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Financial Complexity and Trade*

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Abstract

What are the implications on trading activity if investors are not sophisticated enough to understand and evaluate trades that have a complex payoff structure? Can frictions generated by this type of financial complexity be so severe that they lead to a complete market freeze, like that of the recent financial crisis? Starting from an allocation that is not Pareto optimal, we find that whether complexity impedes trade depends on how investors perceive risk and uncertainty. For smooth convex preferences, such as subjective expected utility, complexity cannot halt trade, even in the extreme case where each investor is so unsophisticated that he can only trade up to one Arrow-Debreu security, without being able to combine two or more in order to construct a complex trade. However, for non-smooth preferences, which allow for kinked indifference curves, such as maxmin expected utility, complexity can completely shut down trade.

JEL-Classifications: D70, G01.

Keywords: Financial Complexity, Financial Crises, Agreeable Bets, Agreeable Trades, No Trade, Betting, Ambiguity Aversion.

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1 Introduction

An implicit assumption when modeling financial markets is that each investor is sophisticated enough to be able to understand and trade any available security, however complex it might be. In reality, however, cognitive limitations do exist. Investors may have limited attention and time, be unaware of certain dimensions of the payoff structure, have difficulty formulating complex plans or lack special training. Moreover, information acquisition about past performance of some securities may be too costly. The Economic Affairs Committee (2009) reports that "It is hard for investors to evaluate complex financial instruments, because difficult risk modeling is required, and because they are often unaware of the details of the asset pool which backs financial securitisations".

To provide an example, consider the following description of security Jayanne 4, which was marketed by Credit Agricole in 2007 (Célérier and Vallée (2017)):

> This is a growth product linked to a basket composed of the FTSE Euro First 80, the FTSE 100, the SMI and the NIKKEI 225. The Annual Performance is set at 5% for the first three years. In the following years, if the performance since the start date of the worst-performing index is positive or null, then the Annual Performance for that year is registered at 5%, otherwise 0%. The Basket Performance since the start date is registered every six months. The Final Basket Performance is calculated as the average of all these six-monthly readings, capped at a maximum basket performance of 100%. After 8 years, the product offers a guaranteed capital return of 100%, plus the greater of either the sum of the Annual Performances, or 100% of the Final Basket performance.

A typical investor probably understands the indices FTSE Euro First 80, FTSE 100, SMI and NIKKEI 225, which are the basic ingredients of the security's payoff structure. In other words, he understands a "simple" bet that pays 1 if FTSE 100 is above 6000 tomorrow and 0 otherwise. However, he may fail to understand the "complex" Jayanne 4, even though it is "just" a combination of these four indices. Moreover, evaluating bets on these four indices separately is computationally much simpler than evaluating bets on all possible combinations of the four indices.

Another example of a complex security is the Collaterized Debt Obligation (CDO), which pools together cash-flow generating assets (mortgages, bonds and loans) and repackages them into discrete tranches. Because each tranche has a different risk profile and usually incorporates hundreds of thousands of underlying assets, it is a complicated task to work out its payoff structure, even though it is easy to understand the payoff structure of each separate underlying asset.

A result of these cognitive limitations is that although investors may be able to trade "simple" securities, they may fail to consider all of their possible combinations when formulating their portfolio. Polkovnichenko (2005) reports data from the Survey of Consumer Finances, showing that many households invest significant fractions of their wealth simultaneously in well-diversified mutual funds and in un-diversified portfolios of individual stocks. Nieuwerburgh and Veldkamp (2010) derive optimal under-diversification in a framework with costly information acquisition. Carlin et al. (2013) show experimentally that complexity makes subjects less inclined to trade.

Is it possible that the effect of these limitations is so large that opportunities for trade cease to exist completely? We study this question in a complete markets environment with general convex preferences, where all Arrow-Debreu (A-D) securities are available (paying 1 if some state occurs and 0 otherwise).

The complexity of a trade is measured by the number of different values it has across the state space. A trade that provides a different payoff at every state is generated by a combination of all A-D securities, hence its complexity is $|S|$, the number of states. A trade whose payoff differs only with respect to whether a state has occurred or not is generated by the respective A-D security, hence its complexity is 2. Cognitively constrained investors cannot formulate trades that are complex, even if their welfare would increase as a result.

Our notion of complexity effectively allows investors to formulate only "coarse" trades, which are measurable with respect to a coarse partition of the state space. Investors who have maximum perceived complexity of the asset structure can only construct coarse trades that are measurable with respect to a two-element partition, consisting of a state and its complement.

Holding preferences and the initial allocation fixed, if there are gains from trade in an environment without any cognitive restrictions (i.e. the allocation is not Pareto efficient), will there always be trade when all investors have maximum perceived complexity of the asset structure? We analyze this question by characterizing the existence of trading opportunities in terms of the investors' subjective beliefs, in Theorem 2. To obtain the characterization we restrict attention to preferences and initial allocations that satisfy a no-arbitrage principle, so that it is not possible for some investors to make a sure profit. In Proposition 1, we show that no-arbitrage is characterized in terms of the investors' subjective beliefs.[1]

[1]The notion of subjective beliefs applies to all convex preferences, not just subjective expected utility, and is defined in Rigotti et al. (2008).

Whether there is trade in an environment with cognitively constrained investors depends on their attitudes towards risk and uncertainty. In particular, we find that if they have smooth preferences at the initial allocation, which do not allow for kinked indifference curves (e.g. subjective expected utility, smooth ambiguity, multiplier and mean-variance preferences), trading will not stop even if investors have maximum perceived complexity of the asset structure. This is a robust result, because it implies that trade will not stop also if the perceived complexity is less severe, so that investors are able to trade more than one simple security at once. However, we also find that if all investors have maximum perceived complexity of the asset structure, trade can completely freeze if preferences are non-smooth, so that the indifference curve of at least one investor has a kink at his initial allocation. We provide such an example in Section 2 with maxmin expected utility (MEU) and a full insurance allocation.

This dichotomy of preference models provides behavioral implications. Suppose that there is trading in an environment where we control the perceived complexity of the asset structure. If we observe that trading stops as perceived complexity increases, it must be that some of the investors' preferences are non-smooth.

The model can also provide an explanation of why trading froze in some markets during the recent financial crisis. Suppose that financial crises generate a lot of uncertainty (Caballero and Simsek (2013), Brunnermeier and Sannikov (2014)) and that investors have non-smooth preferences. If we know that during the crisis the investors' priors about fundamentals do not change significantly but we nevertheless observe a market freeze, this can be interpreted as the investors perceiving the asset structure of this market to be complex.

To provide an example, Acharya et al. (2009) describe how a series of events that was triggered by an unexpected decrease of the US house prices in the first quarter of 2006 led to the freezing of the market for asset-backed commercial paper in 2007, right after BNP Paribas announced that it was suspending redemptions from its structured investment vehicles, which were trading these types of securities. We can interpret the once in a lifetime decline of US house prices as an event that created uncertainty about fundamentals. However, non-smooth preferences were not sufficient for shutting down trade. This happened one year later, exactly when the suspension of redemptions informed everyone that asset-backed commercial paper was no longer easy to price and value. That this second event, which did not convey any information about fundamentals, triggered an immediate suspension of trade, could be explained by investors realizing that these securities have a complex payoff structure and therefore cannot simultaneously trade any subset of them.

We conclude by discussing some aspects of our definition of complexity. First, the investors' maximum perceived complexity of the asset structure is very different (but less restrictive) from a standard incomplete markets structure with one available A-D security. The former allows investors to buy or sell *any* A-D security but not combine two or more, whereas the latter allows them to trade only the unique security that is available. Second, employing a complete market structure is not restrictive. In Section 4, we discuss how we can generalize to an incomplete market structure, where there exists a partition of the state space and a security pays 1 if an event of that partition occurs and 0 otherwise.

Gul et al. (2017) study investors with cognitive limitations that have coarse (final) consumption plans, instead of coarse trades. For example, maximum perceived complexity in their model means that the investor chooses among all final consumption plans that are measurable with respect to a two-element partition of the state space. However, to finance these consumption plans he is allowed to trade any combination of the A-D securities, thus generating a complex payoff structure, according to our terminology. In contrast, in our model an investor having maximum perceived complexity of the asset structure can only execute a trade that is measurable with respect to a two-element partition, so that if his initial endowment is different across all states, then in general so is his final consumption plan. Finally, the focus of Gul et al. (2017) is different from ours, as they show that allocations are riskier and prices are more extreme when compared to the no perceived complexity case, whereas we examine whether trade would occur.[2]

Finally, our approach is not without limitations. We say that a trade is complex if it reallocates wealth across many states. This is certainly plausible if all the A-D securities are available (or more generally securities that take only two values, like betting on whether an index will go up or down), because to construct such a complex trade an investor needs to combine several securities. It is also plausible if we model investors who think about their trading strategy by conditioning on a few events, for example betting on Trump winning the elections, and then delegate to an expert the construction and execution of the trade. However, it is not plausible if all investors have to execute their own trades and the available securities pay differently across all states. Then, constructing a trade that pays 1 if a state occurs and 0 otherwise might require combining several of these securities. Such a trade would be complex, rather than simple.

[2]See Section 5.3 in Gul et al. (2017) for a detailed comparison of the two approaches.

1.1 Relation to literature

Our paper is related to Billot et al. (2000, 2002), who characterize trading, from a full insurance allocation, in terms of disjoint sets of priors in a complete markets setting with a full insurance allocation, using the MEU model of Gilboa and Schmeidler (1989) and the Choquet expected utility (CEU) model of Schmeidler (1989). Rigotti et al. (2008) generalize these results for all convex preferences, encompassing many models with ambiguity averse preferences. Additionally, they characterize trading from any initial allocation. Ghirardato and Siniscalchi (2018) analyze the case of non-convex preferences.

In the MEU model with two investors and a full insurance allocation, Kajii and Ui (2006) show that there exists an agreeable bet on event E if and only if the maximum of the probability of E for one investor is smaller than the minimum of that for the other investor. In the case where each investor's prior is the core of a convex capacity, they show that an agreeable bet on *some* event exists if and only if there is no common prior, hence it is equivalent to the existence of an agreeable trade. Dominiak et al. (2012) extend this result for the CEU model with not necessarily convex capacities. An agreeable bet on E is replicated in our model by an incomplete markets setting with one security that pays 1 if E occurs and 0 otherwise. As we discuss in Section 4, we can generalize our approach to an incomplete markets setting, by having a partition of the state space and each available security paying 1 if an event E of the partition occurs and 0 otherwise. Theorem 2 improves on the results of Kajii and Ui (2006) and Dominiak et al. (2012) by providing a characterization of no-trade for any finite number of investors with general convex preferences and any initial allocation, many available securities but maximum perceived complexity of the asset structure. More importantly, our characterization applies irrespective of whether an agreeable trade (absence of perceived complexity) is equivalent to an agreeable bet (maximum perceived complexity), which is crucial in separating between models with smooth and non-smooth preferences. However, we impose a no-arbitrage condition, whereas Kajii and Ui (2006) and Dominiak et al. (2012) do not.

Our definition of complexity relates to how investors perceive the asset structure. Alternatively, Caballero and Simsek (2013) use ambiguity and the notion of complexity about the structure of cross exposures of banks to explain market freezes.

Rigotti and Shannon (2005) characterize Pareto optima and equilibria in the incomplete preferences model of Knightian uncertainty of Bewley (1986). This model is used also by Easley and O'Hara (2010) to explain no-trade. Rigotti and Shannon (2012) show that generic determinacy is a robust feature of general equilibrium models with ambiguity averse preferences, because kinks are relatively rare, whereas robust indeterminacies arise naturally in the model of Bewley (1986), where kinks are ubiquitous. Although we prove our results only for complete convex preferences, it is straightforward to extend them in the incomplete preferences model of Bewley (1986).[3] In this model, the indifference curve at any endowment has a kink, hence maximum complexity would generically shut down trade, as opposed to some models with ambiguity aversion, where indifference curves are smooth at non full insurance endowments.

Our main difference from these papers is that we use complexity in order to explain no-trade in an environment where there are actually gains from trade. This difference is important, because we suggest that if complexity was lifted then there would be gains from trade, whereas the aforementioned papers suggest that (in the case of no-trade) uncertainty has destroyed all gains from trade. Moreover, our mechanism can help explain the BNP Paribas incident, by suggesting that the market froze not because of the initial event that created uncertainty (and hence ambiguity or incompleteness) about house prices, but due to the investors' realization that the payoff structure was too difficult to understand.

Rigotti and Shannon (2005) provide conditions under which endogenous incomplete markets can arise. Roughly, if investors have different but precise probabilities about some states but similar but imprecise probabilities about the remaining states, then they trade only A-D securities contingent on the former set, so it is as if the latter securities are missing. Our focus is different, because we ask whether trade would still occur in the case of maximum perceived complexity, which is not the same as an incomplete markets structure with one available security. Since investors have different and precise probabilities about at least one state, maximum perceived complexity does not shut down trade in any such setting with endogenous incomplete markets.

Lang (2017) defines first-order and second-order ambiguity aversion and characterizes them in term of whether the indifference curve at the endowment point has a kink or it is smooth. He also provides several economic examples where this dichotomy matters. Using his terminology, the present paper shows that second-order ambiguity aversion implies that maximum perceived complexity does not shut down trade. Mihm (2016) proposes a model of reference-dependent MEU preferences where the indifference curve has a kink at the endowment, so that using our results maximum perceived complexity can shut down trade.

Our paper is related to the growing literature on complex securities (Amromin et al. (2011), Henderson and Pearson (2011), Ghent et al. (2017), Griffin et al. (2014), Hens

[3]See Section 4 for details.

and Rieger (2014), Sato (2014)). Célérier and Vallée (2017) study more than 50,000 securities and show that complexity has increased over time. Simsek (2013) shows that complexity increases opportunities for speculation in a model with heterogeneous beliefs. He uses smooth (mean-variance) preferences for which, according to the present paper, complexity does not shut down trade.

Our notion of complexity specifies that the investor has a coarse understanding of his available trading strategies. Alternatively, several strands of the literature study the coarse understanding of the state space, such as in decision theory (Dekel et al. (2001), Epstein et al. (2007), Ahn and Ergin (2010)), unawareness (Fagin and Halpern (1988), Heifetz et al. (2006), Galanis (2013)) and inattention (Sims (2003), Woodford (2012), Gabaix (2014)).

The paper is organized as follows. In the next section we provide an example which illustrates our approach. Section 3 introduces the model and characterizes the occurrence of trade in the case where all investors have maximum perceived complexity of the asset structure. Section 4 concludes.

2 Illustration

We illustrate our approach using an example. Let $S = \{s_1, s_2, s_3\}$ be the state space, describing the uncertainty about tomorrow. Consider a standard complete markets setting with three A-D securities, each paying 1 if a particular state $s \in S$ occurs and 0 otherwise. The economy consists of two investors, i and j, who have the same endowment $e = (5, 5, 5)$, paying 5 at every state. We call this a full insurance allocation. Their preferences are represented by maxmin expected utility (MEU) with $u^i(x) = x$, $x \in \mathbb{R}_+$. In particular, i's utility from trade $f^i \in \mathbb{R}^3$, where $e + f^i \geq 0$, is

$$V^i(e + f^i) = \min_{q \in C^i} \sum_{s \in S} u^i(e(s) + f^i(s))q(s),$$

where $C^i \subseteq \Delta S$ is a compact and convex set of beliefs.

Suppose that C^i is the convex hull of probabilities $p_1 = (0.2, 0.6, 0.2)$, $p_2 = (0.2, 0.4, 0.4)$ and $p_3 = (0.3, 0.5, 0.2)$, whereas C^j is the convex hull of probabilities $q_1 = (0.4, 0.4, 0.2)$ and $q_2 = (0.3, 0.4, 0.3)$.

These sets are shown in Figure 1. The triangle represents the probability simplex, so that each point represents a probability on $\{s_1, s_2, s_3\}$. A dashed line, which is parallel to a side of the triangle, contains all probabilities that assign the same weight on the state depicted opposite to that side. Set C^i is the triangle formed by p_1, p_2 and

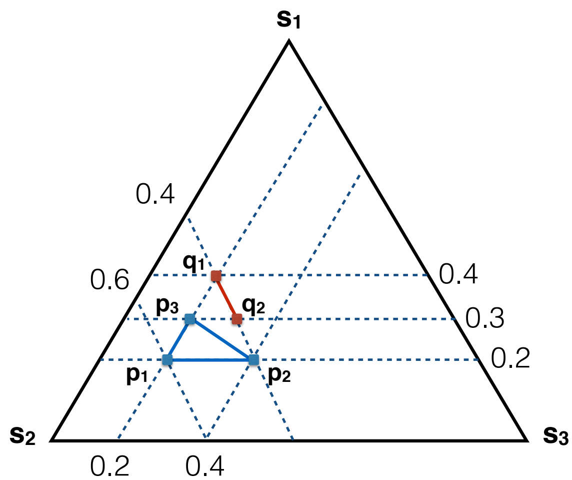

Figure 1: Trade occurs only in the absence of complexity

p_3, whereas C^j is the line formed by q_1 and q_2.

Because C^i and C^j are disjoint, there are trades that will make both strictly better off than consuming their endowment, which gives utility 5. For example, consider $f^i = (-3.2, 2.5, -0.5)$ and $f^j = (3.2, -2.5, 0.5)$, which is a trade because $f^i + f^j = 0$. It is an agreeable trade because both investors strictly prefer it to their endowment, as $V^i(e + f^i) = 5.16$ and $V^j(e + f^j) = 5.11$. Billot et al. (2000) show that in the MEU model with a full insurance allocation, there is a trade (i.e. the initial allocation is not Pareto optimal) if and only if the sets of beliefs are disjoint.

According to our terminology, $\{f^i, f^j\}$ is a complex trade, because it provides a different payoff at every state, hence its construction requires a combination of all three A-D securities.

Suppose now that each investor is cognitively constrained, so that he can buy or sell at most one A-D security and cannot combine two or more to construct a complex trade. In other words, he can only formulate a coarse trade, which is measurable to a two-element partition of S, consisting of a state and its complement. Then, trade between the two investors translates to i betting on state s occurring, hence buying the A-D security, and j betting on s not occurring, hence selling the same security.

Consider an A-D security on state s_1, with price $c < 1$. If investor i sells it, he bets that s_1 will not occur and the trade he gets is $f^i = (c - 1, c, c)$, whereas if j buys it he gets $f^j = (1 - c, -c, -c)$.

Let $\overline{p}^i(s) = \max\limits_{p\in C^i} p(s)$ and $\underline{p}^i(s) = \min\limits_{p\in C^i} p(s)$ be i's maximum and minimum belief on state s. If both investors agree on this simple trade $\{f^i, f^j\}$, then investor i must strictly prefer $e + f^i$ over e, and similarly for j. In particular, $V^i(e + f^i) = 5 + \overline{p}^i(s_1)(c-1) + (1-\overline{p}^i(s_1))c > V^i(e) = 5$ and $V^j(e+f^j) = 5+\underline{p}^j(s_1)(1-c)-(1-\underline{p}^j(s_1))c > V^j(e) = 5$. These inequalities imply $\underline{p}^j(s_1) > c > \overline{p}^i(s_1)$. In other words, i agrees to sell if all of his beliefs place a small weight on s_1 happening, relative to j's beliefs. Similarly, i buys the A-D security on s_1 that j sells if $\overline{p}^j(s_1) < c < \underline{p}^i(s_1)$. Put more compactly, if the two investors agree to bet on s_1 then $[\underline{p}^i(s_1), \overline{p}^i(s_1)] \cap [\underline{p}^j(s_1), \overline{p}^j(s_1)] = \emptyset$ and the converse is also true.

In this example the two investors do not agree to bet on s_1 because $[\underline{p}^i(s_1), \overline{p}^i(s_1)] \cap [\underline{p}^j(s_1), \overline{p}^j(s_1)] \neq \emptyset$, and the same is true for s_2 and s_3. Can we generalize this result, so that we find sets of beliefs, $\overline{C}^i$ and $\overline{C}^j$, such that they are disjoint if and only if the two investors agree to bet on *some* state or, equivalently, to trade a particular A-D security?

Let $\overline{C}^i$ be the set of probability measures p such that $\underline{p}^i(s) \leq p(s) \leq \overline{p}^i(s)$ for all states s, and similarly for $\overline{C}^j$. In Figure 1, $\overline{C}^j = C^j$ is still the convex hull of q_1 and q_2. However, $\overline{C}^i$ is bigger than C^i, as it is the convex hull of p_1, p_2, p_3 and q_2. It is constructed by including all probabilities that are within the respective dashed lines.

It is straightforward that if $[\underline{p}^i(s), \overline{p}^i(s)] \cap [\underline{p}^j(s), \overline{p}^j(s)] = \emptyset$ for some s (so that there is betting on that state), then $\overline{C}^i$ and $\overline{C}^j$ are disjoint. But the converse is not true. A counterexample in Section 3.5 shows that even if $[\underline{p}^i(s), \overline{p}^i(s)] \cap [\underline{p}^j(s), \overline{p}^j(s)] \neq \emptyset$ for all states, so that there is never any betting, $\overline{C}^i$ and $\overline{C}^j$ may still be disjoint. However, it turns out that these cases violate the following no-arbitrage condition: it is not possible for any investor to successfully offer to others a series of bets that others will accept and it will give him a positive payoff at all states. Sets $\overline{C}^i$ and $\overline{C}^j$ have q_2 as a common element, which is consistent with no betting on any state.

We have shown that starting from an allocation where there are gains from trade and as the investors' perceived complexity of the asset structure increases, it is as if their sets of beliefs expand and they are no longer disjoint, leading to a result of no-trade. The example might seem restrictive because, with only three states, there is either maximum perceived complexity ($\mathcal{K} = 2$) or no perceived complexity ($\mathcal{K} = |S|$). The following modification shows that it is easy to construct an example where there is no-trade with maximum perceived complexity, however there are trading opportunities with intermediate perceived complexity ($2 < \mathcal{K} < |S|$). Suppose there are four states and preferences are as before, whereas each investor's endowment is $(5, 5, 5, 5)$. Let C^i be the convex hull of probabilities $p_1 = (0.2, 0.6, 0.1, 0.1)$, $p_2 = (0.2, 0.4, 0.2, 0.2)$ and $p_3 =$

$(0.3, 0.5, 0.1, 0.1)$, whereas C^j is the convex hull of probabilities $q_1 = (0.4, 0.4, 0.1, 0.1)$ and $q_2 = (0.3, 0.4, 0.15, 0.15)$. Compared to the initial example, the probability of s_3 is now equally divided between s_3 and s_4. Similar arguments show that there is no-trade under maximum perceived complexity, because i is not willing to bet with j at any state. However, there are trading opportunities with $\mathcal{K} = 3$. For example, trade $f^i = (-3.2, 2.5, -0.5, -0.5)$, $f^j = (3.2, -2.5, 0.5, 0.5)$ is strictly preferable because $V^i(e + f^i) = 5.16$ and $V^j(e + f^j) = 5.11$.

We also assume MEU with linear u, two investors and a full insurance allocation. Our main result, Theorem 2, shows that the arguments can be substantially generalized to accommodate any allocation and any finite number of investors with preferences that are complete, transitive, strongly monotonic and convex.

Instead of starting from a set C^i of beliefs which are specific to the MEU model, we follow Rigotti et al. (2008) and consider the set of subjective beliefs $\pi^i(e^i)$, which are the prices of A-D securities, normalized to sum to 1, such that i would prefer not trading his endowment e^i.[4] Mathematically, subjective beliefs are the normals of the supporting hyperplanes of the indifference curve at the endowment point. If preferences are smooth at the endowment then $\pi^i(e^i)$ is a singleton, but if there is a kink then it is a general convex and compact set. In the standard case of no perceived financial complexity, Rigotti et al. (2008) show that an allocation is Pareto optimal if and only if there is a common subjective belief.

In the case where investors have maximum perceived complexity of the asset structure, we establish that they agree to bet on a state if and only if the intervals defined by their maximum and minimum subjective beliefs on that state are disjoint. For this result, convexity of preferences is crucial. We then define $\overline{\pi}^i(e^i)$ in the same way as $\overline{C}^i$ and say that there is an $\overline{S}$-common belief if the intersection for all investors is nonempty. Our main result, Theorem 2, specifies that there there is trade in an environment with maximum perceived complexity if and only if there is no $\overline{S}$-common belief. To achieve this characterization, we assume a no-arbitrage condition, only considering initial allocations where it is not possible for some investors to obtain a sure profit by successfully offering a bet to all other investors. Proposition 1 characterizes this no-arbitrage condition in terms of the investors' subjective beliefs.

It is straightforward that trading between investors with maximum perceived complexity of the asset structure implies trading in the absence of perceived complexity. However, the converse is not true, as shown in Figure 1, which means that as complexity increases (or investors become less sophisticated), trade may eventually stop

[4] In the MEU model with a full insurance allocation, $C^i = \pi^i(e^i)$.

completely. Nevertheless, trade in one environment is equivalent to trade in the other environment if each investor's set of subjective beliefs consists of a single probability measure, which is the case of smooth (differentiable) indifference curves at the endowment point. This observation allows us to obtain a dichotomy of models with convex preferences (including most models with ambiguity aversion), in terms of whether the investors' perceived complexity of the asset structure impedes trade.

3 Model

3.1 Set up

Consider a set I of investors with typical element i and a single consumption good. Uncertainty is represented by a finite set of payoff relevant states S, with typical element s. The set of consequences is $\mathbb{R}_+$, interpreted as monetary payoffs. Investor i has binary preference relation $\succsim^i$ on the set of acts $\mathcal{F} = \mathbb{R}_+^S$, which satisfies the following standard axioms.

Axiom 1. *(Preference). $\succsim^i$ is complete and transitive.*

Axiom 2. *(Continuity). For all $f \in \mathcal{F}$, the sets $\{g \in \mathcal{F} : g \succsim^i f\}$ and $\{g \in \mathcal{F} : f \succsim^i g\}$ are closed.*

Axiom 3. *(Convexity). For all $f \in \mathcal{F}$, the set $\{g \in \mathcal{F} | g \succsim^i f\}$ is convex.*

Axiom 4. *(Strong Monotonicity). For all $f \neq g$, if $f \geq g$, then $f \succ^i g$.*

An economy is a tuple $\{\succsim^i, e^i\}_{i\in I}$, where $|I| \geq |S| + 1$ and $\{e^i\}_{i\in I} \in \mathbb{R}_{++}^{SI}$ is the interior initial allocation. An economy is large if each investor i has at least $|S|$ copies.

We assume a complete market with a collection $\{d_s\}_{s\in S}$ of A-D securities, where d_s has price p_s and pays 1 if state s occurs and 0 otherwise. A portfolio $\{a_s\}_{s\in S}$ at prices $\{p_s\}_{s\in S}$ generates net trade $f \in \mathbb{R}^S$ such that $f = \sum\limits_{s\in S} a_s d_s - \mathbf{1} \sum\limits_{s\in S} a_s p_s$, where a_s denotes the units of security d_s which are bought if $a_s > 0$ or sold if $a_s < 0$ and $\mathbf{1}$ pays 1 at every state. Investor i weakly prefers this portfolio to his initial endowment if $e^i + f \succsim^i e^i$.

3.2 Subjective beliefs

Based on Yaari (1969), Rigotti et al. (2008) define investor i's *subjective beliefs at an act f* to be the normals (normalized to be probabilities) of all supporting hyperplanes at f,

$$\pi^i(f) = \{p \in \Delta S : \mathbb{E}_p g \geq \mathbb{E}_p f \text{ for all } g \succsim^i f\},$$

where $\mathbb{E}_p f = \sum_{s \in S} p(s) f(s)$ is the expectation of f given probability measure p. For convex preferences, $\pi^i(f)$ is nonempty, convex and compact.

Rigotti et al. (2008) establish the following two properties for strictly positive acts f and convex preferences.[5] First, $\mathbb{E}_p f \geq \mathbb{E}_p g$ for some $p \in \pi^i(f)$ implies $f \succsim^i g$. Second, $\mathbb{E}_p g > \mathbb{E}_p f$ for all $p \in \pi^i(f)$ implies $\epsilon g + (1-\epsilon) f \succ^i f$ for sufficiently small $\epsilon > 0$.

3.3 Common beliefs

We say that there is a common belief at initial allocation $\{e^i\}_{i \in I}$ if $\bigcap_{i \in I} \pi^i(e^i) \neq \emptyset$. For each $s \in S$, let $\underline{p}^i(s) = \min_{p \in \pi^i(e^i)} p(s)$ and $\overline{p}^i(s) = \max_{p \in \pi^i(e^i)} p(s)$ be i's minimum and maximum subjective belief about s, respectively. Let

$$\overline{\pi}^i(e^i) = \{q \in \Delta S : \underline{p}^i(s) \leq q(s) \leq \overline{p}^i(s) \text{ for all } s \in S\}$$

be the set of probability measures that are within i's minimum and maximum subjective beliefs at e^i, for each $s \in S$.[6] We next define a weaker notion of common beliefs.

Definition 1. *There is an $\overline{S}$-common belief at $\{e^i\}_{i \in I}$ if $\bigcap_{i \in I} \overline{\pi}^i(e^i) \neq \emptyset$.*

Because $\pi^i(e^i) \subseteq \overline{\pi}^i(e^i)$, if there is a common belief then there is a $\overline{S}$-common belief but the converse is not true, as shown in Figure 1. However, if $\pi^i(e^i)$ is a singleton for each i, then the two notions are equivalent.[7] In Section 3.7, we show that there is trade in the presence of maximum perceived complexity if and only if there is no $\overline{S}$-common belief.

3.4 Trades and bets

We say that $f \in \mathbb{R}^S$ is a bet on state $s \in S$ if there exist $a, b \in \mathbb{R}$, $ab < 0$, such that $f(s') = a$ if $s' = s$ and $f(s') = b$ otherwise. A bet on s can be constructed by buying or short selling some units of an A-D security d_s that pays 1 if s occurs and 0 otherwise. A bet on s where $a > 0 > b$ can be generated by buying, at price $-\frac{b}{a-b}$, $a - b$ units of

[5] An act f is strictly positive if $f(s) > 0$ for all $s \in S$.

[6] Note that $\overline{\pi}^i(e^i)$ is a closed and convex polytope, as it is bounded and the intersection of half spaces.

[7] The converse is not true, so that if the set of common beliefs is equal to the set of $\overline{S}$-common beliefs, it is not the case that each $\pi^i(e^i)$ is a singleton.

d_s. If state s does not occur, then the payoff is $(a-b)\frac{b}{a-b} = b < 0$. If s occurs, the payoff is $(a-b)\frac{b}{a-b} + a - b = a > 0$.

Similarly, a bet on s where $b > 0 > a$ can be generated by selling, at price $\frac{b}{b-a}$, $b-a$ units of d_s. If state s does not occur, then the payoff is $(b-a)\frac{b}{b-a} = b > 0$. If s occurs, the payoff is $(b-a)\frac{b}{b-a} - (b-a) = a < 0$.

Tuple $\{f^i\}_{i\in I} \in \mathbb{R}^{SI}$ is a *trade* if $\sum\limits_{i\in I} f^i = 0$ and $e^i + f^i \geq 0$ for all $i \in I$. It is a *bet* if, additionally, each f^i is a bet on some s. Trade $\{f^i\}_{i\in I}$ is *agreeable* if $e^i + f^i \succ^i e^i$ for all $i \in I$. A trade is an *agreeable bet* if it is an agreeable trade and a bet.

3.5 No-arbitrage

Gains from trade exist at an allocation if it is not Pareto optimal. However, in an environment with uncertainty and no common prior, the notion of Pareto improvement is not as compelling as in an environment with certainty.[8] In what follows, we only consider Pareto improvements from initial allocations that satisfy a no-arbitrage condition, so that it is not possible for some investors to make a sure profit by offering a trade or a bet that others are willing to take. We then show in Proposition 1 that this condition imposes a restriction on the investors' subjective beliefs at this allocation.

Definition 2. *Tuple $\{f^i\}_{i\in I} \in \mathbb{R}^{SI}$ is an arbitrage trade at endowment e if it is an agreeable trade and there exist partitions $\{A, B\}$ of I and $\{S_i\}_{i\in B}$ of S such that:*

- *for all $i \in A$, $f^i(s) = k_i > 0$ for all $s \in S$,*
- *for all $i \in B$, $f^i = \sum\limits_{s\in S_i} h_s$, where h_s is a bet on s,*
- *for all $i \in I$ and all $s \in S$, $e^i \succsim^i e^i - h_s$.*

It is an arbitrage bet at e if, additionally, each S_i is a singleton.

In an arbitrage trade there are two types of investors. Each $i \in A$ is an arbitrageur, receiving a positive and fixed payoff k_i at each state, thus making a sure profit. Each $i \in B$ is a bettor, willing to bet on all states $s \in S_i$. That is, he prefers $e^i + \sum\limits_{s\in S_i} h_s$ over e^i. If S_i is a singleton, then he only bets on one state s and receives $e^i + h_s$. Note that two bettors never bet on the same state, but collectively they all bet on the whole state space. Because the securities are in zero net supply, the sure profit of the arbitrageurs (the sum of all k_i's) is equal to $-\sum\limits_{s\in S} h_s$, the opposite side of all the bets made by the

[8]See Gilboa et al. (2014) and Brunnermeier et al. (2014) for a discussion of this issue in the subjective expected utility environment.

bettors. The third condition specifies that no investor is willing to take the opposite side of an individual bet h_s. That is, he prefers his endowment e_i over $e^i - h_s$.[9]

An arbitrageur $i \in A$ who receives k_i may still have maximum perceived complexity of the asset structure, because his trade is constant across all states. However, if all investors have maximum perceived complexity, then they may not be able to formulate arbitrage trades, because these may require that a bettor trades more than one A-D security. In that case, they will still be able to formulate arbitrage bets, where each bettor $i \in B$ bets only on one state and S_i is a singleton.

Definition 3. *There is no arbitrage at e if there are no arbitrage trades at e, or there are no arbitrage bets at e and the economy is large.*

It is important to emphasise that no-arbitrage is a joint restriction on preferences and endowments, not on prices of assets. We require this condition in Theorem 2, which characterizes trading in the case where all investors have maximum perceived complexity of the asset structure. When the economy is large, so there are $|S|$ copies of each investor i, we only need to assume that there are no arbitrage bets at e. As we show below, no-arbitrage is characterized in terms of subjective beliefs. More importantly, it excludes cases where there is no bet on any state, yet there is no $\overline{S}$-common belief.

Fix preferences and the initial endowment e. As we argued in the example of Section 2 and Theorem 2 below generalizes, there is an agreeable bet on state s between i and j if and only if $[\underline{p}^i(s), \overline{p}^i(s)] \bigcap [\underline{p}^j(s), \overline{p}^j(s)] = \emptyset$, implying that either $\underline{p}^i(s) > \overline{p}^j(s)$ or $\underline{p}^j(s) > \overline{p}^i(s)$. In other words, the A-D prices about s that would sustain zero net demand are very different for i and j. More generally, define

$$[\underline{q}(s), \overline{q}(s)] \equiv \bigcap_{i \in I} [\underline{p}^i(s), \overline{p}^i(s)]$$

to be the conjunction of all the constraints about state s that an $\overline{S}$-common belief must satisfy. Then, $[\underline{q}(s), \overline{q}(s)] = \emptyset$ is equivalent to the existence of a bet on s between two investors.

If $[\underline{q}(s), \overline{q}(s)] = \emptyset$ for some state s, then it is straightforward that there is no $\overline{S}$-common belief. However, the converse is not true. It is possible that $[\underline{q}(s), \overline{q}(s)] \neq \emptyset$ for all $s \in S$, so that there is no agreeable bet on any state, yet there is no $\overline{S}$-common belief. However, in that case there are arbitrage opportunities.

To show this, consider the following example with four states and five investors. Investors $i = 1, 2$ have identical preferences and endowments. Their set of subjective beliefs $\pi^i(e^i)$ is the convex hull of the following three probabilities,

[9] If he were, he might be tempted to enter into a bidding war and offer a bet with better odds.

$$(0.2315, 0.0385, 0.2773, 0.4527),$$
$$(0.2306, 0.1668, 0.3948, 0.2078),$$
$$(0.1549, 0.0163, 0.4365, 0.3923).$$

Investors $i = 3, 4, 5$ have identical preferences and endowments. Their set of subjective beliefs $\pi^i(e^i)$ is the convex hull of probabilities

$$(0.0303, 0.5476, 0.3179, 0.1042),$$
$$(0.4765, 0.1215, 0.1492, 0.2528),$$
$$(0.3107, 0.2340, 0.3140, 0.1413).$$

We then have that $[\underline{q}(s_1), \overline{q}(s_1)] = [0.1549, 0.2315]$, $[\underline{q}(s_2), \overline{q}(s_2)] = [0.1215, 0.1668]$, $[\underline{q}(s_3), \overline{q}(s_3)] = [0.2773, 0.3179]$ and $[\underline{q}(s_4), \overline{q}(s_4)] = [0.2078, 0.2528]$. There is no $\overline{S}$-common belief because any p that satisfies the previous four constraints has at most $\sum_{s \in S} p(s) \leq \sum_{s \in S} \overline{q}(s) = 0.969 < 1$.

However, this example is problematic because it allows for arbitrage. Fix $\sum_{s \in S} \overline{q}(s) < 1$ and consider, for each state s, a bet f_{a_s} for investor i with endowment e^i and $\overline{q}(s) = \overline{p}^i(s)$, that pays $a_s - 1$ if s occurs and a_s otherwise, where a_s is bigger but arbitrarily close to $\overline{p}^i(s)$, so that $\sum_{s \in S} a_s < 1$. This bet can be generated by short selling an A-D security d_s at price a_s.

Because the expectation $\mathbb{E}_p(f_{a_s} + e^i) = a_s - p(s) + \mathbb{E}_p e^i > \mathbb{E}_p e^i$ for all $p \in \pi^i(e^i)$, convexity of preferences implies that for small enough $k > 0$, investor i strictly prefers the convex combination $k(f_{a_s} + e^i) + (1-k)e^i = kf_{a_s} + e^i$ to e^i.

Investor 5 (the arbitrageur) can offer bet $kf_{a_{s_1}}$ to investor 1, $kf_{a_{s_2}}$ to investor 2, $kf_{a_{s_3}}$ to investor 3 and $kf_{a_{s_4}}$ to investor 4, such that $\sum_{s \in S} a_s < 1$. Essentially, investor 5 is offering to buy k units of A-D security d_s at price a_s, for each state s. These bets are going to be accepted, because they make each $i = 1, 2, 3, 4$ strictly better off. Moreover, no other investor is willing to take the opposite side of each bet, $-kf_{a_s}$. However, investor 5's payoff at any state s is $-k \sum_{s' \neq s} a_{s'} - k(a_s - 1) = -k(\sum_{s \in S} a_s - 1) = k(1 - \sum_{s \in S} a_s) > 0$. Hence, all conditions of Definition 2 are satisfied.

The following Proposition generalizes this result.

Proposition 1. *Suppose that for each $s \in S$, $[\underline{q}(s), \overline{q}(s)] \equiv \bigcap_{i \in I} [\underline{p}^i(s), \overline{p}^i(s)] \neq \emptyset$. Then, no-arbitrage at e implies $\sum_{s \in S} \underline{q}(s) \leq 1 \leq \sum_{s \in S} \overline{q}(s)$. Conversely, $\sum_{s \in S} \underline{q}(s) \leq 1 \leq \sum_{s \in S} \overline{q}(s)$ implies that there are no arbitrage bets at e.*

Proof. Suppose that for each $s \in S$, $[\underline{q}(s), \overline{q}(s)] = \bigcap_{i \in I} [\underline{p}^i(s), \overline{p}^i(s)] \neq \emptyset$ and $\sum_{s \in S} \overline{q}(s) < 1$. Choose $a_s > \overline{q}(s)$ such that $\sum_{s \in S} a_s < 1$. We will construct an arbitrage trade $\{f^i\}_{i \in I}$, or an arbitrage bet in the case that the economy is large. Note that for each $s \in S$, $\overline{q}(s) = \overline{p}^i(s)$ for some $i \in I$ and $\overline{q}(s) \leq \overline{p}^j(s)$ for all $j \in I$.

Consider bet $f_{s,a}$ that pays $a-1$ at s and a otherwise. For all a such that $a > \overline{p}^i(s) = \overline{q}(s)$, we have that $\mathbb{E}_p(f_{s,a} + e^i) = a - p(s) + \mathbb{E}_p e^i > \mathbb{E}_p e^i$ for all $p \in \pi^i(e^i)$. From the second property of π^i, there exists small enough $k \in (0,1)$ such that $k(f_{s,a} + e^i) + (1-k)e^i = kf_{s,a} + e^i \succ^i e^i$. Therefore, investor i would strictly prefer to get bet $kf_{s,a}$ which pays $ka - k$ at s and ka otherwise. Moreover, this is also true for any $0 < k_0 < k$. Note that investor i is a bettor and belongs to set B of Definition 2.

The third condition of an arbitrage trade is also satisfied because $[\underline{q}(s), \overline{q}(s)] \neq \emptyset$ and $a > \overline{p}^i(s) = \overline{q}(s)$ imply $a > \underline{p}^i(s)$ for all $i \in I$. This means that $\mathbb{E}_p(-f_{s,a} + e^i) = -a + \underline{p}^i(s) + \mathbb{E}_p e^i < \mathbb{E}_p e^i$ for some $p \in \pi^i(e^i)$, hence the first property of the subjective beliefs implies that $e^i \succsim^i e^i - f_{s,a}$. From convexity, we also have $e^i \succsim^i e^i - kf_{s,a}$.

By repeating the same argument for each $s \in S$, we can create a tuple $\{k_s f_{s,a_s}\}_{s\in S}$ of bets. If the economy is not large, there is the possibility that for the same investor $i \in B$ we have $\overline{p}^i(s) = \overline{q}(s)$, $\overline{p}^i(s') = \overline{q}(s')$ and this can be true for more than two states. Then, the same arguments show that investor i would strictly prefer to get $k_s f_{s,a_s} + k_{s'} f_{s',a_{s'}}$. However, in this case i receives an f^i that provides different payoffs in s, s' and $S \setminus \{s, s'\}$, hence it is an arbitrage trade and not an arbitrage bet. If the economy is large, then there are $|S|$ investors with the same preferences as i, hence we can assign to each copy a bet on a different state.

By setting $k = \min_{s\in S} k_s$, the new tuple is $\{kf_{a_s}\}_{s\in S}$, where for each i with $\overline{q}(s) = \overline{p}^i(s)$ we have $f^i = kf_{a_s}$ and $kf_{a_s} + e^i \succ^i e^i$. Because there are at least $|S|+1$ investors, we can assign one investor i^* to take the opposite side of $\{kf_{a_s}\}_{s\in S}$, which yields $\sum_{s\in S} -kf_{a_s}$ and pays $-k\sum_{s'\neq s} a_{s'} - k(a_s - 1) = k(1 - \sum_{s\in S} a_s) > 0$ at all states. In other words, $i^* \in A$ is an arbitrageur. For any j who has not been offered kf_{a_s} for some $s \in S$, we let $f^j(s) = k_j > 0$ for all $s \in S$, where k_j is arbitrarily small. From Axiom 4, each j would strictly prefer to accept f^j and this is affordable because $\sum_{s\in S} - kf_{a_s}$ is strictly positive at all states, so i^* would get a slightly lower payoff at each state and each such $j \in A$ is an arbitrageur as well. Hence, we have created an arbitrage trade, or an arbitrage bet in the case of a large economy. We can create a similar arbitrage opportunity if $\sum_{i\in I} \underline{q}(s) > 1$, with bets that pay $k - ka_s$ at s and $-ka_s$ otherwise, where $\underline{q}(s) > a_s$, for small enough $k > 0$.

Conversely, suppose $\sum_{s\in S} \underline{q}(s) \leq 1 \leq \sum_{s\in S} \overline{q}(s)$ but there is an arbitrage bet $\{f^i\}_{i\in I}$. By definition we have $\sum_{i\in B} f^i < 0$ and $f^i + e^i \succ^i e^i$ for all $i \in B$, where f^i is of the form $f^i(s^i) = a^i$ and $f^i(s) = b^i$ for $s \neq s^i$, $a^i \neq b^i$. From the first property of $\pi^i(e^i)$ and

Axiom 4, we have that $\mathbb{E}_p(e^i + f^i) > \mathbb{E}_p e^i$ for all $p \in \pi^i(e^i)$, all $i \in I$.

We next show that $\mathbb{E}_p f^i > 0$ for all $p \in \overline{\pi}^i(e^i)$ and all $i \in I$. For all $i \in A$ this is obvious, as they receive a fixed positive payoff at all states. Fix $i \in B$ and let $s^i = s$. Note that $\overline{p}^i(s)a^i + \underline{p}^i(S \setminus s)b^i > 0$ and $\underline{p}^i(s)a^i + \overline{p}^i(S \setminus s)b^i > 0$, because $1 - \overline{p}^i(s) = \underline{p}^i(S \setminus s)$ and $1 - \underline{p}^i(s) = \overline{p}^i(S \setminus s)$, where $\underline{p}^i(E) = \min_{p \in \pi^i(e^i)} p(E)$ and $\overline{p}^i(E) = \max_{p \in \pi^i(e^i)} p(E)$, for any event $E \subseteq S$. Take $p \in \overline{\pi}^i(e^i)$. If $p(s) = \overline{p}(s)$ or $p(s) = \underline{p}(s)$ then we are done. Suppose $\underline{p}(s) < p(s) < \overline{p}(s)$ and let $q_1(s) = \underline{p}(s)$, $q_2(s) = \overline{p}(s)$, where $q_1, q_2 \in \pi^i(e^i)$. Let $\lambda \in (0,1)$ such that $\lambda q_1(s) + (1-\lambda)q_2(s) = p(s)$, which implies that $\lambda q_1(S \setminus s) + (1-\lambda)q_2(S \setminus s) = p(S \setminus s)$. We also have that $q_1(s)a^i + q_1(S \setminus s)b^i > 0$ and $q_2(s)a^i + q_2(S \setminus s)b^i > 0$. Multiplying with $\lambda, 1-\lambda$, and adding the two inequalities we have that $\mathbb{E}_p f^i = p(s)a^i + p(S \setminus s)b^i > 0$.

Because $\sum_{s \in S} \underline{q}(s) \leq 1 \leq \sum_{s \in S} \overline{q}(s)$, there exists $a \in [0,1]$ such that $a \sum_{s \in S} \underline{q}(s) + (1 - a) \sum_{s \in S} \overline{q}(s) = 1$. For each $s \in S$, let $p(s) = a\underline{q}(s) + (1-a)\overline{q}(s)$. We then have $\sum_{s \in S} p(s) = 1$, so that p is a probability measure such that $p(s) \in [\underline{q}(s), \overline{q}(s)]$ for all $s \in S$, hence $p \in \bigcap_{i \in I} \overline{\pi}^i(e^i) \neq \emptyset$. Using this p and by adding $\mathbb{E}_p f^i > 0$ over all $i \in B$ we have $\sum_{i \in B} \mathbb{E}_p f^i = \mathbb{E}_p \sum_{i \in B} f^i < 0$, a contradiction.

□

3.6 Absence of complexity

Consider first the standard environment of no perceived complexity, where each investor can understand and trade any combination of the A-D securities. In other words, his trade can be measurable with respect to any partition of S, even the finest one. We say that there is trade in the absence of perceived complexity if there is an agreeable trade $\{f^i\}_{i \in I} \in \mathbb{R}^{SI}$, so that $\{e^i\}_{i \in I}$ is not Pareto optimal. Recall that an agreeable trade is such that $\sum_{i \in I} f^i = 0$ and $e^i + f^i \succ^i e^i$ for all $i \in I$.[10] Proposition 7 in Rigotti et al. (2008) shows that interior allocation $\{e^i\}_{i \in I}$ is not Pareto optimal if and only there is no common subjective belief, so that $\bigcap_{i \in I} \pi^i(e^i) = \emptyset$.[11]

Theorem 1. *There is trade in the absence of complexity if and only if there is no common subjective belief.*

[10]Note that because of Axiom 4, $\{e^i\}_{i \in I}$ is not Pareto optimal ($\sum_{i \in I} f^i = 0$, $f^i + e^i \succsim^i e^i$ for all $i \in I$ and $f^j + e^j \succ^j e^j$ for some $j \in I$) if and only if $\sum_{i \in I} f^i = 0$ and $e^i + f^i \succ^i e^i$ for all $i \in I$.

[11]Theorem 3 in Rigotti and Shannon (2005) proves the same result in the incomplete preferences model of Bewley (1986).

3.7 Maximum perceived complexity

Consider now a setting where preferences and the allocation $\{e^i\}_{i\in I}$ are still the same, however each investor is so unsophisticated or cognitively constraint that he can buy or sell at most one A-D security, instead of any linear combination, as in the previous section. This means that his available trades are coarse, as they need to be measurable with respect to a two-element partition $\{s, S \setminus s\}$ of the state space.

The main result of the paper characterizes the existence of trading opportunities in the case where investors have maximum perceived complexity of the asset structure, under a no-arbitrage condition. We say that there is trade in the presence of maximum perceived complexity if there is an agreeable bet.

Theorem 2. *Under no-arbitrage at e, there is trade in the presence of maximum perceived complexity if and only if there is no $\overline{S}$-common belief.*

Proof. Suppose that $\bigcap_{i\in I}\overline{\pi}^i(e^i) = \emptyset$. We first show that $\bigcap_{i\in I}[\underline{p}^i(s), \overline{p}^i(s)] = \emptyset$ for some $s \in S$. Suppose not, so that for each $s \in S$, $[\underline{q}(s), \overline{q}(s)] \equiv \bigcap_{i\in I}[\underline{p}^i(s), \overline{p}^i(s)] \neq \emptyset$. No-arbitrage and Proposition 1 imply that $\sum_{s\in S}\underline{q}(s) \leq 1 \leq \sum_{s\in S}\overline{q}(s)$. Let $a \in [0,1]$ such that $a\sum_{s\in S}\underline{q}(s) + (1-a)\sum_{s\in S}\overline{q}(s) = 1$. For each $s \in S$, let $p(s) = a\underline{q}(s) + (1-a)\overline{q}(s)$. We then have $\sum_{s\in S} p(s) = 1$, so that p is a probability measure such that $p(s) \in [\underline{q}(s), \overline{q}(s)]$ for all $s \in S$, contradicting that $\bigcap_{i\in I}\overline{\pi}^i(e^i) = \emptyset$.

We therefore have that, for some $i, j \in I$ and $s^* \in S$, $[\underline{p}^i(s^*), \overline{p}^i(s^*)] \bigcap [\underline{p}^j(s^*), \overline{p}^j(s^*)] = \emptyset$. Suppose without loss of generality that $\underline{p}^i(s^*) > \overline{p}^j(s^*)$. Let c be such that $\underline{p}^i(s^*) > c > \overline{p}^j(s^*)$. Define bet f on s^* such that $f(s^*) = 1 - c$ and $f(s) = -c$ for $s \neq s^*$. We then have that $\mathbb{E}_p f = p(s^*)(1-c) - (1-p(s^*))c > 0$ for all $p \in \overline{\pi}^i(e^i)$ and $\mathbb{E}_p(-f) = p(s^*)(-1+c) + (1-p(s^*))c > 0$ for all $p \in \overline{\pi}^j(e^j)$.

Define bet $\{f^k\}_{k\in I}$ such that $f^i(s^*) = f(s^*) - \epsilon$ and $f^i(s) = f(s)$ for $s \neq s^*$, for sufficiently small $\epsilon > 0$ such that the above inequality still holds. For each $k \neq i, j$, let $f^k(s^*) = \frac{\epsilon}{|I\setminus\{i,j\}|}$ and $f^k(s) = 0$ for $s \neq s^*$. Let $f^j = -f$.

By Axiom 4, for all $k \in I$, each $p \in \overline{\pi}^k(e^k)$ has full support on S. Hence, for small enough $\epsilon > 0$ we have, for all $k \in I$, $\mathbb{E}_p f^k > 0$ for all $p \in \overline{\pi}^k(e^k)$ and $\mathbb{E}_p(f^k + e^k) > \mathbb{E}_p e^k$. From the second property of $\pi^k(e^k)$, there exists small enough $\lambda^k > 0$ such that $\lambda^k(e^k + f^k) + (1-\lambda^k)e^k = \lambda^k f^k + e^k \succ^k e^k$. By taking $\lambda = \min_{k\in I}\lambda^k$, we have that $e^k + \lambda f^k \succ^k e^k$ for all $k \in I$. Because $\sum_{k\in I}\lambda f^k = 0$, $\{\lambda f^k\}_{k\in I}$ is an agreeable bet.

Conversely, suppose there is an agreeable bet $\{f^i\}_{i\in I}$. This means that $\sum_{i\in I} f^i = 0$ and each f^i is of the form $f^i(s^i) = a^i$ and $f^i(s) = b^i$ for $s \neq s^i$, where $a^i \neq b^i$.

Moreover, $f^i + e^i \succ^i e^i$ for all $i \in I$. From the first property of $\pi^i(e^i)$ and Axiom 4, we have that $\mathbb{E}_p(e^i + f^i) > \mathbb{E}_p e^i$ for all $p \in \pi^i(e^i)$, all $i \in I$. In the proof of Proposition 1, we show that the same inequality $\mathbb{E}_p f^i > 0$ holds for all $p \in \overline{\pi}^i(e^i)$, all $i \in I$. Suppose there is an $\overline{S}$-common belief p. Then, by adding $\mathbb{E}_p f^i > 0$ over all investors we have $0 < \sum_{i \in I} \mathbb{E}_p f^i = \mathbb{E}_p \sum_{i \in I} f^i = 0$, a contradiction.

□

4 Concluding remarks

We characterize the existence of trading opportunities in two environments which are the extreme opposites in terms of the investors' perceived complexity of the asset structure. The first is the standard setting of no perceived complexity, where each investor is sophisticated enough to be able to understand and trade any combination of the available securities. In the second, each investor has cognitive limitations so that even though he can understand each A-D security separately, he cannot combine two or more and construct a complex trade. In other words, he can only formulate a coarse trade which is measurable with respect to a two-element partition of the state space, consisting of a state and its complement.

Our main question is, can this type of complexity explain market freezes? The answer depends on the investors' attitudes towards risk and uncertainty. We first show that for smooth preferences, even if all investors have maximum perceived complexity of the asset structure, not all trading opportunities are destroyed, meaning that if there is an agreeable trade (no perceived complexity), there is an agreeable bet (maximum perceived complexity). More importantly, trading opportunities are not destroyed even if we were to provide a more detailed but (necessarily) weaker definition of complexity (e.g. generating complexity endogenously through cognitive costs or computational complexity), as long as each investor can trade at least one security. In other words, our result for these preferences that complexity does not impede trade is robust, because it is independent of the particular details of how a complex security is defined. Examples of smooth preferences are subjective expected utility, the smooth variational preferences of Maccheroni et al. (2006) (including, as special cases, the mean-variance preferences of Markowitz (1952) and Tobin (1958) and the multiplier preferences of Hansen and Sargent (2001)), the smooth ambiguity of Klibanoff et al. (2005) and the second-order expected utility of Ergin and Gul (2009).

If preferences are non-smooth, we find that maximum perceived complexity can impede trade. These are preferences for which there are multiple sets of A-D prices that support zero net demand at the initial allocation, because the indifference curve has a kink at the endowment point. Examples are the CEU with convex capacity of Schmeidler (1989), the MEU of Gilboa and Schmeidler (1989), the non-smooth variational preferences of Maccheroni et al. (2006), the confidence preferences of Chateauneuf and Faro (2009) and the uncertainty averse preferences of Cerreia-Vioglio et al. (2011). For instance, in the MEU model with a full insurance allocation, each investor's subjective beliefs coincide with his (non unique) priors and his indifference curve has a kink at his endowment. As shown in Figure 1, in that case maximum perceived complexity of the asset structure completely shuts down trade.

Our results make heavy use of the following property of convex preferences. If the expectation of security $g^i = f^i + e^i$, according to all of i's subjective beliefs at his endowment e^i, is higher than the expectation of e^i, then there exists a convex combination of g^i and e^i that investor i strictly prefers to e^i. This property is also true in the incomplete preferences model of Bewley (1986), which means that our results apply there as well. The distinctive property of this model is that indifference curves have a kink at each endowment, whereas in some well known models with convex and complete preferences, such as MEU, kinks appear only at a full insurance endowment. Our results then predict that maximum perceived complexity would shut down trade at all allocations in the Bewley (1986) model but only at full insurance allocations in the MEU model.

Finally, our results can be generalized to an incomplete markets setting, by having a partition $\mathcal{P}$ of S. Given event $E \in \mathcal{P}$, there exists a security that pays 1 if E occurs and 0 otherwise. A constrained Pareto optimal allocation (i.e. no-trade in the absence of complexity) is then characterized by the existence of a $\mathcal{P}$-common belief, meaning that all investors agree on all events in $\mathcal{P}$ according to one of their subjective beliefs. Similarly, no-trade in the presence of maximum perceived complexity is characterized by a $\overline{\mathcal{P}}$-common belief, so that there exists a probability measure which is between the maximum and minimum probability assigned to each event $\mathcal{P}$, by all investors.

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