Stepwise Green Investment under Policy Uncertainty

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ABSTRACT

We analyse how market price and policy uncertainty, in the form of random provision or retraction of a subsidy, interact to affect the optimal time of investment and the size of a renewable energy (RE) project that can be completed in either a single (lumpy investment) or multiple stages (stepwise investment). The subsidy takes the form of a fixed premium on top of the electricity price, and, therefore, investment is subject to electricity price uncertainty. We show that the risk of a permanent retraction (provision) of a subsidy increases (decreases) the incentive to invest, yet lowers (raises) the amount of installed capacity, and that this result is more pronounced as the size of the subsidy increases. Additionally, we show that increasing the number of policy interventions lowers the expected value of a subsidy and the size of the project. Furthermore, we illustrate that, although an increase in the size of a subsidy lowers the relative value of the stepwise investment strategy, the expected value of a lumpy investment strategy is still lower than that of stepwise investment.

Keywords: investment analysis, capacity sizing, renewable energy, policy uncertainty

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1. INTRODUCTION

Green investments rely crucially on government support, however, the absence of a clear policy framework increases uncertainty in revenue streams. This poses a formidable challenge to firms that must typically determine both the optimal time of investment and the size of a project in the form of installed capacity. For capital intensive projects, such as renewable energy (RE) power plants, such decisions entail considerable risk, since, by installing a large capacity, it may not be possible to recover the investment cost in the case of an unexpected downturn, whereas by installing a small capacity, revenues could be forgone if market conditions suddenly become favourable. Additionally, the inability to contract an investment project after its initial installation due to high cost makes the investment timing and capacity sizing decisions even more crucial. Therefore, we develop an analytical framework in order to determine how such decisions are affected by price and policy uncertainty, in the form of random introduction or retraction of a support scheme, assuming that a project can be completed in either a single or multiple stages. This situation is relevant for both on– and offshore wind park development, where an area can, and often is, developed in stages. Although the impact of policy uncertainty on investment decisions has been analysed from the perspective of carbon prices and the random introduction of a policy scheme (Blyth *et al.* 2007; Boomsma and Linnerud 2014), the implications of repeated provisions and retractions of a support scheme on both the optimal investment timing and capacity sizing decisions as well as the optimal investment strategy have not been analysed thoroughly yet. Additionally, while stepwise investment is more preferable than lumpy investment when a firm has discretion over capacity (Chronopoulos *et al.,* 2014), whether the introduction of a subsidy mitigates this effect remains an open question.

Examples that indicate the impact of policy uncertainty on investment and operational decisions are increasing as the structural transformation of the power sector continues. For instance, uncertainty in the introduction of a support scheme delayed more than half of a series of wind power plants in the UK, that had originally been scheduled for operation by March 2016 (The Telegraph, 2013), as well as a \$509 million wind farm by AGL Energy Ltd., Australia's largest developer of RE projects (Bloomberg, 2013). Also, in Spain, uncertainty regarding the timing and the size of the reduction in feed–in–tariffs has increased downside risk considerably for both existing and new investors (The Economist, 2011). In addition, the absence of a clear policy framework has also reduced the growth in RE capacity and projections indicate that this reduction will continue over the next years unless policy uncertainty is reduced (IEA, 2014). Despite the crucial impact of policy uncertainty on the evolution of RE projects, its implementation in analytical frameworks for stepwise investment and capacity sizing has been limited, and, therefore, models for predicting the level of RE investment remain underdeveloped. Indeed, although uncertainties for commodities such as electricity, natural gas, and oil are reasonably well known, those pertaining to RE technologies, climate change, and regulatory risk are less well understood. For example, learning curves are necessary to model efficiency improvements in existing technologies, yet may be less well specified for the development of RE technologies, that evolve through several stages, and, therefore, their future development path is likely to be different from their progress in the past (Jamasb and Köhler, 2008). We address this disconnect by assuming that a firm has discretion over both the time of investment and the size of the project and that it can adopt a lumpy or a stepwise investment strategy in the light of random provision or retraction of a support scheme. The latter takes the form of a fixed premium on top of the electricity price, and, as a result, the firm is subject to electricity price uncertainty as is the case with one of the widely implemented support schemes, namely premium feed–in tariff.

This policy mechanism has been introduced, for example, in Spain and Portugal, yet, after the financial crisis, tariff levels have been subject to frequent reductions at random points in time. In turn, this has had crucial implications for the viability of private firms. For example, Iberdrola, Spain's biggest power group, reported a 13% decline in profits following a reform of the energy sector that aimed at reducing the tariff deficit (Financial Times, 2014a). Such tariff cuts were also implemented in Portugal, as part of the wider cuts in financial support affecting all electricity producers, in order to reduce the deficit in the generation sector (Wind Power, 2012). Similarly, subsidy cuts in the UK for solar photovoltaic may not only delay the point at which solar could be cost competitive, but also damage broader investor confidence and affect the progress with both deployment and cost reductions (The Guardian, 2015a). Consequently, the contribution of this paper is threefold. First, we develop an analytical framework for stepwise investment under price and policy uncertainty. Second, we analyse how price and policy uncertainty interact to affect the optimal investment timing and capacity sizing decisions as well as the relative value of the two investment strategies, i.e., stepwise and lumpy. Finally, we provide managerial and policy insights based on analytical and numerical results. More specifically, we illustrate how the random provision or retraction of a subsidy impacts not only the time of investment and the size of a project, but also the choice of investment strategy, in terms of lumpy versus stepwise investment. Thus, we derive insights on how policies may be designed not only to incentivise investment in RE projects but also to ensure that the level of investment promotes the viability of decarbonisation targets.

We proceed in Section 2 by discussing some related work. In Section 3, we introduce assumptions and notation and formulate the investment problem under each strategy, i.e., lumpy and stepwise investment, as an optimal stopping–time problem. In Section 4, we analyse the benchmark case of investment and capacity sizing without policy uncertainty and then extend it in Sections 5.1 and 5.2 by allowing for the sudden retraction or provision of a subsidy, respectively. In Section 5.3, we analyse the case of sudden provision of a retractable subsidy, and, in Section 5.4, we allow for infinite provisions and retractions. Section 6 provides numerical examples for each case and illustrates the interaction between price and policy uncertainty in order to enable more informed investment, capacity sizing, and policy decisions. Section 7 concludes the paper and offers directions for further research.

2. RELATED WORK

Despite the extensive literature that illustrates the amenability of real options theory to the energy sector (Lemoine, 2010; Rothwell, 2006), analytical formulations of problems that address investment in RE projects typically do not combine crucial features such as policy uncertainty, discretion over capacity, or flexibility for stepwise investment. An empirical approach for analysing the impact of regulatory risk on investment in generation facilities is presented in Walls *et al.* (2007). They consider regulatory uncertainty with respect to both the timing and pace of restructuring of electricity markets, and find that power plant investment is higher in states that have restructured electricity markets than in states that have taken no restructuring actions. Additionally, they find that greater uncertainty increases the incentive to choose power plant types with lower capital to generating capacity ratios. Blyth *et al.* (2007) and Kettunen *et al.* (2011) analyse how a firm's investment propensity is affected by uncertainty in carbon prices. The former find that carbon price uncertainty creates a risk premium for power generation and that the option to retrofit CCS may accelerate investment in a coal power plant, while the latter use a multistage stochastic optimization model and demonstrate how real options valuation yields substantially different results regarding investment propensities compared to conventional economic analysis.

Linnerud *et al.* (2014) examine how uncertainty in the introduction of RE certificates affects the timing of investments. Their results indicate that while investors with a portfolio of licences act in line with real options theory, i.e., policy uncertainty delays investment rates, investors with a single license act in line with the traditional NPV approach. Boomsma and Linnerud (2014) analyse how investment incentives are affected by the likely termination or revision of a support scheme allowing for electricity and subsidy prices to follow correlated geometric Brownian motions. Their results indicate that, expectations that a support scheme may be terminated, delay investment if it is applied retroactively, but may facilitate investment otherwise. While the aforementioned papers address the impact of various forms of policy uncertainty on a firm's propensity to invest, they ignore both discretion over capacity as well as the flexibility for stepwise investment.

Examples of early work in the area of sequential investment include Majd and Pindyck (1987), who show how traditional valuation techniques understate the value of a project by ignoring the flexibility embedded in the time to build, and Dixit and Pindyck (1994), who develop a sequential investment framework assuming that the project value depreciates exponentially and the investor has an infinite set of investment options. The value of modularity and sequential investment is emphasised in Gollier *et al.* (2005) and Malchow–Møller and Thorsen (2005). The former show that the option value of modularity may trigger investment in the initial module at a level below the now–or–never NPV, while the latter illustrate how the investment policy resembles the simple NPV rule under repeated investment options. More recently, Siddiqui and Maribu (2009) analyse how sequential investment in distributed generation capacity may reduce the exposure of a microgrid to risk from natural gas price volatility and find that the microgrid prefers a direct (stepwise) investment for low (high) levels of volatility. By contrast, Kort *et al.* (2010) show that higher price uncertainty makes a lumpy investment more attractive relative to a stepwise investment strategy by increasing the reluctance to make costly switches between stages, yet Chronopoulos *et al.* (2014) show how this result does not hold if a firm has discretion over capacity. Siddiqui and Takashima (2013) extend the symmetric, non–pre–emptive duopoly of Goto *et al.* (2008) by allowing for sequential capacity expansion in order to explore how sequential decision making offsets the effect of competition. While sequential investment is a crucial feature of RE projects, the scope of these papers is limited as they ignore capacity sizing and policy uncertainty.

Analytical models for investment and capacity sizing decisions include Dangl (1999), who analyses how demand uncertainty impacts the decision to invest in a project with continuously scalable capacity, and shows that, even when demand is high, low uncertainty makes waiting for further information the optimal strategy. Bøckman *et al.* (2008) adopt a similar approach for valuing small hydropower projects that are subject to electricity price uncertainty, while Huisman and Kort (2009) examine the same problem in monopoly and duopoly settings and show how a leader can use discretion over capacity strategically in order to deter a follower's entry temporarily. Relaxing the assumption of risk neutrality, Chronopoulos *et al.* (2012) show how risk aversion facilitates investment by increasing the incentive to build a smaller project. A policy–oriented model that allows for capacity sizing is presented by Boomsma *et al.* (2012), who analyse investment behavior under fixed and premium feed–in tariffs (FIT), RE certificate trading, and changes of a support scheme via Markov switching. They find that the choice of support scheme and any corresponding uncertainty has a crucial impact on both the timing and the size of an investment. However, by modelling subsidy prices via a Markov–modulated geometric Brownian motion, the implications of permanent or temporary termination of a support scheme on investment timing and capacity sizing decisions are not taken into accounted.

More pertinent to our analysis is the working paper of Adkins and Paxson (2013), who analyse investment in a RE facility allowing for uncertainty in the price of electricity and the quantity of electricity produced, as well as policy uncertainty in the form of the random provision or retraction of a subsidy that is proportional to the quantity of electricity produced. They consider the case in which a subsidy may be either retracted or provided permanently at a random point in time, as well as the case in which a subsidy may be introduced and then retracted permanently. In each case, they find that investment thresholds increase with greater quantity uncertainty and decrease with the size of the subsidy, thus implying that either production volume floors or high subsidies might encourage investment. Additionally, the value of the option to invest decreases with greater quantity uncertainty and increases as the correlation between the price of electricity and quantity of electricity produced increases, since this raises the aggregate volatility. Although we do not consider quantity uncertainty, we assume that a firm faces price and policy uncertainty and apart from discretion over the investment strategy, i.e., lumpy versus stepwise, it also has discretion over both the time of investment and the size of the project. We find that the likely provision of a subsidy lowers the relative value of the two strategies, yet stepwise investment still dominates a lumpy investment strategy even when the size of the subsidy increases. Additionally, in line with Boomsma and Linnerud (2014), we show how a retractable subsidy may facilitate investment relative to the case where the subsidy is available permanently, yet lowers the optimal capacity of a project. Moreover, we find that the sudden provision of a permanent subsidy may delay investment relative to the case in which the subsidy is never available, yet raises the amount of installed capacity. Finally, an increasing number of policy interventions facilitates investment and lowers the amount of installed capacity.

3. MODEL

We consider a price–taking firm that holds a perpetual option to invest in a project of infinite lifetime facing both price and policy uncertainty. The firm has the option to either exercise an investment option immediately or delay investment as well as the flexibility to invest in either a single or a sequence of i discrete stages, with $i \in \mathbb{N}$. Policy uncertainty takes the form of the random provision or retraction of a subsidy, that is implemented as a fixed proportion y on top of the electricity price. We let $\zeta \in \{0, 1\}$ indicate the presence ($\zeta = 1$) or absence ($\zeta = 0$) of a subsidy, while m and n denote the number of retractions and provisions, respectively. Also, we assume that $t \geq 0$ is continuous and denotes time and that policy uncertainty is modelled via a Poisson process $\{M_t, t \geq 0\}$, which is defined in (1)

$$M_t = \sum_{d \geq 1} \mathbb{1}_{\{t \geq T_d\}} \tag{1}$$

where $T_d = \sum_{\xi=1}^{d} h_\xi$ and $\left\{h_\xi, \xi \geq 1\right\}$ is a sequence of independent and identically distributed random variables, with $h_\xi \sim \exp(\lambda)$. Hence, M_t counts the number of policy interventions that occur between 0 and t, and h_ξ is the time interval between subsequent policy interventions. Thus, if no policy intervention has occurred for t years, then, with probability λdt, it will occur within the next short time interval dt, i.e:

$$dM_t = \begin{cases} 1 & \text{, with probability } \lambda dt \\ 0 & \text{, with probability } 1 - \lambda dt \end{cases}$$

We assume that the variable production cost is zero and that the long–term electricity price at time t, E_t (in \$/MWh), is independent of $\{M_t, t \geq 0\}$ and follows a geometric Brownian motion (GBM) that is described in (2) (Boomsma *et al.*, 2012). We denote by μ the annual growth rate, by σ the annual volatility, and by dZ_t the increment of the standard Brownian motion. Also, $\rho > \mu$ is the subjective discount rate.

$$dE_t \;=\; \mu E_t dt + \sigma E_t dZ_t, \quad E_0 \equiv E > 0 \tag{2}$$

The capacity of the project in the now–or–never investment case is denoted by $\overline{K}^{(j)}_{\zeta,m,n}$ (in MWh) and by $K^{(j)}_{\zeta,m,n}$ (in MWh) if the firm can delay investment. Also, $\overline{F}^{(j)}_{\zeta,m,n}(\cdot)$ (in million \$) is the expected value of a now–or–never investment opportunity, where $j \in \{\ell, s_i\}$ (denoting lumpy and staged investment, respectively), while $\overline{k}^{(j)}_{\zeta,m,n}$ (in MWh) is the corresponding optimal capacity. For example, $\overline{F}^{(\ell)}_{1,1,0}(\cdot)$ denotes the expected NPV for a lumpy investment when a subsidy is present ($\zeta = 1$) but may be retracted permanently ($m = 1$, $n = 0$) at a random point in time and $\overline{k}^{(\ell)}_{1,1,0}$ is the corresponding optimal capacity. If the option to defer investment is available, then $F^{(j)}_{\zeta,m,n}(\cdot)$ (in million \$) denotes the maximised option value, while $\tau^{(j)}_{\zeta,m,n}$, $\varepsilon^{(j)}_{\zeta,m,n}$, and $k^{(j)}_{\zeta,m,n}$, denote the time of investment, the optimal investment threshold, and the corresponding optimal capacity, respectively. The investment cost, $I(\cdot)$ (in \$), is indicated in (3), where $\gamma_j > 1$ implies that $I(\cdot)$ is a convex function of the capacity, and, consequently, this model is more suitable for describing projects that exhibit diseconomies of scale, e.g., RE power plants. Indeed, the convexity of the investment cost encapsulates the combined impact of features, such as maintenance cost, wake effects, etc., that cause the marginal investment cost to increase with greater capacity (NREL, 2012; Coulomb and Neuhoff, 2006).

$$I\left(K^{(j)}_{\zeta,m,n}\right) = a_j K^{(j)}_{\zeta,m,n} + b_j K^{(j)^{\gamma_j}}_{\zeta,m,n} \quad , \quad a_j, b_j > 0 \text{ and } \gamma_j > 1 \tag{3}$$

Finally, we assume that stepwise investment is more costly than lumpy investment although each stage is less costly than the entire project, as indicated in (4).

$$I\left(K^{(\ell)}_{\zeta,m,n}\right) < \sum_i I\left(K^{(s_i)}_{\zeta,m,n}\right) \quad \text{and} \quad I\left(K^{(s_i)}_{\zeta,m,n}\right) < I\left(K^{(\ell)}_{\zeta,m,n}\right), \forall i \tag{4}$$

The firm's optimisation objective is summarised in (5), where the outer maximisation indicates the firm's decision on whether to invest immediately or delay investment. The first argument of the maximisation indicates that when the firm decides to wait for a small time interval dt, then the value it holds is the discounted expected value of the capital appreciation of the investment opportunity. The second argument represents the value that the firm receives when exercising a now–or–never investment opportunity and indicates that the firm will choose the capacity of the project so that it maximises its expected NPV at investment.

$$F^{(j)}_{\zeta,m,n}(E) = \max\left\{(1-\rho dt)\mathbb{E}_E\left[F^{(j)}_{\zeta,m,n}(E+dE)\right], \max_{\overline{K}^{(j)}_{\zeta,m,n}}\left[\overline{F}^{(j)}_{\zeta,m,n}\left(E, \overline{K}^{(j)}_{\zeta,m,n}\right)\right]\right\} \tag{5}$$

If we denote by $R^{(j)}_{\zeta,m,n} \equiv R\left(E^{(j)}_{\zeta,m,n}, K^{(j)}_{\zeta,m,n}\right)$ the instantaneous revenue of an active project, then the expected value of a project under lumpy investment is described in Figure 1. Notice that the firm can postpone investment until $\tau^{(\ell)}_{\zeta,m,n}$, at which point it must fix the capacity, $K^{(\ell)}_{\zeta,m,n}$, of the entire project. Consequently, $K^{(\ell)}_{\zeta,m,n}$ is a function of the electricity price at time $\tau^{(\ell)}_{\zeta,m,n}$.

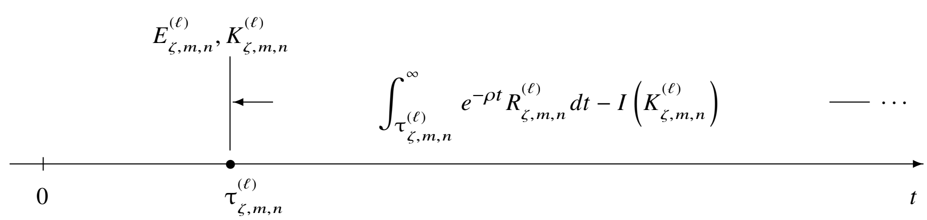

Figure 1: Lumpy investment

Subject to the optimal capacity choice at investment, i.e., the inner maximisation in (5), the firm's optimisation objective when investment is deferred is described in (6).

$$F^{(\ell)}_{\zeta,m,n}(E) = \sup_{\tau^{(\ell)}_{\zeta,m,n} \in \mathcal{S}} \mathbb{E}_E \int_{\tau^{(\ell)}_{\zeta,m,n}}^{\infty} e^{-\rho t} R\left(E^{(\ell)}_{\zeta,m,n}, K^{(\ell)}_{\zeta,m,n}\right) dt - I\left(K^{(\ell)}_{\zeta,m,n}\right) \tag{6}$$

Using the law of iterated expectations and the strong Markov property of the GBM, we can rewrite (6) as in (7), where the stochastic discount factor $\mathbb{E}_E\left[e^{-\rho\tau^{(j)}_{\zeta,m,n}}\right] = \left(\frac{E}{E^{(j)}_{\zeta,m,n}}\right)^{\beta_1}$ and $\beta_1 > 1, \beta_2 < 0$ are the roots of the quadratic $\frac{1}{2}\sigma^2\beta(\beta-1) + \mu\beta - \rho = 0$.

$$\begin{aligned} F^{(\ell)}_{\zeta,m,n}(E) &= \sup_{\tau^{(\ell)}_{\zeta,m,n}\in\mathcal{S}} \mathbb{E}_E\left[e^{-\rho\tau^{(\ell)}_{\zeta,m,n}}\right]\mathbb{E}_{E^{(\ell)}_{\zeta,m,n}}\left[\int_0^\infty e^{-\rho t} R\left(E^{(\ell)}_{\zeta,m,n}, K^{(\ell)}_{\zeta,m,n}\right)dt - I\left(K^{(\ell)}_{\zeta,m,n}\right)\right] \\ &= \max_{E^{(\ell)}_{\zeta,m,n}\geq E}\left(\frac{E}{E^{(\ell)}_{\zeta,m,n}}\right)^{\beta_1}\left[\frac{R\left(E^{(\ell)}_{\zeta,m,n}, K^{(\ell)}_{\zeta,m,n}\right)}{\rho-\mu} - I\left(K^{(\ell)}_{\zeta,m,n}\right)\right] \end{aligned} \tag{7}$$

Next, we consider a stepwise investment strategy, and, without loss of generality, we assume that it comprises of two stages. As indicated in Figure 2, the firm must fix the capacity, $K^{(s_1)}_{\zeta,m,n}$, of the first stage at $\tau^{(s_1)}_{\zeta,m,n}$. Then, it operates the first stage of the project until $\tau^{(s_2)}_{\zeta,m,n}$, at which point it invests in the second stage and fixes the corresponding capacity, $K^{(s_2)}_{\zeta,m,n}$. Once the firm invests in the second stage, it incurs the corresponding cost and receives revenues from both stages.

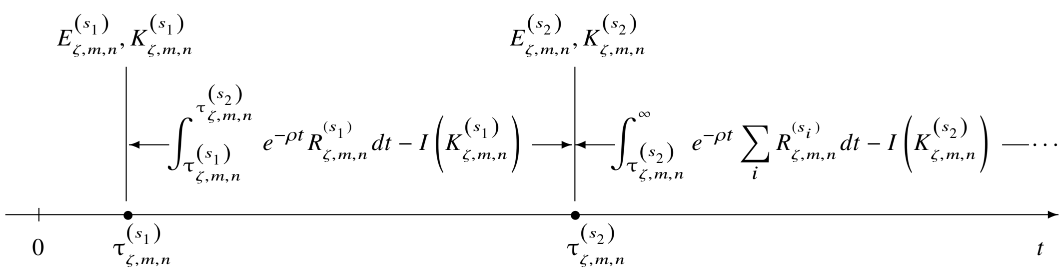

Figure 2: Stepwise investment

Notice that the option to invest in the first stage may be seen as a compound option, since, by completing the first stage, the firm receives the option to proceed to the second. From Kort *et al.* (2010), we know that we can express the investment option as two independent optimal stopping–time problems, as in (8)

$$\begin{aligned} F^{(s)}_{\zeta,m,n}(E) &= \sup_{\tau^{(s_1)}_{\zeta,m,n}\in\mathcal{S}} \mathbb{E}_E\left[\int_{\tau^{(s_1)}_{\zeta,m,n}}^\infty e^{-\rho t} R\left(E^{(s_1)}_{\zeta,m,n}, K^{(s_1)}_{\zeta,m,n}\right)dt - I\left(K^{(s_1)}_{\zeta,m,n}\right)\right] \\ &\quad + \sup_{\tau^{(s_2)}_{\zeta,m,n}\geq\tau^{(s_1)}_{\zeta,m,n}} \mathbb{E}_E\left[\int_{\tau^{(s_2)}_{\zeta,m,n}}^\infty e^{-\rho t} R\left(E^{(s_2)}_{\zeta,m,n}, K^{(s_2)}_{\zeta,m,n}\right)dt - I\left(K^{(s_2)}_{\zeta,m,n}\right)\right] \end{aligned} \tag{8}$$

and, therefore, $F^{(s)}_{\zeta,m,n}(E) = \sum_i F^{(s_i)}_{\zeta,m,n}(E)$, where the expression for the value of the option to invest in each stage is indicated in (9).

$$F^{(s_i)}_{\zeta,m,n}(E) = \max_{E^{(s_i)}_{\zeta,m,n}\geq E}\left(\frac{E}{E^{(s_i)}_{\zeta,m,n}}\right)^{\beta_1}\left[\frac{R\left(E^{(s_i)}_{\zeta,m,n}, K^{(s_i)}_{\zeta,m,n}\right)}{\rho-\mu} - I\left(K^{(s_i)}_{\zeta,m,n}\right)\right] \tag{9}$$

4. BENCHMARK CASE: INVESTMENT WITHOUT POLICY UNCERTAINTY

Here, we assume that the subsidy is already either provided ($\zeta = 1$) or retracted ($\zeta = 0$) permanently. Initially, the firm ignores the option to wait for more information and invests in the project immediately. Thus, it must first solve the inner maximisation in (5). The expected value of the now–or–never investment opportunity under lumpy ($j = \ell$) and stepwise investment ($j = s_i$) for $\zeta \in \{0, 1\}$ is indicated in (10), where $\mathbb{1}$ is the indicator function. Note that the analysis of lumpy and stepwise investment follows the same steps, and, therefore, it is presented in a single framework.

$$\overline{F}^{(j)}_{\zeta,0,0}\left(E, \overline{K}^{(j)}_{\zeta,0,0}\right) = \frac{E\overline{K}^{(j)}_{\zeta,0,0}\left(1 + y\mathbb{1}_{\{\zeta=1\}}\right)}{\rho - \mu} - I\left(\overline{K}^{(j)}_{\zeta,0,0}\right) \tag{10}$$

Since, at investment, E is known, the firm needs to determine only the corresponding optimal capacity. By maximising (10) with respect to $\overline{K}^{(j)}_{\zeta,0,0}$, we obtain the optimal capacity, $\overline{k}^{(j)}_{\zeta,0,0}$, when the firm invests immediately, as indicated in (11).

$$\max_{\overline{K}^{(j)}_{\zeta,0,0}} \overline{F}^{(j)}_{\zeta,0,0}\left(E, \overline{K}^{(j)}_{\zeta,0,0}\right) \Rightarrow \overline{k}^{(j)}_{\zeta,0,0}(P) = \left[\frac{1}{b_j\gamma_j}\left(\frac{E\left(1 + y\mathbb{1}_{\{\zeta=1\}}\right)}{\rho - \mu} - a_j\right)\right]^{\frac{1}{\gamma_j - 1}} \tag{11}$$

Next, we assume that the firm can defer investment. The value of the option to invest under lumpy or stepwise investment is obtained by solving the optimal stopping–time problem (12).

$$F^{(j)}_{\zeta,0,0}(E) = \max_{E^{(j)}_{\zeta,0,0} \geq E} \left(\frac{E}{E^{(j)}_{\zeta,0,0}}\right)^{\beta_1} \left[\frac{E^{(j)}_{\zeta,0,0}K^{(j)}_{\zeta,0,0}\left(1 + y\mathbb{1}_{\{\zeta=1\}}\right)}{\rho - \mu} - I\left(K^{(j)}_{\zeta,0,0}\right)\right] \tag{12}$$

The solution to the unconstrained optimisation problem (12) is described in (13). The endogenous constant, $A^{(j)}_{\zeta,0,0}$, the optimal investment threshold, $\varepsilon^{(j)}_{\zeta,0,0}$, and the corresponding optimal capacity, $k^{(j)}_{\zeta,0,0}$, are determined via value–matching and smooth–pasting conditions between the two branches of (13) together with the condition for optimal capacity choice at investment (11) and are indicated in (A–3), (A–4), and (A–5), respectively (all proofs can be found in the appendix).

$$F^{(j)}_{\zeta,0,0}(E) = \begin{cases} A^{(j)}_{\zeta,0,0}E^{\beta_1} & , \text{ for } E < \varepsilon^{(j)}_{\zeta,0,0} \\ \frac{Ek^{(j)}_{\zeta,0,0}\left(1+y\mathbb{1}_{\{\zeta=1\}}\right)}{\rho-\mu} - I\left(k^{(j)}_{\zeta,0,0}\right) & , \text{ for } E \geq \varepsilon^{(j)}_{\zeta,0,0} \end{cases} \tag{13}$$

As Proposition 1 indicates, the presence of a permanent subsidy increases the incentive to invest and reduces the optimal investment threshold. However, compared to the case where the subsidy is not available, the size of the project remains unaffected. This happens because the subsidy lowers the optimal investment threshold, and, in turn, the corresponding optimal capacity. Intuitively, while a subsidy raises the expected value of the now–or–never investment opportunity, the optimal investment threshold when investment is deferred decreases, thereby lowering the expected project value at investment. Consequently, when evaluating the expected NPV of the project at the optimal investment threshold in the presence of a subsidy, these two opposing forces cancel and the optimal capacity remains unaffected. Notice that, although the cost premium associated with a stepwise investment strategy creates a discrepancy between the quantitative results for lumpy and stepwise investment, the analytical results of Proposition 1 hold for $j = \ell, s_i$.

Proposition 1 $\varepsilon^{(j)}_{1,0,0} < \varepsilon^{(j)}_{0,0,0}$ *and* $k^{(j)}_{1,0,0} = k^{(j)}_{0,0,0}$.

5. INVESTMENT UNDER POLICY UNCERTAINTY

5.1 Retractable Subsidy

We extend the previous framework by assuming that a subsidy is available but may be retracted permanently at a random point in time. The expected NPV of the project in the absence ($\zeta = 0$) or presence ($\zeta = 1$) of a subsidy is indicated in (10). Hence, since the likelihood of subsidy retraction within an infinitesimal time interval dt is λdt, the expected value of the now–or–never investment opportunity in the presence of a retractable subsidy is indicated in (14). Notice that $\lambda = 0$ implies that the subsidy will never be retracted, however, a greater λ raises the likelihood of subsidy retraction and lowers the expected value of the now–or–never investment opportunity.

$$\overline{F}^{(j)}_{1,1,0}\left(E, \overline{K}^{(j)}_{1,1,0}\right) = \frac{E\overline{K}^{(j)}_{1,1,0}\left[1 + (1-\lambda)y\right]}{\rho - \mu} - I\left(\overline{K}^{(j)}_{1,1,0}\right) \tag{14}$$

By maximising (14) with respect to $\overline{K}^{(j)}_{1,1,0}$, we obtain the optimal capacity, $\overline{k}^{(j)}_{1,1,0}$, when investing immediately, as indicated in (15).

$$\max_{\overline{K}^{(j)}_{1,1,0}} \overline{F}^{(j)}_{1,1,0}\left(E, \overline{K}^{(j)}_{1,1,0}\right) \Rightarrow \overline{k}^{(j)}_{1,1,0}(E) = \left[\frac{1}{b_j \gamma_j}\left(\frac{E(1 + (1-\lambda)y)}{\rho - \mu} - a_j\right)\right]^{\frac{1}{\gamma_j - 1}} \tag{15}$$

Next, we assume that the firm has the option to delay investment. As indicated in (16), within an infinitesimal time interval dt, the subsidy may be retracted with probability λdt and the firm will receive the option $F^{(j)}_{0,0,0}(E)$, which is described in (13) for $\zeta = 0$. However, with probability $1 - \lambda dt$ no policy intervention will take place and the firm will continue to hold the option $F^{(j)}_{1,1,0}(E)$.

$$F^{(j)}_{1,1,0}(E) = (1 - \rho dt)\left[\lambda dt \mathbb{E}_E\left[F^{(j)}_{0,0,0}(E + dE)\right] + (1 - \lambda dt)\mathbb{E}_E\left[F^{(j)}_{1,1,0}(E + dE)\right]\right] \tag{16}$$

By expanding the right–hand side of (16) using Itô's lemma, we obtain an ordinary differential equation which we can be solve in order to obtain $F^{(j)}_{1,1,0}(E)$. Since the incentive to invest under a retractable subsidy is greater compared to the case in which the subsidy is absent permanently, we have $\varepsilon^{(j)}_{1,1,0} \leq \varepsilon^{(j)}_{0,0,0}$, and, therefore, the expression for $F^{(j)}_{0,0,0}(E)$ must be taken from the top part of (13). The value of the option to invest under a lumpy or a stepwise investment strategy is indicated in (17), where $\varepsilon^{(j)}_{1,1,0}$, $k^{(j)}_{1,1,0}$, and $B^{(j)}_{1,1,0} > 0$ are determined numerically via value–matching and smooth–pasting conditions together with condition (15) for optimal capacity choice at investment, while $\delta_1 > 1, \delta_2 < 0$ are the roots of the quadratic $\frac{1}{2}\sigma^2\delta(\delta - 1) + \mu\delta - (\rho + \lambda) = 0$. The first term on the top part of (17) is the value of the option to invest in the permanent absence of a subsidy, however, since the subsidy is temporarily present, the first term must be adjusted via the second term. The bottom part of (17) represents the expected NPV of the project.

$$F^{(j)}_{1,1,0}(E) = \begin{cases} A^{(j)}_{0,0,0}E^{\beta_1} + B^{(j)}_{1,1,0}E^{\delta_1} & , E < \varepsilon^{(j)}_{1,1,0} \\ \frac{Ek^{(j)}_{1,1,0}\left[1 + (1-\lambda)y\right]}{\rho - \mu} - I\left(k^{(j)}_{1,1,0}\right) & , E \geq \varepsilon^{(j)}_{1,1,0} \end{cases} \tag{17}$$

As indicated in Proposition 2, the presence of a retractable subsidy increases the incentive to invest compared to the case in which a subsidy is not available, thus lowering the optimal investment threshold and the corresponding optimal capacity. Interestingly, the likelihood of subsidy retraction facilitates investment compared to the case in which a subsidy is available permanently when λ is low. Intuitively, the threat of subsidy retraction increases the incentive to invest in order to take advantage of the subsidy for a longer period.

Proposition 2 $\lambda \geq 0 \Rightarrow \varepsilon^{(j)}_{1,1,0} \leq \varepsilon^{(j)}_{0,0,0}$ *and* $k^{(j)}_{1,1,0} \leq k^{(j)}_{0,0,0}$*, while, for low values of* λ, $\varepsilon^{(j)}_{1,1,0} \leq \varepsilon^{(j)}_{1,0,0}$.

Furthermore, the likelihood of subsidy retraction lowers the expected option value compared to the case in which a subsidy is available permanently. As shown in Proposition 3, for $\lambda = 0$ the subsidy will never be retracted, and, therefore, the relative loss in option value is zero. As λ increases, the relative loss increases, since $B^{(j)}_{1,1,0}E^{\delta_1} \rightarrow 0 \Rightarrow F^{(j)}_{1,1,0}(E) \rightarrow F^{(j)}_{0,0,0}(E)$. Notice that the relative loss in option value will always be below one, since the firm can invest even in the absence of a subsidy.

Proposition 3 $\frac{F^{(j)}_{1,0,0}(E)-F^{(j)}_{1,1,0}(E)}{F^{(j)}_{1,0,0}(E)} \in \left[0, \frac{A^{(j)}_{1,0,0}-A^{(j)}_{0,0,0}}{A^{(j)}_{1,0,0}}\right)$.

5.2 Sudden Provision of a Permanent Subsidy

Here, we assume that a subsidy is not available but may be provided permanently at a random point in time. The provision of a subsidy may be required in order to support green investments, as the increasing replacement of fossil–fuel with RE facilities can result in the deterioration of the financial risk–return performance metrics for incremental investments (Muñoz and Bunn, 2013). The expected NPV of the project in the presence ($\zeta = 1$) or absence ($\zeta = 0$) of a subsidy is described in (10). Hence, by taking into account the likelihood of subsidy provision, the expected value of the now–or–never investment opportunity is indicated in (18)

$$\overline{F}^{(j)}_{0,0,1}\left(E, \overline{K}^{(j)}_{0,0,1}\right) = \frac{E\overline{K}^{(j)}_{0,0,1}(1+\lambda y)}{\rho-\mu} - I\left(\overline{K}^{(j)}_{0,0,1}\right) \tag{18}$$

and, by maximising (18) with respect to $\overline{K}^{(j)}_{0,0,1}$, we obtain the optimal capacity, $\overline{k}^{(j)}_{0,0,1}$, which is described in (19). Contrary to (14), $\lambda = 0$ now implies that the subsidy will never be provided, while an increase in λ raises the likelihood of subsidy provision, and, in turn, both the expected value of the now–or–never investment opportunity and $\overline{k}^{(j)}_{0,0,1}$. By comparing (19) to (11) for $\zeta = 1$, we find that $\overline{k}^{(j)}_{0,0,1} < \overline{k}^{(j)}_{1,0,0}$. Hence, the optimal capacity under a now–or–never investment decision is lower when the provision of the subsidy is uncertain compared to the case in which the subsidy is available permanently, since the expected NPV of the project is lower.

$$\max_{\overline{K}^{(j)}_{0,0,1}} \overline{F}^{(j)}_{0,0,1}\left(E, \overline{K}^{(j)}_{0,0,1}\right) \Rightarrow \overline{k}^{(j)}_{0,0,1}(E) = \left[\frac{1}{b_j\gamma_j}\left(\frac{E(1+\lambda y)}{\rho-\mu} - a_j\right)\right]^{\frac{1}{\gamma_j-1}} \tag{19}$$

Next, we assume that investment can be deferred. As indicated in (20), within a short time interval dt a subsidy may be provided with probability λdt, and, thus, the firm will receive the option $F^{(j)}_{1,0,0}(E)$, which is indicated in (13) for $\zeta = 1$. However, with probability $1 - \lambda dt$ no policy intervention will take place and the firm will continue to hold the option $F^{(j)}_{0,0,1}(E)$.

$$F^{(j)}_{0,0,1}(E) = (1-\rho dt)\left[\lambda dt \mathbb{E}_E\left[F^{(j)}_{1,0,0}(E+dE)\right] + (1-\lambda dt)\mathbb{E}_P\left[F^{(j)}_{0,0,1}(E+dE)\right]\right] \tag{20}$$

By expanding the right–hand side of (20) using Itô's lemma, we obtain an ordinary differential equation which can be solved for $F^{(j)}_{0,0,1}(E)$. Notice that $F^{(j)}_{0,0,1}(E)$ is defined over three different regions of E. If $E < \varepsilon^{(j)}_{1,0,0}$, then, even if a subsidy becomes available, the firm would still have to wait until $E = \varepsilon^{(j)}_{1,0,0}$ in order to invest. Hence, the first term on the top part of (21) is the option to invest in the presence of a permanent subsidy, however, since the subsidy is not available yet, this term must be adjusted via the second term. Next, if $\varepsilon^{(j)}_{1,0,0} \leq E \leq \varepsilon^{(j)}_{0,0,1}$, then investment will take place immediately if a subsidy is provided, and, therefore, the first two terms in the middle part of (21) reflect the project's expected revenues and cost, respectively. The third term corresponds to the probability of investment if the subsidy is not provided and the last term reflects the probability that the price will drop below $\varepsilon^{(j)}_{1,0,0}$. Finally, if $E \geq \varepsilon^{(j)}_{0,0,1}$, then investment will take place immediately.

$$F^{(j)}_{0,0,1}(E) \quad = \quad \begin{cases} A^{(j)}_{1,0,0}E^{\beta_1} + B^{(j)}_{0,0,1}E^{\delta_1} & , E < \varepsilon^{(j)}_{1,0,0} \\ \frac{\lambda E k^{(j)}_{0,0,1}(1+y)}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I\left(k^{(j)}_{0,0,1}\right)}{\rho+\lambda} + C^{(j)}_{0,0,1}E^{\delta_1} + D^{(j)}_{0,0,1}E^{\delta_2} & , \varepsilon^{(j)}_{1,0,0} \leq E \leq \varepsilon^{(j)}_{0,0,1} \\ \frac{E k^{(j)}_{0,0,1}(1+\lambda y)}{\rho-\mu} - I\left(k^{(j)}_{0,0,1}\right) & , E \geq \varepsilon^{(j)}_{0,0,1} \end{cases} \tag{21}$$

The endogenous constants $B^{(j)}_{0,0,1}$, $C^{(j)}_{0,0,1}$, $D^{(j)}_{0,0,1}$, the optimal investment threshold, $\varepsilon^{(j)}_{0,0,1}$, and the corresponding optimal capacity, $k^{(j)}_{0,0,1}$, are obtained numerically via value–matching and smooth–pasting conditions between the three branches of (21) together with the condition for optimal capacity choice at investment, which is indicated in (19). Notice that the temporary absence of the subsidy lowers the value of the project. Consequently, as shown in Proposition 4, the optimal investment threshold and the corresponding optimal capacity when the subsidy is likely to be provided are greater than in the permanent presence of a subsidy. Interestingly, in the light of subsidy provision investment is delayed relative to the case in which the subsidy is never provided for low values of λ. Intuitively, the likelihood of a permanent subsidy increases the incentive to build a bigger project, which raises the optimal investment threshold.

Proposition 4 $\lambda \geq 0 \Rightarrow \varepsilon^{(j)}_{0,0,1} \geq \varepsilon^{(j)}_{1,0,0}$ *and* $k^{(j)}_{0,0,1} \geq k^{(j)}_{1,0,0}$*, while, for low values of* λ*,* $\varepsilon^{(j)}_{0,0,1} \geq \varepsilon^{(j)}_{0,0,0}$.

Unlike the case of sudden subsidy retraction, the relative loss in option value due to policy uncertainty, which is indicated in Proposition 5, decreases with greater λ. Indeed, for $\lambda = 0$ the subsidy will never be provided and the relative loss in option value is maximised, as $F^{(j)}_{0,0,1}(E) = F^{(j)}_{0,0,0}(E)$, whereas, it decreases with greater λ, since the expected value of the project increases.

Proposition 5 $\frac{F^{(j)}_{1,0,0}(E) - F^{(j)}_{0,0,1}(E)}{F^{(j)}_{1,0,0}(E)} \in \left[-\frac{B^{(j)}_{1,0,0}}{A^{(j)}_{1,0,0}}, 0\right)$.

5.3 Sudden Provision of a Retractable Subsidy

Unlike Section 5.2, the sudden provision of a subsidy is now followed by a potential permanent retraction. Therefore, once a subsidy is provided, the firm receives the expected value of a project under a retractable subsidy, which is already determined in (14). By contrast, if the subsidy is not provided, then the firm will hold the value of a project in the absence of a subsidy. Given the likelihood of these two outcomes, the expected value of the active project under sudden provision of a retractable subsidy is described in (22). Notice that, compared to (18), the subsidy will be available for a smaller time period, and, therefore, its expected value is reduced, i.e., $\lambda(1-\lambda)y < \lambda y$.

$$\overline{F}^{(j)}_{0,1,1}\left(E, \overline{K}^{(j)}_{0,1,1}\right) \quad = \quad \frac{E\overline{K}^{(j)}_{0,1,1}\left[1 + \lambda\left(1-\lambda\right)y\right]}{\rho - \mu} - I\left(\overline{K}^{(j)}_{0,1,1}\right) \tag{22}$$

By maximising (22) with respect to $\overline{K}^{(j)}_{0,1,1}$, we obtain the optimal capacity, $\overline{k}^{(j)}_{0,1,1}$, which is indicated in (23). By comparing (23) to (19), we see that $\overline{k}^{(j)}_{0,1,1} < \overline{k}^{(j)}_{0,0,1}$, since the reduction in the value of the subsidy creates an incentive to install a smaller project.

$$\max_{\overline{K}^{(j)}_{0,1,1}} \overline{F}^{(j)}_{0,1,1}\left(E, \overline{K}^{(j)}_{0,1,1}\right) \quad \Rightarrow \quad \overline{k}^{(j)}_{0,1,1} = \left[\frac{1}{b_j\gamma_j}\left(\frac{E(1+\lambda(1-\lambda)y)}{\rho-\mu} - a_j\right)\right]^{\frac{1}{\gamma_j - 1}} \tag{23}$$

Next, we determine the value of the option to invest when investment is deferred. As indicated in (24), within a short time interval dt a subsidy may be provided with probability λdt, and, thus, the firm will receive the option $F^{(j)}_{1,1,0}(E)$, which is described in (17). However, with probability $1-\lambda dt$ no policy intervention will take place and the firm will continue to hold the option $F^{(j)}_{0,1,1}(E)$.

$$F^{(j)}_{0,1,1}(E) \quad = \quad (1-\rho dt)\left[\lambda dt \mathbb{E}_E\left[F^{(j)}_{1,1,0}(E+dE)\right] + (1-\lambda dt)\mathbb{E}_E\left[F^{(j)}_{0,1,1}(E+dE)\right]\right] \tag{24}$$

By expanding the right–hand side of (24) using Itô's lemma, we obtain an ordinary differential equation which can be solved for $F^{(j)}_{0,1,1}(E)$. The expression for $F^{(j)}_{0,1,1}(E)$ is indicated in (25), where $C^{(j)}_{0,1,1}$, $D^{(j)}_{0,1,1}$, $G^{(j)}_{0,1,1}$, $\varepsilon^{(j)}_{0,1,1}$, and $k^{(j)}_{0,1,1}$, are obtained numerically via value–matching and smooth–pasting conditions between the branches of (25) together with (23). Notice that, unlike (21), the extra term on the top part of (25) reflects the reduction in the expected option value due to subsidy retraction. Next, if $\varepsilon^{(j)}_{1,1,0} \leq E \leq \varepsilon^{(j)}_{0,1,1}$, then investment will take place immediately if a subsidy is provided and the first two terms in the middle part of (25) reflect the project's expected profit. The third term, is the probability of investment if the subsidy is not provided and the last term reflects the probability that the price will drop below $\varepsilon^{(j)}_{1,1,0}$. Finally, if $E \geq \varepsilon^{(j)}_{0,1,1}$, then investment will take place immediately even in the absence of a retractable subsidy.

$$F^{(j)}_{0,1,1}(E) \quad = \quad \begin{cases} A^{(j)}_{0,0,0}E^{\beta_1} + B^{(j)}_{1,1,0}E^{\delta_1} + C^{(j)}_{0,1,1}E^{\delta_1} & , E < \varepsilon^{(j)}_{1,1,0} \\ \frac{\lambda E k^{(j)}_{1,1,0}[1+(1-\lambda)y]}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I\left(k^{(j)}_{1,1,0}\right)}{\rho+\lambda} + D^{(j)}_{0,1,1}E^{\delta_1} + G^{(j)}_{0,1,1}E^{\delta_2} & , \varepsilon^{(j)}_{1,1,0} \leq E < \varepsilon^{(j)}_{0,1,1} \\ \frac{E k^{(j)}_{0,1,1}(1+\lambda(1-\lambda)y)}{\rho-\mu} - I\left(k^{(j)}_{0,1,1}\right) & , E \geq \varepsilon^{(j)}_{0,1,1} \end{cases} \tag{25}$$

As it will be illustrated numerically, the likelihood of permanent retraction after the subsidy is provided reduces the amount of installed capacity compared to the case of permanent subsidy provision, i.e., $k^{(j)}_{0,1,1} < k^{(j)}_{0,0,1}$. This happens because the likelihood of subsidy retraction decreases the expected value of the project, thereby increasing the incentive to install a smaller capacity.

5.4 Infinite Provisions and Retractions

Here, we assume that a subsidy can be retracted or provided infinitely many times. In this case, policy uncertainty in the form of extra provisions and retractions does not affect the value of the active project. Indeed, even after several policy interventions have taken place, there still remain infinite provisions and retractions. Taking into account that $\lambda\,(1-\lambda(1-\lambda\,\ldots = \sum_{i=0}^{\infty}(-1)^i\lambda^i = \frac{1}{1+\lambda}$, the expected value of the active project when $\zeta = 0$ is indicated in (26)

$$\frac{E\overline{K}^{(j)}_{0,\infty,\infty}}{\rho-\mu}\left[1+y\lambda\,(1-\lambda(1-\lambda\ldots\right] - I\left(\overline{K}^{(j)}_{0,\infty,\infty}\right) \quad = \quad \frac{E\overline{K}^{(j)}_{0,\infty,\infty}}{\rho-\mu}\left[1+\frac{\lambda y}{1+\lambda}\right] - I\left(\overline{K}^{(j)}_{0,\infty,\infty}\right) \tag{26}$$

whereas for $\zeta = 1$ the expected value of the active project is indicated in (27).

$$\frac{E\overline{K}^{(j)}_{1,\infty,\infty}}{\rho-\mu}\left[1+y\left(1-\sum_{i=1}^{\infty}(-1)^{i+1}\lambda^i\right)\right] - I\left(\overline{K}^{(j)}_{1,\infty,\infty}\right) = \frac{E\overline{K}^{(j)}_{1,\infty,\infty}}{\rho-\mu}\left[1+\frac{y}{1+\lambda}\right] - I\left(\overline{K}^{(j)}_{1,\infty,\infty}\right) \tag{27}$$

Consequently, the optimal capacity of the project when exercising a now–or–never investment opportunity is obtained by maximising (26) and (27) with respect to $\overline{K}^{(j)}_{0,\infty,\infty}$ and $\overline{K}^{(j)}_{1,\infty,\infty}$, respectively.

With the option to defer investment, the value of the option to invest for $\zeta = 0$ is described in

(28), where η is the positive root of the quadratic $\frac{1}{2}\sigma^2 x(x-1) + \mu x - (\rho + 2\lambda) = 0$.

$$F^{(j)}_{0,\infty,\infty}(E) = \begin{cases} A^{(j)}_a E^{\beta_1} - A^{(j)}_b E^{\eta} & , E < \varepsilon^{(j)}_{1,\infty,\infty} \\ \frac{\lambda E k^{(j)}_{0,\infty,\infty}\left(1+\frac{y}{1+\lambda}\right)}{(\rho-\mu)(\rho+\lambda-\mu)} - \frac{\lambda}{\rho+\lambda} I\left(k^{(j)}_{0,\infty,\infty}\right) & \\ +D^{(j)}_{0,\infty,\infty} E^{\delta_1} + G^{(j)}_{0,\infty,\infty} E^{\delta_2} & , \varepsilon^{(j)}_{1,\infty,\infty} < E < \varepsilon^{(j)}_{0,\infty,\infty} \\ \frac{E k^{(j)}_{0,\infty,\infty}}{\rho-\mu} \frac{1+\lambda+\lambda y}{1+\lambda} - I\left(k^{(j)}_{0,\infty,\infty}\right) & , E \geq \varepsilon^{(j)}_{0,\infty,\infty} \end{cases} \tag{28}$$

Also, the value of the option to invest for $\zeta = 1$ is described in (29). The endogenous constants $A^{(j)}_a$, $A^{(j)}_b$, $D^{(j)}_{0,\infty,\infty}$, and $G^{(j)}_{0,\infty,\infty}$, as well as $\varepsilon^{(j)}_{\zeta,\infty,\infty}$ and $k^{(j)}_{\zeta,\infty,\infty}$ are determined numerically via value–matching and smooth–pasting conditions between the three branches of (28) and the two branches of (29).

$$F^{(j)}_{1,\infty,\infty}(E) = \begin{cases} A^{(j)}_a E^{\beta_1} + A^{(j)}_b E^{\eta} & , E < \varepsilon^{(j)}_{1,\infty,\infty} \\ \frac{E k^{(j)}_{1,\infty,\infty}}{\rho-\mu} \frac{1+\lambda+y}{1+\lambda} - I\left(k^{(j)}_{1,\infty,\infty}\right) & , E \geq \varepsilon^{(j)}_{1,\infty,\infty} \end{cases} \tag{29}$$

6. NUMERICAL EXAMPLES

For the numerical examples we assume that $\mu = 0.01$, $\rho = 0.1$, $\sigma \in [0.1, 0.4]$, $b_\ell = b_{s_1} = b_{s_2} = 0.5$, $a_\ell = 30$, $a_{s_1} = 15$, $a_{s_2} = 25$, $\gamma_\ell = \gamma_{s_1} = \gamma_{s_2} = 3$, and $\lambda \in [0, 1]$. Note that the parameter values satisfy assumption (4), i.e., the stepwise investment is more costly than the lumpy investment strategy. Figure 3 illustrates the impact of λ on the optimal investment threshold and the corresponding optimal capacity in the case of sudden subsidy retraction. Without loss of generality, the impact of λ on the optimal investment threshold and optimal capacity under staged investment is omitted in order to improve the clarity of the graphs, since it is qualitatively similar. Notice that if $\lambda = 0$, then the subsidy will never be retracted, and, as a result, $\varepsilon^{(\ell)}_{1,1,0} = \varepsilon^{(\ell)}_{1,0,0}$. However, for low values of λ, the optimal investment threshold decreases as the likelihood of retraction increases and $\varepsilon^{(\ell)}_{1,1,0} \leq \varepsilon^{(\ell)}_{1,0,0}$, as shown in Proposition 2. This happens because the firm wants to take advantage of the subsidy for a longer period and the extra incentive to invest increases as the expected time until retraction decreases. However, beyond a certain high value of λ the subsidy is very likely to be retracted and the extra investment incentive decreases. Indeed, if $\lambda = 1$, then $\varepsilon^{(\ell)}_{1,1,0} = \varepsilon^{(\ell)}_{0,0,0}$ and $k^{(\ell)}_{1,1,0} = k^{(\ell)}_{0,0,0}$, i.e., the optimal investment threshold and the corresponding optimal capacity are the same as in the case in which a subsidy is not available. As the right panel illustrates, $k^{(\ell)}_{1,0,0} = k^{(\ell)}_{0,0,0}$, i.e., the permanent presence or absence of a subsidy does not impact the optimal capacity, as shown in Proposition 1, while, if $\lambda > 0$, then $k^{(\ell)}_{1,1,0} < k^{(\ell)}_{\zeta,0,0}$. Additionally, the impact of λ on $\varepsilon^{(\ell)}_{1,1,0}$ and $k^{(\ell)}_{1,1,0}$ becomes more pronounced as the level of the subsidy increases, since this raises the firm's incentive to invest. Hence, in order to incentivise investment, policymakers should announce a potential subsidy retraction while allowing for a sufficient time interval so that firms can still invest and take advantage of the subsidy. Otherwise, investment will be delayed relative to the case in which the subsidy is always available. The effect of this policy is that it may accelerate investment, yet the amount of installed capacity will be lower.

As the left panel in Figure 4 illustrates, the relative loss in option value due to subsidy retraction, which is described in Proposition 3, increases as the likelihood of subsidy retraction increases. Nevertheless, this result is less pronounced with greater price uncertainty. In fact, the impact of price uncertainty on the relative loss in option value is more pronounced when λ high. Intuitively, greater price uncertainty raises the optimal capacity of the project, thereby making the loss in option value due to subsidy retraction less pronounced. According to the right panel, although both greater price uncertainty and a greater subsidy lower the relative value of the two strategies, $F^{(s)}_{1,1,0}(E)/F^{(\ell)}_{1,1,0}(E)$, they do not present a significant incentive to adopt a lumpy over of a stepwise investment strategy. This is in line with Chronopoulos *et al.* (2014), who show that stepwise investment dominates the lumpy investment strategy when a firm has discretion over capacity.

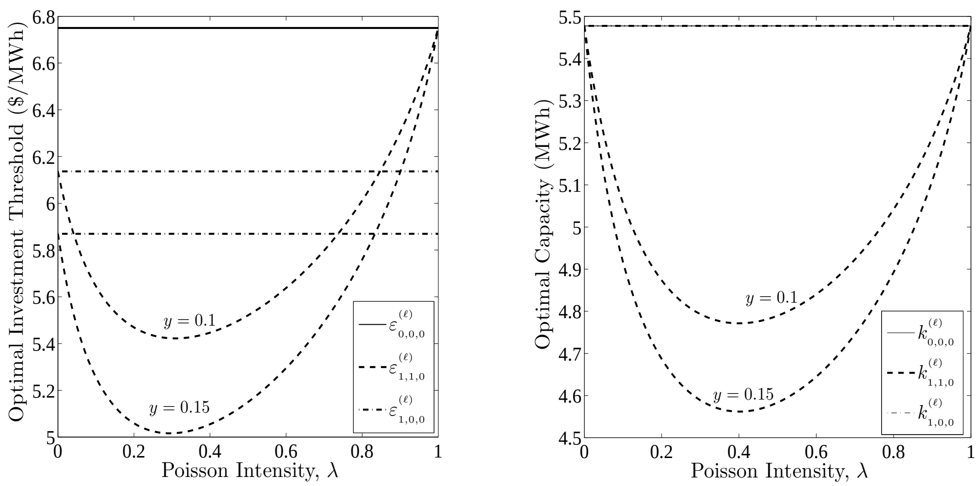

Figure 3: Impact of the likelihood of sudden and permanent subsidy retraction on the optimal investment threshold (left) and the optimal capacity (right) for $y = 0.1, 0.15$ and $\sigma = 0.2$. Greater likelihood of permanent subsidy retraction increases the incentive to invest, yet lowers the amount of installed capacity by decreasing the expected value of the subsidy.

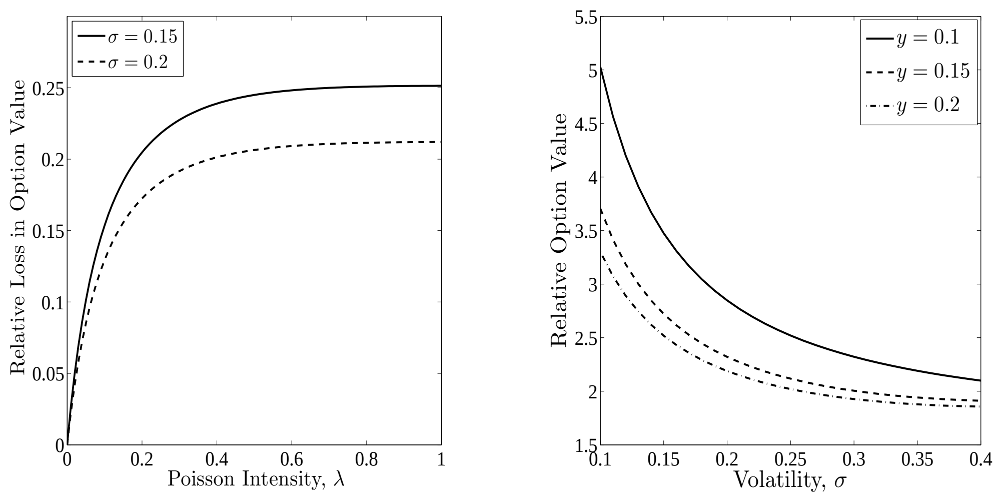

Figure 4: Relative loss in option value due to subsidy retraction versus λ (left) and relative value of the two strategies (lumpy and stepwise investment) versus σ for $\lambda = 0.1$ (right). The relative loss in option value increases as the retraction of the subsidy becomes more likely. Also, stepwise investment dominates a lumpy investment strategy, although the incentive to invest in stages decreases with either greater price uncertainty or as the level of the subsidy increases.

Figure 5 illustrates the impact of λ on the optimal investment threshold and the corresponding optimal capacity in the case of sudden provision of a permanent subsidy. Notice that if $\lambda = 0$, then the subsidy will never be provided, and, as a result, $\varepsilon^{(\ell)}_{0,0,1} = \varepsilon^{(\ell)}_{0,0,0}$. Also, when the likelihood of subsidy provision is small, the incentive to delay investment increases with greater λ and $\varepsilon^{(\ell)}_{0,0,1} \geq \varepsilon^{(\ell)}_{0,0,0}$, as shown in Proposition 4. Indeed, greater λ raises the expected value of the project, and, in turn, the incentive to increase the amount of installed capacity. However, beyond a certain high value of λ the extra incentive to delay investment decreases, and, for $\lambda = 1$ we have $\varepsilon^{(\ell)}_{0,0,1} = \varepsilon^{(\ell)}_{1,0,0}$ and $k^{(\ell)}_{0,0,1} = k^{(\ell)}_{1,0,0}$. Intuitively, greater λ lowers the likelihood of an inaccurate capacity choice, and, in turn, the extra incentive to delay investment. Consequently, while the installation of larger projects brings the industry closer to decarbonisation targets, the expected time until investment is justified moves further into the future, as the optimal investment threshold increases. With this in mind, policymakers should weigh the benefits from the installation of bigger projects against the cost of postponed investment in designing both the level of the subsidy as well as the expected time at which it should be provided.

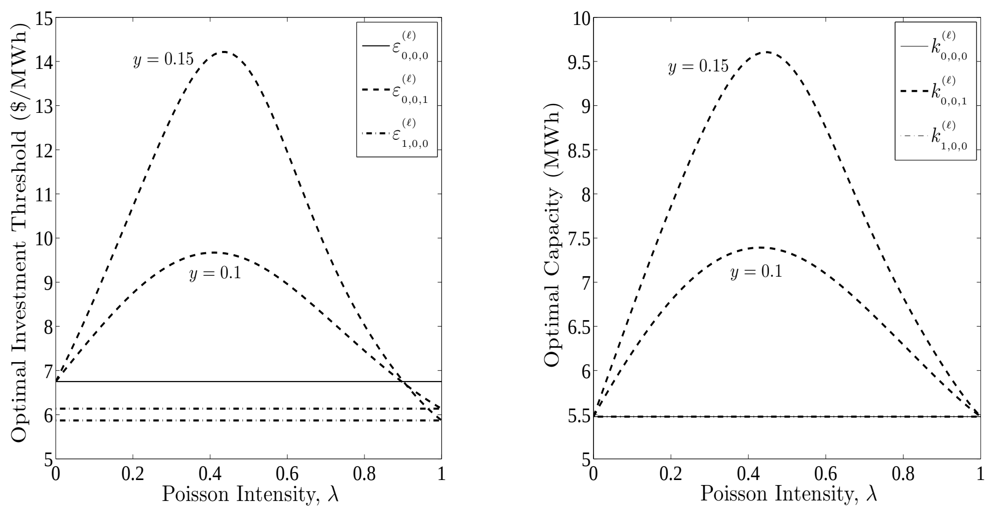

Figure 5: Impact of the likelihood of sudden provision of a permanent subsidy on the optimal investment threshold (left) and the optimal capacity (right) for $y = 0.1, 0.15$ and $\sigma = 0.2$. Greater likelihood of subsidy provision raises the expected value of the subsidy, and, in turn, the incentive to install a bigger project, yet delays investment by increasing the optimal investment threshold.

The left panel in Figure 6 illustrates the impact of λ on the relative loss in option value due to uncertainty in the provision of the subsidy, which is described in Proposition 5. Notice that, as λ increases, the likelihood of subsidy provision increases, and, as a result, the relative loss in option value converges to zero. Like in the case of sudden subsidy retraction, greater price uncertainty lowers the relative loss in option value and this result is more pronounced when the likelihood of subsidy provision is low. As the right panel illustrates, the relative value of the two strategies decreases as price uncertainty and the level of the subsidy increase, nevertheless, the stepwise investment strategy dominates that of lumpy investment.

Figure 7 illustrates the impact of λ on the optimal investment threshold and the corresponding optimal capacity in the case of sudden provision of a retractable subsidy. Notice that, compared to the case of sudden provision of a permanent subsidy, the likely retraction of the subsidy after its initial provision reduces the expected value of the project, and, in turn, lowers both the optimal investment threshold and the corresponding optimal capacity. More specifically, the optimal investment threshold (left panel) and the corresponding optimal capacity (right panel) are greater than in the case of sudden subsidy retraction, yet lower than in the case of sudden subsidy provision. Consequently, as the number of policy interventions increases, a firm may have a greater incentive to invest but the amount of installed capacity decreases. Hence, although an increasing number of policy interventions may be inevitable, it is, nevertheless, possible to design a policy in a way that not only facilitates investment but also raises the amount of installed capacity compared to the case in which the subsidy is never available.

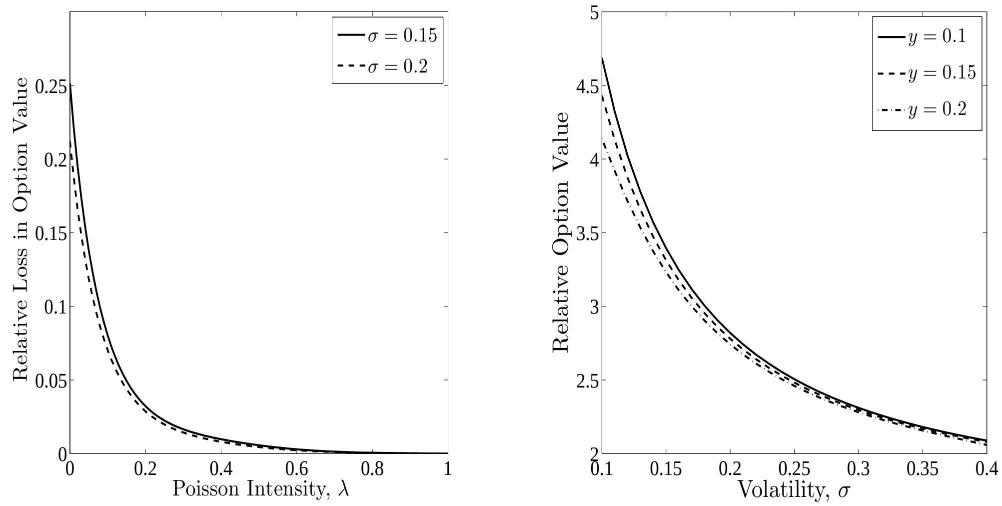

Figure 6: Relative loss in option value due to policy uncertainty versus λ (left) and relative value of the two strategies (lumpy and stepwise investment) versus σ for $\lambda = 0.1$ (right). Greater likelihood of subsidy provision lowers the relative loss in option value by raising the expected value of the subsidy. Also, stepwise investment dominates a lumpy investment strategy, although this becomes less pronounced as either price uncertainty or the level of the subsidy increases.

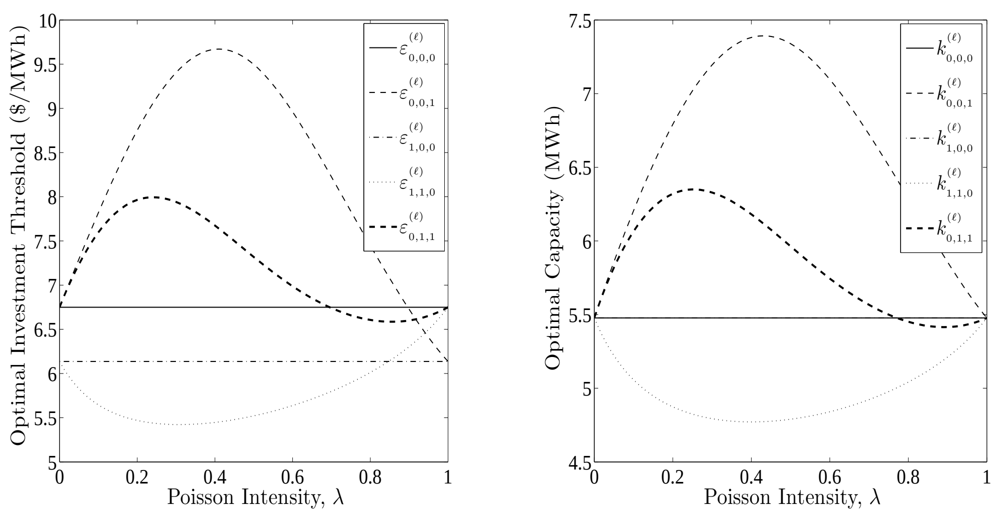

Figure 7: Impact of the likelihood of sudden provision of a retractable subsidy on the optimal investment threshold (left) and the optimal capacity (right) for $\sigma = 0.2$. Increasing number policy interventions lowers the expected value of the subsidy, thereby reducing the amount of installed capacity, and, in turn, the incentive to postpone investment.

As illustrated on the left panel of Figure 8, the optimal investment threshold exhibits a similar non–monotonic behaviour with greater λ under infinite provisions and retractions for $\zeta = 0, 1$. More specifically, if $\zeta = 1$, then the optimal investment threshold is greater compared to the case of permanent subsidy retraction. This happens because the expected value of the subsidy, and, in turn, the value of the project is greater, which, in turn, raises both the incentive to build a bigger project and the optimal investment threshold. Similarly, for $\zeta = 0$ and under infinite provisions and retractions, the expected value of the project, and, in turn, the optimal investment threshold are greater compared to the case of provision of a retractable subsidy, yet lower compared to the case of permanent subsidy provision. Also, as the right panel illustrates, an increasing number of policy interventions lowers the expected value of the subsidy, thereby increasing the firm's incentive to invest in stages.

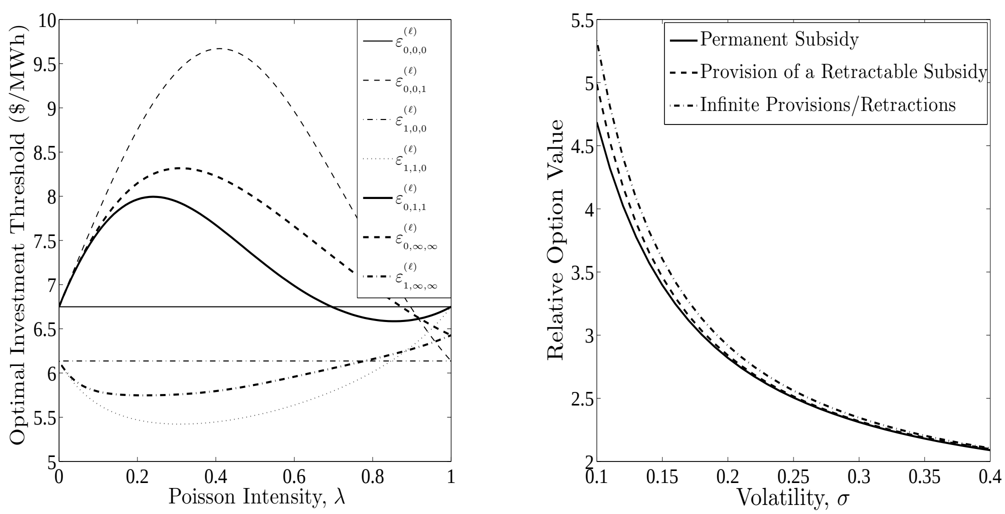

Figure 8: Impact of policy uncertainty on the optimal investment threshold for $\sigma = 0.2$ (left) and relative value of the two strategies (lumpy and stepwise investment) versus σ for $\lambda = 0.1$ (right). Under infinite provisions and retractions, policy uncertainty and investment timing exhibit a non–monotonic relationship similar to the case of finite policy interventions. Also, increasing number of policy interventions raises the relative value of the stepwise investment strategy, thereby mitigating the impact of a greater subsidy.

Figure 9 illustrates the impact of price uncertainty on the optimal investment threshold and the corresponding optimal capacity in the case of sudden provision of a permanent subsidy and under infinite provisions and retractions for $\zeta = 0$. Notice that higher price uncertainty delays investment by increasing the opportunity cost of investing and raising the value of waiting. Also, increasing policy interventions facilitate investment, yet lower the amount of installed capacity. Interestingly, however, the total amount of installed capacity via a stepwise investment strategy is greater than that under lumpy investment. This happens because the endogenous relationship between the electricity price at investment and the size of the project allows the firm to compensate for the extra cost it incurs for the flexibility to invest in stages by adjusting the size of the project so that it offsets the reduction in the value of the investment opportunity (Chronopoulos *et al.*, 2014). This has crucial implications for both policymakers and private firms. Indeed, the former must take into account how policy uncertainty interacts with a firm's ability to respond to a reduction in the expected value of a project through managerial flexibility, e.g., discretion over capacity. Similarly, the latter can exploit the flexibility to scale the capacity of the project in order to offset any extra cost associated with either stepwise investment or policy uncertainty.

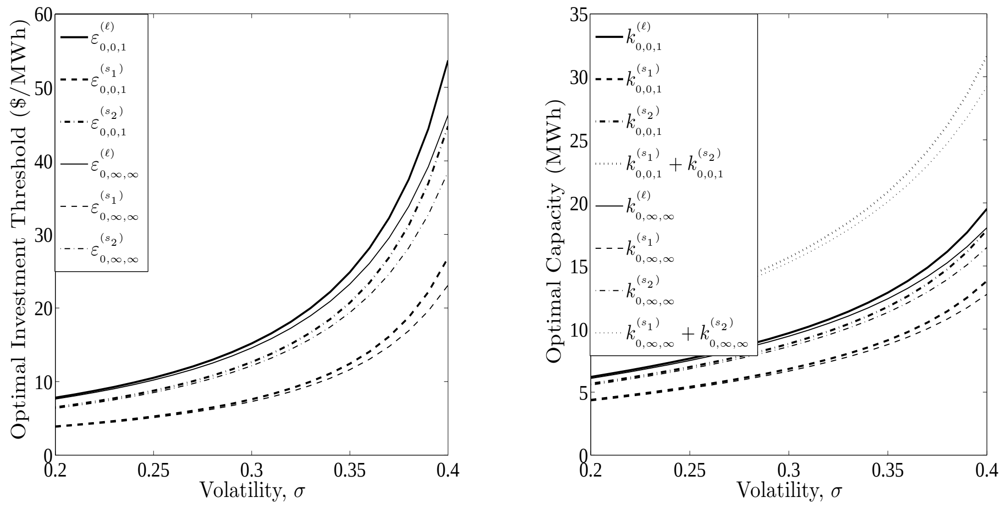

Figure 9: Optimal investment threshold (left) and optimal capacity (right) versus σ for $\lambda = 0.1$. The flexibility to invest in stages entails a cost premium, which may delay investment relative to the lumpy investment strategy, however, discretion over capacity enables the firm to offsets the extra cost by installing a bigger project.

7. CONCLUSIONS

The future development of RE technologies relies crucially on government support, however, frequent changes and revisions of support schemes in combination with price uncertainty complicate investment and operational decisions. As the structural transformation of the power sector continues, it is expected to result in substantial changes in the wholesale market dynamics and will have crucial implications for both market participants and policymakers (Sensfuß *et al.*, 2008). In fact, empirical research based on the UK power market has indicated that a *ceteris paribus* increasing replacement of commodity–based with RE facilities may cause a deterioration of the financial risk–return performance metrics for incremental investments. Consequently, unlike conventional degression trajectories, RE projects may require a progressively higher level of support over time, as they entail high risk and low return (Muñoz and Bunn, 2013). Fluctuating commodity prices may also contribute to this, as decreasing oil prices increase the attractiveness of commodity–based over RE facilities. Notable examples of companies that suffered a, at least temporary, share price fall due to the collapse of oil prices include Vestas, the world's largest wind turbine supplier, Tesla Motors, the US electric carmaker, and the solar panel giant, Yingli Green Energy (Financial Times, 2014b). Within this environment, private firms are required to make accurate investment and capacity sizing decisions, while policymakers must take into account how private firms respond to price and policy uncertainty in order to incentivise green investment decisions.

Therefore, we develop an analytical framework in order to investigate how price and policy uncertainty interact in order to affect the optimal investment timing and capacity sizing decisions assuming that investment can proceed in either a single or multiple stages. Thus, we obtain insights on the combined impact of price and policy uncertainty on both the investors' propensity to invest as well as on the level of investment. We assume that price uncertainty is described via a GBM and implement policy uncertainty by allowing for the random introduction or retraction of a subsidy, that takes the form of a fixed proportion on top of the price of electricity. Results indicate that, in the absence of policy uncertainty the optimal investment threshold is lower when the subsidy is available permanently, yet the corresponding optimal capacity is the same as in the case where the subsidy is not available. Furthermore, the likelihood of sudden provision or retraction of a subsidy has a non–monotonic impact on both the optimal investment threshold and the corresponding optimal capacity. More specifically, the threat that a subsidy may be retracted permanently increases the incentive to invest in order to take advantage of the subsidy for a longer period, however, this lowers the amount of installed capacity.

By contrast, the likelihood of provision of a permanent subsidy raises the incentive to install a bigger project, yet this delays investment as the optimal investment threshold increases. In addition, we illustrate how increasing number of policy interventions lowers the size of the project. Moreover, the sudden provision of a subsidy lowers the relative value of the two investment strategies, yet the stepwise investment strategy dominates lumpy investment. In fact, increasing number of policy interventions raises the relative value of the stepwise investment strategy, and, although a greater subsidy makes the impact of policy uncertainty on investment and capacity sizing decisions more pronounced, it does not impact the choice of investment strategy. This implies that, if a firm has discretion over capacity, then the stepwise investment strategy dominates lumpy investment under both price (Chronopoulos *et al.*, 2014) and policy uncertainty, in the form of random provision and retraction of a subsidy.

These results imply that policymaking decisions may become more efficient if they take into account the opposing forces emerging from the interaction between the different types of uncertainties and a firm's flexibility over not only investment timing and capacity sizing decisions, but also over the choice of investment strategy. Inclusion of such features in the policymaking process may alleviate the complexity underlying the design and implementation of policies for supporting green investments, in terms of the timing of policy interventions and how this influences the risk–return profile of RE projects (The Guardian, 2015b; The Telegraph, 2015). For example, despite the attractiveness of announcing the permanent retraction of a subsidy in terms of accelerating investment, policymakers should consider the implications of a smaller project, as indicated in the left panel of Figure 3. Similarly, the provision of a permanent subsidy may result in a bigger project, however, as indicated in the left panel of Figure 5, this postpones investment by raising the required investment threshold. More importantly, in the case of sequential policy interventions, the non–monotonic impact of policy uncertainty on the optimal investment threshold and the optimal capacity implies a flexibility from a policymaking perspective, in terms of balancing the timing of investments and the size of RE projects. Indeed, as the left panel of Figure 7 indicates, the rate of policy interventions may be adjusted to facilitate the progress of green investments by either promoting bigger projects that take longer to be realised or by accelerating investment in smaller projects. Additionally, although it may be feasible to promote lumpy over stepwise investment by increasing the size of the subsidy when a firm does not have discretion over project scale, this is not the case when the capacity of the RE project is scalable. Indeed, a *ceteris paribus* increase in the subsidy raises the value of the lumpy investment strategy, as it is relatively cheaper than stepwise investment. However, with discretion over capacity, a firm compensates for the extra cost associated with the flexibility to proceed in stages by increasing the amount of installed capacity (Chronopoulos *et al.,* 2014), and, thus, in contrast to Siddiqui and Maribu (2009) and Kort *et al.* (2010), stepwise investment always dominates a lumpy investment strategy.

A limitation of this work is that it assumes that the electricity price is independent of the size of the project. This limitation is particularly obvious when the installation of a very large project is optimal. In order to relax this assumption, we can allow the electricity price to depend on the size of the project via an inverse demand function (Dangl, 1999). This will not only facilitate the analysis of economies of scale, i.e., $\gamma_j < 1$, but will also enable further insights on the impact of policy uncertainty on the total level of investment and how much lower it is relative to the benchmark case of no policy uncertainty. Directions for further research may also include the implementation of a different stochastic process, e.g., mean–reverting process or arithmetic Brownian motion, in order to relax the assumption of a GBM. Moreover, our setup also allows for game–theoretic considerations, and, therefore, it would be interesting to include strategic interactions via duopolistic competition and analyse how the presence of a rival may impact investment and capacity sizing decisions as well as the choice of investment strategy. This will provide further insights on how policy measures may enhance or reduce the competitive advantage of power plants depending on their asymmetries related to cost and operational flexibility. For example, a carbon–price floor can influence the value of operational flexibility, thereby inducing investment in a RE facility by decreasing the value of operational flexibility embedded in a commodity–based facility (Chronopoulos *et al.,* 2014).

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APPENDIX

INVESTMENT WITHOUT POLICY UNCERTAINTY

The value of the option to invest in the presence ($\zeta = 1$) or absence ($\zeta = 0$) of a subsidy is described in (A–1).

$$F^{(j)}_{\zeta,0,0}(E) = \begin{cases} (1-\rho dt)\mathbb{E}_E\left[F^{(j)}_{\zeta,0,0}(E+dE)\right] & , E < \varepsilon^{(j)}_{\zeta,0,0} \\ \frac{Ek^{(j)}_{\zeta,0,0}(1+y\mathbb{1}_{\{\zeta=1\}})}{\rho-\mu} - I\left(k^{(j)}_{\zeta,0,0}\right) & , E \geq \varepsilon^{(j)}_{\zeta,0,0} \end{cases} \tag{A–1}$$

By expanding the first branch on the right–hand side of (A–1) using Itô's lemma, we obtain the differential equation for $F^{(j)}_{\zeta,0,0}(E)$, which, together with its solution for $E < \varepsilon^{(j)}_{\zeta,0,0}$, is indicated in (A–2).

$$\frac{1}{2}\sigma^2 E^2 F^{(j)''}_{\zeta,0,0}(E) + \mu P F^{(j)'}_{\zeta,0,0}(E) - \rho F^{(j)}_{\zeta,0,0}(E) = 0 \quad \Rightarrow \quad F^{(j)}_{\zeta,0,0}(E) = A^{(j)}_{\zeta,0,0}E^{\beta_1} + C^{(j)}_{\zeta,0,0}E^{\beta_2} \tag{A–2}$$

Note that the value of the project becomes very small as $E \to 0$. Since $\beta_2 < 0$, we have $E \to 0 \Rightarrow C^{(j)}_{\zeta,0,0}E^{\beta_2} \to \infty$. Consequently, $C^{(j)}_{\zeta,0,0} = 0$, and, thus, the expression for $F^{(j)}_{\zeta,0,0}(E)$ is indicated in (13). By applying value–matching and smooth–pasting conditions between the two branches of (13), we obtain the expression for the endogenous constant, $A^{(j)}_{\zeta,0,0}$, and the optimal investment threshold, $\varepsilon^{(j)}_{\zeta,0,0}$.

$$A^{(j)}_{\zeta,0,0} = \frac{1}{\varepsilon^{(j)\beta_1}_{\zeta,0,0}}\left[\frac{\varepsilon^{(j)}_{\zeta,0,0}k^{(j)}_{\zeta,0,0}\left(1+y\mathbb{1}_{\{\zeta=1\}}\right)}{\rho-\mu} - I\left(k^{(j)}_{\zeta,0,0}\right)\right] \tag{A–3}$$

$$\varepsilon^{(j)}_{\zeta,0,0}\left(k^{(j)}_{\zeta,0,0}\right) = \frac{I\left(k^{(j)}_{\zeta,0,0}\right)}{k^{(j)}_{\zeta,0,0}\left(1+y\mathbb{1}_{\{\zeta=1\}}\right)}\frac{\beta_1(\rho-\mu)}{\beta_1-1} \tag{A–4}$$

Finally, by inserting (A–4) into (11) we obtain the expression for the optimal capacity.

$$k^{(j)}_{\zeta,0,0} = \left[\frac{a_j}{b_j}\frac{1}{\gamma_j(\beta_1-1)-\beta_1}\right]^{\frac{1}{\gamma_j-1}}, \quad \gamma_j(\beta_1-1)-\beta_1 > 0 \tag{A–5}$$

Moreover, from (A–5) we see that the existence of an optimal solution to the investment problem requires that the cost function is strictly convex, i.e., $\gamma_j(\beta_1-1)-\beta_1 > 0 \Leftrightarrow \gamma_j > \frac{\beta_1}{\beta_1-1} > 1$. ■

Proposition 1 $\varepsilon^{(j)}_{1,0,0} < \varepsilon^{(j)}_{0,0,0}$ *and* $k^{(j)}_{1,0,0} = k^{(j)}_{0,0,0}$.
Proof: In the presence of a subsidy, the value of the option to invest is indicated in (13). The

value–matching and smooth–pasting conditions between the two branched of (13) are:

$$A^{(j)}_{1,0,0}\varepsilon^{(j)\beta_1}_{1,0,0} = \frac{\varepsilon^{(j)}_{1,0,0}k^{(j)}_{1,0,0}(1+y)}{\rho-\mu} - I\left(k^{(j)}_{1,0,0}\right) \quad \text{(A–6)}$$

$$\beta_1 A^{(j)}_{1,0,0}\varepsilon^{(j)\beta_1}_{1,0,0} = \frac{\varepsilon^{(j)}_{1,0,0}k^{(j)}_{1,0,0}(1+y)}{\rho-\mu} + \underbrace{\frac{\varepsilon^{(j)}_{1,0,0}k^{(j)\prime}_{1,0,0}(1+y)}{\rho-\mu} - a_j k^{(j)\prime}_{\zeta,m,n} - \gamma_j b_j k^{(j)\gamma_j-1}_{\zeta,m,n}\,k^{(j)\prime}_{\zeta,m,n}}_{=0} \quad \text{(A–7)}$$

The expression for the endogenous constant, $A^{(j)}_{1,0,0}$, and the optimal investment threshold, $\varepsilon^{(j)}_{1,0,0}$, is indicated in (A–8).

$$A^{(j)}_{1,0,0} = \frac{\varepsilon^{(j)^{1-\beta_1}}_{1,0,0}\,k^{(j)}_{1,0,0}(1+y)}{\beta_1(\rho-\mu)} \quad \text{and} \quad \varepsilon^{(j)}_{1,0,0}\left(k^{(j)}_{1,0,0}\right) = \frac{(\rho-\mu)I\left(k^{(j)}_{1,0,0}\right)}{k^{(j)}_{1,0,0}(1+y)}\frac{\beta_1}{\beta_1-1} \quad \text{(A–8)}$$

Notice that $\varepsilon^{(j)}_{1,0,0} = \varepsilon^{(j)}_{0,0,0}/(1+y)$ and that by inserting the expression for $\varepsilon^{(j)}_{1,0,0}$ into (11) we obtain:

$$k^{(j)}_{1,0,0}\left(\varepsilon^{(j)}_{1,0,0}\right) = \left[\frac{1}{b_j\gamma_j}\left(\frac{\varepsilon^{(j)}_{1,0,0}(1+y)}{\rho-\mu} - a_j\right)\right]^{\frac{1}{\gamma_j-1}} = k^{(j)}_{0,0,0}\left(\varepsilon^{(j)}_{0,0,0}\right) \quad \text{(A–9)}$$

■

INVESTMENT UNDER A RETRACTABLE SUBSIDY

First, we determine the expected value of the active project in the presence of a retractable subsidy. Notice that, within an infinitesimal time interval dt, either the subsidy will be retracted with probability λdt and the instantaneous revenue will decrease by $EK^{(j)}_{1,1,0}y$, or no policy intervention will take place with probability $1-\lambda dt$ and the reduction in the instantaneous revenue will be zero. Consequently, the expected reduction in the instantaneous revenue over a small time interval dt is $\lambda EK^{(j)}_{1,1,0}ydt$ and the expected present value of this reduction is $\frac{\lambda EK^{(j)}_{1,1,0}y}{\rho-\mu}$. By subtracting this from the expected revenues under a permanent subsidy, $\frac{EK^{(j)}_{1,1,0}(1+y)}{\rho-\mu}$, we obtain the expected value of the revenues under sudden subsidy retraction, i.e., $\frac{EK^{(j)}_{1,1,0}[1+(1-\lambda)y]}{\rho-\mu}$. ■

Proposition 2 *$\lambda \geq 0 \Rightarrow \varepsilon^{(j)}_{1,1,0} \leq \varepsilon^{(j)}_{0,0,0}$ and $k^{(j)}_{1,1,0} \leq k^{(j)}_{0,0,0}$, while, for low values of λ, $\varepsilon^{(j)}_{1,1,0} \leq \varepsilon^{(j)}_{1,0,0}$.*
Proof: The value of the option to invest in the presence of a retractable subsidy is indicated in (17). Notice that $\lambda = 0 \Rightarrow F^{(j)}_{1,1,0}(E) = F^{(j)}_{1,0,0}(E)$, and, therefore, $\varepsilon^{(j)}_{1,1,0} = \varepsilon^{(j)}_{1,0,0} < \varepsilon^{(j)}_{0,0,0}$ and $k^{(j)}_{1,1,0} = k^{(j)}_{1,0,0} = k^{(j)}_{0,0,0}$. From (15), we know that $\lambda \nearrow \Rightarrow \overline{k}^{(j)}_{1,1,0} \searrow$, which implies that a higher λ lowers the expected value of the project, and, in turn, both the amount of installed capacity and the optimal investment threshold. Hence, for small values of λ, $\varepsilon^{(j)}_{1,1,0} \leq \varepsilon^{(j)}_{1,0,0}$, whereas $\lambda \to 1 \Rightarrow \varepsilon^{(j)}_{1,1,0} \to \varepsilon^{(j)}_{0,0,0}$. ■

Proposition 3 $\frac{F^{(j)}_{1,0,0}(E)-F^{(j)}_{1,1,0}(E)}{F^{(j)}_{1,0,0}(E)} \in \left[0, \frac{A^{(j)}_{1,0,0}-A^{(j)}_{0,0,0}}{A^{(j)}_{1,0,0}}\right)$
Proof: In the presence of a retractable subsidy, the value of the option to invest is:

$$F^{(j)}_{1,1,0}(E) = A^{(j)}_{0,0,0}E^{\beta_1} + B^{(j)}_{1,1,0}E^{\delta_1} \quad , E < \varepsilon^{(j)}_{1,1,0} \quad \text{(A–10)}$$

If $\lambda = 0$, then the subsidy will never be retracted. This implies that $F^{(j)}_{1,1,0}(E) = F^{(j)}_{1,0,0}(E)$, and, in turn, that the relative loss in option value is zero. By contrast, as λ increases, the likelihood of subsidy retraction increases, and, as a result, $B^{(j)}_{1,1,0}E^{\delta_1} \to 0$, and, in turn, $F^{(j)}_{1,1,0}(E) \to A^{(j)}_{0,0,0}E^{\beta_1}$, which implies that the relative loss in option value is $\frac{A^{(j)}_{1,0,0}-A^{(j)}_{0,0,0}}{A^{(j)}_{1,0,0}}$. ■

INVESTMENT UNDER SUDDEN PROVISION OF A PERMANENT SUBSIDY

The extra instantaneous revenue from subsidy provision is $EK^{(j)}_{0,0,1}y$ and will be realised with probability λdt, whereas with probability $1 - \lambda dt$, no subsidy will be provided. Hence, the expected value of the subsidy is $\lambda EK^{(j)}_{0,0,1}ydt$ and its expected present value is $\frac{\lambda EK^{(j)}_{0,0,1}y}{\rho-\mu}$. Consequently, the expected value of the revenues under sudden provision of a permanent subsidy consist of the expected revenues without the subsidy, $\frac{EK^{(j)}_{0,0,1}}{\rho-\mu}$, and the extra revenues due to the subsidy, $\frac{\lambda EK^{(j)}_{0,0,1}y}{\rho-\mu}$, i.e., $\frac{EK^{(j)}_{0,0,1}(1+\lambda y)}{\rho-\mu}$. ■

Proposition 4 $\lambda \geq 0 \Rightarrow \varepsilon^{(j)}_{0,0,1} \geq \varepsilon^{(j)}_{1,0,0}$ *and* $k^{(j)}_{0,0,1} \geq k^{(j)}_{1,0,0}$, *while, for low values of* λ, $\varepsilon^{(j)}_{0,0,1} \geq \varepsilon^{(j)}_{0,0,0}$.
Proof: The value of the option to invest under sudden provision of a permanent subsidy is indicated in (21). Notice that $\lambda = 0 \Rightarrow F^{(j)}_{0,0,1}(E) = F^{(j)}_{0,0,0}(E)$, and, therefore, $\varepsilon^{(j)}_{0,0,1} = \varepsilon^{(j)}_{0,0,0} > \varepsilon^{(j)}_{1,0,0}$ and $k^{(j)}_{0,0,1} = k^{(j)}_{0,0,0} = k^{(j)}_{1,0,0}$. As λ increases, the likelihood of subsidy provision increases, thereby raising the expected value of the project, and, in turn, the incentive to install greater capacity. Indeed, $\lambda \nearrow \Rightarrow \overline{k}^{(j)}_{0,0,1} \nearrow$, and, therefore, at low values of λ we have $\varepsilon^{(j)}_{0,0,1} \geq \varepsilon^{(j)}_{0,0,0}$. By contrast, at high values of λ it is very likely that the subsidy will be provided, and, therefore, $\lambda \to 1 \Rightarrow \varepsilon^{(j)}_{0,0,1} \to \varepsilon^{(j)}_{1,0,0}$. ■

Proposition 5 $\frac{F^{(j)}_{1,0,0}(E)-F^{(j)}_{0,0,1}(E)}{F^{(j)}_{1,0,0}(E)} \in \left[-\frac{B^{(j)}_{1,0,0}}{A^{(j)}_{1,0,0}}, 0\right)$.
Proof: Under sudden provision of a permanent subsidy, the value of the option to invest is:

$$F^{(j)}_{0,0,1}(E) = A^{(j)}_{1,0,0}E^{\beta_1} + B^{(j)}_{0,0,1}E^{\delta_1} \quad , E < \varepsilon^{(j)}_{1,0,0} \tag{A–11}$$

If $\lambda = 0$, then the subsidy will never be provided. This implies that $F^{(j)}_{0,0,1}(E) = \left(A^{(j)}_{1,0,0} + B^{(j)}_{0,0,1}\right)E^{\beta_1}$, and, in turn, that the relative loss in option value is maximised. By contrast, as λ increases, the likelihood of subsidy provision increases, and, as a result, $B^{(j)}_{0,0,1}E^{\delta_1} \to 0$, and, in turn, $F^{(j)}_{0,0,1}(E) \to A^{(j)}_{1,0,0}E^{\beta_1}$, which implies that the relative loss in option value is zero. ■

INVESTMENT UNDER INFINITE PROVISIONS AND RETRACTIONS

The dynamics of the value of the option to invest under infinite provision and retractions for $\zeta = 0, 1$, are described in (A–12) and (A–13), respectively.

$$\begin{aligned} F^{(j)}_{0,\infty,\infty}(E) &= (1-\rho dt)\left[\lambda dt \mathbb{E}_E\left[F^{(j)}_{1,\infty,\infty}(E+dE)\right] + (1-\lambda dt)\mathbb{E}_E\left[F^{(j)}_{0,\infty,\infty}(E+dE)\right]\right] && \text{(A–12)} \\ F^{(j)}_{1,\infty,\infty}(E) &= (1-\rho dt)\left[\lambda dt \mathbb{E}_E\left[F^{(j)}_{0,\infty,\infty}(E+dE)\right] + (1-\lambda dt)\mathbb{E}_E\left[F^{(j)}_{1,\infty,\infty}(E+dE)\right]\right] && \text{(A–13)} \end{aligned}$$

By expanding the right–hand side of (A–12) and (A–13) using Itô's lemma, we have

$$\begin{aligned} \frac{1}{2}\sigma^2 E^2 F^{(j)''}_{0,\infty,\infty}(E) + \mu E F^{(j)'}_{0,\infty,\infty}(E) - (\lambda+\rho)F^{(j)}_{0,\infty,\infty}(E) + \lambda F^{(j)}_{1,\infty,\infty}(E) &= 0 && \text{(A–14)} \\ \frac{1}{2}\sigma^2 E^2 F^{(j)''}_{1,\infty,\infty}(E) + \mu E F^{(j)'}_{1,\infty,\infty}(E) - (\lambda+\rho)F^{(j)}_{1,\infty,\infty}(E) + \lambda F^{(j)}_{0,\infty,\infty}(E) &= 0 && \text{(A–15)} \end{aligned}$$

and by adding and subtracting (A–14) and (A–15) we obtain (A–16) and (A–17), respectively, where $F_a(E) = F^{(j)}_{1,\infty,\infty}(E) + F^{(j)}_{0,\infty,\infty}(E)$ and $F_b(E) = F^{(j)}_{1,\infty,\infty}(E) - F^{(j)}_{0,\infty,\infty}(E)$.

$$\frac{1}{2}\sigma^2 E^2 F_a''(P) + \mu E F_a'(P) - \rho F_a(E) = 0 \tag{A–16}$$

$$\frac{1}{2}\sigma^2 E^2 F_b''(P) + \mu E F_b'(P) - (\rho + 2\lambda) F_b(E) = 0 \tag{A–17}$$

The solution to (A–16) and (A–17) can be obtained by setting $F_a(E) = A_a^{(j)} E^{\beta_1}$ and $F_b(E) = A_b^{(j)} E^{\eta}$. Thus, we obtain:

$$F^{(j)}_{1,\infty,\infty}(E) = \frac{1}{2}\left[A_a^{(j)} E^{\beta_1} + A_b^{(j)} E^{\eta}\right] \tag{A–18}$$

$$F^{(j)}_{0,\infty,\infty}(E) = \frac{1}{2}\left[A_a^{(j)} E^{\beta_1} - A_b^{(j)} E^{\eta}\right] \tag{A–19}$$

where η is the positive root of the quadratic $\frac{1}{2}\sigma^2 x(x-1) + \mu x - (\rho + 2\lambda) = 0$. ■

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