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Default risk premium in credit and equity markets

Raffaele Corvino*[1] and Gianluca Fusai†[2]

[1]Cass Business School - City University London. 106, Bunhill Row, London
[2]Cass Business School - City University London. 106, Bunhill Row, London, and Dipartimento DISEI, Universita' del Piemonte Orientale, Via Perrone 18, 28100 Novara, Italy

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ABSTRACT

The default risk premium expresses the difference between the actual default risk of a company and the default risk implied by the securities issued by the company. In this paper, we study the simultaneous relationship between the dynamics of the default risk premium and both the dynamics of the stock price and the CDS (Credit Default Swap) spread of a company. We show that an increase in the default risk premium can be associated, at the same time, to either an increase in the stock price and a decrease in the CDS spread, or to a decrease in the stock price and an increase in the CDS spread. We document that the first type of relationship features securities belonging to a consistent risk-return framework, while the second type of relationship describes securities following a counterintuitive risk-return puzzle. We show this result theoretically end empirically, by adopting a contingent claim model. We estimate the model with a non-linear Kalman filter in conjunction with quasi-maximum likelihood, and we shed light on the relationship over time between the default risk premium and both the equity value and the CDS spreads for a sample of non-financial firms.

JEL classification: C4, G12, G32, G33.

*Raffaele.Corvino.1@cass.city.ac.uk
†Gianluca.Fusai.1@city.ac.uk

I. Introduction

The default risk premium is a measure of the difference between the actual default risk of a company, and the evaluation of the default risk that is reflected in the price of the securities issued by the company, such as equity, bonds, or derivatives. The default risk implied by the price of the securities is straightforward to compute, by adopting widely used financial models. The result is the risk-neutral probability of default of the company. We refer to this measure as risk-neutral, as it is the evaluation of the default risk by a risk-neutral agent, such as the market. Instead, the actual default risk of the company, that we refer to as real-world default probability, is generally measured by using the company rating provided by rating agencies, which combine information on firm-specific economic indicators with macroeconomic drivers. Then, a risk premium arises as long as the two default risk measures are not equal. In particular, the premium is defined as the ratio between the risk-neutral default probability and the real-world default probability.

In this paper, we study the relationship between the default risk premium and the price of both the stock and the CDS of a company, over time. We show that an increase in the default risk premium can be associated, at the same time, to either an increase in the stock price and a decrease in the CDS spreads, or to a decrease in the stock price and an increase in the CDS spread.

We use CDS spreads following previous papers that show the better performance of the CDS market to provide a reliable proxy of credit risk with respect to bonds or loans spreads (Blanco, Brennen, and Marsh (2005), Almeida and Philippon (2007)). The CDS spreads are less affected by other sources of risk, in particular by liquidity risk (Longstaff, Mithal, and Neis (2005)). In practice, the CDS spread is the price of an insurance against the firm's default, which pays a given amount to the holder when the firm defaults. The higher is the default risk of the firm, then, the higher should be the price of the insurance (i.e., the CDS spread).

The intuition behind our results is simple. The default risk premium may increase because the risk-neutral default probability (i.e., the market-based evaluation of the default risk) has increased more than the real-world default probability (the actual default risk of the company), that is the numerator of the ratio has increased more than the denominator. However, the default risk premium may also increase because the real-world default probability has decreased more than the risk-neutral default probability (the denominator has decreased more than the numerator). In the first case, the stock depreciates because of the greater risk of default of the company, while the price of the CDS increases (buying an insurance against the default is more costly). In the second case, instead, the stock price moves up (the firm is safer and the equity is worth more), while the CDS depreciates (the price for hedging against default is lower).

On the other hand, the default risk premium may decrease either because the risk-neutral default probability has decreased more than the real-world default probability, or because the real-world default probability has increased more than the risk-neutral default probability.

In the first case, the stock price moves up and the price of the CDS decreases. In the second case, the stock depreciates and the price of the CDS increases.

We show that the first type of relationship describes securities (i.e., firms) belonging to a consistent risk-return framework (we call them as *normal*), where the firm reports an expected rate of growth higher than the risk-free rate, and the actual default probability is lower than the risk-neutral default risk (default risk premium larger than 1). The second type of relationship, instead, describes securities belonging to a counterintuitive risk-return framework (we refer to as *distressed*), where the firm generates expected return even lower than the risk-free rate, and the actual default risk is greater than the risk-neutral default risk (default risk premium lower than 1).

We show this result theoretically and empirically, by adopting a contingent claim model. In particular, we adopt a first-time passage model, where the default occurs as soon as the asset value of the firm crosses from above a default barrier. This framework is an extension of the seminal model of Merton (1974), where the default can occur only at the maturity of the debt. Moreover, in the appendix, we show that the result does not depend on the particular choice of the model, and we prove it in a very simple and general pricing model, that features the pricing of a stock and an insurance, by using both the real-world and the risk-neutral probability. We derive the relationship between the default risk premium and the dynamics of the stock price, then we apply similar proofs to derive the relationship between the default risk premium and the dynamics of the insurance premium (such as a CDS spread).

In the paper, we illustrate this result numerically in the contingent claim model, then we estimate the model with a non-linear Kalman filter in conjunction with quasi-maximum likelihood, by using equity prices and CDS spreads. We first report that our estimates well fit the real data. Buying and selling equity shares and CDS, according to the one-step ahead forecasts based on our estimation of the firms' asset and debt, generates highly significant performance.

Then, we use the output of our estimation to compute both the real-world and the risk-neutral probability of default, for each firm, and we combine the two measures to compute the default risk premium, at each point in time. We show that the equity and the credit markets exhibit a relationship with the default risk premium which is opposite to each other, where the direction of the relationship depends on the type of firm-security (normal, or distressed).

Therefore, our paper makes three contributions. First, we propose a novel methodological approach to estimate a corporate credit risk model, by using only market information, for a sample of non-financial firms, across different geographic regions. We generally find that the firm's market value of the leverage, defined as the ratio between debt and asset, reaches the peak around the 2008-2009 financial crisis. Also, we estimate the actual value of the default boundary for each firm in our sample, and we report values of the default barrier around 75% of the face value of the debt. This result is substantially similar to the findings of Davydenko (2012), who estimates a default boundary around 70% of the debt value, and Wong and Choi (2009), who report a default barrier equal to 74% of the debt.

As second contribution, we address the estimation of the default risk premium by using

only market data, and by gathering the information conveyed simultaneously from the credit and the equity markets. The result is a measure of the distance between actual and risk-neutral default risk that is reflected at the same time in both the financial markets. We find that the default risk premium is substantially time varying. In particular, the default risk premium exhibits the lowest peak around the great recession, and an additional downturn during the sovereign debt crisis. Moreover, the premium is generally higher for the shorter time horizons.

We generally find that the default risk premium is greater than one for most of the firms, that is the risk-neutral default probability is higher than the corresponding real-world measure (*normal* firms). However, we document the presence of a significant number of *distressed* firms. In fact, for the 10-years (5-years) maturity, we find that the 32% (26%) of our sample firms reports an average default risk premium lower than 1.

As last contribution, we bring new insight on the broad literature which links the default risk and the equity returns, and recently the CDS and the equity market. We show that the dynamics of the default risk premium can be associated to diametrically opposite dynamics of the stock price and the CDS spreads, and this completely different relationship depends on the type of the firm-security in terms of risk-return framework.

We split our sample firms in two sub-samples, normal and distressed firms, and we show that the difference in the correlation, between equity prices and default risk premium, across the two sub-samples is positive and very significant, while the difference in the correlation, between CDS spreads and default risk premium, across the two sub-samples is negative and very significant. We corroborate this result by implementing long-short portfolio strategies of equity shares and CDS, based on the one-step ahead forecast generated by the Kalman filter of the default risk premium, for each firm. Then, we buy (sell) equity shares and we sell (buy) CDS when we have a prediction of an increase (a decrease) in the default risk premium, for firms with default risk premium larger than 1 (normal). Instead, we buy (sell) equity shares and we sell (buy) CDS when we have a prediction of a decrease (an increase) in the default risk premium, for firms with default risk premium lower than 1 (distressed). We report highly significant performance for our long-short portfolio strategies, both in-sample and out-of-sample.

The rest of the paper is organized as follows. In the next section we relate our study to the previous literature. In section 3 we detail the model and the estimation methodology. Moreover, we define the default risk premium, and we provide numerical evidence on the relationship between the default risk premium and the market variables. The data are described in Section 4, and Section 5 presents the empirical analysis. The first part of this section reports our results on the contingent claim model estimation, while in the second part we focus on the default risk premium, and the last part documents the construction and the performance of the portfolio strategies that use the premium as key driver. Section 6 concludes and introduces further potential research directions.

II. Related Literature

The first method to estimate a credit risk model $\grave{a}$ la Merton is based on a system of two equations which link the equity value and volatility to the unobservable asset value and volatility. This method, proposed by Jones, Mason, and Rosenfeld (1984), Ronn and Verna (1986), and improved and amended by Bohn and Crosbie (2003), Vassalou and Xing (2004), is known as Volatility-Restriction (VL). Duan (1994) highlights the drawbacks of this approach and constructs a transformed likelihood function to estimate the asset values from the observable equity prices.

Brockman and Turtle (2003) address the estimation of a model with early default risk, and they proxy the value of the asset by adding the equity value to the book value of the debt. They report values of the default boundary above the nominal value of the debt. Ericsson and Reneby (2004), Li and Wong (2008), show that the maximum-likelihood approach of Duan (1994) outperforms both the VL and the market proxy methods, either in the Merton model or in the barrier-dependent model. Perlich and Reisz (2007) attribute the misleading results of Brockman and Turtle (2003), who predicts that the equity holders liquidate the firm when the asset value is more than enough to guarantee the payment of the final debt, to the overvaluation of the market value of the debt when it is replaced by the nominal face value. By using a transformed maximum-likelihood approach to estimate a barrier-dependent model, Wong and Choi (2009) report values of the barrier around 70% of the face value of the debt. As in the original Duan (1994) formulation, they use equity prices to infer the asset value, by constructing a modified likelihood function adapted to the barrier framework. Similar results are documented in Davydenko (2012), where the market value of the asset just prior to the default is estimated for those firms with market value of debt entirely observable. This is the case of those firms that have all liabilities traded on the market. Forte (2011) proposes a calibration approach to derive the asset value from the equity prices, and the default boundary level from the CDS spreads, while Forte and Lovreta (2012) implement a double-stage maximization algorithm that extracts both the asset and the barrier values from the stock prices, under the assumption that the equity value incorporates the optimal level of the barrier. These two last papers estimate barrier values below the nominal debt.

Our paper nests in this literature, with several innovations. We estimate a barrier-dependent model, by using only market data. In contrast with Brockman and Turtle (2003), Perlich and Reisz (2007), Wong and Choi (2009), Forte and Lovreta (2012), we use not only equity prices but also CDS spreads. This approach allows to estimate also the market value of the debt, instead of using the book value of the debt. Moreover, differently from Forte (2011), equity value and CDS spreads enter in our estimation model simultaneously, in order to estimate at the same time the asset value dynamics and the barrier value.

Also, we show that our contingent claim framework fits better the real data than the Merton (1974) model. Buying and selling equity shares and CDS, according to the one-step ahead forecasts based on the estimation implemented with the barrier-dependent model, generates larger and more significant performance than using the one-step ahead forecasts based on the

estimation implemented with the Merton (1974) model.

We assume an exogenous boundary, as in Longstaff and Schwartz (1995). In this case, the barrier may represent certain covenants between creditors and debtors, where managers are committed to maintain some financial ratios above pre-specified levels in order to preserve the possibility of debt financing. Moreover, consistently with the previous literature, we specify a default barrier that is strictly positive but no greater than the book value of the liabilities. The intuition is simple. As long as the asset value is above the default boundary, even if is below the face value of the debt, the shareholders are willing to cover the promised payment and bear these losses in order to keep the firm operating. Otherwise, the firm defaults and the equity holders get nothing.

Moreover, we focus our attention on the dynamics of the default risk premium which captures remarkable attention in both asset pricing and credit risk literature. Different approaches have been proposed to assess this key measure. Hull, Predescu, and White (2005) compute the ratio between the risk neutral and the real world default probability by using corporate bond spreads and historical data, while Driessen (2005) estimates a reduced-form model to derive a default risk premium, constant over time, implied by corporate bond spreads and rating-based default probability, with a Kalman filter. He finds that the premium is statistically and economically significant. In a pioneer work on the CDS-based default risk premium, Berndt, Douglas, Duffie, Ferguson, and Schranz (2008) perform both a panel regression and a reduced form model by using 5-years CDS spreads and Moody's EDFs data, and they show that the premium is time, sector, and rating-varying. Diaz, Groba, and Serrano (2013) adopt a similar approach to Driessen (2005) and Berndt et al. (2008), by using a wider term structure of CDS spreads on European firms. The default risk premium has been recently studied by Huang and Huang (2012), who calibrate different credit risk models with corporate bond spreads and default data from rating agencies. They support the conclusion that the premium decreases with the credit quality, and after the crisis periods.

In terms of magnitude of the premium, Berndt et al. (2008) report a default risk premium roughly equal to two, thus arguing that the investors price twice the expected default loss, evaluated under the actual probability measure. Instead, in case of corporate bonds, the premium describes the amount required by the investors to bear the risk related to the company's default, which indeed will affect the bond repayment. Driessen (2005) estimates that the investors multiply the actual default probability with a factor close to 6, in order to price corporate bonds. Therefore, we report similar but lower premia with respect to Berndt et al. (2008) and Driessen (2005), with the exception of the US sample, which exhibits default risk premia significantly larger.

III. The Model

We define the firm i as an entity fully financed with equity, with market value $E_{i,t}$ at time t, and a zero-coupon debt with a face value of F_i maturing at time T. Let $V_{i,t}$ be the asset

value of the i-th firm and $Z_{i,t}$ its risky zero-coupon bond value at time t. Then, the following condition holds for every point in time, and for every firm:

$$V_{i,t} = E_{i,t} + Z_{i,t}$$

The asset value of the i-th firm is described by a geometric Brownian motion on the filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t : t \geq 0\}, \mathcal{P})$:

$$dV_{i,t} = \mu_{V_i} V_i dt + \sigma_{V_i} V_i dW_{i,t},$$

where μ_{V_i} and σ_{V_i} are the $\mathcal{P}$-drift and diffusion constant coefficients, $W_{i,t}$ is a standard Brownian motion under the physical probability measure $\mathcal{P}$.

We define the i-th firm's market value of leverage as $L_{i,t} = \ln\left(\frac{F_i}{V_{i,t}}\right)$, following an arithmetic Brownian motion,

$$dL_{i,t} = \mu_{L_i} dt - \sigma_{L_i} dW_{i,t}, \tag{1}$$

where $\mu_{L_i} = -\left(\mu_{V_i} - \frac{1}{2}\sigma_{V_i}^2\right)$ is the $\mathcal{P}$-leverage drift coefficient, and $\sigma_{L_i} = \sigma_{V_i}$ is the leverage diffusion component. As result of the inverse relationship between the asset value and the leverage, the minus in front of the diffusion component stands for the perfect and negative correlation between the Brownian motions of the asset value and the leverage.

For pricing the firm's securities, we adopt a first-time passage framework, as in Black and Cox (1976), Longstaff and Schwartz (1995), with a constant and deterministic barrier, that we assume to be below the face value of the debt. Default occurs as soon the asset value of the i-th firm crosses from above the barrier C_i at any time s, with $t \leq s \leq T$, where T is the outstanding firm's debt maturity. Therefore, the firm's equity value $E_{i,t}$ can be considered as a down-and-out European call (DOC) option, written on the firm's asset value, with maturity T. The pricing formula for the equity value in a DOC framework is rearranged in order to express the observable equity value as function of the firm's leverage.[1]

$$E_{i,t} = \frac{F_i}{e^{L_{i,t}}}\Phi(d_1) - F_i e^{-r(T-t)}\Phi(d_1 - \sigma_{L_i}\sqrt{(T-t)}) - \frac{F_i}{e^{L_{i,t}}}\exp\left((K_i + L_{i,t})\left(\frac{2r}{\sigma_{L_i}^2} + 1\right)\right)\Phi\left(d_1^{C_i}\right)$$
$$+ F_i e^{-r(T-t)}\exp\left((K_i + L_{i,t})\left(\frac{2r}{\sigma_{L_i}^2} - 1\right)\right)\Phi\left(d_1^{C_i} - \sigma_{L_i}\sqrt{(T-t)}\right), \tag{2}$$

where

$$d_1 = \frac{-L_{i,t} + \left(r + \frac{1}{2}\sigma_{L_i}^2\right)(T-t)}{\sigma_{L_i}\sqrt{(T-t)}}, d_1^{C_i} = \frac{(2K_i + L_{i,t}) + \left(r + \frac{1}{2}\sigma_{L_i}^2\right)(T-t)}{\sigma_{L_i}\sqrt{(T-t)}}$$

Φ stands for the cumulative distribution function of a standard normal variable, and $K_i = \ln\left(\frac{C_i}{F_i}\right)$. The parameter K_i measures the willingness to the gamble for resurrection of the

shareholders. As the default barrier is below the face value of the debt, K_i assumes only negative values. The larger is the magnitude of the absolute value of K_i, the larger is the distance between the face value of the debt F_i and the default barrier C_i, and the larger is the potential amount the shareholders are willing to take out of their pocket in order to bail out the firm's debt, and to keep the firm operating.

The firm's default risk is also priced in the credit default swaps (CDS) issued with different maturity τ_j, with j going from 1 to J, where the longest maturity τ_J matches the firm's debt maturity T. In a CDS contract, the protection buyer pays a fixed premium each period until either the default event occurs or the contract expires, and the protection seller is committed to buy back from the buyer the defaulted bond at its par value. Therefore, the price of the CDS, i.e. the premium (the spread) paid by the insurance buyer, is defined at the inception date of the contract in order to equate the expected value of the two contractual legs. Then, by assuming the existence of a default-free money market account appreciating at a constant continuous interest rate r, and M periodical payments occurring during one year, the CDS spread γ with time-to-maturity τ_j, priced at $t = 0$, solves the following equation:

$$\sum_{m=1}^{M} T\frac{\gamma}{M}\exp\left(-r\frac{m}{M}\right)\mathcal{E}_0^{\mathcal{Q}}[1_{t^*>\frac{m}{M}}] = \mathcal{E}_0^{\mathcal{Q}}[\exp(-rt^*)\alpha 1_{t^*<\tau_j}],$$

where t^* stands for the time of default, α is the amount paid by the protection seller to the protection buyer in case of default, and $\mathcal{E}_0^{\mathcal{Q}}$ indicates that the expectation is taken under the risk-neutral measure $\mathcal{Q}$. Therefore, $\mathcal{E}_0^{\mathcal{Q}}[1_{t^*<\tau_j}]$ is the probability that the firm defaults at any time before τ_j, that is the probability that the asset value crosses from above the barrier C_i. At t, this probability is equal to:

$$PD_{i,t}^{\mathcal{Q}}(\tau_j) = \Phi\left(\frac{K_i + L_{i,t} - \left(r - \frac{1}{2}\sigma_{L_i}^2\right)(\tau_j - t)}{\sigma_{L_i}\sqrt{(\tau_j - t)}}\right) + \exp\left((K_i + L_{i,t})\left(\frac{2r}{\sigma_{L_i}^2} - 1\right)\right)\Phi\left(\frac{(K_i + L_{i,t}) + \left(r - \frac{1}{2}\sigma_{L_i}^2\right)(\tau_j - t)}{\sigma_{L_i}\sqrt{(\tau_j - t)}}\right), \quad (3)$$

if $\tau_j < T$, otherwise

$$PD_{i,t}^{\mathcal{Q}}(\tau_J) = 1 - \Phi\left(\frac{-L_{i,t} + \left(r - \frac{1}{2}\sigma_{L_i}^2\right)(\tau_J - t)}{\sigma_{L_i}\sqrt{(\tau_J - t)}}\right) + \exp\left((K_i + L_{i,t})\left(\frac{2r}{\sigma_{L_i}^2} - 1\right)\right)\Phi\left(\frac{(2K_i + L_{i,t}) + \left(r - \frac{1}{2}\sigma_{L_i}^2\right)(\tau_J - t)}{\sigma_{L_i}\sqrt{(\tau_J - t)}}\right), \quad (4)$$

as $\tau_J = T$, and we have to consider not only the early bankruptcy risk as in the equation (11), but also the probability of the firm not being able to pay back the outstanding debt F_i at time T, even though the asset value never crossed the default boundary.

The face value of the debt, F_i, appears in the equations (10) and (12) (even not explicitly in (12), where it appears in terms of the parameter K_i), and this is consistent with the market evaluation which takes into account the information released by the company with the book data in order to form a proper assessment of the firm's value, and the firm's riskiness. Nevertheless, we define $V_{i,t}$ as the market value of the i-th firm at time t, as we obtain this estimation by using the information contained in market data.

However, we need to estimate F_i because we are mapping companies which are normally characterized by more complex debt structure, in terms of debt instruments and maturities, into companies with very simple debt structure, with one single zero-coupon bond issue. This simplification does not affect the definition of default risk. In fact, what only matters in terms of default risk is the threshold triggering the default. As long as the asset value is above the default boundary the shareholders are willing to cover the firm's losses, even if the asset value is not enough to meet the promised debt payment at maturity. In practice, the shareholders would buy additional shares in order to bail out the maturing debt holders, still assessing as profitable the investment in the company.

We formulate our problem in a state-space model, where the measurement equations come from (10) and (11-12), and the default probability for different time horizons can be extracted from the CDS spreads according to O'Kane and Turnbull (2003). The firm's leverage $L_{i,t}$ is unobservable, while we observe $E_{i,t}$ on the equity market, and we observe the CDS spreads for four time horizons (1,3,5, and 10 years). Hence, we have five measurement equations. The noise terms associated with the CDS implied-default probability for different time to maturities τ_j are assumed to be uncorrelated, and with equal variance.

$$(I-IV): PD^{\mathcal{Q}}_{i,t}(\tau_j) = g\left(L_{i,t}; K_i, \sigma_{L_i}\right) + \epsilon_{i,t}(\tau_j), [j = 1,3,5,10]$$
$$(V): E_{i,t} = h\left(L_{i,t}; K_i, \sigma_{L_i}, F_i\right) + \varsigma_{i,t},$$

where the time to maturity is expressed in years, and $j = 10$ stands for the maturity T of the outstanding debt F_i (i.e., 10 years). The functions g and h define the non-linear relationships between the observable and the latent variable, and $\epsilon_{i,t}(\tau_j)$ is the measurement noise associated with the CDS implied-default probability equation and the time horizon j. These four measurement noises, for each firm i, are assumed to follow a multivariate normal distribution, with zero mean, and diagonal covariance matrix R_i. We assume a homoscedastic covariance matrix, which is firm-varying. $\varsigma_{i,t}$ is the measurement noise associated with the equity equation, and we assume it to be normally distributed, with zero mean and variance ω_i.

On the other side, the transition equation describes the evolution of the firm's leverage. It follows from the discretization of the stochastic process defined in (9):

$$L_{i,t+\delta t} = L_{i,t} + \mu_{L_i}\delta t + \eta_{i,t+\delta t},$$

where $\eta_{i,t+\delta t} = \sigma_{L_i}(W_{i,t} - W_{i,t+\delta t}) \backsim \mathcal{N}(0, Q_i)$ is the transition error, and $Q_i = \sigma^2_{L_i}\delta t$.

The dynamics of $L_{i,t}$, and the parameters of the model, such as the face value of the debt F_i, the parameters of the leverage $\mathcal{P}$-dynamics (μ_{L_i}, σ_{L_i}), and the willingness to the gamble for resurrection K_i, are then estimated by performing a non-linear Kalman filter in conjunction with quasi-maximum likelihood estimation. For parsimony, the steps to implement the non-linear Kalman filter, and the construction of the likelihood function, are described in details in the Appendix A.

The estimation of the dynamics of the firm's asset value $V_{i,t}$, the dynamics of market value of the debt $Z_{i,t}$, and the level of the default boundary C_i, becomes straightforward, once we have obtained the estimates of the leverage dynamics, and the parameters of the model. In fact, from the definition of $L_{i,t}$, and K_i, it immediately follows that:

$$\begin{aligned} V_{i,t} &= \frac{F_i}{e^{L_{i,t}}}, \\ C_i &= e^{K_i} \times F_i, \\ Z_{i,t} &= V_{i,t} - E_{i,t}. \end{aligned} \tag{5}$$

A. *The Default Risk Premium*

The default risk premium is defined as the ratio between the risk-neutral default probability and the real-world default probability:

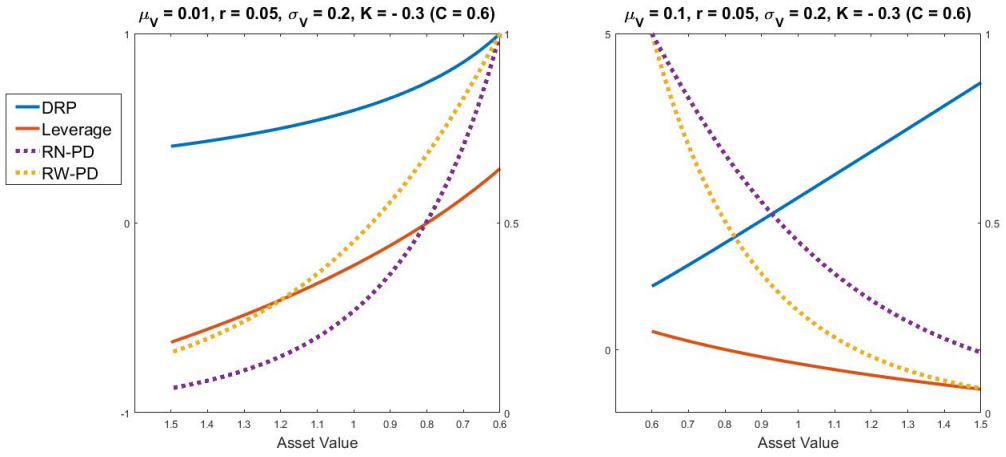
$$DRP_{i,t}(\tau_j) = \frac{PD^{Q}_{i,t}(\tau_j)}{PD_{i,t}(\tau_j)} \tag{6}$$

The default risk premium can be computed for each firm i, each point in time t, and each time horizon τ_j. Our target is to investigate the relationship between the dynamics of the default risk premium and the dynamics of both the stock price and the CDS spread, for a given firm. We know that there is a straightforward relationship between both the stock price, and the CDS, and the state variable of the model, that is the leverage of the firm. The leverage increases, indeed, when the value of the firm decreases. As consequence, the equity value of the firm decreases, and the CDS spread increases. The firm, in fact, is becoming riskier, as the value of the firm is approaching the default boundary.

Therefore, the natural procedure to achieve our target is to study the sensitivity of the default risk premium with respect to the leverage, that means to study the sign of the derivative of the premium with respect to the leverage of the firm. We do not report here the analytical derivation of this derivative. However, we study the sign of the derivative numerically. In particular, we show that, *ceteris paribus*, the sign of this derivative depends on the difference between μ_V and r, that is whether this difference is positive or negative. The plots in figure 1 illustrate this result.

The left plot shows that the default risk premium increases when the leverage increases, therefore the equity value of the firm depreciates, and the CDS spread increases. The default

Figure 1. Default Risk Premium and Leverage. Numerical Example



The left plot shows the variation of the leverage, the default risk premium, and the two measures of default probability when the asset value of the firm decreases, up to the default boundary. The right plot shows the variation of the leverage, the default risk premium, and the two measures of default probability when the asset value of the firm increases, starting from the default boundary.

risk premium increases, in fact, because the risk-neutral probability of default increases more than the real-world probability of default. This is the case of firms with $\mu_V < r$, for which the default risk premium is lower than 1. The right plot, instead, shows that the default risk premium increases when the leverage decreases, therefore the CDS spread depreciates, and the equity value increases. The default risk premium increases, in fact, because the real-world probability of default decreases more than the risk-neutral probability of default. This is the case of firms with $\mu_V > r$, for which the default risk premium is larger than 1.

In a nutshell, an increase in the default risk premium can be associated to diametrically opposite dynamics of the stock price and the CDS spreads, and this completely different relationship depends on the type of the firm-security in terms of risk-return framework. While, in fact, it is natural that a firm has an expected rate of value growth higher than the risk-free rate, since the assets of the firm are risky and may end up in default, it is puzzling that the this expected rate of growth is even lower than the risk-free rate. Nevertheless, we document in our empirical analysis that a significant portion of firms is characterized by a default risk premium lower than 1, and then they show a dynamics of the premium negatively related to the equity value and positively related to the CDS spread.

Moreover, this result does not depend on the particular choice of the model. In the appendix, we present a very simple and general pricing model, where we show the relationship between the default risk premium and the dynamics of the stock price, and the relationship between the default risk premium and the dynamics of the insurance premium (such as a CDS spread).

IV. Data

Our dataset consists of the market variables that we use to estimate the contingent claim model. In particular, we use daily CDS spreads traded on four different maturities (1 year, and

3,5,10 years), which are the most liquid on the CDS market, and daily market capitalization (i.e., the product between the number of outstanding shares and the share price).

The CDS premia are expressed in basis point. In a CDS transaction, the premium, which the buyer of credit risk (i.e. the protection seller) receives, is expressed as an annualized percentage of the notional value of the transaction and this value is recorded as the market price. The details of a CDS transaction are recorded in the CDS contract, which is usually based on a standardized agreement prepared by the International Swaps and Derivatives Association (ISDA), an association of major market participants.[2]

The event triggering the payment of the protection leg in a CDS contract ranges from no-restructuring to full-restructuring. As priority rule, we select, when is possible, the contracts that adopt the no-restructuring (MR) clause, as it is the standard convention after the CDS Big Bang protocol in April 2009. Otherwise, we include the contracts that adopt the modified-restructuring (MR) clause, as it was the standard convention before the protocol. As last options, we include the contracts with full-restructuring or modified-modified restructuring clause.

We take CDS data from Thomson Reuters by Datastream, which provides information on CDS spreads from December 2007. The sample firms are chosen from the reference entities listed on the Markit indexes, where the most liquid companies in terms of CDS transactions are quoted. In particular, we refer to the iTraxx indexes for Australia, Japan, and Europe. In the last index, we select only the Eurozone entities to ensure homogeneity in currency. The CDX North America - Investment Grade index is adopted for the United States sample, where we select only the US companies, for the same homogeneity reason.[3]

First, we keep only the reference entities with active CDS contracts before the 20th December 2007, which is the starting point of our time series. It goes from the 20th December 2007 to the 19th December 2013. The choice of this time window is driven by the so-called International Monetary Market dates.[4]

Moreover, we exclude the financial firms, which are characterized by a peculiar capital structure in terms of assets and liabilities, that would deserve a specific analysis and discussion.

Also, we control for the stale prices, where a price is defined stale when remains equal for many consecutive days. This issue may have an impact on the inversion from the CDS spreads curve to the risk-neutral default probabilities. In fact, a massive discrepancy in the number of active trading days between CDS on the same entity with different maturities may produce non-monotone default probabilities. In this case, for instance, we would have higher probability of defaulting in a time interval going from now to one year than defaulting from now to five years. As a consequence, we would end up with a negative probability of defaulting in the time interval going from one to five years, i.e. negative forward default probability. Instead, the default probability for longer time horizons must be at least equal to the default probability for shorter time horizons, as the longer horizon nests the shorter.

This same finding can be a consequence of an enormous variation in the CDS spread recorded for only one or some of the four maturities. We find 5 companies to show this il-

logical result, and we drop out from the dataset the companies with negative forward-default probabilities for more than the 1% of the entire time series, for at least one time horizon, and we set this negative forward measure to zero for the remaining entities, in order to retrieve a monotone non-decreasing term structure of the default probability. In fact, while it is possible to have a CDS downward sloping term structure when the company is perceived to be riskier in the short term than in the long term (similarly to a downward sloping term structure of interest rates), this curve must be not too steep.

At the end, we are left with a CDS dataset of 172 firms. The data on the daily market capitalization, for these companies, and the same time series, are downloaded from Bloomberg. Finally, we delete from our dataset 8 firms with no data on the market capitalization value, and we are left with a final dataset of 164 firms, geographically divided as follows: Australia (9), Japan (10), United States (89), Europe (56).

As proxy of the risk-free rates, we use the sovereign bonds curve constructed by Bloomberg for Unites States, Australia, and Japan. The European curve is given by an aggregation of the sovereign bonds issued by France, Germany, and Holland. Given the CDS spread, the termination date, the arbitrary assumption on the recovery rate (we follow the common approach of a recovery rate equal to 40% of the notional value), and the risk-free term structure, we can extract from the CDS spread the risk-neutral default probability for the corresponding maturity. In practice, there is only one arbitrage-free PD (for a certain debt instrument, with a certain contractual structure) that is implied by the market default swap spreads, and equates the starting value of the premium leg (protection buyer) and the protection leg (protection seller) in a CDS contract.[5]

A. Summary Statistics

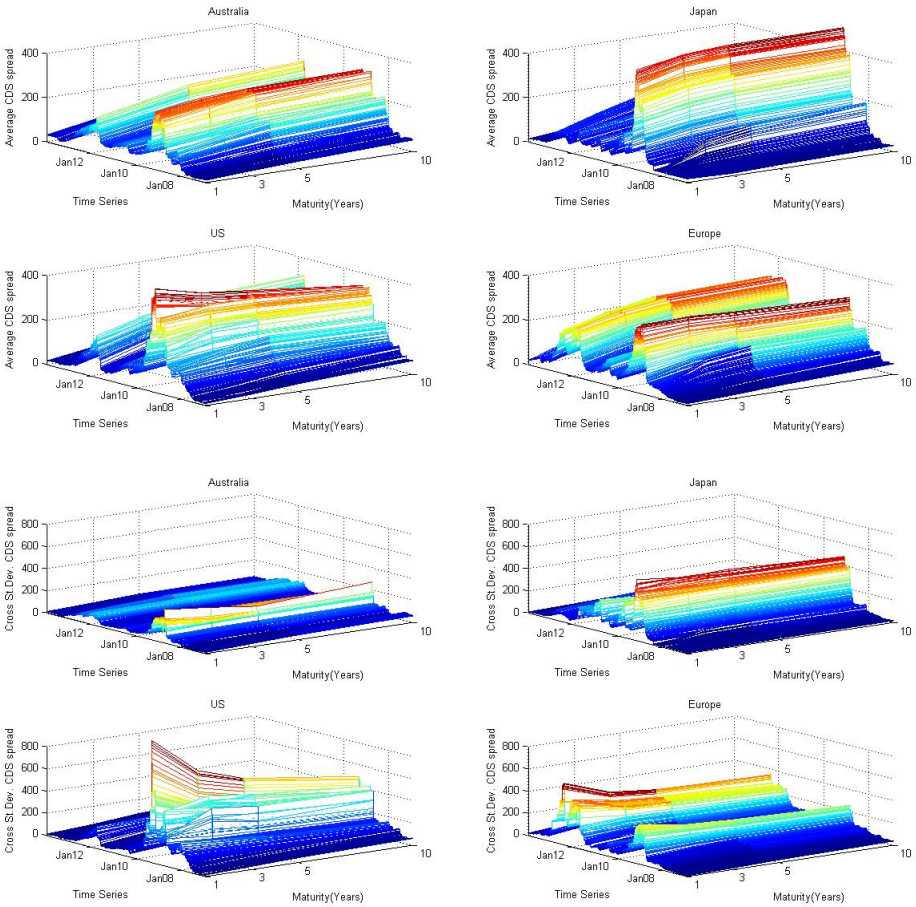
We report in the table 1 the descriptive statistics about the CDS spreads for all the entities in our sample. The statistics for each geographic region are available upon request.

[Insert Tables 1, here]

The upper graph of figure 1 shows the term structure of the average CDS spreads for each region. The worldwide propagation of the 2008-2009 financial crisis is the main source of the peak in the CDS premia around that time interval. The difference in the CDS spreads across years is very pronounced for the shorter time horizons, and in particular for the one-year maturity, while the CDS premia paid on the longer time horizons are more stable over the entire time series. It turns out that the conjecture of the market on the default risk in the short-term dramatically rose up.

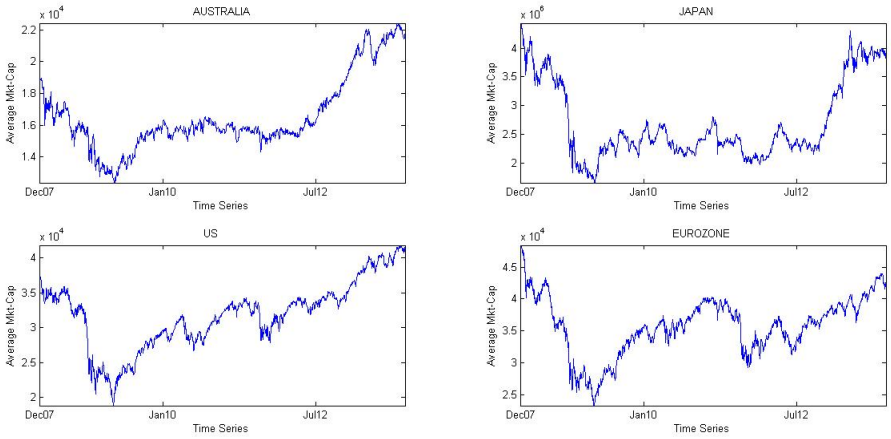
These features are homogeneous across regions. However, the larger premia paid on the US reference entities in the first part of the time window arise from the strong concentration of the financial turmoil in the US market, and the higher spreads quoted for the Eurozone

Figure 2. CDS Spreads. Regional Average and Standard Deviation



The four upper plots show the cross-sectional average of the CDS spread across the four geographic regions, along the sample time series (X-axis), and the four different maturities (Y-axis).The four lower plots show the cross-sectional standard deviation of the CDS spread across the four geographic regions, along the sample time series (X-axis), and the four different maturities (Y-axis). Both the average and the standard deviation are expressed in basis points.

Figure 3. Market Capitalization. Regional Average



The plots show the cross-sectional average of the market capitalization across the four geographic regions, along the sample time series. The firm's market capitalization is calculated as number of outstanding shares times the share price, and is expressed in millions of the local currency.

companies in the last years follow from the European sovereign debt crisis starting in the 2011.

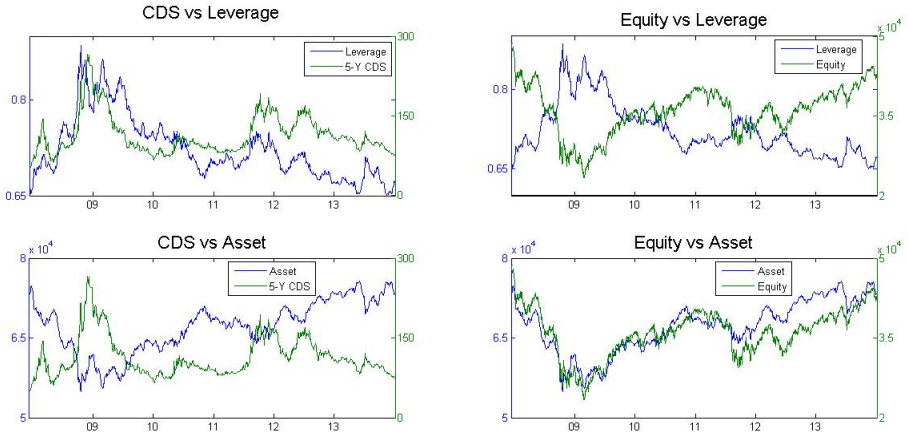
The financial crisis of 2008-2009 is also characterized by a larger cross-sectional standard deviation of the CDS spreads (figure 2, lower graph), showing an increase in the heterogeneity between the CDS premia paid on the different entities. It is evident for the US sample, and is emphasized by the same scale used in the lower part of figure 2. However, the gap across years in this dispersion measure is clearly present in each region. The combination of higher average spreads and higher heterogeneity across spreads reflects the attitude of the insurance sellers, even more emphasized during a crisis period, to ask for a very large premium on the risky entities, and to be willing to accept a lower price on the more safe companies.

The summary statistics for the firms' market capitalization are calculated for each region as this data is expressed in millions of the local currency.

[Insert Table 2, here]

The historical pattern of the equity value generally pairs with the trend described for the CDS spreads. The market value for the shareholders drop in the core of the sub-primes distress, whilst a pretty constant increase starts at the end of 2009. Few peculiarities still appear across regions. The market capitalization swiftly grows up for the US and Eurozone firms from 2009 to 2010, then the US companies keep substantially rising up, while the Eurozone entities move around that level of equity value for the remaining part of the time series. The shift at the end of the financial crisis is not terribly noticeable for the Australian and the Japanese firms. On the other hand, these companies show a big jump during the 2013.

Figure 4. Leverage and Assets - Eurozone



The first plot (CDS vs Leverage) shows the estimated mean firm's leverage against the observed mean CDS spread for the 5-years maturity, along the sample time series, for the Eurozone firms. The leverage is expressed as $e^{L_{i,t}} = \frac{F}{V_{i,t}}$, and the CDS spread in basis points. The second plot (Equity vs Leverage) shows the estimated mean firm's leverage against the observed mean Equity value, for the Eurozone firms, where the equity value is expressed in millions of Euro. The third plot (CDS vs Asset) shows the estimated mean firm's asset value ($V_{i,t}$) against the observed mean CDS spread for the 5-years maturity. The last plot (Equity vs Asset) shows the estimated mean firm's asset value against the observed mean Equity value, for the Eurozone firms.

V. Empirical Results

A. *The dynamics of leverage and the default boundary*

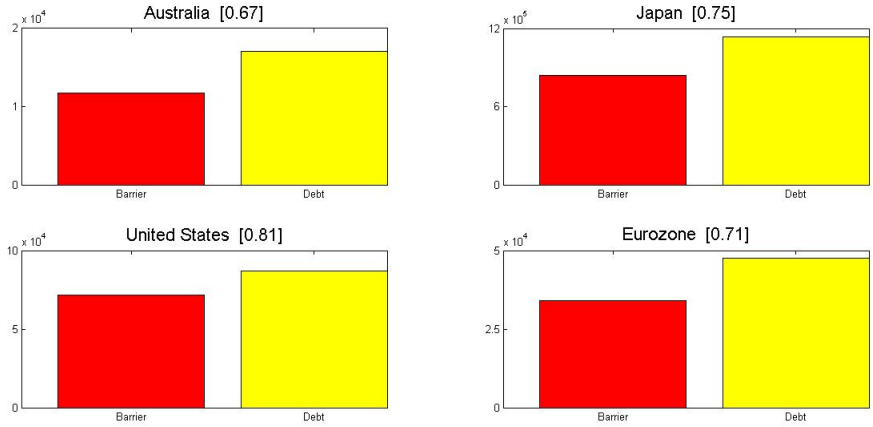
We combine the CDS spread-implied default probability and the equity value to compose the set of observable variables, which appear on the left-hand-side of the equation (I-IV), and (V), respectively. We are gathering information from credit and equity markets to estimate the unobservable dynamics of the market value of the firm's leverage, and the hyperparameters of the model. We perform this estimation for each firm, one-by-one.

The estimation of the state variable dynamics $L_{i,t}$ is achieved by the iteration of the predicting and the updating equations of the non-linear Kalman filter. The hyperparameters are estimated by performing a maximum likelihood algorithm, under the assumption of normality for the measurement errors, given by the difference between the actual and the fitted value of the observable variables. The fitted value of the observable variables is computed from the equations (10) and (11-12), respectively, by using the current estimate of the state variable.

As result, we reconstruct the time series of the market value of the leverage for each firm in our sample. Then, by mean of the equation set (13), we also reconstruct the time series of the market value of the asset, and we provide a quantitative measure of the firm's outstanding debt, and a measure of the default boundary. Figure 3 shows the dynamics of leverage and asset value averaged across firms for the Eurozone sample. The leverage value is expressed in terms of debt-to-asset ratio. The plots for the other regions are reported in the Appendix C. We compare these dynamics against the time series of the market data, such as the equity value and the CDS spreads. In particular, we use the 5-years CDS maturity as benchmark in the following plots.

On average, the market value of the Eurozone firms' leverage exhibits a peak around

Figure 5. Debt and Barrier



The four plots show the cross-sectional average of the debt and default barrier values, across the four geographic regions. The values are expressed in millions of the local currency, and the title of each plot reports the average ratio between the default boundary value and the value of the debt.

the 2009 financial crisis, when the debt counted for more than 80% of the market value of the asset. After a decreasing trend until the beginning of 2011, this ratio has grown up as consequence of the sovereign debt crisis, before starting again to decrease in the last part of the time series. The value of the asset shows indeed the opposite dynamics, with a lowest peak around the great recession, and an increasing trend in the second part of the time series, partially interrupted during the sovereign debt crisis in 2012.

We can also appreciate how the market value of the leverage has a very similar pattern to the CDS spread dynamics. It means that our estimation correctly reflects the evaluation of the credit market in terms of firm riskiness. On the other side, the leverage moves in the opposite direction with respect to the market capitalization of the firm along the entire time period. It turns out that our leverage estimation is also able to reflect the market value of the firm, as reported by the stock market. Symmetrical discussion applies to the asset value estimation, which follows an opposite trend with respect to the CDS spread, and shows a pretty overlapping dynamics with respect to the equity value. We derive similar results for the other geographic regions in our sample. However, the downturn of the asset value (increase in the leverage) is less pronounced around the 2012 for the other regions, as the sovereign debt crisis has produced lower impact on the non-European firms.

A key parameter in our framework is the willingness of the shareholders to rescue the firm in case of distress to avoid default and keep the firm operating. It is measured by K_i, which proxies the distance between the default boundary and the value of the debt. By plugging in the equation set (13) the estimate of K_i, coupled with the estimation of the market value of leverage and asset, we are able to quantify the default barrier and the outstanding debt, for each firm. Figure 4 shows the average barrier and debt for each region, and reports the barrier-to-debt ratio.

We document similar barrier-to-debt ratios for the different geographic areas, between 67% (Australia) and 81% (United States). Intuitively, this result suggests that the Australian firms are more likely to be rescued by the shareholders with respect to the US firms. These findings are very similar to the estimates reported by Davydenko (2012), who argues that the firms on average default when the asset value drops until 70% of the debt value, and by Wong and Choi (2009), who also estimate a Black-Cox default boundary, and they find that this boundary lies around 74% of the debt.

Finally, we also show that our estimation well fits the real data. By using our estimates of the hyperparameters, and the reconstructed dynamics of the unobservable variables, we generate a fitted time series of equity value and CDS spreads. Figure 5 compares our fitted time series of the market data against the actual observations. We show the average fitted 5-years CDS spread and equity value against the average actual CDS spread and equity value, for each region.

In order to test the ability of our estimation results in fitting the real data, we construct daily long-short portfolio strategies, over the sample time series. By using the fitted dynamics of the equity value and CDS spreads, at each day, we buy those stocks and CDS for which we forecast an increase in the price in the next day, and we sell otherwise.

Table 4 reports a highly significant performance achieved by this strategy, for all securities. As robustness, we perform a long-short portfolio strategy with randomly selected firms. At each day, we randomly split the sample in two sub-samples, and we buy the securities for the firms belonging to the first sub-sample, and we sell the securities for the firms belonging to the second sub-sample. We repeat this exercise for 1000 simulations, where each simulation generates a trading strategy performance over the sample time series. Table 4 reports the maximum performance achieved across the 1000 simulations, which is not significant for none of the securities.

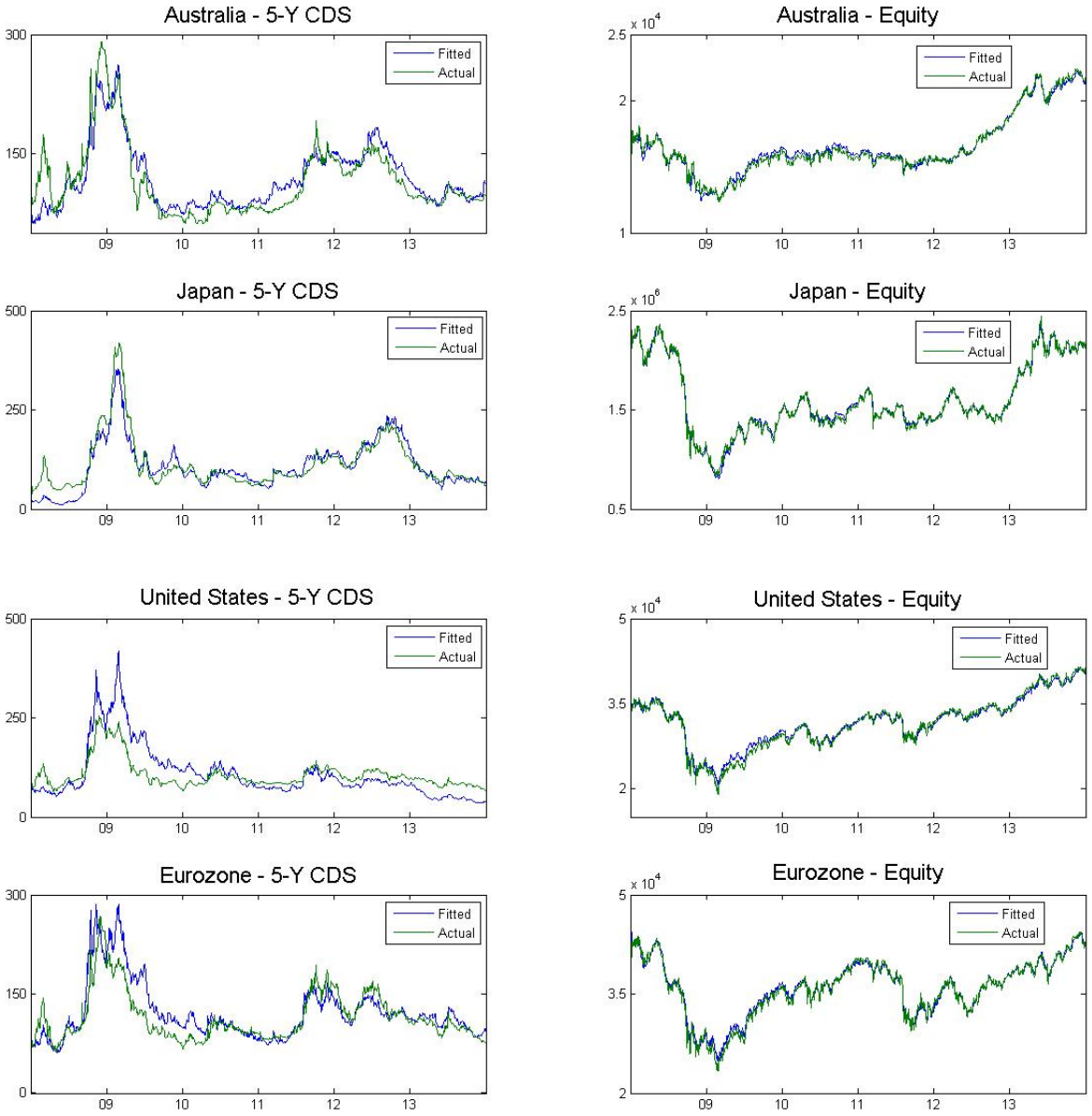
Therefore, we can conclude that our results have significant ability in fitting the real-data, and allow to generate performances that are statistically significant, differently from strategies implemented with random selection of portfolios.

B. The Default Risk Premium

Previous papers have disentangled the estimation of the default risk premium with different approaches, by using either corporate bonds or credit default swaps to extract the risk-neutral probability of default, and the rating agency data to compute the real-world probability of default. Hence, the default risk premium proxies the distance between the actual evaluation of the default risk and the evaluation reflected in the pricing of the securities issued by the company. We extract this measure not only from the credit market, by using CDS spreads, but also form the equity market, by including information on the firm's equity value.

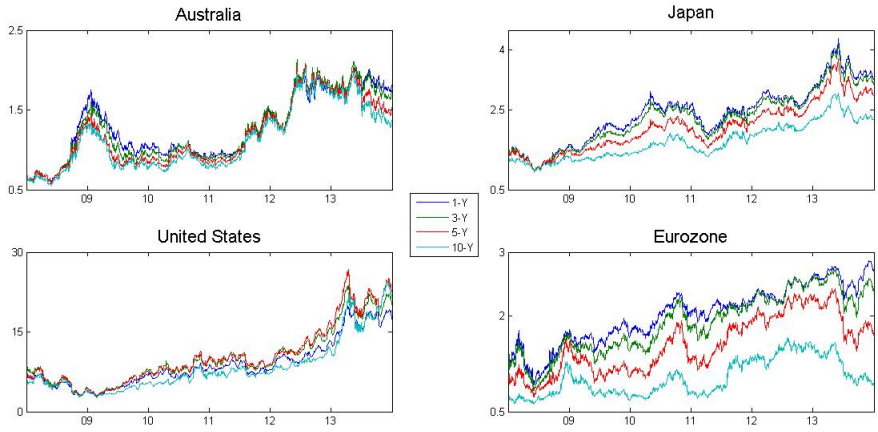
We calculate the default risk premium as the ratio between the risk-neutral and the real-world default probability, for each firm i, for each point in time t, and each time horizon τ_j. The risk-neutral and the real-world measures of default probability are computed by using the equation (11-12), where we plug in the estimates of the hyperparameters, and the estimated

Figure 6. Estimation Performance: Fitting Real Data



The plots compare the fitted dynamics of the 5-years CDS spread and Equity value, by using the structural estimation results, against the observed 5-years CDS spread and Equity value, for the average firm across regions, along the sample time series. The CDS spread is expressed in basis points, and the Equity value is expressed in millions of the local currency.

Figure 7. Term Structure of Default Risk Premia. Median Firm



The plots show the term-structure of the estimated default risk premium, for the median firm across regions, along the sample time series. The default risk premium is expressed as ratio between the risk-neutral and the real-world default probability, which have been both calculated with a daily frequency, and for each firm, by using the structural estimation results.

value of the unobservable variable $L_{i,t}$. The result is a firm-specific daily term structure of the default risk premia.

The difference between the two measures of default probability is given by the drift component of the asset value dynamics. In the equation (11-12), we keep the risk-free rate to estimate the risk-neutral default probability, and we use the actual drift coefficient of the leverage, μ_{L_i}, to estimate the real-world measure of default risk. This coefficient is related to the asset drift coefficient as follows

$$\mu_{V_i} = -\left(\mu_{L_i} - \frac{1}{2}\sigma_{L_i}^2\right),$$

where σ_{L_i} comes from the quasi-maximum likelihood estimation, and μ_{L_i} is the outcome of a matching moment approach.

In fact, after performing the quasi-maximum likelihood algorithm to obtain the estimates of the hyperparameters, we also perform a non-linear squares minimization for calibrating the measurement errors variances to the observed market data, and we reconstruct the dynamics of the market value of the leverage which is able to simultaneously fit the CDS spreads and the equity value. Finally, we compute the mean of the first differences, divided by the time step. Therefore, μ_{L_i} returns a measure of the average growth of the leverage value along the entire time series. It contains information on the growing trend of the leverage coming from both the credit and the equity markets. [6]

Figure 6 shows the term structure of the default risk premia for the median firm in each regional sample. The default risk premium is substantially time varying. The variation over time of the premium is primarily due to the evolution over time of the market value of the firm's asset and leverage. In particular, the premium exhibits a lowest peak around the great recession, and an additional downturn during the sovereign debt crisis, although less pronounced. This pattern is the consequence of the greater actual default risk during the periods

of turmoil, which increases the real-world default probability.

This finding is consistent with Huang and Huang (2012), who argue that the default risk premium decreases as the credit quality declines, as the real-world default risk accounts more for the lower-rating firms. In terms of magnitude, we report similar but lower premia with respect to Berndt et al. (2008) and Driessen (2005), with the exception of the US sample, which exhibits default risk premia significantly larger. Such lower premia are primarily due to our sample period spanning over both the two recent crises, when the actual default risk has substantially increased.

[Insert Table 3, here]

Moreover, the premium is generally higher for the shorter time horizon. In fact, we know that the credit risk models, that do not include jumps in the asset value, tend to underestimate the actual default probability for short time horizons. However, Huang and Huang (2012) argue that including jumps do not substantially affect the fraction of spread explained by credit risk, unless if the model is calibrated with rather extreme values. Second, in particular for investment grade companies, Huang and Huang (2012) also document that the mean reversion of credit quality increases the credit risk for longer maturities.

We generally find that the default risk premium is greater than one, that is the risk-neutral default probability is higher than the corresponding real-world measure (*normal* firms). However, we document the presence of a few firms with estimated actual default probability higher than the corresponding risk-neutral measure, and therefore default risk premium lower than 1 (*distressed* firms).

For instance, narrowing our attention on the 10-years maturity, we find that the 32% of our sample firms (53 out of 164) is characterized by an average default risk premium lower than 1. Looking at the 5-years maturity, instead, the percentage of firms with average default risk premium lower than one drops to 26% of the sample (43 firms). Similar results are obtained for the 3-years (41 firms), and 1-year maturity (40). In the next section we explore the relation between this result and the dynamics of the market data, by implementing a long-short portfolio strategy of equity and CDS, based on the default risk premium.

C. Long-Short Portfolio Strategy

We start linking the default risk premium and the market data dynamics in a descriptive analysis, where we assess the sample correlation between the premium and the market variables. We calculate the average premium for each firm over the sample time series, and we rank the firms according to the average premium in ascending order. In the following graphs we show the correlation coefficient between the default risk premium and both the equity value and the 10-years CDS spread, after sorting the firms.

Figure 7 clearly highlights that the firms with average default risk premium higher than one exhibit positive correlation with the dynamics of the equity value, while the firms with average premium lower than one report opposite relationship with the equity prices. On the

Figure 8. DRP and Equity Prices

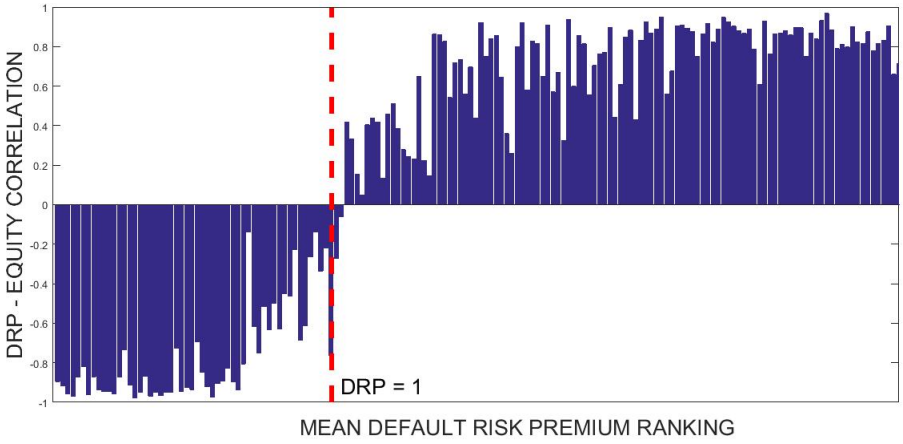


Figure 9. DRP and CDS spreads

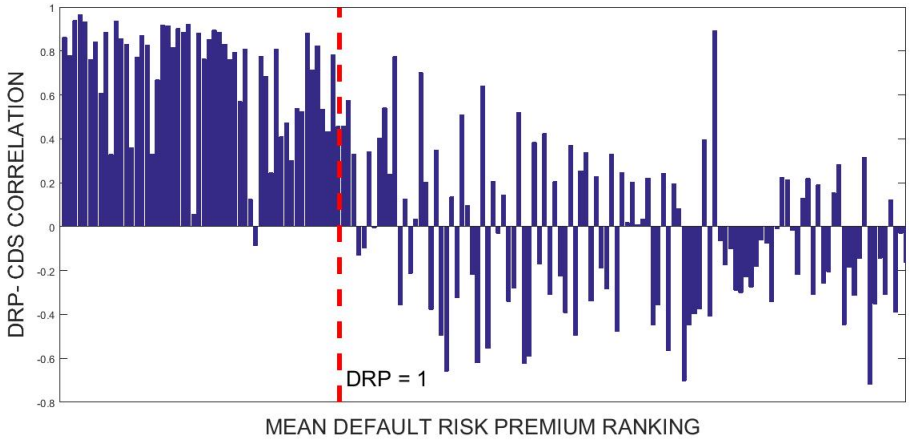


Figure 7 shows the sample correlation between the equity prices and the default risk premium, for firms ranked according to the average premium over the time series. Figure 8 shows the sample correlation between the 10-years CDS spread and the default risk premium, for firms ranked according to the average premium over the time series. In both graphs, the dotted-red line is drawn in correspondence of the first firm with average premium higher than one.

other hand, the low premium firms show positive correlation with the CDS-spread dynamics, as described by figure 8. Then, this correlation substantially decreases as the premium increases, even if this opposite relationship is not as strongly monotone as for the low premium firms.

Further than graphically, table 6 supports these results statistically. The difference in the equity-DRP (Default Risk Premium) correlation between the high and the low risk premium firms is positive and very significant, and the difference in the CDS-DRP correlation between the high and the low risk premium firms is negative and very significant too. The bottom line is that the market variables exhibit a relationship with the default risk premium which is opposite to each other, either if we consider the high or the low premium sub-sample of firms.

We test this empirical hypothesis by constructing long-short portfolios of firm's equity shares and CDS spreads. We implement our trading strategy in the last year of the time series (i.e., 252 daily observations). We start with an in-sample analysis, where we sort the firms according to the t-average default risk premium by using the market prices information up to t, but the estimation of the hyperparameters is based on full sample information, following the approach of Cochrane and Piazzesi (2005).

At each day t, and for each firm, we compute the average default risk premium until $t-1$, by plugging in the equations (11-12) and (14) the estimated value of the state variable ($L_{i,t}$), and the hyperparameters. We split the sample in two sub-samples: the firms with average premium higher than one (HP-firms), and the firms with average premium lower one (LP-firms). For the HP-firms, an expected increase (decrease) in the premium should predict higher (lower) future price of the stock, and lower (higher) future CDS spread. Instead, for the LP-firms, an expected increase (decrease) in the premium should predict lower (higher) future price of the stock, and higher (lower) future CDS spread.

Then, at t, we buy (sell) the stocks and we sell (buy) the CDS of the HP-firms for which our estimation of the premium at $t+1$ is higher (lower) than the current estimate of the premium at t. On the other hand, we sell (buy) the stocks and we buy (sell) the CDS of the LP-firms for which our estimation of the premium at $t+1$ is higher (lower) than the current estimate of the premium at t. We can do that, as the filter generates a forecast for the state variable one-step ahead. Then, by using the equations (11-12), and the estimated hyperparameters, we can produce a forecast for the default probabilities one-step ahead, and thus for the default risk premium by using the equation (6). Table 7 (top-left) reports the average daily performance of the trading strategy, for the equity and the CDS portfolios, respectively. The trading strategy performed by using either the equity shares, or the CDS spread, shows a positive daily mean, which is very significant.

D. Out-of-Sample and Robustness

In the last step we support our results with a set of robustness checks, and an out-of-sample test. First, we start constructing the strategy by excluding, from the feasible basket of firms, those firms with average default risk premium, up to the beginning of 2013, higher

than 1000. Such a big value of the premium is primarily driven by a very low estimation of the real-world default probability, which is the denominator of the ratio. Table 7 shows that the previous results are not affected by narrowing the feasible set of firms for the strategy.

Moreover, we perform the portfolio strategy by trading in the opposite direction with respect to the approach described above. In other words, we buy (sell) the stocks and we sell (buy) the CDS of the LP-firms (instead of the HP-firms) for which our estimation of the premium at $t+1$ is higher (lower) than the current estimate of the premium at t. On the other hand, we sell (buy) the stocks and we buy (sell) the CDS of the HP-firms for which our estimation of the premium at $t+1$ is higher (lower) than the current estimate of the premium at t. The bottom part of table 7 shows that, in-sample, the strategy generates now negative and significant performance for both the portfolios, and even after excluding the outliers.

Finally, we strength our results with an out-of-sample analysis, where we still implement our trading strategy sorting the firms according to the t-average default risk premium, by using the market prices information up to t. However, the estimation of the hyperparameters is based now on the information up to the beginning of the trading strategy window. In practice, we perform again the non-linear filtering in conjunction with quasi-maximum likelihood estimation to extract the model parameters, and to reconstruct the dynamics of the state variable, but now we only use data on the observable market variables up to the beginning of 2013. Though, all the described results are confirmed, with the only exception of the negative performance of the opposite direction trading strategy in the case of the equity portfolio. This performance becomes now positive but not significant.

VI. Conclusion

In this paper we estimate a barrier-dependent contingent claim model, with only market data. By gathering information coming from the credit and the equity markets, we reconstruct the dynamics of the market value of assets and debt, and the default boundary, for a sample of non-financial firms. Then, we exploit the results of our estimation to measure the default risk premium, for each company, that at the same time reflects the information conveyed by the two different markets. This premium arises from the difference between the actual and the market-implied evaluation of the default risk. We find that the default risk premium is time varying, and decreases during the crisis periods because of an increase in the actual risk of default.

Our main finding is that the dynamics of the default risk premium can be associated to completely different dynamics of the stock price and the CDS spreads, and these opposite relationships depend on the type of the firm in terms of risk-return framework.

We believe that our estimation approach may also be addressed for different targets. Combining the result on the default boundary and the market value of the asset, we supply a new insight in the academic research on the corporate credit risk based on the well-known distance-to-default. In addition, we equip the corporate finance research with a new approach for the evaluation of the market value of the debt. It can be then primarily used in the firm's

evaluation framework, and in the firm's profitability assessment debate, such as the discussion on the Tobin's Q.

More importantly, we leave an open question that awaits for further research. Why is it possible to estimate a default risk premium lower than 1, that is an expected rate of growth of the value of the firm even lower than the risk-free rate? Is there any omitted premium component that justifies this counterintuitive result?

Appendix A. A simple asset pricing model

Appendix A.1. Default Risk Premium and Stock Price

Consider two states of the world, *high* and *low*, and a stock that pays out S^h and S^l in the two states, at the payout date T, respectively. We refer to the low state as the default. We define S_t the price of the stock at time t, μ the discounting rate of return under the real-world probability, and r the discounting rate of return under the risk-neutral probability (i.e., the risk-free rate). Hence, at time $t = 0$,

$$S_0 = e^{-\mu} E_0^P[S_T] = e^{-\mu}[p_0 S^l + (1 - p_0)S^h], \tag{7}$$

and

$$S_0 = e^{-r} E_0^Q[S_T] = e^{-r}[q_0 S^l + (1 - q_0)S^h], \tag{8}$$

where E^P indicates the expectation taken under the real-world probability, E^Q indicates the expectation taken under the risk-neutral probability, p is the real-world default probability, and q is the risk-neutral default probability. Therefore,

$$\mu = -\ln\left(\frac{S_0}{E_0^P[S_T]}\right), r = -\ln\left(\frac{S_0}{E_0^Q[S_T]}\right),$$

and

$$p_0 = \frac{S^h - \frac{S_0}{e^{-\mu}}}{S_l}, q_0 = \frac{S^h - \frac{S_0}{e^{-r}}}{S_l}$$

At time $t = 1$,

$$p_1 = \frac{S^h - \frac{S_1}{e^{-\mu}}}{S_l}, q_1 = \frac{S^h - \frac{S_1}{e^{-r}}}{S_l},$$

and

$$\frac{p_1}{p_0} = \frac{e^{-\mu} S^h - S_1}{e^{-\mu} S^h - S_0}, \frac{q_1}{q_0} = \frac{e^{-r} S^h - S_1}{e^{-r} S^h - S_0}$$

Indeed, when $S_1 > S_0$, then $p_1 < p_0$ and $q_1 < q_0$, provided that $S^l < S_t > S^h$. The opposite holds when $S_1 < S_0$.

Moreover, by equating (1) and (2), we obtain

$$\mu - r = \ln\left(\frac{E_0^P[S_T]}{E_0^Q[S_T]}\right)$$

.

Hence, if

$$\mu > r \Leftrightarrow E_0^P[S_T] > E_0^Q[S_T] \Leftrightarrow p(S^l - S_h) > q(S^l - S_h) \Leftrightarrow p < q,$$

as $(S^l - S_h) < 0$. The opposite relationship holds when $\mu < r$.

Let define now Z as the default risk premium, that is the ratio between the risk-neutral and the real-world default probability:

$$Z_0 = \frac{q_0}{p_0},$$

then we have that,

$$\mu > r \Leftrightarrow Z_0 > 1 \tag{9}$$

$$\mu < r \Leftrightarrow Z_0 < 1 \tag{10}$$

We refer to (3) as the *normal* case, and to (4) as the *distressed* one. At every time t,

$$Z_t = e^{r-\mu} \frac{e^{-r} S^h - S_t}{e^{-\mu} S^h - S_t}$$

Hence, the gross rate of growth of the default risk premium $\dot{Z}$ between 0 and 1 is

$$\frac{Z_1}{Z_0} = \frac{e^{-r} S^h - S_1}{e^{-r} S^h - S_0} \cdot \frac{e^{-\mu} S^h - S_0}{e^{-\mu} S^h - S_1} = \dot{q} \cdot 1/\dot{p} \tag{11}$$

Therefore, the default risk premium increases, that is $\dot{Z} > 1$, either because $\dot{q} > \dot{p} > 1$ if $S_1 < S_0$, or because $1/\dot{p} > 1/\dot{q} > 1$ if $S_1 > S_0$. In the first case there is an increase in the risk-neutral default probability higher than the increase in the real-world default probability. This picture is associated to a decrease in the stock price. In the second case there is a decrease in the risk-neutral default probability higher than the decrease in the real-world default probability. This picture is associated to an increase in the stock price.

Moreover, by simply rearranging (5), we can show that

$$\dot{Z} > 1 \Leftrightarrow e^{-r}(S_1 - S_0) > e^{-\mu}(S_1 - S_0)$$

It turns out that $S_1 > S_0$ when $\mu > r$, and $S_1 < S_0$ when $\mu < r$. By combining this result with the statement following the equation (5), we can conclude that an increase in the default risk premium can be associated to an increase in the stock price, when the real-world default probability decreases more than the risk-neutral default probability, and this is the case when $\mu > r$. Otherwise, an increase in the default risk premium can be associated to a decrease in the stock price, when the risk-neutral default probability increases more than the real-world default probability, and this is the case when $\mu < r$.

Appendix A.2. Default Risk Premium and Insurance Price

Consider again the two states of the world, *high* and *low*, and a contingent claim security that pays out B^h and B^l in the two states, at the payout date T, respectively. We refer again to the low state as the default. We think the security B as a type of insurance, that pays out more in case of default, that is $B_l > B_h$. We define B_t the price of the insurance at time t, a the discounting rate under the real-world probability, and c the discounting rate under the risk-neutral probability. Hence, at time $t = 0$,

$$B_0 = e^{-a} E_0^P[B_T] = e^{-a}[p_0 B^l + (1 - p_0)B^h], \tag{12}$$

and

$$B_0 = e^{-c} E_0^Q[B_T] = e^{-c}[q_0 B^l + (1 - q_0)B^h], \tag{13}$$

Then,

$$a = -\ln\left(\frac{B_0}{E_0^P[B_T]}\right), c = -\ln\left(\frac{B_0}{E_0^Q[B_T]}\right),$$

and

$$p_t = \frac{B^h - \frac{B_t}{e^{-a}}}{B_l}, q_t = \frac{B^h - \frac{B_t}{e^{-c}}}{B_l}$$

Hence,

$$\frac{p_1}{p_0} = \frac{e^{-a}B^h - B_1}{e^{-a}B^h - B_0}, \frac{q_1}{q_0} = \frac{e^{-c}B^h - B_1}{e^{-c}B^h - B_0}$$

Indeed, when $B_1 > B_0$, then $p_1 > p_0$ and $q_1 > q_0$, provided that $B^l > B_t > B^h$. The opposite holds when $B_1 < B_0$.
Moreover, by equating (6) and (7), we obtain

$$a - c = \ln\left(\frac{E_0^P[B_T]}{E_0^Q[B_T]}\right)$$

.
Hence, if

$$a > c \Leftrightarrow E_0^P[B_T] > E_0^Q[B_T] \Leftrightarrow p(B^l - B_h) > q(B^l - B_h) \Leftrightarrow p > q,$$

as $(S^l - S_h) > 0$. The opposite relationship holds when $a > c$.

Now, the gross rate of growth of the default risk premium $\dot{Z}$ between 0 and 1 is

$$\frac{Z_1}{Z_0} = \frac{e^{-c}B^h - S_1}{e^{-c}B^h - B_0} \cdot \frac{e^{-a}B^h - B_0}{e^{-a}B^h - B_1} = \dot{q} \cdot 1/\dot{p} \tag{14}$$

Therefore, the default risk premium increases, that is $\dot{Z} > 1$, either because $\dot{q} > \dot{p} > 1$ if $B_1 > B_0$, or because $1/\dot{p} > 1/\dot{q} > 1$ if $B_1 < B_0$. In the first case there is an increase in the risk-neutral default probability higher than the increase in the real-world default probability. This picture is associated to an increase in the insurance price. In the second case there is a decrease in the risk-neutral default probability higher than the decrease in the real-world default probability. This picture is associated to a decrease in the insurance price.

Again, by simply rearranging (8), we can show that

$$\dot{Z} > 1 \Leftrightarrow e^{-c}(B_1 - B_0) > e^{-a}(B_1 - B_0)$$

It turns out that $B_1 > B_0$ when $c < a$, and $B_1 < B_0$ when $c > a$. By combining now with the statement following the equation (8), we can conclude that an increase in the default risk premium can be associated to a decrease in the insurance price, when the real-world default probability decreases more than the risk-neutral default probability, and this is the case when $c > a$. Otherwise, an increase in the default risk premium can be associated to a increase in the insurance price, when the risk-neutral default probability increases more than the real-world default probability, and this is the case when $a > c$.

We can finally state the simultaneous and two-fold relationship between default risk premium and stock price, and between the default risk premium and a contingent claim security such as an insurance (e.g., a CDS spread). An increase in the default risk premium can be associated to an increase in the stock price and a decrease in the insurance price, when the real-world default probability has decreased more than the risk-neutral default probability. This result holds with *normal* securities in the context of the classical risk-return framework, that is $\mu > r$ (a risk-averse investor seeks for an excess rate of return on the top of the risk-free rate), and $a < c$ (a risk-averse investor is willing to pay more to hedge against a risky scenario).
On the other hand, an increase in the default risk premium can be associated to a decrease in the stock price and an increase in the insurance price, when the risk-neutral default probability has increased more than the real-world default probability. This result holds with *distressed* securities, that are in a counterintuitive risk-return framework (ie., $\mu < r$, and $a < c$).

Appendix B. Kalman filter and Quasi-Maximum Likelihood Estimation

In a general formulation, with a non-linear relationship between the measurement and the state variables, the state-space model is defined by two sets of equations, the transition and the measurement equation, respectively:

$$X_{i,t+\delta t} = X_{i,t} + c_i + \epsilon_{i,t+\delta t},$$

$$Y_{i,t+\delta t} = \psi(X_{i,t+\delta t}) + u_{i,t+\delta t},$$

where $X_{i,t+\delta t}$ is the i-th observation of the state variable at time $t+\delta t$, c_i is the time-invariant component driving the evolution of the state variable, $\epsilon_{i,t+\delta t}$ is the transition error on the i-th observation of the state variable at time $t+\delta t$. On the other hand, $Y_{i,t+\delta t}$ is the i-th observation of the measurement variable at time $t+\delta t$, ψ is the measurement function which links the observable and the latent variable, and $u_{i,t+\delta t}$ is the measurement error.

For a Gaussian state-space model, under standard assumptions, the discrete Kalman filter is proved to be the minimum mean squared error estimator. However, in the case of non-linear relation between the measurement and the state variable, the classic linear Kalman filter is not longer optimal. One possible solution is to linearize the estimation around the current estimate by using the partial derivatives of the process and measurement functions.[7] To linearize the measurement process, we need to compute the derivatives of ψ with respect to

(a) the state variable: $H_{i,j} = \frac{\partial \psi_i}{\partial X_j}(\tilde{X}_t, 0)$,

where H is the Jacobian matrix of partial derivatives of the generic measurement function $\psi(\cdot)$ with respect to the state variable X, and $\tilde{X}_t$ is the current estimate of the state.

(b) the measurement noise: $\breve{H}_{i,j} = \frac{\partial \psi_i}{\partial \nu_j}(\tilde{X}_t, 0)$,

where $\tilde{H}$ is the Jacobian matrix of partial derivatives of $\psi(\cdot)$ with respect to the noise term ν.

Once the linearization has been completed, we can implement the discrete Kalman filter in the usual steps. First, we need to set the *initial conditions*:

$$\lambda_{i,0} \quad P_{i,0},$$

where $P_{i,t} := var[X_{i,t} - \lambda_{i,t}]$ is the variance of the estimation error, and $\lambda_{i,t}$ is the estimate of the state at time t based on the information available up to time t. Then, the filter implementation is based upon two sets of equations, the *predicting* equations, and the *updating* equations, that must be repeated for each time step in the data sample.

- *State Prediction*

$$\lambda_{i,t+\delta t/t} = \lambda_{i,t} + c_i,$$

and

$$P_{i,t+\delta t/t} = P_{i,t} + Q_i,$$

where $\lambda_{i,t+\delta t/t}$ is the estimate of the state at time $t + \delta t$ based on the information available up to time t, and Q_i is the covariance of the transition noise.

- *Measurement Update*

$$\lambda_{i,t+\delta t} = \lambda_{i,t+\delta t/t} + P_{i,t+\delta t/t} H'_{i,t+\delta t} Z^{-1}_{i,t+\delta t} \left(Y_{i,t+\delta t} - \psi(\lambda_{i,t+\delta t/t})\right)$$

$$P_{i,t+\delta t} = P_{i,t+\delta t/t} - P_{i,t+\delta t/t} H'_{i,t+\delta t} Z^{-1}_{i,t+\delta t} H_{i,t+\delta t} P_{i,t+\delta t/t}$$

$$Z_{i,t+\delta t} = H_{i,t+\delta t} P_{i,t+\delta t/t} H'_{i,t+\delta t} + R_i,$$

where H stands for the Jacobian matrix of partial derivatives of the generic measurement function ψ with respect to the state variable X, $Z_{i,t+\delta t}$ is the covariance matrix of the prediction errors at time $t + \delta t$. The prediction errors are defined as $v_{i,t+\delta t} = Y_{i,t+\delta t} - \psi(\lambda_{i,t+\delta t/t})$, where $Y_{i,t+\delta t}$ is the observation of the measurement variable at time $t + \delta t$.

The parameters that describe the dynamics of the transition and the measurement equations (i.e., *hyperparameters*) are unknown, and need to be estimated.

Let rewrite the state-space model as follows:

$$(y_{t+\delta t}, x_{t+\delta t}) = (x_t, \{\theta\}), \quad \{\theta\} = \{\theta^{(f)}; \theta^{(g)}\}$$

, where $y_{t+\delta t}$ is the observable variable at time $t + \delta t$, $x_{t+\delta t}$ is the state variable at time $t + \delta t$, $\{\theta^{(f)}\}$ is the set of unknown parameters in the transition equation, and $\{\theta^{(g)}\}$ is the set of unknown parameters in the measurement equation. The measurement and transition equations of the system are:

$$g(y_{t+\delta t}, \alpha) = \varphi(x_{t+\delta t}, \beta) + \epsilon_{t+\delta t}, \quad \epsilon_t \backsim \mathcal{N}(0, \sigma^2_\epsilon)$$
$$x_{t+\delta t} = f(x_t, \gamma) + \eta_{t+\delta t}, \quad \eta_t \backsim \mathcal{N}(0, \sigma^2_\eta)$$

Then,

$$\{\theta^{(f)}\} = \{\gamma, \sigma^2_\eta\}$$
$$\{\theta^{(g)}\} = \{\alpha, \beta, \sigma^2_\epsilon\}$$

We assume that the nonlinear regression disturbance, ϵ_t, is normally distributed:

$$f(\epsilon_t) = \frac{1}{\sqrt{2\pi\sigma_\epsilon^2}} \exp\left[-\frac{\epsilon_t^2}{2\sigma_\epsilon^2}\right]$$

By transformation of variable, the density of y_t is given by

$$f(y_t) = f(\epsilon_t)\left|\frac{\partial \epsilon_t}{\partial y_t}\right|, \quad \frac{\partial \epsilon_t}{\partial y_t} = \frac{\partial g(y_t, \alpha)}{\partial y_t}$$

Then, the density of y_t is

$$f(y_t) = \frac{1}{\sqrt{2\pi\sigma_\epsilon^2}} \exp\left[-\frac{\left(g(y_t, \alpha) - \varphi(x_t, \beta)\right)^2}{2\sigma_\epsilon^2}\right]\left|\frac{\partial g(y_t, \alpha)}{\partial y_t}\right|$$

The log-likelihood function for observation t is

$$\ln \Omega_t \left(y_t; \{\theta\}\right) = -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln(\sigma_\epsilon^2) - \frac{\left(g(y_t, \alpha) - \varphi(x_t, \beta)\right)^2}{2\sigma_\epsilon^2} + \ln\left|\frac{\partial g(y_t, \alpha)}{\partial y_t}\right|,$$

and the log-likelihood function for $t = 1, 2, ..., T$ observations (i.e., $\delta t = 1$) is

$$\ln \Omega = \sum_{t=1}^{T} \ln \Omega_t \left(y_t; \{\theta\}\right) = -\frac{T}{2}\ln(2\pi) - \frac{T}{2}\ln(\sigma_\epsilon^2) - \frac{1}{2\sigma_\epsilon^2}\sum_{t=1}^{T}\left(g(y_t, \alpha) - \varphi(x_t, \beta)\right)^2$$

$$+ \sum_{t=1}^{T} \ln\left|\frac{\partial g(y_t, \alpha)}{\partial y_t}\right|$$

As long as $g(y_t, \alpha) = y_t$, then

$$f(y_t) = f(\epsilon_t) \Rightarrow \ln \Omega_t \left(y_t; \{\theta\}\right) = \ln \Omega_t \left(\epsilon_t; \{\theta\}\right)$$

The last term in the log-likelihood function is equal to zero, and the space of the hyperparameters to be estimated is reduced to:

$$\{\theta^{(f)}\} = \{\gamma, \sigma_\eta^2\}$$
$$\{\theta^{(g)}\} = \{\beta, \sigma_\epsilon^2\}$$

In practice, the iteration of the filter generates a measurement-system prediction error, and a prediction error variance at each step. Under the assumption that measurement-system prediction errors are Gaussian, we can construct the log-likelihood function as follows:

$$\ln \Omega(y_t; \{\theta\}) = \ln \prod_{t=0}^{T-\delta t} p\left(y_{t+\delta t/t}\right) = \sum_{t=0}^{T-\delta t} \ln p\left(y_{t+\delta t/t}\right) =$$

$$= -\frac{N}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=0}^{T-\delta t}\ln|Z_{t+\delta t}| - \frac{1}{2}\sum_{t=0}^{T-\delta t} v_{t+\delta t}{}' Z_{t+\delta t}^{-1} v_{t+\delta t},$$

where N is the number of time steps in the data sample. Finally, this function is maximized with respect to the unknown parameters vector $\{\theta\}$. This is known as the ***Quasi-Maximum Likelihood*** estimation, in conjunction with the non-linear Kalman filter.

Appendix C. Prices Observation Noise: Minimization by Model Calibration

A useful interpretation of the Kalman filtering is to think of it as an updating process, where we first form a prior preliminary guess about the unobservable state variable and then we perform a correction to this guess. It is a typical Bayesian procedure. The correction depends on how good has been the guess in predicting the next observation, and the weight of the correction to update our guess about the state variable primarily depends on how confident we are about the reliability of the observation. It is measured by the variance of the errors in the measurement equation, which links the state and the observable variable.

It follows that we are more confident about the observation of the CDS-PD and the equity prices when lower is the variance of the measurement errors. In the extreme case, when the error associated with the CDS-PD measurement equation has zero variance and the equity equation has high measurement error variance, we take into account only the CDS prices to reconstruct the dynamics of the market value of the leverage, and vice versa. We interpret this variance as a measure of the noise in the observation of the market data. This noise may be a consequence of the market microstructure effects, the model misspecification, or even the recording price procedure for the CDS spreads, which are traded on an OTC market, and the quotes may vary according to the data provider. It turns out that the impact of the market data on the estimated hyperparameters and state variable dynamics is inversely proportional to the observation noise measured by the measurement error variance.

In principle, we could impose an arbitrary prior on the variance of the measurement error either of the CDS-PD equation or the equity equation according to the weight that we want to assign to a particular market variable. It would be the case if we have an ex-ante conjecture on the informativeness of the credit or the equity market. However, we do not impose any prior constraint, and we include the variance of the measurement errors in the hyperparameters that we estimate, by using the quasi-maximum likelihood algorithm based on the non-linear filtering. Then, in a second stage, we implement a non-linear squares minimization to calibrate the values of the measurement error variances associated with the risk-neutral PD and the equity pricing equation, in order to fit simultaneously the observed dynamics of the risk-neutral PD implied by CDS spreads, and the equity value.

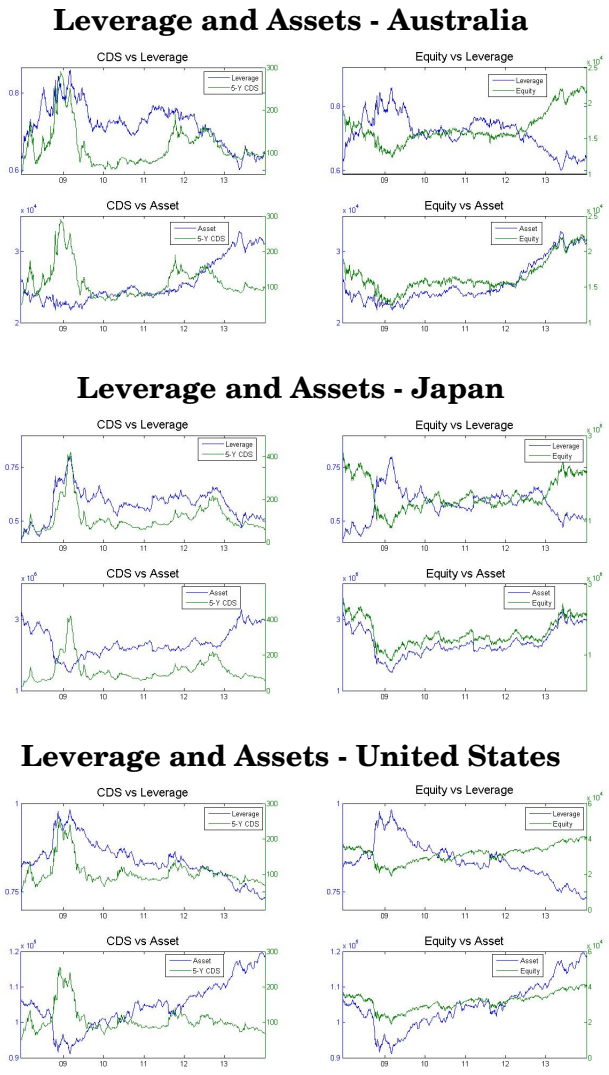
Driessen (2005) adopts a similar algorithm to derive the risk-premium component which translates the real-world probability measure of default, extracted from rating agency data, into the risk-neutral intensity. In practice, he performs a non-linear squares minimization by using the observed and the model-implied default probabilities, after estimated the hyperparameters with a Kalman filter. We search the value of the measurement errors variances which minimize the sum of the squared differences between the observed and the model-implied market data, where the sum is calculated over the sample time series.

$$\min_{R_i,\omega_i} \sum_{t=1}^{T} \alpha_{i,t}^2,$$

where $\alpha_{i,t} = \begin{bmatrix} \hat{PD}^{\mathcal{Q}}_{i,t}(\tau_a) - PD^{\mathcal{Q}}_{i,t}(\tau_a) \\ \hat{E}_{i,t} - E_{i,t} \end{bmatrix}$

Finally, we iterate again the updating and the predicting equations of the non-linear Kalman filter to reconstruct the dynamics of the firm's market value of the leverage. Now, we do not perform the quasi-maximum likelihood estimation as we have already obtained the hyperparameters in the first step, and the measurement errors variances in the second stage. The result is the dynamics of the market value of the leverage which minimizes the distance between the implied and the observed market variables. It may be interpreted as the 'best-we-can-do' to reduce the noise in the prices observation, due to the model misspecification. We can attribute the residual error to the systematic noise implicitly contained in the market quotes observation.

Appendix D. Leverage and Asset against Market data: Other Regions



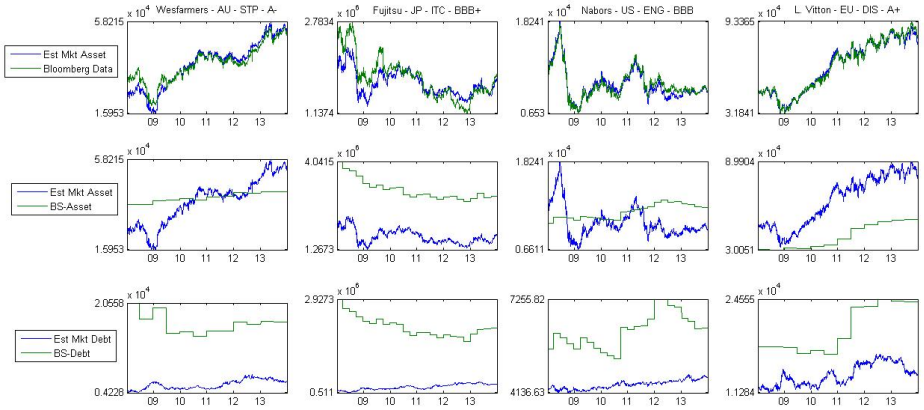
The plots report the same comparison between estimated average leverage and asset value against observed average market prices dynamics, as shown in the figure 3 for the Eurozone sample. See legend of figure 3.

Appendix E. Goodness of Fit: Bloomberg Data

The estimation of the dynamics of the market value of the leverage is performed for each firm one-by-one, and the hyperparameters of the model are firm-specific. As consequence, by using the set of equations (5), we can reconstruct the dynamics of the market value of asset and debt for each company. We can compare these results against the data provided by Bloomberg on the market value of asset, and the balance-sheet data of asset and debt. More precisely, Bloomberg gives information on the market value of the asset by gathering the data on the equity value, the market prices of the debt items traded on the market, and the balance-sheet data of the debt items not traded on the market. Then, especially for that companies with many debt items not traded on the market, Bloomberg needs a big amount of information from the balance-sheet to produce this kind of data. Instead, we use only market variables, which can be easily collected, to provide the same data. We are quicker in the production of the data, and we do not need to rely on any book information.

In the following figures, we show (first row) the comparison between the Bloomberg data on the market value of asset and our estimation of the market value of asset, (second row) the balance-sheet data on the value of the asset and our estimation of the market value of asset, and (third row) the balance-sheet data on the value of the debt and our estimation of the market value of debt. In particular, the first comparison is done between two time series of daily data, whilst the second and the third rows compare daily data (our estimation), and quarterly data (book information). We focus on the description of the first one, where we contrast two equivalent, in principle, sets of information. Nevertheless, the remaining two are useful to give a graphical intuition of the differences, in terms of value and frequency, between market and book evaluation of the same quantity, which we have underlined in the introduction, the motivation, and the contribution of the paper. We choose twelve illustrative firms, which belong to different regions, sectors, and ratings, in order to show that these results are robust across different firms' categories.

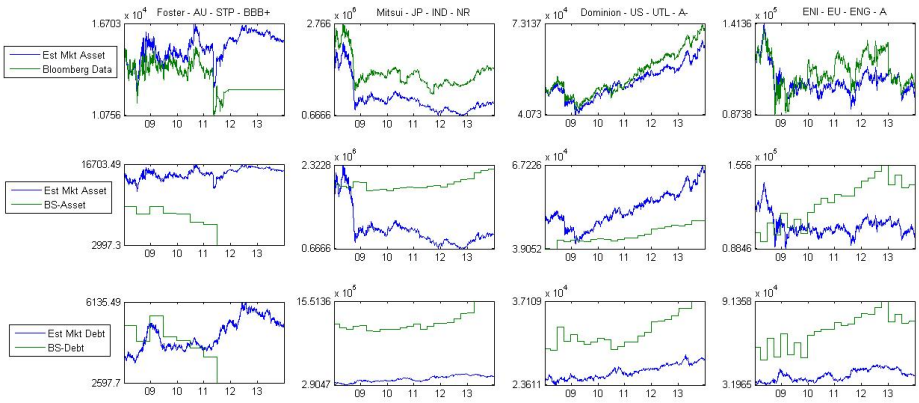
E-I



The figure E-I shows four examples where our model produces an estimation of the market value of the asset that is very similar to the data provided by Bloomberg. For almost the entire

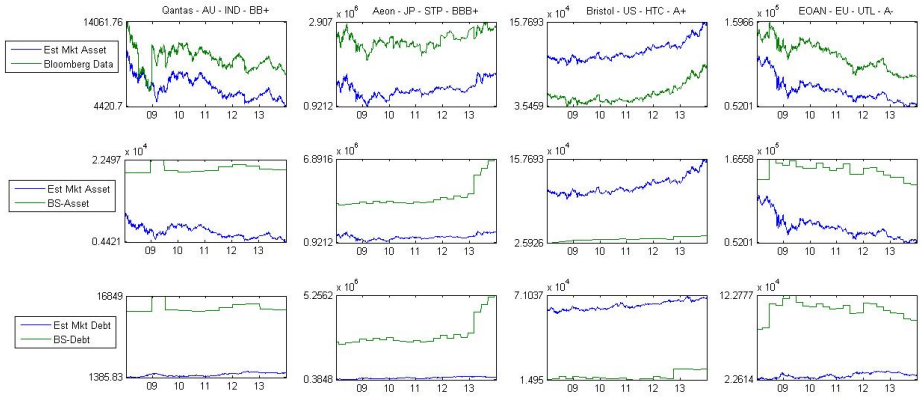
time series, the two dynamics seem to really overlap. It means that we and Bloomberg are providing exactly the same data. Moreover, we are able to be quicker, and totally ignore the book information. However, if they are nearly equal, which is the point to collect data on the equity value and the CDS spreads, construct a proxy of the risk-free rate, and then implement our estimation model, if it would be possible to easily download the same result directly from a Bloomberg terminal? The answer is in the following graphs.

E-II



In the first plot of the figure E-II (Foster's Group), our estimation and Bloomberg data are very similar in the first part of the sample period, then Bloomberg would give stale data, while we keep providing fresh daily information. In the second and the third plot, the two series are similar in the first part, then the Bloomberg data is constantly larger than our estimation. In the last plot, this situation occurs only in specific time windows of the time series. More precisely, the two lines pretty overlap along the wider part of the time series, but Bloomberg reports a few jumps that we do not obtain in our results.

E-III



The figure E-III, finally, shows that the two sets of data can be really different for the entire time interval. The difference is not very noticeable in the trend, as for the both data sets there is a common and equal item (the equity value), but particularly in the levels. Such difference is due to the different evaluation of the firm's debt, which is highlighted in the last row of the figure E-III. When we obtain a market value of debt clearly lower than the book data of debt, we provide data on the market value of asset which is much lower than the Bloomberg data. The opposite case appears in the third plot. In this regard, we need to remind that Bloomberg uses the balance-sheet data when the debt items are not traded on the market. The conclusion is that we may be able to produce a more reliable and fresher information, by ignoring the book data and by taking into account only the market assessment.

Table 1 - Summary Statistics, CDS Spreads

		1 year						3 years					
STAT		**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**
Mean		**86.69**	**140.74**	**43.36**	**39.80**	**41.74**	**18.81**	**107.93**	**151.43**	**69.40**	**72.73**	**84.67**	**52.12**
Standard Deviation	**TS**	71.84	88.29	11.21	19.18	11.12	3.61	65.10	78.97	11.23	23.48	12.13	7.63
	CS	166.38	300.91	37.34	27.09	48.26	30.79	167.02	274.38	50.89	40.43	70.35	44.10
Median	**TS**	60.38	107.44	40.92	28.08	39.78	18.83	86.00	121.56	65.95	57.75	81.14	52.18
	CS	56.81	83.70	34.34	32.98	27.20	13.38	75.61	93.73	54.98	64.14	62.78	41.50
		5 years						**10 years**					
STAT		**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**
Mean		**121.91**	**156.27**	**93.87**	**106.15**	**122.18**	**92.24**	**128.75**	**148.60**	**111.62**	**136.59**	**154.57**	**136.96**
Standard Deviation	**TS**	60.49	70.16	11.36	24.68	12.45	9.24	54.42	58.39	9.79	25.19	12.13	7.74
	CS	159.98	251.92	60.66	52.03	85.45	63.83	143.89	216.25	63.77	58.13	88.68	74.67
Median	**TS**	102.52	127.58	93.34	90.71	118.41	92.56	112.20	119.15	111.89	125.61	154.81	138.43
	CS	89.88	101.93	75.55	94.53	98.07	74.61	99.54	101.27	91.88	122.19	131.77	117.72

Legend: Table 1 reports the summary statistics of the CDS spreads for all the 164 firms in the sample, and for the four time horizons. The statistics are calculated over two dimensions, the time series (TS), and the cross section (CS). The years refer to the time period going from the 20th December of the previous year to the 19th December of that year. The CDS spreads are expressed in basis points as annualized percentage of the notional value of the transaction.

Table 2 - Summary Statistics, Market Capitalization

		Australia						Japan					
STAT		**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**
Mean		**16106.98**	**14269.40**	**15830.27**	**15558.27**	**16615.72**	**20973.81**	**3306770.16**	**2187244.71**	**2374068.84**	**2312945.03**	**2354328.36**	**3650067.03**
Standard Deviation	**TS**	1388.69	1082.65	336.20	330.65	1026.57	934.35	702575.93	249284.28	167946.76	210075.19	150128.26	404024.02
	CS	16552.35	13902.67	14003.61	14192.08	16159.59	20936.58	4449327.34	3234826.62	3015261.81	2841162.46	2981459.13	5360640.13
Median	**TS**	16079.42	13988.59	15836.51	15630.89	16327.32	21121.01	3482359.81	2250207.16	2354792.22	2350307.91	2327871.27	3825030.35
	CS	6971.62	5591.93	7826.98	8502.39	8996.27	10797.14	1594845.22	1047443.75	1301167.07	1355906.53	1270331.88	1559423.72

		United States						Europe					
STAT		**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**
Mean		**31736.19**	**24972.26**	**29632.92**	**31879.01**	**33599.42**	**38801.01**	**37476.06**	**29748.36**	**36045.39**	**36186.81**	**35124.99**	**39957.96**
Standard Deviation	**TS**	4395.72	2568.67	1269.05	1566.59	815.32	1945.23	6002.41	3337.60	1556.23	3476.93	1597.79	1934.45
	CS	38637.15	32837.07	35168.41	36649.62	41794.40	44666.25	42760.72	33453.42	42475.56	42916.63	42344.26	47475.09
Median	**TS**	33456.44	24420.28	29546.20	32264.76	33676.55	39087.60	39113.69	28951.05	35924.19	37473.11	35617.56	39421.99
	CS	18109.02	14066.85	17251.33	18248.14	18756.83	22731.48	21511.27	16238.91	20440.47	19406.24	18639.01	22046.98

Legend: Table 2 reports the summary statistics of the market capitalization across regions. The same structure in terms of time intervals as for the CDS table applies. The market capitalization is expressed in millions of the local currency.

Table 3: Empirical Results - Parameters Estimation

			Volatility	Barr/FV	Drift	Debt	Barrier
Regions	**AUS**	**Mean**	0.19	-0.39	-0.08	17119	11746
		St. Dev.	0.03	0.06	0.04	17378	12133
	JAP	**Mean**	0.19	-0.30	-0.10	841215	1137242
		St. Dev.	0.04	0.10	0.09	676442	474817
	USA	**Mean**	0.08	-0.21	-0.04	71826	86770
		St. Dev.	0.02	0.08	0.02	84963	99737
	EUR	**Mean**	0.15	-0.34	-0.06	33972	47644
		St. Dev.	0.03	0.05	0.04	40132	55945

Legend: The table reports the mean and the standard deviation of real data estimation on 164 firms divided, respectively, in four geographic regions, nine GICS sectors, and three Rating classes, for the unknown parameters σ_{L_i}, K_i, μ_{L_i}, F_i, C_i, R_i, ω_i, by using the univariate non-linear Kalman filter in conjunction with quasi-maximum likelihood estimation. The statistics for F_i, C_i are reported only for the four regions as the respective values are expressed in local currency.

Table 4: Structural Estimation - Fitting Data

	Equity			**5-Y CDS**			**10-Y CDS**		
	SME	Merton	Random	SME	Merton	Random	SME	Merton	Random
Mean	0.011***	-4.76E-05	5.41E-05	0.008***	0.005***	-3.00E-05	0.007***	4.00E-03***	-4.00E-05
t-test	48.47	-0.28	1.03	24.46	16.16	-0.4	21.92	15.88	-0.58

Legend: The table reports the mean, and the t-test statistic, for the daily performance of the long-short portfolio trading strategy described in section 5, for the Equity and the CDS portfolios. The stars on the mean stand for the level of significance according to the t-test outcome (*** = 99% significance level, ** = 95%, * = 90%). The SME columns refer to the portfolio strategy constructed by using the barrier-dependent model estimation, the Merton columns report the performance achieved by using the estimation with the Merton model, while the Random columns refer to the maximum performance achieved across the 1000 randomly constructed portfolios.

Table 5: The Default Risk Premium

	Real-World PD				**Risk-Neutral PD**				**Default Risk Premium**			
	1Y	**3Y**	**5Y**	**10Y**	**1Y**	**3Y**	**5Y**	**10Y**	**1Y**	**3Y**	**5Y**	**10Y**
AUS	0.001	0.031	0.071	0.166	0.001	0.033	0.073	0.179	1.27	1.25	1.18	1.13
JAP	0.000	0.020	0.051	0.139	0.000	0.029	0.074	0.186	2.38	2.29	2.05	1.70
USA	0.000	0.006	0.013	0.032	0.001	0.028	0.068	0.173	8.78	10.03	10.35	8.28
EUR	0.000	0.023	0.063	0.181	0.001	0.035	0.082	0.186	2.02	1.88	1.53	1.02

Legend: The table reports the mean over the entire time series of the estimated median-firm risk-neutral and real-world default probability, and default risk-premium, across the four time horizons (1 year, 3,5, and 10 years), and the four geographic regions.

Table 6: Correlation DRP - Market Data

	5-th	25-th	50th	75-th	95th
corrEQ	-0.95	-0.28	0.31	0.88	0.89
corrCDS	0.77	0.08	-0.35	-0.49	-0.44

	DRP-Equity		DRP-CDS	
	DRP<1	DRP>1	DRP<1	DRP>1
Mean	-0.76	0.68	0.69	-0.03
t-test Diff	-33.39		15.11	

Legend: The upper table reports the sample correlation between the default risk premium and the equity prices, and the 10-years CDS spread, respectively, in correspondence of different firm-percentiles of the default risk premium distribution, after sorting the firms according to the average default risk premium. The lower table reports the mean correlation, across the high and low default risk premium sub-samples, between the default risk premium and the market variables. Moreover, the table reports the t-test outcome on the difference between the mean correlations across the high and low premium samples, for each market variable.

Table 7: Trading Strategy - Long-Short Portfolios

Correct Trading								
	All Firms				**Excluding Outliers**			
	In-Sample		**Out-of-Sample**		**In-Sample**		**Out-of-Sample**	
	Equity	CDS	Equity	CDS	Equity	CDS	Equity	CDS
Mean	0.0015***	6.29E-04***	1.98E-04**	2.55E-04**	7.47E-04***	4.03E-04***	2.76E-04***	2.50E-04***
t-test	16.89	5.34	2.31	2.34	8.29	3.64	3.17	2.49
Opposite Trading								
	All Firms				**Excluding Outliers**			
	In-Sample		**Out-of-Sample**		**In-Sample**		**Out-of-Sample**	
	Equity	CDS	Equity	CDS	Equity	CDS	Equity	CDS
Mean	-0.0023***	-0.0011***	1.75E-04	-6.73E-04**	-8.47E-04***	-8.21E-04***	8.24E-04	-4.97E-04*
t-test	-12.72	-3.65	0.91	-2.13	-5.11	-2.82	0.45	-1.64

Legend: The table reports the mean, and the t-test statistic, for the daily performance of the long-short portfolio trading strategy described in section 6, for the Equity and the CDS portfolios. The stars on the mean stand for the level of significance according to the t-test outcome (*** = 99% significance level, ** = 95%, * = 90%). The table reports the results for the in-sample, and the out-of-sample approach, as described in the paper. The upper part of the table reports the results obtained by following the trading strategy scheme as described in the paper, while the lower part reports the results obtained by trading in the opposite direction. The left-side reports the results for the trading strategy performed by using all the sample firms, while the right-side reports the results for the trading strategy performed after excluding the firms with default risk premium higher than 1000.

Notes

[1]The original formulation for the default probability and the equity value in a first-time passage framework can be found, among others, in Perlich and Reisz (2007)

[2]See Longstaff et al. (2005) for an extensive description of the CDS contractual structure, and functioning

[3]The Markit iTraxx and CDX indices are constructed every six months, according to specified criteria and selections rules, which ensure the eligibility of an entity to be a constituent of the index, especially in terms of CDS contracts liquidity. We refer to the 20th list for the iTraxx indexes, and the 21th list for the CDX index. All have been issued on September 2013

[4]This term is also used for the conventional quarterly termination dates of credit default swaps, which fall on 20 March, 20 June, 20 September and 20 December. From late 2002, the CDS market began to standardize credit default swap contracts so that they would all mature on one of the four days of 20 March, 20 June, 20 September and 20 December. These dates are used both as termination dates for the contracts and as the dates for quarterly premium payments. So, for example, a five-year contract traded any time between 20 September 2005 and 19 December 2005 would have a termination date of 20 December 2010.

[5]See O'Kane and Turnbull (2003) for an extensive overview of the CDS valuation

[6]By adopting this procedure we also avoid the issue related to the insensitivity of the likelihood function with respect to the drift coefficient. Previous works have widely reported huge complexity in estimating the drift component in stochastic processes. We find that even trying to solve this problem by calibration is pretty useless. The impact of changing the drift coefficient of the state variable dynamics on the value of the sum of the squared errors is quite negligible. Instead, the likelihood function is very sensitive to the diffusion component, and we can directly estimate σ_{L_i} in the first step

[7]Instances of application of this filter deal in particular with interest rates term structure estimation. Duan and Simonato (1999) implement this technique to estimate an exponential-affine term structure models, while more recently Duffee and Stanton (2012) prove the robustness of the non-linear Kalman filter for a dynamics term-structure model estimation

REFERENCES

Almeida, H., and T. Philippon, 2007, The risk-adjusted cost of financial distress, *Journal of Finance* 62, 2557–2586.

Berndt, A., R. Douglas, D. Duffie, M. Ferguson, and D. Schranz, 2008, Measuring default risk premia from default swap rates and edfs, Working Paper*, Tepper School of Business at Carnegie Mellon University* .

Black, F., and J. Cox, 1976, Valuing corporate securities: some effects of bond indenture provisions, *Journal of Finance* 31, 351–367.

Blanco, R., S. Brennen, and I. Marsh, 2005, An empirical analysis of the dynamic relationship between investment grade bonds and credit default swaps, *Journal of Finance* 60, 2255–2281.

Bohn, J., and P. Crosbie, 2003, Modeling default risk, *Moody's KMV* .

Brockman, P., and H.J. Turtle, 2003, A barrier option framework for corporate security valuation, *Journal of Financial Economics* 67, 511–529.

Cochrane, J.H., and M. Piazzesi, 2005, Bond risk premia, *American Economic Review* 95, 138–160.

Davydenko, S.A., 2012, When do firms default? a study of the default boundary, *Working Paper* .

Diaz, A., J. Groba, and P. Serrano, 2013, What drives corporate default risk premia? evidence from the cds market, *Journal of International Money and Finance* 37, 529563.

Driessen, J., 2005, Is default event risk priced in corporate bonds?, *Review of Financial Studies* 18, 165–195.

Duan, J.C., 1994, Maximum likelihood estimation using price data of the derivative contract, *Mathematical Finance* 4, 155–157.

Duan, J.C., and J.G. Simonato, 1999, Estimating exponential-affine term structure models by kalman filter, *Review of Quantitative Finance and Accounting* 13, 111–135.

Duffee, G.R., and R.H. Stanton, 2012, Estimation of dynamic term structure models, *The Quarterly Journal of Finance* 2, 1–54.

Ericsson, J., and J. Reneby, 2004, An empirical study of structural credit risk models using stock and bond prices, *Journal of Fixed Income* 13, 38–49.

Forte, S., 2011, Calibrating structural models: A new methodology based on stock and credit default swap data, *Quantitative Finance* 11, 1745–1759.

Forte, S., and L. Lovreta, 2012, Endogenizing exogenous default barrier models: The mm algorithm, *Journal of Banking and Finance* 36, 1639–1652.

Geske, R., 1979, The valuation of compound options, *Journal of Financial Economics* 7, 63–81.

Huang, J.Z., and M. Huang, 2012, How much of the corporate-treasuty yield spread is due to credit risk?, *Review of Asset Pricing Studies* 2, 153–202.

Hull, J., M. Predescu, and A. White, 2005, Bond prices, default probabilities, and risk premiums, *Journal of Credit Risk* 1, 53–60.

Jones, E., S. Mason, and E. Rosenfeld, 1984, Contingent claims analysis of corporate capital structures: An empirical investigation, *Journal of Finance* 39, 611–625.

Li, K.L., and Y.H. Wong, 2008, Structural models of corporate bond pricing with maximum likelihood estimation, *Journal of Empirical Finance* 15, 751–777.

Longstaff, F., S. Mithal, and E. Neis, 2005, Corporate yield spreads: Default risk or liquidity? new evidence from the credit default swap market, *Journal of Finance* 60, 2213–2253.

Longstaff, F.A., and E.S. Schwartz, 1995, A simple approach to valuing risky fixed and floating rate debt, *Journal of Finance* 50, 789–819.

Merton, R., 1974, On the pricing of corporate debt: the risk structure of interest rates, *Journal of Finance* 29, 449–470.

O'Kane, D., and S. Turnbull, 2003, Valuation of credit default swaps, *Fixed Income Quantitative Credit Research - Lehman Brothers* .

Perlich, C., and A. Reisz, 2007, A market-based framework for bankruptcy prediction, *Journal of Financial Stability* 3, 85–131.

Ronn, E.I., and A.K. Verna, 1986, Pricing risk-adjusted deposit insurance: An option-based model, *Journal of Finance* 41, 871–895.

Vassalou, M., and Y. Xing, 2004, Default risk in equity returns, *Journal of Finance* 59, 831–868.

Wong, H.Y, and T.W. Choi, 2009, Estimating default barriers from market information, *Quantitative Finance* 9, 187–196.