Optimal Subsidies for Development Supply Chains

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Problem definition: When donors subsidize products for sale to low-income families, they need to address who to subsidize in the supply chain and to what extent, and whether such supply chain structures as retail competition, substitutable products, and demand uncertainty matter.

Academic/practical relevance: By introducing and analyzing development supply chains in which transactions are commercial but subsidies are needed for affordability, we explore different supply chain structures, with product substitution and retail competition motivated by a field study in Haiti of subsidized solar-lantern supply chains.

Methodology: We incorporate product substitution, retail competition, and demand uncertainty in a three-echelon supply chain model with manufacturers, retailers and consumers. This model has transactions among the donor, manufacturers, retailers and consumers as a 4-stage Stackelberg game and we solve different variations of this game by using backward induction.

Results: The donor can subsidize the manufacturer, retailer or the customer, as long as the total subsidy per unit across these echelons is maintained at the optimal level. Having more product choice and having more retail-channel choice can increase the number of beneficiaries adopting the products; this increase becomes more pronounced as demand becomes more uncertain.

Managerial implications: Donors must coordinate across different programs along the entire supply chain. They should look for evidence in their collective experience for more beneficiaries when subsidizing competing retailers selling diverse substitutable products.

Key words: Subsidies, development supply chains, Haiti, socially responsible products, solar lanterns

1. Introduction

After a 7.0-magnitude earthquake struck Haiti on January 12, 2010, more than 200,000 people were killed, more than 300,000 injured, and 1.5 million people rendered homeless. Donors created subsidy
programs for selling essential products such as solar lanterns to the poor using what we call *development* supply chains, to distinguish from *humanitarian* or *commercial* supply chains. In a field study of solar lantern distribution in Haiti during 2014-2016, we found there was no agreement among donors or other stakeholders as to where subsidies should be provided in the supply chain and whether or not competing products or supply chain entities should be subsidized.

We assume the donor’s goal, subject to a budget, is to maximize the number of beneficiaries who can afford the product. The field observations then motivate the research question: *Where in the supply chain and how much should the donors subsidize, keeping in mind the supply chain structure, product choice, retail competition, and demand uncertainty?* Considering a three-echelon supply chain with manufacturers, retailers, and customers, we analyze different variants of a 4-stage Stackelberg game to seek answers by considering the following settings: (1) the *base setting* with one retailer selling a single product; (2) a *choice setting* with one retailer selling two substitutable products; and, (3) a *competition setting* with two competing manufacturers (retailers) producing (selling) two substitutable products separately. (Sodhi et al., 2017 have shown that the same structural results persist when a single manufacturer sells two substitutable products through two competing retailers separately.) These settings assume exogenous wholesales prices and a known market size, so we add two extensions (4) *endogenous wholesale price*, and (5) *market size uncertainty* to check the robustness of any results from these three settings.

Comparing the corresponding equilibrium outcomes associated with different supply chain settings from our stylized modeling, we find there is a unique optimal total subsidy level for each product in these settings. It is optimal to subsidize any of the manufacturers, the retailers, or the beneficiaries as long as the total subsidy per unit is maintained at the optimal level. Moreover, without increasing its budget, a donor can stimulate more beneficiaries to adopt the lanterns by: (1) encouraging more heterogeneous products with different valuations; (2) offering product-specific subsidies; and (3) encouraging more manufacturers and retailers to enter the market. These results become even more pronounced with growing market size uncertainty. In all settings, the retailers’ profits are positive so subsidy programs, if optimal, create economic value for the micro-retailers, thus meeting the development goals of the donor.
With these findings, we seek to contribute to the literature on the use of subsidies in supply chains with our focus on supply chain structure. We also seek to contribute to the humanitarian operations literature by presenting and analyzing subsidies for post-disaster recovery and development.

Section 2 provides background for this work based on our field study. Section 3 analyzes subsidies (per lantern) in three supply chain settings as the base model in which the wholesale price is exogenous. Sections 4 and 5 extend the base model for two extensions where: (1) the wholesale price is endogenously determined by the manufacturer; and (2) the market size is uncertain. Section 6 highlights our contribution to the literature and implications for practice as conclusion. Proofs for theorems appear in the (online) Appendix.

2. Background Information

The development supply chains for solar lanterns we studied in Haiti (2014-16) have entities at three echelons: (1) OEMs (e.g., d.light) sourcing the lanterns mainly from vendors in China, (2) importers importing the lanterns into Haiti and supplying to distributors who sell through retail chains or micro-retailers (possibly funded by micro-finance institutions), and (3) consumers or beneficiaries. Some importers distribute multiple brands of solar lanterns through multiple channels so there is product substitution and retail competition. Donors such as USAID offer unit subsidies indirectly to micro-entrepreneurs by funding micro-finance institutions (MFIs) who offer lower interest rates than the market on loans to micro-entrepreneurs for buying solar lanterns from distributors. Although in Haiti, donors typically provide lump-sum grants to OEMs, importers or distributors, donor practice in Bangladesh or India is to offer (unit) subsidies sold via cash vouchers.

This paper focuses on unit subsidies and we refer the reader to Sodhi et al. (2017) for an analysis of lump-sum grants. Three observations set the stage for modeling:

1. The wholesale price: The wholesale price of solar lanterns sold in Haiti can be pre-specified (i.e., exogenous to the setting) or negotiated (i.e., endogenous to the setting). For certain brands of solar lanterns imported directly from China, the price tends to be pre-specified for all countries. One senior Manager of Solar Product Company told us that: “The norm is ‘business is business’, here’s our price.”
However, the wholesale price of other brands is negotiable. Indeed, as one interviewee, the founder and CEO of a solar products company put it, “[Negotiation] is country by country – there are times that we’ve negotiated pretty big distribution agreements that have a minimum one container per month.”

2. **Number of brands**: Some importers (distributors or retailers) focus on a single brand while others prefer multiple brands of solar lanterns with different perceived quality levels and price points. One importer in Haiti told us that: “We always try to buy products approved by [World Bank-funded] Lighting Global <www.lightingglobal.org>. It is about forming a relationship with one manufacturer, testing in-country to see how well [the products] are accepted, and then negotiating prices.” However, the CEO of a different importer of solar lanterns told us that: “It’s best to have a range of products – we have 12 different products for different end customers at different price points and different feature sets.”

3. **Donors’ objectives and budget**: Donors use a planned budget to maximize the total adoption. For instance, World Bank had a solar lantern project with a budget of $8.62 million in Haiti. A senior manager at a donor organization said, “What counts is good quality products that are affordable, and that they make a positive change to the beneficiaries.” A common goal for donors is to maximize the number of beneficiaries adopting solar lanterns subject to the donor’s budget.

Despite their common goal of maximizing the total adoption of solar lanterns, we found that donors have divergent views on where to provide subsidies in the supply chain and what to subsidize. Even within the same donor organization such as USAID, different units offer financial supports at different echelons in the supply chain. This divergence motivated us to better understand where and how much should the donor subsidize in a (development) supply chain. To do so, we analyze the following three supply chain settings, which we observed in our field study:

**Supply chain setting 1: Selling a single product through a single retailer.** In Setting 1, a major distributor (Eneji Pwop) imports and sells Nokero’s solar lanterns directly to consumers through micro-entrepreneurs (Figure 1). Nokero is a US-based company who sources its production from China and sells its solar lanterns in over 120 countries. Eneji Pwop received indirect subsidies via zero- or
low-interest loans provided by Kiva.org (a non-profit crowdfunding-based impact investor). These micro-entrepreneurs used micro loans from Kiva to purchase products and resell to low-income end customers around Haiti.

Supply chain setting 2: Selling two substitutable products through a single retailer. Besides Nokero’s solar lanterns, Eneji Pwop sells solar lanterns from another OEM, Greenlight Planet, a “for-profit” social business (Figure 2). Greenlight Planet contracts with a manufacturer in China to produce solar lanterns and solar home systems for sale in 54 countries. Greenlight Planet has received investments from a number of impact investors, including Bamboo Finance, the Overseas Private Investment Corporation (OPIC), and Ashden. Eneji Pwop received indirect subsidies from Kiva.org as stated above. (Notice from Figure 2 that the Greenlight Planet solar lanterns are distributed by Total Haiti - a division of Total, the French oil-and-gas multinational.)

Supply chain setting 3: Selling two substitutable products (produced by two competing manufacturers) separately through two competing retailers. In Haiti, MicamaSoley and Sogexpress are major distributors of two competing brands: d.light and Ekotek solar lanterns (Figure 3). Solar lanterns from d.light have higher perceived quality because they are certified by Lighting Global. MicamaSoley sells d.light solar lanterns at wholesale prices to micro-entrepreneur women who accept Fonkoze (or other subsidized) micro-loans. Fonkoze is a large Washington DC-based non-profit MFI who uses donor grants to supplement revenues from loans, enabling it to offer micro-loans at low interest rates. Sogexpress distributes Ekotek’s solar lanterns using a similar setup.
3. Base Model: Exogenous wholesale price

We develop a stylized three-echelon model of the three supply chain settings in Section 2. In this model, we “aggregate” importers, distributors, and retailers as “retailer(s),” and treat “manufacturer’
Yu et al.: Optimal subsidies for development
Article submitted to Manufacturing & Service Operations Management; manuscript no. MSOM-17-321

and “beneficiaries” as two separate entities. In the base variant of this model, the wholesale price \( w \) is exogenous because the manufacturer has established a common wholesale price \( w \) across different countries, and will not change its wholesale price for the market in question due to concerns on parallel-imports arbitrage. We do not consider import taxes, logistics or distribution costs; however, our analysis can be easily extended to include these.

We assume in the base model that the potential size of the beneficiary market \( M \) is known just as it was known after the 2010 earthquake in Haiti or after an outbreak of malaria in Africa (Taylor and Xiao, 2014). We also assume this potential market size is independent of the selling price, although the consumer’s purchasing decision, and therefore the realized demand, would depend on the selling price.

We scale the size of the beneficiary market \( M \) to 1 for exposition in this section and Section 4 but in Section 5, we will consider uncertain market size with \( E(M) = 1 \).

We model the interactions among the donor, the retailer(s), and beneficiaries as a three-stage Stackelberg game:

1. The donor acts as the leader and, given a planned budget \( K \), selects the retailer subsidy \( s_r \) and the beneficiary subsidy \( s_b \) to maximize product adoption by consumers. (With the wholesale price \( w \) being exogenous in the base model, it is optimal to set the manufacturer subsidy \( s_m = 0 \) because such subsidy will not support the donor’s goal of increasing product adoption.)

2. The retailer acts as the follower who sets its retail price \( p \) to maximize its profit, given the subsidies \((s_r, s_b)\) and the wholesale price \( w \)

3. Beneficiaries with product valuation \( v \geq (p - s_b) \) will purchase the product, given the subsidy \( s_b \) and the retail price \( p \) under the assumption that the valuation of each beneficiary \( v \sim U[0, 1] \).

Sales quantities purchased by the beneficiaries drive the subsidies for beneficiaries, retailers, and manufacturers in this paper, so the retailers’ subsidies are not based on the order quantities ordered by the retailers and the manufacturers’ subsidies are not based on the production quantities produced by the manufacturers. We focus on these end-consumer sales-based subsidies for the following reasons: (a) the sales, order, and production quantities are the same for the case when the market demand is deterministic as presented in Sections 3 and 4; (b) sales-based subsidies enable us to obtain tractable results
especially for supply chain settings 2 and 3 with competing products, competing retailers, and competing manufacturers when the market demand is uncertain as presented in Section 5; (c) sales-based subsidies are commonly used in retailing to avoid retailers to “forward buy” (Dreze and Bell, 2003); and (d) sales-based subsidies can be distributed electronically to reduce processing costs and frauds in developing countries so that donors need to track the total sales only and reimburse all entities accordingly. As reported in Sodhi and Tang (2014), various NGOs distribute electronic vouchers to beneficiaries in the Philippines via mobile phones so that they can use it to purchase necessary items immediately after flood. To reduce processing cost of farm input subsidies, various governments have implemented e-vouchers in various African countries such as Zambia, Zimbabwe, etc. (See: https://www.africanfarming.com/slow-take-off-e-voucher-system/.) In contrast, Taylor and Xiao (2014) promote the use of order-based (or production-based) subsidies and we defer the implications of the different approaches to future research.

We use backward induction to solve different variants of a 3-stage Stackelberg game by considering three different supply chain structures (Figure 4) that captures the key features associated with those three supply chain settings from Haiti depicted in Figures 1, 2, and 3. Before solving the Stackelberg game for each setting (Figure 4, with \( s_{m1} = s_{m2} = 0 \)) for the case when the wholesale prices \((w_1, w_2)\) are exogenous, we formulate the donor’s problem involving two products (taking the single product case in setting 1 as a special case). Let \( D_1 \) and \( D_2 \) be the demand for product 1 and 2 so that the donor’s total subsidy cost is equal to \((s_{r1} + s_{b1})D_1 + (s_{r2} + s_{b2})D_2\). Given a budget \( K \), we shall consider the following formulation of the donor’s problem throughout this paper:

\[
\max_{(s_{r1}, s_{r2}; s_{b1}, s_{b2})} (D_1 + D_2) \quad (1)
\]

subject to \((s_{r1} + s_{b1})D_1 + (s_{r2} + s_{b2})D_2 \leq K. \quad (2)\)

3.1. Setting 1: Selling a single product through a single retailer

In setting 1 (Figure 4-1), the manufacturer sells one product through a single retailer. For any given retail price \( p \) and subsidy \( s_b \), demand \( D = 1 - (p - s_b) \) because the beneficiary valuation \( v \sim U[0,1] \).

Anticipating demand \( D \) and subsidy \( s_r \), the retailer solves:

\[
\pi_r(s_b, s_r) = \max_p \left\{ \left[ p - (w - s_r) \right] \cdot \left[ 1 - (p - s_b) \right] \right\} \quad (3)
\]
which yields optimal retail price $p^*(s_b, s_r)$ satisfying:

$$p^*(s_b, s_r) = \frac{1 + w}{2} + \frac{s_b - s_r}{2} \quad (4)$$

The first term $\frac{1 + w}{2}$ corresponds to the base retail price, and the second term represents the upward (downward) adjustment in retail price in response to the subsidy $(s_r, s_b)$. Using (4), we can retrieve the corresponding $D = \frac{1-w}{2} + \frac{s_b + s_r}{2}$, showing that the beneficiary demand increases with the subsidies $(s_b, s_r)$. (To ensure that demand is always greater than 0 even when there is no subsidy, we assume $w < 1$.) Similarly, we get $\pi_r(s_b, s_r) = \left(\frac{1-w+(s_b+s_r)}{4}\right)^2$. We assume the budget is reasonably low so that the donor cannot use $K$ to set subsidy $s \geq w$ to make the lantern essentially free for beneficiaries. To ensure $s < w$, we assume that $K < \frac{1}{2} \cdot w$. Denoting $s \equiv s_b + s_r$ as the total subsidy, we can express $s = 2D - 1 + w$.

The donor’s problem (1) can be reformulated as:

$$\max_D \quad D \quad \text{s.t.} \quad (2D - 1 + w) \cdot D \leq K \quad (5)$$
Proposition 1. When selling a single product through a single retailer, the budget constraint is binding, with the optimal demand \( D^* = \frac{1-w+\sqrt{(1-w)^2+8K}}{4} \) and the optimal total subsidy \( s^* \equiv s^*_b + s^*_r = \frac{-(1-w)+\sqrt{(1-w)^2+8K}}{2} \) is increasing in \( w \) and \( K \).

Proposition 1 suggests that (1) the budget constraint is binding, and (2) it does not matter whether the donor subsidize the retailer or the beneficiary as long as the total subsidy \( s^* \) is maintained at the optimal level.

Next, we examine whether these two results hold in settings 2 and 3 (Figure 4), and examine which supply-chain configuration results in higher product adoption.

3.2. Setting 2: Selling two substitutable products through a single retailer

We learned from our field study that many retailers sell substitutable solar lanterns of different quality at different price points. In supply chain setting 3 (Figure 3), d.light lantern is known to be of higher quality than Ekotek in terms of durability, ease of use, and maintenance. To capture this quality difference when selling two substitutable products (Figure 4-2), we assume that the two products have different beneficiary valuations, where \( v_1 \sim U[0,1] \), and \( v_2 = \delta \cdot v_1 \) with \( \delta > 1 \). In the event when the quality of one product does not dominate the other, it is possible that the valuations of these products are correlated instead of being proportional. We defer investigation of this general case to future research.

We assume that the product-specific wholesale price \( w_2 > w_1 \) to rule out product 2 dominating product 1 completely as a trivial case. Analogous to setting 1, we also assume \( w_1 < 1 \) and \( w_2 < \delta \) to ensure the demand of either product will not always be 0. In addition to the wholesale price the retail price \( p_i \) and the subsidies \( (s_{r_i}, s_{b_i}) \) are product-specific.

Given retail price \( p_i \) and subsidy \( s_{b_i} \), a beneficiary will buy product 1 if \( v_1 > (p_1 - s_{b_1}) \) and \( v_1 - (p_1 - s_{b_1}) > v_2 - (p_2 - s_{b_2}) \); buy product 2 if \( v_2 > (p_2 - s_{b_2}) \) and \( v_2 - (p_2 - s_{b_2}) > v_1 - (p_1 - s_{b_1}) \); and buy nothing, otherwise. By using \( v_1 \sim U[0,1] \), \( v_2 = \delta \cdot v_1 \), demand \( D_i \) satisfies:

\[
D_1 = \frac{(p_2 - s_{b_2}) - \delta(p_1 - s_{b_1})}{\delta - 1}, \quad D_2 = 1 - \frac{(p_2 - s_{b_2}) - (p_1 - s_{b_1})}{\delta - 1}
\]
Anticipating the demand $D_i$ in (6) along with any given subsidy $s_{r_i}$, the retailer solves: $\pi_i(s_{b_i}, s_{r_i}; i = 1, 2) = \max_{p_1, p_2} \sum_{i=1}^{2} \{ (p_i - (w_i - s_{r_i})) \cdot D_i \}$, which yields the optimal retail price $p_i^*(s_{b_i}, s_{r_i})$ satisfying:
\[
p_1^*(s_{b_1}, s_{r_1}) = \frac{1+w_1}{2} + \frac{s_{b_1} - s_{r_1}}{2}, \quad p_2^*(s_{b_2}, s_{r_2}) = \frac{\delta + w_2}{2} + \frac{s_{b_2} - s_{r_2}}{2},
\]
with the same properties as the optimal price $p^*$ in (4). Substituting $p_i^*$ into $D_1$ and $D_2$ and denoting $s_1 \equiv s_{b_1} + s_{r_1}$ and $s_2 \equiv s_{b_2} + s_{r_2}$, we get:
\[
D_1 = \frac{\delta(s_1 - w_1) - (s_2 - w_2)}{2(\delta - 1)}, \quad D_2 = \frac{(\delta - 1) - (s_1 - w_1) + (s_2 - w_2)}{2(\delta - 1)}.
\]
From (8), $s_1 = 2(D_1 + D_2) + (w_1 - 1)$ and $s_2 = 2(D_1 + \delta D_2) + (w_2 - \delta)$. By considering donor’s budget constraint $s_1 D_1 + s_2 D_2 \leq K$, the donor’s problem (1) can be reformulated as:
\[
\max_{D_1, D_2} D_1 + D_2 \quad \text{s.t.} \quad [2(D_1 + D_2) + (w_1 - 1)] \cdot D_1 + [2(D_1 + \delta D_2) + (w_2 - \delta)] \cdot D_2 \leq K.
\]
By noting that the objective function and the left hand side of are increasing in $D_1$ and $D_2$, we can conclude that the budget constraint is binding so that the optimal $D_i^*$ can be expressed as a function of $D_2$, where $D_i^* = \frac{1}{4} \cdot [1 - w_1 - 4D_2 + \sqrt{(4D_2 - 1 + w_1)^2 - 8[2(D_2 - \delta) + 2D_2^2 \delta - K]}].$ Through substitution, the donor’s problem (9) simplifies as:
\[
\max_{D_2 \geq 0} \frac{1}{4} \cdot [1 - w_1 + \sqrt{(4D_2 - 1 + w_1)^2 - 8[2(D_2 - \delta) + 2D_2^2 \delta - K]}].
\]

**Proposition 2.** When selling two substitutable products through a single retailer, the donor’s optimal subsidy $s_i^*$ and the corresponding optimal $(D_1^*, D_2^*)$ satisfy:

1. When $\delta - w_2 > 1 - w_1$, we have 
\[
(D_1^*, D_2^*) = \left( \frac{w_2 - \delta w_1}{4(\delta - 1)}, \frac{1}{4} \sqrt{\frac{(w_1 - w_2)^2}{4(\delta - 1)} + w_1^2} - 2w_2 + \delta + 8K, \frac{\delta - w_2}{4(\delta - 1)} \right) \text{ and } 
(s_1^*, s_2^*) = \left( \frac{1}{2} (w_1 - 1 + \sqrt{8K + w_1^2}) - 2w_2 + \frac{(w_1 - w_2)^2}{\delta - 1} + \delta \right), \quad \frac{1}{2} (w_2 - \delta + \sqrt{8K + w_1^2} - 2w_2 + \frac{(w_1 - w_2)^2}{\delta - 1} + \delta);
\]

2. When $\delta - w_2 \leq 1 - w_1$, we have
\[
(D_1^*, D_2^*) = \left( \frac{1}{4} (1 - w_1 + \sqrt{8K + (1 - w_1)^2}), 0 \right) \text{ and } 
(s_1^*, s_2^*) = \left( \frac{1}{2} (w_1 - 1 + \sqrt{8K + (1 - w_1)^2}) + \frac{1}{2} (1 - w_1 + \sqrt{8K + (1 - w_1)^2} + w_2 - \delta).
\]
Also, the total demand under setting 2: $D_1^* + D_2^* \geq \frac{1}{4} (1 - w_1 + \sqrt{8K + (1 - w_1)^2})$. 

Like Proposition 1, Proposition 2 implies that the optimal subsidies \((s^{*}_{1}, s^{*}_{2})\) in are not unique but the total subsidy per unit \(s^{*}_{i} = s^{*}_{h_{i}} + s^{*}_{r_{i}}\) for products \(i = 1, 2\) is uniquely defined. The valuations of products 1 and 2 are bounded above by 1 and \(\delta\) and that the retail prices are bounded below by \(w_{1}\) and \(w_{2}\) (without subsidies). We can interpret \((1 - w_{1})\) and \((\delta - w_{2})\) as the maximum consumer surplus for products 1 and 2; respectively. So, when \((\delta - w_{2}) \leq (1 - w_{1})\), the second statement reveals that there is no demand for product 2 in equilibrium. With \(D_{2}^{*} = 0\), the problem reduces to the single product case as in setting 1. Proposition 2 thus implies the two key results for setting 1. In the reverse case with \((\delta - w_{2}) > (1 - w_{1})\), the first statement implies that both products have positive demands in equilibrium and the total demand \(D_{1}^{*} + D_{2}^{*}\) is higher than the demand obtained in the single product case in setting 1. So, even though the products are substitutable, offering product choice to beneficiaries can increase the total demand in line with the donor’s goals.

### 3.3. Setting 3: Two competing manufacturers sell their product separately through two competing retailers

Consider the case when two competing manufacturers sell their substitutable products through separate channels by way of competing retailers (Figure 4-3). Because the wholesales price \((w_{1}, w_{2})\) are exogenous, there is no incentive to offer manufacturer subsidy so that \(s_{m1} = s_{m2} = 0\). Consequently, the competition between manufacturers does not play a role and we focus on the competition between retailers. We deal with manufacturer competition in Section 4 where the wholesale prices are endogenous. Consumer valuations of either product are taken to be \(v_{1} \sim U[0, 1]\) and \(v_{2} = \delta \cdot v_{1}\) with \(\delta > 1\). The beneficiary faces the same situation as in Setting 2, so the demand for each product is as in (6). The retailer \(i\)’s problem can now be formulated as \(\pi_{i}(s_{h_{i}}, s_{r_{i}}; i = 1, 2) = \max_{p_{i}} \{(p_{i} - (w_{1} - s_{r_{i}})) \cdot D_{i}\}\) so the “best response” functions are: \(p_{1}^{*}(p_{2}) = \frac{(p_{2} - s_{h_{2}}) + \delta w_{1}}{2\delta} + \frac{s_{h_{1}} - s_{r_{1}}}{2},\) and \(p_{2}^{*}(p_{1}) = \frac{(\delta - 1) + (p_{1} - s_{h_{1}}) + w_{2}}{2} + \frac{s_{h_{2}} - s_{r_{2}}}{2}\). Considering these two equations simultaneously, we obtain retailer \(i\)’s equilibrium retail price \(p_{i}^{*}\):

\[
p_{1}^{*} = \frac{(\delta - 1) - (s_{2} - w_{2}) - 2\delta(s_{1} - w_{1})}{4\delta - 1} + s_{h_{1}}, \quad p_{2}^{*} = \frac{2\delta(\delta - 1) - 2\delta(s_{2} - w_{2}) - \delta(s_{1} - w_{1})}{4\delta - 1} + s_{h_{2}}
\]

where \(s_{i} = s_{h_{i}} + s_{r_{i}}\). By substituting \(p_{i}^{*}\) into \(D_{1}\) and \(D_{2}\) given in (6), we get:

\[
D_{1} = \frac{\delta(\delta - 1) - \delta(s_{2} - w_{2}) + \delta(2\delta - 1)(s_{1} - w_{1})}{4\delta - 1)(\delta - 1)}, \quad D_{2} = \frac{2\delta(\delta - 1) + (2\delta - 1)(s_{2} - w_{2}) - \delta(s_{1} - w_{1})}{4\delta - 1)(\delta - 1)}
\]
By using \((D_1, D_2)\) given above, we can express subsidy \(s_1 = \frac{2\delta - 1}{\delta} D_1 + D_2 + (w_1 - 1)\) and \(s_2 = D_1 + (2\delta - 1)D_2 + (w_2 - \delta)\). As before, the donor’s problem can be simplified as:

\[
\max_{D_1, D_2} \quad D_1 + D_2 \\
\text{s.t.} \quad \frac{2\delta - 1}{\delta} D_1^2 + 2D_1 D_2 + (2\delta - 1)D_2^2 + (w_1 - 1)D_1 + (w_2 - \delta)D_2 \leq K.
\] (13)

**Proposition 3.** When two competing manufacturers sell two substitutable products separately through two competing retailers, the optimal demand \((D_1^*, D_2^*)\) satisfies \(\frac{2\delta - 1}{\delta} D_1^*^2 + 2D_1^* D_2^* + (2\delta - 1)D_2^*^2 + (w_1 - 1)D_1^* + (w_2 - \delta)D_2^* = K\) so that the corresponding optimal subsidy \(s_i^*\) satisfies the binding budget constraint. Moreover, the optimal subsidy \((s_b^*, s_r^*)\) are not unique, but the total subsidy per unit \(s_i^*\) for product \(i\) is uniquely determined.

For setting 3, Proposition 3 implies that the two key results obtained from the single product case in setting 1 continue to hold: it does not matter whether the donor subsidizes the retailer or the beneficiaries as long as the optimal subsidy is maintained at the optimal level.

By comparing settings 2 and setting 3, we obtain:

**Corollary 1.** Selling two substitutable products produced by different manufacturers through competing retailers instead of a single retailer can achieve a higher total demand \(D_1^* + D_2^*\).

To summarize, our base model offers the following insights: (1) as long as the total subsidy per unit is set at the optimal level, the donor can offer any portion of this per unit subsidy to the retailers and/or to the beneficiaries, (2) the donor can increase the total realized demand of the products by introducing substitutable products; and (3) the donor should support retailer-competition to further boost total demand. These insights rely on the assumptions that the wholesale prices are exogenous and that the market size is fixed \((M = 1)\). To check the robustness of these results, we relax these two assumptions in Sections 4 and 5 respectively.

### 4. Extension 1: Endogenous wholesale price

We now consider the case when the manufacturer’s wholesale price is *endogenous*. We first determine the manufacturer’s optimal wholesale price for any subsidy \(s_m\), and then we solve the donor’s problem.
4.1. Setting 1: Selling one product through a single retailer

Recall from Section 3.1 that, for any given subsidy \((s_m, s_r, s_b)\), the corresponding demand \(D = \frac{1-w}{2} + \frac{s_b + s_r}{2}\). Hence, for any wholesale price \(w\) and unit cost \(c\), and subsidy \(s_m\), the manufacturer solves:

\[
\pi_m = \max_w \left( w + s_m - c \right) \cdot \left( \frac{1-w}{2} + \frac{s_b + s_r}{2} \right),
\]

and obtains the optimal wholesale price \(w^* = \frac{1+c}{2} + \frac{s_b + s_r}{2}\) from which we can retrieve the corresponding optimal price \(p^* = \frac{1}{2}(3 + 3s_b - s_m - s_r)\), the optimal demand \(D^* = \frac{1}{4}(1 - c + s_b + s_r + s_m)\), the optimal retailer’s profit \(\pi_r = \frac{1}{16}(1 - c + s_b + s_r + s_m)^2\), and the optimal manufacturer’s \(\pi_m = \frac{1}{8}(1 - c + s_b + s_r + s_m)^2\). Also, we can further compute the corresponding consumer welfare \(W = \int_{p^* - s_b}^{1} [v - (p^* - s_b)] \, dv = \frac{1}{32}(1 - c + s_b + s_r + s_m)^2\). It is interesting to note that all optimal quantities and the consumer welfare depend only on the total subsidy level \(s' \equiv s_b + s_r + s_m\), not by its split among the manufacturer, the retailer, and beneficiaries.

Next, given that the optimal demand \(D^* = \frac{1}{4}(1 - c + s_b + s_r + s_m) = \frac{1-c}{4} + \frac{s'}{4}\), the donor’s problem can be formulated as:

\[
\max \quad D \equiv \frac{1-c}{4} + \frac{s'}{4} \quad \text{s.t.} \quad s' \cdot \left( \frac{1-c}{4} + \frac{s'}{4} \right) \leq K.
\]

**Proposition 4.** When selling one product through a single retailer and when the wholesale price is endogenously determined by the manufacturer, the budget constraint is binding, the optimal total subsidy \(s'^* = \frac{(1-c) + \sqrt{(1-c)^2 + 16K}}{2}\) and the optimal demand \(D^* = \frac{(1-c) + \sqrt{(1-c)^2 + 16K}}{8}\). Moreover, the corresponding consumer welfare \(W^* = \frac{(1-c) + \sqrt{(1-c)^2 + 16K}}{128}\), retailer’s profit \(\pi^*_r = \frac{(1-c) + \sqrt{(1-c)^2 + 16K}}{64}\), and manufacturer’s \(\pi^*_m = \frac{(1-c) + \sqrt{(1-c)^2 + 16K}}{32}\), and \(\pi^*_m = 2\pi^*_r = 4W^*\).

Proposition 4 is analogous to Proposition 1: the total subsidy per unit \(s'^*\) is uniquely determined but the optimal subsidies \((s^*_b, s^*_r, s^*_m)\) are not. Moreover, “double marginalization” persists: the manufacturer’s profit is twice that of the retailer, and four times of the consumer welfare (i.e., \(\pi^*_m = 2\pi^*_r = 4W^*\)).

4.2. Setting 2: Selling two products through a single retailer

Recall from Setting 2 in Section 3.2 that the retail price \(p_i^*\) is given in (7) and the corresponding demand function \(D_i\) is given in (8). Hence, for any given subsidy \((s_{m1}, s_{m2})\), the manufacturer solves:

\[
\max \quad \sum_{i=1,2} (w_i + s_{m_i} - c_i) \cdot D_i,
\]
and obtain the optimal wholesale price \( w_1^* \) that satisfies:

\[
{w_1^*} = \frac{1}{2}(1 + c_1 + s_1 - s_m), \quad {w_2^*} = \frac{1}{2}(c_2 + s_2 - s_m + \delta)
\]  

Substituting \( w_1^* \) and \( w_2^* \) into (8) and denoting the total subsidy \( s_1' \equiv s_1 + s_m \) and \( s_2' \equiv s_2 + s_m \), we get:

\[
D_1 = \frac{(c_2 - s_2') + (s_1' - c_1)\delta}{4(\delta - 1)}, \quad D_2 = \frac{\delta - 1 + (c_1 - s_1') - (c_2 - s_2')}{4(\delta - 1)}
\]  

Also, we can get \( \pi_m = \frac{1}{2}(1 - c_1 + s_1') \cdot D_1 + \frac{1}{2}(\delta - c_2 + s_2') \cdot D_2 \) and \( \pi_r = \frac{1}{4}(1 - c_1 + s_1') \cdot D_1 + \frac{1}{4}(\delta - c_2 + s_2') \cdot D_2 \) via substitution, from which we can easily find that \( \pi_m = 2 \cdot \pi_r \). Moreover, we can compute the consumer welfare

\[
W = \int_{p_1 - s_1}^{(p_2 - s_2) - (p_1 - s_1)} [v_1 - (p_1 - s_1)] dv_1 + \int_{(p_2 - s_2) - (p_1 - s_1)}^{p_1} [\delta \cdot v_1 - (p_2 - s_2)] dv_1 = \frac{\pi_m}{2}
\]

From (18), we get: \( s_1' = -1 + c_1 + 4(D_1 + D_2) \), \( s_2' = -\delta + c_2 + 4(D_1 + \delta D_2) \) so that we can express the budget constraint \( s_1'D_1 + s_2'D_2 \leq K \) in terms of \( D_i \). Hence, the donor’s problem becomes:

\[
\max_{D_1, D_2} D_1 + D_2 \quad \text{s.t.} \quad [-1 + c_1 + 4(D_1 + D_2)] \cdot D_1 + [-\delta + c_2 + 4(D_1 + \delta D_2)] \cdot D_2 \leq K
\]

Using the same approach as in Section 3.2, the donor’s problem can be simplified as:

\[
\max_{D_2 \geq 0} \quad \frac{1}{8} [1 - c_1 + \sqrt{(-1 + c_1 + 8D_2)^2 - 16(4\delta D_2^2 - K + D_2(c_2 - \delta))}]
\]

**Proposition 5.** When selling two substitutable products through a single retailer and when the wholesale price is endogenous, we get:

1. When \( \delta - c_2 > 1 - c_1 \), \( D_1^* = \frac{c_2 - c_1\delta}{4(\delta - 1)} + \frac{1}{8} \sqrt{c_2^2 - 2c_2 + 16K + \frac{(c_3 - c_2)^2}{4(\delta - 1)}} + \delta \), \( D_2^* = \frac{\delta - c_2 - (1 - c_1)}{8(\delta - 1)} \); and

2. When \( \delta - c_2 \leq 1 - c_1 \), \( D_1^* = \frac{1}{8}[1 - c_1 + \sqrt{(1 - c_1)^2 + 16K}] \), \( D_2^* = 0 \).

Also, the optimal total subsidy level \((s_1^*, s_2^*) = (-1 + c_1 + 4(D_1^* + D_2^*), -\delta + c_2 + 4(D_1^* + \delta D_2^*)) \) and the corresponding manufacturer’s profit \( \pi_m^* = \frac{1}{2}(1 - c_1 + s_1^*) \cdot D_1^* + \frac{1}{2}(\delta - c_2 + s_2^*) \cdot D_2^* \) and retailer’s profit \( \pi_r^* = \frac{\pi_m^*}{2} \) and consumer welfare \( W^* = \frac{\pi_m^*}{2} \).

Analogous to Proposition 2, Proposition 5 suggests that the structural results remain the same even when the wholesale price is endogenous; i.e., (a) the optimal subsidies \((s_{b1}^*, s_{r1}^*, s_{m1}^*) \) are not unique but the total subsidy per unit \( s_{i1}^* \) for product \( i \) is uniquely defined; and (b) the total demand under setting
2 (i.e., $D_1^* + D_2^*$) will always be greater than the total demand under setting 1 with one product (i.e., $D = \frac{1}{4}(1 - c_1 + \sqrt{8K + (1 - c_1)^2})$). Also, Proposition 5 is analogous to Proposition 4 in that the optimal profits of different parties and the consumer welfare depend only on the total subsidy per unit $s^*_i$, not on how its split across the supply chain.

4.3. Setting 3: Two competing manufacturers sell substitutable product separately through competing retailers

Noting that this setting is akin to Setting 3 as presented in Section 3.3, the retailer’s pricing problem is the same as in Section 3.3. So the retail price $p_i^*$ is given by (11) and the corresponding demand function $D_i$ is given by (12), where the total per unit subsidy is $s_1 \equiv s_{b1} + s_{r1}$ and $s_2 \equiv s_{b2} + s_{r2}$. To incorporate manufacturer competition when the wholesale price $(w_1, w_2)$ is endogenous, each manufacturer $i, i = 1, 2$, determines its best response by solving: $\max_w \{(w_i + s_{m_i} - c_i) \cdot D_i\}$ for any given wholesale price $w_j$ for $j \neq i$. By considering the best response of both manufacturers simultaneously, we obtain the optimal wholesale price $w_1^*$ and $w_2^*$ in equilibrium. The corresponding demands satisfy:

$$
D_1 = \left[\delta(2\delta - 1)(2 - 2c_1 - c_2 + 2s'_1 + s'_2 - \delta(8 - 9c_1 - 2c_2 + 9s'_1 + 2s'_2) + 2\delta^2(3 - 4c_1 + 4s'_1))/[(\delta - 1)(4\delta - 1)(4 + \delta(16\delta - 17))]\right],
$$

$$
D_2 = [(2\delta - 1)(2(s'_2 - c_2) + \delta(3 - c_1 + 9c_2 + s'_1 - 9s'_2) + \delta^2(-11 + 2c_1 - 8c_2 - 2s'_1 + 8s'_2) + 8\delta^3)]/[(\delta - 1)(4\delta - 1)(4 + \delta(16\delta - 17))],
$$

where the total subsidy per unit $s'_1 \equiv s_1 + s_{m_1}$ and $s'_2 \equiv s_2 + s_{m_2}$. Through (21), we can express $s'_1$ and $s'_2$ as: $s'_1 = c_1 - 1 + D_2 + D_1 \cdot (4 + \frac{1}{1 - 2\delta} - \frac{2}{\delta})$ and $s'_2 = c_2 - \delta + D_1 + D_2 \cdot (-\frac{5}{2} + \frac{1}{2 - 4\delta} + 4\delta)$ so that the donor’s problem can be formulated as:

$$
\max_{D_1, D_2} \quad D_1 + D_2
$$

s.t. \ \ \ \ [c_1 - 1 + D_2 + D_1 \cdot (4 + \frac{1}{1 - 2\delta} - \frac{2}{\delta})] \cdot D_1 + [c_2 - \delta + D_1 + D_2 \cdot (-\frac{5}{2} + \frac{1}{2 - 4\delta} + 4\delta)] \cdot D_2 \leq K
$$

As before, by showing that the subsidy cost (i.e., left hand side (22)) is increasing in $D_1$ and $D_2$, we get:

**Proposition 6.** When two competing manufacturers sell two substitutable products separately through two competing retailers and when the wholesale price is endogenous,
1. The donor’s budget constraint (22) is binding;
2. The total subsidy per unit $s_i^*$ for product $i$ across the supply chain echelons is uniquely determined, so the optimal total demand $D_1^* + D_2^*$, the corresponding manufacturers’ profit $\pi_{m_i}^*$, retailers’ profit $\pi_{r_i}^*$, and the consumer welfare $W^*$ are not affected by the split of the total subsidy level $s_i^*$ amongst the different parties; and
3. Selling two substitutable products (produced by two competing manufacturers) separately through two competing retailers will generate a higher total demand than selling through a single retailer.

Proposition 6 shows that the donor can achieve a higher product adoption in Setting 3 than in Setting 2. In summary, when the wholesale price is endogenous, the results from the base model continue to hold: (1) the budget constraint is binding; (2) it does not matter who to subsidize as long as the total subsidy $s_i^* = s_{b_i}^* + s_{r_i}^* + s_{m_i}^*$ is maintained at the optimal level; and (3) manufacturing and retail competition can enable the donor to generate a higher product adoption.

5. Extension 2: Market Uncertainty

In the base model, the wholesale price is exogenous so that: (1) there is no incentive for the donor to offer the manufacturer subsidy so that $s_m = 0$; and (2) the manufacturing competition in setting 3 does not play a role. The analysis associated with the case when wholesale price is endogenous is intractable and we shall defer such analysis as future research. Instead of assuming that the market size $M = 1$ in the base model, we now extend our base model to the case when $M$ follows a probability density function $f(m)$ with $m \in (0, \infty)$. Following Taylor and Xiao’s (2014) five steps:

1. The donor determines and announces the subsidies $s_k$ for entity $k = r, b$.
2. The retailer knows the density function $f(m)$ and selects the order quantity $z$.
3. The retailer observes the realized market size $M = m$.
4. The retailer decides on the retail price $p$ by taking the order quantity $z$ and the realized market size $m$ into consideration.
5. The beneficiary demand $D$ is realized.
This extension is more complex than earlier settings because, as noted in Step 2, the retailer selects the order quantity “before” but decides on the retail price “after” the market size is realized in Step 4. To solve the 3-stage Stackelberg game for each of the three settings, we use backward induction beginning with Step 5 and ending with Step 1. In the remainder of this section, we first characterize the optimal total subsidy level for each setting for the case when the probability density function \( f(m) \) of the market size \( M \) is continuous. Then, by considering the case when the market size \( M \) follows the uniform or the normal distribution, we show numerically that the key results obtained from the base model continue to hold.

5.1. Setting 1: Selling one product through one retailer

Consider Setting 1 as depicted in Figure 4-1 (base case with \( s_m = 0 \)). For any given subsidy \( s_b \), any realized market size \( m \) and any retail price \( p \), the beneficiary demand (in step 5) is given by \( D = (1 - p + s_b) \cdot m \).

**Retailer’s pricing problem.** Observe from step 4 that the retailer’s pricing problem takes place “after” the order \( z \) is placed and the market size \( m \) is realized. Therefore, the ordering cost \( w \cdot z \) is sunk, the actual sales \( S = \min\{D, z\} \), where \( D = (1 - p + s_b) \cdot m \), and the retailer’s pricing problem for any given subsidy \( s_r \) is:

\[
\max_p \ (p + s_r) \cdot \min\{(1 - p + s_b) \cdot m, z\}
\]

which can be reformulated as:

\[
\max_p \ (p + s_r) \cdot (1 - p + s_b) \cdot m \quad \text{s.t.} \quad (1 - p + s_b) \cdot m \leq z. \tag{23}
\]

By solving the above problem, the optimal price satisfies:

\[
p^* = \begin{cases} 
1 + \frac{s_b - s_r}{2} & \text{if } m \leq \frac{2z}{1 + s_b + s_r} \\
1 + s_b - \frac{z}{m} & \text{if } m \geq \frac{2z}{1 + s_b + s_r}
\end{cases} \tag{24}
\]

By denoting the total subsidy \( s \equiv s_b + s_r \), the corresponding sale \( S = \min\{D, z\} \) and the retailer’s per unit revenue \( p^* + s_r \) are:

\[
S = \begin{cases} 
\frac{1 + s}{2} \cdot m & \text{if } m \leq \frac{2z}{1 + s} \\
z & \text{if } m \geq \frac{2z}{1 + s}
\end{cases}, \quad p^* + s_r = \begin{cases} 
\frac{1 + s}{2} & \text{if } m \leq \frac{2z}{1 + s} \\
1 + s - \frac{z}{m} & \text{if } m \geq \frac{2z}{1 + s}
\end{cases} \tag{25}
\]
Retailer’s ordering problem. Observe that the sales $S$ and the retailer’s revenue $(p^* + s_r) \cdot S$ depends only on the total subsidy $s$. Hence, it suffices to focus only on $s$ when we examine the retailer’s ordering that takes place “before” the market size is realized as in step 2. For any given per unit total subsidy $s = s_r + s_b$, the retailer’s (ex-post) profit is:

$$
\Pi_r(m) = \left( p^* + s_r \right) \cdot S - w \cdot z = \begin{cases} 
\frac{1+s}{2} \cdot m - w \cdot z & \text{if } m \leq \frac{2z}{1+s}, \\
1 + s - \frac{z}{m} - w \cdot z & \text{if } m \geq \frac{2z}{1+s}.
\end{cases}
$$

To maximize the retailer’s (ex-ante) expected profit, the retailer’s ordering problem is:

$$
\max_z E_m[\Pi_r(m)] = \int_0^{\frac{2z}{1+s}} \left[ \left( \frac{1+s}{2} \right)^2 m - w \right] f(m) dm + \int_{\frac{2z}{1+s}}^{\infty} \left[ 1 + s - \frac{z}{m} - w \right] z f(m) dm. \quad (27)
$$

By differentiating $E_m[\Pi_r(m)]$ with respect to $z$ and by applying the Leibniz rule, we get:

$$
\frac{\partial E_m[\Pi_r(m)]}{\partial z} = \int_{\frac{2z}{1+s}}^{\infty} \left( 1 + s - \frac{2z}{m} \right) \cdot f(m) dm - w.
$$

By considering the first order condition and by using the implicit function theorem, we get:

**Proposition 7.** When selling one product through one retailer, the retailer’s optimal ordering decision $z^*$ satisfies: $\int_{\frac{2z}{1+s}}^{\infty} \left( 1 + s - \frac{2z^2}{m} \right) \cdot f(m) dm = w$. Also, the optimal order quantity $z^*$ is increasing in the donor’s subsidy $s$ and decreasing in the wholesale price $w$.

From this proposition and the retailer’s profit function in (27), we see that the optimal retailer’s expected profit is only affected by the total subsidy level $s$, not by the split of $s$ between the retailer and beneficiaries.

Donor’s problem. By substituting the optimal retailer’s ordering quantity $z = z^*$ given in Proposition 7 into the sales function $S$ as given above, the expected sale $S$ is:

$$
E_M[S] = \int_0^{\frac{2z^*}{1+s}} \left( \frac{1+s}{2} \cdot m \right) f(m) dm + \int_{\frac{2z^*}{1+s}}^{\infty} z^* f(m) dm = \frac{1+s}{2} E[M] - \frac{1}{2} \int_{\frac{2z^*}{1+s}}^{\infty} \left[ (1+s) m - 2z^* \right] f(m) dm,
$$

where $\int_{\frac{2z}{1+s}}^{\infty} \left( 1 + s - \frac{2z^2}{m} \right) \cdot f(m) dm = w$. As such, the donor’s problem in step 1 can be written as:

$$
\max_s E_M[S] \quad \text{s.t. } s \cdot E_M[S] \leq K. \quad (29)
$$
Proposition 8. When selling one product through one retailer, the donor’s budget constraint is binding so that the optimal subsidy $s^*$ satisfies $s^* \cdot E_M[S] = K$. Also, the donor’s optimal subsidy $s^*$ is increasing in the budget $K$ and the wholesale price $w$.

Proposition 8 reveals that even when market size is uncertain, the key results obtained in the base case in Section 3.1 continue to hold.

5.2. Setting 2: Selling two substitutable products through one retailer

We now consider setting 2 (Figure 4-2) corresponding to the base case with $s_{m1} = s_{m2} = 0$. To obtain tractable analytical results, we focus on the following scenario: (a) the donor offers uniform subsidies so that the subsidy is product-independent (uniform subsidies have been considered by Taylor and Xiao (2014), and the conditions for the optimality of uniform subsidies have been established by Levi et al. (2017)); (b) product 1 has a long replenishment lead time so that the retailer needs to place the order $z_1$ “before” the market size is realized. However, product 2 has a short lead time so that the retailer can place the order $z_2$ “after” the market size is realized. This scenario can occur when product 1 is sourced from afar and product 2 is sourced nearby. In Appendix A, we consider a more general scenario when the replenishment lead time is long so that both retailers need to place their orders “before” the market size is realized. We show numerically that having more product choice and having more retailer-manufacturing-channel choice can increase product adoption. In other words, our structural results continue to hold for this general case.

In the remainder of this section, we shall analyze settings 2 and 3 as depicted in Figure 4-2 and Figure 4-3 by focusing on this specific scenario. We begin with the realization of beneficiary demand in step 5. For any given wholesale price, per unit subsidy, market size, and retail price, the demand function is equal to (6) multiplied by $m$: $D_1 = m \cdot \frac{(p_2 - s_{b2}) - \delta (p_1 - s_{b1})}{\delta - 1}$, $D_2 = m \cdot [1 - \frac{(p_2 - s_{b2}) - \delta (p_1 - s_{b1})}{\delta - 1}]$.

Retailer’s pricing problem. Recall that the retailer’s pricing problem in step 4 occurs after the order $z_1$ is placed and the market size $m$ is realized. Hence, the ordering cost for product 1; i.e., $w_1 \cdot z_1$ is sunk, and the actual sales for product 1 is $S_1 = \min\{D_1, z_1\}$, where $D_1 = m \cdot \frac{(p_2 - s_{b2}) - \delta (p_1 - s_{b1})}{\delta - 1}$. However, because product 2 is ordered after the market size is realized, $z_2^* = D_2 = S_2$. Hence, we only need to
determine the optimal order quantity for product 1. For any given subsidy \( s_r \) for the retailer, we can use the same approach as in setting 1 to show that the retailer’s pricing problem is 
\[
\max_{p_1, p_2} \{(p_1 + s_{r_1}) \cdot S_1 + (p_2 + s_{r_2} - w_2) \cdot S_2\},
\]
which can be reformulated as:
\[
\max_{p_1, p_2} \{(p_1 + s_{r_1}) \cdot D_1 + (p_2 + s_{r_2} - w_2) \cdot D_2\} \quad \text{s.t.} \quad D_1 \leq z_1.
\] (30)

By considering the first order condition and by defining a threshold \( M_1 = \frac{2z_1(\delta-1)}{s_{r_1} + w_2 - s_{r_2}} \), the optimal retail price \((p_1^*, p_2^*)\) and the corresponding sale \((S_1, S_2)\) satisfy:
\[
p_1^* = \begin{cases} 
\frac{1}{2}(1 + s_{b_1} - s_{r_1}) & \text{if } m \leq M_1 \\
\frac{1}{2m\delta}[-2z_1(\delta - 1) + m(\delta + w_2 - s_2 + 2s_{b_1})] & \text{if } m > M_1
\end{cases}, \quad p_2^* = \frac{1}{2}(s_{b_2} - s_{r_2} + w_2 + \delta) \tag{31}
\]
\[
S_1 = \begin{cases} 
m \cdot \frac{s_{s_1} + w_2 - w_2}{2(\delta - 1)} & \text{if } m \leq M_1 \\
\frac{1}{4m\delta}[-2z_1(m + s_2 - w_2 + \delta)] & \text{if } m > M_1
\end{cases}, \quad S_2 = \begin{cases} 
m \cdot \frac{1 - s_{s_1} + s_2}{2(\delta - 1)} & \text{if } m \leq M_1 \\
\frac{1}{4m\delta}[-2z_1(m + s_2 - w_2 + \delta)] & \text{if } m > M_1 \tag{32}
\end{cases}
\]

**Retailer’s ordering problem.** From (31) and (32), the retailer’s profit in step 2 can be written as:
\[
\Pi_r(m) = (p_1^* + s_{r_1}) \cdot S_1 - w_1 z_1 + (p_2^* + s_{r_2} - w_2) \cdot S_2
\]
\[
= \begin{cases} 
\frac{m}{4(\delta - 1)}[(s_2 - w_2)(s_2 - w_2 - 2(s_1 + 1) + (s_2^2 - 1 + 2s_2 - 2w_2)\delta + \delta^2) - w_1 z_1] & \text{if } m \leq M_1 \\
\frac{1}{4m\delta}[-4z_1^2(\delta - 1) + m^2(s_2 - w_2 + \delta)^2 - 4mz_1(s_2 - w_2 - \delta(s_1 - w_1))] & \text{if } m > M_1.
\end{cases}
\]

By letting \( \Pi_{r,1}(m) \) and \( \Pi_{r,2}(m) \) be \( \Pi_r(m) \) when \( m \leq M_1 \) and \( m > M_1 \); respectively, the retailer’s (ex-ante) expected profit is:
\[
E_M[\Pi_r(m)] = \int_0^{M_1} \Pi_{r,1}(m) \cdot f(m)dm + \int_{M_1}^{\infty} \Pi_{r,2}(m) \cdot f(m)dm. \tag{33}
\]

Hence, the retailer’s ordering problem is: 
\[
\max_{z_1} E_M[\Pi_r(m)], \text{ and}
\]
\[
\frac{\partial E_M[\Pi_r(m)]}{\partial z_1} = \int_0^{M_1} (-w_1) \cdot f(m)dm + \int_{M_1}^{\infty} \frac{[-2z_1(\delta - 1) m\delta] - 1}{\delta}(s_2 - w_2 - (s_1 - w_1)\delta) \cdot f(m)dm.
\]

From the first order condition and the implicit function theorem, we get:

**PROPOSITION 9.** When selling two substitutable products through one retailer, the retailer’s optimal order quantity for product 1 \( z_1^* \) satisfies 
\[
\int_{z_1^*}^{\infty} \left[ \frac{-2(\delta - 1)z_1^*}{s_{r_1} + w_2 - s_{r_2}} + \frac{\delta s_{s_1} - s_{r_1} - w_2}{\delta} \right] \cdot f(m)dm - w_1 = 0, \text{ where } z_1^* \text{ is increasing in } s_1 \text{ and decreasing in } s_2 \text{ and } w_1.
From Proposition 9, we see that the optimal $z_1^*$ depends only on the total subsidy level of each product (i.e., $(s_1, s_2)$). Therefore, we know that the retailer’s expected profit given by (33) is only affected by the total subsidy level of each product (i.e., $(s_1, s_2)$), not by the split of $(s_1, s_2)$ between the retailer and beneficiaries.

**Donor’s problem.** When the donor offers uniform subsidy so that $s_1 = s_2 = s$, we can use the optimal order quantity $z_1^*$ given in Proposition 9 along with the sales function given in (32) to show that the expected total sales is:

$$E_M[S_1 + S_2] = \int_0^{M_1} \frac{s + 1}{2} \cdot m \cdot f(m)dm + \int_{M_1}^{\infty} \left( \frac{d - 1}{d} z_1^* + \frac{m(s - w_2 + d)}{2d} \right) \cdot f(m)dm.$$  (34)

Given the budget $K$, the donor’s problem in step 1 is:

$$\max_s E_M[S_1 + S_2] \text{ s.t. } E_M[s \cdot (S_1 + S_2)] \leq K$$  (35)

**Proposition 10.** When selling two substitutable products through one retailer, the donor’s budget constraint is binding: the optimal subsidy $s^*$ satisfies $s^* \cdot \left[ \int_0^{2z_1^*(\delta - 1)} \frac{s + 1}{2(s^*(\delta - 1) + w_2)} \cdot m \cdot f(m)dm + \int_{2z_1^*(\delta - 1)}^{\infty} \left( \frac{d - 1}{d} z_1^* + \frac{m(s^* - w_2 + d)}{2d} \right) \cdot f(m)dm \right] = K$.

So, when the market size is uncertain, the key results from the base case in Section 3.2 continue to hold.

5.3. **Setting 3: Two manufacturers sell two substitutable products separately through two retailers**

In Setting 3 (Figure 4-3), the wholesale price is exogenous in the base case, so $s_{m1} = s_{m2} = 0$ and the competition between manufacturers does not play a role. Considering the same scenario in the previous subsection, we can show that for any given wholesale price, per unit subsidy, market size, and retail price, in step 5, the realized demand is equal to that given in (12) multiplied by $m$. Because the order for product 1 (i.e., $z_1$) is placed by retailer “before” the market size is realized, the sales for product 1 is given by $S_1 = \min\{D_1, z_1\}$. However, retailer 2 can postpone it ordering decision of product 2 “after” the market size is realized so that $z_2^* = D_2$ and the sales for product 2 is equal to $S_2 = D_2$. It remains to focus on retailer 1’s order quantity $z_1$. 
Retailers’ pricing problem. By using the same arguments as presented in the last subsection for setting 2, retailers’ pricing problem in step 4 can be formulated as follows:

\[
\begin{align*}
\max_{p_1} \{ (p_1 + s_{r_1}) \cdot D_1 \} \quad \text{s.t.} \quad D_1 &= m \left( \frac{p_2 - s_{b_2}}{\delta - 1} \right) \leq z_1, \quad \text{and} \\
\max_{p_2} \{ (p_2 + s_{r_2} - w_2) \cdot m \cdot [1 - \frac{p_2 - s_{b_2}}{\delta - 1}] \}. 
\end{align*}
\]

By solving the above two pricing problems simultaneously and by defining a threshold for \( m \) as \( M_2 = \frac{z_1 (\delta - 1) (4\delta - 1)}{(1 + 2s_1) \delta^2 - \delta (1 + s_1 + s_2 - w_2)} \), the equilibrium retail price and the equilibrium sales satisfy:

\[
p_1^* = \begin{cases} 
\frac{1 + s_{b_1} + s_2 - w_2 - \delta - 2\delta s_{b_1} + 2\delta s_{r_1}}{1 - 4\delta} & \text{if } m \leq M_2 \\
\frac{m(-1-s_{b_1} - s_2 + w_2 + \delta + 2\delta s_{b_1}) - 2z_1 (\delta - 1)}{m(2\delta - 1)} & \text{if } m > M_2 
\end{cases} \quad \text{and} \quad p_2^* = \begin{cases} 
\frac{s_{b_2} (1-2\delta) + \delta (2+s_1 + 2s_{r_2} - 2w_2 - 2\delta)}{1 - 4\delta} & \text{if } m \leq M_2 \\
\frac{z_1 (1-\delta) + m(s_{b_2} (\delta - 1) + \delta (\delta - 1 - s_{r_2} + w_2))}{m(2\delta - 1)} & \text{if } m > M_2 
\end{cases} 
\]

\[
S_1 = \begin{cases} 
m \cdot \frac{-\delta(1-w_2+s_1+s_2) + (1+2s_1)\delta^2}{(\delta - 1)(4\delta - 1)} & \text{if } m \leq M_2 \\
\frac{m}{z_1} & \text{if } m > M_2 
\end{cases} \quad \text{and} \quad S_2 = \begin{cases} 
m \cdot \frac{w_2 - (2+s_1 + 2s_2) \delta + 2\delta^2 + (2\delta - 1) s_2}{(\delta - 1)(4\delta - 1)} & \text{if } m \leq M_2 \\
\frac{-z_1 + m(s_{b_2} - w_2 + \delta)}{2\delta - 1} & \text{if } m > M_2 
\end{cases} 
\]

Retailers’ ordering problem. Because retailer 2 can postpone its ordering decision of product 2 until after the market size is realized, the order quantity \( z_2^* = D_2 = S_2 \). It remains to solve retailer 1’s ordering problem for product 1 in step 2. For any order quantity \( z_1 \), we can use the retail price \( p_1^* \) and the sales of product 1 \( S_1 \) as stated above to compute retailer 1’s (ex-post) profit for selling product 1, where:

\[
\Pi_{r_1}(m) = (p_1^* + s_{r_1}) \cdot S_1 - w_1 z_1 = \begin{cases} 
\frac{m\delta(1+s_1 + s_2 - w_2 - \delta - 2s_1)\delta^2}{(1-4\delta)^2(\delta - 1)} - w_1 z_1 & \text{if } m \leq M_2 \\
(s_1 + \frac{w_2 - 1 + \delta - s_2}{2\delta - 1}) + \frac{z_1 (1-\delta)}{m(2\delta - 1)} - w_1 \cdot z_1 & \text{if } m > M_2 
\end{cases} 
\]

As before, we use \( \Pi_{r_1,1}(m) \) and \( \Pi_{r_1,2}(m) \) to represent the profit of retailer 1 under the cases when \( m \leq M_2 \) and \( m \geq M_2 \); respectively. Hence, retailer 1’s (ex-ante) expected profit is:

\[
E_M[\Pi_{r_1}(m)] = \int_0^{M_2} \Pi_{r_1,1}(m) \cdot f(m) dm + \int_{M_2}^{\infty} \Pi_{r_1,2}(m) \cdot f(m) dm, \quad (40)
\]

and retailer 1’s ordering problem is: \( \max_{z_1} E_M[\Pi_{r_1}(m)] \).
PROPOSITION 11. When selling two substitutable products through two separate retailers, retailer 1’s optimal ordering quantity $z_1^*$ satisfies

$$
\int_0^\infty \frac{z_1^* (\delta - 1) (4\delta - 1)}{(1+2s)^2-\delta(1+2s^*+s-w_2)} \left( 1 + \frac{m w_2 - s w_2}{2\delta - 1} + \frac{4z_1^* (1-\delta)}{m(2\delta - 1)} \right) \cdot f(m) \, dm - w_1 = 0.
$$

Analogous to setting 2, it is easy to check that both $z_1^*$ and the two competing retailers’ profits depend only on the total subsidy level of each product $(s_1, s_2)$ but not on the split of the total subsidy between the retailer and beneficiaries.

**Donor’s problem.** When the donor offers uniform subsidy with $s_1 = s_2 = s$, we can use sales functions given above to determine the expected total sale (i.e., $E_M[S_1 + S_2]$). From (38) and formulate the donor’s problem in step 1 as:

$$
\max_s \quad E_M[S_1 + S_2] \quad \text{s.t.} \quad E_M[S_1 + S_2] \leq K, \quad \text{where}
$$

$$
E_M[S_1 + S_2] = \int_0^{M_2} \frac{s - w_2 + 2\delta s}{4\delta - 1} \cdot m \cdot f(m) \, dm + \int_0^\infty \frac{2(\delta - 1) z_1^* + m(s - w_2 + \delta)}{2\delta - 1} \cdot f(m) \, dm.
$$

By using the same approach as in Setting 2, we get:

**PROPOSITION 12.** When selling two products through two separate retailers, the budget constraint is binding: the donor’s optimal subsidy $s^*$ satisfies $s^* \cdot \left( \int_0^{M_2} \frac{s - w_2 + 2\delta s}{4\delta - 1} \cdot m \cdot f(m) \, dm + \int_0^\infty \frac{2(\delta - 1) z_1^* + m(s - w_2 + \delta)}{2\delta - 1} \cdot f(m) \, dm \right) = K$.

By reviewing the results presented in Propositions 8, 10, 12 in this section, we can conclude that, when the market size is uncertain, our two key results obtained from the base model continue to hold; namely, the donor can offer per unit subsidy to the retailers and/or to the beneficiaries, as long as the total subsidy per unit is set at a certain optimal level; and the donor’s budget constraint is binding.

To complete our analysis, it remains to examine whether it is still true that the donor can increase the total demand by introducing substituting products (as in Settings 2 and 3) and by supporting a supply chain structure that entails retail competition (as in Setting 3).

Comparisons to examine these issues analytically are intractable as no closed-form expressions are available for general probability distribution $f(m)$ so we investigate numerically.

**Numerical comparison.** We consider four cases: (1) market size $M$ is deterministic with $m = 1$; (2) market size $M \sim N(1, 0.04)$; (3) market size $M \sim N(1, 0.09)$; (4) market size $M \sim U[0, 2]$, noting
that $E[M] = 1$ in all cases. In our numerical analysis, we set the exogenously given wholesale price as $w_1 = 0.5$ (for the single product in Setting 1, and for product 1 in Settings 2 and 3 when there are two products), $w_2 = 0.8$ (for product 2 in Settings 2 and 3), and set the valuation multiplier for product 2 $\delta = 1.2$.

For each of these 4 cases, we compute the optimal uniform subsidy $s^*$ and the optimal expected total sales $S^*$ by varying the budget $K$ from 0 to 0.18. Our numerical results are summarized in Figures 5, 6, 7, and 8. In each figure, we depict the optimal subsidy $s^*$ on the left panel and the total expected sales $S^*$ on the right panel.

![Graph showing optimal subsidy and total sales](image)

**Figure 5** Optimal uniform subsidy (left) and the corresponding total sales (right) when the market size $M$ is deterministic with $m = 1$.

We now observe:

1. The optimal per unit subsidy $s^*$ is the lowest in setting 3, followed by that in setting 2. This implies that the donor can afford to reduce its per unit subsidy $s^*$ when there are more products available in the market (as in Setting 2), and can reduce the subsidy even further when there is retail competition (as in setting 3).

2. Combining observation 1 above with our analytical observation that the budget constraint is binding in all three settings, we see that the optimal total sales is the highest in Setting 3, followed by Setting 2 for any given budget $K$. 
3. Both the optimal subsidy $s^*$ and the total sale are increasing in the budget $K$ under all three supply chain structures.

4. When market size is deterministic, Figure 5 illustrates the finding that when the donor offers uniform subsidy, the donor cannot increase the total demand in setting 2. However, despite the “uniform subsidy” effect, Figure 5 verifies that retail competition in Setting 3 can enable the donor to obtain a higher total demand even when the donor offers uniform subsidy.

5. The variance of the market size $M$ increases as we progress from Figure 6 to Figure 7, and from Figure 7 to Figure 8. Comparing the total demand across Figures 5 through 8, we see that, as the variance of the market size $M$ increases, the donor can obtain a higher total demand by selling two

Figure 6 Optimal uniform subsidy (left) and the corresponding total sales (right) when the market size $M \sim N(1, 0.04)$

Figure 7 Optimal uniform subsidy (left) and the corresponding total sales (right) when the market condition $M \sim N(1, 0.09)$
products through one retailer in setting 2 compared to selling one product through one retailer in setting 1 even when the donor offers uniform subsidies. Hence, **product choice can increase total demand further when the underlying market size is uncertain.** Also, retail competition can enable the donor to obtain a **higher total demand when the market size is uncertain.**

We conclude from these observations that the results obtained in the base model are robust in that they continue to hold even when the market size is uncertain. More importantly, we find the demand increase – in setting 3 over setting 2 and the in setting 2 over setting 1 – becomes more pronounced when the market size becomes more uncertain.

6. **Conclusion**

We introduced development supply chains as hybrids of commercial and humanitarian supply chains. Using three settings of three-echelon stylized supply-chain model, we modeled competition among donor, manufacturers, retailers and consumers as a 4-stage Stackelberg game. These settings incorporated product substitution, retail competition, and demand uncertainty. Results from different variations of this game obtained using backward induction indicated that the donor can subsidize any echelon as long as the total subsidy per unit is maintained at the optimal level. More product choice (especially when the subsidies are product-specific) and more channel choice can increase the number of beneficiaries adopting the products, and this increase is more pronounced as the market size becomes more uncertain.
Our work complements the literature on subsidizing manufacturers or consumers by analyzing product-specific subsidies in different supply-chain settings with substitutable products and retail competition with interactions between manufacturers, retailers, beneficiaries and the donor. Rather than provide only upper bounds, we provide closed-form solutions for optimal subsidy in the deterministic demand case and characterize the solution in the uncertain demand case. Scholars have studied this question in a variety of contexts with different policy goals such as consumer welfare with: private and public firms (cf. Myles 2002; Poyago-Theotoky 2001; Scrimitore 2014); promotion of investment in a particular sector, say telecommunication (cf. Jeanjean 2010); or promotion of environmental sustainability (cf. Bansal and Gangopadhyay 2003). Part of environmental sustainability (and energy security) is adoption of solar technology among consumers (cf. Lobel and Perakis 2011; Cohen, Lobel, and Perakis 2016). There is also the rationale for health of the poor with subsidies for malaria medication in many developing countries (cf. Levi et al. 2017; Taylor and Xiao 2014). Other contexts for subsidies include education (Schultz, 2004); electricity (Goodarzi et al., 2015); food (Peeters and Albers, 2013); housing (Gilbert, 2004); and smallholders farmers (Tang et al., 2015). As regards energy or lighting specifically, there is the question of empirically establishing willingness to pay (Yoon et al. 2016), consumer adoption of alternative lighting products (Uppari et al. 2017), and supply chain coordination for photovoltaic modules (Chen and Su, 2014).

To cite some specific studies, Shen et al. (2016) analyze how the Chinese government should subsidize home appliances for residents in rural areas. We use a three-echelon model in contrast to their two-echelon subsidy model. Their subsidies are a fixed percentage of the retail price across all products whereas we have product-specific subsidies and in three different supply chain settings. Moreover, Shen et al. (2016) determine the optimal percentage that maximizes consumer welfare, whereas we focus on total beneficiary demand, a common measure for the effectiveness of a subsidy program. Furthermore, they assume demand to be known whereas we also include the case of uncertain demand. Goodarzi et al. (2015) study the interaction among the regulator, manufacturer and customers, focusing on the optimal feed-in tariff policies of the regulator to minimize the grid operator’s total cost. In contrast,
we seek to maximize product adoption under different supply-chain structures using unit subsidies. Our paper also complements Taylor and Xiao (2014) by considering three different supply chain settings with two substitutable products and two competing retailers in contrast to their one setting with one product and one retailer. Doing so affords us a broader set of results.

Our secondary contribution is to the humanitarian operations literature with this work having been motivated by the recovery efforts in Haiti following the 2010 earthquake. Supply chains such as one we presented help alleviate poverty, and poverty alleviation in turn reduces vulnerability to future disasters and crises (Sodhi 2016, Sodhi and Tang 2014, Van Wassenhove and Pedraza Martinez 2012). Post-disaster development as well as poverty alleviation in general require supply chains to incorporate local communities (Hall and Matos 2010) to provide jobs and investment, and address institutional voids in low-income markets (Parmigiani and Rivera-Santos 2015).

It is clear from our stylized modeling and from our field study, that donors must map out the supply chains of the products they want to subsidize – echelons, substitutable products, and competition between retailers – and coordinate donor action along these supply chain as well as those of intersecting supply chains. Our work motivates further research with implications for donors to look into their collective experience for evidence on these issues:

1. **Product choice and retailer competition**: Donors must look at whether having more competition in the distribution channels and ensuring a range of substitutable products across the quality-price range has increased or can increase the number of beneficiaries in development efforts (in contrast to immediate relief after a disaster).

2. **Choice of subsidy target**: Donors have had experience in giving subsidies to manufacturers, to distributors, to retailers and more recently directly beneficiaries. Where to apply subsidies in the supply chain for maximum effect is an important issue. Our models indicate that the echelon does not matter as regards increasing the number of those adopting the subsidized products, so donors can look at what factors underlie their choice of where to subsidize and with what result.

3. **Optimum subsidy**: Donors can look at past experience to understand whether more subsidy would have helped or whether there was too much subsidy. This would indicate, as our modeling shows, that
there is an optimal level of subsidy – greater levels may increase the number of potential consumers, but
would only increase the donor’s investment and possibly lower the profitability for the micro-entrepreneur
retailer. Likewise, donors must take care not to over-subsidize certain products to price out substitutes.

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