Adaptive Inspection Regime for Reinforced Concrete Short to Medium Span Bridges with Stochastic Deterioration

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ABSTRACT

Bridge components are subjected to deterioration factors such as aggressive environment, corrosion, chemical attack etc. that can result in a loss of load capacity and life span. In order to keep the safety of the bridge at an appropriate level inspection regimes are followed.

This thesis concern is to establish an adaptive inspection regime for reinforced short to medium span concrete bridges. Our emphasis is mainly on using the information about the deterioration progress to determine efficient inspection regime.

An updatable structural deterioration model that follows the inspection outcomes is developed. A stationary continuous Gamma process is used to develop the structural deterioration model. In order to predict the deterioration profile of bridge slab, deterioration process of a reinforced concrete slab subject to corrosion is modeled using Gamma process through the thesis. Inspection outcomes at specific ages are used to update the deterioration model. The updated deterioration model reflects the latest condition of component at inspection time. Different deterioration condition such as initiation time and deterioration rate are considered in thesis and influence of deterioration condition on deterioration process is represented.

Initially it is assumed that the observed inspection outcomes are perfect. It is identified that the inspection outcomes are associated with uncertainties. In order to characterize the probability of detection and measurement error as inspection outcomes uncertainties, the probabilistic model is implemented. A new probabilistic framework is developed to take into account uncertainties associated with inspection outcomes. The deterioration model is applied following the actual inspection outcomes to reflect the influence of the inspection outcomes uncertainties. Finally a new adaptive inspection regime is established based on the actual deterioration profile. An efficient inspection regime is established as result. The novel probabilistic method is highly flexible and can be implemented in different countries with different environments.

1. Modeling of Ageing Highway Infrastructure

1.1 Introduction

It is widely recognized that a well-managed transport infrastructure is vital to the economic stability, growth and social wellbeing of a country. Bridges and other highway structures are fundamental to the transport infrastructure because they form essential links in the highway network. The management of highway structures in the UK is undertaken by a variety of highway authorities and other owners e.g. local authorities, trunk agencies, Network Rail, etc. (UK Roads Liaison Group, 2005).

Highway bridges are one of the most vital components of transport networks and as it has been indicated in the reports of UK Department of Transport, 80% of the bridges are concrete bridges. (Mahut and Woodward, 2005)

Mallet (1986) classified the data of bridge types and their population in UK. The Table 1.1 presents this classification.

Table 1.1 Classification of UK Bridge (Mallet, 1986)

TYPE	NUMBER
Motorway	5000
Trunk Roads	8000
Local Authorities	129000
Railway	12000
British Waterways	1000
Total	**155000**

The Department of Transport (1987) conducted a survey which indicated that 25% of masonry bridges, 30% of concrete which is equal to 1200 of motorway bridges or 37200 of the total and 46% of steel bridges in the UK have capacity below standard to carry design traffic load.

Highway structures are often subject to destructive effects of material ageing, harsh weather condition, extensive corrosion of reinforcement bars in concrete structures, corrosion of steel structures and components, increasing traffic volume and overloading, or simply overall deterioration and ageing. These factors, accompanied with imperfections of design and construction and accidental damage, initiate the deterioration of highway structures and result in the loss of serviceability and load carrying capacity (Dong et al., 2010).

The deterioration of infrastructure facilities such as highway bridges built in 50s and 60s has raised concerns over objective methodology to quantify the change in their safety level during the service life (Dong et al., 2010)

In order to maintain the safety and serviceability of structure at adequate level, it is important to represent the structural deterioration process as comprehensively as possible in respect to the influence of deterioration factors.

In recent years, modern technology has enabled greater variety of monitoring techniques and therefore availability of data from sensors, video imaging, etc. is increasing. It is established infrastructure inspection processes can be reviewed to reconcile quality and diversity of site-specific data, physical behavior models and technology. However, the non-destructive inspection techniques can bring in additional uncertainty in deterioration model due to the uncertainty of inspection techniques (Ohadi and Micic, 2011).

Even in circumstances when an NDT inspection program has been performed on the entire a component and all defects detected are repaired, the engineer cannot guarantee that there will be absolutely no defects or that defects would be defiantly smaller than a particular size (Tang, 1973).

If the current status of deterioration is to be established on the basis of inspection, it has become evident that quality and consistency of the data needs to be taken into account (Ohadi and Micic, 2011).

The safety of existing bridges is an important research topic owing to ageing process affecting their strength and stiffness as well as need to revise prediction of the maximum loads associated with operation and environmental factors. Many studies have been conducted since 1987 and in 2005 UK Roads Liaison Group published a code of practice to assist bridge managers and practitioners to maintain bridges safe and functional (UK Roads Liaison Group, 2005).

Bridge owner and managers are required to ensure that the structures for which they are responsible serves the purpose for which they were built in safe and maintainable manner. As a result, the requirement to be able to identify the presence of deterioration and to quantify it in terms of its effects on serviceability and carrying capacity is increasing. (Dong et al., 2010)

The deterioration of structures can be presented using deterministic or probabilistic approach. However, considering that the current and future status of structures are associated with many sources of uncertainty the deterministic approach cannot provide an appropriate mathematical model. Instead, probabilistic approach should be considered as more appropriate alternative. (Frangopol et al., 2004).

The probabilistic approach to characterize the capacity of a structural component is a function of available statistics for contributing variables, but also taking into account the errors induced by modeling and scaling effects. The random variable and stochastic processes are two alternative probabilistic models to represent the deterioration process. In the last decades, researchers have focused on the random variable approach (Frangopol et al., 2004).

Among many factors that could lead to poor condition of highway bridges, one factor that has been sometimes neglected is the inadequate inspection and monitoring of existing structures. It is essential to inspect bridges periodically, assess their condition and evaluate their functionality (Ellingwood and Mori, 1993).

In order to provide reliable outcomes for structural assessment, the current inspection regime that is explained in this chapter needs to improve. As majority of bridges around the UK are reinforced concrete, we focus on the inspection regime of reinforced concrete highway bridges here.

1.1.1 Aim

The aim of this research is to develop an adaptive inspection regime based on stochastic deterioration process for active assessment of reinforced concrete bridges. The new regime should enables decision on most suitable inspection type for specific bridge component and deterioration profile during the lifecycle and provide information for efficient bridge management system.

1.1.2 Objectives

In order to establish such inspection regime,

- Develop a new time-dependent stochastic representation of structural deterioration that can be updated over the lifecycle (Det profile). Here, the reduction of flexural moment capacity due to corrosion can be considered as deterioration model.
- Develop a probabilistic model to characterize the imperfect nature of inspection outcomes and take into account in deterioration process(X^a)
- Establish the relative criteria to ensure the structural performance level (Th_f). So that inspection type can be recognized inappropriate when defect size is out of inspection thresholds.
- Develop a framework which includes the structural performance criteria and deterioration model to establish a simple and fully site specific adaptive inspection regime (AI)
- Demonstrate a cost function to compare and provide a clear perspective of total inspection cost over the lifetime (C_T) that take site-specific.

1.2 Structural Deterioration

The ageing infrastructure is gradually becoming a global concern. This is especially true in the case of advanced countries, where a large fraction of critical civil infrastructure systems were built decades ago (Karbhari and Lee, 2011).

A breakdown of most common types of defects is tabulated in Table 1.2.

Table 1.2 Ageing defects and factors (Braverman et al., 2000)

Reinforce Concrete Bridge		Steel Bridge	
Defects	**Factors**	**Defects**	**Factors**
Cracking	Freeze-Thaw, Corrosion	Cracking	Moisture
Spalling	Leaching Chemical Attack	Loss of Material	Temperature-Elevated or Subfreezing
Pop outs	Corrosion of Embedded steel	Reduced Strength	Chemical Attack
Loss of Material	Elevated Temperature Corrosion	Loss of Fracture Toughness	Mechanical Wear
Excessive Deformation	Erosion	Excessive Deformation	Erosion
		Loss of Preload	Mechanical Loads
		Loosening	Organisms
		Rupture	Improper Design
		Plugging	Fatigue

The aging factors that are listed above can directly affect mechanical properties and lead to a loss in component resistance capacity. In order to evaluate the current condition of an existing bridge, the current practice in many countries is to take account of deterioration in some way.

The ageing civil infrastructure systems are often considered structurally deficient, due to aforementioned deterioration and ageing factors. The structural effects of ageing factors on structural degradation integrity are reviewed in the following.

Firstly corrosion, as one of the most destructive factors, can seriously weaken a structure or impair its operation (Bertonili et al., 2013).

The major degrading effects of corrosion on structural member are a loss of sections; buildup of corrosion products at connections and a notching effect that creates stress concentration (Zayed et al., 2002). Figure 1.1 demonstrated loss of reinforcement sections of a reinforced concrete pile due to corrosion.

Figure 1.1 The corroded reinforcement sections of a three span bridge pile in south of Kerman (Constructed in1998)

Brittle fracture is a catastrophic failure that occurs suddenly without prior plastic deformation and, can occur at nominal stress levels below the yield stress. Fracture of a structure occurs when a relatively high stress level is applied to material with relatively low fracture stiffness. (Melchers et al., 2008)

Chemical attack occurs when aggressive liquids are in contact with concrete. Etching or softening of surface may result. Alternatively, the concrete may crack and spall (Tang et al., 2015).

Figure 1.2 illustrates a carked surface of reinforced concrete section.

Figure 1.2 The cracked surface of reinforced concrete in a three span bridge slab in south of Kerman (Constructed in 1998)

Other faults, which can influence the structural strength and stiffness, are insufficient cover to steel, honeycombing or voids in concrete (Tang et al., 2015).

In order to maintain the structural performance at an adequate level in terms of serviceability and safety, the actual structural condition needs to be characterized. It is with the aim of not just knowing that the performance level may have changed, but rather to be able to locate the area of degradation and more importantly to assess remaining performance levels and the remaining life that current work is developed.

1.2.1 Processes Associated with Bridge Structural Deterioration

Deficient bridges are in need to major construction work, since, they restrict commercial trucks and emergency service vehicles. Thus, any public and economic decision of action for maintenance, repair, rehabilitation, upgrading, posting, or decommissioning requires through evaluation on the remaining strength, serviceability and durability (Farhey, 2007).

Unfortunately, bridge deterioration is not often the result of just one of the factors and severe deterioration regularly involves a number of factors.

Due to the different degradation factors, many types of defects can occur on the structure on the basis of the constituent material and the position of a component (Phares et al., 2004).

The origins of deterioration can be sub-divided into three different groups as:

- Deterioration or defects arising from faults in design e.g. low cover, reinforcement congestion, badly located joints, poor drainage system and etc.
- Defects due to construction method errors like poor quality concrete, bad compaction, inadequate curing and etc.
- Defects from external factors like bridge overloading, vehicle impact, carbonation, poor maintenance, freeze-thaw action and fatigue (Woodward et al., 1996).

Bridge evaluation consists of structural condition assessment and structural performance evaluation. When an existing bridge evaluated based on analytical data, the bridge evaluation result is most likely to be different from the actual evaluation result. Therefore, inspection outcomes are required to validate and calibrate the analytical evaluation result (Farhey, 2007).

Effectively section dimensions measured on site or assumed, the material properties are based on material tests or NDT methods. In order to take into account bridge deterioration in bridge evaluation, an inspection regime can be carried out on bridge components while inspection outcomes is used to assess the bridge condition. It is evident that outcomes of such evaluations would depend on the knowledge and experience of the assessment engineer. It is concluded that all structural condition is collected with significant variability via current inspection regime. It was found that there is significant variability in the condition state assignments of bridges and in some cases the condition states cannot applied correctly (Phares et al., 2004)

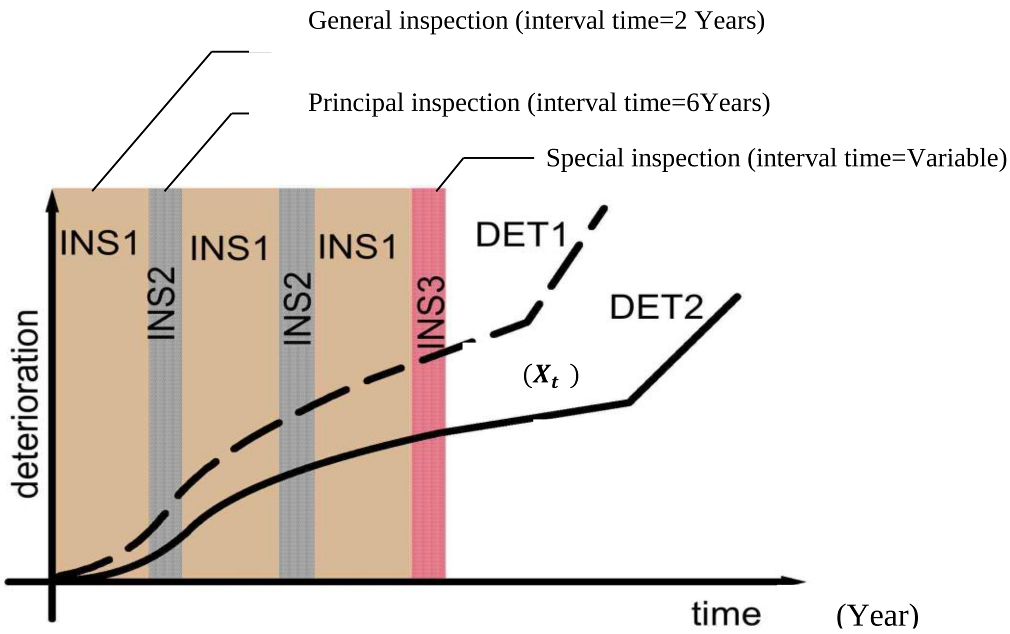

Figure 1.3 Illustration of current UK inspection regime for 3 common inspection types

In Figure 1.3 for illustration the current UK inspection regime for an assumed bridge component with two different deterioration scenarios ($DET1, DET2$) is demonstrated. The current UK inspection regime is superimposed. It is evident that

1. Deterioration profile for the component could be significantly time dependent as demonstrated by the two deterioration profiles
2. Inspection techniques cannot be equally efficient over lifetime, i.e. it would be extremely difficult to detect cracks by visual inspection in early years but once cracking is established it will be feasible to estimate the scale of cracking.
3. As a result of specific scale of deterioration the inspection quantitative outcomes will have varied accuracy.
4. All inspection types in Figure 1.3 are classified (INS1, INS2, INS3…) but their uncertainty content will depend on the inspection technique and the defect properties i.e. time of implementation.

5. The inspection outcomes are thus associated with uncertainties and these inspection outcomes uncertainties have to be taken in to account in order to establish the actual deterioration at some given time.

Time dependent changes in bridge condition are to a large extent random in nature; therefore, condition assessment of existing bridge can be conducted rationally within a probabilistic framework. The mathematical formulation of a probabilistic model can provide data to identify ageing bridge components performance level that may have a key role in improvement of structural condition management (Frangopol et al., 2004).

Due to uncertainties associated with inspection outcomes and deterioration status, some authors identified that random variable is an appropriate form of representation for the deterioration profile (Frangopol et al., 2004).

Ellingwood and Mori (1993) used experimental data to describe the strength of structural member statistically and improve the base for structural assessment. Several research studies have been conducted to identify certain factors that must be included in aging assessment and deterioration mechanisms that may affect concrete structures. They concluded that corrosion of reinforcement is one of the most damaging mechanisms affecting the strength of reinforced concrete structures over time. They represented change of structural capacity in the form of a time-dependent degradation function. Ellingwood and Mori (1993) proposed that time dependent degradation status at time t , $x(t)$**,** is determined by:

$$x(t) = c_p^{\alpha} = c(t - t_I) \qquad (1.1)$$

In which c and α are experimental deterministic corrosion constants and t_I is the initiation time. One aspect of performance is the assessment of failure time. In general, the failure rate of a structure or a component will be time variant as the structure ages. The probability that the structure will fail in the next time interval is the conditional probability. Furthermore, the hazard function is seen to be the rate of change of the conditional probability of failure, given that the structure has survived to time *t*. In order to control the structural safety, time –dependent reliability analysis method (Hazard function) has been implemented (Ellingwood and Mori, 1993).

Several simple parametric time dependent functions have been used to represent degradation of flexural moment capacity and shear capacity over the time. The linear, parabolic and square root functions were utilized to model degradation of strength due to corrosion, sulfate attack and diffusion-controlled degradation respectively and the deterioration model has been used to evaluate the time-dependent reliability of a single component and a series system.

VanNoortwijk and Frangopol (2004a) characterized the structural deterioration of a dike section and a highway bridge due to ageing by two models:

- Lifetime model on the basis of the probability distribution of lifetime or time to failure.
- Deterioration model on the basis of the random variable model.

For the former different standard distribution function such as normal distribution function can be used to take into account uncertainty associated with parameters of deterioration model such as time to damage initiation, deterioration rate and initial condition. (VanNoortwijk and Fangopol, 2004 a)

As mentioned before, the structural deterioration of reinforced concrete structures is not just result of one factor, however it is identified that the corrosion is one of the major destructive factors (Bertonili et al., 2013). Since the corrosion is a long term mechanism for well-designed structures, there is only limited documentation and consistent experience available on which to draw to generate empirical rules.

In order to characterize the structural deterioration of reinforced concrete beams subject to corrosion, Melchers et al. (2008) estimated the ultimate moment capacity and stiffness of reinforced concrete beams under reinforcement corrosion by developing a reliability model that relies on the estimation point in time at which significant corrosion is initiated while it has been assumed that the corrosion rate is constant in time. Linear models used to model degradation of ultimate bending capacity and stiffness of a reinforced concrete single beam have been implemented. It is concluded that the ultimate bending moment capacity and stiffness of a reinforced concrete beam can be obtained as a function of time of exposure. Theoretical results have been compared with experimental data.

It is concluded that the model was able to provide reasonable estimates of deterioration process of ultimate bending moment capacity and deflection stiffness (Melchers et al., 2008). This model cannot take into account the uncertainties associated with degradation process in time as the bending moment capacity and stiffness have been modeled by a linear function.

An alternative method to evaluate the safety of structures can be by using damage processes. When materials, which are used to construct structural components, are subjected to harsh condition such as cold and hot working processes, temperature variations, chemical actions, radiation, mechanical loading, microscopic defects and cracks may develop inside the materials. Such damage causes reduction in strength and stiffness that may lead to failure and shorten the lifetime of structures. Such deterioration process in mechanical properties of the material is known as a damage process (Valliappan and Chee, 2008).

Owing to the major influence of damage on material properties, a number of studies have been conducted on modeling of crack growth in a structure under various loading conditions. Valliappan and his assistants have been one of the leaders in developing numerical methods for the structural analysis using damage processes. For instance, form the view point of variety of damage processes concept, Valliappan and Zhang (1996) addressed the problem of the effect of microscopic defects and cracks within materials in order to study the behavior of structural components under different loading conditions. A formulation for elasto-plastic model of damage process was developed based on the principles of thermodynamics and associated finite element method. (Valliapan and Zhang, 1996). The issue of the proposed model is that the model cannot take into account the current condition of structure in order to represent the deterioration model. However, it is identified that the proposed elasto-plastic model of damage can be implemented to define the realistic structural failure mode.

Due to ageing, degradation of materials accumulates over the time by various damage processes that depend on the specific operating environmental and service conditions. A dynamic two-dimensional finite element method joined with damage process has been developed to assess damage initiation and propagation of an aged mechanical structure (Valliappan & Chee, 2008).

The quantified age-related degradation factor is then included in the damage process and finite element model. The ageing degradation of the component capacity has been formulated as:

$$R(t) = R_0 G(t) \qquad (1.2)$$

Where R_0 is the component capacity in the original state and $G(t)$ is a time dependent degradation function defining the fraction of initial strength remaining at time t (Valliappan & Chee, 2008).

1.2.2 Management of Highway Infrastructure

Due to evolution of deterioration process, the structural behavior of highway structures is not static over the time and as a result there could be loss of structural capacity and serviceability of various components. It is the infrastructure authority's responsibility to be assured of structural safety and serviceability. In addition, the highway network is a dynamic system with changing user demands, some of which may be reflected in changes to relevant code and standards (Das, 1999). Hence, it is necessary to have an adoptable assessment method which can be updated over the structural lifetime and take into account evolving structural condition.

In order to ensure that the serviceability and safety of structures at an acceptable level, actions to slow or stop the deterioration process must be taken in the form of cost effective and sustainable plan that supports the safe operation of the structure while delivering the required levels of service. Two major actions that can deliver improvements are maintenance and repair (Grall et al, 2002).

In order to have an efficient maintenance and repair plan, a bridge or a group of bridges needs to have a lifecycle plan which would describe the long term strategy for managing a group of similar structures with a point to minimizing whole life cost, while providing the required levels of performance and is used to identify maintenance cycles and intervention thresholds. (UK Roads Liaison Group, 2005)

Life cycle plans differ depending on the management strategy. Typical types of strategies can be identified

- Enhancement strategy to enhance the condition and includes upgrading
- Steady strategy to maintain the current condition
- Manage deterioration strategy to manage and control the deterioration so that condition may deteriorate but not fall below a predefined condition level.

Manage deterioration strategy is generally considered if decommissioning or replacement is planned in the near future. (UK Roads Liaison Group, 2005)

1.2.3 Management Processes

Highway structures management system includes four main contexts.

I. Maintenance
II. Repair
III. Inspection
IV. Assessment

The maintenance and repair are explained in this section while the assessment and inspection categories, that have a key role in present research, are described in details in sections 1.2.4.2 and 1.2.4.3, respectively.

I. Maintenance

Maintenance includes actions whose purpose is to slow down or prevent the deterioration process. The maintenance plan is undertaken to identify needs, prioritize maintenance and provide cost effective and sustainable work plan (UK Roads Liaison Group, 2005).

Since the structural condition can change over the lifetime in respect to various ageing factors, a set of different actions as maintenance plan can be taken. The maintenance plans are generally classified into three types in terms of actions efficiency and action interval.

1. Routine maintenance reflects to minor works carried out on a regular or cyclic basis that help to maintain the condition and functionality of the structure and reduce the need for other action. In general, it includes tasks e.g. cleaning of drainage and expansion joints system, greasing of metal bearings or removing vegetation. The experts identified that, whilst many of routine maintenance tasks

are fairly minor in themselves, failure to carry them out may lead to deterioration of the structure and need more costly repairs.

2. Preventive maintenance is work carried out to maintain the condition of structure by protecting it from deterioration or slowing down the rate of deterioration. By timely intervention preventive maintenance reduces the need for essential one. In general, it includes tasks e.g. repainting, minor defect repairs, cathodic protection and re-waterproofing.
3. Reactive maintenance which can be sub-divided as emergency and essential maintenance. The emergency maintenance is a reaction to some emergency accident that happened on the structure and requires an immediate work, while essential maintenance is a major structural repair work and especially work that undertaken when part or all of the structure is considered to be, or about to become, structurally inadequate or unsafe .e.g. major concrete or steelwork repairs and scour repairs (UK Roads Liaison Group, 2005).

However, it is possible to make a maintenance plan for a structure with combination of maintenance types at different time interval.

For instance, the maintenance plan for a structure can be prepared in terms of planed interval in three different stages as

- Complete maintenance plan to covers lifetime
- Forward maintenance plan to covers next 1 to 3 years period
- Annual maintenance plan

The last two sets of actions should be updated every year to take account of the updated information on structural condition and describe the work to be carried out and when. It is evident that the long established inspection process outcomes can be used to reconcile the quality and diversity of structural behavior models. Once prediction of structural deterioration process is available it is possible to revise maintenance plan effectively (Stratt, 2010).

II. Repair

As different bridge components have different life time, alternative plans can be considered to deliver the safety objectives. Some components of bridge have finite service life so they have to be renewed at the relatively short intervals e.g. bearings and expansion joints, while for other components of existing bridges the work needs to bring components up to appropriate current standard e.g. strengthening or waterproofing. Various policies may have resulted in change to standards or change in requirements. When usable life of a component ends, it has to be replaced with a new component (Stratt, 2010).

As dedicated budget for maintenance and repair of a network is often limited, design appropriate maintenance plan to keep the structural safety at an acceptable level is a critical issue. An important concept in maintenance and repair plan modeling is that of life-cycle cost, where the effects and costs of a particular maintenance and repair policy are considered over the total expected lifetime of structures (Yang et al., 2006)

Every maintenance and repair plan tries to reflect the future condition of the structure. Repair solutions can differ depending upon the extent and the type of damage, when repairs of a structure which is intact and useable should be carefully detailed so that they are effective and can be executed safely and with the minimum of disturbance of users of the structure. A wide range of repair techniques –detailed in British standard BD27 and BA35- such as repainting, replacement of concrete for decks, bonding steel plated to concrete bridges deck overlays and deck patching can be implemented on highway bridges in regard to many factors such as accessibility, cost, and efficiency.

III. Maintenance and Repair Plan

In order to design an appropriated maintenance and repair plan, it is necessary to characterize the future performance level of the structure which is associated with various degree of uncertainty. It is often proposed to use a probabilistic approach, in order to take into account structural condition uncertainties (Frangopol et al., 2004). During the last decades, a large number of papers on maintenance optimization models, mainly focusing on the mathematical aspects, have been published as Kwon and Frangopol (2012), Kim et.al. (2013), Kallen and Van Noortwijk (2004).

Farngopol et al. (2004) presented a number of probabilistic models of maintenance and optimization of the life-cycle performance. The maintenance models use the structural deterioration profile to determine the optimal times for maintenance actions and repair. It is identified that maintenance and repair actions can be periodic or aperiodic. The proposed approach in Farngopol et al. (2004) to determine optimal maintenance policy, using the reliability index to estimate structural deterioration profile, has a life-cycle cost function associated with risk ranking. Risk ranking model can be used to identify the most critical bridges in the network while the life-cycle cost function is the preferred model when decision makers are not only concerned with safety, but also with costs. It has been identified that the risk ranking model should be limited to inspection prioritization at the time of evaluation. It does not account for the full life-cycle of the structure (Frangopol et al., 2004).

1.2.4 Current Practice for Ageing Highway Structures

Many practical codes are proposed around the world, in order to provide a guidance on highway structures supervision duties and the development of recognized good management practice. The UK Management of Highway Structures and AASHTO (standard specifications for highway bridges-1983) are the most comprehensive highway structures codes. According to the UK code, the management of highway structures is categorized to three major themes.

- Asset management & resource accounting
- Maintenance and repair planning and management
- Engineering processes

These processes are meant to be supported by appropriate data and information. As mentioned before, we will focus on the engineering processes, particularly on the inspection and monitoring context and assessment.

The engineering process includes five important contexts as:

I. Design
II. Construction
III. Inspection and Monitoring
IV. Structure Assessment
V. Maintenance & repair (UK Roads Liaison, 2005)

All these engineering processes are associated with different types of uncertainty which will be explained in Chapter 2; however it should be noted that types of uncertainty are associated with design and construction processes are different with uncertainty associated with other three stages. It can be assumed that uncertainty associated with design and construction processes are partly due to uncertainty in the primary information about geometry and material properties and partly due to uncertainty of the physical or mechanical model. The value of this type of uncertainty can be reduced by improvement in methodology and standardization (Birolini, 2013). More information on uncertainty modeling and classification will be provided in Chapter 2. In order to identify the structural condition of existing structures in the future, we will just focus on the modeling of uncertainty associated with inspection processes and the methodology to take into account the inspection data uncertainty in deterioration mechanism modeling.

1.2.4.1 Data Available From Inspections

To demonstrate the type of data that is available for an existing structure from inspection, we have considered a reinforced concrete bridge as an example and categorized the inspection outcomes in Tables 1.4. Inspection types and defect types are demonstrated. The Table 1.4 shows information for four components with different inspection types (Frischmann and Partners 1973; DMRB 3, 2009).

Table 1.4 Sample outcome of inspection of a deck and pavement (Frischmann and Partners, 1973; DMRB 3, 2009)

Inspection Type	Time	Deformation	Crack	Rupture	Damage of surface	Leakage /dampness	Discoloration	Insufficient cover of	Honeycombing of concrete	Delamination of concrete	Spalling	Corrosion of reinforcement	Washout concrete
Safety	Weekly	V	V		V	V							
General	Two years	V	V	V	V	V	V		V				
Principal	Six years	V/M	V/M	V	V	V	V	V/M	V	V	V/M		
Special	Var	M	M	J	M	M	J	M	J	M	M	M	J
Routine	Var	V/M	V	V	V				V	V	V		

V=visible defects M=measurable J=expert judgment Var=variable

Annotation in Table 1.4 is used to define the defect features. The visible defects (V) can be detected without using any specific equipment at time of inspection. However, it can be identified that some types of defects can be measured (M). While other types of defects such as discoloration cannot be measured and their extent can be assessed just on the basis of the expert judgment (J).

According to the inspection data classification in the Table 1.4, it can be identified that the many issues regarding the available data are

- The available data is in most cases not quantified; hence they cannot used for probabilistic methods to provide a plausible structural condition model.

- The measured outcomes can be used to establish a certain probabilistic deterioration model.
- The visual inspection outcomes could seldom be used to establish appropriate deterioration model.
- Visible defects whether they are measurable or not are associated with uncertainty
- The visual inspection technique with standard interval for some components, such as the deck that are visible, is an appropriate technique.
- The efficiency of visual inspection results is not usually equally effective for other components such as bearings.
- There is limited valuable structural condition information for specific components for many inspection types if the inspection regime is just considered with the restricted schedule.

1.2.4.2 Structural assessment

The purpose of assessment of highway structure is to determine its ability or capacity to carry the loads which are imposed upon it and which may reasonably be expected to be imposed upon it in the future. The structural assessment is needed when there are significant changes to the usage, loading and/or structural condition. The assessment should consider all available current information, taking account of the known condition of similar structures, their inherent strengths and weakness. The information needed for structural assessment can be derived from inspection outcomes. It can provide valuable information for managing the safety and serviceability of infrastructure (UK Roads Liaison Group, 2005).

1.2.4.3 Inspection and Monitoring

In practice, the primary purpose of inspection and monitoring is to confirm that a structure is safe for use and fit for purpose. The aim of inspection regime is to:

1. Provide data on the current condition, performance and environment of the structure e.g. severity and extent of defects, material properties and loading.

2. Inform analysis, assessments and processes e.g. change in condition, cause of deterioration, rate of deterioration etc.
3. Compile, verify and maintain inventory data e.g. structural type, dimensions and location.

It is identified that data provided by inspection has vital role to develop an efficient management strategy. For this purpose, an inspection regime that could be supplemented by testing and monitoring where appropriate is needed (UK Roads Liaison Group, 2005).

In an ideal inspection regime, the combination of inspection techniques with various frequencies at which they are applied, should be determined by considering adequate criteria in an objective manner. Any defect, that may cause an unacceptable safety or serviceability risk, should be detected by an appropriate inspection technique. However, the current inspection regimes rely on expert judgment and rarely take account on the quality of inspection techniques. (Attoh-Okine and Chajes, 2003; Brodski and Ponomarev, 2006) Furthermore, different types of inspection might be focused on known or suspected areas of deterioration or inadequacy (UK Roads Liaison, 2005).

There are several inspection codes that provide guideline for the infrastructure managers around the world. In this study, the two most comprehensive codes for management of highway structures around the world, UK-DMRB (3) and AASHTO 2011 are considered as representative for industry standard. We particularly focus on the inspection types that are commonly used for highway bridge inspection to demonstrate areas for improvement and uncertainty embedded within processes.

1.2.4.3.1 Frequency of inspection

The frequency of inspections in the UK code is recommended in accordance with the type of inspection. Information about inspection types, used in UK code, is summarized in Table 1.5.

Table 1.5 UK inspection regime (DMRB 3, 2007)

Inspection Type	Interval Time	Description
Safety	Weekly	Identify obvious deficiencies and cursory check of the visible part. It is undertaken at frequencies, which ensure timely identification of safety related defects and reflect the importance of a particular route.
General	Two years	Provide information on physical condition of visible elements without any special equipment. It is recommended to carry out not more than 2 years after the previous General or Principal inspection.
Principal	Six years	Provide information on physical condition of all inspect able parts with close examination. It is undertaken not more than 6 years after previous Principal inspection.
Special	Variable	Provide detailed information on a particular part
Routine	Variable	Provide information required to undertake bridge assessment. The schedule of this type of inspection should be set by the manager.

The recommended inspection interval by UK code is applied for most highway structures but in some circumstances, changes are allowed specifically for General and Principal Inspections, which are undertaken frequently and only for specific components or features and with strict upper limit interval time.

Interval time increments are restricted by UK code i.e. for General inspection interval inspection cannot exceed 3 years and for Principal inspection this increment would be restricted to12 years (DMRB 3, 2007).

1.2.4.3.2 U.S inspection regime

The inspection types in AASHTO 1983 are categorized in five types like UK code but there are some differences such as inspection interval, inspection intensity and type of data in comparison to the UK code. Each type is carried out for specific position with varied outcomes (AASHTO, 1983). Information of AASHTO inspection regime are indicated in Table 1.6. (For more information see Appendix-E)

Table 1.6 AASHTO inspection regime (AASHTO, 1983)

Inspection Type	Interval Time	Description
Initial	Once in lifetime	Identify initial deficiencies which might not have been present at time of construction. It provides a basis for all future inspections and modifications to the bridge.
Routine	Two years	Identify unusual conditions or changes without any special equipment.
In-Depth	Five years	Provide information on physical condition of all inspect able parts with close examination. It can be follow-up routine inspection.
Special	Variable	Monitor new types of structures to develop information database.
Damage	Variable	Provide the information of damage extent after collision, fire, flood, and etc.

In order to establish an improved model of structural deterioration process in this research, the initial inspection outcomes can be considered to estimate the initiation time of a structural deterioration process while the in-depth inspection outcomes can be taken to improve characterization of the deterioration process over the lifetime which depends on the accuracy and interval of inspections. Special inspection outcomes usually can be employed to model the deterioration process of a particular damage at a specific location on the structure.

1.3 Issues of current inspection regime

Having considered information provided by inspections in previous sections it is possible to identify issues of current inspection regimes.

1) Quality of current inspection outcomes is highly variable over the lifetime due to the technique specific characteristics.
2) Current inspection outcomes are difficult to use as quantitative, i.e. once cracking is advanced, the estimate of the scale of cracking will be highly variable if visual inspection is the selected technique, just as demonstrated in AASHTO (2001).
3) There are no current guidelines for effectiveness of alternative inspection techniques.
4) The inspections are prescribed with very strict interval times regardless to the deterioration progress.

1.4 Summary and Conclusions

In order to have an efficient bridge management system, an optimal plan of repair and rehabilitation of bridge system have to be established over the bridge lifetime with respect to the limited dedicated budget. Since the inspection outcomes have key role in the bridge management, in this chapter the current inspection regimes are investigated. The issues of current inspection regimes are summarized. It is identified that the inspection outcomes are associated with uncertainty. Thereby, the probabilistic models should be used to estimate the deterioration process on the basis of the inspection outcomes. Moreover, it is concluded that an adaptive inspection regime has to be used, in order to accommodate diverse sources of uncertainty in an adequate manner. However, it is also identified that various performance criteria can be used due to environmental conditions and prevailing policies.

The benefit of using an adaptive regime would be that the manager would have a choice to use appropriate inspection techniques to provide quantified and usable inspection outcomes and update the estimate of the component condition or structure as a whole over the life cycle.

Chapter Two

2. Uncertainty Modeling for Structures

2.1. Introduction

The concern is modeling of deterioration in existing structures. Since available data is limited and diverse, it is required to establish a method to take into account the uncertainty associated with physical system and inspection process. As large majority of bridges in transport network are reinforced concrete short to medium span bridges, the first step is to identify the types of uncertainty associated with inspection outcomes from reinforced concrete structures and their parameters (Stratt, 2010).

There exists a large number of propositions for the characterization of different types of uncertainty (Haldar and Mahadevan, 1999). It is necessary to differentiate between types of uncertainty due to different sources of uncertainty e.g. aleatoric natural variability and epistemic that reflects modeling and data availability. Another reason to make a distinction between different types of uncertainty is that some types of uncertainty such as epistemic uncertainty might be reduced by collecting more data as this type of uncertainty is caused by a lack of knowledge (JCSS, 2008; Haldar and Mahadevan, 1999).

In order to model the structural deterioration process of a bridge, the deterioration factors have to be considered. If the current status of deterioration is to be established on the basis of inspection outcomes it has become evident that quality and consistency of the data acquired needs to be taken into account (Mahut and Woodward, 2005).

The deterioration of structures can be represented using deterministic or probabilistic approach.

However, when uncertainty is present, the deterministic approach cannot provide an appropriate model and probabilistic modeling should be considered (Frangopol et al. 2004). In this chapter options for probabilistic modeling are reviewed and their suitability evaluated.

2.2 Uncertainty modeling

Uncertainty is defined as a measure of imperfect knowledge or probable error that can occur during data collection process, modeling and analysis of engineering systems. A significant body of research on uncertainty associated with engineering systems is focused on types of uncertainty and proposed models to take into account uncertainty. Uncertainty associated with engineering systems is inherent characteristic that cannot be avoided in defining the construction parameters and main prediction models for the systems. What does it take to take into account uncertainty associated with parameters and system so that it is evaluated on the basis of knowledge of the system and the experience (Lemaire et al., 2009).

Both, aleotoric and epistemic uncertainties, can be subdivided to secondary types. According to Kikuchi and Pursula (1998), fuzzy set theory can be used to represent aleotoric uncertainty while evidence theory can be used to deal with epistemic uncertainty among the classical probabilistic approaches. These theories are complementary to classical probabilistic approaches when dealing with human perception and decision processes. It is important to identify the most suitable mathematical model with respect to the nature of uncertainty (Kikuchi and Pursula, 1998).

Usually it is impossible to find exact approach to characterize types of uncertainty, and concepts like ‘intuition’, ‘expert opinion’ and ‘engineering judgment’ are often used.

The important question still remains if different types of uncertainty can be treated in the same way or different procedures should be implemented. The answer can be attained with respect to the concept of interpretation of probability. The Joint committee of Structural Safety probabilistic model code (JCSS, 2008) recommends three possible approaches:

- The frequentist’s interpretation
- The formal interpretation
- The Bayesian interpretaion

According to JCSS probabilistic code, the frequentist’s interpretation is straightforward and lets only observable data to record the domain of probability theory. It is evident that this interpretation can be used in statistical events (JCSS, 2008).

Owing to insufficient amount of statistical or theoretical evidences in the field of infrastructure management, it should be clear that such an interpretation is not feasible in this application.

The formal interpretation gives full credit to the fact that numbers used in reliability and risk analysis approaches to characterize uncertainty associated with response function are based on ideas and judgment rather than statistical data. Such approach is believed to be more appropriate approach compared to deterministic approach in terms of uncertainty modeling (JCSS, 2008).

It is essential that the values of the probabilistic model have meaning in deterministic model. In the third approach, which is named Bayesian interpretation, probabilities are considered as best possible expression of the degree of belief in the occurrence of a certain event. The results of Bayesian interpretation describes the probability of an event based on prior knowledge of conditions that might be related to the event. With Bayesian interpretation the theorem express how a subjective degree of belief should rationally change to account for availability of related evidence. It should be noted that in this approach, two types of uncertainty (epistemic and aleatoric) are treated in the same way. The benefits of using this approach are, firstly it enables calculation of probability of an event with combination of several sources of evidence, and secondly it provides a fully developed theory of probability at ones disposition for both types of uncertainty (JCSS, 2008, Haldar and Mahadevan, 1999).

In order to calculate a response variable which is generally based or empirical relation between uncertain basic variables, a model can be described as a functional

$$Y = g(X_1, X_2, X_3, \dots, X_n) \tag{2.1}$$

Where Y is the response variable, g () is the model function and X_i are the basic variables. The response variable Y can be predicted without error, if the model function is perfect function with no uncertainty. However, this is not normally the situation. The model function is usually associated with uncertainty. This may be result of lack of knowledge, or reflect simplification of the model. There is difference between the predicted response variable Y and actual value Y', even if it is assumed that the value of basic variables are given.

The actual response variable Y' can be denoted as:

$$Y' = g(X_1, X_2, X_3, \dots, X_n, \varepsilon_1, \varepsilon_2, \varepsilon_3, \dots, \varepsilon_n) \qquad (2.2)$$

The variables ε_i are random variables which reflect the uncertainty associated with the basic variables. Their statistical properties can in some cases be derived from a set of laboratory experiments or measurements in situ (JCSS, 2008, Haldar and Mahadevan, 1999).

2.2.1 Classification of uncertainty

There are various classifications of types of uncertainty. One is to distinguish between 'aleatoric' and 'epistemic' uncertainty. Any type of uncertainties can be referred to in respect to its source. For instance, if the aleatoric uncertainty dominates, redesign or reconstruction may be recommended, while in case that epistemic uncertainty dominates, we can start investigation of the system, process or mechanism to increase the knowledge. It is identified that randomness is an inherent part of nature, it is not possible to reduce the aleatoric uncertainty. However, it is demonstrated that the aleotoric uncertainty can be gradually transformed into an epistemic uncertainty, which may be reduced by measurement and updating procedures (Ayyub, 1997).

Another alternative for uncertainty classification, for the purpose of structural engineering, is

- Physical uncertainty
- Statistical uncertainty
- Model uncertainty

These types of uncertainty are explained in the next sections

2.2.2 Physical uncertainty

The physical uncertainty can be associated with experiments or random temporal or spatial fluctuations inherent to natural phenomena such as loads, material properties, dimensions, etc. However, physical uncertainties in structural analysis can be quantified only by examining sample data. (Haldar and Mahadevan, 1999).

It is identified that physical uncertainties associated with variables can have two different sources. The physical uncertainty can arise from errors in data , measurement inaccuracy e.g. physical uncertainty associated with crack length or inadequacy of data handling and transcription as first sources of this type of uncertainty while uncertainty associated with physical functions can be consider as second sources of physical uncertainty e.g. uncertainty associated with wind speed. It is recognized that physical uncertainty arise from second sources can be reduced by more precise methods (JCSS, 2008).

2.2.3 Statistical uncertainty

To apply probability theory to an engineering process, we study the observed data of that process. The collection of all possible observations of a process is called a statistical population. The population itself often cannot be totally observed, because the process is time dependent. Most often, only a portion of the population is observed which is called a sample.

An observation can be characterized by one or more variables that are, to a certain degree, unpredictable, random variables.

In order to establish a probabilistic model, it is necessary to firstly select an appropriate distribution function, and then calculate the value of distribution parameter. For instance, the identification of a correct distribution function depends very much on the accuracy with which its parameters can be estimated. Parameter uncertainty is caused by lack of data, poor-quality data, or an inadequate method of parameter estimation. However, the data themselves may have associated with physical uncertainty.

The distribution parameters such as mean value can be obtained with consideration to the sample data. There are two approaches to estimate parameters. An estimate of population parameter given by a single number is called point estimate of the parameter. The other approach is an estimate of a population parameter given by two numbers between which the parameter may be considered lie is called an interval estimate of parameter.

The statistical uncertainty reflects the amount of sample data or in general, the amount of data and any prior knowledge. This uncertainty arises solely as a result of lack of information (Haldar and Mahadevan, 1999).

Three sources of statistical uncertainty are defined as:

1. A limited number of observations or test results which cause uncertainties in the estimation of statistical parameters
2. Neglecting systematic variations of observed variables
3. Neglecting possible correlations

The statistical uncertainties can normally decrease by increasing test and observational efforts. (Ayyub, 1997)

For instance, the yield strength of steel can be consider as parameter, which is a random variable and it is associated with statistical uncertainty. It is recognized that the statistical uncertainty arises from parameter estimation from sample that is too small due to the limited observations.

In respect to the focus of current work on reinforced concrete bridge decks, due to the variety of concrete material properties, the compression strength of concrete as a random variable is associated with statistical uncertainty.

2.2.4 Model uncertainty

The performance of a structural system can usually be modeled by physical or mechanical model in conjunction with empirical relations. Engineers use mathematical model with regard to outcome quantities and basic variables. The structural models such as ultimate flexural moment capacity of a reinforced concrete beam, in general, are result of many assumptions. Model uncertainty reflects the inability of the simulation model or design technique to represent precisely the structure's true behavior. Thus, model parameter uncertainties reflect the variability in determining the parameters to be used in a model. (Haldar and Mahadevan, 1999).

In other words, it is frequently not possible to obtain accurate response model for typical structures even if the parameters can be estimated precisely. It is acknowledged that the simplifying assumptions, unknown boundary conditions, unknown effect of other variables and their interaction, which are not considered in the model, are reasons of occurring model uncertainty (Haldar and Mahadevan, 1999).

As focus in this research is on the prediction of ultimate flexural moment capacity of reinforced concrete section, it is necessary to identify the uncertainty types associated with the prediction model. The prediction of the phenomena in the future involves model subject to natural variability, model uncertainty and statistical uncertainty.

The models available for prediction of deterioration of reinforced concrete flexural moment capacity often tend to lose their precision rather fast so can be predicted only with significant uncertainty (Mori and Ellingwood, 1992). Due to discrete and qualitative nature of present measurements such as inspection outcomes physical uncertainty within predictive model is accompanied with model and/or statistical uncertainties. The main future is that uncertainty about the future can generally not be decreased through research, thus certainly not by using inspection outcomes. (JCSS, 2008; CIB report, 1986)

In many cases, it is sufficient to model the uncertain quantities by random variable model with given distribution functions, while distribution parameters estimated on the basis of observed data (JCSS, 2008).

However for structural deterioration modeling, it is identified that random variable model is not the most appropriate model and needs to consider other alternatives e.g. stochastic processes (Frangopol et al., 2004).

2.3 Inspection Data Uncertainty

In recent years, modern technology has enabled greater variety of monitoring and inspection techniques and more precise data. The NDT techniques such as visual inspection, ground penetrating radar, ultrasonic inspection and electrical methods are an essential tool for assessment of a structure, they also can bring in additional uncertainty to the prediction models. During the last decade, a lot of research has been conducted to identify and characterize the uncertainty associated with inspection data.

Zhang et al. (2001), Melchers et al. (2008), and Straub and Der Kiureghian (2010) used the probabilistic method to characterize inspection uncertainty e.g. probability of detection (POD), probability of false alarm (PFA) and measurement error. In 2001 AASHTO carried out an extensive research (AASHTO, 2001) to characterize the uncertainty associated with visual inspection data. This is useful to illustrate the variety of uncertainties associated with specific inspection outcomes.

2.3.1 Variability of Visual Inspection Outcomes (AASHTO, 2001)

As mentioned in Chapter 1, the most common and available inspection type to detect defects on most bridge members is visual inspection with 2 years interval. It was observed that consistency of visual inspection between bridge inspectors does not come naturally and it is a result of training, quality control and shared experiences (AASHTO, 2001).

According to the AASHTO investigation, the factors, which can influence the visual inspection outcomes, are categorized as follows:

1) Subjective factors (visual quality, color vision…)
2) Physical and environmental factors (lighting, background noise…)
3) Task factors (inspection time, viewing area…)
4) Organization factors (number of inspectors, training…)

The In-Depth and Routine inspection outcomes have been collected from different states in US to define reliability of visual inspection technique. It is identified that the accuracy of both inspection types could be further increased by considering the known factors. However, bridge design practices should put more consideration on the comfort with which the bridge could be inspected. The method to evaluate the influence of each factor in such a way that quantitative data can be collected is presented in AASHTO, 2001. It has clearly emerged that the inspection outcomes can be more sensitive to some factors such as rushed level, light intensity, and structure complexity than the others (AASHTO, 2001).

It is recognized, with respect to the survey results, that In-Depth inspection is not likely to detect and identify the specific types of defects for which this inspection is sometimes prescribed. It has been indicated that there is a relationship between In-Depth inspection outcomes accuracy and factors such as, time to complete inspection, inspector comfort with access equipment and heights, structure complexity and accessibility and number of annual bridge inspections. However, further factors still remain, which can influence the inspection outcomes, to be investigated. For instance, it is reported that inspectors who find fewer than the average number of defects found on one bridge are likely to do so, on other bridges (AASHTO, 2001).

In addition, more evidently defined inspection procedure that outlines systematic search criteria and techniques may increase inspection accuracy.

Study has proposed that revising the condition rating system may significantly increase the accuracy and reliability of the Routine inspection outcomes; while the accuracy and reliability of In-Depth inspection could be increased through increasing training of inspectors in types of defects that should be identified and methods that would frequently allow this identification to be possible. More information about the condition rating system is in Appendix-D.

A quick review of the analysis methods of significant information, which are used in the AASHTO report, is presented here to define the variety of uncertainties associated with visual inspection outcomes.

The questionnaire forms have been used to collect quantitative and qualitative inspectors and inspection condition information, and the inspectors have been asked to complete specific tasks. The common statistical methods are used to analyse the collected information. As condition rate is a discrete random variable, the statistical analysis results can represent in form of probability histogram. However, it is identified that direct extrapolation of the data to population is not statistically justifiable.

One means of extrapolating a sample to a population is by using theoretical probability distribution based on data from the sample, normal distribution was proposed (AASHTO, 2001).

It has been identified that there are many factors that can influence the accuracy of visual inspection outcomes. The information on inspection factors influence on the inspection outcomes accuracy might be useful for infrastructure managers. It can established how often and to what extent condition ratings vary from the reference rating. It is indicated that

AASHTO survey data can be used to derive the level of inspector consistency between different elements of a bridge (AASHTO, 2001)

Two inspection factors are considered here to illustrate the variety of uncertainty associated with visual inspection outcomes. Figures 2.1a and 2.1b illustrated the discrepancy between condition inspection ratings and reference condition for two bridge components at three locations. The reference condition is defined as condition that is estimated in laboratory environment. We can observe the influence of the maintenance level of deck and superstructure on condition inspection outcomes. The features of each task in the graphs are tabulated in Table 2.1.

Table 2.1 AASHTO Tasks Information (AASHTO, 2001)

Task	Applied inspection technique	Bridge type	Superstructure type	Width(m)	Span(m)	location
B	Routine	Single-span concrete	T-beam	22.35	6.81	Pennsylvania
C	Routine	Single-span concrete	T-beam	21.34	6.65	Pennsylvania
D	Routine	Single-span concrete	Rigid-frame	33.22	12.88	Pennsylvania

In Figures $\boldsymbol{I_i}$ expresses the inspection influence factor. It is identified that $\boldsymbol{I_i > 0}$ in both cases means that the inspection outcomes are overestimated. $\boldsymbol{I_i = 0}$ means the condition rating is equal to reference condition rate. The regression analysis of condition ratings is presented as inspection influence factor $\boldsymbol{I_i}$. The regression analysis results for predicting bridge condition rating can be considered to determine in three sections. The first, second and third section present the developed regression equation solely in terms of the inspector factors, and combination of inspection and inspector factors, respectively (AASHTO, 2001).

Some interesting trends can be observed in the results. First, when a certain factor was found to only correlate with specific task, the relationship of that factor to the deck, superstructure and substructure condition ratings generally was consistent between the superstructure, deck, and substructure. However, when a factor was found to correlate with two tasks, the influence of that factor was not, in general, consistent for the two tasks. Finally, when a factor was found to correlate with more than two tasks, there was greater consistency in the influence of that factor across the tasks. More information on influence of other factors can be found in the AASHTO report itself (AASHTO, 2001).

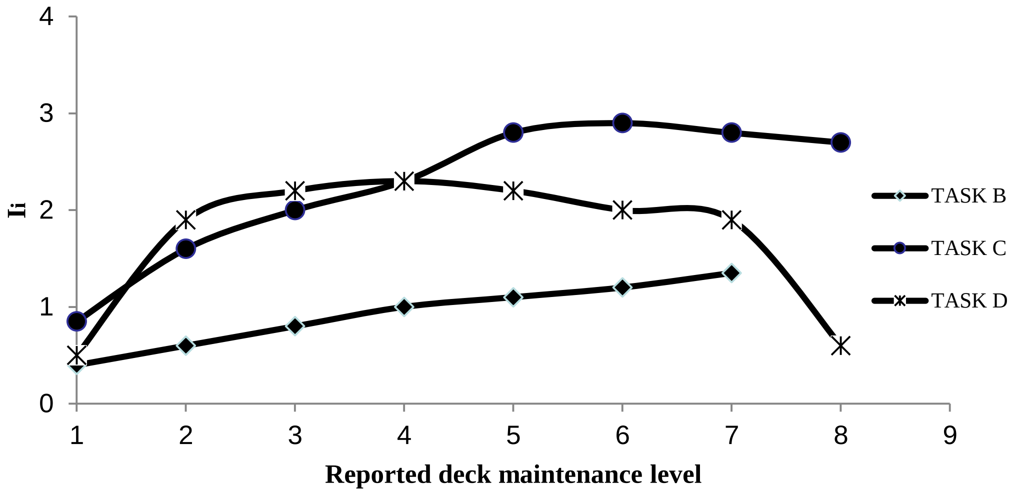

Figure 02.1a Inspection influence (*Ii*) on condition rating with reported maintenance levels on the bridge deck (Maintenance level: 1=very poorly, 9=very well) (AASHTO, 2001)

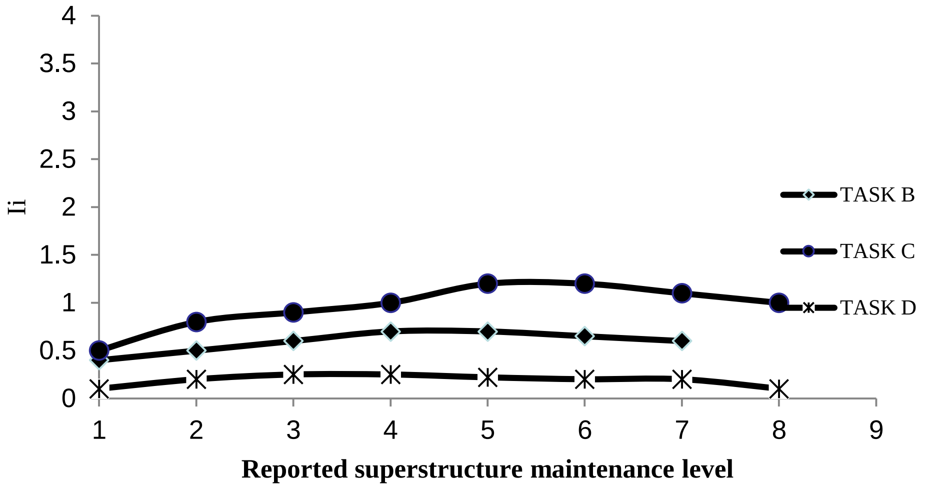

Figure 2.1b Inspection influence for condition rating on bridge superstructure with reported maintenance levels (Maintenance level: 1=very poorly, 9=very well) (AASHTO, 2001)

It is evident that if we consider outcomes in Figure 2.1a and 2.1b and use random variable to represent the condition rating, the mean values and standard deviation of condition rating model will be different between bridge superstructure and bridge deck.

In this particular case variability in outcomes is much greater for the condition ratings on the basis of the deck inspection. Full consideration of modeling inspection outcomes variability for a reinforced concrete bridge will be considered in Chapter 4.

2.4 Probabilistic Structural Analysis

In general, a structure should be designed so that its strength or resistance is greater than the effects of applied load. However, it is identified that all basic variables of load function and resistance function are in reality associated with some uncertainty as it was explained in section 2.2. Here, flexural moment capacity of a reinforced concrete beam is consider as resistance and basic variables such as material properties and geometry are associated with uncertainty.

Initial structural response models, for sake of simplicity, can be assumed deterministic. It means that value of parameters in the formula are considered with certainty and the model is perfect. However, the fact that the structural response model such as flexural moment capacity is characterized deterministic does not mean that the model is assumed to be constant. It only means that characteristics of the model vary according to given rules and not in a random way (JCSS, 2008; Ayyub, 1997).

As it is mentioned before in Section 2.2, most of basic variables are associated with uncertainties. However, in several cases it may be convenient to consider the basic variable as a deterministic variable due to low level of uncertainty such as the yield strength of steel.

It is identified that resistance of a structure (R) and load on structure (S) are random in nature and their randomness can be characterized by using standard probabilistic models. Normally, the standard probabilistic structural analysis such as First-Order methods can be applied to determine probability of failure of a structure or its components.

As mentioned before, any structural response model contains a set of variables such as resistance of the structure (R) and the applied load (S) that have to be evaluated.

Many studies have been conducted to establish methodologies, which take into account the uncertainties associated with the variables (Haldar and Mahadevan, 1999).

Several physical phenomena can cause failure such as yielding, fatigue or large deformations. Each or combination of these phenomena can lead to a failure. The probabilistic model of a failure mode is achieved by defining a function known as limited state function. Note that the limited state function is itself a random, as such:

- Limit state function > 0 defines the structure's safe domain
- Limit state function < 0 define the structure's failure domain
- Limit state function = 0 defines the limit state surface

The structure therefore has two possible states, a fully functional state and a state of failure, separated by a boundary called limited state (Hami and Radi, 2013).

Two major methods to represent the uncertainties associated with the variables of limited state function are. (Frangopol et al., 2004)

- Deterministic and semi probabilistic method
- Probabilistic method

2.4.1 Deterministic and semi probabilistic methods

It is convenient to use the term deterministic for a variable with certain value with no uncertainty. It means in the deterministic method, a function of variables with exact value represents the certain response variable. The characteristic values model is a deterministic model in which the value assigned to a variable usually has a prescribed probability of not being unfavorably exceeded during the applicable reference period. It is identified that the characteristic values could represent a statistical lower/upper bound of an uncertain parameter after consideration in broad range of observed data through inspections. (CIB report, 1985; Bulliet, 2008)

Engineers have always recognized the presence of uncertainty in the analysis and design of structural system. In the case, they need to take into account combination types of uncertainty, it is proposed to apply semi probabilistic method, and for instance classical tools like partial safety factors that are accounting for uncertainties through the use of empirical factors. Safety factors are derived based on past experience but do not absolutely guarantee safety or satisfactory performance. (JCSS, 2008; Bulliet, 2008)

Models for load and structural resistance that are presented in codes can be considered as a semi probabilistic method that is using safety factors to account uncertainty arises from material properties, e.g. nominal capacities and resistance factors and uncertainty that arises from variable loads, e.g. load factors. However, it has been identified that some uncertainties such ageing factors cannot be dealt with using code criteria.

2.4.2 Probabilistic Method in Design and Assessment

The design of structural system utilize the basic concept that the capacity, resistance, or strength of a member or a collection of members should at least satisfy applied loads, load combinations, and their effects. The primary task of design is to ensure satisfactory performance, that is, so ensure that the capacity is greater than demand during structure's useful life. Engineering design is usually a trade-off between maximizing safety levels and minimizing cost. Deterministic method does not provide adequate information to achieve that purpose. In view of the uncertainties in the problem, satisfactory performance cannot be absolutely ensured. Instead, assurance can only be given in terms of probability of success in satisfying some performance criterion. The probabilistic method is an alternative to represent uncertainties associated with parameters in structural assessment model. (Bulliet, 2008)

On the other hand, probabilistic method can supply the required information to optimum assessment process. For this reason, it is recommended to use the probabilistic method to take account for uncertainty in the assessment system (Haldar & Mahadevan, 1999).

Since early in 1960's, different probabilistic method such as structural reliability method, which is based on probabilistic point of view, have been developed to characterize the uncertainty associated with the structural assessment (Thoft-Christensen and Baker, 1982).

The probabilistic approach is based on

- The identification of all variables influencing the expression of safety criterion in respect to the safety policy

- Studying statistically the variability of each of relevant variables sometimes considered to be independent
- Deriving the most appropriate probability distribution for each of variables
- Comparing the probability of occurrence of the event to a acceptable probability of occurrence of that event

In order to provide likelihood of existence of an event, reliability analysis, classical statistics approach, Bayesian approach, etc can be used. They are extremely attractive and have produced a lot of studies and results regarding safety of structures. (Cremona and Gao, 1997; Melchers, 2003)

However, they cannot guarantee that all uncertainties will be taken into account in predicting the structure's ability to withstand the actual loads that will be applied to it, due to the model uncertainty of basic variables .Using the probabilistic method is expected to provide notional information about system behavior, the influence of different uncertain parameters on system performance and the interaction between different system components. Two models are commonly implemented to represent uncertainty.

1. Random variable models
2. Stochastic process

2.4.2.1 Random variable model

As it has been explained, using a random variable is one of the ways to characterize uncertainties. A random variable, which can take on any value, is called a continuous random variable. The probability that a continuous random variable, X is less than or equal a value x , is given by the cumulative distribution function

$$F_x(X) = P(X \leq x) \qquad (2.3)$$

The general illustration of probability density function and cumulative distribution function of a continuous random variable is presented in Figure 2.2.

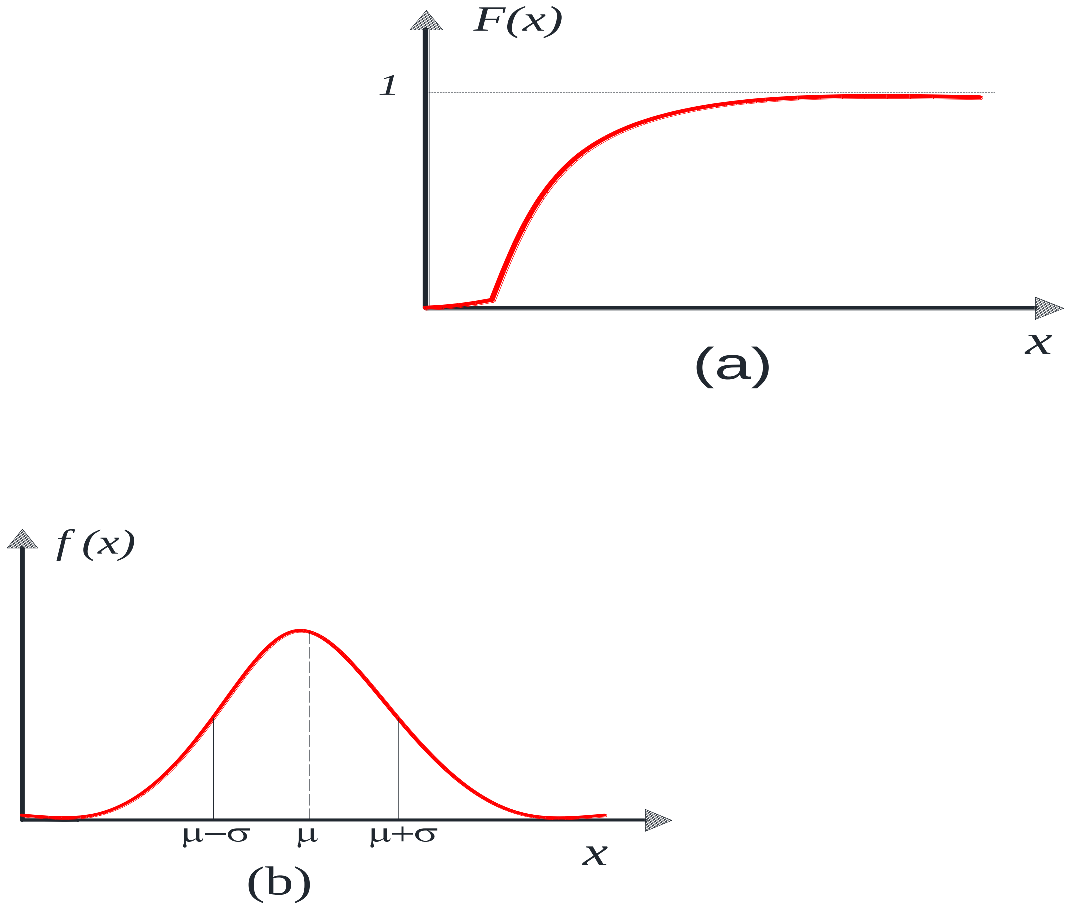

Figure 2.1 General illustration of a random variable (a) cumulative distribution function (b) probability density function (Miller et al., 1990)

For continuous random variable the probability density function is given by

$$f_x(X) = \frac{dF_x(X)}{dx} \tag{2.4}$$

If assigned values for the random variable constitute a finite set or a count ably infinite set which can only be measured as integers we have a discrete random variable. The cumulative distribution function of a discrete random variable is

$$F(X) = P(X \leq x) = \sum_{x_i \leq x} p(x_i) \qquad (2.5)$$

where $p(x_i)$ is the probability mass function given as

$$p(x_i) = P(X = x_i) \qquad (2.6)$$

There are such events whose outcomes are a vector of random variables. These events have in common that there is a relation between the random variables that we measure, and by describing them only one by one, we do not get all the possible information.

In order to define the correlation of random variables, a two dimensional vector is considered. The correlation coefficient is given by

$$\rho(X, Y) = \frac{COV(X,Y)}{\sigma(X)\sigma(Y)} \qquad (2.7)$$

Where $\rho(X, Y)$ is correlation coefficient, $COV(X, Y)$ is covariance of two random variables X, Y which indicates the degree of linear relationship between two random variables and $\sigma(X)\sigma(Y)$ are standard deviations of random variables. The correlation coefficient can only takes values in the$[-1,1]$. If $\rho = 0$ it is implied that there is no linear relationship between two random variables and two random variables can be considered to be uncorrelated, otherwise it should be determine the joint density function in order to represent the uncertainty of vector of random variables. (Olofssn and Andersson, 1963)

If two dimensional vector of discrete random variables (X, Y) is considered then the joint distribution function is given by

$$F(X, Y) = P(X \leq x, Y \leq y) \qquad (2.8)$$

and the joint probability mass function is

$$p(x_i, y_k) = P(X = x_i, Y = y_k) \qquad (2.9)$$

The conditional probability density function and the probability density function of a random variable that belongs to a given area are concepts which are explained later.

2.4.2.2 Stochastic process

In order to model a number of various phenomena where the quantity of interest varies through time, such as deterioration profile, the stochastic process is often recommended. It can be said that the outcome is denoted by function of time and possible outcomes which for any time $t(t \geq 0), X(t)$ is a random variable in the sense of a random variable description and can be considered as the collection of all possible records of variation of the observed quantity in time. (Helstrom, 1984; Olofssn and Andersson, 1963; Ross, 1996).

The general illustration of a stochastic process is presented in figure 2.3.

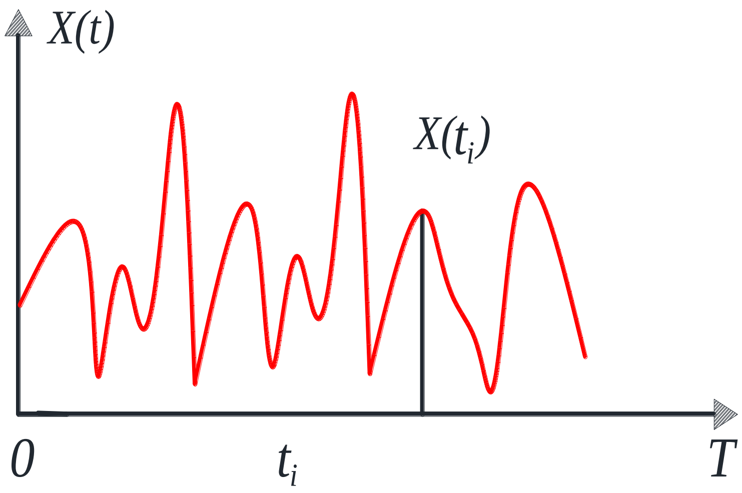

Figure 2.2 General illustration of a stochastic process (Ross, 1996)

A particular outcome of the event which is called a sample function is considered here. In order to imagine the sample function, the process $X(t)$ at any finite number of times is presented in the form $X_1 = X(t_1), X_2 = X(t_2), \dots, X_n = X(t_n)$ that a single possible value of each random variable of stochastic process included in sample function$(-\infty \leq X_i \leq \infty, i = 1, n)$. Their nature of randomness is specified, through a joint probability density function $f_{X_i}(x_1, x_2, x_3, \dots, x_m)$ (Helstrom, 1984; Olofssn and Andersson, 1963,; Ross, 1996).

The stochastic process$\{X(t), t \geq 0\}$ is defined a collection of all such joint probability density functions for all values of *m* and for all possible sampling times. Although here t is considered as the time, in important applications the parameter in the denoting function may represent a spatial coordinate or a number of spatial coordinates, together, perhaps, with a temporal coordinate.. (Helstrom, 1984; Olofssn and Andersson, 1963; Ross, 1996).

2.5 Representation of Uncertainty Associated with Structural Deterioration

Many engineering problems, such as structural deterioration, are associated with uncertainties due to the lack of data and knowledge. As mention in Section 2.3, inspection outcomes can be considered as a data resource for structural assessment, which are associated with significant uncertainty. Moreover, physical models are used to estimate the structural behavior associated with model uncertainty. In general, the structural deterioration process can be associated with uncertainties from different resources. The probability concepts can be used in such cases by taking advantage of experience, judgment, and observational data. In the probabilistic approach, the parameters are considered to be random nature themselves, enabling an engineer to systematically combine subjective judgment based on intuition, experience, or indirect information with observed data to obtain a balanced estimate, and to update the estimate as more information becomes available.

Before the probability of an event can be estimated, the uncertainty in the problem needs to be identified (JCSS, 2008).

2.5.1 Deterministic Representation of Deterioration

The structural deficiencies that occur over the structural lifetime can be divided, in terms of measurement, into two groups as:

- Defects can be measured
- Defects cannot be measured by existing equipment

According to the classification of uncertainty in section 2.1, it can be concluded that the defects can be associated with all type of uncertainties, and the type uncertainty associated with defect can be change over time.

In order to represent the uncertainty associated with deterioration process, the simplest way is to take into account uncertainties by using characteristic values model or partial safety factors. However, it is identified that using these methods may provide overestimate in comparison with the actual condition of the structure, which could increase maintenance and repair costs.

2.5.2 Random Variable Representation Models for Deterioration

In the last two decades, many studies have been carried out using the probabilistic method to represent the structural deterioration process. Owing to the intensive use of the reliability index (see Appendix-C) in code calibration and in reliability-based analysis and design, time-dependent reliability index approach to maintaining safety and optimizing the life-cycle performance of deteriorating structures is often considered. The reliability is not always independent of time; rather it is highly time dependent and reliability of many structural systems reduces with time owing to structural deterioration. Variation of reliability index with time is represented as $\beta(t)$.Variation of reliability index is influenced by various factors and can be characterized by using random variables models. Thoft-Christensen and Sorensen (1987) proposed a reliability-based methodology to optimize inspection, maintenance and repair cost of structural system. Inspection interval and inspection quality have been considered as optimization variables. Failure function is formulated in a form of function with load parameters, mechanical properties of materials, geometrical quantities considered as basic random variables. Probability of failure of one mode in a specific inspection interval is represented in form of a standard normal distribution. To estimate deterioration of the reinforcement of cross-section subject to corrosion, the diameter of reinforcement bar was considered as a function of the chloride concentration on the concrete surface which is a time dependent variable, and its initial diameter. (Thoft-Christensen and Sorensen, 1987). They proposed an optimal strategy to minimize the total cost of inspection and repair in the expected lifetime of the structure when maintain the reliability at acceptable level.

Mori and Ellingwood (1992) developed a probability-based methodology to estimate the reliability of reinforced concrete structures. The method includes models of structural deterioration and mathematical techniques to analyze time-dependent reliability of concrete structures. The sensitivity of structural reliability index to three degradation functions was evaluated, namely deterministic linear, square-root and parabolic functions were used to represent corrosion, sulphate attack and diffusion-controlled deterioration mechanisms, respectively. Time-dependent effects of deterioration factors in situ strength are considered in estimation of the effects of ageing. The effect of the type of degradation function on the limit state probability function are represented, when type of degradation function is assumed deterministic and initial strength of a concrete component is represented as lognormal distribution function. It is concluded that the failure probability associated with square root model for critical components of reinforced concrete structures in nuclear power plants (NPP) is the highest, followed by linear and parabolic models (Mori and Ellingwood, 1992).

Cheung and Kyle (1996) represented a reliability-based system for the service life prediction of reinforced concrete structures by using statistical databases and probability theory, to estimate reliability of a system composed of several components. They have assumed that the performance state of a system has a key role to estimate the reliability level, therefore identification and quantification of the limit state are taken to govern the performance of the structure. Their emphasis is on the flexural strength and punching shear capacity of concrete slab. Every physical limit state function is considered as a function of some basic random variables. Each of limit state function variables is assigned a probability distribution function that reflects the background statistical information. It is assumed that the majority of deterioration of reinforced concrete structures results from corrosion of reinforcing steel. Due to the loss of cross section area bars, the section capacity is predicted to decrease. The loss of reinforcement cross section area subject to corrosion is determined as a deterministic variable. The Monte Carlo simulation method or Second Order reliability method can be used to calculate the reliability. It is concluded that various levels of complexity of deterioration mechanisms and limit state functions can be readily incorporated into reliability-based framework. Specific levels of accuracy of life-cycle models can be achieved by using different limit state functions (Cheung and Kyle, 1996).

So et al. (2009) used a performance-based life-cycle cost management model for a reinforced concrete bridge subject to chloride-induced reinforcement corrosion. The cumulative probabilities of different limit states functions at the time of corrosion initiation and time of severe cracking were simulated by the Monte Carlo simulation method. The service life can be defined as the time at which any of limit states reach or accumulated damage reaching some specified amount, therefore the model proposed in this paper in order to predict the service life is considered as the probability of damage occurring at a particular periods of time. The service life model is based on fixed pre-defined limit states function, either as serviceability control or as ultimate limits control (So et al., 2009).

Marsh and Frangopol (2008) considered the multiple corrosion sensor networks throughout a structural component to improve the quantification of the steel corrosion rate. Two types of sensors were used to measure the chloride diffusion rate and corrosion rate. The corrosion rate sensors used method of linear polarization resistance (LPR) to determine rebar corrosion rate. In this method, the value of corrosion current density (CCD) was directly used to determine corrosion rate. The uncertainty associated with temporal and spatial variability of the corrosion rate can be reduced, if corrosion rate sensors are installed properly. Several of the variables associated with reliability model are affected by uncertainties. The spatial and temporal variability are treated as random variables and described in probabilistic terms. These descriptors include the expected value and the standard deviation have been estimated to generate probabilistic distribution of corrosion rates for each critical section. In order to characterize the uncertainty associated with variables of reliability model, the random variable model is employed. It is assumed that the resistance of a reinforced concrete bridge deck slab is reduced due to the loss of reinforcement steel cross section area over the time. The resistance is calculated using Monte Carlo simulation based on the properties and dimensions of the deck and assumed to be lognormal distributed (Marsh and Frangopol, 2008).

Enright and Frangopol (1998) considered the flexural strength loss in concrete bridge beams due to the corrosion of steel reinforcement as a time-dependent random variable. A model is developed to take into account the uncertainty associated with the loss of steel reinforcement area over the time by a normal distribution function. Different corrosion rate and initiation time were considered to represent the influence of these variables on the deterioration process. It is indicated that the rate of loss of normalized area of bending steel reinforcement in a concrete component is influenced by diameter of reinforcement, the corrosion rate, and the corrosion initiation time. The loss of mean value of normalized reinforcement area generally decreases as the mean value of reinforcement diameter increases, the mean value of corrosion rate increases, and the mean value of corrosion initiation time decreases. It is identified that the mean corrosion rate has a significant influence on the descriptor of normalized area. Also it appears the influence of the coefficient of variation of corrosion initiation time on the descriptor of normalized area is time-dependent (Enright and Frangopol, 1998).

Li (2003) proposed a degradation model of flexural stiffness and flexural strength of a RC component subject to corrosion based on experimental data. This approach can be justified when the development of theories for RC structural design is examined, in which design formulas are based on large quantity of experimental research. It has been identified that reinforced concrete flexural members deteriorate at different rates with stiffness deteriorating faster than strength. A methodology is established based on the deterministic parameter and is compared with the experimental results. It is identified that the differences between theoretical and experimental result are due to uncertainties associated with corrosion process and other factors such as environmental condition, material discrepancy, and workmanship. A general model of structural deterioration like equation (1.2) has been employed. It has been indicated that the theoretical results of strength deterioration of RC members in comparison with the destructive tests are grossly underestimated. The time-dependent deterioration function of flexural strength of the RC member has been represented as a function of random variables. The mean value of time-dependent deterioration function which is determined as multiplies of a deterministic coefficient and deterioration experimental function is represented in the following form

$$\mu_G(t) = G_0 \exp(-\gamma t) \tag{2.10}$$

where $\mu_G(t)$ is mean value of the deterioration function, G_0 is initial deterioration function at time zero, which is one according to the definition of deterioration function and γ coefficient represents the rate of structural deterioration (Li, 2003).

Brodski and Ponomarev (2006) used deterministic and probabilistic methods to compare the deterioration prediction of bridge condition. The study considered different models to determine the deterioration process. First, it is considered a model on the basis of the mean values of structural element condition without taking into consideration specific design features or condition of a specific component and deterioration process. The deterioration of a bridge is represented in form of an exponential function as following

$$I = e^{\lambda t} - 1 \tag{2.11}$$

where t is time and λ is the rating coefficient determined for each bridge on the basis of the boundary condition

$$\lambda = \frac{Ln2}{T_c} \tag{2.12}$$

where T_c is the average life of a known bridge. In the second model, the inspection outcomes have been studied to determine the rating coefficient in agreement with a procedure for adaption of baseline data in terms of statistics. The mean value and standard deviation of a specific inspection finding is taken into account in the deterioration process and it is represented as lognormal distribution. It is identified that the result of expert judgment provides underestimate for the prediction of a bridge condition corresponding to the lower limit of the average statistical scatter of the service lifespan (Brodski and Ponomarev, 2006).

Frangopol et. al. (2004) reviewed three random variable models as a part of an investigation of the bridge maintenance management to represent deterioration process. First model is the failure rate model in which the only random variable is the lifetime itself. Second model is the classical reliability index model, where the life time distribution function follows from a limit state which is a function of one or more random variables. The last is the condition index model where the lifetime distribution function follows results of visual inspections. Unfortunately, these models are described just as functions. No numerical examples were provided to illustrate the application of the models (Frangopol et al., 2004). The significant disadvantages of using these models are that they consider the deterioration process as a time independent process without taking into account uncertainty associated with deterioration process which is propagated forward in time

A condition –based model is implemented to determine the time to failure of a component or structure as a random variable. In order to estimate the structural condition, a Markov chain process is employed based on the assumption that the condition of a component is explained in terms of a limited number of condition states. A transition probability is defined as the probability that a component will move from one state to another, depending on the action taken. It is quite flexible in adapting it to visual inspection data. (Van Noortwijk and Frangopol, 2004a)

However, there are some issues on this model as:

1. Deterioration of a component is described in qualitative terms only
2. Transfer process of condition is considered as a single step function
3. Future condition is only dependent on the current condition not the deterioration history
4. Bridge system condition is not explicitly considered

2.5.2.1 Issues of Random Variable Deterioration Models

Owing to usual lack of failure data, a reliability approach solely based on lifetime distribution and their unobservable failure rates as proposed by Farngopol et al. (2004) is unsatisfactory. According to results of comparison of the random variable and stochastic process models of deterioration process representation, it is concluded that use of random variable model to represent deterioration process is not most appropriate model ((Pandey et al., 2009; Frangopol et al., 2004).

In structural engineering, time-dependent functions are advocated for which the coefficients (such as an average rate of deterioration per unit time) are random quantities. However, as mentioned before, temporal variability is not taken into account in this random-variable model. It is recommended to represent deterioration process in terms of a stochastic process (Frangopol et al., 2004).

It is demonstrated in Figure 1.3 that the variability of deterioration state can be out of step with the inspection intervals. Therefore, it is concluded that the deterioration state estimation has to be as actual as possible. It means that the deterioration process has to be presented in form of a time-dependent variable due to uncertainties associated with deterioration process.

Pandey et al. (2009) compared the random variable model and stochastic process to represent the deterioration process of a structure. It is identified that random variable model cannot reflect temporal variability associated with deterioration process. As a consequence, the deterioration throughout a specific sample deterioration path is effectively deterministic over time in the random variable model, while it varies probabilistically in stochastic process. It is identified that the random variable model tends to underestimate the life-cycle cost (Pandey et al., 2009).

In order to compare the random variable model (RV) and stochastic process, deterioration of structure resistance is considered. Assuming that the linear model used to represent the cumulative deterioration.

$$X(t) = At \tag{2.13}$$

Where $X(t)$ is the cumulative deterioration, where A is deterioration rate. The deterioration rate is considered as a random variable. The following notation is used to define the random variable properties at given time.

$$\mu_{X(t)} = \mu_A t,\ \ \sigma^2_{X(t)} = \sigma^2_A t^2,\ \ COV\big(X(t)\big) = v_{X(t)} = \frac{\sigma_{X(t)}}{\mu_{X(t)}} = v_A \quad (2.14)$$

It is concluded from equation 2.14 that the COV of deterioration is constant over the time as it is demonstrated in Figure 2.4. Since the RV model is inadequate model to represent the structural deterioration, stochastic process will be applied here.

For the stationary gamma process model, the cumulative deterioration $X(t)$ follows a gamma distribution, $Ga(x; \alpha t, \beta)$, with a shape parameter αt and a scale parameter β. The mean, variance and COV of $X(t)$ are determined as (Pandey et al., 2009)

$$\mu_{X(t)} = \alpha\beta t,\ \ \sigma^2_{X(t)} = \alpha\beta^2 t,\ \ \ v_{X(t)} = \frac{1}{\sqrt{\alpha t}} \qquad (2.15)$$

The mean value of deterioration rate and COV of lifetime of two models are compared in Figure 2.4. It is demonstrated that the mean value of deterioration rate is not identical in two models. When the COV of lifetime is small value, the mean rates in both models are quite close. However, as COV of lifetime increase, the GP deterioration rate accelerates much faster than that in RV model (Pandey et al., 2009).

The comparison of COV of cumulative deterioration in two models is illustrated in Figure 2.5. It is time-invariant and nonlinear function in the RV model. In contrast, COV of cumulative deterioration in GP model is a time-dependent parameter, which decreasing over time. Nevertheless, COV of cumulative deterioration in GP model is greater than that of equivalent RV model. Therefore, it is identified that a stochastic process is more appropriate to represent deterioration process (Pandey et al., 2009).

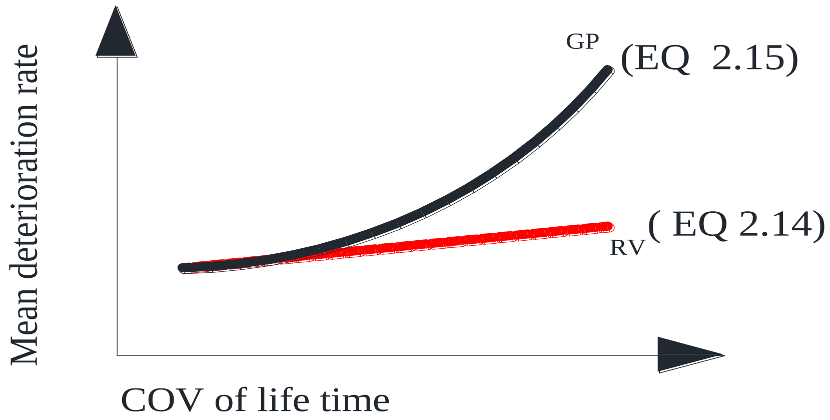

Figure 02.3 Comparison of mean deterioration rate in equivalent RV and GP (Pandey et al., 2009)

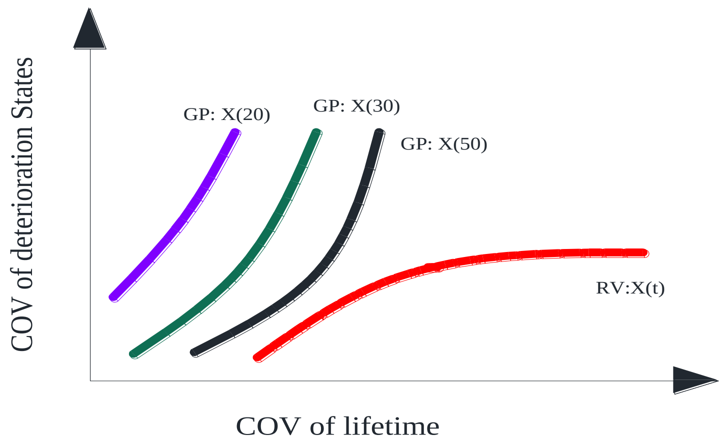

Figure 2.4 Coefficient of variation of deterioration states in equivalent RV and GP (Pandey et al., 2009)

According to the studies which are developed on the basis of random variable models to represent deterioration process, main issues are:

- A sample path for component deterioration is set at the start and does not change over the lifetime
- COV of deterioration model is constant over the time
- After the first inspection, the deterioration modelling is effectively deterministic

An example of a probabilistic time-dependent approach is presented in the following. Integrated approach for deterioration modeling, probabilistic state-based/time-based model and reliability-based mechanistic, is proposed by Morcous et al. (2010). The probabilistic state-based/time-based model is used to represent the deterioration process of network analysis and reliability-based model for prediction of components of project level analysis. A reinforced concrete slab of a girder bridge, which is the most dominant type of structure in Quebec with the inspection outcomes of Transportation Department of Quebec have been used, is selected to model development of deterioration. In this model, elements of the network are categorized to three groups (primary, secondary and auxiliary) dependent on the element's influence on the network safety level (Morcous et al., 2010).

The mechanistic models are used to estimate the analysis of safety-critical structures, while the deterioration is represented by quantitative performance indicators i.e. resistance, deflection, stress. The mechanistic models result can include beginning of deterioration, deterioration propagation, and deterioration impact on the safety and serviceability of structure. Deterioration mechanisms of a bridge result in a complex process with parameters such as structural system behavior, material, etc. Most of these parameters are time-dependent and random in nature, with considerable level of uncertainty. Flexural capacity of corroded reinforced concrete members is used as performance indicator. The ratio of time-variant cross sectional area to initial cross-sectional area is considered as a basic random variable. Monte Carlo simulation is used to estimate the distribution of percentage remaining steel area at different points in time. This quantitative information is valuable for estimating of deterioration propagation (Morcous et al., 2010).

2.6 Summary and Conclusions

Types of uncertainties associated with structural deterioration mechanisms are reviewed in this chapter. Moreover, different methods that can be used to characterize the uncertainty are investigated. It is identified that the deterministic and semi-probabilistic methods cannot provide adequate information to represent uncertainties. Therefore, the probabilistic method is the most appropriate method to characterize the uncertainty. Random variable models and stochastic processes are models that can be used to represent uncertainties through the probabilistic method. It is identified that most common models to represent uncertainties associated with structural deterioration process are random variable models. However, Pandey et al. (2009) identified with consideration to comparison of the random variable and stochastic process that random variable models cannot reflect the temporal variability with deterioration process.

It is concluded that the stochastic process is the most appropriate model to characterize the structural deterioration process. More information about different stochastic processes and their application is described in the next chapter.

Chapter Three

3. Stochastic Process Model for Structural Deterioration

3.1 Introduction

In order to maintain the safety and serviceability of a structure at adequate level, it is important to represent the structural deterioration process as comprehensively as possible in respect to the influence of deterioration factors. The deterioration of structures can be represented using deterministic or probabilistic approach. However, considering that the current and future status of the structure are associated with many sources of uncertainty probabilistic approach should be considered as more appropriate model. In chapter 2, it is identified that a stochastic damage accumulation process model has to be considered to characterize the structural deterioration. More information about different stochastic processes can be find in Appendix-F.

During the last decade many studies have been conducted to represent the structural deterioration as stochastic processes. Campoli and Ellingwood (2002) proposed a time-dependent reliability method to evaluate safety level of concrete structures of a nuclear power plant. The effects of deterioration mechanisms such as corrosion and freezing and thawing on the reinforced concrete structures in the nuclear power plants (NPP) are modeled mathematically, in order to evaluate their impact on time-dependent reliability and structural performance. It has been proposed that the stochastic damage accumulation process can be considered as an alternative to model structural deterioration. It is identified that the stochastic deterioration processes for material properties deterioration can be characterized from

- Mathematical models that describe the effects of ageing on steel and concrete.
- Accelerated life testing.
- Combination of the two approaches.

Since the accelerated life tests often do not scale properly from laboratory to the prototype or the actual in-service condition, this model is not appropriate. From structural engineering point of view, it is important that the models of material deterioration be consistent with the needs of structural engineering calculations. Thus, a mathematical model is needed. The structural damage can be evaluated by an indicator $D(t)$ (represented by a non-negative non-decreasing function) adopted to represent the state of structural component. Therefore, the Markov process has been applied to determine the damage indicator$D(t)$. This requires that cumulative damage at time$t$, which defines the state of the component, is dependent only on the damage state at time $(t - dt)$ and the damage that occurs during dt (Campoli and Elingwood, 2002).

Grall et al. (2002) considered a structure that is subjected to deterioration factors, and is monitored through perfect inspections. In order to establish a condition base deterioration model, a structure subject to a continuous accumulation of deterioration in time is considered. Its condition at time t is assumed to be completely described as a single scalar random variable$X(t)$. The stochastic process $\{X(t); t\epsilon T\}$ corresponds to the deterioration of the structure and satisfies following assumptions.

1. $X(0) = 0$ at time $t = 0$
2. The increments in a time interval are non-negative, stationary and statistically independent.

It is proposed that the deterioration process can be observed only at discrete equidistant times $t_k = k\Delta t$, where interval time Δt is either arbitrarily chosen or imposed by the considered condition state. Natural assumption in a stationary context can be made to consider the average amount of deterioration at time t, in order to characterize the deterioration process better.

It is identified that applying a stochastic process to represent structural deterioration can take into account inspection outcomes in a reasonable manner (Grall et al., 2002).

3.2 Stochastic Processes Model for Deterioration

Owing to the usual lack of failure data, it is recommended to represent structural deterioration in terms of a time-dependent process. Random deterioration rate, Markov processes, Brownian motion with drift and monotonically increasing jump processes are forms of stochastic process and have been reviewed by Van Noortwijk (2009).
They can be used to represent the structural deterioration. The characteristics of stochastic processes are explained in next sections.

3.2.1 Random deterioration rate model

The simplest stochastic process is a time dependent function for which the average rate of deterioration per unit time is a random variable. Reliability methods have been developed on the basis of random deterioration rate. However, the sample path of such model is linear and a single inspection thus directs the future deterioration prediction in advance. This model is recommended to use as an approximation (VanNoortwijk, 2009). Figure 3.1 demonstrates general principles of this method. The deterioration process is represented as a linear function of interval t_i. The interval deterioration rate $C_i(t)$ is assumed as a normal random variable where the mean value is constant over the interval time and standard deviation value is constant over the lifetime as shown in Figure 3.1.

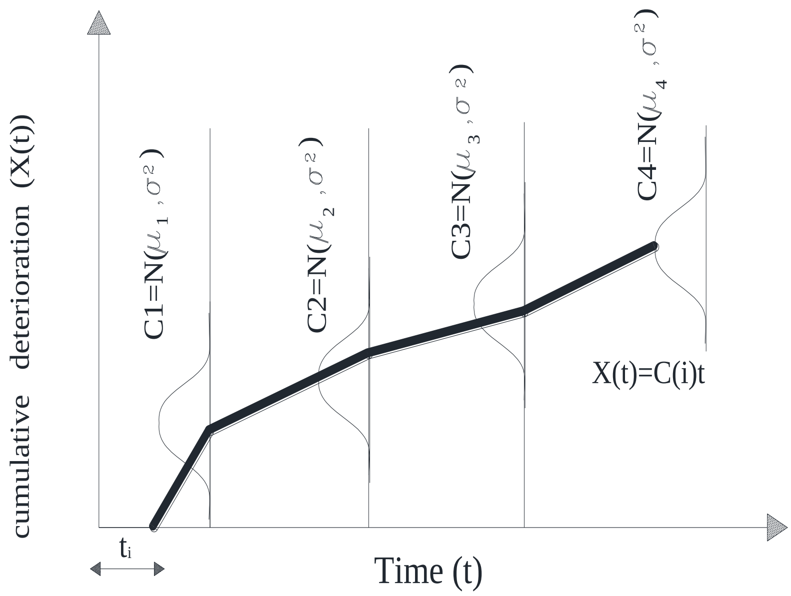

Figure 3.1 General illustration of random deterioration rate (Pandey et al., 2007)

3.2.2 Markov process for deterioration

For the purpose of inspection and maintenance optimization, it is better to consider the deterioration process model that properly characterizes the temporal variability. A Markov process is based on the assumption that the condition of a structural component can be described in terms of a condition state. It is a stochastic process where the condition state at a particular time is just depending on the prior condition state and it is independent of the condition at other times. Classes of Markov processes are discrete-time Markov processes which have finite state spaces (called Markov chain) and continuous-time Markov processes with independent increments (Van Noortwijk, 2009).

3.2.2.1 Discrete Markov (Markov Chain)

Assume that there is a finite countable state space, the condition of a structure or component can be in any one of $N \geq 0$ discrete states. A Markov chain is a discrete time stochastic process $\{X_n, n = 0,1, \dots\}$ for which the Markovian property holds. This property states that the future condition only depends on the current condition. The conditional probability of moving into state j at time $n + 1$ given that at the current time n the object is in the state i is given by:

$$P_{ij} = \Pr\{X_{n+1} = j | X_0 = 0, \dots, X_{n-1} = i_{n-1}, X_n = i\} = \Pr\{X_{n+1} = j | X_n = i\} \qquad (3.1)$$

If this transition probability does not depend on n then the process is called stationary in time (Van Noortwijk, 2009).

A stationary first-order Markov-chain has been developed as an application example of state-based probabilistic deterioration model. Transition probabilities are estimated by solving the non-linear optimization problem which minimizes differences between the deck condition predicted using regression model and the deck condition predicted using the Markov-chain model (Morcous et al., 2010). On the other hand, in time-based model, the transition time is defined as the time needed for an element to change initial state to the next lower state. The information required for development of this model includes the condition state transition events and the corresponding time data. Due to periodic inspection schedule and lack of condition data, it can be obtained only for most common condition states. It is concluded that frequent inspections over a long observation period are required for developing time-based models, while infrequent inspections over a relatively short observation period can be used for developing state-based models (Morcous et al., 2010).

Kallen (2010) reviewed different methods which have been applied for the estimation of Markov chain models in civil engineering problems. Due to variation of information type which is available for engineer, it is concluded that a direct comparison of these models is not possible. He proposed to subdivide data to three major groups as:

- Type I: observation of the state itself and represented by realization value of condition state of the process
- Type II: aggregated data in the form of relative fractions of proportions of a specific condition state
- Type III: count data in the form of the number of condition transitions

It is concluded that there are some important issues concerning the use of Markov chain models (Kallen, 2010)

- The condition state is not continuous, but discrete and finite. This feature of deterioration process works for visual inspection but not for other types.
- Transition probabilities in the transition matrix are difficult to assess and quite subjective.
- The Markov modeling of no memory has often been criticized.

3.2.2.2 Brownian motion with drift process

The Brownian motion with drift is a continuous-time stochastic process with drift parameter μ and variance parameter σ^2 having the following properties

- $X(t)$ is normally distributed with mean μt and variance $\sigma^2\, t$ for all times($t \geq 0$)
- $X(t)$ has an independent increments
- $X(0) = 0$ with probability one

A characteristic of this process, in the structural reliability manner, is a live load model which alternately increases and decreases (Van Noortwijk, 2009).

Park and Padgett (2005) used some accelerated life test models to define the failure model and degradation process. The results are used to compare the actual data with the theoretical representation. It is concluded that the model for failure can be approximated closely with the available data from accelerated test and estimation by Brownian motion with drift (Park and Padgett, 2005).

As the structural deterioration process is monotone incremental process, it is identified that the Brownian motion with drift is inadequate stochastic process. However, other studies have been conducted to establish stochastic structural deterioration using different stochastic processes such as Poisson process.

In general, the service life of deteriorating structures is a progression of reliability states. Therefore, time-dependent reliability index models were developed and applied to extend the service life of deteriorating structures under various maintenance scenarios (VanNoortwijk and Frangopol, 2004b).

In reliability-based model, the time-dependent reliability of a deteriorating structure or a group of structures is considered as function that is influenced by some factors i.e. initial reliability index and the time of deterioration initiation. The advantage of this model is that the reliability is explicitly taken into account. It is concluded that the ideal way is to base a deterioration model on the stochastic processes of mechanical properties, and using this deterioration results to compute the time-dependent reliability function. In order to represent the deterioration model without maintenance, the bi-linear reliability index profile has been applied extensively (VanNoortwijk and Frangopol, 2004b).

$$\beta(t) = \begin{cases} \beta_0 & for\ 0 \leq t \leq t_I \\ \beta_0 - \alpha_1(t - t_I) & for\ t > t_I \end{cases} \tag{3.2}$$

However, a nonlinear model of reliability index deterioration profile was recently proposed as (VanNoortwijk and Frangopol, 2004b)

$$\beta(t) = \begin{cases} \beta_0 & for\ 0 \leq t \leq t_I \\ \beta_0 - \alpha_2(t - t_I) - \alpha_3(t - t_I)^p & for\ t > t_I \end{cases} \tag{3.3}$$

Where $\alpha_1, \alpha_2, \alpha_3$ are deterioration rates, t_I is the deterioration at initial time and p is a parameter related nonlinearity effect in terms of a power low in time. It should be noted that reliability profile in equation (3.2) and (3.3) are not calculated from state functions, but simulated by the Monte Carlo method (VanNoortwijk and Frangopol, 2004b).

Kuniewski et al. (2009) proposed a sampling-inspection strategy for the evaluation of time-dependent reliability of deteriorating steel bridge system. They assumed that the deterioration could initiate at random time and at random location. The bridge safety becomes unacceptable when a defect reaches the critical size, at least. The gamma process is used to represent the defect size growth procedure. Here again, it is the same issues with reliability index evaluation that it can be that every component has a relevant reliability index which cannot be used to evaluate safety of other components.

In general deterioration process of a bridge component is a non-negative, independent and monotonic incremental process over the time in sequence of small increments. The continuous gamma process is considered the most appropriate process to represent the associated degradation.

3.3 Continuous Gamma Process

A gamma process is a continuous-time stochastic process with independent, non-negative increments having gamma distribution with an identical scale parameter. This process is appropriate to model development of phenomena that has a monotone increasing trend over time, such as wear, fatigue, creep, crack, corrosion and, etc. An advantage of modeling deterioration processes through gamma processes is that the required mathematical calculations are relatively straightforward (Van Noortwijk, 2009).

Firstly, it is considered that a random variable X has a gamma distribution with shape parameter $\alpha > 0$ and scale parameter $\beta > 0$ if its probability density function is given by:

$$Ga(x|\alpha, \beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} \exp(-\beta x) \ , x \geq 0 \qquad (3.4)$$

and $\Gamma(a) = \int_{z=0}^{\infty} z^{a-1} e^{-z} dz$ is the gamma function for $\alpha > 0$. Furthermore, let $\alpha(t)$ be a non-decreasing, right-continuous, real valued function for $t \geq 0$,with $\alpha(0) = 0$.

The gamma process with shape function $\alpha(t) > 0$ and scale parameter $\beta > 0$ is a continuous-time stochastic process $\{X(t); t \geq 0\}$ with the following properties:

$$\begin{cases} X(0) = 0 & \textit{with probability one} \\ X(\eta) - X(t) \approx Ga(\alpha(\eta) - \alpha(t), \beta) & \textit{for all } \eta > t \geq 0 \\ X(t) & \textit{has independent increments} \end{cases} \quad (3.5)$$

Let $X(t)$ denote the deterioration at time $t, t \geq 0$, and let the probability density function of $\{X(t); t \geq 0\}$, in accordance with definition of the gamma process, be given by:

$$f_{X(t)}(x) = Ga(x|\alpha(t), \beta) \quad \alpha(t) > 0, \beta > 0, t \geq 0 \quad (3.6)$$

With expectation and variance:

$$E\big(X(t)\big) = \frac{\alpha(t)}{\beta}, Var\big(X(t)\big) = \frac{\alpha(t)}{\beta^2} \quad (3.7)$$

The coefficient of variation is defined by the ratio of the standard deviation and the mean:

$$COV\big(X(t)\big) = \frac{\sqrt{Var(X(t)}}{E(X(t))} = \frac{1}{\sqrt{\alpha(t)}} \quad (3.8)$$

COV decreases as time increases. On the other hand, the ratio of the variance and the mean equals $\frac{1}{\beta}$ and therefore does not depend on time (Van Noortwijk, 2009).

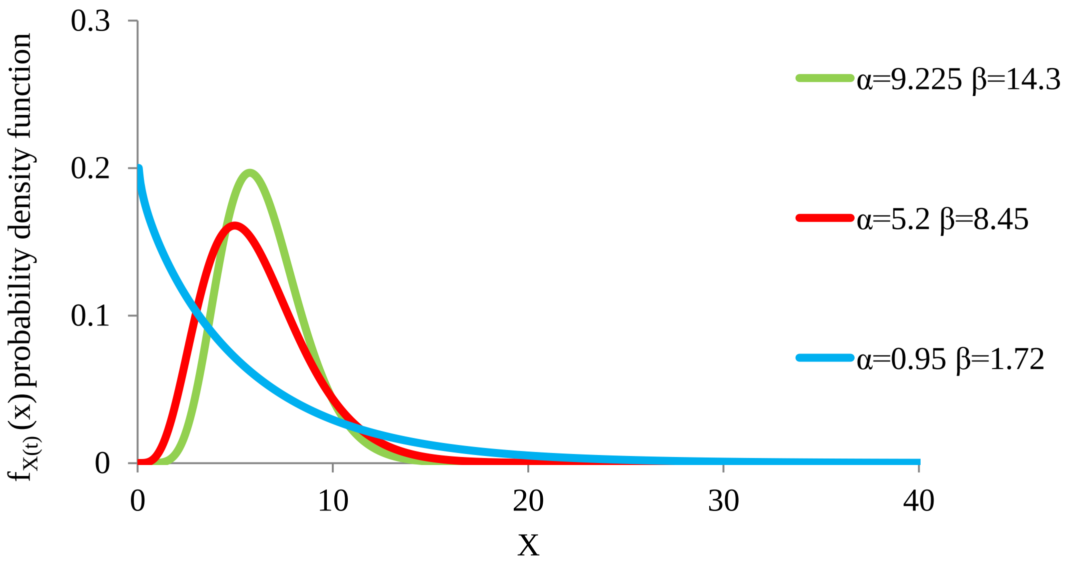

Figure 3.2 General illustration of different Gamma probability density functions

Figure 3.2 illustrates several sample gamma distributions for selected distribution parameters that are function of the time horizon. It is evident that shape and scale parameters need to be established for specific deterioration process. Furthermore, it is identified that they are independent so effectively the distribution for deterioration associated with the site-specific bridge condition only on the basis of current observation can be obtained.

3.3.1 Gamma Process Parameters

In order to represent the reinforced concrete structures deterioration, some researchers have conducted work to establish empirical and mathematical models.

Zdenek and Bazant (1979) developed a theoretical physical model to determine the corrosion initiation time and the time to cracking as function of reinforcement depth and spacing, corrosion rate and certain mechanical properties of the concrete including tensile strength, modulus of elasticity and Poisson's ratio.

This considers a gamma process with shape function $\{\alpha(t) = ct^b\ \alpha(t) > 0, t \geq 0\}$ and scale parameter$\beta > 0$. When there is engineering knowledge about the shape of the degradation process, b might be constant. Some studies recommend b for different factors i.e. corrosion of reinforcement (linear, $b = 1$), sulphate attack (parabolic, $b = 2$) and diffusion-controlled ageing (square root, $b = 0.5$) (Campoli and Ellingwood, 2002).

Here it is assumed that the value of the power b is known, but c and β are unknown. The question which remains to be answered is how expected deterioration increases over time. In order to apply the Gamma process model to practical examples, statistical methods for the parameter estimation of Gamma process are required. In the event of expected deterioration in terms of the parameters c and β they have to be obtained by using observational data. Once the randomness is uniquely defined in terms of the parameters of the distribution, it is used in subsequent probabilistic analysis, assuming the basic characteristics of the random variable remain unchanged. This is generally known as the point estimation of parameters. Two most common methods of point estimation of parameters are (Van Noortwijk, 2009)

- Method of Maximum Likelihood
- Method of Moments

To account for statistical uncertainties, it is proposed to use Bayesian analysis in which the scale parameter of the Gamma process is assumed to have an inverted gamma distribution as prior. This estimation method is called

- Method of Bayesian Statistics

3.3.2 Estimation of Gamma Process Parameters

Once the Gamma process is selected to represent the deterioration model, it is necessary to define it uniquely by evaluating its parameters. The accuracy in estimating these parameters based on the test or observational data determines the success in modeling the uncertainty in the deterioration model.

3.3.2.1 Method of Moments

In statistics, method of moments estimates population parameters, by equating sample moments with unobservable population moments and then solving the equations for the quantities to be estimated.

According to the expected value and variance of the accumulated deterioration at time t, when the power parameter is known, the non-stationary gamma process can be easily transformed to a stationary gamma process by performing a monotonic transformation from the time to transformed or operational time $z(t) = t^b, z(t) \geq 0$ (Van Noortwijk, 2009).

Substituting the inverse time transformation $t(z) = z^{\frac{1}{b}}, t \geq 0$ it's expected value and variance will be:

$$E\left(X\big(t(z)\big)\right) = \frac{cz}{\beta},\ Var\left(X\big(t(z)\big)\right) = \frac{cz}{\beta^2} \qquad (3.9)$$

Similarly, the transformed inspection times are $\{z_i = {t_i}^b, i = 1,2,\dots,n,\ z_i \geq 0\}$. Transformed times between inspections are defined as $\{\forall\ i,\ \omega_i = {t_i}^b - {t_{i-1}}^b,\ and\ \omega_i > 0\}$ and $\{\forall\ i,\ \gamma_i = X_i - X_{i-1}, \text{and}\ \gamma_i > 0\}$.

The deterioration increments $\gamma_i > 0$ have a gamma distribution with shape factor $c\omega_i > 0$ and scale parameter $\beta > 0$ for all, $i = 1,2,\dots,n$. According to Cinlar et al. (1979), the method -of-moments estimates $\hat{c}, \hat{\beta}$ can be solved from:

$$\frac{\hat{c}}{\hat{\beta}} = \frac{\sum_{i=1}^{n} \gamma_i}{\sum_{i=1}^{n} \omega_i} = \frac{x_n}{{t_n}^b} = \bar{\gamma} \qquad (3.10)$$

$$\frac{x_n}{\hat{\beta}}\left(1-\frac{\sum_{i=1}^{n}\omega_i{}^2}{[\sum_{i=1}^{n}\omega_i]^2}\right)=\sum_{i=1}^{n}(\gamma_i-\bar{\gamma}\omega_i)^2 \qquad (3.11)$$

Clearly, the method of moments leads to simple formula for parameter estimation which can be easily computed. However, it is identified that estimated parameters can be biased. When estimating parameters of known family of probability distributions, this method can be replaced by Maximum likelihood method, because Maximum likelihood estimators have higher probability of being close to the quantities to be estimated more often unbiased. Furthermore, in some cases, infrequent with large samples but not so infrequent with small samples, the estimates given by the method of moments are outside of the parameter space and they are not necessarily sufficient statistics (Loeve, 1977). Estimates by method of moments can be used as the first approximation for gamma process parameters.

3.3.2.2 Method of Maximum Likelihood

In general, for a fixed set of data and underlying probability model, the method of maximum likelihood can be used to select the value of parameters that produce the distribution most likely to have resulted in the observed data. The maximum-likelihood estimators of c and β can be obtained by maximizing the logarithm of the likelihood function of the deterioration increments. The likelihood function of the observed deterioration increments $\{\gamma_i = x_i - x_{i-1}, \gamma_i > 0\}$ is a product of independent gamma densities (Van Noortwijk, 2009).

$$\begin{cases}\mathcal{L}(c\omega_i,\beta|\gamma_i)=\prod_{i=1}^{n}f(\gamma_i|c\omega_i,\beta)=\prod_{i=1}^{n}\frac{\beta^{c\left[t_i^b-t_{i-1}^b\right]}}{\Gamma(c\left[t_i^b-t_{i-1}^b\right]}\gamma_i^{c\left[t_i^b-t_{i-1}^b\right]-1}\exp(-\beta\gamma_i)\\ \gamma_i>0, t_i\geq 0, i=1,2,\ldots,n; \beta>0, c>0\end{cases} \qquad (3.12)$$

Cinlar et al (1979) show that the maximum likelihood estimates $\hat{c}$ and $\hat{\beta}$ are as follows:

$$\hat{\beta} = \frac{\hat{c}t_n^b}{x_n}, \sum_{i=1}^{n}\left[t_i^b - t_{i-1}^b\right]\{\psi(\hat{c}\left[t_i^b - t_{i-1}^b\right] - \log\gamma_i\} = t_n^b \log(\frac{\hat{c}t_n^b}{x_n}) \qquad (3.13)$$

where ψ is the first derivative of gamma function $(\frac{d\Gamma(x)}{dx})$. As the cumulative amounts of deterioration are measured, the last inspection contains the most information. It is therefore assumed that the expected deterioration at the last inspection represents the real deterioration.

It is identified that the maximum likelihood method provides a consistent approach to parameter estimation. This means that using this method estimates can be obtained for a large variety of situations. However, the estimators can be heavily biased for small samples. Nevertheless, the maximum likelihood method has desirable mathematical and optimality properties. They become minimum variance unbiased estimators as the sample size is increased. It is indicated that the numerical estimation is usually non-trivial except for a few cases where the maximum likelihood formulas are in fact simple. In addition, the equations of this method need to be specifically worked out for a given distribution and estimation problem (Ash, 1970)

3.3.2.3 Method of Bayesian Statistic

In the framework of estimating the unknown parameters $c>0$ and $\beta > 0$, the Bayesian approach assumes these parameters to have a known probability distribution. Bayes theorem can be then be written as:

$$\pi(c,\beta|\gamma_i) = \frac{\mathcal{L}(\gamma_i|c,\beta)\pi(c,\beta)}{\int_0^\infty\int_0^\infty \mathcal{L}(\gamma_i|c,\beta)\pi(c,\beta)dcd\beta} \qquad \gamma_i > 0, i = 1,2,\dots,n \qquad (3.14)$$

Where $\mathcal{L}(\gamma_i|c,\beta)$ is the likelihood function of the deterioration increment's inspection outcomes $\gamma_1, \dots, \gamma_n$ when the parametric vector (c,β) is given, $\pi(c,\beta)$ is the prior density of (c,β) before observing the inspection outcomes, $\pi(c,\beta|\gamma_i)$ is the posterior density of

(c, β) after observing the inspection outcomes, and $\pi(\gamma_i)$ is the marginal density of the inspection data (Van Noortwijk, 2009).

Using Bayes theorem, we can update the prior distribution to the posterior distribution as soon as the new inspection outcomes become available. First, we focus on the prior distribution of the scale parameter when the parameter c is given. If the prior distribution of the scale parameter is given by a gamma distribution with shape factor $v > 0$ and scale parameter $u > 0$when the value of $c > 0$is given, then the posterior distribution is also a gamma distribution with

shape parameter $v + \sum_{i=1}^{n} c\left[t_i^b - t_{i-1}^b\right] = v + ct_n^b$ and

scale parameter $u + \sum_{i=1}^{n} \gamma_i = u + x_n$.

Bayesian estimation of the scale parameter of the gamma process can be extended to Bayesian estimation of both the scale parameter and shape function. In combination with the prior density of $\pi(c)$, Bayes theorem can be written as

$$\pi(c, \beta|\gamma_i) = \pi(\beta|c, \gamma_i)\pi(c|\gamma_i)$$

$$= \prod_{i=1}^{n} \frac{\beta^{c\omega_i}}{\Gamma(c\omega_i)} \gamma_i^{c\omega_i - 1} \exp(-\beta\gamma_i) \times \frac{u^v}{\Gamma(v)} \beta^{v-1} \exp(-\beta u) \pi(c)$$

$$= Ga(\beta|v + ct_n^b, u + x_n) \times \left[\frac{1}{u + x_n}\right]^{v+ct_n^b} \frac{u^v}{\Gamma(v)} \frac{\Gamma(v + ct_n^b)}{\prod_{i=1}^{n} \Gamma(c\omega_i)} \times \prod_{i=1}^{n} {\gamma_i}^{c\omega_i - 1} \pi(c)$$

$$\gamma_i > 0,\ w_i > 0 \quad i = 1,2, \ldots, n \quad c > 0,\ \beta > 0,\ u > 0,\ v > 0 \quad (3.15)$$

When the parameter c is unknown, the parameter of the prior density of β can be depended on c, that is the prior density of β given c is gamma distribution with shape parameter $v(c)$ and scale parameter $u(c)$. One assumption for shape and scale parameters is $c\tau^b$ and u respectively (Van Noortwijk, 2009).

Under this assumption, the posterior mean of scale parameter of the gamma process β when the value of c is given can be written as

$$E(U|c,\gamma_i) = \frac{v(c)+ct_n^b}{u(c)+x_n} \qquad (3.16)$$

The predictive mean of the cumulative amount of deterioration at time t has the form

$$E\left(\frac{ct_b}{U}\middle|\gamma_1,\dots,\gamma_n\right) = E\left(\frac{C[u(c)+x_n]t_b}{v(c)+Ct_n{}^{b}-1}|\gamma_1,\dots,\gamma_n\right) \quad (3.17)$$

Dufresne et al. (1991) applied this method to determine the posterior distribution of the scale parameter of a stationary gamma process. Kallen and van Noortwijk (2004) extended the Bayesian estimation from perfect to imperfect inspection. However, it is identified that a big sample data is needed in order to rely on the estimation results (Dufresne et al., 1991; Kallen and VanNoortwijk, 2004).

Unfortunately, it is evident that in our model the inspection outcomes which might be available to estimate the deterioration process parameters are scarce and therefore such approach would not be appropriate.

3.4 Reinforced Concrete Deterioration Model by Stationary Continuous Gamma Process

It is identified that for a component subjected to an increasing deterioration with a certain scale of deterioration, at time t, there is a function of at least one parameter that could be inspected representing the cumulative deterioration, random variable $X(t)$. In order to establish such function for a concrete bridge component, it is needed to identify the deterioration mechanism. Variety of defects and mechanisms, which can result in structural deterioration, have been defined and demonstrated in Table 1.2.

In preceding sections, it has been become evident that the Gamma process is the most appropriate stochastic model to characterize the structural deterioration of reinforced concrete bridges. It is necessary to define it uniquely by estimating distribution parameters. The accuracy of estimates for the Gamma parameters is dependent on the available data and determines the success in modeling the uncertainty in deterioration. Firstly in order to estimate the gamma process parameters by method of moments, it is assumed that the inspection outcomes based on expert judgment might provide the defects features on the bridge component. The inspection outcomes can be collected at different times over the bridge lifetime regarding the inspection interval. As more inspection outcomes become available, the outcomes can be used in parameters estimation process to update the Gamma process parameters, which can result in updating of the deterioration profile for concrete bridge component. The application of this methodology is explained in the following section.

3.4.1 Deterioration Model for RC Bridge Deck

As mentioned Chapter 1, predominant deterioration mechanism for reinforced concrete structures is corrosion of the reinforcement. Here it is assumed that embedded bars in concrete are uniformly corroded due to the chloride attack. This could result from exposure of bridge component to de-icing salts. Here, we consider a reinforced concrete bridge deck that is subjected to corrosion to present the structural deterioration process as a continuous gamma process. We implement the new methodology using the inspection outcomes to characterize the deterioration progress. Thus we introduce and enable an updatable deterioration model. The properties and characteristics of the slab are illustrated in Figure 3.3.

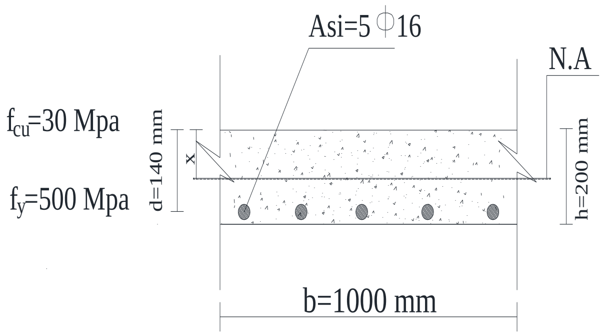

Figure 3.3 Reinforced Concrete Slab Section

3.4.1.1 Corrosion Mechanism Models

The corrosion process consists of two consecutive phases. Robert et al. (2000) suggested an initiation and propagation phase. The initiation phase describes the permeation of aggressive agents through the concrete cover until they reach the reinforcement. The propagation phase describes the development of rust products that induce cracking and spalling of the concrete cover (Robert et al, 2000).

An important part of the corrosion model is the pattern of corrosion. Generally two patterns are considered to model the corrosion propagation as

- Uniform corrosion is commonly assumed when calculating levels of corrosion in reinforced concrete section. In this pattern, a uniform loss of reinforcement bar is assumed (Robert et al, 2000).
- Pitting corrosion is a corrosion pattern of severe but local loss of section that is typically 4-8 times the equivalent uniform corrosion loss. It is likely to affect a more localized area of a bridge deck compared with uniform corrosion (Robert et al, 2000).

It is concluded that the overall effect of localized pitting corrosion on the capacity of a bridge deck may be no more severe than the effects of more widespread uniform corrosion (Robert et al, 2000). In this study only the uniform corrosion model is considered.

3.4.1.2 Deterioration of a Steel Bar Subject to Uniform Corrosion

It should be noted that in this study the initiation time and corrosion rate are considered as deterministic parameters. These parameters are derived from Enright & Frangopol (1998) research which considered an existing reinforced concrete bridge located in Colorado.

The deterioration process of a specific steel bar as part of bridge slab deck subject to uniform corrosion pattern and constant corrosion rate is considered. It is assumed that a periodic inspection scheme is conducted on the bridge to gain corrosion information on site.

In this chapter the inspection outcomes are assumed deterministic, however it is identified that inspection outcomes are associated with uncertainties.

3.4.1.3 Gamma Processes Presentation of Deterioration

Following principles set out earlier in this Chapter continuous gamma process model is implemented. Initial flexural moment capacity of the section is determined on the basis of EC2 (2006) formulations (see Appendix-B). It should be noted that the partial safety factors in this equation are neglected, as the moment capacity is considered as a stochastic process. However, the rectangular block model is assumed to represent flexural capacity.

$$M_0 = A_{S0} f_y (d - 0.4x) \qquad (3.18)$$

Where M_0 is the initial moment capacity of the section, A_{S0} is the initial reinforcement cross section area, f_y is the yield strength of the steel bar, f_{cu} is the compressive strength of concrete, d is the distance between steel bar's center and concrete compression area and x is the location of neutral axis as it has been shown in Figure 3.3. The flexural moment capacity of section at inspection time can be computed on the basis of rectangular stress block model.

$$M_t = A_{St_i} f_y (d - 0.4x)\,, \quad t_i \geq 0 \qquad (3.19)$$

On the other hand the reinforcement cross-section area can be represented as

$$As_{t_i} = \frac{\pi D_{t_i}^2}{4}\ ,\ D_{t_i} = D_0 - r(t_i - T_I), \quad t_i \geq 0 \qquad (3.20)$$

Where D_{t_i} is the steel bar diameter at inspection time, D_0 is the initial steel bar diameter, r is the corrosion rate and T_I is the initiation time. Thus the section deterioration can be represented

$$X(t_i) = M_0 - M_{t_i}, \qquad t_i \geq 0 \qquad (3.21)$$

It is evident that this function is non-negative and continuous.

3.4.1.4 Parameter estimation of gamma process

The method of moments is used to estimate the Gamma process parameters, here. According to proposed model of shape and scale parameters of Gamma process in section 3.3.2.1, the shape factor is a time function. Hence it can be represented as

$$\alpha(t) = ct^b \; \alpha(t) > 0, c > 0\,, t \geq 0 \qquad (3.22)$$

As the corrosion of reinforcement bar is target defect in our model, it is assumed that b is known and equal to one according to the power law model adopted by many authors Campoli and Ellingwood (2002), while parameters β and c are unknown. $X(t_i)$ as deterioration variable as defined by Equation (3.21) and is substituted in Equations (3.10) and (3.11) to estimate the shape and scale factors.

For simplicity, at present all parameters in Equation (3.19) except As_{t_i} are assumed deterministic. Inspection outcomes of three corrosion environments are shown in Tables 3.1-3.3. Subsequently, gamma process parameters for three corrosion environments are calculated using the Equations (3.10) and (3.11) where $X_{t_i}^m$ is the observed cumulative deterioration at inspection time, $\omega_i = t_i - t_{i-1}$ is the interval time, and $\gamma_i = X_{t_i} - X_{t_{i-1}} = M_{t_i} - M_{t_{i-1}}$ is the deterioration increment over the interval and shown in Table 3.4.

In order to provide inspection outcomes for the bridge slab, it is assumed that the principal inspection with respect to the UK highway bridge management code is carried out every six years. The corrosion initiation time is assumed at 9 years, while the corrosion rate for each corrosion environmental condition for simplicity is assumed to be constant over the bridge lifetime. As the inspection outcomes become available, the loss of reinforcement section area and the corresponding loss of flexural moment capacity at inspection time can be obtained by Equations (3.20) and (3.19), respectively. The probability density function associated with specific corrosion rate is determined by equation (3.4). The probability density function of gamma processes for low, medium, and high corrosion rate are shown in Figures 3.4, 3.5, and 3.6, respectively.

Table 3.1 Inspection outcomes for the low corrosion rate environment for RC slab, perfect inspection is assumed (Assumed initiation time=9 years)

Time (year)	Corrosion rate (mm/year)	A before inspection mm^2	A' after inspection mm^2	M before inspection kNm	M' after inspection kNm	X_t^{im} (%)	γ_i(%)	ω_i (year)
0	0	201	201	62.21	62.21	0	-	-
18	0.013	201	200.3	62.21	62.04	0.27	0.27	18
24	0.013	200.3	198.5	62.04	61.54	1.08	0.81	6
30	0.013	198.5	196.6	61.54	61.02	1.91	0.83	6
36	0.013	196.6	194.6	61.02	60.50	2.75	0.84	6

Table 3.2 Inspection outcomes for the medium corrosion rate environment for RC slab, perfect inspection is assumed

Time (year)	Corrosion rate (mm/year)	A before inspection mm^2	A' after inspection mm^2	M before inspection kN.m	M' after Inspection kN.m	X_t^{im} (%)	γ_i(%)	ω_i (year)
0	0	201	201	62.21	62.21	0	-	-
18	0.076	201	197.3	62.21	61.22	1.59	1.59	18
24	0.076	197.3	186.1	61.22	58.16	6.51	4.92	6
30	0.076	186.1	175.3	58.16	55.15	11.35	4.84	6
36	0.076	175.3	164.7	55.15	52.18	16.12	4.77	6

Table 3.3 Inspection outcomes for the high corrosion rate environment for RC slab, perfect inspection is assumed

Time (year)	Corrosion rate (mm/year)	A before Inspection mm^2	A' after Inspection mm^2	M before Inspection kN.m	M' after inspection kN.m	X_t^{im}(%)	γ_i(%)	ω_i (year)
0	0	201	201	62.21	62.21	0	-	-
18	0.254	201	188.5	62.21	58.83	5.43	5.43	18
24	0.254	188.5	153.2	58.83	49.9	19.80	14.37	6
30	0.254	153.2	121.6	49.9	39.6	36.34	16.54	6
36	0.254	121.6	9.37	39.6	31.03	50.12	13.78	6

Table 3.4 Gamma process parameters for different corrosion rate environment

	Corrosion rate					
	0.013 mm/year		0.076 mm/year		0.254 mm/year	
Time (year)	β	c	β	c	β	c
24	0.694	0.031	0.11	0.0314	0.042	0.042
30	0.93	0.059	0.16	0.06	0.050	0.065
36	1.13	0.086	0.19	0.086	0.065	0.089

It is evident that continuous gamma process provides a refined model for deterioration estimates that can provide a valuable and site specific information about the progress of deterioration.

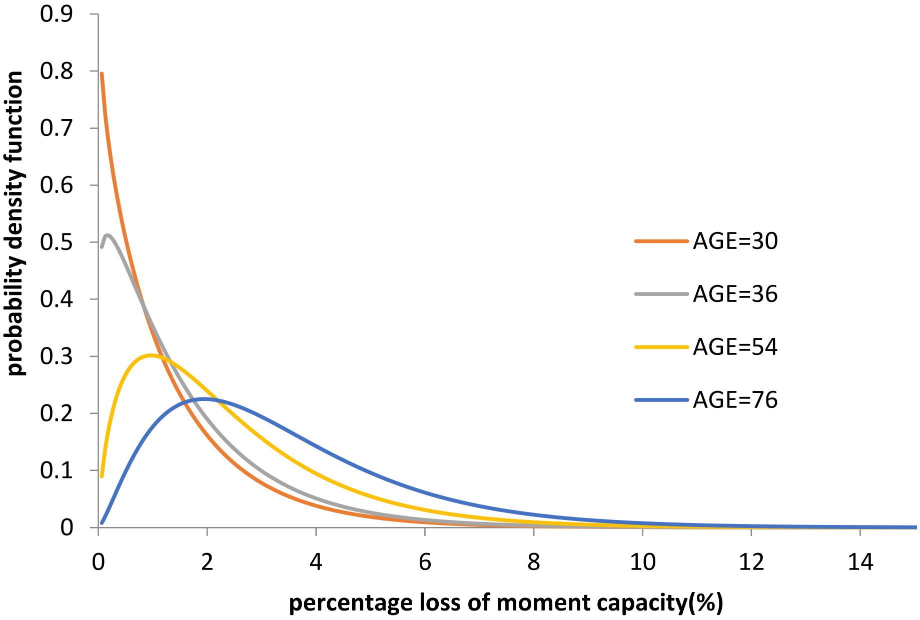

Figure 3.4 Probability density function for the percentage loss of moment capacity for low corrosion rate environment based on the inspection at age 24

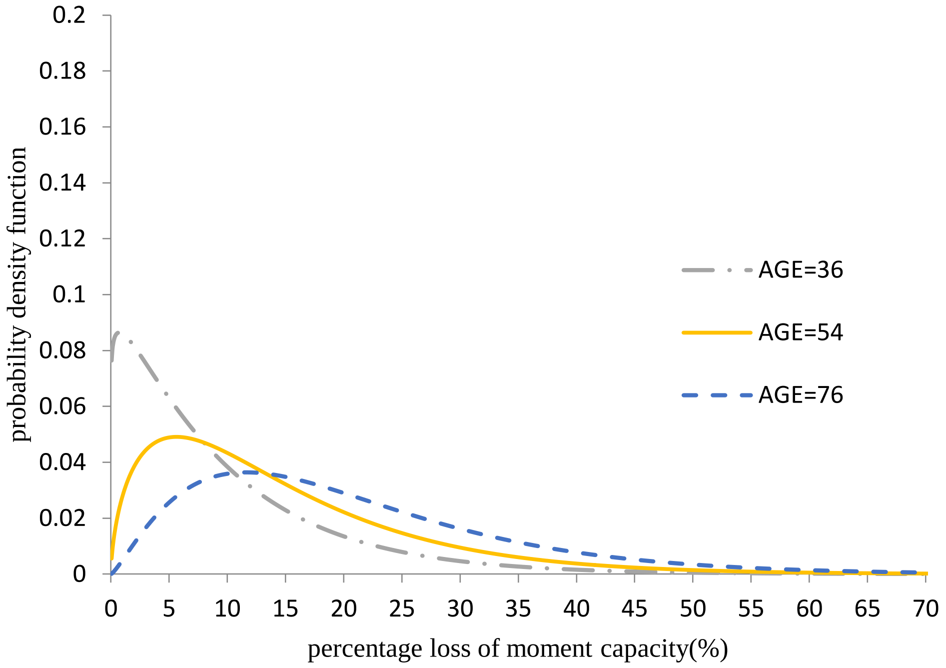

Figure 3.5 Probability density function for percentage loss of moment capacity for medium corrosion rate environment based on the inspection at age24, and 30

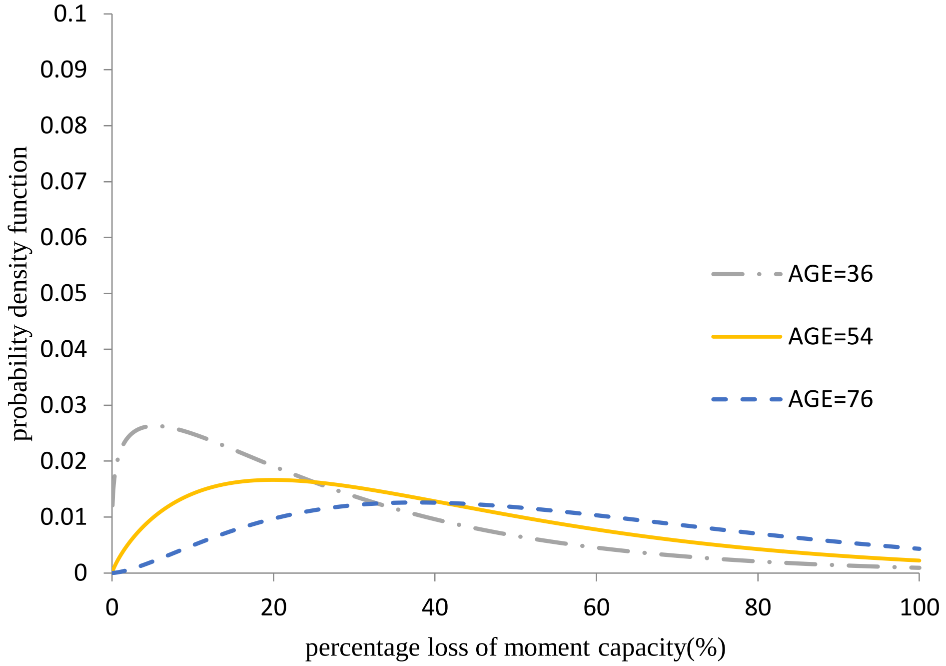

Figure 3.6 Probability density function for percentage loss of moment capacity for high corrosion rate environment based on the inspection at age 24, 30, and 36

X_{t_i} Characterized as a gamma distribution then the cumulative density function of this variable is computed by

$$F\left(x_{t_i} \middle| \alpha(t_i), \beta\right) = \int_0^x f\left(u_{t_i} \middle| \alpha(t_i), \beta\right) du = \frac{\gamma(\alpha(t_i), \beta x_{t_i})}{\Gamma(\alpha(t_i))} \quad (3.23)$$

where $\gamma\left(\alpha(t_i), \beta x_{t_i}\right)$ is lower incomplete gamma function and $f\left(x_{t_i} \middle| \alpha(t_i), \beta\right) = Ga(x|\alpha(t_i), \beta)$

The figures 3.7, 3.8, 3.9 represent the cumulative distribution function for the percentage loss of moment capacity.

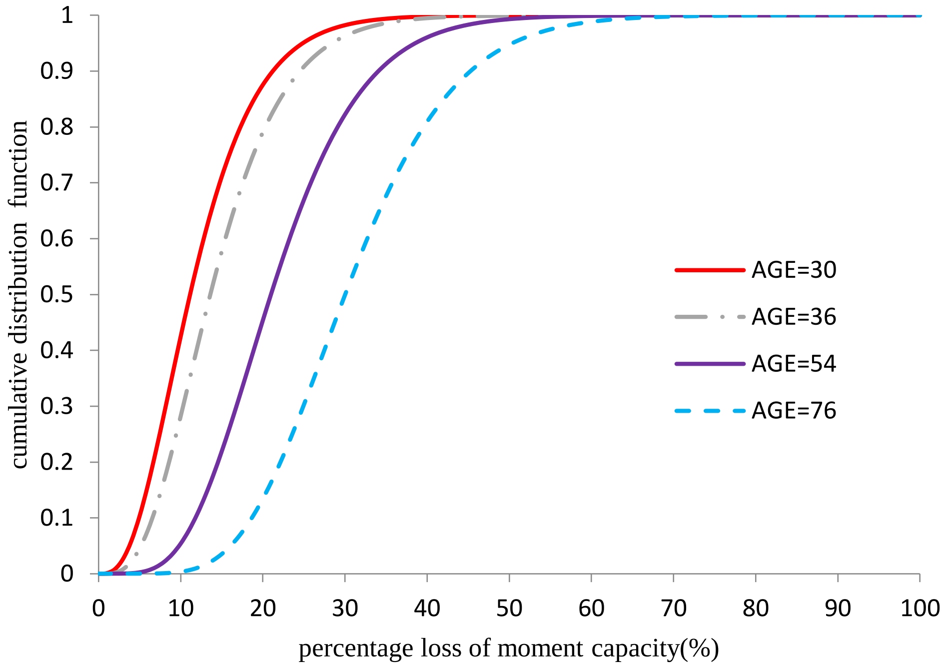

Figure 3.7 Cumulative distribution function for percentage loss of moment capacity for low corrosion rate environment based on the inspection at age 24

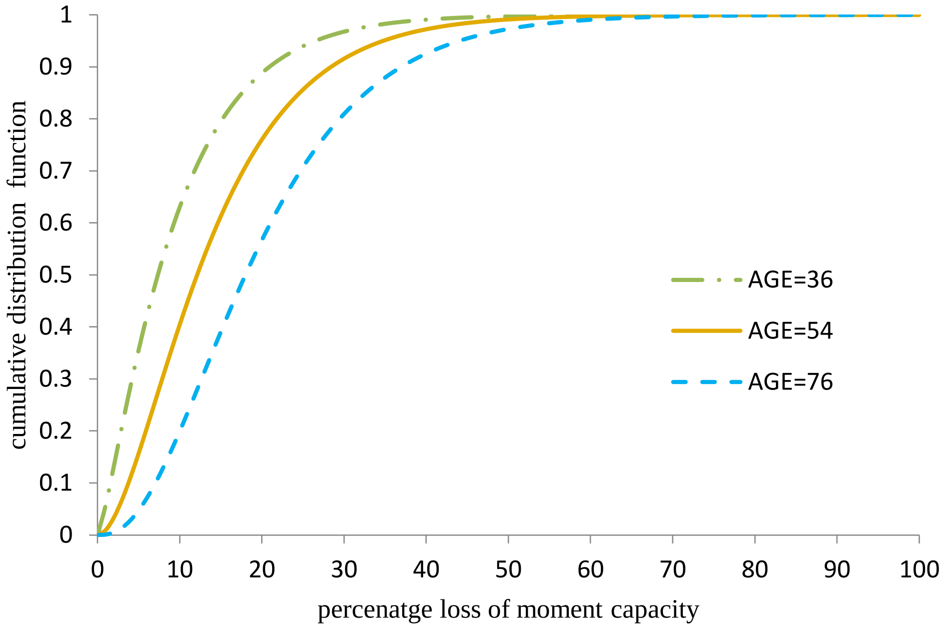

Figure 3.8 Cumulative distribution function for percentage loss of moment capacity for medium corrosion rate environment based on the inspection at age 24, 30

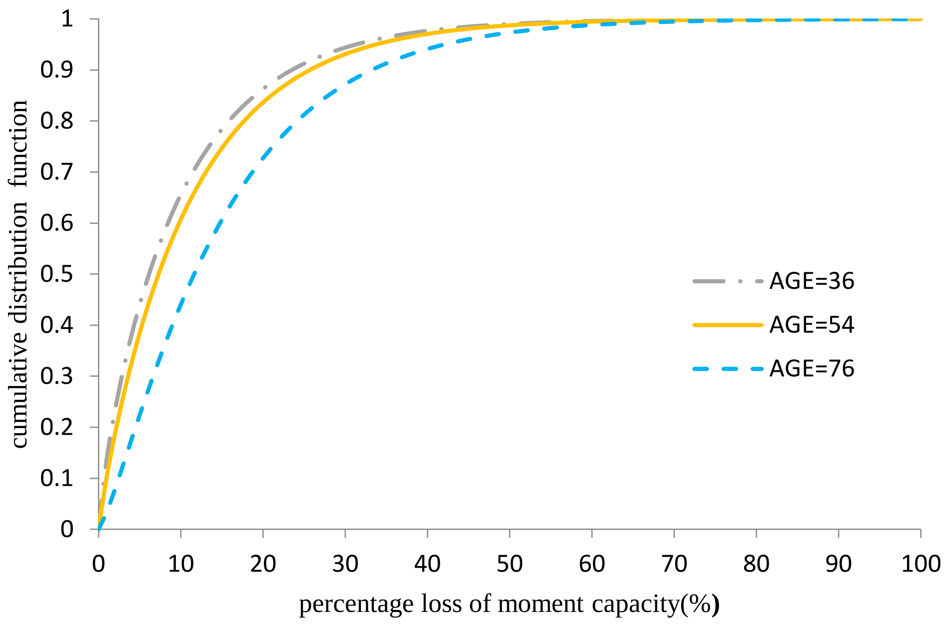

Figure 3.9 Cumulative distribution function for percentage loss of moment capacity for high corrosion rate environment based on the inspection at age 24, 30, and 36

It can be observed that the gamma process parameters decreases as the corrosion rate increases. However, it is indicated that the shape parameter reduction is not remarkable. It is concluded that COV decreases as more inspection outcomes are available which indicates that gamma process is a dynamic process.

3.4.1.5 Demonstration of Degradation Prediction for Selected Time Interval

In this section, the gamma process representation for deterioration of moment capacity for different corrosion rates for selected interval time is considered. Figures 3.10 and 3.11 are showing probability density functions and cumulative density functions for such scenario.

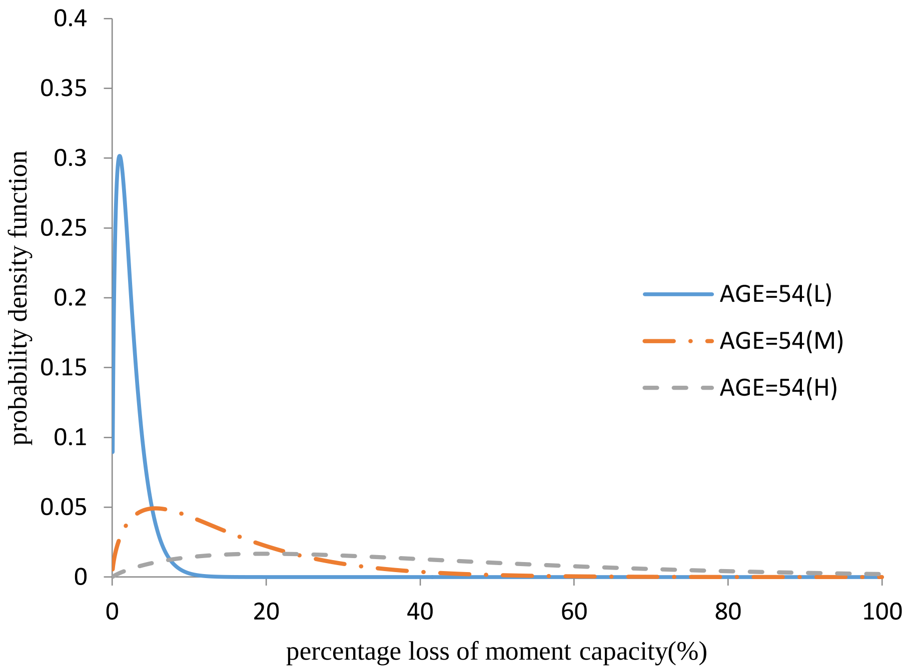

Figure 3.10 Probability density functions of loss of flexural moment for 3 corrosion rate environments for selected interval time (L=Low, M=Medium, H=High)

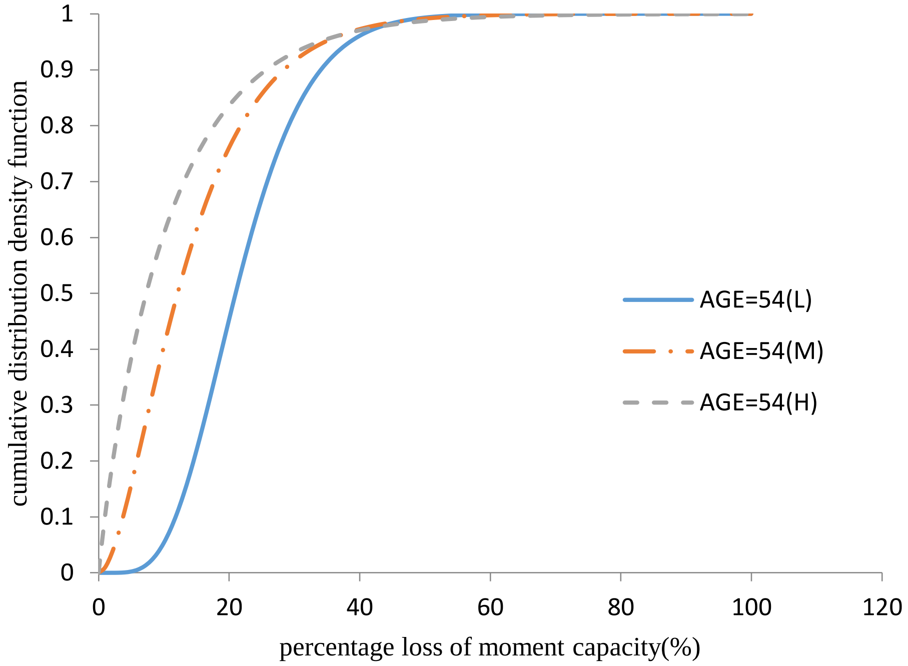

Figure 3.11 Cumulative distribution function of loss of flexural moment for 3 different corrosion rate environments for selected interval time (L=Low, M=Medium, H=High)

It can be concluded from Figure 3.10 that the gamma process parameters increases as corrosion rate increase for selected time while it is indicated in Figure 3.11 that the COV decreases as corrosion rate increase.

3.4.1.6 Demonstration of Degradation Prediction with Variable Initiation Time

Initiation time is assumed to be deterministic in this study. However, due to different environmental and structural conditions it is likely to be variable. In this section, we consider three different initiation times. The functions that reflect the projection for reduction in flexural moment capacity of the reinforced concrete slab for different time horizons in the form of gamma process with different initiation time are illustrated in Figures 3.12-3.14.

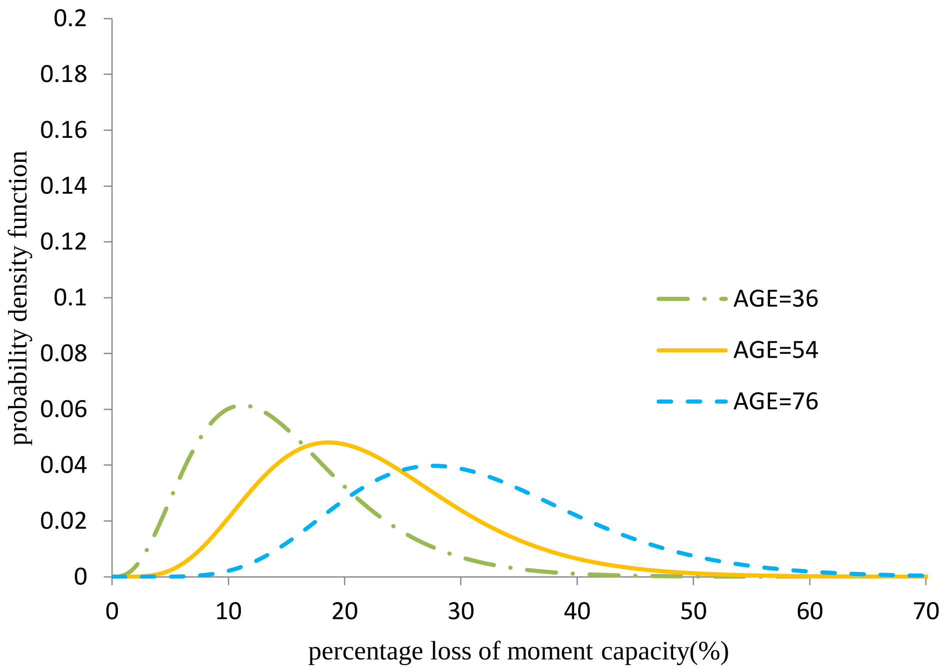

Figure 3.12 Probability density function of loss of flexural moment capacity for medium corrosion rate environment with initiation time 9 years

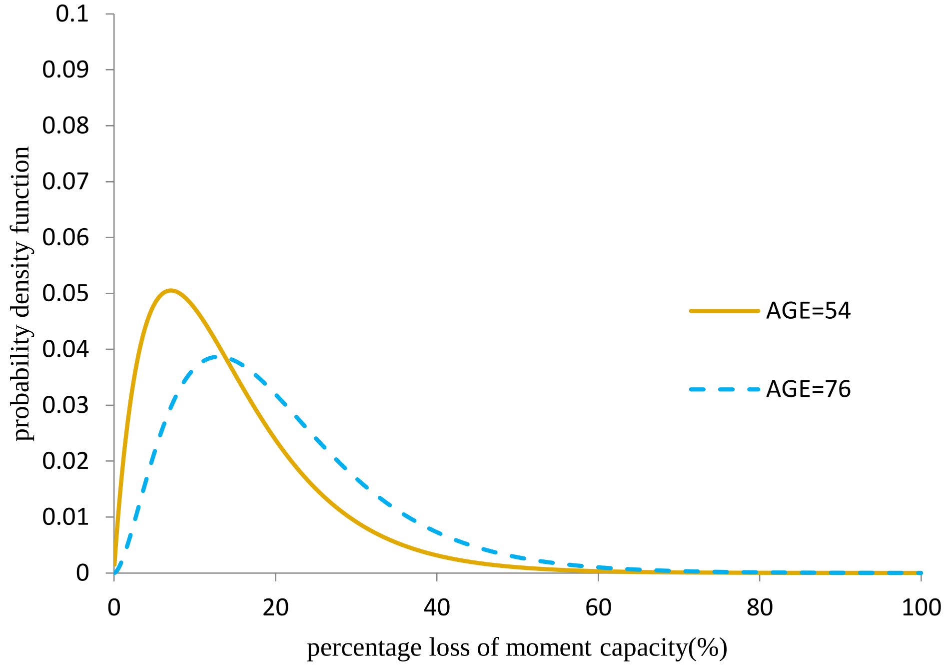

Figure 3.13 Probability density function of loss of flexural moment capacity for the medium corrosion rate environment with initiation time 24 years

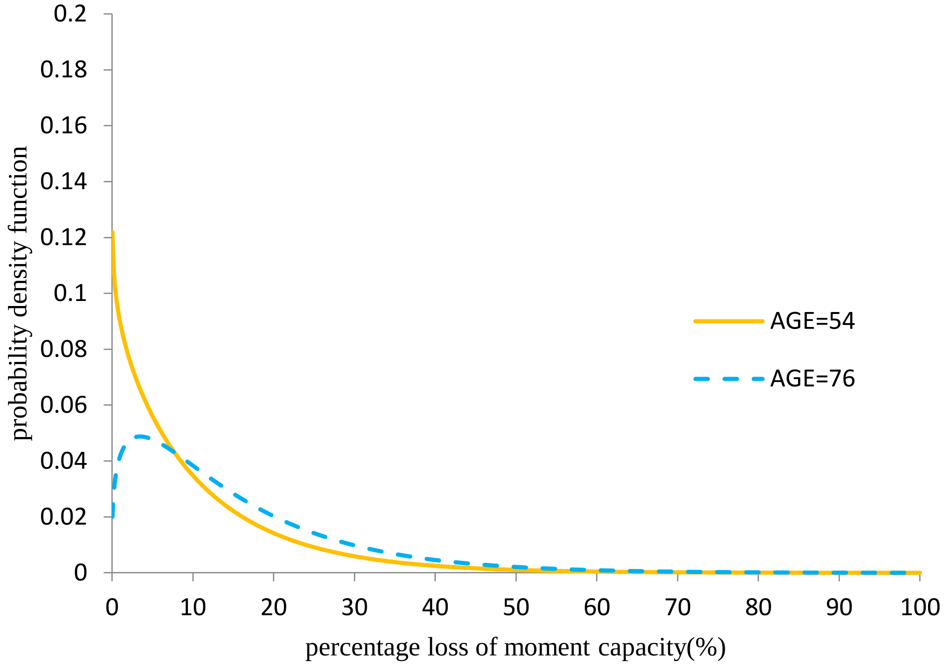

Figure 3.14 Probability density function of flexural loss of moment capacity for the medium corrosion rate environment with initiation time 36 years

As expected the predicted percentage loss of moment capacity for a particular frequency of percentage loss of moment capacity decreases as initiation time increases. However, it is identified that the uncertainty associated with a projected deterioration model in a specific horizon time will increase in respect to COV value. It is concluded that the initiation time and corrosion rate are the parameters that reflect the effect of site-specific environmental and maintenance conditions on the deterioration model.

3.4.2 Summary and Conclusions

In this Chapter, continuous gamma process is used to represent the new updatable deterioration model for flexural capacity of reinforced concrete slab to the level of detail that enables practical implementation.

For a sample of deterioration process taking the advantage of the power law formulation to represent the expected degradation due to corrosion, the simplified approach in the form of the method of moments was applied to obtain the gamma process parameters. It is identified that the shape and scale parameters can be improved as further outcomes are provided. In effect, our projection for deterioration, from the moment of observation should take into account current status but not be concerned by the past events that have preceded the current state. Once the parameters have been defined the deterioration progress for the reinforced concrete slab is predicted for different horizon times.

The new adaptive deterioration model has been implemented to demonstrate how structure specific features influence future structural performance level.

As the degradation of flexural moment capacity measure $X(t_i)$ is associated with uncertainty, the value of the cumulative measure, deterioration state Det_{t_i} of the component at certain time can be evaluated with selected target probability P_{Th}(this probability could be related to current policy). Thus, we have

$$Det_{t_i} = x_{t_i} \; such \; that \; P\left(0 < X_{t_i} \leq x_{t_i}\right) = P_{Th} \qquad (3.24)$$

For example, it is assumed that if the target probability is 50% (P_{Th}=0.5), which means the deterioration state (Det_{t_i}) can be determined as the expected value equation (3.25)

$$Det_{t_i} = E\left(X_{t_i}\right) = \frac{\alpha(t)}{\beta} \qquad (3.25)$$

Figures 3.15 and 3.16 demonstrate the influence of variation of the initiation time and corrosion rate on deterioration states.

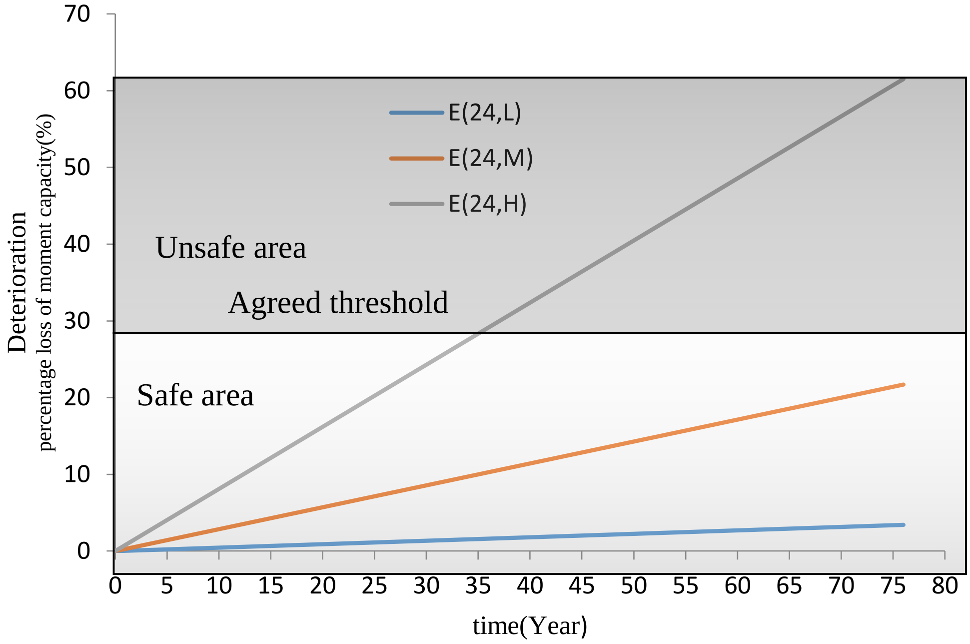

Figure 3.15 Effect of variation of mean corrosion (L=Low, M=Medium, and H=High) rate on expected value of percentage loss of moment capacity based on the same inspection time

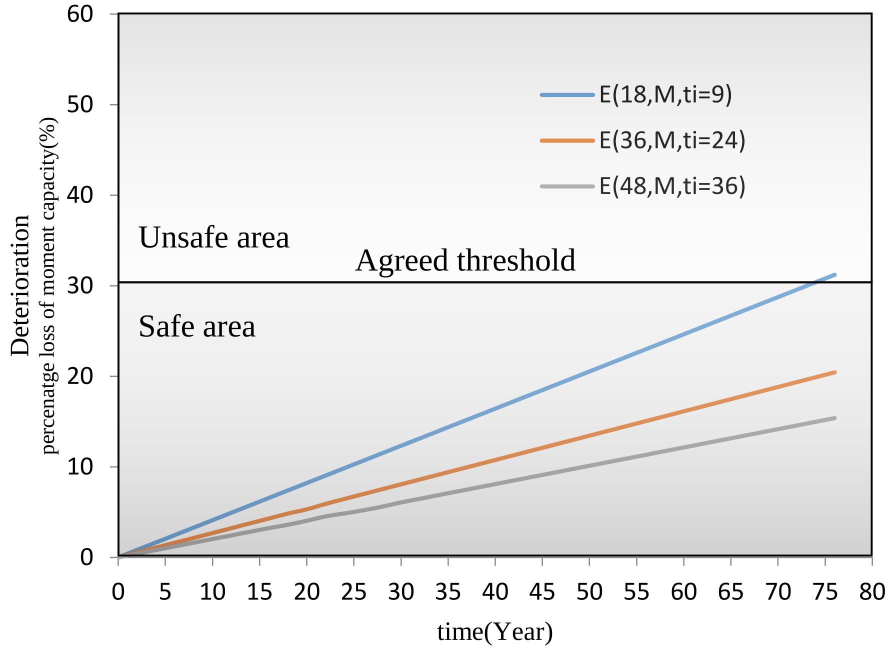

Figure 3.16 Effect of variation of initiation time (t_i= 9, 24, 36 years) on expected value of percentage loss of moment capacity based on different inspection time (18, 36, and 48)

The indicated safety threshold in Figures 3.15 and 3.16 is related to the denoted safety level and could be specific. It is concluded that the new adaptive deterioration profile has a great flexibility and sensitivity to the environmental conditions as it has been demonstrated on the deterioration model graphs due to different corrosion rates. Furthermore, it is demonstrated that the deterioration model can be affected by the effect of structural maintenance in form of initiation time. The information about the deterioration status can be employed with the failure threshold to identify the remaining lifetime of the bridge deck. Once more realistic models for inspection outcomes are included simple deterioration profile graphs will no longer be linear. It is evident that with realistic inspection outcomes, the new updatable deterioration model will be able to capture changes in the deterioration rate after inspection. As the next step, in the next Chapter modeling of inspection outcomes is going to be addressed and subsequently deterioration status graphs that include information about uncertainties associated with inspections will be used to establish an adaptive inspection regime in Chapter 5.

Chapter Four

4 Imperfect Inspection Model for Gamma Process Deterioration Representation

4.1 Introduction

Assessment of existing structures often relies on update on physical properties following an inspection. It is identified that a complete inspection which can detect all types of defects and size of defects is rarely feasible or necessary and may be too costly (Kuniewski et al, 2009). Basically, the operation characteristic of each of the present nondestructive testing (NDT) techniques is governed by a detect ability parameter. Hence, even though an NDT inspection program has been performed on the entire a component and all the defects detected are repaired, the engineer cannot guarantee that there will be absolutely no defects or that defects would be definitely smaller than a particular size. Even most sophisticated NDT techniques are imperfect inspection techniques in practice (Tang, 1973).

Due to natural variability, and the inherent uncertainties associated with NDT techniques, any study on the effect of NDT in determining defect sizes and densities would have to be pursued in the context of the probability theory (Tang, 1973).

During the last decade, the concepts of probability of detection, probability of false alarm, probability of indication have been proved to be suitable parameters to characterize the uncertainties associated with inspection technique (Schoefs et al. 2009).

In order to be able to characterize the deterioration process based on the inspection outcomes, the uncertainty associated with inspection outcomes needs to be characterized and taken into account.

4.2 Characterization of Imperfect Inspection

Uncertainties associated with inspection outcomes reflect the inspection technique features and several parameters can be used to quantify these uncertainties, namely:

- The probability of detection (POD) evaluates the capability of inspection technique to detect a given defect size. Practically, an inspection technique can't detect all sizes of a defect with certainty (Sheils, 2010).
- The probability of false alarm (PFA) is a measure that determines the probability of reporting a defect that does not exists. This measure actually is the value of POD when defect size is equal to zero.
- The report ability threshold is another measure that represents the lowest defect size, which can be detected by a particular inspection technique. This measure characterizes the inspection equipment accuracy and divides the defect's population into two groups as detected and undetected. Report ability factor can be denoted via a detection indicator (D).
- The measurement error represents the factor that is associated with the observed defect size (Tang, 1973; Schoefs et al., 2009; Farngopol et al., 1997; Sheils, 2010). The measurement error has been considered as imperfect inspection parameter, which has been represented as a normal distribution by Kallen and Van Noortwijk (2004).

However, there are different parameters that may be used to assess the uncertainties associated with inspection outcomes, for instance, Schoefs et al (2009) proposed a new parameter as ROC for a set of defects and given NDT tool and operator that couples (POD, PFA) called receiver-operating characteristic (ROC). It is proposed that using the ROC indicator in the case of very harsh conditions of inspection can be very useful. In order to determine the ROC of an inspection technique, the inspection outcomes are assumed as signal-noise model. Two approaches are suggested to define and assess the noise on measurement.

  - The first one consist in considering one independent random variable by level of inspection
  - The second one consists in gathering data by area which leads to get one independent random variable by zone (Schoefs et al.,2009)

All parameters that are described above can be taken into account for the deterioration model characterization.

For clarity, the probability of detection (POD) and the measurement error will be taken into account to characterize the uncertainties associated inspection technique in this study. However, in order to establish the complete deterioration model, it is required to characterize all parameters listed above individually.

4.3 Imperfect Inspection Outcomes

In order to take into account uncertainties associated with inspection outcomes, a new random variable call actual defect size ($X^a_{t_{insp}}$) is presented in this thesis. As mentioned in Chapter 3, the Gamma process is used to represent structural deterioration model while actual defect size need to be considered to estimate the Gamma process parameters. A new model will be developed in the following section to represent the actual defect size respect to the inspection features.

4.3.1 Measurement Error

The measurement error is a well-known inspection uncertainty, which is often presented as a normal random variable with known variance and zero mean value (Zhang & Mahadevan, 2001).

In order to take into account the measurement error with deterioration model in this study, the actual defect size $X^a_{t_{insp}}$ is presented in form of:

$$X^a_{t_{insp}} = X^m_{t_{insp}} + X_\varepsilon \qquad (4.1)$$

Where $X^a_{t_{insp}}$ is the actual defect size at inspection time, $X^m_{t_{insp}}$ is the measured defect size at inspection time, which is presented as a normal random variable here, and X_ε is measurement error considered also as normal random variable. In general the measurement error distribution definitions reflects the inspection technique features.

4.3.2 Probability of Detection

During the last two decades many studies have been conducted to develop a deterioration model where the inspection outcomes uncertainty is taking into account. As mentioned before, the probability of detection (POD) is a parameter that is often addressed.

Zhang and Mahadevan (2001) developed a comprehensive approach to integrate computational reliability methods and used nondestructive inspection outcomes to evaluate reliability due to fatigue. They considered two measures, POD and measurement error to quantify the inspection outcomes uncertainty. The POD has been modeled in form of an exponential function of actual fatigue crack depth while the relationship between the actual and measured crack depth size is expressed with a linear function (Zhang & Mahadevan, 2001).

Pandey (1998) presented a probabilistic framework to estimate the pipeline reliability incorporating the impact of inspection and repair activities planned over the service life. Two parameters, POD and measurement error, have been taken into account to evaluate the uncertainty of in-line inspection outcomes. The POD has been determined by a parametric exponential function. Using the Bayes theorem, the probability density function of detectable defect size has been calculated from the overall defect size distribution (Pandey, 1998).

Maes and Dann (2011) used a Bayesian approach to represent deterioration model of pipelines in respect to the in-line inspection data. In order to evaluate the inspection uncertainties, they used POD, PFA, measurement error, and report ability. These parameters are evaluated using similar model of Zhang & Mahadevan (2001) models, however the hierarchical Bayes model was employed to upgrade the deterioration model (Maes and Dann, 2011).

Orcesi and Frangopol (2011) developed a probabilistic model using the lifetime function to evaluate the probability of failure of bridge components. The possible outcomes with nondestructive inspections are incorporated in an event-tree model. The probability function of failure has been assumed to be Weibull distribution (see Appendix-A). It has shown that for poor-quality inspection outcomes, there is a significant risk to overestimate the probability of safe performance (Orcesi and Frangopol, 2011).

Frangopol et al. (1997) developed a probabilistic framework to optimize planning of inspection and repair of structures that deteriorated over the time. The model incorporates the quality of inspection techniques with different detection capabilities. The cumulative normal distribution function is used to calculate the probability of detection (Frangopol et al., 1997).

The probability of detection (POD) of a particular inspection technique with a given threshold can be expressed as:

$$POD = P\left(X^{D}_{t_i,insp_j} = 1\right) = P(x^{m}_{t_i} > Th^{l}_{j}) \quad (4.2)$$

where $x^{m}_{t_i}, Th^{l}_{j}$ are the observed defect size (physical parameter) at inspection time and the inspection lower threshold, respectively. Here, it is assumed that the *POD* becomes equal to 1 when the defect size is greater than a specific upper threshold. It is denoted as Th^{u}_{j} . Frangopol et al. (1997) proposed that the inspection lower threshold (Th^{l}_{j}) and upper threshold (Th^{u}_{j}) could be calculated for components

$$Th^{l}_{j} = x_{0.5} - 3\sigma \quad (4.3)$$

$$Th^{u}_{j} = x_{0.5} + 3\sigma \quad (4.4)$$

where $x_{0.5}$ is the defect size at which the inspection technique has a 50% probability of detection and σ is the standard deviation value of detectability. Since the coefficient of variation of report ability is an arbitrary value in our model, which is assumed 0.1, then σ is determined as:

$$\sigma = COV \times x_{0.5} = 0.1 \times x_{0.5} \quad (4.5)$$

The probability of detection is thus

$$\begin{cases} POD = 0 & 0 < x_{t_i}^m \leq Th_j^l \\ POD = \Phi\left(\frac{x_{t_i} - x_{0.5}}{\sigma}\right) & Th_j^l < x_{t_i}^m < Th_j^u \\ POD = 1 & x_{t_i}^m \geq Th_j^u \end{cases} \quad (4.6)$$

where $\Phi(.)$ is the cumulative normal distribution function. A general illustration of probability of detection for variety of observed defect sizes with arbitrary thresholds is demonstrated in Figure 4.1.

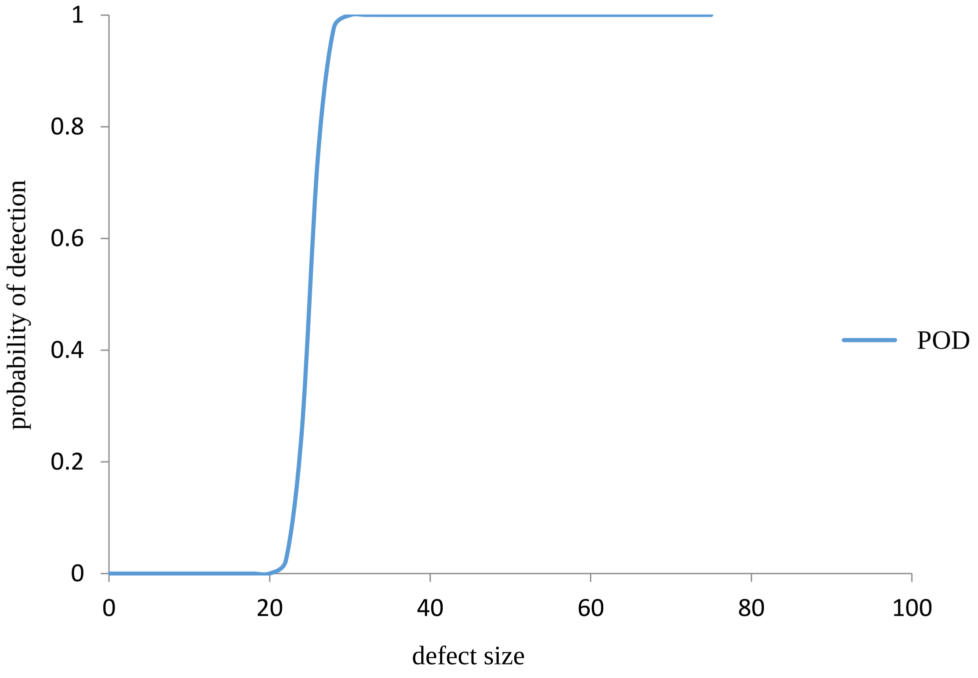

Figure 4.1 General illustration of cumulative distribution function of probability of detection (POD) of a particular inspection technique ($\boldsymbol{insp_j}$)

In order to indicate whether a defect is detected by a particular inspection technique at inspection time or not, a detection indicator is introduced here ($X^{D}_{t_i,insp_j}$). It is a binary random variable taking the value one (*1)* with the probability of detection (POD), when the defect is detected. Detection indicator ($X^{D}_{t_i,insp_j}$) takes the value zero (0), when the defect is not detected. In other words, the indicator acts like a filter that divides the defect population into two groups of detected and undetected defects. A new variable ($X^{SD}_{t_i,insp_j}$) is introduced here to represent the successfully detected defects.

$$\begin{aligned} X^{SD}_{t_i,insp_j} &= X^{m}_{t_i,insp_j} \times X^{D}_{t_i,insp_j} = X^{m}_{t_i,insp_j} \qquad X^{D}_{t_i,insp_j} = 1 \qquad \forall\, x_{t_i,insp_j} > Th^{l}_{j} \\ X^{SD}_{t_i,insp_j} &= X^{m}_{t_i,insp_j} \times X^{D}_{t_i,insp_j} = 0 \qquad X^{D}_{t_i,insp_j} = 0 \qquad \forall x_{t_i,insp_j} \leq Th^{l}_{j} \end{aligned} \tag{4.7}$$

where $X^{D}_{t_i,insp_j}$ is expressed as a discrete binary random variable; Bernoulli distribution function is appropriate probability function to characterize its uncertainty.

The Bernoulli distribution is a distribution function that takes value one (1) with success probability p and value zero (0) with failure probability $q = 1 - p$ (Ugrate et al., 2008).

The probability mass function of $X^{D}_{t_i,insp_j}$ (Bernoulli distribution) is presented

$$f\left(X^{D}_{t_i,insp_j}, p\right) = p^{X^{D}_{t_i,insp_j}}(1-p)^{1-X^{D}_{t_i,insp_j}}\ for\ X^{D}_{t_i,insp_j} \in \{0,1\} \quad (4.8)$$

Here, the value of p is considered a variable, which is represented in form of a cumulative standard normal distribution. Figure 4.2 illustrates the PMF of a Bernoulli distribution.

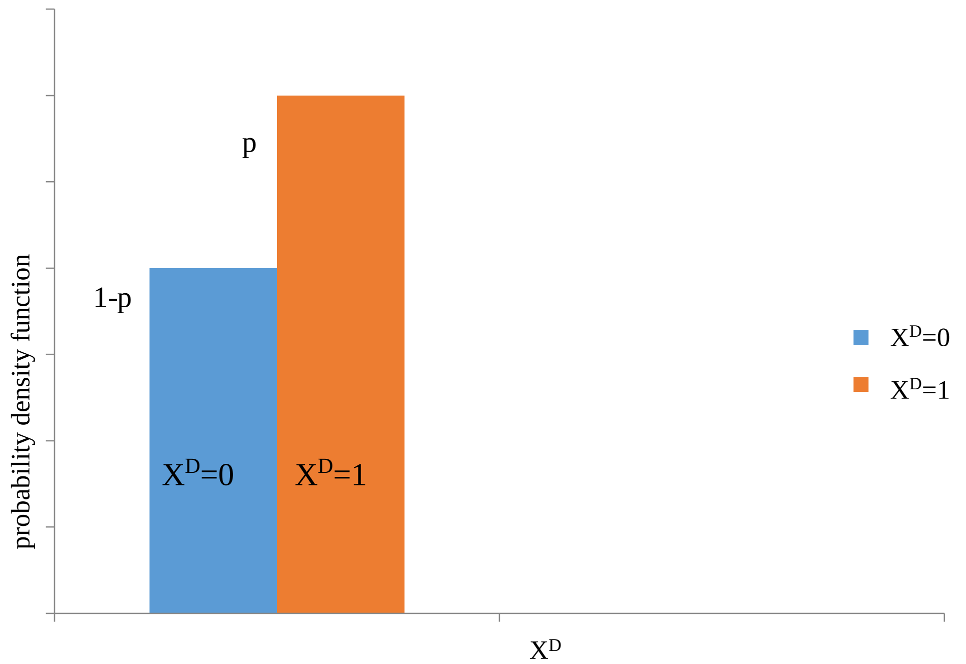

Figure 4.2 General illustration of PMF of detection indicator

It is assumed that the value of p in the former equation is identical to POD.

4.4 Updating of Deterioration Projection Subject to Imperfect Inspection

In order to take into account the probability of detection and measurement error as inspection outcomes uncertainties, the mathematical function that can present the relationship of observed defect size ($X^m_{t_i,insp_j}$) and actual defect size ($X^a_{t_i,insp_j}$) at inspection time is

$$X^a_{t_i,insp_j} = h\left(X^m_{t_i,insp_j}, X^\varepsilon_{insp_j}, X^D_{t_i,insp_j}\right) = X^m_{t_i,insp_j} X^D_{t_i,insp_j} + X^\varepsilon_{insp_j} \quad \forall x_{t_{insp}} \geq 0 \quad (4.9)$$

In order to take into account the detect ability of inspection technique; the total probability law can be applied (Newby and Dagg, 2004).

The joint density function of successfully detected defect size (X^{SD}) that is a function of observed defect size and detection indicator, when a specific inspection technique is carried out, could be represented as

$$P\left(X^{SD}_{t_i,insp_j} \le x_{t_i}\right) = \sum_{n=0}^{1} P\left(X^{m}_{t_i,insp_j} \le x_{t_i}, X^{D_n}\right)$$
$$= P(X^{m}_{t_i,insp_j} \le x_{t_i}, X^{D_0}) + P(X^{m}_{t_i,insp_j} \le x_{t_i}, X^{D_1}) \qquad (4.10)$$

Given $X^{D_1} = 1$ then $X^{SD}_{t_i,insp_j} = X^{m}_{t_i,insp_j}$

$$P\left(X^{m}_{t_i,insp_j} \le x_{t_i}, X^{D_1}\right) = P(X^{m}_{t_i,insp_j} \le x_{t_i} | X^{D_1} = 1) \times P(X^{D_1} = 1) \qquad (4.11)$$

$$P\left(X^{m}_{t_i,insp_j} \le x_{t_i} \middle| X^{D_1} = 1\right) = \frac{P(X^{D_1}=1|X^{m}_{t_i,insp_j} \le x_{t_i}) \times P(X^{m}_{t_i,insp_j} \le x_{t_i})}{P(X^{D_1}=1)} \qquad (4.12)$$

Since $\qquad P\left(X^{D_1} = 1 \middle| X^{m}_{t_i} \le x_{t_i}\right) = P(X^{D_1} = 1) = POD$

The previous equation to obtain the conditional density function can be written as:

$$P\left(X^{m}_{t_i,insp_j} \le x_{t_i} \middle| X^{D_1} = 1\right) = \frac{POD \times P(X^{m}_{t_i,insp_j} \le x_{t_i})}{POD} = P(X^{m}_{t_i,insp_j} \le x_{t_i}) \quad (4.13)$$

Thus, the joint density function of successfully detected defect size at inspection time can be determined as:

$$\begin{cases} P(X^{m}_{t_i,insp_j}, X^{D_0}_{t_i,insp_j}) = P\left(X^{m}_{t_i,insp_j} \le x_{t_i}\right) \times (1 - POD) & \forall x_{t_i} \le Th^l_j \\ P(X^{m}_{t_i,insp_j}, X^{D_1}_{t_i,insp_j}) = P\left(X^{m}_{t_i,insp_j} \le x_{t_i}\right) \times POD & \forall x_{t_i} > Th^l_j \end{cases} \qquad (4.14)$$

Furthermore, it is defined that this feature of inspection technique can be used to indicate whether an inspection technique is adequate, with regard to the prediction model of deterioration process, or not.

The measurement error is another known uncertainty that is identified as a normal random variable while its mean value is zero. In order to take into account the measurement error with deterioration model, the actual defect size $X^{a}_{t_{insp}}$is presented in form of Equation (4.1).

It should be noted that the measured defect size $X^{m}_{t_i,insp_j}$ is replaced by $X^{SD}_{t_i,insp_j}$the successfully detected defect size.

Since the successfully detected defect and measurement error are independent variables, then the joint density function of actual and successful defect size is determined as:

$$P\left(X^{a}_{t_i,insp_j}\right) = P\left(X^{SD}_{t_i,insp_j}, X^{\varepsilon}_{insp_j}\right) = P\left(X^{SD}_{t_i,insp_j}\right) P(X^{\varepsilon}_{insp_j}) \tag{4.15}$$

$$P(X^{SD}_{t_i,insp_j}) = P(X^{m}_{t_i,insp_j}, X^{D_1}_{t_i,insp_j}) = P\left(X^{m}_{t_i,insp_j} \leq x_{t_i}\right) \times POD \qquad \forall x_{t_i} > Th^{l}_{j}$$
(4.16)

Since X_{ε} is presented in form of a normal variable $X^{\varepsilon}_{insp_j} \sim N(0, \nu^2)$ then:

$$P\left(X^{\varepsilon}_{insp_j} \leq x_{t_i} - y\right) = \Phi\left(\frac{x_{t_i} - y}{\nu}\right) \quad , y = Th^{l}_{j} \tag{4.17}$$

$$P\left(X^{a}_{t_i,insp_j}\right) = P\left(X^{SD}_{t_i,insp_j}, X^{\varepsilon}_{insp_j}\right) = P\left(X^{m}_{t_i,insp_j} \leq x_{t_i}\right) P(X^{\varepsilon}_{insp_j}) POD \tag{4.18}$$

Once the inspection technique and its features are identified, the inspection thresholds can be determined by Equations (4.3) and (4.4). In order to calculate the probability of detection (POD), the defect size needs to be available. The inspection technique is implemented to collect the defect size. As mentioned in Chapter 2, it is evident that the inspection outcomes are not deterministic. Therefore, an appropriate distribution function, which is assumed normal distribution here, is assigned to the inspection outcomes. The probability distribution function for successfully detected defect size is formulated by Equation (4.14). Since the defect size and measurement error are assumed independent variables, and then cumulative distribution function of successfully detected defect size and measurement error can be determined by Equation (4.18). The actual defect size with certain confidence level can be derived from the cumulative distribution function of actual defect size. The actual defect size with 50 percent confidence level over the lifetime will be demonstrated in Figure 4.17.

4.5 Application of Different Inspection Types

In order to illustrate the influence of uncertainties associated with inspection outcomes, the reinforced concrete bridge slab subject to corrosion used in Chapter 3 is considered. The corrosion of reinforcement bars results in degradation of flexural moment capacity.

In the previous Chapter, the deterioration model of the RC slab has been determined by a Gamma process while the perfect inspection outcomes were used to estimate Gamma process parameters. However, it is not the case here.

It is assumed that two inspection techniques (inspection type 1 and 2) are carried out to provide observed inspection outcomes. However, it has to be noticed that each inspection technique can be chosen at inspection time with respect to the inspection thresholds and defect size. Thus resulting in different inspection scenarios, where every scenario could be a combination of inspection types or a single inspection type over the lifetime. The features, mean value and standard deviation of two inspection outcomes are listed in Tables 4.1 and 4.2, respectively. Inspections are assumed to have been carried out every 6 years. The inspection feature can be determined based on in-situ outcomes.

Two scenarios are considered here to provide observed inspection outcomes.

- Since the inspection type 2 can cover a wide range of defect size, the inspection 2, solely, will be carried out form 18 to 36 years.

- It is demonstrated that the defect size at 18 years is less than the lower threshold of inspection type 1 therefore; inspection type 1 can be carried out from 24 years to 36 years. It is assumed that there is no data before 24 years in this scenario.

Table 4.1 Inspections features

Inspection type	$x_{0.5}$ (%)	σ_{insp_j}	Th_j^l (%)	Th_j^u(%)	y(%)	σ_ε
INS1	7.41	1.14	3.99	10.83	Variable	0.95
INS2	9	2.5	1.5	16.5	Variable	1.19

Table 4.2 Observed percentage loss of moment capacity (X^m) INS2

Time	Mean value	Standard deviation
18	1.59	0.8
24	6.51	0.8
30	11.35	0.8
36	16.21	0.8

The standard deviation of inspection outcomes is derived from inspection results in AASHTO (2001). As inspection outcomes are associated with uncertainties, a random variable can be considered to present their properties. For the sake of simplicity, normal distribution is used to represent the uncertainty associated with observed inspection outcomes.

The probability density function and cumulative distribution function of observed defect size as a random normal variable is demonstrated in Figure 4.3 and 4.4, respectively.

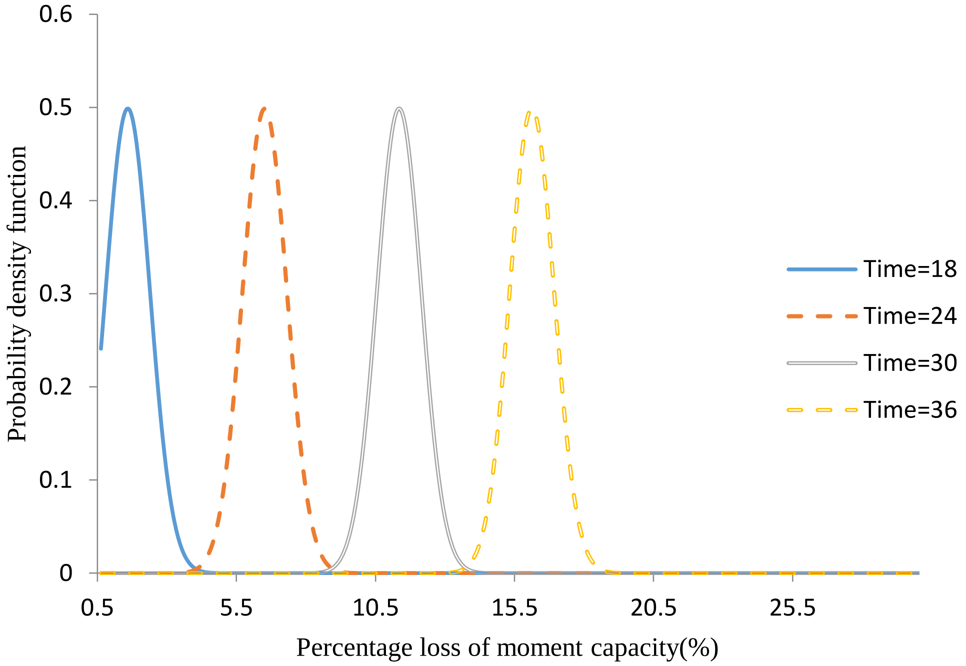

Figure 4.3 Probability density function of the observed inspection outcomes using INS2

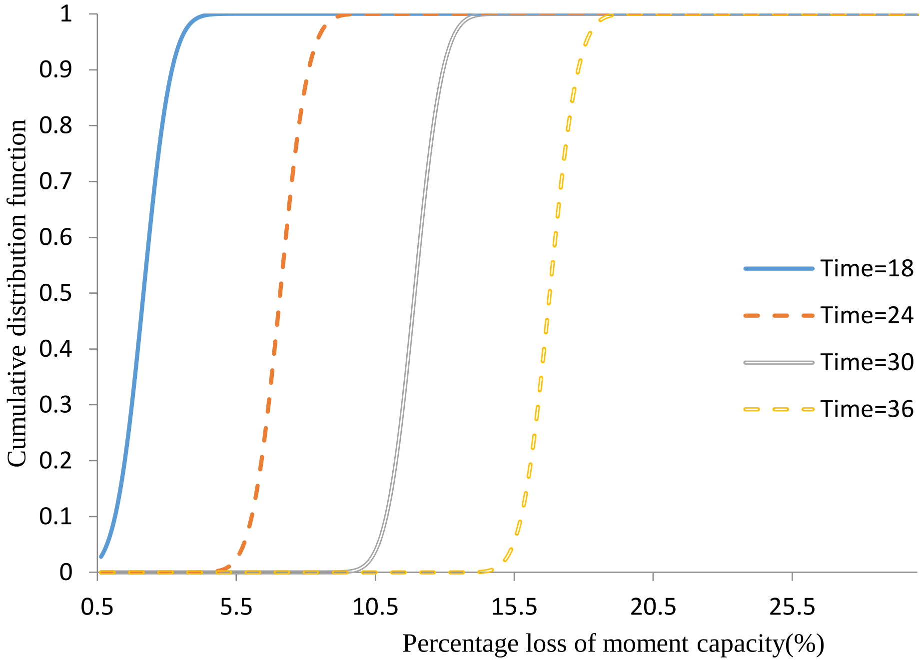

Figure 4.4 Cumulative distribution function for observed inspection outcomes using INS2

In order to obtain the cumulative distribution function of actual percentage loss of moment capacity, the POD has to be established, Equation (4.8).

The cumulative distribution function of successfully detected defect size (X^{SD}) is determined by Equation (4.14). The cumulative distribution functions of successfully detected defects at inspection times are demonstrated in Figure 4.5-4.8.

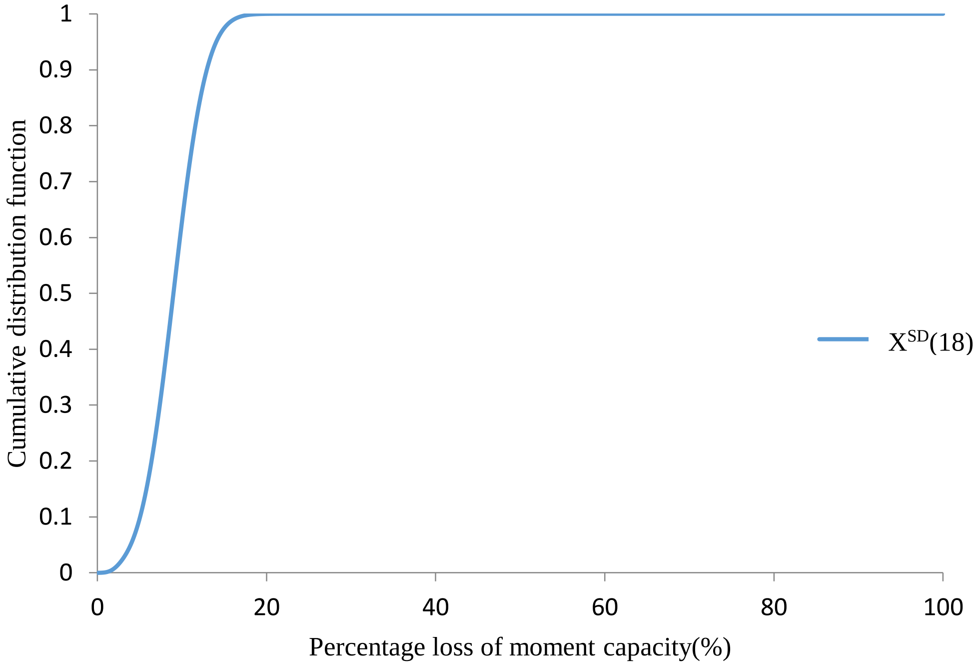

Figure 4.5 Cumulative distribution function for successfully detected percentage loss of moment capacity at age18

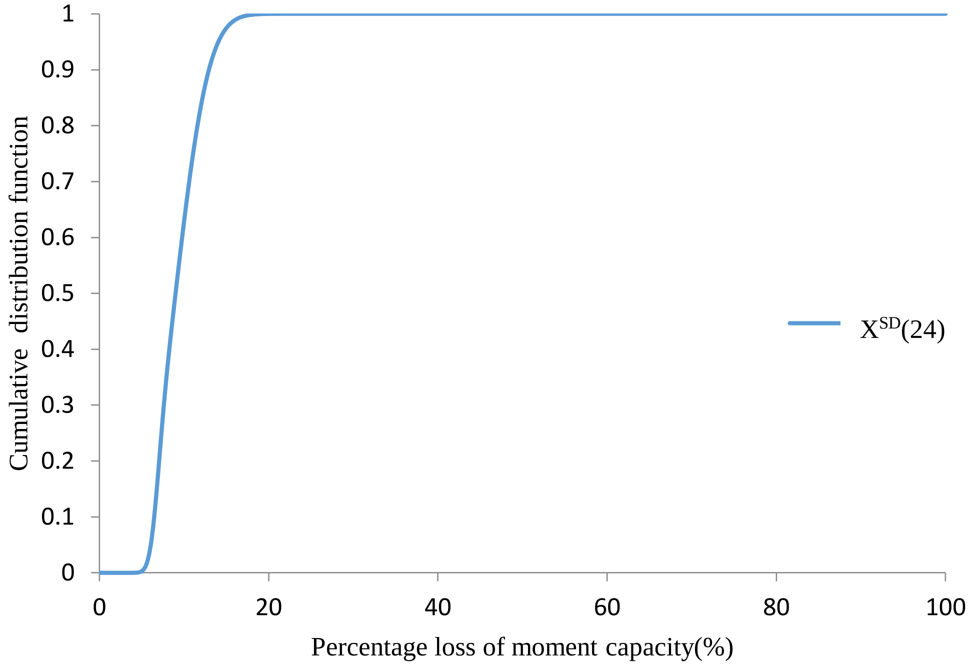

Figure 4.6 Cumulative distribution function of successfully detected percentage loss of moment capacity at age 24

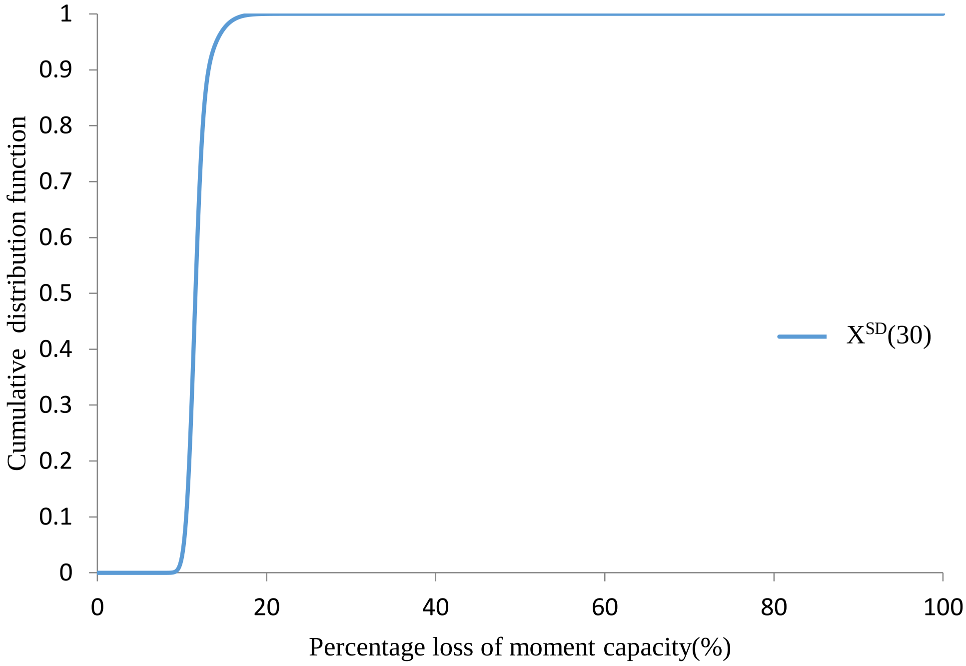

Figure 4.7 Cumulative distribution function of successfully detected percentage loss of moment capacity at age 30

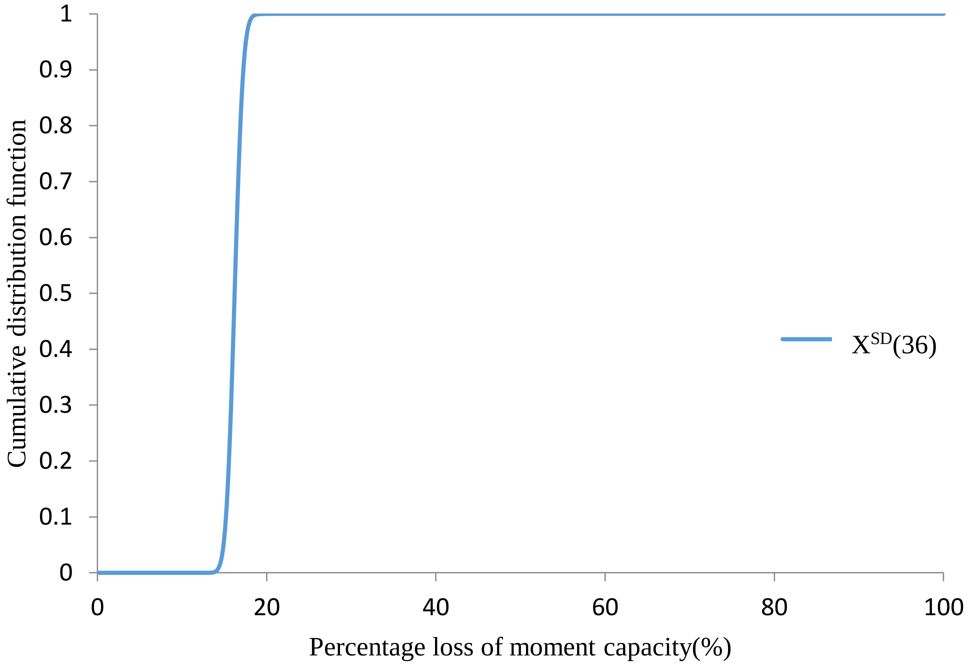

Figure 4.8 Cumulative distribution function of successfully detected percentage loss of moment capacity at age 36

It is demonstrated in Figure 4.9 that the difference between cumulative distributions of observed defect and successfully detected defect will decrease as mean value of the observed defect size increases over the time.

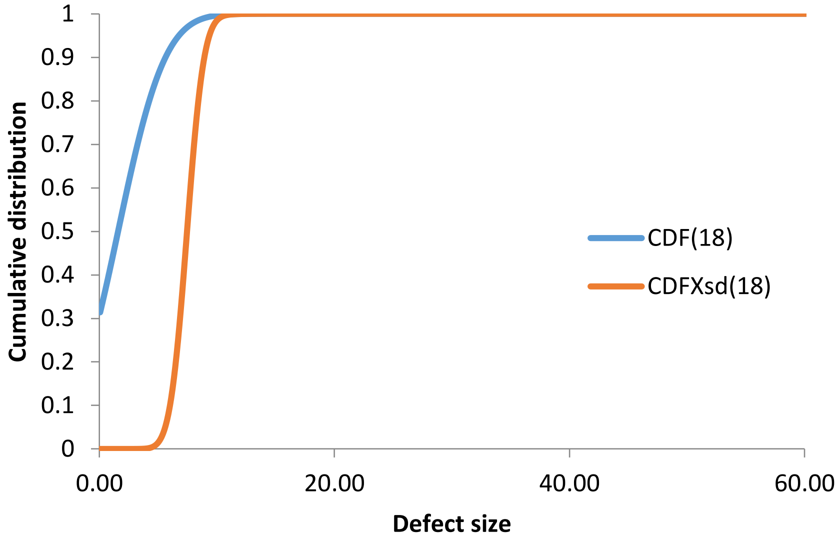

Figure 4.9 Comparison of cumulative distribution function of observed and successfully detected percentage loss of moment capacity at age 18

It is observed in Figure 4.10 that at 36 years the cumulative distributions of observed and successfully detected percentage loss of moment capacity are much closer.

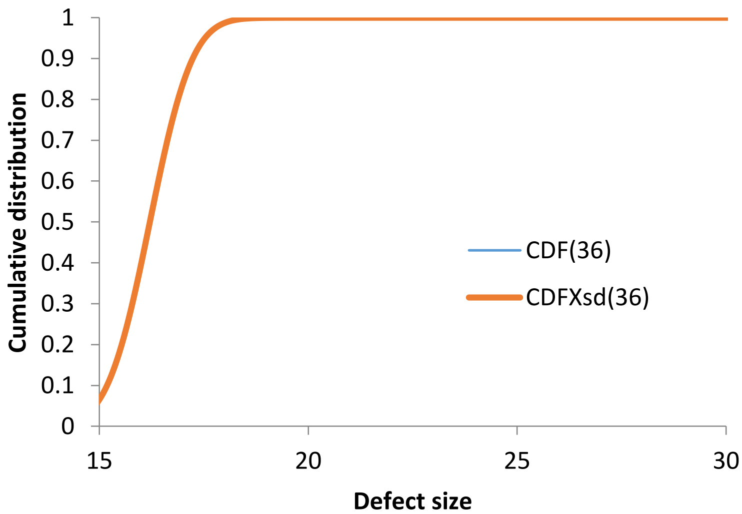

Figure 4.10 Comparison of cumulative distribution function of observed and successfully detected percentage loss of moment capacity at age 36

It can be concluded that applying an inspection technique with lower probability of detection at later age, which may cost lower , can be sophisticated decision due to the growth of the percentage loss of moment capacity. The mean values of successfully detected defect size are obtained from Figure 4.5-4.8 and demonstrated in Table 4.3.

Table 4.3 Mean value of successfully detected percentage loss of moment capacity

Time	$X_{0.5}^{SD}$
18	9.00
24	9.00
30	11.50
36	16.20

Measurement error is defined as another inspection uncertainty associated with inspection outcome. Since the successfully detected defect size and measurement error are independent random variables, then cumulative distribution function of actual percentage loss of moment capacity is determined by Equation (4.15) and (4.16), where y is the successfully detected defect size with 90% confidence level.

The measurement error is assumed as a normal random variable with zero mean value and known standard deviation, which is demonstrated for two inspection types in Table 4.1. The cumulative distribution function of actual defect size is identified by equation (4.18).

The cumulative distribution function of actual defect size at inspection time, which can be used to estimate the Gamma parameters, is demonstrated in Figure 4.11-4.14.

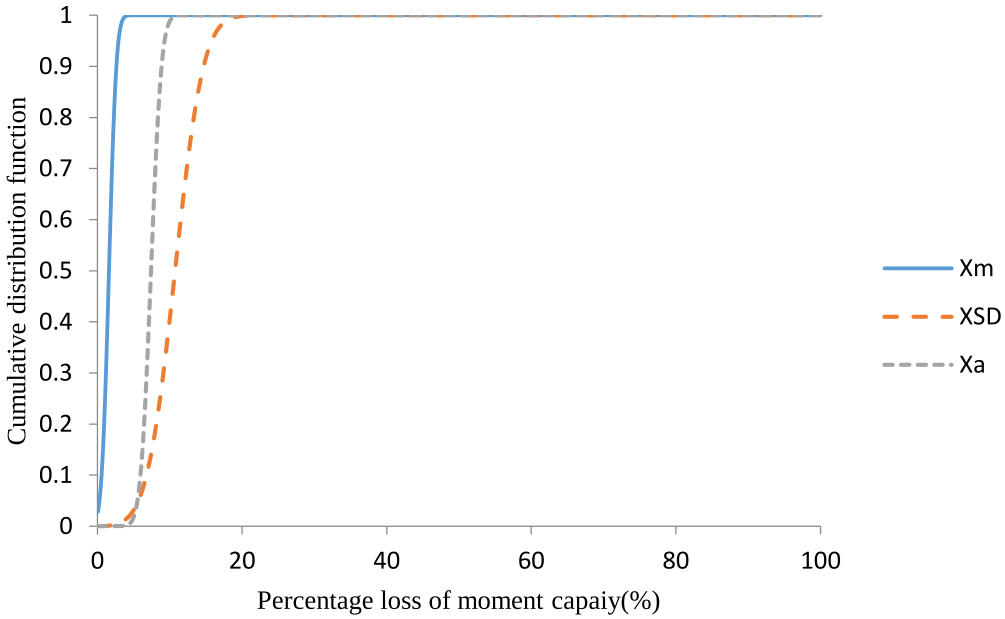

Figure 4.11 Comparison of cumulative distribution function of observed, successfully detected and actual percentage loss of moment capacity at age 18

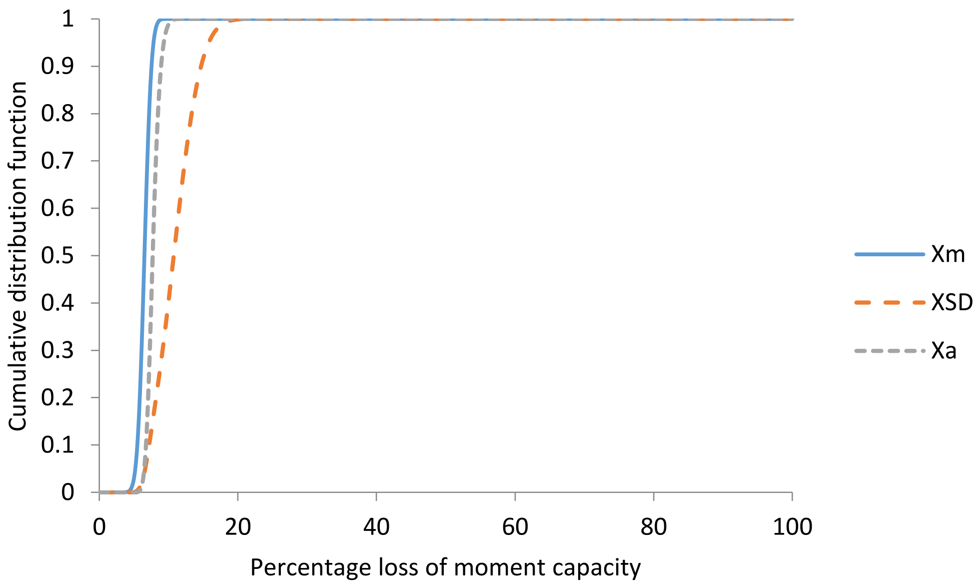

Figure 4.12 Comparison of cumulative density function of observed, successfully detected and actual percentage loss of moment capacity at age 24

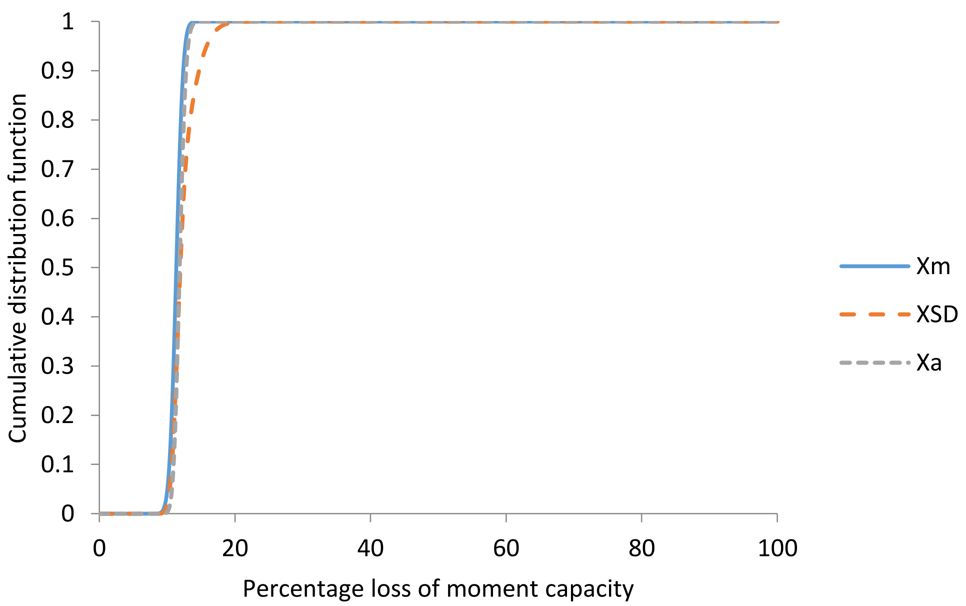

Figure 4.13 Comparison of cumulative density function of observed, successfully detected and actual percentage loss of moment capacity at age 30

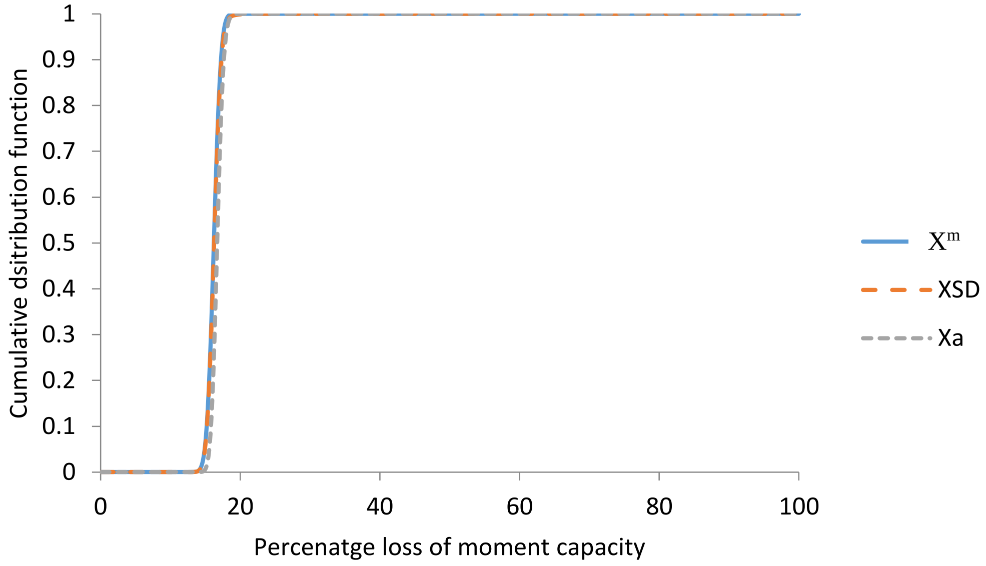

Figure 4.14 Comparison of cumulative density function of observed, successfully detected and actual percentage loss of moment capacity at age 36

In order to estimate the Gamma process parameters, inspection outcomes are used. Rather than using expert opinion (assumption of perfect inspection) the actual information with 50% confidence level is considered in this study. The method of moments is applied to estimate the scale and shape function of gamma process at different inspection time. The results of actual defect size and gamma process parameters are presented in Table4.4 and 4.5, respectively. Where γ^a is the actual deterioration increment over the interval and ω Is the interval time as it has been explained in Chapter 3.

Table 4.4 Actual percentage loss of moment capacity at inspection time with 50% confidence level

Time	X^a	γ^a	ω
18	9.937	9.937	18
24	9.937	0	6
30	12.06	2.12	6
36	16.75	4.69	6

Table 4.5 Gamma process parameters for perfect and imperfect inspection technique 2

Time	β^a	c^a	β^m	c^m
24	0.301	0.125	0.11	0.0314
30	0.511	0.205	0.16	0.06
36	0.780	0.362	0.19	0.086

In the following, the estimated gamma process parameters are used to determine the deterioration profile of the bridge slab for age 46 and 54. The cumulative distribution of gamma processes at 46 and 54, which are estimated with regard to the actual defect size at inspection times, are indicated in Figure 4.15 and 4.16, respectively.

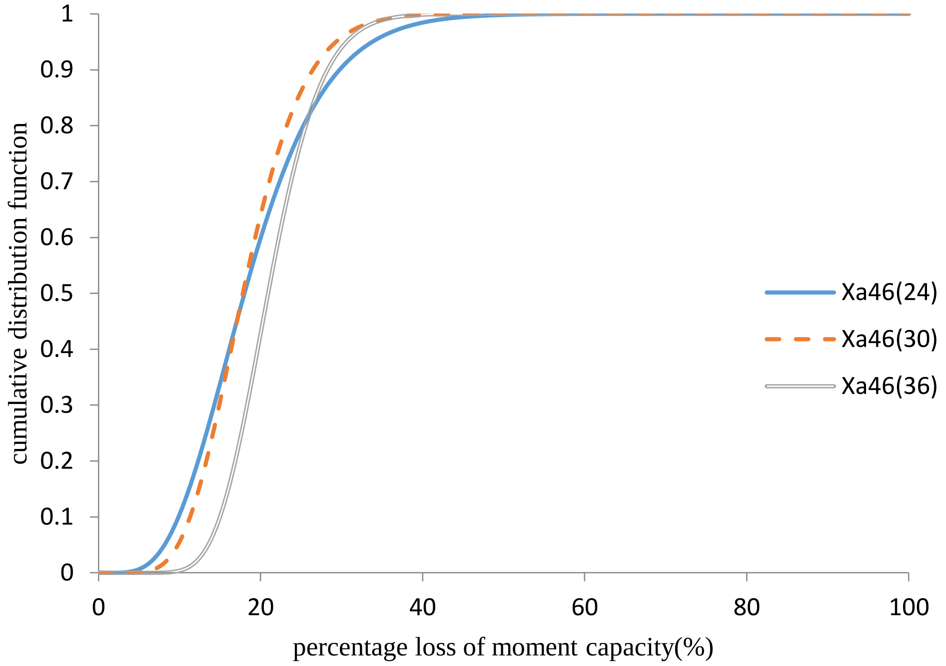

Figure 4.15 Cumulative distribution of Gamma process for percentage loss of moment capacity at age 46 using actual defect measurement form INS2

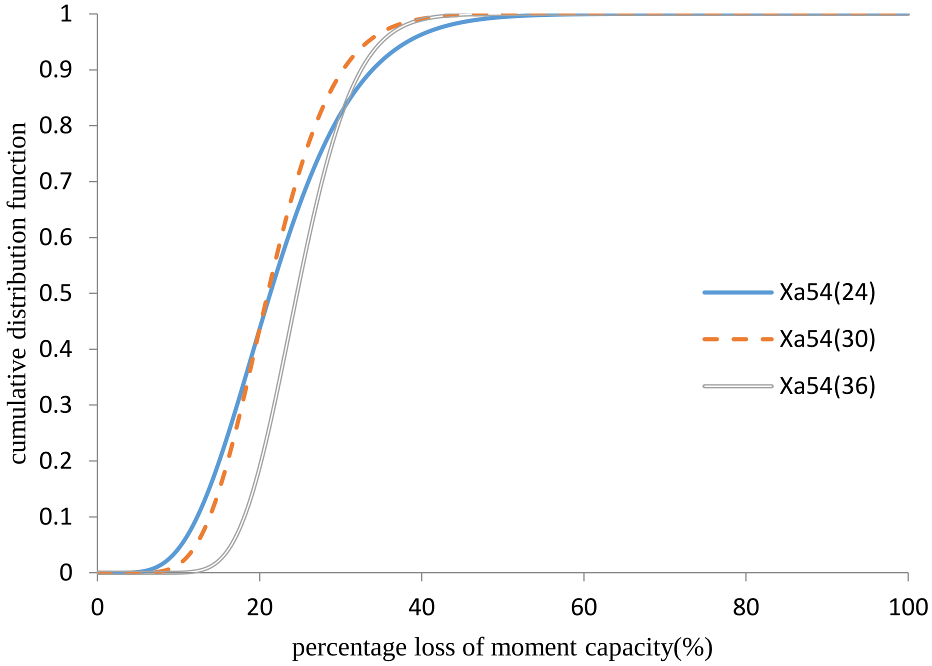

Figure 4.16 Cumulative distribution of Gamma process for percentage loss of moment capacity at age 54 using actual defect measurement form INS2

Once the definitions of inspection uncertainties are provided through the empirical and mathematical models, the continuous gamma process is used to predict the deterioration model. In the same way as for the perfect inspection outcomes the deterioration status of the reinforced concrete slab is obtained. It is possible to establish loss of moment capacity of the slab for the selected time horizons. In Figure 4.17 comparison of the deterioration profile of the slab based on the perfect and imperfect inspection outcomes are demonstrated.

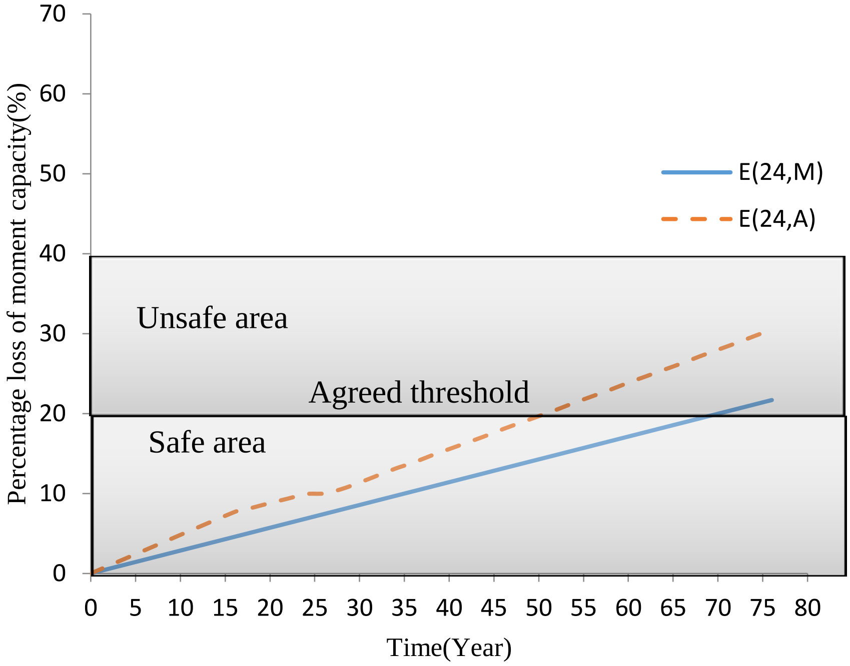

Figure 4.17 Comparison of predication of deterioration status based on the perfect and imperfect inspection type 2 at age 24(M=observed, A=actual)

It can be observed in Figure 4.16 that the uncertainty associated with deterioration process decreases as more inspection outcomes are provided.

It is evident that with inclusion of further inspection technique characteristics a realistic model would emerge and enable a well-informed model of deterioration projections. This model is reflecting the current status of the structure and the current technique quality therefore accounting for multiple sources of temporal variability. Moreover, it has been demonstrated that there is pronounced effect of the inspection technique features. Once further inspection imperfections are consider the gamma process model would represent a rather versatile tool for planning of maintenance and repair.

4.6 Summary and Conclusions

The inspection outcomes can be used to characterize the structural deterioration process. It is identified in Section 4.2 that the inspection outcomes are associated with uncertainties that might affect the deterioration process. The uncertainties associated with inspection outcomes are investigated in this chapter. Parameters such as probability of detection (POD), measurement error, probability of false alarm (PFA) are proposed to characterize the inspection outcomes uncertainties. Selective methods for characterization of inspection uncertainties have been reviewed. In this Chapter, a new probabilistic method is developed to characterize the probability of detection (POD) and measurement error as inspection outcomes uncertainties. The uncertainties are taken into account to determine the actual defect size which it can be used to represent the deterioration progress. In order to reflect the influence of the uncertainties associated with inspection outcomes, the new method is applied on the same sample as in Chapter 3. It is identified that at an early age the influence of inspection uncertainties is pronounced. As the deterioration progresses over the time the risk to underestimate the deterioration status based on the observed inspection outcomes is increased in comparison to deterioration status based on the actual outcomes. The methodology developed here addresses these issues by offering comprehensive adaptive modelling for deterioration progression.

Chapter Five

5 Adaptive Inspection Regime

5.1 Introduction

Due to often limited resources for the performance management of existing bridges at an acceptable level, the infrastructure manager and owner have to use models that optimize the strategies to keep them safe and serviceable. Generally, maintenance actions follow inspections outcomes and current structural condition. The inspection outcomes can be used to determine whether the defect exists, what is the extent of defects, and the type of maintenance action required (UK Roads Liaison Group, 2005).

It is identified by Jandu (2008) that the current UK inspection regime is prescriptive and not most cost-effective inspection regime, and bridge repairs are not always performed with life-cycle cost effectiveness in mind. As a result, over the last decade a lot of research has been conducted into optimization of maintenance management that consider dual constraint of optimal maintenance budget while maximizing efficiency for the required remaining service life. Many of methods assume the quantitative inspection data, rather than qualitative and subjective data (VanNoortwijk and Frangopol, 2004b). However, the inspection outcomes of current inspection regime are mostly qualitative.

For corrosion deterioration mechanism of bridges, lifetime methodologies for planning repair strategies of corroded RC structures were developed by Enright and Frangopol (1999), Estes and Frangopol (1999), Orcesi and Cremona (2009), Faber and Sorensen (2002),So et al. (2009) among others. Furthermore, several probabilistic approaches for optimum maintenance strategies have been developed and applied to steel structures subject to fatigue and corrosion (Kim et al., 2013; Kwon and Frangopl, 2012; Zayed et al., 2002). Inspection and monitoring planning for RC structure under corrosion was investigated by Kim and Frangopol (2011), where the effect of updating the deterioration model parameters after inspection or monitoring actions using Bayesian technique was revealed. The inspection planning is formulated as an optimization problem with objective of minimizing the expected damage delay. This approach was extended to find the optimum combined inspection and monitoring planning.

5.2 Issues of Current Optimization Models

Reviewing the current optimization models that are proposed by Kim et al. (2013), and Morcous and Lounis (2010) it is identified that current optimization models typically have a higher degree of complexity than the current inspection regime. Thus, they usually cannot be readily implemented and can be seen as prototypes for future inspection regime. As was the case for the current inspection regime the objective of these models is to optimize maintenance and inspection decisions. However, it is often the case that current models use the total life-cycle cost as objective function for optimization process. Total life-cycle cost generally includes inspection cost, maintenance cost and failure cost. The failure cost is defined as subjective model and it should be used only to provide an indication of relative benefits of different strategies. This total cost model does not take into account indirect cost such as traffic delay cost. While the general form of the optimization models is similar, these models differs from each other in many aspects such as scope of optimization, decision variations, layout optimization, deterioration model and assumptions about the knowledge of current inspection regimes.

According to issues of current inspection optimization models, a simple and practical model is needed that can take into account the deterioration status at inspection time when minimizing the number of future inspections and their cost. It is identified in Chapter 1 that the current inspection regime with fixed inspection interval is not most efficient strategy. Thus, an adaptive inspection regime is proposed here.

5.3 Adaptive Inspection Features

Schematic illustration of current rather prescriptive inspection strategy has been demonstrated in Figure 1.3. Here, an adaptive inspection program is proposed so that, the inspection time is determined with respect to the deterioration status as it is shown in Figure 5.1.

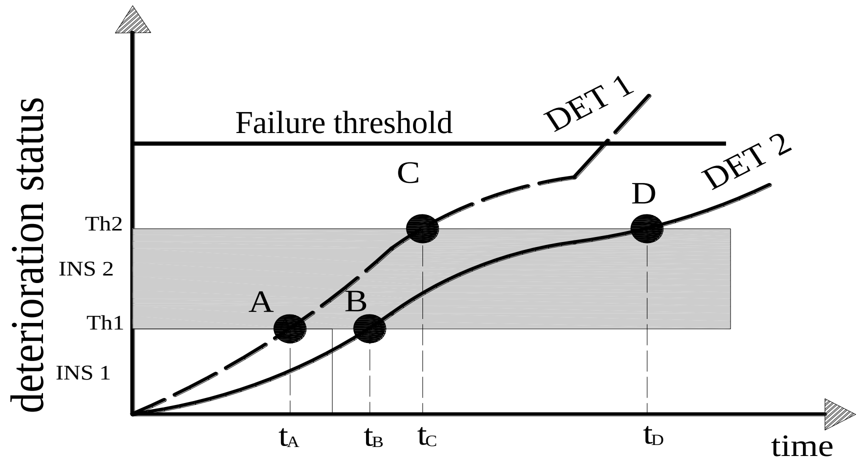

Figure 5.1 General illustration of an adaptive inspection regime

As it is shown in Figure 5.1, in order to obtain an adaptive inspection regime, the prediction for deterioration status of the bridge and present inspection technique's thresholds can be used. Once the prediction of structural deterioration status is achieved, the inspection thresholds and failure threshold – the failure threshold is considered as the defect size that the present inspection techniques cannot detect- has to be identified. The inspection thresholds (Th_i) greatly depend on the inspection type characteristic while failure threshold (Th_f) is related to many factors such as safety policy at location of structure, environmental condition and traffic load. It is identified that the inspection interval can be variable considering the deterioration status and inspection technique. For example, it is shown in Figure 5.1 that the inspection type 1 can be carried out early on the bridge in case of the deterioration status being DET1 while the same inspection type can be carried out later on the bridge with deterioration status DET2.

5.4 Adaptive Inspection for RC Deck

The continuous Gamma process is employed to characterize the deterioration of a reinforced concrete bridge slab subject to corrosion with medium corrosion rate. Different inspection intervals are considered and the inspection outcomes processed to obtain deterioration status as described in previous Chapters. The deterioration status of the reinforced concrete bridge slab subjected to corrosion is used to illustrate the adaptive inspection schedule.

A sample set of cumulative distribution functions of the percentage loss of moment capacity of a reinforced concrete bridge slab for different time horizons from the inspection outcomes are indicated in the Figure 5.2.

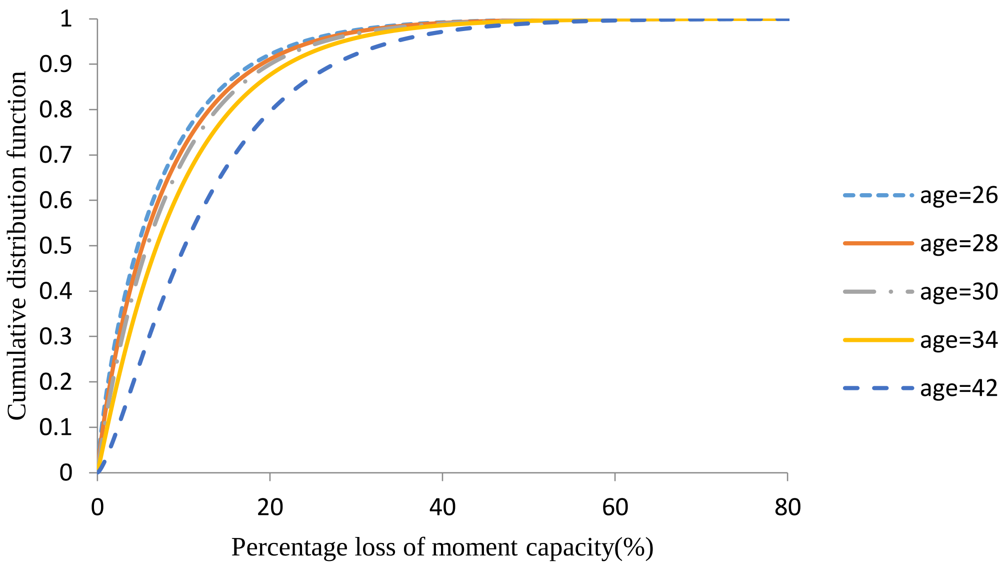

Figure 5.2 Cumulative distribution function of flexural capacity loss based on prior inspection outcomes at age 18 and 24

The deterioration is predicted for 6 years (from 24-30) with the interval 2 years and then the interval is increased to 4 years for another 12 years (from 30-42). In order to establish an adaptive inspection schedule, two different inspection types are considered in this study and their thresholds have to be identified. Thresholds are a matter of expert judgment and safety policy but here it is taken as

$$\begin{cases} Th_1 = X(t) = 10\% \\ Th_2 = X(t) = 16\% \end{cases} \quad (5.1)$$

These thresholds reflect that INS1 and INS2 are acceptable as long as the deterioration is less than 10% and 16%, respectively.

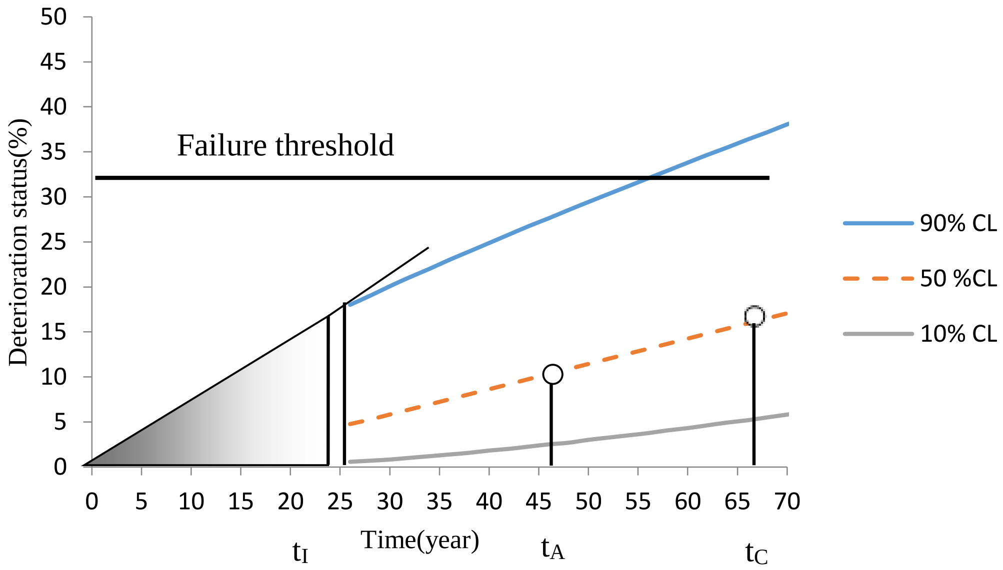

Figure 5.3 Deterioration status with 90%, 50% and 10% confidence level based on prior inspection outcomes at age 18 and 24

Using information illustrated in Figure 5.2 the deterioration status graphs are overlapped to demonstrate the deterioration prediction obtained based on the inspection outcomes at 18 and 24 years for the remaining lifetime with three different confidence levels as 90%, 50% and 10%. These outcomes can now be used to identify appropriate intervals for particular inspection type.

As the deterioration status with 50% confidence level will reach the inspection type 1 threshold (Th_1) 10% at 44 years, it can be concluded that it is appropriate to carry out the inspection type 1 until 44 years on this bridge and from then the inspection type 2 should be used. Furthermore, the inspection type 2 can be used until 66 years when the deterioration status exceedsTh_2. It is assumed that inspection type 1 interval is 2 years while the interval of inspection type 2 might be matter of expert judgment. Nevertheless, the criteria of inspection interval also can be the time interval before the threshold is reached.

It is evident in Chapter 3 that as more inspection outcomes become available, the deterioration prediction can be updated, therefore the inspection strategy also can be updated. Updated deterioration status and the inspection program are shown in the next graph. In order to identify the influence of prior inspection outcomes, the deterioration status with 50% confidence level based on three and four sets of inspection outcomes are compared in the Figure 5.5, 5.6.

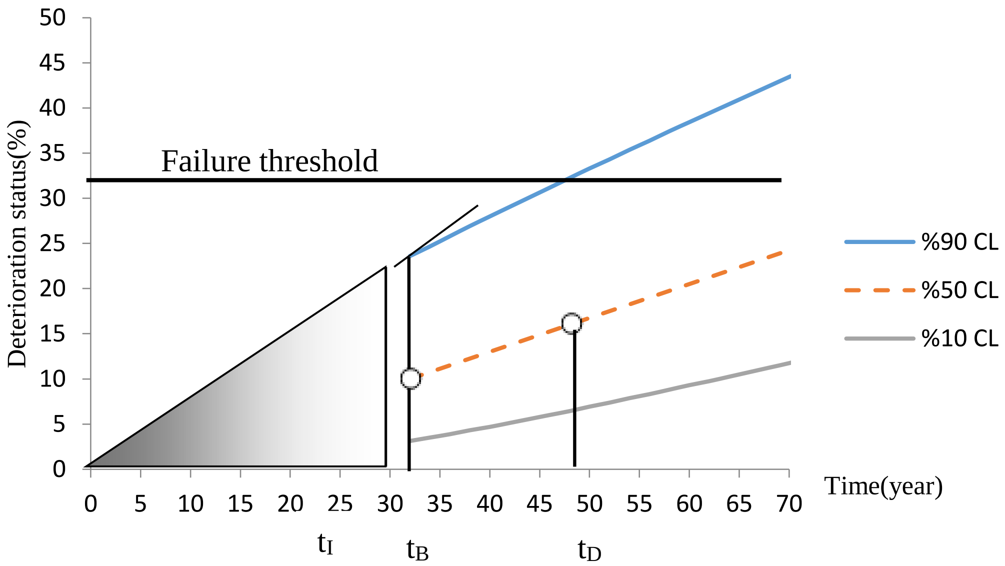

Figure 5.4 Deterioration status prediction graph based on prior inspection outcomes at age 18, 24 and 30

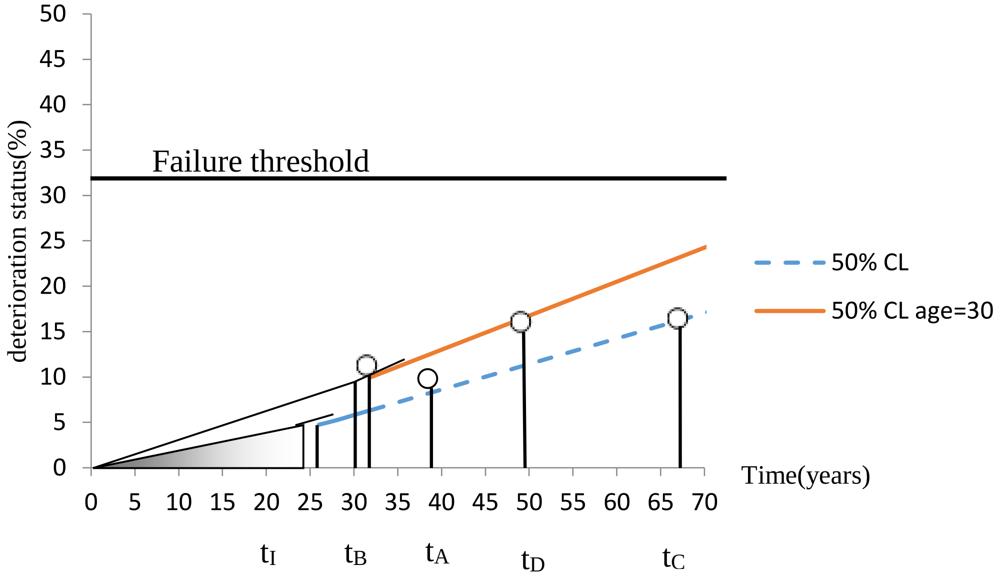

Figure 5.5 Comparison of deterioration status based on prior inspection outcomes at years 24 and year 30

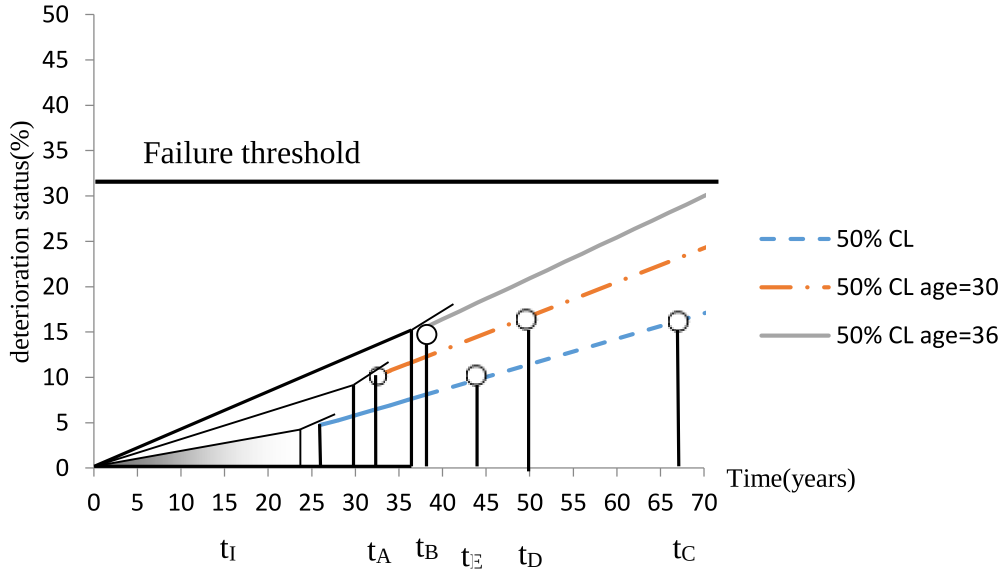

Figure 5.6 Comparison of deterioration status based on inspection outcomes at year 24, year 30 and year 36

Where t_A- t_E are the appropriate interval time when carrying out of a specific inspection type has to be stopped. For instance, inspection type 1 at time t_A (33 years) has to be switch to inspection type 2 in case of inspection data has been provided at age 30.

In the second scenario, it assumed that the inspection type 2- more expensive and accurate than the inspection 1- is carried out at 18 and 24. In order to develop an adaptive inspection program and define the appropriate inspection interval, the deterioration is predicted for next 32 years until 56. The inspection type 2 threshold (Th_2) and failure threshold (Th_f) are indicated in the next graph. It is assumed that the inspection interval should be changed to Δt_2 , $(\Delta t_2 < \Delta t_1)$ as soon as deterioration status with selected confidence level exceedsTh_2. It means that the inspection type 2 is used at every Δt_2 year afterwards. Moreover, as the deterioration status with 90% confidence level exceeds Th_f , the other actions such as maintenance, repair, and etc, need to be considered. The possible decisions for inspection program can be formulated as

- If $\Pr[X(24 + k\Delta t_1) \leq Th_2] > 0.5$, $Th_2 = 16\%, k = 1, \dots, n$ then INS1 is carried out with interval $\Delta t_1$where k is the inspection frequency
- If $\Pr[X(24 + k\Delta t_1) \leq Th_2] = 0.5$, $Th_2 = 16\%, k = 1, \dots, n$ then $K\Delta t_1$ is the time to change the inspection interval to Δt_2
- If $\begin{cases} \Pr[X(24 + K\Delta t_1) \leq Th_2] < 0.5 \\ \Pr\left[X(24 + K\Delta t_1 + j\Delta t_2) \leq Th_f\right] > 09 \end{cases}$ $Th_2 = 16\%, Th_f = 30\%, t\ k = K,$

  then $j = 1, \dots, m$ where j is the inspection frequency with interval Δt_2
- If $\Pr\left[X(24 + K\Delta t_1 + j\Delta t_2) \leq Th_f\right] = 0.9$, $Th_2 = 16\%, Th_f = 30\%, k = 1, \dots, n,$ $j = 1, \dots, m$ then $K\Delta t_1 + J\Delta t_2$ is the time to take other actions
- If $\Pr\left[X(24 + K\Delta t_1 + J\Delta t_2) \leq Th_f\right] < 0.9$, $Th_2 = 16\%, Th_f = 30\%, k = K,\ j = J$ then structure is no longer functional

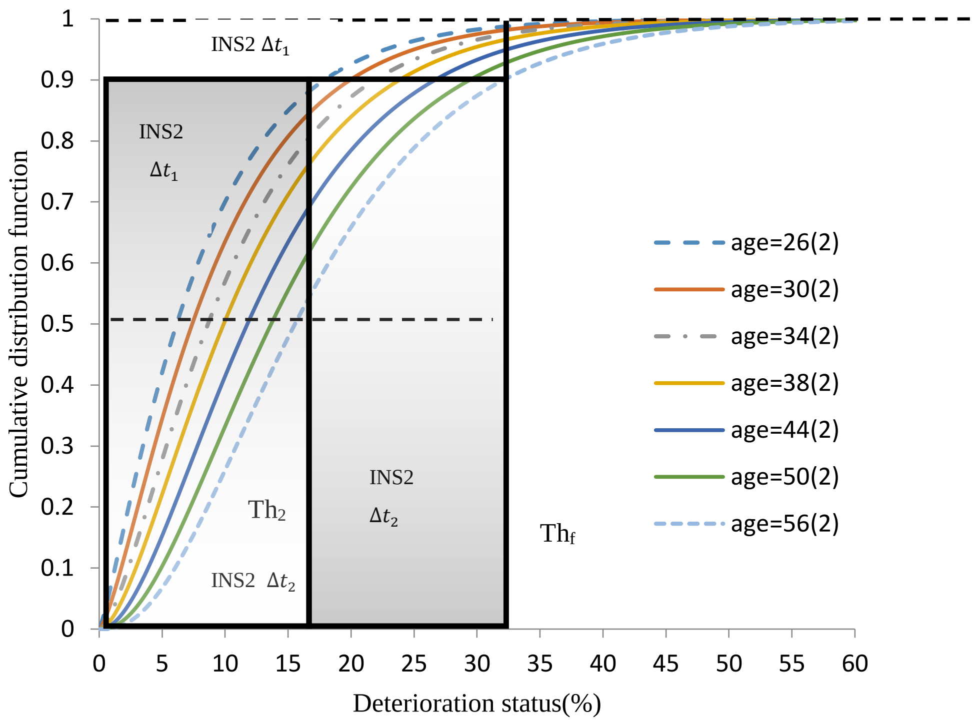

Figure 5.7 Cumulative distribution function of deterioration based on INS2 outcomes at age 18 and 24

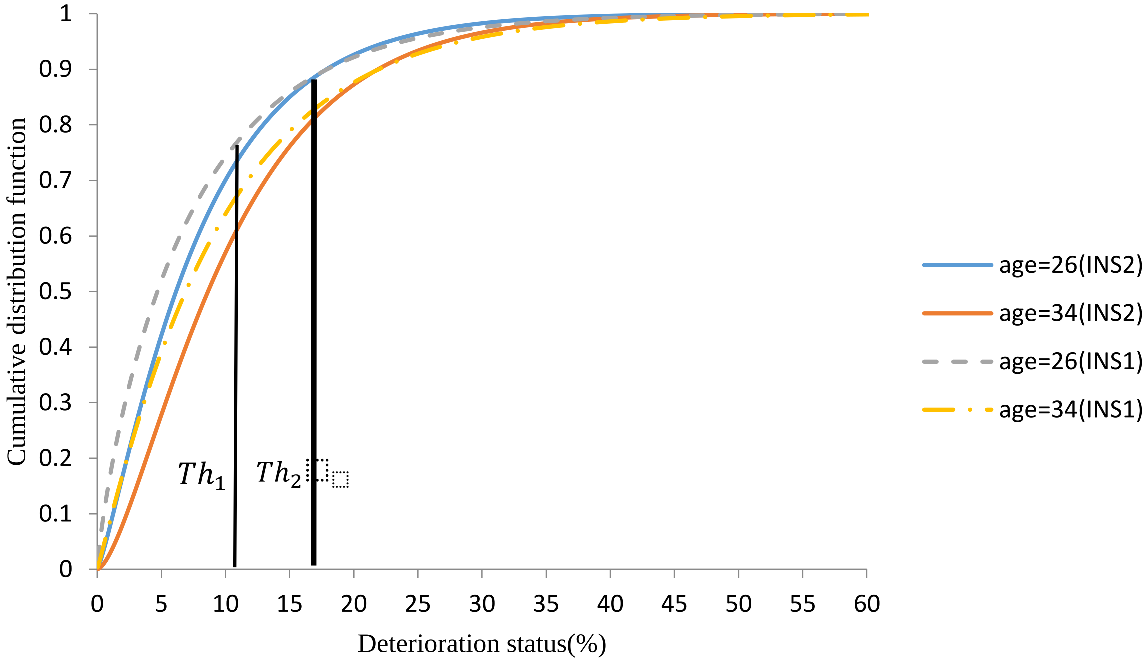

Figure 5.8 Comparison of cumulative distribution function of deterioration based on INS1 and INS2 outcomes at age 18 and 24

Different adaptive inspection scenarios can be considered to reflect the effect of inspection features, inspection interval and outcomes on the inspection program.

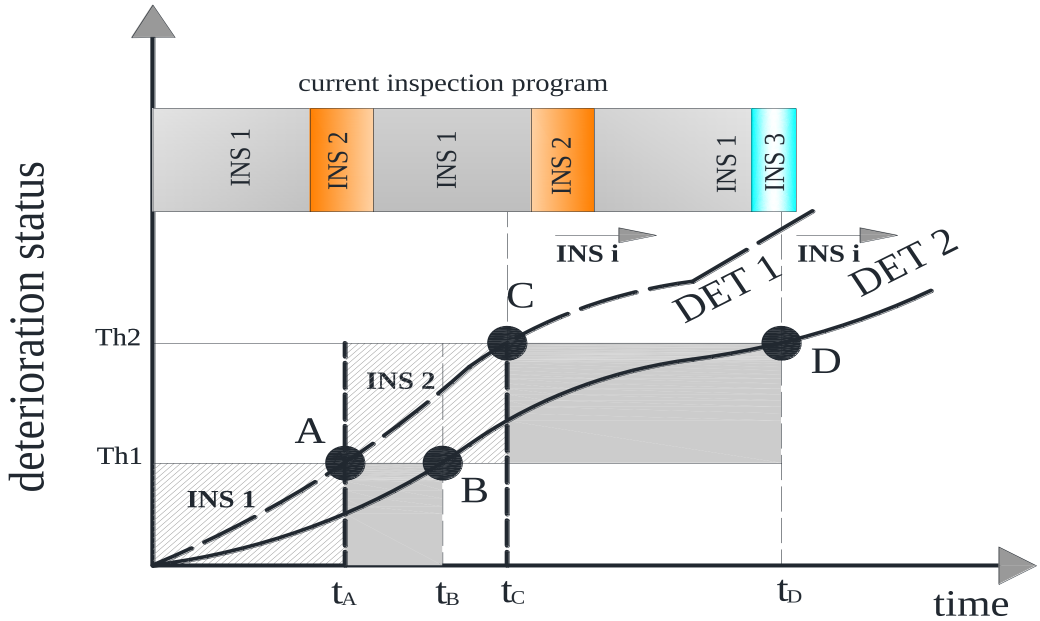

Figure 5.9 Comparison of adaptive and current inspection programs

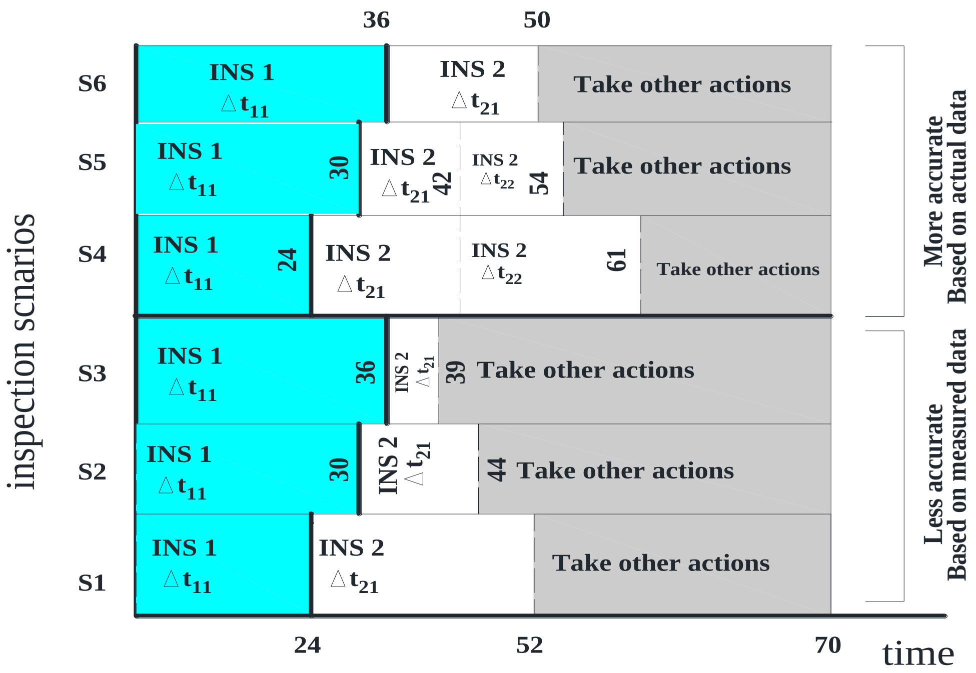

Figure 5.10 Illustration of adaptive inspection scenarios

In Figure 5.10 different inspection scenarios that are named S_1 to S_6 are identified. Scenarios 1-3 have been established using measured data and scenarios 4-6 are developed using the 'actual' data i.e. the uncertainties associated with the inspection techniques have been taken into account. According to $S_1$the inspection type 1 is carried out until age 24 and from that point is onward the predicted deterioration status exceeds the relevant threshold; the inspection type 2 can be carried out every Δt_{21} years until age 52. The inspection outcome at age 30 features in S_2 and on the basis of the updated deterioration profile, the inspection interval is changed and it is identified that inspection type 2 can be carried out until the age 44. It means that structural performance level exceeds the failure threshold 8 years earlier than the previous scenario S_1. However, the outcomes could also be reversed so that inspection type 2 can be carried out for longer than age 52 as inS_1. It should be noted that the measured inspection outcomes are used for deterioration status prediction in S_1 to S_3. In order to demonstrate how the imperfect nature of inspection outcomes can be accounted for, S_4 to S_6 can be considered. Moreover, the actual inspection outcomes, which can be determined by the method that has been explained in Chapter 4, are employed to predict the deterioration status. It is identified that the inspection type 2 with Δt_{21} interval can be carried out until the age 42 when inspection type 2 with Δt_{22} interval can be used until the age 61 in S_4. As inspection type 1 is continued until the age 30, inspection type 2 with Δt_{21} interval can be used until age 42 like S_4 but inspection type 2 with Δt_{22} (Δt_{21} greater or smaller thanΔt_{22}) interval can be carried out until age 54. It is demonstrated that in S_6 the inspection type 1 is carried out until the age 36 and then inspection type 2 with Δt_{21} interval used until age 50 when the deterioration status is predicted to exceed the threshold.

The total inspection cost of these inspection scenarios can be determined in simple terms.

5.5 Total Inspection Cost Function

Many research has been conducted over the last two decades to optimize the life cycle cost and maintenance cost of highway bridges. Bakker et al. (1999) presented a formula to calculate the cost of maintenance. The cost of maintenance roughly has been divided in this paper into four types as:

- Cost of initial investment
- Cost of preventive replacement
- Cost of corrective replacement
- Cost of lifetime-extending maintenance

It is evident that they did not consider the cost of inspection in the study at all.

Enright and Frangopol (1999), Frangopol (2004), Kim et al. (2013) established life cycle cost models to optimize the total maintenance cost. All models are included initial cost, cost of all inspections, cost of all repairs and cost of failure. It is identified however, the cost of inspection has been take into account in the models but cost of inspection assumed as fixed cost regardless to different inspection scenarios which could be applied.

In order to take into account of different inspection scenarios, total inspection cost function of different inspection scenarios with change over the lifetime, as illustrated in Figure 5.1 can be determined by:

$$C_T = \frac{t_I}{\Delta t_{11}} C_1 + \frac{t_{Th_2} - t_I}{\Delta t_{21}} C_2 + \frac{t_{Th_f} - t_{Th_2}}{\Delta t_{22}} C_2 + C_m \qquad (5.2)$$

Where C_T is the total inspection cost, C_1 the inspection type 1 cost, C_2 the inspection type 2 cost, C_m the other action cost, t_I is the last inspection time that the deterioration status prediction is based on, Th_2 the inspection type 2 threshold, Th_f the failure threshold, Δt_{11} the inspection interval type 1, Δt_{21} the inspection interval type 2 until the deterioration status exceeds the upper limit of inspection type 2, and Δt_{22} the inspection interval until the deterioration status exceeds the failure limit. The total inspection cost of the inspection scenarios are demonstrated in table 5.1

Table 5.1 Total inspection cost of different scenarios

Inspection scenario(S_i)	Adapted total inspection cost(C_T)	Current total inspection cost
S_1	$\frac{24}{\Delta t_{11}}C_1 + \frac{28}{\Delta t_{21}}C_2 + C_m$	$\frac{24}{\Delta t_{11}}C_1 + \frac{46}{\Delta t_{21}}C_2$
S_2	$\frac{30}{\Delta t_{11}}C_1 + \frac{14}{\Delta t_{21}}C_2 + C_m$	$\frac{30}{\Delta t_{11}}C_1 + \frac{40}{\Delta t_{21}}C_2$
S_3	$\frac{36}{\Delta t_{11}}C_1 + \frac{3}{\Delta t_{21}}C_2 + C_m$	$\frac{36}{\Delta t_{11}}C_1 + \frac{34}{\Delta t_{21}}C_2$
S_4	$\frac{24}{\Delta t_{11}}C_1 + \frac{18}{\Delta t_{21}}C_2 + \frac{19}{\Delta t_{22}}C_2 + C_m$	$\frac{24}{\Delta t_{11}}C_1 + \frac{46}{\Delta t_{21}}C_2$
S_5	$\frac{30}{\Delta t_{11}}C_1 + \frac{12}{\Delta t_{21}}C_2 + \frac{12}{\Delta t_{22}}C_2 + C_m$	$\frac{30}{\Delta t_{11}}C_1 + \frac{40}{\Delta t_{21}}C_2$
S_6	$\frac{36}{\Delta t_{11}}C_1 + \frac{14}{\Delta t_{21}}C_2 + C_m$	$\frac{36}{\Delta t_{11}}C_1 + \frac{34}{\Delta t_{21}}C_2$

5.6 Summary and Conclusions

In this Chapter we have addressed the implementation adaptive of, site specific, inspection regime that is beyond current practical inspection regime. The new regime has been applied to a reinforced concrete slab which is subject to corrosion. As before, the deterioration process of the reinforced concrete slab is modeled using gamma process. Once the deterioration status exceeds the agreed failure inspection threshold a new inspection type has to be considered. Using the deterioration status prediction, inspection type thresholds and the agreed structural failure threshold it is possible to identify the most appropriate inspection type and inspection interval and ensure that the structure remains functional.

Since the current optimization model employing the simulation techniques to find the optimum inspection planning then the new adaptive inspection program can takes less time in comparison with them to establish an appropriate inspection schedule. Furthermore, this program can be used simply by owners to give them a clear perspective of the inspection scheme and make their management strategy more efficient. The new adaptive inspection program can be site specific as the failure inspection threshold also is site specific. The choice of inspection type and inspection interval for any structure depends largely on the structural deterioration process and inspection features such as lower and upper thresholds in new adaptive inspection program. Moreover, it is possible to combine different inspection types or change the inspection type and inspection interval to develop new inspection scenarios over the structural lifetime. Form the application of adaptive inspection approach, it is concluded that

- It is possible to have an adaptive inspection program that reflects deterioration status at the time of the last inspection and the program can be updated over the time as more inspection outcomes become available. It means that if there is a change in the rate of deterioration process, then relevant actions can be taken.
- Relevant criteria are established to ensure structural performance level. The criteria are represented in the form of failure thresholds which greatly depend on the standard safety of structure's location and the environment.
- This approach has a great deal of flexibility to owner and stakeholder. Different inspection types can be used in this program when the owner makes decisions about the functionality thresholds and type of actions that need to be taken. Furthermore, as mentioned before the deterioration mechanism can impact the inspection program. Hence, it is possible to consider variety of deterioration mechanisms in order to find the most efficient inspection program that will maintain the required safety level.
- The adaptive inspection program could be fully site specific enabling consideration of inspection quality but also environmental conditions
- A simple cost function has demonstrated the benefits of the approach for management decisions. It is evident that the cost has a key role in management strategy due to the often limited budget.
- The adaptive inspection regime also provides analytical format to compare total inspection costs for different scenarios that could include various inspection techniques and inspection intervals.

- It is evident that beyond inspection process adaptive approach provides condition for site specific maintenance and repair planning.

Chapter Six

6 Conclusions and Future Work

It has been identified that using a flexible bridge inspection regime enables the owner and stakeholders to have a more efficient management plan and keep the road network functional within a limited budget. However, the safety and serviceability level of the reinforced concrete bridges are highly variable in regard to environment and safety standard. The benefit of using the new adaptive inspection regime is that the manager is able to use the inspection outcomes to update the prediction about the component condition over the lifetime. Since the inspection outcomes have key role in the bridge management, in this thesis the current inspection regimes are investigated and reviewed. It is identified in Chapter 2 that the inspection outcomes and deterioration model are associated with uncertainties from different sources. Thereby, an updatable Gamma process was developed in Chapter 3 to predict the structural deterioration process over the life time. It is concluded that the Gamma process is the most appropriate model to characterize the structural deterioration.

The results of the application of the updatable deterioration model was demonstrated in that the method reflects the influence of ageing factors such as corrosion initiation time and corrosion rate. The results of the prediction of structural deterioration with certain confidence level is presented in form of the deterioration profile which could be more realistic as more inspection outcomes become available. However, in Chapter 3 inspection outcomes assume perfect inspections which is not the always case.

The uncertainties associated with inspection outcomes in Chapter 4 are identified and characterized. A new probabilistic model is presented in this chapter to take into account inspection outcomes uncertainties. The new parameter that is called the actual defect size si intoduced. The actual defect size from inspection outcomes can be used to estimate Gamma parameters and represent the deterioration process. It can be concluded that the structural deterioration status based on the actual defect size has lower uncertainty in comparison with the deterioration status following the measured defect size. This is important benefit as for ageing structures ever greater uncertainties are accounted for in a rigorous manner.

The issue of establishing an easy approach to implement an adaptive site specific inspection regime, which is beyond current practical inspection regime, has been addressed.

The new adaptive inspection regime is developed in Chapter 5 and is applied to a reinforced concrete slab which is subject to reinforcement corrosion. Using the deterioration status prediction, inspection type thresholds and the agreed structural failure inspection threshold it is possible to identify the most appropriate inspection type and interval to ensure that the structure remains functional.

The new adaptive inspection program takes less time in comparison with the current optimum inspection program to establish an inspection schedule. Furthermore, this program can be used simply by owners to give a clear and relatively long term perspective of the inspection scheme and make their management strategy more efficient. The new inspection program can be adapted to be used in different areas as the failure threshold, which is used to reflect the safety level, depends on the specific acceptable levels based on the safety policy.

The choice of inspection type and inspection interval for any structure depends largely on the structural deterioration process and inspection features (such as lower and upper thresholds) in the new adaptive inspection program. According to the adaptive inspection approach it is concluded that:

- If there is a change in the rate of deterioration, it can be accounted for at the next inspection time.
- Relevant criteria such as functionality threshold can be established to ensure structural performance level.
- Different inspection types can be used in this program and the owner can decide on the functionality threshold and type of actions that need to be undertaken when the deterioration status exceeds failure threshold.
- It is possible to consider a variety of deterioration mechanisms in order to find the most efficient inspection program.
- The adaptive inspection program could be fully site specific enabling consideration of inspection quality but also site-specific environmental conditions.

- The subjective factors that can influence the inspection outcomes such as light intensity, the inspector character, the inspection instrument features can be taken into account.
- A simple cost function has demonstrated the benefits of the approach for management decisions.
- The adaptive inspection regime also provides analytical format to compare total inspection costs for different scenarios that could include various inspection techniques and inspection intervals.
- The new adaptive inspection regime opens an opportunity for optimization of inspection costs firstly but also general infrastructure management that includes maintenance and repair.

However, the adaptive inspection regime has some limitations and there are possibilities for future work as follows:

- In the deterioration process some of the variables are assumed deterministic. These variables can be represented in form of a random variable model or stochastic process to represent the deterioration process.
- The inspection uncertainties can be characterized by experimental data which could be more realistic and compared with current random variable models.
- The adaptive inspection criteria can be categorized for different inspection types.
- The optimum inspection interval of the adaptive inspection can be determined by an optimization model.
- Other inspection uncertainties such as PFA can be taken into account or new terms such as detection delayed can be used to model the inspection uncertainties.
- Parameters such as inspection lower and upper thresholds are considered deterministic. These parameters can be modeled as random variable to take into account uncertainties associated with inspection techniques.
- An analytical method can be used to establish a structural safety threshold.

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APPENDIX A

Suppose X is a random variable and n observations of X are available. The mean or expected value of X can be calculated for the n observations as:

$$Mean = E(X) = \mu_X = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \text{(A.1)}$$

In the mean value equation, no distinction is made between the population and sample mean. In this equation, in fact, it is implicitly assumed that sample size is relatively large. The mean value of small sample size can be used to estimate mean value of population by interval estimation method. Since it is impractical to collect information from all available sources, the information on the sample mean is useful. In this context, if another sample of size n is collected, the sample mean obtained can be somewhat different. In fact, mean value of one of each observation is itself a random variable; therefore it can be denoted as $\bar{X}$ and can be estimated as:

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \text{(A.2)}$$

The mean or expected value of the sample mean can be calculated as:

$$E(\bar{X}) = E\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) = \frac{1}{n}\sum_{i=1}^{n} E(X_i) = \frac{1}{n} n\mu = \mu \qquad \text{(A.3)}$$

The variation of X is denoted as *Var(X)* and can be computed as:

$$Var(X) = \frac{1}{n-1}\sum_{i=1}^{n} (x_i - \mu_X)^2 \qquad \text{(A.4)}$$

The dimensional problem can be avoided by taking the square root of the variance. This is standard deviation, denoted as σ_X and can be calculated as:

$$\sigma_X = \sqrt{Var(x)} \qquad \text{(A.5)}$$

As mentioned previously, it is impractical to estimate *Var(X)* for all information; therefore the interval estimation method is appropriate method to estimate *Var(X)* for small sample size.

Since the mean and the standard deviation values are expressed in the same units, a non-dimensional term can be introduced by taking the ratio of the standard deviation and the mean. This is called the coefficient of variation (*COV*) and will be denoted as *COV(X)* or δ_X.

$$COV(X) = \delta_X = \frac{\sigma_X}{\mu_X} \qquad \text{(A.6)}$$

A smaller value of the *COV* indicates a smaller amount of uncertainty or randomness in the variable.

The Skewness, also known as the third moment, can be calculated as:

$$Skewness = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_X)^3 \qquad \text{(A.7)}$$

A non-dimensional Skewness is known as the Skewwness coefficient and denoted as θ_X. It is calculated as:

$$\theta_X = \frac{Skewness}{\sigma_X^3} \qquad \text{(A.8)}$$

This parameter defines dispersion of the variable about the mean value.

Normal or Gaussian distribution

One of the most commonly used distributions in engineering problems is the normal distribution. The PDF of the distribution can be expressed as:

$$f_X(x) = \frac{1}{\sigma_X\sqrt{2\pi}} exp\left[-\frac{1}{2}(\frac{x-\mu_X}{\sigma_X})^2\right], -\infty < x < +\infty \quad \text{(A.9)}$$

Where the mean μ_X and standard deviation σ_X are the two parameters of the distribution which are estimated from the available data. The corresponding CDF can be expressed as:

$$F_X(x) = \int_{-\infty}^{x} \frac{1}{\sigma_X\sqrt{2\pi}} exp\left[-\frac{1}{2}(\frac{x-\mu_X}{\sigma_X})^2\right] d_x \qquad \text{(A.10)}$$

This distribution has many desirable features. It is applicable for any value of a random variable and is symmetric about mean. Since estimation of probability by integrating equations is not simple, the original random variable can be transformed into standard normal variable as:

$$Y = \frac{X-\mu_X}{\sigma_X} \quad \text{(A.11)}$$

Using the PDF equation of normal distribution and variable transformation technique, the PDF of standard normal can be expressed as:

$$f_Y(y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}y^2\right) and\ F_Y(y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}y^2\right) d_y = \Phi(y) \quad \text{(A.12)}$$

The mean value and standard deviation of transformed variable are 0 and 1, respectively.

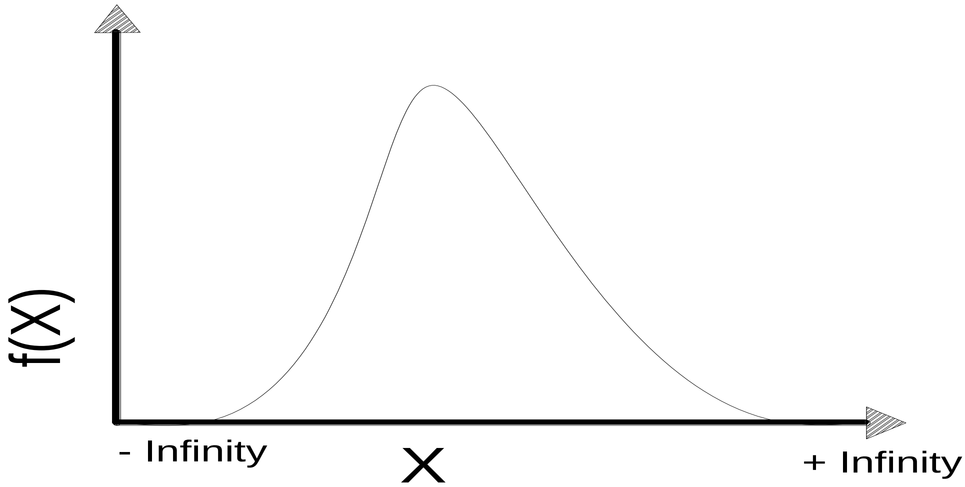

Figure A.0.1 General illustration of probability density function of a standard normal distribution

Lognormal distribution

In many engineering problems, a random variable cannot have negative values due to the physical aspects of the problem. The representation of random variable as a lognormal distribution, the possibility of negative values will be eliminated. If a random variable has a lognormal distribution, then its natural logarithm has a normal distribution. The PDF of lognormal variable is determined as:

$$f_X(x) = \frac{1}{\sqrt{2\pi}\xi_X x} exp\left[-\frac{1}{2}(\frac{lnx-\lambda_X}{\xi_X})^2\right],\ 0 \leq x < +\infty \qquad \text{(A.13)}$$

Where λ_X and ξ_X are the two parameters of the lognormal distribution. Its PDF is unsymmetrical. Some similarities can be observed between the normal and lognormal distribution. The two parameters of the lognormal distribution can be calculated from the information on the two parameters of the normal distribution. It is denoted as:

$\lambda_X = E(lnx) = ln\mu_X - \frac{1}{2}\xi_X^2$ (A.14)

$\xi_X^2 = Var(lnX) = ln\left[1 + (\frac{\sigma_X}{\mu_X})^2\right] = \ln(1 + \delta_X^2)$ (A.15)

If the δ_X is small then $\xi_X \approx \delta_X$. To calculate the probability an event, the method used for the normal variables are still applicable, except that for the lognormal variables, the standard variable Y will be denoted as:

$Y = \frac{lnX - \lambda_X}{\xi_X}$ (A.16)

The probability of a lognormal random variable can be represented as:

$P(a < X \le b) = \Phi\left(\frac{lnb - \lambda_X}{\xi_X}\right) - \Phi\left(\frac{lna - \lambda_X}{\xi_X}\right)$ (A.17)

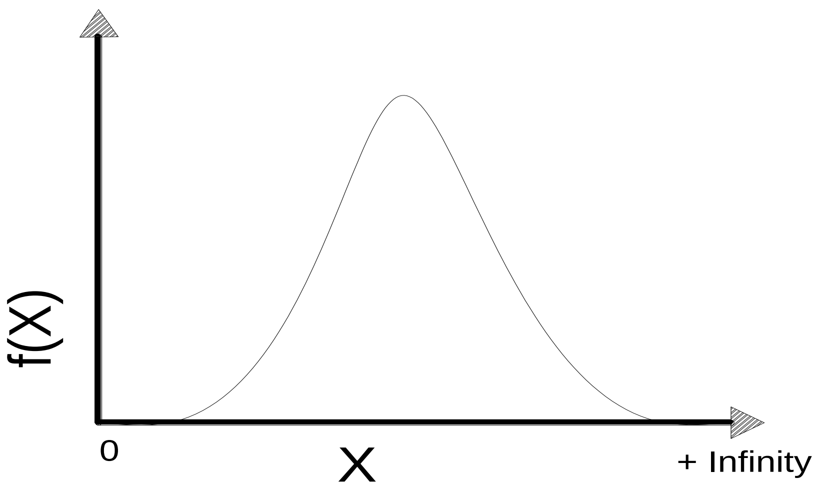

Figure A.0.2 General illustration of probability density function of a log normal distribution

Beta distribution

When a random variable is known to be bounded by two limits, the beta distribution is a very flexible and useful distribution. The PDF of a beta distribution is denoted as:

$$f_X(x) = \frac{1}{B(q,r)} \frac{(x-a)^{q-1}(b-x)^{r-1}}{(b-a)^{q+r-1}} \text{ }, a \le x \le b \qquad \text{(A.18)}$$

Where q and r are the parameters of the distribution and $B(q,r)$ is the beta function. The parameters can be estimated from the mean and standard deviation of the available data using the following equations.

$$E(X) = a + \frac{q}{q+r}(b-a) \qquad \text{(A.19)}$$

$$Var(X) = \frac{qr}{(q+r)^2(q+r+1)}(b-a)^2 \qquad \text{(A.20)}$$

If the upper and lower limits and the mean and variance of a random variable are known, the corresponding parameters of the beta distribution can be estimated. The beta function can be denoted as:

$$B(q,r) = \int_0^1 x^{q-1}(1-x)^{r-1}dx \quad or \; B(q,r) = \frac{\Gamma(q)\Gamma(r)}{\Gamma(\mathrm{q+r})} \qquad \text{(A.21)}$$

Where $\Gamma(\quad)$ is the gamma function. When parameters are both equal one, the beta distribution becomes a uniform distribution. Once the PDF of a beta distribution is defined, the probability of any event can be estimated by numerically integrating the area under PDF corresponding to the upper and lower limits.

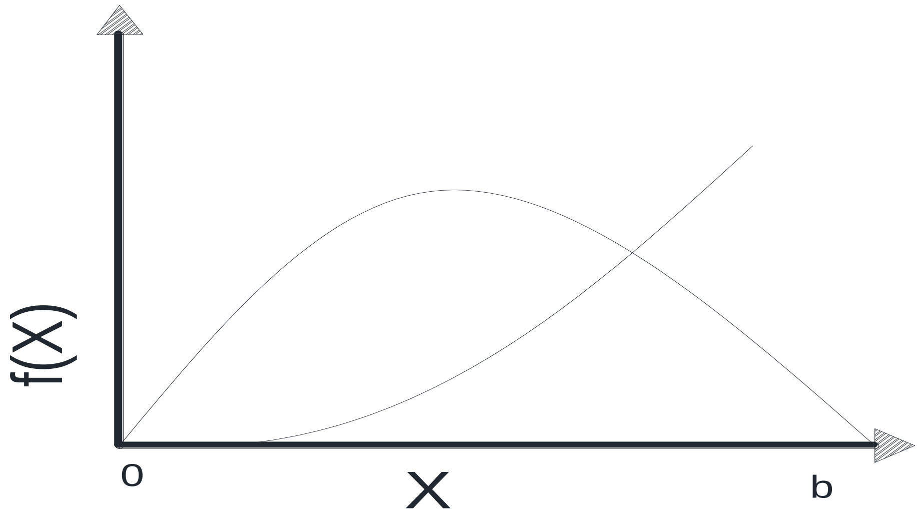

Figure A.0.3 General illustration of probability density function of Beta distribution

Binomial distribution

In many engineering applications, events can be formulated in terms of occurrence or non-occurrence. Only two outcomes are possible which represent the behavior of a discrete random variable. If the probability of occurrence of an event in each trial is *p* and the probability of non-occurrence is *(1-p)*, then the probability of *x* occurrences out of a total of n trials can be described by the PMF of binomial distribution as:

$$P(X = x, n|p) = \binom{n}{x} p^x (1-p)^{n-x} \quad x = 0,1,2,\ldots,n \quad \text{(A.22)}$$

Poisson distribution

Another important distribution which is frequently used in engineering to evaluate the risk of damage is the Poisson distribution. Some events can be occurred at any point in time or space. If they need to be modeled in a Bernoulli sequence at a given time or space, the total space or time needs to be subdivided into very small intervals so that only one occurrence is possible in an interval. Modeling x occurrences in time t in a Bernoulli sequence as n approaches infinity will lead to the Poisson distribution which can be expressed as:

$$P(x\ occurrences\ in\ time\ t) = \lim_{n\to\infty} \binom{n}{x} (\frac{\upsilon t}{n})^x (1 - \frac{\upsilon t}{n})^{n-x} = \lim_{n\to\infty} \left[\frac{(\upsilon t)^x}{x!} (1 - \frac{\upsilon t}{n})^n\right] = \frac{(\upsilon t)^x}{x!} e^{-\upsilon t} \quad \text{(A.23)}$$

Exponential distribution

If events occur according to a Poisson process, then the time T before the first occurrence of the event can be represented by the exponential distribution.

$$P(T > t) = \frac{e^{-\nu t}(\nu t)^0}{0!} = e^{-\nu t} \quad \text{(A.24)}$$

Then, the CDF of T can be obtained of

$$F_T(t) = P(T \leq t) = 1 - e^{-\nu t} \quad \text{(A.25)}$$

And the corresponding PDF of the exponential distribution is

$$f_T(t) = \frac{dF_T(t)}{dt} = \upsilon e^{-\upsilon t}, t \geq 0 \quad \text{(A.26)}$$

APPENDIX B

The design of a reinforced concrete member is generally based on the ultimate limit state which is usually performed for loading corresponding to that state. To design a structure, it is necessary to know the bending moments, torsion moments, shearing forces and axial forces in each member. An elastic analysis is generally used to determine the distribution of these forces within the structure; but because it is identified - to some extent- that reinforced concrete is a plastic material, a limited redistribution of the elastic moments is something allowed. However, a plastic yield-line theory may be used to calculate the moments in concrete slabs. We focus on the bending moment capacity at ultimate limit state to determine the bending moment capacity of bridge slab. Some method of elastic analysis is generally used to calculate forces in a concrete structure, despite the fact that the structure does not behave elastically near its ultimate load.

The assumption of elastic behavior is reasonably true for low stress levels, but as a section approaches its ultimate moment of resistance, plastic deformation will occur. This is recognized in EC2, by allowing redistribution of elastic moments subject to certain limitations. It is assumed that the reinforced concrete section is considered elastic until the steel yields, and then plastic until concrete failure, or more specifically, the concrete failure limits the rotation that may take place at a section in bending. Thus, in an indeterminate structure, once a beam section develops its ultimate moment of resistance, M_u ,it then behaves as plastic hinge resisting a constant moment of that value.

The three most important principles in the reinforced concrete section analysis are

- The stresses and strains are related by the material properties, including the stress-strain curves of concrete and steel.
- The distribution of strains must be compatible with distorted shape of the cross section.
- The resultant forces developed by the section must balance the applied loads for static equilibrium.

Distribution of strains and stresses across a section in bending

The theory of bending for reinforced concrete assumes that the concrete will crack in the regions of tensile strains and that, after cracking, all the tension is carried by the reinforcement. It is also assumed that plane sections of a structural member remain plane after straining, so that across the section there must be a linear distribution of strains. Figure shows the cross-section of a member subjected to bending, and the resultant strain diagram, together with three different types of stress distribution in the concrete:

1. The triangular stress distribution applies when the stresses are very nearly proportional to the strains, which generally occurs at the loading levels encounter under working conditions and is, therefore, used at the serviceability limit state.
2. The rectangular-parabolic stress block represents the distribution of failure when the compressive strains are within the plastic range, and it is associated with design for the ultimate limit state.
3. The equivalent rectangular stress block is simplified alternative to the rectangular-parabolic distribution.

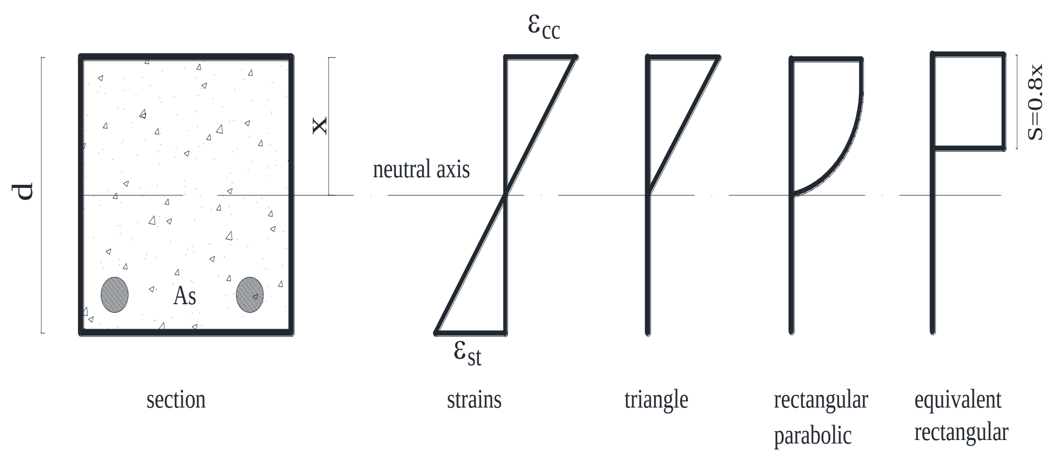

Figure B.0.1 Illustration of stress and strain diagrams for a reinforced concert beam section

The relationships between the depth of neutral axis and the maximum concrete strain(ε_{cu2}) and steel strain(ε_{st}) are given by

$$\varepsilon_{st} = \varepsilon_{cu2}(\frac{d-x}{x}) \qquad \text{(B.1)}$$

Where d is the effective depth of section.

At the ultimate limit state the maximum compressive strain in the concrete is taken as $\varepsilon_{cu2} = 0.0035$ for concrete class$\leq C50/60$

For higher classes of concrete reference should be made to EC2.

To ensure rotation of the plastic hinges with sufficient yielding of tension steel and also to allow for other factors such as the strain hardening of steel, EC2 limits the depth of neutral axis to $x \leq 0.45d$ for concrete class$\leq 50/60$.

Bending and equivalent rectangular stress block

For most reinforced concrete structures it is usual to commence the design for the conditions at the ultimate limit state, followed by checks to ensure that structure is adequate for the serviceability limit state without excessive deflection or cracking of the concrete. For this reason, the analysis is considered the simplified rectangular stress block which can be used for the design at ultimate limit state. The rectangular stress block as shown in figure may be used in preference to the more rigorous rectangular-parabolic stress block. It can be seen from figure that stress block does not extend to the neutral axis of the section but has a depth$s = 0.8x$. Thus the moment of resistance of the section will be similar using calculation based on either of the two stress block. Bending of the section will induce a resultant tensile force F_{st} in the reinforcing steel and a resultant compressive force in the concrete F_{cc} which acts through the centre of the effective area of concrete in compression. For equilibrium, the ultimate design moment,M, must be balanced by the moment of resistance of the section so that.

$$M = F_{cc}z = F_{st}z \qquad \text{(B.2)}$$

Where z is the lever arm between the resultant forces F_{cc}, F_{st}.

$$F_{cc} = 0.567 f_{ck} bs \ , \qquad z = d - s/2 \qquad \text{(B.3)}$$

Then M is calculated as

$$M = 0.567 f_{ck} bsz = 1.134 f_{ck} b(d - z)z \qquad \text{(B.4)}$$

Rearranging and substituting $k = \frac{M}{bd^2 f_{ck}}$

$$(\frac{z}{d})^2 - \left(\frac{z}{d}\right) + \frac{k}{1.134} = 0 \qquad \text{(B.5)}$$

Solving the quadratic equation:

$$z = d[0.5 + \sqrt{(0.25 - \frac{k}{1.134})}] \qquad \text{(B.6)}$$

In order to calculate A_s

$$F_{st} = \left(\frac{f_y}{\gamma_s}\right) A_s \ , \quad \gamma_s = 1.15 \quad and \quad F_{st} = 0.87 f_{yk} A_s \qquad \text{(B.7)}$$

Hence

$$A_s = \frac{M}{0.87 f_{yk} z} \qquad \text{(B.8)}$$

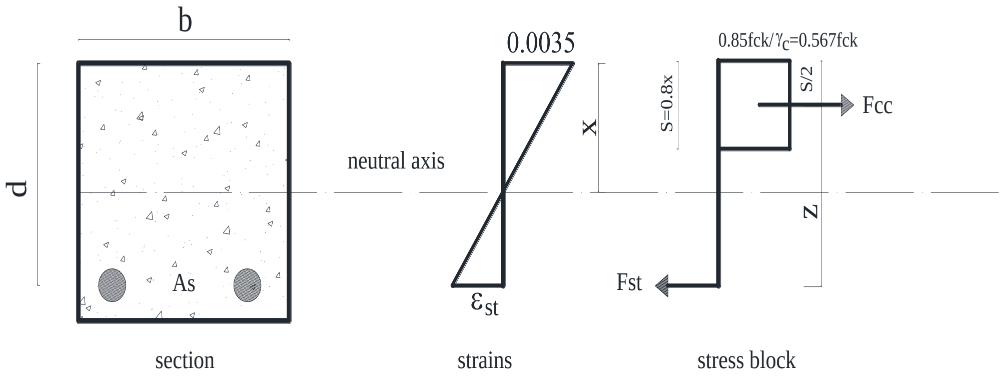

Figure B.0.2 Illustration of strain and rectangular stress diagrams of reinforced concrete beam under ultimate bending moment

Shear in slabs

The shear resistance of a solid slab may be calculated by the procedure like a beam. Experimental work has indicated that, compared with beams, shallow slabs fail at slightly higher shear stresses and this is incorporated into the values of the ultimate concrete shear resistance V_{Rd}as given by

$$V_{Rd,c} = [0.12k(100\rho_1 f_{ck})^{\frac{1}{3}}]b_w d \qquad \text{(B.9)}$$

Where $V_{Rd,c}$is the design shear resistance, b_w the smallest width of the section in tensile area and

$$k = \left(1 + \sqrt{\frac{200}{d}}\right) \leq 2.0\ with\ d\ expressed\ in\ mm \qquad \text{(B.10)}$$

$$\rho_1 = \frac{A_{s1}}{b_w d} \leq 0.02 \qquad A_{s1} = the\ area\ of\ tensile\ reinforcement\ \text{(B.11)}$$

Calculations are usually based on a strip of slab 1m wide. Since shear stresses in slabs subject to uniformly distributed loads are generally small, shear reinforcement will seldom be required and it would be usual to design the slab such that the design ultimate shear force,V_{Ed}, is less than the shear strength of the unreinforced section,$V_{Rd,c}$.

As for beams, the section should also be checked to ensure that V_{Ed} does not exceed the maximum permissible shear force$V_{Rd,max}$. Localised 'punching' actions due to heavy concentrated loads may, however, cause more critical conditions.

APPENDIX C

Structural reliability theory

The manner in which an engineer structure will respond to loading depends on the type and magnitude of the applied loads and structural strength and stiffness. Whether the response is considered satisfactory depends on the requirements which must be satisfied. These include safety of the structure against collapse, limitation on damage, or on deflections or other criteria. Each such requirement may be termed a limit state.

The study of structural reliability is concerned with calculation and prediction of the probability of limit state contravention for a structural system at any stage during it life. The probability of occurrence of an event such as limit state contravention is a numerical measure of the chance of its occurrence. This measure either may be obtained from measurements of the log-term frequency of occurrence of the event for generally similar structures, or may be simply a subjective estimate of the numerical value. However, in practical it is seldom possible to observe for a sufficiently long period of time, and a combination of subjective estimates and frequency observations for structural components and properties may be used to predict the probability of limit state contravention for the structure.

The structural safety of a structure can be estimated by three methods as

- Deterministic assessment such as safety factor
- Semi-probabilistic assessment such as return period
- Probabilistic assessment

The probabilistic assessment method is explained in the subsequent. In general, the loads which are applied to a structure vary with time and are of uncertain value at any one point in time. This is carried over directly to the load effect$S$. Somewhat, similarly the structural resistance R will be function of time (but not variation one) owing to deterioration and similar action. Loads have a tendency to increase, and resistance to decrease with time. It is usual also for the uncertainty in both these quantities to increase with time. This means that the probability density functions $f_s()$ and $f_R()$ become wider and flatter with time and that the mean values of S and R also change with time.

The safety limit state will be contravened whenever, at any time t

$$R(t) - S(t) < 0 \quad \text{OR} \quad \frac{R(t)}{S(t)} < 1 \qquad \text{(C.1)}$$

The probability that this occurs for any one load application is the probability of limit state violated, or simply the probability of failure p_f. Roughly, it may represented by the amount of overlap of the probability density function f_S and f_R. Since overlap may vary with time, p_f also may be a function of time. However, in some situations, it is convenient to assume that neither Q or R is a function of time. This will be the case if the load Q is applied once only to the structure and the probability of failure is sought for that load application only.

If this done, the effect of time may now be ignored in the reliability calculations. This approach is not satisfactory when more than one load is involved or when the resistance changes with time.

The basic reliability problem

The basic structural reliability problem considers only one load effect S resisted by one resistance R. Each is described by a known probability density function, $f_S()$ and $f_R()$, respectively. As noted, S may be obtained from the applied loading Q through a structural analysis. It is important that R and S are expressed in the same units.

For convenience, but without loss of generality, only the safety of a structural member will be considered here and as usual, that structural member will be considered to have failed if its resistance R is less than the stress resultant S acting on it. The probability of failure p_f of the structural member can be stated in any of the following ways:

$$p_f = P(R \leq S) = P(R - S \leq 0) = P\left(\frac{R}{S} \leq 1\right) = P(LnR - LnS \leq 0) \quad \text{(C.2)}$$

or in general form

$$p_f = P[G(R,S) \leq 0] \qquad \text{(C.3)}$$

Where $G()$ is termed the ‘limit safe function ‘and the probability of failure is identical with the probability of limit state violation. If the R and S assume the continuous variables then the probability of failure can be calculated by

$$p_f = P(R - S \leq 0) = \iint_D f_{RS}(r,s)drds \qquad \text{(C.4)}$$

When R and S are independent $f_{RS}(r,s) = f_R(r)f_S(s)$

$$p_f = P(R - S \leq 0) = \int_{-\infty}^{+\infty}\int_{-\infty}^{s \geq r} f_R(r)f_S(s)drds = \int_{-\infty}^{+\infty} F_R(x)f_s(x)dx \qquad \text{(C.5)}$$

This is also known as a ‘convolution integral’. Its meaning easily is explained in next figure.

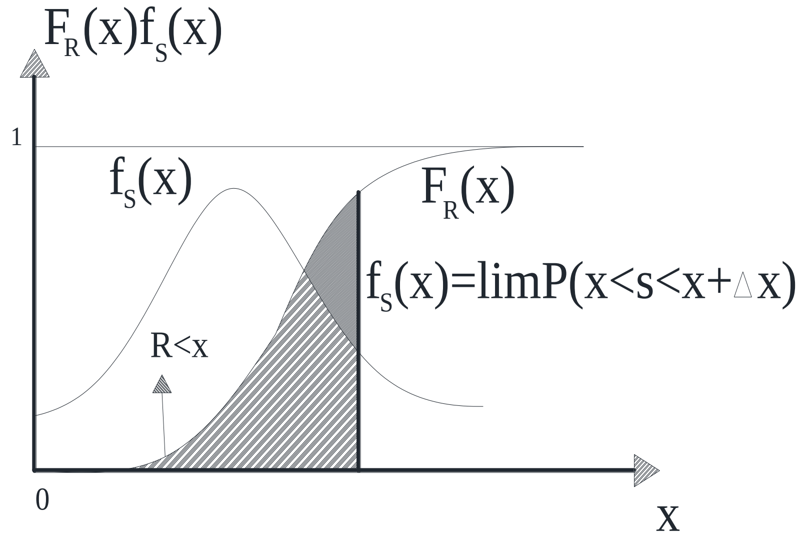

Figure C.1 Basic R-S problems: $F_R()$ $f_S()$ representation

$F_R(x)$ is the probability that $R \leq x$ or the probability that the actual resistance R of the member is less than some value x. Let this represents failure. The term $f_S(x)$ represents the probability that the load effects S acting in the member has a value between x and $x + \Delta x$ in the limit as $\Delta x \xrightarrow{yields} 0$. By considering all possible values of x, i.e. by taking the integral over all x, the total failure probability is obtained.

This also seen in next figure where the density functions f_R and f_S have been drawn along the same axis.

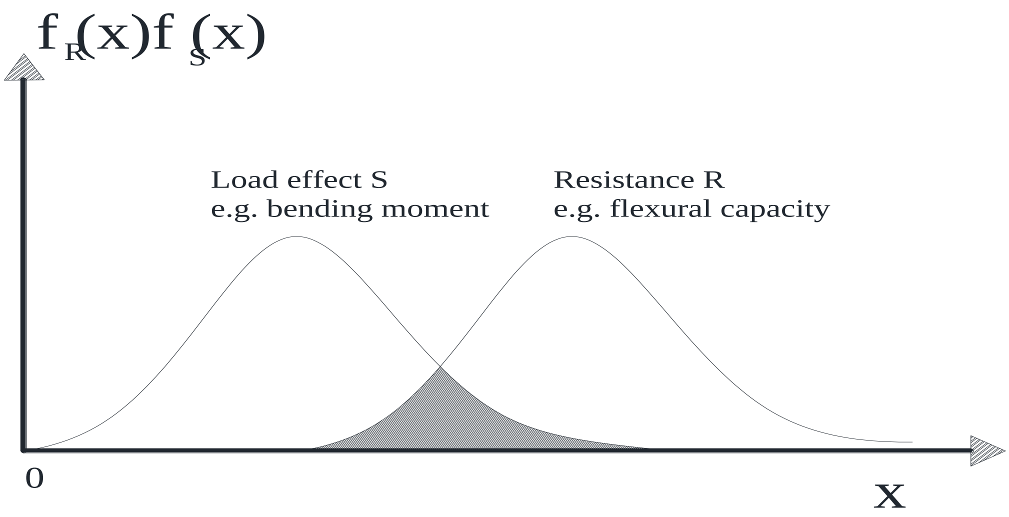

Figure C.2 Basic R-S problem: $f_R()$ $f_S()$ representation

The lower limit of integration may not be totally satisfactory, since a negative resistance usually is not possible. The lower limit of integration should be strictly zero.

Generalized reliability problem

For many problems the simple formulation as indicated above are not adequate, since it may not possible to reduce the structural reliability problem to a simple R versus S formulation with R and S independent random variables. In general, R is a function of material properties and member or structure dimensions while S is a function of applied loads Q, material densities and perhaps dimensions of structure, each of which may be a random variable. Also, R and S may not be independent, such as some loads act to oppose failure (e.g. overturning) or when the same dimensions affect both R and S.

In this case, it is not valid to use the convolution integral. It is also not valid when there is more than one applied stress resultant acting at a section. A more general formulation is required. The simple $R - S$ form of the limit state needs to replace with a generalized version expressed directly in terms of basic variables. Let the vector X represent all the basic variables involved in the problem. Then the resistance R can be expressed as $R = G_R(X)$ and loading or load effect as $S = G_S(X)$. Since the functions G_R and G_S may be non-linear, the cumulative distribution function $F_R(\quad)$, for example, must be obtained by multiple integration over the relevant basic variable.

$$F_R(r) = \int_r \dots \int f_X(x)dx \qquad \text{(C.6)}$$

A similar expression would apply for S and $F_S()$.

It is seldom necessary to follow this approach. The limit state function $G(R,S)$ can also be generalized. When the functions $G_R(X)$ and $G_S(X)$ are used in $G(R,S)$, the resulting limit state function can be written simply as $G(X)$, where X is the vector of all relevant variables and $G()$ is some function expressing the relationship between the limit state and basic variables. The limit state equation $G(X) = 0$ now defines the boundary between satisfactory or safe domain $G > 0$ and unsatisfactory or unsafe domain $G \leq 0$ in n-dimensional basic variables space. Usually the limit state equation is derived from the physic of the problem.

With limit state function expressed as $G(X)$, the generalization of probability of failure function becomes:

$$p_f = P[G(X) \leq 0] = \int \dots \int_{G(X)\leq 0} f_X(x)dx \qquad \text{(C.7)}$$

Here $f_X(x)$ is the joint probability density function for the n-dimensional vector X of basic variables. Note that the resistance R and load effect S are no longer involved in the formulation and may even not be explicit-generally they are implicit in X. If the basic variables themselves are independent, the formulation is simplified as

$$f_X(x) = \prod_{i=1}^{n} f_{X_i}(x_i) = f_{X_1}(x_1) f_{X_2}(x_2) \dots f_{X_n}(x_n) \qquad \text{(C.7)}$$

With $f_{X_i}(x_i)$ the marginal probability density function for the basic variable X_i.

Time-dependent reliability

In general, the basic variables X will be function of time. This comes about, for example, because loading changes with time and because material strength properties change with time, either as a direct result of previously applied loading or because of some deterioration mechanism. Fatigue and corrosion are typical examples of strength deterioration. The elementary reliability problem in time-variant terms with a resistance $R(t)$ and load effect $S(t)$, at time t becomes

$$p_f(t) = P[R(t) \leq S(t)] \quad \text{(C.8)}$$

OR

$$p_f(t) = \int_{G[X(t)]} f_{X(t)}\,[X(t)]dX(t) \quad \text{(C.9)}$$

There are several methods to calculate the probability of failure such as numerical solutions which provide the approximate results, simulation methods and the method of the First-order Second-moment theory.

APPENDIX D

Bridge condition rating

A condition state categorizes the nature and extent of damage or deterioration of abridge element. It has been established to measure the state of bridge components over time in a consistent and uniform manner. The AASHTO Guide Manual for Bridge Element Inspection, first edition 2011, provides detailed information on bridge components and their corresponding condition states. General condition ratings are used to describe the existing in-place bridge or culvert as compared to as-built condition. The materials used in the bridge are considered as well as physical condition of the deck, superstructure and substructure components. The information used to determine GCRs on a numerical scale that ranges from 0(failed condition), to 9(excellent condition) as described in the FHWA coding guide. These ratings provide an overall characterization of the general condition of the entire component being rated; the condition of specific individual bridge components may be higher or lower. The bridge condition rating in more detail is described in the next table.

Table D.1 National Bridge Inventory general condition rating

Code	Description	Commonly Employed Feasible Actions
9	Excellent condition	Preventive maintenance
8	Very good condition No problems noted	Preventive maintenance
7	Good condition Some minor problems	Preventive maintenance
6	Satisfactory condition Structural components show Some minor deterioration	Preventive maintenance And/or repair
5	Fair condition All primary structural Components are sound but May have some minor section loss, cracking, spalling or scour	Rehabilitation or replacement
4	Poor condition Advanced sections loss, deterioration, spalling or scour	Rehabilitation or replacement
3	Serious condition Loss of section, deterioration, spalling or scour has seriously affected primary structural components. Local failures are possible. Fatigue cracks in steel or shear cracks in concrete may be present	Rehabilitation or replacement
2	Critical condition Advanced deterioration primary structural components. Fatigue cracks in steel or shear cracks in concrete may be present or scour may have removed substructure support. Unless closely monitored the bridge may have to be closed until corrective action is taken	Rehabilitation or replacement
1	Imminent failure condition Major deterioration or section loss present in critical structural components or obvious vertical or horizontal movement affecting structure stability. Bridge is closed to traffic but corrective action may put back in light service	Rehabilitation or replacement
0	Failed condition Out of service-beyond corrective action	Rehabilitation or replacement

From information collected through the inspection process, assessment are performed to determine the adequacy of a structure to service the current structural and functional demands; factors considered include load-carrying capacity, deck geometry, clearance, waterway adequacy, and approach road alignment. Structural assessment together with ratings of physical condition of key bridge's components determines whether a bridge should be classified as 'structurally deficient'. Functional adequacy is assessed by comparing the existing geometric configurations and design load carting capacities to current standards and demands. Disparities between the actual and preferred configurations are used to determine whether a bridge should be classified as 'functionally obsolete'.

APPENDIX E

AASHTO Inspection Regime

A. **Initial inspection**

Initial inspection can be carried out on new bridges or when existing bridges are first entered into the database. This inspection provides a basis for all future inspections or modifications to the bridge. Initial deficiencies are noted which might not have been present at the time of construction. Changes in condition of the site might be noted such as erosion, scour and slopes.

The final bridge completion checklist includes the notification to the District Bridge inspection coordinator when the bridge is opened to traffic and available for use by permit vehicle.

B. **Routine inspection**

The Routine inspection usually is undertaken every two years for most bridges. Routine inspection is regularly scheduled and recorded in accordance with all the procedures based on bridge record rule and the instruction-coding guide.

A specific Routine inspection which is performed approximately every six months on most structures to identify unusual conditions or changes is named Brief inspection. It doesn't need to review all points and members done in a normal Routine inspection. Unusual conditions or changes will often result in a follow-up in other types.

C. **Damage inspection**

In result of collision, fire, flood, significant environmental changes, loss of support and etc, Damage inspection will be undertaken. It is sometimes called Emergency inspection and is performed on as-needed basis.

D. **In-Depth inspection**

In order to identify better any deficiencies, In-Depth inspection can be carried out as a follow-up inspection to an Initial, Routine or Damage inspection. Sometimes Load testing may be performed as part of an in depth inspection. This is regularly performed every five years.

E. **Special inspection**

The purpose of this type of inspection is to monitor new types of structures, structural details, or materials. A special inspection may also be used to develop an information database. (The manual for condition evaluation of bridges; AASHTO, 2011)