Joint time-frequency representation of simulated earthquake accelerograms via the adaptive chirplet transform

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ABSTRACT: Seismic accelerograms are inherently nonstationary signals since both the intensity and frequency content of seismic events evolve in time. The adaptive chirplet transform is a signal processing technique for joint time-frequency representation of nonstationary data. Analysis of a signal via the adaptive chirplet decomposition in conjunction with the Wigner-Ville distribution yields the so-called adaptive spectrogram which constitutes a valid representation of the signal in the time-frequency plane. In this paper the potential of this technique for capturing the temporal evolution of the frequency content of strong ground motions is assessed. In this regard, simulated nonstationary earthquake accelerograms compatible with an exponentially modulated and appropriately filtered Kanai-Tajimi spectrum are processed using the adaptive chirplet transform. These are samples of a random process whose evolutionary power spectrum can be represented by an analytical expression. It is suggested that the average of the ensemble of the adaptive chirplet spectrograms can be construed as an estimate of the underlying evolutionary power spectrum. The obtained numerical results show, indeed, that the estimated evolutionary power spectrum is in a good agreement with the one defined analytically. This fact points out the potential of the adaptive chirplet analysis for as a tool for capturing localized frequency content of arbitrary data- banks of real seismic accelerograms.

Keywords: chirplet decomposition, joint time-frequency analysis, non stationary random process, evolutionary power spectrum, earthquake accelerograms

1 INTRODUCTION

The non-stationary features of strong ground motions have long been recognized by the earthquake engineering community by studying accelerograms pertaining to actual seismic events. The dispersion of the propagating seismic waves reflects on the evolving frequency composition and intensity of seismic signals in time. Ordinary Fourier analysis provides only the average spectral decomposition of a signal, and thus cannot adequately represent the time- dependent frequency content of seismic signals. Clearly, the use of a joint time- frequency analysis is a more reliable approach for the study of such signals, and leads to a better capturing of their non-stationary nature.

In recent years the wavelet transform has become a potent signal processing tool that can be used to yield valid time-frequency representations of nonstationary signals; see for instance Mallat (1998) and Spanos and Failla (2005). However, a more specialized signal analysis scheme for the purpose is the so-called adaptive chirplet transform (ACT) (Qian, 2001). The latter method resembles the standard wavelet transform in the sense that it also utilizes oscillatory analyzing functions of localized energy in time, namely Gaussian chirplets, for the decomposition of signals. Nevertheless, the ACT is capable of representing a greater variety of signals since Gaussian chirplets are more versatile functions incorporating more than wavelets "degrees of freedom" (Baraniuk and Jones, 1996). It is also more efficient enabling more economical representations of signals with fewer terms. This is due to the fact that Gaussian chirplets form frames, and thus a set of these functions is over-determined (Chen et al., 1998 and Qian, 2001). Different subsets from a collection of Gaussian chirplets are chosen for the representation of different signals that echoes on the adaptive character of the method.

Furthermore, the ACT scheme circumvents the obscure, in many practical cases, signal representation on the time-scale domain (scalogram), which is interwoven with the wavelet transform. Instead, it directly leads to the adaptive chirplet spectrogram: an amenable to physical interpretation distribution of the energy of the original signal on the time-frequency plane (Qian, 2001 and Wang et. al., 2002). This is achieved by exploiting the appealing mathematical properties of the Wigner-Ville distribution (Cohen, 1995).

In light of the above, the main motivation of the present study is to assess the appropriateness of the ACT for capturing the localized frequency content of strong ground motions, and for tracing the temporal variation of their spectral composition. To this end, a uniformly modulated non-stationary stochastic process as introduced in Priestley (1965) is considered. It is characterized by an analytically defined evolutionary filtered Kanai-Tajimi power spectrum (Clough and Penzien, 1993). Non-stationary time histories representing artificial seismic accelerograms compatible with this evolutionary power spectrum are generated and analyzed via the ACT.

To extend the applicability of the ACT for the study of random processes, it is proposed to treat the average of the adaptive chirplet spectrograms of the individual simulated accelerograms as an estimate of the underlying evolutionary power spectrum. Obviously, a comparison of the analytic evolutionary power spectrum with its estimate provided by the averaged adaptive chirplet spectrograms serves as an indication of the effectiveness of the ACT for the undertaken purpose.

2 THE ADAPTIVE CHIRPLET TRANSFORM

The adaptive chirplet transform (ACT) is a signal analysis technique specifically developed for joint time-frequency representation of nonstationary data. The process of any finite energy non-stationary signal via this method is performed in two stages. In the first stage, the signal is decomposed onto a set of analyzing functions of special structure, the Gaussian Chirplets. This is accomplished by making use of a special numerical procedure, the Matching Pursue (MP) algorithm. Subsequently, the Wigner-Ville distribution (WVD) of the decomposed signal is computed to yield the so-called Adaptive Spectrogram (AS). The AS constitutes an image of exceptional resolution of how the energy of the signal is distributed on the time-frequency plane (Qian, 2001). The remainder of this section discusses briefly the most pertinent of the mathematical details of the ACT.

2.1 *Signal decomposition via the Matching Pursue Algorithm*

Consider the Gaussian function of unit energy and standard deviation, centered at t= 0. That is,

$$g(t)=\sqrt[4]{\frac{1}{\pi}}\exp\left(-\frac{1}{2}t^2\right) \quad (1)$$

The Gaussian chirplet $h_k(t)$ is a four- parametered function described by the equation

$$h_k(t)=\sqrt[4]{\frac{a_k}{\pi}}\exp\left\{-\frac{a_k}{2}(t-t_k)^2+ i\left(\frac{\beta_k}{2}(t-t_k)^2+\omega_k(t-t_k)\right)\right\} \quad (2)$$

It is constructed by introducing four successive transformations on the Gaussian function namely: scaling in time by α_k, shifting in time and in frequency by t_k and ω_k, respectively, and multiplying by a linear frequency modulation signal of chirprate β_k (Mann and Haykin, 1995). Gaussian chirplets attain finite effective support both in the time and in the frequency domain as illustrated in Figure 1. Thus, they are capable of capturing the local characteristics of highly non-stationary signals in both domains.

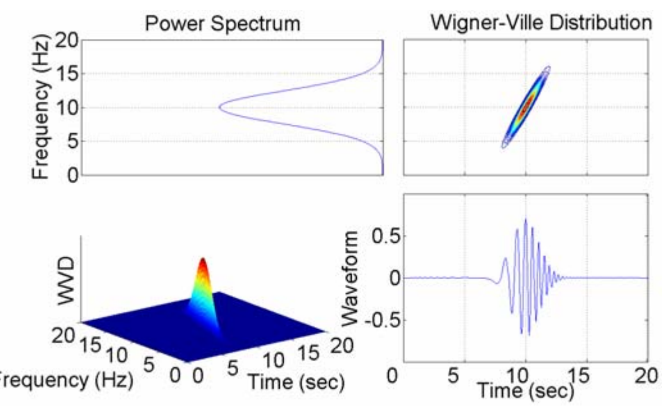

Figure 1. Wigner-Ville distribution of a Gaussian Chirplet with parameters α_k= 0.8; t_k = 10; ω_k = 10; β_k = 3

Consider a non-stationary signal $x(t)$ of finite energy satisfying the condition

$$E=\|x(t)\|^2=\int_{-\infty}^{\infty}|x(t)|^2\,dt<\infty, \quad (3)$$

where E is the total energy of the signal. Developed independently by Mallat and Zhang (1993), and Qian and Chen (1994), the matching pursue (MP) algorithm, allows the decomposition of any signal satisfying Equation 3 into a linear combination of any set of analyzing functions (dictionary) (Chen et al., 1998). Employing a dictionary consisted of Gaussian chirplets the MP algorithm yields the following decomposition of the signal $x(t)$:

$$x(t)=\sum_k A_k h_k(t)+r(t), \quad (4)$$

where A_k are the expansion coefficients determined sequentially by solving the optimization problem

$$\left|A_k\right|^2 = \max_{h_k} \left| \int_{-\infty}^{\infty} x_k(t) h_k(t)\, dt \right|^2 , \quad (5)$$

and *r(t)* is the final residual.

For k=0 the original signal $x(t) = x_k(t)$ is projected onto all the functions of the dictionary, and the coefficient A_0 is determined from Equation 5. The residual $x_{k+1}(t)$ is then computed by the equation

$$x_{k+1}(t) = x_k(t) - A_k h_k(t) . \quad (6)$$

The same procedure is repeated for the residual iteratively, and the algorithm terminates when the energy of the final residual drops below a predefined level.

By considering unit energy functions as in Equation 2, that is

$$\left\| h_k(t) \right\|^2 = \int_{-\infty}^{\infty} \left| h_k(t) \right|^2 dt = 1 , \quad (7)$$

it can be readily proved that the energy of the signal can be expressed as (Qian, 2001)

$$\left\| x(t) \right\|^2 = \sum_{k=0}^{\infty} \left| A_k \right|^2 . \quad (8)$$

This fact shows that the previously described algorithmic process conserves the energy of the signal.

The algorithm outlined in Equations 4~6 is based on a prescribed dictionary, whose size suggests a trade-off between the achieved accuracy of the representation of the original signal, and the computational cost. Clearly, more accurate representations require larger dictionaries, and thus excessive computations. In the present study an efficient refinement scheme introduced by Yin et al. (2002), is adopted.

2.2 *Wigner-Ville distribution of the decomposed signal*

The WVD of an arbitrary signal $x(t)$ is defined by (Cohen, 1995)

$$WVD_x(t,\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x\left(t + \frac{1}{2}\tau\right) x^*\left(t - \frac{1}{2}\tau\right) \exp(-i\omega\tau)\, d\tau \quad (9)$$

The Gaussian chirplet is the most general form of function whose WVD

$$WVD_{h_k}(t,\omega) = \frac{1}{\pi} \exp\left\{ -a_k (t - t_k)^2 + i\left(-\frac{1}{a_k} \left[(\omega - \omega_k) - \beta_k (t - t_k) \right]^2 \right) \right\} \quad (10)$$

is non-negative everywhere (Cohen, 1995). Thus, the WVD leads to a physically meaningful distribution of the energy of the Gaussian chirplet on the time-frequency domain (Figure 1).

Assuming the energy of the final residual to be negligible, application of the WVD in both sides of Equation 4 yields

$$WVD_x(t,\omega) = \sum_k \left| A_k \right|^2 WVD_{h_k}(t,\omega) + \sum_{k \neq q} A_k A_q WVD_{h_k h_q}(t,\omega) \quad (11)$$

where the first summation corresponds to the auto-WVD terms of the analyzing chirplet functions, and the second summation corresponds to their cross-WVD terms. Making use of the energy conservation properties of the MP algorithm and of the WVD (Cohen, 1995), it can be proved that the energy of the cross-WVD terms in the above equation vanishes (Qian, 2001). That is,

$$\sum_{k \neq q} A_k A_q WVD_{h_k h_q}(t,\omega) = 0 . \quad (12)$$

Hence, the cross-term free adaptive spectrogram (AS) of the decomposed signal $x(t)$ using the MP is defined as (Qian and Chen, 1994)

$$AS_x(t,\omega) = \sum_k \left| A_k \right|^2 WVD_{h_k}(t,\omega) . \quad (13)$$

The AS is always non-negative, Equation 10, and preserves the energy of the original signal. Clearly, it is a powerful tool of representing any signal in the joint time-frequency domain.

3 EVOLUTIONARY FILTERED KANAI-TAJIMI POWER SPECTRUM

Under the assumption of a slowly-varying time-dependent modulation (envelope) function *A(t)*, the evolutionary power spectrum $S(t,\omega)$ of a uniformly modulated non-stationary random process $y_{ns}(t)$ can be analytically expressed as (Priestley, 1965)

$$S(t,\omega) = \left| A(t) \right|^2 S(\omega) , \quad (14)$$

where $S(\omega)$ is the power spectrum of a stationary random process $y(t)$. Then, the underlying separable non-stationary random process is given by the equation (Priestley, 1965)

$$y_{ns}(t) = A(t)\,y(t)\,. \tag{15}$$

It is noted that the preceding definition of the non-stationary process implies that all frequency components exhibit exactly the same temporal variation. This a special form of the more general case where the modulation function varies both in time and in frequency.

Herein, the commonly used Bogdanoff-Golberg-Bernard (BGB) envelope function (Bogdanoff et al., 1961)

$$A(t) = ct\exp\left\{-\frac{t}{t_p}\right\} \tag{16}$$

is adopted, where t_p is the time instant for which the function attains its peak value and c is a normalization parameter such that $A(t_p) = 1$.

The stationary part $y(t)$ of the non-stationary process of Equation 15 is defined by an appropriately filtered Kanai-Tajimi power spectrum. It is extensively used for ground motion spectral representation (Clough and Penzien, 1993). According to this model the seismic fault is assumed to be a source of stationary seismic waves, with unbiased frequency content, statistically described by a band-limited white noise power spectrum. The propagating seismic waves are first filtered by a high-pass filter which is meant to capture the impact of the geological formations of the crust of the Earth (the so-called bedrock). Then the Kanai-Tajimi filter is used in cascade to account for the relatively soft surface soil deposits (Kanai, 1957 and Tajimi, 1960).

Mathematically, the resulting spectrum is given as the product of the previously discussed filters with white noise input of intensity S_o and cut-off frequency ω_b. That is,

$$S(\omega) = S_o \frac{\left(\dfrac{\omega}{\omega_f}\right)^2}{\left(1-\left(\dfrac{\omega}{\omega_f}\right)^2\right)^2 + 4\zeta_f^2\left(\dfrac{\omega}{\omega_f}\right)^2} \cdot \frac{1+4\zeta_g^2\left(\dfrac{\omega}{\omega_g}\right)^2}{\left(1-\left(\dfrac{\omega}{\omega_g}\right)^2\right)^2 + 4\zeta_g^2\left(\dfrac{\omega}{\omega_g}\right)^2} \quad ;\omega \le \omega_b \tag{17}$$

where the ω_f and ζ_f are the natural frequency and damping ratio of the bedrock, while the ω_g and ζ_g are the respective parameters for the soil deposit.

4 ARMA SIMULATION

A discrete stationary stochastic process $\tilde{y}$ compatible with a predefined (target) power spectrum with cut-off frequency ω_b can be generated as the response of a linear time-invariant autoregressive moving average (ARMA) digital filter subject to band-limited white noise excitation (Spanos and Mignolet, 1986). In this respect, the r-sample of an ARMA(p,q) process is computed as a linear combination of the previous p samples plus a convolution term involving the white noise input as follows

$$\tilde{y}[r] = -\sum_{k=1}^{p} b_k \tilde{y}[r-k] + \sum_{l=0}^{q} c_l w[r-l] \tag{18}$$

where the b_k and c_l are the coefficients of the ARMA filter. The symbol w denotes a discrete white noise process band-limited to ω_b with autocorrelation function

$$E\{w[i]w[j]\} = 2\omega_b \delta_{ij} \tag{19}$$

where $E\{\cdot\}$ is the operator of mathematical expectation and δ_{ij} is the Kronecker delta. To avoid aliasing the sampling period of the discrete process T should be related to the cut-off frequency through the Nyquist relation

$$T = \frac{\pi}{\omega_b} \tag{20}$$

The objective is to determine the filter coefficients b_k and c_l such that the squared modulus of the frequency response of the filter

$$S_{\tilde{y}\tilde{y}}(\omega) = \left|H\left(e^{i\omega T}\right)\right|^2 \tag{21}$$

matches the target spectrum. In this equation H is the transfer function of the ARMA filter which in Z-transform notation reads as

$$H(z) = \frac{\sum_{l=0}^{q} c_l z^{-l}}{1+\sum_{k=1}^{p} b_k z^{-k}} \tag{22}$$

In the ensuing analysis the auto/cross-correlation matching (ACM) procedure was used to determine the unknown coefficients. The main idea is to first construct a relatively long autoregressive (AR) filter, in the context of the standard linear prediction theory, to closely approximate the target spectrum. Then, matching of both the output auto-correlation and the input/output cross-correlation functions between this preliminary AR and the final ARMA model is enforced. Eventually, the b_k and c_l coefficients are calculated by solving a p+q by p+q system of linear equations. More details on the ACM procedure can be found in Spanos and Zeldin (1998).

5 NUMERICAL RESULTS

To demonstrate the effectiveness of the ACT for capturing the temporal evolution of the frequency content of earthquake accelerograms, artificial non-stationary time histories compatible with an appropriately defined evolutionary power spectrum were generated and processed.

Specifically, the ARMA method discussed in the previous section was used for the simulation of a discrete stationary random process taken to be compatible with the filtered Kanai-Tajimi spectrum (hereafter target spectrum), as given by Equation 17. The adopted values of the required parameters for the complete definition of the target spectrum were ω_f = 0.40Hz, ω_g = 4.0Hz and $\zeta_f = \zeta_g = 0.60$ which correspond to relatively stiff soil conditions. The intensity of the white noise S_o was taken equal to 398cm^2/sec^3 so that the variance of the target spectrum to be 10^{4}cm^2/sec^4 and the cut-off frequency was taken equal to 40π rad/sec. The thus defined target spectrum is shown in Figure 2. Note that the maximum spectral value is attained at approximately 3.3Hz.

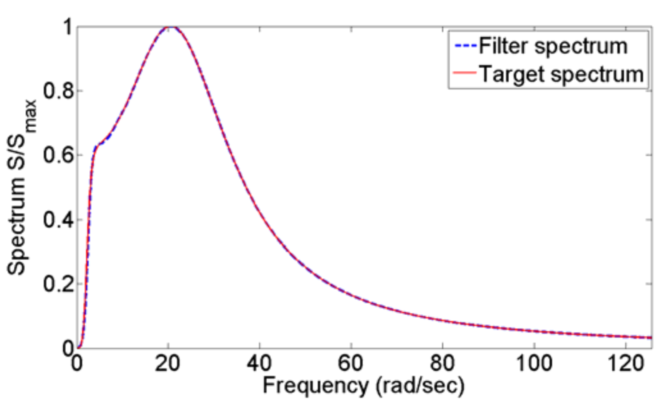

Figure 2. Target filtered Kanai-Tajimi power spectrum for stiff soil conditions and its ARMA(40,40) filter approximation

An ARMA model of order (40,40) provided virtually perfect matching between the target spectrum and the frequency response of the ARMA filter as is seen in Figure 2.

A collection of 250 stationary time histories of 25sec duration each were generated using Equation 18 sampled at T= 0.025sec as Equation 20 requires. The associated non-stationary simulated seismic acceleration records were synthesized by the discrete form of Equation 15 adopting the BGB modulating function discussed earlier. For the purposes of the present study t_p was chosen equal to 4sec, and the resulting shape of the BGB envelope is shown in Figure 3.

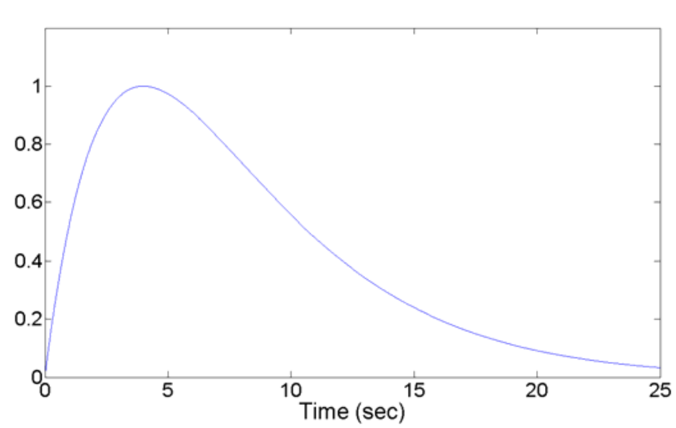

Figure 3. The Bogdanoff-Golberg-Bernard modulating function for t_p= 4sec

The acquired non-stationary time histories are compatible with the evolutionary power spectrum which is mathematically expressed by the closed analytic formula of Equation 14. Shown in Figure 4 is the three-dimensional surface of this theoretically obtained spectrum along with its contour plot.

Subsequently, the adaptive spectrograms (ASs) of the non-stationary records considered were calculated by Equation 13. Obviously, if the ACT is capable for providing valid representations of seismic accelerograms in the time-frequency plane, one should expect that the average of the ASs of the individual non-stationary samples should constitute an adequate approximation of the analytically known underlying evolutionary power spectrum. Implicit in this claim is the assumption that the number of the records processed (250) is sufficiently large to yield statistically dependable results. Indeed, the average of the above mentioned ASs together with its corresponding contour plot presented in Figure 5 is found to be in a good agreement with the analytically obtained evolutionary power spectrum of Figure 4.

In Figure 6, cross-sections of the surfaces of Figures 4 and 5 along the frequency axis at various time instants (instantaneous spectra) are superimposed to facilitate the comparison. A relatively small discrepancy between the analytical instantaneous spectra and those obtained from the averaged ASs of the simulated records at times when the maximum amplitude of the time histories is expected (around the 4th second), can be observed. In particular, it is noted that in the second case the peak values are at-

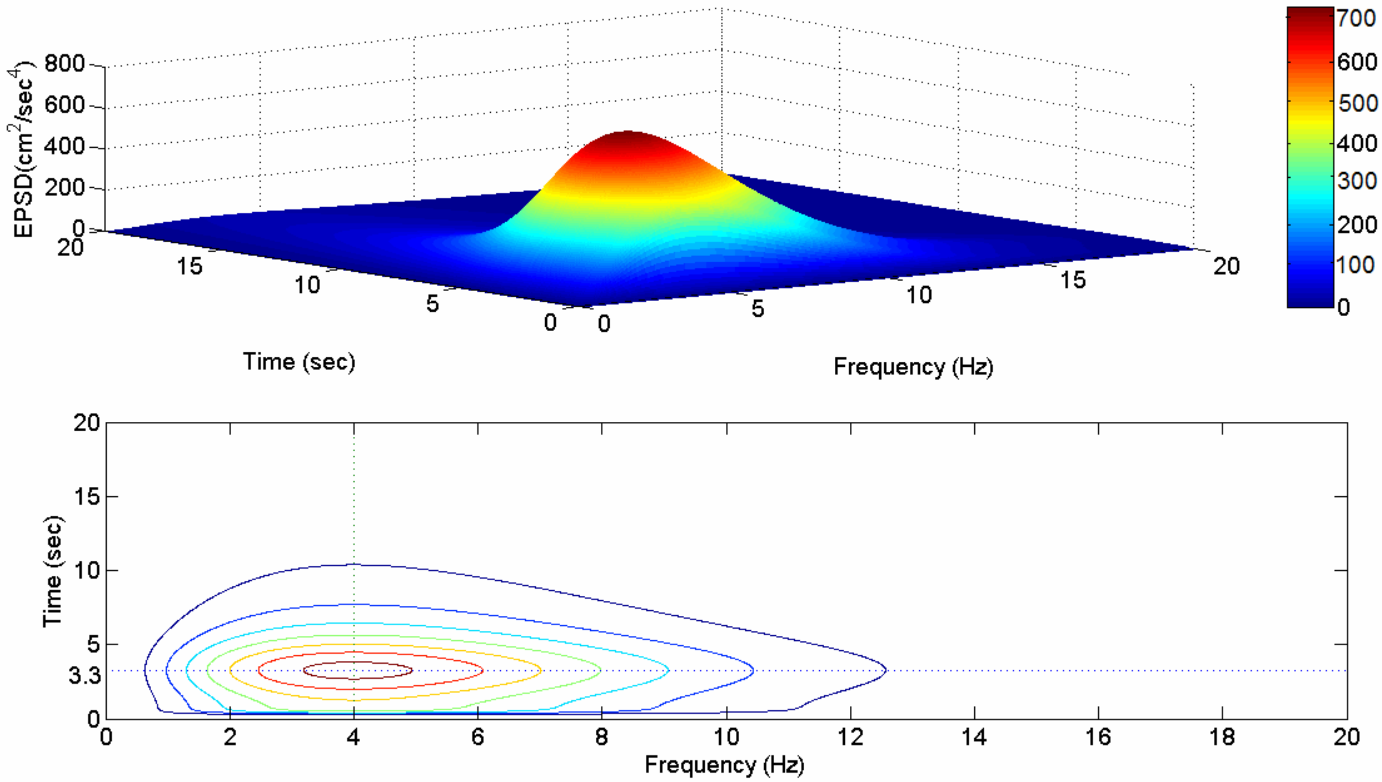

Figure 4. Evolutionary filtered Kanai-Tajimi power spectrum as obtained by equation 14

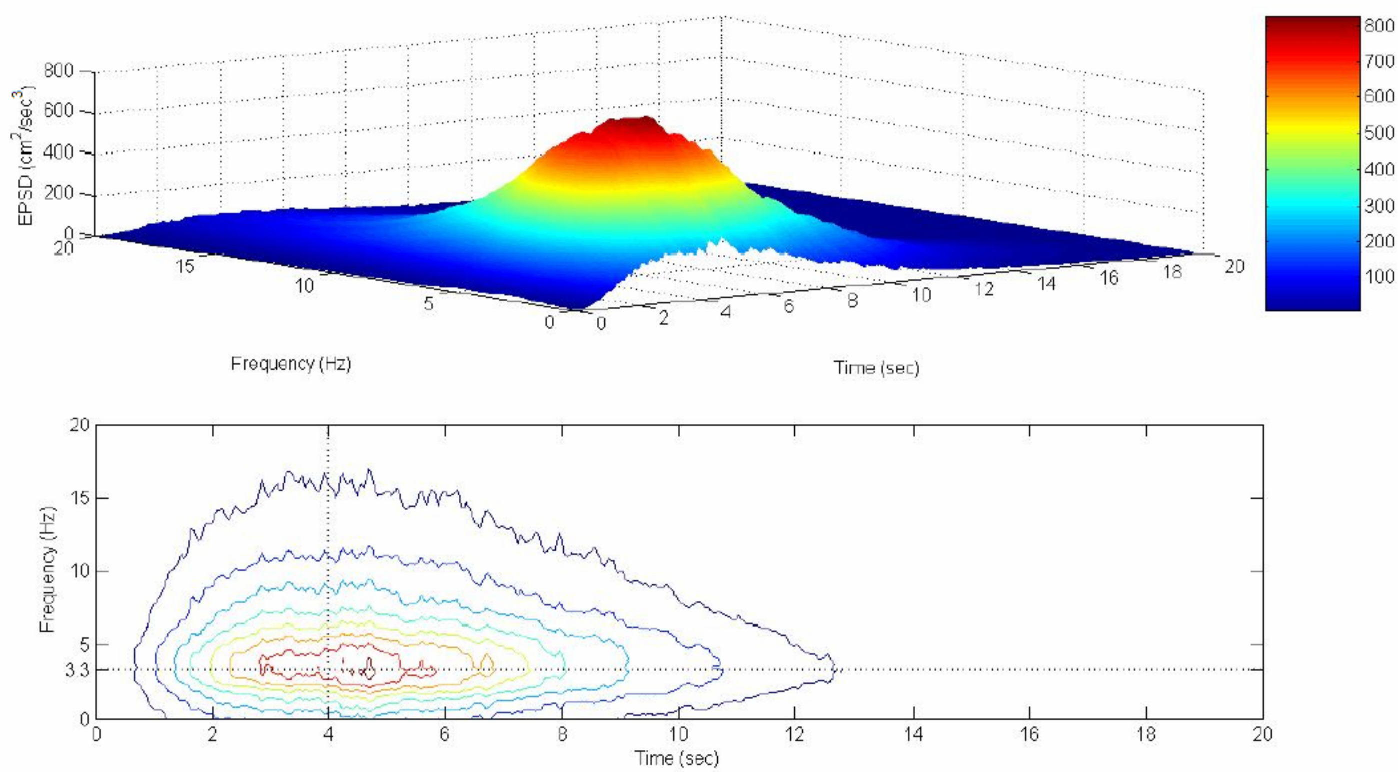

Figure 5. Estimated evolutionary filtered Kanai-Tajimi power spectrum given by the average of the adaptive spectrograms of the simulated non-stationary accelerograms

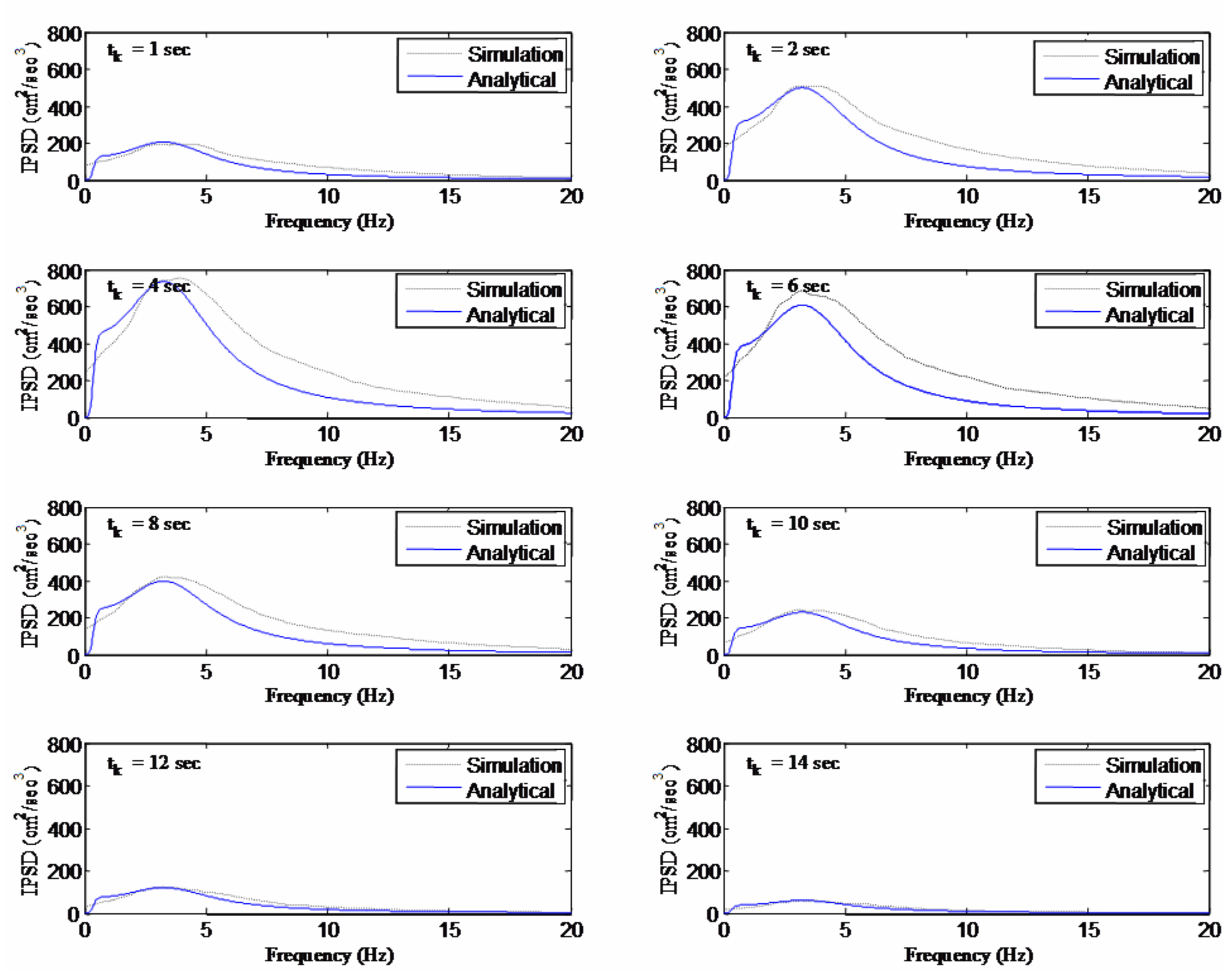

Figure 6. Instantaneous power spectra of the simulated non-stationary accelerograms and of the analytically defined evolutionary filtered Kanai-Tajimi power spectrum

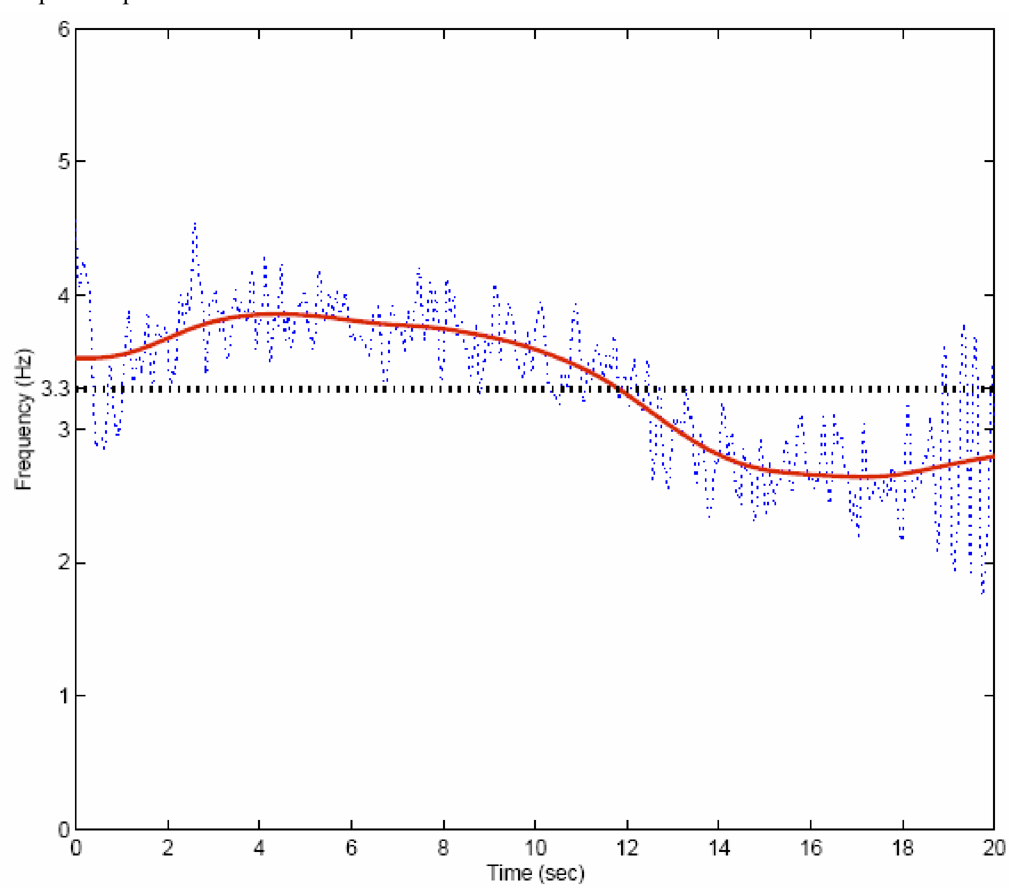

Figure 7. Position of the maximum attained values of the estimated evolutionary filtered Kanai-Tajimi power spectrum on the time-frequency plane (Ridgeline of the surface of Figure 4)

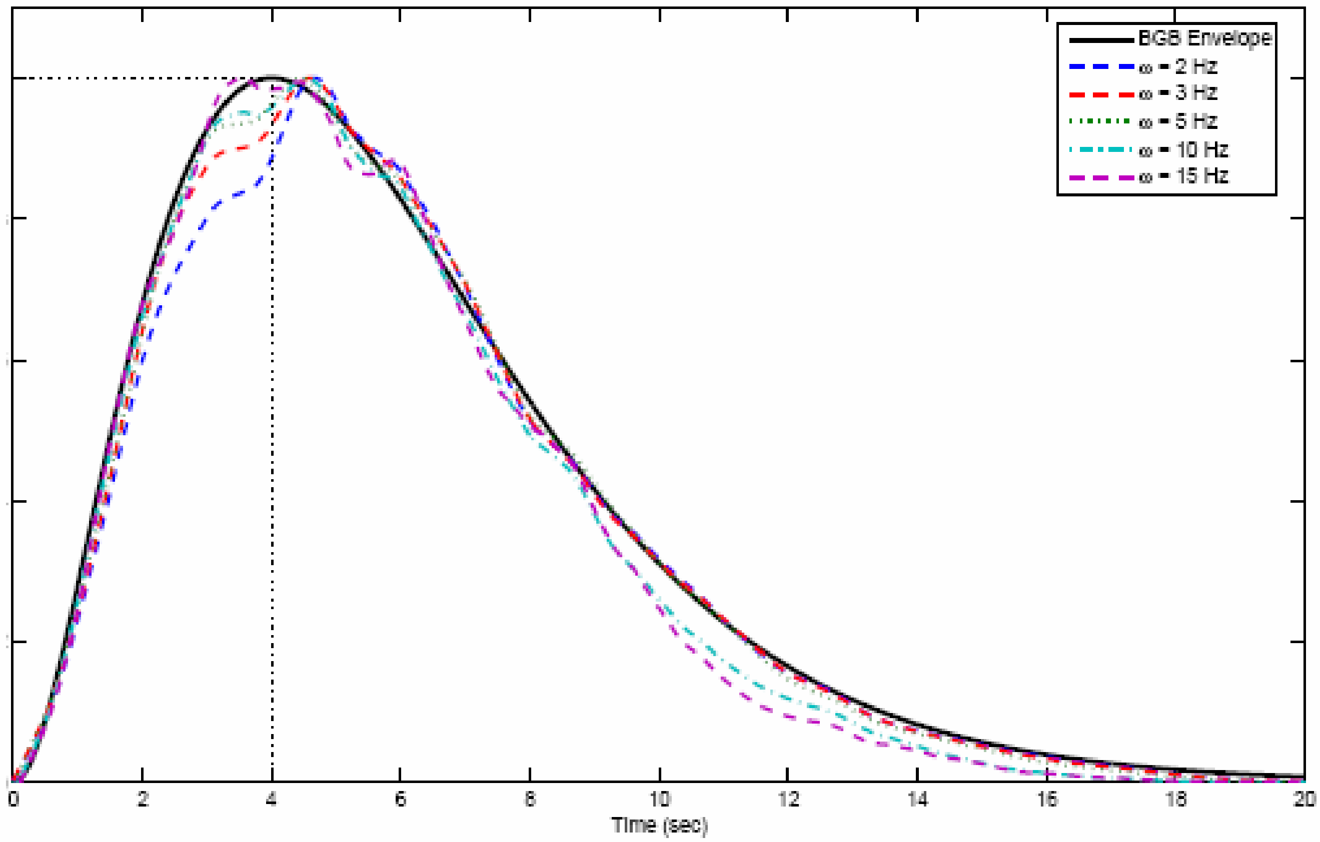

Figure 8. Estimated modulating envelopes for various frequency levels plotted together with the analytically defined Bogdanoff-Goldberg-Bernard modulating envelope

tained at slightly higher frequencies than the theoretically predicted 3.3Hz where the analytically defined filtered Kanai-Tajimi spectrum attains its maximum value at all times. This observation is confirmed by examining the "ridgeline" of the surface of Figure 5 as provided in Figure 7 (highly oscillating dotted line), which fluctuates around a mean value of approximately 3.3 Hz. To facilitate the interpretation, Figure 7 also provides a smooth (continuous) line obtained by low-pass filtering the highly oscillating ridgeline. Clearly, the maximum deviation of this smoothed ridgeline from the mean value of 3.3Hz is exhibited between the 3rd and the 6th second. After the 12th second the observed peak values of the averaged ASs occur at lower frequencies than 3.3 Hz. However, this last part of the plot is not crucial in the sense that the accelerograms have already decayed significantly and most of the energy of the signal has already been released.

Furthermore, by considering cross-sections of the surface of Figure 5 along the time axis the modulating envelopes at various frequency levels as estimated by the averaged ASs are obtained, as shown in Figure 8. The same figure accommodates the theoretical modulating envelope $|A(t)|^2$ (continuous line), which remains the same at all frequencies. In general, the estimated envelopes are reasonably close to the BGB modulating function used in defining the non-stationary stochastic process under consideration. Nevertheless, there is a noticeable shift to the right of the lower frequency components during the first 5 seconds while the higher frequency components tend to delay after the 9th second. These trends suggest that although the simulated records were generated as samples of a uniformly modulated non-stationary random process, there is a certain frequency dependence of the actual modulating envelopes. It is not a coincidence that this occurs at the time intervals where the rate of change of the adopted modulating function becomes significant and the assumption of the slowly-varying function becomes less reliable.

6 CONCLUSIONS

Earthquake accelerograms are inherently non-stationary as their intensity and frequency content evolve with time. An effort to capitalize on recent advances in the field of joint time-frequency analysis for the representation of such signals has been made.

Specifically, the adaptive chirplet transform (ACT) has been employed for processing a collection of simulated seismic accelerograms characterized by an evolutionary power spectrum which admits a known analytical expression. It has been suggested that the average of the obtained adaptive chirplet spectrograms of the individual accelerograms can be viewed as an estimate of the underlying evolutionary power spectrum. In general, beyond minor discrepancies, the shape of the averaged adaptive spectrograms has been found to be adequately close to the analytical expression of the evolutionary power spectrum. In this manner, the appropriateness of the adaptive chirplet analysis for capturing the evolutionary nature of the spectral content of appropriately simulated artificial seismic signals is partially confirmed. Obviously, this assessment can be extended to include real recorded strong ground motions, as well.

Furthermore, it has been noted that although the signals considered were realizations of a uniformly modulated random process, the numerical results indicate slight frequency-dependent deviations of the actual modulating envelopes from the adopted theoretical one. The latter suggests that caution should be exercised when adopting the Priestley (1965) model for the simulation of non-stationary data as realizations of separable (uniformly modulated) non-stationary random processes. In cases which the assumption of a slowly-varying time envelop is severly violated the proposed analytical expression for the evolutionary power spectrum is not valid; It cannot effectively represent the temporal evolution of the frequency content of the process. Clearly, the ACT can serve as a validation tool to assess the "slowly-varying" feature of certain modulation envelopes which have been extensively used over several years for the definition of separable non-stationary stochastic process models in earthquake engineering.

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