Department of Economics

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Andrea Gurgone
Macro-Financial Division,
Central Bank of Ireland, Dublin

Giulia Iori[1]
Department of Economics,
City, University of London

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[1] Corresponding author: Giulia Iori, Department of Economics, City, University of London, Northampton Square, London EC1V 0HB, UK.
Email: G.Iori@city.ac.uk

A multi-agent methodology to assess the effectiveness of systemic risk adjusted capital requirements

Andrea Gurgone*

Central Bank of Ireland, Financial Stability Directorate
andrea.gurgone@centralbank.ie

Giulia Iori

Department of Economics, City, University of London

Abstract

We propose a multi-agent approach to compare the effectiveness of macro-prudential capital requirements, where banks are embedded in an artificial macroeconomy. Capital requirements are derived from alternative systemic-risk metrics that reflect both the vulnerability or impact of financial institutions. Our objective is to explore how systemic-risk measures could be translated in capital requirements and test them in a comprehensive framework. Based on our counterfactual scenarios, we find that macro-prudential capital requirements derived from vulnerability measures of systemic-risk can improve financial stability without jeopardizing output and credit supply. Moreover, macroprudential regulation applied to systemic important banks might be counterproductive for systemic groups of banks.

* *Corresponding author*

1. Introduction

The concept of *systemic risk* (SR) is relatively recent in economic and financial literature. The first appearance in scientific articles dates back to the early '90s, even if citations reveal that most of these contributions have been revived after 2008, when the term regained strength with the crisis. *ECB (2009, p. 134)* provide a general definition: *"it refers to the risk that financial instability becomes so widespread that it impairs the functioning of a financial system to the point where economic growth and welfare suffer materially."* The European Systemic Risk Board (ESRB) was established by the EU on 16 December 2009, based on the recommendation of the *"de Larosière report"* of bringing the European Union forward. The ESRB has a macroprudential mandate whose objective is to prevent and mitigate systemic risk in the EU. The recommendation of ESRB have shaped the conduct of macroprudential policies in EU countries and provided guidance for its implementation through a set of macroprudential policy tools *(ESRB, 2014a,b)*. Within this framework, the systemic risk buffer (SRB) is designed to prevent and mitigate structural systemic risks of a long-term, non-cyclical nature that are not covered by the Capital Requirements, including excessive leverage. The SRB is an additional capital requirement imposed on credit institutions, proportional to their total risk exposure, to cover unexpected losses and keep themselves solvent in a crisis. The introduction of a capital buffer applies to all systemically important institutions, both at the global (G-SIIs) and national (O-SIIs) levels. While for some instruments authorities have recommended to use prescriptive measures (such as the credit-to-gdp gap for the countercyclical capital buffer), considerable differences across countries exist regarding the level, range and calculation basis of the SRB. There is no maximum limit for the SRB, but authorisation from the European Commission is required for buffer rates higher than 3%. Caps on the SRB have been under the spotlight as often perceived as being too low to mitigate the risk some institutions pose to the financial system. Furthermore, SRB are hard to implement, *inter alia* because they need to be computed from a reliable measure of systemic risk: it is however unclear which metric performs better and under what circumstances. The task is more intricate given that systemic events are observed infrequently, as a banking crisis is observed on average every 35 years for OECD countries *(Danielsson et al., 2018)*.

In this article we propose a methodology to explore the effectiveness of capital surcharges implemented in the form of a systemic-risk buffer derived from different systemic risk measures. Banks are required to maintain a level of common equity tier 1 adequate to meet a systemic-risk weighted share of their assets. By assuming that banks adopt different capital rules within a multi-agent macro-economic model, we quantify the impact of such policies in a stress-test scenario-based analysis. Many techniques have been proposed so far to measure systemic risk, but there is no consensus among scholars on which is most appropriate. We consider two alternative classes, namely *market-based* and *network* approaches. Each one can measure systemic-risk in terms of both vulnerability or impact. Vulnerability focuses on the effect of a systemic event on the capital of a given bank, while impact captures the losses to produced by the distress of one, or few, institutions on the rest of the financial system.

We conduct counterfactual policy experiments in an agent-based model (ABM) of the economy based on *Gurgone et al. (2018).* The original model is expanded to allow banks to employ systemic-risk measures to determine their capital requirements.[1] In the first set of experiments we assume that capital requirements are set on the basis of vulnerability metrics, so that fragile banks are required to hold more equity capital than sound banks. However, this might not be satisfactory, as it does not operate on systemic impact of banks. Hence in the second set of experiments capital requirements depend on the impact of banks on the system, or the extent of externalities they produce in case of default.

We find that systemic-capital requirements based on vulnerability are able to stabilize the economy. Having them in place is preferable to a standard rule that determines regulatory equity as a fixed fraction of assets. On the other hand, systemic-capital requirements based on impact may lead to suboptimal outcomes and produce detrimental effects on financial stability. This is relevant when systemic-risk is not concentrated in few superspreaders, but is diluted in groups of banks with similar behavior and exposures to risk. Moreover, both market and network policies turn out to be procyclical. They also differ in some aspects: the former exhibit a regime switch during the first period of a crisis, while the latter can better capture the evolution of systemic risk but are highly correlated with the exposures to equity ratio prevailing in the financial system.

This paper is the first attempt to: (i) compare systemic risk measures recently proposed in the literature from both the perspectives of vulnerability of single institutions to system wide shocks and the individual impacts of institution distress on the financial system overall; (i) suggest how to incorporate heterogeneous systemic-risk metrics into banks' capital requirements; (iii) analyse the impact of the SRB macroprudential tool by means of simulated data generated by a multi-agents model, rather than empirically observed data that, given the rare occurrence of systemic crisis, are scant. Our simulated economy produces data on returns on equities of banks and at the same time includes a network structure of interlocked balance sheets, thus it allows for a double comparison.

The usage of an ABM allows to apply both network and market-based techniques to measure systemic risk. Financial networks between banks and firms and within the interbank sector arise endogenously as a consequence of interaction in ABMs. This feature can be employed to run network-based algorithms as DebtRank. This would not be feasible in an aggregate macroeconomic model. Moreover, working with a model rather than a dataset permits to design how to

[1] Note that we do not consider in our model the full range of capital buffers typically used by macroprudential authorities e.g. countercyclical capital buffers, liquidity buffer ratio, etc

make comparisons, and explore counterfactual scenarios that generate artificial data.

The paper is organized as follows: Section 2 presents the related literature. Section 3 describes the modelling framework, distress dynamics, systemic-risk measures and macro-prudential policies. Section 4 goes through the results of the simulations and the policy experiments. Conclusions are in Section 5.

2. Related literature

Our paper contributes to a vast, post-crisis, literature that focusses on empirical testing and comparison of systemic risk methodologies. The most common measures of systemic risk used in the literature are: Marginal Expected Shortfall (MES), defined as the expected daily percentage decrease in equity value of a financial institution when the aggregate stock market declines by at least 2 percent on a single day; Long Run Marginal Expected Shortfall (LRMES) defined as the expected equity loss, over a given time horizon, conditional on a sufficiently extreme phenomenon (such as an hypothetical 40% market index decline over a six months period); SRISK introduced by *Acharya et al. (2012)* and *Brownlees and Engle (2016)* which measures the expected capital shortfall, or the capital a firm is expected to have, conditional on a prolonged market decline (SRISK can be expressed in terms of LRMES). The sum of SRISK across all firms provides the total systemic risk of the system and can be thought of as the capital required by the system in the case of a bailout; CoVaR *Adrian and Brunnermeier (2016)* and *Chun et al. (2012)* which is defined as the risk (VaR) of the financial system conditional on an institution being in distress, i.e. at its own VaR level; Delta conditional value at risk ($\Delta CoVaR$) which measures the risk materializing at the system level if an institution is in distress relative to a situation where the same institution is at its median; Codependence risk (CoRisk) *(Giudici and Parisi, 2018)* the change in the survival probability of an institution when potential contagion deriving from all other institutions is included; Lower Tail Dependence (LTD) introduced by *Zhou (2010)* is estimated from the joint probability return distributions of individual financial institutions and the industry index, and aims to measure the probability of a simultaneous extreme, lower tail event in the financial sector as a whole and the equity values of individual financial institutions. A large part of the empirical literature has focussed on empirical testing and comparison of alternative systemic risk methodologies by means of econometric methods. *Benoit et al. (2013)* provide a theoretical and an empirical comparison of three market-based measures of systemic risk, namely MES, SRISK and $\Delta CoVaR$. They find that there is no measure able to fully account for multiple aspects of systemic risk, but SRISK is better than $\Delta CoVaR$ for describing both the too-big-to-fail and too-interconnected-to-fail dimensions. This may be possibly be because SRISK is a combination of market and balance sheet metrics and as such not purely a market-based measure given the inclusion of leverage. *Kleinow et al. (2017)* empirically compare four widespread measures of systemic risk, namely MES, Co-Risk, $\Delta CoVaR$ and LTD using data on US financial institutions. Their estimates point out that the four metrics are not consistent with each other over time, hence it is not possible to fully rely on a single measure. *Rodríguez-Moreno and Peña (2013)* consider six measures of systemic risk using data from stock, credit and derivative markets. They quantitatively evaluate such metrics through a “horse race", exploiting a sample composed of the biggest European and US banks. Their results favour systemic risk measures based on simple indicators obtained from credit derivatives and interbank rates, rather than more complex metrics whose performance is not as satisfactory. Similarly *Pankoke (2014)* opposes sophisticated to simple measures of systemic risk and concludes that simple measures have more explanatory power. Overall these papers find that different systemic risk measures focus on different characteristics of systemic risk and do not appear to capture its complex multidimensional nature, resulting in different rankings. *Nucera et al. (2016)* and *Giglio et al. (2016)* both apply principal component analysis to a range of systemic risk measures in the attempt to capture the multiple aspects of systemic risk. A useful discussion on the difficulty in finding a measure that can capture all aspects of systemic risk can be found in *Hansen (2013)*.

Other studies assume that the regulator is disposed to tolerate a systemic-wide risk level and aims to reach the most parsimonious feasible capitalization at the aggregate level. Such objective is formally translated into a constrained optimization problem, whose solution includes both the unique level of capital in the banking system and its distribution across banks. *Tarashev et al. (2010)* find that if capital surcharges are set in order to equalize individual contributions to systemic risk, then a lower level of aggregate capital is needed to reach the system-wide risk objective. *Webber and Willison (2011)* find that optimal systemic capital requirements increase in balance sheet size and in the value of interbank obligations. However, they are also found to be strongly pro-cyclical.

Another set of contributions presents network approaches to quantify systemic risk. *Battiston et al. (2016)* propose a network-based stress test building on the DebtRank algorithm. The framework is flexible enough to account for impact and vulnerability of banks, as well as to decompose the transmission of financial distress in various rounds of contagion and to estimate the distribution of losses. They perform a stress-test on a panel of European banks. The outcome indicates the importance of including contagion effects (or indirect effects) in future stress-tests of the financial system, so as not to underestimate systemic risk. *Alter et al. (2014)* study a reallocation mechanism of capital in a model of interbank contagion. They compare systemic risk mitigation approaches based on risk portfolio models with reallocation rules based on network centrality metrics and show that allocation rules based on centrality measures outperform credit risk measures. *Gauthier et al. (2012)* compare capital allocation rules derived from five different measures of systemic risk by means of a network-based model of interbank relations applied to a dataset including the six greatest banks of Canada. They also employ an iterative optimization process to solve the optimal allocation of capital surcharges that minimizes total risk, while keeping constant the total amount of capital to be kept aside. The adopted framework leads to a reduction of the probability of systemic crises of about 25%; however, results are sensitive to including derivatives and cross shareholdings in the data. *Poledna et al. (2017)* propose to introduce a tax on individual transactions that may lead to an increase in systemic risk. The amount of the tax is determined by the marginal contribution of each transaction to systemic risk, as quantified by the DebtRank methodology. This approach reduces the probability of a large-scale cascading event by re-shaping the topology of the interbank networks. While the tax deters banks from borrowing from systemically important institutions, it does not alter the efficiency of the financial network, measured by the overall volume of interbank loans. The scheme is implemented in a macro-financial agent-based model, and the authors show that capital surcharges for G-SIBs could reduce systemic risk, but they would have to be substantially larger than those specified in the current Basel III proposal in order to have a measurable impact.

Finally our paper provides a contribution to the literature that estimates macroprudential capital requirements using systemic risk measures. *Brownlees and Engle (2016)* and *Acharya et al. (2012)* have estimated the capital shortfall of an institution given a shock in the system. *Gauthier et al. (2012)* compare five approaches to assigning systemic capital requirements to individual Canadian banks based on each bank's contribution to systemic risk while *van Oordt (2018)* apply market-based measures to calculate the countercyclical capital buffer.

3. The model

3.1. Macroeconomic Model

The macroeconomy is based on an amended version of the agent-based-model (ABM) in *Gurgone et al. (2018)*. The economy is composed of several types of agents: households, firms, banks, a government, a central bank and a special agency. The (discrete) numbers of households, firms and banks are N^H, N^F , N^B respectively. Interactions take place in different markets: firms and households meet on markets for goods and for labour, while firms borrow from banks on the credit market and banks exchange liquidity on the interbank market. The CB buys government-issued bills on the bond market. The role of the government is to make transfer payments to the household sector. The governmental budget is balanced, namely the transfers are funded by taxes while the level of the public debt is maintained at a steady level. The CB generates liquidity by buying government bills and providing advances to those banks that require them; it furthermore holds banks' reserve deposits in its reserve account. Households work and and buy consumption goods by spending their disposable income.[2] It is made up of wage and asset incomes after taxes and transfers. In the labour market, households are represented by unions in their wage negotiations with firms, while on the capital market, they own firms and banks, receiving a share of profits as part of their asset income. Firms borrow from banks in order to pay their wage bills in advance, hire workers, produce and sell their output on the goods market. The banking sector provides credit to firms, subject to regulatory constraints. In each period every bank tries to anticipate its liquidity needs and accesses the interbank market as a lender or a borrower. If a bank is short of liquidity, it seeks an advance from the CB.

The special agency was not present in *Gurgone et al. (2018)*. It has been introduced as a convenient way to model the secondary market for loans. It acts as a liquidator when banks default or when banks exceed the regulatory constraint and thus must de-leverage. The assets in its portfolio are then put on the market and can be purchased by those banks that have a positive credit supply. Further details about the working of the special agency are described in the section below.

Banks		Firms	
Assets	Liabilities	Assets	Liabilities
L	Dep	Dep	L
I^l	I^b		
R	Adv		
	nw^B		nw^F

Table 1: Balance sheets of banks (left) and firms (right). Loans to firms (L), interbank lending (I^l), liquidity (R), deposits (Dep), interbank borrowing (I^b), advances from the central bank (Adv).

3.2. Distress dynamics

Banks and firms default if their equity turns negative. Distress propagates through defaults in the credit and interbank markets and banks' deposits. The transmission begins when firms cannot re-pay loans due to a negative outcome in the goods market. Shocks propagate from firms to banks, within the interbank

[2]The reference model *(Gurgone et al., 2018)* does not include households' borrowing since it is mainly focused on credit to firms and on the interbank market.

market and from banks to firms.[3] The process is illustrated in Fig. 1 and terminates only when there are no new losses. The balance sheets of firms and banks are illustrated in Table 1.

$$
\begin{tikzcd}
\text{firm-bank} \arrow[d] \\
\text{bank-bank} \arrow[d] \\
\text{bank-firm} \arrow[uu, bend left]
\end{tikzcd}
$$

Figure 1: Diagram of the distress transmission. The distress is transmitted through the credit market (firm-bank), the interbank market (bank-bank) and banks' deposits (bank-firms).

Liquidation of assets The contagion dynamic is enhanced by the forced liquidation of assets sold by defaulted banks in order to repay creditors. The role of liquidator is operated by a special agency that buys the assets of bank i at price p:

$$p_\tau = p_{\tau-1}\left(1 - \frac{\Delta q_{i,\tau}}{q_t}\frac{1}{\epsilon}\right) \tag{1}$$

where $\Delta q_{i,\tau}$ is the quantity of loans that bank i needs to liquidate,[4] ϵ is the asset price elasticity, q_t is the total quantity of loans in period t. Banks that need liquidity enter the market in a random order represented by the subscript τ; we assume that at the end of each period of the simulation, the initial asset price is set again at $p_0 = 1$. The assets purchased by the agency are then put on sale before the credit market opens (lending to firms). Banks with positive net worth and complying with regulatory leverage rate can buy them at their net present value.

Recovery rates The effective loss on a generic asset A_{ij} owed by j to i is $A_{ij}(0)(1-\varphi_{ij}(t))$, where φ is the recovery rate. Each of j's creditors can recover

[3] If the net worth of a bank is negative, it defaults on its liabilities including the deposits of firms and households. A deposit guarantee scheme is not implemented.

[4] Banks first determine their liquidity need, then compute the fair value of their portfolio loan by loan. Next they determine Δq taking into account eq. (1). Lastly, they choose which loans should be liquidated to reach their objective.

The loans for sale are evaluated at their fair market value by discounting cash flows:

$$L_{ij}^{fv} = \frac{L_{ij}(1+Sr^f)(1-\rho_j^f)}{\underline{r}^S}$$

where $L_{i,j}$ is the book value of the loan of bank i to firm j, S is the residual maturity, r^f is the interest rate on the loan, ρ^f is the default probability of firm j, and $\underline{r}$ is the risk-free rate.

$\varphi_{ij} = \frac{\mathcal{A}_j}{\mathcal{L}_j}$, *i.e.* the ratio of borrower's assets ($\mathcal{A}$) to liabilities ($\mathcal{L}$). However, the nominal value of illiquid assets is not immediately convertible in cash and must be first liquidated to compensate creditors. We denote the liquidation value of the assets of bank j with $\mathcal{A}_{j,t}^{liq}$, with $\mathcal{A}_{j,t}^{liq} \leq \mathcal{A}_{j,t}$. The actual recovery rate can be written as:

$$\varphi_{ij} \equiv \frac{\mathcal{A}_j^{liq}}{\mathcal{L}_j}$$

Furthermore, we assume that there is a pecking order of creditors, so that they are not equal from the viewpoint of bankruptcy law: the most guaranteed is the central bank, then depositors and finally banks with interbank loans. For instance, those creditors who claim interbank loans towards the defaulted bank j recover the part of j's assets left after the other creditors have been compensated. The recovery rate on an interbank loan, can be expressed as:

$$\varphi_{ij} = \max\left(0,\ \frac{\mathcal{A}_j^{liq} - A_j^{CB} - D_j}{\mathcal{L}_j - A_j^{CB} - D_j}\right) \tag{2}$$

where A^{CB} are central bank's loans to j and D are j's deposits. It is worth noticing that *loss given default* is $LGD \equiv 1-\varphi$, so that the net worth of creditor i updates as $nw_{i,t}^B = nw_{i,t-1}^B - LGD_{ij,t} I_{i,t}^l$.

3.3. Measuring systemic risk

Before defining *systemic risk adjusted capital requirements* (SCR) we clarify how we measure SR. We do it along two dimensions, that is vulnerability and impact. Vulnerability should be understood as the sensitivity of banks to a system-wide shock in terms of reduction in their equity. Conversely, impact measures the equity losses of the financial system originated from the distress of a chosen bank. Two distinct techniques are adopted to quantify vulnerability and impact, that is network and market-based approaches.

Network approach: DebtRank

DebtRank is a systemic-risk measure and an algorithm introduced by *Battiston et al. (2012).* It is conceived as a network measure inspired by feedback centrality with financial institutions representing nodes. Distress propagates recursively from one (or more) node to the other, potentially giving rise to more than one round of contagion. Despite DebtRank is a measure of impact in strict sense, the algorithm can provide both measures of vulnerability and impact (see Section 6.2 for details), that we denote respectively by DR^{vul} and DR^{imp}.

When accounting for vulnerability, we impose a common shock on the balance sheets of all banks and let that the algorithm computes how the equities were affected after the shock had died out. Individual vulnerabilities produced by the stress test are expressed in terms of the relative equity loss of each bank (h) at the last step of the algorithm ($\tau = T$) after we impose a shock on assets.

$$h_{i,T} \equiv \frac{nw_{i,T}^B - nw_{i,0}^B}{nw_{i,0}^B} \tag{3}$$

If impact is considered, we impose the default of one bank at a time and observe the effects on equities of all the other. The impact of each bank on the rest of the system is the overall loss in capital produced by the default of bank i. The value for each institution (g) are obtained by imposing its default at the beginning of the algorithm.

$$g_i = \sum_{j=1}^{N^b} h_{j,T}\ nw_{i,0}^B \tag{4}$$

Each measure is computed by repeating DebtRank 1000 times for vulnerability and 500 for impact.[5] In each run recovery rates are randomly distributed between 0 and 1. At the end, the value of SR indexes is determined by an expected-shortfall, that is by computing the average over the observations exceeding the $99th$ percentile. Finally, the items on the balance-sheets of firms and banks that are the input of the algorithm are entered as weighted averages over the last 30 periods. This avoids excessive time volatility of SR measures which would occur if DebtRank were computed with period-by-period inputs. Further details and the calibration procedure are detailed in Sections 6.2 and 6.1.

Market-based approach: LRMES and ΔCoVaR

Long Run Marginal Shortfall or $LRMES$ *(Brownlees and Engle, 2012)* describes the expected loss of equity conditional on a prolonged market decline. The last represent a systemic event which is defined as a drop of 40% of the market index over a period of six months. Considering this, we interpret $LRMES$ as a measure of vulnerability. Following *Acharya et al. (2012)*, $LRMES$ is computed as an approximated function of Marginal Expected Shortfall (MES)

$$LRMES_{i,t} = 1 - \exp\{-18MES_{i,t+h|t}^{Sys}\} \tag{5}$$

where $MES_{i,t+h|t}^{Sys} = E_t\left(r_{i,t+h|t}|r < \Omega\right)$ is the tail expectation of the firm equity returns conditional on a systemic event, that happens when i's equity returns r from $t-h$ to t are less than a threshold value Ω. Further details can be found in Section 6.3. Banks compute their $LRMES$ based on the last 200

[5] The number of repetitions is lower in DR-imp to contain its computational time: by imposing the default bank-by-bank we end up with 500xN^B runs of DebtRank for each period in the simulation.

observations starting from 50 periods prior the external shock. The required information are individual and market monthly returns. The first are computed as returns on equity (ROE) of bank i, that correspond to the relative change in i's net worth during each step of the simulation.[6] The same logic is applied to obtain market returns, which are weighted by the net worth of each bank. Being $LRMES$ a function of the individual and market cross-correlation, $LRMES$ accounts somehow the interconnectedness of banks in the financial system.

Another well-known measure of systemic risk is $\Delta CoVaR$, which quantifies the systemic distress conditional to the distress of a specific financial firm, namely it accounts for the impact of a bank on the financial system.

CoVaR is implicitly defined as the VaR of the financial system (sys) conditional on an event $C(r_{i,t})$ of institution i:

$$Pr\left[r_{sys,t} \leq CoVaR^{sys|C(r_i)} \mid C(r_{i,t})\right] = \alpha \tag{6}$$

where r represents ROE and the conditioning event $C(r_i)$ corresponds to a loss of i equal or above to its Var^i_α level.

$\Delta CoVaR$ is a statistical measure of tail-dependency between market returns and individual returns, which is able to capture co-movements of variables in the tails and account for both spillovers and common exposures. $\Delta CoVaR$ is the part of systemic risk that can be attributed to i: it measures the change in value at risk of the financial system at α level when the institution i shifts from its normal state (measured with losses equal to its median Var) to a distressed state (losses greater or equal to its Var).

$$\Delta CoVaR^{sys|i}_\alpha = CoVaR^{sys|r_i=VaR_{i,\alpha}}_\alpha - CoVaR^{sys|r_i=VaR_{i,0.5}}_\alpha \tag{7}$$

A flaw of $\Delta CoVaR$ is its (at best) contemporaneity with systemic risk: it fails to capture the build-up of risk over time and suffers of procyclicality. Furthermore, contemporaneous measures lead to the "*volatility paradox*" *(Brunnermeier and Sannikov, 2014)*, inducing banks to increase the leverage target when contemporaneous measured volatility is low. A workaround would be to substitute contemporaneous with a forward-looking version of $\Delta CoVaR$ *(Adrian and Brunnermeier, 2016, p.1725)*. The latter is obtained by projecting on the regressors of $\Delta CoVaR$ their estimated coefficients, where the independent variables include individual banks' characteristics and macro-state variables. Nevertheless our model lacks of the wide range of variables that can be employed in empirical works, as a results our measure of forward $\Delta CoVaR$ turns out to be strongly proportional to the VaR of banks, thus failing to capture the build up of systemic risk.

[6] We account the final value of the new worth before a bank is recapitalized, otherwise returns would be upwards biased by shareholders' capital.

3.4. Adjusted Capital Requirements

In the benchmark case, *i.e.* without employing any SR measures, banks comply with a standard regulatory capital requirements. The net worth must be greater or equal than a fraction $\frac{1}{\lambda} = 4.5\%$ of their risk-weighted-assets (RWA).[7]

$$nw^B_{i,t} \geq \frac{1}{\lambda} RWA_{i,t} \tag{8}$$

Differently, Systemic-risk adjusted Capital Requirements (SCR) are derived from measures of SR. These metrics are then mapped into a coefficient that can be interpreted as weighting the total assets by systemic-risk.[8] In other words, banks must hold a minimum net worth equal to a fraction of their assets given by the risk-weight coefficient ψ.

$$nw^B_{i,t} \geq \psi_{i,t} \mathcal{A}_{i,t} \tag{9}$$

where $\psi_{i,t} \equiv \frac{\frac{1}{\lambda}}{1-(1-\frac{1}{\lambda})sr_{i,t}}$ and sr is a generic SR index.[9]

If a $sr = 0$, then $\psi = \frac{1}{\lambda}$ and a bank must have a capital greater or equal than a standard regulatory threshold. When $sr = 1$, then $\psi = 1$ and capital requirements are as strict as possible, so that equity should equal assets, $nw^B = \mathcal{A}$.

Banks manage their balance sheet to meet capital requirements by setting their lending to firms and banks (which is limited upwards by (8) or (9)) and passively raising new capital by cumulation of profits.

Equation (9) can be obtained starting from the approach of *Acharya et al. (2012)* and setting the expected capital shortfall (CS) equal to zero.[10] CS is the capital needed to restore capital adequacy ratio to the value set by the regulator: it is the difference between minimum regulatory capital expressed as a fraction $\frac{1}{\lambda}$ of assets and the value of equity in case of a crisis. Following *Acharya et al. (2012)*, to obtain (10) we assume that debt and liquidity are unchanged in case

[7] We assign a weight $\omega_1 = 100\%$ to loans to firms and $\omega_2 = 30\%$ to interbank lending. Liquidity is assumed to be riskless, hence its weight is $\omega_3 = 0$. Risk weighted assets of bank i can be expressed as $RWA_{it} = \omega_1 L^F_{it} + \omega_2 I^l_{it} + \omega_3 R_{it} = L^F_{it} + \omega_2 I^l_{it}$.

[8] We do not define an objective in terms of macroprudential policy, but each bank is subject to capital requirements as a function of its measured systemic-risk.

[9] SR metrics (sr) are normalized in the interval $[0, 1]$.

[10] We consider the nominal value of equity rather than its market value to accommodate for the characteristics of the macroeconomic model. If the market values is considered, CS corresponds to $SRISK$.

of a systemic crisis, hence $E_t\left[\mathcal{L}_{i,t+\tau}|\ crisis_{t+\tau}\right] = \mathcal{L}_{i,t}$.

$$\begin{aligned} CS_{i,t+\tau|t} &= E_t\left[\frac{1}{\lambda}\mathcal{A}_{i,t+\tau} - nw^B_{i,t+\tau}|\ crisis_{t+\tau}\right] \\ &= E_t\left[\frac{1}{\lambda}\mathcal{L}_{i,t+\tau}|\ crisis_{t+\tau}\right] - E_t\left[\left(1-\frac{1}{\lambda}\right)nw^B_{i,t+\tau}|\ crisis_{t+\tau}\right] \\ &= \frac{1}{\lambda}\mathcal{L}_{i,t+\tau} - E_t\left[\left(1-\frac{1}{\lambda}\right)nw^B_{i,t+\tau}|\ crisis_{t+\tau}\right] \end{aligned} \tag{10}$$

In other words, (9) determines the minimum level of capital that a bank should hold in order that its expected capital shortfall conditional to a systemic event equals zero.

Vulnerability adjusted capital requirements

Adjusted capital requirement based on vulnerability are obtained under the assumption that the conditional value of net worth is determined by a vulnerability measure:

$$E_t\left[nw^B_{i,t+\tau}|\ crisis_{t+\tau}\right] = (1 - vul^j_{i,t})nw^B_{i,t} \tag{11}$$

where $j = \{LRMES,\ DR^{vul}\}$.

Capital requirements for bank i are then obtained in (12) by imposing $CS = 0$, so that it should always maintain a capital buffer great enough to avoid recapitalization during periods of distress.

$$nw^B_{i,t} \geq \frac{\frac{1}{\lambda}}{1-(1-\frac{1}{\lambda})LRMES_{i,t}}\mathcal{A}_{i,t} \tag{12}$$

Impact-adjusted capital requirements

We adopt a top-down approach to ensure consistency with the previous rule. Otherwise stated, capital requirements are determined to zero expected capital shortfall, which is computed top-down proportionally to the impact of each agent. Adjusted capital requirements are defined by deriving the equity values that each bank must satisfy to offset aggregate capital shortage. The idea is that banks contribute to the aggregate CS in proportion to their systemic impact. To this end, we rewrite CS in aggregate terms as the sum of the individual capital shortages. To keep internal consistency and to avoid aggregation issues we also assume that the individual capital shortages values are computed with the same procedure in Sect. 3.4 (respectively by $LRMES$ and DR^{vul}).

Each bank should contribute to expected capital shortage in proportion to its systemic importance. We follow the approach in *Gauthier et al. (2012)*, but rather than determining the equity capital that should be reallocated to bank i

from the total capitalization of the system, the left-hand side of (13) states the extra amount of CET1 capital as a fraction of the aggregate CS. This means that the additional capital required for each bank is:

$$nw_{i,t}^{+} = \frac{imp_{i,t}^{j}}{\sum_{i=1}^{N^b} imp_{i,t}^{j}} \sum_{i=1}^{N^b} CS_{i,t+\tau|t} \tag{13}$$

where $j = \{\Delta CoVaR,\ DR^{imp}\}$.

$$imp_{i,t} = \begin{cases} \frac{\Delta CoVaR_t^{sys|i}}{CoVaR^{sys|r_i=VaR_{i,\alpha}}} & if\ j = \Delta CoVaR \\ \frac{DR_{t,i}^{imp}}{\sum_i nw_{i,t}^B + \sum_k nw_{k,t}^F} & if\ j = DR^{imp} \end{cases}$$

Hence the target level of capital for bank i is given by the minimum regulatory level of capital plus the additional capital,

$$nw_{i,t}^{tag} = \frac{1}{\lambda}\mathcal{A}_{i,t} + nw_{i,t}^{+} \tag{14}$$

We can write adjusted capital requirement in the same form of (12).

$$nw_{i,t}^{tag} \geq \frac{\frac{1}{\lambda}}{1-(1-\frac{1}{\lambda})\zeta_i}\mathcal{A}_{i,t} \tag{15}$$

with $\zeta_i = \frac{nw_{i,t}^{+}}{(1-\frac{1}{\lambda})(\frac{1}{\lambda}\mathcal{A}_{i,t}+nw_{i,t}^{+})}$

4. Results

This section presents the results of simulations and policy experiments. We compare the benchmark scenario, where all banks are subject to the same fixed regulatory ratio of RWA, to those where SCR are derived from measures of vulnerability or impact of financial institutions, as described in Section 3.4. We run a set of 100 Monte Carlo simulations for each scenario under different seeds of the pseudo-random numbers generator.

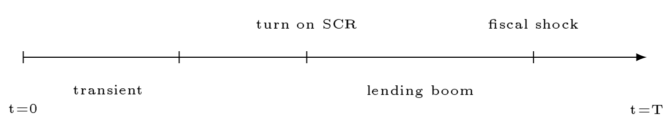

Figure 2: Timeline of the simulations.

The simulations are based on a variant of the macroeconomic model in *Gurgone et al. (2018)* in which the wage-price dynamics is dampened by setting the wage rate constant, so that business-cycle fluctuations are eliminated and the model converges to a quasi-steady-state after a transient period. Moreover, we supply to the lack of fluctuations of credit by simulating a lending boom, that is increasing the credit demand of firms in the periods before an external shock. It increases the exposures of banks and contributes to the build-up of the risk. Note that despite the elimination business cycles, the baseline dynamics produces a series of defaults and bankruptcies of firms and banks. These have a very lower extent before the shock than after. The presence of such financial distress helps systemic risk measures to better capture the characteristics of banks. We turn on systemic-capital requirements at the beginning of the lending boom, so that macroprudential regulation becomes binding. We finally impose a fiscal-shock of 10 periods that consists in a progressive reduction of transfers to the household sector. The purpose of the shock is to reduce the disposable income of households, that in turn affects consumption and firms' profits. Firms with negative equity then cannot repay their debts to the banking sector, thus the initial shock triggers a series of losses through the interlocked balance sheets of agents. At the time of the shock transfers are reduced by 20% and then by an additional 1% per period with respect to the period before the shock. Fig. 2 summarizes what happens during each simulation.

The behaviour of SR measures over time is shown in 4.1. Autocorrelation is analysed in 4.2, and the effects of SCR are presented in Section 4.3.

4.1. SR measures over time

In the next lines we conduct a qualitative analysis of the behaviour of SR metrics over the shock. For this purpose SCR are not active, rather the results show the evolution of risk measures to understand their differences.

Fig. 3 show a comprehensive representation of the time pattern of SR metrics. The evolution of impact and vulnerability presents a parallel trend within market and network-based measures. This reflects their construction: for market-based measures, conditional volatility of returns, which is estimated by a TGARCH model, is employed to construct $LRMES$ and $\Delta CoVaR$ (see Sect.6). Market-based measures exhibit a regime switch during the initial phase of the shock, persistently shifting from lower to higher values and exceeding the network-based counterparts in the immediate aftermath of the crisis. On the other side, network-based measures reflect the leverage dynamics of banks's balance sheets, that is the increase in credit demand prior to the shock and reduced equity after it. Their trend is approximated by the exposure to equity ratio of the economy.

Some observation can be inferred with the help of Fig. 3. First, all measures are pro-cyclical. SR metrics are not able to anticipate the forthcoming crisis before the shock, hence they cannot be used as early warning signals. This could partially depend on the exogenous nature of the shock imposed in our simplified framework, whereas alternatively a crisis might arise from the endogenous developments in the system. Moreover, measured systemic risk adjusts only after the beginning of the shock. Of course, this descends from the construction of our variables. In particular, the behaviour of network-based measures is sen-

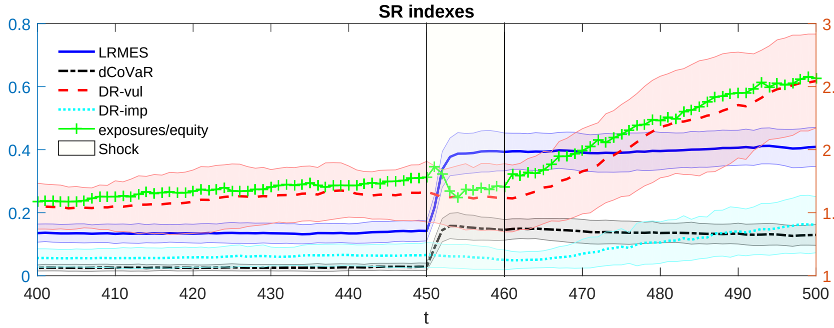

Figure 3: Time average and standard deviation of SR indexes.

sitive to the length of the time windows considered to input the past values of balance-sheet items, as there is a trade-off between shortening the windows and the volatility of network-based indexes. Second, network metrics have a smooth adjustment process, while market indicators show an “off-on" pattern. Therefore, the first should be preferred because it would be more desirable to conduct macroprudential policy smoothly than suddenly imposing restrictions on banks’ capital requirements, even more so if the change cannot be easily anticipated. Third, a stylized behavior of SR indexes can be characterized despite the time series are computed for the average. Vulnerability and impact of network-based measures are higher before the shock and lower after compared to market-based. This is clear looking at $t \in [450, 460]$ in Fig. 3, or at the individual breakdown represented in Fig. 4. The latter is also useful to point out the limits of our approach: capital requirements are determined separately for vulnerabily and impact. Instead, they could be considered jointly, because otherwise low-vulnerability but high-impact banks would be penalized by capital requirements built on impact and viceversa.

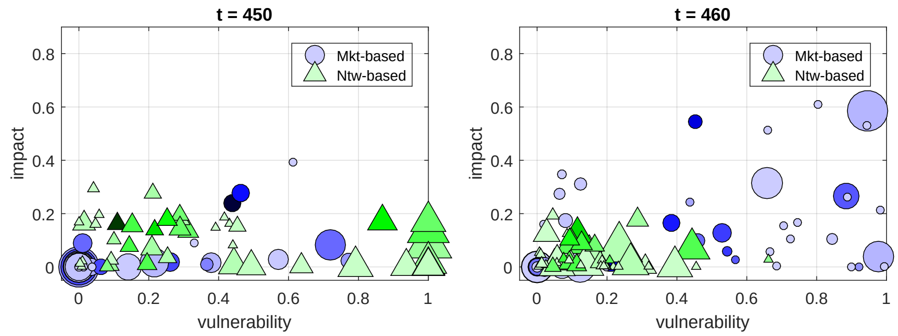

Figure 4: Market and network-based SR measures over the shock. Sizes represents assets, colour is total equity, with dark (light) corresponding to the highest (lowest) value.

4.2. Rank correlation

Banks behavior could be consistent with the objective of macroprudential regulation if such policies are based on stable values of the variables measuring SR. Therefore, a desirable property of SR measures is stability over time, that is the ranking of systemically important financial institutions has no high variability and identifies the same set of subjects in a given time span absent substantial changes in the financial environment. We study the auto-correlation of SR metrics to understand how stable they are.

We consider a measure of rank correlation, Kendall's tau (τ^k), which is a non-parametric measure of correlation between pairs of ranked variables with values between -1 and 1. If two variables are perfectly correlated $\tau^k = 1$, otherwise if there is no correlation at all $\tau^k = 0$.

$$\tau^k = \frac{C - D}{n(n-1)/2}$$

where C and D are the total number of concordant and discordant pairs and n is the sample size. Moreover when two variables are statistical independent, a z statistics built on τ^k tends to distribute as a standard normal, therefore it can be tested the null of no correlation versus the alternative of non-zero correlation. We compute τ^k between the rank of SR measures of each bank and its lagged values. Results are reported in Tab. 2. When market-based measures are considered, the ranking has a high and persistent autocorrelation. On the other hand network-based measures are autocorrelated to a lower extent. The difference could be explained in terms of construction, as market-based measures are obtained from conditional variances (or conditional VaR), which in turn are estimated through a TGARCH model, where conditional variances are assumed to follow an autoregressive process (see Section 6.3). Conversely, network-based measures do not assume any dependence on past values, rather they depend on the network structure and credit-debt relationships, so that the outcome of the DebtRank algorithm might change as a result of small variations in configuration of the network.

Kendall's tau

	Lags			
SR metric	+1	+5	+10	+15
LRMES	0.834	0.616	0.465	0.340
	(0.000)	(0.000)	(0.020)	(0.160)
$\Delta CoVaR$	0.807	0.618	0.457	0.334
	(0.000)	(0.000)	(0.020)	(0.140)
DR-vul	0.784	0.598	0.423	0.286
	(0.000)	(0.000)	(0.080)	(0.300)
DR-imp	0.875	0.681	0.493	0.343
	(0.000)	(0.000)	(0.040)	(0.180)

Table 2: Kendall's correlation coefficients. Reported statistics refers to the average of τ^k computed for each bank. Permutations p-values are reported in parenthesis, that is $p = 1 - \frac{\#successes}{\#experiments}$, where *successes* is the number of times when $p < 0.01$.

4.3. Policy Experiments

We present here the results of the policy experiments obtained under the four scenarios with active SCR and the benchmark case. Results for each policy are elaborated out of 100 Monte-Carlo runs. We cleaned the data to remove the outliers by trimming the observations above (below) the third (first) quartile plus (minus) 3 times the interquartile range.

We start by focusing on the macroeconomic performance under SRC in Fig.s 5 and 6. Within the vulnerability-based rules, market and network measures have approximately the same behavior for credit and output. They produce dynamics similar to the benchmark prior to the shock and yield a deterioration after. Most certainly the prociclicality of SR measures leads to a restriction in the credit supplied to the real economy after $t = 450$ and consequently to the lowered output. Looking at impact-based measures, they do worse than the benchmark even before the shock. In this case *DR-imp* produces a slighty better performance than $\Delta CoVaR$ on average, but in both cases with remarkable volatility. We have hyphotesized several reasons at the roots of the pattern for impact-based rules. The first is that the map from SR measures to SCR might non achieve an optimal distribution of capital: for instance, demanding to hold extra capital in proportion to impact only does not account for the actual default probabilities, so that financially sound banks might be

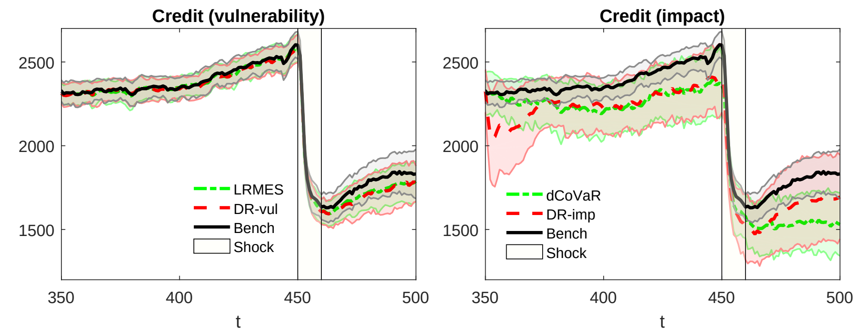

Figure 5: Time average and standard deviation of credit. (*Left*) Measures of vulnerability. (*Right*) Measures of impact.

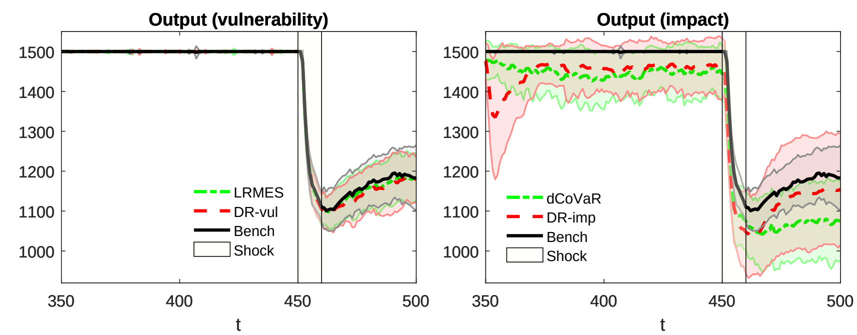

Figure 6: Time average and standard deviation of output. (*Left*) Measures of vulnerability. (*Right*) Measures of impact.

required to further increase their capital. This results in hindering the lending activity. Another reason is that the model dynamics might be defective of the emergence of high-impact systemic-important banks: impact-based capital requirements would work better if applied to few highly systemic banks than to many banks which are systemic to a lower degree. SCR would allow to isolate the first group without impairing too much lending. The second group of banks, which seems prevailing in our simulations, can be defined *"systemic as a herd" (Adrian and Brunnermeier, 2016)* because its members show moderate values of impact but present similar behaviors and exposures to risk. Thus, SCR can be conter-productive because they limit lending capacity of a part of the financial system. Following this line of thinking, SCR based on impact lead to an increase in the variance of the distribution of equity (Fig. 9), as they affect the profitability of some banks but allow others for high exposures. As a result, under impact-SCR the capitalization of the financial system as a whole is worse-off (Fig. 8). In light of this, the probability of contagion is greater under rules based on impact, as in Fig. 7. The greater financial fragility of the banking sector makes it more likely that at least 10% of all banks (or firms) are simultaneously in bankruptcy. Conversely, vulnerability based policies decrease the likelihood of contagion.

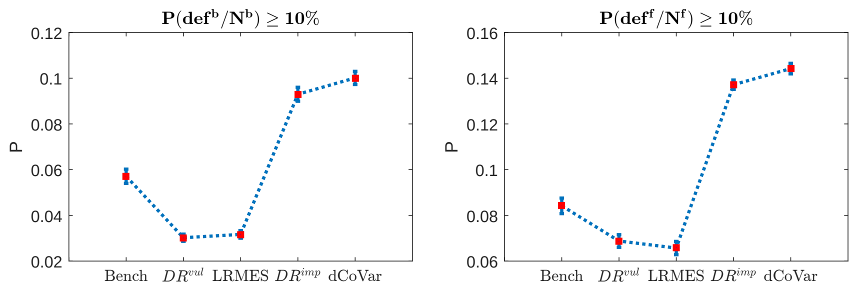

Figure 7: Probability that at least 10% of all banks *(left)* or firms *(right)* are in bankruptcy.

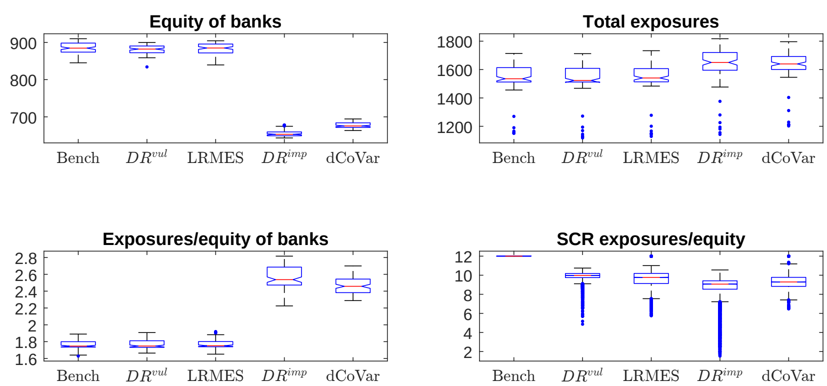

Figure 8: (*Top-left*) Aggregate equity of banks. (*Top-right*) Aggreate exposures of banks. (*Bottom-left*) Aggregate exposures/equity ratio of banks. (*Bottom-right*) Maximum aggregate exposures/equity ratio allowed under SCR.

Fig.s 10 and 9 illustrates the feasibility of SCR. Demanded capital requirements cannot be attained by a part of those banks with lower values of equity, which are represented above the 45° line in Fig. 9. This is more marked in the case of *DR-imp*. However the scatter plots do not provide an adequate representation of density, so we compare the feasibility of SCR by means of a CDF in Fig. 10. About 92% of observations have a $CR/equity < 1$ in the benchmark case, around 88% under $LRMES$ and $DR\text{-}vul$, 79% under $\Delta CoVar$, and 76% under $DR\text{-}imp$. So, it is less likely that banks comply with rules based on impact compared to rules based on vulnerability.

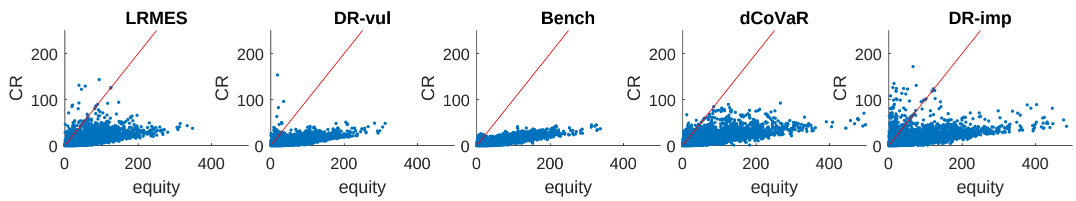

Figure 9: Capital Requirements (CR) versus equity of banks under different rules. The x and y-axis represent respectively the left and right-hand sides of eq. (9).

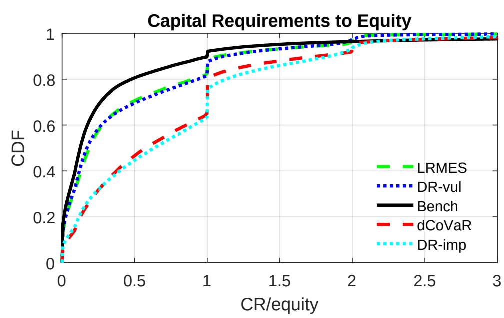

Figure 10: CDF of capital requirements to equity ratio.

We conclude that SCR built on vulnerability minimize the probability of a contagion and achieve a macroeconomic performance comparable to the benchmark case before the shock. Due to their prociclicality, all SCR bring about credit rationing and reduced output after the crisis. This calls for a relaxation of macroprudential rules after the shock. Despite capital requirements based on impact should reduce the damages caused by systemic banks, we do not observe an improvement with respect to the benchmark case. This could descend from the construction of impact-based SCR, or because the model dynamics rarely let arise *"too big-to-fail"* or *"too interconnected-to-fail"* banks, but rather financial institutions are *"systemic as a herd"*. Hence, imposing restrictions based on impact affects the lending ability of a number of banks and in turn their net-worth, reducing financial soundness and paving the way to instability.

5. Concluding remarks

We presented a methodology to compare a set of lender-targeted macro-prudential rules in which banks are subject to capital requirements built on systemic risk measures. Four metrics are considered: the first set is composed by two market-based measures (*LRMES* and $\Delta CoVaR$), while the second one includes network-based measures (*DR-vul* and *DR-imp*). Each set contains a metric for vulnerability, which states how much a financial institution is systemically vulnerable to an adverse shock, and one measure for impact, which accounts for the effects of distress of single banks on the financial system. Capital requirements are derived in Section 3 so that required capital is proportional to each bank's expected (or induced) capital shortage, which in turn depend on the SR measures. The construction and the calibration of SCR aims to ease the comparison within each set of market and network based measures.

In Section 4 we employ an agent-based macroeconomic model to analyse and compare qualitatively and quantitatively macroprudential rules. We find that all systemic-risk measures are prociclical to some degree. While market-based metrics display a regime switch after the exogenous shock, the network-based ones smoothly adjust with the exposures to equity ratio of the banking sector. This suggests that they lack of predictive power and thus cannot be used to build early warning systems. In particular the performance of market-based measures is sensitive to the past values of return-to-equity of financial institutions. If the time series of each bank is volatile enough, the SR measures can capture the dependency between individual and market changes and reflect the true systemic-risk. Otherwise, systemic-risk is underestimated. On the other hand, network-based measures exhibit a trade-off between pro-cyclicality and variance: the longer the time windows of past input balance-sheet data, the lower the variance. Using alternative calibrations, network-based measures could capture better the build-up of systemic-risk but the ranking of individual institutions would show lower autocorrelation. This translates in less reliable measures and a more difficult implementation of macroprudential policy.

Another key results is that SCR based on vulnerability are able to reduce contagion and to achieve a macroeconomic performance similar to the benchmark case before the aggregate shock. After it they should be relaxed to accommodate credit demand from firms. Despite procyclicality, the map from vulnerability to capital requirement provides an improvement with respect to the benchmark case. This can be interpreted as evidence that the individual measured values reflect the actual vulnerability of banks in case of a systemic event.

Differently, SCR based on impact cannot beat the benchmark. This result is specific to our model and have several interpretations: while SCR based on vulnerability are derived assuming that banks must be recapitalized depending on its expected losses conditional to a systemic-event, this is not true using a measure of impact. In this case capital requirements depends on the individual contribution to the expected aggregate shorfall, which is not directly connected to the equity of banks. Even though it is widely accepted that systemic banks can be identified and regulated conditional to the impact on the financial system, this logic does not work well in our framework. One explanation is that raising additional capital to comply with regulation is easier for banks with high equity than for small ones, being equal their impact. This puts small banks at a disadvantage by impairing their lending ability and creates a less equal equity distribution, and a lower aggregate capitalization of the banking system than in the other scenarios. Moreover, results suggest that macroprudential policy should treat differently *"too-big"* or *"too-interconnected to-fail"* and *"systemic as a herd"* institutions. In the first case the impact of one bank have critical effects on the financial system, hence it is rational to impose capital surcharges. In the latter case -as emerges in our model- banks are part of a homogeneous group in terms of indiviudal impacts, behavior, risk exposures. When hit by a common shock, the herd might produce systemic-effects. However, imposing capital requirements based on individual impacts may not be efficient at the macroeconomic level because it affects lending of a relevant part of the financial system, reduces profits and equity and makes the financial system more fragile.

This work can be extended in several ways. The regulation of systemic groups of banks, as opposite to SIFIs, can be studied in-depth; macroprudential rules could be built to combine indicators of both impact and vulnerability to derive SCR; the analysis of systemic risk can be repeated in a model capable to generate endogenous crisis without any exogenous shocks; finally, the model can be feed with real data for an empirical comparison.

References

Acharya, V., Engle, R., and Richardson, M. (2012). Capital shortfall: A new approach to ranking and regulating systemic risks. *The American Economic Review*, 102(3):59–64.

Adrian, T. and Brunnermeier, M. K. (2016). Covar. *The American Economic Review*, 106(7):1705–1741.

Alter, A., Craig, B., and Raupach, P. (2014). Centrality-based capital allocations and bailout funds. *International Journal of Central Banking, Forthcoming*.

Bardoscia, M., Battiston, S., Caccioli, F., and Caldarelli, G. (2015). Debtrank: A microscopic foundation for shock propagation. *PloS one*, 10(6):e0130406.

Bardoscia, M., Caccioli, F., Perotti, J. I., Vivaldo, G., and Caldarelli, G. (2016). Distress propagation in complex networks: the case of non-linear debtrank. *PloS one*, 11(10):e0163825.

Battiston, S., Caldarelli, G., D'Errico, M., and Gurciullo, S. (2016). Leveraging the network: a stress-test framework based on debtrank. *Statistics & Risk Modeling*, 33(3-4):117–138.

Battiston, S., Puliga, M., Kaushik, R., Tasca, P., and Caldarelli, G. (2012). Debtrank: Too central to fail? financial networks, the fed and systemic risk. *Scientific reports*, 2:541.

Benoit, S., Colletaz, G., Hurlin, C., and Pérignon, C. (2013). A theoretical and empirical comparison of systemic risk measures. *HEC Paris Research Paper No. FIN-2014-1030*.

Brownlees, C. and Engle, R. F. (2016). Srisk: A conditional capital shortfall measure of systemic risk. *The Review of Financial Studies*, 30(1):48–79.

Brownlees, C. T. and Engle, R. (2012). Volatility, correlation and tails for systemic risk measurement. *Available at SSRN*, 1611229.

Brunnermeier, M. K. and Sannikov, Y. (2014). A macroeconomic model with a financial sector. *The American Economic Review*, 104(2):379–421.

Chun, S. Y., Shapiro, A., and Uryasev, S. (2012). Conditional value-at-risk and average value-at-risk: Estimation and asymptotics. *Operations Research*, 60(4):739–756.

Danielsson, J., Valenzuela, M., and Zer, I. (2018). Learning from history: Volatility and financial crises. *The Review of Financial Studies*, 31(7):2774–2805.

ECB (2009). Financial stability review.

Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics*, 20(3):339–350.

ESRB (2014a). The esrb handbook on operationalising macro-prudential policy in the banking sector.

ESRB (2014b). Flagship report on macro-prudential policy in the banking sector.

Gauthier, C., Lehar, A., and Souissi, M. (2012). Macroprudential capital requirements and systemic risk. *Journal of Financial Intermediation*, 21(4):594–618.

Giglio, S., Kelly, B., and Pruitt, S. (2016). Systemic risk and the macroeconomy: An empirical evaluation. *Journal of Financial Economics*, 119(3):457–471.

Giudici, P. and Parisi, L. (2018). Corisk: Credit risk contagion with correlation network models. *Risks*, 6(3):95.

Gurgone, A., Iori, G., and Jafarey, S. (2018). The effects of interbank networks on efficiency and stability in a macroeconomic agent-based model. *Journal of Economic Dynamics and Control*.

Hansen, L. P. (2013). Challenges in identifying and measuring systemic risk. In *Risk topography: Systemic risk and macro modeling*, pages 15–30. University of Chicago Press.

Kleinow, J., Moreira, F., Strobl, S., and Vähämaa, S. (2017). Measuring systemic risk: A comparison of alternative market-based approaches. *Finance Research Letters*, 21:40–46.

Koenker, R. and Bassett Jr, G. (1978). Regression quantiles. *Econometrica: journal of the Econometric Society*, pages 33–50.

Nucera, F., Schwaab, B., Koopman, S. J., and Lucas, A. (2016). The information in systemic risk rankings. *Journal of Empirical Finance*, 38:461–475.

Pankoke, D. (2014). Sophisticated vs. Simple Systemic Risk Measures. Working Papers on Finance 1422, University of St. Gallen, School of Finance.

Poledna, S., Bochmann, O., and Thurner, S. (2017). Basel iii capital surcharges for g-sibs are far less effective in managing systemic risk in comparison to network-based, systemic risk-dependent financial transaction taxes. *Journal of Economic Dynamics and Control*, 77:230–246.

Rabemananjara, R. and Zakoian, J.-M. (1993). Threshold arch models and asymmetries in volatility. *Journal of Applied Econometrics*, 8(1):31–49.

Rodríguez-Moreno, M. and Peña, J. I. (2013). Systemic risk measures: The simpler the better? *Journal of Banking & Finance*, 37(6):1817–1831.

Tarashev, N. A., Borio, C. E., and Tsatsaronis, K. (2010). Attributing systemic risk to individual institutions. *BIS Working Papers*, No 308.

van Oordt, M. R. (2018). Calibrating the magnitude of the countercyclical capital buffer using market-based stress tests. Technical report, Bank of Canada Staff Working Paper.

Webber, L. and Willison, M. (2011). Systemic capital requirements. *Bank of England Working Papers*, No 436.

Zhou, C. C. (2010). Are banks too big to fail? measuring systemic importance of financial institutions. *International Journal of Central Banking*, 6(4):205–250.

6. Appendix

6.1. Calibration of DebtRank

In general, our approach is similar to that adopted in *Battiston et al. (2016)*, but we have adapted the algorithm to account for the structure of the underlying macro-model, as described in greater detail in Sect. 6.2. Given that the macro-environment includes firms, we first impose the shock on firms' assets to compute the systemic vulnerability index DR^{vul}. Next the induced distress transmits linearly to the assets of creditors (i.e. banks). This allows to capture the specific dynamics of the distress process.

Our calibration strategy aims to compare market and network-based measures on a common ground. To do so, we apply to DebtRank the definition of systemic crisis employed in the SRISK framework. SRISK is computed by LRMES, which represents the expected equity loss of a bank in case of a systemic event. This is represented by a decline of market returns of 40% over the next six months. We run 100 Monte-Carlo simulations of the macro-model, record the market ROE and the firms' losses to equity ratio. Then we compute the change in market ROE over the past 180 periods (approximately six months). Finally we construct a vector of the losses of firms to their equities in those periods where the ROE declined at least by -40%.

To compute vulnerabilities by DebtRank we randomly sample from the vector of the empirical distribution of losses/equity at each repetition of the algorithm. Finally we obtain DR^{vul} for each bank as an average of the realized values, after removing the $1st$ and the $99th$ percentiles.

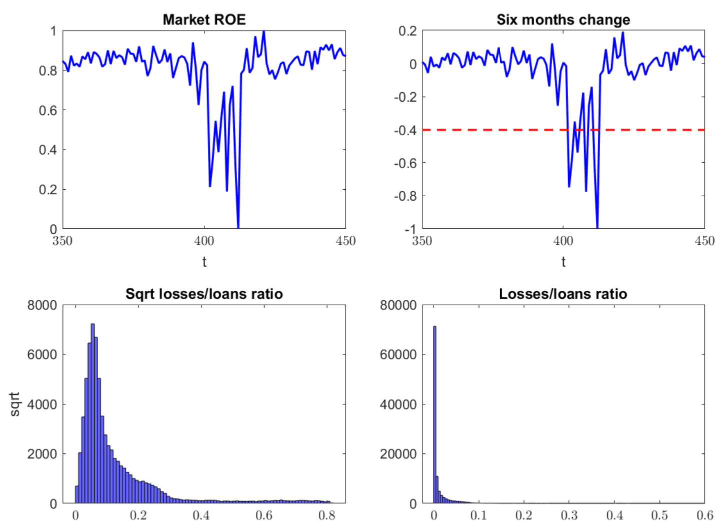

Figure 11: *(Top-left)* rescaled market ROE from a random Monte-Carlo run. *(Top-right)* Six month chance of market ROE. The red dashed line represents the threshold of -40%. *(Bottom-left)* Histogram of the square root of the losses/loans ratio of firms, where values equal to zero are ignored. *(Bottom-right)* Histogram of the losses/loans ratio of firms.

6.2. DebtRank

We employ a differential version of the DebtRank algorithm in order to provide a network measure of systemic risk. Differential DebtRank *(Bardoscia et al., 2015)* is a generalization of the original DebtRank *(Battiston et al., 2012)* which improves the latter by allowing agents to transmit distress more than once. Moreover our formulation has similarities with *Battiston et al. (2016)*, where it is assumed a sequential process of distress propagation. In our case we first impose an external shock on firms' assets, then we sequentially account for the propagation to the banking sector through insolvencies on loans, to the interbank network and to firms' deposits.

The relative equity loss for banks (h) and firms (f) is defined as the change in their net worth (respectively nw^B, and nw^F) from $\tau = 0$ to τ with respect to their initial net worth. In particular the initial relative equity loss of firms happens at $\tau = 1$ due to an external shock on deposits:

$$h_i(\tau) = \min\left[\frac{nw_i^B(0) - nw_i^B(\tau)}{nw_i^B(0)}\right]$$
$$f_j(\tau) = \min\left[\frac{nw_j^F(0) - nw_j^F(\tau)}{nw_j^F(0)}\right]$$

The dynamics of the relative equity loss in firms and banks sectors is described by the sequence:

- Shock on deposits in the firms sector:

$$f_j(1) = \min\left[1,\ \frac{D_j^F(0) - D_j^F(1)}{nw_j^F(0)}\right] = \min\left[1,\ \frac{loss_j(1)}{nw_j^F(0)}\right]$$

- Banks' losses on firms' loans:

$$h_i(\tau+1) = \min\left[1,\ h_i(\tau) + \sum_{j\in J} \Lambda_{ij}^{fb}(1 - \varphi_j^{loan})(p_j(\tau) - p_j(\tau-1))\right]$$

- Banks' losses on interbank loans:

$$h_i(\tau+1) = \min\left[1,\ h_i(\tau) + \sum_{k\in K} \Lambda_{ik}^{bb}(1 - \varphi_k^{ib})(p_k(\tau) - p_k(\tau-1))\right]$$

- Firms' losses on deposits:

$$f_j(\tau+1) = \min\left[1, f_j(\tau) + \Lambda_{jk}^{fb}(1 - \varphi_k^{dep})(p_k(\tau) - p_k(\tau-1))\right]$$

Where p_j is the default probability of debtor j and $\varphi^i,\ i = \{loan, ib, dep\}$ is the recovery rate on loans, interbank loans and deposits. Recovery rates on each kind of assets are randomly extracted from a vector of observations generated by the benchmark model.

For the sake of simplicity we can define it as linear in f_j (h_k for banks), so that $p_j(\tau) = h(\tau)$ [11]. Λ is the exposure matrix that represents credit/debt relationships in the firms-banks

[11] In a more realistic setting the default probability could be written as

$$p_j(\tau) = f_j(\tau)\exp(\alpha(h_j(\tau)) - 1)$$

where if $\alpha = 0$ it corresponds to the linear DebtRank, while if $\alpha \to \infty$ it is the Furfine algorithm *(Bardoscia et al., 2016)*. Moreover we can assume that deposits are not marked-to-market, but they respond to the Furfine algorithm, in other words the distress propagates network. It is written as a block matrix, where Λ^{bb} refers to the interbank market, Λ^{bf} refers to deposits, Λ^{fb} refers to firm loans and Λ^{ff} is a matrix of zeros.

$$\Lambda = \begin{bmatrix} \Lambda^{bb} & \Lambda^{bf} \\ \Lambda^{fb} & \Lambda^{ff} \end{bmatrix}$$

The exposure matrix Λ represents potential losses over equity related to each asset at the beginning of the cycle, where each element has the value of assets at the numerator and the denominator is the net worth of the related creditor. in our specification firms have no intra-sector links, hence $\Lambda^{ff} = 0$. In case there are $N^b = 2$ banks and $N^f = 3$ firms, the matrix Λ looks like:

$$\Lambda = \begin{bmatrix} 0 & \frac{Ib_{12}}{nw_2^B} & \frac{D_{13}}{nw_1^F} & \frac{D_{12}}{nw_2^F} & \frac{D_{15}}{nw_3^F} \\ \frac{Ib_{21}}{nw_1^B} & 0 & \frac{D_{23}}{nw_1^F} & \frac{D_{24}}{nw_2^F} & \frac{D_{25}}{nw_3^F} \\ \frac{L_{31}^f}{nw_1^B} & \frac{L_{32}^f}{nw_2^B} & 0 & 0 & 0 \\ \frac{L_{41}^f}{nw_1^B} & \frac{L_{42}^f}{nw_2^B} & 0 & 0 & 0 \\ \frac{L_{51}^f}{nw_1^B} & \frac{L_{52}^f}{nw_2^B} & 0 & 0 & 0 \end{bmatrix}$$

6.3. SRISK

SRISK *(Brownlees and Engle, 2012)* is a widespread measure of systemic risk based on the idea that the latter arises when the financial system as a whole is under-capitalized, leading to externalities for the real sector. To apply the measure to our model we follow the approach of *Brownlees and Engle (2012)*. The SRISK of a financial firm i is defined as the quantity of capital needed to re-capitalize a bank conditional to a systemic crisis

$$SRISK_{i,t} = \min\left[0,\ \frac{1}{\lambda}\mathcal{L}_i - \left(1 - \frac{1}{\lambda}\right) nw_{i,t}^B (1 - MES_{i,t+h|t}^{Sys})\right]$$

where $MES_{i,t+h|t}^{Sys} = E\left(r_{i,t+h|t}|r < \Omega\right)$ is the tail expectation of the firm equity returns conditional on a systemic event, that happens when i's equity returns r from $t-h$ to t are less than a threshold value Ω.

Acharya et al. (2012) propose to approximate MES^{Sys} with its *Long Run Marginal Expected Shortfall* (LRMES), defined as a

$$LRMES_{i,t} = 1 - \exp\{-18MES_{i,t}^{2\%}\}$$

LRMES represents the expected loss on equity value in case the market return drops by 40% over the next six months. Such approximation is obtained through extreme value theory, by means of the value of MES that would be if the daily market return drops by -2%.

The bivariate process driving firms' (r_i) and market (r_m) returns is

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only in case of default of the debtor. For deposits it might be reasonable to assume

$$p_j^D(\tau - 1) = \begin{cases} 1 & if\ h_k(\tau - 1) = 1 \\ 0 & otherwise \end{cases}$$

$$r_{m,t} = \sigma_{m,t}\epsilon_{m,t}$$
$$r_{i,t} = \sigma_{i,t}\rho_{i,t}\epsilon_{m,t} + \sigma_{i,t}\sqrt{1-\rho_{i,t}^2}\xi i,t$$
$$(\xi_{i,t}, \epsilon_{m,t}) \sim F$$

where $\sigma_{m,t}$ is the conditional standard deviation of market returns, $\sigma_{i,t}$ is the conditional standard deviation of firms' returns, $\rho_{i,t}$ is the conditional market/firm correlation and ϵ and ξ are i.i.d. shocks with unit variance and zero covariance ϵ and ξ are i.i.d. shocks with unit variance and zero covariance.

$MES^{2\%}$ is expressed setting $\Omega = -2\%$:

$$MES_{i,t-1}^{\Omega} = \sigma_{i,t}\rho_{i,t}E_{t-1}\left(\epsilon_{m,t}|\epsilon_{m,t} < \frac{\Omega}{\sigma_{m,t}}\right) + \sigma_{it}\sqrt{1-\rho_{i,t}^2}E_{t-1}\left(\xi_{i,t}|\epsilon_{m,t} < \frac{\Omega}{\sigma_{m,t}}\right)$$

Conditional variances $\sigma_{m,t}^2$, $\sigma_{i,t}^2$ are modelled with a TGARCH model from the GARCH family *(Rabemananjara and Zakoian, 1993)*. Such specification captures the tendency of volatility to increase more when there are bad news:

$$\sigma_{m,t}^2 = \omega_m + \alpha_m r_{m,t-1}^2 + \gamma_m r_{m,t-1}^2 I_{m,t-1}^- + \beta_m\sigma_{m,t-1}^2$$
$$\sigma_{i,t}^2 = \omega_i + \alpha_i r_{i,t-1}^2 + \gamma_i r_{i,t-1}^2 I_{i,t-1}^- + \beta_i\sigma_{i,t-1}^2$$

$I_{m,t}^- = 1$ if $r_{m,t} < 0$ and $I_{i,t}^- = 1$ when $r_{i,t} < 0$, 0 otherwise.

Conditional correlation ρ is estimated by means of a symmetric DCC model *(Engle, 2002)*. Moreover to obtain the MES it is necessary to estimate tail expectations. This is performed with a non-parametric kernel estimation method *(see Brownlees and Engle, 2012)*.

Open-source Matlab code is available thanks to *Sylvain Benoit*, and *Gilbert Colletaz*, *Christophe Hurlin*, who developed it in *Benoit et al. (2013)*.

6.4. **ΔCoVaR**

Following *Adrian and Brunnermeier (2016)* $\Delta CoVaR$ is estimated through a quantile regression *(Koenker and Bassett Jr, 1978)* on the α^{th} quantile, where r_{sys} and r_i are respectively market-wide returns on equity and bank i's returns. Quantile regression estimates the α^{th} percentile of the distribution of the dependent variable given the regressors, rather than the mean of the distribution of the dependent variable as in standard OLS regressions. This allows to compare how different quantiles of the regressand are affected by the regressors, hence it is suitable to analyse tail events. While *Adrian and Brunnermeier (2016)* employ an estimator based on an augmented regression, we further simplify the estimation of $\Delta CoVaR$ following the approach in *Benoit et al. (2013)*, which is consistent with the original formulation.

First we regress individual returns on market returns:

$$r_{sys,t} = \gamma_1 + \gamma_2 r_{i,t} + \varepsilon_{\alpha,t}^{sys|i}$$

The estimated coefficients (denoted by $\frown$) are employed to build CoVaR. The conditional VaR of bank i ($Var_{\alpha,t}^i$) is obtained from the quasi maximum likelihood estimates of conditional variance generated by the same TGARCH model described above *(see Benoit et al., 2013, p.38)*.

$$CoVar_{\alpha,t}^{sys|i} = \widehat{\gamma}_1 + \widehat{\gamma}_2 Var_{\alpha,t}^{i}$$

Finally $\Delta CoVar$ is obtained from the difference between the α^{th} and the median quantile of $CoVar$.

$$\Delta CoVar_{\alpha,t}^{sys|i} = CoVar_{\alpha,t}^{sys|i} - CoVar_{0.5,t}^{sys|i}$$
$$\Delta CoVar_{\alpha,t}^{sys|i} = \hat{\gamma}_2 \left(VaR_{\alpha,t}^{i} - VaR_{0.5,t}^{i} \right)$$