Roles of inventory and reserve capacity in mitigating supply chain disruption risk

Abstract

This research focuses on managing disruption risk in supply chains using inventory and reserve capacity under stochastic demand. While inventory can be considered as a speculative risk mitigation lever, reserve capacity can be used in a reactive fashion when a disruption occurs. We determine optimal inventory levels and reserve capacity production rates for a firm that is exposed to supply chain disruption risk. We fully characterize four main risk mitigation strategies: inventory strategy, reserve capacity strategy, mixed strategy and passive acceptance. We illustrate how the optimal risk mitigation strategy depends on product characteristics (functional versus innovative) and supply chain characteristics (agile versus efficient). This work is inspired from a risk management problem of a leading pharmaceutical company.

Keywords: Supply chain resilience, Supply chain management, Disruption management, Inventory management, Stochastic models

1. Introduction

Boosted by recent high impact disasters, like the nuclear catastrophe in Japan, the topic of supply chain resilience has emerged as an important business issue. Practitioners are increasingly being challenged to build resilient supply chains (WEF, 2013; Snyder et al., 2012). The impact of supply chain disruptions on the financial performance of a company can be severe. Hendricks and Singhal (2005) use an empirical approach to quantify the effect of supply chain disruptions on long-run stock price performance. Analyzing a time period starting one year before the disruption and lasting until two years after the disruption, they find that the average abnormal stock return after announcing a supply chain disruption is nearly -40%.

To mitigate the negative consequences of supply chain disruptions, companies often adopt the practice of building up supply chain resilience using risk mitigation inventory (RMI) and reserve capacity (Tomlin, 2006). RMI is extra inventory that is designed to be used to meet customer demand in the event of a supply chain disruption (Simchi-Levi et al., 2014; Lücker et al., 2016). It is different from the operational safety stock which is held to cope with demand uncertainty. Reserve capacity refers to reserving free capacities that can be used for production in the event of a supply chain disruption (Chopra and Sodhi, 2004; Lücker and Seifert, 2016).

Take for example a pharmaceutical company that produces life saving cancer drugs such as Roche's Avastin. The production of the biological compound of the drug is exposed to substantial risks such as a biological contamination at a production site or a fire, resulting in a shut down of the production site for several months. After such an incident, the production site can only be re-used after regulatory approval, which can be time consuming. Roche generated with this drug 6.8bn CHF revenue in 2016. Besides the regulatory requirement of reliably delivering drugs to the patient, there is a high profit margin, providing the firm incentives to build up RMI and/or reserve capacity.

In this paper we focus on understanding the optimal use of RMI and reserve capacity to deal with disruption risk at a single location under stochastic demand. An important objective in this research is to understand and describe factors that lead to increasing RMI or reserve capacity levels. To simplify our models, we ignore safety stock and focus entirely on RMI, reserve capacity and supply chain disruption risk. Holding RMI causes inventory holding costs. The reserve capacity is associated with fixed costs for reserving the capacity as well as emergency production costs that are incurred when the capacity is deployed. There is a cost for stocking out.

We derive theoretical insights related to the optimal use of RMI and reserve capacity under supply chain disruption risks. Our analytical results demonstrate that the optimal reserve capacity increases with the coefficient of variation of demand, whereas the optimal RMI either decreases or increases, depending on the inventory holding costs. We also show that under certain conditions the RMI level is constant in the penalty cost.

The remainder of this paper is structured as follows. In Section 2 we review the relevant literature, focusing mainly on reserve capacity strategies, inventory policies and statistical risk measures. In Section 3 we present our mathematical model, followed by managerial insights (Section 4). Finally, we provide concluding remarks and envision future research topics in Section 6.

2. Literature Review

Our paper is related to the studies that focus on the role of reserve capacity and/or inventory management in mitigating the disruption risk. We also refer the reader to Chopra and Sodhi (2004); Snyder et al. (2012) for extensive reviews of alternative risk mitigation strategies against supply chain disruptions.

Research on the use of RMI (also known as speculative capacity) and reserve capacity (also known as reactive capacity) mainly focuses on dealing with demand uncertainty under different settings such as multi-product newsvendor (Reimann, 2011), unexpected demand surges (Huang et al., 2016), and heavy-tailed demand (Biçer, 2015). These papers are based on the work by Cattani et al. (2008), who provide a general solution procedure for models with speculative and reserve capacity in the fashion industry. Biçer and Seifert (2017) develop an analytical model that allows optimization of inventory and capacity levels over time when demand forecasts are updated according to an additive or a multiplicative process. The common assumption in these papers is that there is no supply disruption. We extend the models studied by these researchers by simultaneously considering the demand risk and the disruption risk.

The literature on the supply chain disruption risks focuses on the supply risks, generally ignoring the impact of demand uncertainty on the risk mitigation strategies. Tomlin (2006) investigates dual sourcing and reserve capacity scenarios in the presence of supply chain disruption risk. His model is based on a reliable but more expensive supplier and an unreliable yet cheaper supplier.

He characterizes high-level risk mitigation strategies, but does not jointly optimize RMI and reserve capacity decisions under stochastic demand. Lücker and Seifert (2016) study a model in which a pharmaceutical firm determines optimal RMI levels under supply chain disruption risk and deterministic demand. Further related papers focus on the role of dual sourcing in mitigating the disruption risk under deterministic demand (Parlar and Perry, 1996; Gürler and Parlar, 1997). We contribute to this literature stream by jointly optimizing RMI and reserve capacity levels under stochastic demand and deriving novel structural insights.

The impact on the supply chain networks of supply disruptions is widely studied by different scholars (Schmitt et al., 2015; Liberatore et al., 2012; Berger et al., 2004; Ruiz-Torres and Mahmoodi, 2007; Li et al., 2010; Yu et al., 2009; Sarkar and Kumar, 2015; Niknejad and Petrovic, 2016). Schmitt et al. (2015) analyze the role of inventory to safeguard against supply chain disruptions in a multi-location supply chain. The propagation of disruption in a network is analyzed by Liberatore et al. (2012). Berger et al. (2004) and Ruiz-Torres and Mahmoodi (2007) present a decision tree approach that helps to determine the optimal number of suppliers under disruption risk. In Li et al. (2010) the authors align the sourcing strategy with the pricing strategy of a firm that is exposed to supply chain disruption risk. Closely related is the work of Yu et al. (2009) who analyzes dual sourcing decisions for non-stationary and price-sensitive demand under disruption risk. Behavioral factors in multi-echelon supply chains that are prone to supply chain disruptions are studied by Sarkar and Kumar (2015). They find that supply chain disruptions may cause higher order variability compared to the base case without disruptions. Niknejad and Petrovic (2016) propose a risk evaluation method for global production networks that is based on a dynamic fuzzy model. However, this research stream lacks the optimality structures for the joint use of RMI and reserve capacity.

In summary, our paper contributes to the literature by providing structural insights into optimal RMI and reserve capacity decisions under stochastic demand and the disruption risk. We illustrate how the optimal risk mitigation strategy depends on product characteristics (functional versus innovative) and supply chain characteristics (agile versus efficient).

3. Mathematical Model

In this section we present a stylized mathematical model that is based on a single product and a single location subject to supply chain disruptions. In the event of a supply chain disruption the firm can instantaneously use the available RMI and the reserve capacity to meet customer demand. The reserve capacity is characterized by its production rate that determines how many goods can be produced in a given time. The research problem is to find the optimal combination of RMI and reserve capacity production rate under stochastic demand.

Since RMI levels are decided before a disruption has occurred, there is a risk of keeping either too much RMI (overage cost) or too little (underage cost). The overage costs are the RMI holding costs h, which are incurred as long as no disruption takes place. In the event of a supply chain disruption, only excess inventory is charged with the holding cost h during the disruption time τ. The reserve capacity production rate a is decided before a disruption has occurred. The actual production volumes given a specific reserve capacity, however, are only decided after a disruption has occurred, and hence this mitigation strategy provides more flexibility. In particular, there is no risk of overproduction and hence no overage cost due to using the reserve capacity. The reserve capacity is associated with an upfront fixed component for reserving the capacity, denoted by $\hat{c}_A$, and a variable production cost of c_A, which is incurred based on actual production volumes. The underage costs for unmet demand during the disruption time τ are the penalty costs p (e.g., unit selling price minus unit production cost plus goodwill). The firm minimizes its expected costs by deciding for RMI levels I and reserve capacity production rate a.

As a simplification we assume that only one disruption of the length τ occurs at a given point in time with probability ω_τ. This assumption is reasonable for applications in the pharmaceutical industries where the deterministic disruption time represents a worst-case scenario which the pharmaceutical company considers for risk mitigation (e.g. mitigating longer disruptions are out of scope of the company). Demand during the disruption time τ is characterized as a non-negative, continuous random variable X with the distribution $F_\tau(\,\cdot\,)$ and the probability density $f_\tau(\,\cdot\,)$.

Our optimization problem can be written as follows:

$$
\begin{aligned}
\min_{I\geq 0, a\geq 0} \quad & L(I,a) \\
& = \omega_\tau \Bigg(p \int_{I+a\tau}^{\infty} \big(x - I - a\tau\big) f_\tau(x)dx \\
& + h \int_0^I (I-x) f_\tau(x)dx + c_A \int_I^{I+a\tau} (x-I) f_\tau(x)dx \\
& + c_A a\tau \Big(1 - F_\tau\big(I + a\tau\big)\Big)\Bigg) + \big(1-\omega_\tau\big)hI + \hat{c}_A a. \qquad (1)
\end{aligned}
$$

In the objective function, the first term (starting with ω_τ) represents the penalty, inventory holding and reserve capacity production costs in case a disruption occurs. Penalty costs are only incurred for demand larger than $I + a\tau$. Holding costs are incurred if the demand is smaller than I. Costs for emergency production are incured if demand is larger than I. The second term ($(1-\omega_\tau)hI$) gives the inventory holding costs in cases where no disruption occurs. The reservation costs for the reserve capacity $\hat{c}_A$ are incurred for all time, independent of the occurrence of disruptions. A complete list of all parameters is given in Table 1. Proposition 1 characterizes the optimal risk mitigation strategy.

Decision variables	
$I \geq 0$	RMI level
$a \geq 0$	Reserve capacity production rate
Parameters	
$\tau > 0$	Disruption time
$\omega_\tau \in (0,1)$	Probability of disruption of length τ
$p > 0$	Penalty cost
$h > 0$	Inventory holding cost, $h < p$
$c_A > 0$	Reserve capacity variable unit cost, $c_A < p$
$\hat{c}_A > 0$	Reserve capacity fixed cost

Table 1: Decision variables and parameters of the model

Proposition 1. *The optimal RMI I^* and reserve capacity production rate a^* are as follows:*

I: Inventory strategy: If $\hat{c}_A \geq \Delta_1$ and $p > \frac{\hat{h}}{\omega_\tau}$, then
$I^* = F_\tau^{-1}\Big(\frac{\omega_\tau p - \hat{h}}{(p+h)\omega_\tau}\Big)$ *and* $a^* = 0$.

II: Mixed strategy: If $\Delta_1 > \hat{c}_A > \Delta_2$, then
$I^* = F_\tau^{-1}\Big(1 - \frac{h\tau - \hat{c}_A}{(h+c_A)\omega_\tau\tau}\Big)$ *and* $a^* = \frac{F_\tau^{-1}\Big(1-\frac{\hat{c}_A}{(p-c_A)\omega_\tau\tau}\Big)-I^*}{\tau}$.

III: Process flexibility strategy: If $\hat{c}_A \leq \Delta_2$ and $(p-c_A)\omega_\tau\tau > \hat{c}_A$, then
$a^* = \frac{1}{\tau}F_\tau^{-1}\big(1 - \frac{\hat{c}_A}{(p-c_A)\omega_\tau\tau}\big)$ *and* $I^* = 0$,

IV: Passive acceptance: If $p \leq \frac{\hat{h}}{\omega_\tau}$ and $\{\hat{c}_A \geq \Delta_1$ or $\hat{c}_A \leq \Delta_2\}$, then
$I^* = 0$ *and* $a^* = 0$,

where $\hat{h} \triangleq (1-\omega_\tau)h$, $\Delta_1 \triangleq \tau\frac{h}{p+h}(p-c_A)$ and $\Delta_2 \triangleq \tau(\hat{h} - \omega_\tau c_A)$.

All proofs are provided in the appendix. If the reserve capacity fixed cost $\hat{c}_A$ exceeds the threshold Δ_1, the inventory strategy (I) is preferable (for sufficiently large penalty costs) because the reserve capacity is too expensive. The threshold depends on various model parameters, including RMI holding cost and reserve capacity variable unit cost. If the reserve capacity fixed cost $\hat{c}_A$ is below the threshold Δ_2 (and the fixed cost for the reserve capacity is not too high), reserve capacity strategy (III) is preferred, since RMI is becoming too expensive compared to sourcing from the reserve capacity. In between these two cases, the mixed strategy (II) is optimal. Otherwise, a passive acceptance of supply chain disruption risk is optimal (IV).

The threshold Δ_1 can be interpreted as the effective penalty costs (e.g., penalty costs reduced by the actual reserve capacity production costs) that are gauged from the relative holding costs to total penalty and holding costs. The threshold Δ_2 can be interpreted as the expected effective inventory holding costs (e.g., the expected inventory holding costs reduced by the expected reserve capacity production costs). The dependence of Δ_1 and Δ_2 on $h, \hat{c}_A$ and c_A as key parameters is presented in Figures 1 and 2 (with $\tau = 10,\ \omega_\tau = 0.05$ and $p = 40$). Sections I, II, and III indicate the areas where inventory strategy, mixed strategy or reserve capacity strategy, respectively, are optimal. Clearly, for holding costs h and reserve capacity fixed costs $\hat{c}_A$ that are too high (upper right-hand corner of the graphs), a passive acceptance of the risk is optimal ($I^* = 0$ and $a^* = 0$).

According to Proposition 1, for the inventory strategy, optimal RMI levels depend on the holding cost h, penalty cost p and probability of a disruption

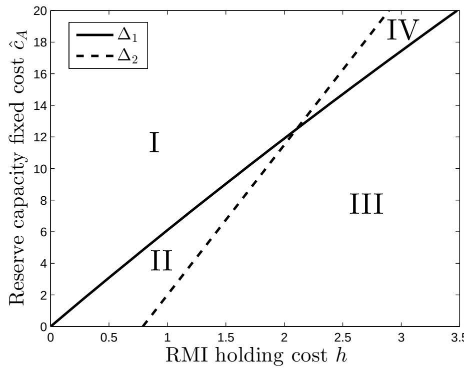

Figure 1: Phase space for the thresholds Δ_1, Δ_2 and $c_A = 15$

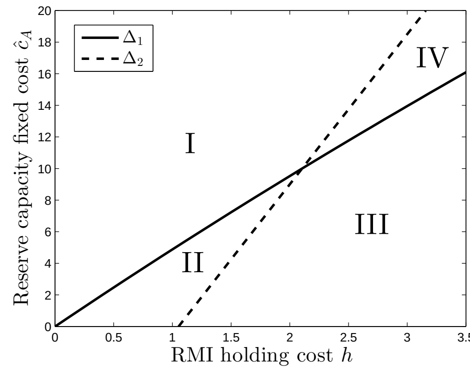

Figure 2: Phase space for the thresholds Δ_1, Δ_2 and $c_A = 20$

ω_τ. For the mixed strategy, the optimal RMI level depends on the expected additional cost of production through the reserve capacity $(\omega_\tau c_A - \hat{h})$ as well as the reserve capacity reservation cost $\hat{c}_A$. The lower this expected additional cost and the cheaper the reservation cost for the reserve capacity, the lower the optimal RMI I^* and vice versa. The firm's optimal reserve capacity production rate a^* depends on the lost profit (e.g., the difference between penalty cost and production cost $(p - c_A)$) as well as the reserve capacity reservation cost $\hat{c}_A$. The smaller the lost profit and the cheaper the reservation cost for the reserve capacity, the larger the optimal reserve capacity production rate a^*.

Regarding a sensitivity analysis, the following lemma holds:

Lemma 1. *Let the mixed strategy be optimal. Then: 1) I^* is constant in p and a^* increases with p, 2) a^* decreases with τ for sufficiently large τ if $\hat{h} - \omega c_A \leq 0$ and $p > \frac{\hat{c}_A + c_A \omega_\tau \tau}{\omega_\tau \tau}$, 3) $\exists \omega_{\tau,\epsilon} > \omega'_\tau = \frac{\tau h - \hat{c}_A}{\tau h + c_A}$ such that a^* decreases with ω_τ on the interval $(\omega'_\tau, \omega_{\tau,\epsilon})$ if $\hat{c}_A < \tau h$.*

Regarding the sensitivity in the coefficient of variation of demand CV, we find:

Lemma 2. *Let the mixed strategy be optimal and let demand follow a normal distribution $F_\tau(\,\cdot\,)$. Then: 1) I^* decreases (increases) with CV if $h > \frac{c_A \omega_\tau \tau + 2\hat{c}_A}{\tau(2-\omega_\tau)}$ ($h < \frac{c_A \omega_\tau \tau + 2\hat{c}_A}{\tau(2-\omega_\tau)}$), 2) a^* increases with the coefficient of variation of demand CV.*

In other words, it is best to deal with increasing demand volatility by building up more reserve capacity and by holding more RMI as long as inventory holding costs are not too high. If inventory holding costs are high, it is best to deal with increasing demand volatility by holding less RMI. Further managerial insights based on these findings are discussed in the subsequent section.

4. Managerial Insights

A manager is typically concerned with two main questions: First, which risk mitigation strategy is optimal for which products? Second, what are optimal RMI and/or reserve capacity levels?

To address the first question, we refer to the typology in Figure 3, where we identify high (low) inventory holding costs with functional (innovative) products and high (low) fixed cost for the reserve capacity with an efficient (agile) supply chain. In the automotive industry, for example, profit margins are generally low, so the inventory holding costs are relatively high compared to the total revenue generated. The industry is also capital intensive, resulting in high fixed costs for the reserved capacity. Therefore, automotive supply chains are generally efficient. For an automotive company such as Renault, the passive acceptance of supply chain disruption risks is optimal, given the high inventory holding costs and efficient supply chains. In contrast, for the innovative pharmaceutical segment of the drug manufacturer Pfizer, the 2015 annual report lists an average cost of sales of 11.2% of revenues (`http://www.pfizer.com/investors`). Clearly, for such innovative products, either an inventory strategy, or a mixed strategy is optimal, depending on the agility of the supply chain.

Another way to analyze Figure 3 is to identifying the y-axis with demand uncertainty. An agile [efficient] supply chain corresponds in this scenario to high [low] demand uncertainty. Our risk mitigation classification matrix can then be seen as an extension of the classical push-pull process to supply chain disruption risk where- depending on the product's characteristic we have to decide for the right risk mitigation strategy, besides the push-pull boundary (Simchi-Levi et al., 2004).

To address the second question, we provide structural insights on optimal RMI and reserve capacity levels. From Lemma 1 and 2 we find the following main insights:

- The optimal RMI level is constant in the penalty cost.
- The optimal production rate of the reserve capacity may decrease with the disruption probability.
- While the optimal production rate of the reserve capacity always increases with the coefficient of variation for normally distributed demand, the optimal RMI level may decrease or increase with the coefficient of

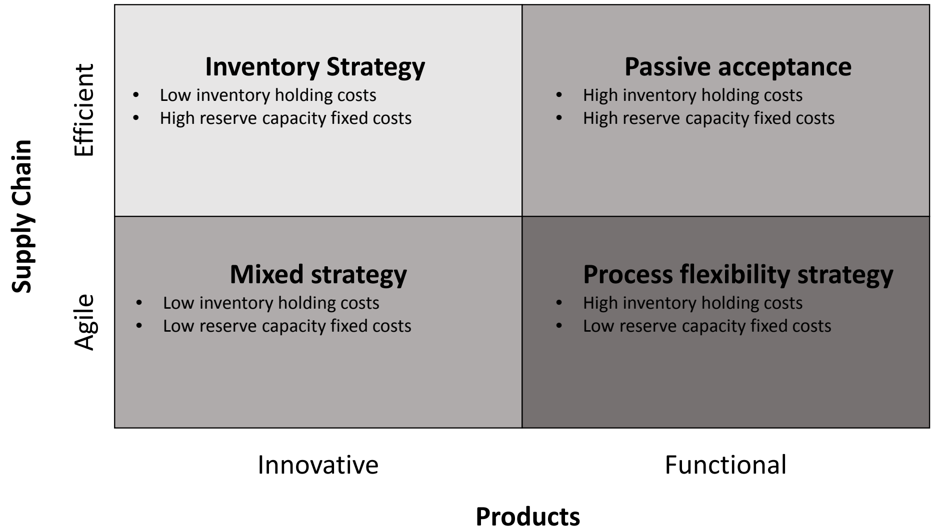

Figure 3: Four risk mitigation strategies

variation.

In the following we discuss each insight in detail.

The optimal RMI level is constant in the penalty cost.

This insight is interesting as one might expect that the RMI level increases with the penalty cost. However, keeping in mind that we apply a mixed strategy, we observe that only the production rate of the reserve capacity increases with the penalty cost, and the RMI level remains constant. Once a certain threshold for the penalty cost is passed, building up reserve capacity becomes more cost-efficient than building up RMI. We illustrate this insight in Figure 4, where we show the impact of the penalty cost on RMI I^* and reserve capacity production rate a^*. The solid curve represents RMI I^* (levels are given on the left y-axis) and the dashed curve represents the reserve capacity production rate a^* (values are given on the right y-axis). We have divided the graph in two sections. Section I shows the inventory strategy with $\hat{c}_A \geq \Delta_1$. Section II shows the mixed strategy with $\Delta_1 > \hat{c}_A > \Delta_2$. The inventory strategy (I) is optimal in the approximate range $19 < p < 25$, where I^* increases with p. For $p > 25$ we pass the breaking point and I^* remains constant while a^* increases with p (mixed strategy II). This plot and the following ones are based on the following parameters: $p = 40$, $c_A = 20$, $\hat{c}_A = 2$, $\tau = 10$, $\omega_\tau = 0.05$ and a normally distributed demand with $\mu = 1$ and $\sigma = 0.3$. In this context, it is important to discuss when the transition from the inventory strategy to the mixed strategy occurs. From Proposition 1 we read that the longer the disruption, the more likely the disruption, or the cheaper the reserve capacity the more likely it is to transit from strategy I to II.

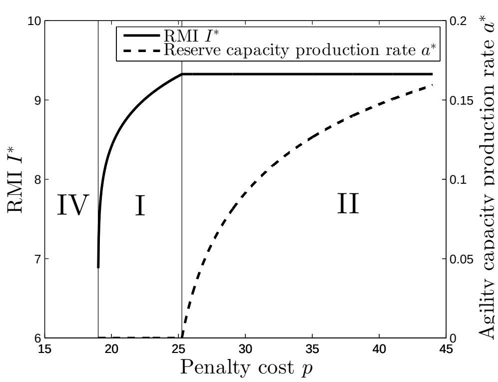

Figure 4: Penalty cost p

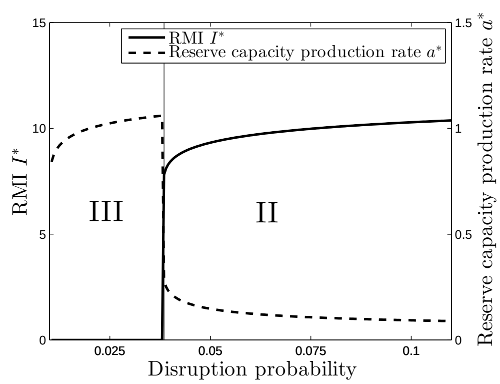

Figure 5: Probability of disruption ω_τ

The optimal production rate of the reserve capacity may decrease with the disruption probability.

This insight states that RMI and reserve capacity do not necessarily both increase with the disruption probability. Low probability risks are efficiently mitigated with reserve capacity only (under some mild assumptions). As the disruption probability increases, RMI becomes an efficient mitigation lever and the increase in RMI may cause a decrease in reserve capacity. Figure 5 shows how I^* and a^* depend on the disruption probability ω_τ. Clearly, for low disruption probabilities ($\omega_\tau < 0.04$), reserve capacity is the preferred risk mitigation strategy (section III). The expected overage costs of RMI are too high compared to the reserve capacity costs. For $\omega_\tau > 0.04$, the mixed strategy is preferred and we observe that a^* decreases with ω_τ while I^* increases with ω_τ. This result may be interpreted such that it is not cost effective to keep inventory and be exposed to high excess inventory charges when disruptions are rare. Instead, reserving capacity is more cost effective than building up RMI if disruptions are less likely.

Let us shortly discuss this insight in the context of the pharmaceutical company. For the pharmaceutical supply chain upstream sites that may involve complex biological manufacturing processes are more likely to be disrupted than downstream sites that rather focus on simple packaging procedures. Thus, the higher disruption probability at the upstream sites may induce the firm to hold less RMI and more reserve capacity upstream, whereas the downstream sites with the lower disruption probability are better-off with holding RMI only.

While the optimal production rate of the reserve capacity always increases with the coefficient of variation for normally distributed demand, the optimal RMI level may decrease or increase with the coefficient of variation.

This insight reveals a key difference between RMI and reserve capacity. As RMI is decided prior to the occurrence of a disruption, there is the risk of incurring overrage costs (inventory holding costs) or underage coasts (penalty costs). Clearly, if inventory holding costs are high [low], it is optimal to hold less [more] RMI as demand uncertainty increases (compare discussion of the newsvendor problem). RMI is an on-going decision that can be adapted to the demand uncertainty of the product (for example due to life-cycle changes). In contrast reserve capacity is a design decision that is likewise decided prior to the occurrence of a disruption. However, the production rate that is used in the event of a disruption can be adapted after the occurrence of the disruption. Thus, there is no additional overrage cost in the event of a disruption. Thus, the reserve capacity increases as demand uncertainty increases.

In Figure 6 we show how I^* and a^* depend on the disruption time τ. As expected, RMI increases with the disruption time τ. In contrast, the reserve capacity shows a non-trivial result. For $4 < \tau \leq 15$, a^* and I^* increase with τ. Both risk mitigation levers are complements. For $\tau > 15$, a^* decreases with τ whereas I^* increases with τ. The decrease of a^* can be explained by the additional production costs through the reserve capacity.

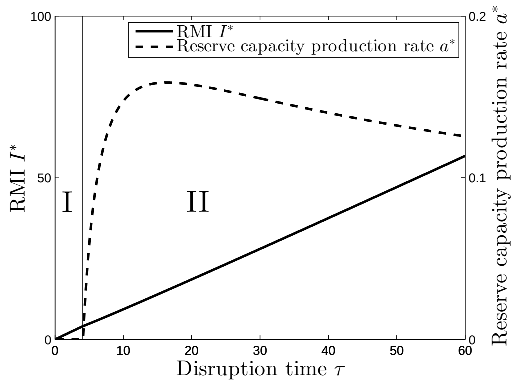

Figure 6: Disruption time τ

5. Conclusion and Outlook

We have examined optimal RMI and reserve capacity decisions under supply chain disruption risk and stochastic demand. We quantify four main risk mitigation strategies: inventory, mixed and reserve capacity strategy, and derive structural insights. We illustrate how the optimal risk mitigation strategy depends on product characteristics (innovative vs functional) and supply chain characteristics (agile versus efficient).

A main limitations of our modeling framework is the assumption of a zero lead time. Clearly, this assumptions allow us to focus the analysis entirely on the role of disruptions risk when determining optimal RMI and reserve capacity quantities. However, by doing so, we neglect potential synergies between safety inventory, which is neglected due to zero lead time, and RMI.

As an avenue for future research we suggest to expand this framework to include multi-echelon supply chains with product transformation at each echelon. Given that various factors push RMI and reserve capacity up- and downstream, it would be interesting to study the optimal location and quantity of RMI and reserve capacity in such systems.

Appendix A. Proof of Proposition 1

Proof. The firm minimizes the expected loss $L(I,a)$ by determining the optimal RMI I and reserve capacity production rate a, which are non-negative:

$$\begin{aligned}\min_{I\geq 0, a\geq 0} \quad & L(I,a) \\ & = \omega_\tau \Bigg(p \int_{I+a\tau}^{\infty} \big(x - I - a\tau\big) f_\tau(x)dx \\ & + h \int_0^I (I-x) f_\tau(x)dx + c_A \int_I^{I+a\tau} (x-I) f_\tau(x)dx \\ & + c_A a\tau \Big(1 - F_\tau\big(I + a\tau\big)\Big)\Bigg) + \big(1-\omega_\tau\big)hI + \hat{c}_A a. \end{aligned} \tag{A.1}$$

We introduce the Lagrangian multipliers λ_I and λ_A to satisfy the constraints $I \geq 0$ and $a \geq 0$. The Karush-Kuhn-Tucker condition leads to four cases:

- $I = 0, a \geq 0$ with $\lambda_I \geq 0, \lambda_a = 0$ (Process flexibility strategy)
- $I \geq 0, a \geq 0$ with $\lambda_I = 0, \lambda_a = 0$ (Mixed strategy)
- $I \geq 0, a = 0$ with $\lambda_I = 0, \lambda_a \geq 0$ (Inventory strategy)
- $I = 0, a = 0$ with $\lambda_I \geq 0, \lambda_a \geq 0$ (Passive acceptance of risk)

For the mixed strategy we find:

$$0 = \frac{\partial L(I,a)}{\partial I} = \hat{h} - \lambda_I + \omega_\tau \Big(-p + F_\tau\big(I + a\tau\big)(p - c_A) + F_\tau(I)(h + c_A)\Big) \tag{A.2}$$

where $\hat{h} = \big(1 - \omega_\tau\big)h$, and

$$\begin{aligned} 0 &= \frac{\partial L(I, a, \tau)}{\partial a} \qquad (A.3) \\ &= \hat{c}_A - \lambda_A + \omega_\tau\Big(c_A\tau - p\tau + F_\tau\big(I + a\tau\big)(p - c_A)\tau\Big). \end{aligned}$$

Both solutions are unique. We have:

$$F_\tau(I + a\tau) = \frac{\omega_\tau(p - c_A)\tau - \hat{c}_A}{\omega_\tau(p - c_A)\tau} \qquad (A.4)$$

and

$$F_\tau(I) = \frac{\omega_\tau[p - F_\tau(I + a\tau)(p - c_A)] - \hat{h}}{\omega_\tau(h + c_A)}, \qquad (A.5)$$

which leads to

$$F_\tau(I) = \frac{(\omega_\tau c_A - \hat{h})\tau + \hat{c}_A}{\omega_\tau\tau(h + c_A)}. \qquad (A.6)$$

For the inventory strategy we note that $a = 0$ for $\hat{c}_A \geq \Delta_1$. We find from Eq. (A.2):

$$F_\tau(I) = \frac{\omega_\tau p - \hat{h}}{(p + h)\omega_\tau}. \qquad (A.7)$$

For reserve capacity we note that $I = 0$ for $\hat{c}_A \leq \Delta_2$. We find from Eq. (A.4):

$$F_\tau(a\tau) = \frac{\omega_\tau(p - c_A)\tau - \hat{c}_A}{\omega_\tau(p - c_A)\tau}. \qquad (A.8)$$

Second-order condition: We calculate the matrix elements of the corresponding Hessian matrix: $\frac{\partial^2 L}{\partial I \partial a} = \omega_\tau\tau(p - c_A)f_\tau(I + a\tau)$, $\frac{\partial^2 L}{\partial I^2} = \omega_\tau(p - c_A)f_\tau(I + a\tau) + (h + c_A)f_\tau(I)$, and $\frac{\partial^2 L}{\partial a^2} = \omega_\tau\tau^2(p - c_A)f_\tau(I + a\tau)$. The determinant of the

Hessian is given by:

$$
\begin{aligned}
|H| &= \omega^2(\tau)\begin{vmatrix} (p-c_A)f_\tau(I+a\tau)+(h+c_A)f_\tau(I) & \tau(p-c_A)f_\tau(I+a\tau) \\ \tau(p-c_A)f_\tau(I+a\tau) & \tau^2(p-c_A)f_\tau(I+a\tau)\end{vmatrix} \\
&= \omega^2(\tau)\tau^2(p-c_A)f_\tau(I+a\tau)\begin{vmatrix} (p-c_A)f_\tau(I+a\tau)+(h+c_A)f_\tau(I) & 1 \\ (p-c_A)f_\tau(I+a\tau) & 1\end{vmatrix} \\
&= \omega^2(\tau)\tau^2(p-c_A)(h+c_A)f_\tau(I+a\tau)f_\tau(I) \\
&\geq 0. \qquad (A.9)
\end{aligned}
$$

□

Appendix B. Proof of Lemma 1

Proof. It is sufficient to look at the following sensitivities: Sensitivity of I^* with h:

$$\frac{d}{dh}\Big(1-\frac{h\tau-\hat{c}_A}{(h+c_A)\omega_\tau\tau}\Big) = -\frac{c_A\omega_\tau\tau^2+\hat{c}_A}{\big((h+c_A)\omega_\tau\tau\big)^2} < 0.$$

Sensitivity of I^* with ω_τ:

$$\frac{d}{d\omega_\tau}\Big(1-\frac{h\tau-\hat{c}_A}{(h+c_A)\omega_\tau\tau}\Big) = \frac{h\tau-\hat{c}_A}{(h+c_A)\tau\omega_\tau^2}.$$

This term is greater than zero because $\hat{c}_A < \Delta_1 = \tau\Big(h\frac{p}{p+h} - \frac{hc_A}{p+h}\Big) \leq \tau h$ for the mixed strategy. Sensitivity of I^* with c_A:

$$\frac{d}{dc_A}\Big(1-\frac{h\tau-\hat{c}_A}{(h+c_A)\omega_\tau\tau}\Big) = \frac{h\tau-\hat{c}_A}{(h+c_A)\omega_\tau\tau}\omega_\tau\tau = \frac{h\tau-\hat{c}_A}{(h+c_A)} > 0. \qquad (B.1)$$

Sensitivity of a^* with c_A:

$$\frac{d}{dc_A}\big(1-\frac{\hat{c}_A}{(p-c_A)\omega_\tau\tau}\big) = -\frac{\hat{c}_A}{\big((p-c_A)\omega_\tau\tau\big)^2}\omega_\tau\tau = -\frac{\hat{c}_A}{\big((p-c_A)\big)^2\omega_\tau\tau} < 0. \tag{B.2}$$

Sensitivity of a^* with p:

$$\frac{d}{dp}\big(1-\frac{\hat{c}_A}{(p-c_A)\omega_\tau\tau}\big) = \frac{\hat{c}_A}{\big((p-c_A)\omega_\tau\tau\big)^2}\omega_\tau\tau = \frac{\hat{c}_A}{(p-c_A)^2\omega_\tau\tau} > 0. \tag{B.3}$$

Regarding the sensitivity of a^* with ω_τ, we assume that $\hat{c}_A < \tau h$. For $\omega_\tau = \omega'_\tau = \frac{\tau h - \hat{c}_A}{\tau h + c_A}$ we have $a^*_0 > 0$ and $I^* = 0$ (risk mitigation strategy III). For $\omega_\tau > \omega'_\tau$ we have $a^*_0 > 0$ and $I^* > 0$, and $\frac{dI^*}{d\omega_\tau} > 0$ (risk mitigation strategy II). Therefore, $\forall \epsilon > 0 \quad \exists \omega_\tau = \omega_{\tau,\epsilon} > \omega'_\tau : \; I^*(\omega_\tau = \omega_{\tau,\epsilon}) = \epsilon$. Then:

$$\frac{d(I^* + a^*\tau)}{d\omega_\tau}|_{\omega_\tau=\omega_{\tau,\epsilon}} = \frac{\hat{c}_A}{f_\tau(I^* + a^*\tau)(p-c_A)\omega_\tau^2\tau}|_{\omega_\tau=\omega_{\tau,\epsilon}} = \frac{\hat{c}_A}{f_\tau(\epsilon + a^*\tau)(p-c_A)\omega_{\tau,\epsilon}^2\tau} = K. \tag{B.4}$$

and

$$\frac{dI^*}{d\omega_\tau}|_{\omega_\tau=\omega_{\tau,\epsilon}} = \frac{h\tau - \hat{c}_A}{f_\tau(I^*)(h+c_A)\tau\omega_\tau^2}|_{\omega_\tau=\omega_{\tau,\epsilon}} = \frac{h\tau - \hat{c}_A}{f_\tau(\epsilon)(h+c_A)\tau\omega_{\tau,\epsilon}^2} = K'$$

Since $f_\tau(x)$ is a smooth and positive functions for $x > 0$ with $f_\tau(0) = 0$, we can choose a $\omega_{\tau,\epsilon} > \omega'_\tau$ such that $K < K'$ and therefore $\frac{d(I^*+a^*\tau)}{d\omega_\tau}|_{\omega_\tau=\omega_{\tau,\epsilon}} < \frac{dI^*}{d\omega_\tau}|_{\omega_\tau=\omega_{\tau,\epsilon}}$. Therefore $\frac{da^*}{d\omega_\tau} < 0$ on the interval $(\omega'_\tau, \omega'_\tau + \omega_{\tau,\epsilon})$. □

Appendix C. Proof of Lemma 2

Proof. If the demand distribution is given by a normal distribution $\mathcal{N}(\mu,\ \sigma)$, we have:

$$I^* = F^{-1}(\beta) = \mu + \sigma\sqrt{2}\ \mathrm{erf}^{-1}(2\beta - 1) \tag{C.1}$$

with $\beta = \frac{(\omega_\tau c_A - \hat{h})\tau + \hat{c}_A}{(h+c_A)\omega_\tau \tau}$. Using the Maclaurin series for the inverse error function erf^{-1}, we have:

$$I^* = F^{-1}(\beta) = \mu + \sigma\sqrt{2}\sum_{k=0}^{\infty} \frac{c_k}{2k+1}\Big(\frac{\sqrt{\pi}}{2}(2\beta - 1)\Big)^{2k+1} \tag{C.2}$$

where $c_0 = 1$ and $c_k = \sum_{m=0}^{k-1} \frac{c_m c_{k-1-m}}{(m+1)(2m+1)}$. Since $c_k \geq 0\ \forall k \in \{1, .., \infty\}$ we have that I^* increases with CV, if $\beta > \frac{1}{2}$ and I^* decreases with CV, if $\beta < \frac{1}{2}$. Otherwise, I^* remains constant.

If the demand distribution is given by a normal distribution $\mathcal{N}(\mu,\ \sigma)$, we have:

$$a^* = \frac{1}{\tau}\Big(F^{-1}(\alpha) - F^{-1}(\beta)\Big) = \sigma\frac{\sqrt{2}}{\tau}\ (\mathrm{erf}^{-1}(2\alpha - 1) - \mathrm{erf}^{-1}(2\beta - 1)) \tag{C.3}$$

with $\alpha = \frac{(p-c_A)\omega_\tau \tau - \hat{c}_A}{(p-c_A)\omega_\tau \tau}$. Using the Maclaurin series for the inverse error function erf^{-1}, we have:

$$a^* = \sigma\frac{\sqrt{2}}{\tau}\sum_{k=0}^{\infty} \frac{c_k}{2k+1}\Big(\frac{\sqrt{\pi}}{2}\Big)^{2k+1}\Big((2\alpha - 1)^{2k+1} - (2\beta - 1)^{2k+1}\Big) \tag{C.4}$$

Since $a^* > 0$ we have $\alpha > \beta$. Therefore, a^* is increasing with CV. □

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