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**Citation:** White, M. & Sayma, A. I. (2018). A preliminary comparison of different turbine architectures for a 100 kW supercritical CO<sub>2</sub> Rankine cycle turbine. Paper presented at the The 6th International Supercritical CO<sub>2</sub> Power Cycles Symposium, 27 - 29 Mar 2018, Pittsburgh, Pennsylvania.

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## **A preliminary comparison of different turbine architectures for a 100 kW supercritical CO<sub>2</sub> Rankine cycle turbine**

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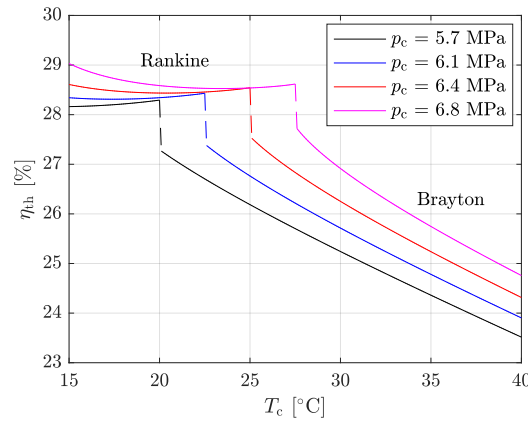
### **ABSTRACT**

The aim of this paper is to conduct a preliminary comparison of different turbine architectures for a small-scale 100 kW supercritical CO<sub>2</sub> Rankine cycle. The turbine is required to expand supercritical CO<sub>2</sub> from 650 °C and 170 bar, down to 50 bar. For such an application, it is not immediately clear which turbine architecture is the most suitable design when considering both aerodynamic and mechanical design constraints. Within this paper, three different turbine architectures are considered, namely radial-inflow, single-stage axial, and two-stage axial turbines. For each architecture, a preliminary design model is constructed which is based on conventional turbomachinery design parameters such as the loading coefficient, flow coefficient and degree of reaction. Using this model, a parametric investigation on the effect of the rotational speed on the required rotor diameter and blade height is conducted and the different turbine architectures are compared. This is completed with the view of establishing the feasible design space for a small-scale supercritical CO<sub>2</sub> turbine. For all three architectures, it is found that in order to obtain feasible blade heights it is necessary to maximise the loading coefficient whilst minimising the flow coefficient, and design the turbine with the minimum allowable diameter. Typically, this results in a turbine design with a rotor diameter of 30 mm, a rotor-inlet blade height in the range of 1.74 to 2.47 mm, and a rotational speed between 150 and 250 kRPM for a single-stage radial or axial turbine, and 75 and 175 kRPM for a two-stage axial turbine. Ultimately, nine candidate turbine designs have been identified, which should be studied further using more advanced 3D CFD and FEA simulations.

### **INTRODUCTION**

In the past decade, the search for the next generation of high-efficiency thermal power cycles has led to a significant interest in supercritical carbon dioxide (sCO<sub>2</sub>) as a working fluid. One of the primary benefits in operating a thermodynamic cycle with sCO<sub>2</sub> is that high efficiency can be obtained at moderate turbine inlet temperatures (in the region of 500 °C), which makes sCO<sub>2</sub> an interesting option for applications such as nuclear power [1], concentrated-solar power [2] and high efficiency coal-fired power plants [3]. Moreover, sCO<sub>2</sub> cycles can also be considered for waste-heat recovery applications for heat-source temperatures exceeding 450 °C [4]; at this temperature, organic Rankine cycles, which would typically be considered for waste-heat recovery, are not suitable due to factors such as fluid decomposition and low efficiency, whilst sCO<sub>2</sub> have the advantage of compactness and simplicity compared to steam Rankine cycles. At present, the primary focus of sCO<sub>2</sub> research and development activities has been on medium-to large-scale applications, with power outputs in the order of 1 MW<sub>e</sub> and above, for example in Refs. [5,6]. However, given the potential thermodynamic benefits, sCO<sub>2</sub> cycles can also be considered for small-scale applications with power outputs in the order of 100 kW<sub>e</sub>, and systems of this size are already under investigation [7-9].

As recently discussed in Ref. [10], there are a large number of variants of sCO<sub>2</sub> cycle, and each variant has a varying degree of complexity. However, for a small-scale application a simple recuperated cycle, comprising of compressor/pump, heat-addition heat exchanger, turbine, recuperator and heat-rejection heat exchanger, is probably the most suitable cycle based on both cost and technical considerations. Depending on the inlet conditions to the compression process this simple recuperated cycle can either operate as a Rankine cycle or a Brayton cycle. The effect of the compression inlet temperature  $T_c$ , and compression inlet pressure  $p_c$ , on the operating condition and performance of a sCO<sub>2</sub> cycle is shown in Fig. 1. As observed from the analysis shown in Fig. 1, operating with a low inlet temperature to the compression process corresponds to operating a Rankine cycle, and this can achieve a higher thermal efficiency than the Brayton cycle operating under the same assumptions. Therefore, operating a Rankine cycle is preferential from a thermodynamic perspective. However, since the critical temperature of CO<sub>2</sub> is 31 °C, it is only possible to operate a sCO<sub>2</sub> Rankine cycle in areas with a low ambient temperature. However, there has been recent research into mixing CO<sub>2</sub> with small amounts of other fluids to obtain a CO<sub>2</sub> blend that has a higher pseudo-critical temperature [11,12]. Using a blend could therefore realise the potential of the sCO<sub>2</sub> Rankine cycle in areas with higher ambient temperatures.



**Figure 1.** Effect of the inlet conditions of the compression process (inlet temperature  $T_c$ , and inlet pressure  $p_c$ ) on the thermal efficiency of a sCO<sub>2</sub> cycle, assuming a turbine inlet temperature of  $T_{01} = 650$  °C, pressure ratio of  $PR = 3$ , compression and expansion isentropic efficiencies of  $\eta_{c,is} = 0.7$  and  $\eta_{t,is} = 0.8$  respectively, and a recuperator effectiveness of  $\epsilon_r = 0.8$ .

Another advantage of sCO<sub>2</sub> is that it has a high density, which facilitates the miniaturisation of system components, enabling a high power density compared to conventional power plants. Whilst this is advantageous from the point of view of reducing the size, and cost of the plant, it presents challenges with regards to turbomachinery design. More specifically, to achieve a high efficiency the turbomachinery components become very small and must rotate at high speed. This results in low aspect-ratio blades (blade height to chord ratio), and increased relative clearance gaps, which give rise to increased secondary flow and over the tip gap losses. Furthermore, the combined effects of a high power density, high operating pressures, high rotational speeds and small component dimensions mean that the design of the bearing-shaft arrangement needs to be carefully considered, in addition to considering the axial thrust that is exerted by the rotor on the bearings. Moreover, for the relatively low pressure-ratio and mass-flow rate, it is not immediately obvious whether a radial-inflow or axial-flow turbine is the most suitable design when considering both aerodynamic and mechanical design constraints.

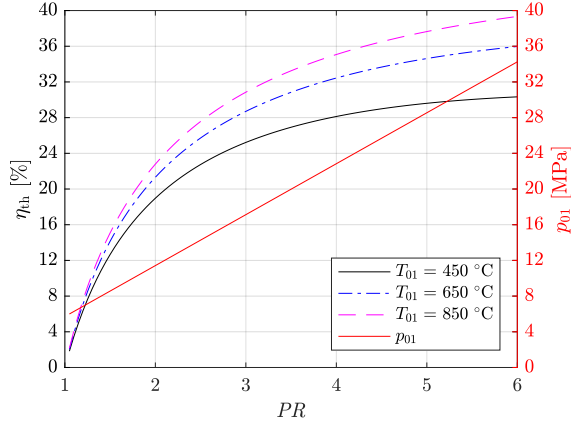
The aim of this paper is to conduct a preliminary comparison of different turbine architectures for a small-scale sCO<sub>2</sub> Rankine cycle system, and establish the feasible design space for different turbine types. Focus is given here on the turbine based on the premise that the future use of CO<sub>2</sub>-blends could realise the potential to operate a Rankine cycle, thus removing the need for a compressor.

## THERMODYNAMIC SPECIFICATION AND PRELIMINARY TURBINE SIZING

The results from a thermodynamic analysis of a recuperated sCO<sub>2</sub> Rankine cycle are shown in Fig. 2. For

this analysis, a compressor inlet temperature of 20 °C is assumed, and the effect of the turbine inlet temperature  $T_{01}$  and pressure ratio  $PR$  on the thermal efficiency  $\eta_{th}$  and turbine inlet pressure  $p_{01}$  has been investigated. These calculations have been completed assuming fixed polytropic efficiencies of 70% and 80% for the pump and turbine respectively, and a recuperator effectiveness of 80%.

It is observed from Fig. 2 that as  $PR$  increases,  $\eta_{th}$  increases, but at the expense of higher operating pressures. For  $T_{01} = 650$  °C,  $PR = 3$  has been selected as this is considered to be reasonable compromise, corresponding to  $\eta_{th} = 28\%$ , and  $p_{01} = 17$  MPa. The turbine design specification is summarised in Table 1.



**Figure 2.** Effect of pressure ratio  $PR$  and turbine inlet temperature  $T_{01}$  on thermal efficiency  $\eta_{th}$  and turbine inlet pressure  $p_{01}$  of a recuperated sCO<sub>2</sub> Rankine cycle ( $T_c = 20$  °C,  $\eta_{c,p} = 0.7$ ,  $\eta_{t,p} = 0.8$ ,  $\epsilon_r = 0.8$ ).

**Table 1.** Design specification of the sCO<sub>2</sub> turbine.

$T_{01}$	650	°C
$p_{01}$	17.0	MPa
$PR$	3	-
$\dot{m}$	0.65	kg/s
$\dot{W}$	100	kW
$\eta_{t,is}$ (target)	0.8	-

Based on the information provided in Tab. 1 a preliminary assessment of a single-stage turbine, either radial-inflow or axial-flow, can be completed using the specific speed  $\omega_s$ :

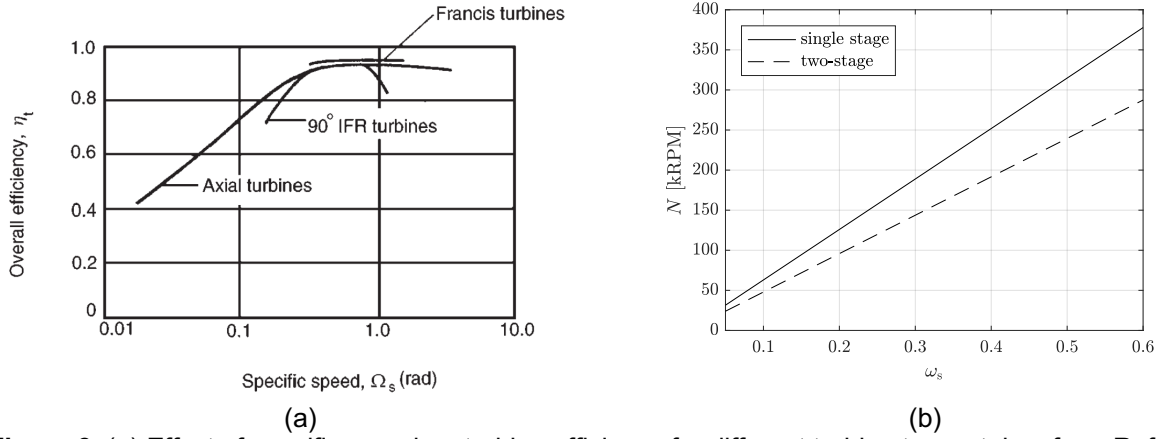
$$\omega_s = \frac{\omega \dot{V}_3^{\frac{1}{2}}}{\Delta h_{ss}}, \quad (1)$$

where  $\omega$  is the rotor rotational speed in rad/s,  $\dot{V}_3$  is the volumetric flow rate at the rotor outlet in m<sup>3</sup>/s and  $\Delta h_{ss}$  is the isentropic enthalpy drop across the expander, given in J/kg. In general, optimal turbine efficiency occurs within a certain range of values for  $\omega_s$ , typically  $0.1 < \omega_s < 1.0$  [13] (Fig. 3a), and therefore  $\omega$  can be found by rearranging Equation 1. This calculation can be completed using the information provided in Tab. 1, if the total and static conditions at the rotor outlet are assumed equal.

For a two-stage turbine Equation 1 is still valid, but instead applies to each individual stage. In this case the total enthalpy drop across the entire turbine is equally divided across the two stages, and each stage is assumed to operate with the same isentropic efficiency (*i.e.*,  $\eta_{t,is} = 80\%$ ). It is assumed that both stages are mounted on the same shaft, and therefore both stages rotate at the same speed. Therefore, the second stage will have a higher specific speed than the first, owing to the lower density. In this instance, the quoted value for  $\omega_s$  refers to the first stage only. For the case study in question, the specific speed of stage two is then 27% higher than stage one. For a first-stage specific speed below 0.5, the second-stage specific speed will therefore be below 0.64. Therefore, since this is still within the limits previously stated, a penalty in the second-stage efficiency is not expected.

The required rotational speed (in kRPM) for a single- and two-stage turbine for  $\omega_s$  ranging between 0.05 and 0.6 is shown in Fig. 3b. For a single-stage turbine to operate at an optimal efficiency, the rotor must rotate at a speed in the region of a few-hundred kRPM, which introduces challenges with regards to the mechanical and electrical design of the turbine. Reducing the speed could simplify these aspects, but at the penalty of a reduced aerodynamic efficiency. Comparatively, a two-stage design facilitates a higher

specific speed to be obtained at a lower rotational speed, but at the complexity of requiring multiple stages. Clearly, a trade-off exists between aerodynamic performance, and mechanical/electrical constraints.



**Figure 3.** (a) Effect of specific speed on turbine efficiency for different turbine types, taken from Ref. [13]; (b) the required rotational speed for a single- and two-stage turbine as a function of the desired rotor specific speed  $\omega_s$ . For the two-stage turbine,  $\omega_s$  refers to the specific speed of the first stage.

## RADIAL TURBINE DESIGN

To obtain a preliminary radial-turbine design, a design model has been developed, which is coupled to NIST REFPROP [14] to obtain the thermodynamic properties of CO<sub>2</sub>. The selected design parameters are the rotational speed  $N$ , blade-loading coefficient  $\Psi$ , and flow coefficient  $\Phi$ , which are defined as:

$$\Psi = \frac{\Delta h_0}{u_2^2}; \quad (2)$$

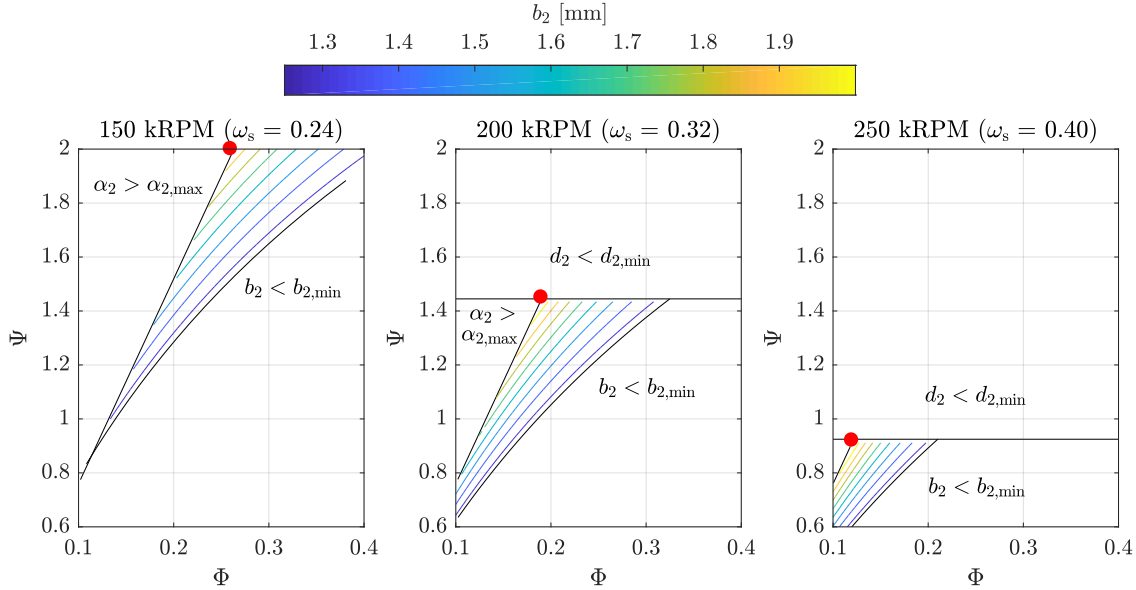
$$\Phi = \frac{c_{m3}}{u_2}, \quad (3)$$

where  $\Delta h_0$  is the total enthalpy drop across the turbine,  $u_2$  is the rotor-inlet blade velocity and  $c_{m3}$  is the meridional velocity at the rotor outlet. Using the defined target efficiency, the total enthalpy drop across the turbine can be determined since  $\Delta h_0 = \eta_{t,is} \Delta h_{ss}$ , from which  $u_2$  follows. Moreover, although a small amount swirl at the rotor outlet is often applied in practice, for simplicity the rotor outlet is designed for zero swirl. Therefore, the absolute tangential velocity at the rotor inlet  $c_{\theta 2}$  can be determined since the stage enthalpy drop reduces to  $\Delta h_0 = u_2 c_{\theta 2}$ . Then, assuming a constant meridional velocity (*i.e.*,  $c_{m3} = c_{m2}$ ), the rotor inlet velocity triangle is fully defined. Finally, if a loss coefficient is applied to account for losses within the stator, the density at the rotor inlet and, in turn, the rotor-inlet dimensions (*i.e.*, blade height  $b_2$ , and diameter  $d_2$ ) are found.

To design the rotor outlet an additional design parameter must be defined. However, at this stage of the design process it is not clear whether a  $90^\circ$  radial-inflow turbine, a radial-inflow turbine with a non-zero inlet blade angle, or even a highly-loaded impulse-type turbine would be the most suitable choice. However, since the rotor-inlet design is a function of only  $N$ ,  $\Psi$  and  $\Phi$ , and is therefore independent of the rotor-outlet design, a parametric investigation concerning these three parameters can be conducted. For this investigation constraints were placed on the minimum allowable rotor diameter ( $d_2 \geq 30$  mm), maximum rotor-inlet absolute flow angle ( $\alpha_2 \leq 82.5^\circ$ ), and minimum allowable blade height ( $b_2 \geq 1.25$  mm). Of these constraints, the rotor-inlet blade height is expected to be the most significant, owing to the relative size of clearance gaps, and values in the region of 1 to 2 mm are currently considered to be close to the limit of practicality [15,16]. Alongside these constraints, a stator loss coefficient of  $\zeta_n = 0.075$  is assumed. Fixing  $\zeta_n$  means there is no consideration of the effect that changing the rotor-inlet velocity has on the stator performance, however this is not expected to have a significant effect on the final rotor-inlet dimensions.

Results for three rotational speeds, namely 150, 200 and 250 kRPM, are shown in Fig. 4. These three

speeds correspond to specific speeds of 0.24, 0.32 and 0.40 respectively. The analysis was also conducted for 100 and 300 kRPM, however no design could be found that met all of the imposed constraints. In general, it is observed that increasing  $\Psi$  corresponds to a reduction in  $d_2$ , but an increase in  $b_2$ . Moreover, for a specified  $\Psi$ , reducing  $\Phi$  increases  $b_2$  further, but at the cost of increasing  $\alpha_2$ . Ultimately, the result of this interplay between the design parameters and constraints means that in order to maximise the blade height, it is necessary to design a turbine with small diameter, and a high absolute flow angle.



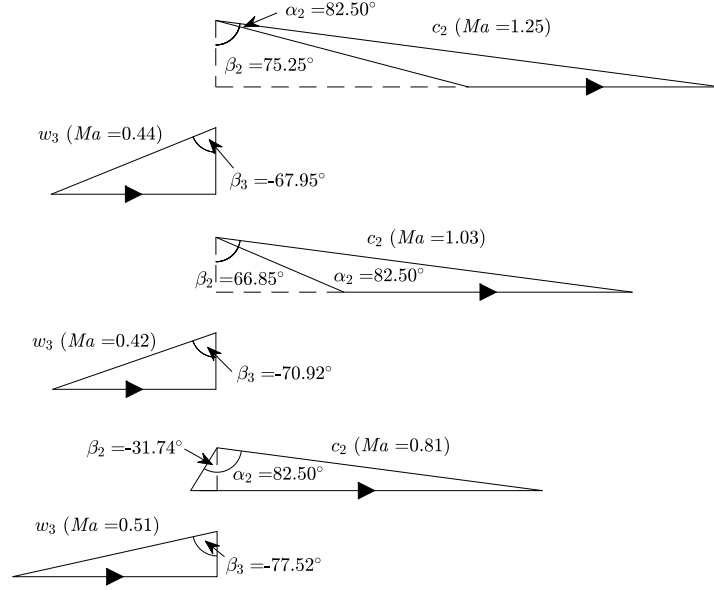
**Figure 4.** Effect of rotational speed, blade-loading coefficient  $\Psi$ , and flow coefficient  $\Phi$  on the rotor-inlet blade height of a radial-inflow turbine. The red dots correspond to the designs summarised in Tab. 2.

For each rotational speed, the designs that maximise  $b_2$  are summarised in Tab. 2, and these designs are shown by the red dots in Fig. 4. The velocity triangles for these designs are presented in Fig. 5. From these results, it is observed that a different type of radial-inflow turbine is required as the rotational speed is reduced. At 250 kRPM the blade-loading coefficient is below unity, implying a 90° radial-inflow turbine design. In order to maximise the blade height ( $b_2 = 2.13$  mm), a flow coefficient of 0.12 is required, corresponding to  $\alpha_2 = 82.5^\circ$ , and a relative flow angle of  $\beta_2 = -31.7^\circ$ , which is within the recommended range of  $-40^\circ$  to  $-20^\circ$  [13]. Increasing  $\Phi$  to 0.16, reduces  $\alpha_2$  by  $2.5^\circ$ , and the  $b_2$  to 1.60 mm. On the other hand, increasing the diameter to 35 mm, increases  $\beta_2$  to  $-74.4^\circ$ , which is no longer within the recommended limits.

As the speed is reduced to 200 kRPM, it is necessary to have a blade-loading coefficient above unity, and therefore a rotor with a non-zero rotor inlet blade angle is required. Then, as the rotational speed is further reduced to 150 kRPM, a high blade-stage loading coefficient, approaching 2, is required to obtain a blade height close to 2 mm. This implies a highly-loaded rotor, with a degree of reaction approaching zero. In this case, the rotor is an impulse turbine, with the majority of the expansion occurring in the stator. It is also noted that for the 150 kRPM design, the feasible design space is not constrained by the minimum diameter (30 mm), hence the design that maximises the blade height has a slightly larger diameter.

**Table 2.** Summary of radial turbine rotor design that maximises the rotor-inlet blade height for the three different rotational speeds considered.

	$N$ [kRPM]	$\Psi$	$\Phi$	$d_2$ [mm]	$b_2$ [mm]	$\alpha_2$ [°]	$\beta_2$ [°]	$Ma_2$	Rotor type
Radial 1	150	2.00	0.26	34.00	1.98	82.50	75.25	1.25	Impulse
Radial 2	200	1.45	0.19	30.00	2.09	82.50	66.85	1.03	Backswept
Radial 3	250	0.92	0.12	30.00	2.13	82.50	-31.74	0.81	90° IFR



**Figure 5.** Velocity triangles for the three radial-turbine rotor designs given in Tab. 2. From top to bottom: design 1 (150 kRPM); design 2 (200 kRPM); design 3 (250 kRPM). The rotor-outlet velocity triangles are obtained by assuming rotor diameter ratios ( $d_3/d_2$ ) of 0.65, 0.55 and 0.55 for design 1, 2 and 3 respectively.

### SINGLE-STAGE AXIAL TURBINE

The aerodynamic design of an axial turbine is very similar to the radial turbine, and can once again be designed by defining  $N$ ,  $\Psi$ , and  $\Phi$ . The axial velocity is assumed constant (*i.e.*,  $c_{m2} = c_{m3}$ ), and since there is no change in the meanline radius between the inlet and outlet, there is no change in the blade velocity (*i.e.*,  $u_2 = u_3$ ). However, unlike the radial turbine, the rotor outlet tangential velocity  $c_{\theta 3}$  is not assumed to be zero. Therefore, to complete the design, the degree of reaction  $\Lambda$  is also defined as an input:

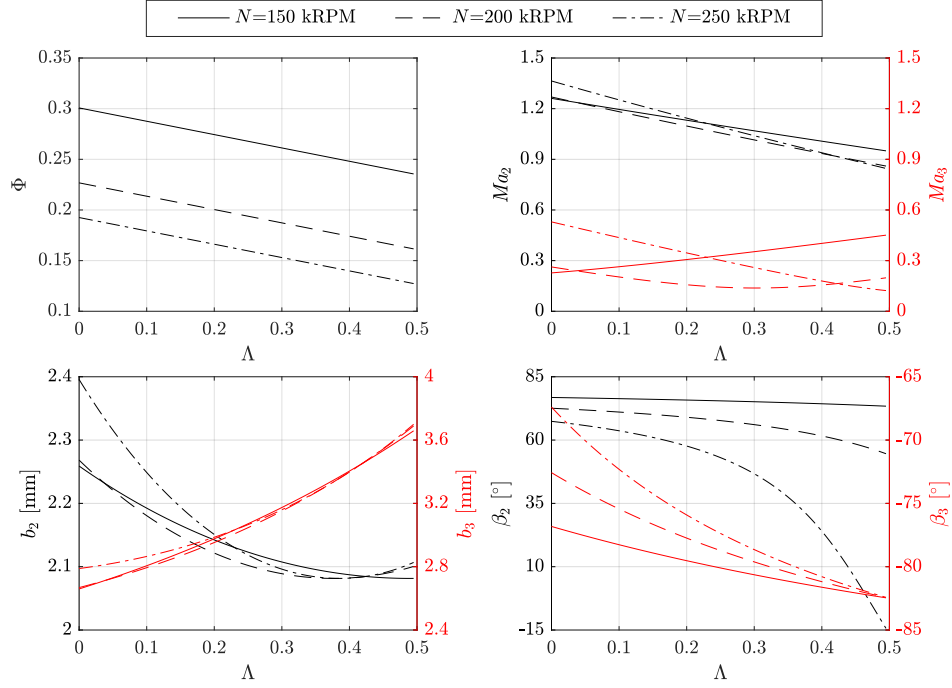
$$\Lambda = \frac{h_2 - h_3}{h_{02} - h_{03}}, \quad (4)$$

where  $h_2$  and  $h_3$  are the static enthalpies, and  $h_{02}$  and  $h_{03}$  are the total enthalpies at the rotor inlet and outlet respectively. Both the inlet and outlet conditions are taken at the rotor mid-span. It should be noted that a non-zero  $c_{\theta 3}$  allows  $\Psi$  to be greater than 2, and that by defining  $\Lambda$  the rotor-outlet velocity triangle is fully defined and therefore the rotor outlet can be sized.

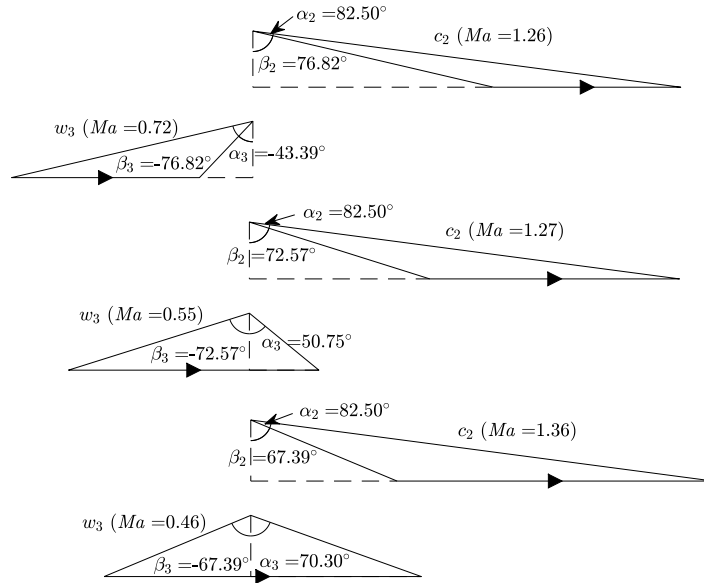
For the single-stage axial turbine design, another parametric investigation into the effects of the design parameters on the design of the turbine rotor can be conducted. For this investigation, the same stator loss coefficient of  $\zeta_n = 0.075$  is assumed. Referring to Fig. 3a, it is observed that, compared to radial turbines, axial turbines can achieve higher efficiencies at lower specific speeds. However, a preliminary study found that reducing the rotational speed reduced the maximum possible rotor-inlet blade height; for example, at 100 kRPM ( $\omega_s = 0.16$ ) the maximum blade height was only 1.6 mm. For this reason, the following analysis considers the same rotational speeds as the those considered for the radial turbine, namely 150, 200 and 250 kRPM.

Since the analysis of an axial turbine is very similar to a radial turbine, it can be inferred from the previous section that selecting the minimum allowable rotor diameter ( $d_2 = 30$  mm), and maximum absolute rotor-inlet flow angle ( $\alpha_2 = 82.5^\circ$ ), will result in the largest rotor-inlet blade height. Therefore, a parametric investigation into the effect of  $\Lambda$  on the turbine design, using these values has been conducted, and the results are presented in Fig. 6. To be consistent with the constraint applied to the rotor-inlet absolute flow angle,  $\Lambda$  is only varied between 0 and 0.5. This is because  $\Lambda > 0.5$  corresponds to a rotor-outlet relative flow angle that is greater than the rotor-inlet absolute flow angle (*i.e.*,  $\beta_3 > \alpha_2$ ), which would violate the maximum flow angle constraint (*i.e.*,  $\beta_3 > 82.5^\circ$ ).





**Figure 6.** Effect of the degree of reaction  $\Lambda$  on the design of a single-stage axial turbine for different rotational speeds. Top-left: flow coefficient  $\Phi$ ; top-right: rotor inlet absolute Mach number  $Ma_2$  and rotor outlet relative Mach number  $Ma_3$ ; bottom-left: rotor inlet  $b_2$  and rotor outlet  $b_2$  blade heights; bottom-right: rotor inlet  $\beta_2$  and rotor outlet  $\beta_3$  relative flow angles.



**Figure 7.** Rotor velocity triangles for the three single-stage axial turbine rotor designs. From top to bottom: design 1 (150 kRPM); design 2 (200 kRPM); design 3 (250 kRPM).

**Table 3.** Summary of single-stage axial turbine rotor designs that maximise the rotor-inlet blade height.

	$N$ [kRPM]	$\Psi$	$\Phi$	$\Lambda$	$d_2$ [mm]	$b_2$ [mm]	$\alpha_2$ [°]	$\beta_2, \beta_3$ [°]	$Ma_2$
Axial-1 1	150	2.57	0.30	0.0	30.0	2.26	82.50	76.82	1.26
Axial-1 2	200	1.45	0.23	0.0	30.0	2.27	82.50	72.57	1.27
Axial-1 3	250	0.92	0.19	0.0	30.0	2.40	82.50	67.39	1.36

From Fig. 6 it is observed that for all three rotational speeds, a low degree of reaction is favourable to increase  $b_2$ . Unsurprisingly, the result of having the majority of the expansion within the stator is a high rotor-inlet Mach number, and a small change in volume through the rotor, resulting in a smaller rotor-outlet blade height compared to a higher degree of reaction. For each rotational speed, the design that maximises  $b_2$  (*i.e.*,  $\Lambda = 0$ ) is summarised in Tab. 3 and the velocity triangles are shown in Fig. 7. The stage loading coefficients are 2.57, 1.45 and 0.92 for the 150, 200 and 250 kRPM designs respectively. For the 200 and 250 kRPM rotational speeds, these are the same as for the radial turbine. For the 150 kRPM design, the stage loading for the axial turbine is higher than the radial turbine, owing to the non-zero value for  $c_{\theta 3}$ . It is also noted that  $\Lambda = 0$  corresponds to  $\beta_2 = \beta_3$ , and this angle is reduced as the rotational speed is increased.

## TWO-STAGE AXIAL TURBINE

For the two-stage axial turbine, the same procedure outlined for the single-stage axial turbine is applied. The only difference is that the total enthalpy drop is divided equally across the two-stages (*i.e.*,  $\Delta h_{\text{stage}} = \Delta h_0/2$ ). Once the stage enthalpy drop is known, the process outlined in the previous section can be completed. For this preliminary design, the analysis is only applied to the first stage. This is because the volumetric flow rate through the machine increases as the pressure reduces. Therefore, the smallest blade dimensions will be at the rotor inlet of the first stage.

For the two-stage design, the same stator loss coefficient is assumed ( $\zeta_n = 0.075$ ), and three rotational speeds are considered, namely 75, 125 and 175 kRPM, which correspond to first-stage specific speeds of 0.16, 0.26 and 0.37 respectively. Following on from the previous discussion regarding the single-stage axial turbine design, a preliminary study found that reducing the rotational speed below 75 kRPM resulted in very small rotor-inlet blade heights. Moreover, following from the analysis in the previous sections, it can again be stated that designs with the minimum rotor diameter ( $d_2 \geq 30$  mm), and maximum absolute rotor-inlet flow angle ( $\alpha_2 \leq 82.5^\circ$ ) will correspond to the largest rotor-inlet blade heights. Therefore, another parametric investigation considering the effect of  $\Lambda$  on the rotor design can be completed. For brevity, the results from this analysis are not reported in this paper, but ultimately, it is observed that, unlike the single-stage axial turbine design, a high degree of reaction ( $\Lambda = 0.5$ ) results in the largest rotor-inlet blade height. This is because in the two-stage turbine the enthalpy drop per stage is lower, and therefore the conditions at the rotor inlet remain subsonic for all values of  $\Lambda$ . However, in the single-stage design, the rotor inlet conditions are supersonic for  $\Lambda < 0.35$  (Fig. 6). Since the mass flux ( $\rho c$ ) is always at a maximum at  $Ma = 1$ , the blade height can be maximised by increasing  $\Lambda$  for subsonic designs, and reducing  $\Lambda$  for supersonic designs. This also explains why  $b_2$  increases for  $\Lambda > 0.35$  in Fig. 6.

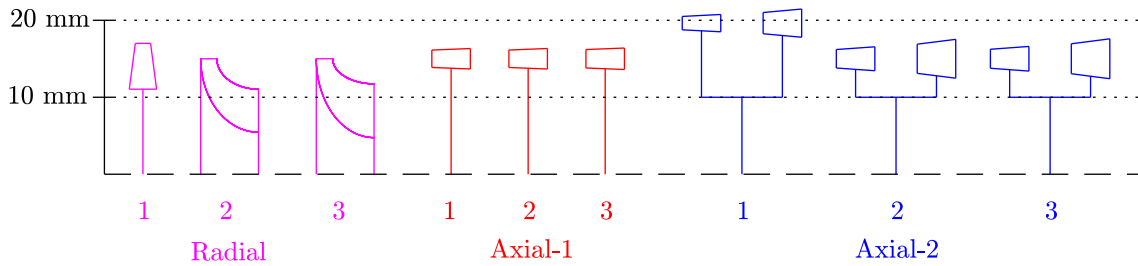
The final designs for the first-stage of a two-stage axial turbine are summarised in Tab. 3. It is noted that for the 75 kRPM design the rotor has a diameter of 39.26 mm, which is larger than the minimum diameter of 30 mm. This is because the stage-loading coefficient is capped at 3.0, in accordance with the range of values typically quoted within the literature (*i.e.*, the Smith chart [13]). The design of the second-stage is completed assuming the second stage has the same mid-span radius as the first stage, and that the flow coefficient, loading coefficient and degree of reaction are the same in both stages. This is considered to be a reasonable assumption for the preliminary design phase, since the second-stage will always have a larger blade height and is only of secondary interest. However, this assumption should be refined during future studies.

**Table 3.** Summary of the two-stage axial-turbine rotor designs that maximise the rotor-inlet blade height for the three different rotational speeds considered. The results reported correspond to the first stage.

	$N$ [kRPM]	$\Psi$	$\Phi$	$\Lambda$	$d_2$ [mm]	$b_2$ [mm]	$\alpha_2$ [°]	$\beta_2, \beta_3$ [°]	$Ma_2$
Axial-2 1	75	3.00	0.26	0.5	39.26	1.74	82.50	75.25	0.67
Axial-2 2	125	1.85	0.19	0.5	30.00	2.40	82.50	66.17	0.61
Axial-2 3	175	0.94	0.13	0.5	30.00	2.47	82.50	-12.44	0.58

## COMPARISON OF THE DIFFERENT TURBINE ARCHITECTURES

In the previous sections, three preliminary designs have been obtained for three different turbine architectures, namely radial, single-stage axial and two-stage axial turbines. In Fig. 8, the nine preliminary turbine designs are shown, as viewed from the meridional plane. For all three architectures, it is found that the maximum rotor-inlet blade height increases as the rotational speed is increased. Moreover, to ensure the rotor diameter is greater than 30 mm, the stage-loading coefficient must reduce, and this is also accompanied by a reduction in the flow coefficient. Comparing the different architectures, it is observed that the single-stage axial turbine design can facilitate a slightly larger blade height than a radial turbine rotating at the same speed. The blade height can be further increased by moving to a two-stage design, and reducing the rotational speed. It is also found that when moving from a single-stage axial design to a two-stage axial design, the degree of reaction changes. More specifically, for the single-stage turbine a zero reaction (impulse type) design should be selected, whilst the degree of reaction should be increased to 0.5 for the two-stage design.



**Figure 8.** Comparison between the nine preliminary rotor designs. For the Radial/Axial-1 turbines ‘1’, ‘2’ and ‘3’ correspond to rotational speeds of 150, 200 and 250 kRPM respectively. For Axial-2 ‘1’, ‘2’ and ‘3’ correspond to rotational speeds of 75, 125 and 175 kRPM respectively.

Ultimately, from Fig. 8 it can be observed that most of the designs all have similar dimensions, with an inlet diameter of 30 mm, and an inlet blade height around 2 mm. With a rotor this small, it is expected that all of the rotor designs will experience relatively large clearance losses, and increased viscous losses. Unfortunately, existing loss correlations for predicting turbine performance are not validated for sCO<sub>2</sub> applications. Therefore, the next stage of this research will require 3D CFD studies of the nine turbine designs shown in Fig. 8 to assess the aerodynamic performance that can be achieved by each design.

## MECHANICAL AND ELECTRICAL DESIGN CONSIDERATIONS

The challenging operating conditions of a sCO<sub>2</sub> cycle mean that alongside considering the aerodynamic design of the turbine, the mechanical and electrical design must also be considered during the turbine design phase. This includes decisions such as choosing whether a hermetic turbine-generator assembly is used, which can be subject to high windage losses owing to the high density, or whether an external generator is used, in which case suitable shaft seals are required. Moreover, for a directly-coupled turbine and generator, expensive power electronics are required to match the grid frequency, or alternatively a gearbox with a high gear ratio is required. However, these challenges, despite their importance, pertain to any turbine architecture at the given scale.

Of more relevance when comparing different turbine architectures are factors such as the rotor-dynamics, axial thrust generated by the rotor and the blade stresses. For example, due to the high power density and high pressure, the stresses on the rotor blade may be too large to use a radial-inflow turbine with a non-zero blade angle, as in the case for Radial-2 design. Alternatively, due to the moderate tip speed, which ranges between 235 and 392 m/s for the single-stage turbine designs, means that shrouded rotors may be considered to improve aerodynamic performance, but may have consequences in terms of the rotor-dynamics of the shaft. Ultimately, these effects are hard compare at the preliminary level that is applied within this paper, and can only be compared after more detailed simulations are carried out. Therefore, an important next step in this research is to conduct rotor-dynamic and FEA simulations of the nine turbine designs identified, in addition to the already mentioned CFD simulations, to allow for a more detailed comparison. Moreover, it will be necessary to consider different bearing-shaft arrangements for

each design, in addition to giving consideration to the thermal management of the assembly to ensure suitable cooling is available for the bearings and high-speed generator [17].

## CONCLUSIONS

In this paper, the design of the turbine for a small-scale (100 kW) supercritical CO<sub>2</sub> Rankine cycle with a turbine inlet temperature of 650 °C has been evaluated. To identify the most suitable designs, design models for radial-inflow, and single- and two-stage axial turbines have been developed, which use, as inputs, the loading coefficient, flow coefficient, and degree of reaction. The objective of the design study is to identify turbine designs that result in feasible blade heights. It is found that to obtain optimal efficiencies, a single-stage turbine must rotate at speeds in excess of a few hundred kRPM, and for the three different turbine architectures considered the resulting turbine has a diameter of 30 mm, and a minimum rotor blade height in the range of 1.74 to 2.47 mm. More generally, it is found that to obtain a feasible blade height, it is necessary to design the turbine with a high loading coefficient, and low flow coefficient, corresponding to a high absolute flow angle. Moreover, when considering axial turbines, an impulse-type turbine, with a low degree of reaction, is most suitable for a single-stage design, and this results in supersonic conditions at the rotor inlet. On the other hand, for a two-stage axial turbine, a higher degree of reaction is more suitable, whilst conditions at the rotor inlet remain subsonic.

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## NOMENCLATURE

$\alpha$	absolute flow angle, °	$p$	pressure, Pa
$\beta$	relative flow angle, °	$PR$	pressure ratio
$\epsilon_r$	recuperator effectiveness	$T$	temperature, K
$\zeta_n$	stator loss coefficient	$u$	blade velocity, m/s
$\eta$	efficiency	$w$	relative velocity, m/s
$\Lambda$	degree of reaction	$\Delta h_0$	stage enthalpy drop, J/kg
$\Phi$	flow coefficient	$\Delta h_{ss}$	isentropic enthalpy drop, J/kg
$\Psi$	blade-loading coefficient		
$\omega$	rotational speed, rad/s		
$\omega_s$	specific speed, rad/s		
$b$	blade height, m		
$c$	absolute velocity, m/s		
$d$	diameter, m		
$h$	enthalpy, J/kg		
$\dot{m}$	mass-flow rate, kg/s		
$\dot{W}$	power, J/s		
$\dot{V}$	volumetric-flow rate, m <sup>3</sup> /s		
$Ma$	Mach number		
$N$	rotational speed, RPM		

## Subscripts

$\theta$	tangential
0	total conditions
1	turbine inlet
2	rotor inlet
3	rotor outlet
c	compression process inlet/compressor
is	isentropic
m	meridional
p	polytropic
th	thermal

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