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A Hybrid Stabilization Technique for Simulating Water Wave – Structure Interaction by Incompressible Smoothed Particle Hydrodynamics (ISPH) Method

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ABSTRCT

The Smoothed Particle Hydrodynamics (SPH) method is emerging as a potential tool for studying water wave related problems, especially for violent free surface flow and large deformation problems. The incompressible SPH (ISPH) computations have been found not to be able to maintain the stability in certain situations and there exist some spurious oscillations in the pressure time history, which is similar to the weakly compressible SPH (WCSPH). One main cause of this problem is related to the non-uniform and clustered distribution of the moving particles. In order to improve the model performance, the paper proposed an efficient hybrid numerical technique aiming to correct the ill particle distributions. The correction approach is realized through the combination of particle shifting and pressure gradient improvement. The advantages of the proposed hybrid technique in improving ISPH calculations are demonstrated through several applications that include solitary wave impact on a slope or overtopping a seawall, and regular wave slamming on the subface of open-piled structure.

Keywords:

Hybrid stabilization; ISPH; minimum pressure; particle shift; wave impact

1. Introduction

The Smoothed Particle Hydrodynamics (SPH) technique is a Lagrangian mesh-free numerical method, which was originally introduced by Lucy (1977), and Gingold and Monaghan (1977) to solve the astrophysical problems. In recent years, the SPH method has been successfully used in free surface flow simulations. In an SPH computation, the particles are discretized by the moving nodes and they carry field variables such as the pressure, density and velocity. The smoothing kernels are used to approximate a continuous flow field.

The incompressibility of fluid can be imposed in two different ways in an SPH numerical scheme. Originally, the simulation of incompressible fluid flows was through a weakly compressible SPH formulation (WCSPH), in which the water was considered as slightly compressible and its pressure was related to the density through an equation of state. Thus an artificially specified sound speed has to be introduced (Monaghan, 1994). The WCSPH approach has quite a few advantages, such as that it is easy to program and does not need to solve the pressure boundary value problem. However, at least two weaknesses emerged during its application to the water wave problems (Lee et al., 2008; Rafiee et al., 2012): (a) the use of very small time steps; and (b) significant spurious pressure fluctuations in the spatial and temporal domains.

To overcome the limitation of WCSPH, a strictly incompressible SPH (ISPH) approach has been proposed by Shao and Lo (2003) based on the SPH projection method initiated by Cummins and Rudman (1999) to simulate the free surface flows. In ISPH approach the water is considered as truly incompressible with a constant density. The method projects the intermediate velocity field to a divergence-free space by solving a Poisson equation of pressure (PPE). It employs a strictly incompressible SPH formulation, and thus the CFL condition is based on the fluid velocity rather than the speed of the sound. Therefore, the pressure is not an explicit thermodynamic variable obtained through an equation of the state such like in WCSPH, but obtained through a hydrodynamic equation. For the ISPH modeling techniques, there are mainly two types of the formulation, i.e. the density-invariant ISPH (Shao and Lo, 2003) and velocity divergence-free ISPH (Lee et al., 2008). The ISPH has also been widely applied in the field of water wave dynamics (Khayyer et al., 2008; Lind et al., 2012). According to the comparative studies carried out by Lee et al. (2008) and Violeau and Leroy (2015), the time step used for the ISPH can be five times larger. In addition, the computational results from ISPH could be much more stable and accurate than those from the WCSPH without extra smoothing techniques (Zheng et al., 2014). However, Shadloo et al. (2011; 2012) and Hughes and Graham (2010) noted that the inclusion of certain numerical treatments could significantly enhance the performance of WCSPH. On the other hand, we should also realize that the turbulent flows involve more complex particle convections and free surface deformations, which has more stringent requirement on the pressure solution schemes. In addition, as indicated by Gotoh and Khayyer (2016), one distinct advantage of ISPH corresponds to its superior volume conservation properties. It should be realized that the SPH approaches have been recently expanded to solve the shallow-water equations (SWEs) where the flow is over large domain and the vertical variation of parameters of interest is not demanding (Chang et al., 2016; Chang et al. 2017).

The wave impact loadings on structure constitute an important practical problem with highly distorted free-surface motion. For the SPH application in this field, considerable progress has been made in the investigation of wave-structure interactions, such as documented by Khayyer and Gotoh (2011), Rudman and Cleary (2016) and Ren et al. (2016). According to the extensive computations in engineering practice, it has been found that the homogeneity of particle distributions plays an important role in the accuracy and robustness of the SPH models. The formation of ill particle distributions could significantly degrade the SPH numerical accuracy and lead to the failure of correct solutions.

There have been some remedies which were proposed to address this issue. For example, Monaghan (2000) introduced an additional set of stress node at the points other than the SPH particle locations to address the tensile instability, which was mainly proposed for WCSPH. As for ISPH, Khayyer and Gotoh (2011) and Gotoh et al. (2014) proposed an error compensating scheme to minimize such numerical errors. Following the similar concept, to maintain a more uniform particle distribution, Sriram and Ma (2012) proposed that the pressure of reference particle should be replaced by the minimum pressure of all neighboring particles when calculating the pressure gradient, based on the original idea of Koshizuka and Oka (1996) and improved by Khayyer and Gotoh (2013) in the Moving Particle Semi-implicit (MPS) method.

Another numerical scheme to improve the particle distribution is through the shifting of particle positions directly. Xu et al. (2009) initially used this idea to correct the non-uniformity of particle distributions. Recently a more efficient method based on the Fick's law for adjusting the particle distributions has been introduced by Lind et al. (2012) and Skillen et al. (2013). Besides, Shadloo et al. (2012) also proposed a particle fracture repairing procedure and a corrected SPH discretization scheme to eliminate the instability induced by the particle clustering. The improved model performance has been demonstrated in the benchmark water wave propagations and wave-body interactions. However, we have found from various tests of violent water wave impact on fixed structures, especially those involve longer simulation time, the above-mentioned approaches could face some challenges at the free surface because the shifting scheme is a function of the gradient of concentration field. This challenge is highlighted by Khayyer et al.(2017a), where a correction for elimination of shifting normal to the free-surface is proposed. Despite that the particle shifting algorithm may partially violates the principle of volume conservation for free-surface flows (Nair and Tomar, 2015; Pahar and Dhar, 2016), the issues of particle non-homogeneity have been well resolved.

To make full use of the potentials of available practice, this paper introduces a hybrid ISPH model by combining the particle shifting algorithm of Xu et al. (2009) and minimum pressure idea of Sriram and Ma (2012). The improved numerical scheme would be expected to effectively eliminate the particle clustering/stretching issues and make the particle/pressure distributions more stabilized in wave impact simulations.

2. Review of ISPH Methodology

The governing equations used to solve the fluid problems in an ISPH method are the mass and momentum conservation equations. As there is no major improvement in the fundamental ISPH theory in present paper, Tab.1 briefly summarizes the ISPH solution algorithms, spatial derivative approximations and boundary treatments.

3. Hybrid Particle Stabilization Scheme

This section first reviews the available stabilization approaches, followed by the proposal of a hybrid technique. Then a benchmark test is done to validate the accuracy of this new method.

3.1. Existing stabilization techniques

Among a variety of the particle stabilization algorithms reported in the literature, we have found the minimum pressure (MP) approach of Sriram and Ma (2012) provided an effective solution. When computing the pressure gradient, the minimum pressure $P_{\min}$ as illustrated in Fig. 1 in the influence domain of reference particle i is used instead of P_i, which is shown in Eq. (1). Here this approach is named as ISPH_MP.

$$\nabla P_i = \sum_{j=1,\, j\neq i}^{N} \frac{n_{i,x_m} B_{ij,x_m} - n_{i,xy} B_{ij,x_k}}{n_{i,x} n_{i,y} - n_{i,xy}^2} (P_j - P_{min}) \tag{1}$$

Nevertheless, we should realize that the force exerted on particle i by particle j, and on particle j by particle i, would not be the same, and thus the momentum is not exactly conserved even if the number of particles in the sub-domain is identical and also whether it is uniformly or irregularly distributed.

On the other hand, Xu et al. (2009) introduced an artificial particle displacement (APD) method to prevent the particle clustering, which is named as ISPH_APD in this paper. In this approach the trajectory of particles is re-distributed by adding a small artificial displacement δr_i^{ζ} to the advection of the particles as

$$\delta r_i^{\zeta} = \beta \sum_{j=1}^{N} \frac{r_{ij}^{\zeta}}{r_{ij}^{3}} r_0^{2} V_{max} \Delta t \tag{2}$$

where β is a problem-dependent parameter; ζ is the direction component; $r_0 = \sum_{j=1}^{N} r_{ij} / N$ is the cut-off distance; and V_{max} is the largest particle velocity in the computational system. Here, N is the number of neighbours for particle i in its support domain. The problem-dependent parameter β was recommended to be 0.01 ~ 0.1 by Xu et al. (2009). It should be noted that β must be selected carefully such that it should be small enough not to affect the physics of the flow, but large enough to prevent the occurrence of particle clustering and fracture in SPH simulation. The artificial particle displacement approach has also been used by Shadloo et al. (2011), where β was kept constant as 0.01. Fig. 2(a) gives the comparison between the experimental data and ISPH_APD results for the pressure time history of a solitary wave impacting on the vertical wall (detailed in Section 4.2.1) with the parameter β = 0.01. From the stability in the pressure results and reasonable agreement with the experimental data, we could fix this value in other simulations as well.

Moreover, Lind et al. (2012) proposed another approach based on the Fick's law for adjusting the particle distribution. This was further improved by Skillen et al. (2013), in which a particle displacement vector $\delta \mathbf{r}_s$ was used to update the particle position

$$\delta \mathbf{r}_s = -Ah \|\mathbf{U}\|_i \Delta t \nabla C \tag{3}$$

where a value of $A = 2$ has been found to provide good compromise in Lind et al. (2012), $\|\mathbf{U}\|_i$ is the velocity amplitude of particle i, and $\nabla C = \sum_{j=1}^{N} V_j \nabla_i W(\mathbf{r}_{ij})$ is defined, in which V_j is the volume of particle.

Fig. 2(b) gives the comparison between experimental data and SPH results for the same case as Fig. 2(a) but using the particle shifting method of the Fick's law. It is shown that this approach still generates some spurious oscillations in the pressure time history. As mentioned before, the reason could be attributed to that the shifting scheme is based on the function of the concentration gradient, which cannot be accurately calculated near the free surface. Therefore, we would use ISPH_APD as a viable approach in this work.

3.2. A hybrid stabilization scheme

In order to further improve the ISPH modelling capacity, here we introduce a hybrid particle stabilization technique to improve the numerical stability through correcting the irregular particle distributions, by combining the ISPH_MP and ISPH_APD in Section 3.1. In principle it uses the minimum pressure in the influence domain of reference particle i to replace the actual pressure of this particle for calculating the pressure gradient, and meanwhile adds a small artificial displacement $\delta \mathrm{r}_{\mathrm{i}}^{\zeta}$ to the advection of the particle. This hybrid approach is named as ISPH_MPAPD in the paper. After some numerical trials, it has been found that a value of β = 0.001 ~ 0.01 for $\delta \mathrm{r}_{\mathrm{i}}^{\zeta}$ would be appropriate for modelling the violent water wave impact. It has also been noted that since the physical velocity of a particle is different from the velocity with which the particle position is shifted with $\delta \mathrm{r}_{\mathrm{i}}^{\zeta}$, we should interpolate the physical velocity to the new position of the particles in the next computational cycle. The same interpolation technique as used by Xu et al. (2009) is also adopted here as

$$\mathbf{u}_{\mathrm{i}'} = \delta \mathbf{r}_{\mathrm{i}'\mathrm{i}} \mathbf{u}_{\mathrm{i}} \tag{4}$$

where i and i' refer to the old and new values, respectively; and $\delta \mathbf{r}_{\mathrm{i}'\mathrm{i}}$ is the distance vector between the two particles.

To examine whether or not Eq. (4) still satisfies the pressure Poisson equation PPE, Fig. 3(a) and (b) give the time history of the averaged velocity divergence and the impact pressure, computed with and without the SPH interpolation technique. Meanwhile, the analytical solutions and experimental data (Zheng et al., 2015) are also provided for the validation purpose. The numerical test is for the solitary wave propagation which will be detailed in Section 4.2.1. It can be seen that there is almost no difference observed between the two ISPH results. So we could judge that this interpolated velocity field should still satisfy the PPE.

3.3. Model test on vortex spin-down

To validate the proposed hybrid method, a vortex spin-down simulation following Xu et al. (2009) is conducted. In this study a vortex is bounded by the four walls and placed in the middle of the domain, as shown in Fig. 4. The initial velocity field is given by $u = U_0(y - 0.5)$ and $v = U_0(0.5 - x)$ inside a unit square, where D = 1.0 m is the width of the square and U_0 = 1 m/s is the velocity scale. The kinematic viscosity ν is taken 0.001 m^2/s and the vortex spin-down process is simulated for the Reynolds number Re = 1000.

Fig. 5(a) - (d) show the comparisons of particle distribution computed by using the standard ISPH, ISPH_MP, ISPH_APD and ISPH_MPAPD, respectively, at time t = 1.0 s. The particle number in the x direction is N_x = 60. The traditional ISPH model cannot achieve the converged result and the computation breaks at t = 0.53 s. From the comparisons between three particle stabilization methods, the result of ISPH_APD and ISPH_MP still demonstrates particle clustering and stretching patterns near the corner region, as clearly demonstrated by the enlarged portion of the particle distributions at $0 < x < 0.25$ and $0 < y < 0.25$. In contrast, the hybrid ISPH_MPAPD computation has obtained the most satisfactory particle distributions.

In order to quantify the accuracy of different particle stabilization methods, Fig. 6(a) gives the comparison of horizontal velocity components at x = 0.5 m and t = 1.0 s. Here the particle number in the x direction is N_x = 200. The reference value of the velocity component was provided by Xu et al. (2009) using the STAR-CD. It shows that all ISPH computations achieved good agreement with the STAR-CD results. Besides, Fig. 6(b) gives the convergence test on the horizontal velocity component, where N_t is the total particle number at different values of 3600, 6400, 10000 and 40000, respectively. The relative error Err is defined as

$$Err = \frac{1}{N_y}\sum_{j=1}^{N_y}\sqrt{(u_j - u_{j,s})^2} \tag{5}$$

where u_j and $u_{j,s}$ are the horizontal velocity components computed by ISPH and STAR-CD, respectively, N_y is the particle number in the y direction. It is shown that the hybrid ISPH_MPAPD computation achieved the smallest errors as compared with either ISPH_MP or ISPH_APD results. However, we should also realize that all three ISPH numerical schemes are below first-order accurate in the convergence behaviour when the particle distribution becomes disordered, in spite of the use of various correction techniques.

To demonstrate the time history of velocity variations, Fig. 7(a) gives the maximum velocity computed by different ISPH particle stabilization methods with N_x = 200, in which $u_{max} = \max(|U_i|)$ is defined and i is the index of particle. It shows that the ISPH_MP computations demonstrate some kinds of oscillation in the velocity time histories, while both the ISPH_APD and ISPH_MPAPD results are quite stable and smooth. To further investigate the convergence behaviour of ISPH_MPAPD, Fig. 7(b) gives the comparison of maximum velocity time histories for different particle numbers at N_x = 60, 80, 100 and 200, respectively. Again the close overlap of four computational curves and the noise-free velocity profiles indicate the convergence of the model.

Since pressure field is the most sensitive one to the particle disorder and instability, Fig. 8(a) - (c) give the comparisons of pressure distribution computed by ISPH_MPAPD at time t = 1.0 s with different total particle numbers of N_t = 3600, 10000 and 40000, respectively. It shows that with an increase in the particle number, the pressure distributions become much more reasonable. This is further supported by the enlarged portion near the corner regions. Besides, Fig. 9 gives the comparison of pressure profiles at x = 0.0 m between different ISPH results with N_x = 200 and the STAR-CD computation made by Xu et al. (2009). From this it is shown that ISPH_MPAPD can get the best agreement with STAR-CD, while ISPH_MP and ISPH_APD significantly underestimate the pressure values in the centre domain.

To study the computational efficiency, Fig. 10 gives the comparisons of CPU time versus total particle number N_t for different particle stabilization schemes, where T is the CPU time measured in seconds. It demonstrates that ISPH_MP consumes the longest CPU time especially at high particle numbers, since it requires more iterations to solve the pressure Poisson equation under particle clustering or stretching. On the other hand, the irregular particle distributions have less influence on the numerical iterations in an ISPH-APD scheme, which takes similar CPU expenses as the ISPH_MPAPD.

4. Model Applications in Wave Impact

In this section, to test the effectiveness of the hybrid ISPH_MPAPD on modelling the violent water wave impact, we consider five practical applications. These include a dam break flow, solitary wave impact on the vertical and inclined walls, wave overtopping of an impermeable structure, and wave slamming on subface of an open-piled structure. The enhanced performance of ISPH_MPAPD will be demonstrated through the quantitative comparisons with standard techniques such as ISPH_MP and ISPH_APD, as well as the experimental data.

4.1. Dam-break flow impact on a vertical wall

In this test a rectangular column of water is confined between the two vertical walls as shown in Fig. 11**.** The width of water column is L and the height is H . At beginning the dam is instantaneously removed and water is allowed to flow out along the dry horizontal bed. D is the length of horizontal section of water tank and a pressure sensor P_1 is located on the right wall at a vertical distance of h_1 from the bottom. In the interpretation of numerical result, all variables and parameters are non-dimensionalised by the characteristic dam height H and gravitational acceleration g.

The following parameters are studied here: $L = 0.5$ m, $H/L = 2.0$ and $D = 4L$. To show the convergence of ISPH_MPAPD model results, the time history of impact pressures at P_1 computed by using different time steps and particle numbers are presented in Fig. 12(a) and (b), respectively. Here it should be mentioned that the computed pressures are obtained by the particle nearest to the measuring location which does not involve the samplings from neighbouring particle. It is shown from Fig. 12 that as the time step or particle spacing becomes smaller (i.e. when the particle number becomes larger), the difference between two adjacent numerical results becomes smaller. Also the numerical results become smoother and less fluctuating, following the refinement in spatial and temporal resolutions. These have clearly evidenced the convergence of numerical results in the temporal and spatial domains.

Besides, Fig. 13 gives the comparisons of wave front and water column height of dam break flow computed by three alternative ISPH methods. The numerical results are compared with the experimental data of Martin and Moyce (1952). It seems that very minor differences are found between them, which may imply that the water surface profiles are not very sensitive to the particular choice of particle stabilization schemes as compared with the impact pressure.

In order to further quantify the accuracy of different particle stabilization schemes, another benchmark dam break flow as documented by Colagrossi and Landrini (2003) is considered, where the dimensions $L = 2.0$ m, $H = 0.5L$ and $D = 5.3667L$ are used in Fig. 11. On the right wall, there is also a pressure sensor point P_1 with height $h_1 = 0.14H$ to record the impact pressure time history. For all controlled SPH simulations in this case, the particle numbers keep the same at 120 × 60 corresponding to a particle size of 0.0167 m. The time step is taken to be constant $dt = 0.003$ s. Fig. 14 illustrates the particle distributions by using different ISPH stabilization methods and the snapshots were extracted at time $t = 2.775$ s. We could observe that there is a slight particle strip distribution in the ISPH_MP results as shown in Fig. 14(a), and the particle distribution becomes disordered in the ISPH_APD results as shown in Fig. 14(b). Overall speaking, the particle distributions computed by ISPH_MPAPD seem to be most satisfactory as shown in Fig. 14(c).

The time histories of pressure at P_1 computed by using different ISPH particle correction methods (with total particle number $N_t = 7200$) are compared with the experimental data of Zhou et al. (1999) in Fig. 15. It shows that the pressure obtained by ISPH_MPAPD is much better than that from the other two methods, i.e. ISPH_MP or ISPH_APD. The ISPH_MP result exhibits a more obvious phase shift in the second pressure peak, while the ISPH_APD result demonstrates a much larger pressure oscillation. For the three ISPH results, their major differences appear after the second pressure peak. One reason could be due to the lack of two-phase water-air modelling, since the influence of air becomes increasingly significant during the second violent wave impact when the water column plunges down onto the surface and forms a cavity region. It has been recorded that the CPU expense (Intel i7 3.4 GHz with RAM 8 GB) of present simulation is 324 s by using ISPH_MP, 332 s by ISPH_MP and 326 s by ISPH_MPAPD, respectively.

4.2. Solitary wave impact on a vertical wall

In order to further evidence the effectiveness of improved particle stabilization technique, the analysis of numerical results of solitary wave impact on a vertical wall is provided below. The experiment of solitary wave propagation and its impact on a vertical wall was carried out by Zheng et al. (2015) in a 3-D wave flume with piston wave maker in Harbin Engineering University (HEU). The schematic diagram of the wave tank is shown in Fig. 16**.** The wave tank is 10 m long and the water depth is d = 0.25 m. The solitary wave height is h = 0.15 m, thus the wave nonlinearity is $\varepsilon = h/d$ = 0.6. A measurement point P_1 is located on the right wall at a distance of 0.05 m from the tank bottom to monitor the pressure time history. In ISPH computation the initial particle spacing is 0.01 m and the time step is 0.001 s.

Fig. 17 illustrates the particle distributions with pressure contour by using the original ISPH (Shao and Lo, 2003) and improved ISPH with different particle stabilization methods. The snapshots were extracted at time t = 1.2 s after the wave is initiated. Under such a high wave-to-depth ratio, it would be very easy to generate the particle clustering in standard ISPH computation, which is illustrated in Fig. 17(a). On the other hand, it can be seen that these abnormal particle distributions can be corrected effectively by using the different stabilization techniques as shown in Fig. 17(b) - (d). However, we could still find that there is a slight particle strip distribution in ISPH_MP result as shown in Fig. 17(b). Besides, the particle distribution is slightly disordered in ISPH_APD result as shown in Fig. 17(c). Overall speaking, the distribution of particles in ISPH_MPAPD result is the most desirable, as shown in Fig. 17(d), which demonstrates its superiority in predicting the pressure fields.

To investigate the conservation of volume for all ISPH models, Fig. 18 shows the time history of water particle volume variations during the wave propagation. It can be seen that ISPH_MP and ISPH_APD cannot satisfy the strict volume conservation, namely the mass conservation, while the proposed ISPH_MPAPD has the best conservation performance. By analysis it was found that the relative volume errors are about 1.45% for ISPH_MP, 1.24% for ISPH_APD and only 0.71% for ISPH_MPAPD in Fig. 18. Besides, the comparisons of wave surface profile at two time instants of t

= 2.0 s and 3.1 s are shown in Fig. 19(a) and (b), respectively, which shows that all ISPH simulated free surfaces have an overall agreement with the analytical solution, although there are some differences in the wave crest. Here the relative errors in wave height are about 1.013% for ISPH_MP, 5.153% for ISPH_APD and 0.433% for ISPH_MPAPD in Fig. 19(a), while they are 5.31% for ISPH_MP, 2.5% for ISPH_APD and 2.86% for ISPH_MPAPD in Fig. 19(b). Generally speaking, ISPH_MPAPD computation also shows the best accuracy and stability in the wave surface profiles.

Furthermore, the comparisons of wave impact pressure at sensor point P_1 between the experimental data (Zheng et al., 2015) and numerical results by using different ISPH particle stabilization methods, are illustrated in Fig. 20(a) - (d). It should be mentioned that Fig. 20(a) is the superposition of all the data, while Fig. 20(b) - (d) is the comparison with each individual ISPH correction scheme. It is shown that in Fig. 20(b) there appear spurious oscillations around the ISPH_MP pressure peak. In Fig. 20(c) the pressure peaks computed by ISPH_APD are larger than the experimental data. Again the proposed ISPH_MPAPD achieves the best agreement in both the pressure peak and its evolutions, as shown in Fig. 20(d). Comparing Fig. 20 with Figs. 17-19, it can be understood that the impact pressure simulations can best demonstrate the superiority of ISPH_MPAPD than the other illustrations, such as the particle snapshot and volume and free surface profile.

4.3. Solitary wave impact on a slope wall

In this section, the ISPH method with improved particle stabilization technique is used to the simulation of solitary wave impacting on a slope with angle of 150°. The computational domain is the same as that used in the laboratory experiment of Zheng et al. (2015), so a direct comparison can be made. Four pressure sensors, labelled as $P_1 - P_4$, are placed along the slope at a distance of 0.05 m from the bed and subsequent intervals of 0.1 m upward. The schematic diagram of the domain is shown in Fig. 21.

As shown in Fig. 21 a solitary wave with wave amplitude $h/d = 0.6$ is studied. The water depth is $d = 0.25$ m and the length of horizontal section is $L = 10.0$ m. The initial particle spacing is 0.01 m and approximately 25000 particles are involved in the ISPH computations.

Fig. 22 illustrates the process of solitary wave running up and down the slope at different times computed by ISPH_MPAPD, whose particle snapshots coincide well with the laboratory photographs. It can be seen from Fig. 22(a) that the wave front reaches its maximum climbing point at time $t = 6.5$ s. Then the run-down process starts and the main flow retreats from the slope. It is shown in Fig. 22(b) that a violent backflow occurs near the original shoreline at $t = 7.0$ s, which explains the abrupt pressure drop in its time history (as shown in later Fig. 24**)**. Generally the agreement between numerical and experimental free surfaces is quite satisfactory.

Fig. 23 illustrates the particle distributions with pressure field computed by using different particle stabilization methods. The snapshots were extracted at time t = 7.1 s and t = 7.25 s after the model was run. It can be seen from Fig. 23(a1) and (a2) that there exist particle clustering and disorders in the pressure field, which was computed by using ISPH_MP. In Fig. 23(b1) and (b2), the pressure fields computed by ISPH_APD displayed obvious local chaos, especially at later stage of the wave impact. On the other hand, the distribution of particles and their pressure fields in ISPH_MPAPD result shows much more stable and uniform patterns, as indicated in Fig. 23(c1) and (c2).

To quantify the accuracy of ISPH_MPAPD, Fig. 24(a) - (d) show the comparisons of wave impact pressure at four measurement point (P_1 - P_4) between the experimental data and different ISPH correction results. It is shown that good agreement has been found in spite of some discrepancies, due to that the pressure fields are always difficult to predict by any numerical model. Similar to experimental data, the computed pressures at P_1 and P_2 which are located below the surface of water, share similar evolution features. That is to say, the impact pressure first reaches its maximum value when the wave runs up to the maximum point, and then it gradually decreases to negative pressure as the wave runs down freely, until to the minimum pressure point. However, all ISPH computations exhibit much larger pressure oscillations than the experimental observations. It is also promising to note ISPH_MPAPD computation demonstrates much less pressure noise and shows better agreement with the experiment. This conclusion has been further strengthened by the zoomed sub-figures of Fig. 24(a1 - a3) and (b1 - b3) with separate comparison with each ISPH model, which shows that ISPH_MPAPD is superior to either ISPH_MP or ISPH_APD in obtaining the stable and accurate pressure predictions.

On the other hand, as shown in Fig. 24(c) and (d), the computed pressures at sensor point P_3 and P_4, which is on and above the still-water shoreline, exhibit much more stable pressure patterns as compared with those at P_1 and P_2. Both pressures increase rapidly to the maximum value when the solitary wave impacts on the slope and then fall to zero without generating the negative pressures. Again the numerical results of ISPH_MPAPD show an overall better agreement with the experiment.

Since maximum pressure generated during the wave impact is quite important for the safety and reliability of marine structures, we carry out an error analysis and find out that the relative errors are around 10.25% for ISPH_MP, 10.69% for ISPH_APD, and only 0.3% for ISPH_MPAPD, as compared with the experimental peak pressure in Fig. 24(a). In contrast these errors are about 11.83%, 9.24% and 5.6%, respectively, in Fig. 24(b)**.**

4.4. Solitary wave overtopping on an impermeable seawall

Here another robust test is carried out to investigate the tsunami-like solitary wave impinging and overtopping on an impermeable trapezoidal seawall located on a 1:20 sloping beach. The numerical computation was based on the benchmark physical experiment documented by Hsiao and Lin (2010). In the study, the wave nonlinearity $\varepsilon = \mathrm{h}/\mathrm{d}$ is 0.35 and other relevant parameters are shown in Fig. 25(a) inside the wave tank. For analysis, the relative time $\mathrm{t}' = \mathrm{t} - \mathrm{t}_{\mathrm{MR}}$ is used, where t_{MR} is the time of maximum wave run-up against the wall.

The ISPH computation used a particle spacing of 0.01 m and constant time step of 0.001 s, involving 21360 particles. The solitary wave was generated by pushing a solid wave paddle on the offshore boundary. The numerical simulations were carried out to 10.0 seconds of the wave propagation. The experimental data of water surface profile and wave impact pressure are used to validate the ISPH results and evaluate the accuracy of different particle stabilization schemes. The measurement points of water surface “G” and impact pressure “P” are shown in Fig. 25(b). It should be noted that only selected results from the experiment of Hsiao and Lin (2010) are used here for the model comparisons.

Fig. 26 shows the particle snapshots with pressure field during the wave impinging and overtopping on the trapezoidal caisson at t' = 3.19 s, computed by all ISPH particle correction schemes. It is shown that as the wave overtops over the seawall an overtopping tongue develops on the crown. In addition, the experimental photo and measured free surface profiles (Hsiao and Lin, 2010) indicated by the black dots are superimposed on the ISPH particle snapshots, quantifying the good accuracy of numerical simulations. From the enlarged portion of the sub-figures, we could observe that there is a slight particle strip distribution near the run-up boundary in ISPH_MP results as shown in Fig. 26(a). On the other hand, the particle distribution seems to be noisy in ISPH_APD results as shown in Fig. 26(b). In comparison, the distribution of particles and pressure patterns in ISPH_MPAPD results are still the most satisfactory as shown in Fig. 26(c).

Fig. 27(a) - (d) show the time histories of free surface variation compared between experimental data (Hsiao and Lin, 2010) and numerical results at four wave gauging points (see Fig. 25(a)). Although the computed free surface elevations seem to be generally higher than the experimental values, the overall good agreement is quite promising. For Fig. 27(a) - (b) the ISPH_APD gives a slight overestimation of the peak elevation as compared with the ISPH_MP and ISPH_MPAPD, while the time histories of ISPH_MPAPD computation are much more stable than the ISPH_APD and ISPH_MP results as shown in Fig. 27(c) - (d). Besides, the small and narrow spread of free surface profile in Fig. 27(d) indicates that only a small portion of water overtops on the impermeable seawall, thus explaining the oscillation in numerical free surfaces at G_{37} and the slightly larger discrepancy in predicting the maximum wave height, in contrast to the situations at G_3, G_{10} and G_{28}.

Furthermore, Fig. 28(a) - (d) shows the time histories of experimental (Hsiao and Lin, 2010) and numerical impact pressures computed by using different ISPH particle correction schemes, at pressure gauge of P_1, P_4, P_7 and P_8 on the weather side of trapezoidal structure (see Fig. 25(b)). It is shown that the general trend of impact pressures computed by all ISPH models follows good consistency with the experimental measurement, in spite of unavoidable discrepancies due to the complication of the physical problem. The pressure time history of ISPH_MP and ISPH_MPAPD is much more stable than that of ISPH_APD, in which larger pressure oscillations are observed. Also it is found that ISPH_MP computation generates more pressure noises than the ISPH_MPAPD, especially in Fig. 28(a) at the first pressure measuring point.

Although all ISPH computations underestimate/overestimate the peak pressures to some extent, the relative errors are about 34.77% for ISPH_MP, 43.6% for ISPH_APD and 32.55% for ISPH_MPAPD in Fig. 28(a). On the other hand, these errors are around 20.2% for ISPH_MP, 41.9% for ISPH_APD and 5.6% for ISPH_MPAPD, respectively, in Fig. 28(c). Overall speaking, the present wave overtopping simulation further provides the indication that the hybrid ISPH_MPAPD stabilization technique is superior to existing ones in accurately predicting the wave impinging and overtopping process.

4.5. Regular wave slamming on subface of an open-piled structure

To finally validate the computational accuracy and stability of the hybrid ISPH_MPAPD model again, the simulation of a regular wave slamming on the subface of an open-piled structure is investigated in this section. The schematic setup of computational domain is shown in Fig. 29(a), where the wave flume is 14.0 m long with a wavemaker being located at $x = 0.5$ m. The incident wave is a regular wave with a wave height $H = 0.15$ m and wave period $T = 1.2$ s. A horizontal platform is fixed at $0.1H$ above the still water surface and 8.0 m away from the left-hand-side of the flume. Eleven pressure measuring points (P_1 - P_{11}) on the subface of the horizontal structure are shown in Fig. 29(b). The detailed information on the physical experiment is illustrated in Ren and Wang (2005) and Gao et al. (2012). Similar problems have also been addressed in the benchmark work of Gomez-Gesteira et al. (2005).

By using a particle spacing of 0.015 m and totally 36000 particles, the ISPH simulations are carried out. The particle distributions with pressure field computed by different particle stabilization methods are shown in Fig. 30 at time t = 11.67 s. It can be seen from Fig. 30(a) that there is a slight particle strip distribution in the ISPH_MP results, such that a small blank area around the left corner of the platform is observed.

By examining Fig. 30(b), the particle distributions under the platform demonstrate irregularity and there also exists an obvious separation zone with the structure in the ISPH_APD results. On the other hand, the distribution of particles in the ISPH_MPAPD results is again much more stable and uniform than the other two results, as shown in Fig. 30(c). In addition, the comparisons of experimental (Gao et al., 2012) and ISPH wave profiles are also shown in Fig. 30 and the general agreement is acceptable, since there are unavoidable discrepancies found especially in the upper region of the platform.

Fig. 31(a) and (b) shows the time histories of experimental and ISPH impact pressures computed by different correction methods at pressure gauges P_2 and P_8 (see Fig. 29(b)), respectively. The numerical pressure at each measuring point is obtained by the spatial averaging of the pressures of neighboring fluid particles within a radius of three-time particle spacing. It can be seen that the computed impact pressures by all ISPH models reasonably coincide with the experimental data of Gao et al. (2012), in spite of the unavoidable discrepancies. Besides, the pressure history of ISPH_MPAPD is much more promising than that of ISPH_APD, which shows larger pressure oscillations, also more reliable than that of ISPH_MP, which demonstrates severe pressure noises, especially in Fig. 31(a) at the measuring point P_2. The present regular wave slamming simulations once again evidence that the improved ISPH_MPAPD stabilization technique has great potentials in wider wave application fields.

5. Conclusions

In this paper an improved hybrid particle stabilization scheme of ISPH is proposed to simulate violent wave impact with coastal structure. The method adopts an ISPH_MPAPD approach, which combines the ISPH_MP and artificial particle displacement ISPH_APD algorithms to reduce particle clustering and instability so as to improve the ISPH modeling capacity. To validate the accuracy and stability of the model, ISPH_MPAPD is applied to study five benchmark cases of wave-structure interaction, including the dam break flow and solitary wave impact on a vertical wall, solitary wave impact on a slope, solitary wave overtopping on an impermeable seawall and regular wave slamming on the subface of an open-piled structure. According to the comparison between numerical results computed by ISPH_MPAPD, ISPH_MP and ISPH_APD and experimental data, the performance of ISPH_MPAPD is found to be most satisfactory in view of its accuracy, stability and efficiency in dealing with the instabilities caused by the particle clustering and fracturing. Future work is needed to improve the method for more challenging applications in the wave interactions with a movable structure.

However, as documented in the benchmark study of Nair and Tomar (2015) and Pahar and Dhar (2016), any particle shifting technique can violate the conservation of volume. The sensitivity test on the particle volume for solitary wave case in Fig. 18

disclosed that the relative volume errors are 1.45% for ISPH_MP, 1.24% for ISPH_APD and 0.71% for ISPH_MPAPD, respectively, but this small deviation of the volume could significantly improve the stability of numerical results by effectively regularizing the particle distributions. So the benefit of shifting scheme well outweighs the drawback caused by the particle volume errors. On the other hand, as for ISPH_MP, it may violate the momentum conservation but only to some extent. In the context of particle methods, it would be impossible to satisfy both the momentum conservation and the Taylor-series consistency at the same time. ISPH_MP tends to provide approximate pressure gradient, i.e. not perfectly momentum conservative, but being closer to the Taylor-series consistency. Recently, it has been found that the Taylor-series consistency appears to be more important than the exact local conservation of the momentum (Khayyer et al., 2017b).

Besides, we should also be aware that the present SPH accuracy is influenced by various factors. Turbulence is one of the issues whose influence is case-dependent. In present study the main objective is to evaluate the combined correction scheme. Also, in the numerical simulations the effect of sub-particle-scale turbulence on the macroscopic hydrodynamics, such as water surface deformation and impact pressure, seems to be trivial due to the use of sufficiently small particle size. However, if the coarser particles are used in larger practical domains, the SPS turbulence modelling must be considered due to the significant increase of turbulence levels.

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