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Large-Eddy and Wall-Modeled Simulations of Turbulent Flow over Two-Dimensional River Dunes

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Abstract

In this research, models of the Detached-Eddy Simulation (DES) family are used for calculations of turbulent flow over a two-dimensional dune geometry at \( \text{Re}_b = \frac{U_b H_b}{\nu} = 50000 \). This case is neither a fully attached flow nor a massively separated flow and seems to be a good candidate to investigate the cost and accuracy of DES wall-models in such cases. Results are compared to the well-validated LES database. Compared to attached, equilibrium flows or flows with a mild separation, a higher accuracy was achieved in prediction of the mean and second order statistics using all the present hybrid methods. All the methods saved up to 97% of the total computational cost (compared to the LES simulations based on the required CPU time). Moreover, considering both the first and second order statistics, as well as the flow physics included in the instantaneous turbulent field of the simulation, the IDDES model resulted in a higher level of accuracy compared to the DES and DDES methods. However, the same grid density as the two other models might lead to generation of small-scale uncorrelated structures over the reattachment region. To resolve this issue, one can bypass the near-wall region and model its effects on the outer flow in a statistical sense; that can result in saving of more than 90% of the grid points expended to resolve less than 10% of the total volume in the near-wall area. This approach is known as Wall-Modeled LES (WMLES) and extends the capabilities of large-eddy simulation technique to high Reynolds number wall-bounded flows. In this regard, the hybrid Reynolds-Averaged Navier-Stokes (RANS)/LES techniques blend a RANS-type turbulent eddy-viscosity near the wall and a LES sub-grid eddy-viscosity in the rest of the domain and solve the filtered equations on a single mesh, resulting in a strong coupling between the inner and outer-layer flow.

A unified hybrid RANS/LES model should have the following properties: (i) result in a RANS (or URANS when a separation of scales is present in the flow) solution when applied along with a coarse mesh; (ii) the effect of explicit filtering on the model should be negligible (a usual consequence of numerical discretization as well as the order of accuracy of the numerical integration); (iii) the subgrid-scale model should automatically turn off if the grid resolution is of the order of DNS; (iv) statistical and instantaneous features of the solution should be recovered, independent of the initial condition used for the simulation.

The hybrid simulation strategy was firstly prompted by the required computational power for LES of airplane wings at Reynolds numbers near flight values. Spalart & Allmaras proposed the DES model that reduces to RANS-like solution in the attached boundary layers and LES after separation. The new formulation modifies the turbulence length scale \( d \) (distance to the nearest wall) in the base RANS eddy-viscosity equation as

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follows,
\[
\tilde{d} = \min(d, C_{DES} \Delta), \quad \Delta = \max(\Delta x, \Delta y, \Delta z)
\] (1)
where \( C_{DES} = 0.65 \) is the model constant and \( \Delta x, \Delta y, \) and \( \Delta z \) are the grid spacings in wall-parallel and wall-normal directions. This approach was originally intended for massively separated flows where the instability of the shear layer after separation is strong enough to accelerate the eddy-generation at the RANS/LES interface. Therefore, the problem associated with the “grey area” is no more present (the grey area of the hybrid RANS/LES techniques is the region where the model switches between RANS and LES behaviors. There, the Reynolds shear stress is usually under-predicted; this can lead to the log-layer mismatch or under-prediction of the wall shear stress).

The Sparalrt-Allmaras RANS eddy-viscosity equation (hereinafter referred to as SA-RANS [1]) is given by the following formula, equation 2.
\[
\frac{D\tilde{\nu}}{Dt} = \frac{c_{1S} \tilde{V}}{\text{production}} - \frac{c_{w} f_{w} \tilde{V}}{\text{destruction}} + \frac{1}{\sigma} \nabla \cdot (\tilde{V} + \tilde{V}) \nabla \tilde{V} + c_{p2} (\nabla \tilde{V})^2
\] (2)
Substituting 1 into 2 results in a RANS-type eddy-viscosity near the wall. The model length-scale reduces to \( \tilde{d} = C_{DES} \Delta \) in the outer-layer, however. This leads to [3] which is the basis for Smagorinsky-type subgrid scale models.
\[
\text{production} \approx \text{destruction} \implies \tilde{V} \propto \Delta^2
\] (3)
Here, \( S = (2S_{ij} S_{ij})^{1/2} \) where \( S_{ij} \) is the strain rate tensor. DES model suffers from the problem associated with the “grey area” when applied to attached, equilibrium flows [8][10]. Besides, it may lead to premature separation due to modeled-stress depletion when applied to a grid which is neither close to LES mesh density nor coarse enough to be considered as a RANS grid. These are some of the prevalent consequences of hybrid RANS/LES methods and notable effort has been spent so far to completely remove them or reduce their effect.

Piomelli et al. [10] noticed that the log-layer mismatch (LLM) can be completely removed from mean velocity profiles of turbulent channel flows if a correct value of stochastic forcing is applied in the inner-layer along with the hybrid calculation. Keating & Piomelli [11] proposed a dynamic method to estimate its magnitude with minimal user inputs. In addition to numerical instabilities, Davidson & Peng [12] observed that physical instabilities such as interaction of three-dimensional structures generated through a shear-layer may reduce LLM and lead to more accurate results. Furthermore, Spalart et al. [3] suggested to use a Delayed version of DES (DDDES) which transfers the RANS/LES interface farther from the wall using a delay function, \( f_{\tau} \), to avoid premature separation in cases with developing boundary layers or when local grid refinements are inevitable. Also, Shur et al. [4] introduced a new LES length-scale (\( l \)) that depends on the distance to the wall as well as the grid spacing, leading to a uniform distribution of \( l \) near the wall and far from the wall, and a linear transition in between. This is the key element to the improved DDES (IDDES) model. This new model is also claimed to resolve LLM.

In this study, the DES, DDES, and IDDES models are used for calculations of turbulent flow over a two-dimensional dune geometry at \( Re_h = U_{b} H_{b} / \nu = 50000 \). This case is neither a fully attached flow nor a massively separated flow and seems to be a good candidate to investigate the cost and accuracy of DES wall-models in such cases.

## 2 Problem Formulation

The filtered conservation of mass and momentum are the governing equations as follows [4] and [5]. The equations are discretized and solved on a structured curvilinear grid.
\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0
\] (4)
\[
\frac{\partial u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = - \frac{1}{\rho} \frac{\partial p}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + \nu \frac{\partial^2 u_i}{\partial x_i \partial x_j}
\] (5)
Here, \( x_1, x_2, \) and \( x_3 \) (or \( x, y, \) and \( z \), respectively) are the streamwise, wall-normal, and spanwise directions. To calculate the effect of unresolved scales on the flow, the dynamic procedure, proposed by Germano et al. [13] with Lagrangian averaging of Meneveau et al. [14] is used for the LES simulations in the present work. Furthermore, the SA-RANS is utilized as the base RANS model. The required modifications of the DES, DDES, and IDDES are then applied to the RANS equation for the purpose of hybrid calculations [2][3][4]. The computer code is second-order accurate in time and space which is well-validated through many publications [5][15][16].

The present test-case is a two-dimensional river dune, shown in figure [1]. Here, \( H_{b} = 3.5h \) is the average dune height. Also, \( U_{b} \) is the bulk velocity at the location with \( H = H_{b} \). The simulations are performed at \( Re_h = U_{b} H_{b} / \nu = 50000 \) (equivalent to \( Re_{\tau} = u_{\tau} H_{b} / \nu = 2500 \)
where \( u_\tau \) is the friction velocity at the location with \( H = H_b \), adequately high to analyze performance of the wall-layer models. Moreover, the boundary conditions are as follows: periodic in the \( x \) and \( z \) directions, no-slip at the wall, and zero shear stress (free-slip) on the top.

### 3 Results

#### 3.1 Turbulence statistics

The RANS/LES interface (where \( \langle uv \rangle_{\text{model}} = \langle uv \rangle_{\text{resolved}} \)) and the wall shear stress at the dune bed are shown in Figure 2. We could not specify any interface close to the wall over the recirculation region (\( 0.5 < x/h < 4.5 \)) and the majority of the total shear stress (\( \langle uv \rangle_t = \langle uv \rangle_{\text{model}} + \langle uv \rangle_{\text{resolved}} \)) is supported by the LES branch of the hybrid models (note that the possible intersection of \( \langle uv \rangle_{\text{model}} \) and \( \langle uv \rangle_{\text{resolved}} \) far from the wall is a result of the shear layer rather than the RANS-to-LES transition). Near the reattachment point, all the models predict a similar interface location due to the interaction of the three-dimensional structures, generated in the shear layer, with the interface (\( 4.5 < x/h < 7 \)). Farther downstream, the difference between the models becomes more visible; As expected, IDDES switches to LES closer to the bed while the DDES model shows a delayed transition as a result of a thicker RANS zone near the wall.

The aforementioned features have their own advantages and disadvantages when considering \( \tau_w \) distribution over the dune bed. RANS models such as SA-RANS are not calibrated to account for geometry-induced scales, generated in the shear layer. As a result, all the hybrid models better predict \( \tau_w \) compared to a pure RANS calculation on the same grid. Comparing different hybrid models, the reattachment point (where \( \tau_w = 0 \)) is better predicted by the IDDES (separation is fixed because of the sharp edge on the lee side). Moreover, over the recirculation region as well as the acceleration area near the next dune crest, IDDES results in a better consistency when \( \tau_w \) is compared to the wall-resolved data. However, the two other models better estimate this quantity on the stoss side due to the quasi-equilibrium state of the flow over this region (the RANS models account for these effects).

The streamlines, and the mean separation and reattachment locations for each model are shown in Figure 3. The reattachment length is calculated based on zero mean wall shear stress criterion (\( \tau_w = 0 \)). The mesh applied for these calculations resolves the step with an acceptable resolution (\( \Delta s/h \approx 0.05 \), this value is comparable to a coarse wall-resolved LES simulation at this Reynolds number with \( \Delta s/h \approx 0.045 \)). The reattachment location, depends on the state of the boundary layer at separation, and is predicted quite differently using the three hybrid models. The reattachment length is over-predicted by 13%., 16%, and 10% us-
Figure 3: Recirculation bubble and average streamlines shown on the contours of the mean streamwise velocity (contour levels are the same as Figure 1(c)); (a) DES; (b) DDES; (c) IDDES; (d) wall-resolved LES; the symbols, ■ and ● denote the mean separation and reattachment locations, respectively; 0, zero-velocity line ($U = 0$); $Re_b = 50000$.

Figure 4: (a) the normalized velocity profile at $x/h = 18.5$; (b) Comparison of the total Reynolds shear stress at different streamwise locations; - - - - - - - - , DES; --- ---, DDES; -- -- -- --, IDDES; ● wall-resolved LES; $Re_b = 50000$.

Figure 5: Contours of the resolved Turbulent Kinetic Energy ($k$) over the domain for different models.

The normalized mean velocity profile at $x/h = 18.5$ is shown in figure 4(a). Here, $y^+ = u_c y / \nu$ and $U^+ = U / u_c$ ($u_c$ is the local friction velocity computed through each individual calculation). Considering the local RANS/LES interface for different models (shown by vertical lines), it is observable that the IDDES model’s interface is located inside the buffer layer; a region with non-equilibrium effects. As a result a model with more LES content better estimates the velocity profile in this area (the over-prediction of the velocity in the free stream by IDDES is a result of under-prediction of local $u_c$).

All the hybrid methods estimate similar distribution of $k$ in the shear layer and around the reattachment point, however, IDDES prediction is in better agreement with the wall-resolved LES data (under-predicted by a fac-
tor of 15%). We observed that the maximum $k$ value is lower by 20% when comparing the DES and DDES results to the wall-resolved LES simulation, presumably because of the larger filter width. Over this region ($0.5 < x/h < 4.5$), although the LES branch of all the hybrid models is active (look at Figure 2(b)), the effect of the new-born eddies becomes important and only those comparable to the filter width can be resolved while the subgrid scales carry large values of energy. As a result, LES becomes grid dependent no matter where the interface of RANS and LES regions is located.

3.2 Flow structures

The dominant geometry-induced structures in the flow over 2D dunes, such as those generated due to the instability of the shear layer at separation, are usually large enough to be resolved by a LES grid in the outer layer. As a result, neither the grid resolution, especially in wall-parallel directions, nor the wall-model itself applied to the near-wall area, should not affect these large scales. As observed in the previous section, the inner-outer layer interactions due to generation of these large scales can lead to improvements in prediction of the mean quantities using hybrid techniques.

Contours of the streamwise velocity fluctuations ($u'$) are shown in Figure 6. Extraction of the data is performed in a plane parallel to the bed at $y_w/h = 0.04$ where $y_w$ is the normal distance to the wall. At this distance, the DES and DDES models are still in the RANS mode while IDDES is in the RANS-to-LES transition over majority of the streamwise locations (all the methods result in a similar pattern for $u'$ far from the wall).

Flow over 2D river dunes includes some important physics. The reattachment location fluctuates around a mean value, as a result of 3D, spanwise-oriented eddies. Farther downstream, acceleration of the flow on the stoss side forms large streamwise-oriented structures; their signature is observable on the bed as the stripes of high-speed streaks alternating with the low-speed ones.

Comparison of the outer-layer structures at Reynolds of 18900 using 640\times 180\times 640 grid points in $x$, $y$, and $z$ directions, respectively. This mesh was fine enough to make it possible to observe all the types of structures present in this physic: (i) rollers are generated at the crest due to the Kelvin-Helmholtz instability; (ii) horseshoe-like structures generated along the shear layer that either convect downstream or rise up to the surface; (iii) kolk vortices around the reattachment point; and (iv) large streamwise-oriented vortices on the stoss side. These outer-layer structures are mainly generated through the shear layer and are Reynolds number independent, subsequently, they must be also observed at all the higher Reynolds numbers.

Comparison of the outer-layer structures at Reynolds of 50000, predicted by different wall-layer models as well as the wall-resolved LES calculation, is shown in Figure 7. We did not observe the kolk vortices at this Reynolds number using $p = -0.02$ (the kolk vortices are strongly rotating structures, similar to tornadoes, generated in the regions of high velocity gradients) using any of the simulations techniques. They
are observed for smaller $p'$, however. Note that the wall-resolved LES calculation for this study utilizes a mesh size of $961 \times 301 \times 769$ which is comparable to the intermediate simulation of Omidyeganeh & Piomelli [5] considering the grid resolution. We estimate that in order to finely resolve and be able to visualize all the structures, at least we would need up to $1500 \times 450 \times 1500$ grid points in the streamwise, wall-normal, and spanwise directions, respectively. However, the other three types of structures are usually large in size and can be resolved using a coarse grid. To do so, we extracted the instantaneous data for a duration equal to two flow-through times ($t^* = 40$ where $t^* = T_{Ub}/h$) for each simulation technique. We successfully identified the horseshoe-like structures, rollers, and the streamwise vortices for each calculation.

A qualitative comparison of the DES and DDES simulations reveals that they have similar behavior in the recirculation region (a similar size of the resolved structures), nonetheless on the stoss side, as the flow convects downstream, DDES model predicts larger streamwise vortices which is also observable in Figure 6 where their signature on the bed is clearly seen as larger stripes of high-speed, low-speed streaks. In comparison with DES and DDES models, a larger range of scales is resolved using IDDES. This is attributed to a substantially thinner RANS area and a lower eddy-viscosity all over the domain predicted by the model, especially around the shear layer. However, some uncorrelated small-scale structures (also observed in Figure 6 (c)) are present in the isosurfaces of the pressure fluctuations all over the domain (Figure 7 (d)). We believe that this is because of insufficient turbulence dissipation near the reattachment point. Over this area, the large spanwise-oriented eddies touch the dune bed. Grid refinements can fix this issue.

4 Conclusion and Future Work

Wall-Modeled Large-Eddy Simulations (WMLES) were used for turbulent flow over two-dimensional river dunes using models of the Detached-Eddy Simulation family, including DES, DDES and IDDES methods. Simulations were carried out at $Re_{b} = 50000$ based on bulk velocity and average dune height. Wall-resolved LES calculations were also used to investigate the accuracy of the three wall-layers. Performing hybrid calculations using coarse meshes resulted in 98% and 97% of the mesh density and CPU time, respectively (the wall-resolved LES simulation required...
Figure 7: Instantaneous isosurfaces of the pressure fluctuations \( (p' = -0.02) \) colored with height; (a) LES; (b) DES; (c) DDES; (d) IDDES; \( Re_b = 50000 \).

276000 CPU hours for removing transient effects as well as statistical convergence).

Considering the instantaneous flow field, all the present wall-layers are able to resolve the coherent structures present in the flow over river dunes at high Reynolds numbers (wall-layers are generally designed for flow simulations at very high Reynolds numbers). However, IDDES may require a finer grid in the wall-parallel directions \((x\) and \(z\) directions) compared to the two other models, DES and DDES, wherever a strong interaction is present between the large structures and the dune bed (e.g. over the reattachment location). This model switches to its LES content faster than the two other methods and its predicted eddy-viscosity is smaller than DES and DDES. Therefore, the dissipation of the structures interacting with the bed must be fed into the model by increasing the resolved dissipation using a finer grid. If the mesh density is not sufficiently fine, IDDES may over-estimates the normal Reynolds stresses near the bed.

Future work may involve the following directions: (i) revision of the DES-based methods to accelerate the eddy-generation process over the interface of the RANS and LES zones by imposing random noises in the solution of the subgrid eddy-viscosity; (ii) applying IDDES model to more realistic flow conditions for two-dimensional dunes in order to investigate the Reynolds number effects; (iii) studying sediment transport at high Reynolds numbers using present wall-models and eventually, simulating the movement of mobile sand beds.

All in all, the hybrid strategy seems to be a good candidate for turbulence simulation of flows with physical and geometry-induced instabilities (e.g. separation and flow over convex geometries).

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