City Research Online

City St George's, University of London

Citation: Rickayzen, B. D. (1997). A sensitivity analysis of the parameters used in a PHI multiple state model (Actuarial Research Paper No. 103). London, UK: Faculty of Actuarial Science & Insurance, City University London.

This is the unspecified version of the paper.

This version of the publication may differ from the final published version. To cite this item please consult the publisher's version.

Permanent repository link: https://openaccess.city.ac.uk/id/eprint/2233/

Copyright and Reuse: Copyright and Moral Rights remain with the author(s) and/or copyright holders. Copies of full items can be used for personal research or study, educational, or not-for-profit purposes without prior permission or charge, unless otherwise indicated, provided that the authors, title and full bibliographic details are credited, a hyperlink and/or URL is given for the original metadata page and the content is not changed in any way. For full details of reuse please refer to City Research Online policy.

City Research Online: http://openaccess.city.ac.uk/ publications@citystgeorges.ac.uk

A SENSITIVITY ANALYSIS OF THE PARAMETERS USED IN A PHI MULTIPLE STATE MODEL

by

B D RICKAYZEN

Actuarial Research Paper No. 103

Department of Actuarial Science and Statistics
City University
London

December 1997

ISBN 1 901615 20 0

"Any opinions expressed in this paper are my/our own and not necessarily those of my/our employer or anyone else I/we have discussed them with. You must not copy this paper or quote it without my/our permission".

Abstract

This paper presents a multi-state model for Permanent Health Insurance. It includes more states than are normally found in such a model to act as a proxy to duration spent in a particular state. This enables a Markov approach to be adopted. Lapses are incorporated within the model and the net premium for a particular policy is tested for sensitivity to the various parameters used, including the lapse rate. The most significant conclusion is that the net premium is insensitive to changes in the lapse rate.

Key words

PHI benefits; Markov chain; lapses.

"A sensitivity analysis of the parameters used in a PHI multiple state model"

1. Introduction

When Permanent Health Insurance (PHI) is modelled using a multi-state process, the approach is usually to consider only 3 states: healthy, sick and dead.

Furthermore, in order to make the calculations tractable, the transition forces are usually assumed to be constant, at least over each year of age of the policyholder.

The problem with this approach is that the model is over simplified since, in practice, the probability that a policyholder moves out of the sick state (into the dead state or back into the healthy state) will depend on the time spent in the sick state. In other words, the transition probabilities depend on duration in a particular state as well as the age of the policyholder.

It would be possible to incorporate within the model the duration-dependence aspect described above by adopting a semi-Markov, as opposed to Markov, model. However, it is generally accepted (Jones 1994) that semi-Markov models are complicated and do not lead to convenient expressions for the transition probabilities required to obtain actuarial values.

In this paper, we adopt an alternative approach suggested by Jones (1994) whereby we introduce more states into the model as a means of dealing with duration-dependence whilst maintaining the Markov as opposed to semi-Markov property of the model. Increasing the number of states makes the state space more complicated but maintaining the Markov process keeps the calculations tractable.

Having introduced our model, including a status for lives who have lapsed their policies, we use the data contained within Continuous Mortality Investigation Report No 12, published by the Institute and Faculty of Actuaries, to test the sensitivity of the parameters involved.

2. The Model

We set out below a diagrammatic representation of the multi-state model adopted.

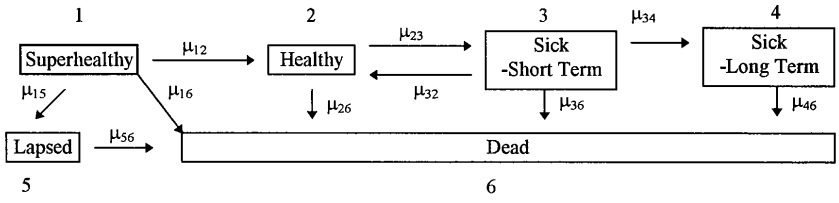

Figure 1. Outline of PHI model

The model has 6 states. State 1 is the state in which policyholders enter the model when their policy commences. Since they will have needed to provide satisfactory medical evidence, they

are deemed to be select lives and therefore healthier than other insured lives of the same age. We describe these lives as "superhealthy". It is likely that, in time, the selection effect will wear off and that the lives will move to an unstable form of the healthy state (State 2) from which they may become sick enough to make a claim under the PHI policy.

As with "healthy", we have a sub-divided the "sick" state between "short term sick" (State 3) from which it is possible to recover (and therefore return to State 2) and "long term sick" (State 4) from which it is assumed that recovery is impossible.

We have also introduced a lapsed state (State 5) into the model. We assume that policyholders will only enter this state from State 1, on the grounds that policyholders in any other state would find it worthwhile to continue their PHI policy.

The approach adopted here is to assume, initially, that the forces of transition between the states are all constant (i.e. not dependent on age). We then relax this assumption by using force of transition functions which are piecewise constant (i.e. constant only within age bands).

3. Constant Forces of Transition

As implied above, as a first step, it is possible to use a time-homogeneous continuous time Markov process to represent the transitions between states. Hence, we assume that a person in any state is subject to a constant force of progression into the other states which it is possible to enter according to Figure 1.

Let $p_{ij}(t)$ be the probability that a life currently in state i will be in state j in t years time. The constant force of transition between states i and j, μ_{ij}, is defined as

$$\mu_{ij} = \underset{t \to 0+}{Lim} \frac{p_{ij}(t)}{t} \quad i \neq j$$

It is convenient also to define, for each i,

$$\mu_{ii} = -\sum_{j:j \neq i} \mu_{ij}$$

where the summation is over the states j which are linked directly to state i.

The constant transition intensities are depicted in Figure 1. The fact that these intensities are constant means that the length of time already spent in the current state has no effect on the future length of time which the policyholder will remain in the state. Hence, a "memoryless" property exists (Haberman, 1992).

A common approach used to set up equations for the required transition probabilities (for example, Jones (1994) and Haberman (1995)) is to use the Chapman-Kolmogorov backward system of difference-differential equations (Cox and Miller, 1965). The fact that the transition intensities are assumed to be constant means that explicit solutions to the equations can be found.

The backward system of equations is derived by considering the interval $(0, t+h]$ as being comprised of subintervals $(0, h]$ and $(h, t+h]$ and then letting $h \to 0$.

These equations lead to a set of difference-differential equations. For illustration purposes, some of the differential equations are set out below:

$$\left.\begin{aligned}
&\frac{d}{dt}p_{11}(t) = -\left(\mu_{12}+\mu_{15}+\mu_{16}\right)p_{11}(t)\\
&\frac{d}{dt}p_{12}(t) = -(\mu_{12}+\mu_{15}+\mu_{16})p_{12}(t)+\mu_{12}p_{22}(t)\\
&\frac{d}{dt}p_{22}(t) = -(\mu_{23}+\mu_{26})p_{22}(t)+\mu_{23}p_{32}(t)\\
&\frac{d}{dt}p_{23}(t) = -(\mu_{23}+\mu_{26})p_{23}(t)+\mu_{23}p_{33}(t)\\
&\frac{d}{dt}p_{33}(t) = \mu_{32}p_{23}(t)-(\mu_{32}+\mu_{34}+\mu_{36})p_{33}(t)\\
&\frac{d}{dt}p_{32}(t) = \mu_{32}p_{22}(t)-(\mu_{32}+\mu_{34}+\mu_{36})p_{32}(t)\\
&\frac{d}{dt}p_{44}(t) = -\mu_{46}p_{44}(t)\\
&\frac{d}{dt}p_{46}(t) = -\mu_{46}p_{46}(t)+\mu_{46}p_{66}(t)\\
&\frac{d}{dt}p_{55}(t) = -\mu_{56}p_{55}(t)\\
&\frac{d}{dt}p_{56}(t) = -\mu_{56}p_{56}(t)+\mu_{56}p_{66}(t)\\
&\frac{d}{dt}p_{66}(t) = 0
\end{aligned}\right\} \quad (1)$$

The most straightforward way to solve these differential equations is to follow the method outlined by Jones (1994).

The forces of transition and transition probability functions are expressed in matrix form such that:

Q is the 6 × 6 matrix with (i, j) entry μ_{ij}

$P(t)$ is the 6 × 6 matrix with (i, j) entry $p_{ij}(t)$

The Chapman-Kolmogorov forward and backward systems of equations may then be written respectively

$$P'(t) = P(t).Q \quad (2)$$

and

$$P'(t) = Q.P(t) \quad (3)$$

where $P'(t)$ is the matrix with (i, j) entry $\frac{d}{dt}p_{ij}(t)$ and with boundary condition $P(0) = I$, the identity matrix.

Equations (2) and (3) have the solution

$$P(t) = e^{Qt} \tag{4}$$

Now, as noted by Cox and Miller(1965), if Q has distinct eigenvalues $d_1, d_2, \ldots, d_6$ then

$$Q = A.D.A^{-1} \tag{5}$$

where $D = diag\ (d_1, \ldots, d_6)$ and the ith column of A is the right-eigenvector associated with d_i.

Furthermore,

$$P(t) = A.diag(e^{d_1 t}, \ldots e^{d_6 t}).A^{-1} \tag{6}$$

The transition probability matrix, $P(t)$, can then be obtained from equation (6) using standard computer software, for example S-PLUS.

We should then note that $p_{ij}(t)$ is the (i, j) element of the matrix $P(t)$.

4. Piecewise Constant Forces Of Transition

In Section 3. above, we assumed that the forces of transition were constant with respect to time which enabled us to obtain the transition probabilities in a relatively straightforward manner. However, when considering Permanent Health Insurance it is unrealistic to assume that the forces of transition do not depend on age. To deal with this problem, we again adopt the general methodology described in Jones(1994).

By assuming that the forces of transition are piecewise constant, we are able to use the theory described in Section 3, whilst permitting the forces of transition to vary by age. For illustrative purposes in this paper, we have assumed that the forces of transition are constant over successive 5-year age bands (i.e. age 30-34, 35-39, ...60-64).

We now define $P_x(1)$ to be the transition probability matrix $P(1)$ applicable to the year of age x to $x + 1$. In other words, the (i, j) element of $P_x(1)$ is the probability that the policyholder currently aged x and in State i will be in State j in one year's time (aged $x + 1$).

If we let $p_{ij}^x(t)$ be the probability that a life currently aged x and in State i will be in State j in t years time then

$p_{ij}^x(t) = (i, j)$ element of the matrix resulting from the matrix product:

$$\prod_{k=0}^{t-1} P_{x+k}(1) \qquad \text{provided } t \text{ is an integer.}$$

5. PHI Data

Initially, we wish to test the effect which the force of transition from state 1 to state 5 (described here as the lapse rate) has on the net premium emerging from the model. In order to do this, we need to assign parameter values to all the forces of transition in the model and allow the lapse rate, μ_{15}, to vary.

The data used in the investigation has been taken from Report No 12 of the Continuous Mortality Investigation Reports published by the Institute and Faculty of Actuaries in 1991 ("CMIR 12").

As mentioned in Section 4 above, we have assumed that the parameter values applicable to the initial age of each 5-year age band (ie ages 30, 35, ..., 60) are constant over the whole of the 5-year age band.

6. Parameter Values

We set out below the way in which each parameter value has been chosen.

μ_{23}, **Unstable Healthy** ⟶ **Short Term Sick**

We consider the sickness inception rate, σ_x, described in Part C of CMIR 12. We could have used values assuming a deferred period of 1 week; however, in view of the fact that this would include trivial sickness claims, we have instead used values for a deferred period of 4 weeks which are found in Table C16 of CMIR 12 (page 74).

μ_{16}, **Superhealthy** ⟶ **Dead**
μ_{26}, **Unstable Healthy** ⟶ **Dead**

CMIR 12 assumes the morality rate amongst healthy lives to be that of Male Permanent Assurances 1979 - 82, duration 0. The rates are shown in Table E17 (page 132) under the column headed $m(x)$.

In our model, we have sub-divided healthy lives into "Superhealthy" and "Unstable healthy" states. Since the latter will experience higher mortality rates than the former, we have decided to assume:

μ_{16} = 80% of the mortality rates for Male Permanent Assurances of 1979-82, duration 0.
μ_{26} = 120% of the mortality rates for Male Permanent Assurances of 1979-82, duration 0.

The overall effect can therefore be considered to be broadly consistent with CMIR 12. As suggested by Cordeiro (1995), net premium values are likely to be insensitive to the parameter values chosen for the forces of mortality.

μ_{32}, **Short Term Sick** ⟶ **Unstable Healthy**

We consider the recovery rates described in Section 3, Part B of CMIR 12. Page 34 of CMIR 12 sets out values of $\rho_{y+z,z}$ at various ages where:

$\rho_{y+z,z}$ = The transition intensity from sick to healthy at current age $y + z$ and current duration of sickness z.

It is clear that these recovery rates vary considerably by duration of sickness (measured in weeks). In view of the relatively simplistic approach deliberately adopted in our model, we will use parameter values which we consider to be reasonable. Initially we will assume a constant value of 2.0 at all ages. We will then analyse the effect of assuming $\mu_{32} = 3.0$ at all ages instead in Section 8.4.

μ_{36}, Short Term Sick ⟶ Dead

We consider the mortality intensities described in Section 6, Part B of CMIR 12. Page 39 of CMIR 12 sets out values of $\nu_{y+z,z}$ at various ages where:

$\nu_{y+z,z}$ = The transition intensity from sick to dead at current age $y + z$ and current duration of sickness z.

For our calculations, we will use the values at 15 weeks duration of sickness which is when the values reach their peak. Interpolated values have been used where necessary.

μ_{34}, Short Term Sick ⟶ Long Term Sick

As a proxy for this force of transition, we use the sickness inception rate, σ_x, for a 26 week deferred period (i.e. a suitably long period of deferment). As with μ_{23}, the rates can be found in Table C16 of CMIR 12 (page 74).

μ_{46}, Long Term Sick ⟶ Dead

We can again consider the mortality intensities $\nu_{y+z,z}$ which were described under μ_{36} above. It seems appropriate to use these intensities at a suitably long sickness duration. We will use the values at duration 5 years which are shown on page 39 of CMIR 12.

μ_{56}, Lapse ⟶ Dead

In view of the fact that we are assuming that only "Superhealthy" policyholders lapse their policies, we assume that :

$$\mu_{56} = \mu_{16}$$

μ_{12}, Superhealthy ⟶ Healthy

It is clear that CMIR 12 will not be able to provide explicit parameter values for μ_{12}. However, it seems reasonable to ensure that our estimates of μ_{12} should be such that the aggregate mortality rates implied within the model approximately reflect the mortality table Male Permanent Assurances 1979-82, duration 0. We have, therefore, chosen values for μ_{12} which meet this constraint. This has been done by inspection, since more sophisticated techniques would lead to spurious accuracy. For the purpose of carrying out this exercise, we assume that μ_{15}, the lapse rate, takes the value 0.01 at all ages.

μ_{15}, **Superhealthy** ⟶ **Lapse**

Finally, having fixed the other parameters, we will let the lapse rate, μ_{15}, vary in order to investigate the effect on the net premium.

We set out a summary of the parameter values, other than μ_{15} in Appendix 1. Appendix 2 shows the number of lives in each state at sample ages given 100 healthy lives entering the superhealthy state (State 1) at age 30. It can therefore be seen for example that, by age 65, 17.4% of the lives have died, 19.9% have lapsed and 13% are still in the "superhealthy" state.

7. Valuation Functions

For illustrative purposes, we calculate the annual net premium required in respect of the following PHI policy:

We consider a 35 year term PHI policy issued to a life aged 30. A payment of £1,000 is paid at the end of each year if the policyholder is sick at that time. Premiums are waived during periods of sickness and there is no deferred period. The benefits and premiums cease at the age of 65.

We use a valuation rate of interest of 6% pa.

In order to calculate the annual net premium, we need to set up the standard equation of value, based on the equivalence principle:

Expected present value of net premiums = Expected present value of benefits

We note that a premium is only payable if the policyholder is either in State 1 ("Superhealthy") or State 2 ("Unstable healthy") at the start of the policy year in question. Hence the expected present value of the premium is

$$P\left(\ddot{a}^{11}_{30:35\,\overline{|}} + \ddot{a}^{12}_{30:35\,\overline{|}}\right)$$

where P = annual net premium

and $\ddot{a}^{1j}_{30:35\,\overline{|}} = \sum_{k=0}^{34} p^{30}_{1j}(k)v^k \qquad j = 1, 2$

We also note that the PHI benefit is only payable if the policyholder is in either State 3 ("Short Term Sick") or State 4 ("Long Term Sick") at the end of the policy year in question. Hence, the expected present value of the PHI benefits is

$$1000\left(a^{13}_{30:35\,\overline{|}} + a^{14}_{30:35\,\overline{|}}\right)$$

where $a^{1j}_{30:35\,\overline{|}} = \sum_{k=1}^{35} p^{30}_{1j}(k)v^k \qquad j = 3, 4$

Therefore, we can find P from

$$P = \frac{1000\left(a^{13}_{30:35\,\overline{|}} + a^{14}_{30:35\,\overline{|}}\right)}{\left(\ddot{a}^{11}_{30:35\,\overline{|}} + \ddot{a}^{12}_{30:35\,\overline{|}}\right)}$$

8. Results

8.1 Sensitivity to μ_{15}, the lapse rate

We set out in Figure 2 below a graph showing how the theoretical net premium varies as the lapse rate, μ_{15}, takes values between 0 and 1.

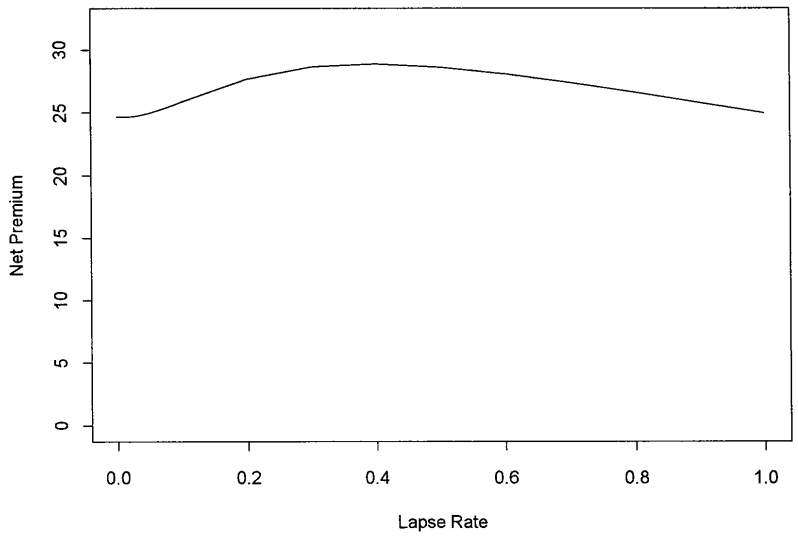

Figure 2: Sensitivity of net premium to the lapse rate, μ_{15}

It can be seen that, perhaps surprisingly, the net premium is relatively insensitive to the lapse rate. The net premium remains fairly constant at around £25 per annum despite large changes in the lapse rate.

The net premium increases from £24.67 per annum to £28.86 per annum as μ_{15} increases from 0 to 0.4. The reason for this is that when the lapse rate is small, there are a large number of lives in the system who are in the "superhealthy" state and therefore continue to pay premiums without receiving any PHI benefit payments. This tends to suppress the net premium averaged over all the policyholders in the system. As the lapse rate increases, more of the "superhealthy" lives leave the system by lapsing which will tend to increase the average premium payable in respect of the remaining, relatively unhealthy, insured population.

It is then necessary to explain why the net premium decreases as the lapse rate increases beyond 0.4. The reason for this counter-intuitive result is that, at very high lapse rates, most of the population will lapse in the first few years of the policy which makes the total premium paid in advance at the outset by all the insured lives a significant amount. The fact that a large amount of premium income is received at the start of the policy, but that the insured population then reduces dramatically over the first few years, will tend to reduce the average premium required.

It should be noted that lapse rates of more than 0.4 would appear to be unrealistic. For example, it can be shown that if $\mu_{15} = 0.4$, over 83% of the insured population aged 30 at the outset would have lapsed their policy during the first 5 years of the policy.

Finally, before discussing other sensitivity issues, it is worth comparing the net premiums calculated using the model described in this paper with those derived from the data in CMIR 12.

The data contained in Table F1 on page 228 of CMIR 12 suggests that the net premium for a policy similar to that described in Section 7, but with premium and benefit payments made continuously and with a deferred period of 1 week, should be £24.24 per annum. The net premium figures shown in Figure 2 are clearly of the same order of magnitude and hence provide some comfort that our model (including the parameter values chosen) is consistent with the model described in CMIR 12.

8.2 Sensitivity to μ_{12}

We set out in Figure 3 below a graph showing how the net premium changes when the parameter values for μ_{12} given in Appendix 1 are increased or decreased by 10%.

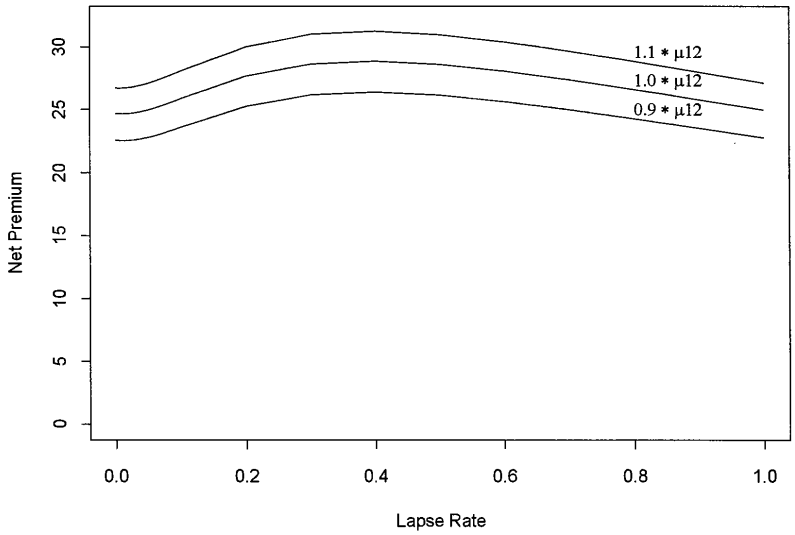

Figure 3: Sensitivity of net premium to μ_{12}

The net premium increases by 8.5% when the μ_{12} values are increased by 10% and decreases by 8% when the μ_{12} values decrease by 10%. In both cases, it can be seen that the effect is insensitive to the lapse rate, μ_{15}.

It is to be expected that the net premium moves in the same direction as μ_{12}. This is because an increase in μ_{12} causes more lives to move from the "superhealthy" to the "healthy" state

where they are exposed to the risk of sickness, which in turn will lead to an increase in the premium required.

8.3 Sensitivity to μ_{23}, the sickness inception rate

We set out in Figure 4 below a graph showing how net premium changes when the parameter values for μ_{23}, the sickness inception rate, are altered by 10%.

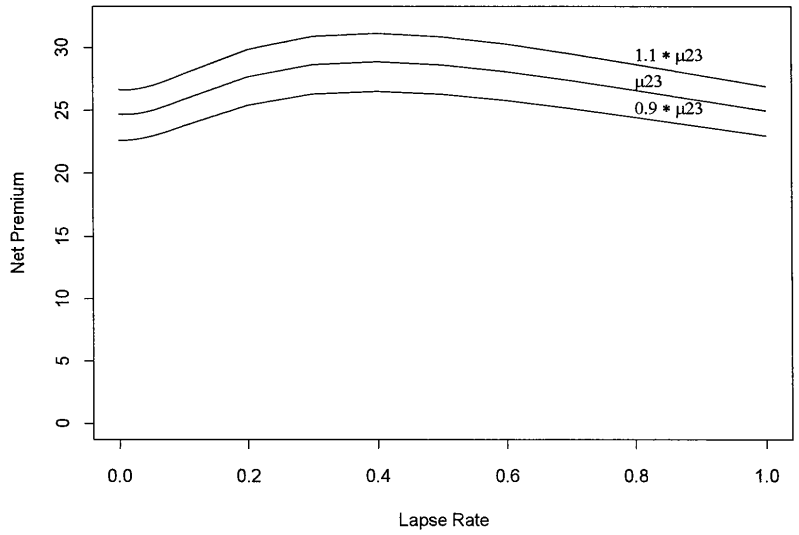

Figure 4: Sensitivity of net premium to sickness inception rate, μ_{23}

The net premium increases by 8% when the μ_{23} values are increased by 10%, and decreases by 8% when the μ_{23} values are decreased by 10%. Again, the results are largely unaffected by the level of lapse rate assumed.

As expected, an increase in the sickness inception rate causes an increase in the net premium required.

8.4 Sensitivity to μ_{32}, the recovery rate

We set out in Figure 5 below a graph showing how the net premium changes when the parameter value for μ_{32}, the recovery rate, is increased or decreased by 10% (i.e. changed from 2.0 at all ages to 2.2 or 1.8, respectively).

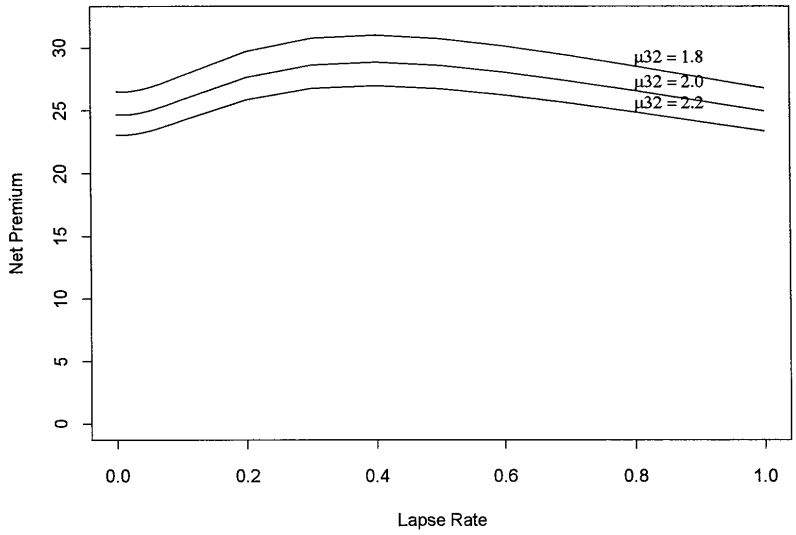

Figure 5: Sensitivity of net premium to the recovery rate, μ_{32}

The net premium increases by 7.5% when the recovery rate is reduced by 10%, and decreases by 6.5% when it is increased by 10%. Again, the level of lapse rate has very little effect on these results.

It is to be expected that an increase in the recovery rate should lead to a reduction in the amount of PHI premium required.

8.5 Sensitivity to μ_{34}

We set out in Figure 6 below a graph showing how the net premium changes when the parameter values for μ_{34} are increased or decreased by 10%.

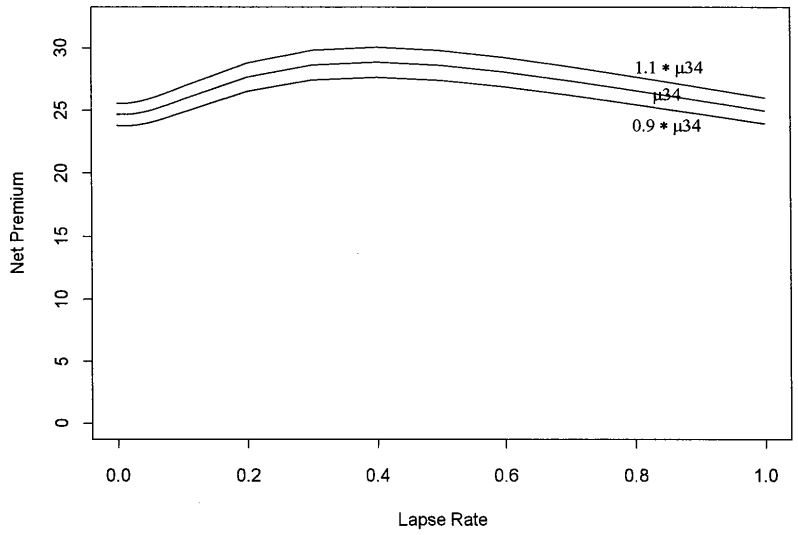

Figure 6: Sensitivity of net premium to μ_{34}

It can be seen that the net premium is relatively insensitive to changes in μ_{34} since a 10% increase/decrease in the latter causes only a 4% increase/decrease in the net premium.

As expected, an increase in the long term sickness inception rate leads to an increase in the net premium required.

8.6 Analysing the relationship between μ_{12} and μ_{32}

In Section 6, we explained how the parameter values for μ_{12} were chosen so that the aggregate mortality rates within the model broadly reflected the Male Permanent Assurances 1979-82, duration 0.

We now analyse how sensitive the values of μ_{12} are to a change in the other parameters, in particular to a 50% increase in the recovery rate, μ_{32}. In other words, we retain all the parameter values summarised in Appendix 1 except for μ_{32}, which we increase from 2.0 at all ages to 3.0, and μ_{12} which we need to recalibrate in order to ensure that the aggregate mortality rates still reflect the mortality table mentioned above.

As before, we derive the values for μ_{12} by inspection and the results can be summarised as follows:

Table 1: Comparison of μ_{12} values

	$\mu_{32} = 2.0$	$\mu_{32} = 3.0$
Age	μ_{12}	μ_{12}
30-34	0.022	0.026
35-39	0.005	0.009
40-44	0.015	0.021
45-49	0.015	0.035
50-54	0.055	0.070
55-59	0.085	0.110
60-64	0.129	0.430

It can be seen that a 50% increase in μ_{32} requires an increase in μ_{12} of approximately the same order of magnitude in order to leave the aggregate mortality rates within the model unaltered.

It can be seen from Figure 7 below that the changes in the two sets of parameter values would leave the net premium at approximately the same level as before (i.e. approximately £25 per annum).

Figure 7 below shows the effect on the net premium of changing the values of μ_{32} and μ_{12} from the first column of Table 1 to the second column.

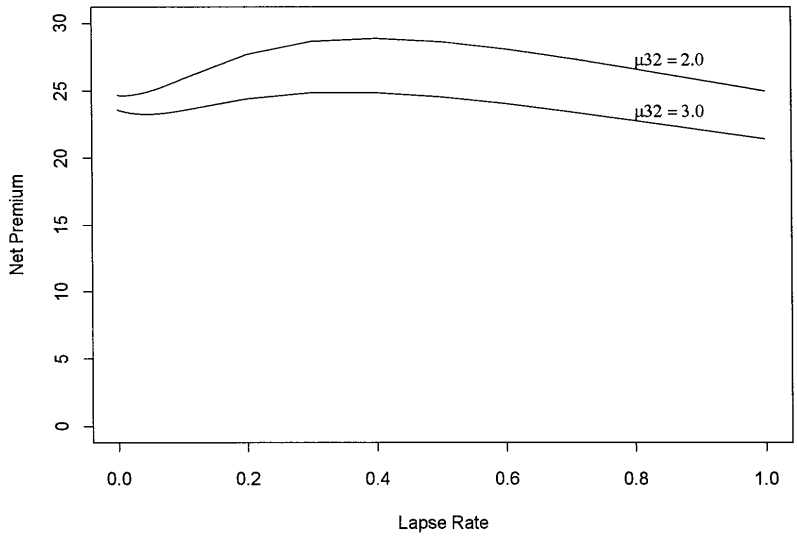

Figure 7: Effect of increasing μ_{32}

It can be seen that the net premium values are very similar when the lapse rate, μ_{15}, is very small. This can be explained by the fact that both sets of values of μ_{12} are calibrated when μ_{15} is set equal to 0.01. The net premiums diverge slightly at higher lapse rates, although they are clearly still of the same order of magnitude (i.e. £25 per annum). As noted in Section 8.1, this level of premium is consistent with the net premium calculated using the model described in CMIR 12.

9. Conclusions

In modelling Permanent Health Insurance, one of the main difficulties that needs to be overcome is that the forces of transition between different states may depend not only on the age of the policyholder but also on the time spent in the current state (e.g. sickness state). This usually leads to a semi-Markov model being used in which case convenient expressions for the transition probabilities are hard to obtain.

In this paper, we have adopted a model where the problems of duration-dependence described above are dealt with by increasing the number of states to differentiate between short-term and long-term stays in a particular status. This enables the model to be Markov, rather than semi-Markov, and therefore leads to tractable solutions. We have also taken the opportunity to consider lapses within the model.

We have tested the sensitivity of the net premium for a particular policy to changes in the model parameter values. The most significant result is that the net premium is insensitive to changes in the lapse rate.

REFERENCES

Continuous Mortality Investigation Report No 12 (1991). Institute and Faculty of Actuaries

Cordeiro I M F (1995). Sensitivity analysis in a Multiple State Model for Permanent Health Insurance. Continuous Mortality Investigation Report No 14. Institute and Faculty of Actuaries.

Cox DR and Miller HD (1965). The Theory of Stochastic Processes. Chapman and Hall, London.

Haberman S.(1995), HIV, AIDS, Markov Chains and Health and Disability Insurance. Journal of Actuarial Practice, 3, 51-75.

Haberman S (1992). HIV, AIDS and the Approximate Calculation of Life Insurance Functions, Annuities and Net Premiums. Journal of Institute of Actuaries, 119, 345-364

Jones B J (1994). Actuarial Calculations Using a Markov Model. Transactions of the Society of Actuaries, 46, 227-250.

Appendix 1: Summary of Parameters

Age	$\mu_{56}=\mu_{16}$	μ_{26}	μ_{46}	μ_{36}	μ_{32}	μ_{34}	μ_{23}	μ_{12}
30-34	0.0003	0.0005	0.0172	0.1108	2.0000	0.1183	0.2346	0.0220
35-39	0.0004	0.0006	0.0190	0.1180	2.0000	0.1065	0.2240	0.0045
40-44	0.0006	0.0010	0.0215	0.1251	2.0000	0.1007	0.2059	0.0150
45-49	0.0011	0.0017	0.0239	0.1379	2.0000	0.1000	0.1901	0.0150
50-54	0.0019	0.0028	0.0271	0.1507	2.0000	0.1042	0.1834	0.0550
55-59	0.0031	0.0046	0.0303	0.1694	2.0000	0.1140	0.1929	0.0850
60-65	0.0049	0.0073	0.0343	0.1880	2.0000	0.1310	0.2302	0.1290

Appendix 2: Percentage of Lives in Each State at Sample Ages

Age	State					
	1	**2**	**3**	**4**	**5**	**6**
30	100	0	0	0	0	0
31	96.9	2.0	0.1	0	1.0	0
32	93.7	3.9	0.3	0	2.0	0.1
.						
.						
.						
.						
50	61.0	16.6	1.4	1.6	15.6	3.8
.						
.						
.						
65	13.0	40.6	4.0	5.1	19.9	17.4

CITY University

ACTUARIAL RESEARCH CENTRE

DEPARTMENT OF ACTUARIAL SCIENCE AND STATISTICS

Actuarial Research Papers

1. Haberman S. Long Term Prognosis after Cerebral Infarction. 78 pages, 1984 (out of print).
ISBN 1 874 770 01 8

2. Haberman S. Long Term Prognosis after Subarachnoid Haemorrhage. 43 pages, 1984 (out of print).
ISBN 1 874 770 02 6

3. Haberman S. Age Variation in Incidence of Cerebrovascular Disease. 36 pages, 1985 (out of print).
ISBN 1 874 770 03 4

4. Kaye G.D. Taxation of Pension Funds. 11 pages, 1985.
ISBN 1 874 770 04 2

[Also: Haberman S. and Bloomfield D.S.F. Social Class Differences in Mortality Around 1981. SSRU Working Paper No. 55. 41 pages, 1987.]

5. Kaye G.D. Life Offices' Expenses Regression Analyses I, 98 pages, 1988. (Limited circulation).
ISBN 1 874 770 05 0

6. Kaye G.D. Life Assurance Taxation - Comments in Response to the Board of the Inland Revenue's Consultative Document Concerning the Taxation of Life Insurance. 11 pages. 1988.
ISBN 1 874 770 06 9

7. Verrall R.J. Bayesian Linear Models and the Claims Run-Off Triangle. 32 pages. 1988.
ISBN 1 874 770 07 7

8. Kaye G.D. Life Offices' Expenses Regression Analyses II. 12 pages. 1989. (Limited circulation).
ISBN 1 874 770 08 5

9. Bloomfield D.S.F. and Haberman S. The Assessment of Damages and the Measurement of Work time Lost to Sickness, Unemployment and Stoppages. 100 pages. 1989.
ISBN 1 874 770 09 3

10. Adams A.T. Pricing Property Investment Company Shares. 14 pages. 1989.
ISBN 1 874 770 10 7

11. Booth P.M. History of Exchange Controls in the EC with Particular Reference to the United Kingdom. 39 pages. 1989.
ISBN 1 874 770 11 5

12. Verrall R.J. Chain Ladder and Maximum Likelihood. 12 pages. 1989.
ISBN 1 874 770 12 3

13. Verrall R.J. On the Unbiased Estimation of Reserves from Loglinear Models. 13 pages, 1989.
ISBN 1 874 770 13 1

14. Adams A.T. An Analysis of Investment Trust Shares, 1990. 31 pages.
ISBN 1 874 770 14 X

15. Haberman S. Demographic Trends Towards 2000. 16 pages. 1989.
ISBN 1 874 770 15 8

16. Haberman S. British Life Insurance: The Role of the Actuary and 1992. 30 pages. 1990.
ISBN 1 874 770 16 6

17. Verrall R.J. Bootstrapping, Saddlepoint Approximations and the Probability of Ruin. 10 pages. July 1990.
ISBN 1 874 770 17 4

18. Booth P.M. Theoretical Aspects of the Taxation of Real Returns from Short Term Deposits. 26 pages. June 1990.
ISBN 1 874 770 18 2

19. Booth P.M. Practical Aspects of the Taxation of Real Interest. 38 pages, 1990.
ISBN 1 874 770 19 0

20. Puzey A.S. Actuarial Funding in Unit-linked Life Assurance - Derivation of the Actuarial Funding Factors. 19 pages. July 1990.
ISBN 1 874 770 20 4

21. Kaye G.D. Economies of Scale in the United Kingdom Insurance Industry. A report for the Association of British Insurers. Limited Circulation. October 1990.
ISBN 1 874 770 21 2

22. Adams A.T. & Venmore-Rowland P. Direct or Indirect Property Investment. 71 pages. September 1990.
ISBN 1 874 770 22 0

23. England P.D. Statistical Modelling of Excess Mortality. 124 pages. September 1990.
ISBN 1 874 770 23 9

24. Verrall R.J. & Li Z. Negative Incremental Claims: Chain Ladder and Linear Models. November 1990.
ISBN 1 874 770 24 7

25. Haberman S. State Pension Provision and Social Security in the United Kingdom. Presented at a Conference in Lisbon on "Social Security and Private Provision; What are the Prospects?". 40 pages. October 1990.
ISBN 1 874 770 25 5

26. Renshaw A.E. Graduation by Generalized Linear Modelling Techniques. 42 pages. February 1991.
ISBN 1 874 770 26 3

27. England P.D. Statistical Modelling of Excess Mortality No. 2. 170 pages. June 1991.
ISBN 1 874 770 27 1

28. Adams, A.T. The Market Pricing of Composite Insurance Shares. 37 pages. July 1991.
ISBN 1 874 770 28 X

29. Haberman, S. The Effect of Autoregressive Rates of Return on the Variability of Pension Contributions and Fund Levels. 58 pages. September 1991.
ISBN 1 874 770 29 8

30. Puzey, A.S. An Investigation into the Effect of Averaging Age Nearest Birthday at the Beginning of a Rate Year to Exact Age. October 1991. 33 pages.
ISBN 1 874 770 30 1

31. Chadburn, R.G. Bias in Select Mortality Investigations where Data are Subdivided by Age at Death. November 1991. 69 pages.
ISBN 1 874 770 31 X

32. England P.D. and Verrall R.J. Dynamic Estimation for Models of Excess Mortality. January 1992. 19 pages.
ISBN 1 874 770 32 8

33. Verrall R.J. A State Space Formulation of Whittaker-Henderson Graduation, with Extensions. January 1992. 13 pages.
ISBN 1 874 770 33 6

34. Verrall R.J. Graduation by Dynamic Regression Methods. January 1992. 37 pages.
ISBN 1 874 770 34 4

35. Gerrard R.G. and Haberman S. Stability of Pension Systems when Gains/Losses are Amortized and Rates of Return are Autoregressive. March 1992. 12 pages.
ISBN 1 874 770 35 2

36. Haberman S. HIV, AIDS and the Approximate Calculation of Life Insurance Functions, Annuities and Net Premiums. April 1992. 28 pages.
ISBN 1 874 770 36 0

37. Cooper D.R. Savings and Loans: An Historical Perspective. May 1992. 29 pages.
ISBN 1 874 770 37 9

38. Verrall R.J. Dynamic Bayes Estimation for AIDS Delay Tables. May 1992. 16 pages.
ISBN 1 874 770 38 7

39. Kaye G.D. Data Sources Relating to Life Assurance Expenses. May 1992. 39 pages. Presented to the Staple Inn Actuarial Society and Royal Statistical Society - February 1992.
ISBN 1 874 770 39 5

40. Renshaw A.E., and Haberman S. On the Graduation Associated with a Multiple State Model for Permanent Health Insurance. May 1992.
ISBN 1 874 770 40 9

41. England P.D. Statistical Modelling of Excess Mortality Number 3. June 1992. 163 pages.
ISBN 1 874 770 41 7

42. Bloomfield D.S.F. and Haberman S. Male Social Class Mortality Differences Around 1981: An Extension to Include Childhood Ages. 21 pages. June 1992.
ISBN 1 874 770 42 5

43. Berg M.P and Haberman S. Trend Analysis and Prediction Procedures for Time Nonhomogeneous Claim Processes. 33 pages. June 1992.
ISBN 1 874 770 43 3

44. Booth P.M. The Single Market for Insurance, Free Capital Movements and attempts to Fix Exchange Rates. October 1992. 28 pages.
ISBN 1 874 770 44 1

45. Verrall R.J. Chain Ladder with Varying Run-off Evolutions. February 1993. 15 pages.
ISBN 1 874 770 45 X

46. Gavin J., Haberman S. and Verrall R.J. Moving Weighted Average Graduation using Kernel Estimation. November 1992. 14 pages.
ISBN 1 874 770 46 8

47. Gavin J., Haberman S. and Verrall R.J. On the Choice of Bandwidth for Kernel Graduation. November 1992. 21 pages.
ISBN 1 874 770 47 6

48. S. Haberman. Pension Funding with Time Delays and the Optimal Spread Period. May 1993. 13 pages.
ISBN 1 874 770 48 4

49. S. Haberman. Stochastic Investment Returns and the Present Value of Future Contributions in a Defined Benefit Pension Scheme. May 1993. 22 pages.
ISBN 1 874 770 49 2

50. A. Zimbidis and S. Haberman. Delay, Feedback and Variability of Pension Contributions and Fund Levels. May 1993. 19 pages.
ISBN 1 874 770 50 6

51. S. Haberman. Pension Funding: The Effect of Changing The Frequency Valuations. June 1993. 19 pages.
ISBN 1 874 770 51 4

52. S Haberman. HIV, AIDS Markov Chains and PHI. June 1993. 15 pages.
ISBN 1 874 770 52 2

53. S Haberman. A Consideration of Pension Credit and Termination Insurance. June 1993. 22 pages.
ISBN 1 874 770 53 0

54. M Z Khorasanee. Survey of Actuarial Practice in the Funding of UK Defined Benefit Pension Schemes. July 1993. 19 pages.
ISBN 1 874 770 54 9

55. P M Booth, R G Chadburn and A S K Ong. A Utility Maximisation Approach to Asset Allocation. September 1993. 40 pages.
ISBN 1 874 770 55 7

56. R G Chadburn. Bias in Select Mortality Investigations. August 1993. 62 pages.
ISBN 1 874 770 56 5

57. M Z Khorasanee. A Comparison of Pension Funding Strategies by means of Simulations for a Model Scheme. August 1993. 43 pages.
ISBN 1 874 770 57 3

58. A E Renshaw, P Hatzopolous and S Haberman. Recent Mortality Trends in Male Assured Lives. June 1993. 23 pages.
ISBN 1 874 770 58 1

59. E Pitacco. Disability Risk Models: Towards A Unifying Approach. September 1993. 33 pages.
ISBN 1 874 770 59 X

60. M Boskov and R J Verrall. Premium Rating by Geographic Area Using Spatial Models. September 1993. 14 pages.
ISBN 1 874 770 60 3

61. R G Chadburn. Managing Mutual Life Offices: The Use of an Expense Ratio in New Business Decision Making and Expense Control. October 1993. 21 pages.
ISBN 1 874 770 61 1

62. Haberman S. Pension Funding Modelling and Stochastic Investment Returns. 56 pages. March 1994.
ISBN 1 874 770 62 X

63. Renshaw A E and Verrall R J. The Stochastic Model Underlying the Chain-Ladder Technique. 25 pages. April 1994.
ISBN 1 874 770 63 8

64. Haberman S and Sung J-H. Dynamic Approaches to Pension Funding. 22 pages. April 1994.
ISBN 1 874 770 64 6

65. Renshaw A.E. On the Second Moment Properties and the Implementation of Certain GLIM Based Stochastic Claims Reserving Models. 36 pages. September 1994.
ISBN 1 874 770 65 4

66. Booth P.M., J.N. Allan, and J.W. Jang. An Evaluation of the UK Life Insurance Mismatching Test. September 1994.
ISBN 1 874 770 66 2

67. Booth P.M. and Stroinski K. Insurance and Investment Markets in Poland. September 1994. 35 pages.
ISBN 1 874 770 67 0

68. Ong A. A Stochastic Model for Treasury-Bills: An Extension to Wilkie's Model. September 1994. 12 pages.
ISBN 1 874 770 68 9

69. Bloomfield D.S.F. Moving on from Undergraduate Exams to Professional Exams: Actuaries. November 1994. 22 pages.
ISBN 1 874 770 69 7

70. Huber P. A Review of Wilkie's Stochastic Investment Model. January 1995. 22 pages.
ISBN 1 874 770 70 0

71. Renshaw A.E. On the Graduation of `Amounts'. January 1995. 24 pages.
ISBN 1 874 770 71 9

72. Renshaw A.E. Claims Reserving by Joint Modelling. December 1994. 26 pages.
ISBN 1 874 770 72 7

73. Renshaw A.E. Graduation and Generalised Linear Models: An Overview. February 1995. 40 pages.
ISBN 1 874 770 73 5

74. Khorasanee M.Z. Simulation of Investment Returns for a Money Purchase Fund. June 1995. 20 pages.
ISBN 1 874 770 74 3

75. Owadally M.I. and Haberman S. Finite-time Pension Fund Dynamics with Random Rates of Return. June 1995. 28 pages.
ISBN 1 874 770 75 1

76. Owadally M.I. and Haberman S. Stochastic Investment Modelling and Optimal Funding Strategies. June 1995. 25 pages.
ISBN 1 874 770 76 X

77. Khorasanee M.Z Applying the Defined Benefit Principle to a Defined Contribution Scheme. August 1995. 30 pages.
ISBN 1 874 770 77 8

78. Sebastiani P. and Settimi R. Experimental Design for Non-Linear Problems. September 1995. 13 pages.
ISBN 1 874 770 78 6

79. Verrall R.J. Whittaker Graduation and Parametric State Space Models. November 1995. 23 pages.
ISBN 1 874 770 79 4

80. Verrall R.J. Claims Reserving and Generalised Additive Models. November 1995. 17 pages.
ISBN 1 874 770 80 8

81. Nelder J.A. and Verrall R.J. Credibility Theory and Generalized Linear Models. November 1995. 15 pages.
ISBN 1 874 770 81 6

82. Renshaw A.E., Haberman S. and Hatzopoulos P. On The Duality of Assumptions Underpinning The Construction of Life Tables. December 1995. 17 Pages.
ISBN 1 874 770 82 4

83. Chadburn R.G. Use of a Parametric Risk Measure in Assessing Risk Based Capital and Insolvency Constraints for With Profits Life Insurance. March 1996. 17 Pages.
ISBN 1 874 770 84 0

84. Haberman S. Landmarks in the History of Actuarial Science (up to 1919). March 1996. 62 Pages.
ISBN 1 874 770 85 9

85. Renshaw A.E. and Haberman S. Dual Modelling and Select Mortality. March 1996. 30 Pages.
ISBN 1 874 770 88 3

86. Booth P.M. Long-Term Care for the Elderly: A Review of Policy Options. April 1996. 45 Pages.
ISBN 1 874 770 89 1

87. Huber P.P. A Note on the Jump-Equilibrium Model. April 1996. 17 Pages.
ISBN 1 874 770 90 5

88. Haberman S and Wong L.Y.P. Moving Average Rates of Return and the Variability of Pension Contributions and Fund Levels for a Defined Benefit Pension Scheme. May 1996. 51 Pages.
ISBN 1 874 770 91 3

89. Cooper D.R. Providing Pensions for Employees with Varied Working Lives. June 1996. 25 Pages.
ISBN 1 874 770 93 X

90. Khorasanee M.Z. Annuity Choices for Pensioners. August 1996. 25 Pages.
ISBN 1 874 770 94 8

91. Verrall R.J. A Unified Framework for Graduation. November 1996. 25 Pages.
ISBN 1 874 770 99 9

92. Haberman S. and Renshaw A.E. A Different Perspective on UK Assured Lives Select Mortality. November 1996. 61 Pages.
ISBN 1 874 770 00 X

93. Booth P.M. The Analysis of Actuarial Investment Risk. March 1997. 43 Pages.
ISBN 1 901615 03 0

94. Booth P.M., Chadburn R.G. and Ong A.S.K. Utility-Maximisation and the Control of Solvency for Life Insurance Funds. April 1997. 39 Pages.
ISBN 1 901615 04 9

95. Chadburn R.G. The Use of Capital, Bonus Policy and Investment Policy in the Control of Solvency for With-Profits Life Insurance Companies in the UK. April 1997. 29 Pages.
ISBN 1 901615 05 7

96. Renshaw A.E. and Haberman S. A Simple Graphical Method for the Comparison of Two Mortality Experiences. April 1997. 32 Pages.
ISBN 1 901615 06 5

97. Wong C.F.W. and Haberman S. A Short Note on Arma (1, 1) Investment Rates of Return and Pension Funding. April 1997. 14 Pages.
ISBN 1 901615 07 3

98. Puzey A S. A General Theory of Mortality Rate Estimators. June 1997. 26 Pages.
ISBN 1 901615 08 1

99. Puzey A S. On the Bias of the Conventional Actuarial Estimator of q_x. June 1997. 14 Pages.
ISBN 1 901615 09 X

100. Walsh D. and Booth P.M. Actuarial Techniques in Pricing for Risk in Bank Lending. June 1997. 55 Pages.
ISBN 1 901615 12 X

101. Haberman S. and Walsh D. Analysis of Trends in PHI Claim Inception Data. July 1997. 51 Pages.
ISBN 1 901615 16 2

102. Haberman S. and Smith D. Stochastic Investment Modelling and Pension Funding: A Simulation Based Analysis. November 1997. 91 Pages.
ISBN 1 901615 19 7

103. Rickayzen B.D. A Sensitivity Analysis of the Parameters used in a PHI Multiple State Model. December 1997. 18 Pages.
ISBN 1 901615 20 0

Statistical Research Papers

1. Sebastiani P. Some Results on the Derivatives of Matrix Functions. December 1995. 17 Pages.
ISBN 1 874 770 83 2

2. Dawid A.P. and Sebastiani P. Coherent Criteria for Optimal Experimental Design. March 1996. 35 Pages.
ISBN 1 874 770 86 7

3. Sebastiani P. and Wynn H.P. Maximum Entropy Sampling and Optimal Bayesian Experimental Design. March 1996. 22 Pages.
ISBN 1 874 770 87 5

4. Sebastiani P. and Settimi R. A Note on D-optimal Designs for a Logistic Regression Model. May 1996. 12 Pages.
ISBN 1 874 770 92 1

5. Sebastiani P. and Settimi R. First-order Optimal Designs for Non Linear Models. August 1996. 28 Pages.
ISBN 1 874 770 95 6

6. Newby M. A Business Process Approach to Maintenance: Measurement, Decision and Control. September 1996. 12 Pages.
ISBN 1 874 770 96 4

7. Newby M. Moments and Generating Functions for the Absorption Distribution and its Negative Binomial Analogue. September 1996. 16 Pages.
ISBN 1 874 770 97 2

8. Cowell R.G. Mixture Reduction via Predictive Scores. November 1996. 17 Pages.
ISBN 1 874 770 98 0

9. Sebastiani P. and Ramoni M. Robust Parameter Learning in Bayesian Networks with Missing Data. March 1997. 9 Pages.
ISBN 1 901615 00 6

10. Newby M.J. and Coolen F.P.A. Guidelines for Corrective Replacement Based on Low Stochastic Structure Assumptions. March 1997. 9 Pages.
ISBN 1 901615 01 4.

11. Newby M.J. Approximations for the Absorption Distribution and its Negative Binomial Analogue. March 1997. 6 Pages.
ISBN 1 901615 02 2

12. M. Ramoni and P. Sebastiani. The Use of Exogenous Knowledge to Learn Bayesian Networks from Incomplete Databases. June 1997. 11 Pages.
ISBN 1 901615 10 3

13. M. Ramoni and P. Sebastiani. Learning Bayesian Networks from Incomplete Databases. June 1997. 14 Pages.
ISBN 1 901615 11 1

14. P. Sebastiani and H.P. Wynn. Risk Based Optimal Designs. June 1997. 10 Pages.
ISBN 1 901615 13 8

15. R. Cowell. Sampling without Replacement in Junction Trees. June 1997. 10 Pages.
ISBN 1 901615 14 6

16. R.A. Dagg and M.J. Newby. Optimal Overhaul Intervals with Imperfect Inspection and Repair. July 1997. 11 Pages.
ISBN 1 901615 15 4

17. P. Sebastiani and H.P. Wynn. Bayesian Experimental Design and Shannon Information. October 1997. 11 Pages.
ISBN 1 901615 17 0

18. L.C. Wolstenholme. A Characterisation of Phase Type Distributions. November 1997. 11 Pages.
ISBN 1 901615 18 9

ARU-ARP/arcdsk

ISBN 1 901615 20 0

Department of Actuarial Science and Statistics

Actuarial Research Club

The support of the corporate members

Commercial Union
Coopers & Lybrand
Government Actuary's Department
Guardian Insurance
Hymans Robertson
KPMG
Munich Reinsurance
Swiss Reinsurance

is gratefully acknowledged.