of funds. The interaction of the mechanisms above leads to rationing in the interbank market, which becomes more pronounced when supply declines, as it is clear from Fig. 11. The area in red represents the sum of the differences between total interbank demand and the realised borrowing of the demanding banks at each t, which is a measure of interbank rationing. As can be seen from the diagram, (i) interbank supply goes down while interbank demand goes up during recessionary periods; (ii) rationing can occur even when interbank supply exceeds interbank demand (see the right panel of Fig. 11) but (ii) rationing increases as interbank supply decreases during recessions.

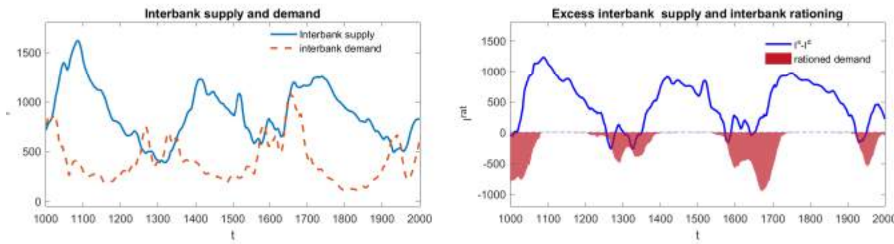


Figure 11: *Left:* interbank supply versus interbank demand. *Right:* interbank supply net of demand and rationing. The displayed series are 30-periods moving averages to improve legibility.

3.2 The role of the interbank market.

The interbank market produces spill over effects on the real sector as a consequence of rationing of interbank funds. The beginning of the process can be attributed to an increase of expected shortfall due to losses on the credit market as firms default due to adverse outcomes in the goods market. This effect is related to the endogenous business cycle dynamics, as described in Sect. 3.1, in other words the trigger of the process is the real sector.

The increase in expected shortfall leads to a decrease in the maximum leverage that banks can undertake and a rise in illiquid banks facing rationing on the interbank market. Rationed banks must borrow from the central banks at high rates, as a result the cost of funds soars. Along with this, those banks that can access loans on the interbank market are charged higher rates (see Fig.s 12-13).

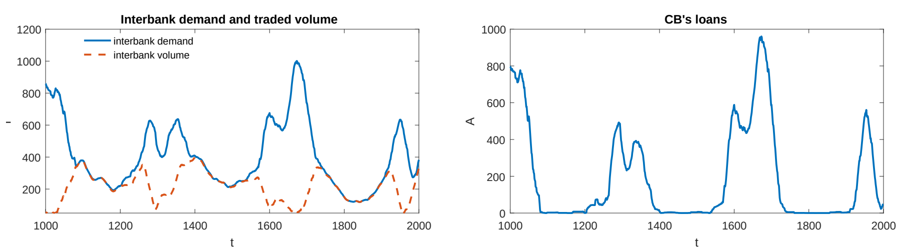


Figure 12: *Left*: aggregate interbank demand and total exchanged volume. *Right*: advances from the central banks. The displayed series are 30-periods moving averages to improve legibility.

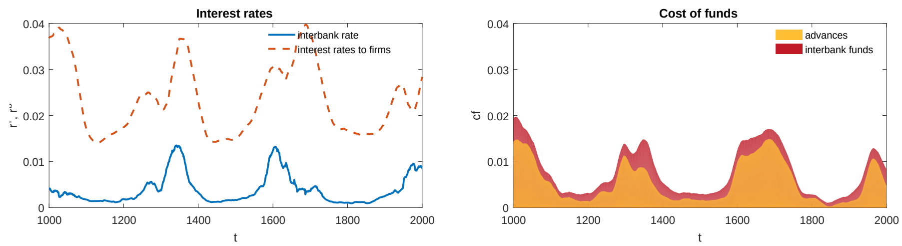


Figure 13: *Left*: time evolution of interbank rate and interest rate to firms. *Right*: time pattern of cost of funds. CB's loans (yellow), interbank funds (red). The displayed series are 30-periods moving averages to improve legibility.

The increased cost of funds spills over to the interest rate charged on new loans to firms, whose balance sheets are in turn weakened, thus generating instability. Moreover the cost of borrowing from the CB reduces profits in the banking sector and causes the default of banks in the lower part of the wealth distribution. Such losses in turn contribute to weaken the balance sheets of creditors, as the distress transfers to the business sector through a loss of the deposits that they held in failing banks.

3.3 Robustness checks: the auto- and cross-correlation structure

In this section we investigate the lagged correlation structure of the economy. We start with the autocorrelation of output with its own lagged values and go on to examine the cross-correlation of other key variables with lagged output. Our aim is to verify that the time series plots presented in section 3.1 are representative of the model's dynamics, despite having been extracted from a single simulation. The cyclical dynamics were tested through spectral analysis of the simulated time series. We employed a *Discrete Fourier Transform* to switch from the time to the frequency domain of the signals. Results were averaged on a sample of 40 independent Monte Carlo repetitions of 2000 periods each.

Fig. 14 shows the amplitude spectrum of output: the x-axis represents the frequency domain (in periods), while the y-axis reports the amplitude of the signal (in terms of output) at each frequency. The dominant frequencies are those at which the signal shows the greatest amplitude and they correspond to the peaks of the business cycle. The cycles occur on average once every 400 periods. The spectral analysis of the main aggregate variables is reported in the last two columns of Tab. 1, which shows that they too are cyclical with a periodicity that is also approximately 400. These results suggest that the model generates a recurring cyclical pattern, irrespective of the initial seed of the pseudo-random number generator.[21]

[21] A few individual variables, namely the net worth of firms and the interbank interest rate,

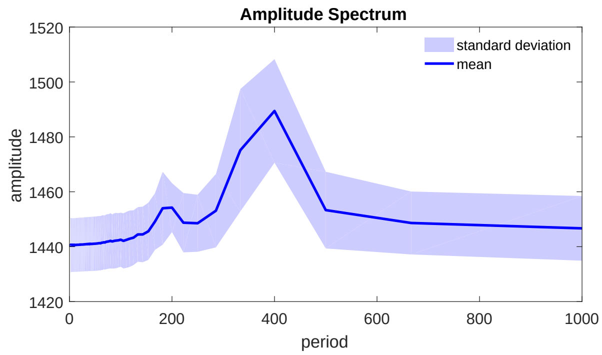


Figure 14: *A*mplitude spectrum of output. The average and standard deviation are computed on a sample of 40 Monte Carlo trials.

Examining Tab. 1 we see that consumption (C) has a maximum correlation of 0.96 with output at t: this suggests that consumption rises and falls in almost perfect synchronicity with output. Both prices (P) and wages (W), on the other hand, have a maximum negative correlation of 0.73 with output at t-25 suggesting that they are counter-cyclical and lead output by 25 periods. In other words the trough in output is reached 25 periods after prices and wages peak. This is in line with the time series behaviour of prices shown in Fig. 3 and the cost-push explanation offered for it in Sect. 3.1. Similarly credit demand L^d and the stock of loans to firms L are counter-cyclical with absolute correlations with output, peaking at 0.44 and 0.73 respectively, somewhere between $t+25$ to $t+50$. This suggests that peaks in these variables arise 25-50 periods after a trough in output. This is in line with the explanation offered in sec. 3.1. Note that the net worth of firms is pro-cyclical with a lag of approximately 25 periods and that credit demanded by firms tends to rise as their net worth falls, as was noted in explaining the behaviour of credit demand in Fig. 8. On the interbank market I^d, I^s and I^l follow the liquidity target, which is largely determined by past losses. When output is low, banks experience high losses: deficit banks increase their liquidity demand while surplus banks reduce their liquidity supply, implying that rationing might occur during a crisis. One interesting result is that while firms' net worth lags output in a pro-cyclical fashion, banks' net worth is also pro-cyclical but appears to *lead* output by 25 periods. Thus downturns in banks' net worth can be predictors of a downturn in economic activity. Finally interest rates on loans to firms (r^f) and banks (r^b) as well as banks' expected shortfall (ES) are counter-cyclical and synchronous with output.

display standard deviations in excess of 100, suggesting that their periodicity is less smooth than is the case of other variables.

Table 1: Cross correlation between the main aggregate variables and lagged values of output. The last two columns refer to the average period of the time-series (from spectral analysis) and to the standard deviation (std) across 40 Monte Carlo trials.

	-200	-150	-100	-50	-25	t	$+25$	$+50$	$+100$	$+150$	$+200$	period	std
Y^s	-0.44	-0.49	-0.16	0.33	0.61	1.00	0.61	0.33	-0.16	-0.49	-0.44	391.67	36.79
Y^d	-0.52	-0.41	0.01	0.52	0.75	0.92	0.53	0.16	-0.37	-0.60	-0.37	397.50	38.03
C	-0.49	-0.45	-0.06	0.46	0.70	0.96	0.56	0.23	-0.29	-0.56	-0.40	391.67	36.79
P	0.45	0.17	-0.25	-0.65	-0.73	-0.64	-0.27	0.15	0.58	0.56	0.17	394.17	40.57
W	0.45	0.16	-0.26	-0.66	-0.73	-0.63	-0.26	0.16	0.59	0.55	0.17	394.17	40.57
U	0.44	0.49	0.16	-0.33	-0.61	-1.00	-0.61	-0.33	0.16	0.49	0.44	391.67	36.79
L^d	0.13	0.37	0.32	-0.01	-0.24	-0.44	-0.44	-0.41	-0.23	0.13	0.34	394.17	40.57
L^s	-0.59	-0.31	0.21	0.64	0.73	0.67	0.37	0.03	-0.54	-0.71	-0.32	401.67	54.93
L	0.21	0.64	0.62	0.05	-0.29	-0.58	-0.73	-0.73	-0.36	0.23	0.58	394.17	40.57
I^d	0.44	0.53	0.26	-0.26	-0.58	-0.64	-0.56	-0.39	-0.01	0.42	0.46	391.31	44.03
I^s	-0.55	-0.28	0.16	0.60	0.71	0.66	0.39	0.02	-0.54	-0.66	-0.26	401.67	54.93
I	-0.41	-0.19	0.04	0.10	0.31	0.34	0.22	0.07	-0.15	-0.27	-0.08	387.14	41.28
nw^H	-0.55	-0.37	-0.02	0.39	0.64	0.62	0.43	0.23	-0.25	-0.54	-0.29	400.00	41.34
nw^F	-0.32	-0.67	-0.55	0.05	0.43	0.72	0.78	0.69	0.23	-0.33	-0.59	409.17	104.05
nw^B	-0.58	-0.14	0.38	0.66	0.68	0.52	0.17	-0.19	-0.65	-0.66	-0.16	401.67	54.93
r^f	0.36	0.34	0.10	-0.20	-0.38	-0.59	-0.47	-0.23	0.22	0.41	0.31	380.64	77.63
r^b	0.36	0.67	0.40	-0.14	-0.46	-0.65	-0.65	-0.56	-0.18	0.36	0.57	414.64	140.92
ES	0.39	0.50	0.23	-0.27	-0.53	-0.70	-0.60	-0.37	0.14	0.52	0.47	394.17	40.57

Legend: Y^s output, Y^d aggregate demand, C consumption, P price level, W wage rate, U unemployment rate, L^d loan demand, L^s loan supply, L loans to firms, I^d interbank demand, I^s interbank supply, I interbank loans, nw^F net worth of firms, nw^B net worth of banks, r^f interest rate to firms, r^b interest rate to banks, ES expected shortfall.

4 Interbank connectivity

In this section we present the results of increasing connectivity within the interbank market on both the financial markets and the real economy. By connectivity we mean the number of potential banks with which a given bank can exchange credit on the interbank market. We simulated the model with different levels of interbank connectivity and check for network effects on the system.[22]

As discussed in Appendix 6.D, the interbank network is obtained by growing a preferential-attachment model with m initial nodes, adding one node with n edges at each iteration. The initial number of nodes represents the core of the network, where every node is connected with the other. As new nodes are added, they tend to form the peripheral part of the structure. In order to check the effects of increased interconnectivity, we generate nine networks by increasing the number of edges for new nodes. The values of m and n are chosen to monotonically increase the density of the network between 0.08 (least connected graph) and 0.34 (most connected graph). Values are reported in the bottom part of Tab. 2.[23] Furthermore we add the connectivity level $d0$, which corresponds to the absence of an interbank market, so that the adjacency matrix of the interbank network is null. Note that in going from d0 to d1, the number of nodes jumps from 0 to 9; thereafter, it increases by adding one node at a time, thus the results comparing outcomes in d0 with outcomes in d1 should be interpreted in this context.

Another point to note is that in running Monte Carlo simulations at each

[22] We conducted 40 Monte Carlo replications for each level of connectivity. Each simulation lasts for 3000 periods (where the first 1000 periods are discarded to eliminate the transient).

[23] The resulting structures define a set of fixed network topologies that constrain the maximum degree of each node. Realised degrees of nodes observed throughout the simulations can be at most equal to such determined potential degrees.

level of connectivity, d1 to d9, the network was held fixed at each level of connectivity (in order to minimise noise across simulations at a given level of connectivity). Because the network is randomly generated at each level, fixing it across simulations makes it more likely that not all network properties will change monotonically with increasing connectivity (see, *e.g* the behaviour of max degree in Tab. 2) so the interpretation of results should also bear this mind.[24]

Network statistics	Connectivity								
	d1	d2	d3	d4	d5	d6	d7	d8	d9
Density	0.08	0.11	0.14	0.17	0.21	0.24	0.27	0.30	0.34
Avg degree	4.12	5.20	7.00	8.32	10.16	11.68	13.16	14.76	16.52
Median degree	2.00	2.50	3.00	5.00	6.00	7.00	8.00	9.00	10.00
Max degree	25.00	24.00	33.00	37.00	43.00	43.00	48.00	43.00	48.00
Avg path length	2.37	2.17	1.91	1.84	1.76	1.73	1.70	1.66	1.63
Initial nodes (m)	9	9	9	9	9	9	9	9	9
New edges (n)	1	2	3	4	5	6	7	8	9

Table 2: Network statistics for the interbank network

4.1 Macroeconomic outcomes and connectivity

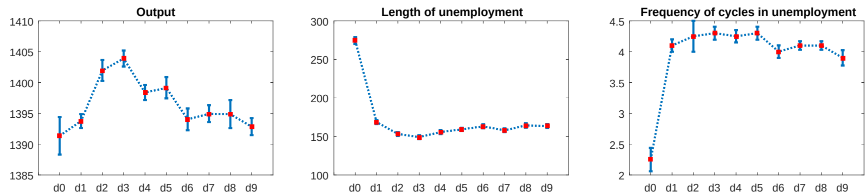
We start by examining the effects of increasing connectivity on the macroeconomy. The top panels of Fig. 15 shows the effect of connectivity on (i) output; (ii) duration of unemployment spells and (iii) frequency of cycles in unemployment across the different simulations at each level of connectivity. The dots are averages and bars represent standard errors.

The behaviour of output (which can be interpreted as a measure of economic efficiency) is surprisingly non-monotonic. At first this is puzzling, given the prior expectation that increasing interbank connectivity should make funds more readily available from surplus banks to firms (recall that direct bank:firm links are never 100%) and thus improve economic efficiency. However as we see from the bottom right plot the number of firm defaults also tends to change non-monotonically with connectivity. It seems that adding an interbank market makes firms default increase, then moderate increases from d1 to d3 help reduce firm defaults slightly (although the errors are fairly large), but increasing connectivity further makes defaults tend to rise again.

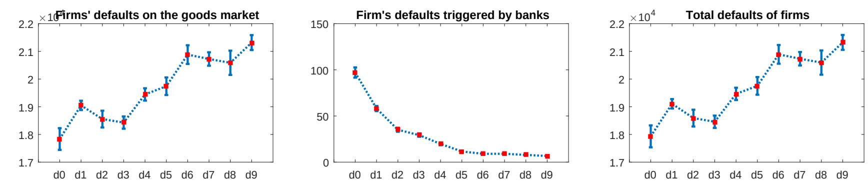
Going deeper, the top right plot shows that introducing the interbank market almost doubles the frequency of business cycles, while it can be seen from the top centre plot that their duration reduces by a similar factor.[25] Further increasing

[24] In any case, a monotonic evolution of network properties can never be guaranteed in this type of random network.

[25] The length of unemployment is computed as the average value per Monte Carlo of at least 10 consecutive periods where the unemployment rate is greater than 0.05. Frequency of cycles in unemployment is the average per Monte Carlo of the dominant frequencies obtained through spectral analysis of moving average series of unemployment rate. We employ the Fast Fourier Transform algorithm to compute the Discrete Fourier Transform which converts the time series from the time domain to the frequency domain. Next the frequency associated with the highest power spectrum is identified as the dominant frequency.



(a)



(b)

Figure 15: The effects of increasing connectivity on average output, cycles and firm defaults

connectivity from d1 to d3 slightly reduces the duration of recessions while going from d3 to d9 tends to increase them. The average frequency appears to move in the opposite direction, although given that there is much more noise in that plot it seems that increasing connectivity from d1 upwards does not have a significant effect on frequency.[26] The overall picture appears to be that adding an interbank market makes the economy more volatile compared to no interbank market, as firms are able to borrow greater amounts. While the latter enhances production the former leads to more business cycles.

As we can see from the middle right panel of Fig. 19, which is introduced a bit further on, increasing interbank connectivity from d1 onwards does not have a significant effect on the overall volume of credit flowing from banks to firms; thus the efficiency-enhancing effect remains relatively stagnant. On the other hand, the changing nature of the economic business cycle affects the rate of firm defaults in a non-monotonic way. From the bottom centre plot we can see that firm defaults caused by failing banks go down monotonically with connectivity; however, defaults arising from losses on the goods market jump up as connectivity increases from d0 to d1, due to the upward jump in frequency of cycles, but then increasing further from d1 to d3, makes the cycles slightly shorter without changing their frequency very much; hence defaults fall over this range. Increasing connectivity further enhances the duration of cycles, again without significant impact on frequency so defaults go up again.

Putting the above observations together, we see that (i) adding an interbank

[26] Comparing time series across connectivity levels it was also hard to tell them apart in terms of frequency of cycles.

market makes the flow of credit increase considerably along with volatility in the goods market as firms become more leveraged and thus more likely to fail; (ii) increasing connectivity further does not increase the flow of funds much more but initially reduces firm failures, most likely because the effect of easier credit availability helps failing firms to avoid default more than it increases the vulnerability of other firms, and (iii) eventually increases firm failures as more and more firms feel themselves over-leveraged relative to their ability to repay. In this respect, increases in interbank connectivity produce an outcome resembling what the growth of what some authors have called *zombie firms* that add little to overall productivity in the economy but are prone to failure due to their inability to repay the debt that they so easily incur *(see Acharya et al., 2017; McGowan et al., 2017)*. The reason output behaves non-monotonically is that each time a firm fails, the firm that replaces it takes time to build up its own capacity and hire workers. In fact, the positive correlation that we see between the behaviour of firm defaults and the length of unemployment spells is because the more firms that fail the longer it takes the labour market to return to full employment.

Fig. 16 shows how competition in the real sector changes with connectivity. It displays the Herfindahl-Hirschman index (HF) which measures concentration.[27] The value of HF varies between approximately 0.00482 and 0.00494 over different connectivities. It rises from 0.00483 to its maximum value as the interbank market is opened, but then it decreases steadily, down to approximately 0.00482, as further connectivity is added. The explanation appears to be that with no interbank market, all firms are equally (un)able to access funds while once a market is opened, inequalities emerge as some firms are better able to access funds via the interbank channel than others. But with further increases in connectivity and more rapid failures of incumbent firms, market shares tend towards relative equality again both because access spreads more widely across incumbent firms and because there are more new, small firms.

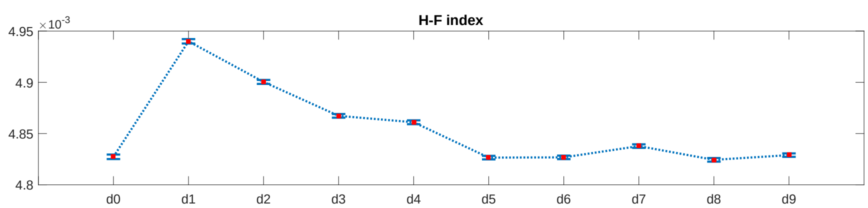


Figure 16: The Herfindahl-Hirschman index and connectivity.

[27] HF is computed as the sum of the square of each firms' market share. With 250 firms, if each firm were to have the same share, the HF would be 0.004. If on the other hand there was a single monopolist it would be 1.

4.2 The banking sector and connectivity

We have seen from the above that while an interbank market enhances efficiency it also has the potential to destabilise. In particular we saw that business cycles can increase in frequency and duration as a result of increasing the interbank market and/or increasing connectivity within an existing market. In this subsection we shall explore further the mechanisms through which interbank connectivity affects financial fragility via its effects on bank balance sheets.

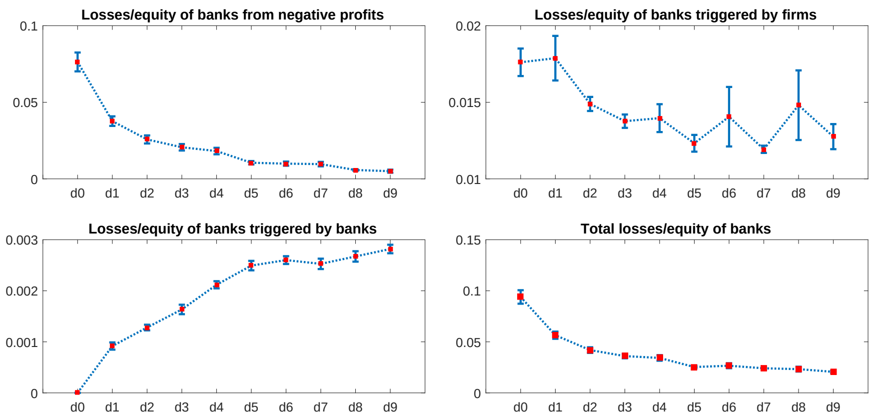
Part (a) of Fig. 17 shows bank losses per unit of equity, which is a more precise measure of fragility than absolute losses. It shows how aggregate losses per equity change with connectivity (bottom right panel), along with the three main sources of such losses: negative profits from having to borrow from the Central Bank (top left), losses arising due to debtor firm defaults (top right), and those from interbank borrower defaults (bottom left). Part (b) shows similar plots for bank defaults. The behaviour of each measure of bank fragility is similar as connectivity increases.

It can be seen that while increasing connectivity makes banks safer on the whole, this is mainly because banks have to resort less frequently to Central Bank advances to meet their own liquidity needs. Thus losses and defaults arising from Central Bank advances decline monotonically while losses arising from interbank defaults rise in the same fashion. Neither is surprising since banks now switch from Central Bank advances to interbank borrowing. Losses arising from borrower firms defaulting tend to behave in an irregular fashion, with apparent turning points at d1 and d3, and there is much more noise in this dimension. This is possibly because the greater access of firms to funds via the interbank market makes them more likely to default. Note that the turning points in the upper right plot of Fig. 17(a) reflect the behaviour of firm defaults in the right panel of Fig. 15(b).

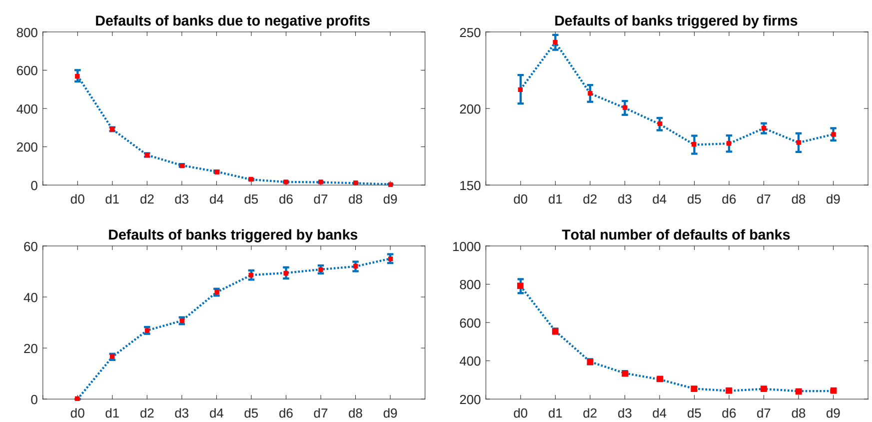
Fig. 18 examines behaviour of extreme losses and defaults. To be precise we look at how the 99^{th} percentile of losses per equity and numbers of banks defaulting at the same time (*i.e.* in a single period) changes as connectivity increases. The purpose is to see whether extreme events behave differently from the average.

As the plot shows, while the right tail tends to show the same qualitative variation as the mean of the distribution, a few differences can be observed. Large losses arising from firm defaults tend to become even more irregular and volatile, thus defaults arising from such losses tend no longer to decline monotonically with connectivity. Similarly the possibility of a large number of banks defaulting simultaneously goes up with connectivity. This adds to the insight that increasing connectivity can increase financial fragility; in this case, at the right tail of the distribution.

We next see how interbank connectivity affects average performance in both the banking and the real sectors. Bank profits tend to increase with connectivity, this is not surprising since banks are able to increase their intermediation activity as the interbank network expands . Firm profits decrease sharply when the interbank market is introduced but then do not change significantly as con-

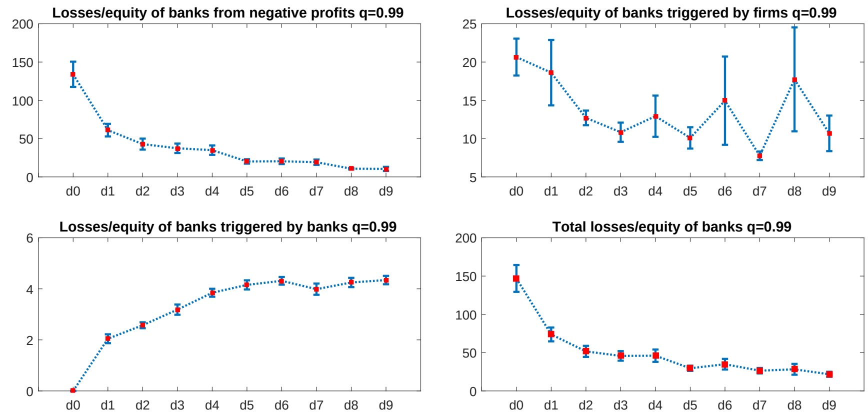


(a)

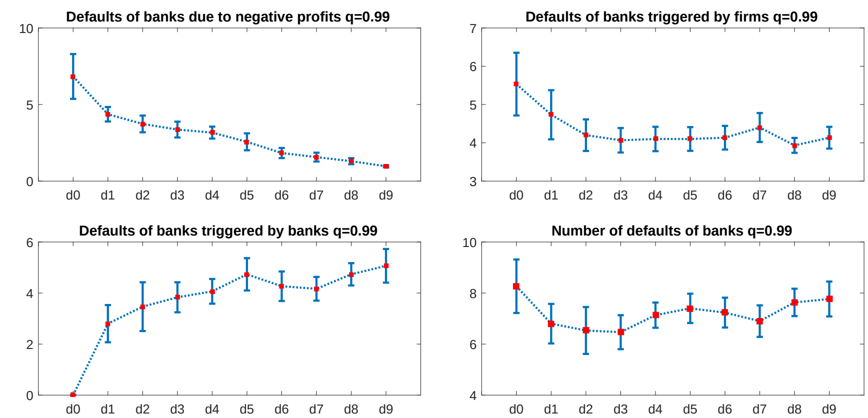


(b)

Figure 17: The behaviour of bank losses and defaults and their sources.



(a)

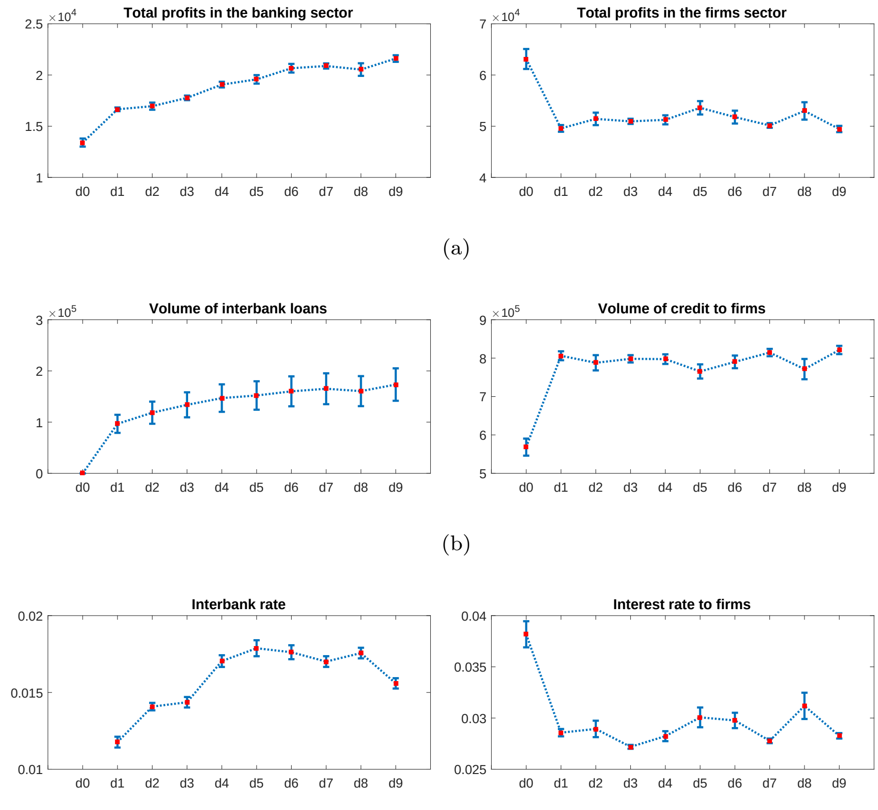


(b)

Figure 18: The sources and behaviour of extreme values of bank losses and defaults per period.

nectivity increases. Recall the conflicting effects of enabling firms greater access to banking sector funding as the interbank network opens: firms become more likely to be able access liquidity in order to meet production targets but there are also more failures. Beyond this, it appears the conflicting forces that shaped firm defaults in Fig. 15 tend to keep average profits stable since, while surviving firms are able to access more credit in order to meet their production targets, more firms fail especially after d3 so averages tend to stay relatively constant.

Fig. 19(b) shows how lending volumes evolve with connectivity. As expected interbank lending volumes go up as banks are better able to avoid using expensive Central Bank advances. Credit to firms increases sharply but then tends to



(c)

Figure 19: The effect of connectivity on bank and firm profits, credit flows and interest rates.

stay roughly the same. But note that since defaults start increasing especially after d3, this means that fewer firms benefit from these funds at least until their replacements come up to speed.

Finally part (c) looks at how interest rates evolve. Interbank rates tend to increase monotonically (after d1, since at d0 the interbank rate is not defined) which reflects the increasing likelihood of default on the interbank market. However, the rate of increase appears to be diminishing as connectivity increases, which indicates that the increasing probability of default is being offset by a countervailing force, namely increasing competition among lenders. Eventually after crossing d5 they tend to go down (with some noise at d8).

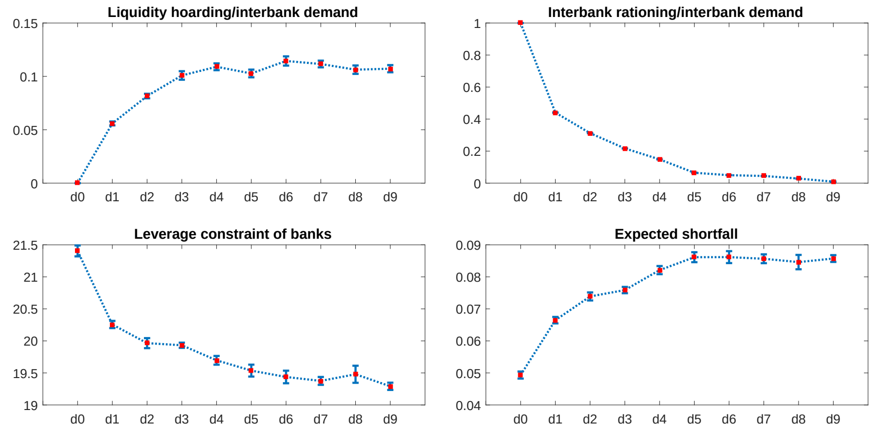
The interest rates charged to firms decrease sharply in going from d0 to d1 but then they follow an irregular and somewhat noisy (across simulations) pattern. The explanation seems related to the behaviour of firm defaults on the one hand (which behave non-monotonically between d1 and d9, first decreasing and then increasing) and the cost of funds within the banking system (which go up from from d1 to d5 and then tend to decrease), the combination of these effects drives the pattern on the right panel of Fig. 19(c).

We last consider Fig. 20. This looks at various measures that determine the overall supply of funds in the interbank market. The first panel looks at liquidity hoarding which increases monotonically with connectivity,[28] despite interbank rationing decreases. This pattern is explained by the behaviour of the variables in the bottom part of the subfigure (a), that is Expected Shortfall (ES) and the leverage constraint of banks (λ^{max}), that determine interbank supply. Banks adopt an internal risk mechanism based on historical losses. Perceived risk in turn is measured by ES, as the average of losses over the loan portfolio exceeding the historical VaR during the last n periods at 97.5% confidence level. The more extreme events affect banks' portfolio, the higher is their expected shortfall. Increasing connectivity enhances the volumes of credit to firms and banks (see Fig. 19), but also the likelihood of large losses. In other words what matters are not losses themselves, but their volatility. Moving to a state with higher volatility makes bank more careful. This is reflected in their expected shortfalls that in turn determine the liquidity target, interbank supply and the hoarded liquidity. The greater volatility of losses could be explained by the larger exposure of the banking sector to firms and banks along with connectivity. When connectivity grows more banks offer competitive interest rates to borrowers, while if connectivity is low banks charge lenders with the cost of CB's funds. The better access to credit allows borrowers to satisfy their demand from fewer banks, that is multiple loans reduce with connectivity (Fig. 20 (b) bottom-left). As a result the credit market is characterized by an higher concentration of borrowers per bank and lower diversification. Furthermore the number of active interbank linkages goes up with connectivity, thus easing interbank contagion (see Fig. 17). In the end growing connectivity leads to increasing levels of liquidity hoarding.

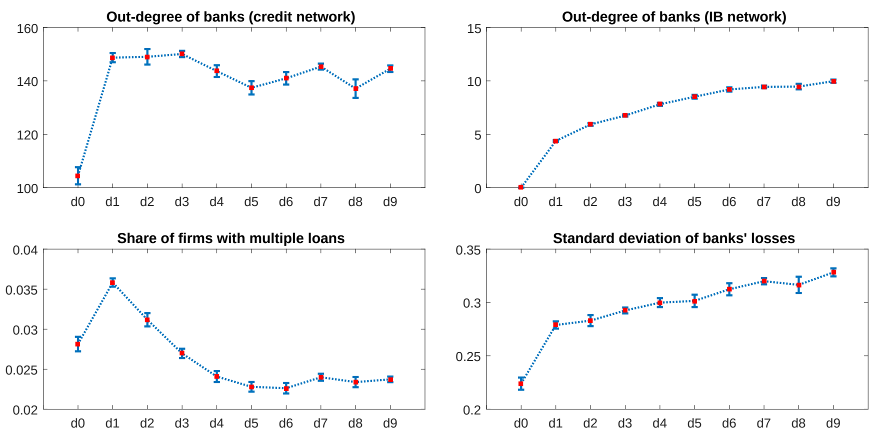
[28] Liquidity hoarding is measured as the sum over the difference between all borrower-lender pairs in the interbank market of the liquidity demanded by the borrower and the loan offered by the lender conditional on the lender holding sufficient reserves to satisy the demand. In mathematical terms it can be written as

$$\sum_{i \in I^B} \sum_{h \in I^L} (I^d_{ih,t} - I^s_{hi,t}) \quad if \ I^s_{hi,t} < I^d_{ih,t} \leq R_{h,t}$$

where i represents a borrower, h a lender, I^B is the set of borrowing banks, I^L is the set of lending banks, I^d_{ih} is the loan demand from i to h, I^s_{hi} is the loan offered by h to i and R_h is the available liquidity of lender h. Liquidity hoarding arises not just because liquidity-surplus banks face leverage constraints but because they hold liquidity buffers in order to satisfy their own estimated liquidity needs.



(a)



(b)

Figure 20: (a) Liquidity hoarding and interbank rationing to interbank demand ratios, leverage constraints and expected shortfall of the banking system.
(b) Total out-degrees of banks in credit and interbank networks, share of firms with multiple loans and standard deviation of banks' total losses.

5 Concluding remarks

We developed an agent-based model that incorporated both real and financial sectors, including an interbank market which complemented the credit market in facilitating the provision of liquidity by banks to the real economy. The set of potential interactions among lenders and borrowers was governed by static network structures in each market. A distinctive feature of the model consisted in the way in which the effects of prudential regulation were accounted for: in particular minimum capital requirements and the maximum allowed financial leverage depended upon the expected shortfall computed by each bank on the basis of the losses experienced in the recent past. Moreover we generated heterogeneity in banks' loan maturities by assuming a lagged structure of maturities of loans to the firm sector.

The model's dynamics were explored through simulations. We first showed that endogenous business cycles can occur, driven by price and wage dynamics that result from the behaviour of firms and unions rather than through any external disturbance. In the boom phase, unemployment is low, demand is high, firms increase output, while prices rise both because unions set higher wages and because firms increase mark-ups. The latter sows the seeds for a future downturn as at some point there is over-production of output and firms begin to experience losses.

Second, we showed that the effects of firm closures in the real sector are amplified by the financial side of the economy via a financial accelerator. In particular, a recession creates spill-over effects in financial markets, such as the reduction of credit availability and higher interest rates. Our analysis shows that these are mainly driven by prudential regulations that lead banks to deleverage as well as to increase their own precautionary liquidity buffers, resulting in both cases in liquidity hoarding.

Third, the financial amplification mechanism is strictly related to the procyclical effects of prudential regulation. In order to comply with capital regulations, banks must hold enough capital to cover potential losses as estimated by applying the expected shortfall measure, which in turn is based on its VaR estimates over a fixed number of past periods. The key here is the possibility of risk misperception at different stages of the business cycle. In good times, observed losses are low so the VaR and expected shortfall measures are low *ex ante*. Capital constraints are therefore generous. As a slump begins, VaR and expected shortfall measures rise *ex post* and the constraints tighten, contributing to the credit crunch which aggravates the slump. While the model is not primarily aimed at policy analysis, these results support the Basel III aim of designing prudential regulation that focuses on limiting credit expansion during booms, in order to reduce the extent and severity of the downturn.

We also found that that increasing connectivity has ambiguous effects on both the efficiency and stability of the real sector. Efficiency, as measured by output levels, increases as a result of greater connectivity but then it decreases. The reason appears to be that business cycles and firm failures can also be increased by greater connectivity, especially after a threshold of connectivity is crossed. As a result of more firm failures there is greater disruption and longer spells of unemployment following a recession. Greater connectivity however seems to increase bank profits while having ambiguous effect of those on the firm sector.

In future research, we plan to improve the performance of the model by introducing dynamic elements into the real economy, via capital accumulation or, as an alternative, endogenous growth in labour productivity (via learning by doing) as in *Delli Gatti et al. (2011)*. In addition trades on the interbank market could include other kinds of assets with different maturities. We would also wish to model firm strategies in a way that allows firms choice of capital structure to be both endogenous and to play a greater role in the business cycle. Finally the behaviour of the Central Bank could be endogenised so that it actively manages the interest rate corridor to ensure macroeconomic stability by means of monetary policy.

References

Acharya, V. V., Eisert, T., Eufinger, C., and Hirsch, C. W. (2017). Whatever it takes: The real effects of unconventional monetary policy. *CEPR Discussion Paper*.

Acharya, V. V. and Merrouche, O. (2010). Precautionary hoarding of liquidity and interbank markets: Evidence from the sub-prime crisis. Technical report, National Bureau of Economic Research.

Allen, F. and Carletti, E. (2008). The role of liquidity in financial crises. *Available at SSRN: or http://dx.doi.org/10.2139/ssrn.1268367*.

Allen, F., Carletti, E., and Gale, D. (2009). Interbank market liquidity and central bank intervention. *Journal of Monetary Economics*, 56(5):639–652.

Allen, T. W. and Carroll, C. D. (2001). Individual learning about consumption. *Macroeconomic Dynamics*, 5(02):255–271.

Ashraf, Q., Gershman, B., and Howitt, P. (2017). Banks, market organization, and macroeconomic performance: An agent-based computational analysis. *Journal of Economic Behavior & Organization*, 135:143–180.

Assenza, T., Delli Gatti, D., and Grazzini, J. (2015). Emergent dynamics of a macroeconomic agent based model with capital and credit. *Journal of Economic Dynamics and Control*, 50:5–28.

Bénassy, J.-P. (2002). *The macroeconomics of imperfect competition and nonclearing markets: a dynamic general equilibrium approach.*

Berardi, S. and Tedeschi, G. (2017). From banks' strategies to financial (in) stability. *International Review of Economics & Finance*, 47:255–272.

Bianconi, G. and Barabási, A.-L. (2001). Competition and multiscaling in evolving networks. *EPL (Europhysics Letters)*, 54(4):436.

Caiani, A., Godin, A., Caverzasi, E., Gallegati, M., Kinsella, S., and Stiglitz, J. E. (2016). Agent based-stock flow consistent macroeconomics: Towards a benchmark model. *Journal of Economic Dynamics and Control*, 69:375–408.

Carroll, C. D. (2001). A theory of the consumption function, with and without liquidity constraints. Technical report, National Bureau of Economic Research.

Chiarella, C., Flaschel, P., Hartmann, F., and Proaño, C. R. (2012). Stock market booms, endogenous credit creation and the implications of broad and narrow banking for macroeconomic stability. *Journal of Economic Behavior & Organization*, 83(3):410–423.

Chiarella, C., Iori, G., et al. (2002). A simulation analysis of the microstructure of double auction markets*. *Quantitative finance*, 2(5):346–353.

Chiarella, C., Iori, G., and Perelló, J. (2009). The impact of heterogeneous trading rules on the limit order book and order flows. *Journal of Economic Dynamics and Control*, 33(3):525–537.

De Masi, G. and Gallegati, M. (2012). Bank–firms topology in italy. *Empirical Economics*, 43(2):851–866.

Delli Gatti, D., Desiderio, S., Gaffeo, E., Cirillo, P., and Gallegati, M. (2011). *Macroeconomics from the Bottom-up*. Milano: Springer Milan.

Delli Gatti, D., Gallegati, M., Greenwald, B., Russo, A., and Stiglitz, J. E. (2010). The financial accelerator in an evolving credit network. *Journal of Economic Dynamics and Control*, 34(9):1627–1650.

Delli Gatti, D., Gallegati, M., Greenwald, B. C., Russo, A., and Stiglitz, J. E. (2009). Business fluctuations and bankruptcy avalanches in an evolving network economy. *Journal of Economic Interaction and Coordination*, 4(2):195–212.

Dosi, G., Fagiolo, G., Napoletano, M., and Roventini, A. (2013). Income distribution, credit and fiscal policies in an agent-based keynesian model. *Journal of Economic Dynamics and Control*, 37(8):1598–1625.

Dosi, G., Fagiolo, G., and Roventini, A. (2010). Schumpeter meeting keynes: A policy-friendly model of endogenous growth and business cycles. *Journal of Economic Dynamics and Control*, 34(9):1748–1767.

Fazzari, S., Ferri, P., and Greenberg, E. (2008). Cash flow, investment, and keynes–minsky cycles. *Journal of Economic Behavior & Organization*, 65(3):555–572.

Gabbi, G., Iori, G., Jafarey, S., and Porter, J. (2015). Financial regulations and bank credit to the real economy. *Journal of Economic Dynamics and Control*, 50:117–143.

Georg, C.-P. (2013). The effect of the interbank network structure on contagion and common shocks. *Journal of Banking & Finance*, 37(7):2216–2228.

Godley, W, L. M. (2007). Monetary economics: an integrated approach to credit, money, income production and wealth.

Grilli, R., Tedeschi, G., and Gallegati, M. (2014). Bank interlinkages and macroeconomic stability. *International Review of Economics & Finance*, 34:72–88.

Heider, F., Hoerova, M., and Holthausen, C. (2009). Liquidity hoarding and interbank market spreads: The role of counterparty risk. *EBC Discussion Paper*, 2009.

Hicks, J. R. et al. (1974). Crisis in keynesian economics.

Iori, G., De Masi, G., Precup, O. V., Gabbi, G., and Caldarelli, G. (2008). A network analysis of the italian overnight money market. *Journal of Economic Dynamics and Control*, 32(1):259–278.

Iori, G., Jafarey, S., and Padilla, F. G. (2006). Systemic risk on the interbank market. *Journal of Economic Behavior & Organization*, 61(4):525–542.

Jordà, Ò., Schularick, M. H., and Taylor, A. M. (2011). When credit bites back: leverage, business cycles, and crises. Technical report, National Bureau of Economic Research.

Layard, P. R. G., Layard, R., Nickell, S. J., and Jackman, R. (2005). *Unemployment: macroeconomic performance and the labour market*. Oxford University Press on Demand.

Lux, T. (2015). Emergence of a core-periphery structure in a simple dynamic model of the interbank market. *Journal of Economic Dynamics and Control*, 52:A11–A23.

McGowan, M. A., Andrews, D., and Millot, V. (2017). The walking dead? zombie firms and productivity performance in oecd countries. *OECD Economics Department Working Papers*, (1372).

McMorrow, K. (1996). *The wage formation process and labour market flexibility in the Community, the US and Japan*. Number 118. Directorate-General for Economic and Financial Affairs, European Commission.

Modigliani, F. and Brumberg, R. (1954). Utility analysis and the consumption function: An interpretation of cross-section data. *Franco Modigliani*, 1.

Montagna, M. and Kok, C. (2013). Multi-layered interbank model for assessing systemic risk. Technical report, Kiel Working Paper.

Popoyan, L., Napoletano, M., and Roventini, A. (2017). Taming macroeconomic instability: Monetary and macro-prudential policy interactions in an agent-based model. *Journal of Economic Behavior & Organization*, 134:117–140.

Postel-Vinay, F. and Turon, H. (2010). On-the-job search, productivity shocks, and the individual earnings process. *International Economic Review*, 51(3):599–629.

Tedeschi, G., Mazloumian, A., Gallegati, M., and Helbing, D. (2012). Bankruptcy cascades in interbank markets. *PloS one*, 7(12):e52749.

van der Hoog, S. and Dawid, H. (2017). Bubbles, crashes, and the financial cycle: The impact of banking regulation on deep recessions. *Macroeconomic Dynamics*, pages 1–42.

6 Appendix

Appendix 6.A Calibration and initial values

The parameters of the model are reported in Tab. 3. It is worth noticing that the value of transfers is obtained from the steady state solution of a backbone model. It ensures the steady state level of full employment when the wage rate W and the mark-up rate μ are at their initial values:

$$G = \frac{W^0 N^{full} \left[1 + \mu - (1-\theta)\left(c_1 + c_2 \frac{1-c_1}{c_2 - r^D}\right)\right]}{\left(c_1 + c_2 \frac{1-c_1}{c_2 - r^D}\right)}$$

Table 3: Calibration of the baseline model

Parameter	Description	Value
T	Length of the simulation	2000
N^F	Number of firms	250
N^H	Number of households	750
N^B	Number of banks	50
α	Labour productivity	2
W^0	Initial wage rate	2
θ	Tax rate	0.4
δ	Dividend share	0.5
c_1	Marginal propensity to consume out of income	0.8
c_2	Marginal propensity to consume out of savings	0.2
r^L	Interest rate on reserves	0
r^D	Interest rate on deposits	0
r^B	Interest rate on bills	0
r^H	Interest rate on advances	0.05
rr	Reserve coefficient	0.03
v_f	Sensitivity of r^f to the default probability	0.12
v_b	Sensitivity of r^b to the default probability	0.69
λ	Maximum leverage rate banks	24
τ	Length of firms' and banks' memory	10
τ^w	Length of unions' memory	120
τ^{ES}	Length of losses memory (ES)	100
σ_1	Sensitivity of the wage rate to unemployment	0.05
σ_2	Sensitivity of the wage rate to hysteresis	0.15
u^*	Full-employment rate of unemployment	0.03
Fh	Share of firms observed on the goods market	0.2
μ	Initial mark-up rate	0.01
rev	Equity/assets ratio of the recapitalized banks	0.03
L^{max}	maximum duration of loans	10
L^{min}	minimum duration of loans	2
t^{recap}	minimum time between bankrupt and recapitalization of banks	5
G	transfers	465

Appendix 6.B Accounting

The equations of the model are divided in the behavioural and accounting ones. The model includes both *stock* and *flow* variables. A stock-flow consistent accounting system can verify consistency among them. It is composed by a *transactions flow matrix* (Tab. 4) and a *balance sheet matrix* (Tab. 5). The former describes the changes in the stock variables between the beginning and the end of any time period while the latter indicates the level of the stock variables at a given time, resulting in the description of the model from an accountancy viewpoint.

6.B.1 Aggregate balance sheet and transactions matrix

Tab. 4 represent the aggregate balance sheet of the economic system. The sum of each row and column is zero and each element for a class of agents balances with the corresponding one. Since it is assumed that (i) there is no physical capital and (ii) inventories are perishable, the firms' accounts sum to zero and the sum of all the net worth is zero, so that the government has a negative net worth.[29]

$$\sum_{i \in N^H} nw_i^H + \sum_{j \in N^F} nw_j^F + \sum_{h \in N^B} nw_h^B + nw^G = 0$$

Tab. 5 represents the aggregate transactions taking place in the system. Each flow should move from a class of agents to another (the intra-class flows are not displayed at the aggregate level) as it is reported on the rows. The aggregate flows occurring within a class of agents is represented on the columns and may be divided in current (CA) and capital accounts (KA). The current account describes the current inflows and outflows due to payments or earnings, while the capital account describes the changes in the balance sheet of the agents, that is the change in assets or liabilities.

[29] In a model with physical capital and/or inventories the sum of the latter plus the sum of the net worth should be zero.

Table 4: Aggregate balance sheet

	HH	FF	BB	CB	Gov	$\sum$
Deposits	$+D^H$	$+D^F$	$-D^B$			0
Loans		$-L^F$	$+L^F$			0
Bills				$+B$	$-B$	0
Reserves			$+R$	$-R$		0
Advances			$-A$	$+A$		0
Net worth	$-nw^H$	$-nw^F$	$-nw^B$		$-nw^G$	0
$\sum$	0	0	0	0	0	0

Variables measured at current prices. Assets(+), liabilities(-). Households (HH), firms (FF), banks (BB), central bank (CB), government (Gov).

Table 5: Aggregate transactions flow matrix

	HH	FF		BB		CB		Gov	$\sum$
		CA	KA	CA	KA	CA	KA		
Consumption	$-C$	$+C$							0
Transfers	$+G$							$-G$	0
Production		Y							0
Wages	$+WN$	$-WN$							0
Taxes	$-T^H$	$-T^F$				$-T^B$		$+T$	0
Profits Firms	$+\delta\Pi^F$	$-\Pi^F$	$+(1-\delta)\Pi^F$						0
Profits Banks	$+\delta\Pi^B$			$-\Pi^B$	$+(1-\delta)\Pi^B$				0
Profits CB						$-\Pi^{CB}$		$+\Pi^{CB}$	0
Deposits interest	$+r^D D^H$	$+r^D D^F$		$-r^D D$					0
Loans interest		$-r^f L^f$		$+r^f L^f$					0
Bills interests						$+r^B B$		$-r^B B$	0
Reserves interests				$+r^R R$		$-r^R R$			0
Advances interests				$-r^H A$		$+r^H A$			0
ΔLoans			$+\Delta L$		$-\Delta L$				0
ΔBills							$-\Delta B$	$+\Delta B$	0
ΔReserves					$-\Delta R$		$+\Delta R$		0
ΔDeposits	$-\Delta D^H$		$-\Delta D^F$		$+\Delta D$				0
Δ Advances					$+\Delta A$		$-\Delta A$		0
$\sum$	0	0	0	0	0	0	0	0	0

Variables measured at current prices. Sources of funds(+), uses of funds(-). Households (HH), firms (FF), banks (BB), central bank (CB), government (Gov).

Appendix 6.C Correlation structure

Tab. 6 reports the average correlation coefficients across 40 Monte Carlo simulations of 2000 periods and different initial seeds of the pseudo-random number generator. Averages are computed setting to zero those coefficients with a p-value greater than 0.05

Table 6: Average cross-correlations between the main aggregate variables. Standard deviations in parenthesis.

	Y^s	Y^d	Y	C	P	W	U	L^d	L^s	L	I^d	I^s	I	A	nw^H	nw^F	nw^B	B	Π^F	Π^B	r^f	r^b	ES
Y^s	1.00	0.91	0.96	0.96	-0.64	-0.64	-1.00	-0.38	0.67	-0.57	-0.58	0.50	0.27	-0.55	0.59	0.71	0.53	0.86	0.48	-0.46	-0.55	-0.62	-0.75
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Y^d	0.91	1.00	0.98	0.98	-0.89	-0.88	-0.91	-0.21	0.84	-0.38	-0.47	0.69	0.29	-0.48	0.78	0.54	0.77	0.82	0.66	-0.27	-0.59	-0.46	-0.76
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
C	0.96	0.98	1.00	1.00	-0.80	-0.79	-0.96	-0.29	0.79	-0.47	-0.54	0.62	0.29	-0.53	0.68	0.62	0.68	0.85	0.65	-0.36	-0.57	-0.54	-0.76
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
P	-0.64	-0.89	-0.80	-0.80	1.00	0.99	0.64	-0.00	-0.84	0.06	0.22	-0.76	-0.21	0.25	-0.75	-0.22	-0.86	-0.58	-0.70	-0.00	0.49	0.16	0.60
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
W	-0.64	-0.88	-0.79	-0.79	0.99	1.00	0.64	-0.00	-0.85	0.00	0.22	-0.77	-0.22	0.26	-0.75	-0.21	-0.87	-0.58	-0.71	-0.00	0.48	0.16	0.60
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
U	-1.00	-0.91	-0.96	-0.96	0.64	0.64	1.00	0.38	-0.67	0.57	0.58	-0.50	-0.27	0.55	-0.59	-0.71	-0.53	-0.86	-0.48	0.46	0.55	0.62	0.75
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
L^d	-0.38	-0.21	-0.29	-0.29	-0.00	-0.00	0.38	1.00	-0.00	0.61	0.53	0.00	0.06	0.38	-0.00	-0.58	0.12	-0.45	-0.18	0.56	0.23	0.58	0.29
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
L^s	0.67	0.84	0.79	0.79	-0.84	-0.85	-0.67	-0.00	1.00	-0.10	-0.38	0.87	0.34	-0.42	0.75	0.28	0.93	0.66	0.73	-0.15	-0.52	-0.40	-0.74
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
L	-0.57	-0.38	-0.47	-0.47	0.06	0.00	0.57	0.61	-0.10	1.00	0.67	0.00	0.00	0.50	-0.12	-0.96	0.14	-0.78	-0.32	0.78	0.35	0.73	0.50
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
I^d	-0.58	-0.47	-0.54	-0.54	0.22	0.22	0.58	0.53	-0.38	0.67	1.00	-0.14	-0.39	0.93	-0.32	-0.72	-0.23	-0.73	-0.50	0.39	0.35	0.79	0.47
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
I^s	0.50	0.69	0.62	0.62	-0.76	-0.77	-0.50	0.00	0.87	0.00	-0.14	1.00	0.27	-0.21	0.63	0.12	0.81	0.47	0.59	-0.00	-0.44	-0.19	-0.70
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
I	0.27	0.29	0.29	0.29	-0.21	-0.22	-0.27	0.06	0.34	0.00	-0.39	0.27	1.00	-0.70	0.36	0.07	0.42	0.25	0.24	0.29	-0.08	-0.18	-0.14
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
A	-0.55	-0.48	-0.53	-0.53	0.25	0.26	0.55	0.38	-0.42	0.50	0.93	-0.21	-0.70	1.00	-0.39	-0.58	-0.34	-0.66	-0.48	0.18	0.30	0.68	0.41
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
nw^H	0.59	0.78	0.68	0.68	-0.75	-0.75	-0.59	-0.00	0.75	-0.12	-0.32	0.63	0.36	-0.39	1.00	0.26	0.76	0.59	0.53	0.00	-0.48	-0.18	-0.57
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
nw^F	0.71	0.54	0.62	0.62	-0.22	-0.21	-0.71	-0.58	0.28	-0.96	-0.72	0.12	0.07	-0.58	0.26	1.00	0.00	0.89	0.44	-0.77	-0.47	-0.79	-0.64
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
nw^B	0.53	0.77	0.68	0.68	-0.86	-0.87	-0.53	0.12	0.93	0.14	-0.23	0.81	0.42	-0.34	0.76	0.00	1.00	0.48	0.65	0.15	-0.40	-0.14	-0.50
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
B	0.86	0.82	0.85	0.85	-0.58	-0.58	-0.86	-0.45	0.66	-0.78	-0.73	0.47	0.25	-0.66	0.59	0.89	0.48	1.00	0.67	-0.60	-0.60	-0.75	-0.79
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Π^F	0.48	0.66	0.65	0.65	-0.70	-0.71	-0.48	-0.18	0.73	-0.32	-0.50	0.59	0.24	-0.48	0.53	0.44	0.65	0.67	1.00	-0.29	-0.45	-0.51	-0.59
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Π^B	-0.46	-0.27	-0.36	-0.36	-0.00	-0.00	0.46	0.56	-0.15	0.78	0.39	-0.00	0.29	0.18	0.00	-0.77	0.15	-0.60	-0.29	1.00	0.34	0.80	0.50
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
r^f	-0.55	-0.59	-0.57	-0.57	0.49	0.48	0.55	0.23	-0.52	0.35	0.35	-0.44	-0.08	0.30	-0.48	-0.47	-0.40	-0.60	-0.45	0.34	1.00	0.41	0.63
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
r^b	-0.62	-0.46	-0.54	-0.54	0.16	0.16	0.62	0.58	-0.40	0.73	0.79	-0.19	-0.18	0.68	-0.18	-0.79	-0.14	-0.75	-0.51	0.80	0.41	1.00	0.57
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
ES	-0.75	-0.76	-0.76	-0.76	0.60	0.60	0.75	0.29	-0.74	0.50	0.47	-0.70	-0.14	0.41	-0.57	-0.64	-0.50	-0.79	-0.59	0.50	0.63	0.57	1.00
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Appendix 6.D Networks generation and statistics

In this part of the appendix we describe in detail the generation of the networks and their characteristics.

1. We create deposits networks (households and firms) following the steps below: the household-bank deposit network is built through a preferential attachment algorithm with a randomly generated fitness measure. Each household i enters the matching process and has a $p_{i,b}$ probability to link with a randomly chosen bank b, where the total degree of the bank is $degree_b$ and its fitness measure is $fitness_b$:

$$p_{i,b} = \frac{degree_b fit_b}{\sum degree_b fitness_b}$$

   If a new link is formed, the household exits the algorithm and the degree of b is updated, otherwise it repeats the matching process until it connects to a node. At the end each household has a link, so that the total degree of the network is equal to the number of households.

   The creation of the firm-bank deposit networks is similar, but the attachment probability is only determined by the degree of banks in the household-bank network, so that banks have a similar out-degree in both networks.

2. The credit network is generated through the same preferential attachment mechanism above, where the fitness measure is inversely proportional to the normalized degree of banks in the deposit networks. In this case when a new link is created, the firm does not exit from the matching process but meets the next bank in the list. The algorithm is repeated 5 times.

3. The interbank network is generated through a *Bianconi-Barabasi* model *(Bianconi and Barabási, 2001)*, where the fitness measure is the degree of banks in the firm-bank credit network.

This approach results in the creation of a network structure in which those banks with many links with depositors are potential lenders on the interbank market. They also have a low interbank degree, so that they form the peripheral part of the interbank network and their size in terms of net worth is negatively correlated with their degree in the deposits networks. At the opposite those banks with a high degree in the credit market have few links with depositors but are densely connected in the interbank network, being the core. They borrow funds from the peripheral banks and their size is positively correlated with their degree in the firm-bank credit market. Fig. 21 and Tab.s 7 and 8 show the distribution of banks' out-degrees, network statistics and the cross-correlation of out-degrees with selected variables. The statistics for the interbank network are in Tab. 2 in Sect. 3, where the reference connectivity level employed in the simulations is $d4$.

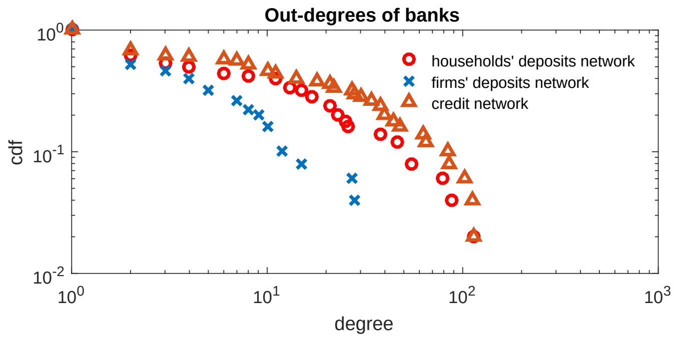


Figure 21: Loglog plot of the cumulative distribution function (cdf) of banks' out-degrees on deposits and credit networks.

	hb	fb	credit
Avg degree	15	5	22.64
Median degree	2	2	8
Max degree	113	28	113
Number of nodes	750	250	1132

Table 7: Descriptive statistics for the households-banks deposits network (hb), firms-banks deposits network (fb) and firms-banks credit network (credit).

	hb	fb	credit	ib	credit volume	ib lend	ib borr	bank size
hb	1.000 (1.000)	0.965 (0.000)	-0.386 (0.000)	-0.286 (0.000)	-0.400 (0.000)	0.513 (0.000)	-0.290 (0.000)	-0.269 (0.000)
fb	0.965 (0.000)	1.000 (1.000)	-0.410 (0.000)	-0.302 (0.000)	-0.428 (0.000)	0.527 (0.000)	-0.308 (0.000)	-0.300 (0.000)
credit	-0.386 (0.000)	-0.410 (0.000)	1.000 (1.000)	0.941 (0.000)	0.933 (0.000)	-0.299 (0.000)	0.916 (0.000)	0.903 (0.000)
ib	-0.286 (0.000)	-0.302 (0.000)	0.941 (0.000)	1.000 (1.000)	0.863 (0.000)	-0.209 (0.000)	0.923 (0.000)	0.842 (0.000)
credit volume	-0.400 (0.000)	-0.428 (0.000)	0.933 (0.000)	0.863 (0.000)	1.000 (1.000)	-0.289 (0.000)	0.914 (0.000)	0.982 (0.000)
ib lend	0.513 (0.000)	0.527 (0.000)	-0.299 (0.000)	-0.209 (0.000)	-0.289 (0.000)	1.000 (1.000)	-0.216 (0.000)	-0.204 (0.000)
ib borr	-0.290 (0.000)	-0.308 (0.000)	0.916 (0.000)	0.923 (0.000)	0.914 (0.000)	-0.216 (0.000)	1.000 (1.000)	0.901 (0.000)
bank size	-0.269 (0.000)	-0.300 (0.000)	0.903 (0.000)	0.842 (0.000)	0.982 (0.000)	-0.204 (0.000)	0.901 (0.000)	1.000 (1.000)

Table 8: Cross-correlation of selected network-related variables (p-values in parenthesis).
The first set of variables represents the total out-degree of each bank in the household-bank deposits network (hb), firm-bank deposit network (fb), firm-bank credit network (credit) and interbank network (ib).
The second set represents total credit lent by each bank (credit volume), interbank lending (ib lend) and borrowing (ib borr) and the size of banks (bank size) measured by their net worth. Correlations are computed on the cumulated normalized values of each bank over each simulation, for 10 Monte Carlo repetitions.